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CONTENTS

PAPERS

No. PAGE

2015 **THE DESIGN OF ROCK-FILL DAMS.**
By **J. D. Galloway** 1

Discussion:
By Cecil E. Pearce 25
H. B. Muckleston 27
Harold K. Fox 28
Charles H. Paul 29
A. Floris 30
Howard F. Peckworth 30
Oren Reed 31
Walter L. Huber 33
Samuel B. Morris 35
L. F. Harza 39
Paul Baumann 39
O. W. Peterson 40
George W. Howson 42
John E. Field 45
John H. Wilson 47
Frederick H. Fowler 49
I. C. Steele and Walter Dreyer 53
F. Knapp 64
Ralph J. Reed 64
F. J. Sanger 65
C. S. Jarvis 66
L. R. East 69
Francisco Gomez-Perez and Miguel Jinich 75
J. D. Galloway 83

MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION.
By **A. M. Rawn, A. Perry Banta, and Richard Pomeroy** 93

Discussion:
By Edward W. Moore 120
Willem Rudolfs and H. Heukelekian 122
Hermann Bach 125
C. E. Keefer 128
A. M. Rawn, A. Perry Banta, and Richard Pomeroy 129

2017 **EARTHQUAKE STRESSES IN AN ARCH DAM.**
By **Ivan M. Nelidov and Harold E. von Bergen** 133

Discussion:
By A. Floris 143
Cecil E. Pearce 143
F. W. Hanna 144
A. W. Fischer 147
Ray L. Allin 148
Ivan M. Nelidov and Harold E. von Bergen 148

2018 **GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS.**
By **Frank B. Campbell** 150

Discussion:
By Donald M. Burmister 156
A. J. Weinig 159
Carl H. Kadie, Jr. 159

No. PAGE

Carlton S. Proctor 161
T. T. Knappen 163
Jacob Feld 164
Howard F. Peckworth 164
Joel D. Justin 165
L. B. Olmstead 166
T. A. Middlebrooks 168
Frank E. Fahlquist and Waldo I. Kenerson 169
E. W. Lane 174
F. J. Sanger 177
F. Knapp 178
Charles H. Lee 178
Frank B. Campbell 182

2019 **ECONOMIC PIPE SIZES FOR WATER DISTRIBUTION SYSTEMS.**
By **Thomas R. Camp** 190
Discussion:
By Clinton L. Bogert and J. M. M. Greig 203
Charles M. Mower, Jr. 203
A. C. Michael 204
Ellwood H. Aldrich 205
M. H. Klegerman 207
F. Knapp 209
Thomas R. Camp 210

2020 **GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT.**
By **George E. Hubbell** 214
Discussion:
By G. M. Ridenour 230
Thomas R. Camp 232
W. E. Howland 240
George E. Hubbell 242

2021 **STRUCTURAL BEHAVIOR OF BATTLE-DECK FLOOR SYSTEMS.**
By **Inge Lyse and Ingvald E. Madsen** 244
Discussion:
By H. N. Hill and R. L. Moore 267
Jonathan Jones 272
Inge Lyse and Ingvald E. Madsen 274

2022 **FLOOD ROUTING.**
By **Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder** 275
Discussion:
By Cecil S. Camp 295
R. D. Goodrich 295
E. L. Myers 298
Ralph W. Powell 298
William T. Collins 302
George E. Townsend 304
Waldo E. Smith 307
Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder 309

2023 **PIN-CONNECTED PLATE LINKS.**
By **Bruce G. Johnston** 314
Discussion:
By Harold D. Hussey 337
Bruce G. Johnston 339

No. PAGE
2024 **WATER-HAMMER PRESSURES IN COMPOUND AND BRANCHED PIPES.**
By **Robert W. Angus** 340

Discussion:
By K. J. DeJuhasz 377
Harold A. Thomas 378
F. Knapp 388
Martin A. Mason, Pierre F. Danel, and Antoine Craya 391
A. A. Kalinske 395
Robert W. Angus 397

2025 **NATURAL PERIODS OF UNIFORM CANTILEVER BEAMS.**
By **Lydik S. Jacobsen** 402

Discussion:
By K. Bert Hirashima 432
Merit P. White 433
Lydik S. Jacobsen 438

2026 **INCREASING THE TRAFFIC CAPACITY AND SAFETY OF THOROUGHFARES: A SYMPOSIUM** 440

CAUSES OF FAILURE IN HANDLING TRAFFIC.
By **Lewis W. McIntyre** 441

REMEDIES TO RELIEVE TRAFFIC CONGESTION.
By **Donald M. McNeil** 461

IMPROVEMENTS TO REDUCE TRAFFIC ACCIDENTS.
By **Arnold H. Vey** 474

ECONOMICS OF ALIGNMENT, GRADE, AND WIDTH.
By **E. L. Schmidt** 485

Discussion:
By John A. Oakey 497
L. W. Mahone 500
Fred Lavis 501
Hugh A. Kelly 502
Sidney J. Williams 504
Harold Nevin Carey 506
W. W. Crosby 507
Roger L. Morrison 508
W. R. Flack 509
Hawley S. Simpson 511
Charles M. Noble 513
Lewis W. McIntyre 517
Arnold H. Vey 517

2027 **RELATIVE FLEXURE FACTORS FOR ANALYZING CONTINUOUS STRUCTURES.**
By **Ralph W. Stewart** 521

Discussion:
By Frederick Shapiro 531
Dean F. Peterson, Jr. 531
George W. Housner 534
Homer M. Hadley 536
Adolphus Mitchell 537
John B. Wilbur 539
Leon Blog 542
A. Floris 544
D. B. Hall 545
Ralph W. Hutchinson 546
Arthur B. McGee 548
E. Neil W. Lane 551
Ralph W. Stewart 554

No. PAGE

2028 **DEOXYGENATION AND REOXYGENATION.**
By **C. J. Velz** 560

Discussion:
By Earle B. Phelps 573
C. J. Velz 575

2029 **PRELIMINARY DESIGN OF SUSPENSION BRIDGES.**
By **Shortridge Hardesty and Harold E. Wessman** 579

Discussion:
By A. W. Fischer 609
Jacob Karol 610
Glenn B. Woodruff and Norman C. Raab 614
Hardy Cross 617
A. A. Eremin 618
A. Fraser Rose and William A. Rose 619
Leon S. Moisseiff 622
O. H. Ammann 624
Leon Blog 626
Shortridge Hardesty and Harold E. Wessman 629

2030 **ENGINEERING ECONOMICS AND PUBLIC WORKS: A SYMPOSIUM.** 635

ADVANTAGES OF ORDERLY PLANNING.
By **Frederic H. Fay** 636

INFLUENCE OF PUBLIC OPINION.
By **Daniel W. Mead** 652

HAZARDS OF UNECONOMICAL CONSTRUCTION.
By **Henry Earle Riggs** 668

AN APPEAL TO REASON.
By **William J. Wilgus** 690

Discussion:
By J. K. Finch 701
Elliott J. Dent 705
C. Frank Allen 706
Bradley G. Seitz 707
Alfred Allen Stuart 708
Pierce P. Furber 708
R. F. Bessey 713
Donald M. Baker 720
Philip W. Henry 726
H. K. Barrows 727
Harry A. Wiersema 727
J. D. Galloway 734
E. S. Martin 739
K. Bert Hirashima 741
Louis E. Ayres 742
John H. Meursinge 747
Samuel B. Folk 751
Daniel W. Mead 756
William J. Wilgus 757

2031 **DESIGN OF PILE FOUNDATIONS.**
By **C. P. Vetter** 758

Discussion:
By Hibbert M. Hill 779
Odd Albert 784
August E. Niederhoff 788
A. A. Eremin 790
Jacob Feld 791
John M. Coan, Jr. 792

No. PAGE

A. AGATZ 795
D. P. KRYNINE 803
C. P. VETTER 806

2032 **STREAM POLLUTION IN THE OHIO RIVER BASIN: A SYMPOSIUM** 812

FOREWORD.
By **George E. Barnes** 814

SURVEYS FOR STREAM POLLUTION CONTROL.
By **Harold W. Streeter** 815

LOW-FLOW REGULATION BY PYMATUNING RESERVOIR.
By **Charles E. Ryder** 832

SANITATION PROBLEMS INCIDENTAL TO FLOODS.
By **W. L. Stevenson** 844

PROGRESS IN POLLUTION CONTROL IN THE OHIO RIVER BASIN.
By **E. S. Tisdale** 852

PLANNING FOR POLLUTION CONTROL AT PITTSBURGH, PENNSYLVANIA.
By **D. E. Davis** 860

STREAM POLLUTION PROBLEMS AT CINCINNATI, OHIO.
By **J. E. Root** 868

WHAT CAN BE DONE ABOUT STREAM POLLUTION?
By **Abel Wolman** 874

Discussion:
By D. E. DAVIS 878
L. S. MORGAN 879
C. A. HOLMQUIST 882
ROBERT SPURR WESTON 883
KARL IMHOFF 885
HENRY D. JOHNSON, JR. 885
CHARLES M. REPPERT 887
EDWIN K. MORSE 889
JAMES H. LEVAN 889
JOHN C. H. LEE 891
W. H. WISELY 892
F. C. DUGAN 894
G. M. RIDENOUR 895
ARTHUR W. BAUM, JR. 897
G. R. SCOTT 898
DON E. BLOODGOOD 901
W. L. STEVENSON 902
D. E. DAVIS 902

2033 **PRACTICAL METHODS OF RE-ZONING URBAN AREAS.**
By **Hugh E. Young** 904

Discussion:
By ROBERT D. KOHN 928
MYRON D. DOWNS 928
HARLAND BARTHOLOMEW 930
BENJAMIN SALTZMAN 933
G. H. HERROLD 936
HUGH E. YOUNG 938

2034 **ECONOMIC ASPECTS OF ENERGY GENERATION: A SYMPOSIUM** 942

PROGRESS IN THE GENERATION OF ENERGY BY HEAT ENGINES.
By **Geo. A. Orrok** 943

HYDRO-GENERATION OF ENERGY.
By **Frank H. Rogers** 952

No. PAGE

IMPROVEMENTS IN THE UTILIZATION OF ENERGY.
By **Joel D. Justin** 971

COST OF GENERATION OF ELECTRIC ENERGY.
By **Philip Sporn** 984

SOCIAL IMPLICATIONS OF TECHNOLOGICAL TRENDS IN THE POWER INDUSTRY.
By **Ralph E. Freeman** 995

ELECTRIC POWER IN ECONOMIC PERSPECTIVE.
By **B. Alden Thresher** 1001

Discussion:
By W. S. Finlay, Jr. 1009
R. M. Riegel 1014
Ralph Bennett 1015
J. M. Mousson 1018
Ben C. Sprague 1019
V. M. Marquis 1022
R. L. Sackett 1023
D. J. McCormack 1024
I. E. Moultrop 1025
M. M. Samuels 1026
E. W. Kramer 1031
Paul E. Gisiger 1034
F. A. Dale 1036
Ray S. Quick 1037
F. Knapp 1039
D. S. Jacobus 1041
Joel D. Justin 1044
Philip Sporn 1045

2035 COST OF ENERGY GENERATION: SECOND SYMPOSIUM ON POWER COSTS .. 1050

ELEMENTS OF COST.
By **John C. Page** 1051

HEAT-GENERATED ENERGY.
By **C. F. Hirshfeld and R. M. Van Duzer, Jr.** 1060

HYDRO-GENERATED ENERGY.
By **H. K. Barrows** 1088

COMBINED ENERGY GENERATION.
By **Ezra B. Whitman** 1115

DEPRECIATION AND OBSOLESCENCE.
By **Maurice R. Scharff** 1127

RECAPITULATION.
By **W. F. Uhl** 1139

Discussion:
By Joel D. Justin 1149
Frank H. Mason 1152
W. V. Burnell 1153
Daniel W. Mead 1154
A. G. Christie 1160
William E. Rudolph 1164
Louis Elliott 1166
H. G. Gerdes 1167
Leverett S. Lyon 1173
John T. Madden 1177
R. L. Thomas 1181
Theodore B. Parker 1183
Philip Sporn 1186
L. N. McClellan 1188

No. Page
James C. Bonbright 1192
R. Robinson Rowe 1196
V. B. Libbey 1197
Edwin D. Dreyfus 1200
R. H. Parsons 1203
F. Knapp 1205
John C. Page 1206
C. F. Hirschfeld and R. M. Van Duzer, Jr. 1207
H. K. Barrows 1208
Maurice R. Scharff 1209

2036 **ECONOMICS OF SEWAGE TREATMENT.**
By **George J. Schroepfer** 1210

Discussion:
By Charles Gilman Hyde 1239
Frederic Bass 1242

2037 **LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION.**
By **Y. L. Chang** 1246

Discussion:
By Hans Kramer 1285
E. W. Lane 1286
Joe W. Johnson 1287
J. E. Christiansen 1293
W. H. Huang 1297
Lorenz G. Straub 1298
Hunter Rouse 1303
Y. L. Chang 1308

2038 **THE THREE-POINT PROBLEM IN A CO-ORDINATED FIELD.**
By **R. Robinson Rowe** 1314

Discussion:
By Oscar S. Adams 1325
George D. Whitmore 1326
John R. Jahn 1328
R. Robinson Rowe 1332

2039 **TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE.**
By **G. W. Howard** 1334

Discussion:
By Fred R. Brown 1349
Josef E. Montgomery 1350
Elliott J. Dent 1351
David L. Neuman 1357
Morrough P. O'Brien and R. G. Folsom 1359
R. L. Vaughn 1362
M. P. Durepaire 1369
Pierre F. Danel 1372
H. S. Gladfelter 1374
G. W. Howard 1376

2040 **PRINCIPLES APPLYING TO HIGHWAY ROAD-BEDS.**
By **Ira B. Mullis** 1381

Discussion:
By A. F. Greaves-Walker 1402
H. Z. Schofield 1403
W. H. Campen 1403
Bernard E. Gray 1405
W. J. Turnbull 1411

No. Page
J. E. Christiansen 1757
Samuel Shulits 1760
Gerald Lacey 1761
Glenn W. Holmes 1764
Hunter Rouse 1767
S. G. Bauer 1768
C. C. Inglis 1769
W. M. Griffith 1771

2053 **SIPHONS AS WATER-LEVEL REGULATORS.**
By **J. C. Stevens** 1787

Discussion:
By A. Griffin 1802
H. P. Currin and D. M. Umphrey 1803
T. J. Corwin, Jr. 1807
Henry R. King 1808
J. C. Stevens 1811

2054 **FRONTIERS OF ENGINEERING: ADDRESS AT THE ANNUAL CONVENTION, SAN FRANCISCO, CALIFORNIA, JULY 26, 1939.**
By **Donald H. Sawyer** 1815

2055 **FINAL REPORT OF THE COMMITTEE OF THE IRRIGATION DIVISION ON INTERSTATE WATER RIGHTS: INTERSTATE WATER PROBLEMS** 1822

MEMOIRS OF DECEASED MEMBERS

	Page
Harrison Prescott Eddy, Past-President, Am. Soc. C. E.	1867
Algernon Brown Alderson, M. Am. Soc. C. E.	1871
Walter Hinds Allen, M. Am. Soc. C. E.	1872
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THE DESIGN OF ROCK-FILL DAMS

By J. D. GALLOWAY,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Cecil E. Pearce, H. B. Muckleston, Harold K. Fox, Charles H. Paul, A. Floris, Howard F. Peckworth, Oren Reed, Walter L. Huber, Samuel B. Morris, L. F. Harza, Paul Baumann, O. W. Peterson, George W. Howson, John E. Field, John H. Wilson, Frederick H. Fowler, I. C. Steele and Walter Dreyer, F. Knapp, Ralph J. Reed, F. J. Sanger, C. S. Jarvis, L. R. East, Francisco Gomez-Perez and Miguel Jinich, and J. D. Galloway.

Synopsis

The design of rock-fill dams is discussed in this paper. The term is defined, and a short history is given of the origin and evolution of the type, together with a description of some of the important dams that have been built. The major elements of a rock-fill dam include the foundation, the nature of the rock of which the dam is constructed, the dimensions of the loose rock-fill, settlement, the impervious element on the water face, the intermediate rubble cushion, and expansion joints. The limiting conditions under which such dams should be built are considered.

During the past 75 years there has been developed, in California and other Western States, a dam of a distinctive type which is of sufficient importance to warrant description. This is the rock-fill dam, which in recent years has been built to heights such as 270 ft at Dix, in Kentucky, or 328 ft at Salt Springs, in California. Although there is a general agreement among engineers as to the principal features of this type of dam there are sharp differences of opinion regarding details of construction. It has been thought desirable to give a description of the elements of rock-fill dams in order that the present stage of this evolution may be presented and the differences among engineers discussed.

The description includes a definition of the term, "Rock-Fill Dams," an historical sketch of the evolution of the type, with details of construction, of some representative dams, the elements of the design, the material and methods used in construction, the limiting conditions under which such dams may be built, and the behavior of the dam after completion and when subjected to the water load.

Note.—This paper was published in October, 1937, *Proceedings*.

[1] Cons. Engr., San Francisco, Calif.

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THE DESIGN OF ROCK-FILL DAMS

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Synopsis

The design of rock-fill dams is discussed in this paper. The term is defined, and a short history is given of the origin and evolution of the type, together with a description of some of the important dams that have been built. The major elements of a rock-fill dam include the foundation, the nature of the rock of which the dam is constructed, the dimensions of the loose rock-fill, settlement, the impervious element on the water face, the intermediate rubble cushion, and expansion joints. The limiting conditions under which such dams should be built are considered.

During the past 75 years there has been developed, in California and other Western States, a dam of a distinctive type which is of sufficient importance to warrant description. This is the rock-fill dam, which in recent years has been built to heights such as 270 ft at Dix, in Kentucky, or 328 ft at Salt Springs, in California. Although there is a general agreement among engineers as to the principal features of this type of dam there are sharp differences of opinion regarding details of construction. It has been thought desirable to give a description of the elements of rock-fill dams in order that the present stage of this evolution may be presented and the differences among engineers discussed.

The description includes a definition of the term, "Rock-Fill Dams," an historical sketch of the evolution of the type, with details of construction, of some representative dams, the elements of the design, the material and methods used in construction, the limiting conditions under which such dams may be built, and the behavior of the dam after completion and when subjected to the water load.

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Definition and Principles

The principles governing the design of rock-fill dams are the result of an evolution from the first ones built by the hydraulic miners in California, following the discovery of gold in 1848. The general form in plan and cross-section, as far as design is concerned, is almost entirely empirical. Various engineers have introduced different elements into the design until a fairly uniform cross-section has been evolved. Some dams have failed, thus furnishing information as to what should not be done. Many have been constructed that have given years of service and are to be considered entirely safe. It must be borne in mind, however, that although the design of rock-fill dams is mainly based upon experience certain definite theoretical principles apply to such structures which should always be considered.

Rock-fill dams do not represent the most stable and certain type, but there are a number of reasons for their use. They are generally built in relatively remote locations where the engineer uses the material at hand for his structure. In such locations the cost is usually less than for other types, such as gravity or buttressed dams. Foundation conditions are often a determining element in selecting a rock-fill dam as the best type to be built. Judgment, which determines so much in engineering, must be depended upon in the final decision as to what type to adopt.

Rock-fill dams were evolved from the log crib dam in which a series of log cribs, filled with rock and faced with timber, formed the dam. Such structures have been built in many countries where timber is, or was, abundant. Many dams of the log crib type were built throughout the United States in the period of early development, the height ranging from 5 ft to 25 ft.

Definition.—Rock-fill dams, as now built, are composed of three elements: A loose rock-fill forming the mass of the dam; an impervious face next to the water; and a rubble cushion between the two.

The characteristic that differentiates rock-fill dams from other types is that the element resisting the thrust of the water pressure is of loose rock of varying sizes, placed as a fill at an appropriate site. In almost every case, the rock is dumped loosely in position and there is no attempt at orderly arrangement of the individual rocks; nor is there any other material introduced to bind the rocks together. The mass of rock is somewhat consolidated when placed in position and further consolidation takes place by settlement under load and the action of the weather.

Resistance to the forces imposed by water is obtained from the weight of the mass of rock in the dam. There can be no arch action; nor can there be any action such as the cantilever effect of a gravity masonry dam.

As the mass of loose rock permits the free passage of water it is necessary to provide the dam with an impervious element to make it water-tight. Several different arrangements have been tried. A combination of an earth fill backed by a rock-fill has been used, but this arrangement is not properly a rock-fill dam. In a few cases a diaphragm has been placed in the center of the dam, examples being the old Lower Otay Dam in California, with its steel diaphragm encased in concrete, or the Crane Valley Dam (California) with its masonry core wall. The most usual arrangement is to place the impervious element as a facing on the water side of the dam. This facing in the earlier dams was made of timber, but in the larger and more recent structures it has been made of concrete, usually reinforced. Sheet steel has also been used. On one structure, the Morena Dam, in California, the facing was made of uncoursed rubble masonry set in concrete. Asphalt concrete has also been proposed for this purpose.

Early in the history of rock-fill dams it was recognized that on account of the unequal settlement of the rock mass, it was necessary to interpose between the facing and the rock-fill a layer of dry uncoursed rubble laid up by hand and derrick. This rubble acts as a semi-rigid element as it is subject to less settlement than the rock-fill and permits a better distribution of the water load to the rock-fill. The dry rubble is now recognized as one of the three important elements in the rock-fill dam.

HISTORICAL

As the design of rock-fill dams, as now practiced, has been the result of an evolution, it is of interest to sketch briefly the history of the principal dams in order to illustrate the empirical methods used. Table 1 gives a list of some of the important rock-fill dams.

First Bowman Dam, California.—The first dams built by miners in California were log crib dams of the type with which they were familiar in their Eastern homes. Probably the highest one was the English Dam on the Middle Yuba River, more than 100 ft high. It was built in 1856 to a height of 79 ft, enlarged and repaired in 1876, and destroyed in 1883. It preceded the Old Bowman Dam on Canyon Creek, a tributary of the South Yuba River, built by the late Hamilton Smith, Jr., M. Am. Soc. C. E. The first part of the Old Bowman Dam was constructed in 1872 as a log crib dam about 70 ft high. In

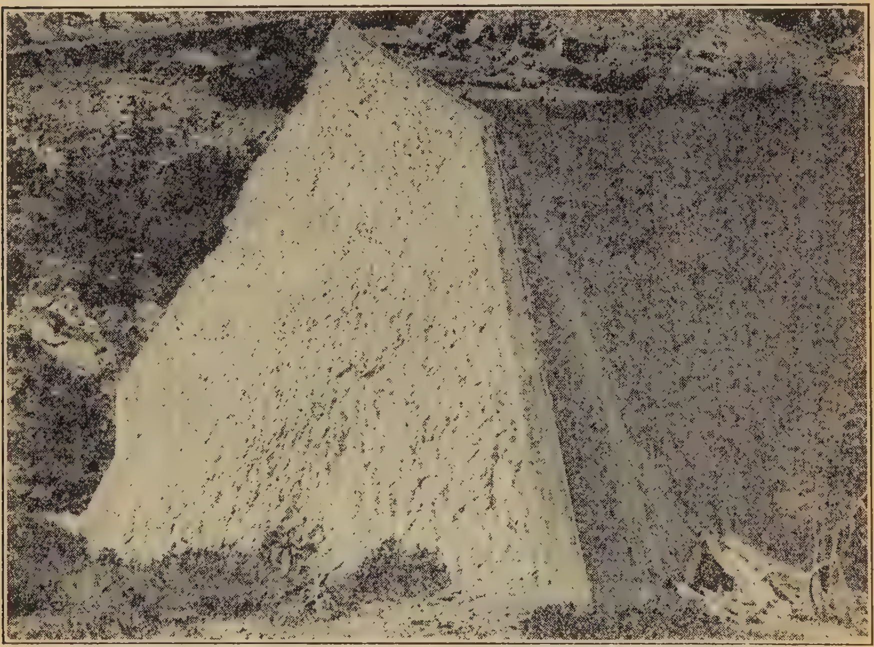

FIG. 1.—FIRST BOWMAN DAM; CONSTRUCTED IN 1876 AND REMOVED IN 1926

TABLE 1.—IMPORTANT ROCK-FILL DAMS IN THE UNITED STATES

Item	Dam	River	State	Date constructed	Height, in feet	Crest length, in feet	RATIO OF HORIZONTAL TO VERTICAL		Facing	THICKNESS OF RUBBLE, IN FEET		Volume, in cubic yards
							Down stream	Water face		At bottom	At top	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	First Bowman	South Yuba	California	1872–76	100	425	1 : 1	1 : 1	Timber	18	6	55 000
2	First Fordyce	South Yuba	California	1873	75	650	1 : 1	1 : 1	Timber	. .	. .	. . . .
3	French Lake	South Yuba	California	1873	68	250	0.5 : 1	. . . .	Timber	. .	. .	. . . .
4	Walnut Grove	Hassayampa	Arizona	1887–88	110	400	1 : 1.55	1 : 2.35	Timber	14	4	46 000
5	Castlewood	Cherry Creek	Colorado	1890	70	600	1 : 1	1 : 10	Rubble	6	4	. . . .
6	Chatsworth	Mormon Creek	California	1896	41	159	1 : 1.15	1 : 1.75	Concrete	2		6 025
7	Escondido	San Elijo Creek	California	1895	76	380	1 : 1 and 1.25 : 1	1 : 2	Timber	15	5	37 159
8	Lower Otay	Otay	California	1894–97	130	615	1.5 : 1	1.5 : 1	Steel core	None		. . . .
9	East Canyon	East Canyon Creek	Utah	1899–1902	93	160	2 : 1	⅔ : 1	Steel core	None		39 000
10	Bear River	Mokelumne	California	1900	80	748	1 : 1; 0.75 :1; and 0.5 : 1	0.75 : 1 and 0.5 : 1	Timber	16	8	43 543
11	Meadow Lake	Mokelumne	California	1899–1903	61	775	0.5 : 1 and 1 : 1	0.75 : 1 and 0.5 : 1	Timber	6	7	46 148
12	Crane Valley	San Joaquin	California	1910	130	1 880	1.3 : 1	2 : 1	Concrete core	. .	. .	. . . .
13	Sabrina	Bishop Creek	California	1907–09	70	1 065	1.25 : 1	0.75 : 1	Timber	. .	. .	47 023
14	South Lake	Bishop Creek	California	1909–10	80	650	1.25 : 1	0.75 : 1	Timber	. .	. .	74 759
15	Morena	Cottonwood	California	1909–12	167	520	1.5 : 1	9 : 10 and 1.2 : 1	Rubble in cement mortar	50	16	306 000
16	Relief	Stanislaus	California	1907–10	140	505	1.5 : 1	0.5 : 1	Reinforced concrete	108	13	136 994
17	Cucharas	Cucharas	Colorado	1911	125	550	1.5 : 1	0.5 : 1	Reinforced concrete	30		195 000
18	Swift	Birch Creek	Montana	1914	165	. .	1.25 : 1 and 1.5 : 1	1 : 1	Reinforced concrete	6	4	. . . .
19	Strawberry	Stanislaus	California	1913–16	140	612	1.5 : 1	1.2 : 1 and1 : 1	Reinforced concrete	16	4	325 640
20	Beaver Park	Beaver Creek	Colorado	1914	87	370	1.5 : 1	0.5 : 1	Reinforced concrete	5		. . . .
21	Dix	Dix	Kentucky	1924–25	270	1 032	1.4 : 1	1.2:1; 1.1:1; and 1:1	Reinforced concrete	22	7	1 885 000
22	Second Fordyce	Yuba	California	1925–26	140	410	1.3 : 1	1 : 1	Reinforced concrete	6	4	417 000
23	New Bowman	Yuba	California	1926–27	168	700	1.5 : 1	0.75 : 1 and 0.5 : 1	Reinforced concrete	20	5	300 000
24	Bucks	Feather	California	1926–28	118	1 200	1.5 : 1	1.4 : 1	Reinforced concrete	7	3	347 000
25	Salt Springs	Mokelumne	California	1928–30	328	1 300	1.4 : 1	1.3 : 1	Reinforced concrete	15		3 171 500
26	Bonito	Rio Bonito	New Mexico	1931	102	440	1.4 : 1	7 : 6	Reinforced concrete	20	11	140 000
27	San Gabriel No. 2	San Gabriel	California	1932–33	280	600	1.5 : 1	1.35 : 1	Laminated concrete	15	6	1 200 000
28	Skaguay	. . . .	Colorado	1901	70	405	0.8 : 1	0.5 : 1	Steel	. .	. .	. . . .
29	Penrose-Rosemount	Beaver Creek	Colorado	1932	100	580	1.4 : 1	0.5 : 1	Steel	12.5	4	. . . .

1876, the dam was raised to a height of 100 ft. The log crib portion was left in place, but the crib idea was definitely abandoned in favor of a loose rock-fill with a dry rubble wall on both faces (see Fig. 1). The timber facing was extended to the top of the dam, being held by log stringers fastened to the rubble wall.

This dam in its major part represented the modern plan of rock-fill dams. The dry rubble wall on the down-stream face, which retained the interior fill, may have been built as a measure of economy owing to the cost of blasting the rock. The published plans of this dam show the down-stream rubble wall of a thickness varying from 14 ft to 5 ft, but when the dam was removed in 1926, the rubble was found to be only 2 ft to 3 ft thick. The loose rock was small in size and earth and quarry waste was mixed with it. The timber logs of the old crib were found in good condition and the wrought-iron drift-bolts showed little rust. The dam stood for more than 50 yr, being replaced in 1926 by a higher dam.

French Lake and First Fordyce Dams, in California.—At about the same time the Bowman Dam was built, two other rock-fill dams were constructed on the South Yuba River in California. The Eureka or French Lake Dam was built by dry rubble stone with a slope of 1 : 1 on the water face and about 0.5 : 1 on the down-stream face. There may be some loose rock in the heart of the dam. The water face was covered with timber. This dam is still in service (1937) after a life of more than 60 yr.

The First Fordyce Dam, about 75 ft high, was built of dry rubble, about 60 ft thick at the base and with slopes of about 0.25 : 1. The facing was of timber. This dam was in service about 40 yr. In 1911 an earth and loose rock-fill was placed in front of the old dam, a concrete cut-off wall built in the fill, and a concrete face on a slope of 1 : 1 substituted for the timber face. In 1926, the dam was enlarged to a height of 140 ft by a loose rock-fill on the down-stream side with a slope of 1.3 : 1, the concrete water face being continued on the slope of 1 : 1.

The aforementioned three dams (Items (1), (2) and (3), Table 1) represent a change from the older log crib type to one in which the rock-fill alone resisted the forces of the water.

Walnut Grove Dam, Arizona.—This dam on the Hassayampa River, in Arizona, was built in 1888 of dry rubble with a narrow base and steep slopes. It was 110 ft high. The section was too frail, but apparently it held water for two years. It was washed out in February, 1890, in a flood in which the dam was overtopped by the water, the spillway being too small.

Escondido Dam, California.—This dam on San Elijo Creek, in California, built in 1895 as part of an irrigation system, was probably the first of the best rock-fill type. A loose rock-fill supported an ample dry rubble wall which was faced with timber. The slope on the water face was made 0.5 : 1 and on the down-stream face, 1.25 : 1 and 1 : 1. The dam is still in service (1937).

Lower Otay Dam, California.—This dam, 130 ft high, on Otay River, in California, was built at the same time as the Escondido Dam. The dam was designed with ample slope,[2] but the impervious element was a membrane of steel plates riveted together, enclosed in concrete, and connected to a cut-off wall on the bottom and sides of the canyon. The membrane was placed directly in the center of the dam. The dam was in service for about 18 yr when it was overtopped in a flood on January 27, 1916, and washed out. The spillway was insufficient to carry the flood.

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[2] *Engineering News*, March 10, 1898.

Bear River and Meadow Lake Dams, California.—These two dams on the Mokelumne River, in California, built 1900–1903, were about the last rock-fill dams to have the down-stream face protected by a dry rubble wall. There is a saving in the quantity of loose rock by the method, but on account of higher unit costs of the paving of the down-stream slope, it makes a more expensive dam. Fig. 2, a view of the Bear River Dam, illustrates a section common to early dams, in which both faces were laid up in rubble. The section of such dams was too frail. In one flood this dam was overtopped about 18 in. and some damage was done. Later, it was reinforced with a loose rock-fill on the down-stream side.

Some preliminary plans made by the writer for the Sabrina and Hillside Dams, on Bishop Creek, California, included a rubble facing for the down-stream slope which was not used when the dams were built. The original timber facing of Sabrina Dam is shown in Fig. 3, revealing the distortion due to settlement at places where the facing was laid on loose rock-fill. In 1929, after 20 yr in service, the facing was replaced by laying timber sleepers, between rubble, on the rock-fill.

Relief Dam, California.—This dam on the Stanislaus River, California, 140 ft high, marked what is believed to be the first use of a reinforced concrete facing. In 1906, when it was planned, Charles D. Marx, Past President and Hon. M. Am. Soc. C. E., and the writer were of the opinion that a steel-plate membrane should be enclosed in the concrete. The experiment of depending upon concrete, 36 in. thick, under a 140-ft head of water, was tried and was entirely successful.

The dam has a dry rubble wall, 100 ft thick at the bottom and 13 ft at the top, the front 5 ft being very carefully laid up and 2 ft thick next this concrete set in cement mortar. In the light of subsequent experience, these dimensions of the dry rubble seem excessive.

Morena Dam, California.—The Morena Dam, built in 1909–1912, has a height of 167 ft above the stream bed, but the cut-off wall was carried 112 ft below in a narrow gorge in the granite. The dam is of ample proportions and is noteworthy as having a facing made up of rubble stone laid in concrete 6 ft thick at the stream-bed level and decreasing to about 3 ft at the top. In addition, a layer of reinforced concrete 1 ft thick was placed on the rubble masonry rising to a point 42 ft above stream bed, above which the rubble masonry is relied upon for water-tightness.

The dam was raised 5 ft in 1916–17 and 10 ft in 1922–1923. At one time it was overtopped, but it withstood the water without damage.

Recent Dams.—Table 1 gives a list of dams with main dimensions, which have been built in the past twenty-five years. Practically all these dams are of the same type as defined above, the result of the evolution of the previous

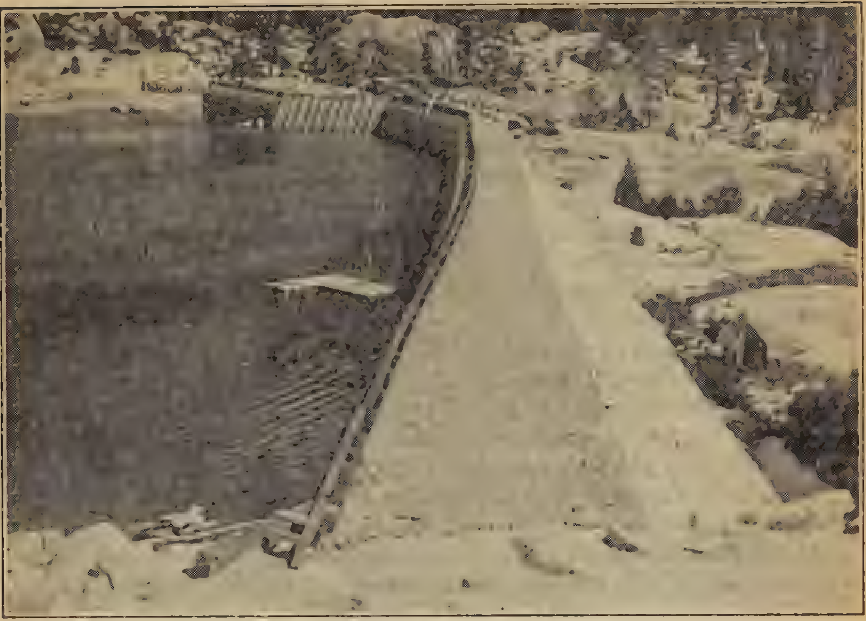


Fig. 2.—View of Bear River Dam, California

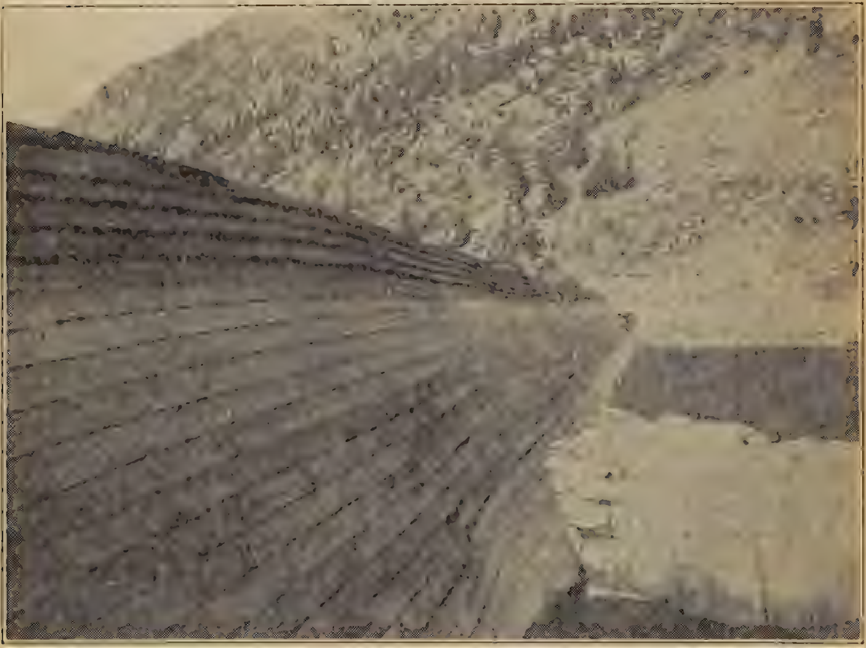


Fig. 3.—View of Sabrina Dam, Bishop Creek, California

half century. The Salt Springs Dam on the Mokelumne River, California, 328 ft in height, represents the present maximum height of rock-fill dams.

DESIGN

The design of rock-fill dams is based upon the empirical knowledge gained in the past. The important elements are as follows.

Foundation.—To a great extent, the criteria that obtain for other dams may be applied to rock-fill dams. Rock is the best foundation but hard material, such as cemented gravel or indurated sand, will sustain the weight of such a structure. On the assumption that water is kept from the foundations by the cut-off wall and the facing, the nature of the foundation rock is not of as great importance as in other dams.

Usually, the dam site is covered with stream gravel and boulders and with hill wash. Salt Springs Dam rests upon granite, but it was necessary to remove about 400 000 cu yd of gravel and boulder over-burden. Bucks Dam rests in one part upon granite and at the north end upon a hard indurated granite sand from which an overburden 10 to 20 ft thick was removed. Most of the dams in California rest upon native granite, as they were built in the higher Sierra Nevada.

The important element in the foundations is to obtain a material that will not easily erode, will not settle, and will permit the construction of a safe cut-off wall.

Rock-Fill.—The character of the rock from which the fill is made is of the utmost importance. In granite regions the rock varies from a hard durable stone capable of standing great stresses to stone that will crumble into sand either under pressure or the action of weather. Granite will often be so coarse-grained that freezing water will disintegrate it. Hard durable granite is often found close to belts or zones where the crystalline structure has been broken down by stresses resulting from earth movements. It is not an unusual circumstance to find a hard rock cliff of excellent rock, behind which is a mass of rock wholly unsuited for the purpose. In large dams the opinion of a competent geologist familiar with rocks should be obtained from time to time.

Other igneous rocks are sometimes suitable for rock-fill dams and often they are not. Andesites that have been completely metamorphosed into hard tough greenstones, or the diorites, are good material, but the andesites may be of such recent origin as to be unsuitable.

Sedimentary rock, such as sandstone or limestone, may furnish material suitable for a rock-fill dam. The Dix Dam was made of limestone. The writer is unaware of any dam built of sandstone.

The criterion of rock for the fill is that it shall be sound and not subject to rapid disintegration under weather conditions. Many rocks, while apparently sound, are defective in that they are cut by numerous shear planes due to block faulting or other earth stresses. The rock in a dam of even moderate size must resist considerable stresses due to the weight of the rock above and, in addition, the forces arising from the weight of water. Under such conditions any rock, however hard, that will split, crush, or otherwise disintegrate, is unsuitable for a rock-fill dam.

The nature of the rock-fill is one upon which difference of opinion will develop. It is believed that the fill should be composed of individual rocks of fairly uniform size, one rock bearing directly upon another, usually expressed as "rock to rock." Any wide divergence in size will cause excessive and unequal settlements, something to be avoided wherever possible. To illustrate the idea, if one selects a sample from a mass of broken stone of fairly uniform size, it will be found extremely difficult to force the sample into the mass by added pressure. On the contrary, if a large rock, which may rest upon small gravel or sand, is given a load, it will settle into the mass by displacing the smaller grains. In the usual method of dumping rock into the fill the larger ones roll to the bottom and the smaller ones remain at the top. A graded fill results. At Salt Springs, rocks as heavy as 25 tons were used, but the usual size was about 10 tons. At Bucks (see Fig. 4), where the rock was handled entirely by trucks, the maximum size was about 3 tons.

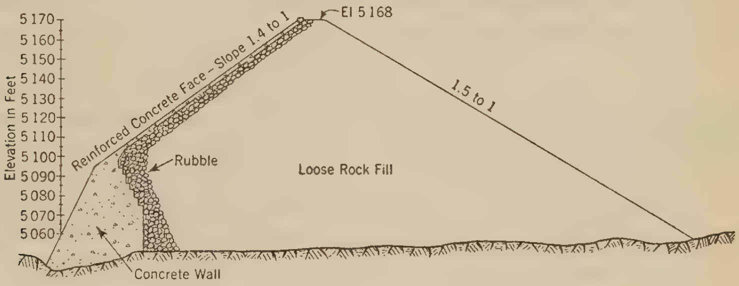


FIG. 4.—CROSS-SECTION OF BUCKS DAM

In all quarry work there is a certain amount of earth, spalls, and dust that results from the operations. Some of this waste should be rejected, but a certain quantity finds its way to the dump of the fill. It is necessary to wash this quarry waste into the fill and not allow it to accumulate in any one zone or position on the slope of the fill. At every place where the rock is dumped, there should be at least two standard fire streams of water under 100 lb pressure, constantly washing the fines into the rock. It is inevitable that the smaller rocks and most of the quarry waste will accumulate at the top of the dump. The necessary roadways require a surfacing of quarry waste. If the dam is built in vertical stages, the accumulated layer of small material at the top of any stage should be washed into the mass, but as this is rarely possible, it should be discarded. If left in place, such a layer of small material invites large local settlement, especially when the water load is applied.

It is realized that a heavier mass will result if fine material can be washed into the interstices of the fill. Such a course is advisable, only after the fill of rock has been made. The procedure should not result in a fill composed of rock in some places and masses of small material in others.

To the extent that it is possible in construction, the equipment for moving and placing the rock should be arranged to handle rock of about the same size should it be found necessary to open more than one quarry. Fills made in stages should have each stage fairly well complete before the next above is placed. Usually, in large dams, work is carried on from both sides of the canyon, in which case there should not be any great difference in the height of the stages.

Settlement.—All rock-fill dams settle. The settlement is due to the crushing of the bearing points of the rocks under increased load or to the displacement of small rocks by large ones. If the fill is made as described, the point of one rock will rest, in general, upon a surface of the rock below. As pressure increases, the rock point will be crushed, and, finally, when the water pressure is on the dam, the maximum forces will be exerted and the rock points will be crushed until a balance is established between the compressive forces and the resistance of the rock.

Weather effects will operate to increase the disintegration of the rock and this is especially true in regions with cold winters. However, in the interior mass of a dam, the effects of weather are not so great as upon the surface.

The maximum settlement of a rock-fill occurs in the first few months after being placed. Settlement of as much as 5% of the height may be expected; and it continues as the dam increases in height. After completion the settlement continues for years. In the first Bowman Dam, the vertical settlement after completion continued for 50 yr and amounted to about 1.25% of the height. This settlement was quite uniform for about 20 yr with a total of 1 ft in 100 ft of height and then it was much more gradual.

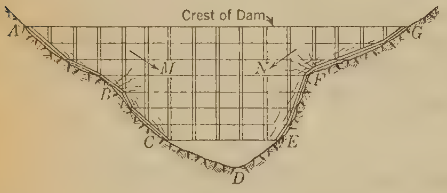


Fig. 5

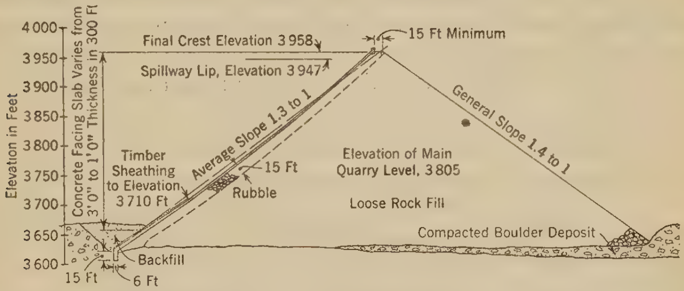
Settlement in a dam is not uniform; nor is it vertical. Under usual conditions, the rock is dumped into the canyon from both sides, the fill advancing toward the center. There is a settlement of the rock toward the center as indicated by the arrows, *M* and *N*, in Fig. 5. This movement continues after the fill is complete. Fig. 5 indicates a typical profile across a canyon in which a dam may be built. The presence of a cliff, *E-F*, results in different settlements within a short distance. The configuration of the canyon wall determines to a large extent the difference in settlement.

Settlement due to the water load is an important factor in rock-fill dams. At the Dix Dam, the vertical settlement in the central portion 3 yr after completion amounted to 1.75 ft and the horizontal settlement was 1.50 ft, the height of the dam being 270 ft. This represented a movement approximately at right angles to the water-face of the dam which is mostly on a slope of 1 : 1. At the Strawberry Dam the settlement in the central section, 140 to 145 ft in height, was 1.62 ft vertical and 0.95 ft horizontal in a period of about 7 yr, indicating a movement at an angle of 75° to the slope of the water-face which is mostly 1 : 1. These measurements were taken at the top of the dam. The general indications are that the vertical settlement will continue a longer time than the horizontal, the latter being due to water pressure.

There are indications that, sometimes, movements will occur that are greater than those indicated above. This is especially true where the impervious element is placed in the center of the dam. In this location it is subjected to full water pressure. In the Crane Valley Dam, with a core wall of concrete, a horizontal movement of several feet has occurred at the top.

In the Salt Springs Dam (see Fig. 6), provision was made for ultimate settlements of 6 ft vertically and 4.2 ft horizontally.

The character and location of the settlements of this rock-fill are important as determining the design of the impervious element of the dam. The movements from the side toward the center, indicated by arrows, *M* and *N* (Fig. 5) will result in closing the expansion joints in the central section and opening those on the flanks. Dams such as Dix, Strawberry, and Salt Springs, show this effect. Again, where differential settlement is possible as at Point *F*, Fig. 5, cracks may be formed in the concrete facing. In large dams the necessity of expansion joints where the facing joins the rigid rock of the canyon wall is also indicated.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3 800
3 750
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3 600
Elevation in Feet
Concrete Facing Slab Varies from 3'0" to 1'0" Thickness in 300 Ft
Final Crest Elevation 3 958
15 Ft Minimum
Spillway Lip, Elevation 3 947
Average Slope 1.3 to 1
General Slope 1.4 to 1
Timber Sheathing to Elevation 3 710 Ft
15 Ft
Rubble
Elevation of Main Quarry Level, 3 805
Loose Rock Fill
Compacted Boulder Deposit
Backfill
15 Ft
6 Ft

FIG. 6.—CROSS-SECTION OF SALT SPRINGS DAM IN CALIFORNIA

In the end, the depth of settlement and the question as to whether it is uniform throughout the mass of the rock-fill will be determined largely by the character of the fill, as discussed above. If layers of quarry spoil or gravel are permitted to accumulate in the rock-fill, excessive local settlements may be expected.

Cross-Section.—In the first rock-fill dams, the down-stream face was built on a slope of 1 : 1, or steeper. The up-stream slope was usually the same which resulted in a width of base of about twice the height. These steep slopes made it necessary to protect each face with rubble in order to retain the loose rock-fill.

Escondido Dam of 1895 may have been the first one to be built in which the down-stream face was composed of the unconfined loose rock. Probably Bear River and Meadow Lake Dams (Items 10 and 11, Table 1) were the last ones built with the down-stream face of rubble. In all the older dams there was a tendency to build the upper portion with steep slopes such as 0.75 : 1 and 0.5 : 1. The undoubted reason was to reduce the rock contents of the dam and the resulting cost.

There does not seem to be any good reason, except a possible saving in cost, for making the down-stream face other than the natural slope of rock dumped from cars. It is possible to have this slope as steep as 1 : 1 with large rocks only. However, in the ordinary construction of such dams, the rock varies in size and a loose rock-fill will generally assume a slope of 1.3 : 1 or

1.4 : 1 when the rocks are allowed to roll down the face of the fill. To make the slope steeper than 1.4 : 1 will usually require a second handling of the rock and there is no reason why this should be done.

The slope of the up-stream face is a subject upon which there is considerable difference of opinion. This is evidenced by the different slopes adopted by various engineers, as shown in Table 1. One of the controlling elements is found in the necessity of sufficient width of base to withstand the horizontal component of the water pressure. Experience with the Bowman Dam indicates that a base width equal to twice the height will stand without failure. Other dams noted in Table 1 have approximately the same ratios of base-to-height. However, in later dams, the ratios of width of base to the height have been increased, ranging from 2.5 : 1 up to 3 : 1. In low dams, the width of crest adds to the width of base a distance that is a factor in the total. This value decreases as the height increases. A crest width of 15 ft is a minimum, and usually a road is required across the dam. On a dam 100 ft high a 15-ft crest would add about 6% to the width of base, while on a dam 300 ft high the proportion would be about 2 per cent. The cross-sections adopted in recent dams have the ratios of base width to height, indicated in Table 2.

TABLE 2.—Ratio of Base Width to Height

No.	Dam	Height, in feet	Width of base, in feet	Ratio: Height to base
20	Dix	270	691	1 : 2.56
22	New Bowman	168	375	1 : 2.23
23	Bucks	118	340	1 : 2.90
24	Salt Springs	328	930	1 : 2.84

On an assumption of a dam 200 ft high, with a crest 15 ft wide, a down-stream slope of 1.4 : 1 and a unit weight of 100 lb per cu ft for the loose rock and with the water load, ratios of height to base of 1 : 2.25; 1 : 2.5, and 1 : 3 would have sliding factors (ratios of weight of rock to water pressure) of 4.50, 5.14, and 6 : 45, respectively. These ratios are practically constant for all heights of dams. Within the limits of the ratio of height to base width, mentioned, all such dams have ample safety against sliding. It is possible, therefore, to adopt any slope for the up-stream face that may be desired.

In Table 1 the principal face slopes are given for a number of dams. The writer believes that the slope should not be less than 0.75 : 1 and that a slope of 0.5 : 1 is too steep. Much depends upon the amount of rubble rock behind the facing. In the Relief Dam, 140 ft high, the rubble section is 100 ft thick at the bottom and 13 ft thick at the top. Such a section is stable in itself, and the slope of 0.75 : 1 on the up-stream face is ample. However, the cost of the rubble indicated that a flatter slope and less rubble would have been more economical.

In any slope steeper than the natural slope of the loose rock, it is necessary to carry up this rubble facing with the loose rock-fill in order to retain the latter Under these conditions it is necessary to maintain the derricks by which the rubble is placed, upon the face of the rubble already laid, usually a difficult operation.

In the case of the Salt Springs Dam it was decided after discussion, to make the up-stream slope of the loose rock-fill 1.3 : 1, or what was practically the natural slope (see Fig. 7). The principal reasons for this conclusion are as follows:

(1) The loose rock-fill could be built in advance of the rubble facing. To a certain extent this eliminated danger to workmen from rocks rolling down the slope. However, the controlling motive was the fact that time was given for the loose rock to settle. As settlement is a function of time and of superincumbent weight, building up the loose rock ahead of the rubble permitted the fill to take its initial settlement before the rubble was placed upon it. The settlement is usually considerably greater in the first three or four months than thereafter.

(2) The design of the rubble facing permitted the use of caterpillar derricks resting upon the rubble as it was built up. Rock for the rubble was brought in trucks and dumped down the loose rock-fill at convenient points, the caterpillar derricks moving about as required.

It is recognized that a steep slope for the up-stream face reduces the quantity of all parts of the dam—loose rock, rubble and facing—but the aforementioned advantages are believed to outweigh the increased cost of the flatter slope. This is especially true of the element of initial settlement.

It is the common custom to make the up-stream slope a curve concave to the water. This prevents buckling of the concrete facing under the settlement. Usually, the result is obtained by varying the slope. At the Strawberry Dam, several slopes, varying from 1.2 : 1 at the bottom to 1 : 1 at the top, were used. At the Salt Springs Dam, at the central section, the up-stream face was made a curve, concave to the water. Measured on a horizontal line from the theoretical slope of 1.3 : 1 (the rock at the crest elevation) the offset was 13.6 ft up stream, and at an elevation of 200 ft below, the offset was 10.5 ft down stream from the theoretical slope. In plan, the Salt Springs Dam has a crest slightly curved up stream. The face is a warped surface.

Summarizing the elements that control the design of the cross-section, the down-stream slope may be made of dumped rock, unconfined, 1.3 : 1 or 1.4 : 1; the crest width should be at least 15 ft; and the up-stream face should preferably be the natural slope, which can be made 1.3 : 1. This will result in a stable dam with base width 2.7 times the height, to which is added the crest width. To prevent buckling of the facing, the profile of the up-stream face should be made concave to the water.

Rubble Wall.—By this term is meant the layer of rock interposed between the impervious facing of the dam and the loose rock of the main fill. It is variously referred to as hand-laid or packed rock, derrick-laid rock, placed rock, etc. The term, rubble, may be open to criticism.

The function of the rubble is to act as a semi-rigid member between the rigid facing and the loose fill which is subject to settlement in several directions. The rubble also helps to bridge inequalities in the loose rock and to furnish a uniform bearing to the facing.

The rubble wall is an important part of the dam. It should be built of large rocks which are placed in positions in bearing contact with each other. The remaining spaces should be carefully filled with small rock and spalls wedged together to form a compact mass. The rock may be laid in horizontal planes, but made interlocking between layers by projecting rocks. Large rectangular rocks on the outer surface should be placed with faces roughly parallel with the slope.

If it is well built, a rubble wall is capable of resisting distortion or overturning stresses comparable to masonry walls. Note is made of well-built rubble retaining walls on the Southern Pacific Railroad where the line crosses the Sierra Nevada. A wall at the summit of the mountains, with a batter about 0.25 : 1 has carried, for many years, a double-track railroad upon which is operated the heaviest locomotives. Other instances will occur to the reader.

Table 1 gives the thickness of rubble in different dams. With the exception of Morena and Relief Dams, there is a rough correspondence in the dimensions in that the thickest part is at the bottom which narrows to a nominal thickness at the top. The idea seems to have been to vary the thickness in accordance with water pressure. In some dams the rubble was made of only nominal thickness as in the Beaver Park Dam. The thickness given for the first Bowman Dam is incorrect as the rubble facing was found to be only 2 to 3 ft thick when the dam was demolished.

At Salt Springs (Fig. 8) the writer believed that there were two controlling factors in determining the thickness. Water pressure indicated a maximum at the bottom. However, settlement of the loose rock-fill, known to be unequal, is a maximum at the crest of the dam. This factor required a considerable thickness at the crest. As built, the thickness of the rubble was made uniform from top to bottom, being 15 ft normal to the face, or about 25 ft wide on the horizontal. A similar design was followed in the Cucharas Dam where the rubble was made 30 ft thick normal to the face slope of 1 : 1.

An argument in favor of the uniform thickness of rubble of reasonable dimensions is that the derricks used to lay the rock can operate upon the wall itself without change from base to crest, which is an important item in reducing cost (see Fig. 9).

In the final analysis, the thickness of the rubble is a matter of judgment. Experience indicates that the dams built in recent years have been provided with rubble walls of sufficient thickness to perform the necessary function of distributing the water load to the loose rock-fill and of resisting excessive distortion under unequal settlement.

Cut-off Wall.—The cut-off wall is an important element in rock-fill dams, as upon it depends the connection of the facing with the bottom and sides of the canyon walls. It should be of dense concrete, wider than the facing if of concrete; and the depth into the rock of the canyon should be sufficient to cut off all visible seams in the rock and also to provide resistance to the grout injected under pressure. Grout holes should be drilled through the concrete and should penetrate deeply enough into the rock so that grout under a pressure of at least 100 lb per sq in. may be forced into any seams not closed by the wall. The design and dimensions will be determined by each locality.

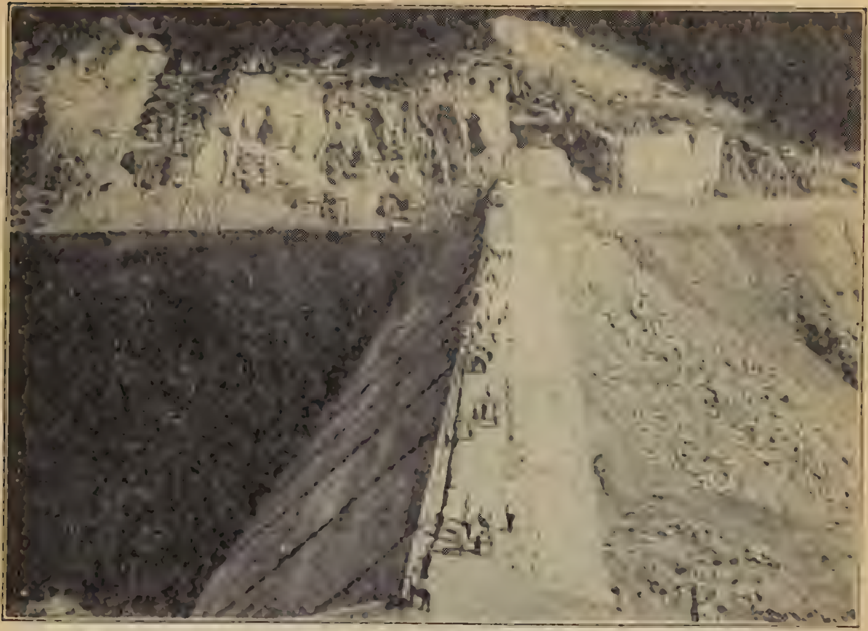


Fig. 7.—General View of Salt Springs Dam, with Spillway on Left Bank

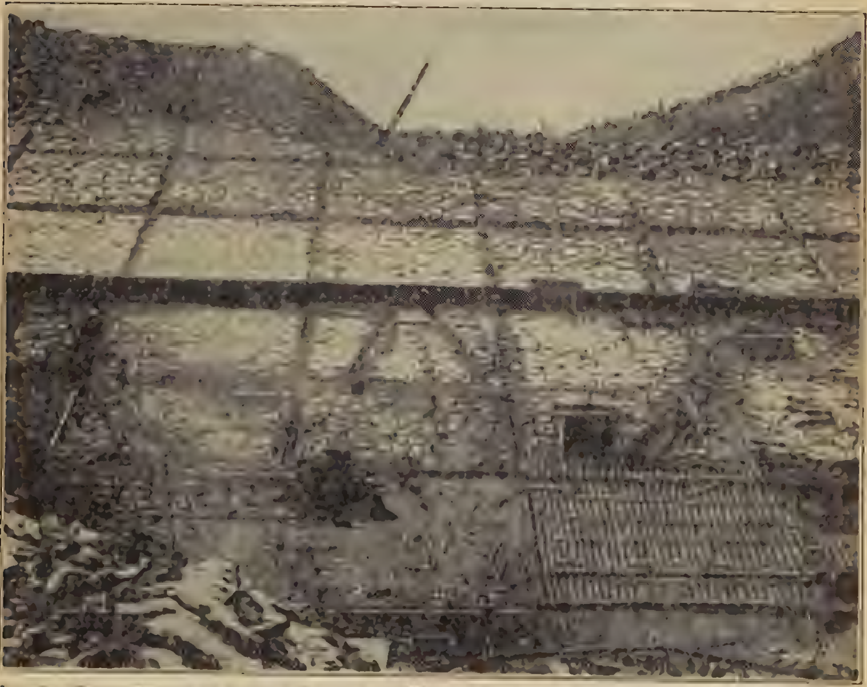


Fig. 8.—Construction of Rubble Cushion and Timber Sheathing at Bottom of Salt Springs Dam

Facing.—Dams of the type under review must be provided with an impervious element to resist the water. As previously mentioned, the facing is the first and only resistance to the water as the loose rock and rubble offer practically no obstruction to the flow.

The position of the impervious element is important. A few dams have been built with the steel or concrete diaphragm in the center of the dam. The most conspicuous example of the method was the Lower Otay Dam. There are a number of objections to placing the impervious element in the body of the dam. All the rock-fill up stream from the diaphragm is submerged, and its function of resisting the forces of the water is nullified by partial flotation of the rock and by the fact that the water pressure is exerted where the up-stream portion of the rock cannot resist it. Again, settlement of the loose rock is not vertical but approximately along a resultant of the forces of gravity and of the water. In settling, the rock distorts the diaphragm and possibly ruptures it, and, as it is inaccessible, inspection and repairs are not possible. Other disadvantages may be mentioned, but those given should be sufficient to cause the rejection of this type of design.

It is believed that the only proper place for the impervious element is on the water-face of the dam, directly in contact with the water. In such a design all the rock acts to resist the water, the facing is open for inspection and repair, and its construction can follow as far behind the rock-fill as possible.

In the design of the facing of a rock-fill dam of medium or large size the engineer is confronted with the problem of uniting a fairly rigid facing to a mass of rock in which settlement is expected. The facing is exposed to changes of length due to temperature changes. In the spring the reservoir will be filled. In the late summer it will be partly or entirely emptied, which condition will prevail through the winter. The facing will then be exposed to ranges of temperature of 100° F, or more. In dams the facing may have a horizontal length from 500 ft to 1 000 ft, or more, and a width on the slope of from 200 ft to 500 ft. If made of concrete or steel and if it is free to move, the facing may expand and contract from 2 to 10 in. under the conditions named. However, movement cannot take place where the facing joins the walls of the canyon, which do not move.

The first factor in the problem (that of settlement which may be of the order of several feet) would indicate that the facing should be separated from the rock mass and left free to slide on the rock mass as it settles. The second factor, annual changes due to temperature would call for movements which, it is believed, cannot be permitted. Horizontal movements would take place, but in any possible plane of sliding the vertical movements, or rather movements along the inclined face of the dam, would not take place. Under low temperatures the facing (if made of steel or concrete) would contract and the movement would be downward. With rising temperatures, the movement would tend to slide the facing upward which would be resisted by the weight of the facing, friction on the sliding plane, and resistance where the facing joins the side walls of the canyon. It is believed that in any practicable case the vertical movement would not take place and that the facing, if made of steel or concrete, would buckle or crush under the forces to which it would be subjected. Timber facing, which is adapted only to relatively low dams, is sufficiently flexible to adjust itself to the changes due to settlement or to temperature changes.

It should be noted that the rubble cushion behind the facing, if properly constructed, forms a semi-rigid rock mass that resists, to a large extent, the deformation due to settlement. The differential movements between the rock-fill and the rubble wall with the facing resting upon it, is believed to take place in that zone of contact rather than on the plane between the facing and the rubble.

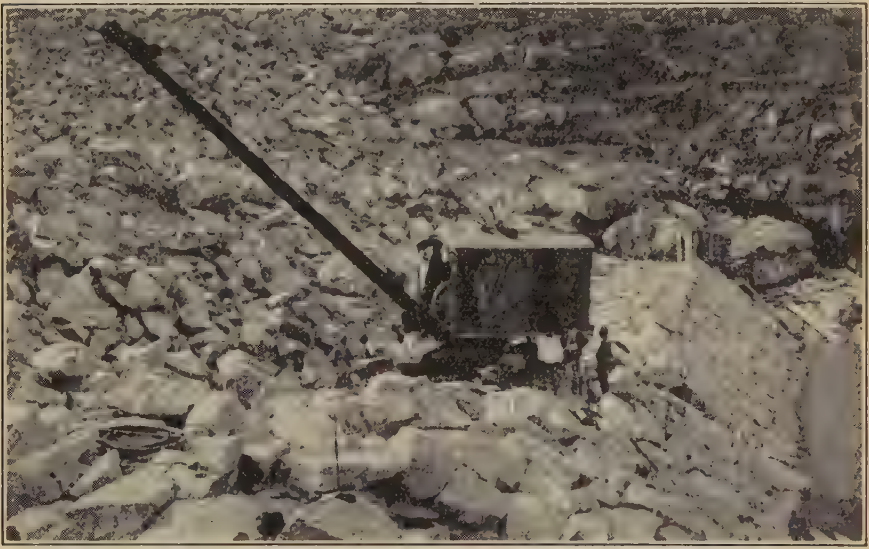


Fig. 9.—Construction of Rubble Cushion, Salt Springs Dam

The plan generally followed is to fasten a steel or timber facing directly to the rock of the rubble wall. Taking account of the various factors involved and basing the opinion on experience, the writer believes that the best method is to pour a concrete facing directly upon the rubble wall, taking care that interstices are left in the rock work into which the concrete is poured in order to secure a firm bond between the two elements. In addition, grooves or chases should be built in the rubble at the joints in the concrete facing to support the slabs over the joints, thus giving a closer bond. In this manner the facing is secured to the rubble and temperature changes are resisted. In settlement, the facing follows the rubble and stresses caused by local settlement can be resisted by the reinforcement. It is possible that the facing may be crushed in places, but experience shows that this does not often occur. Reference is again made to Bucks Dam (see Figs. 3 and 10) where a concrete facing more than 1 000 ft long, poured directly on the rubble and containing no expansion joints, has not cracked in the eight years since it was constructed. At Salt Springs (see Fig. 11), the facing was poured directly on the rubble and although some minor crushing has taken place, repairs are easily made at low water. On the contrary, at the

Strawberry Dam, the design provided for a complete separation between the two elements in the upper portion of the facing, about 100 ft wide along the slope. The rubble was made smooth by a covering of cement mortar and roofing paper. The facing is in good condition but the expansion joints have opened wide at the ends of the dam and closed together at the center.

It must be recognized that these conclusions have been reached after consideration of the several contradictory factors that enter into the problem of constructing an impervious face of rigid concrete upon a rock-fill subject to continued settlement and to temperature changes.

The problem of a dam with a sheet steel facing is similar to that with a concrete face. In the two rock-fill dams listed in Table 1, with steel facing (Items 28 and 29), the steel plates were fastened by bolts to the rubble wall and were provided with expansion joints. In the highest of the two dams, the Penrose-Rosemont Dam in Colorado, the dry rubble is faced with rubble masonry laid in cement mortar 4 ft to 2 ft thick, with a plaster face next to the steel facing.

Timber Facing.—With the exception of the small Chatsworth Dam (Item 6, Table 1), the facing of the older dams was made of timber. Timber sills, 12 by 12 in., or larger, or made of hewn timbers, are buried in the rubble wall on the face of the dam. It was common practice to space them 6 ft, center to center, the sill being in vertical planes at right angles to the water-face. Such sills were fastened to the rock with large bolts. In the lower half of the facing, three layers of 2-in. or 3-in. planks, made as wide as available, were spiked to the sills, the central layer being placed at right angles to the other two. In the upper section, two layers of planks were used. Small spaces between the sheathing and the rubble were carefully filled with spalls to give as much support as possible to the facing. All joints were caulked with oakum. The result was a good water-tight facing that lasted, with minor repairs, for at least 20 yr. Some of the timbers of the first Bowman Dam lasted 50 yr. On the Sabrina Dam the timber face lasted about 20 yr, after which it was replaced by another timber face in 1929. In remote locations, where concrete is costly and timber plentiful it is good practice to use timber facing. It has the advantage of cheapness, which is often an important factor in new enterprises. Moreover, it is flexible, and its use at first permits the rock-fill to take most of its settlement before a final facing of concrete is applied.

Concrete Facing.—On the higher dams, built in recent years, the common practice has been to use reinforced concrete for facing. The small dam at Chatswork Park, in Southern California, built in 1896, was probably the first dam in which concrete was used as a facing. The Relief Dam (1907–1910) is believed to be the first of the larger dams in which reinforced concrete was used.

There was a question as to whether the concrete would be water-tight, but it was there proved that concrete would be practically water-tight under pressures up to 140-ft. Experience at Dix and Salt Springs Dams indicates that water pressures up to 300 ft can be resisted safely by concrete. There is hardly any way in which the thickness can be determined. At Dix Dam the slab is 18 in. thick under a head of about 260 ft. At Salt Springs, the thickness was made 3 ft, under a 300-ft head. The element of settlement and cracking of the concrete

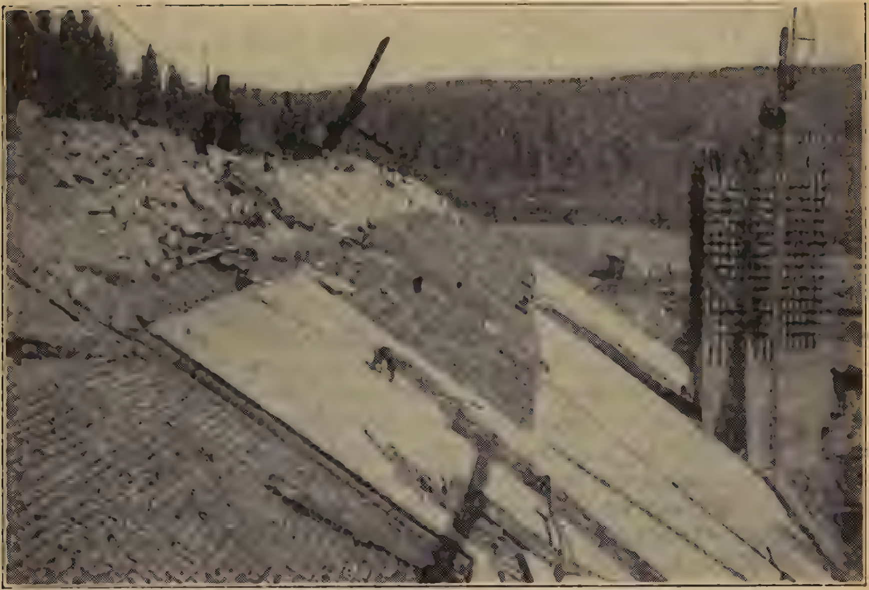


Fig. 10—Construction of Monolithic Concrete Face, Bucks Dam, in California

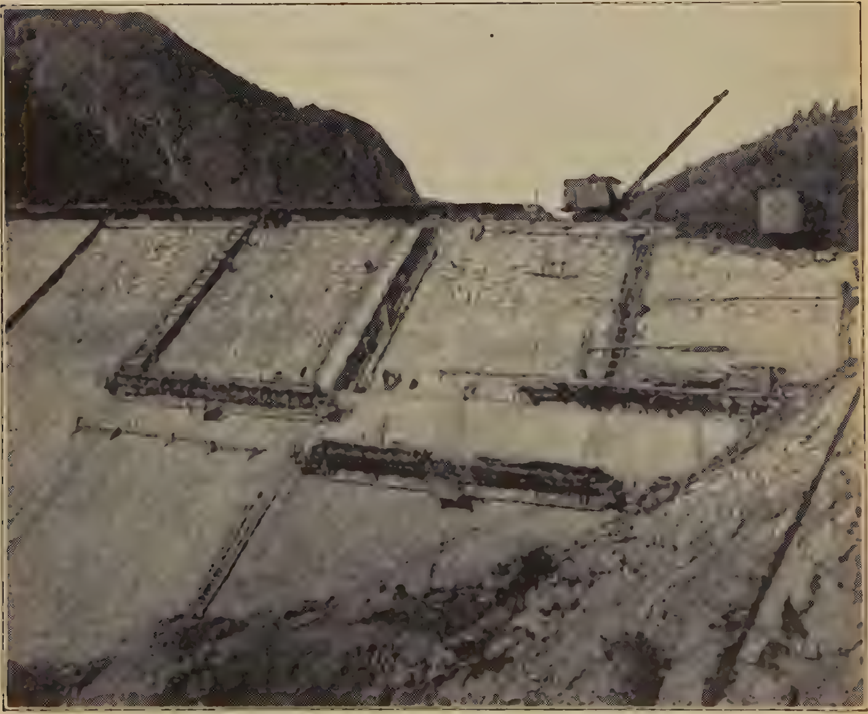


Fig. 11.—Construction of Concrete Facing, Salt Springs Dam

seems to indicate a substantial thickness of slab. It is believed that a slab thickness of 1% of the head of water will provide ample thickness. It is also believed that the slab should not be less than 12 in. thick at the top and preferably thicker where climatic conditions may lead to disintegration from frost.

At about the time the Relief Dam was under construction, a concrete facing was projected for the Morena Dam. The rubble backing was set in cement mortar which was found to be so water-tight that the concrete slab was only installed in the lower sections of the dam.

Concrete facing of rock-fill dams has usually been placed in slabs ranging from 50 to 60 ft square. Expansion joints permit adjustment under contraction and settlement of the fill. However, in the Bucks Storage Dam, the concrete facing was built without expansion joints and the reinforcement was extended across the construction joints so that the slab is practically monolithic. It may seem inconsistent to have approved two different forms of facing in two important dams being built at the same time, as was done by the writer at Bucks and at Salt Springs. However, the conditions are different. At Bucks, owing to the construction of a gravity concrete dam across the main stream as a part of the structure, the maximum width of the concrete facing measured up and down the slope is about 127 ft, while the crest length is 1 200 ft. In plan, the dam has two angles dividing it into three sections, the angles being convex to the water. Construction joints were spaced 60 ft apart horizontally and 31 ft 10 in. apart up and down the slope. The conditions were believed to warrant the omission of expansion joints. At Salt Springs, on the other hand, the canyon was relatively narrow and of V-shape. With a crest length of 1 300 ft, the height at the center was 328 ft and the width of the facing slab, measured on the slope, was about 550 ft. Much greater settlement was expected than at Bucks, and it was deemed advisable to build the slab with expansion joints, which were placed on 60-ft centers, making the slabs 60 ft square.

Reinforcement of the slab of Bucks Dam consisted of one layer of deformed bars, in the center of the slab. The vertical bars were 1 in. square spaced 12 in. center to center, and $\frac{3}{4}$ in. square bars spaced 9 in. apart, the latter being used in the upper reaches of the slab. The horizontal bars were $1\frac{1}{4}$ in. square in the lower reaches, spaced 11 in. to 12 in. apart, and 1 in. square in the upper reaches, spaced 9 in. and 11 in. apart, the bars being about 0.75% of the slab area. Reinforcement of the slab of Salt Springs Dam was made with one layer of bars in the center of the slab down to an elevation 184 ft below the spillway lip, where the slab was 27 in. thick, and two layers below that elevation. Bars are 1 in. square and 1 in. round, spaced from 8 in. to 14.5 in., center to center, depending upon position. The reinforcement was the same in both directions and did not pass either the construction joints or the expansion joints. In general, the area of bars was made about 0.5% of the area of the slab.

An exact calculation of the percentage of reinforcement is not possible although the stress can be estimated under temperature changes. The concrete of the slab is poured directly upon the rubble wall that supports it. The concrete enters into the interstices of the rock, fills the inequalities and adheres to the rock. The concrete is thus held in position, and expansion or contraction is prevented. It might be questioned whether the concrete should not be formed so as to be separated from the rock in order to permit movement when the settlement of the loose rock-fill takes place. There does not seem to be any valid reason why this should be done and there are a number of reasons why it should not. Experience has shown that concrete slabs formed on well-laid rubble do not crack from expansion and contraction of the concrete. Some settlement cracks are to be expected. However, the slab of Buck Dam (elevation more than 5 000 ft) has passed through eight winters with temperatures below zero and does not show any cracks. The leakage is negligible. The experience at other dams such as Relief or Strawberry is the same. The rubble masonry face of Morena Dam, which has been in service more than 20 yr (see Item 15, Table 1), has proved ample for the purpose.

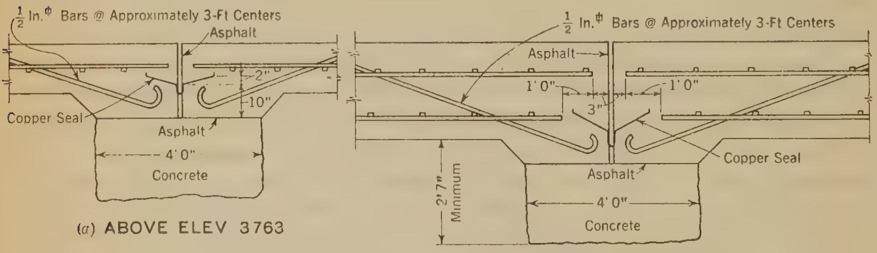
Steel Facing.—This type of facing has been used on two dams in Colorado with success. The Skagway Dam built in 1901 is 70 ft high with the up-stream face on an angle of 30° from a vertical. The crest is 405 ft long. Steel plates 5 ft wide and 15 ft long and varying in thickness from 0.5 in. at the bottom to 0.25 in. at the top. Horizontal joints were made with butt straps and the vertical joints were made with 5 by 4-in. angles, with 5-in. legs outstanding; 2-in. by $\frac{3}{8}$-in. filler bars were riveted between the toes of the adjacent angles, temperature adjustment taking place by the flexure of the angles. The dam has been in use 35 yr (see Item 28, Table 1). The facing has performed its function, and the damage due to corrosion was found, in 1932, to be negligible.

The Penrose-Rosemont Dam, built in 1932, is 100 ft high, 580 ft long on the crest, and was built with a water-face on a slope of 0.5 : 1. The steel plates are 8.5 ft wide by 20 ft long, varying in thickness from $\frac{3}{8}$ in. at the bottom to $\frac{1}{4}$ in. at the top. Horizontal joints were made by lapping the plates, bolting them together every 18 in., and welding. Vertically, two plates were connected by a T, making a plate about 40 ft long. These lengths were connected by a contraction joint made of plate steel rolled to a semi-circular form. The plate facing was fastened to the rubble with bolts. Steel facing is entirely practicable and may have an increasing use in the future.

Construction and Expansion Joints.—The difference between the two types of joints is one of function. Construction joints representing the boundary of a concrete slab poured at one time have the concrete of the next pour directly in contact with the first. At Bucks Dam both horizontal and vertical joints were construction joints. In practically all cases, all horizontal joints become construction joints as such joints, being always in compression, never tend to open although they may be subjected to shear. Vertical joints, on the contrary, are subject to movements of the mass of the dam in which case they may either open or close. As the movements of the rock-fill under settlement are toward the center of the dam from the flanks, the joints in the central portion will tend to close and those at the abutments will open. Where there is an abrupt change in the profile of the canyon wall, as at Point *F* in Fig. 1, the joints will open. For these reasons it is advisable to build the expansion joints at the center wide open and those at the abutments closed.

In dams of some height there is a tendency for cracks to appear where the concrete facing joins the canyon walls at the cut-off wall. These cracks are due to settlement and the extent of the cracking depends upon the care used in placing the rock-fill and the rubble. At the line of contact between the concrete face and the concrete cut-off wall it is necessary to connect the two with reinforcing bars. Movement of the facing under settlement will cause bending stresses at the junction line. This movement will not all take place at one line, but there will be a zone along the contact line where the movement changes from nothing at the cut-off wall to a maximum in the body of the dam. Provision should be made in such cases for additional expansion joints which will permit motion without rupture of the concrete.

When the pressure is not great construction joints may be made in the concrete without seals, offsets being made to form keys. Such joints will generally be kept tight if the reinforcement is carried across the joint. However, it is common practice in important dams to provide some form of seal, both in construction and in expansion joints. In the latter, where movement is expected which will cause the joint to open or close, there must be a seal against the passage of water. The designs take various forms. In timber facing no such joints are necessary as there is in effect a joint at each board of the facing. In concrete, the joint usually consists of sheet copper extending into the adjoining slabs with a U-shaped section covering the open joint. The open part of the joint is usually filled with asphalt. At Salt Springs (see Fig. 12) the sheet copper was 33 in. wide, the sheets being brazed together at junctions. In steel facings, the expansion joints may be made in different ways, as illustrated in the two Colorado dams cited.



(a) ABOVE ELEV 3763

(b) BELOW ELEV. 3763

FIG. 12.—VERTICAL EXPANSION JOINTS, SALT SPRINGS DAM

Other elements of rock-fill dams, such as outlets, valves, spillways, etc., that are similar to other types of dams, do not require discussion herein. However, it should be borne in mind that a rock-fill dam may be destroyed by the passage of a flood over the crest, as in the case of the Lower Otay Dam, where a masonry dam would not have been injured. This points to the necessity of ample spillway capacity which should be free from all gates, flash-board structures, and such other construction that might cause the spillway to clog at the time of flood.

CONCLUSIONS

The paper is limited to the design of rock-fill dams. A certain amount of historical and statistical data is included because the type has been the result of evolution extending over a course of years. Rock-fill dams are not the most stable type, but there are a number of reasons for their adoption.

A rock-fill dam is defined as one made up of loose rock forming the mass of the dam, an impervious face next to the water that may be made of timber, steel, or concrete, and a mass of rubble placed between the loose rock and the face. Dams made partly of rock and partly of earth are not included.

The foundation should preferably be on solid rock, but the dam may be partly on compacted material, provided that the impervious face is connected to a cut-off wall on the bottom and sides of the canyon that will effectually shut off water.

The material composing the body of the dam should be a hard, solid rock, free from joints that would permit the rock to fracture under load, and where the crystalline structure has not been broken down by earth stresses. The material must not weather or be broken by freezing. Granite and limestone have been used, but no examples are known of the use of sandstone. The rock material must be placed in the dam in a way such that rock bears against rock. Fine material may be sluiced between the rocks of the mass after they have been placed, but not before or in such manner that rocks will rest upon the fines.

Dams have stood where the base was little more than twice the vertical height, but the section should be such that the ratio is greater. The downstream slope should be that of free falling rock, generally about 1.30 : 1 to 1.40 : 1. The slope of the water-face may range from 0.75 : 1 to 1.30 : 1, but the latter slope is believed the best for constructional and settlement purposes. The sliding factors will then be ample.

In plan, the dam should be somewhat convex to the water and the up-stream profile in section should also be a curve that is concave to the water.

The rubble cushion between the rock-fill and the impervious face should be of uniform thickness throughout its height. The thickness of the rubble is a matter of judgment and Table 1 gives the data for several dams. It is believed that a minimum thickness of 15 ft is necessary, and a greater thickness is advisable. The rubble should be carefully laid and packed with spalls.

The facing may be of timber, concrete, or steel. Timber of two and three layers of plank fastened to sleepers built in the rubble is sufficient for dams of moderate height. Concrete should be reinforced and of a thickness at least 1% of the height, or more. On low dams of considerable length, with the dam convex to the water in plan, the concrete may be laid without expansion joints, provided it is reinforced against temperature changes. In dams in narrow canyons and of considerable height, the facing should be divided into squares of approximately 50 ft on a side, by expansion joints and construction joints. Grooves or chases should be formed in the rubble and filled with concrete upon which the joint may rest. Copper seals should be used and the joints filled with asphaltum. All rock-fill dams settle over a long period of years, although the major settlement takes place in the first years. Settlement is roughly at right angles to the water-face and also along the axis of the dam from the canyon walls toward the center. It is not possible to construct a concrete face that will move independently of the settlement of the rock mass. The concrete should be poured directly on the rubble face and attached thereto by filling the openings in the rubble with concrete. On account of axial settlement, expansion joints at the central section should be built wide open and those at the ends, closed. Steel facing, made of plates and shapes, forms an excellent and durable facing which will fulfill all the functions of the other methods. Experience with steel facing has been limited to two dams, but is believed to be satisfactory.

Other details of design are not discussed because they are the same nature as those of other types. One element should be emphasized—on account of the danger of destruction if the dam is overtopped with water, the spillway should be of ample proportions and left entirely free from all obstructions, such as flash-boards, gates, etc.

The design of rock-fill dams is a matter of judgment, based upon experience. Opinions will differ, and the writer hopes that discussion will be directed to matters of design, that being the subject-matter of this paper.

DISCUSSION

Cecil E. Pearce,[3] Assoc. M. Am. Soc. C. E. (by letter).—There has been no good compendium on the subject of rock-fill dams and, therefore, the paper by Mr. Galloway is a timely one. The writer wishes to make several comments on it.

The sliding factors mentioned in the paragraph following Table 2 have been checked, and, at the same time, it is found that the cases which have sliding factors of 4.50, 5.14, and 6.45 have up-stream slopes of (ratios of base to height) 0.775 : 1, 1.02 : 1, and 1.54 : 1, respectively. Although it may be true that these values represent safe sliding factors in the sense usually applied to gravity dams, it may also be true that the stability of the up-stream slope is a more important criterion. If dumped rock assumes a natural slope, then does not a natural phenomenon offer a perfect index of what should be done? Carrying the case to an extreme illustrates the point. Assume that the up-stream face is vertical for a high dam (say, 300 ft high). Obviously, it would not stand; the entire vertical face would tumble, slide, and roll until equilibrium was reached at its natural angle of repose. Any slope steeper than such natural slope would mean that there were still some forces at work trying to arrive at such a slope. Any slope as flat as the angle of repose, or flatter, would mean that the material had reached a state satisfactory to the forces of Nature, and that there was no longer a condition that would cause rocks to roll down the slope.

Although it is true that Man may place these rocks somewhat carefully, and gain certain advantages in friction and interlocking, which will permit the face to stand steeper, it is also true that it is difficult to appraise the degree of success of such operations. On the other hand, one cannot assume that an up-stream face, placed as flat as the angle of repose, or flatter, has much tendency, if any, to become still flatter.

The author has stated (see heading, "Definition and Principles") that "although the design of rock-fill dams is mainly based upon experience certain definite theoretical principles apply to such structures which should always be considered." It is the writer's contention that the slope of the up-stream face is one of these points which should be based upon the rational. Considering the angles of repose which have been observed for dumped rock, the writer would recommend that the up-stream slope of important rock-fill dams be not less than 1 on 1.3, as recommended by the author in his "Conclusions."

In discussing construction and expansion joints, Mr. Galloway states that (see heading: "Design: Construction and Expansion Joints"): "In practically all cases, all horizontal joints become construction joints as such joints, being always in compression, never tend to open although they may be subjected to shear." The statement that the horizontal joints will always be in compression is quite debatable. In fact, in discussing the concrete facing, the author states (see heading, "Design: Concrete Facing"): "The concrete of the slab is poured directly upon the rubble wall that supports it. The concrete enters into the interstices of the rock, fills the inequalities, and adheres to the rock. The concrete is thus held in position, and expansion or contraction is prevented." It is agreed that the slab should be poured in this manner. Such being the case, if the job has been properly done the slab is truely supported by the fill, so that the horizontal joints will or will not be in compression, depending on whether or not the settlement shortenings overtake the original contraction of the concrete on cooling. In fact, one of the most important reasons for pouring the concrete facing as recommended is to prevent the inclined thrust down the slab and the consequent possible shearing across the corner of the joint.

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[3] Senior Hydr. Engr., Hiwassee Project, Tennessee Valley Authority, Knoxville, Tenn.

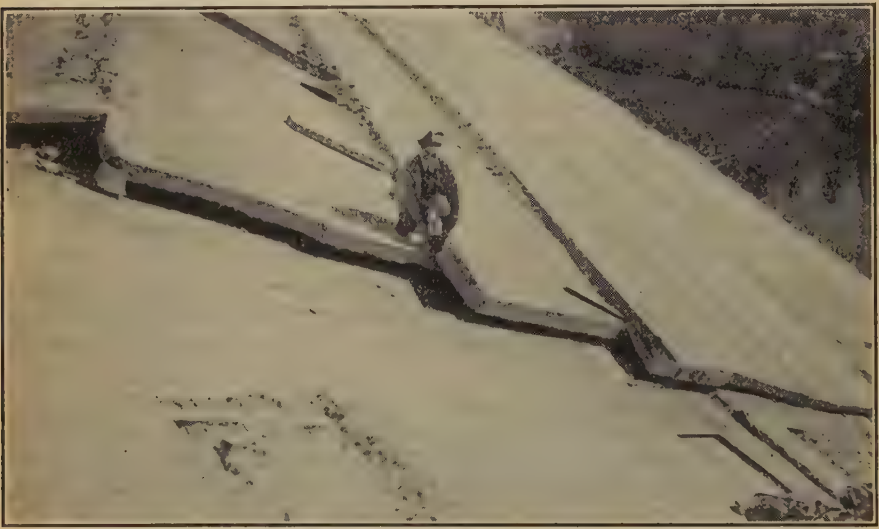

FIG. 13.—FAILURE OF CONCRETE FACING, SAN GABRIEL DAM NO. 2, IN CALIFORNIA.

Another question that arises in connection with this paper is the lamination of face slabs. From time to time various engineers have either recommended or built the facing on rock-fill dams laminated instead of in a single slab. San Gabriel Dam No. 2, built by the Los Angeles County Flood Control District on the San Gabriel River in 1930, was constructed with laminated slabs. Fig. 13 shows what happened. The settlement of the dam was irregular, and the upper slabs were offset enough at the joints so that they did not bear across the joint against the slab below. Hence, the slabs slipped over or under each other, and the corners sheared and spalled off.

If the facing had been a single slab instead of several slabs the same offset at the joints would still have left a considerable proportion of the joint area for transmitting thrust down the slope to the adjacent slab below. However, the idea of the laminated slab runs into still other undesirable conditions. It becomes difficult to anchor such a slab to the slab beneath it, even with reinforcement steel, as was done in the case shown in Fig. 13, so that there is a decided tendency for the top slab to slip on the plane top surface of the slab below. If the facing is poured as a single concrete slab on the rubble cushion, as recommended by Mr. Galloway, the slab is fixed to the rubble by bond, and by the shear in the concrete which fills the interstices in the rock so that the tendency to slide down hill is minimized. The possibility of the slab sliding down the slope has led to a general practice of establishing horizontal benches at intervals on the up-stream face of the rubble cushion, which benches are filled with concrete and act as anchors or thrust-blocks for the concrete slab.

The author's contention that the rock-fill should be sluiced while placing is a very important one, because if the fill is placed otherwise, the later rains wash the fines downward, settlement is excessive, and trouble is experienced with the concrete facing slab.

The author states that at Salt Springs the sheet-copper water-seals were brazed together. Experience with gravity dams has shown that the heat requirements of brazing seem to be such as to cause a burning of the copper, and a piling of braze upon braze, because of the rapid cooling and consequent poor job, until a kind of stiffener effect is formed.

The brazing metal, on cooling, becomes brittle, and repeated bending tests caused the braze to crack and break away from the copper. A better method has been to rivet and solder. Tinners' copper rivets, about $\frac{1}{8}$ in. in diameter, are spaced on $\frac{3}{4}$-in. centers in each of two rows, being staggered, say, $\frac{3}{4}$ in. and $1\frac{1}{2}$ in. from the end of each sheet of copper. The sheets of copper, as shipped, are tinned on the ends for about 2 in., and can be soldered easily and thoroughly. A thin film of solder results, by sweating, over the 2 in. of contact, using a soldering iron, and also a blow torch to heat the entire contact area and sweat it together. The resultant joint is as thin as possible and, consequently, stands repeated deflections without cracking.

H. B. MUCKLESTON,[4] M. AM. SOC. C. E. (by letter).—Apparently the only recorded failures of the rock-filled type of dam were not due to defective design, but to inadequate spillway capacity. The Walnut Grove Dam, cited by the author in Table 1, Item No. 4, was designed with an up-stream slope of 20 : 47. The rear slope was 70 : 108, with a talus of loose rock on 1 : 1 slope for about one-half the height. It stood successfully for two years and would doubtless be still standing had the spillway been sufficient. The Escondido Dam (Item No. 7, Table 1) nearly as frail, is still (1937) doing its work after forty years. In both dams the slope of the water-face was much steeper than the angle of repose for loose rock, and dry rubble retaining walls were necessary to hold them. Both dams are excellent illustrations of the "great Hydraulic Principle" enunciated in 1912 by Mr. George L. Dillman[5]: "Construct one impervious surface, and build the rest of the structure to support that surface."

In solution of that principle, the author advocates reinforced concrete for the impervious surface and a backing or cushion of dry rubble masonry to minimize the effects of unequal settlement within the rock mass which is the supporting structure. Experience seems to show that this manner of construction meets the requirements successfully, but it also seems to show that it is not the only manner of construction that will do so. In every case, the designer must find the answer to the question: What type of construction will "make the owner's dollar earn the most interest?"

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[4] Cons. Engr., Vancouver, B. C., Canada.

[5] *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 52.

The object of the dry rubble backing is to take up and distribute the inequalities of settlement within the rock mass so that the impervious surface will not be ruptured, either directly by water pressure, or indirectly by unequal settlement. If this end can be achieved by cheaper means than dry rubble, the end will justify its adoption. Dry rubble work is expensive and, moreover, the type of skilled labor needed for its construction seems to be nearly extinct in many localities.

The cause of settlement in a loose rock mass is mainly crushing of the individual rocks at points of contact; and inequality of settlement is mainly due to localized arching. Both can be greatly minimized if the voids in the rock mass are filled with a fairly well graded but coarse material, such as a mixture of coarse sand and quarry waste. Such a material could be sluiced into the mass from the up-stream face and allowed to take its own slope through the fill. The escaping water would find its way out through the down-stream face taking the undesirable fines with it. The resulting mass should be stable; it would be sufficiently pervious; and settlement should be relatively small and much more nearly uniform. Thus constructed, there would be no need for the rubble backing, and the thickness of the impervious surface could be fixed by consideration of impermeability alone.

For the impervious surface, many materials are available. If the dam is for head only and if storage is not a factor, a well-built timber surface should be almost everlasting. If storage is a great factor, the life of a timber face may be short but, even so, it may be the cheapest in the end.

If an adequate supply of material were available, a thick blanket of puddled clay, loaded with rock for protection, and also to confine it, would meet all requirements for impermeability. It would be sufficiently plastic to stand considerable deformation without rupture, and it should be much cheaper than concrete. If clay is not available, a blanket of well-graded sand, impregnated with a heavy asphaltic road oil, would serve as well. It probably is not possible to consolidate clay by artificial means as thoroughly as Nature has sometimes done it. The writer knows of one clay deposit (underlying the Bassano Dam, in Alberta, Canada) which was so dense that a disk 6 in. in diameter by 1 in. thick, under a head of 50 ft scarcely passed sufficient water in 24 hr to fill a tea-cup. Asphaltic concrete would be impervious—more so than cement concrete—and it would be flexible to a high degree. It would have to be loaded, however, to hold it on a slope of $1\frac{1}{2} : 1$. On one small dam, 30 ft high in a V-shaped gorge, the writer used old brick coated with, and laid up in, asphalt. It has been in service for twenty years and is apparently good for a hundred years more.

Harold K. Fox,[6] Esq. (by letter).—Core walls in the center of the fill are inadvisable, as Mr. Galloway states, the principal reason being their inaccessibility.

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[6] Dist. Engr., San Joaquin Light and Power Corp., Bakersfield, Calif.

The writer questions the advisability of using too much so-called fines in the fill because there is a hazard of obstructing drainage and also because of the possibility of the fines acting as a lubricant of the larger material and thereby facilitating slides. He has had the experience of sinking a shaft 140 ft from the top of an old rock-fill dam through the fill to bed-rock, and bases his belief on that experience.

The writer does not agree that gates cannot be used in the spillway because there are cases in which it can be done with safety, due to the character of the run-off and the possibility of maintaining an attendant to operate the gates. Gate control will often add materially to the usefulness of the project.

In regard to the stability of a rock-fill dam, the writer cannot overlook an experience with a small one about 50 ft high which was built by a logging crew. Rock was obtained from the surface at the abutments, and the facing was of lumber. Cross-sections indicated that the rock had been piled up as steeply as possible instead of allowing it to take its angle of repose. The lumber facing had been projected to form about 3 ft of flash-boards above the crest, and there is plenty of evidence of overtopping. Of course, no engineer would recommend any such conditions; however, there is value in knowing that a dam of the rock-fill type has stood under such conditions for more than forty years. It leaves little doubt as to the stability of the modern well built structures.

Charles H. Paul,[7] M. Am. Soc. C. E. (by letter).—As would be expected, the author has presented, in convenient and concise form, a discussion of the more important features of design and construction of rock-fill dams, which should be of value to all engineers having direct or indirect interest in such structures. The writer is in general agreement with the author's conclusions, and with the precautionary remarks that appear throughout the text.

Under "Definition," an earth-fill backed by a rock-fill is eliminated from the rock-fill classification, which seems to be proper. The writer infers that a rock-fill faced with earth or gravel is also eliminated, and that raises this question: When a rock-fill section is stable in itself, and when an earth or gravel facing is used simply to secure water-tightness, should not such design be included under the rock-fill definition? The Minidoka Dam on Snake River, in Idaho, is an example, in which it was economical to face the rock-fill with gravel and earth, allowing this facing to wash into the voids of the rock as the placing of the facing automatically raised the water in the reservoir. That is a design that may not often be used, perhaps, but it seems to the writer that it should be classified as a rock-fill dam.

The paper states that the English Dam, built in 1856, was destroyed in 1883; it would be of interest to know how it was destroyed—whether deliberately for some reason, or by natural causes and, if so, what.

The writer would be inclined to give rock-fill dams more credit for stability than the author appears to do. Assuming good design and construction, it seems as if no question as to stability should arise, except perhaps in connection with overflow, and even in such cases it has been the writer's experience that rock-fill dams will stand considerable overflow without damage, or even danger, to the structure. It is fair to assume that the capacity and design of the spillway is an integral part of the design of a dam. Even in case of concrete dams, it is often of vital importance to have sufficient spillway capacity to prevent overflow of the main structure, not necessarily because of danger of failure, but for other reasons. Assuming a spillway with sufficient capacity, which should be provided in any case, it is the writer's opinion that where conditions are favorable, a rock-fill dam takes its place, without handicap, among the various types of approved dam designs.

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[7] Cons. Engr., Dayton, Ohio.

The writer would not absolutely prohibit the use of spillway gates. Whether or not they should be permitted would depend largely upon the design of the gates themselves, including their control mechanism; whether or not a reliable operating crew is to be maintained at the dam at all times; whether access to the spillway gate control is such that it would not be cut off in case of emergency; and the value or importance of gate control as a feature of the project. In many cases, spillway gates are being operated and maintained with perfect safety, and are fully justified.

A. FLORIS,[8] ESQ. (by letter).—In this clear, concise, and authoritative paper the statement is made that rock-fill dams resist water pressure by their own weight only. It is also true that in most rock-fill dams, the weight of water contributes to their stability. It is well known that the most important economic advantage in the design of multiple-arch dams is gained by sloping their arch barrels. The weight of water above these barrels compensates for the deficiency in the weight of multiple-arch dams against the water thrust.

With the exception of the older dams (Lower Otay[9] and East Canyon[10] constructed with a steel core-wall), and the Oued-Kébir Dam[11] with a hollow, multiple-arch core diaphragm, the dam section is made water-proof by means of wooden, masonry, concrete, or steel facing. Hence, assuming a water-tight facing and a cut-off wall at the up-stream face, the weight of water above the slope of the dam will add materially to the resistance of the structure against sliding.

HOWARD F. PECKWORTH,[12] M. AM. SOC. C. E. (by letter).—The definition of rock-fill dams, as given by Mr. Galloway, does not include those with a facing of impervious rolled-earth fill. It is perfectly evident, from a historical viewpoint, why this was done but the writer believes that a rock-fill dam with an impervious rolled-earth blanket should come under the heading of rock-fill dams especially as the rock-fill in both cases is serving the same function—that is, as weight to support the impervious section. Naturally, the grading of the rock behind the impervious section may be different, but the materials and functions of the rock-fill are the same in both cases. Mr. Galloway's paper is especially useful in defining the basic principles of design for this type of dam as developed in the West because there is no doubt that in the near future this type will be used in many variations in connection with rolled-earth fill in the South and East as well as in the West.

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[8] Dipl.-Ing., Los Angeles, Calif.

[9] "A Rock-Fill Dam with a Steel Heart-Wall at Otay, Cal.," by W. S. Russel, *Engineering News*, March 10, 1898, p. 157; also, "Reservoirs for Irrigation, Water Power, and Domestic Water Supply," by the late James Dix Schuyler, M. Am. Soc. C. E., New York, 1905, p. 19; and "Otay Rock Fill Dam Failure," by Charles Whiting Baker, *Engineering News*, February 3, 1916, p. 236.

[10] A Rock-Fill Dam with Steel Core Across East Canyon Creek, Utah," by W. P. Hardesty, *Engineering News*, January 2, 1902, p. 14.

[11] "Core Wall in Rock-Fill Dam Tilts When Reservoir Fills," by the late F. A. Noetzli, M. Am. Soc. C. E., *Engineering News-Record*, November 3, 1932, p. 529; also, *La Révue Industrielle*, September, 1931; and "Algerian Dams of Placed Rockfill," by I. Gutmann, *Engineering News-Record*, December 2, 1937, p. 889.

[12] Civ. Engr., Pacific Constructors, Inc., Shasta Dam, Redding, Calif.

At present, a graded sandstone supporting an impervious rolled-earth blanket is being used in the construction of Inland Dam, near Oneonta, Ala., for the City of Birmingham. This is one case of the use of sandstone, although not in the exact type described by Mr. Galloway; and as sandstone will undoubtedly be used some day in the type described, it is of interest to note the following characteristics of a sandstone quarry:

(1) The line of demarcation between the over-burden and the stone may be very indistinct, the over-burden grading gradually from top soil through clay, sandy clay, and decomposed or unconsolidated sandstone to good, hard, suitable sandstone. The question will arise as to exactly where suitable stone begins and many arguments can be avoided by including in the specifications, "suitable stone is that which rings under a hammer or passes a compression test of (state allowable)."

(2) In a sandstone quarry there is likely to be an excess of quarry waste; decomposed or unconsolidated sandstone, broken down by blasting to sand or fines; and included with this material may be horizontal seams of shale or vertical seams of clay or top soil. In the quarrying operations this must be expected and must be taken care of by suitable methods of wasting.

The writer checks Mr. Galloway's ratio of 1.3 to 1 for free falling sandstone rock, 3 in. to 1 in. in size, and for sandstone screenings, 0.5 in. to zero. He also checks the values, 1.3 to 1 and 1.4 to 1, for free falling sandstone rock, 15 in. to 3 in. in size.

OREN REED,[13] M. AM. SOC. C. E. (by letter).—As stated by the author, the design of rock-fill dams was developed primarily in California. Experience has established certain rules of design, which have been used in the construction of several notable structures such as the Salt Springs Dam, in California. Economy by the use of native materials is ordinarily the controlling factor in determining the selection of a rock-fill dam for any site.

Rock-fill dams are not common in Europe. However, there are a few structures of this type, one of the largest being the Porjus Dam, in Sweden, which was inspected by the writer in 1930. The dam at Porjus, 30 miles north of the Arctic Circle, consists of an earth and rock-fill, two concrete spillway sections over the natural channels, and a short concrete gravity section, giving a total length of 4 100 ft. The dam, completed in 1912, was constructed largely of earth and rock-fill because of the high cost of concrete in this remote location. Like the Crane Valley Dam, in California, the section consists of a rock-fill down stream from a center core-wall of reinforced concrete, and gravelly earth-fill, covered with rip-rap, on the up-stream side.

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[13] Prin. Civ. Engr., Project Planning Div., TVA, Knoxville, Tenn.

The down-stream face was built on a slope of 1 on 2 and the up-stream slope varies from 1 cn 2 at the top to 1 on 3 at the toe. The crest width is 15.4 ft. The normal free-board is 5.2 ft. The material for the rock-fill was obtained from the excavation for the spillway and intake tunnel.

The two spillway sections have a total length of 648 ft, each consisting of concrete arches between concrete buttresses. The crest of the spillway sections is regulated by timber needles 15 ft long, supported by hinged steel bents, and a roller gate 16 ft in diameter and 39 ft long. The roller gate is remotely controlled from the power-house, providing close regulation of the pond level for smaller fluctuations of flow.

Due to better transportation facilities and increased knowledge of concrete construction, a multiple-arch dam was given preference when the regulation of the Sourva Lakes was under consideration. The dam at Sourva provides regulation on Lule River to increase the normal low-water flow at Porjus. This dam, completed in 1924, has a maximum height of 75 ft and a length of 785 ft.

Two more recent rock-fill dams in Sweden are situated at Sillre and Torrön. The dam at Sillre, completed in 1933, has a length of 920 ft and is constructed of rock from the power tunnel excavation. The dam is located on a marshy and morainic foundation. The down-stream face has a slope of 1 on 1.5 and the up-stream face, a slope of 1 on 1.6. The crest width is 9.85 ft, and the maximum height is about 26 ft. The water-tight part consists of stiff puddled clay, placed on the sloping up-stream face of the dam. The puddled clay has a normal thickness of 2.0 ft at the crest of the dam and 4.9 ft at the foundation. The clay blanket is joined at the bottom to a sheet-piling cut-off, which extends 13 ft to 16.5 ft in the hard, but pervious, foundation. The clay blanket has an up-stream slope of 1 on 2 and is separated from the rock-fill by a thin layer of gravel, which acts as an inverted filter. An insulating layer of peat was placed on the face of the clay blanket and heavy rubble rip-rap was constructed on the insulation material to prevent erosion by wave action. The development at Sillre is a pumped-storage project and the capacity of the reservoir is large in relation to the run-off. There is no spillway, but a sluice-way is provided which is controlled by a slide gate, 4 ft by 9 ft in section.

The dam at Torrön, completed in 1936, is a true rock-fill with a timber facing, which is fastened directly to the rock of the rubble wall. The up-stream face was built on a slope of 1 on 1, whereas the down-stream face has a slope of 1 on 1.5. The crest width is 10 ft. The timber deck was selected instead of a concrete deck because it was expected that settlement of the rock-fill might be great enough to cause dangerous cracking of a concrete deck. A low concrete cut-off wall is provided to connect the timber facing to the rock foundation.

The writer considers that a well-designed rock-fill dam is a relatively stable structure and in locations where cold temperatures are experienced, a fill dam may be preferred to a concrete structure.[14] In some cases, as noted previously, the spillway may be provided with gates with perfect safety. However, spillway regulation should be of a positive type and operated by experienced crews.

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[14] "Preventing the Disintegration of Dams," by Oren Reed, *Civil Engineering,* January, 1933.

WALTER L. HUBER,[15] M. AM. SOC. C. E. (by letter).—At least a partial remedy for the lack of any published compilation of authentic data on rock-fill dams is found in this paper. The rock-fill dam is a type of structure which has been widely used in the western part of the United States. The author has confined his discussion to true rock-fill dams which, as he states, consist of three elements: "A loose rock-fill forming the mass of the dam; an impervious face next to the water; and a rubble cushion between the two." From this true type many variations have been developed, including important structures built in recent years wherein a very substantial section adjacent to the impervious face is of rolled fill, back of which is a loose graded rock-fill (such as San Gabriel Dam No. 1 of the Los Angeles County Flood Control District, with a maximum height exceeding 350 ft). As is indicated in the paper, the true rock-fill type is the result of an evolution from the rock-filled log crib type and from an intermediate type wherein the section was largely of uncemented, hand-laid, rubble masonry.

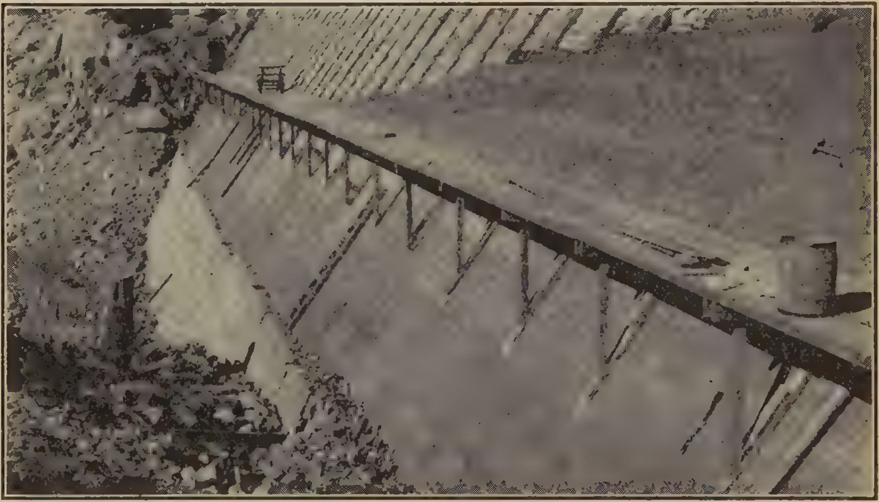

FIG. 14.—REFACING SABRINA DAM, OCTOBER, 1929

The author mentions the Bear River and Meadow Lake Dams, on the Mokelumne River, in California, built from 1900 to 1903, as about the last rock-fill dams to have the down-stream face protected by a dry rubble wall. Attention is directed to the Utica Dam, of the Utica Mining Company, on the head-waters of the North Fork of Stanislaus River, in California, which was completed to a height of 60 ft in 1908 and which has a back face of dry rubble.

Mention is made of Sabrina and Hillside Dams, on Bishop Creek, California, both of which are of the true rock-fill type, 70 and 80 ft in height, respectively. Fig. 3 is a view of the timber face of Sabrina Dam after the rock-fill had acquired most of its settlement. This is in no way indicative of the present condition of the structure. As the author has noted, the timber face was replaced after twenty years of service. Fig. 14 is offered to show the present condition of this face. The original facing was of native lodge-pole pine timber, admittedly an inferior grade of lumber, but because of the inaccessibility of the site of both Sabrina and Hillside Dams from 1908 to 1910 this was used for reasons of economy. The timber face was not laid directly on the loose fill as noted by the author, but the cushion of hand-placed rock was comparatively thin.

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[15] Cons. Engr., San Francisco, Calif.

After twenty years of service, the original timber faces of both dams were showing signs of deterioration and leakage was increasing. Accordingly, these faces were removed and, in each case, the cushion of hand-placed rock was brought up to an even surface. Pressure-creosoted, 8-in. by 8-in. Douglas fir vertical stringers were placed in the reconstructed face of the rock cushion at intervals of 5 ft 4 in. and securely anchored by 1-in. round steel anchors which were spaced at 6-ft intervals and secured to sections of rails embedded in the dam. Directly across the stringers is a plank facing of redwood which forms the water-tight element. These planks are 3 in. by 12 in. Redwood splines, 0.5 in. by 1 in., are placed in the horizontal joints, and the end joints are splined with $\frac{1}{8}$-in. by 2-in. galvanized flat steel splines. The redwood facing varies from three layers of. 3-in. planking at depths exceeding about 38 ft to a single layer for a depth of less than about 11 ft. Fig. 15 shows the face of Hillside Dam when the work of resurfacing was practically completed in September, 1930. It is believed that the life of the new faces constructed of redwood and pressure-treated Douglas fir will greatly exceed that of the original faces constructed of native timber of inferior quality. It is interesting to note that estimates indicate that reconstruction of timber faces, as required

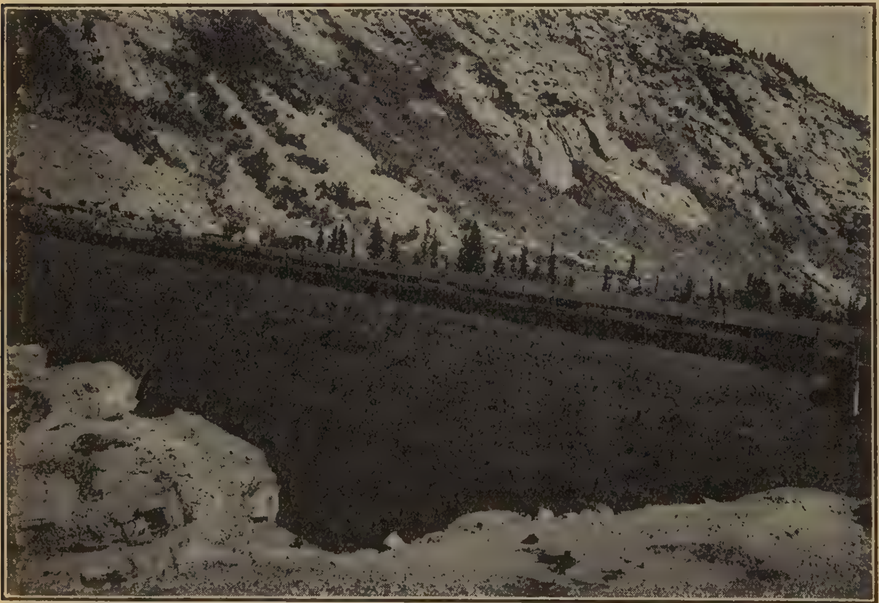

FIG. 15.—REFACING OF HILLSIDE DAM PRACTICALLY COMPLETED, SEPTEMBER, 1930

from time to time, is more economical than the construction of a so-called permanent face under the difficulties imposed at the sites by inaccessibility and by short seasons due to high altitude.

Another advantage secured is that the flexibility of the original timber faces allowed them to follow the unequal settlement of the rock-fills and, at the same time, remain comparatively water-tight. The new faces have been applied since the settlement is substantially completed and these facings will not be subjected to the same warping. Although this type of facing has advantages for dams of lesser height, the designers of some of the very high rock-fill dams have sought a facing of more permanence and have applied it with the original construction. In each instance a degree of rigidity has resulted which has given some, although not always serious, trouble when the inevitable settlement of the rock mass occurred. In the case of San Gabriel Dam No. 2, of the Los Angeles County Flood Control District, a timber face has been installed as a temporary expedient after settlement of the rock-fill had seriously distorted the laminated concrete slab face.

A disadvantage of timber facing is the possibility of destruction by fire when the water is drawn down in the reservoir. Serious damage from this source at Sabrina Dam was traced to the carelessness of a fisherman. The owners have since enclosed all their timber-faced dams with high and substantial wire fences with locked gates.

Samuel B. Morris,[16] M. Am. Soc. C. E. (by letter).—The historical development of the rock-fill dam has been well presented in this paper and the author has outlined the general criteria for its design and construction, to the present maximum height of slightly more than 300 ft. There is one aspect relative to rock-fill dams with which the writer has long been concerned. How will the average, well-constructed, high, rock-fill dam behave during violent earthquake shocks? Earthquake accelerations up to 0.1 g are generally being applied to dams, school buildings, and other structures in regions where earthquakes of such magnitude are likely to occur.

More recent designs of concrete or masonry dams of all types in California and other parts of the United States liable to earthquakes have made provision for resistance to earthquake shock. Such designs are now made to provide for two principal types of forces acting upon a dam during such shock: (1) The forces due to acceleration acting upon the mass of the dam; and, (2) the hydrodynamic forces resulting from the motion of the dam against the still water of the reservoir bearing upon its up-stream face.

There is probably no type of structure built by Man which offers so great a potential hazard to life and property as a large dam and reservoir up stream from a heavily populated area. Proper consideration of safety is paramount. In connection with the design of a high dam so situated the writer, several years ago, took occasion to visit a large number of existing dams and to study different types from the standpoint of safety in a region where earthquakes are to be expected.

The rock-fill dam has been suggested to the writer by some engineers as being particularly suitable for use in regions liable to earthquakes, on account of its large mass and flexible character. The writer has been inclined to question the behavior of high rock-fill dams during earthquake shocks.

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[16] Dean, School of Eng., Stanford Univ., Stanford University, Calif.

The settlement of fills during earthquakes has been general observation and common knowledge. Railway and highway fills, particularly those at bridge approaches, have settled substantially in all major earthquakes. In the Long Beach earthquake of March, 1933, a sand-spit peninsula more than 1 000 ft wide and several thousand feet long settled, and the land surface widened, causing cracks in paving aggregating as much as 5 ft. There has been serious settlement and longitudinal cracking in a large hydraulic-fill dam during an earthquake of such moderate proportions that practically no other damage to structures of any kind occurred. The reservoir was empty at the time. During the Santa Barbara earthquake of 1925, the Sheffield Reservoir was lost through complete failure of the small earth-fill dam which created it. Elsewhere, earth-fill dams have survived severe earthquakes.

During the Long Beach earthquake there was general settlement and consolidation of land surface in deep alluvial-filled valleys in the vicinity. The consolidation of alluvial materials resulted in a sudden rise of water in wells, and where ground-water was near the surface, of a large number of small jets of water flowing like little geysers, and leaving their record in fumarole-like deposits and erosions at the cracks in the ground surface through which the water flowed.

Of course, none of the physical conditions related to the foregoing observations is very closely related to the type of fill represented in properly constructed rock-fill dams. However, the settlement of mine and tunnel dumps has been observed as the result of earthquakes. Here, there may be a very marked resemblance to the normal loose rock-fill dam. The absence of placed rock, and a concrete blanket on one side of the fill, such as that used at the up-stream face of rock-fill dams, is not important in making such comparison. Of course, most of these mine or tunnel dumps will have too much fine material or will be too soft a grade of rock, and will not have included the important sluicing during rock placing. It is believed, however, that observation of such dumps may reveal some that closely resemble the loose fill of rock-fill dams. This subject is mentioned in the hope that engineers may be on the lookout for such rock-fills which may have settled during earthquakes, or which may be checked later if such earthquakes should occur in their vicinity.

With the hope that the author will comment further on the resistance of high rock-fill dams to earthquake shock the writer presents this discussion.

Mr. Galloway describes the cause and manner of settlement of rock-fill dams adequately, and yet briefly, in his first paragraph on settlement (see heading "Design: Settlement").

In discussing the cross-section (see heading, "Design: Cross-Section"; third paragraph) the author presents the accepted practice from which the typical rock-fill dam is shown to be composed of loose rock with a minimum top width of 15 ft and slopes equivalent to the angle of repose of from 1 on 1.3 to 1 on 1.4. For a thickness as great as 15 ft at the up-stream face, derrick-laid or packed rock is placed to form a semi-rigid layer to support the concrete or other impervious face.

In the case of a high dam, settlement of as much as 5% of the height may be expected owing to the spalling or crushing of point contacts between adjacent rocks in the loose rock-fill. As successive layers of rock are placed and the dam is built higher and higher these point contacts continue to spall for some time after completion of the structure until, finally, the points of contact between the rocks have sufficient area to carry the stresses involved in the dam standing without load.

These areas of contact are only sufficient to carry the normal mass of the dam itself. Many of the points of contact between the rocks will have just crushed sufficiently to carry this load at the point of incipient failure. The result is that when the water load is applied there is an immediate increase in unit stress, and the many small areas of contact already stressed to point of failure cannot carry the additional water load; renewed and further spalling takes place until the areas of these point contacts have increased sufficiently to be able to carry the forces due to the mass of the dam plus the water load on the up-stream face. This settlement may continue for months or years depending upon the rate of deterioration of the individual rocks composing the loose rock-fill.

Assume, now, that the reservoir is filled to its maximum high-water level and a severe earthquake occurs—not an earthquake involving any fault movement or permanent displacement of the rocks in the dam foundation, but merely the normal earth waves to be expected. These waves will give an oscillating motion to the foundations, exerting new and additional forces upon the rock mass. Point contact between rocks which have already spalled, first as a result of stresses due to the mass of the dam, and, second, to an additional extent, due to the water load, will immediately be stressed beyond the yield point. Additional spalling and crushing will result in immediate accumulated settlement.

This settlement may easily be much more extensive than may be computed from comparable known loading conditions. The vibrations and oscillations of the earthquake will likely tend to rotate the individual rocks in such a manner as to cause them to slide on their crushing points of contact, thus bringing about a denser compact mass of rock. Such immediate settlement and compaction during earthquakes have already been noted as a common phenomenon in all other types of fill, such as railway, highway, mine and tunnel dumps, and natural alluvial fills and sand-spits. Otherwise stable mountain slopes may yield to severe sliding, rolling, and sloughing of rock and rock masses. Such conditions were actually photographed on the slopes of San Jacinto Mountain during the San Jacinto-Hemet earthquake of 1918.

In other types of dams the vertical component of acceleration during earthquake vibrations is easily taken care of by the usual factors of safety in design. In the rock-fill dam even the vertical acceleration must be considered as it will add to the unit stresses in the point contacts, causing rock spalling and dam settlement.

The effect of sudden settlement of the rock mass upon the concrete lining on the up-stream face may easily be much more disastrous than the slow settlement that occurs during construction, during the filling of the reservoir, and thereafter. Reinforced concrete has generally been used for the highest rock-fill dams, and, therefore, reference to its use is made herein. Because of its high modulus of elasticity and lack of flexibility some failure of lining is likely to occur unless ample provision has been made in the design for such settlement.

The oscillations of the earthquake motion tend to free the rock at the face of the dam. If this rock is placed as steeply as the angle of repose there may be some likelihood of rocks being freed at the surface to roll, slide, or, possibly, to cause bulging.

The total settlement that may reasonably occur during an earthquake is difficult to foresee. Probably the settlement during construction, during reservoir filling, and for several years thereafter, will be some indication. That there will be such immediate settlement seems certain. It is quite conceivable that the settlement during earthquake shock may exceed all other settlement in magnitude. Some dams having too little free-board may settle sufficiently to cause over-topping. Fortunately, most rock-fill dams have been provided with large spillways and liberal free-board as it is well known that they cannot be over-topped with safety.

In a number of the rock-fill dams observed by the writer the irregular settlement of the up-stream face has been a substantial problem in maintaining water-tight conditions. The author suggests "that the fill should be composed of individual rocks of fairly uniform size, one rock bearing directly upon another" and that "any wide divergence in size will cause excessive and unequal settlements." Presumably, the important factor is that the size grading shall be uniform and of uniform quality of rock to avoid unequal settlement, rather than that all rock should be of the same size.

In constructing a rock-fill dam it is the general practice to sluice the fine material from the larger rock as it is placed. This fine material then tends to fill the interstices of the lower parts of the fill. As construction proceeds and the structure rises the lower part of the dam may become quite fully consolidated with fine materials in the interstices between the larger rock. The increasing load will then cause the points of contact of the larger rock carrying the load to spall off. If this process should continue sufficiently the resulting consolidation would cause an indeterminate part of the load to be carried on the fine material.

Under early normal static conditions such condition may not be entirely undesirable. However, with the passing of time, the smaller particles will become more disintegrated. Earthquake forces and vibrations may then tend to cause some greater tendency toward sliding either in whole or in part. Possibly this condition will be offset by more rapid consolidation of the small material than of the larger load-bearing rocks.

There are certain qualities inherent in high rock-fill dams which the writer believes should be considered in the design and construction of such dams. He has enumerated some of these in the hope that the author will discuss the resistance of high rock-fill dams to earthquake shock, and state the factors which in his study and experience should be provided in the design of such dams in regions where earthquakes are likely to occur.

L. F. HARZA,[17] M. AM. SOC. C. E. (by letter).—The rock-fill dam has proved its value in many places where transportation was difficult, concrete materials expensive, and other factors favorable. However, it should be pointed out that a rock-fill dam with concrete face will inevitably require some repair of the face after the initial period of settlement has passed.

This limits the proper application of this type of dam to storage reservoirs which are emptied at intervals in connection with their use, thus exposing the face for repairs. Application of the rock-fill type with concrete face is not satisfactory for dams where a large sacrifice in earnings would be entailed by emptying the reservoir, as in the case of power dams.

PAUL BAUMANN,[18] M. AM. SOC. C. E. (by letter).—In his interesting and very valuable paper the author mentions, among other dams, San Gabriel Dam No. 2 of the Los Angeles County Flood Control District. This structure was built of relatively large rock of granitic gneiss secured from a quarry approximately two miles from the dam site. The rock was hauled by means of 10-yd trucks and was placed dry, that is, without the application of sluicing water.

During the so-called New Year's storm of 1934 which swept over Southern California, heavy rainfall occurred which, in the vicinity of this dam, yielded as much as 12 in. in 24 hr. Due to the sluicing effect of this rainfall, settlement occurred in the dam, which at that time was nearly completed as far as the rock-fill proper was concerned. These settlements amounted to a maximum of about 4 per cent. Subsequent sluicing (which continued for several months) increased the maximum settlement to about 6 per cent.

During the construction of the dam a truck count was kept and recorded, from which it was possible to determine the shrinkage of the rock-fill during construction. Fig. 16 shows a curve that was plotted from the ratio of actual quantity in the dam and truck count as established at the end of each month during construction. From this diagram it is apparent that, although at the beginning of the job (that is, for the first 25-ft lift), each truck load of rock yielded 11.3 cu yd of rock-fill, it yielded only 9.4 yd at the end of the job. Hence the shrinkage due to self-compaction that occurred during construction was roughly 18 per cent. Data are from estimates and inspectors' reports.

As mentioned previously, the additional shrinkage due to natural and artificial sluicing amounted to about 6%; hence, the total shrinkage of this dam amounted to approximately 24 per cent.

During the construction of San Gabriel Dam No. 1, which consists of 6 236 000 cu yd of loose rock-fill and 3 206 000 cu yd of rolled rock-fill, the total shrinkage of the loose rock-fill amounted to about 26%, and the total shrinkage in the rolled rock-fill amounted to about 34 per cent. The loose rock-fill was placed wet; that is, with the application of approximately 2 cu yd of water to 1 cu yd of rock. The analysis shows that the application of water caused an initial shrinkage (that is, at the time of placing) of from 4 to 6 per cent. It is interesting to note that this corresponds substantially to the shrinkage in San Gabriel Dam No. 2 when water was applied after the completion of the rock-fill.

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[18] Senior Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

From the shrinkage measurements due to self-compaction, it follows that since the density of the rock-fill is directly proportional to the shrinkage, the average weight probably increases from 100 lb per cu ft of fill at the time of placing to approximately 125 lb per cu ft of fill at the time of completion for dams of such magnitude. This, in turn, means that the porosity of rock-fill dams of such height, particularly near the base, is relatively small due to the break-down of the contact points of the rock.

O. W. Peterson,[19] M. Am. Soc. C. E. (by letter).—A comprehensive presentation of the design of modern rock-fill dams is presented in this paper. Nevertheless, it contains several statements with which the writer is not in full accord. Mr. Galloway believes that a fill composed of individual rocks of fairly uniform size is best and that "any wide divergence in size will cause excessive and unequal settlements, something to be avoided wherever possible" (see heading, "Design: Rock-Fill"). Quarry rock ranges from fines to the maximum size, the latter depending on the physical characteristics of the rock mass and the method of quarrying. As Mr. Galloway states, the maximum size placed in Salt Springs Dam was about 25 tons, that being the size to which the larger rock fragments were reduced by secondary drilling and blasting so that they could be handled by the equipment provided. Fully 50% of the rock volume consisted of pieces heavier than 10 tons, which were loaded over the top of the power shovel buckets;

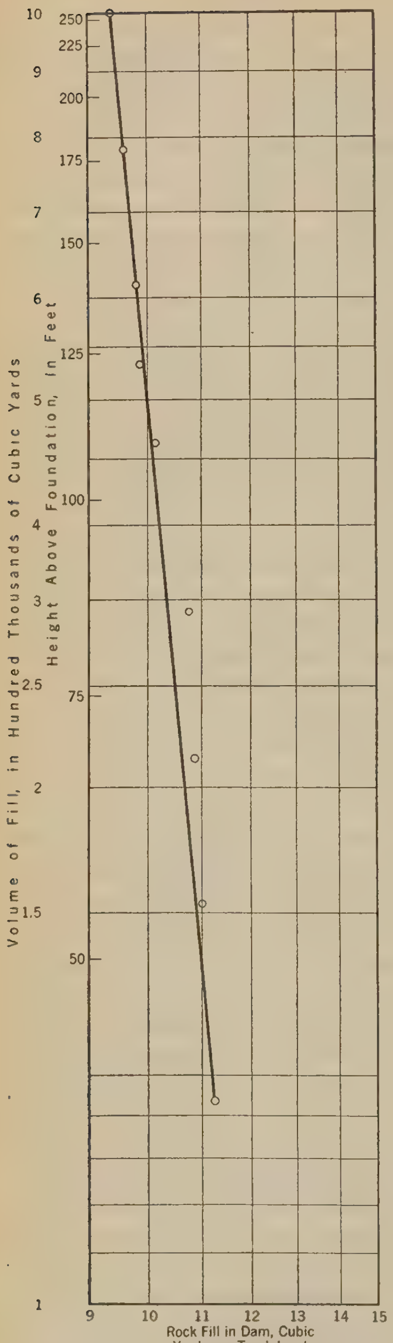

Fig. 16.—San Gabriel Dam No. 2, Shrinkage of Rock-Fill After Placing in Dam

but there was no usual size, as Mr. Galloway states. In the writer's opinion an unsegregated quarry-run mass, with excess fines wasted, will give the highest degree of contact, rock to rock, and the greatest resulting fill density. Such a fill will have ample rock-to-rock contact and will still have sufficient voids to provide adequate drainage.

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[19] Engr. of Gen. Constr., Pacific Gas & Elec. Co., San Francisco, Calif.

Instead of stressing the importance of the nozzle pressure of the streams of water used for washing the finer rock particles into the voids between the larger rock, the writer believes the volume of water supplied should be emphasized as being of critical importance in reducing the frictional resistance to the movement of individual rocks against each other and in causing the fines to lodge in void spaces. The lubricating effect of water is invaluable in obtaining maximum settlement of a rock-fill mass during the construction period. This effect has been utilized in flattening the slopes of high tunnel dumps of hard rock free from earth, thereby stabilizing the mass. At Salt Springs Dam, as many as six 2-in. to 3-in. streams, each delivering about 600 gal per min, were used. Another advantage of the low-pressure stream is that it can be applied more economically than that of a higher pressure stream, which requires more man-power as well as more power for pumping.

At Salt Springs Dam the fines that accumulated at the top of each lift were loosened by light blasting and then washed into the voids of the underlying larger rock. This blasting was done with small charges of powder without damage to the adjacent larger rock; such vibration as occurred to the fill mass was considered beneficial in assisting settlement.

Moderately high lifts are both economical and beneficial in producing a compact fill. For a fill of 200 to 300 ft, or more, lifts of 60 to 90 ft, or more, in height can be placed successfully without undue segregation of rock. The impact of well-lubricated rock masses sliding from such heights helps to break weak or fractured rock, to remove points and thin edges, and to ram the rock firmly into stable position.

Mr. Galloway states that each stage or lift should be fairly well complete before the next above is placed. The writer can see no reason for delaying a subsequent lift after the previous one has been carried sufficiently forward to assure safety to men and equipment operating at lower levels. If the slopes are kept at a moderate angle through the use of ample sluicing water, this condition will also provide an ample margin of stability for the rock masses. Under this plan the up-stream and down-stream faces of each new lift should approximate the required final positions of the loose fill. The face that is advancing across the stream from either abutment should, in the interest of safety to the workmen, be kept back from the forward edge or crest of the underlying lift a distance about equal to twice the height of the upper fill or lift. To minimize unequal settlement, lifts advanced from the two sides of the canyon should be kept reasonably uniform in height and at the same general level.

The writer sees no reason why gates, if of conservative design and of generous proportions, should not be installed on the spillway lip of a rock-fill dam. This conclusion presupposes the necessary vigilance and intelligent operation of the gates.

George W. Howson,[20] Assoc. M. Am. Soc. C. E. (by letter).—A very thorough and complete paper on the rock-fill dam has been presented by the author. The writer was connected with the construction and operation of three of the dams listed in Table 1 and offers the following comments as the result of this experience. Three sub-headings are discussed: the "Rock-Fill," "Settlement," and the "Facing."

Rock-Fill.—At Relief and Strawberry Dams the rock-fill was spoken of as the "drop fill section," the specifications calling for the rock to be dropped into place from a height of not less than 10 ft. The idea was to hammer the rock into place. At Strawberry Dam most of the rock was transported by cableway and automatically dumped. The dropping was distributed over the entire area of the fill. It was found that drops in excess of 50 ft damaged the rock. There was a limited amount of sluicing, but several hundred cubic feet of water per second passed through the fill during a flood at the time of construction.

At Dix Dam each 16-yd car was classified as to the quality of the rock, the larger and better rock going to the up-stream half of the dam. All dirt and decomposed rock was discarded.

During the construction of a rock-fill dam all three phases of the work are generally done at the same time. As the loose fill is being placed, the rubble section is laid on the up-stream slope, and below this the concrete ribs and apron are being poured. The rubble section is built simultaneously with the loose fill and follows directly below. Progress and workmanship are best with rocks of suitable size and with one or more good faces. The selected rocks are easily delivered to the masons on the rubble wall by dumping down the up-stream slope. The rubble wall should always be extended back far enough to have a good bearing against the rock-fill section. The fill should be built to the proper lines and not carried out to meet the back of the rubble wall already built. The rubble must be supported everywhere by the compacted rock-fill.

Settlement.—All settlement is the result of crushing of the rock-to-rock bearing points, as described by Mr. Galloway. The larger the rock the fewer will be the bearing points for any given depth of fill resulting in less settlement, but construction equipment and methods generally limit the size of rock.

A fill of good granite or limestone, made as described by Mr. Galloway and properly sluiced, should not exceed 1% or 2% at the most. A reasonable settlement is to be expected and can be designed for, provided it is uniform. A settlement of 2% of the height would be reasonable. However, a rock-fill dam having a semi-rigid facing may be a partial failure with moderate settlements if the settlement of adjacent areas is not fairly regular. It is difficult to design a concrete facing to accommodate itself to large, sharp, irregular movements.

Large rocks are generally least damaged in quarrying, and have the soundest surfaces and points upon which the rock-to-rock load, as described by Mr. Galloway, is carried; consequently they offer the greatest resistance to crushing, thereby reducing settlement. Very good results may be expected if the bearing rocks weigh 2 tons and more. Smaller rock subsequently sluiced into the interstices of the bearing rocks adds weight to the fill and supporting power as the bearing points of the large rock crush.

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[20] Berkeley, Calif.

The beneficial effect of sluicing cannot be over-stressed. Even in a fill of clean, large rock, sluicing wets all bearing surfaces, which gives a certain degree of lubrication, and assists the slipping and readjustment of the rocks to a better and more permanent position. Settlement is aided.

In considering the settlement of a rock-fill dam one generally means that settlement which occurs after the dam is completed. However, a considerable settlement is taking place in the lower parts of a dam during the construction period as the latter assumes the increased weight of the fill above. These movements are of interest to the engineer.

The rubble section settles as it follows the movement of the supporting loose fill. In most large dams with a concrete face there are vertical supporting ribs under each vertical expansion joint. The ribs are poured in advance of the concrete face. If the ribs are poured too close to the top of the rubble wall as it is being built up, the initial settlement may be sufficient to crush the green concrete ribs. At Dix Dam, it was found advisable to keep the concrete ribs about 50 ft below the top of the rubble wall.

The settlement, to a reasonable depth, is not so much a criterion of the quality of the fill as is the relative settlement of adjacent areas. In a dam in which the total settlement may be only 2%, trouble may be expected if such settlement is uneven; and uneven settlement may be expected in a fill with poor distribution of material, insufficient sluicing, or where a considerable time element exists between placing adjacent areas.

During the construction period at Dix Dam three temporary points were established on the 690-ft level (where the fill was 185 ft high) and maintained 41 days when they were covered by rock for the next lift above. During this period a flood piled up about 60 ft in front of the partly completed dam. Much water flowed freely through the rubble wall and loose fill, emerging at the base of the down-stream slope. It was impossible to measure the quantity of water passing through the dam, but the fill received a perfect sluicing below the hydraulic gradient. Fig. 17 shows a record of the settlement of these three points and is of especial interest as it shows the extent of the early settlement, the settlement caused by the flood, and the increase in settlement toward the down-stream slope of the dam. The increase in settlement toward the down-stream slope is believed to be due entirely to the fact that the poorer material was placed in the down-stream half of the dam. This section of the dam may have had some small rock separating the rock-to-rock bearing points of the larger rocks, and the small rocks were displaced by the flood. Settlement varied from 0.32% at the up-stream slope to 1.02% of the height of the fill at the down-stream slope. All these points were on the loose fill section of the dam.

Another temporary point set on the face of the rubble wall showed the following movement during the 70-day period of record before it was covered by the concrete facing. There was a movement toward the center of the canyon of 0.28 ft, an up-stream movement of 0.07 ft, and a vertical settlement of 0.28 ft. The dam at this point was 100 ft high and on the side of the canyon. Consideration should be given these movements when establishing lines for the work.

At Strawberry Dam a similar flood occurred during construction. It is believed these floods were of material benefit to both dams.

Mr. Galloway gives settlement data for Dix Dam during the first three years after its completion. Later measurements, made eleven years after completion, show the maximum settlement at the crest to be: Vertical, 2.48 ft; horizontal, 1.93 ft; and down on the face of the dam, 55 ft below the crest, where the depth of fill is 215 ft, there is a vertical settlement of 2.28 ft and a horizontal settlement of 1.44 ft. The maximum vertical settlement at the crest is 0.90% of the height of fill.

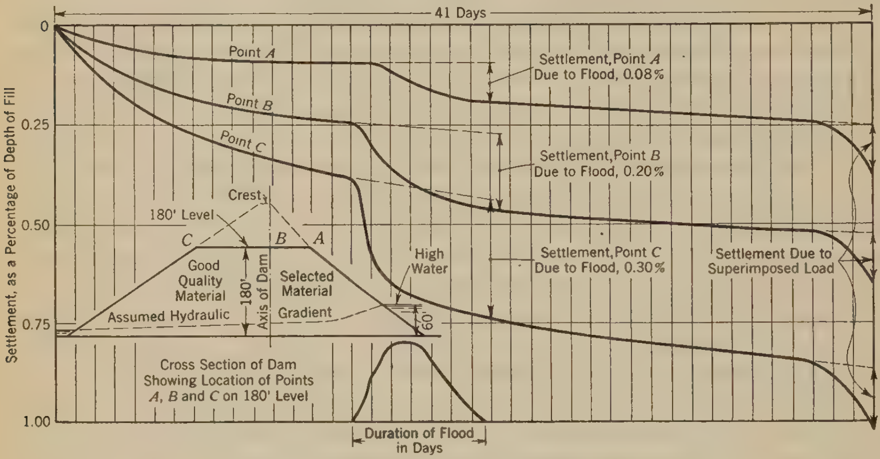

FIG. 17

Facing.—No one standard design of facing can be claimed to be best for all conditions. Each dam is its own problem.

Strawberry Dam has an apron of reinforced concrete sliding leaves over the upper two-thirds of the dam, each approximately 12 in. thick, 60 ft wide, and 135 ft long, fastened at the bottom but free to slide upon a smooth plastered surface covered with roofing paper. Over the lower one-third of the structure the concrete facing was poured directly against the dam, but not until all cracks between the rocks had been plastered to prevent the concrete from running in between the rocks. This was thought to be a measure of economy. There is no evidence that the leaves over the upper two-thirds of the dam have ever moved.

It is interesting to note that the expansion joints at Strawberry Dam have no seal. The original design called for the joints to be packed with asphalt or oakum. High water overtook the water before the seal was installed but the joints proved tight and were never packed. These joints were constructed about 2 in. wide and terminated at the back against the underlying concrete rib.

At Dix Dam concrete was poured directly against the face of the rubble section of the dam and worked in between the rocks. Vertical and horizontal expansion joints were provided. At the east abutment large differential settlements were anticipated and the spacing of expansion joints was decreased. The bottom 180 ft of water in this dam has never been emptied, but inspection at the most critical point, which is above the draw-down, after 10 yr of service indicates a relative movement of about 2 in. at the expansion joint, with some twisting. There are no open cracks in the slabs that are visible. Joints have taken all movements. There were no construction joints as each panel was completed in one continuous pour.

The writer believes that if a concrete facing is adopted it should be poured directly against the dam and between the face rock, with adequate expansion joints. Spacing of these joints should be such as to accommodate the larger settlement occurring over sections of the dam where there is an abrupt change in the depth of fill. Vertical joints should be constructed open or closed according to their position in the dam.

JOHN E. FIELD,[21] M. AM. SOC. C. E. (by letter).—The author is to be commended for bringing this newer and less known rock-fill dam to the attention of the profession, for the very practical suggestions he has made, and for suggesting dimensions based on his experience for criticism and further development and change. Although the paper is quite complete and very instructive, there are experiences among individual engineers which, if presented, would be illuminating. With the growing appreciation of the value of drainage in dams of all types, the rock-fill dam should find a wider and more general acceptance.

Assuming proper design in all cases the writer lists the three types of dam in point of economy of construction, thus: (1) The earth-fill; (2) the rock-fill; and (3) the concrete dam. All must be 100% safe because less than absolute safety cannot be tolerated.

Some designers assume too hastily that, since dumped rock assumes a natural slope, this natural phenomenon offers a perfect guide in design. This assumption may, or may not, be correct, depending on what is meant by a "natural" slope; whether it is the slope on which the rock will stop moving when dumped, or the slope on which, when at rest, the rock will start moving. Observed natural slopes are: For the lee side of sand dunes, 1 on 1.5, and sliding will start when a small quantity of sand is excavated from the foot of the slope; in rock slides composed of small flat and shell-like rock, the angle is generally less than 30°, or about 1 on 1.8; and in rock slides of large boulders the observed slope is often as much as 40°, or about 1 on 1.4. These slides were made by rock falling from cliffs which, by reason of their inertia, would develop flat slopes; also, the action of frost and the elements over long periods of time have flattened them. In excavating a bench across such rock slides or old mine dumps, or cutting through them, the side slopes of the cut are generally made much steeper than the surface slope although at present the tendency is to flatten the slope to prevent rock, loosened by frost or rain, from rolling into the cut. Too many stable and successful dams have been built with slopes less than 1 on 1.3 (see Table 1) to justify the adoption of that slope as a minimum. Although the writer does not disagree with the suggested practical 1 on 1.3 slope for rock-fills he feels that steeper slopes are preferable unless structural conditions and operations require the flatter slope. The steepest slope permissible is that at which movement would start but, as far as the writer knows, this has not been determined by research, or by experiment with different materials, placed by different methods.

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[21] Engr.-Examiner, PWA, Washington, D. C.

The difficulty encountered in connecting the impervious facing to the abutments suggests making a temporary connection of some flexible material, such as lumber, which would last well beyond the period of settlement.

In a rock-fill dam, with all the uncertainties of a reliable, permanent, impervious facing, one must assume that water may pass into the fill and the presence of any erodible material should be avoided.

Two of the dams listed in Table 1 have failed. The Castlewood Dam, in Colorado, was washed out in 1933 by over-topping. The spillway was in the center of the dam, about 200 ft long and 4 ft below the crest at either end; the lower slope was in steps built up of large rectangular rock and then grouted. Many smaller floods had passed through the spillway and over the stepped lower slope, but in the 43 yr the grouting, never first class, had largely disintegrated, the flood of 1933 had probably loosened and dislodged some of the steps, and the material in the body of the dam was rapidly washed away. This material was composed of rock, spalls, and sand; probably the entire material from the quarry, with some of the over-burden, was used. On three occasions prior to 1904, also, piping had washed out the foundation material. After the third wash-out an earth blanket on a slope of 1 on 3 was placed up stream from the dam and was effective in preventing further trouble. The design or dimensions of this rock-fill dam were not at fault, but poor foundations and poor workmanship were to blame for the three wash-outs and the final collapse.

The Beaver Park Dam, in Colorado, has been an economic failure because water in large volume—130 cu ft per sec—passed beneath and around the dam, through the abutments. Apparently, the canyon in which the dam was built had no rock in place, either at the bottom or on the side slopes. It appears that a slide had occurred from the high cliff to the east and choked the ancient river channel, and water passing through this broken up mass of rock had eroded a new channel. After the serious leaks appeared, the bottom and east slope were drilled and grouted but, due to lack of funds, were not extended sufficiently into the old slide and the leak at present is greater than the normal flow of the stream. It has been suggested that an earth blanket be placed in the area above the dam both against it and on the hill slope to a point up stream and beyond the area of the old slide. The crest of the dam appears to have moved about 3 ft down stream and to have settled nearly as much; the rock used was rather soft and friable. In this case, a lack of proper borings and ignorance of the geological conditions are quite apparent.

Fig. 18 shows the section of a dam in which the rock is graded; such grading will probably decrease the percentage of settlement. The plan differs from those of most rock-fill dams in that the Class *A* is large rock and that Class *B* next up stream is from 3 in. to 15 in., and the next is 3 in. and smaller, with a filtering medium of sand (Class *D*) between Class *C* and the earth-fill. Classes *B*, *C*, and *D* are put through a crusher and screened. If the fill is made in horizontal lifts, it is apparent that the line of contact between the several classes of rock can be altered—made steeper or flatter as desired. The lower slope (1 on 2) seems flatter than necessary, and being flatter than the natural dump slope, probably requires some extra work in placing (see Fig. 18).

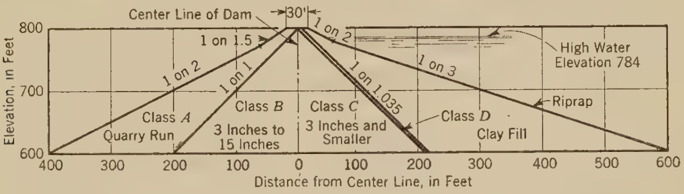

FIG. 18

Core-walls in the center of a dam are inadvisable not only on account of inaccessibility but more because the full hydrostatic pressure is at the center of the structure, leaving to that part above the core-wall the sole duty of supporting the rip-rap and the core-wall. With settlement of the structure and the raising and lowering of the water in the reservoir, changing stresses are imposed on the core, and, if it is an earth-fill, the part above the core becomes saturated, with a decided tendency to slide into the reservoir. Resultant stresses, first on one side and then on the other, tend to rupture the core-wall. A core composed of screened gravel, however, would be of benefit in lowering the line of saturation, and the rock-fill dam can be considered an extreme case in which the "core" occupies almost the entire section of the dam.

Where, for any reason, it is desirable to use all the materials from the quarries, including the generally condemned "fines," one can visualize a core of screened rock obviating any danger of lack of proper drainage in the fill.

Under some conditions gates and collapsible stop-planks can be used in spillways; for example, if the discharge capacity has been determined for a 200-yr period on a 100-yr record automatically collapsible stop-planks could be installed to meet the 1 000-yr probability. Gates placed in a spillway are generally undesirable and to rely on the watchman or attendant at the dam is dangerous; if, however, there is a power-house or factory at the dam where numerous and competent men are always at hand, gates in spillways could be tolerated.

Acknowledgment.—Fig. 18 was derived from the original plans of Oneonta Dam, in Alabama.

JOHN H. WILSON,[22] ASSOC. M. AM. SOC. C. E. (by letter).—Observation of the sandstone in a dam near Oneonta, Ala., will be of interest to those who are contemplating the use of sandstone for rock-fill purposes. By definition Mr. Galloway excludes this earth and rock-fill type; but the character of the rock and the quarrying methods are those that would be encountered at this site regardless of the type of dam.

The rock-fill design contemplated the placing of four gradings of stone.

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[22] Dist. Representative, R. G. Le Tourneau, Inc., Peoria, Ill.

The construction plant was set up to meet the original specification of (quoting):

> "348,072 Cu. Yds. Class (A) shall be run of quarry. It is expected that 50% will be larger than one-man stone.
> "314,584 Cu. Yds. Class (B) shall be crushed stone of from 3 inches to 15 inches in size.
> "336,552 Cu. Yds. Class (C) shall be crushed stone $\frac{1}{2}$ inch to 3 inches in size.
> "36,916 Cu. Yds. Class (D) shall be screenings $\frac{1}{2}$ inch or less in size."

The site of the quarry is adjacent to the dam area, the elevation of the quarry floor being approximately 50 ft higher than the crest elevation of the dam. Over-burden of clay was removed by drag scrapers and carried down to a sandstone which was of decomposed character, yet too hard to excavate by such equipment. A dragline was used to remove more of the decomposed stone. Test specimens of the material uncovered were classified as "argillaceous sandstone," the bonding material being ferric oxide. Under this stone is a harder, more highly consolidated material.

Primary blasting was conducted by use of well-drill holes spaced about 15 ft on centers and loaded with an average charge of 950 lb of dynamite. The detonating agent used was cordeau.

The resulting fragmented rock was loaded by 4-yd steam shovels and hauled in 12-yd side-dump tractor wagons to a 48-in. by 60-in. jaw crusher. From the primary crusher the stone went on to a grizzly that removed the material smaller than 8 in. to a vibrating screen which made the final classification of the material. A secondary 20-in. gyratory crusher received a part of the stone that passed over the grizzly, and fed the fine material to the same grizzly and vibrating screen for reclassification.

Stone too large to be manipulated by the 4-yd shovels is reduced in size by continuous secondary drilling and blasting. The operations described have resulted in the introduction of a large percentage of fines smaller than 0.5 in. The primary blasting and secondary blasting have produced a large quantity of these fines, and the crushers have added to the total. Between 15% and 20% of the total yardage produced will screen smaller than 0.5 in. The quantity of rock broken down by blasting, to sand or fines, is so excessive that it has not been possible to load the larger stone to the exclusion of this fine material. The loads of quarry-run stone containing, by visual inspection, too great a proportion of fines have been wasted.

In order to accommodate the use of the fines produced at the screen, the thickness of the up-stream blanket of Class D rock was increased from 5 ft to 8 ft. A further effort was made to include the fine material in the dam by introducing thin horizontal blankets over the Class B and Class C rock.

Mr. Galloway has requested that discussion be directed to problems of design. The writer's purpose in relating the methods used in the construction of this sandstone rock-fill dam is to direct the attention of designing engineers to conditions that may exist in another sandstone formation similar in character to that described. In the writer's opinion sluicing of fines into the larger rocks would produce a more dense fill than has resulted in the past. This might give greater stability to the structure, but it also might prove objectionable from the standpoint of open drainage for water which seeps past the facing.

The exclusion of fines from a quarry operation in rock of this nature will increase the cost of construction. Not only by reason of rehandling, but during rainy weather the fines form a gritty mud which clogs screening equipment, resulting in serious retardation of construction operations.

The writer would suggest alternate methods of writing specifications covering construction that involves the placing of sandstone fills:

(1) Eliminate all attempt to classify the sandstone by crushing and screening, placing the run-of-quarry stone in the fill. Sluice the fine material to prevent any accumulation of fines in a local area. In order to provide a section that would be stable, both the up-stream and down-stream faces of the rock-fill would probably have slopes of 1 on 2, or flatter. The resulting increase in quantity would not necessarily mean an increase in price or cost due to the lowered unit cost of producing the fill material.

(2) If it is anticipated that a part of the quarried stone may be unsuitable for use, it seems to the writer that the basis of payment for rock-fill should be yardage in cut; or yardage in fill, plus an allowance for yardage placed in a designated waste dump. This would permit the construction engineer more latitude in his selection of suitable stone for the fill. This kind of clause in the specification should be reflected in the price bid for the work by permitting the bidder to reduce his allowance for coverage of ambiguous contract provisions.

In considering the design of a rock-fill dam with rock of questionable character, due regard should be taken of the construction problems which will influence the cost of construction and thereby affect the economy of design.

FREDERICK H. FOWLER,[23] M. AM. SOC. C. E. (by letter).—The definition of a "rock-fill dam" adopted by the author, well describes the type of structure that has reached its maximum development in the Sierra Nevada of California. It does not, however, include the many important variants of the rock-fill type that have developed elsewhere, particularly in regions where foundation conditions are more difficult and common labor cheaper than in the California mountain districts.

Even in the rock-fill structures of the Sierra there has been a rapid evolution and, in the later dams, such as Bucks Creek and Salt Springs, in California, the placed-rock sections are far thinner in relation to the greater heights, and conform more closely to the author's definition than did the earlier Relief Dam, in California. At Relief, for example, with a maximum height of only 140 ft, the combined horizontal thickness of the facing and placed-rock section at the base of the dam was 100 ft. At Salt Springs, with a maximum height of 328 ft, the adopted combined horizontal thickness of the facing and placed-rock was only 25 ft. At Relief, in other words, the base thickness of the placed-fill section was more than 70% of the total height of the dam; at Salt Springs, it was less than 8 per cent.

These designs thus tended toward a simplicity of structure, an increasing proportion of drop-fill, and a corresponding decrease in rubble fill and mortar-placed facing, an evolution that may be considered as typical of practice in the Sierra Nevada.

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[23] Cons. Civ. Engr., San Francisco, Calif.

It will be noted, however, that no design reached the ultimate limit of the drop-fill type, namely, a complete drop-fill structure as in the Cogoti Dam, in Chile,[24] with no derrick-placed cushion. Nor have the designers of dams in the Sierra of California been driven by difficult foundation conditions, or tempted by a plentiful supply of cheap labor, to increase the proportion of derrick-laid rock or to abandon drop-fill entirely.

Drop-fill, however, has been definitely abandoned in Algeria.[25] The most recent technique in hand-placing and derrick-placing of fill and the design of facing, etc., for four large Algerian dams, is set forth in an article[26] published since the author's paper. This article gives (in convenient form and in English translation) material hitherto scattered in various publications in several foreign languages.

From a western viewpoint, the Algerian designs are notable for the care and thoroughness used in the construction of the entire main fill of derrick-placed rock of the largest sizes practicable, with the crevices well packed with smaller rock and quarry spalls. Settlement in the body of the dam is reduced to a minimum by care in placing the fill, and further reduction and equalization of settlement, due to poor foundations, are sought by the construction of thick rubble foundation slabs, on which the rock-fill rests. Under these slabs are very complete systems of under-drains. Heavy cut-off walls, provided with inspection and work galleries, run the full length of the up-stream base of the dams, and deep grout curtains at the up-stream sides of the cut-offs were used as an additional precaution against seepage. Highly specialized laminated facings, varying in detail at the several dams, are characteristic of all four structures. Although none of these dams conforms to the author's very specific definition of a rock-fill dam, they can scarcely be placed under any other general classification.

An even more specialized rock-fill structure is the Shing Mun Dam,[27] 275 ft in height, designed to supplement the water supply of Hong Kong and Kowloon. According to the description, the dam consists of five parts. Its face is a relatively thin diaphragm built of rich reinforced concrete and divided, by vertical bitumen-filled joints, into segments, so that it can stand considerable distortion without destruction; a cut-off wall is formed by an extension of the face downward to impervious rock and built of rich unreinforced concrete placed for maximum density. Back of the face of the dam and the cut-off wall is a "thrust-block" of low-grade mass concrete, resting on the surface rock and designed to transmit the water pressure to the rock-fill behind. Finally, in order to insure a uniform transmission of pressure between the thrust-block and the rock-fill, a wedge of dry sand is inserted between these elements.

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[24] "Earthquake-Proof Dam in Chile," *Engineering News-Record,* November 5, 1931, p. 725.

[25] "Note sur le Masque en Béton Armé du Barrage de Bakhadda," by M. Drouhin, *Annales des Ponts et Chaussées,* 1935, VIII, pp. 281–291.

[26] "Algerian Dams of Placed Rockfill," by I. Gutmann, *Engineering News-Record,* December 2, 1937, p. 889.

[27] "Steep Concrete Face on Rock-fill Dam in China," *Engineering News-Record,* November 12, 1936, p. 677; also, Second Congress on Large Dams, Washington, D. C., September, 1936.

The rock-fill was hand picked and brought up in 2-ft layers sloping toward the thrust block. Drains, and inspection pits and galleries, are provided. The Shing Mun design is "a far cry" from the simple designs outlined by the author, and is probably on the border line between what may, by broadest definition, be called a rock-fill, and a composite structure of uncertain classification. It demonstrates to a marked degree the changes in a basic idea that may be brought about by vastly different costs of labor and material, in combination with more difficult foundation conditions.

Through the migratory tendency that is common to engineering designs, Algerian practice in placing fill and foundation treatment has invaded Mexico, and Chilean practice in laminated concrete facing has traveled to California.

The use of derrick-placed and hand-placed fill in the Mexican project (the Tepuxtepec Dam, on Rio Lerma, Michoacan) probably resulted from the employment of an Italian consultant, thoroughly familiar with North African practice. Its use was made economically possible by the large available supply of cheap labor. Derrick-placed fill, rubble-masonry foundation slab, drainage, and deep grout curtain follow, closely, Algerian practice, under generally similar conditions. The design of the backing for the up-stream facing, however, departs from Algerian and other practice, since it consists of a series of horizontal blocks (with inclined up-stream edges) extending back varying depths into the fill, and tending to tie the fill and facing support together. This unusual block design is merely another example of the many attempts to secure a proper adjustment of the impervious facing to the progressive settlement of the rock-fill body of the dam—one of the main problems in the successful design of this general type of structure.

What is termed the "Chilean type" of laminated concrete facing is that used on the Cogoti Dam in that country. In the article[28] describing its details, it was stated that the same type of facing was to be used on seventeen major rock-fill dams then proposed by the Chilean Bureau of Reclamation; however, according to the most reliable information now available, construction of all these dams, except the Cogoti Dam, has been postponed on account of adverse conditions in world finance, and no details are available concerning the results secured through its use on the Cogoti Dam. That structure was to be built entirely of loose fill, with slopes conforming to the angle of repose of the material used in the structure. A facing of concrete was to be poured against a gravel blanket on the up-stream slope and the laminated facing of reinforced concrete laid directly on this base slab. This plan to secure a stable and watertight structure without using any expensive derrick-laid rubble is naturally very attractive, if practicable, and it is to be hoped that a full description of the measure of success attained at Cogoti may be forthcoming.

The results attained in Chile are of particular interest, since a very similar type of laminated facing was used initially on San Gabriel No. 2 Dam, in California, and was extremely unsatisfactory, with the settlement that developed in that structure. Different methods of construction might have reduced the settlement in the fill, and with a reduced settlement the facing might have held. An answer to the question as to whether the failure of the facing was due to excessive settlement or to an inherent weakness in the laminar type of facing may be had when the results in Chile are known.

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[28] *Engineering News-Record*, November 5, 1931, p. 725.

Selection of construction methods that will produce a fill (of either placed or dropped rock) with a relatively small and generally uniform settlement, and a type of facing of sufficient flexibility to follow such settlement in the fill as cannot be avoided, constitute two of the main problems in design. Under Algerian conditions, with relatively poor foundations and high value of water, difficulties of maintaining a tight face with large settlement in drop-fill, have led to the definite abandonment of that type of placement.

Even where drop-fill is to be used there remains either much to be learned concerning the best construction methods, or a very considerable adjustment of divergent engineering views as to what is accomplished by certain methods already generally accepted. All authorities appear agreed, for instance, that plentiful sluicing of the fill during construction will facilitate and accelerate settlement, thus making it possible to construct the final facing at an earlier date and with less fear of harmful distortion than would otherwise be possible. However, aside from the general agreement that lubrication of the larger pieces is one of the most beneficial results of sluicing, there are divergent views concerning its purpose. One description states that the purpose of sluicing is "to wash the fines into the fill," and another states with equal conviction that the purpose is "to wash the fines out of the fill." Possibly more detailed descriptions of local conditions and construction methods in these particular projects would reconcile these apparently discordant statements—in the meantime, the reader is somewhat at sea.

No more valuable contribution could be made to the discussion of rock-fill dam design than a thorough analysis of the settlement that has taken place, both during construction and after completion, in various dams now actually in operation. This is particularly true of large and unusual structures such as Dix, in Kentucky, Salt Springs, Bucks Creek, San Gabriel No. 2, and Cogoti Dams. Measurements are already available in brief form for Bakhadda Dam,[29] of the Algerian derrick-placed type; these measurements show not only the settlement that took place during the initial filling of the reservoir, but also the elastic recovery that occurred upon emptying the reservoir.

It would also be interesting to know the settlement that has been recorded, and the action of the welded-steel facing, in the El Vado (gravel-fill) Dam, which, so far as the writer knows, is one of the largest steel-faced fill dams now (1938) actually completed.

The difficulties that have arisen in designing concrete or other permanent facing that will conform without rupture to the considerable initial settlements of very large rock-fill dams now commonly constructed, warrant favorable consideration of timber facing; this, although of relatively short life, will last well through the early period when settlement in the rock-fill is largest and most rapid. By the time that it becomes necessary to renew the timber facing most of the settlement will have taken place, and far greater freedom may be used in the selection of facing design.

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[29] *Engineering News-Record,* December 2, 1937, p. 892, Fig. 5.

I. C. STEELE[30] AND WALTER DREYER,[31] MEMBERS, AM. SOC. C. E. (by letter).—The following discussion of Mr. Galloway's excellent paper will be principally confined to the subject of the design and settlement of Salt Springs Dam, in California, the principal design features of which are described by Mr. Galloway. Reference is also made to a paper[32] by O. W. Peterson, M. Am. Soc. C. E., which includes a description of the construction methods used in building the dam, and to an article,[33] "High Rock-Fill Dam with Jointed Concrete Face," by I. C. Steele, which includes a plan of the dam, a table showing slab thicknesses and reinforcing, and a table of elevations and offsets used in laying out the vertical and horizontal curvatures of the up-stream face. Fig. 19, a view of the up-stream face shortly before completion of the rubble facing, shows various features and elevations pertinent to this discussion and to the aforementioned references.

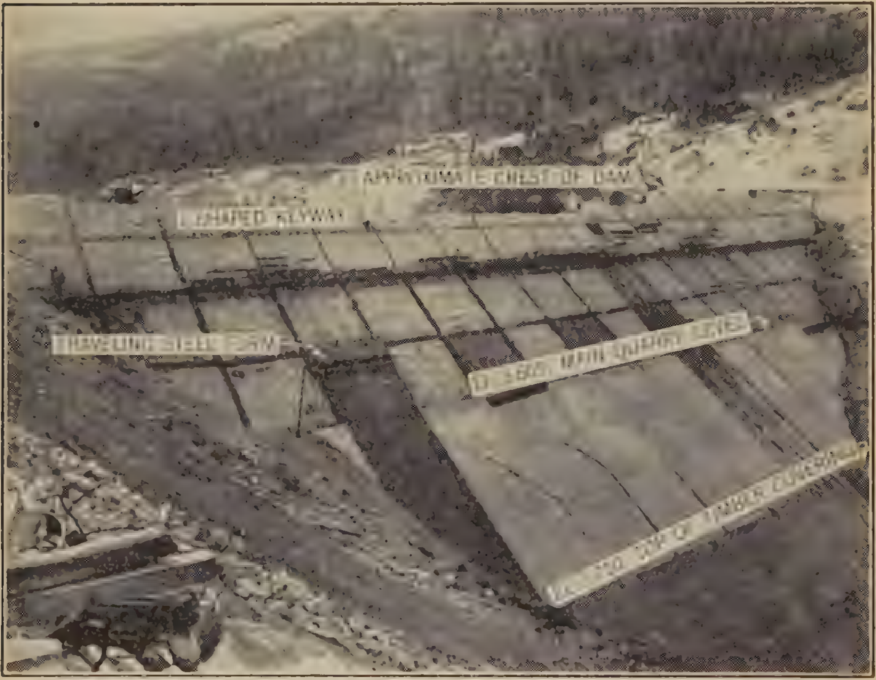

FIG. 19.—UP-STREAM FACE DURING CONSTRUCTION, SALT SPRINGS DAM

A brief summary of the controlling features of design and construction should be sufficient for the purpose of this discussion.

Factors Influencing Selection of Rock-Fill Type of Dam.—Several factors influenced the decision to build a rock-fill type of dam at Salt Springs. The dam site offered a choice between the rock-fill and concrete gravity types. Preliminary estimates indicated a saving of about $2 000 000, including indirect and overhead charges, for the rock-fill structure. The intended height, 328 ft above general foundation level, did not greatly exceed the height of the successful Dix Dam in Kentucky. Unlimited quantities of good quality rock were readily available for quarrying at desired elevations. The construction of a large capacity spillway, separate from the dam, was feasible. Studies of available data, and experience with other dams, indicated that a rock-fill dam of the desired height could be built and operated with the utmost safety.

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[30] Chf., Div. of Civ. Eng., Dept. of Eng., Pacific Gas & Electric Co., San Francisco, Calif.

[31] Asst. Chf., Div. of Civ. Eng., Dept. of Eng., Pacific Gas & Electric Co., San Francisco, Calif.

[32] *Proceedings*, Am. Soc. C. E., August 1930, p. 1319.

[33] *Engineering News-Record*, Vol. 104, p. 92.

Consideration Given to Settlement in the Design Assumption.—Settlement at any point on the face was estimated to be the vector sum of the vertical settlement due to the weight of rock in a vertical column of rock extending from face to foundation and the movement due to water pressure acting on an inclined column of rock normal to the face and having a length equal to the distance between the face and the foundation. An arbitrary value was given to the coefficient of settlement or compaction for this purpose.

Design of Face.—Details of face design are shown in Fig. 20 which also shows methods subsequently adopted for the repair of cracks and joints. A face concave in vertical section was selected to insure compressive forces acting toward the fill in the event that settlement differed greatly from the design assumption. A convex face might be pushed outward especially if there was an abrupt change in direction. Consideration was given to several methods of designing the concrete face for the dam. The several designs considered and the reason for eliminating some and selecting the one are:

(1) Sliding Type Face.—This design consisted of a single thickness of slab supported on a rubble face which has been made smooth with concrete and on which asphalt has been applied. The concrete face would be separated from the rubble and, theoretically, would be free to move independently from the body of the dam. This plan was discarded after some study because it was believed that with water pressure acting on the dam, the face could not move as it was intended to, because construction inequalities and settlement during construction would make the supporting face quite irregular. There also was objection to the relatively slim column to withstand compressive forces developed by movement of the dam, particularly when water pressure was not acting on the face.

(2) Laminated Face.—This type was never given serious consideration because of the desire to make the face quite substantial. A laminated face cannot be anchored successfully to horizontal keyways unless the keyways, in turn, are supported on rigid struts extending upward from the cut-off in the plane of the face.

(3) A Monolithic Slab.—A slab of substantial thickness, and with the concrete worked well back into the interstices of the first layer of dry rubble, was built so the face would have sufficient rigidity to withstand compressive forces resulting from settlement of the rock-fill on which the face was laid. During the early stages of construction the interstices in the exposed surface of the first layer of rubble were left open. This method was later changed and all large openings, and many of the smaller ones, were chinked in order to prevent

undue waste of concrete and to lessen the possibility of movement of fresh concrete from the slab proper into voids in the rock section as the traveling steel forms were moved ahead.

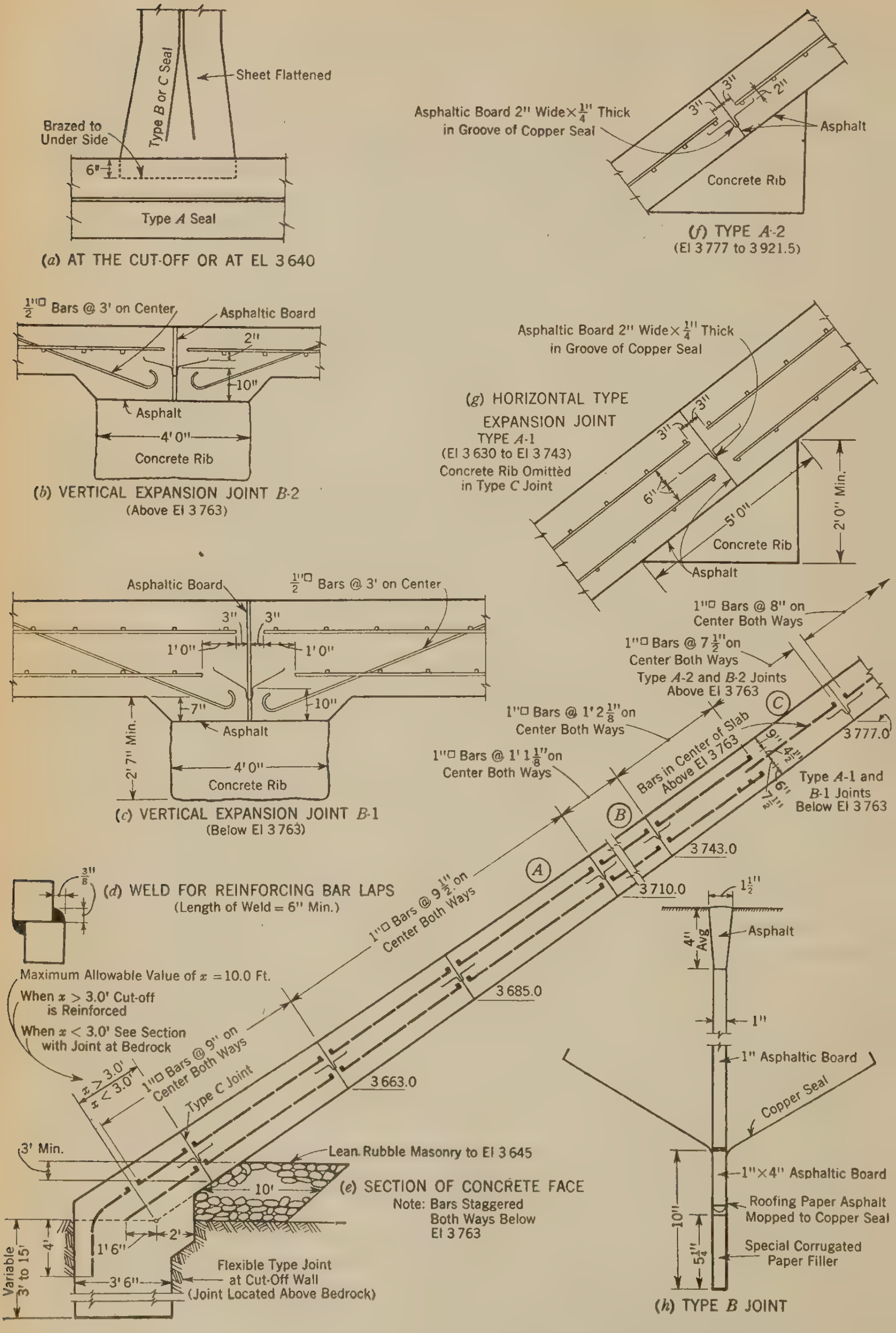
Joints in Face.—Consideration was given to a continuously reinforced concrete face, but one consisting of panels about 60 ft square, with joints along the periphery, was selected partly because of anticipated movement of the fill from each bank toward the center of the canyon in a direction parallel to the axis of the dam, which movement would induce very heavy compressive stresses in the face, and also because studies showed that if the dam settled in the manner and to the degree which was estimated, the stresses would be so great that the face would fail structurally. The adopted type is essentially a number of panels approximately 60 ft square with copper water-stops in the construction joints. Horizontal joints (see Fig. 20) are designed without an elastic filler, so the downward compressive forces can be transmitted directly from one panel to another. Inclined joints are made with a 1-in. thickness of elastic material to permit some movement parallel to the face. Other dams have placed two joints at each inclined rib, but one was considered sufficient for this dam. The copper seal was designed to have sufficient strength to withstand a separation with a maximum head of 4 in. without exceeding usual working stresses. Horizontal keyways were made triangular in shape to avoid overhanging rock during construction. They also served as walkways and work platforms during this period.

Settlement of Face.—The only data available with respect to settlement of rock-fill dams was for the settlement of the crest. The settlement obtained under the design assumptions described in the foregoing indicated that the greatest movement that would be likely to occur after the face was poured was in the lower elevations of the dam. Therefore, reference points were set in the concrete at a corresponding corner of each panel and the movement of these reference points was measured at frequent intervals during the first three years after setting and once each season thereafter.

Fig. 21, with Table 2, shows selected settlements at each measuring point for indicated periods. The extent of vertical, horizontal, and diagonal movement of any point may be obtained by scaling. Initial readings are plotted along straight lines at elevations which are averages of the two end points in each row. Joint opening or closing, as measured along the line of points in Row *H* and caused by lateral movement of the rock-fill, is shown at the top.

The maximum movement in a direction normal to the face was 5.41 ft at Point *D*, Station 4+60. Of this amount, 1.41 ft occurred prior to the first filling of the reservoir, and 4.0 ft occurred during and subsequent thereto. The latter value approximately represents the normal settlement caused by water pressure. Settlement almost equal to the foregoing occurred at Points *C* and *D*, Stations 5+20 and 5+80.

Fig. 22 shows settlements at various measuring points along the face for four vertical sections parallel to the river. The vertical distances between spillway crest level and points of maximum measured settlements for the several sections of fill which exceed 150 ft in height average about 60% of the corre-



(*a*) AT THE CUT-OFF OR AT EL 3 640

(*f*) TYPE *A*-2 (El 3 777 to 3 921.5)

(*b*) VERTICAL EXPANSION JOINT *B*-2 (Above El 3 763)

(*g*) HORIZONTAL TYPE EXPANSION JOINT TYPE *A*-1 (El 3 630 to El 3 743) Concrete Rib Omitted in Type *C* Joint

(*c*) VERTICAL EXPANSION JOINT *B*-1 (Below El 3 763)

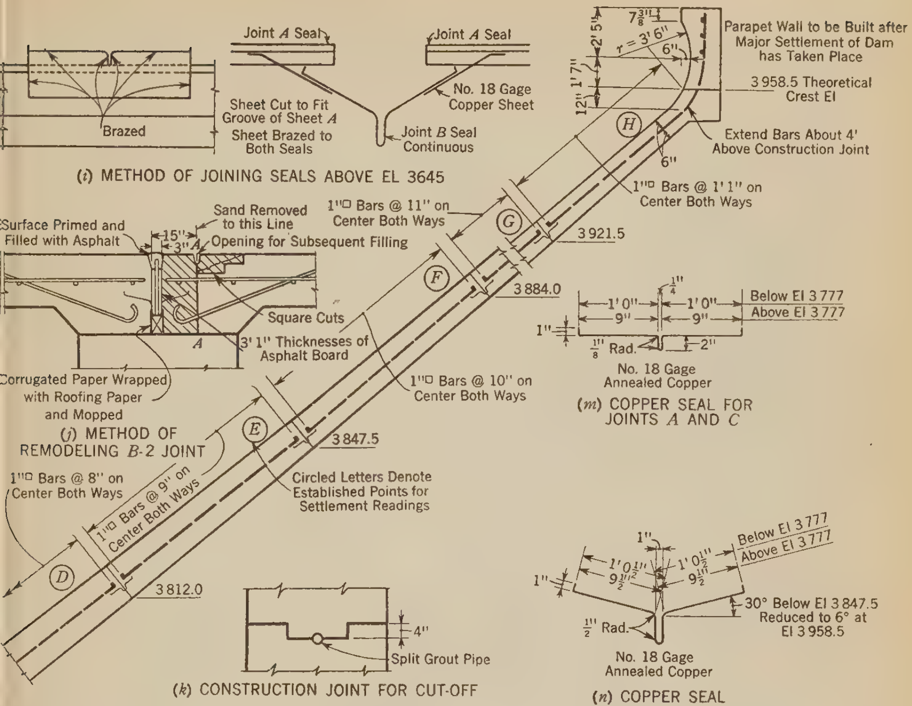
(*d*) WELD FOR REINFORCING BAR LAPS (Length of Weld = 6" Min.)

(*e*) SECTION OF CONCRETE FACE Note: Bars Staggered Both Ways Below El 3 763

(*h*) TYPE *B* JOINT

Sheet Flattened
Type B or C Seal
Brazed to Under Side
6"
Type A Seal
Asphaltic Board 2" Wide × 1/4" Thick in Groove of Copper Seal
3"
2"
3"
Asphalt
Concrete Rib
1/2"□ Bars @ 3' on Center
Asphaltic Board
2"
10"
Asphalt
4' 0"
Concrete Rib
Asphaltic Board 2" Wide × 1/4" Thick in Groove of Copper Seal
3"
3"
6"
5' 0"
2' 0" Min.
Concrete Rib
Asphalt
Asphaltic Board
1/2"□ Bars @ 3' on Center
3"
3"
1' 0"
1' 0"
7"
10"
2' 7" Min.
Asphalt
4' 0"
Concrete Rib
3/8"
1"□ Bars @ 8" on Center Both Ways
1"□ Bars @ 7 1/2" on Center Both Ways
Type A-2 and B-2 Joints Above El 3 763
C
1"□ Bars @ 1' 2 1/8" on Center Both Ways
Bars in Center of Slab Above El 3 763
3 777.0
1"□ Bars @ 1' 1 1/8" on Center Both Ways
6"
4 1/2"
6"
7 1/2"
Type A-1 and B-1 Joints Below El 3 763
B
3 743.0
A
1"□ Bars @ 9 1/2" on Center Both Ways
3 710.0
1 1/2"
Asphalt
4" Avg
Maximum Allowable Value of x = 10.0 Ft.
When x > 3.0' Cut-off is Reinforced
When x < 3.0' See Section with Joint at Bedrock
3 685.0
1"
1" Asphaltic Board
Copper Seal
x > 3.0'
x < 3.0'
1"□ Bars @ 9" on Center Both Ways
Type C Joint
3 663.0
3' Min.
Lean Rubble Masonry to El 3 645
10'
1"×4" Asphaltic Board
Roofing Paper Asphalt Mopped to Copper Seal
10"
5 1/4"
Special Corrugated Paper Filler
Variable 3' to 15'
4'
1' 6"
2'
3' 6"
Flexible Type Joint at Cut-Off Wall (Joint Located Above Bedrock)

FIG. 20.—DETAILS OF CONCRETE FACE AND


Joint A Seal
Joint A Seal
Brazed
Sheet Cut to Fit Groove of Sheet A
Sheet Brazed to Both Seals
No. 18 Gage Copper Sheet
Joint B Seal Continuous
Surface Primed and Filled with Asphalt
15"
3"
A
Sand Removed to this Line
Opening for Subsequent Filling
Square Cuts
A
3' 1" Thicknesses of Asphalt Board
Corrugated Paper Wrapped with Roofing Paper and Mopped
7$\frac{3}{8}$"
2' 5"
r = 3' 6"
6"
1' 7"
12"
Parapet Wall to be Built after Major Settlement of Dam has Taken Place
3 958.5 Theoretical Crest El
H
Extend Bars About 4' Above Construction Joint
6"
1"□ Bars @ 1' 1" on Center Both Ways
1"□ Bars @ 11" on Center Both Ways
G
3 921.5
F
3 884.0
1"□ Bars @ 10" on Center Both Ways
E
3 847.5
Circled Letters Denote Established Points for Settlement Readings
1"□ Bars @ 8" on Center Both Ways
1"□ Bars @ 9" on Center Both Ways
D
3 812.0
4"
Split Grout Pipe
$\frac{1}{4}$"
1' 0"
9"
1' 0"
9"
Below El 3 777
Above El 3 777
1"
$\frac{1}{8}$" Rad.
2"
No. 18 Gage Annealed Copper
1"
1' 0$\frac{1}{2}$"
9$\frac{1}{2}$"
1' 0$\frac{1}{2}$"
9$\frac{1}{2}$"
Below El 3 777
Above El 3 777
1"
30° Below El 3 847.5 Reduced to 6° at El 3 958.5
$\frac{1}{2}$" Rad.
No. 18 Gage Annealed Copper

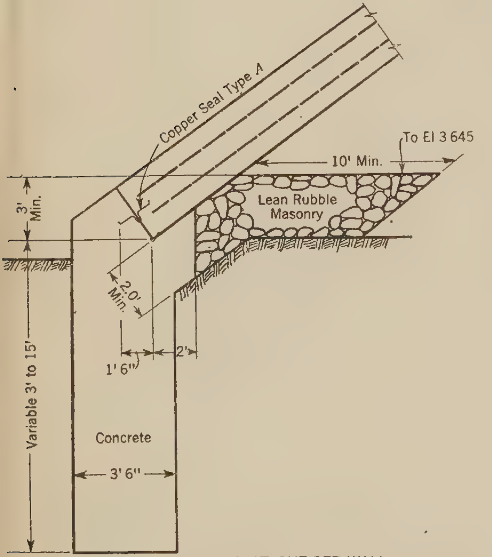
(i) METHOD OF JOINING SEALS ABOVE EL 3645

(j) METHOD OF REMODELING B-2 JOINT

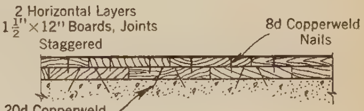
(k) CONSTRUCTION JOINT FOR CUT-OFF

(m) COPPER SEAL FOR JOINTS A AND C

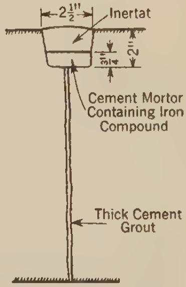
(n) COPPER SEAL FOR JOINT B


Copper Seal Type A
To El 3 645
10' Min.
Lean Rubble Masonry
3' Min.
2.0' Min.
1' 6"
2"
Variable 3' to 15'
Concrete
3' 6"

(l) FLEXIBLE TYPE JOINT AT CUT-OFF WALL
(Joint Located at Bedrock)

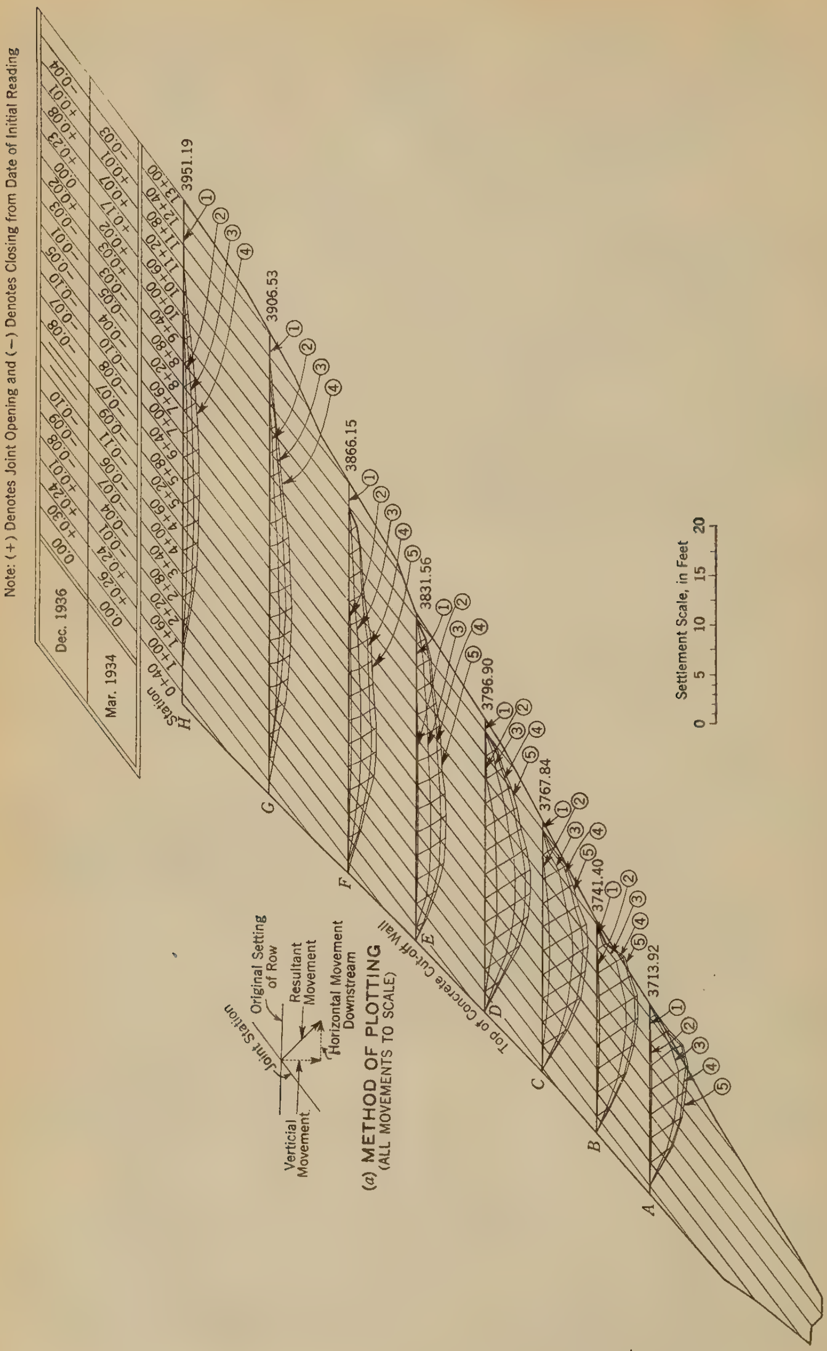

2 Horizontal Layers 1$\frac{1}{2}$" × 12" Boards, Joints Staggered
8d Copperweld Nails
20d Copperweld Anchor Nails

(o) METHOD OF ATTACHING TIMBER FACE BELOW EL 3 710


2$\frac{1}{2}$"
Inertat
2"
$\frac{3}{4}$"
Cement Mortor Containing Iron Compound
Thick Cement Grout

(p) METHOD OF CRACK REPAIR

CUT-OFF WALL, SALT SPRINGS DAM


Note: (+) Denotes Joint Opening and (−) Denotes Closing from Date of Initial Reading

Dec. 1936
Mar. 1934
Station 0+40
1+00
1+60
2+20
2+80
3+40
4+00
4+60
5+20
5+80
6+40
7+00
7+60
8+20
8+80
9+40
10+00
10+60
11+20
11+80
12+40
13+00
0.00
+0.26
+0.24
−0.01
−0.04
−0.07
−0.06
−0.11
−0.09
−0.07
−0.08
−0.10
−0.04
−0.05
−0.03
+0.03
+0.02
+0.17
+0.07
+0.01
−0.01
−0.03
0.00
+0.30
+0.24
+0.01
−0.08
−0.09
−0.10
−0.08
−0.07
−0.10
−0.05
−0.01
−0.03
+0.02
+0.00
+0.23
+0.08
+0.01
−0.04
3951.19
3906.53
3866.15
3831.56
3796.90
3767.84
3741.40
3713.92
①
②
③
④
⑤
H
G
F
E
D
C
B
A
Top of Concrete Cut-off Wall
Settlement Scale, in Feet
0
5
10
15
20
Original Setting of Row
Resultant Movement
Horizontal Movement Downstream
Joint Station
Verticial Movement
(a) METHOD OF PLOTTING
(ALL MOVEMENTS TO SCALE)

FIG. 21.—DIAGRAM SHOWING SETTLEMENT OF UP-STREAM FACE OF SALT SPRINGS DAM

sponding vertical distances from spill crest to foundation level. The point of actual maximum settlement in each vertical section no doubt occurs at some level between measuring points.

TABLE 2.—NUMBER AND DATE OF OBSERVATIONS PLOTTED IN FIG. 21

Row	ORIGINAL SETTING		BEFORE FIRST FILLING		AFTER FIRST FILLING*		AFTER SECOND FILLING		AFTER THIRD TO FIFTH FILLINGS	
	Observation No.	Date	Observation No.	Date	Observation No.	Date	Observation No.	Date	Observation No.	Date
H	1	May 9, 1931	..	....	2	February 15, 1932	3	September 1, 1932	4	December 1, 1936
G	1	March 10, 1931	..	....	2	February 17, 1932	3	September 1, 1932	4	December 3, 1936
F	1	February 10, 1931	2	April 9, 1931	3	February 18, 1932	4	October 4, 1932	5	December 3, 1936
E	1	February 10, 1931	2	April 24, 1931	3	February 17, 1932	4	November 1, 1932	5	December 3, 1936
D	1	May 28, 1930	2	April 10, 1931	3	September 28, 1931	4	November 21, 1932	5	February 28, 1937
C	1	May 8, 1930	2	April 4, 1931	3	November 2, 1931	4	December 27, 1932	5	February 28, 1937
B	1	May 28, 1930	2	March 6, 1931	3	December 10, 1931	4	March 16, 1933	5	February 20, 1935
A	1	May 1, 1930	2	March 6, 1931	3	February 15, 1932	4	March 16, 1933	5	February 14, 1934

* Reservoir rose only to maximum elevation 3 910, in 1931.

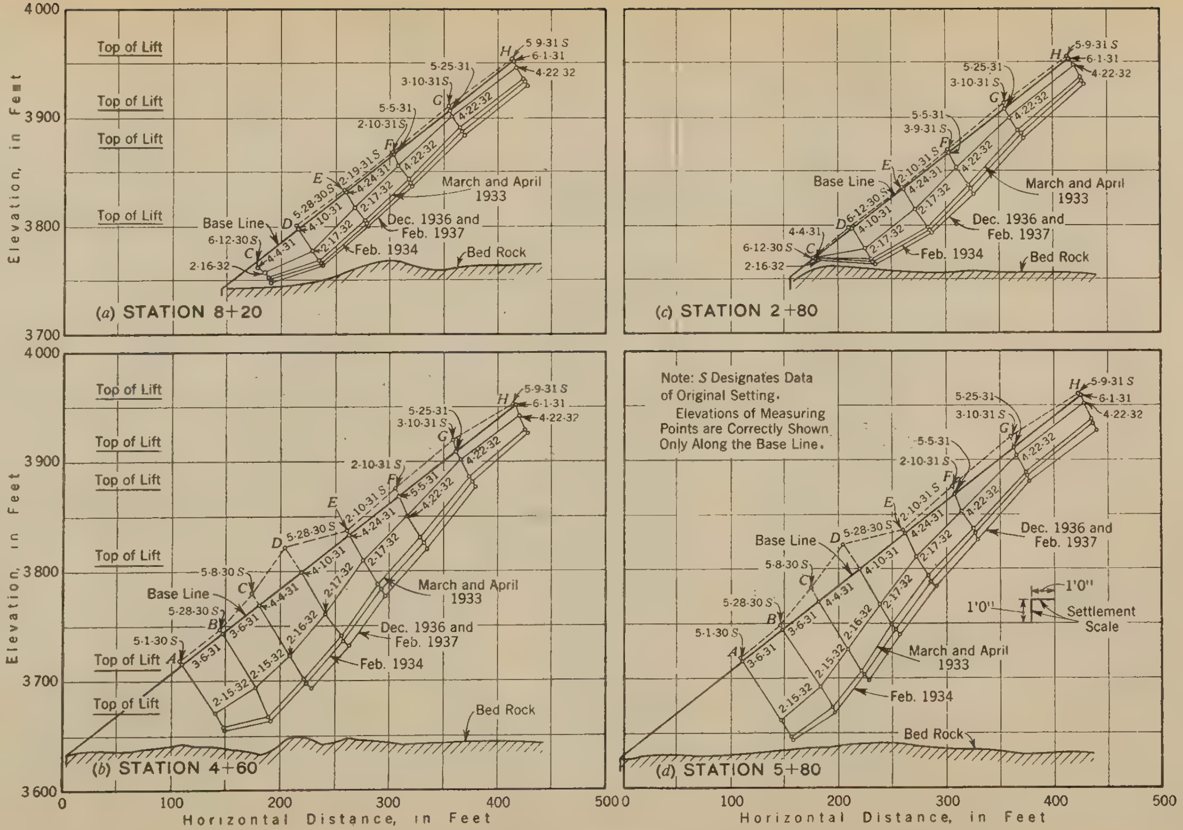
As mentioned previously, the settlement at any point on the face was assumed to be the vector sum of two components, one the vertical settlement due to weight of the fill in a vertical column of rock extending from the face to the foundation and the other movement due to water pressure acting on a column of rock extending from the face to the foundation in a direction normal to the face and having a length equal to the distance between the face and the foundation.

The greater part of the first component occurs while the fill is being made and prior to the construction of the concrete face and, therefore, is not an important factor in affecting the structural behavior of the slab. The settlement that occurred after the concrete face was installed has been carefully measured and analyzed. Maximum settlement of the crest under the first component and for a height of dam of about 330 ft has been nearly 2 ft. The vertical settlement at any other point on the face due to weight of rock only should vary as the square of the vertical distance from the face to the foundation. This component of settlement at any point on the face, therefore, can be expressed as $2.0\left(\frac{h}{H}\right)^2$, in which h = height of point above the foundation and H = maximum height corresponding to the settlement of 2.0 ft.

If the coefficient of settlement could be taken as a constant, the vertical displacement due to water pressure could be expressed by the factor,

$$S = \frac{P\,h}{K} \qquad (1)$$

in which P is the vertical component of water pressure and K is a "modulus of settlement" similar to Young's modulus for an elastic substance. Owing to storage operation it is not always possible to measure the settlement of all points on the face of the dam. At the time of the latest complete observation



Note: *S* Designates Data of Original Setting.
Elevations of Measuring Points are Correctly Shown Only Along the Base Line.

FIG. 22.—PROGRESSION OF SETTLEMENT OF SELECTED SECTIONS, SALT SPRINGS DAM

the settlement of the crest was 1.75 ft. If the vertical settlement due to the weight of the fill only at other points is deducted from the total settlement by using the formula, $1.75\left(\frac{h}{330}\right)^2$, the resulting amount would be due to water load only, and if the foregoing assumptions are valid, the factor, K, can be determined.

The result of this determination is shown graphically in Fig. 23, which has lines of equal resistance to vertical settlement under water load. Each line has the same value for the "modulus of settlement"; hence, the total settlement due to water load can be computed by Equation (1).

It will be noted that around the periphery of the canyon, the modulus has values of 300 000 to 400 000 lb, but is considerably higher at the center of the canyon above Elevation 3 800, which was approximately the main quarry level. Below this elevation the fill was well seasoned, so the settlement was relatively less there than in other parts of the dam.

Effect of Settlement on Concrete Face.—During construction and prior to pouring the face it was found that the settlement of the fill made it somewhat of a risk to construct the rubble face too far above the elevation of the concrete face. Tremendous forces are transmitted from the loose fill to the more rigid rubble face and in a few cases massive granite stones 4 to 5 ft thick at the bottom cracked, or were slightly displaced, under these loads when the rubble wall was carried approximately 100 ft above the concrete face. It is best to let the fill be well seasoned before placing either the rubble or the concrete in order to reduce these stresses to a minimum. In one case these forces were so great that a concrete slab about 2.5 ft thick and with two layers of reinforcing steel, split in the plane of the upper layer over a triangular shaped area 50 ft long and having a maximum width of 17 ft. Similar action occurred over several other small areas in the immediate vicinity.

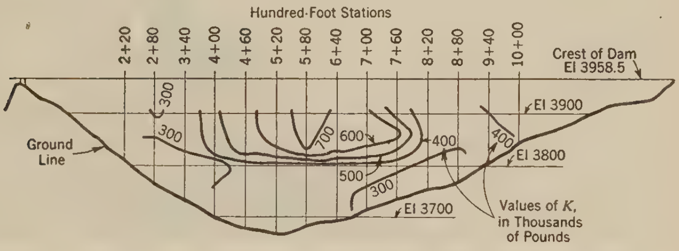


FIG. 23.—VERTICAL PROJECTION OF FACE OF SALT SPRINGS DAM, SHOWING LINES OF EQUAL RESISTANCE TO SETTLEMENT UNDER WATER LOAD

Settlement under water load demonstrated that greater flexibility was necessary adjacent to the canyon walls and in future work the design would provide for three or more joints about 5 ft apart instead of only the one joint at the cut-off wall which was used at Salt Springs. A number of small cracks developed in this region, some of which are shown, after repairs, in Fig. 24. With few exceptions these cracks lie within 60 ft of the abutments and most of them lie within a zone about 20 ft wide and roughly parallel to the cut-off wall.

The limits of this zone are approximately 10 ft and 30 ft away from the cut-off wall. As many as nine cracks, closely spaced, appear at some elevations. The uppermost crack of this nature does not extend closer than 60 ft, vertically, below the crest of the dam. Cracks vary in width from hair-line dimensions to a maximum of 0.5 in. in one or two instances. Most of the cracks were about $\frac{1}{16}$ in. wide. It has been necessary to chip out and fill these cracks, the method of repair being shown in Fig. 20. These repairs were made in 1932, 1933, and 1935, at a total cost of about $15 000, a nominal sum in view of the magnitude of the dam. At present writing (1938) leakage charts indicate that few additional cracks have developed since last repairs were made.

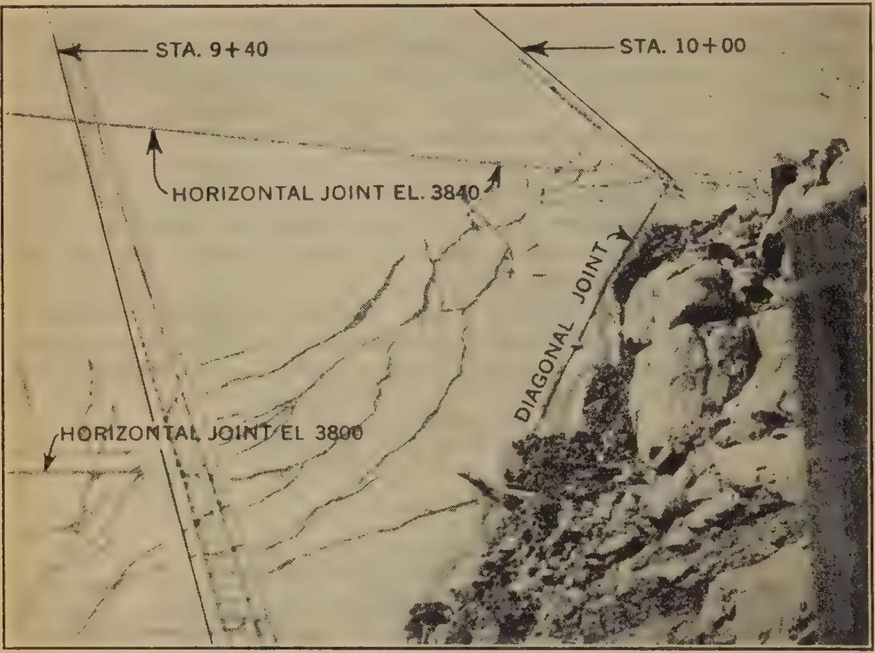


FIG. 24.—VIEW SHOWING CRACK REPAIRS IN CONCRETE FACE, SALT SPRINGS DAM

Movement of the fill toward the center of the canyon caused eight of the inclined joints to close completely, three to such an extent that the compressive stress caused spalling in the upper levels. It was necessary to reconstruct partly three of these joints. The uppermost panel at Stations 3+40 and 4+60, and the two upper panels at Station 5+80 were repaired in the manner indicated in Fig. 20 at a cost of less than $4 000. In future work of this nature the space between adjacent panels will be increased to 2 in. or more in those joints that are likely to be compressed.

Thickness of Rubble Face.—Mr. Galloway mentioned two items given particular consideration when the thickness of the rubble face was being determined for the design, namely, that water pressure indicated that the thickness should be greatest at the bottom; and settlement under vertical load, being greatest at the top, indicated that the rubble should be thickest at the crest, so that a uniform thickness finally was adopted. In the light of the experience at Salt Springs during construction and since, the writers, who participated in the original discussion and agreed to the selection of the uniform thickness, now believe that the thickness near the bottom should be greater than at the crest and that the thickness at the crest could, in a well constructed dam, be less than was used at Salt Springs. The reason for this opinion is that most of the settlement of the crest occurs before the rubble wall and concrete face are constructed, so it is not necessary to have a heavy rubble wall at that level to protect the concrete face. The maximum total movement of the face occurs at about one-third the height of the dam and the greatest differential movement occurs at the bottom and around the periphery of the face near the canyon walls. The greatest protection, therefore, should be had at these critical points.

Furthermore, as brought out by the late D. C. Henny, M. Am. Soc. C. E., in his discussion[34] of Salt Springs Dam, the local sliding factors in the rock adjacent to the up-stream face are high in relation to the coefficient of friction of the rock. In the case of a dam like Salt Springs, the sliding factors at the bottom level are 77% at the face, reducing to about 60% at the back of a 25-ft thick rubble wall (thickness measured normal to the face). It appears desirable in this part of the dam to have a rubble wall with the rock well keyed together so that horizontal movement need not be resisted by frictional forces entirely. This prompts the writers, in the light of all the foregoing factors, to suggest, in the interest of consistent design of high rock-fill dams, a rubble face having a thickness normal to the face of 10 ft at the crest and increasing about 5 ft for each 100 ft of height. It may be advisable to go even further than this by arbitrarily increasing the thickness of the rubble face for some distance out from the canyon walls, as this area of the face is almost certain to develop some cracks.

Conclusions.—It is believed that the crest settlement at Salt Springs Dam is less in proportion to its height than the measured settlement of any other structure of similar cross-section. In the opinion of the writers, the principal reasons for this and for the successful performance of the structure throughout are:

(1) Adoption of essentially natural slopes of loose rock-fill for the up-stream and down-stream faces;

(2) Construction of the loose fill, with quarry-run rock of good quality granite;

(3) Placement of loose fill from high lifts (75 ft or more) resulting in a well compacted fill;

(4) Liberal sluicing of loose fill during placement operations;

(5) Delayed construction of a well-built dry rubble section to form a cushion of reasonable thickness between the impervious membrane and the loose fill; and,

(6) Delayed construction of a monolithic type reinforced concrete jointed impervious membrane of substantial proportions on the up-stream face of the dam.

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[34] *Proceedings*, Am. Soc. C. E., August 1930, p. 1325.

F. Knapp,[35] Esq. (by letter).—The art of designing rock-fill dams, as developed during nearly a century of progress, and based on the experience gained by American engineers, is contained in this valuable paper. Abroad, notably in Algeria, several dams of the type discussed by the author have been built by French engineers, following, in general, the design principles stated in the paper. The innovation introduced consists in vibrating the rock-fill for the purpose of keeping down the voids and, therefore, diminishing the settlement of the structure. The Bakhada Dam, for example, with a height of 45 m (147.6 ft), an up-stream slope of 1 on 0.86 at the top, 1 on 1 at the bottom, and 1 on 1.25 for the down-stream slope, showed a horizontal deflection at the crown of 0.30 m (11.8 in.) and a vertical settlement of about the same magnitude, or 0.7% of the height. The rock-fill consisted of blocks with an average weight of 5 metric tons, resulting in 32% of voids. Increasing the size of the blocks to 10 tons reduced the voids to 26% as evidenced by the dams of Bon-Hanifia and Grib.

The Bakhada Dam, designed originally with a double slab of reinforced concrete and only vertical joints, the lower one with a thickness of 30 cm (11.8 in.), suffered only the small settlements mentioned, with the result that the lower slab, with the reservoir filled, showed neither cracks nor displacements. The drainage amounted to 15 liters per sec (0.5 cu ft per sec) diminishing to 0.1 liter per sec after the second slab was built. It is interesting to note that the space below the joint of the upper slab is being drained by means of small channels.

The concrete slabs of rock-fill dams should be made as elastic as possible in order to be able to follow the settlements of the rubble cushion, and the speed of placing the rock-fill should be kept down to an economic limit for the purpose of allowing the fill to settle under its own weight. Vibrating the fill and sluicing contribute to a possible increase of the speed of construction.

Ralph J. Reed,[36] M. Am. Soc. C. E. (by letter).—The successful service of the typical rock-fill dam depends upon the maintenance of an impervious facing supported by the up-stream slope of the fill. Although small settlements can be provided for in the design, any considerable settlement of this facing may result in such failure of it as to affect seriously the structure of the dam, or at least to result in excessive leakage. It seems to the writer, therefore, that such provisions should be made in the design and such measures should be taken in the construction of a rock-fill dam as will result in the larger part of the dam's settlement taking place before the permanent facing is completed. In accomplishing this result it is obvious that the character of the rock used is of primary importance. The selection of rock that is hard, sound, and does not readily disintegrate, is emphasized by the author. The usual quarry operation results in the production of rock varying considerably in size, so that, without resorting to measures which increase the cost of production, it is usually not feasible to produce or to select rocks of fairly uniform size; and it is questioned whether such selection may be at all necessary. It is believed that a grading from larger to smaller rocks, with the rejection of very small rocks and spalls, and the placing of the fill in low lifts extending completely along the length of the dam, from abutment to abutment, will be advantageous in accomplishing the larger part of the settlement of the fill as construction goes forward, and in diminishing the settlement which is to be expected when the full water load is taken and continues. Even if great care is taken to reject fine material —earth, spalls, and dust—at the quarry, a certain amount of rather fine material will be present in the fill due to chipping and breaking of the rock during transportation from the quarry and handling in the fill.

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[35] São Paulo, Brazil.

[36] Cons. Engr., Los Angeles, Calif.

Sluicing the dumped rock in order to wash this finer material into the fill and to prevent its accumulation in any particular zone, is important; and there may also be a lubricating effect from sluicing, resulting in somewhat better settlement of rocks into position upon their supporting rocks. The performance of San Gabriel Dam No. 2 of Los Angeles County Flood Control District is illustrative in this regard. This high rock-fill dam was built in 1932-1933, the fill being made "in the dry," without sluicing. The specifications provided that the rock used in the main fill should be:

> "* * * a well-graded mixture of sound, angular, quarry-run rock, forty (40) per cent of which shall vary in weight from quarry chips to one thousand (1000) pounds, thirty (30) per cent shall vary in weight from one thousand (1000) pounds to three thousand (3000) pounds, and thirty (30) per cent shall vary in weight from three thousand (3000) pounds to fourteen thousand (14000) pounds."

The rock was not to contain more than 3% of total weight of quarry dust. Although the placing of the laminated concrete slab facing of the dam was delayed until the fall of 1933, when the fill was practically completed, the sluicing effect of very heavy rainfall during the following winter months produced such large settlement of the fill and accompanying deformation of the up-stream face of the dam that the facing was seriously damaged. Subsequent sluicing during the summer of 1934 through openings drilled through the slabs resulted in further settlement, but to a lesser degree. The condition of the facing was such, however, that its complete removal became finally necessary; and the dam has since been provided with a laminated timber facing. It is believed to be clearly indicated that generous sluicing of the fill during the construction of this dam would have resulted in such settlement at that time as would have made possible the building of a concrete face and its maintenance without the failure that occurred.

F. J. SANGER,[37] ASSOC. M. AM. SOC. C. E. (by letter).—In his able summary of the historical development of rock-fill dams, the author shows that large rock-fill dams are rare outside the Western States, and it is thought that the record of such a dam in Australia which partly failed, and the measures taken to remedy the fault would be of interest. Rupert Grenville Knight[38] describes the case fully, and only a very brief précis of his paper is given.

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[37] Head, Dept. of Eng. and Building, The Lester School and Henry Lester Inst. of Technical Education, Shanghai, China.

[38] "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon Reservoir, Australia," by Rupert Grenville Knight, *Journal*, Inst. of Civ. Engrs., March, 1938.

The Eildon Reservoir and Dam were completed in 1927. The dam, including a spillway section (734 ft), is 3 529 ft long. It partly failed in 1929, after the water level had been lowered 46.5 ft, and remedial measures required six years and cost the equivalent of $2 000 000.

The core-wall is of reinforced concrete, 137 ft high, and from 6 ft to 2 ft thick; it is reinforced with 0.5-in. bars at 12-in. centers in grids and has expansion joints at 50-ft intervals. The site is composed of variable shales with some slates and sandstones, with about 20 ft of clay above for two-thirds of the length of the dam. On the up-stream side of the wall a mass of clay was deposited, of base thickness from 27 ft to 37 ft and with a batter of 1 on 6. The rock-fill is of material similar to the bed-rock and weighs 90 lb per cu ft. The slopes were 1 on 2.

Failure began with a slip, exposing the core-wall as well as deflecting it for a length of 700 ft in the middle of the dam, and then extended to a 1 200-ft width when the top 26 ft of the wall was exposed and the fill moved 55 ft into the reservoir; subsidence continued slowly for some weeks afterward, and the maximum wall deflection was 4 ft 8 in., causing many serious cracks. Borings showed that the clay had been squeezed out into the fill. In the discussion of the paper several theories were advanced for the failure, and the most probable reason seems to have been a clay slip in the foundation material.

Remedial measures included adding rock-fill on the toe to prevent sliding and extra deflection of the wall, repair of cracks, drainage, and the reconstruction of the spillway, outlets, etc. In addition, the frictional resistance of the down-stream side was increased by adding rock-fill and by under-drainage. An additional 700 000 cu yd of fill were needed, making the total in the dam 2 000 000 cu yd.

C. S. Jarvis,[39] M. Am. Soc. C. E. (by letter).—Examination of rocky bars intruding into or across stream channels will disclose the fact that these are rock-fill dams with slopes naturally adjusted to withstand over-flow. Many of the rapids prevailing in torrential stream channels are only the down-stream faces of such natural barriers. As might have been expected, such sites have been chosen for intakes of power or other diversion canals. Favoring such locations are the ease and effectiveness with which the current can be deflected by merely piling stones to form wing-walls, then stowing brush, straw, or litter to close the interstices; both initial construction cost and cost of maintenance were reduced thereby.

The development of the basic idea of rock-fill dams may be traced through many centuries of agricultural practices, particularly in relation to terracing on hillside slopes. Dry rubble walls with pronounced batter provided the required stability; rock spalls or stony soil strewn against the inner face of the wall formed both a matrix to hold the stones in place, and a porous screen to retain the fertile soil while permitting adequate drainage.

As the need became apparent for conserving nearly all rainfall in place on these terraces, to bridge over the periods of deficiency so far as practicable, the terrace walls were raised and trench-like pockets similar to those provided by modern contour listing were formed to increase water storage capacity. To prevent wholesale loss of stored water through overtopping and drainage from one basin to another, the terraces were divided transversely into sections of moderate length, each with considerably greater capacity than that ordinarily required to take care of local rainfall and resulting pondage.

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[39] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

Other progressive steps in the use of rock-fill dams provided mill ponds and stock-watering ponds, and larger storage basins behind either combination rock-fill and earth dams, or rock-filled timber cribs. Where suitable soil for earth-fill was scarce or required long haulage, and durable rock was abundant, the natural result was a puddled core in an earthen fill, retained between loose rock shoulders, consisting of either dry rubble walls or more voluminous fills of quarry-run stone dumped to natural slope.

There are in existence to-day, in widely scattered locations, within the United States and neighboring countries, examples of minor rock-fill dams with either earth cores or earth aprons to serve as the relatively impervious element. Among those stock-watering, farm, mill, or wild-life ponds located in regions subject to accelerated erosion, sedimentation has reduced storage volume and necessitated expensive maintenance or enlargement from time to time—or ultimate abandonment. Many of these ponds antedate the legislation requiring approval by the State Engineer's Office; some even date from early colonization by Europeans; but they have served as important factors in the settlement and development of American frontiers.

The writer has never ceased to wonder at the volume of dry rubble and other stone masonry construction, backed by rock-fill, or by a combination of earth and rock, along the network of navigation canals, logging channels, dams, and spillways of the last century or more, including the canal constructed by George Washington around Great Falls of the Potomac River, near Washington, D. C. Some of these structures are still in service, and many others could be restored to working condition at moderate cost, if sufficient demand arose for such uses as they originally served. However, it seems improbable that the nation will ever revert to such extensive projects to serve slow methods of transportation, now that rapid transit seems to be the main objective and has been so widely attained.

There is ample opportunity for future use of low rock-fill dams on minor projects. For greater storage depths the factors of safety should likewise increase, because of the danger of high velocities and sudden widespread destruction as a result of overtopping or failure from any other cause.

Advancement in the art of dam construction has apparently awarded preference to some form of cohesive integral masonry structure for great storage depths. In order to displace masonry structures from what is admittedly their own province, as may doubtless be advisable at times, proponents of the competing type should be required to show overwhelming advantages in first cost, maintenance, or both, with no sacrifice of necessary safety factors. Small differences should be waived in favor of the type which could withstand over-flow and could continue to function even in cases of catastrophic floods, far beyond the probable limits as contemplated by the design.

The position of the water-tight element of a rock-fill dam may be anywhere within the limits of the up-stream face, in the writer's opinion (see Fig. 25), provided the design of cross-section conforms to the position so chosen. The pavement, diaphragm, or apron used as a water-seal may or may not be parallel to the up-stream surface. A core-wall, if installed at all, should stand vertically with either stepped or battered faces to account for thinning toward the top. Certainly, a well-constructed core-wall with its footings properly established in bed-rock, and the superstructure well reinforced against cracking, sandwiched between the up-stream third and the down-stream two-thirds of the entire fill, should be a satisfactory water-seal, if properly encased or cushioned against irregular concentrations of pressure (as from large, angular boulders during the inevitable settlement and compaction of materials). Likewise, an equal volume of masonry or durable mastic material, spread over the greater area presented by the up-stream slope; or well-fabricated sheet metal in a similar position, but protected from both corrosion and abrasion by suitable covering, may satisfactorily serve the same purpose, and may both permit and require inspection, repair, or replacement more frequently than the core-wall.

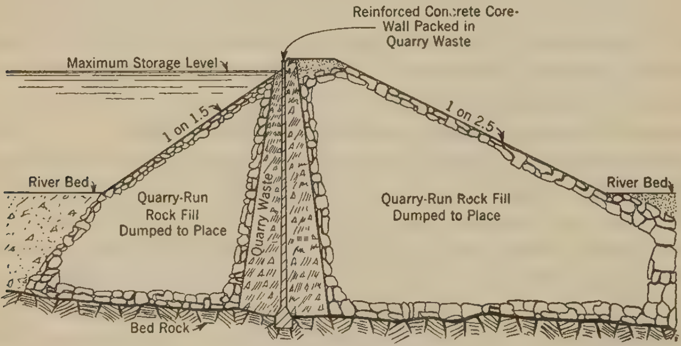
Reinforced Concrete Core-Wall Packed in Quarry Waste
Maximum Storage Level
1 on 1.5
1 on 2.5
River Bed
River Bed
Quarry-Run Rock Fill Dumped to Place
Quarry Waste
Quarry-Run Rock Fill Dumped to Place
Bed Rock

FIG. 25

The author has contributed much of both historical and scientific interest relating to the design of rock-fill dams and to fundamental principles, considerations, and requirements influencing the final choice of structural type for a given location. His definition of rock-fill dams departs somewhat from that accepted in general practice, in that it would apparently exclude such structures as depend on either puddle cores or earth aprons as the impervious face. Thus, his definition would exclude the Castlewood Dam (Table 1, Item No. 5), which had an "earth embankment added to upstream face, 8 feet wide at crest with 3:1 slope faced with 12-inches of riprap."[40] Likewise the Avalon Dam, in New Mexico, both the Milner and Minidoka Dams, in Idaho, and the Zuni Dam, in New Mexico, listed as "rock-fill,"[41] would have been excluded by the author's restrictive definition. Moreover, it would exclude a number of rock-fill dams with puddled earth cores described by the writer[41] in 1915.

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[40] American Civil Engineers' Handbook, Fifth Edition, 1930, p. 1547.

[41] "Provo Reservoir Company's Irrigation Project," by C. S. Jarvis, *Engineering News*, August 26, 1915, p. 394.

Whether the impervious element is in the form of timber, metal plate, masonry, or dense plastic material used, respectively, as facing or as diaphragm core, cut-off, or up-stream apron, a structure dependent on a loose rock-fill for stability seems to qualify as a rock-fill dam. When the earth-fill for either the core-wall or the up-stream apron comprises a volume nearly equal to that represented by the rock-fill, it would appear fitting to include mention of this material in the designation, as "earth and rock-fill dam." However, when the earth apron represents only a minor percentage of the total volume, and is placed mainly to increase the resistance to seepage through bed-rock and abutment formation, by adding to length of travel and thus reducing the pressure gradient, the structure may still be designated as a rock-fill dam. There would be fully as great justification for mentioning the timber, the steel plates, the concrete facing, or the masonry core-wall as to mention the earth-fill used for the same specific purpose, namely, as a seal against seepage.

L. R. East,[42] M. Am. Soc. C. E. (by letter).—Owing to his association with an inquiry into the causes of the partial failure of a large rock-fill dam at Eildon Reservoir, Victoria, Australia, in 1929, the writer has found this paper of particular interest. This dam comprised a rock-fill embankment extending for a distance of 2 525 ft across a comparatively wide valley to a mass concrete spillway of the side-spillway type providing for overflow along a weir section, 682 ft long.

In the river section abutting the concrete spillway, the embankment is approximately 140 ft high, and the rock-fill is founded on bed-rock which is composed of fairly hard metamorphic rock with sandstones and slates dipping almost vertically and with strike diagonally across the line of the dam. Across the river flats, the rock-fill was placed directly upon the natural surface, with from 22 to 24 ft of clay and gravel wash and about 2 ft of sand overlying the bed-rock. On this side of the valley the rock available for fill was shales and mudstones of a less durable character, with bands of slate.

For water-tightness a central vertical core-wall of reinforced concrete was provided, with a clay wall on the up-stream side to assist in staunching any possible leakage through the concrete. This wall varied in thickness from 6 ft at bed-rock (into which it was keyed to a depth of 5 ft) to 2 ft at crest level. The clay wall, of very stiff puddle trafficked with horses and drays during construction, averaged about 30 ft in thickness at the natural surface.

The rock-fill was quarried on both ends of the dam, loaded by hand on trucks (the stone being broken to "one-man" size) and side-tipped into position. The stone from the river quarry was comparatively hard and durable, but that from the other quarry was soft, easily crushed, and weathered rapidly.

Owing to financial difficulties, construction was slow, and the embankment was not brought to its designed height until practically twelve years after the commencement of the fill. At no time, however, did construction actually cease. There was time, therefore, for considerable settlement and consolidation during this period, particularly as water was stored to as high a level as possible during construction.

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[42] Chairman, State Rivers and Water Supply Comm., Melbourne, Victoria, Australia,

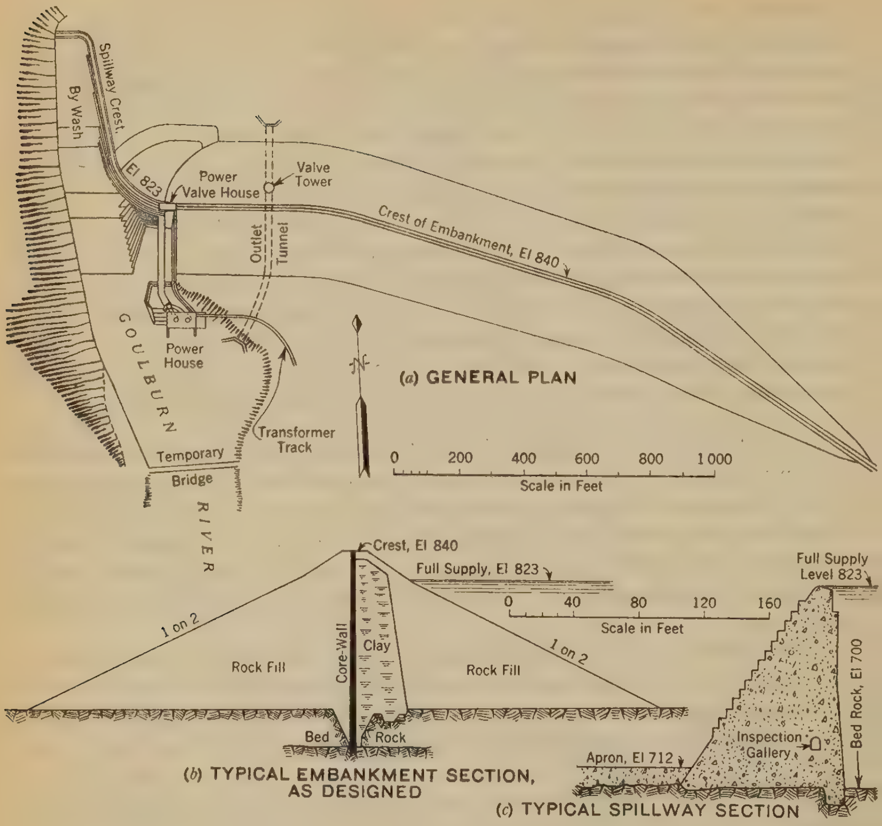
Spillway Crest
By Wash
El 823
Power
Valve House
Valve
Tower
Outlet
Tunnel
Crest of Embankment, El 840
Power
House
GOULBURN
RIVER
Transformer
Track
Temporary
Bridge
(a) GENERAL PLAN
0 200 400 600 800 1 000
Scale in Feet
Crest, El 840
Full Supply, El 823
0 40 80 120 160
Scale in Feet
1 on 2
1 on 2
Rock Fill
Rock Fill
Core-Wall
Clay
Bed
Rock
(b) TYPICAL EMBANKMENT SECTION, AS DESIGNED
Full Supply
Level 823
Bed Rock, El 700
Inspection
Gallery
Apron, El 712
(c) TYPICAL SPILLWAY SECTION

FIG. 26.—TYPICAL CROSS-SECTIONS, EILDON RESERVOIR

Fig. 26 shows the plan and typical cross-sections of embankment and spillway as designed, the basic data being as follows:

Description		Quantity
Height of bank above river bed, in feet		140
Height of spillway above river bed, in feet		123
Length of core-wall, in feet		2 300
Length of spillway crest, in feet		682
Storage capacity, in acre-feet		306 400
Area submerged, in acres		8 000
Length of waterway, in miles:		
Delatile River	11.5	
Goulburn River	10.5	
Total		22
Length of water frontage, in miles		70

Fig. 27 shows a typical cross-section of the embankment as completed in 1927 when the reservoir was filled to its full capacity of 306 000 acre-ft for the first time in August of that year. It was not drawn down very far the next year and in the spring of 1928 it was again filled to capacity, the water flowing over the spillway to a depth of 4 ft 2 in. in October. The irrigation season then resulted in a heavy demand on the storage and in four months, to April 1929, the reservoir was drawn down to the extent of 50 ft.

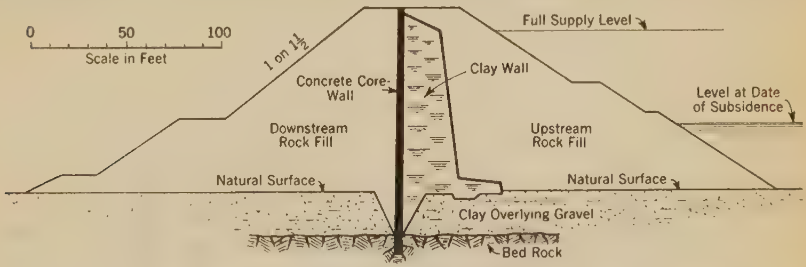
0
50
100
Scale in Feet
1 on 1½
Concrete Core-Wall
Clay Wall
Full Supply Level
Level at Date of Subsidence
Downstream Rock Fill
Upstream Rock Fill
Natural Surface
Natural Surface
Clay Overlying Gravel
Bed Rock

FIG. 27.—TYPICAL CROSS-SECTION OF EMBANKMENT, EILDON RESERVOIR, AS COMPLETED IN 1927

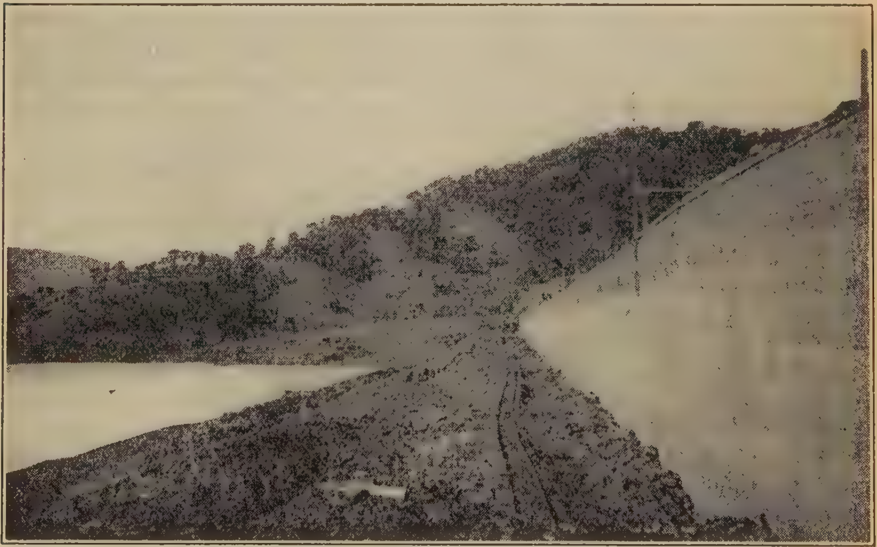

FIG. 28.—VIEW OF EXPOSED CORE-WALL, JUNE 16, 1929

At that stage, without warning, the rock-fill on the up-stream side of the core-wall subsided suddenly and the concrete core-wall itself was exposed over a length of about 1 200 ft to a maximum depth of 26 ft (see Fig. 28). The greatest subsidence occurred against the core-wall itself which was deeply scored or scratched by the rock-fill sliding against it. The exposed top of the core-wall showed deflections of as much as 4 ft 8 in. in lengths of 400 ft and 800 ft originally constructed as straight lines.

Fig. 29 shows the line of the core-wall as constructed, its position in September, 1929, after the subsidence, and its position in 1933, since when movement has been very slight—the movement during the 5 yr since 1933 being less than 2 in. at the point of maximum deflection. Fig. 30 shows a typical cross-section of the embankment after the subsidence, the position of the clay wall having been determined by borings which enabled it to be located within a few inches. Samples showed the clay to be in excellent condition and to have the consistency of very stiff putty into which a pencil could be forced by hand only with difficulty. The moisture content was about 23 per cent.

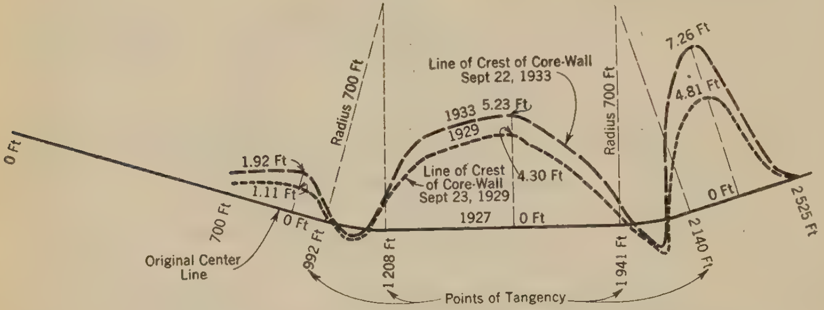

FIG. 29.—DIAGRAM SHOWING LATERAL DEFLECTION OF CORE-WALL, EILDON RESERVOIR

There appeared to have been little or no movement of the foundation clays and gravels upon which the rock-fill had been built. The exposed core-wall showed several vertical cracks, notably at the junction with the massive wing-wall at the river end, and later investigation showed it to have fractured horizontally for some distance along its junction with the rock foundation. There was, however, practically no leakage through the horizontal crack and very little through the vertical cracks which were fairly easily staunched.

The Eildon Weir Inquiry Board was appointed by the Governor in Council on May 24, 1929, to report on the failure. After careful investigation the Board concluded that the subsidence had arisen directly as a result of the action of the clay wall; that the clay had acted as a stiff fluid and pushed the rock-fill out into the water; and, at the same time, had exerted great pressure down stream causing the concrete core-wall to deflect. Although there was little leakage through the concrete core-wall, the clay underlying the down-stream rock-fill was very wet, and as the rock-fill was much lighter than an earth bank (being only 90 lb per cu ft) there was considerable apprehension as to the risk of the entire structure failing by sliding of the down-stream part of the bank. The remedies suggested by the Board were:[43]

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[43] Rept. of the Eildon Weir Inquiry Board, 1929, p. 15, Govt. Printer, Melbourne, Victoria, Australia. (This report may be examined at Engineering Societies Library, 33 West 39th Street, New York, N. Y.)

"(*a*) To restore the subsidence of rock-fill on the up-stream side by deposition of more loose rocky material to provide for greater weight and stability.

"(*b*) To provide drainage under the base of the down-stream side of the embankment, including a tunnel to carry off such water as may pass through the core-wall, and to add a large quantity of rock-fill to this part of the embankment for the purpose of increasing its supporting power.

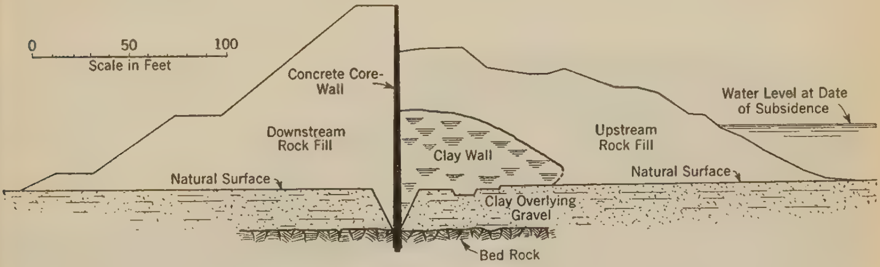
0
50
100
Scale in Feet
Concrete Core-Wall
Downstream Rock Fill
Clay Wall
Upstream Rock Fill
Water Level at Date of Subsidence
Natural Surface
Natural Surface
Clay Overlying Gravel
Bed Rock

FIG. 30.—TYPICAL CROSS-SECTION OF EMBANKMENT, EILDON RESERVOIR, AFTER SUBSIDENCE, APRIL, 1929

"(*c*) To repair all cracks of importance in the concrete core-wall and to provide, where necessary, staunching material on its up-stream side in place of the clay formerly in position for that purpose."

The Board also made recommendations in regard to outlet works and spillway, but the writer proposes to confine his discussion to the rock-fill dam. Fig. 31 shows the additional rock-fill proposed by the Board.

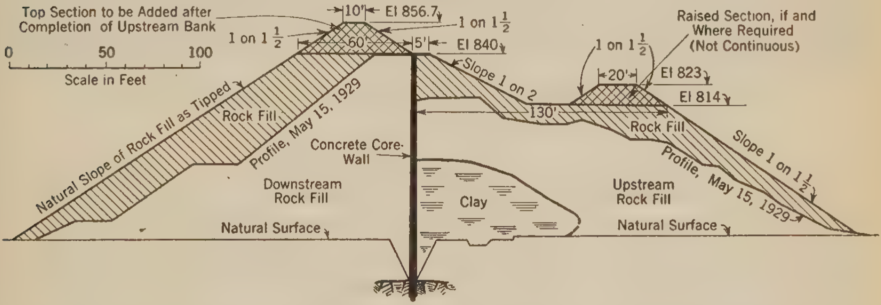
Top Section to be Added after Completion of Upstream Bank
0
50
100
Scale in Feet
10'
El 856.7
1 on 1½
1 on 1½
60'
5'
El 840
Slope 1 on 2
Raised Section, if and Where Required (Not Continuous)
1 on 1½
20'
El 823
El 814
130'
Natural Slope of Rock Fill as Tipped
Rock Fill
Profile, May 15, 1929
Concrete Core-Wall
Rock Fill
Profile, May 15, 1929
Slope 1 on 1½
Downstream Rock Fill
Clay
Upstream Rock Fill
Natural Surface
Natural Surface

FIG. 31.—TYPICAL CROSS-SECTION SHOWING ADDITIONAL ROCK-FILL PROPOSED BY BOARD, EILDON RESERVOIR

All the remedial measures proposed by the Board have been completed at a cost of £A380 000 (Australian pounds = $1 500 000), approximately £A206 000 ($825 000) was for the placement of additional rock-fill, and £A36 000 ($144 000) for the drainage of the foundation under the down-stream section of the embankment. The remainder of the expenditure was on outlet works and spillway.

Regular observations have been made to detect bank movements or deflections of the core-wall, as well as of outflow from the drainage system; and there has been little movement since the completion of the remedial works in 1933. Fig. 32 shows the dam after the completion of the remedial works.

It might be mentioned that the total leakage through the entire structure (in which there is an area of 280 000 sq ft of core-wall) does not exceed 50 gal per min, three-fourths of which comes from one definitely located small fracture at the junction of the core-wall and the outlet tunnel.

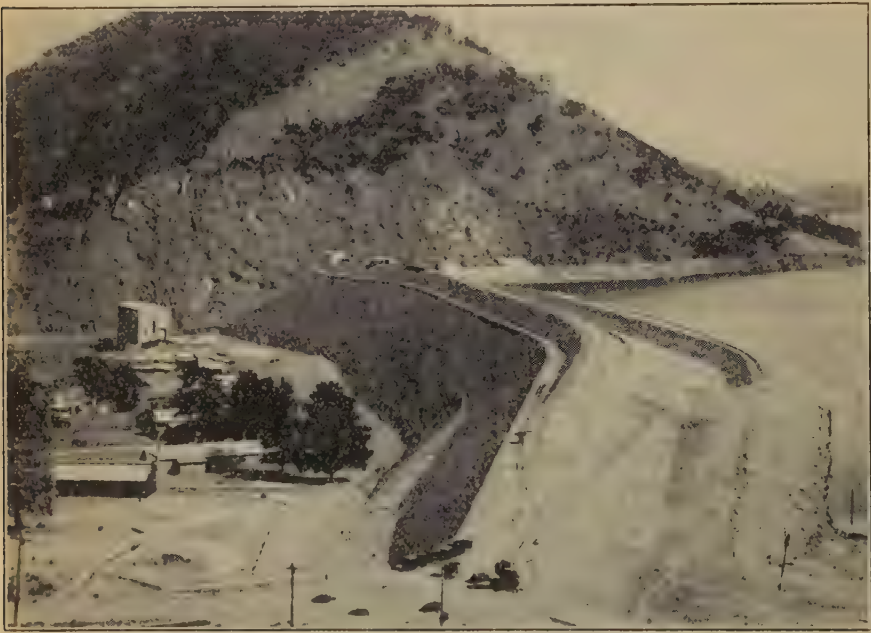

FIG. 32.—VIEW OF EILDON RESERVOIR AFTER REMEDIAL WORKS HAD BEEN COMPLETED. FEBRUARY, 1936

The writer is in complete agreement with the principles of design indicated by the author, departure from which was undoubtedly responsible for the failure at Eildon. The dam was not founded on rock, much of the material of the fill was unsuitable in that it was likely to crush and disintegrate, and the up-stream half of the embankment was made a liability instead of an asset by placing the impervious element in the center of the dam. Experience at Eildon shows that the use of a thick blanket of puddled clay, loaded with rock to protect, and confine it, might have serious consequences unless steps are taken to provide for the enormous pressures exerted by clay in the mass.

A rock-filled dam very similar in design to that at Eildon was constructed at Melton, Victoria, on the Werribee River, in 1916, and has given very little trouble. The embankment of this dam, however, was constructed of harder and more durable stone (basalt from the spillway excavation) and was founded on a rock foundation in a comparatively narrow gorge, so that the resistance against sliding of the up-stream part of the rock-fill embankment would be much greater than in the case at Eildon Reservoir.

A comprehensive description of the failure at Eildon and of the remedial measures to the embankment, spillway, and outlet works, has been published by Mr. R. G. Knight.[44]

FRANCISCO GOMEZ-PEREZ,[45] ASSOC. M. AM. SOC. C. E., AND MIGUEL JINICH,[46] ESQ. (by letter).—The very complete paper presented by Mr. Galloway on the design of rock-fill dams is based on the historical development of this type of structure. Limiting the scope of the historical summary he presents details of the construction of various dams of the rock-fill type that have been built during the last half century, mainly in the western part of the United States.

Any type of structure must necessarily be adapted to the conditions prevailing in the region where it is to be built and the rock-fill type of dam is no exception to the rule. Conditions in Mexico make low-cost labor available and, at the same time, make equipment that is generally imported a high-cost item of the construction project. These two conditions combined have influenced the development of the rock-fill type of dam in Mexico. In general, the trend has been toward using smaller rocks than would have been the case if mechanical equipment could have been used to a greater extent.

As a result of the use of smaller rocks that necessarily have a larger percentage of voids, other parts of the structure have also been affected and, therefore, the size of the panels that constitute the up-stream slab has been greatly reduced in comparison with the size generally accepted in the United States. Furthermore, and considering the possibility of seismic disturbances, a laminated type of slab has been included in one of the Mexican rock-fill dams completed in 1933. Other elements of current practice in Mexico are presented in this discussion.

General Considerations.—In sites that are far away from roads and railroads, for which the transportation of manufactured products, such as cement, steel, and timber, is very expensive—as well as those sites where such manufactured products are relatively expensive, but where labor is relatively cheap—the most economic type of dam is generally that in which the engineer can use to maximum advantage the materials found on the site. Such is the case of earth and rock-fill dams, and the combination of both. This explains why the rock-fill type has been very advantageous for many sites in Mexico. The Mexican National Commission of Irrigation has made comparative studies of various types of dams for several sites and has adopted the rock-fill type; and it has been built successfully both from the economic and from the technical point of view.

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[44] "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon Reservoir, Australia," by R. G. Knight, *Journal*, Inst. C. E., March, 1938.

[45] Chf., Control Div., Designing Dept., National Comm. of Irrig., Govt. of Mexico, Mexico.

[46] Chf. Designer, Designing Dept., Mexican National Comm. of Irrig., City of Mexico, Mexico.

The following rock-fill dams have been completed in Mexico: Charcas Dam, 49 ft high, in the State of Guanajuato in 1933; Taxhimay, 137 ft high, and Madero, 153 ft high, both in the State of Hidalgo (the former in 1934 and the latter in 1938); San Ildefonso Dam, 189 ft high, in the State of Querétaro, designed in 1937 (under construction in 1938); and the Tepuxtepec Dam, 126.5 ft high, in the State of Michoacán, in 1934.

Foundation Requirements.—There is no doubt that sound, durable rock is the ideal material for the foundation of a rock-fill dam, as for any other types; but, in certain cases, it is necessary to use this type at sites where the rock may not be so sound. In the latter case, the site should fulfill the following requisites:

(1) Even when saturated, the material on which the dam is to be founded must be capable of resisting all the load transmitted to it both by the fill and by the hydrostatic pressure when the dam is filled, without any appreciable settlement. The saturated condition must be considered even if it is assumed that all precautions must be taken to prevent filtration, by means of cut-off walls and an up-stream slab.

(2) The material must resist chemical solution and all tendencies to become eroded.

The Charcas Dam had been designed as a gravity structure, but for economic reasons (and mainly due to the character of the foundation rock) it was decided to change the design to a rock fill. The rock was a rhyolite in an advanced state of disintegration that did not furnish the necessary support for a gravity dam; but it complied with the aforementioned requirements for a rock-fill dam.

The Taxhimay Dam (shown in Fig. 33) was a small rock-fill structure that had been built in 1918. When it was taken over by the Commission, in 1927, to be used as a reservoir for one of its irrigation districts the engineers decided to enlarge its capacity by increasing the height of the structure, and it became necessary to study the problem relative to including the old structure within the section of the new one. The old dam was 61 ft high, had been built originally without the necessary precautions for cleaning the river bed perfectly, and the material that had slid down the side-hills had not been completely removed before the original dam was built. The old dam consisted of a loose rock-fill with a down-stream slope of 1 on 1, and up-stream slope of 1 on 2. The impermeable part consisted of a cement-bound masonry zone 6.5 ft thick at the bottom and 1.33 ft thick at the crown, and resting directly on the loose rock-fill. The defects of the original design, together with the foundation defects, caused severe settlements that almost made the structure fail.

In designing the new dam, for the purpose of raising the water surface elevation to a new height of 134 ft it was necessary to remove all the silt and loose materials and to deposit the new fill on the sound solid rock. This structure includes the old loose rock-fill dam and has been finished and in service since 1934 with very satisfactory results.

At the site of the Madero Dam, in the State of Hidalgo, the rock is also sound. It has been classified as a very resistant rhyolitic tuff in the river bed but the resistance gradually diminishes as the elevation increases. When the preliminary design was being prepared, it had been decided to use an arch dam, but on account of the poor resistance of the rock at the higher elevations in the site it was decided to build a rock-fill dam.

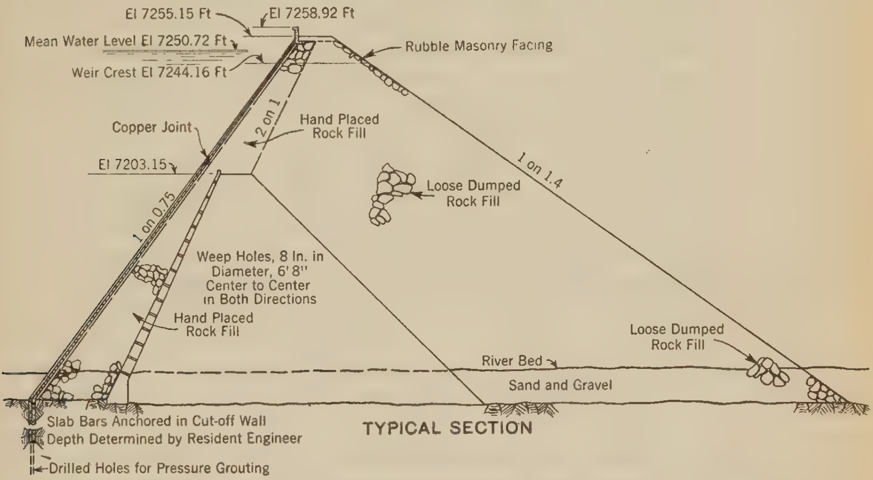

El 7255.15 Ft
El 7258.92 Ft
Mean Water Level El 7250.72 Ft
Weir Crest El 7244.16 Ft
Rubble Masonry Facing
Copper Joint
2 on 1
Hand Placed Rock Fill
El 7203.15
1 on 1.4
Loose Dumped Rock Fill
1 on 0.75
Weep Holes, 8 In. in Diameter, 6' 8" Center to Center in Both Directions
Hand Placed Rock Fill
Loose Dumped Rock Fill
River Bed
Sand and Gravel
Slab Bars Anchored in Cut-off Wall
Depth Determined by Resident Engineer
Drilled Holes for Pressure Grouting
TYPICAL SECTION

FIG. 33.—CROSS-SECTION OF TAXHIMAY DAM, STATE OF HIDALGO, MEXICO

The Material.—Almost always the material for the construction of rock-fill dams comes from near-by quarries. It may be of volcanic origin, it may be granitic, or it may be either sandstone, or limestone, but the main requirement is that it be sound.

The size of the rock used varies from about 4 in. to the largest size that is feasible to move with the equipment available. It is always better to use rocks of the largest dimensions possible, and to limit the volume of small fragments such as quarry spalls, to a quantity that partly fills the voids and is not so large as to prevent easy drainage through the rock mass, nor to favor local concentrations of the material which, because of the larger percentage of voids, may cause important settlements.

It is evident that water-jets make it possible to use a larger proportion of this quarry-run material, depending on the methods of placing the loose rock-fill. It is also evident that water causes the material to be slushed into the voids between large rocks, without endangering the easy drainage of the mass. It may be possible even to distribute the large rocks better by the use of water-jets. .

Cross-Section.—The down-stream slope adopted in the dams built by the Commission has been 1 on 1.4, slightly steeper than the natural average slope for the loose rock-fills. Of course, this slope may vary within limits depending on the size of the rock used and mainly on the degree of arrangements obtained for the rock-fill.

In the aforementioned dams, in view of the low cost of labor, and in order to give them a better appearance, a "finish" layer has been built on the downstream slope. In the Charcas Dam it consists of the formation of steps, and, in the others, in leaving a more or less smooth rubble masonry surface. This finish improves the appearance of the dam greatly at a relatively low cost.

The main function of the hand-placed rock-fill zone is to serve as a semirigid transition section between the outermost layer and the loose rock zone, as the latter is subject to uneven settlement. It gives the outer layer a smooth surface that corrects the irregularities of the loose rock-fill. It also stabilizes the slopes by using them as retaining walls to hold back the loose rock-fill making it possible to use such steep slopes as 1 on 0.75. Such an arrangement (which is recommended by several authorities) seems to offer some economy, but in Mexico this has not been the case on account of the limitations of the equipment generally available for moving rocks of large dimensions.

The original design of Charcas Dam included an up-stream slope of 1 on 0.75 with a zone of hand-placed rock. This zone varied in thickness from 61 ft at the base to 6 ft at the top. After construction had begun it was realized that it was not economical to build the zone of hand-placed rock-fill, which was almost a dry masonry, with 20 to 25% voids. An economic comparative study was made to determine the actual cost of a zone of hand-placed rock-fill, fulfilling all the aforementioned specifications, and the cost of making the slope less steep, adopting a ratio of 1 on 1.25. This was the average natural slope for a rock-fill that would not require perfect work to make it act as a retaining wall. It needed much less thickness, therefore, thus becoming reduced to a nearly superficial layer placed on the slope only for the purpose of supporting the slab.

The dam was built with this slope (1 on 1.25) without any particular attention being paid to the hand-placed rock zone, thus making it possible to obtain additional economy in the pouring of the concrete, because it was placed without the use of forms, and by using a concrete with s slump of 0.5 in. to 1 in.

The difficulties that were met in the construction of the hand-placed rock-fill zone at the Charcas Dam were attributed to the lack of equipment and it was on that account that Taxhimay Dam was built with an up-stream slope of 1 on 0.75 as designed, with a thick mass of hand-placed rock-fill (in which large rocks were used) but with a small percentage of voids. The construction of this type was considered economical because of the possibility of using steam-shovels, derricks, and railroad spurs that were available for its construction. However, it was also difficult to use this design since its cost threatened to be very great if it was to be built in accordance with all the required specifications; and, furthermore, it also required unusually high-class and permanent inspection. As a result of the foregoing experience the up-stream slope for the Madero Dam was made 1 on 1.2.

For the San Ildefonso Dam the slope adopted was 1 on 1.4 both up stream and down stream. When construction was begun it was foreseen that a large portion of the material available in the quarries would be wasted, because it consisted of alternate layers of sound basalt and very finely crushed basaltic breccia. This condition dictated a change in the design (see Fig. 34) and it was decided to build the dam with a central zone to include the material that otherwise would have been wasted; the specifications required that this material be perfectly wetted and rolled, which seems to be in accordance with the tendency shown in the latest rock-fill dams built in the United States, such as Salt Spring and San Gabriel No. 2, both in California.

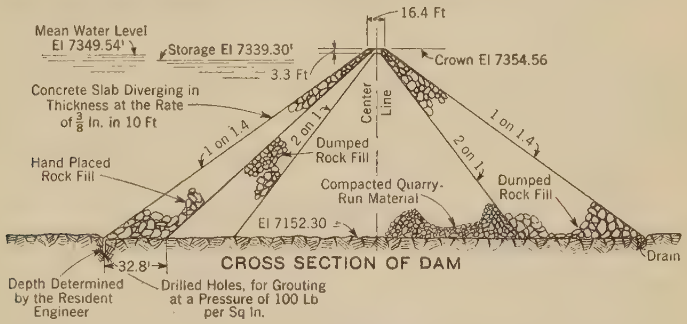

16.4 Ft
Mean Water Level
El 7349.54'
Storage El 7339.30'
3.3 Ft
Crown El 7354.56
Concrete Slab Diverging in Thickness at the Rate of 3/8 In. in 10 Ft
1 on 1.4
2 on 1
Center Line
Dumped Rock Fill
Hand Placed Rock Fill
Compacted Quarry-Run Material
2 on 1
1 on 1.4
Dumped Rock Fill
El 7152.30
Drain
32.8'
CROSS SECTION OF DAM
Depth Determined by the Resident Engineer
Drilled Holes, for Grouting at a Pressure of 100 Lb per Sq In.

Fig. 34.—Cross-Section of San Ildefonso Dam, near San Juan del Rio, State of Queretaro, Mexico

An exception to the aforementioned examples of slope is found in Tepuxtepec Dam, with slopes of 1 on 0.7 up stream and 1 on 0.8 down stream. This entire dam is practically a dry massonry structure with a percentage of voids varying between 20 and 30. It was built by the Southwest Mexico Power Company, and required masonry labor as skilled as would have been required if it had been built as an ordinary masonry structure. Large rocks transported by cableway were used, each individual piece being hand-placed; and all the voids were filled with spalls.

The cost was greater than would have been the case if the dam had been built with flatter slopes and with most of the rock dumped in place (except a thin zone of hand-placed rock to support the up-stream slab). The type was adopted because of the danger of earth shock in that region. It must be stated that the results obtained at the Tepuxtepec Dam have been splendid, and that no settlement has been observed.

Water-Proof Element.—Modern practice requires the construction of a water-proof element for rock-fill dams that consists of a rather thin concrete slab placed on the up-stream slope. In some rock-fill dams, a steel plate has been used instead of the concrete slab, but this has been the exception.

There is no doubt that the most important problem encountered in the design of the impermeable slab is to make it as water-proof as possible, and, at the same time, sufficiently flexible to follow the unavoidable settlements that occur in this type of dam, without excessive cracking. Just as difficult of solution are the problems relative to the joint at the place where the flexible slab connects with the rigid cut-off walls. It is also important to foresee the changes in length to which the slab may be subject due to changes in temperatures.

The type of slab built in sections, separated by elastic joints, seems to be the most satisfactory solution, and it is the one that has been adopted by the

Mexican National Commission of Irrigation. The up-stream element of the Taxhimay Dam consists of panels about 15 ft wide by 24 ft to 32 ft long, measured on the slope and connected by copper-plate contraction joints. The steel reinforcement placed on the center of the slab does not cross the vertical joints, but it does cross the horizontal joints in order to act as articulating units in the respective columns or strips in which the slab as a whole is divided (see Fig. 35).

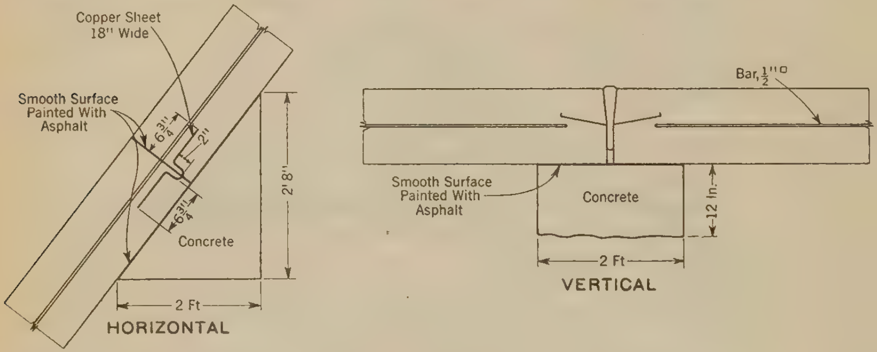


FIG. 35.—TYPICAL SECTIONS OF PANEL FACINGS, TAXHIMAY DAM

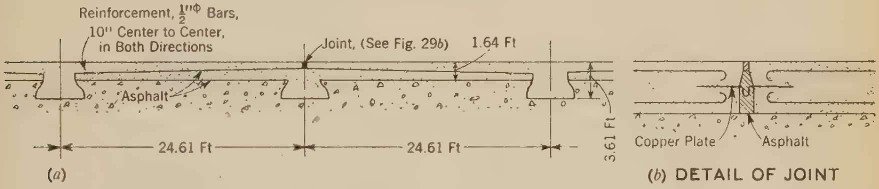
Whereas the tendency in the United States seems to be to divide the up-stream slab into rather large panels (about 50 ft or 60 ft square), the Mexican National Commission of Irrigation has adopted much smaller panels, even smaller than the ones used at Taxhimay, in order to make them more flexible, at the same time keeping them within economic limits.

At Madero Dam the panels were built 13 ft wide and 16 ft long and at San Ildefonso Dam they were made 10.5 ft by 17.5 ft. For these two structures neither the horizontal nor the vertical reinforcement crosses the copper-plate elastic joints. The size of the panels was determined by selecting the size of the pre-fabricated forms that would be simplest to build and move around on the slope.

As an exception to this rule Charcas Dam may be cited, in which, because the structure was low and long, and was built by a method that guaranteed a minimum of settlement, the slab was placed in alternate strips about 15 ft wide, leaving only construction joints through which the reinforcing steel placed in the central part of the slab was extended. The dam was inspected about two years after it was built, and it was observed that the joints had opened slightly, forming very fine cracks which, however, do not endanger the stability of the dam.

To make the up-stream slab even more flexible, in some dams it has been built in the form of thin slabs or layers of concrete placed one on top of the other so that the joints are staggered. The most interesting example of this type in Mexico is the Tepuxtepec Dam, in which two layers were used, each being 14 in. thick at the base and 10 in. thick at the top. The slabs were built in 50-ft strips, continuous from the river bed to the crown, and the joints between strips on the upper layer were provided with a copper, water-tight seal (see Fig. 36).

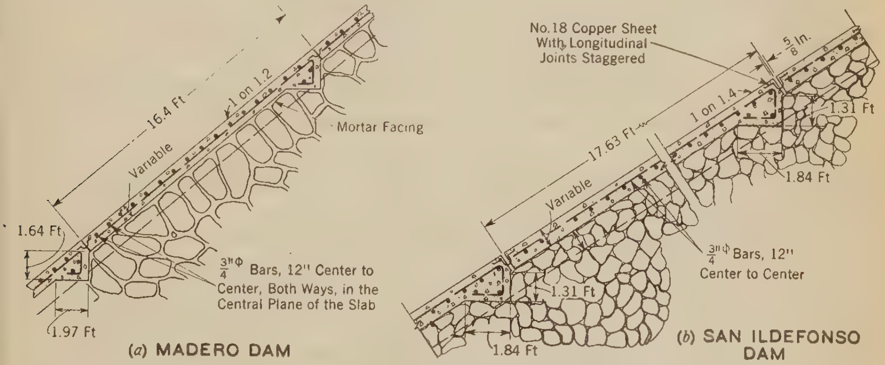
An improvement in this laminated type was also used at the Cogoti Dam in Chile. It is believed that the idea is fundamentally sound; but the cost is rather high and it is justified, therefore, only in special cases where important settlements are expected or where there is a possibility of earth movements, such as is the case at Tepuxtepec Dam, in Mexico, or at Cogoti Dam, in Chile.


Reinforcement, ½"Φ Bars,
10" Center to Center,
in Both Directions
Joint, (See Fig. 29b)
1.64 Ft
Asphalt
24.61 Ft
24.61 Ft
3.61 Ft
Copper Plate
Asphalt

(a) (b) DETAIL OF JOINT

FIG. 36.—REINFORCED CONCRETE FACING, TEPUXTEPEC DAM

Thickness and Reinforcement of the Up-Stream Slabs.—The up-stream slabs in the dams built by the National Commission of Irrigation (see Fig. 37) are designed with a thickness of 8 in. at the top, with a batter of ⅛ in. for every 40 in. measured downward along the up-stream slope. The reinforcement is designed for temperature variations and consists, in general, of a centrally located grid of rods placed in the center of the slab.

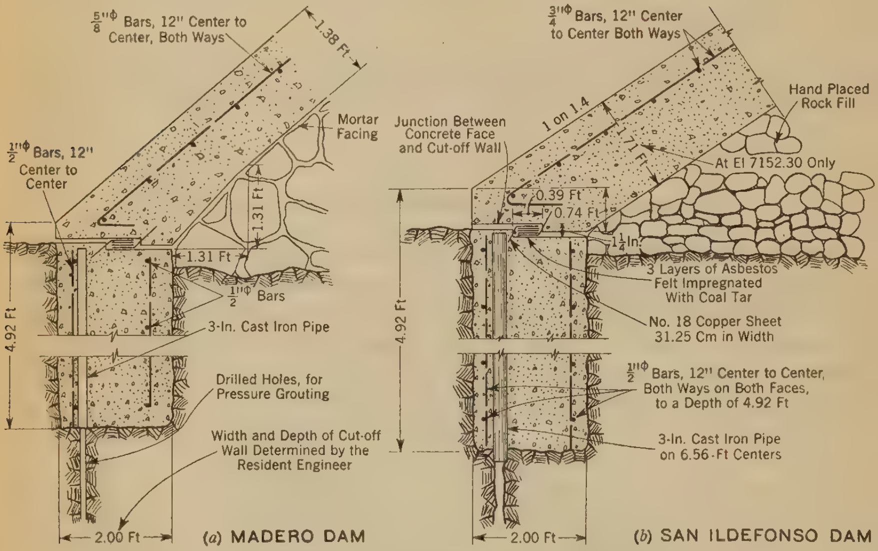

16.4 Ft
Variable
1 on 1.2
Mortar Facing
1.64 Ft
¾"Φ Bars, 12" Center to Center, Both Ways, in the Central Plane of the Slab
1.97 Ft
No. 18 Copper Sheet With Longitudinal Joints Staggered
5/8 in.
1 on 1.4
1.31 Ft
17.63 Ft
Variable
1.84 Ft
¾"Φ Bars, 12" Center to Center
1.31 Ft
1.84 Ft

(a) MADERO DAM (b) SAN ILDEFONSO DAM

FIG. 37.—TYPICAL SECTIONS OF CONCRETE FACING

The slab is placed directly over the facing of the hand-placed rock-fill, taking care, however, to fill all the apparent hollow spaces with spalls so as to obtain a nearly smooth surface as a bed for the concrete face, thus preventing the mortar from seeping into the voids. The slab is attached to the rock-fill by means of 6-in. dowels, 3 ft to 4 ft long, one end of which is hooked. The hook is embedded in a concrete block; that is, it is left within the hand-placed rock-fill zone; the other end projects from the facing, and is also made into a hook that is left within the slab and tied to the steel reinforcement. These hooks are also used to hold the forms in case they are used for pouring the slab. Furthermore, in order to give the slabs a better support, steps or berms about 15 in. to 18 in. wide are left on the facing. Those steps are very convenient for the use of workmen and help to build the slab in such a way that its upper part fills the step as shown in Fig. 37.

Bottom Joint.—To design the joint at the connection of the flexible slab with the rigid cut-off wall is another problem that requires careful consideration. At Madero Dam, the joint was made flexible by means of two copper sheets and a layer of asbestos felt (see Fig. 38(*a*)). This same type of joint (see Fig. 38(*b*)) has been specified for the San Ildefonso Dam.



$\frac{5}{8}$"Φ Bars, 12" Center to Center, Both Ways
1.38 Ft
Mortar Facing
$\frac{1}{2}$"Φ Bars, 12" Center to Center
1.31 Ft
1.31 Ft
$\frac{1}{2}$"Φ Bars
3-In. Cast Iron Pipe
4.92 Ft
Drilled Holes, for Pressure Grouting
Width and Depth of Cut-off Wall Determined by the Resident Engineer
2.00 Ft
$\frac{3}{4}$"Φ Bars, 12" Center to Center Both Ways
Hand Placed Rock Fill
Junction Between Concrete Face and Cut-off Wall
1 on 1.4
At El 7152.30 Only
0.39 Ft
0.74 Ft
$1\frac{1}{4}$In.
3 Layers of Asbestos Felt Impregnated With Coal Tar
4.92 Ft
No. 18 Copper Sheet 31.25 Cm in Width
$\frac{1}{2}$"Φ Bars, 12" Center to Center, Both Ways on Both Faces, to a Depth of 4.92 Ft
3-In. Cast Iron Pipe on 6.56-Ft Centers
2.00 Ft

(*a*) MADERO DAM (*b*) SAN ILDEFONSO DAM

FIG. 38.—TYPICAL DETAILS AT CUT-OFF WALL

Placing Rock-Fill.—The more or less horizontal layers of a rock-fill are constructed by dumping the material. The height of such layers has varied between 6 ft and 12 ft (as in the case in Charcas Dam) to between 18 ft and 24 ft in the more important structures, such as Taxhimay and Madero. It is believed that the use of thinner layers tends to create a more uniform distribution of the rock; and, furthermore, by placing some of the rocks by hand, settlement is reduced quite considerably because the individual particles have a more uniform bearing pressure, one on the other. Deep layers require a more careful classification of material, a condition that is undesirable, because the large rocks roll downward, whereas the smaller ones remain on top. The use of water-jets makes deeper layers possible and tends to produce a better arrangement and a more compact rock mass.

The results obtained by the Mexican National Commission of Irrigation in building dams in rather thin layers and with a slight arrangement of the rock through the entire section, have been very satisfactory.

Conclusion.—The purpose of this discussion has been to present additional information that might not be easily obtainable and that is of interest to engineers connected with the design and construction of rock-fill dams, mainly with the intention of showing how economic conditions influence both the design and the methods of construction.

In accordance with Mr. Galloway's desire that discussion be directed to matters of design, special reference has been made to the use of small rock fragments and also to the use of small concrete panels hung on narrow berms in the water-proofing element; the writers are in agreement with other conclusions of Mr. Galloway's paper.

J. D. GALLOWAY,[47] M. AM. SOC. C. E. (by letter).—It is gratifying to note that a number of engineers have found time to discuss this paper and to contribute valuable information. It was to be expected that differences of opinion would develop, but it is to be noted that the discussion refers mostly to details rather than to the basic elements of design.

The statement that rock-fill dams should consist of a loose fill with slopes on both faces closely approximating natural slopes, with an impervious facing on the up-stream side between which and the rock-fill there should be placed a cushion of dry rubble, was not questioned. In this feature it may be assumed that there is unanimity of opinion. No one advocated placing a core-wall in the center of the dam, as experience has shown that the center of the cross-section is not the place for the impervious element.

There was some discussion of the definition of a rock-fill dam. The writer purposely restricted the definition as given, although that is a matter of opinion. Dams have been built with an earth section, supported on the down-stream side by a rock-fill. It would seem that such a dam is really an earth-fill. The resistance to the water is taken by the earth-fill in accordance with the design of that type of dam, and the rock-fill below is little more than earth of larger grains. It is sufficient for the purposes of the paper to limit the discussion to dams as described.

Mr. Pearce gives a good reason for making the up-stream slope approximately the natural slope of the rock-fill. Steeper slopes have been used, but in all cases, it was necessary to retain the loose rock in place by carrying up the dry rubble at the same time. This was done in the earlier dams, but the up-stream slopes of recent dams have rendered this procedure largely unnecessary. The suggestion was made that the horizontal joints might not be in compression, due to the natural shrinkage of the concrete in setting. This condition might obtain for a time, but it is believed that the settlement of the rock will proceed at a rate greater than the concrete will contract, thus placing the concrete in compression at the horizontal joints.

Among other points mentioned by Mr. Muckleston, he suggests the possibility of using puddled clay loaded with rock as the impervious element. It is the opinion of the writer that nothing of the nature of clay should ever be used in a rock-fill dam. When wet, as it would be, the clay will moisten and will slide from its position. Any rock loaded on top of the clay will be squeezed into it and will cause undue settlement. The same objection may be made to any form of asphalt concrete for the impervious element, as suggested by Mr. Muckleston. Although it is flexible, it will tend to flow under pressure and, in time, will lose its life from contact with water or under the influence of the sun's heat.

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[47] Cons. Engr., San Francisco, Calif.

Mr. Paul refers to the English Dam and asks how it happened to be destroyed. Of this, the writer has no information except that given by the late A. J. Bowie,[48] M. Am. Soc. C. E. The general opinion was that the initial failure of the dam was caused by dynamite, as there was an active contest by different mining companies for water on the Yuba River at that time. However, the failure may have been due to the decay of the log cribs within the structure.

Mr. Floris calls attention to the fact that on an inclined face the weight of water adds to the stability of the dam. It was the writer's intention to convey the same idea, a fact that is common to all dams where the impervious element is next to the water. It was so expressed in the paper (see heading, "Design: Cross-Section") where the sliding factors are given and the water load included as a vertical force that acts by increasing friction to resist the horizontal component tending to slide the dam. In stating that rock-fill dams resist water pressure by their weight only, it was intended to mean that anything of the nature of arch or cantilever action, such as may be present in masonry dams, is not possible.

Dean Morris, in his interesting discussion of the paper, draws attention to the possible effect of earthquakes upon a rock-fill dam and suggests the possibility of large and unequal settlements taking place when the dam is shaken by the waves. Earthquake effects may be of two different kinds. Practically all earthquakes are tectonic; that is, they are caused by an actual rupture of the earth's crust, that may or may not extend to the surface of the ground. Differential movements along the earthquake rift may be large. In the California earthquake of 1906 the maximum differential movement along the San Andreas fault was about 22 ft and the maximum vertical movement, about 3 ft. It is obvious that a dam of the type under discussion would be badly broken if situated on the rift, especially if built with any form of facing mentioned in the paper, providing the movement took place under the dam. It is more than probable that the dam would be destroyed. Earthquake rifts, as a rule, are not single cracks in the earth, but there is usually a zone that may be of a width up to half a mile or so, where, in the past, the earth has been subject to crushing and movement. On account of the crushing of the rock, earthquake rifts or valleys are usually the site of streams where reservoirs are required for storage. The Coast Mountains of California, including the San Gabriel Range, are of this type where numerous earthquake rifts are found. It is the writer's firm opinion that rock-fill dams should never be built in such places. The type does not lend itself to the inclusion of a movable joint such as the one built on the splendid masonry dam for Pasadena, Calif., on the San Gabriel River.

The other earthquake effect mentioned by Dean Morris is that of the earthquake waves that move outward from the rift and which are of sufficient energy to shake a large part of the earth surface with sensible effects. It is true that such waves might cause some settlement in a rock-fill dam. Two factors determine the magnitude of the effects. One is the intensity of the waves and the other, and more important, one is the nature of the foundation material upon which the dam rests. There has been no case that has come to the writer's knowledge where a rock-fill dam has been close to the origin of the waves.

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[48] *Transactions*, Technical Soc. of the Pacific Coast, Vol. II,

In the California earthquake of 1906, the Sierra Nevada was shaken, but the waves were dampened by the granite core of the mountains. There were several rock-fill dams in the mountains subjected to the waves (Items 1, 2, 3, 10, and 11 of Table 1, among others) but no damage was reported on these dams. They were situated about 120 miles distant from the earthquake rift on the San Andreas fault. It is believed that the behavior of the earth dams located close to the fault line will convey some information of the behavior of a rock-fill dam. A description of the earthquake effect on several earth dams is given in the report to the Society by the Committee on the Effect of the Earthquake on Water-Works Structures.[49] With a few minor cracks, the dams were uninjured, and the same is true of the large concrete dam of the Crystal Springs Reservoir, situated within a few hundred feet of the fault line.

The foundation material is of the utmost importance in determining the movement of a dam in an earthquake. The amplitude of the earth movements may vary from less than 1 in. in solid rock to as much as 12 in. in loose alluvial soils saturated with water. As the rock-fill dam should only be built on solid rock, or material that approximates solid rock, the dam will not be subjected to much movement. In practically all instances of rock-fill dams that have come under the writer's observation in the Sierra Nevada, the foundation material is granite.

The settlements cited by Dean Morris, of fills on alluvial soils, mine dumps, and débris on steep mountain sides, do not represent the situation of a rock-fill dam located on bed-rock between two canyon walls. Mine dumps are usually found on side hills or in valleys with a cover of soil of some thickness. They are of small-sized rock, mixed with fine material. They would probably be shaken by an earthquake and some settlement would likely occur. The same is true of the miscellaneous débris on a mountain side that might be disturbed in such cases. The small earth dam that failed at Santa Barbara was located on an alluvial soil and was undoubtedly badly shaken, as the earthquake rift passed close to the dam.

It is the opinion of the writer that properly built, rock-fill dams, situated in mountains where a solid rock foundation is available, will not be distorted to a dangerous extent. The loose rock, when in place, is difficult to displace, excepting possibly on the down-stream face. This opinion is based upon the nature of the structure, the behavior of earth dams in an earthquake, and the fact that such dams as have been subjected to earthquakes have not been injured. However, the subject is an important one and the questions raised by Dean Morris are very pertinent. Until more information is obtained, the subject must remain an open one.

Mr. Peterson's statement regarding the rock used at Salt Springs Dam agrees with the idea that the writer desired to convey when the statement was made that individual rocks of fairly uniform size are desirable. More than one-half the rock was somewhat over 10 tons in weight; the excess fines were wasted, and as most of the fines that reached the dam were sluiced into the interstices, the rock that forms the main part of the dam was of fairly uniform size. It is a matter of description. The writer must differ from Mr. Peterson in the use of water in sluicing. When rock from the quarry is dumped into the dam, the small rock and fines will accumulate at the top. Merely to allow water to run into the mass will not carry the fines into the dam as well as to have a stream of water from a nozzle under pressure directed against the mass. The water will then break up the fines and force them into the rock. If this is not done, the fines will accumulate as a layer and will not be properly sluiced into the dam. The writer agrees that the volume of water is very important. In addition to moving the fines to position, the water acts as a lubricant, assisting in the necessary settlement.

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[49] *Transactions,* Am. Soc. C. E., Vol. LIX (December, 1907), p. 245.

The data, given by Mr. Howson, of three important dams with which he was connected, are of value as indicating methods of construction and the resulting settlement. It should be noted that the vertical settlement of Dix Dam, after eleven years, is slightly less than 1% of the height, which represents an excellent fill.

Mr. Harza raises an important point regarding the adaptability of a rock-fill dam to be used to create head for a power plant. On account of the nature of the rock-fill dam, repairs to the facing are generally necessary, whether the facing is of wood, concrete, or steel. This makes it necessary to draw down the reservoir. The writer is in complete agreement with Mr. Harza that the rock-fill dam is not a satisfactory type to be used for the purpose of creating head for a power plant.

In practically all dams of the rock-fill type, the reservoirs formed by the dam are used for storage only, the water being drawn from the reservoir to augment the low-water flow of the stream. At Salt Springs there is a relatively small hydro-electric power plant that operates by the flow of the stream when the reservoir is full or when water is being drawn from storage. However, the real function of the reservoir, at an elevation close to 4 000 ft above sea level, is to provide water for a chain of power plants on the river below the reservoir. Below the plants, the water is used for irrigation. Sabrina and South Lake Reservoirs (Items 13 and 14, Table 1) also provide water for a chain of power plants, after which it is used for irrigation. The reservoirs are at elevations close to 9 000 ft. Bucks Reservoir, at Elevation 5 160 above sea level, performs the same function for power and irrigation. Some of the reservoirs designated in Table 1 are used for irrigation. It is believed that rock-fill dams, in general, should be used to form storage reservoirs only.

Mr. Huber has described in detail the timber facing of the Sabrina and Hillside or South Lake Dams on Bishop Creek. These dams emphasize the merits of timber facing. On the dams as originally built, there was only a thin layer of dry rubble. The timber facing conformed to the settlement of the loose rock-fill without damage and when the facing was replaced, after twenty years of service, the new timber facing was placed against a fill in which the major settlement had taken place. The new facing may be expected to serve for many years.

Mr. Fowler has added some interesting data on rock-fill dams in regions outside the United States and has given some comment upon the evolution of the type in California. Mention is made of the Relief Dam in this connection. Relief Dam can hardly be cited as illustrating one step in the evolution of dams in California. It was designed by the Consulting Board with a section different from the one built. There was no need for the large section of dry rubble, and as the work was done after the Board was no longer employed, it represented only the ideas of men unfamiliar with California practice. The evolution of the type used in California begins, as stated in the paper, with the dams on the Yuba River, notably, the Bowman Dam, which was built over thirty years before Relief Dam. In the evolution of the type, the principal change has been to abandon the rubble facing of the down-stream slope and to increase the thickness of the rubble on the up-stream slope. The face slopes of recent dams have also been made with flatter slopes. At Relief, the major advance was the introduction of a concrete face.

The Cogoti Dam, in Chile, was mentioned as the one where the facing was laid directly on the loose rock-fill without an intermediate cushion of dry rubble. The Sabrina Dam, built over twenty years previously, was substantially of this type, the facing being of timber resting on a thin layer of rubble that was little more than an arrangement of stones to a uniform surface. It is illustrated in the paper, and additional information may be found in Mr. Huber's discussion.

The behavior of the laminated facings of the Algerian dams throughout the years will be of considerable interest. Contrasted with the California dams, the design of which is based upon the experience of over sixty years, the Algerian dams represent a radical departure. The writer may be allowed to express doubt regarding the permanence of laminated facings built of asphalt concrete, especially where it is necessary to spray the face with water during hot weather. It might happen that on some warm day the water spray will be omitted.

It may be questioned whether a citation of the design of dams built in different parts of the world should be given comparable weight with those dams which have had a considerable life history and in which time has been given to develop such defects as may occur during service. The dams cited by Mr. Fowler, in Algeria, Chile, and in China, are of recent construction. Information regarding their behavior during a few years of service will be of interest and may form the basis for conclusions differing from the ideas held when the dams were designed.

Messrs. Steele and Dreyer have contributed, in their excellent joint discussion, the most important data on the settlement of a large rock-fill dam that has ever appeared; that at Salt Springs Dam. The measurements extend over the five years of the life of the dam and (what is of great value) the measurements of settlement apply to areas of the dam face below the crest. It should be noted that the settlement under the combined rock weight and water load is not a maximum at the crest but is found on the slope of the dam. Settlement due to the weight of the rock occurs progressively during construction and will continue for years after the dam is placed in service. It is difficult to obtain the cumulative effect by direct measurement. The settlement under water load can be determined as stated in the discussion. Above the level of the main quarry (Elevation 3 805), there was a fairly close agreement between the calculated settlement when the design was made and the measured settlement. Below that elevation the settlement was considerably greater than that assumed. It may be doubted whether any exact calculations can be made, as so much depends upon the nature of the rock and the methods used in the construction. However, the theoretical treatment is of value as indicating what may be expected at any time when the crest settlement is known or assumed. In time the crest settlement will increase and the face of the dam will tend to straighten.

The necessity for additional thickness of rubble at the bottom and on the sides of the canyon walls is well stated, as is also the connection of the concrete face to the cut-off wall. The statements represent the experience of the engineers who have been charged with the care of the dam. The writer is in agreement with the conclusions expressed in the discussion.

Mr. Sanger has presented some data on the failure of the Eildon Dam, in Australia, and Mr. East has given a more extended description of the regrettable failure. The writer had occasion to discuss the paper by Mr. Knight,[50] and certain statements were made in that connection.

The presence of clay in the bed of the stream indicated that the material should have been removed. It may have been an important factor in the horizontal movement of the up-stream section of the dam. The concrete core-wall acted as others have done. Under the water load it deflected down stream over considerable lengths, the maximum movement at the top, to 1933, being over 7 ft. The thickness of the concrete core-wall was sufficient to have held the water, but there was added a vertical wall of clay, averaging 30 ft thick, an unnecessary feature. The rupture and movement of the up-stream rock-fill were undoubtedly due to the distortion of this clay, aggravated by the rock being squeezed into the clay under its own weight, and to the lubrication of the clay when submerged.

The writer is in agreement with the conclusions expressed by Mr. East except that the fracture of the core-wall could not be attributed to the pressure of the clay wall in front of it. Even if the clay wall had been absent, the movement of the rock mass under water pressure would probably have produced the same result. It is one more example of a failure showing that a core-wall in the center of a dam is a feature to be avoided.

Messrs. Gomez-Perez and Jinich have described in detail the building of five important rock-fill dams in Mexico, the design of which follows, in general, that of California dams. Some comment may be offered on certain features of design.

There seems to be a tendency in the later dams to use a flatter slope on the up-stream side, the reason being that, on the whole, this form is cheaper. The dams were built in layers of relatively small height. There seem to be good reasons why thicker layers would be better, as referred to later. Opinions will differ on this idea, as there are arguments to be presented on both sides. The writer is impressed with the small dimensions of the facing slabs, as experience elsewhere indicates that larger slabs can be used. In general, it would seem that the Mexican engineers have built dams that are adapted to the location, where there is cheap labor and where the importation of expensive machinery was not possible.

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[50] "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon Reservoir, Australia," by R. G. Knight, *Journal,* Inst. C. E., March, 1938.

Mr. Reed has presented interesting data regarding San Gabriel Dam No. 2—the causes and results of the partial failure and the destruction and removal of the laminated concrete facing.

Other discussers have contributed important data and information of a general nature that does not call for detailed comment. There are certain other elements referred to in the several discussions, upon which the following observations are made.

A number of those who discussed the paper have disagreed with the writer regarding his statement that no gates of any kind should be placed in the spillway. The conclusion, as expressed, was reached after a study of dams that have been destroyed, either earth or rock-fill. The first principle that must be borne in mind is that safety of the structure must, at all times, be assured as far as it is humanly possible to do so. The danger to life and property that exists on every stream below the dam is the major consideration that should guide an engineer in making the design. It is true, as Mr. Fox states, that rock-fill dams are very stable and will resist some water over the crest without failure. Bear River Dam (Item 10, Table 1) was overtopped in one flood without failure. However, three fairly large dams were destroyed by being overtopped, due to inadequate spillways: Walnut Grove, Lower Otay, and Castlewood Dams (Items 4, 5, and 8, Table 1). It is obvious that a sufficient quantity of water over a rock-fill dam will wash it out, and that in three cases at least, this destruction has occurred.

It will be argued that the gates will be opened in time of flood and thus preserve the dam. There are reasons why the gates may not function at the proper time. The operating force will grow negligent and will fail to draw down the reservoir at the proper time, if at all. Storms and resulting floods are sudden, especially in mountain areas where a warm rain falling on snow will bring down a maximum flood in a few hours. Again, rock-fill dams, as a rule, are built in remote locations and usually only one or two men are in attendance at the dam. The failure of a power line, or the failure or inability of the attendants to operate the assumed gates, may cause the overtopping of the dam.

A further argument is found in the fact that the exceptional flood will bring into the reservoir trees and brush that have accumulated over the years, many of large size. This is a common result of the exceptional flood, noted in many instances. The débris will accumulate against the gate structures, in spite of fender logs. The spillway thus clogged will fail to function and the water will go over the dam. Many years ago the writer was connected with the rebuilding of an earth dam that had been washed out due to clogging of the spillway by débris brought into the reservoir in the flood caused by an exceptional storm. The spillway was formed by openings provided with gates placed between masonry piers. It had functioned for over fifty years without damage but failed in the exceptional storm and flood and the dam was washed out.

In a storage reservoir created by a rock-fill dam there is no need for gates. The dam and the reservoir capacity can be ascertained from a study of stream flow and the requirements of storage. The spillway can be located at the proper elevation and gates to regulate the elevation of water are unnecessary. However, the major factor in reaching a decision is found in the fact that rock-fill dams have been, and can be, destroyed by being overtopped by water. Gates in the spillway have failed to function at times, and since no effort should be spared to make the structure safe in view of the possible loss to life and property, an open spillway of large size, capable of passing all floods with attendant débris, should be provided at all dams. The writer emphatically repeats the statement of the paper that no rock-fill dams should have a spillway dependent upon gates of any kind.

The subject of settlement requires further explanation. The statement of the writer that settlement might be 5% of the vertical height referred to settlement of the loose rock-fill during construction and before the building of the rubble cushion and the impervious face. This was made to emphasize the necessity for deferring the construction of the other two elements of the dam for as long a period as possible after placing the loose rock-fill. If the dam has been properly constructed, the final settlement may be less than 1% of the height, although it will continue to settle for a long period of years. Much depends upon the manner in which the fines have been sluiced into the dam. An extreme case is found in San Gabriel Dam No. 2, where no sluicing was done during construction. During the first heavy storm the dam settled about 12 ft, wrecking the facing. The data of settlement at San Gabriel Dam No. 2, given by Mr. Baumann, seem to refer more to the relation between the quarried rock and the volume within the dam. In general, the settlement discussed herein refers to the settlement during construction and especially after the completion of the dam.

The problem of settlement is one of the most difficult to solve in the construction of rock-fill dams. The higher the dam, the more important becomes the problem. Settlement is intimately connected with the methods used in construction. In some dams, as in Algeria, the engineers have gone to the extreme of making the entire dam of rubble. In others, the loose rock-fill has been deposited in relatively thin layers. There are reasons for so doing, but there seem to be better reasons for the construction of the dam in deep layers. The loose rock, dumped at the top of a fill (say, 100 ft in height), rolls down the slope and by its impact, beds itself among the other rock. It does not merely rest upon a point, because the rock is crushed to a bearing. The fill in such a case is not merely a mass of rock loosely dumped together. The rocks are wedged among each other and form a compact mass, differing greatly from a loose mass. It is quite true that the smaller rock accumulates at, or near, the top of the slope, but the general effect is a compacted mass.

On the other hand, the definition of rubble contains a wide variety of ideas, which are difficult to describe. It is quite possible to build such a wall that will have but little settlement and also to build one that will act in the reverse manner. Merely to place the rock by hand or derrick will not necessarily produce a compact mass. The final results may furnish a means of comparing the loose rock-fill with one of rubble. In Algeria the Ghrib Dam was built entirely of a rubble fill, and a settlement of 0.8% of the height took place, presumably in the first year, while on the contrary, the loose rock-fill at Salt Springs had settled at the crest, only 0.6% of the height in five years. In eleven years the Dix Dam has settled less than 1.0% of its height. The sluicing of the fines into the mass of the dam is an important factor in reducing the total settlement.

The position of the impervious element seems to be agreed on, but some dams have recently been built with the element placed as a vertical wall in the center of the dam. At the Eildon Dam, the top of the vertical core-wall, about 140 ft high, was moved down stream, a distance of about 7 ft as a maximum when the reservoir was filled. At one of the dams in Tunis, with a hollow concrete central wall, about 115 ft high, the crest moved down stream about 2 ft 8 in. on the first filling of the reservoir. The wall was ruptured. The plan of using a central core-wall seems to have arisen from the idea that the weight of the rock-fill is vertical and ignores the effect of water pressure. The actual forces are made up of the rock weighing about 100 lb per cu ft, which acts vertically, and the weight of water at 62.5 lb per cu ft, which acts horizontally. The two forces are of the same order of magnitude and the resultant causes a compression of the material of the dam in a direction that is represented by the resultant of the vertical and horizontal forces. The movement off the core-wall down stream and its possible rupture may be foreseen.

The material of the impervious face was described. In the case of a concrete facing it is believed that a laminated face is unnecessary and will tend to be ruptured. At San Gabriel Dam No. 2 the laminated face was badly ruptured, and it was necessary to install a wooden facing to take its place. It should be recognized that the concrete facing is a very thin element compared to the dimensions of most dams. It is subjected to many forces during settlement which requires that it be of sufficient thickness of integral concrete to withstand distortion. A laminated facing will not perform this function to any great extent. Although much of the discussion has centered around concrete as a material for facing, the writer is of the opinion that timber is an excellent material for the purpose. It will follow the distortion due to unequal settlement and may be renewed, as mentioned by Mr. Huber in his description of the renewal of the facing of Sabrina and Hillside Dams. In this remote location, a timber face is cheaper and is all that is required. The writer also believes that a steel facing has similar good qualities to recommend it. It is permanent and the cost will be a determining factor.

The discussion has centered around some of the higher dams, of which Salt Springs, with a height of 328 ft at the center, is the highest yet built. The higher the dam, the greater the total settlement. The rigid facing, resting as it does on the rock-fill, must move during the settlement. At the bottom and sides of the canyon it must be attached to the rigid cut-off wall. The result is that expansion joints must be installed. Their design is difficult. The comment of Messrs. Steele and Dreyer on the experience at Salt Springs illustrates this point. Although individual situations will always determine the height and selection of the type of dam, it is the belief of the writer that rock-fill dams of a height approximating 200 ft represent about the advisable maximum.

The writer desires to express to those who have discussed the paper his appreciation of their kindness in so doing and for the data given. It is believed that the entire discussion will furnish information of value to those who may have before them the problem of designing dams.

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MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION

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With Discussion by Messrs. Edward W. Moore, Willem Rudolfs and H. Heukelekian, Hermann Bach, C. E. Keefer, and A. M. Rawn, A. Perry Banta and Richard Pomeroy.

Synopsis

The results of a study of four years' operation of a multiple-stage sludge digestion system are presented in this paper. Satisfactory results have been obtained with a detention period of ten days. The division of the process into stages has afforded an opportunity to obtain data on the component phases of digestion. Part I includes a quantitative study of operating data. This is followed, in Part II, by a report of research on some chemical phases of sludge digestion. Gas, sludge, and sludge-liquor analyses and a discussion of chemical equilibria are included. The presence of traces of unusual compounds in sludge and sludge gas has been verified by micro-analyses.

Introduction

Two batteries of multiple-stage digestion tanks were placed in operation in July, 1931, by the Los Angeles (Calif.) County Sanitation Districts at their Joint Disposal Plant near Harbor City, Calif. The tanks were designed on basic data obtained from experimental work conducted by the Districts and incorporated certain new principles of operation and control. The principal departure from conventional design and operation is the advantage taken of a feature described as gravimetric segregation. Digesting sludge particles will tend to float or to remain in suspension due to adhering or included gas particles. As the digestion of a solid approaches completion, it will sink and remain near the bottom of the mass. Practically all inorganic particles will likewise sink,

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thus a combination will be found in the lower strata consisting largely of digested particles and inorganic solids.

In the design of the tanks advantage was taken of this automatic segregation or selection to separate out, and pass ahead, the lower strata to successively lower tanks, at the same time, permitting the upper or active strata to lag and thus to utilize more fully the tank capacity for its intended purpose.

The two batteries of stage digestion tanks are identical in all their principal features. Each battery is rectangular in plan, 56 ft wide and 150 ft long, with an effective depth of about 15 ft. Three transverse walls divide each battery into four rectangular tanks, each 56 ft by 37.5 ft in plan (see Fig. 1). Each battery is built so that the bottom has a slope of 1 on 12. Thus, the sludge surfaces of successive tanks differ by 3 ft. Each tank is equipped with a hopper at its lower end, which is placed directly against the transverse wall dividing it from the next tank in succession. The tanks have concrete covers and each is equipped with two longitudinal baffle-walls extending from within a foot of the bottom, to the cover of the tank, dividing it into three bays of equal capacity.

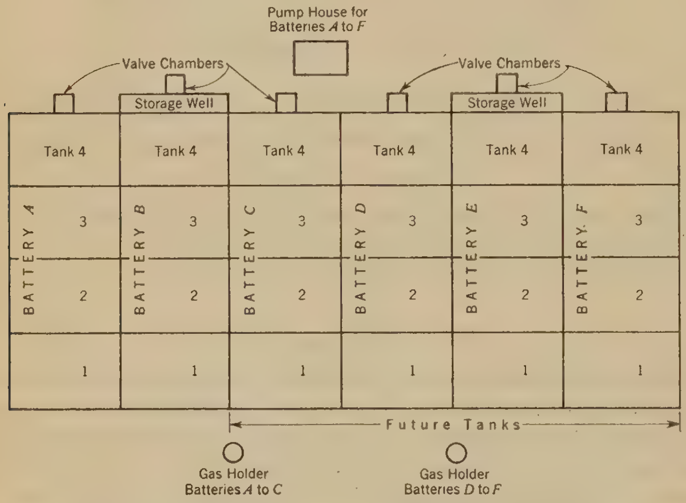
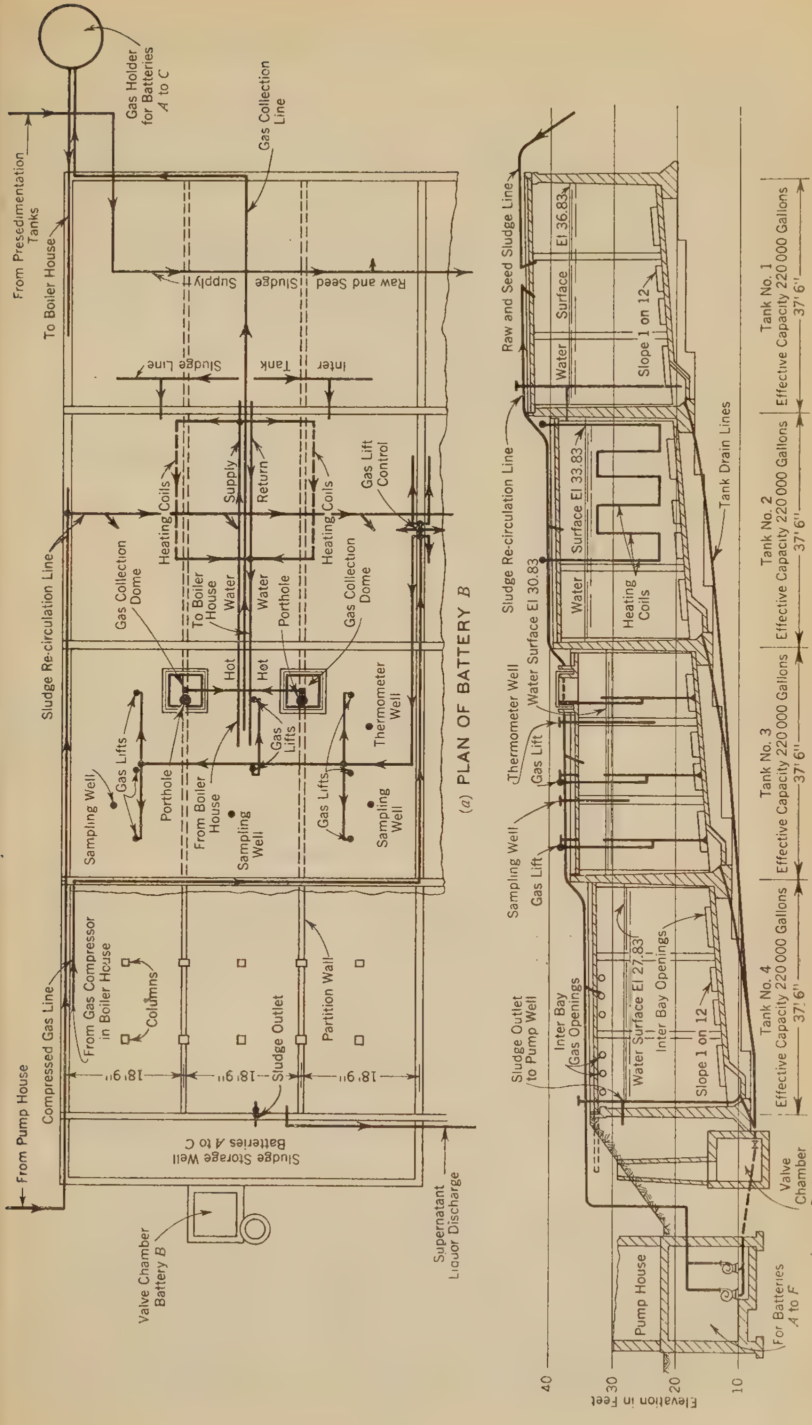

Pump House for Batteries A to F
Valve Chambers
Storage Well
Valve Chambers
Storage Well
Tank 4
Tank 4
Tank 4
Tank 4
Tank 4
Tank 4
BATTERY A
BATTERY B
BATTERY C
BATTERY D
BATTERY E
BATTERY F
3
2
1
Future Tanks
Gas Holder Batteries A to C
Gas Holder Batteries D to F

Fig. 1.—General Layout of Sludge Digestion Tanks

Referring to Fig. 2, man-ways, gas collection, re-circulation and heating lines, sampling and thermometer wells are common to each tank; gas lifts are common to Tanks Nos. 1, 2, and 3; and raw and seed sludge lines go to Tank No. 1 only.

The process of operation is as follows (see Figs. 2 and 3): Primary and activated sludges, plus approximately 20%, by volume, of digested sludge, are introduced into the outer bays of Tank No. 1 in either of the two batteries, and the temperature of the mass is raised to approximately 85° F. To reach the central bay of Tank No. 1, solids must sink to the bottom of the mass in the outer bays and pass under the baffle-walls. To reach the discharge riser pipes, leading from Tank No. 1 to the two outer bays in Tank No. 2, solids must accumulate in the hopper at the lower end of the center bay in Tank No. 1.


Gas Holder for Batteries A to C
From Presedimentation Tanks
To Boiler House
Gas Collection Line
Supply
Sludge
Raw and Seed
Sludge Line
Inter Tank
Sludge Re-circulation Line
Heating Coils
Supply
Return
Heating Coils
Gas Lift Control
Gas Collection Dome
To Boiler House
Water
Water
Porthole
Gas Collection Dome
Hot
Hot
Gas Lifts
Thermometer Well
Sampling Well
Gas Lifts
Porthole
From Boiler House
Sampling Well
Gas Lifts
Sampling Well
Compressed Gas Line
From Gas Compressor in Boiler House
Columns
Sludge Outlet
Partition Wall
18' 9"
18' 9"
18' 9"
From Pump House
Sludge Storage Well Batteries A to C
Valve Chamber Battery B
Supernatant Liquor Discharge
Raw and Seed Sludge Line
El 36.83
Water Surface
Slope 1 on 12
Tank No. 1 Effective Capacity 220 000 Gallons 37' 6"
Sludge Re-circulation Line
Surface El 33.83
Water
Heating Coils
Tank Drain Lines
Tank No. 2 Effective Capacity 220 000 Gallons 37' 6"
Sludge Re-circulation Line
Water Surface El 30.83
Thermometer Well
Gas Lift
Sampling Well
Gas Lift
Tank No. 3 Effective Capacity 220 000 Gallons 37' 6"
Sludge Outlet to Pump Well
Inter Bay Gas Openings
Water Surface El 27.83
Inter Bay Openings
Slope 1 on 12
Tank No. 4 Effective Capacity 220 000 Gallons 37' 6"
Valve Chamber Battery A
Pump House
For Batteries A to F
40
30
20
10
Elevation in Feet

(a) PLAN OF BATTERY B

(b) LONGITUDINAL SECTION, BATTERY A

FIG. 2.—A TYPICAL SLUDGE DIGESTION TANK

Thus, a process of gravimetric selection is operative in the flow of the sludge, and this selection is repeated in each successive tank until the sludge is discharged into the storage well.

An open pit was constructed adjacent to the digestion tanks which, with a capacity roughly equal to the tanks themselves, serves as a balancing reservoir during poor drying weather, promotes the removal of supernatant liquor, and incidentally adds detention time.

In 1933, the tanks, which had been designed for a detention period of about twenty-five days, were heavily overloaded. Good digestion results were obtained despite a detention period approximating ten days. Beginning in April, 1933, and continuing to date, a detailed study of the performance of the tanks was made. The results of this study over the 4-yr period, May, 1933, to April, 1937, are presented in two sections—the first reporting operation data, and the second, results of research and some of the chemical changes undergone by the sludge during digestion.

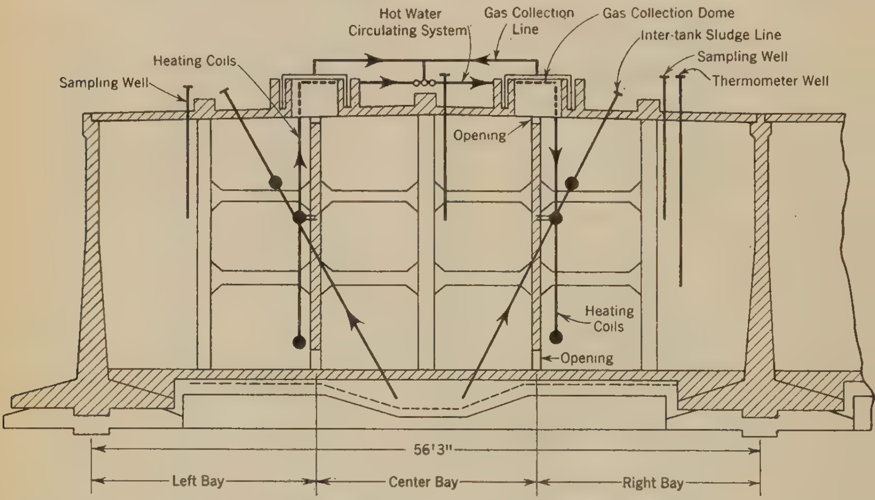

Hot Water Circulating System
Gas Collection Line
Gas Collection Dome
Inter-tank Sludge Line
Sampling Well
Thermometer Well
Heating Coils
Sampling Well
Opening
Heating Coils
Opening
56'3"
Left Bay
Center Bay
Right Bay

Fig. 3.—Cross-Section of a Sludge Digestion Tank

Operation Data

Sludge Quantities.—No routine method of sufficient accuracy was available to measure, directly, the volumes and solids contents of the sludges pumped to the digesters. Two indirect methods were used, however, to obtain these quantities.

The first of these two methods makes use of the known volume of sewage flow and the sewage analyses. The primary sludge is calculated from the difference between the analyses of the suspended solids of the influent and settled sewage, and the activated sludge calculation is made from the difference between the non-volatile part of the total solids in the settled sewage and final effluent, together with data on the volatile content of the activated sludge. The second method makes use of digested sludge analyses. An accurate method of measuring the volume of digested sludge, recirculated digested sludge (seed), and supernatant liquor, was developed, and from such data the volume of new sludge pumped to the digesters was computed. Determinations of the ash content of the digested sludge, seed sludge, supernatant liquor, and in the sand and scum accumulations, provide data on the daily input of ash to the digesters. These results, when divided by the average ash content of the solids of the raw sludges, yield values for the input of raw solids.

Calculations of sludge quantities were made by both methods over a 2-yr period ending April 30, 1935. Since that time only the first method has been used. Results of the calculations reduced to quarterly periods are shown in Table 1.

TABLE 1.—Analysis of Data by Quarterly Periods

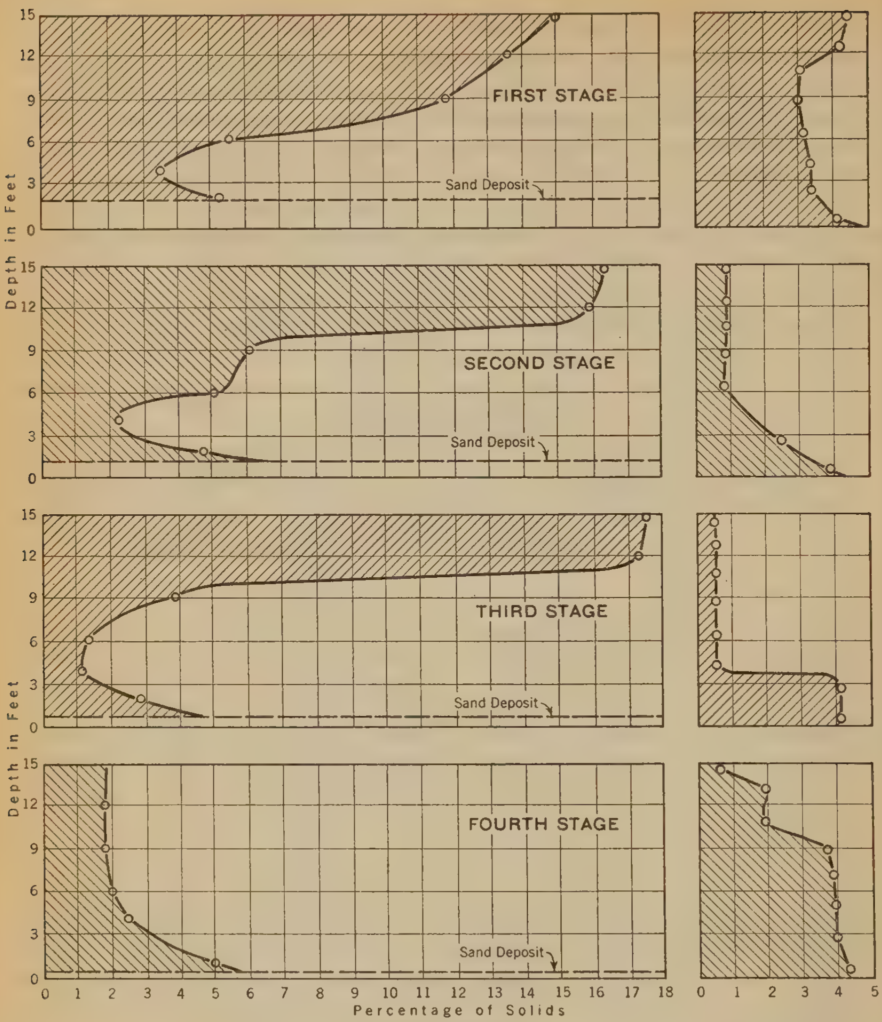
Three-month period ending: (1)	Average sewage flow, in million gallons daily (2)	(a) Average Weight of Raw Solids Pumped to Digesters, in Pounds per Day; Computed from:		(b) Percentage of Organic Matter Digested in Tanks		
		Sewage analysis and flow (3)	Digested sludge ash (4)	Percentage, Volatile Matter in:		Percentage digestion of organic matter (7)
				Raw sludge (5)	Digested sludge (6)	
1933:						
July 31	14.1	48 700	54 600	70.61	54.66	49.8
October 31	13.7	48 300	47 800	71.48	57.24	46.6
1934:						
January 31	14.1	44 200	47 200	70.39	56.31	45.8
April 30	14.7	43 200	43 900	71.42	58.33	44.0
July 31	14.7	41 700	40 100	70.14	56.12	45.6
October 31	15.4	49 200	46 200	73.22	58.33	48.8
1935:						
January 31	15.7	50 300	48 800	71.74	57.76	46.1
April 30	15.5	45 700	47 700	67.87	53.97	44.6
2-yr. average	14.7	46 400	47 000	70.86	56.59	46.4
1935:						
July 31	16.9	....	....	68.96	54.18	46.8
October 31	18.2	....	....	70.88	56.34	46.9
1936:						
January 31	17.7	....	....	71.79	57.67	46.5
April 30	18.6	53 100	....	71.10	55.87	46.3*
July 31	18.6	51 000	....	72.58	56.22	50.6*
October 31	18.2	61 100	....	73.21	56.34	50.5*
1937:						
January 31	18.1	63 300	....	71.19	55.62	47.2*
April 30	18.6	61 000	....	67.62	52.34	44.8*
2-yr. average	18.1	....	....	70.97	55.57	47.4

* Corrected for influence of seed from the pit.

The two methods of calculation, when averaged over the 2-yr period, differ by only 1.3%, although individual quarterly periods differ by as much as 11 per cent.

The average solids content of all new sludge pumped to the digesters is 4.7 per cent. Of the new solids added to the digesters, 33% are in the primary sludge and 67% in the activated sludge.

Quarterly averages of the volume of new sludge pumped to the digesters vary from 122 000 gal per day to 160 000 gal per day. The nominal detention time (that is, the design capacity of the tanks divided by the volume of new sludge added per day) varies from 13.5 days to 10.3 days. Actually, the time



(*a*) SURVEY OF AUGUST, 1933

(*b*) SURVEY OF MAY, 1934

Fig. 4

TABLE 2.—Composition of Sludge by Stages

Description	Survey of August, 1933*				Survey of May, 1934†			
	Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
Solids.........	9.38	8.74	6.93	2.65	3.47	1.60	1.72	3.19
Volatile........	62.8	60.7	59.6	58.1	64.1	61.6	60.5	58.2

* After 25 months of continuous operation. † Seven weeks after tanks were cleaned.

of passage of the liquid component of the sludge through the tanks is about 20% less than the nominal detention time, due to the admixture of seed sludge. The time of passage of the solids may be greater or less than this fluid detention time, due to the tendency of the solids either to remain in the upper part of the tanks or to settle to the bottom and thus to pass more quickly through the tank system; it is also increased by the occasional removal of supernatant liquor from the third stage.

Scum Accumulation.—During August, 1933, when the tanks had been in operation for 25 months, composite samples of the sludge were obtained from various elevations of each tank and were analyzed for total and organic solids. A similar survey was made of Battery A between May 11, and May 15, 1934, about seven weeks after all the tanks had been emptied and cleaned.

The percentage of organic matter tends generally to be higher at the higher elevations in the tanks, as shown in Fig. 4. The percentage of volatile matter for the various stages is shown in Fig. 5 and Table 2. The formation of a thick scum layer is well shown in the results of the first test, but seven weeks after the tanks had been entirely emptied there was only a slight tendency toward concentration near the sludge surface in the first tank, with no significant tendency for flotation in the succeeding tanks. The high solids content of the scum layer, together with the observed presence of seasonal seeds at least several months old, as well as quantities of match sticks, hair and bits of rubber, suggest the possibility that, after twenty-five months of operation, the scum layer was largely made up of semi-permanent, indigestible matter, and, as such, would be expected to reduce, greatly, the effective capacity of the tanks. In order to create some idea of the quantity of coarse material, samples of the scum were composited from each battery and washed on an 8-mesh screen, the retained matter being weighed. The results of these determinations are shown in Table 3. It is estimated that the screening from the scum amounted to 12% of the total weight of sludge solids in the tank.

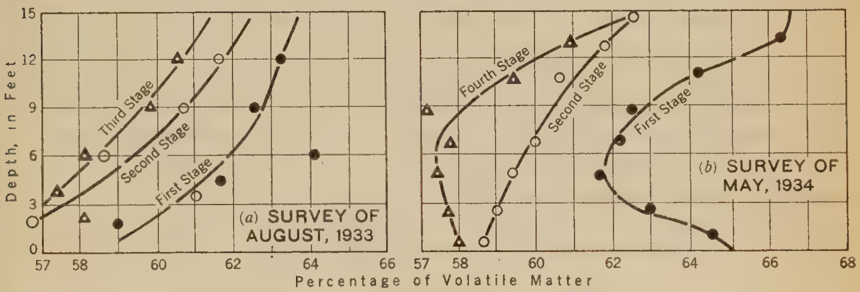


FIG. 5

TABLE 3.—ANALYSES OF SCUM

Description	First stage	Second stage	Third stage
Total solids, percentage of wet weight	16.90	16.04	16.50
Percentage of solids retained on 8-mesh screen	17.9	19.8	21.7
Percentage of organic matter in dry screening	61.8	63.8	67.7

In order to evaluate the effect of the scum accumulation on tank efficiency, experiments were conducted to determine the gas production from samples of the scum. The samples, as taken from the tanks, were placed in gallon bottles and no liquid was added, so that the conditions of tank digestion were simulated as nearly as possible. The results are shown in Table 4.

TABLE 4.—DIGESTION OF SCUM

Stage	Weight, in grams	INITIAL PERCENTAGES:		CUMULATIVE TOTAL GAS PRODUCED, IN CUBIC CENTIMETERS, ON THE FOLLOWING SUCCESSIVE DAYS:							
		Of solids	Of organic matter	1	2	3	4	5	6	7	15
1	1 027	13.97	68.6	5 140	10 420	15 580	20 730	24 020	26 140	27 460	29 760
2	1 034	16.86	62.0	2 720	4 780	6 540	7 970	9 170	....	....	....

For the first four days after removal from the tank, the scum from the first stage produced gas at the rate of 5.0 liters per day per liter of scum. This is well in excess of the averages that have been noted for any of the tanks. Even the sample from the second stage produced gas at an initial rate of 2.6 liters per liter of scum. These results show that, in spite of the accumulation of semi-permanent matter in the scum, there was, nevertheless, enough digestible sludge temporarily retained to make it the seat of very active digestion.

Reduction in Organic Matter and Gas Production.—The percentage of organic matter destroyed in the tanks is obtained from the formula:

$$\text{Fraction digested} = 1 - \frac{W_d}{W_n} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which W_d is the ratio of the weight of organic matter to the weight of ash, for digested sludge; and W_n is the corresponding ratio for new sludge.

The average for the four years ending April, 1937 (see Table 1), shows a reduction in the tanks of 47% of the organic matter added. In weight per day, this equals 17 800 lb.

Computations during the second year of the period covered by this paper showed reduction in the pit of 14% of the organic matter added to the pit; the tank system showed a reduction of 46.3% of the organic matter added, so that the tanks and the pit together effected a 53.8% reduction of the organic matter of sludge which passed through the tanks and the pit. Of the total reduction, 86% occurred in the tanks. The average detention time in the pit was 27 days. The temperature in the pit is lower than that in the tanks.

Gas measurements by stages were made during four separate periods of the first year of the tests. The limited number of meters available prevented the making of simultaneous measurements on all tanks, but single tanks were metered over 10 to 15-day periods. Table 5 gives the totals for both batteries.

In the first three periods the temperature increased in successive tanks. In the fourth period heat was applied only to the first tanks, and the temperature varied but slightly throughout the process. This increasing temperature in the first three periods of measurement probably accounts for the fact that, in two of the periods, the gas production is greater in the third stage than in the second. In the fourth period the gas production is greater in the fourth stage (when full) than in the third, probably because of the greater concentration of solids in the fourth tanks (see Fig. 2) resulting from the removal of supernatant liquor from the third tanks.

TABLE 5.—GAS PRODUCTION BY STAGES

Description	Stage 1	Stage 2	Stage 3	Stage 4*	Measured total
May 22 to June 19, 1933:					
Temperature, in degrees Fahrenheit...	80.5	80.5	83.2	84.5	....
Gas Production, in:					
Cubic feet per day...............	109 100	63 900	65 100	51 600	264 500
Cubic feet per cubic feet of tank...	1.95	1.14	1.16	0.74	1.18
August 8 to September 5, 1933:					
Temperature, in degrees Fahrenheit...	80.0	83.5	85.0	85.0	....
Gas Production, in:					
Cubic feet per day...............	114 400	53 400	35 700	26 000	219 900
Cubic feet per cubic feet of tank...	2.05	0.96	0.64	0.47	0.94
October 4 to November 2, 1933:					
Temperature, in degrees Fahrenheit...	79.0	81.5	81.5	82.5	....
Gas Production, in:					
Cubic feet per day...............	103 800	57 000	62 800	54 500	261 500
Cubic feet per cubic feet of tank...	1.86	1.02	1.12	0.98	1.17
May 5 to May 31, 1934:					
Temperature, in degrees Fahrenheit...	85.6	85.3	85.3	85.3	....
Gas Production, in:					
Cubic feet per day...............	174 100	62 700	38 300	49 200	307 700
Cubic feet per cubic feet of tank...	3.11	1.12	0.69	0.88	1.37
Average production, in cubic feet per day	125 300	59 300	50 400	45 000	263 400

* Tanks in Stage 4 were only partly filled; the values in this column were computed as for full tanks, in order to permit comparison with other stages.

In the fourth period of measurement (that is, after the tanks were emptied and cleaned), the gas production in the first tanks was greater than during the other periods, which might seem to indicate an improvement resulting from the removal of the floating accumulation. This effect, however, may be accounted for adequately by two other factors, the first being the reduction of effective capacity by 13%, due to sand accumulation before cleaning, and the other being the higher temperature after cleaning.

The gas metered is slightly less than the actual production, since there is some tank leakage. On the basis of tests made when the tanks were new, the leakage under conditions of metering would be expected to be about 7 000 cu ft per day, which, added to the average daily production shown in Table 5, indicates an average daily total of 270 000 cu ft.

The gas production was measured at an average pressure of 765 mm of mercury and at 25° C. The average density of the gas under these conditions was determined experimentally to be 1.047 grams per liter. Hence, the weight of gas actually recovered was 17 400 lb per day. The average weight of organic matter destroyed over this period was 16 300 lb. Thus, the weight of gas produced was 107% of the quantity described as "weight of organic matter destroyed." This value (107%) takes no account of carbon dioxide (CO_2), ammonia (NH_3), and volatile acids lost from samples of new or digested sludge upon drying during analysis. If these losses were taken into account a still higher value would result.

In order to determine what additional gas production might be expected from longer detention times, samples of sludge from the fourth tanks were placed in bottles in the laboratory and the subsequent gas production was measured. These tests were continued until the rate of gas production had dropped to roughly one-tenth of the initial rate for the samples from the fourth tanks, or to about 2% of the rate in the first digestion tanks. In the digestion tanks the sludge yielded an average of 343 cu cm of gas per gram of solids added, or 485 cu cm per gram of organic matter added. Table 6 gives the gas yields on the same basis for the additional periods of detention.

TABLE 6.—Gas Production After Sludge Leaves Tanks

Stage	Date started	Detention, in days	Average temperature, in degrees Fahrenheit	Gas Produced			
				In Cubic Centimeters per Gram of Original:		Total produced, plus subsequent digestion, in cubic centimeters per gram of organic matter added	Percentage of gas produced in tanks
				Solids	Organic matter		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	August 14......	20	80.5	64	91	576	84
2	August 15......	19	80.5	60	85	570	85
3	November 1....	23	86.0	80	113	598	81
4	November 2....	22	86.0	77	108	593	82

Detailed results of the analyses of the gas will be given subsequently. The average carbon dioxide content is about 32%, the remainder being nearly pure methane gas. The content of hydrogen sulfide, H_2S, rarely exceeds 0.02 per cent. About 50 000 cu ft per day is burned in a water-heating boiler used to heat the digestion tanks. After the boiler had been in service for about eight months, it was observed that corrosion of the flues was taking place, leading to the deposition of plugs of ferrous sulfate in the lower ends of the flues. Use of a forced draft eliminated the difficulty by preventing condensation of moisture in the flues.

A 200-hp gas engine, which was installed in May, 1935, uses about 110 000 cu ft per day of the gas. The engine has run 99.3% of the time since installation, and has generated an average of 182 hp. The cost of producing this power, in terms of equivalent electrical units, is 0.18 cts per kw-hr. This includes 10-yr amortization at 6% interest, and all operation and maintenance costs. The cylinders show no corrosion attributable to the gas. Valves are ground two or three times each year; rings were changed at the end of 23 months.

The piping of the gas-collecting system has shown no deterioration. The concrete and metal work in the digestion tanks was found to be in good condition except that in the fourth tanks the concrete above the sludge line shows moderate surface deterioration. In these tanks some air occasionally becomes mixed with the gas, and probably oxidizes hydrogen sulfide to sulfuric acid. In a few places the gas that diffused out through small cracks in the concrete has caused detectable deterioration on the exterior of the tanks where the concrete is exposed to the air. The concrete of the tops of the tanks, which are covered with a coat of tar and 12 in. of soil, is apparently unaffected, but electrical conduits laid in the soil have been practically destroyed in some places. Free sulfur has been deposited on the surface of the soil in a few small areas. It is surprising to observe these effects from such a small percentage of hydrogen sulfide in the gas.

Heating System.—The tanks are heated by means of 2-in., extra heavy, wrought-iron coils arranged vertically along the partition walls of one side of each of the first three tank compartments. Individual runs of pipe of each coil were placed in a vertical position to prevent the settlement of solids on the pipe. The coils of the first tanks were designed to supply enough heat to raise the temperature of the new sludge to the operating temperature (85° F). In the succeeding tanks only enough coil area was provided to compensate for estimated heat loss by radiation.

Soundings taken with a thermocouple indicate that there is no temperature stratification and that the tanks are quite uniform in temperature throughout the depth of each unit (see Table 7). When the tanks were emptied and cleaned after thirty months of service, approximately 0.5 in. of limey scale was removed from the heating coils. Prior to the cleaning it was necessary to use all the coils in the first three tanks or stages in order to raise the temperature to 85° F, but subsequent to the cleaning, hot water has been circulated only through the coils of the first tanks. Some minor difficulty has been experienced with air becoming trapped in the bends in the coil system. The tanks are handling more than twice the designed capacity, and additional heating surface would be desirable in the first tanks.

TABLE 7.—SLUDGE TEMPERATURES, IN DEGREES FAHRENHEIT, IN THE FIRST TANKS

Distance from top of Tank No. 1, in feet	Battery A			Battery B		
	East bay	Center bay	West bay	East bay	Center bay	West bay
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	81.7	84.0	83.7	82.8	84.4	84.7
4	....	84.0	83.6	....	....	....
7	80.9	84.1	83.6	85.2	84.2	84.7
11	....	84.0	83.7	....	....	....
15*	81.4	84.0	83.0	84.5	83.5	84.0
Average for bay	81.3	84.0	83.6	84.2	84.0	84.5
Average for tank	....	83.0	....	....	84.0	....

* Bottom of tank.

A study of the sludge-heating data shows that 675 ft of 2-in., extra heavy, wrought-iron pipe in the first tanks transmitted heat at the rate of 84 Btu per day per ft of pipe per degree of temperature differential. This value was obtained after thirty months of service when the coils were covered with lime scale. After the scale was removed and the condition of the pipes was equivalent to new, the conductance rate was 347 Btu.

From May 15, to June 11, 1934, observations were made on the quantity of heat supplied to the digestion system. The heat transmitted daily to the tanks was 13 900 000 Btu as calculated from the sludge-heating water. New sludge at the rate of 129 000 gal per day, and at 73.2° F, was pumped to the digesters. It is calculated that 13 400 000 Btu per day was required to raise this quantity to the temperature of the first tanks. The difference of 500 000 Btu per day between the heat supplied and the heat appearing in the sludge is due: (*a*) To experimental error; (*b*) to radiation from the tanks; and (*c*) perhaps to endothermic requirements of the digestion process; or (*d*) to a combination of Causes (*a*), (*b*), and (*c*).

The difference between the free heat content of the sludge added to, and leaving, the digesters was 12 700 000 Btu per day. The heat added to the sludge was 13 900 000 Btu per day; there was left 1 200 000 Btu, to account for radiation and endothermic requirements of the entire process. C. E. Keefer, M. Am. Soc. C. E., and Herman Kratz[4] found through laboratory experiments that the digestion process in general is endothermic, their experiments showing a net heat absorption varying from 154 to 433 calories per gram of organic matter destroyed. The total destruction of organic matter in the digestion tanks of the Districts, over the corresponding period, averaged 13 300 lb, or 6 030 kg per day. Using the minimum value cited by Messrs. Keefer and Kratz, this destruction should require 929 000 kg-cal, or 3 690 000 Btu, a quantity of heat three times as great as that which appears to be available (1 200 000 Btu).

A similar conclusion may be reached by comparing the temperatures in the second and fourth stages during the period when no heat was being applied beyond the first stage. On the basis of the minimum values given by Messrs. Keefer and Kratz for the endothermic requirements in the latter stages of digestion, it is calculated that the sludge temperature should fall 2.3° F even with no loss by radiation. Actually, the cooling, including that due to heat loss by radiation, was less than 0.5° F.

Foaming.—Foaming is occasionally observed in the first and second tanks. Gas space of 6 ft in the first tanks and 4 ft in the other tanks is provided to accommodate the foam, and for four years after the tanks were placed in service no trouble was encountered. In January, 1936, the foaming became serious, rising to the tops of the tanks and filling the gas lines. Seed sludge was being taken at that time from the fourth tanks. It is believed that this sludge was not far enough advanced in digestion to serve as satisfactory seed, an indication of this being the fact that the content of volatile acids was 600 ppm to 850 ppm (as acetic acid). Seed was then taken from the pit, whereupon the foaming subsided. However, the difficulty has occasionally recurred since that time although the seed is now regularly taken from the pit. It is now believed that the trouble lies in the failure to secure adequate mixing of the seed with the new sludge. It has not been convenient to pump seed to the tanks more than once each day. Between pumpings of seed, one of the first tanks may receive a volume of new sludge nearly as great as the volume of the tank. The pumping of a quantity of seed into the outside bays may not permit proper seeding of the entire mass. When the foam rises to a danger point, some of the raw sludge is by-passed to subsequent tanks until the foam subsides. This procedure has effectively prevented operating difficulty.

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[4] "The Interchange of Heat During Sludge Digestion," by Messrs. C. E. Keefer and Herman Kratz, *Sewage Works Journal*, Vol. 5 (1933), p. 3.

Stirring.—The tanks were equipped with gas lifts for the purpose of stirring the sludge, but a comparison of the performance of one battery operated with the gas lifts as against the other battery operated without stirring, failed to indicate any difference. With the degree of mixing made possible by the tank arrangement it is doubtful whether any additional stirring is warranted.

Sludge Drying.—The sludge drying beds comprised 2.34 acres of underdrained sand beds, 6.0 acres of diked field, and 2.9 acres of shallow lagoons. The diked fields are constructed by grading open fields so that the sludge depth is about 2 ft. The soil is a tight adobe that leaves little chance for seepage or under-drainage. Data indicate that sludge was dried at the rate of 663 tons per acre per yr on the sand beds and at the rate of 342 tons per acre per yr on the "field." About 25% of it was wasted directly from the digestion tanks to the drying beds. For convenience in operation the other 75% was pumped to the pit before passing to the beds or field. No difference in drying characteristics has been noted in the two sludges. Both types become dry without odor nuisance or the breeding of house flies.

Measurements of the rates of draining of sludge pumped to the sand beds gave rather erratic results, indicating considerable variation in the drainability of sludge from time to time. The most that can be concluded, from the twenty beds measured, is that, in general, when the sludge from this installation is pumped to the beds with a solids content of 4½%, it will lose about one-half its moisture by drainage in the first 48 hr. The remainder of the moisture is lost by evaporation.

TABLE 8.—COST OF DIGESTION, SLUDGE DRYING, AND SUPERNATANT LIQUOR FILTRATION FOR THE YEAR 1934

Description	COSTS, IN DOLLARS					WEIGHT OF SLUDGE, IN TONS		UNIT COST, IN DOLLARS PER TON	
		Cost of Operation†							
	Plant cost*	Power, labor, and maintenance	Supervision	Interest and depreciation‡	Total	Raw	Digested	Raw§	Digested
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Digestion	132 000	3 075	380	10 580	14 035	8 470	4 800	1.66	2.92
Drying on sand beds	18 500	2 460	330	1 480	4 270	2 730	1 550	1.57	2.75
Drying on fields or lagoons	3 000	500	45	250	795	5 740	3 250	0.14	0.25
Filtration of supernatant liquor	3 300	440	55	260	755	6 770	3 840	0.11	0.20

* Does not include cost of land. † Does not include cost of removing sludge from beds. ‡ 8% of Column (2). § Column (6) ÷ Column (7). || Column (6) ÷ Column (8).

Costs.—Table 8 was prepared to illustrate costs for digestion and for drying on underdrained beds and open fields or lagoons. Since approximately 75%

of all the digested sludge is transferred to the pit prior to drying, the cost of the pit is included with the tank costs. It has little influence on unit costs for digestion. The cost of land on which tanks, pit, and drying areas are laid out is not included. Sludge is delivered wet, on the beds, to a fertilizer contractor who assumes the cost of its removal. The contractor pays the District $2 per ton of dry solids delivered on the beds and lagoons.

The gas engine, operated on gas from the digesters, has effected a net saving in power costs of $480 per month. No credits for this have been applied to the costs in Table 8.

CHEMICAL STUDIES

Multi-stage digestion as applied in this installation affords an excellent opportunity to study, under plant-scale operation, the chemical changes occurring in the digestion process. Although some of the information presented is abstract, its inclusion is thought to be of value in helping to furnish a clearer concept of the fundamental nature of the digestion process. Basic information obtained through studies of this kind may be expected to point the way to the solution of some of the problems of sludge digestion. The following phenomena were given attention in a chemical study carried on principally during the year, May, 1933, to April 1934: Changes occurring in the sludge liquor, changes of the ether-soluble matter, nitrogen balance, and composition of the gas.

Changes in the Sludge Liquor.—Analyses were made on samples of clear liquor separated from the sludge leaving each stage of the digestion process. No complete analyses were made on liquor from the pit; but instead, two samples of sludge from the fourth tanks were retained in bottles at 86° F for an additional period of 23 days and analyzed at the end of that time. These latter samples represent approximately the results that might be expected from a total digestion time of 32 days.

Most of the analyses were made on samples taken during two test periods: August 11 to 22, 1933, and most of the month of October, 1933. No essential differences appeared between the results covering the two periods, except in the case of phosphate. Table 9 shows the average results of all valid analyses, except that the phosphate values for the two test periods are tabulated separately.

The "mixture" is computed from the known proportions by volume of new and seed sludges added to the first stage. Analyses of filtered sewage are taken as representative of the liquor accompanying the new sludge. Actually, the composition of the liquor begins to change as soon as the sludge settles from the sewage. The method of calculation has the effect of including in the change attributed to the first stage that which occurs in the sludge in the bottom of the settling tanks.

Bicarbonate values in Table 9 were computed from the ion balance, as bicarbonate titrations were not made at the time of these liquor analyses. Some subsequent work on the bicarbonate titration indicates that, in liquor from digested sludge, the computed value is less than the experimental value by an average of about 290 ppm as HCO_3. The pH-values are derived from the computed bicarbonate concentrations, the carbon dioxide content of the gas,

TABLE 9.—SUMMARY; CHEMICAL CONTENT OF SLUDGE LIQUORS, IN PARTS PER MILLION

Description	Sew-age	Seed sludge	Mix-ture*	SAMPLE TAKEN FROM TANKS:				Content after additional detention of 23 days	Total number of individual analyses
				No. 1	No. 2	No. 3	No. 4		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Fixed solids	510	1 138	648	1 016	1 085	1 096	1 138	..	46
Volatile solids	315	1 092	486	1 815	1 687	1 122	1 092	..	46
Ammonia, as N	22	753	183	444	583	692	757	1 026	60
Organic nitrogen, N	8	116	32	148	93	64	77	113	12
Volatile acids, as acetic acid	26	177	59	829	790	470	228	57	46
Bicarbonate, HCO_3*	..	..	1 306	2 124	3 059	3 929	4 502	5 867	..
Silicon dioxide, SiO_2	32	49	35	59	103	80	100	39	12
Sulfate, SO_4:	154	10	122	8	8	22	15	17	23
Phosphate, PO_4									
August analysis	5	72	20	52	57	67	72	..	9
October analysis	6	70	20	82	61	31	24	35	15
Chloride, Cl	136	(155)	140	..	..	..	155	149	9
Calcium, Ca	73	304	124	190	251	277	282	290	25
Magnesium, Mg	22	64	31	43	51	60	56	56	24
Sodium, Na	147	164	151	..	..	166	..	153	7
Potassium, K	16	63	26	..	..	66	..	89	7
Iron, Fe	7	10	8	35	55	34	36	16	10
Hydrogen-ion concentration,* pH	..	..	..	6.74	6.93	7.06	7.14	7.26	..

* Computed.

and the ionization constant of carbonic acid.[5] The difference between computed and experimental bicarbonate values for digested sludge liquor would introduce a difference of 0.03 in the pH-value.

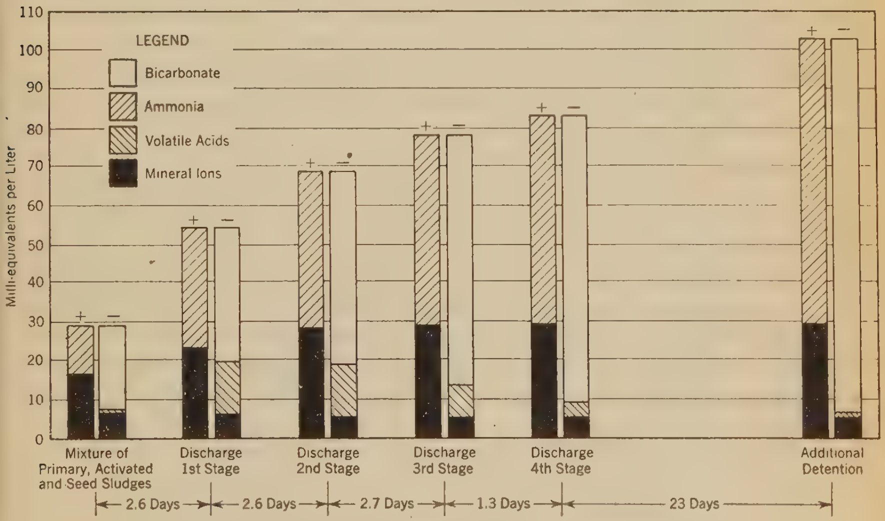

FIG. 6.—APPROXIMATE DETENTION TIME OF SLUDGE LIQUOR

Fig. 6 shows, graphically, the analyses reduced to reaction equivalents by dividing the parts per million by the reacting weights of the ions in question.

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[5] "Hydrogen Ion Concentration and Bicarbonate Equilibrium in Digesting Sludge," by A. P. Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, *Sewage Works Journal*, Vol. VI, 1934, p. 234.

Further consideration of some of the constituents reveals a number of interesting relationships, discussed in the following paragraphs.

Sulfate.—The rapid reduction of sulfate in the first stage is undoubtedly caused by bacterial de-oxidation with attendant liberation of sulfide ion. However, there is practically no sulfide in solution, the sulfur going to form ferrous sulfide. The apparent increase in sulfate in the later stages may have been due to chance variation of samples.

Phosphate.—Table 9 shows that in August the phosphate concentration increased regularly through the digestion tanks whereas in October the phosphate was at a maximum in the first tank and decreased progressively thereafter. It was thought that the decrease in the second period might be explained by precipitation of magnesium ammonium phosphate. This seemed likely in view of another observation. A flume that carries supernatant liquor from the pit has accumulated a crust of crystalline material, in some places to a thickness of an inch. Analyses of two samples of this material showed it to be about 96% $Mg\,NH_4\,PO_4 \cdot 6\,H_2O$.

In order to determine if this compound may precipitate in the digestion tanks, two solubility tests were made by saturating liquor from digested sludge with the pure compound, keeping the solution in equilibrium with sludge gas of known carbon dioxide content. The results yielded solubility products of of 1.06×10^{-12} at 25° C and 1.33×10^{-12} at 30° C.

On the basis of these results it is calculated that the digested liquor from the tanks, having the hydrogen-ion concentration and the content of magnesium and ammonium shown in Table 9, would be able to hold 305 ppm of phosphate in solution before becoming supersaturated. After 32 days of detention the liquor could hold 157 ppm. Hence, it does not seem possible that magnesium ammonium phosphate could precipitate in the digestion tanks, and the peculiar behavior of the phosphate in the October series of tests remains unexplained. The deposition in the flume can be explained as due to the loss of carbon dioxide and the resultant rise in pH. Such loss of carbon dioxide would be expected because of turbulence in the flume in question. If this loss allows the pH-value of the pit liquor to rise to 8.0, the proportion of phosphate that can be held is reduced to about 18 ppm (at 25° C); any excess over this concentration should precipitate.

Volatile Organic Acids.—The increase of organic acids in the first stage and their subsequent destruction supports the observations of other investigators. An attempt was made to analyze completely the volatile organic acids of sludge liquor from the first tank. A satisfactory conclusion was not reached, but by modification of an ether-extraction procedure,[6] tentative results were obtained, as follows:

Acid	Percentage
Formic	0
Acetic	63
Propionic	33
Butyric	0.1
Valeric	4

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[6] "Determination of Formic, Acetic, and Propionic Acids in a Mixture," by O. L. Osborn, H. G. Wood and C. H. Werkman, *Journal of Industrial Engineering Chemistry*, Vol. 25, 1933, p. 247.

A part of the fraction reported as valeric acid may be caproic acid. The large percentage of acetic acid undoubtedly came from decomposition of carbohydrates, whereas the presence of valeric acid suggested the de-aminization of amino acids formed by the bacterial hydrolysis of proteins. Tests for acids of the phenol group showed about 0.7 ppm (as phenol) in the liquor from the first stage, with smaller quantities in subsequent tanks.

Calcium.—The possible saturation of the sludge liquor with calcium carbonate is of some interest. In a solution saturated with calcium carbonate and in equilibrium with a gas phase containing carbon dioxide at a partial pressure, $p\ CO_2$, the following relationship should hold:

$$\frac{(Ca^{++})\ (H\ CO_3)^2}{p\ CO_2} = K \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The value of K is dependent upon the ionic strength of the solution according to the equation:

$$\log K = \log K_0 + 2\ \sqrt{\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which μ is the ionic strength and K_0 is the value of K at zero ionic strength. The values of K corresponding to the ionic strengths of the various sludge liquors have been derived from published data.[7] Results of calculations of the concentrations of calcium necessary to saturate the various sludge liquors are shown in Table 10. In all cases the calcium found is greater than the proportion calculated to be necessary to saturate the solution. The excess is probably accounted for as supersaturation of the liquor with respect to calcium carbonate. That the liquor is supersaturated is supported by observations made when the tanks were opened for cleaning. It was found that the walls, even in the first stage, were partly covered with a hard thin deposit of lime.

TABLE 10.—SOLUBILITY OF CALCIUM CARBONATE, $Ca\ CO_3$, IN SLUDGE LIQUOR

Liquor	Ionic strength	Values of K (concentrations, in mols per liter)	Values of partial pressures, $p\ CO_2$, in atmospheres	Bicarbonate, $HC\ O_3$ in parts per million	Ca required to saturate solution in parts per million calculated	Ca, in parts per million, observed
First tank........	0.062	4.4×10^{-6}	0.345	2 124	50	190
Second tank......	0.079	5.1×10^{-6}	0.313	3 059	25	251
Third tank.......	0.088	5.6×10^{-6}	0.303	3 929	16	277
Fourth tank......	0.094	5.9×10^{-6}	0.292	4 502	13	282
Digested 32 days..	0.113	6.6×10^{-6}	0.29	5 867	8	290

It is surprising that sufficient precipitation does not occur to cause a decline of calcium concentration before the sludge leaves the tanks, but there is evidence that such a decline does occur in the later stages of prolonged digestion. This evidence appeared in the samples which were subjected to the 23 days of additional digestion. A composite of the liquor from these two samples taken at the beginning of the experiment showed 332 ppm of calcium. At the end, the two samples showed, respectively, 316 and 263 ppm. Table 9 does not show any decrease for the reason that chance variations of individual samples obscure an effect which might be revealed by averages of greater numbers of samples.

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[7] *Journal*, Am. Chemical Soc., Vol. 51, p. 2082 (1929).

Iron.—The solubility of ferrous carbonate in sludge liquor is governed by equilibrium expressions similar to those applying to calcium carbonate. On the basis of solubility data,[8] it is calculated that if the sludge liquor from the fourth stage contained 0.02 ppm of iron as ferrous ion, the solution would be saturated with ferrous carbonate. The fact that much larger concentrations prevail suggests that the iron may be in the form of un-ionized or colloidal compounds. Supersaturation with respect to ferrous carbonate may also be a factor.

In one instance when a very large percentage of ferrous iron was present in the sludge going to the digestion tanks, the green color of the iron compounds persisted throughout the digestion process. A sample of this green sludge, when treated with acid, liberated a quantity of carbon dioxide that could be accounted for only by assuming that the iron was present as ferrous carbonate.

Digestion of Ether Soluble Matter.—Determinations of ether soluble matter were made on samples of primary, activated, tank, and pit sludges. (An ether-soluble determination is made as follows: A weighed sample of dried (105° C), pulverized sludge is placed in a weighed Gooch filter; washed with hydrochloric acid (H Cl), and water; dried and re-weighed; extracted for 3 hr with ether under a reflux condenser; dried and re-weighed, the loss in weight being reported as a percentage of ether soluble material on the basis of the original dry weight. By this procedure the insoluble soaps are represented by free fatty acids in the ether extract.) All analyses but one were made on samples taken between May 15 and June 10, 1933. The results are shown in Table 11.

TABLE 11.—PERCENTAGE OF ETHER SOLUBLE MATTER IN SLUDGES

Type of sludge	Number of analyses	Maximum	Minimum	Average	Type of sludge	Number of analyses	Maximum	Minimum	Average
Primary	6	20.7	16.0	18.2	Digested	7	11.6	7.8	9.1
Activated	6	22.7	13.9	20.3	Pit seed	2	8.5	6.7	7.6

Composites of the material separated by ether extraction were analyzed for various constituents. In the case of the ether soluble material taken from the digested sludge, the free acids, after being separated from the remainder and dried, changed in part to a material that could be re-dissolved neither in fat solvents nor in dilute alkali. The change probably involved oxidation and polymerization. A repetition of the extraction and drying of the part that was soluble in potassium hydroxide caused more of it to become insoluble, so that it is estimated that at least one-half the free acids in the ether soluble material of the digested sludge were of an unstable type; the true fatty acids may be only a small proportion of the total acid fraction. In view of this behavior, it seems likely that the original digested sludge contains substances which, in the original state, would be soluble in ether, but which change to insoluble forms when the sludge is dried, acid treated, and re-dried during analysis.

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[8] *Journal*, Am. Chemical Soc., Vol. 40, p. 879.

Table 12(*a*) shows the results of the analyses. Some of the acid fraction may have been phenolic acids. A qualitative test for phenolic acids in the ether extract from the digested sludge gave a strong positive result; and yet, as noted previously, only traces of phenols are present in the liquor.

TABLE 12.—ETHER-SOLUBLE MATTER

Constituent	(*a*) ANALYSES OF ETHER EXTRACTS			(*b*) DIGESTION OF ETHER-SOLUBLE MATTER		
	Primary	Acti-vated	Di-gested	Original sludge, in pounds	Sludge leaving the tanks, in pounds	Per-centage lost
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Total weight of ether soluble material, in pounds	....	....	....	100.0	29.3	71
Free acids*	66%	62%	26%	63.4	7.6	88
Saponifiable fats	28%	20%	28%	22.9	8.2	64
Non-saponifiable matter	6%	18%	46%	13.7	13.5	2
Phosphate P_2O_5	0.11%	0.06%	0.07%	....	....	..
Equivalent weight of acids	306	309	272	....	....	..

* Insoluble soaps in sludge.

Each 100 lb of ether soluble material entering the digestion tanks contains the quantities of various constituents shown in Table 12(*b*); the fates of these constituents are also shown. The "fatty acids" are present in the sludge in combination with calcium and magnesium as insoluble soaps. Taking into account the foregoing observations, indicating that very little true fatty acids are present in the digested sludge, it appears that the insoluble soaps are destroyed quite completely in the digestion process. The very low reduction of the non-saponifiable matter (mineral oils, etc.) is to be expected.

Calculations based upon the equivalent weights of the fatty acids and the other data as given, indicate that the quantities of acids destroyed per day is equivalent to 316 lb of calcium. An equivalent quantity of bases should be released. From the liquor analyses it is calculated that 208 lb of calcium and 33 lb of magnesium per day appear in the sludge liquor. If the magnesium is converted to the calcium equivalent and added to the value for calcium, the result is 263 lb of calcium equivalent. The agreement of this result with 316 lb from the grease analyses, is considered good in view of the many sources of error. The difference is in the direction that would be accounted for by the loss of some of the acids in the digested-sludge analysis through oxidation and polymerization, and by the precipitation of calcium carbonate in the tanks.

It was thought that phosphorous-containing fats might be a significant source of the phosphate that appears in the sludge liquor. However, calculations indicate that only 7 lb per day of phosphate would be released by the digestion of the ether soluble matter, whereas the sludge liquor analyses of August, 1933, showed 70 lb per day going into solution.

Nitrogen Balance.—Nitrogen analyses were made on samples of wet sludges during May, and June, 1933, with results as shown in Table 13(*a*). On the basis of estimates of the weights and volumes of the various sludges pumped during May and June, 1933, calculations have been made of the "through-put" of nitrogen, with results as shown in Table 13(*b*). The quantities of nitrogen entering and leaving the tanks are in good agreement, indicating that there is no appreciable loss. Since this is a rather important point, a careful experiment was performed to determine if any loss of nitrogen could be detected. A laboratory sample of a mixture of strained raw and activated sludges was seeded, and carefully sampled; and five total nitrogen determinations were made by the Kjeldahl method. The mixture was incubated for 33 days, sampled, and analyzed as before. The nitrogen balance in digestion, in parts per million of nitrogen, was found to be as follows:

TABLE 13.—NITROGEN BALANCE

Item No.	Sludge	(*a*) NITROGEN ANALYSES OF WET SLUDGE				(*b*) NITROGEN "THROUGH-PUT" FOR MAY AND JUNE, 1933		
		Number of analysis	Total Nitrogen, Percentage of Dry Weight			Dry weight, in pounds	Nitrogen Analysis	
			Maximum	Minimum	Weighted average		Percentage of dry weight	Pounds per day
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Primary	5	2.50	2.21	2.40	21 950	2.40	527
2	Activated	11	6.51	4.30	5.21	31 770	5.21	1 655
3	Pit seed	4	6.11	5.57	5.84	10 100	5.84	590
4	Total input	..	....	....	....	....	....	2 772
5	Digested sludge	9	6.36	5.15	5.85	45 290	5.85	2 648
6	Stratified liquor	..	....	....	750*	69 000	750*	52
7	Scum and sand accumulation	..	....	....	....	650†	6.5‡	42
8	Total output and accumulation	..	....	....	....	....	....	2 742§

* 750 ppm; third tanks estimated. † Organic matter. ‡ 6.5% of organic matter (estimated).
§ Compared Items Nos. 4 and 8.

	Before digestion	After digestion
	2 089	2 157
	2 084	2 114
	2 105	2 105
	2 070	2 077
	2 054	2 086
Average	2 080	2 108
Corrected for loss of weight during digestion	2 080	2 073
Average difference from mean	15	26
Probable error of averages	7	7
Probable error of difference	10	

The average concentration of nitrogen after incubation is 28 ppm greater than in the original mixture. When a correction is made for the decrease in weight of the sludge due to loss of gas, the quantity of nitrogen in the final mixture, on the basis of the original weight, is less than in the original mixture by 7 ppm, or 0.34 per cent. On the basis of deviations of the individual determinations it is estimated that the probable error of the difference is 10 ppm; hence, the apparent loss is not significant. It is probable that if an actual loss did occur, it did not exceed 17 ppm of Kjeldahl nitrogen. These results are in accord with conclusions of Messrs. A. M. Buswell and C. S. Boruff.[9]

As the sludge originally separates from the sewage, 98.6% of the nitrogen in it is held in the solids, and 1.4% is in the form of ammonium compounds or organic nitrogen in solution. When the sludge leaves the tanks, 56.8% of the original nitrogen is still held in the solids, and 43.2% is in solution. At the end of 23 days of additional digestion, the original nitrogen still held in the solids is reduced to 41.3 per cent.

If the sludge from the tanks is dried on sand beds, two-thirds of the water originally present with the sludge will have been removed as supernatant liquor and as drainage from the beds by the time the water ceases draining from the sludge. The water remaining with the sludge will hold 14.2% of the nitrogen originally present in the sludge. Prolonged and thorough drying of the sludge removes the ammonium nitrogen almost completely, but when the sludge is dried, as is usual, to about 50% moisture, 15% of this ammonium nitrogen, and presumably all the soluble organic nitrogen, remain in the moisture of the sludge. Of the nitrogen originally present 60% will still be available for fertilizer value. If the sludge is digested for an additional 23 days and then dried to 50% moisture, 46% of the original nitrogen will still be available. Of the nitrogen fertilizer value remaining after digestion in the tanks, 23% is lost due to digestion in the pit. These considerations point to the desirability of a short digestion period when the sludge is to be used for fertilizer. It must be noted, however, that a substantial amount of digestion occurs in the sludge from the tanks during drying; this is not taken into account in the foregoing calculations.

The digestion of nitrogenous matter roughly parallels the digestion of other organic matter, with the result that the percentage of nitrogen in the solid organic matter does not show any great change. If the insoluble nitrogen is calculated as percentage of the solid organic matter, the value is 5.79% for the new sludge, 6.51% for the sludge from the tanks, and 6.09% for the sludge after 32 days of digestion. The total nitrogen in the sludge from the pit (including the soluble portion) dried on the sand beds, as calculated from the analyses of the new sludges and the analyses of the sludge liquors, should be 6.54 per cent. Samples of dried pit sludge taken in May, 1934, showed a nitrogen content corresponding to 6.49% of the organic matter (average of eight analyses from two beds). Thus, again, all the nitrogen is accounted for. The average organic content of this sludge, lowered in part by inclusion of sand, was 47.5%; hence the nitrogen was 3.08% of the dry weight of the sludge. Nitrogen analyses at other times indicate that this represents average conditions.

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[9] "The Relation Between Chemical Composition of Organic Matter and the Quality and Quantity of Gas Produced During Sludge Digestion," by Buswell and Boruff, *Sewage Works Journal*, Vol. IV, 1932, p. 450.

Summary of Changes in the Solids.—If the proteins of the sludge solids are assumed to be 6.25 times the nitrogen percentage, the organic matter may be divided into ether soluble matter, protein, and cellulose and miscellaneous, and the fates of these three types of organic matter may be calculated. Results of such calculations, on the basis of 100 lb of original organic matter, are shown in Table 14.

TABLE 14.—ANALYSIS AND FATE OF ORGANIC MATTER IN SLUDGE

Sludge	ORGANIC MATTER		ETHER SOLUBLE MATTER		PROTEIN		CELLULOSE AND MISCELLANEOUS	
	Total weight, in pounds	Percentage destroyed	Total weight, in pounds	Percentage destroyed	Total weight, in pounds	Percentage destroyed	Total weight, in pounds	Percentage destroyed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
New sludge	100	....	27.6	..	36.2	..	36.2	..
Sludge from tanks	52.1	47.9	8.1	71	21.2	41	22.8	37
Sludge after thirty-two days of digestion	42.2	57.8	6.5	77	16.1	56	19.6	46

Gas Analyses.—Samples of gas from various stages of the digestion process and from the laboratory supply, a mixture from all the tanks, were analyzed for various constituents. The results, subject to certain limitations, as described subsequently, are shown in Table 15.

Carbon Dioxide.—Several analyses for carbon dioxide were made by the usual absorption method. In other cases, the density of the gas was determined and the carbon dioxide percentage was estimated from an experimentally determined correlation between density and percentage of carbon dioxide.

From the known solubility of carbon dioxide in water, it is calculated that the sludge liquor from the last tanks contains 373 ppm of free CO_2 in solution. When sludge is dried for analysis, a large additional quantity of this gas is lost by decomposition of the bicarbonates, especially ammonium bicarbonate. This loss would amount to an average of 3 030 ppm for all the liquor from the tanks. In volume, the free carbon dioxide and the bicarbonate, CO_2, that would be lost upon evaporation would amount to 30 100 cu ft per day, a quantity one-third as great as that of carbon dioxide in the gas evolved from the tanks.

Hydrogen Sulfide.—Various methods were tried for the H_2S-determination. Precipitation of lead sulfide, Pb S, from a lead acetate solution containing 2% acetic acid, with filtration and weighing of the precipitate, gave fairly consistent results; but this method is open to theoretical objections. Results considered to be more reliable were obtained by passing the gas through bromine water, and precipitating and weighing the resulting sulfate as barium sulfate.

In the discussion of iron in the sludge liquor, evidence was given showing the presence of ferrous carbonate in the sludge on at least one occasion. The iron content of the sewage at this plant is usually high due to trade wastes. The sludge contains an average of 2.46% of iron (as Fe). The efficiency with which the sulfide, formed by reduction of sulfate and decomposition of organic

TABLE 15.—GAS ANALYSES

Description (1)	PERCENTAGE DETERMINATIONS						Pounds per day (8)	Cubic feet per day (9)
	First stage (2)	Second stage (3)	Third stage (4)	Fourth stage (5)	Laboratory (6)	Weighted average (7)		
(a) CARBON DIOXIDE, CO_2								
Number of determinations	14	9	8	4	3	....	....	....
Content: Maximum.....	39.7	33.2	31.5	31.4	35.4	....	....	....
Minimum......	31.7	29.0	28.7	28.4	32.5	....	....	....
Average.......	34.5	31.3	30.3	29.7	33.8	32.4	9 910	87 500
(b) HYDROGEN SULFIDE, H_2S								
Number of determinations	5	2	3	1	6	....	....	....
Content: Maximum.....	0.023	0.0014	0.0039	....	0.0530	....	....	....
Minimum......	0.00014	0.0001	0.00009	....	0.00058	....	....	....
Average.......	0.0068	0.0007	0.0026	0.0035	0.012	0.0043	1.0	11.6
(c) CARBON MONOXIDE, C O								
Number of determinations	2	....	....	2	....	....	....	....
Content: Maximum.....	0.047	....	....	0.032	....	....	....	....
Minimum......	0.015	....	....	0.017	....		....	....
Average.......	0.03	....	....	0.02	....	Not more than 0.03	5.8	80
(d) HYDROGEN, H_2								
Number of determinations	2	....	....	2	....	....	....	....
Content: Maximum.....	0.018	....	....	0.018	....	....	....	....
Minimum......	0.018	....	....	0.00	....		....	....
Average.......	0.018	....	....	0.01	....	Not more than 0.02	0.3	50
(e) OXYGEN, O_2								
Content...............	....	....	....	....	0.004	....	....	....
(f) PHOSPHINE, PH_3								
Number of determinations	1	....	....	1	1	....	....	....
Content...............	0.00001	....	....	0.0001	0.0001	....	....	....
(g) AMMONIA, NH_3								
Content: Computed.....	0.00010	0.00025	0.00043	0.00062	....	0.00025	0.030	0.68
Observed......	....	....	....	0.00088	....	....	....	....
Observed......	....	....	....	0.00093	....	....	....	....
(h) METHANE, ETC. (BY DIFFERENCE)								
Content...............	65.5	68.7	69.7	70.3	....	67.6	7 760	182 400

matter, is converted to ferrous sulfide, indicates that the iron is in the form of a quite reactive compound. It seems quite probable that the sludge usually contains ferrous carbonate.

Equilibrium considerations lead to some interesting conclusions regarding the percentage of hydrogen sulfide in the gas from a sludge containing both ferrous carbonate, $Fe\,CO_3$, and ferrous sulfide, $Fe\,S$. The following equilibrium may be considered to exist:

$$Fe\,S + CO_2 + H_2O = H_2S + Fe\,CO_3 \ldots\ldots\ldots\ldots\ldots (4)$$

The equilibrium constant expression is:

$$\frac{p\,H_2S}{p\,CO_2} = K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

From data on the solubilities of ferrous carbonate [9] and of ferrous sulfide,[10] the value of K is computed to be 10^{-4}. This may be in error by as much as a factor of two due to uncertainty in the solubility of ferrous sulfide.

If the gas and sludge liquor were really in equilibrium with both ferrous sulfide and ferrous carbonate, and if the gas contains about 30% of carbon dioxide, the concentration of hydrogen sulfide should be about 0.003 per cent. Higher values might be expected if there is insufficient opportunity for reaction of the sulfides to gain equilibrium with the $Fe\,CO_3$. Low values might be found if a trace of air should get into the digestion tanks, because conditions would be favorable for a rapid oxidation of the hydrogen sulfide.

Referring to Table 15(*b*), it is seen that one sample of laboratory gas gave a value seventeen times as great as the computed value, and that, in other cases, the hydrogen sulfide was less than one-thirtieth of the computed value. Fortuitously, the average concentration of hydrogen sulfide is in fair agreement with the value of 0.003 per cent. The most that can be concluded is that, under the conditions of these observations, the equilibrium serves as a moderating factor, holding the percentage of hydrogen sulfide within limits.

According to theory, the concentration of hydrogen sulfide should be independent of the hydrogen-ion concentration as long as both iron compounds are present. Hence, it should be the same at all stages of digestion, except for small changes in the percentage of carbon dioxide. The data show no pronounced trend in the percentages of hydrogen sulfide by stages.

Analyses for mineral sulfides in the sludge show an increase of 153 lb per day in the digestion process. If all this sulfur formed hydrogen sulfide instead of being combined with iron, it would be equivalent to 0.7% of the gas. However, even if no iron were present, approximately one-half this hydrogen sulfide would remain in solution in the sludge liquor.

From the known solubility of hydrogen sulfide in water, it is calculated that sludge liquor in equilibrium with gas containing the average concentration of hydrogen sulfide (0.0043%) would contain about 0.13 ppm of free hydrogen sulfide. The total dissolved sulfides, calculated from the pH-value and the ionization constant [11] of hydrogen sulfide, should be about 0.3 ppm. The gas which, in one case, was found to contain 0.058% of hydrogen sulfide must have come from liquor containing about 4 ppm of sulfides.

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[10] *Zeitschrift für Anorganische Chemie*, Vol. 65, p. 136 (1909).

[11] "Relationship Between Free Hydrogen Sulfide and Total Sulfides in Sewage," by Richard Pomeroy, *California Sewage Works Journal*, Vol. VI, No. 2 (1934), p. 54.

Carbon Monoxide, Hydrogen, and Phosphine.—Carbon monoxide was determined by the iodine pentoxide method, and hydrogen by combustion with palladium catalyst and weighing of the water. These methods do not positively establish the presence of either gas; they do establish upper limits for the concentrations. Phosphine was determined by oxidation with bromine water and subsequent determination of the phosphoric acid. The presence of phosphine in sludge gas has been suggested by other writers, but apparently has not previously been determined. The quantity is a mere trace.

Ammonia.—From the ionization constant of ammonium hydroxide, the concentration of ammonium ion in the liquor, and the pH-value, it is possible to compute the concentration of ammonium hydroxide, NH_4OH, in the liquor. From the known solubility of ammonia one may compute the equilibrium percentage of NH_3 in the gas. Table 16 shows the calculated NH_3 content of the gas in equilibrium with the various liquors which were analyzed.

TABLE 16.—COMPUTED VALUES OF AMMONIUM CONTENT

Liquor discharge	Ammonium nitrogen, in parts per million of nitrogen	Hydrogen ion concentration, pH	Ammonium hydroxide, NH_4OH, in parts per million of nitrogen	Percentage of ammonia, NH_3, in the gas
First tank	444	6.74	1.33	0.00016
Second tank	583	6.93	2.74	0.00033
Third tank	692	7.06	4.43	0.00053
Fourth tank	757	7.14	5.90	0.00071
Twenty-three days additional digestion	1 026	7.26	10.26	0.00123

The analyses of sludge liquors were made in all cases on the liquor flowing from one tank to the next. Hence, they do not represent the average composition of the liquors in the tanks. The gas from any tank would be expected to be intermediate in composition between the gases in equilibrium with the liquors entering and leaving that tank. It is upon this basis that values shown in Table 15 were estimated.

Ammonia in gas from the fourth stage was determined by bubbling a large volume through acid solutions and subsequently adding potassium hydroxide, KOH, and distilling and titrating. Two determinations gave 0.00088% and 0.00093% as against the computed value of 0.00062 per cent.

Olefines.—A volume of 1 800 liters of gas from the first stage was passed at a rate of about 20 liters per hr through bromine water and then into a 2-liter flask cooled with ice. At the end, the excess bromine was allowed to evaporate and the reaction tube and receiving flask were washed with ether. Evaporation of the ether solution left 2 or 3 grams of strongly lachrymatory solid and liquid bromine compounds.

If the gas contained ethylene, ethylene dibromide should be present in the product. An attempt was made to remove ethylene dibromide by vacuum distillation, but none could be obtained. If it was present, it certainly did not exceed 0.01%, and probably was not more than 0.001 per cent. From the solid portion of the mixture a small quantity of a compound was crystallized, which was identified as 1, 2, 3, 4 tetrabromo-butane, showing the presence of buta-diene ($CH_2 : CH \cdot CH : CH_2$) in the gas. However, the proportion of this olefine in the gas did not exceed 0.1% and possibly was not more than 0.001 per cent.

Other Hydrocarbons.—A sample of gas was analyzed by the research laboratory of a local oil company to determine the possible presence of hydrocarbons other than methane. Results were reported as follows:

Ethane, C_2H_6	0.04%
Propane, C_3H_8, and butane, C_4H_{10}	0.01%
Pentane, C_5H_{12}, and heavier	0.07%

Nitrogen.—The usual method of combustion in a combustion pipette is not satisfactory for determining quantities of nitrogen of the order of 2%, or less. No apparatus was available for more suitable methods.

Facts previously cited lead to the belief that very little nitrogen is present in the gas. Analyses of sludges to and from the digestion system did not indicate any loss of nitrogen from the sludge. Laboratory experiments indicated that the loss, if any, probably did not exceed 17 ppm. If this loss went to elemental nitrogen, it would amount to 0.12% by volume of the gas. Atmospheric nitrogen dissolved in the sewage (not included in the nitrogen analyses) would be gassed out, and under the conditions prevailing at this installation would yield about 0.09% of nitrogen in the gas. No other sources of nitrogen in the gas are apparent, except atmospheric contamination, and ordinarily such contamination is effectively excluded in this installation.

Methane.—Samples of the gas were passed through a train of caustic potash absorbers to remove the carbon dioxide, and the density of the residual gas was determined. The density of pure methane relative to air is 0.5544. Eleven determinations of the density of the methane fraction gave values from 0.562 to 0.577, with no consistent differences between the different stages. The average is 0.5678, 2.4% greater than the density of pure methane. The higher hydrocarbons known to be present should raise the density of the methane fraction about 0.6 per cent. The remainder of the effect is doubtless due to various other unindentified constituents.

SUMMARY AND CONCLUSION

(1) Results of a 4-yr study of the operation of a four-stage digestion system disclose that, with a nominal detention time of about 12 days, a reduction of 47% of the organic matter added was achieved. The sludge produced dried satisfactorily without odor or fly nuisance, and showed no appreciable difference in drying characteristics when compared with sludge digested three times as long.

(2) The results are also quite conclusive in determining that agitation by mechanical equipment is not needed in separate sludge digestion.

(3) Controlled seeding of raw solids with digested sludge, as accomplished in the multi-stage process, may be a factor in efficient digestion.

(4) The design of the tanks permitted a large accumulation of scum without reducing the efficiency of the system.

(5) The survey reveals that no appreciable liberation or absorption of heat occurred during the digestion process.

(6) The presence of relatively large quantities of iron salts in the sludge appears to have no deleterious effect upon the digestion process, and is beneficial in holding the concentration of hydrogen sulfide in the gas to a very low value.

(7) No nitrogen is lost in the digestion process. A portion is converted into soluble form and appears in the sludge liquor. The quantity remaining in the solids is approximately proportional to the organic matter remaining.

(8) The gas from the first stage is not substantially inferior to gas from the other stages as regards utilization for fuel; 47% of all gas was produced in the first stage.

DISCUSSION

EDWARD W. MOORE,[12] ESQ. (by letter).—In that it not only treats of a interesting development in the technique of sludge digestion, but also pionee in the application of certain principles of physical chemistry to the study the behavior of digesting sewage sludge, this paper appeals to the writer as particularly valuable contribution to the literature of sludge digestion. T value of the concepts and methods of physical chemistry to the practisin engineer is as yet recognized too infrequently.

Another physio-chemical concept which yields fruitful results when applie to the decomposition of organic matter both under aerobic and anaerob conditions is that of reaction velocity. The course of the decomposition organic matter under both aerobic and anaerobic conditions can be formulate approximately as a uni-molecular reaction, the rate of which is given by constant, k, known as the reaction velocity constant. For aerobic decom position at 20° C, k has a value of 0.1, as shown by the work of Professor Ea B. Phelps, Victor L. Streeter, Jun. Am. Soc. C. E., E. J. Theriault, and other The writer, in collaboration with G. M. Fair, M. Am. Soc. C. E., has bee engaged in collecting data from which the value of k for anaerobic decompos tion at various temperatures may be computed. For 85° F the value of obtained from the data thus far collected, is 0.066 for digestion of slud seeded with material previously digested at this temperature. From t bottle experiments made by the authors, the ultimate total gas productio of the Los Angeles sludge may be assumed to be about 625 cu cm per gra of volatile matter, and the volume produced (say, in 10 days' detention tim is 485 cu cm. From these data and assumptions a value of $k = 0.065$ indicated.

It is of interest to the writer to note that, in sludge digestion studies, tw questions perpetually recur, namely, the relationship between the weight the gas produced and the weight of the volatile matter destroyed, and t evolution or absorption of heat by the reactions of digestion. The autho find that the weight of the gas produced is 7% more than the weight of t volatile matter destroyed, despite the fact that their total gas production p unit of volatile matter added to the tanks is very low (485 cu cm per gram volatile matter added). The work of A. M. Buswell and his associates,[13] a that of Professor Fair, and the writer[14] has indicated that the weight of t gas may exceed the weight of the volatile matter destroyed by as much 28 per cent. This increase in the weight of the decomposition products ov the weight of the original material is explained by Buswell as due to the fa that water enters into the reactions by which the gas is formed. Wille Rudolfs, M. Am. Soc. C. E.,[15] has held that the weight of the gas does n exceed that of the volatile matter destroyed.

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[12] Instructor in San. Chemistry, Harvard Graduate School of Eng., Cambridge, Ma

[13] *Sewage Works Journal, No. 4,* p. 454 (1932).

[14] *Loc. cit., No. 4,* p. 756 (1932).

[15] *Loc. cit., No. 4,* p. 444 (1932).

Sludge digestion consists of a complex of reactions, some of which are exothermic (heat-evolving) and others endothermic (heat-absorbing). The over-all heat effect (that is, the actually observed heat absorption or evolution during the course of digestion) is the sum of the effects of the individual reactions. The work of Messrs. Keefer and Kratz showing the over-all process to be endothermic was based on a few experiments in which gas production per unit of volatile matter was relatively high. Prior work by Professor Fair, and the writer had indicated on theoretical grounds that since the endothermic reactions of the digestion process were concerned with dissolution and evolution of the gas, the over-all process would be expected to be endothermic when gas production per unit of volatile matter added was high, and exothermic when it was low. The results reported in the paper would be expected to be exothermic rather than endothermic on account of the low gas yield.

Despite the layer of lime scale formed on the heating pipes of the digestion tank, the value of the coefficient of heat transfer from water to sludge is about 13 Btu per hr, per sq ft of pipe surface per degree of temperature differential, which is as good as most values reported in practice. It would be of interest to know the average temperature of the hot water entering the coils, since this is believed to control the degree of incrustation of the pipes. It would also be interesting to calculate the heat transfer coefficient from the tank to its surroundings from the data following Table 7 and the surface area of the tank, since the matter of the rate of heat loss from digestion tanks still requires considerable study.

Some sludge digestion studies have indicated the presence of hydrogen and carbon monoxide in the gas produced. No dependable evidence of the presence of these gases in significant quantities has been obtained in the Harvard University laboratories, although hundreds of analyses have been made. The very delicate methods used by the authors show only traces of these gases, and, as the authors state, do not definitely establish their presence. It is possible that much of the hydrogen and carbon monoxide reported in sludge digestion studies is due to errors in the determination of methane by combustion. The determinations of hydrogen sulfide are of practical as well as theoretical interest, since the corrosiveness of the combustion products of the gas is determined largely by the quantity of hydrogen sulfide it contains. The writer believes that some of the sulfur present in the gas evolved from digesting sewage sludge is present in the form of organic homologues of hydrogen sulfide, particularly in the case of thermophilic digestion. Some of the foul odors carried by these gases cannot be accounted for by the presence of such small percentages of hydrogen sulfide.

The fact that none of the nitrogen contained in the digesting sludge appears in the gas as elemental nitrogen is confirmed also by the results of the Harvard studies on sludge digestion. This conclusion, however, cannot be extended to the decomposition of other organic deposits, such as those found on the bottoms of lakes and streams. Studies of the decomposition of such deposits by Allgeier, Peterson, Juday, and Birge[16] and by Professor Fair and the writer,[17] indicate the production of substantial quantities of nitrogen gas. Since these deposits are similar to partly digested sludges, the radical difference in the behavior of the two materials has not as yet been explained.

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[16] *International Revue der Gesamten Hydrobiologie und Hydrographie,* Vol. 26, p. 444 (1931–32).

[17] *Civil Engineering,* December, 1937, p. 846.

The disappearance of phosphate from the sludge liquor during the second test period (see Table 9), might possibly be accounted for by absorption of the phosphate by organisms carried down with the solid matter of the sludge. Bio-chemists often use the decrease in soluble phosphate as a measure of the growth of organisms in a medium.

WILLEM RUDOLFS,[18] M. AM. SOC. C. E., AND H. HEUKELEKIAN,[19] ESQ. (by letter).—The attempt to make use of the observed phenomenon of automatic segregation of solids during digestion by withdrawing particles of digested sludge from the bottom and allowing the more active mass to remain longer in the tanks is of unusual interest. It is well known that a portion of properly seeded fresh solids is rapidly decomposed, another portion will sink to the bottom of a tank, and still another part of the fresh solids (including fatty material, hair, match sticks, soap, etc.) remains floating for a considerable time while undergoing decomposition. Withdrawal of the material segregated at the bottom causes a gradual increase in the quantity, and frequently in the density, of the floating mass. This inherent behavior of the sludge is well illustrated by the authors in Fig. 4. Inability of the scum to move with the separated sludge to the next tank results in scum formation and this material may stay in the primary, secondary, or even tertiary tank for a long time, while the inorganic and partly digested solids pass on. This method of segregation is undoubtedly theoretically correct, because it permits the material most difficult to digest to remain longest in the tanks. On the other hand, the method results automatically in increasing quantities of scum in the first tanks, unless the digestion time allowed is ample to take care of the accumulating mass, or distribution of sludge; that is, unless so-called sludge circulation is practiced. If the digestion time is insufficient the mass of scum eventually becomes so large that the capacity of the tanks is reduced materially and the scum must be taken care of by other means. The results graphically shown in Fig. 4 seem to indicate that after twenty-five months of operation the capacities of Tanks 1, 2, and 3 had been reduced (on account of scum formation) by 25 to 30 per cent. The tanks were emptied and cleaned, and it would be of interest to know what was done with this scum, which apparently digested actively when placed in suitable containers.

Of considerable interest is the fact that the average solids content of the new sludge pumped to the digesters was 4.7 per cent. The sludge mixtures consisted of 33% of primary and 67% of activated sludge by volume. This rather high solids concentration may be a contributory factor for the short digestion period in the digesters. In considering the short digestion period of 10.3 to 13.5 days, the open pits, serving as balancing reservoir, storage tank, and secondary digestion unit, are of importance. The scum formation, occasional foaming, and density of the subnatant and supernatant liquors indicate that the digesters were pushed to the limit and occasionally, possibly, beyond the limit. The foaming probably caused by the combination of using occasional poorly digested sludge for seeding and rapid gasification in some of the units, is perhaps the best indication of over-loading, together with the fact that supernatant liquor removal was promoted in the pits after storage.

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[19] Associate, Div., Water and Sewage Research, State Agri. Experiment Station, New Brunswick, N. J.

The fact that good digestion results were obtained during the short digestion period in the tanks, followed by after-digestion in the pits, is of practical interest, but is not surprising. Similar results have been obtained elsewhere in the laboratory, in pilot plant trials, and in full-scale plant operation. Where no after-digestion facilities were available, sooner or later the difficulty was to produce persistently a rapidly draining, digested sludge. In a large part of the country open-air drying of sludge, which retains its moisture tenaciously, is limited to summer or to warm, dry seasons. The importance of secondary digestion should not be over-looked and, admittedly, it is a factor in the case under discussion. The over-all results obtained in the tanks, a reduction of 46 to 47% of volatile matter, indicate satisfactory digestion. This fact, together with the separation of water in the storage pits, seems to leave no doubt that the process utilized was successful as far as volatile matter reduction and gasification were concerned.

Actively digesting masses of sludge do not show temperature stratifications. The mass is in constant movement and, as has been stated previously, fresh and actively digesting solids entrain gases more readily than old or digested solids. The proportion of solids carried up by entrained gas depends, therefore, on the relative quantities of fresh solids. In an actively digesting mass of sludge the solid particles are in constant motion, either up or down, depending upon whether or not they carry entrained gas, so that the liquid and its contents are in a state of dynamic equilibrium. Since the sludge masses in the tanks were constantly stirred by the gas formation and the flow of sludge from tank to tank, only little, if any, temperature stratification would be expected. In cases where sludge is digested and stored in a single tank, the picture is different, depending upon the mass of actively digesting material and the capacity of the tank. When the storage capacity of the tank is large and considerable quantities of ripe sludge are retained, temperature stratification is likely to be encountered.

The data submitted on heat transfer, although limited, are of interest. The difference of 500 000 Btu between the heat applied and the heat required to raise the temperature of the new sludge was only 3.6% of the total heat input. Without having detailed results it is difficult to state how much of the loss can be accounted for by experimental error. Unfortunately, no results are available for possible radiation losses. The information on this phase of the problem is scarce, but seems to indicate that the difference in free-heat content of the raw and digested sludge and its subtraction from the total heat input, could be accounted for by radiation losses.

The general statement of the authors, that the 25% of sludge wasted directly from the digestion tanks to the drying beds showed no difference in drying characteristics from the 75% of sludge pumped from the pits, does not show what the cause was of the "rather erratic results, indicating considerable variation in drainability of sludge from time to time." In general, the 4.5% sludge lost about one-half its moisture in 48 hr. Extensive investigations with partly and well-digested sludges have shown that the quantity of water drainable on sand beds during the first 24 hr varies greatly with the degree of digestion. Well-digested sludge (52% solids) loses as much as 85% of its total moisture (at 74° F) by drainage, and the remainder is lost by evaporation. With increasing temperatures, evaporation increases. The drainage from incompletely digested solids is much less and slower. Even when 80% of the possible gas production has evolved, drainability is comparatively slow, amounting to from 40 to 50% of the total moisture content in 24 hr.

Nitrogen balances made some years ago[20] showed that there was no reduction in the total nitrogen content during digestion of sewage solids. On the basis of original solids the percentage of nitrogen increases in the digested sludge on account of the decomposition and loss of other materials into gases. The authors reach the conclusion that "the digestion of nitrogenous matter roughly parallels the digestion of other organic matter, with the result that the percentage nitrogen in the solid organic matter does not show any great change." This is similar to the conclusion reached previously. As the authors state, this does not mean that the nitrogen in the raw sludge does not undergo transformation, converting the insoluble nitrogen into soluble forms. During digestion, active hydrolysis of nitrogenous materials in the sludge takes place, resulting in an increase in ammonia, amino acids, and other soluble forms of nitrogen. The possibility of liberation of nitrogen gas from organic matter under anaerobic conditions does not seem great. Nitrates, if present, would be reduced, under anaerobic conditions, with the liberation of nitrogen gas. Nitrogen may also be formed by the interaction of nitrites with amino acids, but under anaerobic conditions neither nitrates nor nitrites could be produced. The loss of nitrogen from digesting tanks really occurs by the discharge of supernatant liquor, containing the various soluble nitrogen compounds. A second, and material, loss occurs when the digested sludge is placed on sand drying beds. The soluble nitrogenous materials disappear with the water drained. If these soluble nitrogenous materials could be retained, the digested sludge would have considerable value as a fertilizer. Activated sludge contains more nitrogen because a large part of the soluble nitrogen is transformed into protoplasmic nitrogenous substances, but the loss of total organic matter is less. When the soluble nitrogenous substances are retained in digested sludge, the nitrogen content should not only be higher on the basis of dry solids, but the nitrogen would be more available for plant food. At present, processes are known and in operation by which at least a part of the soluble nitrogen can be held in the digested sludge.

The outstanding contribution of the work presented by the authors is that with proper operation an apparent digestion schedule of fifteen days can be maintained at a temperature of 85° F, utilizing a system of multi-stage digestion for the gradual and partial destruction of organic solids, producing the usual quantities of gas, leaving a drainable residue, without undue difficulties in operation.

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[20] *Industrial Engineering Chemistry*, Vol. 20, p. 177 (1928).

Short digestion periods are of practical interest and may be obtained with modifications in environmental conditions. With the present state of knowledge the most important environmental factor for properly seeded sludge is temperature. When the temperature is raised high enough to obtain thermophilic digestion, the time required is materially reduced, but the principal disadvantage is that the sludge digested in a period of a few days does not de-water as rapidly as well-digested mesophyllic sludge. Results obtained on a system consisting of consecutive digestion at high temperature (130° F) for a few days, the digestion at low temperatures (80 to 85° F) for the destruction of remaining finely divided material and compacting of sludge, indicate that this is a practical and economical plan. Such a method must include adequate stirring mechanisms to obtain consistently good results. In a system of multi-digestion units, the basic principles of decomposition are not different from single-stage, two-stage, or consecutive digestion. It becomes mainly a question of economics of construction and operation, therefore, to determine which system should be chosen. The fact that the authors have shown that short-time digestion periods can be obtained with a multi-stage digestion unit, which does not require excessively careful operation, is of practical importance to those who must select a system. The greater the variety of methods, the wider the scope.

HERMANN BACH,[21] ESQ. (by letter).—The authors have rendered valuable work in contributing to the bio-chemical and physico-chemical reactions involved in the digestion of sewage sludge. The idea of stages, especially, is basically appealing as it is obvious that the more complicated a process is the more easily it may be simplified by sub-dividing it into convenient working stages, planned so as to favor the particular factor or factors segregated in each stage. In planning such stages, the designer should avoid the temptation of sub-dividing the process into more parts than may be justified economically, in order that the advantages of the separate stages may not be overbalanced or canceled by excess construction costs or unreasonably high operating cost. This kind of foresight is amply evident in the authors' description of the treatment plant constructed for the Los Angeles County Sanitation District. Although two-stage sludge digestion plants using a separate tank construction for each stage are in common use, the authors' description of a four-stage digestion system is remarkable not only because of the multiple stages, but also because, in solving the construction problem, the designers have avoided separate digesters for each stage, by confining the four stages to one structure. This device cleverly avoids the danger of excess first cost and increased operating cost.

The paper raises a logical question as to the number of stages that may actually be necessary in a given case. In the present case, why were there four instead of three, or five, or eight stages? It seems impossible, theoretically, to predict the number of stages that would be most advantageous and, therefore, one must be content to trust his judgment from the results obtained after the plant has been in operation. Selecting gas production as a "yardstick," the data presented in Table 5 afford an opportunity to determine the extent to which the Los Angeles plant has fulfilled expectations. Placing the total gas production (see last column of Table 17) as 100% of the "technical digestion range" (that is, omitting from consideration, the potential surplus of gas production listed in Table 6), the percentage of gas production by stages will be as shown in Table 17. Thus, the proportion of gas production (that is, the digestion work output in Stages 1 to 4) appears to be 1 : 0.46 : 0.40 : 0.36. Since the capacities of the four compartments are equal, whereas Table 17 indicates that the output is not equal (it is less in each stage), the base sub-division in the over-all tank apparently is not consonant with the relative efficiency of the four stages. Whereas, the sub-divisions were made equal in capacity, Table 17 indicates that, relatively, they should have been in the proportion of 1 : 2.1 : 2.48 : 2.8, with the capacity of the first stage assumed equal to 1. In the reverse order, with the fourth stage assumed equal to 1, the relative sub-divisions should have been 0.36 : 0.75 : 0.89 : 1. The dimension of each compartment in that case would have been proportional to the efficiency of the separate stages, thus fully justifying the construction cost.

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[21] Cons. Chemist, Berlin, Germany.

TABLE 17.—AVERAGE PRODUCTION OF GAS, LOS ANGELES COUNTY SANITATION DISTRICTS, CALIFORNIA

Units	Stage 1	Stage 2	Stage 3	Stage 4*	Total, Stages 1 to 4
Thousands of cubic feet per day................	125.3	59.3	50.4	45.4	280.0
Percentage of total.........................	45	21	16	18	100

* The tanks in Stage 4 were only partly filled; the values given in Table 5 were computed as for full tanks.

The need for increasing the dimensions of the compartment in a multiple digestion plant is due to the fact that, in the first period, the sludge decomposes very quickly. As shown in Table 5, after the new sludge in Stage 1 has been substantially decomposed, there is a sudden decrease in the digestion rate, after which subsequent digestion is at a comparatively slow rate tapering to a point beyond which it ceases to be within the "technical digestion range" (a term proposed by Dr. Karl Imhoff). As far as possible, these peculiarities of the material dealt with should be given appropriate consideration in designing the structures for multiple-stage digestion.

The difference in gas production between Stages 3 and 4 is so slight as to indicate that the rate of decomposition had become uniform in each of them. Consequently, it may be unnecessary to construct more than three compartments if the third compartment is made proportionately larger than the first two. On the other hand, however, the more stages that are provided, the greater the opportunity will be for the plant to profit by the "gravimetric segregation," which is an exceedingly valuable feature of the process described in the paper. Indeed, this action may be, to a large extent, the explanation for the astonishingly short time period required to prepare the sludge for the drying bed.

Table 6 indicates that the sludge leaving the last digestion compartment is still capable of producing about 80% more gas if the digestion is extended to approximately twice that of the preceding four stages. It is to be regretted that this volume of gas is lost, which also raises the question as to whether the sludge has actually been digested sufficiently if it still contains some organic substance capable of such gas production. On the other hand, gas production is not the principal purpose of sludge digestion, the gas being only a by-product. The principal purpose, really, is to convert the sludge into a form that can be readily disposed of without creating a nuisance. As this objective seems to have been attained satisfactorily in a brief time at the Los Angeles plant, the designers were probably not justified in providing for an extension of the process merely to produce gas. Furthermore, there would be the potential danger that prolonged digestion would make the sludge too thick to handle in the pumps and on the sludge bed.

Scum Accumulation.—Scum is formed by fat particles which, because of their low specific weight, rise to the surface, carrying other particles with them. The percentage of fat and oil in raw sewage depends on local conditions such as the characteristic food consumed, the presence of industrial plants, automotive traffic, etc. The percentage of fatty material may vary greatly from plant to plant and within the same plant, from season to season. The intensity of scum formation in the sludge digestion tanks may vary accordingly. The so-called mineral oils, such as lubricating oil and grease, gasoline, etc., are highly resistant to bacterial decomposition. Nevertheless, the various fats of animal and plant origin, including the fat contained in greases, are very susceptible to anærobic decomposition, being an important source of sludge gas production as shown in Table 12. To produce and maintain such an anaerobic bacterial action, two conditions are necessary: Sufficient moisture and isolation from the atmosphere. In digestion tanks of the usual construction, including the Los Angeles plant described in this paper, these two conditions are not fulfilled satisfactorily. As may be noted in Table 3, the scum is too dry, containing less than 85% moisture. Such relatively small water content makes the unrestrained action of bacteria more or less difficult. In addition, the scum is freely exposed to the "air." Although, in this particular case, the "air" is not the freely circulating atmosphere, it is sludge gas, which is highly unfavorable to bacterial action because it is made up of products of bacterial metabolism.

In order to accelerate the digestion of fatty substances which form scum layers, means should be provided for moistening the scum throughout its total mass, and preventing its contact with circulating atmosphere or with sludge gas. An improvement in the process of draining the sludge could be devised by placing an elevated sludge tank between the digester or the sludge pit and the sand bed, thus making it possible to discharge the sludge on the bed by gravity. Pumping releases the gas from the sludge, thus raising its specific gravity. The result is that the particles drop quickly to the sand, clogging it against the supernatant. This unfavorable condition may be remedied by retaining the pumped sludge in the elevated tank for an additional digestion period of a half day or so. When the sludge has regained some gas content and is then discharged on the sand bed, the sludge will drain better as its particles will show a tendency to remain on top while the free water below flows through the filter.

The writer was especially interested in Table 9 which contains some very instructive data, the form in which phosphorous occurred is peculiar indeed. Some years ago, the writer observed a surprisingly high phosphate content in the scale deposited on heating coils in a separate sludge digestion tank operated as a whole at mesophilic temperatures. In investigating this strange case, he concluded that the thermophilic bacteria thriving in the immediate vicinity of the heated coils absorb phosphorous compounds to a higher degree than do the mesophilics in the remaining part of the digestion tank. After the thermophilic bacteria on the surfaces of the coils had died, there remained a residue accumulating slowly and being converted to mineral phosphates. It would be interesting to know whether a similar condition occurs in the scale deposited on the heating coils of the plant described by the authors.

C. E. Keefer,[22] M. Am. Soc. C. E. (by letter).—A group of interesting experiments is reported in this paper, which should stimulate further investigations of the separate digestion of sludge. It would appear that the type of tanks used, which permitted the segregation of the solids into scum and settled material with facilities for seeding the raw sludge and digesting the material in several stages, was partly responsible for the unusually rapid digestion obtained. It is hoped that further experiments will be conducted with a similar type of tank in some other locality in order to compare and check the results.

A study was made of the heat added to the digestion tank for a period somewhat less than a month. The authors calculated that the heat added to the sludge was 13 900 000 Btu per day, and that the heat required for radiation and endothermic requirements was 1 200 000 Btu daily. Based on some laboratory experiments conducted by Herman Kratz, Jr. and the writer,[4] the authors calculate that the endothermic requirements alone would be 3 690 000 Btu, a quantity three times as great as the heat available for this purpose. They also state that the fall in temperature of the sludge from the second to the fourth tank was less than 0.5° F, whereas it should have been 2.3° F if the findings of Mr. Kratz and the writer were correct.

It is unfortunate that the authors have not given more of the data upon which their calculations are based. It would be interesting to know how the temperature in the tanks was obtained, how many thermometers were used in each tank, and the location of each of them. What type of thermometer was used? and to what degree of accuracy could they be read? The statement that the difference between the average temperatures of the second and the fourth tanks was less than 0.5° F would, in general, seem to be open to considerable question. It is doubtful whether the average temperature of the sludge in a digestion tank of any appreciable size can be obtained within 2° to 4° F as there is usually a considerable difference of temperature in various parts of the tank. Sludge in the zone immediately adjacent to heating coils will often be from 40° to 60° F hotter than the incoming cold sludge. Sludge near the walls and the bottom of a tank far removed from heating coils will be cooler than the sludge near the coils. To obtain accurate results a large number of thermometers would be required. The complexity of the problem can be appreciated when it is remembered that it is difficult to keep the temperature in specially designed and well-insulated constant-temperature rooms that are thermostatically controlled and provided with means such as fans to prevent the stratification of air within much less than 1° F. As it is much more difficult to determine the average temperature of sludge in a digestion tank, there is considerable question as to the advisability of drawing conclusions regarding the endothermic requirements of sludge digestion from large-scale experiments.

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[4] "The Interchange of Heat During Sludge Digestion," by Messrs. C. E. Keefer and Herman Kratz, Jr., *Sewage Works Journal*, Vol. 5 (1933), p. 3.

It would seem wiser to conduct experiments of this special nature in a calorimeter, in which the temperature can be determined within about 0.001° C. The disadvantage of such an investigation is that a small quantity of the sludge must be used.

In discussing the alleged difference in the results obtained in the laboratory at the sewage works at Baltimore and at the disposal plant near Harbor City, Calif., it should be stated that the Baltimore sludge was from a preliminary sedimentation tank, whereas the material from the latter plant contained 67% activated sludge. Furthermore, more seeding sludge was used at Baltimore. The fact that the Baltimore sludge digested much more slowly may have had a bearing on the difference in the results.

A. M. RAWN,[23] M. AM. SOC. C. E., A. PERRY BANTA,[24] ASSOC. M. AM. SOC. C. E., AND RICHARD POMEROY,[25] ESQ. (by letter).—Replying to Mr. Moore's question about water temperature in the heating coils: When the rate of heat conductance was 13 Btu per hr, per sq ft of pipe surface per degree Fahrenheit of temperature differential, the test records show that the temperature of heating water into and out of the heating coils was 180° and 143° F, respectively. After the coils were cleaned and the conductance rate was 54 Btu, the temperatures were 151° and 113° F, respectively. The writers' observations confirm Mr. Moore's comment that higher temperatures of the circulating water accelerate lime deposit on the coils. Design data for the District tanks contemplated a maximum temperature in the coils of 135° F, but the tanks are handling more than twice the sludge originally contemplated, and it is necessary to raise the circulating water temperature correspondingly.

The writers deemed it impractical to attempt to derive coefficients of heat loss through the outside tank walls, as suggested by Mr. Moore, because the gross heat loss of 1 200 000 Btu per day includes, inseparably, the radiation losses and the endothermic (or exothermic) requirements of the bacterial process. The mild climate of California further complicates the problem.

Mr. Keefer questions the temperature measurements. The quantity of heat added to the sludge by the heating water was determined by metering the flow, and by measuring the temperature of the water entering and leaving the tanks, with a mercury thermometer that could be inserted into the stream at several locations. Temperatures were read to 0.5° F and averages of sixteen days were taken. It is believed that the error in the value for heat input is probably less than 3 per cent. Temperature of the raw sludge was assumed to be the same as sewage temperature, the latter being read three times daily.

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Temperatures in the tanks are read daily to the nearest degree by means of thermo-couples placed near the middle of one of the side bays of each tank. In the first tank there is some uncertainty as to the values due to local temperature differences, as presented by Mr. Keefer; consequently, these values are not relied upon in attempting to ascertain the heat effect of the process. However, the differences are not great, as Table 7 shows. When the measurements in Table 7 were made, readings were also taken at three depths in the west bay of Tank No. 1, Battery A, at a distance of 1 ft from the heating coils, giving values of 85.2°, 85.2°, and 86.8° F. The average is only 2.1° higher than at the middle of the bay, accounted for by agitation due to vigorous digestion, and to the absence of extensive stratification of solids at that time (see Fig. 4(*b*)).

Concerning the second, third, and fourth stages, the rapid flow of the sludges, the vigorous digestion, and the fact that no heat was added and little was lost, are reasons for believing that the temperature must have been quite uniform throughout those tanks. Hence, the routine thermo-couple readings are probably representative of the average temperatures. Sixty-six readings in the third and fourth stages, taken during the period of the heat-effect study, all showed 85° F. In the second stage there were twenty-seven readings of 85° F, four of 86° F, and one of 87° F. Thus, the indicated loss from the second to the fourth stage was 0.19° F; but since a consistent effect as great as 0.5° might escape notice, it was reported that the fall in temperature "was less than 0.5° F."

The estimated difference in free heat between the raw sludge and the digested sludge depends upon the raw sewage temperatures, the temperatures in the fourth tanks, and the measurements of the quantity of sludge pumped. The latter quantity was obtained from pressure readings, and thus represents the actual weight of sludge pumped, free from any error due to entrained gas. Seeding had no effect upon these calculations, since seed was taken from the fourth tank. It is believed that the error in the free-heat difference is probably less than 10 per cent. On the basis of experimental error, it would be very difficult to account for the difference between the possible endothermic effect in this digestion system and the results of Messrs. Keefer and Kratz.

It was not intended that any question should be cast upon the validity of the results reported by Messrs. Keefer and Kratz. As has been noted in the discussions by Mr. Moore and by Mr. Keefer, different behavior in respect to the heat balance may be expected from different sludges. The data were presented to contribute some further information obtained under different conditions. Unfortunately, the data do not allow an exact evaluation of the heat effect, since the heat lost to the surroundings cannot be determined.

The results do show that the process was not accompanied by an endothermic effect as large as that observed by Messrs. Keefer and Kratz.

The writers wish to minimize the significance of the greater weight (7%) of gas produced when compared with the weight of organic matter destroyed. The observational basis of the indicated difference is weak and, therefore, the experimental error may be relatively large. Furthermore, this value (7%) takes no account of the carbon dioxide, ammonia, and volatile acids lost from samples of new or digested sludge upon drying during analysis. The theoretical basis of such a difference is well established but the derived value (7%) should not be considered a precise empirical value.

Regarding scum accumulation, Messrs. Rudolfs and Heukelekian state: "If the digestion time is insufficient the mass of scum eventually becomes so large that the capacity of the tanks is reduced materially and the scum must be taken care of by other means." That this situation does not apply to the installation under discussion is witnessed by the digestion experiments conducted on samples of the scum. It was found that, for the first four days after removal from the tank, scum from the first stage produced gas at a rate of 5.0 liters of gas per liter of scum, whereas the highest measured rate of gas production from the first stage taken as a whole was 3.11 liters of gas per liter of sludge (see Table 5). It seems quite likely that after twenty-five months of operation, conditions regarding scum accumulation may be considered static (see Fig. 4). Tanks were emptied because of the condition of the heating coils, not because of scum accumulation.

No pronounced difference has been noted in the drainability of sludge from the tanks and from the pit. Detail results are at hand relating to fourteen beds of sludge drawn during the period of these tests. Five beds from the pit, averaging 4.49% solids, lost from 16% to 37% of their water in 16 hr, the average loss being 28 per cent. Nine beds from the tanks, averaging 4.77% solids, lost an average of 26% of their water in the same length of time—the slight difference being in the direction that would be expected from the variation in solids content. No explanation is apparent for the erratic variation of individual beds.

Exactly at what point a sludge is "well-digested" is a moot question. The writers believe that a well-digested sludge is one that drains and dries without attendant nuisance, in a reasonable length of time. Freedom from nuisance in drying seems to be closely connected with the content of volatile organic acids. Experience at the plant under discussion indicates that sludge that has less than 200 ppm of volatile acids can be placed on the drying beds with positive assurance that it will not breed house flies and that there will be no objectionable odors. As for drainability, it is doubtless true that, in general, prolonged digestion will produce a sludge which drains rapidly. However, in a separate sludge digestion system it is not economical to aim for the ultimate in drainability. Sludge beds are usually cheaper than digestion tanks.

In Mr. Bach's discussion of the optimum relative size of stages in a digestion system, the writers do not concur in the desirability of proportioning the stages so that an equal volume of gas is produced from each stage. Admittedly, there is little technical information on which to base the choice and, in the installation under discussion, judgment and operating and structural considerations have dictated.

Mr. Bach has misinterpreted Column (7) of Table 6. This column was meant to indicate that, of all the gas produced in the digestion tanks, plus the gas produced with 21 days additional detention time, about 83% was produced in the tanks and 17% was produced in the 21 days of additional detention time.

Scum at this plant contains entrained sludge particles, seeds, matches, hair, bits of rubber, and other light débris. The fact that the moisture content is only 85% and that the surface is in contact with sludge gas does not seem to inhibit active digestion as is suggested by Mr. Bach. That it is the seat of active digestion has been demonstrated previously. The sludge gas is saturated with moisture and has no drying effect.

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EARTHQUAKE STRESSES IN AN ARCH DAM

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Assoc. Members, Am. Soc. C. E.

With Discussion by Messrs. A. Floris, Cecil E. Pearce, F. W. Hanna, A. W. Fischer, Ray L. Allin, and I. M. Nelidov and H. E. von Bergen

Synopsis

The forces of inertia manifested in a structure during an earthquake are ordinarily taken into account in the design. In this paper formulas are derived for determining the stresses due to the horizontal effect of an earthquake on an arch dam for two different conditions: First, the acceleration of the earthquake is assumed to occur along the canyon, that is, horizontally up stream; and, second, the acceleration is assumed to occur across the canyon, that is, horizontally at right angles to the first condition. The vertical effect of the earthquake is omitted from consideration because of its minor importance. Arches with hinged ends as well as with fixed ends are considered, and illustrative numerical examples are given (see Table 1). The derivation of the formulas is based on Castigliano's Theorem of Least Work.

Introduction

Stresses in thin circular arches due to water load have been investigated at length, but not much has been written of the stresses induced by an earthquake in the arch due to its own inertia. In this paper an attempt is made to investigate these stresses and to find their relative importance in the design of an arch dam. Algebraic symbols are defined where first introduced and are assembled, for convenience of reference, in the Appendix. The formulas derived are for moments, thrusts, and shears induced in an arch by an earthquake for two conditions: (1) Acceleration is assumed to occur along the canyon; and (2) it is assumed to occur across the canyon.

Assumption (1) Applied to Arches with Fixed Ends.—Consider an element within an arch ring, 1 ft deep and having an elementary weight, dW, equal to (see Fig. 1):

$$dW = w_c \, t \, r \, d\theta \qquad (1)$$

in which w_c = unit weight of concrete, in pounds per cubic foot; t = uniform

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Note.—Published in December, 1937, *Proceedings*.

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radial thickness of the arch, in feet; and r = the radius of the center line of the arch, in feet. With the earthquake acting horizontally and the plane of the arch ring inclined at an angle, ψ, with the vertical, as in the case of a multiple arch, the elementary force, dP, exerted upon each arch element in the plane of the arch ring will be,

$$dP = \frac{a}{g}\, dW \sin \psi \ldots\ldots\ldots (2)$$

in which a is the acceleration of the earthquake, and g is the acceleration of gravity. The force, dP, acts through the center of gravity of the arch element, which lies slightly outside the neutral axis at a distance, r', from the arch center. In this case, it can readily be shown that $r' = r\left(1 + \frac{k^2}{r^2}\right)$, in which k^2 is substituted for $\frac{t^2}{12}$. Since, for the symmetrically loaded arch, there is no rotation of the crown section under load and no translation of the crown section in the direction of the crown thrust, the partial derivatives of Castigliano's equation for internal elastic work with respect to the crown moment, M_0, and the crown thrust, P_0, are equal to zero.[3] The derivation of the formulas for moment, thrust, and shear for this case is similar to that for the dead load of the arch,[3] and is omitted herein.

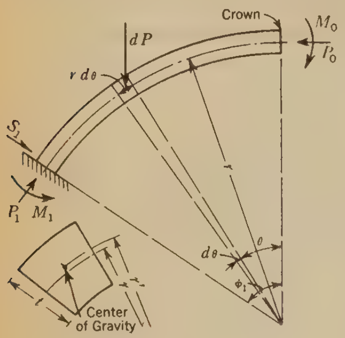

FIG. 1

With the introduction of r', $\frac{a}{g}$, $w_c' = w_c \sin \psi$, and $C = \frac{a}{g}\, w_c'\, t\, r$, the thrust, moment, and shear at the crown are, respectively,

$$P_0 = C\, Z' \ldots\ldots\ldots (3a)$$

$$M_0 = C\, r\, Z_2' \ldots\ldots\ldots (3b)$$

and,

$$S_0 = 0 \ldots\ldots\ldots (3c)$$

and at the abutments,

$$P_1 = C\, Z_1' \ldots\ldots\ldots (4a)$$

$$M_1 = C\, r\, Z_3' \ldots\ldots\ldots (4b)$$

and,

$$S_1 = C\, Z_4' \ldots\ldots\ldots (4c)$$

in which,

$$\begin{aligned} Z' = \Bigg[& 2\left(1 + \frac{r'}{r}\right)\frac{\sin^2 \phi_1}{\phi_1} - \frac{\sin 2\,\phi_1}{2}\left(\frac{5}{2} + \frac{r'}{r}\right) + \frac{\phi_1 \cos 2\,\phi_1}{2} \\ & - \phi_1 \frac{r'}{r} + (2.88 - 1)\frac{k^2}{r^2}\left(\frac{\sin 2\,\phi_1}{4} - \frac{\phi_1 \cos 2\,\phi_1}{2}\right)\Bigg] \\ & \div \left[\left(1 + \frac{k^2}{r^2}\right)\left(\phi_1 + \frac{\sin 2\,\phi_1}{2}\right) - \frac{2 \sin^2 \phi_1}{2} \right. \\ & \left. + \frac{2.88\, k^2}{r^2}\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)\right] \ldots\ldots\ldots (5) \end{aligned}$$

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[3] "Stresses in Inclined Arches of Multiple Arch Dams," by George E. Goodall and Ivan M. Nelidov, Assoc. Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1208.

and,

$$Z_1' = Z' \cos \phi_1 + \phi_1 \sin \phi_1 \tag{6a}$$

$$Z_2' = \left(Z' + \frac{r'}{r} \right) \left(1 - \frac{\sin \phi_1}{\phi_1} \right) - \frac{\sin \phi_1}{\phi_1} + \cos \phi_1 \tag{6b}$$

$$Z_3' = \left(Z' + \frac{r'}{r} + 1 \right) \left(\cos \phi_1 - \frac{\sin \phi_1}{\phi_1} \right) + \phi_1 \sin \phi_1 \tag{6c}$$

and,

$$Z_4' = \phi_1 \cos \phi_1 - Z' \sin \phi_1 \tag{6d}$$

Assumption (1) Applied to Arches with Hinged Ends.—The derivation of the formulas for moment, thrust, and shear for this case is similar to that for dead load, as previously mentioned, except, again, for r', $\frac{a}{g}$, $w_c' = w_c \sin \psi$, and C. The moment, thrust, and shear at the crown are, respectively,

$$P_0 = C\,Z'' \tag{7a}$$

$$M_0 = C\,r\,Z_2'' \tag{7b}$$

and,

$$S_0 = 0 \tag{7c}$$

and at the abutments,

$$P_1 = C\,Z_1'' \tag{8a}$$

$$M_1 = 0 \tag{8b}$$

and,

$$S_1 = C\,Z_4'' \tag{8c}$$

in which:

$$Z'' = \left[\frac{3}{4} \sin 2\,\phi_1 + \frac{3\,r' \sin 2\,\phi_1}{2\,r} - \phi_1 \left(\frac{3}{2} \cos 2\,\phi_1 + \phi_1 \sin 2\,\phi_1 + \frac{r'}{r} (2 + \cos 2\,\phi_1) \right) + (2.88 - 1) \frac{k^2}{r^2} \left(\frac{\sin 2\,\phi_1}{4} - \phi_1 \frac{\cos 2\,\phi_1}{2} \right) \right] \div \left[\phi_1 (2 + \cos 2\,\phi_1) - \frac{3}{2} \sin 2\,\phi_1 + \frac{k^2}{r^2} \left(\phi_1 + \frac{\sin \phi_1}{2} \right) + \frac{2.88\,k^2}{r^2} \left(\phi_1 - \frac{\sin 2\,\phi_1}{2} \right) \right] \tag{9}$$

and,

$$Z_1'' = Z'' \cos \phi_1 + \phi_1 \sin \phi_1 \tag{10a}$$

$$Z_2'' = \left(Z'' + \frac{r'}{r} \right) (1 - \cos \phi_1) - \phi_1 \sin \phi_1 \tag{10b}$$

and,

$$Z_4'' = \phi_1 \cos \phi_1 - Z'' \sin \phi_1 \tag{10c}$$

In the foregoing equations, if $\frac{r'}{r} = 1$, the value of Z' and Z'' will be identical to those derived[3] elsewhere for the dead load of the arch. For a ratio, $\frac{t}{r} = 0.3$, the value of $\frac{r'}{r} = 1.0075$ and decreases with the decrease of $\frac{t}{r}$, which indicates the error involved by such a substitution.

Assumption (2) Applied to Arches with Fixed Ends.—It is assumed for this case that the acceleration of the earthquake is from right to left, acting parallel to a line drawn through the abutments. The forces producing deformation in the arch ring, induced by the inertia of each elementary part of the arch, act toward the right, opposite in direction to that of the acceleration.

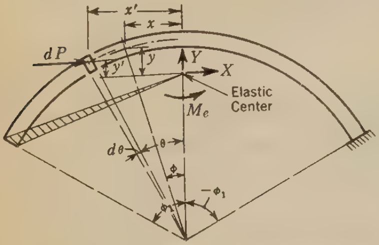

FIG. 2

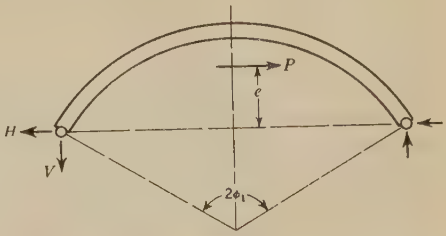

FIG. 3

Since the loading on the arch ring is not symmetrical about the crown and, therefore, the rotation and translation of the crown section are unknown, it is necessary to consider the entire arch ring (see Fig. 2). Assume the arch ring to be fixed, at its right abutment, to the canyon wall, and the left abutment to be held by an imaginary rigid bracket connected to the elastic center of the arch and kept stationary by the forces, X and Y, and the moment, M_e. For a circular arch ring of uniform thickness the elastic center lies at the center of gravity of the neutral axis.

The total internal elastic work in the entire arch ring is:

$$W = \int_{-\phi_1}^{\phi_1} \frac{M^2\,ds}{2\,E\,I} + \int_{-\phi_1}^{\phi_1} \frac{P^2\,ds}{2\,E\,A} + 2.88 \int_{-\phi_1}^{\phi_1} \frac{S^2\,ds}{2\,E\,A} \quad \ldots\ldots\ldots(11)$$

and since there is no rotation or translation of the rigid bracket at the elastic center the partial derivative of the work with respect to X, Y, and M_e is equal to zero, or:

$$\frac{\partial W}{\partial X} = \int_{-\phi_1}^{\phi_1} \frac{M}{E\,I}\frac{\partial M}{\partial X}\,ds + \int_{-\phi_1}^{\phi_1} \frac{P}{E\,A}\frac{\partial P}{\partial X}\,ds + \frac{2.88}{E\,A}\int_{-\phi_1}^{\phi_1} S\,\frac{\partial S}{\partial X}\,ds = 0 \ldots\ldots(12a)$$

$$\frac{\partial W}{\partial Y} = \int_{-\phi_1}^{\phi_1} \frac{M}{E\,I}\frac{\partial M}{\partial Y}\,ds + \int_{-\phi_1}^{\phi_1} \frac{P}{E\,A}\frac{\partial P}{\partial Y}\,ds + \frac{2.88}{E\,A}\int_{-\phi_1}^{\phi_1} S\,\frac{\partial S}{\partial Y}\,ds = 0 \ldots\ldots(12b)$$

and,

$$\frac{\partial W}{\partial M_e} = \int_{-\phi_1}^{\phi_1} \frac{M}{E\,I}\frac{\partial M}{\partial M_e}\,ds + \int_{-\phi_1}^{\phi_1} \frac{P}{E\,A}\frac{\partial P}{\partial M_e}\,ds + \frac{2.88}{E\,A}\int_{-\phi_1}^{\phi_1} S\,\frac{\partial S}{\partial M_e}\,ds = 0 \ldots(12c)$$

At any radial section in the arch ring (x, y), the moment, thrust, and shear are,

$$M = M' + Xy + Yx + M_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13a)$$

$$P = P' + X\cos\phi + Y\sin\phi \ldots\ldots\ldots\ldots\ldots\ldots(13b)$$

and,

$$S = S' + X\sin\phi - Y\cos\phi \ldots\ldots\ldots\ldots\ldots\ldots(13c)$$

in which M', P', and S' are the moment, thrust, and shear at the section (x, y), produced by the external loads on the arch ring to the left of the section. From Equations (13),

$$\frac{\partial M}{\partial X} = y \qquad (14a)$$

$$\frac{\partial M}{\partial Y} = x \qquad (14b)$$

$$\frac{\partial M}{\partial M_e} = 1 \qquad (14c)$$

$$\frac{\partial P}{\partial X} = \cos\phi \qquad (15a)$$

$$\frac{\partial P}{\partial Y} = \sin\phi \qquad (15b)$$

$$\frac{\partial P}{\partial M_e} = 0 \qquad (15c)$$

and,

$$\frac{\partial S}{\partial X} = \sin\phi \qquad (16a)$$

$$\frac{\partial S}{\partial Y} = -\cos\phi \qquad (16b)$$

$$\frac{\partial S}{\partial M_e} = 0 \qquad (16c)$$

Substituting Equations (13) to (16) in Equations (12), and solving for X, Y, and M_e, the results are as follows:

$$-X = \frac{\int_{-\phi_1}^{\phi_1} \frac{M'}{I} y\, ds + \int_{-\phi_1}^{\phi_1} \frac{P' \cos\phi}{A} ds + 2.88 \int_{-\phi_1}^{\phi_1} \frac{S' \sin\phi}{A} ds}{\int_{-\phi_1}^{\phi_1} \frac{y^2\, ds}{I} + \int_{-\phi_1}^{\phi_1} \frac{\cos^2\phi\, ds}{A} + 2.88 \int_{-\phi_1}^{\phi_1} \frac{\sin^2\phi\, ds}{A}} \qquad (17a)$$

$$-Y = \frac{\int_{-\phi_1}^{\phi_1} \frac{M'}{I} x\, ds + \int_{-\phi_1}^{\phi_1} \frac{P' \sin\phi\, ds}{A} - 2.88 \int_{-\phi_1}^{\phi_1} \frac{S' \cos\phi\, ds}{A}}{\int_{-\phi_1}^{\phi_1} \frac{x^2}{I} ds + \int_{-\phi_1}^{\phi_1} \frac{\sin^2\phi\, ds}{A} + 2.88 \int_{-\phi_1}^{\phi_1} \frac{\cos^2\phi\, ds}{A}} \qquad (17b)$$

and,

$$M_e = \frac{\int_{-\phi_1}^{\phi_1} \frac{M'\, ds}{I}}{\int_{-\phi_1}^{\phi_1} \frac{ds}{I}} \qquad (17c)$$

It is now necessary to find the values of M', P', and S', so that they may be substituted in Equations (17). The force acting on each element of the arch through its center of gravity, induced by the earthquake, is,

$$dP = \frac{a}{g}\,dW = \frac{a}{g}\,w_c\,t\,r\,d\theta = C\,d\theta \ldots\ldots\ldots\ldots (18)$$

and the mathematical summation through the angle, $\theta = \phi$ to $\theta = \phi_1$ of these forces, times their respective lever arms, and the mathematical summation of the proper components, will produce M', P', and S', the moment, thrust, and shear, respectively, of the "external" loads at the point, x, y. Since:

$$x = r \sin \phi \ldots\ldots\ldots\ldots (19a)$$

$$x' = r' \sin \theta \ldots\ldots\ldots\ldots (19b)$$

$$y = r\left(\cos \phi - \frac{\sin \phi_1}{\phi_1}\right) \ldots\ldots\ldots\ldots (19c)$$

$$y' = r' \cos \theta - r\,\frac{\sin \phi_1}{\phi_1} \ldots\ldots\ldots\ldots (19d)$$

then,

$$dM' = C\,r\left(\cos \theta - \frac{r'}{r} \cos \theta\right) d\theta \ldots\ldots\ldots\ldots (20)$$

and,

$$M' = C\,r\left[(\phi_1 - \phi) \cos \phi - \frac{r'}{r}(\sin \phi_1 - \sin \phi)\right] \ldots\ldots (21a)$$

Likewise,

$$P' = C \cos \phi\,(\phi_1 - \phi) \ldots\ldots\ldots\ldots (21b)$$

and,

$$S' = C \sin \phi\,(\phi_1 - \phi) \ldots\ldots\ldots\ldots (21c)$$

Substituting Equations (21) into Equations (17), replacing ds by $r\,d\phi$, and integrating:

$$X = -\,C\,\phi_1 \ldots\ldots\ldots\ldots (22a)$$

$$\begin{aligned} Y = -\,C\,\phi \Bigg[& \left(\frac{1}{2} - \frac{\sin 2\,\phi_1}{4\,\phi_1} - \sin^2 \phi_1\right)\left(1 + \frac{k^2}{r^2} - 2.88\,\frac{k^2}{r^2}\right) \\ & + \frac{r'}{r}\left(1 - \frac{\sin 2\,\phi_1}{2\,\phi_1}\right)\Bigg] \div \left[\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)\left(1 + \frac{k^2}{r^2}\right)\right. \\ & \left. + 2.88\,\frac{k^2}{r^2}\left(\phi_1 + \frac{\sin 2\,\phi_1}{2}\right)\right] = -\,C\,C_Y \ldots\ldots\ldots (22b) \end{aligned}$$

and,

$$M_e = -\,C\,r\left(1 - \frac{r'}{r}\right) \sin \phi_1 \ldots\ldots\ldots\ldots (22c)$$

Transfer the forces, X and Y, and the moment, M_e, to the abutment and the following moments, thrusts, and shears in the arch are obtained:

At the crown ($\phi = 0$),

$$M_0 = 0 \ldots\ldots\ldots\ldots (23a)$$

$$P_0 = 0 \ldots\ldots\ldots\ldots (23b)$$

and,

$$S_0 = -\,C\,C_Y \ldots\ldots\ldots\ldots (23c)$$

At the left abutment ($\phi = \phi_1$),

$$M_1 = C\,r\left[\left(\frac{r'}{r} - C_Y\right)\sin\phi_1 - \phi_1\cos\phi_1\right] \qquad (24a)$$

$$P_1 = C\,(-\,\phi_1\cos\phi_1 - C_Y\sin\phi_1) \qquad (24b)$$

and,

$$S_1 = C\,(-\,\phi_1\sin\phi_1 + C_Y\cos\phi_1) \qquad (24c)$$

and, at the right abutment ($\phi = -\,\phi_1$),

$$M_2 = -\,M_1 \qquad (25a)$$

$$P_2 = -\,P_1 \qquad (25b)$$

and,

$$S_2 = S_1 \qquad (25c)$$

Assumption (2) Applied to Arches with Hinged Ends.—In the case of an arch with hinged ends (and also with fixed ends), it may be readily proved that $P = 2\,H$ (see Fig. 3) in which P is the total force induced by the earthquake acting through the center of gravity of the arch. The problem of finding the stresses in a hinged arch is then rendered statically determinate. From Fig. 2,

$$P = 2\frac{a}{g}w_c\,t\,r\,\phi_1 = 2\,C\,\phi_1 \qquad (26a)$$

$$H = C\,\phi_1 \qquad (26b)$$

$$e = r'\frac{\sin\phi_1}{\phi_1} - r\cos\phi_1 \qquad (26c)$$

and,

$$V = \frac{P\,e}{2\,L} \qquad (26d)$$

in which L = one-half the span length. The moment, thrust, and shear for any value of ϕ within the arch is,

$$M = M' - H\,r\,(\cos\phi - \cos\phi_1) + V\,r\,(\sin\phi_1 - \sin\phi) \qquad (27a)$$

$$P = P' - H\cos\phi - V\sin\phi \qquad (27b)$$

and,

$$S = S' - H\sin\phi + V\cos\phi \qquad (27c)$$

At the crown ($\phi = 0$):

$$M_0 = 0 \qquad (28a)$$

$$P_0 = 0 \qquad (28b)$$

and,

$$S_0 = C\left(\frac{r'}{r} - \phi_1\cot\phi_1\right) \qquad (28c)$$

At the left abutment ($\phi = \phi_1$):

$$M_1 = 0 \tag{29a}$$

$$P_1 = - C \frac{r'}{r} \sin \phi_1 \tag{29b}$$

and,

$$S_1 = C\,\phi_1 \left(\frac{r' \cos \phi_1}{r\,\phi_1} - \frac{1}{\sin \phi_1} \right) \tag{29c}$$

and, at the right abutment ($\phi = - \phi_1$):

$$M_2 = 0 \tag{30a}$$

$$P_2 = - P_1 \tag{30b}$$

and,

$$S_2 = S_1 \tag{30c}$$

Numerical Example.—Assume an arch with a thickness, t = 19 ft; mean radius, r = 171 ft; central angle, $2\,\phi_1$ = 103°; and depth of water, h = 70 ft. Let: $\frac{a}{g} = 0.1$; w_c = 150 lb per cu ft; $r' = r$; and ψ = 90 degrees.

The stresses due to uniform water load are computed from known formulas[4] to serve as a basis for comparison to the earthquake stresses, as shown in Table 1. The stresses designated as s_e, s_i, and s_s are combined bending and direct stresses at the extrados and the intrados of the arch and shear stresses, respectively.

TABLE 1.—COMPARISON OF EARTHQUAKE STRESSES, IN POUNDS PER SQUARE INCH

(The minus sign indicates tension)

Load	CROWN			LEFT ABUTMENT			RIGHT ABUTMENT		
	Extrados, s_e	Intrados, s_i	Shear, s_s	Extrados, s_e	Intrados, s_i	Shear, s_s	Extrados, s_e	Intrados, s_i	Shear, s_s
(*a*) ARCH WITH FIXED ENDS									
Uniform water load	341	127	0	33	447	−14	33	447	14
Earth acceleration along the canyon	26	2	0	18	26	− 2	18	26	− 2
Earth acceleration across to canyon	0	0	3	−104	80	−11	104	−80	−11
(*b*) ARCH WITH HINGED ENDS									
Uniform water load	353	217	0	286	286	−27	286	286	27
Earth acceleration along the canyon	27	3	0	22	22	2	22	22	2
Earth acceleration across to canyon	0	0	5	−14	−14	− 9	+14	14	− 9

CONCLUSIONS

The example chosen represents an arch ring at about the mid-height of an average arch dam and is assumed to be a typical one. It will be noted that an earthquake which accelerates the dam up stream produces stresses that seldom exceed about 10% of the water-load stresses; but for an earthquake acting across the canyon the stresses from this cause may be quite large. Of course, these stresses do not include the possible restraining effect of adjacent arch rings, but this effect would probably tend to reduce their magnitude, as the trial-load method of analysis indicates for water-load stresses.

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[4] "The Circular Arch Under Normal Loads," by the late William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., LXXXV (1922), p. 233.

Furthermore, it will be noted by observation that the less the central angle is, the less vulnerable will be the arch for the cross-wise acceleration. Therefore, it is fortunate perhaps that the thick lower parts of some arch dams subtend a much smaller central angle than the thin upper parts.

APPENDIX

NOTATION

The following symbols, defined where first introduced in the paper, are rearranged herein for convenience of reference. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Material"[5] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

a = linear acceleration of an earthquake;
C = a coefficient (see Equations (3)); C_Y = coefficient, C, when forces are translated to the elastic center of the arch (see Equation (23c));
e = eccentricity; lever arm of Load P, measured vertically from the springing;
g = acceleration due to gravity;
I = moment of inertia;
k = a constant ratio $= \frac{t^2}{12}$;
L = one-half the span length of an arch;
M = moment; M_1 = bending moment at the springing due to the elemental external load, dP; M_c = bending moment at the crown, due to the elemental external load, dP; M_e = bending moment translated, and assumed acting at the elastic center of an arch; and M' = moment at the section, x, y, produced by the external loads on the arch ring to the left of the section;
P = a concentrated force; total force induced by an earthquake, acting through the center of gravity of the arch; P_1 = the normal thrust at the springing due to an elemental external load, dP, on the arch; P_c = the normal thrust at the crown due to an elemental external load, dP, on the arch; and P' = the thrust at the section, x, y, produced by the external loads on the arch ring to the left of the section;

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[5] A S A—Z10a—1932.

p = pressure per unit area; pressure due to the head, y; p_0 = pressure due to the head, h_0;

r = radius of the center line of the arch; r_e = radius of the extrados of an arch; r_i = radius of the intrados of an arch; and r' = radial distance to the center of gravity of an arch element;

S = shearing component; S_1 = total shear at the springing; S' = shear at the section, x, y, produced by the external loads on the arch ring to the left of the section;

s = stress; s_e = combined direct stress and bending at the extrados; s_i = combined direct stress and bending at the intrados; and s_s = shear stress;

t = uniform, radial thickness of an arch;

W = total, internal, elastic work;

w = unit weight of water; w_c = unit weight of concrete; and w_c' = component unit weight of concrete = $w_c \sin \psi$;

X = horizontal load assumed acting at the elastic center of an arch;

Y = vertical load assumed acting at the elastic center of an arch;

Z = a coefficient; also Z' and Z'' (see Equations (3) to (10), inclusive);

θ = polar co-ordinate; angular distance; angle between the tangents to the arch ring, at the crown and any point; and $d\theta$ = angle between the radial sides of an elemental segment of an arch of depth, t;

ϕ = the central angle of an arch; ϕ_1 = one-half the central angle of an arch;

ψ = inclination of the plane of the arch ring.

DISCUSSION

A. Floris,[6] Esq. (by letter).—Earthquake stresses produced in an arch dam slice, fixed or hinged at the abutments, but free to move laterally, are analyzed in this paper. Only the influence of the inertia forces of the arch is taken into account, those of the water behind the dam and of the canyon walls being omitted. Under these simplifying assumptions the seismic analysis of the structure is complete.

However, the question arises as to whether such an analysis can be considered as criterion of the seismic resistance of an arch dam. It is true that buildings are analyzed for earthquake shocks by considering their inertia forces only. These structures are usually free to move in all directions above the ground, the resistance of the air being negligible. In an arch dam the inertia force of water, in view of the lightness of the structure, is an important item. In buildings, the dead and live loads are the gravity forces that mainly determine the design. The weight, the only gravity force exerted by arch dams, is neglected in the stress analysis as unimportant. The available theories pertaining to the influence of water upon the dam, presuppose a straight structure with a vertical up-stream face, true only in the case of a gravity dam.[7] Moreover, the arch dam is enclosed in the narrow canyon between the mass of rock. What will be the effect of all the surrounding inertia forces upon the dam? The present analysis fails to yield an answer.

Bearing these limitations in mind, the paper may be judged as an interesting contribution to the aseismic design of arch dams.

Cecil E. Pearce,[8] Assoc. M. Am. Soc. C. E. (by letter).—The effects of earth shock upon an arch dam are analyzed in this interesting investigation. It has ordinarily been stated by enthusiasts that an arch dam can take an earthquake overload better than any other type because all that results is a slight increase in compression, with no alarming effects in regard to sliding or overturning, and the claim is probably correct. These statements refer to an earthquake in which the dam is accelerated up stream.

However, when the direction of the acceleration is at right angles to the stream, the results are different. The authors have shown that these effects are more serious than the up-stream acceleration usually investigated. This checks the statement made by Mononobe:[9]

> "The effect of earthquake upon ordinary masonry dams is most pronounced when the transverse seismic wave propagates parallel to the longitudinal axis of the dam. The seismic wave causes a forced displacement around the perimeter of the dam's base. If the body of the dam is of one mass, the different parts of the dam's foundation bed exercise relative displacements.

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[6] Dipl.-Ing., Los Angeles, Calif.

[7] "Earthquake-Proof Construction of Masonry Dams," by Nagaho Mononobe, M. Am. Soc. C. E., *Proceedings*, World Eng. Congress, Tokyo, 1929, Vol. IX, Pt. I, p. 275; and "Water Pressure on Dams During Earthquakes," by H. M. Westergaard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

[8] Senior Hydr. Engr., TVA, Knoxville, Tenn.

[9] "Earthquake-Proof Construction of Masonry Dams," by Nagaho Mononobe, M. Am. Soc. C. E., World Eng. Congress, Tokyo, Japan, 1929.

This will cause to develop vertical and horizontal crosswise cracks and leakage due to separation of the base of the dam from the bed rock. To remedy these defects and to prevent excessive stresses, vertical crosswise joints or expansion joints may be employed."

Mononobe further suggested that the contraction joints be spaced far enough apart so that the different blocks of the dam can be made stable against accelerations in the direction parallel to the longitudinal axis. In high dams, however, this procedure would conflict with the spacing requirements of contraction joints dictated by shrinkage coincident with the cooling of the concrete.

It would seem that the worst case would be for neither of the two conditions investigated by the authors but for the condition of reservoir empty and an acceleration of the dam down stream. For flood control reservoirs (which should be empty as much as possible), and other reservoirs which are well drawn down part of the year, this case is about as likely to occur as the "reservoir-full" cases investigated. Without computation, it seems plausible that the relatively thin arch dam section would be far less stable against a down-stream acceleration than the top part of a gravity dam, because in this direction there is no arch action in the dam.

It should be emphasized that the formulas developed bear considerable resemblance to those developed by the late William Cain, M. Am. Soc. C. E.[10] The value 2.88 is recognized as the factor for the inclusion of the elastic work of shear.

The writer has computed and checked numerical values for such expressions as: $\phi_1 \left(\phi_1 + \frac{\sin 2\,\phi_1}{2} \right)$; $2 \sin^2 \phi_1$; $2.88\, \phi_1 \left(\phi_1 - \frac{\sin 2\,\phi_1}{2} \right)$; $2 \sin \phi_1$; $\left(1 - \frac{\sin \phi_1}{\phi_1} \right)$; and, $\left(\frac{\sin \phi_1}{\phi_1} - \cos \phi_1 \right)$, for every value of $2\,\phi_1$ from 1° to 180°, by 1° increments which appear in published form.[11] These values are listed in from five to seven decimal places, and are sufficiently accurate for machine computation. Their use shortens the work considerably.

It should always be borne in mind that although formulas such as those developed by the authors seem somewhat complicated at first glance, essentially, they are simply formulas for finding the bending moment, M (in a horizontal plane), and the horizontal thrust, P, at any point in the arch. With these values once obtained, the stresses may be calculated by the simple beam formula, $S = \frac{P}{A} \pm \frac{M\,c}{I}$, the notation for which needs no explanation.

F. W. HANNA,[12] M. AM. SOC. C. E. (by letter).—The effects of earthquakes on dams, of both the arch and the gravity types, are important in regions where earthquakes are prevalent. The authors have presented an important element for the determination of these effects in thin circular arches of uniform thickness both with fixed and with hinged ends. The solutions presented are capable of amplification and also of extension, of which the authors are no doubt aware.

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[10] "The Circular Arch Under Normal Loads," *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 233.

[11] "The Analysis of Circular Arch Dams by Cain's Method, Including Shear," *Western Construction News*, September 10, 1927.

[12] Cons. Engr., Ankeny, Iowa.

The earthquake impulse under Assumption (1) is assumed to act horizontally in a vertical plane parallel to the sides of the canyon and, therefore, parallel to a vertical plane through the crown of the arch rings of an arch barrel of the arch dam. The authors have treated only the down-stream impulse in their paper. This impulse may act either in a down-stream or in an up-stream direction.

For example let Fig. 4 represent a semi-section of a symmetrical ring of uniform thickness of a multiple-arch dam, the plane of the ring being inclined at an angle of ψ with the vertical. Let $C\,D$ be the trace, on the plane of the arch ring, of a vertical plane parallel to the vertical plane through the crown section of the arch ring. Assume the impulse of an earthquake shock to act horizontally either down stream or up stream on the arch in a direction parallel to this vertical plane. At the left of Point C in Fig. 5 is shown a down-stream impulse, dF, on an elementary volume of the arch ring, depicted in section in Fig. 4, resolved into its component forces, dR acting along CD in the arch ring and dN acting upward along the arch barrel, in the vertical plane.

Now, dR is resisted by arch action of the arch ring and dN is carried mainly by beam action of the arch ring to the adjacent piers and tends to overturn them down stream. Also, to the right of Point C in Fig. 5 are shown an up-stream impulse of dF on the elementary volume and its components, dR and dN. In this case also dR is resisted by the arch ring and dN is carried mainly by beam action to the adjacent piers and tends to weaken their resistance against overturning up stream. From these considerations it is evident that the algebraic signs of dF and dR (and, therefore, of the thrust, moment, and shear in the arch ring) are reversed for an earthquake acceleration acting up stream from those of one acting down stream. It also follows, therefore, that an arch ring designed for customary water pressure down stream is well suited to resist the stresses due to an earthquake shock with a down-stream accelera-

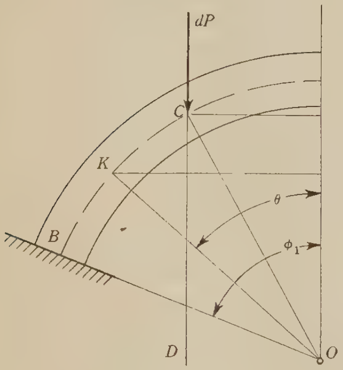

FIG. 4

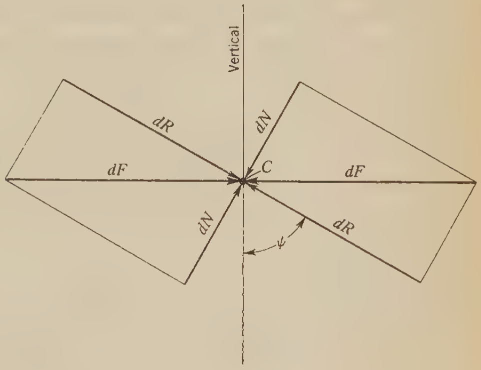

FIG. 5

tion; but that an arch ring so designed may not be suitable to resist stresses due to an earthquake shock with an up-stream acceleration. Steel should be introduced to take the tensional stresses produced by this latter kind of shock.

In the case of an earthquake acceleration parallel to the canyon walls, the force, F, in Fig. 5 will be different for each arch ring of an arch barrel depending on its position relative to the bottom of the arch. Neglecting the effects of the piers or canyon walls, the value of F for each arch ring may be computed approximately by the formula used for buildings[13] or for vertical slices of gravity dams.[14] For multiple or single arches relatively high for their lengths, the effects of the piers or canyon wall abutments are of considerable consequence. Impulses conveyed to a high single arch in a narrow canyon may even exceed that conveyed to it by the canyon floor and may be so great as to prevent the proper use of the building formula for finding the impulse force, F, for arch rings of different elevations. The stresses in such an arch would be more nearly approximated by considering it to be a plate supported on three edges.

In the case of an empty reservoir, the effects of earthquake shocks must be applied to the masonry of the structure only; but for a full or partly full reservoir the influence of the movement of the reservoir water on the masonry stresses must be taken into consideration. This subject has been treated in previous publications[15] and is referred to here only for the purpose of making the category of forces caused by earthquake shocks complete.

The authors have limited their considerations to thin circular arches, but the principle used in their analyses may readily be applied to thick circular arches and also to arches of variable thickness, both of the symmetrical and unsymmetrical types, merely by substituting the earthquake load for the usual water pressure and then following the analysis otherwise applicable. In this, as in all cases, it is best to avoid complications by assuming that r' is equal to r. The distance from the center of gravity of the arch ring is,

$$r' = \frac{\int_{r-\frac{t}{2}}^{r+\frac{t}{2}} r\,dr\,r\,d\phi}{\int_{r-\frac{t}{2}}^{r+\frac{t}{2}} dr\,r\,d\phi} = r\left(1 + \frac{t^2}{12\,r^2}\right) \quad\ldots\ldots\ldots\ldots\ldots (31)$$

and, therefore,

$$\int \frac{r'}{r} = 1 + \frac{t^2}{12\,r^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

For $r = 2\,t$, $\frac{r'}{r} = 1 + \frac{1}{48} = 1.002$, which is negligible in the face of the general uncertainties of the entire problem.

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[13] "Reinforced Concrete and Masonry Structures," by Hool and Kinne, p. 600.

[14] "The Design of Dams," by Hanna and Kennedy, p. 109.

[15] "The Design of Dams," by Hanna and Kennedy, pp. 109–113; and "Water Pressures on Dams During Earthquakes," by H. M. Westergaard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 418–472.

Solution of Equation (31).—Integrating,

$$r' = \frac{\int_{r-\frac{t}{2}}^{r+\frac{t}{2}} r\,dr\,r\,d\phi}{\int_{r-\frac{t}{2}}^{r+\frac{t}{2}} dr\,r\,d\phi} = \frac{\frac{d\phi}{3}\Big[\,r^3\,\Big]_{r-\frac{t}{2}}^{r+\frac{t}{2}}}{\frac{d\phi}{2}\Big[\,r^2\,\Big]_{r-\frac{t}{2}}^{r+\frac{t}{2}}} \quad\ldots\ldots\ldots\ldots(33)$$

Canceling $d\phi$ and substituting limits,

$$r' = \frac{2}{3}\,\frac{\left(r^3 + \frac{3}{2}r^2\,t + \frac{3}{4}r\,t^2 + \frac{t^3}{8} - r^3 + \frac{3}{2}r^2\,t - \frac{3}{4}r\,t^2 + \frac{t^3}{8}\right)}{\left(r^2 + r\,t + \frac{t^2}{4} - r^2 + r\,t - \frac{t^2}{4}\right)}$$

$$= \frac{2}{3}\,\frac{\left(3\,r^2\,t + \frac{t^3}{4}\right)}{2\,r\,t} = \frac{2 \times 3\,r^2\,t\left(1 + \frac{t^2}{12\,r^2}\right)}{3 \times x\,r\,t} = r\left(1 + \frac{t^2}{12\,r^2}\right)$$

and, therefore:

$$\frac{r'}{r} = \left(1 + \frac{t^2}{12\,r^2}\right) \quad\ldots\ldots\ldots\ldots(34)$$

Let r in the right member of Equation (34) equal $2\,t$; then $\frac{r'}{r} = 1 + \frac{1}{48}$ $= 1.002$.

A. W. Fischer,[16] Esq. (by letter).—The subject of earthquake stresses is one of much importance to every engineer who designs structures in localities where earthquakes may occur, and the authors have contributed a valuable paper for calculating certain stresses in an arch dam.

The subject of earthquake stresses in an arch dam either hinged or fixed is treated at quite some length on the assumptions made by the authors and on their basis of calculations. The maximum stresses (as given in Table 1) are equal to ± 104 lb per sq in. for the arch with fixed ends. For the arch with hinged ends the maximum stress, as given in Table 1, is 27 lb per sq in., compression. The writer doubts if the stress given for the hinged-end condition is the maximum that could occur from an earthquake. Neglecting rib-shortening and assuming that due to an earthquake the abutment is displaced only 0.08 ft outward, and then substituting the values given in the numerical example (with E = 3 000 000 lb per sq in.), the fundamental equation for the horizontal thrust, on a slice 1 ft high, in a segmental two-hinged arch with a constant, I, is,

$$H = \frac{3\,000\,000\,(144)\,(1/12)\,(0.08)\,(19)^3}{(171)^3\,[(51.5/57.3)\,(2 + \cos 103°) - 1.5 \sin 103°]} = 29\,500 \text{ lb}$$

For the particular arch the moment at the crown equals 29 500 (171) vers 51° 30′ = 1 904 000 ft-lb, and from this moment the stresses at the crown equal ± 220 lb per sq in. The stress from direct load equals 11 lb per sq in., tension. The stresses are based on the assumption that the arch is a monolith and has no vertical expansion joints.

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[16] Care, Construction Service, Veterans Administration, Washington, D. C.

Authorities might question the possibility of the abutment moving 0.08 ft, but the writer believes that in some localities this could happen and in extreme cases it could be greater than 0.08 ft, and stresses due to slight movements of the abutments should not be overlooked. The stresses in an arch dam with fixed ends will not be as large as in a two-hinged arch for the same displacement of the abutments; but in all arch dams, either two-hinged or with fixed ends, the stresses due to slight movement of the abutment should be considered.

RAY L. ALLIN,[17] ASSOC. M. AM. SOC. C. E. (by letter).—The earthquake effects on the stresses in an arch dam are clearly illustrated in this interesting paper.

Since, ordinarily, no tension can exist between a rock abutment and a concrete arch unless special anchorage has been provided, the cases noted by the authors must be for an arch, loaded prior to the earthquake shock with an intensity of compression stress greater than the intensity of tensile stresses indicated in the author's Table 1; or they must have assumed an equivalent compression distribution that will give the same moment and thrust at the abutment as those determined by the authors.

Where no arch thrust exists prior to earth shock, such as for an empty reservoir with horizontal arches, the case where the acceleration is acting parallel to a line through the abutments is similar to that where the arch is stood on one end with a body force equal to that due to earthquake acceleration. This case would place all the reaction at the abutment where the earth shock is transmitted to the dam, and would cause stresses of twice those values given by the authors at the corresponding abutment; and, at the crown, it would cause a direct thrust equal to one-half the abutment thrust, and a bending moment equal to the product of this crown thrust and its corresponding lever arm normal to the line through the center of gravity of each half of the arch. This case is not as vital to safety as that of a dam with full reservoir; yet it may be necessary to stiffen a slender arch to satisfy this condition.

Those engineers interested in the design of arch dams are indebted to the authors for their contribution on this subject of earthquake stresses in arch dams.

I. M. NELIDOV[18] AND H. E. VON BERGEN,[19] ASSOCIATE MEMBERS, AM. SOC. C. E. (by letter).—The several discussions have been gratifying in that they have expanded and clarified the work of the writers. Since many forces and actions come into play, and act upon a dam and its foundation and abutments during an earthquake, it is possible that the title of the paper has been somewhat misleading as to scope. It was the intention, however, to consider the fundamental force of inertia which is created whenever an earthquake tends to accelerate a structure in any direction. The writers consider only the single arch ring as a separate entity, being either a part of a single arch dam or a multiple-arch dam, realizing at the same time that yielding of abutments, stability of buttresses, differential movements of the abutments, hydrodynamic pressures, etc., must be considered to arrive at a complete solution of the problem.

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[17] Hydr. Engr., Hetch Hetchy Project, San Francisco, Calif.

[18] Senior Engr. of Hydr. Structure Design, State Div. of Water Resources, Sacramento, Calif.

[19] Asst. Hydr. Engr., State Dept. of Public Works, Sacramento, Calif.

Although the hydrodynamic pressures on dams as proposed by Dean Westergaard[20] are not strictly applicable to arch dams, as stated by Mr. Floris, it may be possible to apply these forces to the face of an arch dam, by some rational method.

Mr. Pearce has directed attention to the unstable condition of a dam in the event of a down-stream acceleration with the reservoir empty. This is especially true for a single arch of the variable-radius design since a dam of this type often is provided with up-stream buttresses to prevent toppling up stream when the reservoir is empty even under static conditions.

The writers wish to express their appreciation to Mr. Hanna for clarifying the action of the elementary inertia force and its various components in the third and fourth paragraphs of his discussion and as shown in Figs. 4 and 5. Mr. Hanna has also described the use of the equations of moment, thrust, and shear for thick arches as well as thin arches. The error involved in substituting r' for r may be calculated from Equation (34). Another error involved is the appreciable displacement of the neutral axis as the $\frac{t}{r}$-ratio increases above 0.3.[21]

Mr. Fischer discussed the movement of an abutment which was somewhat beyond the scope of the paper. The yielding or movements of the abutments of an arch is a study in itself, not to mention the differential or relative movements of the two abutments during an earthquake.

Mr. Allin mentions the possibilities in the event of an empty reservoir during a lateral earth shock, when the abutments are unable to take tension. He rightly states the effect on the resisting abutment and the crown section and the resulting stresses. These conditions would have to be considered in the design and either special anchorage or increased strength provided.

In conclusion, the writers wish to thank those who have contributed discussions to the paper and thereby have enhanced its value. There is still much to be investigated in regard to forces and other phenomena produced by earthquakes which affect the stability of an arch dam.

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[20] "Water Pressure on Dams During Earthquakes," by H. M. Westergaard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

[21] "Stresses in Thick Arches of Dams," by B. F. Jakobsen, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 483.

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GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS

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WITH DISCUSSION BY MESSRS. DONALD M. BURMISTER, A. J. WEINIG, CARL H. KADIE, JR., CARLTON S. PROCTOR, T. T. KNAPPEN, JACOB FELD, HOWARD F. PECKWORTH, JOEL D. JUSTIN, L. B. OLMSTEAD, T. A. MIDDLEBROOKS, FRANK E. FAHLQUIST AND WALDO I. KENERSON, E. W. LANE, F. J. SANGER, F. KNAPP, CHARLES H. LEE, AND FRANK B. CAMPBELL.

SYNOPSIS

In view of the lack of uniformity in proposals for the graphical representation of grain-size distribution, this paper is presented in an effort to correlate various existing methods and concentrate them into one method which the writer believes would make a good standard for graphical representation. A semi-logarithmic co-ordinate system is used with the grain sizes plotted as abscissas on a logarithmic scale and cumulative percentages by weight plotted as ordinates. With curves so plotted, the grain-size distribution curves have been conventionalized by introducing straight lines. Such lines can be designated briefly by their slopes and intercepts so that data from various soils may be tabulated or plotted on drawings for purposes of comparison. The value of mechanical analysis is discussed briefly. Further recommendation is made for nomenclature of soil fractions. In doing so there is an attempt to conform closely to the widely accepted classification used by the Bureau of Chemistry and Soils, United States Department of Agriculture, but to make the separations at recurring cycles of log 2 and log 6. Other classifications are also discussed.

INTRODUCTION

On considering the various types of soils which may be encountered in Nature, the writer has attempted to develop some simplified designation for both mean diameter and gradation of particle size. It is often desirable to adopt a set of symbols on a plan or profile which indicates the type of grain-size distribution. In the past, such drawings have usually indicated the soils as clay, sand, or gravel with many variations. Such descriptions depend upon

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NOTE.—Published in December, 1937, *Proceedings*.

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the conception of an individual as to the meaning of such terms. These conceptions vary widely. Sometimes, a set of symbols or numbers is used, in which case, a student of the drawing must learn a different system of designations each time he encounters a separate report. In certain studies, the engineer wishes to make a tabular comparison of a group of analyses of various samples. The method used in the past has been to list the quantity of each soil fraction, or to separate them in columns. Although the information thus recorded is definite, it is difficult for one to obtain a unified "picture" of the comparative grain-size distributions.

Mechanical Analyses Graphs

The importance of the mechanical analysis graph, or grain-size distribution curve, to technical literature lies in the fact that it is a method of international expression, or a universal language. The ordinate is the percentage by weight of the total sample, smaller than a corresponding particle diameter. Percentages involve no physical unit and are often called a ratio or a pure number. The millimeter is used as a unit of particle size. Technically trained men are familiar with the millimeter as a unit of length, and it proves to be a convenient scale for particle diameters of soils. The use of the English inch involves so many ciphers before the first significant figure as to be cumbersome. The purpose of the semi-logarithmic scale is obvious when the nature of grain-size distribution of soils is considered.

In discussing methods of graphical representation of the mechanical analysis of soils, other methods in occasional use should be mentioned. The older method of plotting the weight of soil separates or grain-size fractions, in percentages of the total weight, against particle diameter, has certain advantages of analysis as shown by Mr. Arthur J. Weinig,[2] in 1933. The tri-linear method of plotting mechanical analyses of soil is convenient for an approximate classification of soils according to texture. The method is explained by Messrs. R. O. E. Davis and H. H. Bennett.[3]

Recently some engineers have used repeating cycles of log 1, log 3, and log 5 for the abscissa lines. There is an increasing use in technical literature of log 2 and log 6. It should be noted that since $\log_{10} 2 = 0.301$ and $\log_{10} 6 = 0.778$, the scalar difference between successive abscissa lines is 0.477 and 0.523. These two scalar values offer a spacing of approximately equal distance as may be seen in Fig. 1.

The writer has plotted mechanical analyses selected from several hundred samples from widely different localities and types. Six types are shown in Fig. 1, as follows: Sample (*a*) is a black clay taken from the up-stream face of a dam which failed partly by sliding into the reservoir; Sample (*b*) is a red sandy clay that exhibits a broad variation of grain sizes; Sample (*c*) is an aeolian soil or "blow sand," and is a typical uniform grain-size material; Sample (*d*) represents a residual soil derived from sedimentary rock and has an excellent distribution of soil particles; Sample (*e*) is similar to Sample (*d*), except that it is coarser and indicates that the disintegration process has not been carried so far; and

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[2] "A Functional Size-Analysis of Ore Grinds," by Arthur J. Weinig, *Quarterly*, Colorado School of Mines, Vol. XXVII, No. 3, July, 1933.

[3] "Grouping of Soils on Basis of Mechanical Analysis," by R. O. E. Davis and H. H. Bennett, *Department Circular No. 419*, U. S. Dept. of Agriculture.

Sample (*f*) is a coarse gravel which contains some sand. It was selected from a dam that is quite stable, although it is very porous and continues to leak.

It should be mentioned that the shape of the mechanical analysis curve is often an indication of type; that is, whether it is residual, alluvial, or aeolian; and, if it is residual, the curve indicates its degree of maturity. The term, "residual," is used in the sense of a soil that has been derived by the disinte-

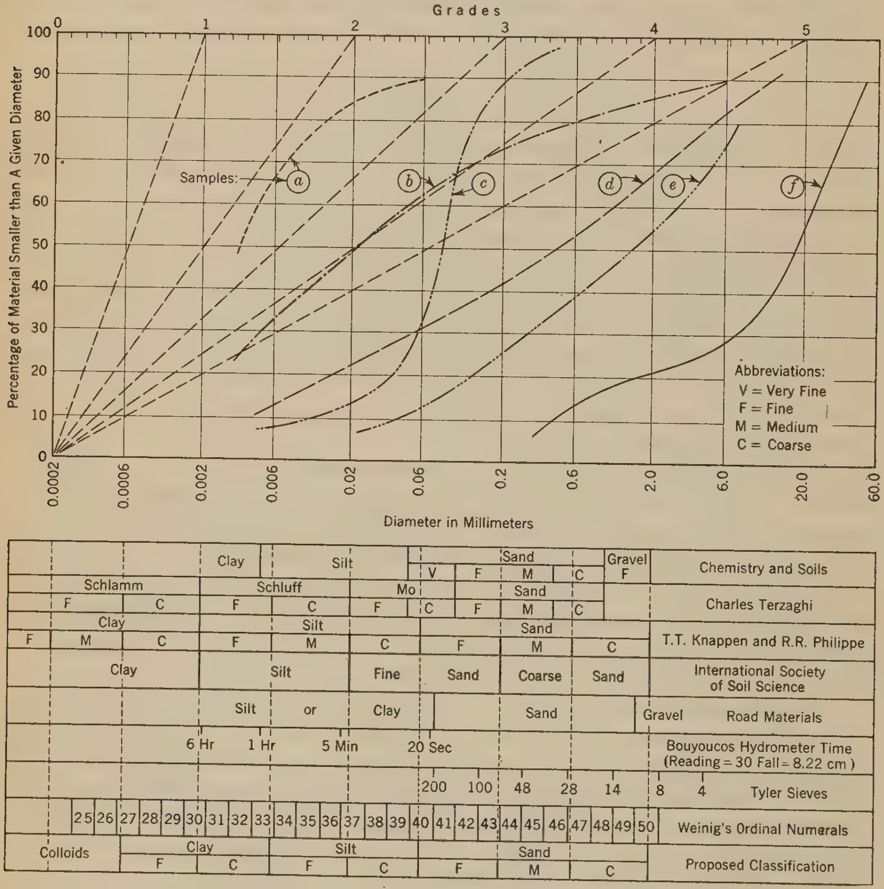

FIG. 1.—COMPARISON OF MECHANICAL ANALYSES AND NOMENCLATURE OF SOILS

TABLE 1.—DIAMETERS AND GRADE CHARACTERISTICS OF SAMPLES IN FIGURE 1

Sample	Mean diameter, in millimeters	Grade line deviation*	Grade, 20% to 80%	Diameter–grade designation	Sample	Mean diameter, in millimeters	Grade line deviation*	Grade, 20% to 80%	Diameter–grade designation
a	0.0038	+0.0002	1.9†	D 0.0038 – G 1.9	*d*	0.47	− 0.19	4.2	D 0.47 – G 4.2
b	0.022	+0.021	3.9	D 0.022 – G 3.9	*e*	1.45	− 0.62	3.3	D 1.45 – G 3.3
c	0.081	−0.022	1.0	D 0.081 – G 1.0	*f*	17.6	−10.0	2.4	D 17.6 – G 2.4

* Convexity is positive. † Estimated by extrapolation.

gration of igneous and certain metamorphosed rock. The shape of the curve may or may not be indicative of the degree of maturity of a soil derived from sedimentary rock. Certain alluvial and aeolian soils are difficult to classify as mature or immature from the standpoint of the disintegration process. In general, the principle follows that immature soils have a size distribution curve that is concave upward, and mature soils show a curve that is convex upward.

By the same reasoning, a soil that is represented by a straight line is often intermediate between maturity and immaturity. Such a soil usually contains particles ranging in size from clay to gravel. It should not be confused with soil of uniform grain size which is a vertical line on the graph. Such a soil is usually an alluvium or an aeolian type. Many observers have noted that water and wind are excellent sorting agents. In view of the foregoing facts, the writer believes that mechanical analysis graphs have a definite value in determining the history of a soil. It is often helpful to know the history of the soil development so that certain other chemical and physical characteristics may be anticipated and tests made for such qualities.

The need of a simplified classification according to grain size has often been recognized. The writer has attempted to develop one, first, by expressing the mean grain diameter, and second, by expressing the gradation of particle size according to the slope of the line on the mechanical analysis graph. The mean grain size is defined as that diameter, in millimeters, of which 50% of the sample by weight has larger diameters and 50% has smaller diameters. The gradation is assumed to be the slope of the straight line passing through the 20% smaller, and the 80% smaller, points of any one curve. The grade lines originating at the lower left-hand corner are shown in Fig. 1. A grade of 0 means that all particles have the same diameter. Grade 1 slopes from 0 to 100%, spanning one logarithmic cycle horizontally; Grade 2 spans two logarithmic cycles, etc. A decimal scale is shown along the top so that the grade may be read to tenths. The method is easily applied by two triangles for transferring the straight line through the 20% and 80% points to a parallel position through the origin at the lower left-hand corner of the graph. The grade can then be read from the horizontal scale at the top.

Table 1 shows the mean diameters and grades of the samples in Fig. 1. The letter designations of the curves on the graph are listed in the first column. The mean diameter is shown in the second column. The "Grade Line Deviation" is the difference between the curve and the grade line measured along the 50% ordinate. The writer has included this column to show the diameter difference, in millimeters, between the actual curve and the grade line. A minus deviation indicates a convex curve. However, the magnitude of the deviation is scarcely an indication of degree of convexity or concavity in a Cartesian sense, for the reason that the scale is logarithmic. The fourth column is the grade, or slope of the grade line, determined by the method described in this paper. The last column shows an abbreviated designation which includes both mean diameter, in millimeters, and the grade. Such a set of symbols may be utilized by plotting it on a soil map or profile.

Various nomenclatures of soil-size groups, or rather soil separates, are shown below the mechanical analysis graph. The most widely used classification is that of the Bureau of Chemistry and Soils, U. S. Department of Agriculture. Charles Terzaghi,[4] M. Am. Soc. C. E., has given a classification of soil separates in use in Germany. Recently, T. T. Knappen, M. Am. Soc. C. E., and R. R. Philippe,[5] Assoc. M. Am. Soc. C. E., have published a classification which they credit to the Massachusetts Institute of Technology. According to this classification, the upper limit of clay corresponds to that of the German "Schlamm." The classification advanced by the International Society of Soil Science is used by some soil scientists both in America and abroad.[6] The Society's Committee on Road Materials has defined the lower limit of sand as a 200-mesh Tyler sieve opening. Some engineers have adopted this conception. The Bouyoucos hydrometer times, corresponding to certain sizes, are shown. It serves to emphasize the lengthening interval of time involved in any sedimentation method when it is desired to determine the content of particles much smaller than 0.006 mm. The corresponding location of Tyler sieve openings illustrates the limits of screen analysis in common use. Mr. Weinig's[2] ordinal numerals are size groups developed for graphical analysis of soil separates. The coincidence of these size groups with the Tyler sieve openings may be seen. The uniform spacing of the Massachusetts Institute of Technology classification, Tyler sieve openings, and Weinig ordinal numerals on a logarithmic scale, are worthy of note.

The writer proposes a classification based upon the log 2 and log 6 division which is not widely different in nomenclature from the much used classification developed by the Bureau of Chemistry and Soils. Although similar to the Terzaghi and Massachusetts Institute of Technology classifications in the use of log 2 and log 6 divisions, the upper limit of clay is moved to 0.006 mm. Since the particles smaller than 0.0006 (the lower clay limit of the writer's classification) are beyond the limits of practical sedimentation analyses and ordinary microscopic technique, such fines should properly be relegated to the fields of colloidal chemistry and ultra-microscopy. For this reason, particles less than 0.0006 mm in diameter are termed colloids. By placing the upper limit of clay at 0.006 mm, fair agreement is had with the 0.005-mm limit of the Bureau of Chemistry and Soils, and the time required for sedimentation analysis is reduced to allow efficient laboratory working schedules. The proposed silt limits of 0.006 mm to 0.06 mm embrace practically all the silt class as defined by the Bureau of Chemistry and Soils, overlapping a little into very fine sand. The proposed fine, medium, and coarse sands correspond roughly to the Chemistry and Soils groups although the ranges are extended somewhat. The proposed coarse sand class overlaps the present fine gravel class, which has an upper limit of 2.0 mm. The new classification presented herein has the distinct advantage of facilitating the transfer of tabular data on soil separates to a mechanical analysis graph sheet for reproducing its curve.

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[4] "Die Prüfung von Baumaterielien fürgewältze Erddämme," by Charles Terzaghi, Premier Congrès des Grandes Barrages, Stockholm (1933), p. 73.

[5] "Practical Soil Mechanics at Muskingum," by Theodore T. Knappen and R. R. Philippe, *Engineering News-Record*, March 26, 1936, p. 455.

[6] See "Soils," by G. W. Robinson, published by Murby, London, 1936.

ACKNOWLEDGMENTS

The writer expresses appreciation to E. W. Lane, M. Am. Soc. C. E., and other former associates in the Bureau of Reclamation, U. S. Department of the Interior, for curve data used in this discussion and to his present associates of the Soil Conservation Service, U. S. Department of Agriculture, for helpful criticism.

DISCUSSION

DONALD M. BURMISTER,[7] ASSOC. M. AM. SOC. C. E. (by letter).—The purpose of the mechanical analysis is two-fold: First, to determine and record the distribution of grain sizes of a soil; and, second, to interpret the results into terms of soil character. The author has given an excellent summary of the commonly recognized classifications of soil fractions or separates. The classification based on the log 2 divisions, which the author proposes, has certain decided advantages, because it is a continuation of the familiar Tyler sieve series.

In order to interpret the results of the grading analysis into terms of soil character, the size characteristics should be described in such a manner as not only to reveal the characteristic features of the soil, but also to yield practical information in relation to general soil behavior. Mr. Campbell has stated that the tri-linear chart, used by the U. S. Bureau of Public Roads, is a convenient one for the approximate classification of soils as to texture.

The most interesting and significant statement in the paper is that the shape of the grading curve is often an indication of whether the soil is of residual, alluvial, or æolian origin, and if it is found to be residual, the curve indicates the degree of maturity. The writer proposes to carry this significant and important fact to its logical conclusion in this discussion, and, therefore, wishes to present a line of reasoning[8] different from that usually followed in evaluating the grading analysis in terms of numerical values.

Although arbitrary definitions of the mean grain size are often convenient, such as the 50% size suggested by the author, or the familiar 10% effective size described by the late Allen Hazen, M. Am. Soc. C. E.,[9] yet such definitions must represent only temporary adaptations, which cannot be truly representative of such a diverse material as soil. A simple consideration of statistical principles will show that neither the median (50%), or the arithmetic mean can be used, logically, to define a characteristic mean for a semi-logarithmic plotting. An analysis[8] involving and extending the ideas of Krey, Kozeny, and Abrams shows that the degree of fineness of the soil may be expressed adequately by a mean grain size that is defined by the length of the equivalent rectangular area which is easily obtained by means of a planimeter or by simple summation of elementary areas, as shown in Fig. 2. This mean, which is the logical one to use for a semi-logarithmic plotting, takes the entire curve into account, and is actually a weighted geometric mean. It may be shown readily, that the mean grain size is also that having the geometric mean surface area. The most significant fact about the grading curve is that the area under it is a logarithmic function of the specific surface area of the soil analyzed. The importance of surface area, as particle size decreases, is recognized in the chemical and metallurgical industries, and practical use of this idea is made in the Gates "crushing surface diagram"[10] in ore-crushing operations.

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[7] Asst. Prof. of Civ. Eng., Columbia Univ., New York, N. Y.

[8] In "A Study of the Physical Characteristics of Soils—with Special Reference to Earth Structures." (Not published.)

[9] Annual Repts., Massachusetts State Board of Health, 1892.

[10] "The Handbook of Ore Dressing," by Taggart, p. 496.

It is further convenient to express the degree of fineness in terms of a size factor, as shown in Fig. 2, which is easily computed from the grading curve,

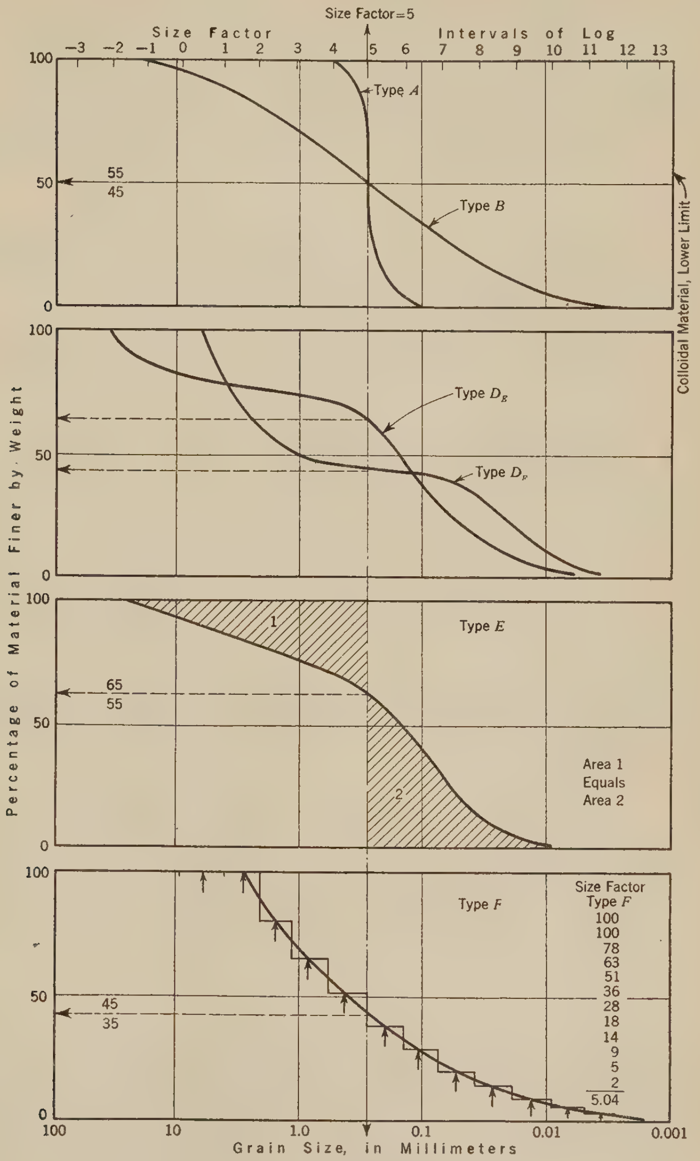

FIG. 2.—SIZE CHARACTERISTICS OF SOILS: (*a*) DEGREE OF FINENESS; (*b*) TYPE OF GRADING CURVE; AND (*c*) RANGE OF PARTICLE SIZES

the origin of this size factor designation of log 2 intervals being taken at the $\frac{3}{8}$-in. sieve, where there seems to be definite change in the capillary tendencies of soils. An advantage of the size-factor method is that the size factors of mixtures of soils may be easily obtained by simple proportion, the size factor as a logarithm being additive, and hence the mean size can be computed.

A description of grading curves as to type is proposed by the writer, based on statistical considerations. Not only does the shape of the curve give an indication of the geological origin of the soil, as the author suggests, but it also has a greater significance, because it furnishes a basis for correctly interpreting the important influence of the distribution of grain sizes on the density of soils, whether in the natural state, or placed in some artificial state in embankments. The classification as to type of grading curve is as follows (see Fig. 2):

Type A.—Single-size, narrow-range materials, or uniform materials are represented by a nearly vertical line on the grading diagram (these materials are always the least dense and can be compacted very little);

Type B.—The normal statistical distribution of particle sizes has a nearly symmetrical **S**-shaped curve (such soils can be compacted to quite a dense state);

Type C.—This is a straight-line distribution with equal proportions of all sizes (this is a poor grading; in general, less dense than Type B);

Type D.—This type is a mixture of two fractions, one fine and the other coarse (such materials have a decided hump in the grading curve; Type D_E is a mixture of some weathered or unweathered rock fragments in a matrix of much fine silty or clayey soil; and Type D_F is a mixture of much gravel and sand in a matrix of some very fine silty or clayey soil; these materials are typical of the so-called boulder clay of glaciated regions; they pack well and may be very dense);

Type E.—In this type the grading curve is skewed toward the fine fractions (that is, it is predominantly fine with some coarse material; the density is determined by the character of the fine material and is usually less than that of Type *B*); and,

Type F.—The grading curve in Type *F* is skewed toward the coarse fractions and is predominantly coarse with some fine material. This type approaches the ideal grading curve for maximum density (the Talbot grading) and conforms to the requirements for the best stabilized earth road mixtures of the U. S. Bureau of Public Roads. Type *F* material in its loosest state may have a greater unit weight than any of the other types in their most dense states.

An interesting and significant fact is that the percentage corresponding to the mean size is not now an arbitrary constant quantity, but varies in a characteristic fashion with the type of grading curve, and, in reality, represents a skew factor. In Types *A*, *B*, and *C* the percentage may vary from about 45 to 55, and the median approximately coincides with the geometric mean. Since Types *E* and D_E are skewed to the fines, the percentage corresponding to the mean is considerably greater than 50, usually from about 55 to 65. In contrast, the percentage for Types *F* and D_F is considerably less than 50, usually from 35 to 45.

In an attempt to describe the grading curve by the slope of the curve or any part of it, there has been a confusion of two separate and distinctly independent ideas—the shape of the curve and the range of particle sizes—which cannot be expressed by any single factor. With degree of fineness defined by a geometric mean size or size factor, and the shape of the curve defined by the type, the size characteristics are finally defined simply in terms of the range of particle sizes, with no reference to slope, as such. The designation of range to take in the entire grading curve may be expressed conveniently as the ratio of the 95% size to the 5% size, or by the number of size factor units (intervals of log 2) between these two extreme sizes. For practical purposes the lower limit of the designation is taken at 0.001 mm and all materials finer are considered as colloidal materials.

The range of particle size also has a most important affect on density. For the same type of grading curve the greatest density is found in the material having the greatest range of particle sizes, and the least dense is always the narrow range Type *A* material.

On these bases it seems logical to define the size characteristics of soil in terms of the three independent factors, which not only have statistical but also physical significance in the grading-density relations,[11] and to define the soil fractions or soil separates in terms of a log 2 series, which is a continuation of the Tyler sieves.

A. J. Weinig,[12] Esq. (by letter).—The comparison of mechanical analyses and nomenclature of soils offered in this paper is particularly valuable in view of the many different systems included, and Mr. Campbell's proposed plans would seem to be a logical, general outcome of all these classifications.

In the matter of diameter-grade designation, a suggestion is made that if the type of distribution curve be represented by a letter, an instant mental picture of the soil type would be had. Thus, for example, if (C) represents convex; (K), concave; (S), S-type *n*-curve; and (L), linear, then the diameter-grade designation, of the samples given in the paper, for example, would be:

Sample:	Diameter-grade designation
(*a*)	D-0.0038–G-1.9 C
(*c*)	D-0.081 –G-1.0 S
(*e*)	D-1.45 –G-3.3 K

Carl H. Kadie, Jr.,[13] Jun. Am. Soc. C. E. (by letter).—The author has developed a method of tabulating, in a simple form, a mass of information regarding the nature of the particle-size distribution of a soil sample. With the increasing use of soil classification and soil tests in the investigations of building foundations, earth dams, highway sub-grades, and erosion control, a need has been felt for a simple method of recording pertinent data on soil profiles and maps. Mr. Campbell's grade line, with the deviation therefrom, and the use

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[11] See p. 364.

[12] Director, Experimental Ore Dressing and Metallurgical Plant, Colorado School of Mines, Golden, Colo.

[13] Asst. Engr., U. S. Bureau of Reclamation, Boca, Calif.

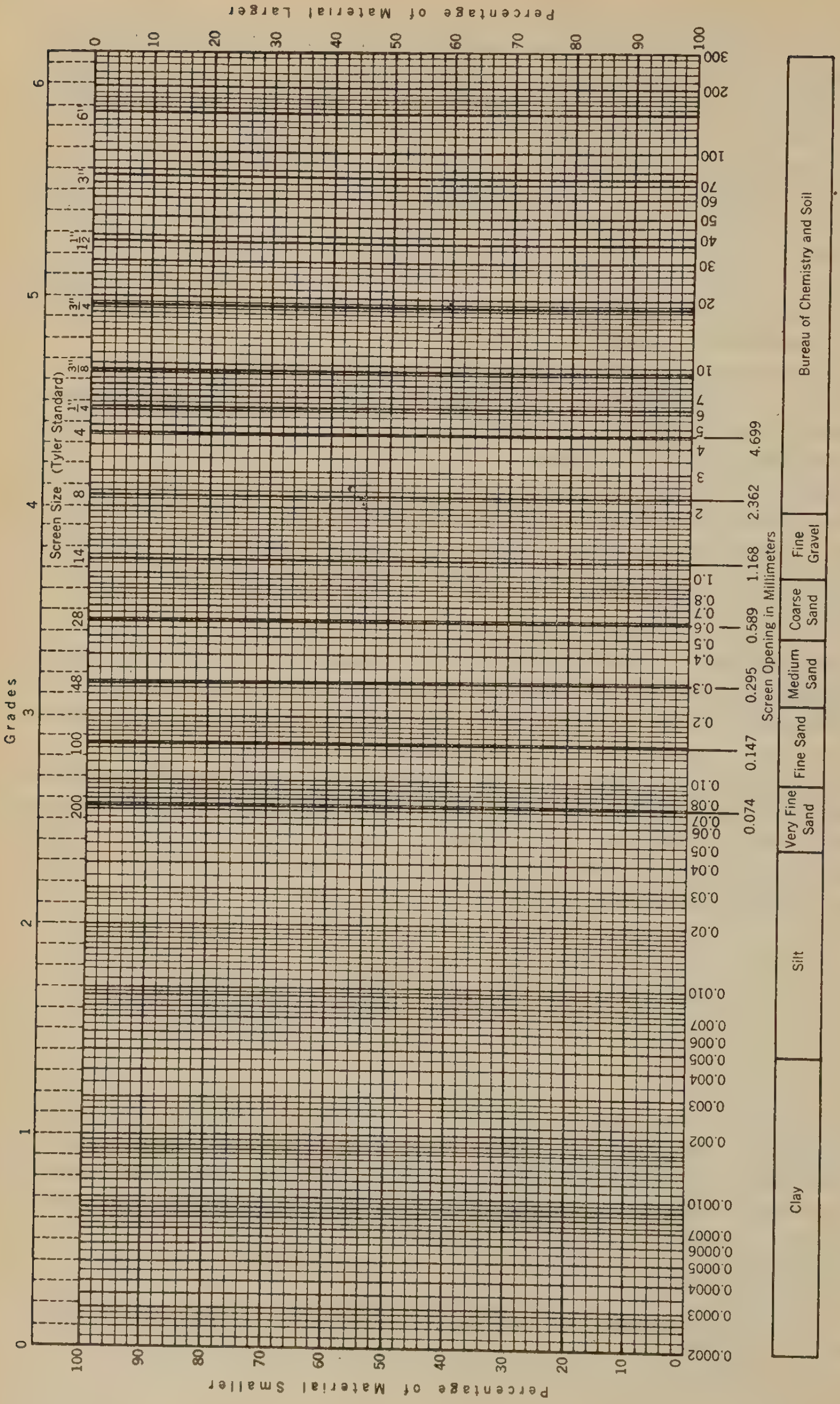
Percentage of Material Larger
0
10
20
30
40
50
60
70
80
90
100
Percentage of Material Smaller
100
90
80
70
60
50
40
30
20
10
0
Grades
0
1
2
3
4
5
6
Screen Size (Tyler Standard)
200
100
48
28
14
8
4
1/4"
3/8"
3/4"
1 1/2"
3"
6"
Screen Opening in Millimeters
0.074
0.147
0.295
0.589
1.168
2.362
4.699
0.0002
0.0003
0.0004
0.0005
0.0006
0.0007
0.0010
0.002
0.003
0.004
0.005
0.006
0.007
0.010
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.10
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.0
2
3
4
5
6
7
10
20
30
40
50
60
70
100
200
300
Bureau of Chemistry and Soil
Fine Gravel
Coarse Sand
Medium Sand
Fine Sand
Very Fine Sand
Silt
Clay

FIG. 3

of the mean diameter, provides a means of supplying all the necessary information regarding the particle-size distribution and eliminates the ambiguous and difficult-to-read symbols generally used in such soil maps and profiles. The symbolic representation of soil classifications, when appearing on drawings which have been reduced in scale, is generally unintelligible. The writer believes that the method of grade lines is a remedy for the aforementioned difficulty, but to be useful it must be standardized. It would seem of little importance which sizes are chosen to correspond to the numbers, 1 through 5, but it is important that all data recorded by this method use the same system.

The need for standardization has been felt by many who have had occasion to read the reports from various engineering offices and Government bureaus. For example, it is of no importance whether the smallest particle size is found to the right or to the left of the mechanical analysis curve. Both conventions will be found in regular use. Probably more curves showing mechanical analyses of soils have been plotted with the small sizes at the left of the sheet, whereas concrete sands and aggregate are generally reversed. There are a great many exceptions to this, however. Although it is a small point, the standardization of this convention, regardless of the use made of the curve or of the material, seems highly desirable.

The writer does not agree that any material benefit may be derived from the proposed change in the limits of sizes in the various soil classifications. Since clay cannot be defined by size alone, the division between clay and silt must be of an arbitrary nature. Division by assigning a larger size to the upper limit of clay would save time in sedimentation analyses in reaching this point. The time, however, required to reach a particular size decreases with an increase in temperature or specific gravity. Each investigator can readily determine how much time he is justified in spending on the sedimentation analyses.

The United States Bureau of Chemistry and Soils has adopted size limits for each classification of soil. These limits have come into wide use, and in view of the fact that no soil class can be entirely identified by size alone, it seems desirable to adopt these limits as a standard.

The transfer of tabular data on soil separates to a mechanical analysis graph sheet for reproducing its curve, can be facilitated by the simple method shown in Fig. 3. The screens used to accomplish this separation are designated by number and a line is drawn on the graph sheet at the position of the size of their openings. The intermediate lines facilitate the determination of mean size and "grade line deviation." The "grade numbers" are those suggested by Mr. Campbell, and the soil classifications are those of the Bureau of Chemistry and Soils.

CARLTON S. PROCTOR,[14] M. AM. SOC. C. E. (by letter).—A timely subject is treated in a refreshingly lucid style in this paper. Mr. Campbell is to be complimented for an excellent contribution to a subject on which there is general engineering agreement as to its immediate importance; because, certainly, an engineering reference to a particular type of soil should be subjected to a uniform general interpretation and such is not now the case.

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[14] Cons. Engr. (Moran, Proctor & Freeman), New York, N. Y.

The existing confusion in soil classification standards is largely a confusion in terminology, and a standard classification is to be expected only after the establishment of a standard terminology. The first step in this direction should be agreement as to the number of classifications to apply to each soil group.

Mr. Campbell proposes three classes of sand, whereas the U. S. Bureau of Chemistry and Soils includes four, and the classification listed in Fig. 1 as the "Tyler sieve" group gives five sand classes. A Tyler sieve classification was proposed by William P. Kimball, Assoc. M. Am. Soc. C. E., in 1936, based on the result of a practical experiment.[15] Professor Kimball submitted a series of bottled soil samples of Tyler sieve grain sizes to various experienced foundation engineers and found a close agreement in classification, in terms of silts, very fine, fine, medium, coarse, and very coarse sands, etc. Because the five grades of sand were invariably used and because the variations in classification occurred at the margins of silt to very fine sand, and sand to fine gravel, a Tyler sieve classification was proposed. A similar sub-division of sands was presented by T. A. Middlebrooks,[16] Assoc. M. Am. Soc. C. E., which covered essentially the same grain size ranges for each of the five sub-divisions. This classification was reported as having been used at the Fort Peck Dam, in Montana. The five-group sand classification confirms a wealth of experience of practicing foundation engineers in that for most reports received from the field, five classifications of sand are used. Therefore, the single criticism that can be made of Mr. Campbell's paper lies in his use of only three sand classifications and in the fact that the abscissa scale of Fig. 1 does not lend itself to four or five sub-divisions.

The soil classification that will be ultimately accepted as standard must be equally practicable of application in both field and laboratory and must follow the terminology familiar to, and accepted by, the contractor who will be guided by the engineer's plans and specifications. The scale proposed meets this condition except for sand, but the fact that Tyler concrete sieves are normally found in the ready equipment of engineers, architects, and contractors should not be overlooked in any consideration of a standard soil classification.

The importance of agreement in terminology of soil classifications lies in the practical consideration that most soil classifications are, and will probably continue to be, made as a result of experienced visual examination only, occasionally correlated by trial mechanical analysis to a preferred classification. Mechanical analysis of all, or nearly all, the samples in a large exploration project is too time-consuming and expensive for many operations.

The paper, as a whole, is highly commendable. The proposed method of recording and tabulating mean diameter, grade line deviation, and grade is ingenious and may well prove of real practical value. This is a logical development from the work of the late Allen Hazen, M. Am. Soc. C. E., in the field of filter sands.

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[15] *Proceedings,* International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., June, 1936, Vol. 1, pp. 302 to 305, inclusive.

[16] *Loc. cit.,* p. 145.

T. T. KNAPPEN,[17] M. AM. SOC. C. E. (by letter).—Grain-size distribution graphs give the engineer or technician a picture that is useful in suggesting, in a general manner, what the properties of a soil may be. The author has indicated a possible method of classifying grain-size curves for statistical purposes. Presumably, such classification might prove to be useful in coordinating physical properties with grain-size distribution. The author remarks that the grain-size curve is frequently an indication of type—that is, residual, alluvial, or æolian—and that it may be an indication of maturity. Otherwise, he does not suggest much use for his method of representing grain-size curves numerically. Possibly he can make the value of this method of classification more apparent.

In the past the writer has considered various methods, including several similar to that suggested by the author, for statistical comparison of the properties of soils, but abandoned them as useless. The engineer is interested in many properties of soils in their natural and remoulded states. Some of these properties are: Permeability; cohesion; internal friction; consolidation characteristics; structure; porosity, or void ratio; water content; chemical composition; specific gravity; etc. So many factors other than grain-size distribution are vital in predicting the behavior of a soil under certain conditions, that one wonders if any useful purpose can be served by the proposed method of classification. Its value must be less than the grain-size curve itself since it is only an approximate method of describing such a curve. To illustrate, the 50% size gives little if any indication of permeability, and even the 10% and 20% sizes may be of little use for this purpose. Unless there are more values than are apparent, it is the writer's belief that the introduction of such a method of classification would be only one more confusing factor in an already "muddled" field.

With his tabulation of the nomenclature used for various soil separates, the author illustrates the confusion now present in this field. He suggests still another method of classifying the separates. Arthur Casagrande, Assoc. M. Am. Soc. C. E., once suggested to the writer that these various efforts to apply well known soil names to these fractions were not only idle but also harmful. Remembering this remark the writer consulted a dictionary to see what these words meant to the authorities on language. Clay is defined[18] as,

> "A widely distributed earth, plastic and tenacious when moist and hardening when baked, consisting of pure kaolin or, more commonly, of a mixture of kaolin with more or less finely eroded material, chiefly quartz, feldspar, and mica. * * * Dry clay consists essentially of a compound of alumina, silica, and the elements of water. When it is subjected to a baking heat water is formed and passes off as steam and a material quite different from the original clay is left behind. It is very hard, and its powder, mixed with water, is not plastic. * * *"

Silt is defined as: "(*a*) Mud or fine earth suspended in running or standing water; (*b*) a deposit of such mud or fine earth." Sand is "a loose material consisting of small but easily distinguishable grains (most commonly of quartz), resulting from the disintegration of rocks. * * *"

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[17] Engr. Mgr., Parsons, Klapp, Brinkerhoff & Douglas, New York, N. Y.

[18] Webster's New International Dictionary, G. & C. Merriam Co., 1930.

It might be best to forget the various attempts to apply these names to soil fractions and retain them as a common language for the engineer and layman. Are not such terms as "blow sand," "sticky clay," "soft silt," "loess," "gumbo," "coarse sand," "hard-pan," "boulder clay," etc., more descriptive for conveying to the engineer or layman the general properties of a soil than "D 0.022-G 3.9-Grade line deviation + 0.021"?

JACOB FELD,[19] M. AM. SOC. C. E. (by letter).—Separation of soil particles by means of square mesh screens is the oldest method of analysis for the purpose of classification. The author's proposed standardization of sizes to match grades covering the usual soil fraction names is complete and should be adopted. However, it must be carefully noted that the separation by screens does not distinguish between particles of different shapes. It is perfectly possible to have identical gradation curves, both for sizes and for percentages in each grade, for two very dissimilar soils. The differences in physical action of large grains, above the grade of fine sand, of different shapes can be disregarded. For smaller grains, ratio of surface area to mass becomes an important factor, because of the importance of surface tension and other cohesive forces in small-grained soils.

The effect of shape can best be pictured by the following tabulation of grains, all of which will be retained on a square-mesh screen with a width of opening slightly smaller than unity:

(1) A cube with unit side, has a volume of one, surface area of 6, and a ratio of surface to volume of 6.

(2) The cube split into halves by a plane parallel to, and midway between, two opposite faces; each grain has a volume of 0.5, a surface area of 4, and a ratio of surface to volume of 8.

(3) The cube split into halves by a plane, through two opposite edges; each grain has a volume of 0.5, a surface area of 4.41, and a ratio of surface to volume of 8.8.

(4) The cube split into three equal parts by two planes parallel to opposite faces; each grain has a volume of $\frac{1}{3}$, a surface area of 3.33, and a ratio of surface to volume of 10.

(5) The cube split into three equal parts by planes joining one corner with two edges meeting at the opposite corner (forming pyramids with a square base, two normal and two oblique faces); each grain has a volume of $\frac{1}{3}$, a surface area of 4.83, and a ratio of surface to volume of 14.5, almost 2.5 times that of the cube.

The process can be continued indefinitely, but the general picture shows why sand and clay or basically cubical and flat grains act so differently. When particles are in the silt and clay grades, shape is at least equal in importance with size.

HOWARD F. PECKWORTH,[20] M. AM. SOC. C. E. (by letter).—The concentration of graphical representation of grain-sized distribution into a recognized standard, as presented by Mr. Campbell, is an excellent proposal. If experience shows that his proposed system meets with general approval an effort would be made to secure its adoption as a standard.

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[19] Cons. Engr., New York, N. Y.

[20] Civ. Engr., Pacific Constructors, Inc., Redding, Calif.

As the author proposes in part to fix the definition of "sand," "silt," "clay," and "colloid" in relation to grain size, and as it is necessary to distinguish between the popular usage of these words and their uses in relation to grain size, it is unfortunate that the meanings of the words, "silt" and "clay," are sometimes in such sharp disagreement with their meanings as proposed and as used in the grain-size distribution charts.

An example of this disagreement was demonstrated by a very thin bituminous coal seam running below a borrow-pit which supplied material for a rolled-earth dam. When the steam shovel, digging into the borrow-pit, broke through into this entirely unsuitable coal seam it picked up fine-grained, broken, coal which, when examined with the earth, fell between the grain sizes, 0.0006 and 0.06 mm; and yet this coal could never be called clay or silt by any stretch of the imagination.

Another example is presented by fines, dust, and waste from a sandstone crushing operation being piled next to a borrow-pit to be used for earth for a rolled-earth dam. When wet it was sometimes difficult to distinguish between the material in the borrow-pit and the dust from the waste pile. Some of the material in the waste pile would fall between the sizes, 0.0006 and 0.06 mm, on the grain-size distribution chart; and yet it was 90% pure silica particles, which again could not be called clay or silt.

If this proposed classification is to be adopted as a standard, the writer would like to suggest that the names of the gradations be changed from "sand," "silt," "clay," and "colloid," to "sand," "fines," "superfines," and "colloid," or some similar wording that would not bring such a sharp disagreement between the popular dictionary usage of the words, "clay" and "silt," with materials actually dealt with in soil mechanics other than clay and silt which nevertheless fall in sizes between 0.0006 and 0.06 mm.

Joel D. Justin,[21] M. Am. Soc. C. E. (by letter).—The author presents an interesting method of classifying soils. The semi-log scale, shown in Fig. 1, is in quite general use for plotting the results of mechanical analyses. Some technicians, however, plot with the small sizes at the left and the larger sizes at the right as the author has done in this case, whereas others plot with the large sizes at the left and the small sizes at the right. This seems like a small matter, but when one becomes familiar with the meaning of curves plotted in a certain manner and another technician appears, who insists on doing it differently, it takes some time before the curves mean very much.

There would seem to be no sound reason why the semi-log scale and its ratio for plotting results of mechanical analyses, as well as the direction of plotting, should not promptly be standardized. The writer does not care which way it is done as long as every one does it the same way.

The author proposes a soil classification based on two factors: (1) The mean grain size, the size than which 50% of the material by weight is smaller;

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[21] Cons. Engr., Philadelphia, Pa.

and (2) a gradation factor based on the slope of the curve as more specifically defined by the author.

Such a method of classification would have its advantages, because a soil for which the mechanical analysis curve had a certain slope and a certain "mean grain size" would have fairly uniform physical properties common to other soils falling into the author's same classification.

The writer doubts, however, that the classification proposed by the author would have any practical advantage over the Kendorco classification and particularly over the modified Kendorco[22] classification as utilized by the U. S. Army Engineers of the Providence, R. I., District, and elsewhere.

In addition to any system of classification, such as the Kendorco, or the one advocated by the author, it is desirable to use a very general classification so that names, such as clay, silt, sand, and gravel, may be applied to soils with particle sizes that fall between certain limits. The so-called M. I. T. classification is satisfactory for this purpose.

The Kendorco classification has been used by the engineers of the Metropolitan District Water Supply Commission of Massachusetts and also in modified form by the U. S. Army Engineers of the Providence, R. I., District. Under this plan soils are classified by numbers. Even numbers indicate soils of relatively uniform grading, and odd numbers represent soils of relatively variable grading. Coarse soils are represented by the lower numbers and fine soils by the higher numbers. Thus, 2 in the modified Kendorco classification represents a uniform coarse, to medium, sand and 12 a uniform clay.

Synthetic samples representing the different classes (from 1 to 13) of soils may be made up. With a little experience one can, quite successfully, determine by visual inspection into what classification a given soil will probably fall. As soils falling into any given classification have quite uniform physical characteristics, and permeabilities are known within reasonable ranges in advance of tests on the actual samples, the idea has definite practical advantages.

The writer considers the modified Kendorco classification to be one of even greater practical usefulness to the engineer who must deal with soil as an engineering material, but who is not himself a soils technician. He hopes that some one of the engineers who have developed this method of classification to its present degree of usefulness will embrace the opportunity afforded by this discussion to describe it more fully.

L. B. Olmstead,[23] Esq. (by letter).—In the mechanical analysis of soil materials texture is expressed as percentages of several size classes. Numerous attempts have been made to increase the usefulness of these data by varying the size limits so that some simple mathematical relationship may exist between the diameters, areas, or volumes of the particles in the different classes. Others have attempted to increase the usefulness of existing data by various forms of curve plotting. Mr. Campbell has attempted both.

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[22] "Permeability Determinations, Quabbin Dams," by Stanley M. Dore, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 682–711.

[23] Physicist, Div. of Soil Chemistry and Physics Research, Bureau of Chemistry and Soils, U. S. Dept. of Agriculture, Washington, D. C.

In addition to semi-logarithmic plotting of size limits against summation percentages, Mr. Campbell has picked out three points on the graph, presumably at such positions as he believes will usually furnish the best criteria of the shape and position of the curve. These points are called diameter-grade designation and grade-line deviation. Whether this contribution by the author succeeds in increasing the application of the mechanical analysis data will depend upon the use that will be made of it. Thus far, little use has been found in the laboratory of the U. S. Bureau of Chemistry and Soils, for derived data, or for graphical representations of mechanical analysis data. This is due in part, no doubt, to the fact that the chemical nature of the finer soil material, as well as the size distribution, affects the physical properties.

In attempting to draw conclusions from mechanical analysis data, it is necessary to take account of the validity of the data. Determinations made by the hydrometer and the pipette methods may not check, and sieves used in different laboratories for the same size limit may differ by more than 50% in area of mesh opening.

In his plan for size classification, Mr. Campbell proposes to adopt the limits, but not always the same grade names, used by the International Society of Soil Science for materials between 2 microns and 2 mm. He extends the range at each end of the scale, and subdivides each class once. Since, in this classification, the integers, 2 and 6, recur alternately, the ratios between class limits are 3 and 3⅓, alternately applied. There are two other rhythms holding fast to the International size limits that might be considered if their practicality is ignored. One involves the ratios of 4 and 2.5, alternately applied. The class limits would be 0.2, 0.8, 2, 8, 20, 80, * * * microns. For most soils the separation at 8 microns is quite likely to give a more even division of the silt class than when the separation is made at 6 microns. Since the logarithms of 2 and 8 are 0.301 and 0.903, respectively, the mantissa differences are almost exactly 0.60 and 0.40, a circumstance that facilitates semi-logarithmic plotting. This plan is open to the same objection as that proposed by Mr. Campbell. It provides too many classes of the small grains and apparently too few of the large ones.

The other and more flexible system would retain the decimal classes of 2, 20, 200 and 2 000 microns, of the International System, and extend it at both ends of the scale, as Mr. Campbell proposes. Then, in regions where the number of size classes are inadequate for the needs of a particular laboratory, additional optional classes might be inserted, based upon the ratio of $\sqrt{10}$ between limits. If further separations should be required, the ratio $\sqrt[3]{10}$ or $\sqrt[4]{10}$ could be used, depending upon the number of sub-divisions needed.

This classification possesses a rhythm, which the mathematical mind appears so emphatically to demand, and makes a flexible provision for a sufficient number of size classes; but is it workable? There are no means of separating clastic materials strictly according to size. About all that can be done is arbitrarily to define equivalent diameters in terms of settling velocities and sieve specifications. If screening values are to yield equivalent sizes close enough to satisfy this classification, special screens, no doubt, will have to be provided. Even if the necessary special sieves might possibly be obtained, the selection of a classification which requires them is open to serious objection.

The writer suggests modifying this plan, in so far as it is necessary, in order to use standard or other accurate and easily obtainable screen materials. Sieves having screening values nearest the nominal sizes of the extended International Classification would be adopted and always used in the region where they apply. Highway engineers probably would omit the smaller sized groups and agronomists the larger ones. In addition, supplementary intermediate classes, utilizing standard screens, should be set up for use when more size classes are needed.

If conventional size designations are desired for the sieved material, they must be defined arbitrarily in terms of mesh openings, because the screening values of clastic materials varies with the shape factor. The present lack of accepted sieve specifications partly nullifies the benefits to be derived from the International Classification. In the smaller sized classes the relation of particle size to settling velocity permits sub-divisions of the material at almost any point. However, there is no fixed relation between settling velocity or particle size and the readings at stated time intervals of the soil hydrometers in common use. These hydrometers were originally calibrated for one particle size distribution, and their design is such that they cannot detect some unusual distributions no matter how many class limits are provided.

The fact that many institutions would not adopt a standard size classification of soil materials at this time is no reason why such a standard should not be set up and made available for all who are free to use it. Conditions may arise that permit laboratories to make gradual changes from the classification in use to a standard system. Much work and many years were required for the adoption of a classification by the International Society of Soil Science, and that classification should be used if possible as the basis of an amplified system.

The size limit of 2 microns appears to have been well chosen by the International Society. Accurate determinations cannot easily be made upon all kinds of soil materials below this limit, with the methods of dispersion and analysis now available. It is difficult to make satisfactory determinations even at this point upon some lateritic soils. In the U. S. Bureau of Chemistry and Soils the upper size limit of extracted colloids is not far from 0.2 micron, but the separation is seldom carried to completion. The upper limit for clay is now set at 2 microns, the material between 2 and 50 microns being classed as silt. An additional pipette sample is now taken for a 20-micron class limit, and a sieve separation is made at 0.2 mm, so that the results of analysis can be expressed in either the Bureau, or the International, Classification.

T. A. MIDDLEBROOKS,[24] ASSOC. M. AM. SOC. C. E. (by letter).—With all due respect to the author for his conscientious attempt to obtain uniformity in soil classification, it is believed that more could be done by withdrawing a number of classifications now in use, rather than recommending new ones to confuse the field further. Little can be gained by additional soil classificationsthat vary only slightly from standard accepted classifications. The classification offered by the author does not vary in any important features from the most widely used classification of the U. S. Bureau of Chemistry and Soils. Therefore, why change?

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[24] Senior Engr., Office, Chf. of Engrs., Washington, D. C.

Grain-size dimensions in the silt and sand ranges are arbitrarily set and have no particular significance as to the soil characteristic. Whether the division between sand and silt is set at 0.05 mm or 0.06 mm is of no consequence. Engineers merely need time to become familiar with the classification agreed upon. On the other hand, the division between silt and clay is of great importance and it cannot be set satisfactorily on any particular grain size, but must be determined by mineralogical examination for the different types of clays encountered. Rock flour, which is finely ground rock mineral, may have a large percentage of grains within the clay range of grain sizes, but it will not have the characteristics of a clay mineral. The writer has found montmorillonite clays in which the division between silt and clay is approximately 0.005 mm and bedilite clay in which this division is approximately 0.002 mm; therefore, it is apparent that no definite limit can be set for clay.

Experience of the writer in various sections of the United States has shown that either the Massachusetts Institute of Technology, or the U. S. Bureau of Chemistry and Soils, classification is applicable for all types of soil provided the division between silt and clay is determined by mineralogical examination.

Frank E. Fahlquist,[25] Assoc. M. Am. Soc. C. E., and Waldo I. Kenerson,[26] Esq. (by letter).—A new method for graphically presenting the grain-size distribution of a soil is proposed by the author. He also submits a soil classification based on the mean grain size and gradation of soil particles. Acceptable standards for describing the grain-size distribution of soils are needed, and the author should be commended for presenting his ideas to the profession. The use of such expressions as uniform coarse sand or uniform fine sand conveys definite impressions of particular types of uniformly textured soil. However, the use of such expressions as sandy silt, silty sand, clayey silt, silty gravel, clayey loam, etc., conveys an inadequate conception of non-uniformly graded materials. Such terms are often interchanged and incorrectly used by engineers. This results in vagueness and considerable confusion.

The Bureau of Chemistry and Soils, U. S. Department of Agriculture, has devised a classification that has been in general use for many years. More recently, the so-called Massachusetts Institute of Technology (MIT) soil classification has been used increasingly among engineers and soil technicians. Both these classifications are included in Fig. 1 of the paper. The writers prefer the MIT classification because it is easier to remember and is more logically constructed in that successive decrease in arbitrary grain-size limits follows a definite pattern. However, neither the MIT nor the Bureau of Soils classifications are adequate for describing non-uniformly graded materials correctly. Furthermore, in analyzing fine-textured sediments, such as silts or clays, there is difficulty in recognizing the limiting sizes of each type of material. Soil scientists do not agree where the line should be drawn in separating colloids from the coarser fractions. Notwithstanding this fact, there appears to be no reason why arbitrary limits, such as are shown in the MIT classification, should not be used. The writers believe that use of the terms, "clay" or "fine silt," for instance, should be based not only on a grain-size analysis, but what is much more important from an engineering viewpoint, on the behavior characteristics of the material.

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[25] Geologist, War Dept. at Large, U. S. Engr. Office, Providence, R. I.

[26] Chf., Soils Testing Laboratory, U. S. Engr. Office, Providence, R. I.

The author proposes to classify soils according to a mean diameter size (grain-size diameter at 50% ordinate) and five arbitrary grade lines as shown in Fig. 1. This is essentially an improved adaptation of the soil constants, "effective size" and "uniformity coefficient," introduced by the late Allen Hazen, M. Am. Soc. C. E. These new constants partly fulfill a long desired improvement growing out of appreciation by engineers of the inadequacy of Hazen's coefficients to classify soils properly.

A soil classification based on grain size only, however, is not sufficient. Engineers who have investigated the finer grained sediments have long recognized this point. Charles Terzaghi, M. Am. Soc. C. E., has stated:[27]

> "The properties that determine the behavior of the soil in the foundation pit directly are not the uniformity, nor the mineralogical composition, nor the water content. They are:
>
> "(1) The volume change produced by an increase of the pressure acting on the soil; * * *.
> "(2) The permeability of the soil; * * *.
> "(3) The cohesion or the shearing resistance of the soil under zero load; * * *."

The extent to which these properties are affected by the mineral and water content and natural structure must be determined in the soils laboratory before final classification is made.[28] In addition, a soil classification to be of practical use in the field must be adaptable to reasonably accurate classification of natural sedimentary deposits.

The Providence (R. I.), District, U. S. Engineers, has developed a soil classification which meets the requirements imposed by conditions in the field and by the behavior characteristics of the sediments. During the two years, 1936-1937, this idea has been fully tested and found reliable. More than 5 000 mechanical analyses performed in the Soils Laboratory have proved its adequacy both as to gradation and size limitation of the individual classes. It is based, but with important modifications, upon the Kendorco classification described by Stanley M. Dore, M. Am. Soc. C. E.[29] Rigidly standardized descriptive terms are used in the Providence classification as follows:

Class	Description of Material
1	Clean Gravel: Contains little coarse to medium sand.
2	Uniform Coarse to Medium Sand: Contains little gravel and fine sand.
3	Variable—Graded from Gravel to Medium Sand: Contains little fine sand.
4	Uniform Medium to Fine Sand: Contains little coarse sand and coarse silt.

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[27] "The Science of Foundations—Its Present and Future," by Charles Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 296.

[28] *Loc. cit.*, p. 297.

[29] "Permeability Determinations, Quabbin Dam," by Stanley M. Dore, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 682–711.

Class	Description of Material
5	Variable—Graded from Gravel to Fine Sand: Contains little coarse silt.
6	Uniform Fine Sand to Coarse Silt: Contains little medium sand and medium silt.
7	Variable—Graded from Gravel to Coarse Silt: Contains little medium silt.
8	Uniform Coarse to Medium Silt: Contains little fine sand and fine silt.
9	Variable—Graded from Gravel to Medium Silt: Contains little fine silt.
10	Uniform Medium to Fine Silt: Contains little coarse silt and coarse clay. Possesses behavior characteristics of silt.
10C	Uniform Medium Silt to Coarse Clay: Contains little coarse silt and medium clay. Possesses behavior characteristics of clay.
11	Variable—Graded from Gravel or Coarse Sand to Fine Silt: Contains little coarse clay.
12	Uniform Fine Silt to Medium Clay: Contains little medium silt and fine clay (colloids). Possesses behavior characteristics of silt.
12C	Uniform Clay: Contains little silt. Possesses behavior characteristics of clay.
13	Variable—Graded from Coarse Sand to Clay: Contains little fine clay (colloids). Possesses behavior characteristics of silt.
13C	Variable Clay: Graded from sand to fine clay (colloids). Possesses behavior characteristics of clay.

Soils are divided into thirteen general classes. Distinctions between soil classes are based upon grain-size limits as shown graphically in Fig. 4. An important feature to be noted in this diagram is the flexibility allowed for the limiting diameters of fine silt and coarse clay. Either of these materials is allowed to range between 0.006 mm and 0.0006 mm. This flexibility is introduced by the writers in recognition of the fact that natural depositions of fine-textured sediments, such as glacial silts (rock flour), behave differently from clays despite similarity of grain-size distribution. It is interesting to note that this same flexibility is recognized by the Society's Committees on Road Materials, as shown by the author in Fig. 1.

Grain-size limits of the even numbered classes have been chosen arbitrarily so that the particle diameters at the 90% and 10% ordinates conform to the limiting sizes as recognized in the MIT classification. Investigations in the laboratory have disclosed that grain-size curves of variable depositions vary over a wider range in their upper parts as compared to their lower parts when attempts are made to conform to this classification. Therefore, the idealized and limiting curves for Classes 3, 5, 7, 9, 11, and 13 are discontinued at the 50% ordinate. Heavy full lines indicate idealized depositions, whereas broken lines indicate limiting boundaries within which classified depositions may vary.

Sediments which do not conform to either of the two principal idealized slopes can be easily classified by this system. For instance, a sediment can be classified as Class 7–5. This designation means that the upper part of the curve lies within the Class 7 range and the lower part within the Class 5 range. Likewise, a Class 5–7 material indicates an extremely variable soil in which the upper part of the curve lies within the Class 5 range and the lower part within the

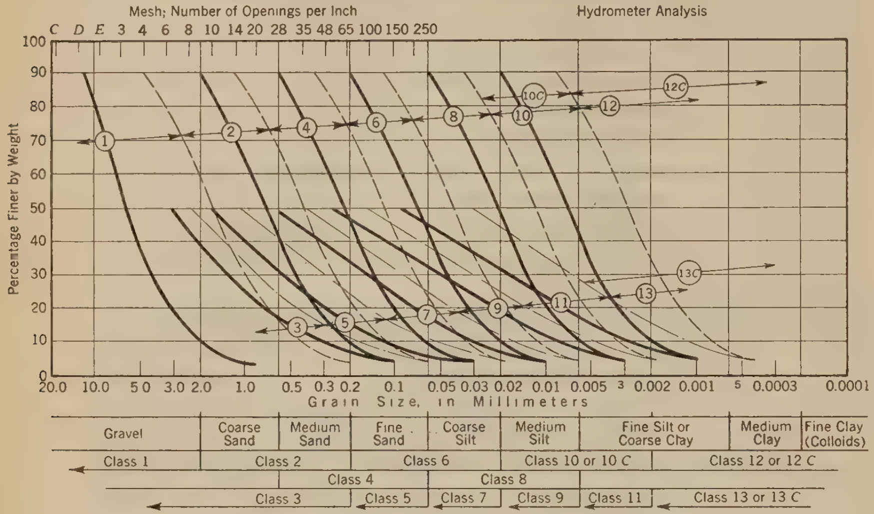


FIG. 4.—DIAGRAM SHOWING LIMIT OF SOIL CLASSES

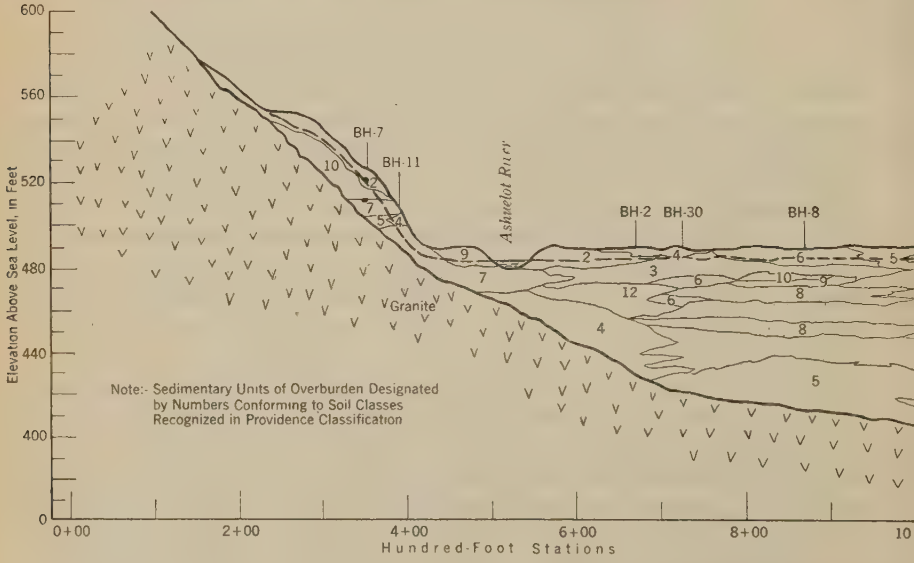

Elevation Above Sea Level, in Feet
600
560
520
480
440
400
0
BH-7
BH-11
Ashuelot River
BH-2
BH-30
BH-8
10
2
7
5
4
9
2
4
6
5
3
6
10
9
12
6
8
8
8
7
4
5
Granite
Note:- Sedimentary Units of Overburden Designated by Numbers Conforming to Soil Classes Recognized in Providence Classification
0+00
2+00
4+00
6+00
8+00
10
Hundred-Foot Stations

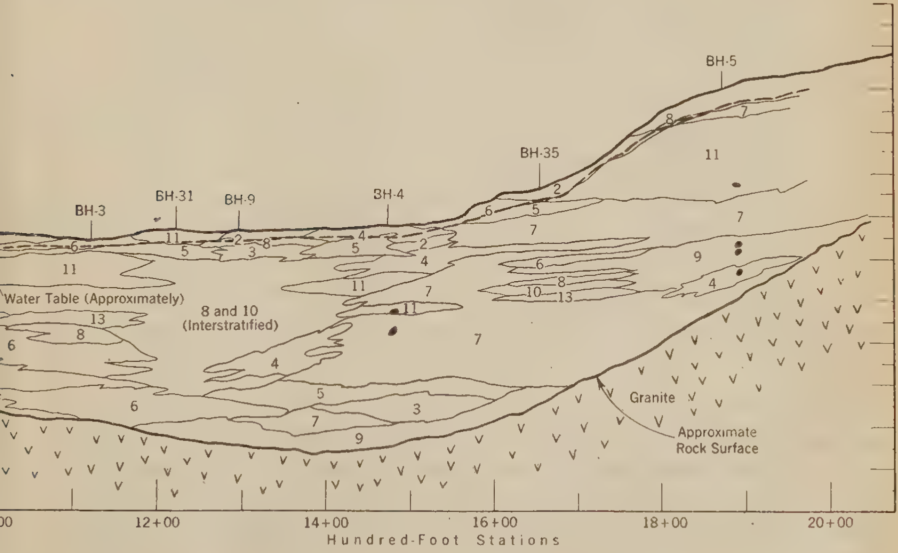
FIG. 5.—PROVIDENCE CLASSIFICATI

Class 7 range. This same procedure is applied to variations in gradations of uniformly textured soils. Thus, a Class 6–4 or Class 4–6 is recognized.

There is surprising conformity between the author's grade lines as shown in Fig. 1 and the several possible gradations recognized in the Providence classification: The author's Grade 1 conforms very closely to the slope of the idealized curves for Classes 1, 2, 4, 6, 8, and 10; Grade 2 conforms to the slope of variations in the foregoing sediments recognized as Classes 2–4, 4–6, 6–8, 8–10, 10–12; Grade 3 conforms to the Providence Classes 5–3, 7–5, 9–7, 11–9, 13–11; Grade 4 conforms to Classes 3, 5, 7, 9, and 11; and Grade 5 conforms to Classes 3–5, 5–7, 7–9, 9–11, and 11–13.

It is to be noted that the Providence classification does not rely upon certain soil constants, such as the 50% mean diameter in Mr. Campbell's classification or the grade lines, as obtained from the 20% and 80% sizes. Instead it utilizes the entire mechanical analysis curve. This is accomplished with the aid of a transparent sheet of celluloid on which is inked a full set of classification curves similar to those in Fig. 4. The master plat is placed and oriented over an unclassified grain-size curve. The class range into which this curve falls is immediately apparent and is recorded directly on the curve sheet. This method makes for rapidity and accuracy.

The behavior of fine-grained sediments particularly general Classes 10, 12, and 13 is markedly different when considered from a regional standpoint. Thus, in the glaciated New England area, fine-grained sediments derived by glacial abrasion and occurring in glacial lake deposits are found, by tests, to be angular in shape of grain, capable of being consolidated rapidly, slightly compressible (depending partly upon the mica content), and possessing a high angle of internal friction. These sediments can be properly designated as silts


BH-5
8
7
BH-35
11
2
BH-3
BH-31
BH-9
BH-4
6
5
7
6
11
2
8
4
5
3
5
2
4
9
11
6
8
10
13
4
Water Table (Approximately)
11
7
11
13
8 and 10
(Interstratified)
8
6
7
4
5
6
3
7
9
Granite
Approximate
Rock Surface
00
12+00
14+00
16+00
18+00
20+00
Hundred-Foot Stations

LIED TO GEOLOGIC SECTION

or rock flours. In contrast, alluvial sediments of similar fineness from the Mississippi Valley have decidedly different properties. These sediments are often composed of flat and scale-like clay particles, together with coarser fractions derived from rock decay. They consolidate slowly, are highly compressible, and possess a low angle of internal friction. It is readily seen that to classify these materials, as Class 10, for example, without further qualification would lead to misunderstanding on many important and critical properties. In such cases the Providence classification provides for referring to the former as Class 10, and to the latter as Class 10C, the letter C indicating behavior characteristics of clay.

The author's statements as to the geologic significance of the mechanical analysis curve are valuable. In the past, the geologic aspect of soils has been too little appreciated by engineers and soil technicians. The ease with which soils classified under the Providence System can be indicated in geologic sections is shown in Fig. 5. The character of the material in a formation and the mode of occurrence are generally reliable criteria as to the geological history of that formation. Certain sedimentary types are more prominently developed in one section of the country than in another. For example, glacial deposits are prominently developed in the northern part of the United States and Canada. Loess deposits, probably of wind-blown origin, occur throughout a large part of the Mississippi Valley, particularly in Nebraska, Wisconsin, and Arkansas. Soils in the South and Southwest are largely of residual origin. In river valleys throughout the United States there are large accumulations of alluvial deposits of both fine and coarse texture. A few examples of the accumulations in certain recognized geologic formations are:

Geology	Classes
Alluvial deposits (coarse-textured)	1, 3, 5, 7, 9, 2, 4
Alluvial deposits (fine-textured)	4, 6, 8, 10, 10C, 12, 12C, 11, 13, 13C
Glacial deposits—till	9, 11, 13, sometimes Class 7
Glacial deposits—outwash	1, 3, 5, 7, 2, 4, 6
Glacial lake deposits	6, 8, 10, 10C, 12, 12C
Aeolian (wind-blown) deposits	4, 6, 8
Loess deposits	Chiefly Class 8
Residual soils	3, 5, 7, 9, 11, 13, 13C

The author's classification has considerable merit. His paper comes at an opportune time, and it is felt that it will result in considerable clarification as to acceptable definitions of soil separates.

E. W. Lane,[30] M. Am. Soc. C. E. (by letter).—In this brief but very enlightening paper, Mr. Campbell has discussed three principal points: (1) The use of semi-logarithmic paper for plotting mechanical analyses; (2) a method of indicating the variability of the size graduation; and (3) a suggested division of soils into classes according to the size of the particles of which they are composed, with class names. In the following paragraphs the writer will discuss each of these three points in turn.

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The use of the semi-logarithmic type of gradation curves advocated by the author has much to commend it and, particularly in the field of earth-dam material, it is much superior to any other method of graphical representation of particle sizes with which the writer is acquainted. Another method extensively used, particularly by geologists, is the so-called histogram, which is simply a bar graph, the length of each bar of which represents the percentage of the total material between two sizes. The number of bars depends on the number of fractions into which the sample was divided. Another type is a summation curve similar to that advocated by Mr. Campbell, but for which a Cartesian scale of co-ordinates is used for the grain diameter in place of a logarithmic one.

The semi-logarithmic plotting of a summation curve is much superior to either of these methods. The disadvantages of the histogram method are that it is not standardized since the widths of the bars are all the same, but each bar may represent any size range selected by the investigator. The shape of the histogram also depends upon the number of fractions into which the sample is divided. It is not possible to determine the percentage of the material smaller or greater than any specified size, except such sizes as the limits of a particular fraction, and then only by a process of adding the lengths of the bars between that point and one end of the diagram. In the semi-logarithmic plotting, the curve will be practically the same regardless of what limits are used for the various fractions, and it is possible to determine the percentage greater or less than any specified size and also to determine the median size or size for which 50% is greater and 50% smaller. This size forms a convenient single value to express the size of the material.

In the form of summation curves using the Cartesian co-ordinates for the grain diameter, unless the chart is very long, the smaller sizes are difficult to read and plot accurately. Very fine sizes particularly come so close to the zero size line that it is very difficult, if not impossible, to read them. In the semi-logarithmic plotting, the scale of the smaller size is enlarged, and it is possible to read accurately down to the finest particle used. For purposes of comparison, it is possible, using the semi-logarithmic paper, to plot on the same sheet, using the same co-ordinates, the size distributions of a number of samples with much less confusion than is possible using the other methods.

In the plotting of sizes of material moved by flowing water there is frequently considerable uniformity of particle size and the curves often take the shape of Mr. Campbell's Sample (*c*). They frequently have a distinct S form, showing that the greater part of the material is of nearly uniform size, but that a small part has a considerable size range larger than the main body and that another small part has likewise a considerable range smaller than the principal part. As these small fractions may prove to have a considerable bearing on the transportation of the material, it is desirable that the method of graphical representation used indicate their proportions accurately. A system of co-ordinates which would expand the extremities of the "per cent larger" scale would aid in this desirable end. The writer is experimenting on the development of such a form of paper.

Mr. Campbell's comparison of the various classifications—clay, silt, sand, etc.—serves to emphasize the lack of agreement among the workers in this field. Mr. W. W. Rubey[31] has mentioned several others, including those in Table 2, Columns (1) (2), and (3), each of which has had considerable use. Still another classification is that used by L. G. Straub, Assoc. M. Am. Soc. C. E. It is the same as that of Mr. C. K. Wentworth except that it divides silt into four classes. Mr. Campbell's classification has much to recommend it, but the writer fears that no classification recommended by an individual, regardless of how eminent he may be, will be universally accepted. It seems probable that the advantages of any reasonable classification, if widely accepted, would more than offset any disadvantages that might occur in case the standard was not the best possible one, and, therefore, considerable advantage would result from the general adoption of a single standard. That classification should be adopted which seemed to be most generally acceptable to all groups, and if no agreement can be reached with other groups, it is suggested that civil engineers at least adopt one standard. Any one who had definite opinions on the superiority of another classification would still be free to use the grouping of his choice. Perhaps an agreement on certain features of the method of plotting could also be standardized, such as whether the size range should increase or decrease toward the right, and whether the percentage smaller

TABLE 2.—COMPARISON OF SOIL CLASSIFICATION STANDARDS
(Units are Millimeters)

Udden* (1)	Atterberg† (2)	Wentworth‡ (3)	Straub§ (4)
	Boulders	Boulder	Boulder gravel
256	2 000	256	256
Boulders	Blocks	Cobble	Cobble Gravel
16	200	64	64
Gravel	Pebbles	Pebble	Pebble Gravel
1	20	4	4
Sand	Gravel	Granule	Granule Gravel
1/16	2	2	2
Silt	Sand	Very Coarse Sand	Very Coarse Sand
1/256	0.2	1	1
	Powder Sand	Coarse Sand	Coarse Sand
....	0.02	1/2	1/2
	Silt	Medium Sand	Medium Sand
....	0.002	1/4	1/4
	Clay	Fine Sand	Fine Sand
....	....	1/8	1/8
		Very Fine Sand	Very Fine Sand
....	....	1/16	1/16
		Silt	Coarse Silt
....	....	1/256	1/32
		Clay	Medium Silt
....	....	....	1/64
			Fine Silt
....	....	....	1/128
			Very Fine Silt
....	....	....	1/256
			Clay

* "Mechanical Composition of Clastic Sediments," by J. A. Udden, *Bulletin*, Geological Soc. of America, Vol. 25, 1914, pp. 655–744.

† "Die rationelle Klassifikation der Sande und Kiese," by A. Atterberg, *Chem. Zeitung*, Jahrgang 29, 1905, pp. 195–198.

‡ "A Scale of Grade and Class Terms for Clastic Sediments," by C. K. Wentworth, *Journal of Geology*, Vol. 30, p. 382, 1922.

§ "Missouri River," H. R. Doc. No. 238, 73d Cong., 2d Session, Appendix XV, February, 1934.

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[31] *Professional Paper 165A*, U. S. Geological Survey, p. 23.

than or larger than a given size should be used for the ascending scale of ordinates.

The standard of gradation or uniformity suggested by Mr. Campbell, the writer believes to be a very good one. Although it has a purely empirical basis, so far as known no classification on a more rational basis has been suggested. It has the great advantage of being easy to obtain. No such standard will be widely used which for each sample requires the expenditure of considerable time for its determination. The writer has long recognized the desirability of some easily determined coefficient of variability for convenience in expressing in numerals, roughly, the composition of samples of granular material. He believes that the two numbers, median size (50% of the weight composed of larger particles and 50% of smaller particles) and the Campbell variability coefficient, together make possible a fair representation of the size and gradation of a granular material.

It is believed, however, that it would be possible to improve the proposed gradation coefficient. Mathematically, the value of this coefficient is 10/6 of the logarithm (Base 10) of the ratio of the "80% smaller" size to the "20% smaller" size. To use 6/6, the logarithm of this ratio, instead of 10/6, would have several advantages. The coefficient would then have a simpler physical meaning, which is always an advantage, especially in defining it. Knowing the 80% and 20% sizes, it would be possible to determine approximately the coefficient by mental calculations, and an accurate computation could be readily made with a slide-rule, requiring one less operation than the form proposed. Graphically, it would be the number of cycles of the semi-logarithmic scale between the 20% smaller and 20% larger point. This distance could be measured directly without drawing the slope line on the graph. The value of this proposed revised coefficient in every case would be 0.6 of that of the one proposed by Mr. Campbell. Some thought should also be given to the possibility that the use of, say, the 15% and 85% points on the 10% and 90% might be preferable to the 20% and 80% points.

F. J. SANGER,[32] ASSOC. M. AM. SOC. C. E. (by letter).—It is a well established fact that the only way to describe the grading of a soil is graphically, and the best chart is probably the one commonly used, having the log scale at the bottom for grain sizes. The terminology used in describing grains of particular grain sizes cannot be justified. To take existing, well understood terms describing natural soils and to use them for more precise meanings very near, but very different from, their previous meanings, is unscientific and misleading. The exact divisions are artificial and hence can never be standardized to agree with all points of view. Why not, then, discard these confusing divisions? If it really is necessary to subdivide the horizontal scale according to present practice, other terms should be used, and probably a numbering system is best. The grain-size distribution of a particular natural soil, as specified in the chart used by the U. S. Bureau of Public Roads, can better be shown by an area bounded by two curves, inside which a soil of the particular class must lie. Engineers are not in the habit of defining structures and machines by lengthy descriptions;

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[32] Head, Dept. of Eng. and Building, The Lester School and Henry Lester Inst. of Technical Education, Shanghai, China.

but rather by the use of drawings. Why not describe the grain-size distribution that way and not bother about printing percentages?

With regard to grain shape, is it not probable that the moisture and plasticity tests, as given in A. S. T. M. publications, take full account of the important effects?

This means that to describe a natural soil, a chart and some moisture and plasticity data are necessary—not a list of percentages, which are difficult to appreciate as they stand. Natural moisture content, specific weight of soil particles, and natural voids ratio from which density of the mass can be calculated, are also desirable, of course. The point is that the list of percentages should be superseded by the curve and that the existing terms for sub-divisions should be discontinued. The importance of the geologic history of the soil is now being appreciated and a description of the soil should include such a history, where it is known.

F. KNAPP,[33] ESQ. (by letter).—The proposal of the author to designate a particular soil by means of the grain size and the grade is most helpful and, in combination with Fig. 1, serves to give an immediate idea as to the type of the soil. Furthermore, this proposal makes generic designations, such as clay, sand, etc., superfluous. The writer would suggest that a more complete diagram of the several hundred samples mentioned by the author be included in the closing discussion.

The graphical representation of analyses of soils and their definitions by the mean grain size and grade should be helpful in permitting a more logical interpretation of a hydraulic problem, namely, the permissible maximum velocity in earth canals in order to prevent erosion. Although other factors still enter into the problem (such as the cross-section of the canal and the velocity distribution) it is clear that the properties of the soil are of outstanding importance.

Russian engineers[34] have published a table of permissible velocities in clayey soil, subdivided into four classes according to porosity. From the few indications given in regard to grain size, the writer plotted the distribution as proposed by the author. The product of mean grain size and grade multiplied by 10^3 may be plotted as abscissa, the velocities as ordinates, and the porosities as parameters. With some imagination it was found that a curve could be fitted to the various points, considering especially that the properties of soil in Nature do not change abruptly. Such a representation serves to indicate more clearly the inter-relation of the permissible velocity and the properties of the soil.

CHARLES H. LEE,[35] M. AM. SOC. C. E. (by letter).—Two objectives are definable in this paper: First, to standardize graphical representation of grain-size distribution; and second, to simplify and systematize the interpretation of such graphs. Both undertakings are opportune and it is to be hoped that the first, especially, can be carried through to an early definite conclusion. The second is more a matter of individual experience, technical background, and judgment, and cannot be reduced to set rules. The author's method of simplification is an interesting one.

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[33] With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[34] "Die zulaessigen Hoechstwerte der mittleren Geschwindigkeit in Gerinnen," by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, 1937, No. 21.

[35] Cons. Hydr. Engr.; Chf. of Div., Water Supply and Sanitation, Dept. of Works, Golden Gate International Exposition, San Francisco, Calif.

As for a graphical standard, the writer agrees with the author in favoring the one with the ordinate as percentage by weight of total sample plotted to natural scale, and the abscissa as particle size, in millimeters, plotted to a logarithmic scale, with small sizes at the left and large sizes at the right. In plotting the results of standard sieve and hydrometer analyses, however, it is found more convenient to have every number from 1 to 9 in each log cycle represented by a vertical line, and, in addition, to have the half lines between 1 and 2 and the standard sieve size lines. The use of log 2 and log 6 or any other combination, although giving a more open diagram, is not convenient for plotting or for picking off end points of soil fractions.

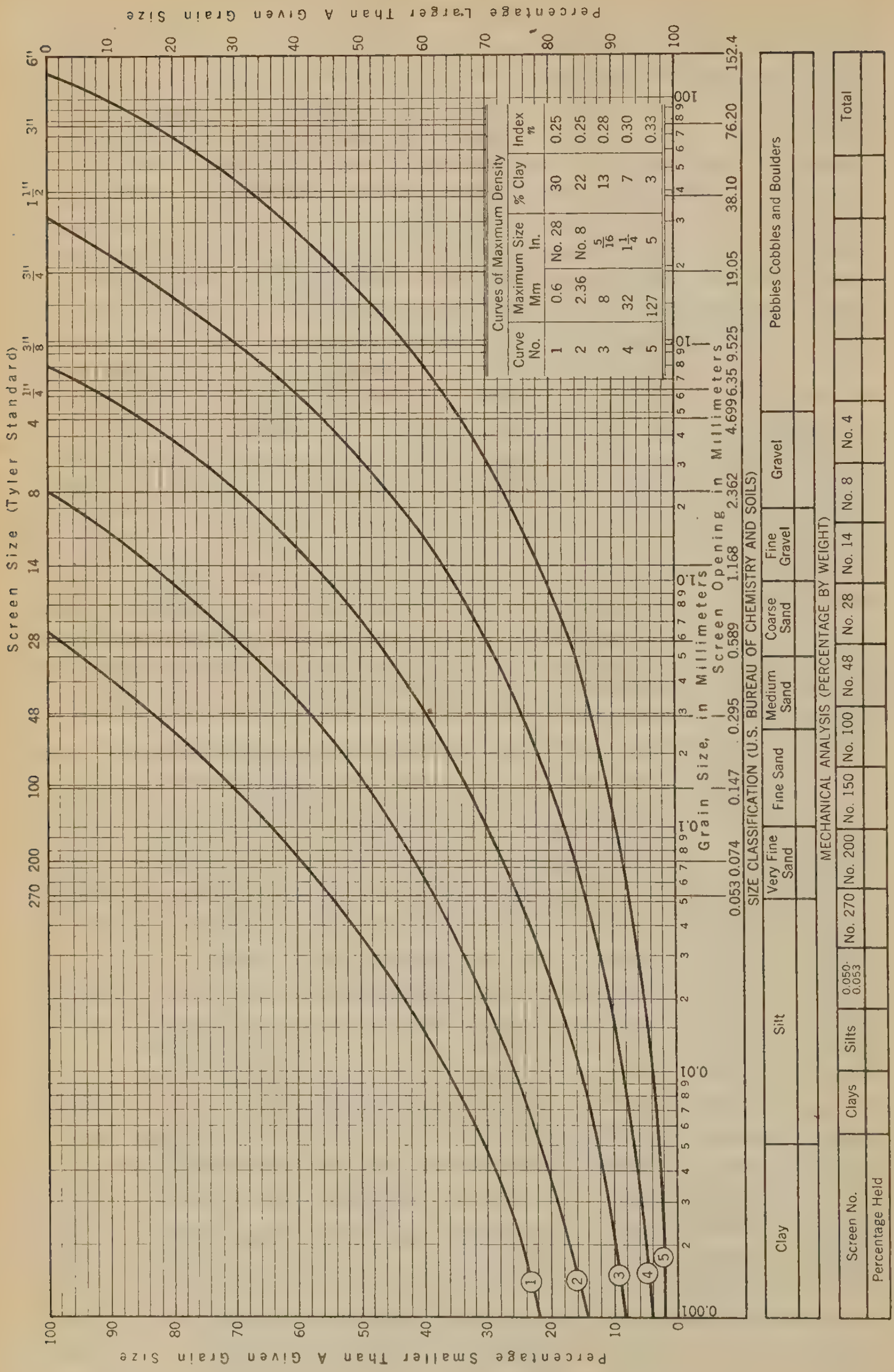
A form of graphical diagram which the writer has developed by trial through the ten years, 1928–1938, is presented in Fig. 6. It includes five cycles ranging in size from 0.001 to 152.4 mm which covers all ordinary requirements. Its one limitation is exclusion of smaller clay sizes extending into the colloids.

Fig. 6 also includes tabular spaces for entering standard Tyler sieve percentages that are held, and size classification according to the U. S. Bureau of Chemistry and Soils. This form has been found to have a wide appeal because it is easily understood, both by those who are familiar with separation by standard Tyler sieves and by those who use the U. S. Bureau of Soils classification. It is the writer's observation that the vast majority of American engineers are familiar with one or the other of these classifications, and that the engineering use of other classifications is confined to a relatively few specialists in soil technology.

The classification favored by the writer for an engineering standard is that of the U. S. Bureau of Chemistry and Soils. The reasons for such choice are: (1) It is generally recognized and widely used both by engineers and by scientific research workers; (2) the fractions conform reasonably well with distinctive physical characteristics resulting from particle size; (3) it has proved reasonably satisfactory in general use; and (4) it can be readily taken from a graph plotted from the results of any sieve and hydrometer, pipette, or similar analysis which a great variety of established laboratories are equipped to make.

The author's generalization that immature soils have size-distribution curves that are concave upward and mature soils convex upward is not supported by inspection of such curves from broad geographic areas. Curves that are concave upward represent a material in which the larger sizes predominate, whereas those in which small sizes predominate have curves that are convex upward. If the original rock contains non-weathering grains of relatively large size, such as quarts or silica, the curve may be concave upward even if weathering has reached a stage of maturity. Such a condition exists in regions of granitic rocks such as granite, diorite, etc. Rocks of this type comprise large areas in all parts of the world and contribute non-weathering constituents to both residual and transported earth materials.

Alluvial materials may vary from the uniform grain-size type to the well-distributed type, depending upon the velocity conditions at the point of deposition. Alluvial débris deposited by swiftly flowing streams at a point of



Curves of Maximum Density

Curve No.	Maximum Size Mm	Maximum Size In.	% Clay	Index n
1	0.6	No. 28	30	0.25
2	2.36	No. 8	22	0.25
3	8	$\frac{5}{16}$	13	0.28
4	32	$1\frac{1}{4}$	7	0.30
5	127	5	3	0.33

FIG. 6

grade change have size-distribution curves that are straight and sloping, or concave upward, rather than vertical. Flowing water is an excellent sorting agent only if the velocity is uniform, such as in streams traversing broad valleys or shore currents in lakes or in the ocean; but in streams that drain steep topography, the velocity changes rapidly, affording little opportunity for sorting.

The mean grain size, as defined by the author, is entirely arithmetical and has no physical meaning. Mean grain size, as a weighted geometric mean, can be determined readily from a size distribution diagram, and it is not a step in advance to adopt an arbitrary definition. The attempt to express gradation by the slope of the line passing through the 20% and 80% points on the curve is equally arbitrary, and will lead to confusion rather than simplification. The entire curve must be taken into consideration in studying the size distribution characteristics of a material.

The nomenclature proposed by the author for designating these arbitrary values is too complicated to be remembered or readily grasped mentally, and is impractical for use. The proposed size classification with log-2 and log-6 division is also arbitrary and differs from those already in use. There are no compelling reasons advanced in its favor and its adoption would add to the confusion in an already over-confused field.

In classifying size-distribution graphs the writer gives consideration to physical characteristics more or less controlled by size distribution, such as porosity, permeability, and density. At one end of the scale are uniform grain-size materials, with high porosity and permeability, but low density. Such materials are sought after as sources of well water. At the other end are well-graded materials whose graphs follow closely a curve with an equation of the form:

$$p = \frac{(d)^n}{D} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which p = the proportion by weight passing a given screen or sieve opening; d = size of opening; D = maximum particle size; and n = an exponent ranging from 0.25 to 0.40. Such material has a low porosity and permeability, and high density, and is excellent for use in constructing earth embankments. An ideal group of five such curves with maximum sizes of particles equal to 0.6, 2.36, 8, 32, and 127 mm, percentages of clay equal to 30, 22, 13, 7, and 3, and values of n equal to 0.25, 0.25, 0.28, 0.30, and 0.33, are shown in Fig. 6.

Superimposed upon this sequence of grain-size distribution is the effect of size, which in fine silt and clay produces greater porosity than in coarser fractions, lower density, and far less permeability. In conjunction with water the fines also introduce the new characteristics of shrinkage and slow consolidation under load. The limiting curves of the group in Fig. 6 include the ranges of size distribution and of size, which experience has shown are satisfactory for construction of impervious sections of rolled-fill earth dams. It is to be noted that Curves *a* and *f* in Fig. 1 are both beyond the limiting curves of the group, and both were unsatisfactory for this type of construction.

FRANK B. CAMPBELL,[36] ASSOC. M. AM. SOC. C. E. (by letter).—It is felt that the paper has been worth while if only for the interesting and valuable discussion that it has provoked. There appears to be a general desire that something should be done to bring order out of the multitude of systems in current use for the recording and the nomenclature of particle-size distribution in soils. The paper was not presented (as some one has suggested) to add another method to an already "muddled" field, but rather as a logical compromise of the two principal existing systems.

The paper presented four proposals or conceptions regarding the mechanical analyses of soils:

(1) A proposal for the standardization of the form of the mechanical analysis graph;

(2) A proposal for the standardization of the nomenclature of soil fractions based upon a compromise between the American, or Whitney, scheme and the German, or Atterberg, scheme;

(3) A method for expressing approximate distributions of particle sizes and the mean size for use on maps, cross-sections, or tabulations (this proposal is offered as a tool to be used only when found convenient for the sorting of a large number of mechanical analyses); and,

(4) A suggestion as to the interpretation of the mechanical analysis graph in terms of the history of the soil.

Standardization of the Graph.—It is indeed fortunate that the vertical ordinate of percentage by weight smaller than a given particle size, plotted with increasing magnitude upward, has found fairly common acceptance. This leaves, then, only the sense, vectorially speaking, of the horizontal ordinate or abscissa to be considered.

In closing, it should be emphasized that some engineers and scientists plot their abscissas with increasing magnitude to the right, whereas others plot them with increasing magnitude to the left. As Mr. Justin has so aptly expressed it, "This seems like a small matter, but when one becomes familiar with the meaning of curves plotted in a certain manner and another technician appears, who insists on doing it differently, it takes some time before the curves mean very much." Professor Burmister and Messrs. Fahlquist and Kenerson have shown graphs with increasing magnitude toward the left. Messrs. Kadie and Lee as well as the writer, have plotted grain size as abscissas with increasing magnitude toward the right.

For the purpose of this discussion the writer will define a positive sense of the abscissa as increasing magnitude toward the right and a negative sense as increasing magnitude toward the left. There may be a good reason for plotting with a negative sense but, unfortunately, no one has offered such reason. The writer feels that originally some one may have had the idea that the upper limit of particle sizes in soils is definite and that the lower limit is indefinite, due to the advancing technique of modern science, which may permit the measurement of smaller diameters in the future. There is probably a definite lower limit if sedimentation methods are to be used. Mr. R. C. Thoreen[37] has clearly defined the limitations of Stokes' law. It would appear then, that the limitation of practical methods of measuring small sizes would indicate plotting the smallest sizes at the left. Considering the fact that both Europeans and English speaking people read with a positive sense from left to right, it would seem more in agreement with their sense of direction if data were plotted with increasing magnitudes toward the right. Furthermore, even the layman is familiar with the classical "first quadrant" of plane geometry. There would probably be less mental confusion for the technician who plots with a negative sense, to change to a graph using a positive sense, than for the technician who has already employed a positive sense, to reverse his procedure. For this reason the writer is presenting a suggested standard form, shown in Fig. 7. It would be somewhat inconvenient for the engineer or scientist who only occasionally plots grain-sized distribution to use a negative sense, in that the semi-logarithmic paper usually available on the market is constructed with a positive sense. Although there is no zero to the logarithmic scale, the left-hand margin of the graph could be started at 0.002 mm, or 2 microns, for all practical purposes. It is fairly positive that sedimentation methods will not be used to determine diameters smaller than 0.6 micron. The expensive and tedious methods of microscopy may yield determinations smaller than the aforementioned limit, but this remains for scientists of the future to determine.

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[36] Associate Engr., War Dept., U. S. Engrs. Office, Denison, Tex.

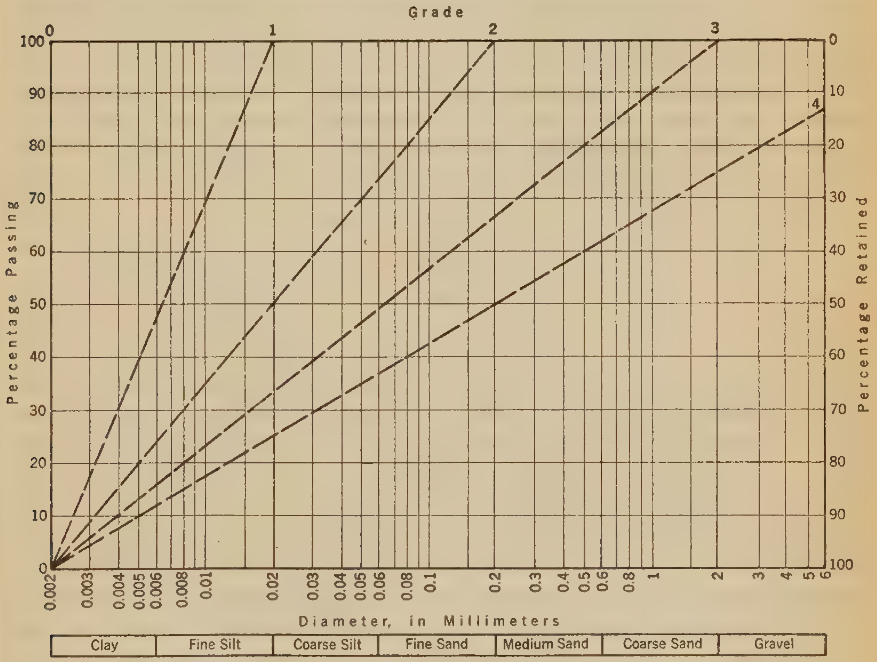

FIG. 7

Messrs. Kadie and Lee have called attention to the fact that Fig. 1 does not have an adequate number of abscissa lines for close plotting. Their contention is true; Fig. 1 was included for demonstration purposes only. The writer is accustomed to using graph paper similar to that shown by Mr. Kadie for purposes of making an accurate record of mechanical analysis data. The method of using heavy lines to correspond to sieve openings of the Tyler series has been found to be very convenient for the purpose of plotting the sieve-analysis data.

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[37] "Comments on the Hydrometer Method of Mechanical Analysis," by R. C. Thoreen, *Public Roads*, Vol. 14, pp. 93–105.

Grade Line-Mean Diameter.—During the winter of 1934–1935 the Platte Valley Public Power and Irrigation District employed a number of inspectors in its Soils Laboratory under the direction of the writer. With adequate assistance it was possible to make about 700 mechanical analyses on borrow-pit material as well as other tests, such as Proctor compaction and percolation tests, before the ground thawed and construction was begun in the spring. At that time, the writer devised a classification which can be called an A B C classification for the wind-blown or aeolian material in Western Nebraska. This classification was used merely as a tool for plotting the results of mechanical analyses on a plan of the dam. Fig. 8 was a part of one of four borrow-pit maps used to guide the engineers and inspectors in directing the contractor's excavating equipment. The following tabulation is a grade-line-mean-diameter designation of the A B C classification:

Local classification	Proposed Designation: Grade line	Proposed Designation: Mean diameter
A	0.60	0.041
B	0.85	0.080
C	1.60	0.145

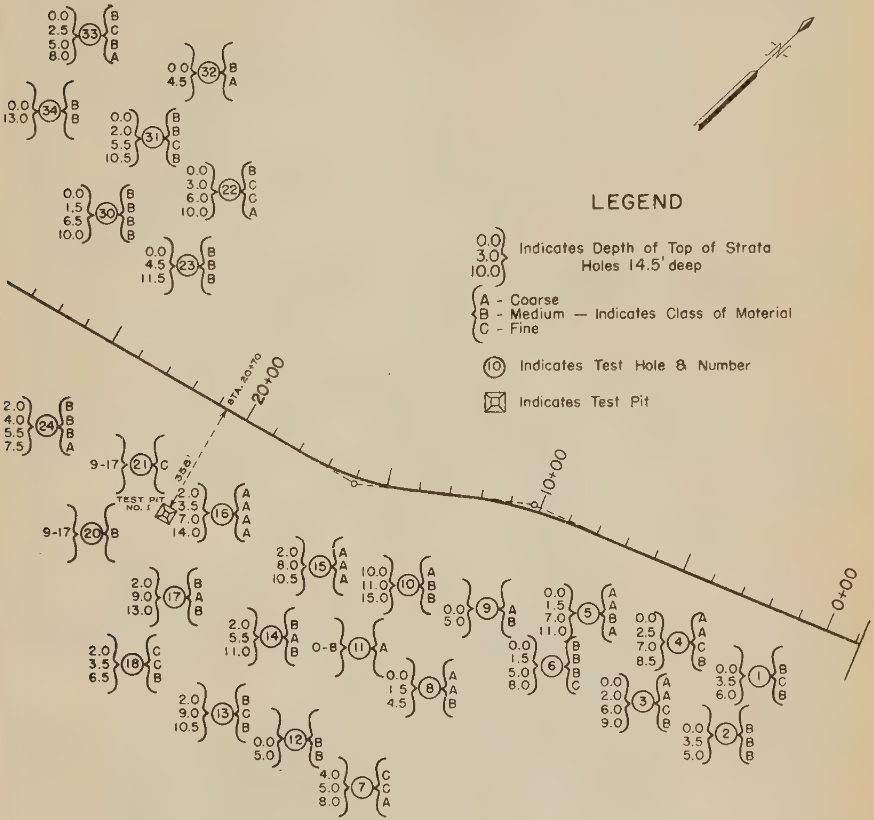
By studying Fig. 8, it may be seen that the A B C classification was very convenient for designating fine, medium, and coarse material. One might also choose to designate the three classes by the initial letters, F M C. The designations, A B C, may be confused with soil horizon designations by soil surveyors.

The Kendorca[37] classification which is preferred by Messrs. Justin, and Fahlquist and Kenerson, was developed for the same purpose as the A B C classification except that the Kendorca was made to fit the local conditions at Quabbin Reservoir. The disadvantages of the Kendorca system are as follows:

(*a*) Although it is more flexible than the A B C system the Kendorca is not as flexible as the proposed Grade-Line-Mean-Diameter system; nor is it any more precise; and,

(*b*) The system is incomplete in that odd numbers do not indicate the existence of any material between the 100% and 50% lines.

Regardless of the two foregoing disadvantages the writer can readily understand that the Kendorca classification may have been very valuable as a tool for the materials problem which the originators had to solve. However, it is believed that if one becomes accustomed to working and speaking with the terms of the 20–80 Grade-Line-Mean-Diameter system it will be found more flexible than establishing some fixed line on a mechanical analysis graph. The modifications proposed by Professor Lane are well worth considering. The writer believes that the use of 10% and 90% intersection points for establishing a grade line would be workable for soils that have the curve symmetrical about the 50% axis. The disadvantage of using 10% and 90% would be the large deviation introduced at 50% mean diameter when the distribution curve has either exaggerated convexity or concavity. Professor Lane's suggestion of using 6/6 of the ratio of the interval rather than 10/6 would be merely another mathematical designation of the grain-sized distribution. It would have the advantage, as he suggests, of enabling the computer to make a hurried mental calculation of the grade instead of resorting to graphical methods. Professor Weinig's method of adding symbols to the diameter grade designation to indicate convexity, concavity, *S*-type, and linear curve, is certainly an excellent suggestion. It would probably add much to the value of the data on a plan or cross-section if the soil presented these four different types of particle-size distribution.



LEGEND
0.0 3.0 10.0 Indicates Depth of Top of Strata
Holes 14.5' deep
A - Coarse
B - Medium — Indicates Class of Material
C - Fine
⑩ Indicates Test Hole & Number
Indicates Test Pit
N
STA. 20+70
20+00
10+00
0+00
35.0'
TEST PIT NO. 1
0.0 2.5 5.0 8.0 (33) B C B A
0.0 4.5 (32) B A
0.0 13.0 (34) B B
0.0 2.0 5.5 10.5 (31) B B C B
0.0 3.0 6.0 10.0 (22) B C C A
0.0 1.5 6.5 10.0 (30) B B B B
0.0 4.5 11.5 (23) B B B
2.0 4.0 5.5 7.5 (24) B B B A
9-17 (21) C
9-17 (20) B
2.0 3.5 7.0 14.0 (16) A A A A
2.0 8.0 10.5 (15) A A A
10.0 11.0 15.0 (10) A B B
2.0 9.0 13.0 (17) B A B
0.0 5.0 (9) A B
0.0 1.5 7.0 11.0 (5) A A B A
2.0 5.5 11.0 (14) B A B
0-8 (11) A
0.0 2.5 7.0 8.5 (4) A A C B
2.0 3.5 6.5 (18) C C B
0.0 1.5 5.0 8.0 (6) B B B C
0.0 1.5 4.5 (8) A A B
0.0 2.0 6.0 9.0 (3) A A C B
0.0 3.5 6.0 (1) B C B
2.0 9.0 10.5 (13) B C B
0.0 5.0 (12) B B
0.0 3.5 5.0 (2) B B B
4.0 5.0 8.0 (7) C C A

FIG. 8.—CLASSIFICATION OF BORROW-PIT MATERIAL, SUTHERLAND RESERVOIR DAM

Nomenclature of Soil Fraction.—There appears to be an attitude, as expressed by Messrs. Knapp, Knappen, and Sanger, that the generic names of soil fractions on separates should be discarded entirely. However strongly one may feel toward discarding the English words—colloids, clay, silt, sand, and gravel—

they have grown to have a definite meaning of magnitude through the widespread use of mechanical analyses. The general difficulty appears to arise in attempting to reconcile the German *schlamm*, *schluff*, *mo*, and *sande*, to the conception of English-speaking people. If it should be agreed by an international commission on soil nomenclature to establish names for soil separates, with due consideration to the generic meaning of both German and English terms, it may be desirable to compromise by designating the fractions by the letters of the Roman alphabet which is acceptable in both languages. For instance, one could call colloids the *a* fraction, clay the *b* fraction, silt the *c* fraction, sand the *d* fraction, and gravel the *e* fraction. There would then remain only the problem of standardizing the size limits. Another alternative would be to use the Greek alphabet in the manner that radio-active rays are designated. On the other hand, the American and English lexicographers would probably appreciate precise definitions of generic words which have heretofore been used very loosely by scientists, engineers, and laymen alike.

Mr. Middlebrooks raises a question as to the reason for changing from the Bureau of Chemistry and Soils classification. The writer believes that the reason for changing is that a large group of engineers and scientists have already begun to swing toward the more logical scheme of Atterberg by setting the division lines at recurring logarithmic cycles in multiples of 2. As stated by Professor Lane, Atterberg's[38] original classification called for the upper limit of clay to be drawn at 2 microns. The same limits established by Atterberg have been followed by the International Classification.

Few engineers and scientists in America have become accustomed to the International, or the Atterberg, designation until recently. In the meantime the Americans have used the U. S. Bureau of Chemistry and Soils Classification for many years. The writer is indebted to Mr. Olmstead[39] for a brief history of the Bureau of Chemistry and Soils Classification:

> "The earliest publication I have seen that gives the Bureau grain size classification is the report of Professor Milton Whitney in the Fourth Annual Report of the Maryland State Agricultural Experiment Station, 1891, pages 249–296. *Bulletin 4* of the Division of Agricultural Soils, published in 1896, states that it is the practise of the Division to separate mineral soils into eight grades, according to diameter of grains, and gives the same limits as does the Maryland station report. No author's name appears on this *bulletin*. Osborne employs some of the same size limits in his mechanical analyses of soils published in the Connecticut Agricultural Experiment Annual Reports for 1886–7. I have seen no discussion of the origin of the Bureau classification but I believe it was devised by Professor Whitney some time before he came to the Department of Agriculture as the first chief of the Bureau of Soils."

The Bureau of Soils classification, therefore, is distinctly American, having its origin sometime during the latter part of the Nineteenth Century. Many English-speaking scientists have become accustomed to it. On the other hand, there is the more rational classification as far as limits or divisions of the soil fractions, which apparently originates with Atterberg.

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[38] "Die rationelle Klassifikation der Sande und Kiese," von A. Atterberg, *Chemische Zeitung*, Jahrgang 29, 1905, pp. 195–198.

[39] Personal letter to the writer from L. B. Olmstead, Soil Physicist, U. S. Bureau of Soils, Washington, D. C., dated October 12, 1938.

As mentioned in Mr. Olmstead's discussion, the engineers would probably prefer to have more divisions in the coarse range whereas the agronomist would prefer more divisions in the fine range. In the proposed classification, an attempt has been made to establish limits for fine, medium, and coarse sand, which roughly approximates that of the Whitney scheme except log 2 − log 6 has been used for the limits. More divisions than the old Whitney system have been made available in the range of clay and silt. For these reasons it should be usable by both engineers and agronomists.

A comparison of the advantages and disadvantages of the several systems can be outlined briefly as follows:

I.—Atterberg or International Classification:

- (*A*) Advantages.—
  - (1) The class limits are set at 2, 20, 200, and 2 000 microns making a good arrangement for semi-logarithmic plotting.
- (*B*) Disadvantages.—
  - (1) There is some difficulty of applying the German terminology to the American concepts of size classification.
  - (2) There are too few divisions in the coarser range.
  - (3) The upper limits of clay at 2 microns is too high for efficient laboratory determination where a large number of samples are involved. The laboratory time required for the Bouyoucos hydrometer is approximately 6 hr.

II.—Whitney or Bureau of Chemistry and Soils Classification:

- (*A*) Advantages.—
  - (1) There is a widespread familiarity with this system among American professional men and perhaps also laymen.
  - (2) The numerous divisions in the larger materials make it convenient for macroscopic examination of soils in the field by engineers and soil surveyors.
  - (3) It establishes a very practical upper limit for clay whether the pipette or hydrometer method is used.
- (*B*) Disadvantages.—
  - (1) The size limits of the class do not conform to evenly divided logarithmic spacings. Whitney's proposal was probably originated before the common use of semi-logarithmic plotting.
  - (2) Too few divisions in the smaller size are included. This is a disadvantage to the agronomist principally, although others may soon be interested, with the advance of the study of soil mechanics.

III.—Massachusetts Institute of Technology Classification:

- (*A*) Advantages.—
  - (1) Sand is divided into three commonly accepted classifications; namely, fine, medium, and coarse.
  - (2) Sand classes roughly approximate those of the Bureau of Chemistry and Soils.

(*B*) Disadvantages.—

(1) There are three classes, each, of silt and clay. The three classes of clay have as yet proved to be of little use to the engineer.

(2) This has the same disadvantage as the International in that the upper limit of clay is depressed to 2 microns instead of at the proximity of 5 microns according to the Whitney system.

IV.—Proposed or Log 2 – Log 6 Classification:

(*A*) Advantages.—

(1) This is a classification to agree roughly with the Whitney system which has grown to be the American conception of size classification.

(2) It is a division of soil fraction based upon log 2 – log 6 which is patterned after the Atterberg system.

(3) An upper limit of clay is set at 6 microns instead of 2 or 5 microns. Such a definition allows clay to be determined in less than 1 hr with a Bouyoucos hydrometer as compared with more than 6 hr for the Atterberg definition. For the investigation of any particular borrow-pit, where the number of samples to be examined is known, this advantage can be evaluated in terms of dollars and cents. With the more cumbersome pipette method, a shorter distance of settlement can be used, but accuracy is sacrificed. The Bouyoucos hydrometer method is usually conceded to be a more efficient method, although the pipette is desired by some technicians.

(*B*) Disadvantages.—

(1) The writer feels that a distinct difficulty exists in that a compromise is suggested between the two schools of thought; namely, those who are experienced with the Whitney system and those who have become accustomed in recent years to the Atterberg system with its modifications.

History of Soils.—Professor Burmister and Mr. Sanger obviously concur with the writer that the mechanical analysis is an indication of the history of the soil.

Mr. Lee dissents with a statement that the premise given by the writer may not be true in the case of resistant quartz or silica. Although quartz may be more resistant to weather than other minerals, it has been observed that even such hard crystals are vulnerable to breakage by freezing and thawing of interstitial water. Even diurnal variation of temperature in the warmer latitudes may cause the disintegration of quartz. This is widely evidenced in the Piedmont region of Southeastern United States. However, Mr. Lee's argument should be considered because the writer's suggestion as to maturity and immaturity of soils is probably more applicable to the podzols of the northern latitudes.

Messrs. Feld and Sanger have mentioned the shape of particles. This fact is important in that, the determination of mechanical analyses by a combination of sieving and sedimentation methods does not take the shape into account. Some one has well defined a diameter determined by sedimentation methods as the "equivalent diameter of a perfect sphere falling in a viscous fluid." As a general rule the presence of angular and plate-shaped particles increase the stability of embankments, although more effort may be required for the mechanical compaction of such soils. The angular particles may require more fines to fill the voids and produce imperviousness. It has been the writer's practise to carry a pocket microscope giving magnification to approximately 60 diameters, when working in the field. Thus, the shape of particles of approximately 20 microns may be inspected visibly. This diameter is the lower limit of coarse silt on the proposed classification.

General Consideration.—In closing, the writer suggests that individuals, interested societies, governmental agencies, and educational institutions should work together in drafting a standard form of mechanical analysis graph, which, in effect, is a language used by many professional men. Consider the confusion if one were to attempt to write the words "mechanical analyses" backward as follows: "sesylana lacinahcem."

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TRANSACTIONS

Paper No. 2019

ECONOMIC PIPE SIZES FOR WATER DISTRIBUTION SYSTEMS

By Thomas R. Camp,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Clinton L. Bogert and J. M. M. Greig, Charles M. Mower, Jr., A. C. Michael, Ellwood H. Aldrich, M. H. Klegerman, F. Knapp, and Thomas R. Camp.

Synopsis

The method for determining the economic size of a single pipe line is well established and has been used for some time. In the past some efforts have been directed toward the extension of this method to a water distribution system, but the procedure to be used was not sufficiently developed for practical application. Had the method been available, it would have been of little use in design because of the difficulty of determining the distribution of flow in a pipe network. Practical methods for analyzing the distribution of flow in pipe networks have been published by the writer elsewhere[2] as well as by Hardy Cross,[3] M. Am. Soc. C. E., and James J. Doland,[4] M. Am. Soc. C. E. In order to apply one of these methods to the actual design of a distribution system, it becomes desirable to give further thought to the economic sizes of pipes. It is the purpose of this paper to develop methods for determining the proper sizes of pipes in distribution systems for best economy.

Introduction

The general method for determining the economic size of pipe is to express the total cost of the system as a function of the pipe sizes and find the pipe sizes that will make this cost a minimum. The total cost consists of the fixed cost, capital plus depreciation, and the operating cost which is the cost of pumping. Two general types of systems are encountered in practice, namely,

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Note.—Published in December, 1937, *Proceedings*.

[1] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[2] "Hydraulic Analysis of Water Distribution Systems by Means of an Electric Network Analyzer," by T. R. Camp, M. Am. Soc. C. E., and H. L. Hazen, *Journal*, New England Water Works Assoc., December, 1934, p. 383.

[3] "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, *Bulletin 286*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[4] "Simplified Analysis of Flow in Water Distribution Systems," by J. J. Doland, *Engineering News-Record*, October 1, 1936, p. 475.

those with pumped supplies, and gravity systems in which there is no cost of pumping. Mr. George W. Tuttle[5] has shown that in some gravity systems the available head is so low that it is more economical to pump. In other words, a gravity system may not be the most economical type for a given case. Nevertheless, once a gravity system is decided upon, the cost of pumping ceases to be a part of the total cost.

Most modern American distribution systems consist largely of cast-iron pipe. The cost of mains in terms of the pipe diameter is presented hereafter only for such pipe. If some of the mains are constructed of other materials, or are built as tunnels, the economic sizes may be derived by methods similar to those presented, provided the costs of such mains can be stated in terms of their sizes. In the following development it is assumed that the cost of pumping, which necessarily includes capital cost of station, equipment, and operator's wages as well as power cost, is directly proportional to the head and to the discharge. It is also assumed that the cost of elevated storage will not be influenced by changes in pipe sizes and that this cost, therefore, need not be considered.

In the design of a distribution system, the location of the mains is usually determined by the location of the consumers. Preparatory to the determination of pipe sizes, the system is divided into districts, and the domestic and fire drafts are estimated for each district. The system is then "skeletonized" to show only the larger mains which play important parts in the delivery of water to each critical point in the system. The domestic and fire loads for each district are assumed to be concentrated at some point or points on the skeleton system. In simplifying the system, the errors introduced by neglect of small pipes must be examined and corrections made if they are found to be too large. Moreover, the location of a fire load in one district may require a modification of the skeleton system used for the fire load in another district.

By the trial-and-error method of, alternately, selecting pipe sizes for the skeleton system and computing corresponding pressure drops, a set of pipe sizes may be found which produces the desired residual pressures at each critical point in the system. The set of pipe sizes thus selected will be only one set of an infinite number each of which may produce the desired residual pressures. In order to obtain the most economical system, it will be necessary to examine the relation of the sizes for best economy as the trial-and-error process is in progress.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix.

Cost in Terms of Pipe Diameter

The first cost of laying cast-iron pipe, in cents per linear foot, based upon cost data cited by the late Dabney H. Maury,[6, 7] M. Am. Soc. C. E., may be expressed as follows:

$$C_c = B\,D = [(0.06 + 0.02\,d)\,C_w + 0.19\,C_l + 0.007\,C_y]\,D \ldots\ldots (1)$$

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[5] "The Economic Velocity of Transmission of Water Through Pipes," by George W. Tuttle, *Engineering Record*, September 7, 1895, p. 258.

[6] *Engineering News-Record*, May 11, 1922, p. 779.

[7] "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, Members, Am. Soc. C. E., McGraw-Hill, 1931, p. 424.

in which,

$$B = (0.06 + 0.02\,d)\,C_w + 0.19\,C_l + 0.007\,C_y \ldots\ldots\ldots\ldots (2)$$

d = the depth of the trench, in feet; C_w = the wage rate for common labor, in cents per hour; C_l = the cost of lead in cents per pound; C_y = the cost of yarn, in cents per pound; and, D = the diameter of the pipe, in inches.

The costs given by Mr. Maury apply to trenches in ordinary soil without pumping, sheeting, rock, or other unpredictable costs. One joint was assumed for every 11 ft of pipe. As Mr. Maury's data made no allowance for contractor's profit, engineering, insurance, and interest during construction, the cost given by Equation (1) has been made approximately 20% greater than Mr. Maury's cost, in order to cover these items. The unpredictable costs may be omitted safely from the equation inasmuch as they will not be affected much by the pipe size. The cost of hydrant installations is also independent of the size of the main and may be omitted from the equation.

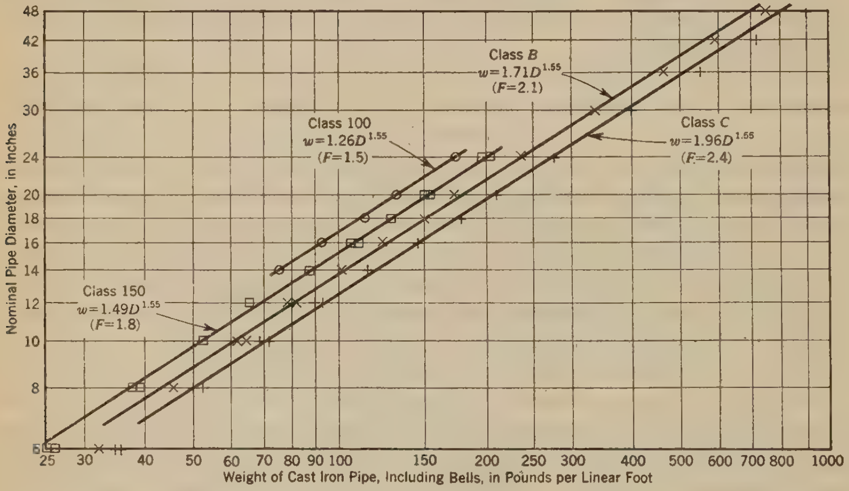

FIG. 1

A study of the weights of standard bell-and-spigot cast-iron pipe (Fig. 1) indicates that the weight, in pounds per linear foot, varies as the 1.55 power of the diameter with errors not exceeding 10% for any single pipe size. A study of the weights of fittings, which are frequently paid for by the pound, indicates that these weights may be assumed to vary as $D^{1.55}$. A study of valve costs shows that they also may be assumed to vary as $D^{1.55}$ for practical purposes. A study of the cost of linings, as given by Mr. E. T. Killam,[8] indicates that either cement or bitumastic enamel lining will cost about 7% of the cost of unlined cast-iron pipe. Therefore, the first cost of materials, including cast-iron pipe, valves, fittings, and pipe lining, in cents per linear foot of pipe, may be expressed by the following empirical equation:

$$C_m = F\,C_f\,D^{1.55} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

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[8] "Economic Significance of Pipe Linings," by E. T. Killam, *Water Works and Sewerage*, March, 1933, p. 73.

in which C_f = the cost of cast-iron pipe, in cents per pound, with freight allowed; and, F = a coefficient giving the cost of different types of pipe at the trench, including appurtenances and 20% allowed for contractor's profit, engineering, etc. Typical values of F are given in Table 1.

TABLE 1.—VALUES OF F, EQUATION (3)

Class	Unlined Pipe		Lined Pipe (Cement or Bitumastic Enamel)	
	Pipe only	Pipe with one valve, one tee, and one cross for each 500 ft	Pipe only	Pipe with one valve, one tee, and one cross for each 500 ft
100	1.50	2.05	1.65	2.20
150	1.80	2.35	1.95	2.50
B	2.10	2.65	2.25	2.80
C	2.40	2.95	2.55	3.10

The annual cost, in cents, of pumping, or of water power, is:

$$C_a = 236\, C_p\, Q\, h \qquad (4)$$

in which C_p = the cost, or the value, in cents, of raising 1 000 000 gal 1 ft; Q = the discharge, in cubic feet per second; and, h = the head, in feet. If h is the friction head for a single pipe, it may be expressed as a function of the slope of the hydraulic grade line and the pipe length. The slope may be expressed as a function of the pipe size by using any convenient formula for pipe friction. The value of the slope, S, from the Williams-Hazen formula is:

$$S = \left(\frac{1\,594}{\mathsf{C}}\right)^{1.85} \frac{q^{1.85}}{D^{4.87}} \qquad (5)$$

in which C = the Williams-Hazen coefficient; and q = the discharge through the pipe, in cubic feet per second.

GRAVITY SYSTEMS

A gravity system is defined as one in which the total head available for loss through pipes is fixed by the topography. Flow is by gravity from a distributing reservoir, the elevation of which has been fixed without regard to the economic size of pipes in the distribution system. In such a system the cost of pumping, if pumping is required, is fixed arbitrarily by the elevation of the distributing reservoir and, hence, is independent of the size of pipes in the distribution system.

The economic pipe sizes for a gravity system will be one set that utilizes all the available head in friction when delivering the peak flow to each critical point in the system. If the system consists of a single pipe line, as is the case of a gravity water supply line, the determination of the economic size is direct and simple. When there are several pipes in series, however, and the costs per foot differ, there are an infinite number of ways in which the total head may be distributed among the pipes. Only one of these ways will produce a minimum cost for the entire series.

Let the cost of a series of gravity pipe lines be:

$$C = c_1 + c_2 + c_3 + \cdots c_n = f_1(h_1) + f_2(h_2) + f_3(h_3) + \cdots f_n(h_n) \ldots (6)$$

in which $c_1, c_2, c_3 \cdots c_n$ = the cost of Sections 1, 2, 3, $\cdots n$; and, $f_1(h_1), f_2(h_2), f_3(h_3) \cdots f_n(h_n)$ = these costs expressed as functions of the head loss in each section. The cost of any two connecting pipes in the series may be made a minimum for any given total head for these two pipes by equating to zero the first derivative of this cost with respect to the head loss in either section, thus:

$$\frac{d(c_1 + c_2)}{dh_1} = f_1'(h_1) - f_2'(h_2) = 0 \ldots\ldots (7)$$

since $dh_1 = -dh_2$. Hence, for minimum cost of the first two sections:

$$\frac{dc_1}{dh_1} = \frac{dc_2}{dh_2} \ldots\ldots (8)$$

Since this is the condition of minimum cost for any given total head in the first two sections, it is also the condition of minimum cost for the total head consistent with minimum cost for the entire series. A similar relation exists for each two connecting pipes in the series. Therefore, for minimum cost of the entire series:

$$\frac{dc_1}{dh_1} = \frac{dc_2}{dh_2} = \frac{dc_3}{dh_3} = \cdots \frac{dc_n}{dh_n} \ldots\ldots (9)$$

and,

$$h_1 + h_2 + h_3 + \cdots h_n = H \ldots\ldots (10)$$

in which H = the total head available for loss in the series. A graphical solution of this problem for an aqueduct is given in an example by Messrs. Babbitt and Doland.[9] For cast-iron pipes in a gravity series, the cost is,

$$C = [B_1 D_1 + F C_f D_1^{1.55}] l_1 + [B_2 D_2 + F C_f D_2^{1.55}] l_2 + \cdots \ldots (11)$$

in which l_1, l_2, etc., equal the lengths of Sections 1, 2, etc. From the Williams-Hazen formula,

$$D = 16.5 \frac{q^{0.38} l^{0.205}}{\mathrm{C}^{0.38} h^{0.205}} \ldots\ldots (12)$$

Substituting this value of D for each pipe size in Equation (11) and differentiating as in Equation (9), the following result is obtained:

$$3.38 \frac{B_1 q_1^{0.38}}{\mathrm{C}_1^{0.38} S_1^{1.205}} + 24.5 F \frac{C_f q_1^{0.59}}{\mathrm{C}_1^{0.59} S_1^{1.318}} = 3.38 \frac{B_2 q_2^{0.38}}{\mathrm{C}_2^{0.38} S_2^{1.205}} + 24.5 F \frac{C_f q_2^{0.59}}{\mathrm{C}_2^{0.59} S_2^{1.318}} \ldots\ldots (13)$$

Equation (13) gives the relation between the slopes of the hydraulic grade lines of any two connecting pipes in a gravity series for best economy. It may be solved by trial and error, but the process is difficult and time-consuming.

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[9] "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, Members, Am. Soc. C. E., McGraw-Hill, 1931, p. 237.

By assuming that the exponent of the slope in every term of Equation (13) is 1.25 an approximate solution may be had, as follows:

$$\frac{S_1}{S_2} = \left[\frac{3.38\,\dfrac{B_1\,q_1^{0.38}}{\mathrm{C}_1^{0.38}} + 24.5\,F\,\dfrac{C_f\,q_1^{0.59}}{\mathrm{C}_1^{0.59}}}{3.38\,\dfrac{B_2\,q_2^{0.38}}{\mathrm{C}_2^{0.38}} + 24.5\,F\,\dfrac{C_f\,q_2^{0.59}}{\mathrm{C}_2^{0.59}}}\right]^{0.8} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

Results obtained by means of Equation (14) will usually vary by not more than 2% from those obtained by Equation (13). In applying it to the design of a distribution system, every pipe series should be considered that delivers water to a critical point for the condition of peak flow to that point. For another loading condition (that is, with the fire load at another critical point) other pipe series will come into play. The main feeders, however, will be parts of series for several loading conditions. The economic sizes of these mains, as determined for the several loading conditions, will not be identical, and it will be necessary to select some standard pipe size which comes nearest to satisfying all the conditions.

Classification of Pumped Supplies

A number of cases arise with pumped supplies which affect the manner of determining the economic pipe sizes. In all these cases, it will be assumed that the minimum head is fixed at the delivery end of the pipe or pipe series at the required residual value for peak flow. The pumping head over and above this static head is chargeable to pipe friction.

If the discharge is constant, this additional pumping head has only one value consistent with best economy for the system; and there is only one set of pipe sizes that will give the least total cost. A distribution reservoir may be inserted in this system without affecting the economic pipe sizes, provided its elevation does not disturb the hydraulic grade lines.

If the discharge is variable, the pumping head may be either fixed or variable, depending upon whether a distribution reservoir is provided near the pump station. Each case will entail a different set of economic pipe sizes, and both sets will differ from the economic sizes for constant discharge.

Case I.—Pumped Supply with Constant Discharge.—If the system consists of a single pipe line discharging only at the lower end, the total annual cost of pipe and pumping against friction per linear foot of pipe is:

$$C_{at} = B\,r\,D + F\,C_f\,r\,D^{1.55} + 236\,C_p\left(\frac{1\,594}{\mathrm{C}}\right)^{1.85}\frac{q^{2.85}}{D^{4.87}} \quad \ldots\ldots (15)$$

in which r = the annual rate of interest plus depreciation of the pipe. Differentiating this cost with respect to the diameter and equating the result to zero, the following relation is obtained:

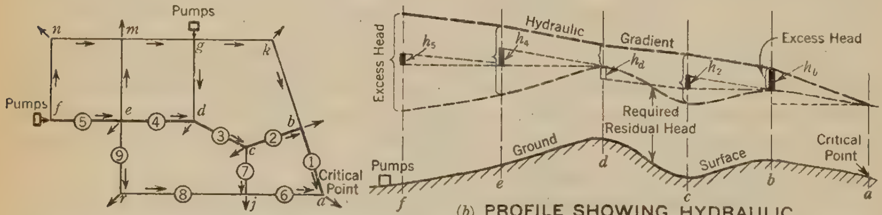
$$B\,r\,D^{5.87} + 1.55\,F\,C_f\,r\,D^{6.42} = 1\,150\,C_p\left(\frac{1\,594}{\mathrm{C}}\right)^{1.85} q^{2.85} \quad \ldots\ldots (16)$$

Equation (16) may be solved by trial and error for the economic diameter of pipe. However, if the exponent of D in both terms is assumed to be 6.15, following the procedure of Babbitt and Doland,[10] the following approximate solution for the economic diameter is obtained:

$$D_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{C_p}{r\,(B + 1.55\,F\,C_f)} \right]^{0.163} q^{0.46} \qquad (17)$$

The coefficient in Equation (17) has been adjusted to 27.4 to compensate for the error introduced by changing the exponents. This formula gives values of the economic pipe diameter within 3% of those given by Equation (16).

If the system consists of a network of pipes with a number of take-offs, the pressure at only one take-off point may be made to correspond with the desired residual pressure. At each of the other take-off points there is an excess of pressure, and there is a power loss, therefore, corresponding to the draft from the system and this excess pressure. Some of the power loss at take-offs is due to the topography of the city and is not influenced by pipe sizes; but some of it is properly chargeable to pipes inasmuch as larger pipes down stream from a take-off point would reduce the excess pressure at the point.



(*a*) PLAN OF SYSTEM

(*b*) PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE

FIG. 2.—ILLUSTRATING CASE I, PUMPED SUPPLY WITH CONSTANT DISCHARGE THROUGHOUT

In the simple network represented by Fig. 2(*a*) all the excess head at Point *b* is chargeable to Pipe 1. The part of the excess head at Points *c*, *e*, and *f*, corresponding to the lost head in Elements 2, 4, and 5, is chargeable, respectively, to those pipes. The excess head at Point *d* is due to Pipes 1 and 2; and the difference between the total head charged to these pipes and the excess at Point *d* is chargeable to Pipe 3. Since this value is negative in the example, the take-off power charge for Pipe 3 is a credit. To formulate a rule:

> (1) The head corresponding to the take-off power loss chargeable to any pipe is the head lost by friction in the pipe, or the difference between the excess head at the upper end of the pipe and the sum of the heads chargeable to the pipes down stream, whichever is the smaller.

The take-off power loss chargeable to Element 1, Fig. 2, is due to the draft from the system at Points *b* and *k*, and a part of the draft from Points *c*, *d*, *e*, *m*, and *n*. Since the loss due to the take-off at Point *c* is chargeable to Elements 6 and 7 as well as to Pipes 1 and 2, it is convenient to apportion the loss by distributing the draft at Point *c*, between the two routes. A similar distribution of the draft from other points should be made between the routes down stream from these points. To formulate a rule:

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[10] "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, Members, Am. Soc. C. E., McGraw-Hill, 1931, p. 425.

> (2) The discharge corresponding to the take-off power loss chargeable to any pipe is the sum of the drafts from the system at all points up stream from the pipe, a suitable apportionment of drafts being made between parallel routes.

For any pipe of a network through which water is pumped at a constant rate, the total annual cost, per linear foot of pipe, of pipe and power loss due to pipe friction and take-offs is given by:

$$C_{at} = B\,r\,D + F\,C_f\,r\,D^{1.55} + 236\,C_p \left(\frac{1\,594}{\mathrm{C}}\right)^{1.85} (q + m\,Q_t)\,\frac{q^{1.85}}{D^{4.87}} \quad \text{..(18)}$$

in which q = the discharge through the pipe, in cubic feet per second; Q_t = the total take-off discharge chargeable to the pipe, in cubic feet per second, estimated as described herein; and, m = the ratio of the take-off head to the head lost by friction in the pipe, the take-off head being estimated as described herein.

Following the procedure used in the derivation of Equation (17), the economic diameter of pipe for any element in a network through which water is pumped at a constant rate, is given by the following equation:

$$D_{ec} = \frac{27.4}{\mathrm{C}^{0.30}} \left[\frac{C_p\,(q + m\,Q_t)}{r\,(B + 1.55\,F\,C_f)}\right]^{0.163} q^{0.30} \quad \text{.............(19)}$$

Case II.—Pumped Supply with Varying Demand and Varying Pumping Head; No Storage in System.—If throughout the design period, the discharge and head loss for any element vary such that q_a, q_b, q_c, etc. = the discharge for times, t_a, t_b, t_c, etc., and h_a, h_b, h_c, etc. = the corresponding friction heads, the average annual cost of the power loss is:

$$C_a = 236\,C_p \left[q_a\,h_a\,\frac{t_a}{t} + q_b\,h_b\,\frac{t_b}{t} + q_c\,h_c\,\frac{t_c}{t} + \cdots\right]$$

$$= 236\,C_p\,q\,h \left[\frac{q_a\,h_a}{q\,h}\,\frac{t_a}{t} + \frac{q_b\,h_b}{q\,h}\,\frac{t_b}{t} + \frac{q_c\,h_c}{q\,h}\,\frac{t_c}{t} + \cdots\right] = 236\,C_p\,A\,q\,h \quad \text{..(20)}$$

in which q and h = the average discharge through the element and the corresponding head, respectively, over the design period; t = the design period; and,

$$A = \left[\frac{q_a\,h_a}{q\,h}\,\frac{t_a}{t} + \frac{q_b\,h_b}{q\,h}\,\frac{t_b}{t} + \cdots\right] \quad \text{...............(21)}$$

Using the Williams-Hazen formula,

$$A = \left(\frac{q_a}{q}\right)^{2.85} \frac{t_a}{t} + \left(\frac{q_b}{q}\right)^{2.85} \frac{t_b}{t} + \left(\frac{q_c}{q}\right)^{2.85} \frac{t_c}{t} + \cdots \quad \text{.........(22)}$$

For a particular case of a single pipe line with a 20-yr design period and with a constant pumping rate throughout any one day, the writer estimated a value of 2.1 for A. For the same pipe line designed to care for hourly fluctuations in demand. the value of A was estimated at 2.9.

The economic diameter of any pipe in a system through which water is pumped at a varying rate and against a varying head (Fig. 3) is given by the following equation:

$$D_{ec} = \frac{27.4}{\mathbf{C}^{0.30}} \left[\frac{C_p\, A\, (q + m\, Q_t)}{r\, (B + 1.55\, F\, C_f)} \right]^{0.163} q^{0.30} \ldots\ldots\ldots\ldots (23)$$

in which q = the average discharge through the pipe throughout the design period; and m and Q_t = values corresponding to q estimated as described for Equation (18).

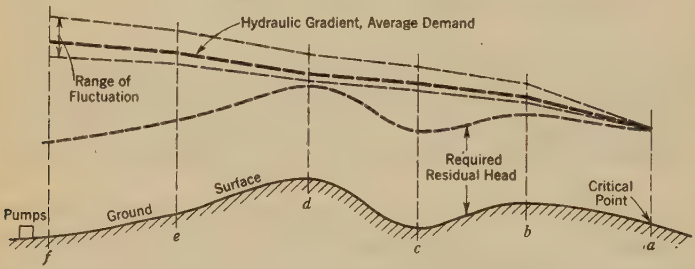
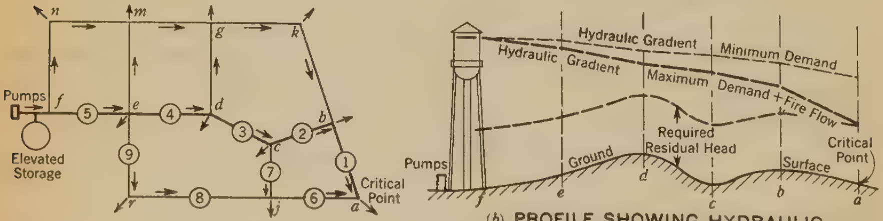

Hydraulic Gradient, Average Demand
Range of Fluctuation
Required Residual Head
Critical Point
Pumps
Ground
Surface
f
e
d
c
b
a

Fig. 3.—Profile Showing Hydraulic Gradient Along One Route, for Case II, Pumped Supply with Varying Demand and Varying Pumping Head; Plan of System Same as Fig. 2; No Storage

The foregoing analysis is based upon the assumption that the flow is distributed throughout the system in the same ratios at all times. Fire flows are excluded since they account for a negligible part of the yearly pumping cost.

It is assumed in the foregoing derivations that for each discharge there is a corresponding friction loss for each pipe. This is not true throughout the design period for pipes in which the friction coefficients change with age. Since the economic diameter of a pipe varies inversely as $\mathbf{C}^{0.30}$, a reduction in the value of $\mathbf{C}$ during the design period as great as from 130 to 80, for example, will produce a variation in the economic diameter as computed from the foregoing equations of only 15 per cent. If the average value of $\mathbf{C}$ expected throughout the design period is used in the equations, the error in the computed economic pipe size will be small.


n
m
g
k
Pumps
f
5
e
4
d
3
2
b
c
1
Elevated Storage
9
7
r
8
j
6
a
Critical Point
Hydraulic Gradient
Minimum Demand
Hydraulic Gradient
Maximum Demand + Fire Flow
Required Residual Head
Critical Point
Pumps
Ground
Surface
f
e
d
c
b
a

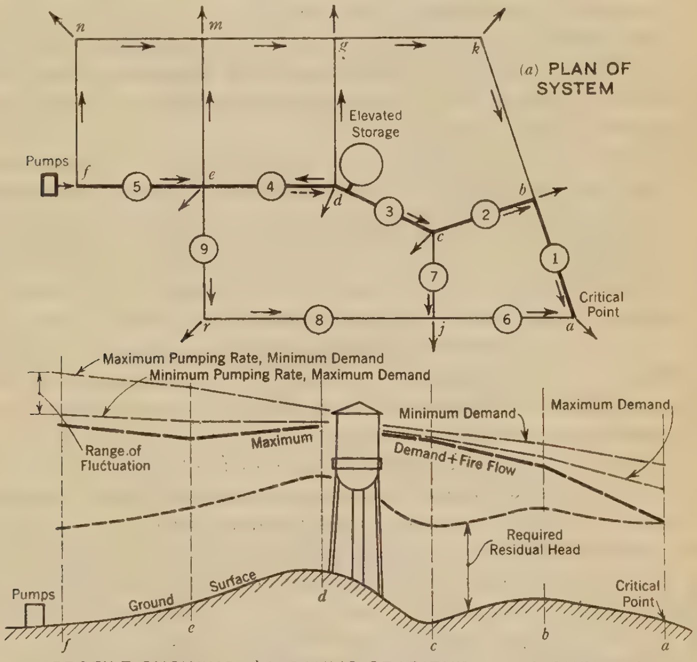
(a) PLAN OF SYSTEM

(b) PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE

Fig. 4.—Illustrating Case III, Supply with Varying Demand and Constant Pumping Head. Elevated Storage at Pumps

Case III.—Pumped Supply with Varying Demand and Constant Pumping Head; Elevated Storage at Pumps.—In this case (Fig. 4) the pumping head will be fixed by the peak flow through the system, including fire flow, and will be maintained constant by a distribution reservoir or elevated tank placed near the pumps at the elevation for best economy. The annual cost of pumping is not affected by the rate of pumping.

The power loss chargeable to each pipe for friction and take-offs is determined from the maximum friction head and from the average discharge over the design period. The elevation of the tank is established by the maximum friction head through the system.


(a) PLAN OF SYSTEM

(b) PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE.

n
m
g
k
Elevated Storage
Pumps
f
e
d
c
b
a
r
j
1
2
3
4
5
6
7
8
9
Critical Point
Maximum Pumping Rate, Minimum Demand
Minimum Pumping Rate, Maximum Demand
Minimum Demand
Maximum Demand
Range of Fluctuation
Maximum
Demand + Fire Flow
Required Residual Head
Ground Surface
Pumps
Critical Point

FIG. 5.—ILLUSTRATING CASE IV, PUMPED SUPPLY WITH VARYING DEMAND AND VARYING PUMPING HEAD. ELEVATED STORAGE REMOTE FROM PUMPS

The economic diameter of any pipe in the system for a varying discharge and constant pumping head is given by the following equation:

$$D_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{C_p\,(q + m\,Q_t)}{r\,(B + 1.55\,F\,C_f)} \right]^{0.163} q_{\max}{}^{0.30} \qquad (24)$$

in which q = the average discharge in the pipe throughout the design period; and, $q_{\max}$ = the peak discharge, including fire flow, and other quantities are the same as for Equation (23).

Case IV.—Pumped Supply with Varying Demand and Varying Pumping Head; Elevated Storage Remote from Pumps.—In this case (Fig. 5) the elevation of the distributing reservoir will be determined by the maximum friction loss through the pipes delivering water from the reservoir to the critical point. This will occur with the maximum draft from the system, including a fire load at the critical point. Presumably, the pumps would be operating at their maximum capacity during a fire, as this practice would make for the greatest economy in the design of the system.

The variation in discharge for normal demand is not similar for all pipes in a system of this type. In some of the pipes, as in Pipe 4 of Fig. 5, the direction of flow reverses. For others, such as Pipes 1, 2, and 3, the flow is always in the same direction for normal demand. Although the pumping head varies, the manner of variation is controlled by both the rate of pumping and the demand, which do not vary similarly and are not equal. The increased cost of pumping due to variable head is chargeable to friction loss in the pipes between the pumps and the elevated storage, and is substantially independent of the pipes down stream from the elevated storage.

In so far as the manner of determining economic sizes is concerned, three types of pipes may be distinguished in a system of this class, as follows:

(1) Pipes down stream from the elevated storage on direct routes to the critical point, such as Pipe 2, Fig. 5. These pipes may be considered as falling in Case III, and their sizes may be determined by means of Equation (24).

(2) Pipes on direct routes between pumps and elevated storage, such as Pipe 5, Fig. 5, may be considered as falling in Case II and their sizes determined by Equation (23). The values of A and q can be approximated only very roughly since they are influenced by both demand and pumping rates.

(3) Pipes that are similar in nature to both Types 1 and 2, such as Pipe 4, Fig. 5, which is down stream from the storage for the fire load at Point a and is en route from pumps to storage for minimum demand. Both Equations (23) and (24) may be used for estimating the economic size of pipes of this type. Equation (23) should be given greater weight for Pipe 4 since the influence of this pipe on the fluctuations of the pumping head is greater than its influence upon the established elevation of the storage. About equal weight should probably be given to both equations for finding the size of Pipe 8.

In the application of Equation (23) to the determination of the size of pipes in which there is a reversal of flow, such as in Pipe 4, the effect of the reversal is to increase the range of fluctuation of the pumping head. Since negative friction losses correspond with savings in pumping head, however, they should be credited to the pipe by using negative values of the corresponding terms in Equation (22) for the solution of A, which value will thus be small for pipes in which there is a reversal of flow for normal demands.

Economic Velocity

The discussion of economic pipe sizes in terms of economic velocity has been purposely avoided in this paper. Notwithstanding the fact that many engineers have become accustomed to thinking of the economics of pipes in terms of velocities, the practice is valid only in very special cases.

If the system consists of a single pipe through which water is pumped at a constant rate at all times (Equation (17)), the velocity does bear a definite relation to the diameter for best economy for a given set of costs. Even for this special case, which is unusual in practice, the relation varies with the unit costs.

If the pumping rate is constant and the system consists of a network of pipes, even for a single set of unit costs, there is no single relation between the economic pipe size and the velocity. The size is determined by both the discharge through the pipe and the take-offs from the system.

If the layout is a gravity system, there is no relation between the velocity and the pipe size for best economy which is independent of the head available. There is no such thing, therefore, as an economic velocity for a given pipe size in a gravity system.

If the system consists of pipes through which water is pumped at a variable rate, the question arises immediately as to which discharge rate should correspond with the economic velocity. If it is assumed that the economic velocity should be determined from the average discharge through the pipe, it is found that it does not bear the same relation to the diameter as in the case of constant discharge. Moreover, since the economic diameter is affected by the manner of variation of discharge, the corresponding velocity is also affected in a similar manner. In this case, again, the take-offs from the system affect the relation of diameter to velocity for best economy.

APPENDIX

NOTATION

The symbols introduced in this paper are defined as follows:

A = a substitution constant (see Equation (21));

B = a substitution constant (see Equation (2));

C = cost of a series of gravity pipe lines; C_a = annual cost, in cents, of pumping, or of water power; C_c = first cost of laying cast-iron pipe, in cents per linear foot; C_f = cost of cast-iron pipe, in cents per pound, with freight allowed; C_l = cost of lead, in cents per pound; C_m = cost of materials, including cast-iron pipe, valves, fittings, and pipe lining, in cents per linear foot; C_p = cost or value, in cents, of raising 1 000 000 gal 1 ft; C_w = the wage rate for common labor, in cents per hour; C_y = cost of yarn, in cents per pound; C_{at} = total annual cost of pipe and pumping, in cents per linear foot of pipe;

$\mathbf{C}$ = Williams-Hazen coefficient;

c = cost of a section of pipe = $f(h)$;

D = diameter of pipe, in inches;

d = depth of pipe trench, in feet;

F = a coefficient giving the cost of different types of pipe at the trench, including contractors' profit, engineering, etc.;

f = function of: $f(h)$ = cost, c, expressed as a function of the head loss of the section;

H = total head available for loss in a series of pipe lines;

h = head loss through a single pipe or element;
l = length of sections of pipe;
m = ratio of take-off head to head lost by friction in the pipe;
Q = discharge, in cubic feet per second; Q_t = take-off discharge;
q = discharge through a single pipe or element, in cubic feet per second;
r = annual rate of interest plus depreciation of the pipe;
S = slope;
t = the design period; and,
w = weight, in pounds per linear foot.

DISCUSSION

Clinton L. Bogert[11] and J. M. M. Greig,[12] Members, Am. Soc. C. E. (by letter).—The clever formulas derived in Professor Camp's interesting paper exhibit to the writers the danger of carrying formulary regimentation into the realm of practical engineering. The writers question the economy of such formulas in office use. The formulas give a designer the exact diameter to fit a given set of fixed conditions; but, unfortunately, conditions do not remain fixed and, more important, cast-iron pipes are not made in the fractional diameters called for by the economical analysis, the designer being constrained to accept stock sizes which increase in diameters by relatively large steps.

A typical example, first solved by substituting in the formulas, and again by the usual trial methods with a Williams-Hazen slide-rule, for a compound line of two sizes and carrying diminishing quantities, required 2 hr to derive economical sizes of 23.2 in. and 19.0 in., respectively, and less than 10 min by slide-rule to arrive at stock sizes of 24 in. and 18 in., respectively.

Moreover, several of the "constants" assumed by Professor Camp as non-variables in the differentiation, are by no means constant in the practical sense:

(1) He assumes the price of cast-iron pipe as constant; bids taken by the writers' office in November were at least 30 cts per ft lower than they would have been if taken in August, 1937, due to the drop in cast-iron pipe prices;

(2) He assumes the quantity of water as a definite forecast, whereas adverse zoning restrictions, or the advent of a large industry, may upset the "picture"; and,

(3) He assumes that the Williams-Hazen-constant, C, is definitely known for a given time and place, whereas the quality of the water or corrective treatments of the water initiated at some future date may alter the values of C and the entire economic "picture" framed by the formula. The writers grant that these uncertainties also prevail in slide-rule analysis, but are less important as less time is involved.

They would qualify the assumption (see "Introduction") that the cost of elevated storage will not be influenced by changes in pipe sizes. It would not hold good in all cases. A certain water company has 17 tanks and stand-pipes and 41 wells spread over an area of 35 sq miles; some tanks are near wells and some distant. In this system the cost and allocation of future storage is influenced directly by the size and system of mains between one or more tanks and the centers of supply and the centers of demand, which are, of course, not collocated.

Charles M. Mower, Jr.,[13] M. Am. Soc. C. E. (by letter).—The often used "economic velocity" is not the correct standard for attaining true economy in distribution system design. The author has shown this fact clearly. The economic diameter of a pipe depends upon both discharge and take-offs from the system and upon unit costs of both pumping and pipes. In a system with a pumped supply, the two sources of power loss are loss of head in the pipes and take-offs at higher pressures than are required; but in a gravity system, the head loss in pipes costs nothing. The only losses which add to the cost of the gravity system are take-offs at pressures higher than those required where the excess is provided by pipes that are larger than necessary.

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[11] Cons. Engr. (Sanborn & Bogert), New York, N. Y.

[12] Designer (Sanborn & Bogert), New York, N. Y.

[13] Hydr. Engr., Glenn D. Holmes, Syracuse, N. Y.

The author's assumption that pumping cost is directly proportional to head and discharge may be questioned since the capital cost of station and equipment and the operator's wages in a particular system are fixed regardless of considerable fluctuation in head and discharge. Since power is the only part of total pumping cost that bears a direct relationship to head and discharge, it would seem more logical to omit all fixed pumping costs from the equation just as the author omitted unpredictable pipe costs, and the cost of hydrant installations which were not affected by pipe sizes. However, in a practical problem, the result is not greatly changed by the use of power costs instead of total pumping cost, because pumping cost decreases much more rapidly with an increase in pipe diameter than pipe cost increases. Pumping cost varies inversely as $D^{4.87}$, whereas the cost of pipe laying varies directly as D and the cost of pipe materials, as $D^{1.55}$.

For the same reason, the total cost for a pipe one size larger than the most economical size is much less than the total cost for a pipe one size smaller. Thus, when computations indicate an odd pipe size, the next larger standard size should be used.

The methods of determining the pipe sizes for best economy for the different classifications of pumped supplies are just as applicable when a design is made on the basis of field tests of an existing system instead of the method suggested by the author, of dividing the system into districts and estimating domestic and fire drafts. In Case II particularly, there would be an advantage in having field measurements of the variations in flow for determining the value of A in Equation (23).

A. C. Michael,[14] Assoc. M. Am. Soc. C. E. (by letter).—For his attempt to rationalize the economic solution of complicated pipe-design problems, in which there are such a large number of variables, the author deserves to be commended. The mathematical derivations are very cleverly executed, except that certain assumptions and required adjustments might be questioned. The coefficients of the formulas are rather complicated, and, no doubt, considerable estimation would be required for their determination.

For very small systems, where the character of the water demand is simple, and where the piping arrangement is not complex, the method proposed probably could be used to advantage. The writer feels, however, that mathematical analyses of the problems of a water distribution system have a very limited application, and this is particularly true for a system of any size or complexity.

Of the variables considered in the formulas, many are extremely indeterminate. Costs vary over wide ranges; the cost of labor for laying parallel lines of the same diameter has been known to vary 50%, or more. The cost of pumping varies widely with changes in costs of fuel and labor and many other variables. The cost of excavation is usually an unknown quantity until after the work is completed.

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[14] Asst. Mech. Engr., Dept. of Water Supply, City of Detroit, Dearborn, Mich.

The most difficult problem, which the author does not discuss, is the determination of average and maximum flow in a proposed pipe line. If a line is in a newly developed section of the city it is impossible to predict the ultimate water demand to be supplied. In many cases, a proposed pipe line is designed and ready for construction when it is learned that a new industry, using large quantities of water, is about to locate in the area to be served; it then becomes necessary to re-design the proposed pipe line and possibly parts of the existing system. This condition will continue until city zoning and planning ordinances have "teeth." The recent and rapid increase in air-conditioning installations is an example of how conditions change suddenly to cause existing piping systems to become inadequate. The careful development and study of population growth curves, per capita consumption, etc., may often be of little value after a depression in the business cycle has wrecked the trend of the established curves.

In addition to these factors, the variations in the daily load curve must be considered; the ratio of maximum hourly demand to the average daily demand is quite large, at times attaining a value of 3.0 or more. It is conceivable that a pipe having an economic diameter determined by the method described by the author would not have sufficient capacity to supply the maximum hourly demand.

Seldom is the engineer confronted with the problem of designing a complete water-distribution system. These systems grow gradually by the addition to, or extension of, existing lines. The usual problem is to select pipe sizes that will meet average flow conditions with normal pipe velocities, and maximum flow conditions with a respectable pressure at the consumer's outlets. The only suitable method open to the designer is to collect all the information possible, use all the tools and intuition he has at his command, and estimate the future average and maximum demand on the line or system. A size of line may then be selected such that, at maximum flow conditions, the pressure loss will be suitable and the velocity will not exceed a reasonable value.

Ellwood H. Aldrich,[15] M. Am. Soc. C. E. (by letter).—In so far as it extends previous studies by Professor Camp and others into the analysis of water distribution problems, this paper should prove helpful in focusing attention on the economics of pumping and pipe installations. It sets forth a trial-and-error method to determine the economical sizes of pipe in water-works systems.

The various examples given in the paper cover, quite completely, the variable conditions that may prevail. The basic premise in all of them is that the economic installation depends upon balancing the cost of pipe line against the cost of pumping in comparative sizes of pipe. In the examples, apparently, trial-and-error methods must be utilized, first, to solve the distributional flow and then to determine the economic sizes of pipe under the assumed distribution. Quantities, depth of trench, and m, the ratio of take-off head to friction loss in the pipe (which, in turn, requires an assumption of pipe size), must be assumed and then rechecked.

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[15] Cons. Engr., Reeves Newsom, New York, N. Y.

Several assumptions in the paper do not appear to be in accord with current practice. Granted that pumping is required, the only variable of any practical consequence in the cost of pumping is the cost of power or fuel. Capital costs of station and equipment are unaffected by the quantity pumped or the pumping head, within the ranges commonly applicable. Likewise, operating costs, exclusive of prime power, such as supplies and operating labor, are relatively stable and are little affected by quantity or head conditions. For the purposes used herein, if it is assumed that distribution storage is adequate to equalize hourly fluctuations in flow, accuracy will not be greatly sacrificed if the cost of pumping included only the cost of fuel or power.

One of the principal purposes of storage is to permit a decrease in pipe capacity. An economic study of the distribution of water cannot neglect the cost, effective elevation, location, and amount of storage in the system.

In some localities the so-called unpredictable costs of pipe laying, such as rock excavation, wet work, removal and relaying of pavement, and avoidance of obstructions, commonly add as much as 50% to the total cost of the work. Machine excavation, when applicable, and the use of composition joints and longer pipe lengths will lessen the cost of installing pipe. These unpredictable costs are not constant but vary with the size of pipe in about the same ratio as does the cost of laying. Therefore, they will affect the economic sizes of pipe appreciably since the higher cost of pipe construction will make more economical the use of smaller pipe with increased pumping charges.

In determining the economic size of a main over a period of years, consideration must be given to the total cost over the period. For instance, in comparing total costs of two sizes of mains with increasing demand over a period of years, the smaller main will have less total cost during the early years, due to lesser fixed charges, and more during the later years, due to increased pumping costs. For a true comparison between the total costs of the two mains, the savings of one over the other in the early years is greater than would be indicated by the formula in the paper because of the effect of compound interest. Furthermore, if water sales which will result from the installation of the smaller main are sufficient to pay the total charges in the early years of its use, substantial earnings above these charges will result from increased sales during the later years. The increased charges due to higher pumping charges, therefore, will be partly or entirely offset by increased earnings. This fact will justify the use of the smaller pipe with later duplication as required. This duplication will be largely paid for by the savings due to decreased capital costs in the early years and increasing earnings throughout the period.

The paper indicates that methods for analyses of distributional flow in pipe systems and for determination of the economic sizes of pipe have hitherto not been available and that in the past designers have not had practical methods for the solution of such problems. Graphical analysis of these problems, the fundamentals of which were illustrated by the writer in 1937,[16] have been in use by him for more than ten years and have proved quite adequate for the practical treatment of these problems. The determination of the economical sizes of pipe similarly yields to simple graphical analysis.

M. H. KLEGERMAN,[17] ASSOC. M. AM. SOC. C. E. (by letter).—There is an admitted reticence on the part of designing engineers toward the use of design methods involving intricate—or apparently intricate—mathematical procedures. Accordingly, they find their attention directed, from time to time, to certain design procedures, the advantages of which are acclaimed because of their "freedom from calculus," or "because they can be solved by simple arithmetic," etc. From the foregoing viewpoint one might, at first glance, relegate Professor Camp's formulas for the design of economic pipe networks, with their derivatives, integration, and exponential equations, to the "intricate mathematical type." That conclusion, however, bears no justification, as is disclosed by a little careful examination.

Much has been written about the ease of solving flow problems, for example, by means of the Williams-Hazen formula; but certainly, the solution of this formula, prior to the introduction of special slide-rules and tables for this purpose, was by no means simpler than any of the formulas for the determination of economic pipe sizes, as presented by Professor Camp. The same might be said for most of the existing flow formulas, such as those of Kutter, Manning, etc., the use of which has been simplified by means of tables, charts, and curves, so that the computer scarcely realizes that he is using rather "complicated" formulas. Therefore, one should not evaluate a new design method solely on the basis of time involved in its solution, by comparing to the time involved for the solution of an existing method, for which, as a product of long usage, special aids have been provided.

Simplification of design methods is indeed a virtue. Such simplification, however, is sometimes brought about through approximations, and under some conditions the degree of inaccuracy in the final solution of a problem may become very much in doubt, depending on the extent of such approximations. For example, in the solution of flow problems of pipe distribution systems, it is not uncommon practice to eliminate from the computations, for the purpose of simplicity, certain of the lines that a computer may "consider" of minor importance; or pay little, if any, attention to the effect on the flow distribution and pressure drops resulting from minor (but cumulative) take-offs. The extent of inaccuracies thus introduced is unknown. They may be insignificant under one set of conditions, but by no means so under another.

To overcome the foregoing objections, the designer is now provided with such devices as the Hardy Cross method[3] and others, the excellence and simplicity of which have already been well illustrated.[4]

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[16] "Solution of Transmission Problems of a Water System," *Proceedings*, Am. Soc. C. E., October, 1937, pp. 1511–1533.

[17] Associate, Alexander Potter, New York, N. Y.

Similarly, Professor Camp has presented a tool that should prove of value in the economic selection of sizes comprising pipe networks, to which past efforts have been confined to "cut and try," and "experience," due to the admitted complexity of the problem.

Selecting the economic size of a single force main is a practise that is certainly followed by every engineer. Be that main relatively long, or even quite short (of the magnitude of 500 ft) a selection of diameter is based, among other factors, upon a consideration of the annual cost of financing the construction (including pipe, jointing material, laying, pumping equipment, etc.) and of the annual energy costs, these factors being plotted against pipe diameters resulting in a **U**-type of curve. Certainly, if such practise is warranted in the case of force-main design, there can be no question of the value of economic selection in the problem of distribution system piping, which represents, in total, vastly greater length and tonnage than the average force main.

The writer believes that the method presented by Professor Camp, used in combination with an exact method of determining flow distribution, has particular merit and application in the analysis of existing distribution systems for valuation purposes. For example, in the determination of a purchase price for public utilities (and the same may be said for rate determinations) one factor at least, in which the higher Courts have been in close accord, is the principle that it is the value of the property now, for which compensation must be paid now. As a result of the widely accepted principle, that "the expenditure required to produce a property as nearly as possible like the one valued, throws an important light upon value," most water supply valuation procedure, in so far as physical plant is concerned, involves the computation of "reproduction cost." This, in turn, requires the determination of a schedule of the property and consideration of the item of depreciation.

There are many cases of the valuation of water-works properties involving principally cast-iron pipe distribution systems, wherein reproduction cost has been determined by applying unit prices which were considered to be representative at the time of the valuation, to the inventory of the property involved, and deducting therefrom an allowance for depreciation. Thus, considering cast-iron pipe alone, having determined the length or tonnage, and adopted a unit price for these items, its "present worth" (deducting depreciation) was established.

The question, however, which did not receive adequate (if any) consideration in this case was: Does the schedule or tonnage of cast-iron pipe present in the distribution system, represent the least tonnage required to serve the same streets at equivalent hydraulic capacities? If it does not, why should the purchaser be required to pay for unnecessary pipe tonnage? Particularly, if he were to reproduce the system, he would, it is properly assumed, design the most economical distribution system possible for the hydraulic conditions represented by the system being valued.

Perhaps this viewpoint is contrary to the generally accepted meaning of the term, "reproduction cost," which refers to the cost of a structure in all respects identical with the old one; nevertheless, it may safely be stated that if the enterprise being valued were completely destroyed, the chances of replacing it exactly as it was before would be remote. In other words, the worth of a new plant of equal capacity, efficiency, and durability, with proper discounts for defects in the old plant (and, of course, depreciation for use) should be the measure of value, rather than the cost of exact duplication.

The drawback to the acceptance of this viewpoint, calling, in effect, for new design, has undoubtedly been due to the fact that in the absence of analytical design procedures, engineers would differ widely in such "equivalent" designs.

It is realized that "excellence of design" receives consideration in valuation work. However, without the relatively recent methods of computing flow distribution, quickly and accurately, in various parts of the water supply distribution system, and in the absence of a method of determining the most economical network of pipe sizes required in it, such weighting of "excellence of design" in determining value is relegated, necessarily, to the realm of opinion. To eliminate this opinion (which tends to vary with individuals) and to substitute for it an analytical method capable of identical solution by independent computers, would be a step forward in the reduction of the indeterminate factors that make valuation work controversial.

In applying the various Camp formulas to a study of an existing system (that is, for determination of least tonnage for an hydraulically equivalent system) the availability of flow records and billings, particularly in metered systems, would make possible fairly accurate estimates of take-offs. Fire flows for the determination of maximum flow conditions, as is required in some of the cases presented, could be determined from field tests. Friction coefficients could also be determined from field tests, the work being confined to representative mains in order to classify coefficients of the mains for various age groups. The extent of this determination may be carried to any degree required. However, as the author indicates, inasmuch as a reduction of the value of C, as great as from 130 to 80, will produce a variation in economic diameter of only 15%, probably typical coefficient values only need be determined by field tests. Power costs at the time of valuation can be determined definitely.

Thus, many of the factors entering into the use of the Camp formulas (as applied to analysis of existing systems) are quite capable of practical determination. Other factors, as to laying costs, etc., can probably be revised, if necessary, to apply to specific conditions.

F. KNAPP, ESQ.[18] (by letter).—There has been no good compendium on the subject of economic sizes of pipes for water distribution systems and, therefore, the paper by Professor Camp is a timely one. The author is to be commended, especially, for his attempt to classify the several cases met with in practice.

As far as the economic diameters for a gravity system are concerned, the writer understands that the method proposed by the author consists in simplifying the network so that only the larger and important mains are shown. To this series of pipes, Equation (13) or Equation (14) is applied, thus determining the best diameters. Applying this procedure to a distribution system, such as that shown in Fig. 4 (*a*), the mains would consist of Pipes 1, 2, 3, 4, and 5. With these diameters fixed, the sizes of the pipes of the remaining network are determined in such a manner that all, or nearly all, the available head is utilized. With the critical point at *k* instead of *a*, the procedure is repeated, but as the economic diameters for the several loading conditions will not be identical, some standard size of pipe will be selected which comes nearest to satisfying all the loading conditions.

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[18] With the São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

This method assumes that the sub-branches are relatively small as compared with the main pipes. Assuming, however, that the take-offs of a network system are distributed in such a manner as to require pipes of about the same size, it is certainly not admissible to "skeletonize" the system. In this case, the flow in each pipe is known only after the pipe system has been analyzed, assuming or estimating the diameters. How would the author proceed in such a case?

The writer is unable to follow Rules (1) and (2) as formulated in the text following Fig. 2, and as the treatment of the economics of pumped supplies makes direct use of these rules, the writer hopes that the closing discussion of the author will clear up these points.

Thomas R. Camp,[19] M. Am. Soc. C. E. (by letter).—In the preparation of this paper, it was the writer's purpose to contribute in a small way toward the development of effective and reliable means for the design of water distribution systems and reinforcing mains. As is well known, the analysis of flow in distribution systems is extremely complicated. Since reliable methods for the solution of the many hydraulic problems relating to distribution networks have been made available only recently, it is small wonder that so little has been done regarding the rational approach to methods of selecting the most economical sizes of pipes. Nevertheless, the selection of economic pipe sizes is of as much importance to the engineer in the design of distribution systems as is the correct solution of the hydraulic problems.

A most necessary part of engineering design is the selection of sizes and shapes for best economy. In the design of most engineering works, considerable time and effort are expended to obtain a workable layout at a minimum cost. The engineer has prided himself on being able to "build for one dollar what any other fool can build for two dollars." The *raison d'être* of the engineer, particularly the consulting engineer who engages in the design and supervision of construction, is his ability to produce a workable design at a minimum cost of construction and operation.

Messrs. Bogert and Greig profess to see in the writer's formulas a "danger of carrying formulary regimentation into the realm of practical engineering," presumably because of the lack of "economy of such formulas in office use." It is not necessary to use the formulas themselves for office use. They merely express the fundamental relations. Any engineer who desires to apply economic principles to the design of pipe networks may simplify the use of the formulas, or other similar ones, by preparing charts or slide-rules. This is what has long since been done for many of the complicated pipe-friction formulas, as has been so well pointed out by Mr. Klegerman. The time required for the use of the formulas for economic pipe size will be quite short as compared to the time that will be required to solve for the flow distribution after the selection of each trial set of economic pipe sizes. The writer is not attempting to submit a method of design that is easier than methods now in use. He is submitting fundamentals underlying a procedure which heretofore has not been used at all for complex networks.

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[19] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

The client can well afford to pay the cost of many times the office work usually required for the design of distribution system reinforcements if this additional work results in substantial savings in over-all costs. That substantial savings are possible cannot be doubted, because the sums involved in distribution system construction are tremendous. C. Maxwell Stanley,[20] Assoc. M. Am. Soc. C. E., has shown that the costs of a number of distribution systems in Iowa represent between 41% and 85% of the total values of the water-works. According to George A. Sampson[21] the distributing pipes represent about 90% of the value of small New England water-works; and according to W. W. Brush,[22] M. Am. Soc. C. E., about 50% of New York City's water-works investment of a half billion dollars is in the distributing system. Since the cost of water-works is from about \$24 to about \$70 per person served, the enormous investments involved in distribution pipes are quite apparent.

Messrs. Bogert and Greig, and Mr. Michael, criticize the writer's formulas because the coefficients, costs, and demand rates are not constant and are difficult to estimate. This is true, of course, for any method of design which involves predictions of future conditions. Nevertheless, it is necessary to make such predictions in order to proceed. Fortunately, the effect of errors in these estimates is not so great upon the economics as it is upon the workability of the design. The cost of a pipe line and pumping, plotted against the diameter, results in a U-shaped curve which is quite flat in the immediate vicinity of the most economic diameter. The cost, therefore, is not affected appreciably by the selection of the commercial diameter nearest the economic size. Similarly, errors in the estimation of any of the coefficients or other factors to be introduced into the formulas do not result in errors of the same magnitude in the diameter or cost.

An error of only 5% in the economic diameter of pipe requires, for its production, much larger errors in each of the factors in the formulas. The estimated discharge must be in error by at least 11%; the friction factor, C, by at least 18%; the pumping cost by at least 35%; the rate of interest plus depreciation by at least 35%; and the cost of laying, the cost of cast iron, or the value of Factor F, must be in error by more than 35 per cent. Hence, precision in the use of the formulas is not warranted.

For the same reason it is of little consequence whether total pumping cost, or only power cost, is used; and the error involved in the assumption that total pumping cost is directly proportional to the head and to the discharge is of little moment. These points have been recognized by Mr. Mower and by Mr. Aldrich.

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[20] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1609.

[21] Paper presented at the September, 1938, meeting of the New England Water Works Assoc. (Not yet published.)

[22] *Loc cit.*

Messrs. Bogert and Greig, and Mr. Aldrich, are concerned regarding the assumption that the cost of elevated storage is unaffected by changes in pipe size. As stated by Mr. Aldrich, "one of the principal purposes of storage is to permit a decrease in pipe capacity." This is correct for the pipes from the supply to storage, but if the storage is more than adequate for hourly fluctuations in demand, the quantity of storage is not related to pipe sizes. Hence, the cost of storage is not related to pipe size, except as the cost is influenced by the elevation of the stored water. The foregoing considerations are based upon the assumption that the locations of the reservoirs are fixed by the topography.

If the locations of the distributing reservoirs are not fixed by the topography, a much more complicated economic problem arises. In flat terrain where elevated storage must be provided in elevated tanks, the choice of the number of tanks and their locations is unlimited. In such a case economic pipe sizes are certainly affected by the location of the storage. The economic considerations leading to the location of distributing reservoirs are outside the scope of the paper. It is manifestly impossible, in one paper, to consider all the possible cases and conditions that might arise in the design of such complex works as distribution systems.

Mr. Michael suggests that "it is conceivable that a pipe having an economic diameter determined by the method described by the author, would not have sufficient capacity to supply the maximum hourly demand." Since the capacity of a pipe depends upon the head provided for flow and the writer's method fixes the head by fixing the elevation of the storage, this difficulty will not arise. It will be found occasionally, however, for cases with widely fluctuating draft, that pressures in the mains will be so great that the computations must be revised using heavier pipe than was originally contemplated. In some cases it will also be found that pumping heads are so great that it will be necessary to revise the computations with the additional pumping equipment costs taken into account.

Mr. Aldrich states that the so-called unpredictable costs, such as rock excavation, wet work, removal and relaying of pavement, and avoidance of obstructions are not independent of pipe size as claimed by the writer, but vary with the size in about the same ratio as the cost of laying. He is quite correct in the case of pavement, but the cost of pavement removal and replacement is predictable and may well be included in the equations of cost. The cost of wet work (sheeting, bracing, and pumping) and avoidance of obstructions is, in most cases, influenced very little by differences in pipe sizes. The cost of rock excavation is influenced by pipe size, but it is not directly proportional to pipe size. According to Mr. Sampson[21] the rock excavation for a number of New England systems averaged 150 cu yd per mile. The cost of this quantity of rock is less than 8% of the total cost of the mains. Where rock excavation is known to be required in considerable quantities, it can be included in the equations of cost.

The effect of compound interest upon the economic pipe size is of little consequence. It has been accounted for in the writer's equations in the form of depreciation. The factor, r, is the rate of interest plus depreciation. Large errors in the assumed value of r are required to produce appreciable errors in the economic pipe size.

Mr. Klegerman has performed a service in emphasizing the possibilities of using the methods presented in this paper for valuation work. It is probable that many difficulties would arise in securing the acceptance of the procedure outlined by Mr. Klegerman in Court cases, but as the method of hydraulic and economic analysis of distribution networks is perfected and simplified, it is certain to find a place in valuation work.

Mr. Knapp, quite properly, questions the validity of the practice of "skeletonizing" the distribution system when there is little difference in size between the trunk lines and the branch pipes. This criticism applies to the hydraulic analysis only, since the size of the branch pipes is commonly fixed at a minimum consistent with good fire protection (6 in. in the United States) without regard to economic diameter. In spite of the fact that the writer has quite naively described the "skeletonizing" process as a part of the method of attack in design, and despite the fact that much has been written by others relative to "skeletonizing" and many engineers have resorted to this practice, it is certainly not permissible to "skeletonize" any system without some knowledge of the magnitude of the errors that will thus be introduced into the hydraulic computations. So far as the writer is aware, no practical method has yet been presented for determining the magnitude of the errors due to the neglect of the smaller pipes in the hydraulic computations of a grid system. This problem appears to be one which has not yet been properly examined.

Rules (1) and (2) for estimating the take-off power loss are somewhat involved and have doubtless not been stated as clearly as is desirable. Two rules are required since the power is a function of both head and discharge and estimates of each must be made. The excess head at Point b is represented by the height of the heavy line, h_b, on Fig. 2(b). This head is all chargeable to Pipe 1, for if this pipe were made large enough, this head loss could be avoided. No more than this amount of head loss could be avoided by such procedure, however, without infringing upon the required residual head at Point b. The heads chargeable to the other pipes are shown on Fig. 2(b) as h_2, h_d, h_4, and h_5 at the up-stream ends of the pipes. The power losses corresponding to each of these heads are estimated from the average rates of discharge which must be pumped against these heads and which are taken out of the system at points up stream from the corresponding pipes.

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Paper No. 2020

GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT

By George E. Hubbell,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. G. M. Ridenour, Thomas R. Camp, W. E. Howland, and George E. Hubbell.

Synopsis

In designing the grit chambers for the Sewage Treatment Plant, at Detroit, Mich., in 1936, several problems were solved by means of a small-scale model of the tank and large-scale tests of two existing tanks. The small model was used to observe flow distribution and grease removal, and the large-scale tests revealed the percentage removal of sand of various sieve sizes for any given length of tank. This paper presents actual data on the settling of sand in flowing water, under field conditions, in a form which greatly simplifies the computations involved in grit-chamber design. This is not a theoretical approach to the problem of sedimentation in flowing water, but rather a presentation of data in a practical and usable form heretofore not available to the sanitary engineer.

Introduction

Cost estimates of the grit chambers at Detroit indicated that a saving of $167 000 could be made by adopting a design consisting of grit chambers, 150 ft long by 15 ft deep, instead of the customary tanks, 75 ft long by 7.5 ft deep. The unusual depth of the proposed chambers raised a question as to their effectiveness for grit removal; and also as each unit was designed to handle a flow of 250 cu ft per sec, the matter of entrance configuration in changing from a velocity of 4 ft per sec to 1 ft per sec required careful consideration. Since the effluent from the grit chambers was to be transported to the settling tanks through submerged conduits, the problem of coercing grease and floating material into the conduits arose. Recourse was made to the performance of a small-scale model for flow distribution and grease removal, and field tests were made on two existing grit chambers for the data on sedimentation of sand in flowing water.

Note.—Published in December, 1937, *Proceedings*.

[1] Cons. Engr. (Hubbell, Roth & Clark, Inc.), Detroit, Mich.

LABORATORY TESTS OF A SMALL-SCALE MODEL

Description of the Model.—A one-fifteenth scale model was constructed of wood and equipped with a glass side wall for observation of flow distribution. The principal dimensions of the prototype are given in Fig. 1. The model was operated in accordance with the Froude model law,[2] the actual flow in the model being 130 gal per min with an average velocity in the model of the grit chamber proper of 0.26 ft per sec. The supply was from a constant head tank entering through a calibrated valve. In the prototype the screen or rack consisted of bars, 3 in. by 0.5 in., with 0.75-in. openings. This screen was modeled of brass strips, 0.2 in. by 0.031 in., with a clear opening of 0.05 in.

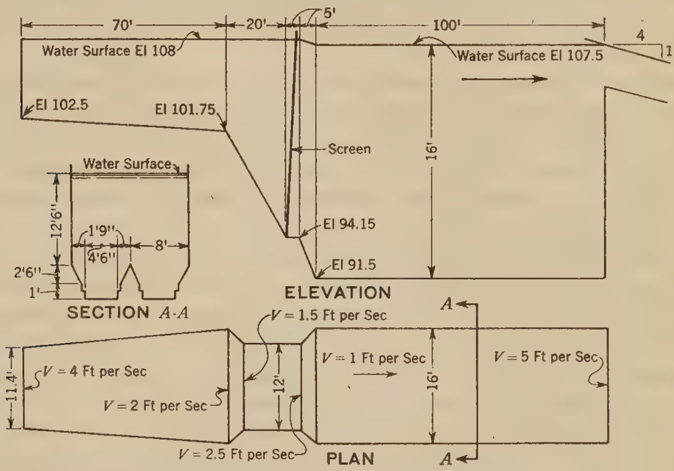
70'
20'
5'
100'
4
1
Water Surface El 108
Water Surface El 107.5
El 102.5
El 101.75
Screen
16'
Water Surface
12'6"
1'9"
4'6"
8'
2'6"
1'
El 94.15
El 91.5
SECTION A-A
ELEVATION
A
V = 1.5 Ft per Sec
11.4'
V = 4 Ft per Sec
V = 2 Ft per Sec
12'
V = 1 Ft per Sec
16'
V = 5 Ft per Sec
V = 2.5 Ft per Sec
PLAN
A

FIG. 1.—PRINCIPAL DIMENSIONS OF GRIT CHAMBER PROTOTYPE MODELED AT 1 : 15 SCALE; Q = 250 CUBIC FEET PER SECOND; AND V = 1 FOOT PER SECOND

Flow distribution was observed by introducing coloring matter, sawdust, and fine sand into the model. Observations indicated the following:

(1) There were no dead areas in the grit chamber;

(2) Flow followed the divergence angle of 2 on 5 in passing from the screen section, 12 ft wide, to the grit chamber section, 16 ft wide, without creating back eddies;

(3) Flow followed the bottom configuration in the approach to the rack, the slope being 2 ft on 5.26 ft;

(4) Flow followed the bottom configuration from the rack to the grit chamber, the slope being 2 ft on 3.77 ft;

(5) Coloring matter moved with an approximately vertical front in a remarkably uniform manner through the grit chamber (the first color flowed through in 74% of the theoretical time of 25.6 sec, the main body in about 90%, and the last color in 130%, of the time); and,

(6) No unusual horizontal or vertical velocity was observed. Coloring matter placed on one side of the channel remained on that side throughout the length of flow.

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[2] "Hydraulic Laboratory Practice," Edited by the late John R. Freeman, Past-President and Hon. M., Am. Soc. C. E., A. S. M. E., New York, N. Y., 1929.

The grit chamber actually adopted (see Figs. 2 and 3) conforms closely to the dimensions of the prototype shown in Fig. 1, except that the length was increased to 150 ft.

Screen.—The excellent distribution obtained is attributed to the inclined bar screen or rack. The tests were conducted with a rack having a slope of 1 horizontal to 3.44 vertical. Variation in the angle of slope of the screen had little effect on flow distribution.

Oil and Scum Removal.—Floating matter was removed from the surface of the model by means of the sloping top of the outlet conduit. A slope of 1 vertical to 4 horizontal is required to prevent undue accumulation of oil and grease. In the model, the draw-off velocity was 1.29 ft per sec, corresponding to a velocity of 5 ft per sec in the prototype. Since the effluent from the grit chambers is carried in submerged conduits to the sedimentation tanks, and since no mechanical means for grease removal will be installed in the grit chamber, it is important that all such matter be forced into the outlet conduit.

Settling of Sand in the Model.—When the investigation was first undertaken, it was felt that perhaps data could be obtained from the model relative to the manner in which sand would settle in the prototype. Direct application of the Froude law of model similitude would indicate that volumes to be used in a one-fifteenth scale model should be 3 375 times as small as the corresponding volumes in Nature. This would indicate that a 0.2-mm sand grain in the prototype would be the finest of dust in the model. In arriving at a proper grain diameter to use in the model, the following assumptions were made:

(1) Particles with diameters ranging from 0.2 mm to 0.1 mm settle in still water at constant temperatures in accordance with the expression,

$$V_s = k\,D^n \dots\dots\dots\dots\dots\dots (1)$$

in which V_s = velocity, in millimeters per second; D = diameter of particle, in millimeters; and, k and n = constants to be determined experimentally.

(2) The horizontal velocity, V_h, of the water in the model varies in accordance with the Froude law, $\frac{1}{\sqrt{l}}$, of the velocity in the prototype, in which l = a scale ratio for length $= \frac{L_n}{L_m}$; L_n = any linear dimension in the prototype (in Nature); and L_m = any linear dimension in the model.

(3) That it is desired to settle a particle having a settling velocity, V_s, a vertical depth, d, while the same particle moves forward with a horizontal velocity V_h a distance, L, in which d = the depth of a tank, in millimeters; and, L = the length of a tank, in millimeters.

Let the settling time, $t_s = \frac{d}{V_s}$; and, the time for corresponding horizontal movement, $t_h = \frac{L}{V_h}$. Then, for $t_s = t_h$:

$$V_s = \frac{V_h\,d}{L} \dots\dots\dots\dots\dots\dots (2)$$

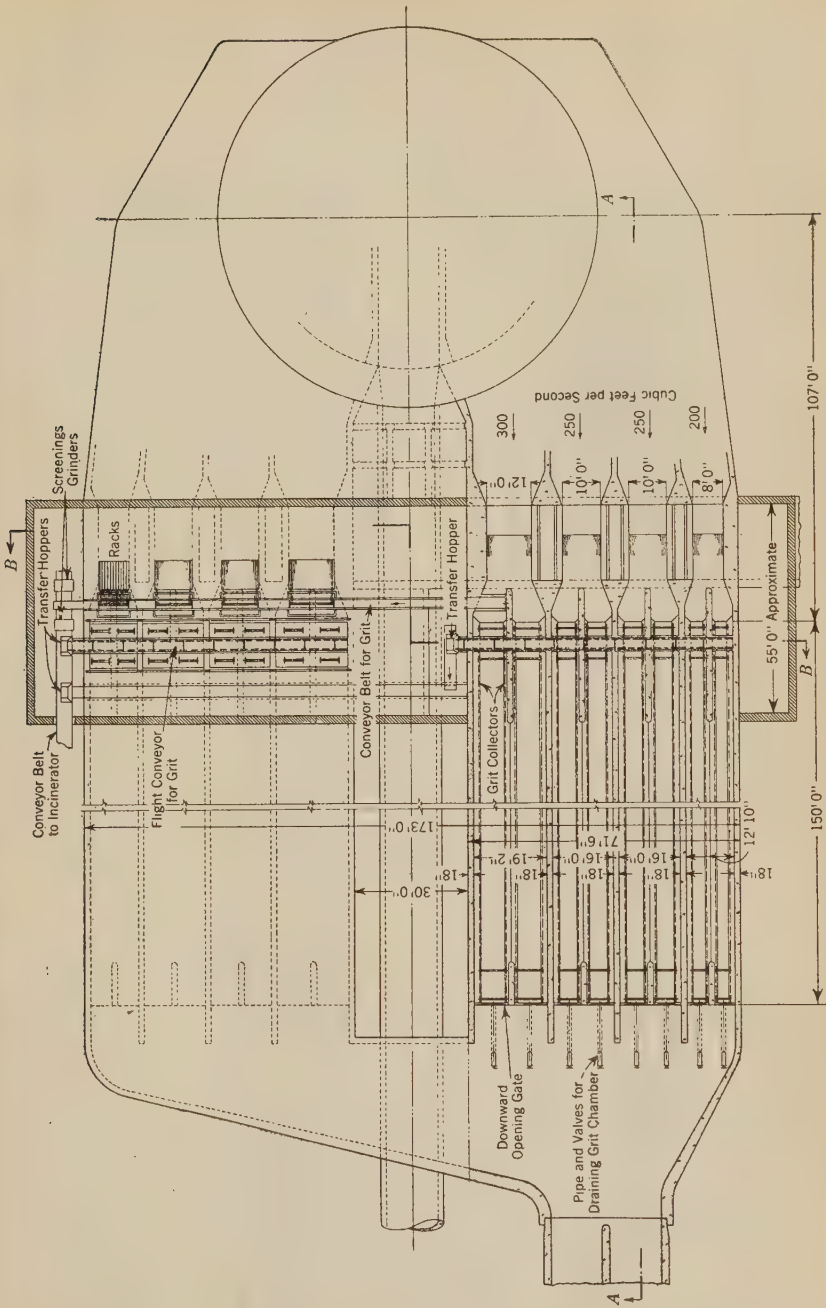
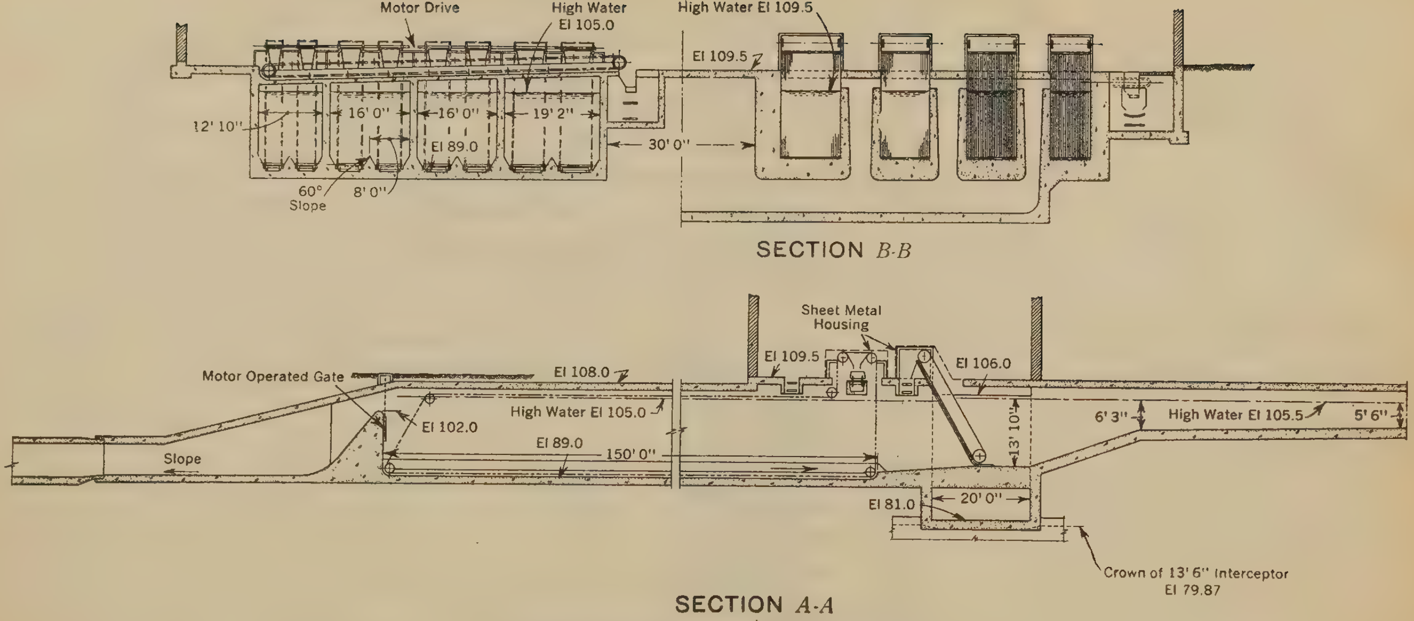

A
B
Screenings Grinders
Transfer Hoppers
Racks
Conveyor Belt to Incinerator
Flight Conveyor for Grit
Conveyor Belt for Grit
Transfer Hopper
Grit Collectors
Cubic Feet per Second
300
250
250
200
12'0"
10'0"
10'0"
8'0"
55'0" Approximate
107'0"
150'0"
173'0"
71'6"
12'10"
19'2"
16'0"
16'0"
16'0"
18"
30'0"
Downward Opening Gate
Pipe and Valves for Draining Grit Chamber
B
A

FIG. 2.—PLAN OF RACKS AND GRIT COLLECTORS, DETROIT SEWAGE TREATMENT PROJECT


Motor Drive
High Water El 105.0
High Water El 109.5
El 109.5
12' 10"
16' 0"
16' 0"
19' 2"
El 89.0
30' 0"
60° Slope
8' 0"
Sheet Metal Housing
El 109.5
El 106.0
Motor Operated Gate
El 108.0
High Water El 105.0
El 102.0
El 89.0
150' 0"
Slope
6' 3"
High Water El 105.5
5' 6"
13' 10"
20' 0"
El 81.0
Crown of 13' 6" Interceptor El 79.87

SECTION *B-B*

SECTION *A-A*

FIG. 3.—SECTIONS OF RACKS AND GRIT COLLECTORS

To obtain the ratio of grain diameter in the prototype to grain diameter in the model, equate the ratios of Equations (1) and (2): $\left(\frac{D_p}{D_m}\right)^n = \frac{l\,\sqrt{l}}{l}$ and,

$$D_m = \sqrt[n]{\frac{{D_p}^n}{l}} = \frac{D_p}{l^{1/2n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Although the foregoing development appears to be rational, the application has various limitations. For example, it is assumed that a logarithmic expression can be written for the manner in which sand settles in still water (Assumption (1)). If the particle size remained in the region governed by Stokes' law, or Newton's law, this might be done; but sand 0.2 mm in diameter lies in the critical region where the law of subsidence changes.

Stokes' law (see Assumption (2)) is true for particles settling in quiescent water. It does not hold for settlement in moving water in which, obviously, certain eddy currents occur due to turbulent flow conditions. Much information is available on the transportation of sediment by flowing water[3] but little is available on the sedimentation of material in flowing water at the velocity encountered in grit-chamber practice. In the prototype the average horizontal velocity is 1 ft per sec whereas in the model it is 0.26 ft per sec. Thus, it would appear that model results indicate greater removal than would actually occur in the prototype.

To operate the model in order to obtain data on the removal of sand, 0.2 mm in diameter, in the prototype (Assumption (3)) requires that sand having a diameter (by Equation (3)) of 0.10 mm be obtained for use in the model. Tyler sieves of 150 and 140 mesh have nominal openings of 0.104 and 0.107 mm, respectively. In the prototype, this range of size would correspond to 0.212 mm and 0.218 mm. Quartz sand, 0.1 mm in diameter, is not spherical, but rather is composed of angular particles of various shapes. In arriving at the required model size, it is assumed that the particles are spherical and of uniform size. Both these assumptions are difficult to achieve.

Field Tests of Large-Scale Models

Because of the inherent difficulties involved in the conduction and interpretation of small-scale tests of sand deposition, recourse was had to two existing grit chambers where tests could be conducted at a velocity in the grit chamber of approximately 1 ft per sec. Although the foregoing comment shows that it is possible to test small-scale models, it is quite evident that the use of a large-scale model in which the velocity ratio is 1.0, materially reduces the number of required assumptions for a transfer of results from model to prototype. Two tests were conducted, one at the sewage treatment works in Grand Rapids, Mich., and one at the East Side Sewage Treatment Plant, in Dearborn, Mich. Both tests were run to obtain data on the manner in which sand of various particle sizes settled in water flowing with an average velocity of 1.0 ft per sec. Actually, due to conditions surrounding the tests, the average velocity was

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[3] "Transportation of Detritus by Flowing Water," by F. T. Mavis, M. Am. Soc. C. E., Chitty Ho, Assoc. M. Am. Soc. C. E., and Yun Cheng Tu, Univ. of Iowa *Bulletin No. 5;* and "Transportation of Débris by Running Water," by G. K. Gilbert, *Professional Paper No. 86, U. S. Geological Survey.*

0.9 ft per sec. Two tests were necessary in order to substantiate the following theoretical development as to interpretation of test results.

Referring to the development of Equation (2), $\frac{L}{V_h} = \frac{d}{V_s}$; or, in terms of ratios: When the V_h-ratio = 1 and the V_s-ratio = 1, the L-ratio is $l = \frac{1.0}{1.0} \times h$-ratio. That is, the length ratio must be equal to the height ratio. Thus, in the model, if the height ratio is 2.0, and 60% of the sand of a given size is removed in 100 ft of tank, then in the prototype the same percentage removal of sand can be expected to occur in 200 ft. Since, in the settling of sand, there is no coagulation action, this conclusion appears rational and is substantially in accord with observations.

Grand Rapids Test.—Fig. 4 shows the general plan of the grit chamber tested. The channel selected was thoroughly cleaned for the test. Under normal flow conditions, the average depth in the channel is 4.7 ft. In order to secure a greater depth, the treatment plant was shut down and the sewage allowed to store in the influent sewer. When it had reached the lip of the overflow, all the sewage was passed through the selected channel, giving a depth at the mid-point of the grit chamber of 6.13 ft at the beginning of the test to 6.00 ft at the end of the test. When flow conditions had become constant, as evidenced by a uniform water level and Venturi meter readings, sand was fed into the grit chamber with shovels by two workmen standing on the platform at Elevation 18.0, dropping the sand into the water at Elevation 15.2. The sand was placed in the narrow end of the curved entrance channel (4 ft wide by 4.5 ft deep). The velocity in this approach section averaged 2.4 ft per sec. The sand was measured in a container having a volume of 0.77 cu ft which held 74.5 lb of sand, measured moist. No determination of moisture content was made. The same container was used to measure the sand removed by the chamber. The test was made on Friday and the grit chamber allowed to drain and dry until the following Monday. It has been assumed that bulking was the same during both sets of measurements. Fig. 5 shows the weight of

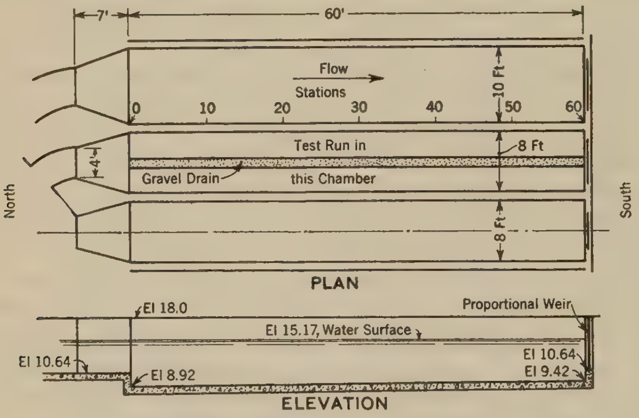
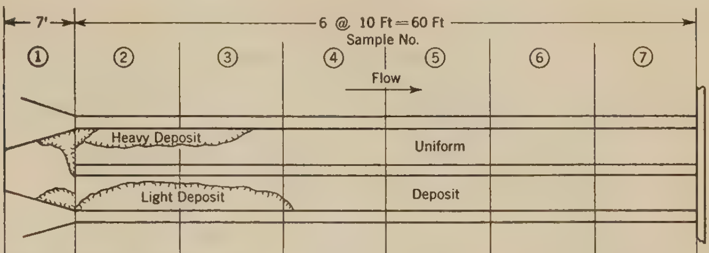


FIG. 4.—PLAN OF GRIT CHAMBER, GRAND RAPIDS, MICHIGAN

sand deposited in each 10-ft section of the chamber. The sand was carefully scraped into piles, a sample was taken in accordance with the A.S.T.M. standard method of sampling, and the volume determined. It was estimated


7'
6 @ 10 Ft = 60 Ft
Sample No.
①
②
③
④
⑤
⑥
⑦
Flow
Heavy Deposit
Uniform
Light Deposit
Deposit

	①	②	③	④	⑤	⑥	⑦	Total
Pails	3.0	11.250	6.393	3.536	1.750	1.080	0.571	27.580
Pounds	224	838	476	263	131	78	43	2053.0
On Drain Channel		← 0.5 Pails @ 74.5 Lb →						37.0
Pounds	Sand Removed From Channel							2090.0
Pounds	Sand Placed in Channel, (30×74.5 Lb)							2235.0
Pounds	Sand Passing Out of Channel							145.0

Fig. 5.—Weight and Location of Sand Deposited in Grit Channel

that 0.5 pail of sand was deposited on top of the central gravel drain that runs the entire length of the chamber. This half pail of sand has been prorated to the various sections. Fig. 6 shows the velocity variation in the channel, the average of all readings being 0.901 ft per sec. There was an unequal distribution over the cross-section, the velocity on the east side being consistently greater than that on the west side.

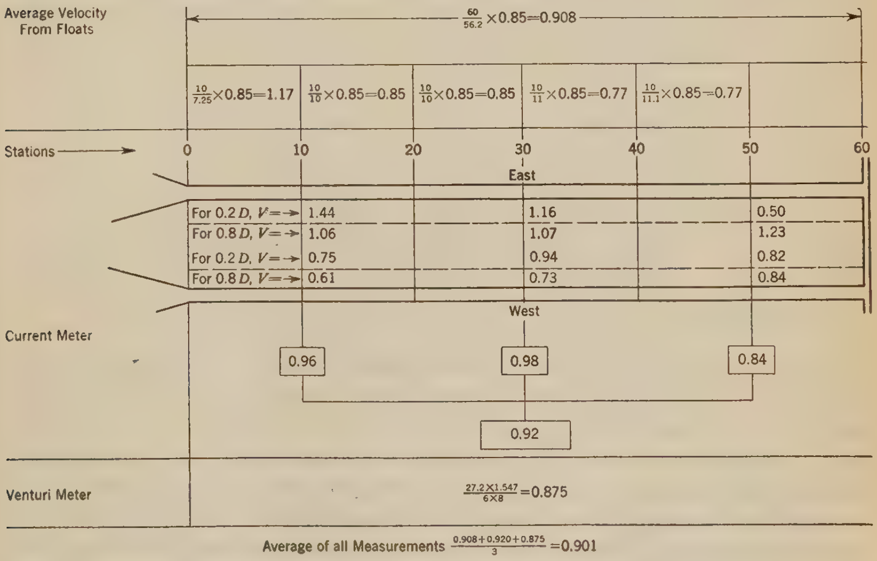

Average Velocity From Floats
$\frac{60}{56.2}\times 0.85 = 0.908$
$\frac{10}{7.25}\times 0.85 = 1.17$
$\frac{10}{10}\times 0.85 = 0.85$
$\frac{10}{10}\times 0.85 = 0.85$
$\frac{10}{11}\times 0.85 = 0.77$
$\frac{10}{11.1}\times 0.85 = 0.77$
Stations
0
10
20
30
40
50
60
East
For 0.2 D, V = 1.44
For 0.8 D, V = 1.06
For 0.2 D, V = 0.75
For 0.8 D, V = 0.61
1.16
1.07
0.94
0.73
0.50
1.23
0.82
0.84
West
Current Meter
0.96
0.98
0.84
0.92
Venturi Meter
$\frac{27.2\times 1.547}{6\times 8} = 0.875$
Average of all Measurements $\frac{0.908+0.920+0.875}{3} = 0.901$

Fig. 6.—Velocity Computations, in Feet per Second

Samples of the sand placed in the tank, and of the sand removed in the various parts of it, were thoroughly dried and sieved for a period of 10 min in a

Tyler type of shaker, using standard Tyler sieves. No attempt was made to calibrate these sieves and the mesh openings are as indicated by the manufacturer. The sand retained on each sieve (see Table 1) was weighed on a balance accurate to 0.10 gram, and the percentage composition of each sample was determined. Knowing the percentage composition of each sample and the actual weight of sand from which the sample was taken, the quantity of sand of each sieve size removed in each 10-ft section of the grit chamber was computed. (The average water depth was 6.0 ft; the temperature of the water was 65° F; the nominal velocity of the water was 0.90 ft per sec; and the specific gravity of the sand was 2.65.) The relation between the percentage removal of sand, of a given sieve size, and the length of tank required to obtain this removal, is best shown by means of the semi-logarithmic plat. Fig. 7 treats the data for the plant at Grand Rapids in this manner. It appears that these curves can be extrapolated as straight lines, all values beyond the 60-ft length being so obtained.

TABLE 1.—SIEVE ANALYSIS OF SAND REMOVED FROM, AND ADDED TO, THE GRIT CHAMBER

SIEVE SIZES		MATERIAL, IN GRAMS,* RETAINED ON A SIEVE, IN ANALYSIS OF SAMPLES NOS.:							
Meshes per inch	Opening, in millimeters	1	2	3	4	5	6	7	8†
Station		....	0–10	10–20	20–30	30–40	40–50	50–60	....
8	2.362	74.7 (13.5)	35.6 (6.8)	7.2 (1.4)	10.6 (1.8)	61.8 (9.3)	80.5 (12.0)	17.2 (2.5)	45.4 (2.6)
9	1.981	62.0 (11.2)	13.8 (2.6)	1.0 (0.2)	0.3 (0.5)	0.9 (0.1)	1.2 (0.2)	0.7 (0.1)	44.6 (2.5)
20	0.833	248.2 (45.1)	151.6 (29.0)	50.0 (9.8)	18.9 (3.2)	11.9 (1.8)	8.2 (1.2)	7.1 (1.1)	362.1 (20.8)
28	0.589	73.8 (13.4)	120.95 (23.1)	117.1 (22.9)	84.8 (14.2)	42.4 (6.4)	20.5 (3.1)	13.3 (2.1)	319.1 (18.3)
35	0.417	50.5 (9.2)	108.05 (20.7)	165.4 (32.3)	200.7 (33.7)	163.5 (24.6)	111.4 (16.6)	81.2 (12.5)	400.8 (22.8)
48	0.295	29.4 (5.3)	66.7 (12.7)	121.5 (23.8)	189.6 (31.9)	242.6 (36.5)	259.8 (38.8)	278.3 (43.0)	344.1 (19.8)
65	0.208	8.3 (1.5)	19.75 (3.8)	37.7 (7.4)	67.1 (11.3)	111.1 (16.7)	148.0 (22.1)	192.9 (29.7)	164.9 (9.4)
100	0.147	2.7 (0.5)	5.5 (1.1)	10.2 (2.0)	16.8 (2.8)	27.4 (4.1)	34.8 (5.2)	49.8 (7.7)	57.9 (3.3)
150	0.104	0.5 (0.1)	0.6 (0.1)	0.7 (0.1)	1.5 (0.3)	2.4 (0.3)	3.3 (0.5)	5.1 (0.8)	5.5 (0.3)
200	0.074	0.2 (0.1)	0.25 (0.0)	0.2 (0.0)	0.4 (0.1)	0.5 (0.1)	0.7 (0.1)	1.3 (0.2)	1.1 (0.1)
Pan	....	0.5 (0.1)	1.05 (0.1)	0.4 (0.1)	1.0 (0.2)	0.8 (0.1)	1.5 (0.2)	2.2 (0.3)	1.8 (0.1)
Total.......	....	550.8 (100.0)	523.85 (100.0)	511.4 (100.0)	591.7 (100.0)	665.3 (100.0)	669.9 (100.0)	649.1 (100.0)	1 747.3 (100.0)

* Numbers in parentheses are percentages.

† Sample of sand added to chamber.

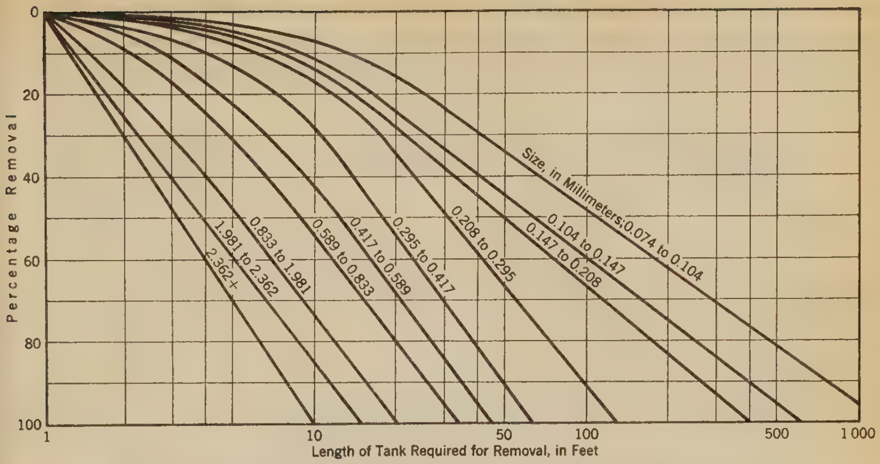


Fig. 7.—Percentage Removal of Sand of Various Sizes, by Grit Chamber, at Grand Rapids, Michigan

Dearborn Test.—Fig. 8 shows the general plan of the grit channel tested at the East Side Plant. The sewage is raised to the chamber by two pumps having a combined capacity of 6.5 cu ft per sec so that it was possible to adjust the effluent weir to obtain an average velocity of 0.9 ft per sec, the average depth being 2.8 ft. The actual velocity during the test, as obtained by float measurements, varied from 0.89 ft per sec to 0.96 ft per sec and averaged 0.93 ft per sec.

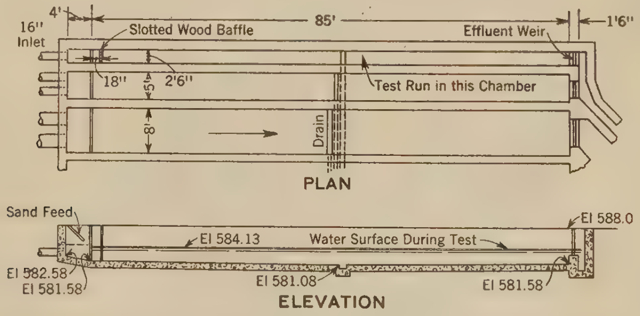
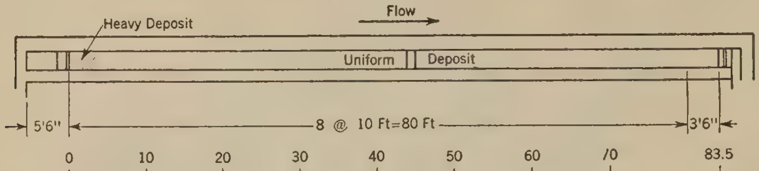

4'
85'
1'6"
16" Inlet
Slotted Wood Baffle
Effluent Weir
18"
2'6"
4'5"
Test Run in this Chamber
8'
Drain
PLAN
Sand Feed
El 588.0
El 584.13
Water Surface During Test
El 582.58
El 581.58
El 581.08
El 581.58
ELEVATION

Fig. 8.—Plan of Grit Chamber, Dearborn, Michigan

The high entrance velocity was dissipated by means of two baffles, the first consisting of a solid plank 24 in. high placed at the bottom, the second being a vertically slotted arrangement made of 1-in. by 3-in. strips. However, turbulence persisted in the first 20 ft of the chamber, but beyond that point flow conditions were uniform.

In order to obtain a wide variation in the composition of the sand placed in the chamber, 990 lb of coarse sand, 673 lb of medium sand, and 1 587 lb of fine sand, were fed to the channel on an inclined board so that the drop to the water surface was 6 in. In this test all of each grade of sand was fed continuously, in the following order: Coarse, medium, and fine. In the future, this procedure will be avoided as it gives a stratified deposit which is difficult to sample. The weight and location of the sand deposited in the grit chamber are shown in Fig. 9.



Sample No.	1	2	3	4	5	6	7	8	Total
Boxes of Sand Removed	16.09	14.00	8.00	5.60	2.18	2.52	1.27	1.50	51.16
Pounds of Sand Removed	885	770	440	308	120	140	83	70	2816
	Total Pounds of Sand Placed in Chamber								3250
	Total Pounds of Sand Passing Out of Chamber								434

Fig. 9.—Percentage and Location of Sand Deposited in Grit Chamber

The sieve analysis of the various samples is shown in Table 2. The computations for percentage removal of sand of various sieve sizes have been platted in Fig. 10 in the same manner as the Grand Rapids test data.

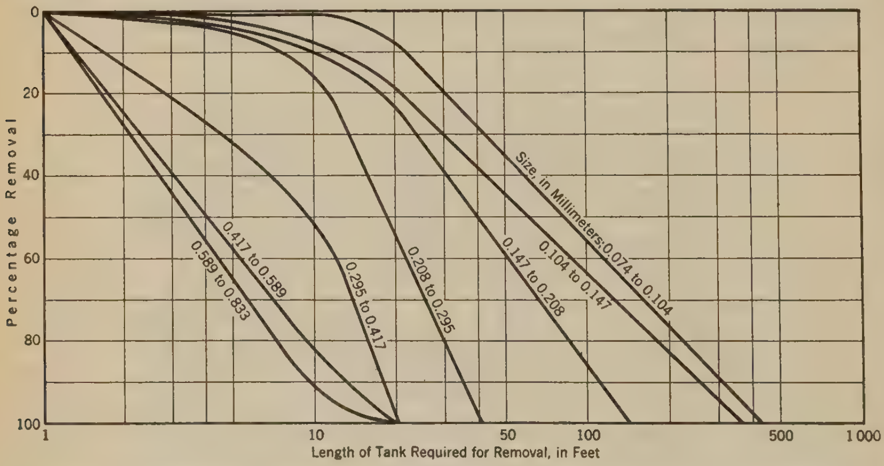


Fig. 10.—Percentage Removal of Sand in Tank 2.8 Feet Deep, by Grit Chamber, at Dearborn, Michigan

Results of Tests.—The laboratory and field model tests were conducted to enable the prediction of the grit removal to be expected from a tank having an effective depth of 15 ft and a length of 150 ft.[4] Based on a test run of the

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[4] It is recognized that, in general, the term, "grit," is all inclusive and covers all material settling out in the grit chamber. As here used, it refers only to material similar to sand, abrasive in nature, the Detroit grit chamber functioning only to protect the mechanical equipment for sludge handling. Sand embedded in the sludge removed from grit chambers may not have the same mechanical analysis as the sand used in the tests reported on. This, however, has no effect on the removals for a given sand size.

TABLE 2.—SIEVE ANALYSIS OF SAND REMOVED FROM AND ADDED TO GRIT CHAMBER

SIEVE SIZES		MATERIAL, IN GRAMS, RETAINED ON A SIEVE IN ANALYSIS OF SAMPLES NOS.:										
Meshes per inch	Opening, in millimeters	1	2	3	4	5	6	7	8	9	10	11
Station		0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–83.5	Sand added to chamber		
8	2.362	163.1 (14.1)	..	..	..	..	..	..	..	42.8 (6.8)	126.0 (19.5)	..
9	1.981	60.1 (5.2)	..	..	..	..	..	..	..	9.5 (1.5)	50.7 (7.8)	..
20	0.833	305.8 (26.3)	19.0 (2.2)	..	..	..	..	..	..	55.6 (8.9)	194.1 (30.0)	0.2 (0.0)
28	0.589	149.3 (12.8)	31.2 (3.6)	2.9 (0.3)	1.8 (0.2)	0.2 (0.0)	...	..	..	38.5 (6.2)	55.4 (8.5)	0.6 (0.1)
35	0.417	149.6 (12.8)	59.8 (6.8)	14.0 (1.6)	3.3 (0.4)	0.7 (0.1)	0.9 (0.1)	0.5 (0.1)	..	50.6 (8.1)	43.1 (6.6)	4.0 (0.7)
48	0.295	186.4 (16.2)	217.1 (24.8)	116.0 (13.0)	38.8 (5.2)	13.5 (1.6)	4.7 (0.5)	1.7 (0.2)	3.1 (0.5)	110.0 (17.6)	62.0 (9.6)	30.3 (5.1)
65	0.208	93.4 (8.0)	314.7 (35.9)	404.0 (45.3)	327.2 (44.2)	278.4 (33.2)	139.0 (14.6)	23.6 (3.5)	12.4 (1.9)	137.0 (22.0)	51.8 (8.0)	143.7 (24.4)
100	0.147	38.6 (3.3)	158.3 (18.1)	214.6 (24.1)	215.4 (29.0)	384.3 (45.8)	531.3 (55.5)	361.4 (53.5)	257.0 (39.3)	101.7 (16.3)	39.5 (6.1)	195.4 (33.2)
150	0.104	10.2 (0.9)	49.2 (5.6)	87.8 (9.8)	94.7 (12.8)	83.0 (9.9)	144.6 (15.2)	158.5 (23.5)	195.3 (29.8)	44.0 (7.0)	13.8 (2.1)	114.4 (19.3)
200	0.074	2.6 (0.2)	18.8 (2.1)	39.0 (4.3)	44.3 (6.0)	58.0 (6.9)	91.1 (9.6)	86.6 (12.9)	127.7 (19.4)	18.9 (3.0)	4.8 (0.7)	60.4 (10.2)
Pan	....	2.1 (0.2)	7.9 (0.9)	14.0 (1.6)	16.5 (2.2)	21.1 (2.5)	42.7 (4.5)	42.3 (6.3)	59.7 (9.1)	16.6 (2.6)	7.0 (1.1)	41.0 (7.0)
Total....	....	1 161.2 (100)	876.0 (100)	892.3 (100)	742.0 (100)	839.2 (100)	954.3 (100)	674.6 (100)	655.2 (100)	625.2 (100)	648.2 (100)	590.0 (100)

laboratory model using sand that will pass a 140-mesh sieve and be retained on a 150-mesh screen, Table 3 has been prepared showing the expected percentage removal of sand having a prototype size of 0.2 mm to 0.3 mm. Results based on the field tests are also shown.

TABLE 3.—COMPARISON OF PREDICTED REMOVALS OF SAND, 0.2 MM TO 0.3 MM IN DIAMETER, IN A CHAMBER OF 15-FOOT DEPTH, BASED ON VARIOUS MODEL TESTS

Length of tank, in feet	PERCENTAGE OF SAND REMOVED			Length of tank, in feet	PERCENTAGE OF SAND REMOVED		
	True, small-scale model, laboratory test	Field test, Dearborn, Mich.	Field test, Grand Rapids, Mich.		True, small-scale model, laboratory test	Field test, Dearborn, Mich.	Field test, Grand Rapids, Mich.
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Scale	1 : 15	1 : 5.35	1 : 2.5	Scale	1 : 15	1 : 5.35	1 : 2.5
60	72	..	40	120	90	63	65
80	79	37	50	140	94	73	70
100	85	50	59	150	96	77	73

As might be expected the laboratory model indicates a greater percentage of removal than the field models. This can be explained in part as due to the reduced turbulence corresponding to a lower velocity in the small model. The results based on the field tests for this sand size are in close agreement for the greater length values. For other sand sizes, the variation is greater, the Dearborn test generally indicating greater lengths for the same percentage removal. It is assumed that velocity-variation effects were the same in both tests and will be the same in the prototype. Undoubtedly, the distribution in the prototype will be more uniform, with consequent greater removals. Fig. 11 gives the expected removal for a tank of 15-ft effective depth. The removals shown are based on the Grand Rapids test and represent the writer's judgment in interpreting the available data.

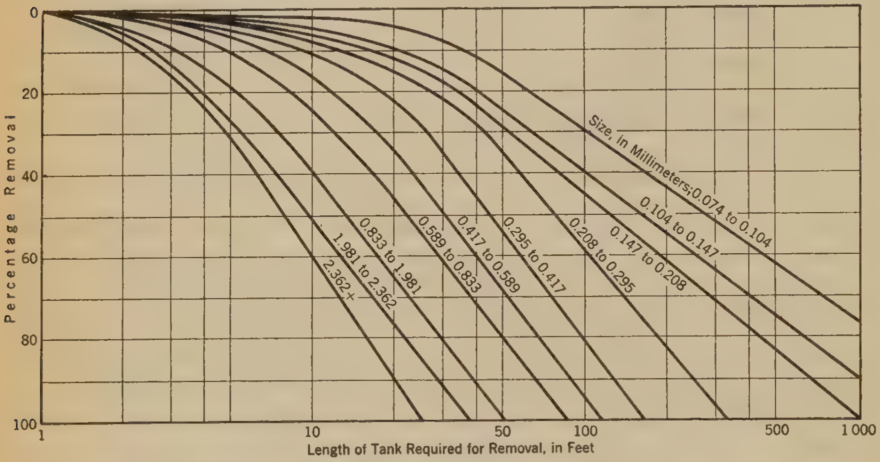

FIG. 11.—EXPECTED PERCENTAGE REMOVAL OF SAND IN TANK, 15 FEET DEEP, BY DETROIT GRIT CHAMBERS

Size of Sand Reaching the Settling Tank.—In setting up the requirements for the grit chamber the Board of Consulting Engineers (Clarence W. Hubbell, M. Am. Soc. C. E., the late Harrison P. Eddy, Past-President, Am. Soc. C. E., and the late John H. Gregory, M. Am. Soc. C. E.) stated that "tests should be run to determine: (1) The possibility of depositing all sand greater than a 0.2-mm effective size; and (2) the length of flow required for such deposition." This may be interpreted as meaning that it would be desirable to deposit all sand having a size greater than 0.2 mm.[5] Table 4 gives a sieve analysis of the sand content of sludge or grit removed from the Grand Rapids grit chamber and from sludge obtained from the final digestion tank. An analysis is included of a catch sample of sand found at the curb of a city street where the soil is clay, with well kept lawns.

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[5] "Grit Chamber Practice": A Symposium, *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 495.

Of the sample of grit removed from the digested sludge, 54.5% was retained on a 65-mesh screen, or was larger than 0.2 mm. Analysis was made possible by treating the sludge with acid to remove the organic material. Apparently, this grit does not cause excessive wear on the mechanisms in the plant. Presumably, then, it is not necessary to remove all the grit of this size in a grit chamber. The field tests indicate that the Detroit chambers will not remove all the grit coarser than 0.2 mm.

TABLE 4.—SIEVE ANALYSIS OF SAND OBTAINED FROM GRIT AND SLUDGE, GRAND RAPIDS SEWAGE TREATMENT PLANT AND TYPICAL DETROIT STREET SAND

Sieve mesh	GRIT CHAMBER, IN THE BOTTOM SLUDGE				DIGESTER		TYPICAL DETROIT STREET GRIT	
	10 Feet from Influent End; Material Retained		50 Feet from Influent End; Material Retained		Removed from the Digested Sludge			
	In grams (1)	Percentages (2)	In grams (3)	Percentages (4)	In grams (5)	Percentages (6)	In grams (7)	Percentages (8)
8	0.10	0.3	5.8*	23.2	0.15	0.1	8.1	1.7
9	0.10	0.3	0.9*	3.6	0.10	0.1	1.5	0.3
20	0.55	1.6	2.4*	9.6	1.95	1.4	18.0	3.7
28	0.70	2.1	0.8	3.2	2.15	1.5	17.3	3.6
35	1.10	3.2	0.6	2.4	4.40	3.1	24.8	5.1
48	5.40	15.9	0.9	3.6	19.60	14.0	63.9	13.2
65	11.60	34.2	1.6	6.4	47.95	34.3	107.2	22.3
100	8.20	24.1	2.5	10.0	40.20	28.5	102.7	21.3
150	2.80	8.2	1.75	7.0	13.70	9.7	56.9	11.8
200	1.30	3.8	1.50	6.0	5.20	3.7	31.9	6.6
Pan	2.15	6.3	6.25	25.0	5.00	3.6	50.1	10.4
Total	34.00	100.0	25.00	100.0	140.40	100.0	482.4	100.0

* Large particles of silt cemented together.

In analyzing the Detroit grit chambers, Eugene A. Hardin, M. Am. Soc. C. E., prepared the material in Table 5, based on the foregoing tests and an analysis of an assumed sample of grit (Column (3)), which indicates that the Detroit chamber, 15 ft deep by 150 ft long, will remove 17% more grit than one 100 ft long, and 72% more than one 50 ft long. Comparing this tank with one of equal length, but 6 ft deep: At 150 ft the 15-ft tank removes 19% less than the 6-ft tank; at 100 ft, 26% less; and at 50 ft, 38% less. The economic length of the grit chamber would appear to be not more than 100 ft since the extension of the tank to 150 ft will increase the total removal only 17% and effect this in the range of grit smaller than 0.3 mm in diameter. It was estimated that the increase in length from 100 ft to 150 ft would cost $75 000. This expenditure seemed warranted in Detroit for the following reasons: The tanks were unusually deep, with no similar tanks available for comparison; and the increased length would reduce the solids load on the settling tanks to some degree and would act as a factor of safety insuring a minimum of grit passage to the sludge disposal mechanism. However, the writer believes that a grit chamber, 150 ft long, is not necessary.

CONCLUSION

It is hoped that this brief description of a method of approach to the grit-chamber problem will result in other similar investigations. Theoretical computations for sand removal in grit chambers are uncertain, due to the difficulty of evaluating the various factors involved, whereas the results obtained from tests are substantially correct and practical in application.

TABLE 5.—ESTIMATED PERCENTAGE REMOVAL OF SAND IN VARIOUS LENGTHS: NORMAL VELOCITY, 0.9 FOOT PER SECOND

Sieve No.	DIAMETER OF PARTICLES, IN MILLIMETERS		Analysis of Detroit grit; percentage retained	PERCENTAGE REMOVAL IN:					
				50 Feet		100 Feet		150 Feet	
	From (1)	To (2)	(3)	Percentage of a given size present (4)	Percentage in Detroit grit (5)	Percentage of a given size present (6)	Percentage in Detroit grit (7)	Percentage of a given size present (8)	Percentage in Detroit grit (9)
				(a) CHAMBERS 15 FEET DEEP					
9	2.0	2.4	2.0	100	2.0	100	2.0	100	2.0
20	0.8	2.0	3.7	100	3.7	100	3.7	100	3.7
28	0.6	0.8	3.6	80	2.9	100	3.6	100	3.6
35	0.4	0.6	5.1	68	3.5	95	4.8	100	5.1
48	0.3	0.4	13.2	54	7.1	81	10.7	97	12.8
65	0.2	0.3	22.3	35	7.8	59	13.1	73	16.3
100	0.15	0.2	21.3	28	6.0	45	9.6	55	11.7
150	0.1	0.15	11.8	24	2.8	40	4.7	48	5.6
200	0.07	0.10	6.6	16	1.0	30	2.0	38	2.5
Pan	Dust		10.4	0	0.0	0	0.0	0	0.0
Totals	....	....	100.0	....	36.8	....	54.2	....	63.3
				(b) CHAMBERS 6 FEET DEEP					
9	2.0	2.4	2.0	100	2.0	100	2.0	100	2.0
20	0.8	2.0	3.7	100	3.7	100	3.7	100	3.7
28	0.6	0.8	3.6	100	3.6	100	3.6	100	3.6
35	0.4	0.6	5.1	100	5.1	100	5.1	100	5.1
48	0.3	0.4	13.2	91	12.0	100	13.2	100	13.2
65	0.2	0.3	22.3	67	14.9	91	20.8	100	22.3
100	0.15	0.2	21.3	50	10.6	67	14.3	76	16.2
150	0.1	0.15	11.8	45	5.3	60	7.1	69	8.1
200	0.07	0.10	6.6	34	2.2	49	3.2	57	3.8
Pan	Dust		10.4	0	0.0	0	0.0	0	0.0
Totals	....	....	100.0	....	59.4	....	73.0	....	78.0

A laboratory approach to the problem has been indicated which may be of value, although it is felt that more reliable information is gained by eliminating the velocity factor.

Excellent flow distribution can be obtained by means of bar screens having $\frac{3}{4}$-in. openings. Such screens introduce sufficient loss of head to insure uniform velocity conditions during grit deposition. Grease can be induced successfully into submerged conduits by using a draw-off velocity of 5 ft per sec and conduit roofs having slopes of less than 14 degrees. In large installations, deep grit chambers proportioned so that uniform velocity conditions exist, are more economical to construct, and will perform with the same degree of efficiency as grit chambers of one-half the depth and length and twice the width.

Acknowledgments

The writer wishes to acknowledge the valuable suggestions offered by Albert Roth, Assoc. M. Am. Soc. C. E., Arthur B. Morrill and Eugene A. Hardin, Members, Am. Soc. C. E.; and, for making the field work possible, he is grateful to James R. Rumsey, M. Am. Soc. C. E., former Sanitary Engineer for the City of Grand Rapids, and Austin Habemehl, former Chief Chemist at the Dearborn Plant.

DISCUSSION

G. M. RIDENOUR,[6] ASSOC. M. AM. SOC. C. E. (by letter).—An interesting approach to a more rational and scientific development of grit chambers is presented in this paper. The similarity to large-scale results in the model, operated in accordance with the model law, is encouraging to further investigation along this line, and the author is to be commended for introducing this method of study.

The difficulty of comparing the effect of depth on the performance of large-scale grit chambers which at the same time vary in so many other respects, such as surface area, volume of flow, distribution of velocities, and ratios of length to width and depth, however, would appear to be great. This seems especially true due to the sensitivity of settling sand to various influences and the exacting relationships of economic calculations.

In view of the seemingly variable conditions between the Grand Rapids, Mich., and Dearborn, Mich., units, the writer wonders whether it is not possible to arrive at conclusions that differ from those presented in the paper, depending on interpretation, and tending to show that the shallower grit chambers might be the more efficient, even at the sacrifice of losing the equality of the theoretical relationship of $\frac{d}{v_s}$ to $\frac{1}{v_h}$ which seems to be the basis for the conclusions in the paper.

From the data presented, the writer has compared the efficiencies of the two units in three different ways: (*a*) On the basis of total percentage of grit removed; (*b*) with respect to the average size of sand removed in succeeding sections of the tank; and (*c*) comparative performance on the basis of equivalent surface areas. The computations are shown in Table 6.

The sieve analysis of the materials removed in each section of the grit chamber shows that, in general, the shallower grit chamber at the Dearborn plant removes a sand of much less average size in equivalent sections of the tank. This action holds throughout the tank length with the exception of the first section which is admittedly adversely influenced by the turbulence of the pumps. Table 6 also shows that the percentage of total sand removed is slightly greater in the Dearborn plant in each succeeding section of the channel, again with the exception of the first 10 ft.

Another observation that can be made from the data is the critical size at or below which the percentage of small sand in the composite sample increases toward the outlet end, indicating that point at which the $\frac{1}{v_h}$-ratio exceeds the $\frac{d}{v_s}$-ratio. In the Dearborn plant this size lies between 0.104 and 1.47 mm. In the Grand Rapids plant, the critical size lies between 0.295 and 0.417 mm.

Additional comparisons have been made in Table 6 on the basis of efficiency in relation to equivalent surface area. A sand, 0.2 to 0.3 mm in size, has been used for the calculations, the same as that used for the design of the Detroit plant. These computations seem to confirm the previous generalizations that a greater efficiency can be expected from the shallower unit, provided exception is made of the poor performance in the first 20 ft of the Dearborn chamber due to pumping disturbances. The end of this section is situated at the equivalent surface area of 256 sq ft of the Dearborn unit. The efficiency of the shallower unit is greater beyond this point.

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[6] Asst. Prof., Dept. of Water Supplies and Sewage Disposal, Rutgers Univ.; and Research Engr., Dept. of Water and Sewage Research, Agri. Experiment Station, New Brunswick, N. J.

TABLE 6.—COMPARISON OF EFFICIENCIES ON BASIS OF EQUIVALENT SURFACE AREAS

Length	Surface Area			Percentage Removal of Sand, 0.2 to 0.3 Millimeter in Size		Average Size of Sand Removed		Percentage of Total Sand Removed	
	Grand Rapids	Dearborn	Grand Rapids equipment	Grand Rapids	Dearborn	Grand Rapids	Dearborn	Grand Rapids	Dearborn
0–10	40	12.5	86	16.9	11.5	0.73	0.85	37.6	27.2
10–20	120	37.5	256	33.7	56.5	0.47	0.25	21.2	23.7
20–30	200	62.5	428	47.9	89.0	0.43	0.19	11.8	13.6
30–40	280	87.5	600	58.3	110.0	0.52	0.17	5.9	9.5
40–50	360	....	....	65.5	....	0.55	0.16	3.5	3.7
50–60	440	....	....	71.6	....	0.33	0.14	1.9	4.3

All these observations would seem to indicate a superior efficiency of the shallower Dearborn unit in contrast to Mr. Hubbell's conclusions that a slightly greater removal efficiency can be expected from the deeper tank. Admittedly, the writer has a disadvantage in the interpretation of the results through lack of detailed data on the investigation, but it does seem that on this basis the author's conclusions are not entirely reconcilable with the data presented. If the writer has not erred in interpretation and if Mr. Hubbell's conclusions are also correct, it then seems that other factors, due to variable conditions in the two units, have played an important part. One such factor which the writer has always considered of importance is the effect of ratios of length to depth on the efficiency of grit chambers. Theoretically, the chamber should be designed so that the value of $\frac{1}{v_h} = \frac{d}{v_s}$ for the smallest particle to be removed. This presupposes, however, that this and all larger particles will settle without turbulence and that the sweeping effect of unequal velocities is not to be present. In actual conditions, this quiescent state does not exist and any given size particle in the liquid is carried farther toward the outlet end of the tank. The distance to which larger sized particles are carried along the tank length is proportionally less, however, than any given decrease in the depth of the tank. The advantage of this fact is gained in a shallower tank. It seems to the writer that the results in Mr. Hubbell's paper also give evidence in this direction.

Thomas R. Camp,[7] M. Am. Soc. C. E. (by letter).—This paper describes an excellent experimental work on grit chambers. The author states in the "Synopsis" that his paper "is not a theoretical approach to the problem of sedimentation in flowing water, but rather a presentation of data in a practical and usable form heretofore not available to the sanitary engineer." These data have assisted the engineers of the Detroit project in preparing efficient designs for the grit chambers. The data are of even greater importance, however, in furnishing, for the first time, full-scale experimental checks of the validity of the rational theory of sedimentation of discrete particles in flowing water.

The Detroit engineers have previously used models successfully for studies of the flow distribution in the settling tanks of the Springwells water purification plant.[8] For this purpose model studies appear to produce reliable results. The conclusions reached by Mr. Hubbell, regarding the passage of oil and scum, however, seem questionable. In this case surface tension and viscosity probably come into play and neither is accounted for by the Froude model law. It is probably justifiable to conclude that if a draw-off velocity of 1.29 ft per sec in the model will remove scum, a somewhat higher velocity in the prototype will do likewise; but there seems to be little reason for concluding that the corresponding prototype velocity for scum removal is 5 ft per sec. In fact, it seems likely that grease will be removed at about the same velocity in both model and prototype since there is no method of reducing the scale of the grease.

Studies of the settling characteristics of suspended matter can be made better and more easily at full scale than with models. The particular difficulty in the use of models for this purpose arises in selecting and obtaining the proper size of material to settle in the model. The difficulty may be overcome in the case of discrete particles, but it is insurmountable with flocculent suspensions. In Equations (1), (2), and (3), the author attempts to show how the proper size of particle may be found for the model; but since Equation (1) is based upon the assumption that the settling velocity varies with some power of the particle size (an assumption which the author admits is incorrect) the method is rejected. This method of attack is much simplified and may be made correctly if the settling velocities are considered without regard to particle size. Settling velocities are readily measured by quiescent settling tests, but it is difficult to estimate them accurately from sieve analyses.

For both model and prototype the path of a settling particle will be the direction of the vector sum of its own settling velocity and the velocity of the liquid in which it is settling. If the velocity of the liquid is taken as the nominal or mean tank velocity, it follows from Equation (2) that the particle settling velocities should bear the same ratio as the tank velocities for equal removal. That is, the settling velocity for the model grit particles should be $\frac{1}{\sqrt{l}}$ times the settling velocity of the prototype grit. Since no quiescent settling tests were made for the Detroit grit, it is necessary to estimate the settling velocities from the particle size.

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[8] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 344.

Fig. 12 shows the settling velocities of spheres with a specific gravity of 2.65 in water at 10° and 20° C. The values on the two curves are for the region beyond the range of validity of Stokes' law, and were computed from the curve of experimental values presented by Schiller[9] in terms of the Reynolds number. Fig. 12 also shows a curve for the settling velocity of sand at 18° C in terms of the grain size. It was estimated by the writer from experiments[10] of Richards, Hazen, and others. The position of this curve is only approximate, of course, and will vary with the shape of the particles. Very angular grains will settle at slower velocities, the retardation being greatest for the larger particles. For the critical grit size of 0.2 mm selected for Detroit, the corresponding settling velocity from Fig. 12 is about 2.3 cm per sec. The

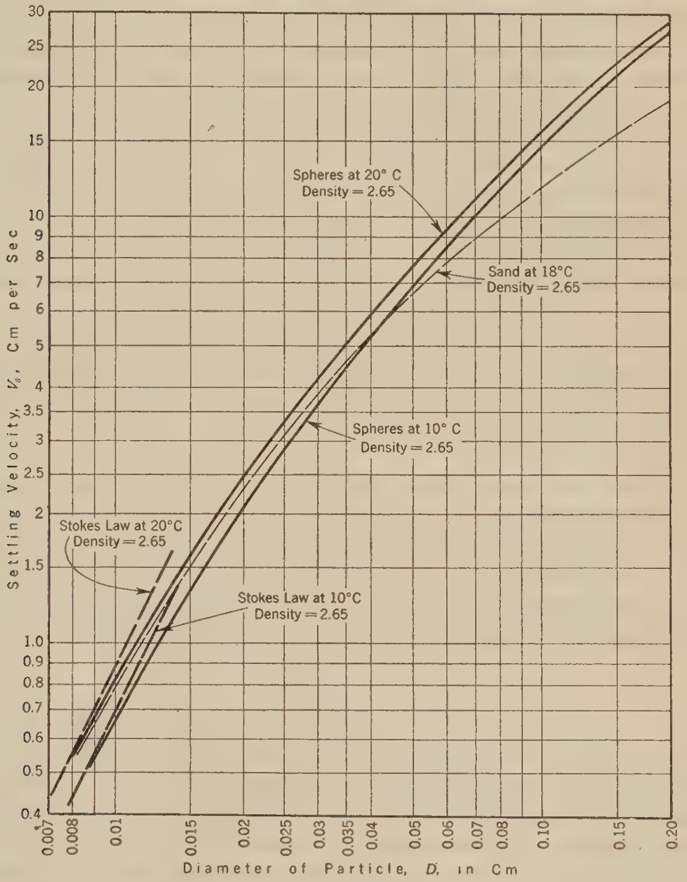

FIG. 12

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[9] "Handbuch der Experimental Physik," von Schiller, Band 4, 2 Teil, 1932, p. 369; see, also, *Sewage Works Journal*, September, 1936, pp. 743–744.

[10] *Transactions*, Am. Inst. Mining Engrs., 1907, p. 283; also, *Transactions*, Am. Soc. C. E., Vol. LIII (December, 1904), p. 45.

proper settling velocity for the model, therefore, would be $\frac{2.3}{\sqrt{15}}$, or 0.6 cm per sec, which corresponds with a grain size of about 0.085 mm.

The author is incorrect in the statement that Stokes' law does not hold for particles settling in moving water. The settling velocity of a particle is a function only of the particle size, shape, and density, and of the viscosity and density of the fluid. When these are constant the settling velocity is constant both within the range of validity of Stokes' law and beyond. The settling velocity of a particle must be considered with relation to the surrounding fluid, however, and the direction and velocity of motion of a particle with relation to the tank correspond with the vector sum of the settling velocity and the velocity of the fluid with respect to the tank.

Theory of Sedimentation of Discrete Particles.—A simple theory[11] of the removal of discrete particles in continuous-flow, rectangular settling tanks has been presented by the writer which involves the following assumptions: (1) The direction of flow is horizontal, and both direction and velocity are the same in all parts of the tank; (2) the concentration of suspended particles of each size is the same at all points in the vertical plane perpendicular to the direction of flow at the inlet end of the tank; (3) all suspended particles maintain their size, shape, and individuality during settling, and the temperature of the fluid remains constant throughout the tank; hence, each particle is assumed to settle at constant velocity; and, (4) a particle is removed when it strikes the bottom.

In view of these assumptions, all particles settling at velocities such that their paths will be parallel to, or steeper than, the path from the liquid surface at the tank inlet to the bottom at the outlet end will be removed. The settling velocity corresponding to this path is known as the tank "overflow rate," and it is equal to the discharge per unit of surface area. The removal of particles which settle at any velocity less than the overflow rate is,

$$r = \frac{V}{V_0} = \frac{A\,V}{Q} \quad \text{.........................} (4)$$

in which r = the removal as a ratio; V_0 = the overflow rate = $\frac{Q}{A}$; Q = the discharge; and, A = the surface area of the tank.

Equation (4) was first presented by the late Allen Hazen,[12] M. Am. Soc. C. E., and represents sedimentation in a basin in which, according to Hazen, the water is absolutely quiet. For a continuous flow basin it represents the condition of no "vertical mixing" of suspended matter. According to Hazen, "in an actual settling basin the water is mixed more or less from top to bottom in the process, with the result that the sediment does not go down in the manner indicated by the assumption." The nearest approach to this "theoretical maximum" removal which may be obtained in an actual basin, according to Hazen's theory, is represented approximately by Curve C of Fig. 1 in Hazen's paper. For conditions which produce a removal of 100% as given by Equation (4), Hazen's Curve *C* gives only 66% removal. It has been previously shown by the writer[11] that, except for the proposition corresponding to Equation (4), Hazen's theory is fundamentally unsound. The effect of vertical currents upon the retardation of settling of grit is very much less than is indicated by Hazen's theory, as will be shown subsequently in comparing the results of the Detroit experiments with removals computed by means of Equation (4).

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[11] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 306; and *Sewage Works Journal*, September, 1936, p. 742.

[12] "On Sedimentation," by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (December, 1904), p. 45 (see Curves C and A, Fig. 1).

In order to apply Equation (4) to a suspension of particles of varying settling velocity, it is necessary to have a "settling velocity analysis curve" analogous to a sieve analysis curve of sand. Such an analysis may be obtained by quiescent settling tests of the suspension. Lacking settling tests, the curve may be estimated for grit from the sieve analysis by means of Fig. 12. Curve 1 of Fig. 13 is prepared in this manner from the sieve analysis of the sand used in the experiment at Grand Rapids, Mich. As has been shown[11] previously, the total removel for a given overflow rate is,

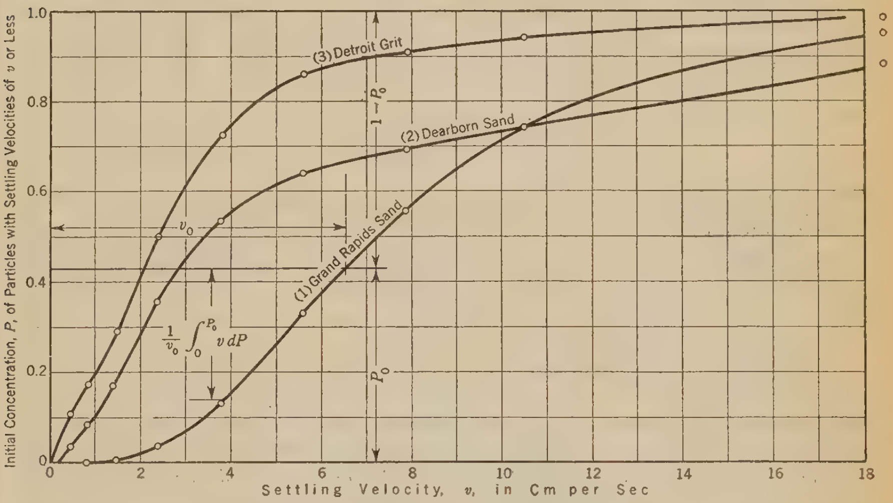

FIG. 13.—SETTLING VELOCITY ANALYSES

$$r = 1 - P_0 + \frac{1}{V_0}\int_0^{P_0} V\,dP \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which P_0 = the value of P from the settling velocity analysis curve corresponding to V_0.

Curve 1, Fig. 13, indicates how this removal may be evaluated graphically from the settling-velocity analysis curve. The last term of Equation (5) is equal to the average vertical distance between the curve and the horizontal line for $P = P_0$.

Correlation of Theory with Results of Hubbell Experiments.—The "count and weigh" sizes of sand grains are normally 10 to 15% greater than the sizes of sieve openings. This is due to the grain shape, and the more the shape departs from the spherical the greater this discrepancy. On the other hand, the more the shape departs from the spherical the less the settling velocity will be. As the "count and weigh" size, and, hence, the grain shape, was not given for the Hubbell tests, the writer has assumed for an approximation that these two effects are balanced; and the grain size has been taken as the size of sieve opening.

TABLE 7.—REMOVAL IN GRIT CHAMBERS

Section	1	2	3	4	5	6	7	8
(*a*) GRAND RAPIDS, MICHIGAN								
Cumulative surface area, in square feet	42	122	202	282	362	442	522	....
V_0, in feet per second	1.035	0.357	0.215	0.154	0.12	0.098	0.083	....
V_0, in centimeters per second	31.6	10.9	6.56	4.7	3.66	2.99	2.53	....
P_0	....	0.765	0.43	0.23	0.12	0.07	0.045	....
Computed removal, r	....	0.67	0.86	0.934	0.969	0.982	0.99	....
Actual removal	....	0.475	0.687	0.806	0.863	0.90	0.917	....
Efficiency	....	0.71	0.80	0.86	0.89	0.92	0.93	....
(*b*) DEARBORN, MICHIGAN								
Cumulative surface area, in square feet	25	50	75	100	125	150	175	203
V_0, in feet per second	0.26	0.13	0.087	0.065	0.052	0.043	0.037	0.032
V_0, in centimeters per second	7.94	3.97	2.65	1.98	1.59	1.31	1.13	0.98
P_0	0.695	0.55	0.40	0.28	0.196	0.15	0.12	0.10
Computed removal, r	0.542	0.725	0.827	0.881	0.92	0.942	0.954	0.962
Actual removal	0.272	0.509	0.645	0.74	0.776	0.82	0.845	0.868
Efficiency	0.50	0.70	0.78	0.84	0.85	0.87	0.89	0.90

Table 7(*a*) shows the results of computations for removal in the Grand Rapids grit chamber using Equation (5) and Curve 1, Fig. 13. The results show cumulative removals up to, and including, each section of the grit chamber in which the deposit was weighed in the Hubbell experiments. For comparison, the actual removals as measured by Hubbell (the ratios of the weights of deposited sand to the total weight of sand added) are also shown. The ratio of the actual removal to that computed by means of Equation (5) is shown under the caption, "Efficiency." Table 7(*b*) shows the comparison of the computed values for removal with the actual removal obtained in the Dearborn grit chamber. Curve 2, Fig. 13, gives the settling velocity analysis for this sand.

The sand used in the model of the Detroit grit chambers was quite uniform in size, being retained between 140-mesh and 150-mesh sieves. The writer has assumed a constant settling velocity for this sand which he has estimated from Fig. 12 at 0.85 cm per sec. The theoretical removal in the model is given by Equation (4) and is equal to $\frac{0.85}{V_0}$. The predicted removals in the Detroit grit chambers of sand 0.2 to 0.3 mm in size, based upon removals obtained in the model, are given in Table 3. Presumably these removals are the same as the actual values obtained in the model. Table 8 shows the removals computed by means of Equation (4) for the model compared with those shown in Table 3.

Table 9 shows the removals computed by means of the theory for the Detroit grit chambers compared with the removals estimated by Mr. Hubbell. Curve 3, Fig. 13, gives the settling velocity analysis for the assumed Detroit grit.

The correlation of the results obtained by the theory with those obtained in all three sets of experiments is very good for the entire length of the chamber, the discrepancy being less than 10 per cent. The errors are greatest for the shortest lengths considered, which may well be expected because of the difficulty of distributing both sand and water uniformly over the cross-section of the chamber at the inlet end and for other reasons which will be explained subsequently. The errors for the chambers complete, although small, are explainable in terms of factors not accounted for by the theory.

TABLE 8.—REMOVAL IN MODEL GRIT CHAMBER

Length, in feet	$\frac{60}{15}=4$	$\frac{80}{15}=5.33$	$\frac{100}{15}=6.67$	$\frac{120}{15}=8$	$\frac{140}{15}=9.33$	$\frac{150}{15}=10$
Cumulative surface area, in square feet	4.26	5.68	7.12	8.53	9.95	10.67
V_0, in feet per second	0.068	0.051	0.041	0.034	0.029	0.027
V_0, in centimeters per second	2.07	1.56	1.25	1.04	0.89	0.82
Computed removal, r	0.41	0.545	0.68	0.818	0.955	1.0
Actual removal	0.72	0.79	0.85	0.90	0.94	0.96
Efficiency	1.75	1.45	1.25	1.10	0.985	0.96

TABLE 9.—ESTIMATED REMOVAL IN PROPOSED DETROIT, MICHIGAN, GRIT CHAMBERS WITH NOMINAL VELOCITY OF 0.9 FOOT PER SECOND

Length, in feet	50	100	150	50	100	150
Cumulative surface area, in square feet	800	1 600	2 400	800	1 600	2 400
	(*a*) CHAMBERS 15 FEET DEEP			(*b*) CHAMBERS 6 FEET DEEP		
V_0, in feet per second	0.27	0.135	0.09	0.108	0.054	0.036
V_0, in centimeters per second	8.24	4.12	2.74	3.3	1.65	1.1
P_0	0.91	0.765	0.56	0.65	0.335	0.215
Computed removal, r	0.364	0.577	0.715	0.655	0.827	0.878
Hubbell's estimated removal	0.368	0.542	0.633	0.594	0.73	0.78
Efficiency	1.00	0.94	0.89	0.91	0.88	0.89

Factors Unaccounted for in Theory.—Reference to the assumptions made in the development of the theory will indicate that only one of the assumptions is fully met in the settling of sand; namely, that the particles do not coalesce in settling. Actual sewage grit contains a variety of substances other than sand, and some of the particles are mixed, or coated, with grease or organic matter. Some flocculation or coalescence in settling, therefore, probably does occur with sewage grit. Nevertheless, sewage grit is the only suspension commonly dealt with by sanitary engineers which approximates the assumption of no flocculation. It is the only suspension, therefore, which can be considered in terms of the foregoing theory. It is not possible to make a settling-velocity analysis of a flocculent suspension, because the particles are changing their settling velocities continuously.

The errors introduced by the assumption of uniform horizontal velocity in the chamber arise from two sources, short-circuiting and turbulence.

The effect of short-circuiting is to send some of the liquid through the chamber in less than the detention period and to retain some of it for longer periods. The overflow rate, therefore, is different for different portions of the contents of the chamber. A correction for this effect may be made if dye studies are available for the chamber provided, of course, that the velocity is great enough to maintain stable flow. The author presents results of dye studies for only the model grit chamber. These results are shown approximately by the writer in the form of a curve (see Fig. 14). Since the effect of short-circuiting is to reduce the removal given by the theory, and since the actual removal in the model was greater than that given by the theory, this curve cannot be used to explain the discrepancies for the model chamber.

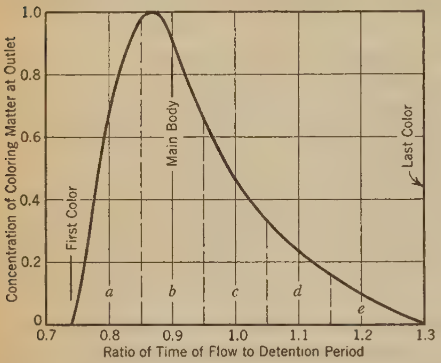

FIG. 14.—DYE CURVE TO SHOW SHORT CIRCUITY IN MODEL GRIT CHAMBER

The writer is at a loss to state precisely why the actual removal in the model exceeded the theoretical, whereas in both the Grand Rapids and Dearborn tests it was less than the theoretical. He suggests, however, that the manner of adding the sand at the inlet of the model might have been such as to carry much of it directly to the bottom. The second of the assumptions upon which the theory is premised may thus have been wide of the truth for the model and approximately true for both the Grand Rapids and Dearborn experiments. Since no measure of the accuracy of this assumption was made, this explanation must be regarded as hypothetical.

In order to explain the procedure to be used in correcting for short-circuiting, let it be assumed that the short-circuiting in the Dearborn chamber is also represented by Fig. 14 for the chamber as a whole. The time of settling of the suspension may be assumed equal to the time of passage of the dye through the chamber. The portion represented by *a*, Fig. 14, is approximately 26.6% of the whole, and it passed through the chamber in 81.5% of the detention period. The overflow rate for this portion is, therefore, $\frac{1}{0.815}$ times the average value of V_0 for the chamber, or 1.20 cm per sec. The corresponding removal from Curve 2, Fig. 13, is 94.8 per cent. These values are given in Table 10 with similar values for the other parts of the dye curve of Fig. 14. The weighted average removal corrected for short-circuiting is 95.8%, as compared with 96.2% without the correction. Hence, it may be concluded that the effect of short-circuiting is very small in a chamber in which the increase in removal is very small with increased length. This is particularly true if the short-circuiting is of as little magnitude as that represented by Fig. 14.

The effect of turbulence in retarding settling in the author's experiments should be examined for the full length of the chambers only, inasmuch as the deposit in any part of the chamber is influenced by transportation of sand along the bottom after it has settled. Transportation out of either the Grand Rapids or the Dearborn chamber could have occurred only if the material were held in suspension. It is reasonable to assume that most of the discrepancy between the actual removal in these two chambers and the removal given by the simple theory can be charged to transportation in suspension.

TABLE 10.—EFFECT OF SHORT-CIRCUITING IN DEARBORN, MICHIGAN, CHAMBER.

Area under dye curve	*a*	*b*	*c*	*d*	*e*	Total
Ratio to total	0.266	0.376	0.198	0.097	0.063	1.0
Time ratio	0.815	0.895	0.995	1.095	1.2	0.9
V_0, in centimeters per second	1.20	1.095	0.985	0.895	0.817	....
P_0	0.13	0.115	0.10	0.09	0.08	....
Removal ratio	0.948	0.956	0.962	0.967	0.973	....
Fractional removal	0.252	0.360	0.191	0.094	0.061	0.958
Removal neglecting short-circuiting	....	....	....	....	....	0.962

The phenomena of transportation of matter in suspension due to turbulence is just beginning to be understood. The recent developments in the theory of fluid turbulence in pipes make available much of value in the study of sedimentation. In turbulent flow, the shearing forces due to velocity gradients are accompanied by eddy currents. The magnitude of these eddies may be stated in terms of the "mixing length" and "mixing velocity." The effect of these eddies in retarding settlement is nil if the concentration of suspended matter is uniform throughout. If, however, there exists a concentration gradient, the effect of an eddy is to carry material from regions of higher concentration to regions of lower concentration in a manner precisely analogous to the process of diffusion. If, then, the concentration of grit in suspension is greater near the bottom of the chamber than above it, the effect of turbulence is to retard settling. If the concentration gradient with depth is just such that the downward motion due to settling is balanced by the upward motion due to turbulence the particles will be held in suspension indefinitely.

von Kármán, Ippen, and Rouse[13] have shown the application of the mechanics of fluid turbulence to the transportation of sediment in suspension, and have presented an equation for the concentration gradient at equilibrium in terms of the settling velocity of the particle and the so-called "friction velocity." Since the "friction velocity" may be stated in terms of the mean chamber velocity, this equation may be given in terms of the ratio of the particle settling velocity to the chamber velocity. Mr. Rouse also presents a diagram, first developed by Ippen, for the rapid solution of the equation.

The writer has examined this diagram for the grit-chamber experiments presented by Mr. Hubbell, in order to estimate what portion of the sand might be subject to permanent transportation in suspension if the grit chambers were long enough for equilibrium to be reached. The examination reveals that about 1% of the Grand Rapids sand, 32% of the Dearborn sand, and 35% of the Detroit grit (in chambers, 15 ft deep) are small enough to be subject to such suspension. This statement does not mean that the actual transportation in suspension will be anything like these values; because, for transportation to take place, the proper concentration gradient must exist before the particles settle out. The extension of the turbulent flow theory to cover the effect of turbulence in retarding settling before equilibrium has been reached has not yet been made.

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[13] *Transactions,* Am. Soc. C. E., Vol. 102 (1937), pp. 534–537.

The fourth assumption made in the development of the simple theory of sedimentation of discrete particles (namely, that a particle is removed when it strikes the bottom) is correct for a tank as a whole, provided the outlet is above the bottom. For a correlation of the Hubbell experiments with the theory, however, the assumption is not correct except for the whole chambers. Movement of particles along the bottom by traction, which was observed by the author to take place in at least one case, results in less deposit near the inlet and greater deposit near the outlet. The removal "efficiency" of the sections near the inlet for both the Grand Rapids and the Dearborn chambers, it will be noted, was less than that of the entire chamber. A part of this effect is doubtless due to "bed movement." Many studies have been made in recent years of the factors influencing stream-bed transportation, but the results thus far obtained are not sufficiently consistent to be used in predicting bed movement.

Conclusion.—A thorough understanding of the mechanism of settling of non-flocculent particles is essential before a rational approach can be made to the theory of settling of flocculent suspensions. The forces at work in the settling of discrete particles are also at work in the clarification of flocculent suspensions, but, in addition thereto, is the phenomenon of coalescence.

Data such as those presented by Mr. Hubbell are of great value in the development of the sedimentation theory. Many data have been collected upon the clarification of flocculent suspensions in full-scale settling tanks and in small pilot plants, but practically all these data are valueless in the development of the sedimentation theory because of the failure of the experimenters to measure all the variables. Most of the experiments that have been made, have had as their purpose the solution of particular sedimentation problems. The writer submits that if all the variables are measured in such experiments the particular problem may be solved with greater facility, and what is more important, the work will be of value to others. The ultimate goal is a theory which will enable engineers to design tanks intelligently from small-scale quiescent settling tests made upon the suspension to be clarified. In practically all experiments thus far made, settling characteristics, as distinct qualities of the suspensions, have not been measured. These qualities have been obscured by the characteristics of the basins in which the experiments have been run.

W. E. Howland,[14] Assoc. M. Am. Soc. C. E. (by letter).—The perfection of a technique for model testing of sedimentation basins is of great importance, and the studies reported in this paper should be of considerable value for this purpose.

In trying to account for the lack of agreement in results of the tests of percentage removal of fine sand obtained in the different models, several explanations may be given. The author suggests that the reduced turbulence in the smaller model (due presumably to a reduced Reynold's number and a nearer approach to the stream-line condition of flow) may account for the observed high percentage removal of the finer sand by this model. This may be a valid explanation notwithstanding the data presented by L. G. Straub,[15] Assoc. M. Am. Soc. C. E., which shows that the so-called (and mis-named) efficiency of sedimentation above a certain limiting value of the Reynold's number is independent of that function. This "efficiency" (defined as the ratio of the flowing-through time to the detention period probably first measured in sedimentation basins by C. H. Capen,[16] M. Am. Soc. C. E., in 1927), may be judged on theoretical grounds to have a doubtful effect upon the percentage removal of sediment in the basin—as has already been argued.[17]

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[14] Associate Prof., San. Eng., School of Civ. Eng., Purdue Univ., La Fayette, Ind.

One other matter that deserves further emphasis is the so-called "transporting power of the stream." In a recent article,[18] Mr. W. M. Griffith considers this matter at length and gives a criterion for the silt transporting power of a given stream as $\frac{v}{d^{0.57}}$, in which v is the mean velocity and d the mean depth of the stream. Of course, one cannot be sure that this criterion is valid for the conditions met in a grit chamber since it was developed empirically for different conditions and it may not be at all applicable to a small scale model. Nevertheless, its consideration may throw some light upon the behavior of grit chambers. It would not be surprising to find some kind of an inverse relation between the percentage removal of sediment in a settling basin and the "transporting power" of the stream in the basin if, as in the case of these models, the values of $\frac{a}{t}$, as required by Hazen's theory of sedimentation,[19] are the same in all cases. (The letter symbol, a, is the mean time that a particle is in the basin, and t is the time theoretically required for the particle to settle from top to bottom of the basin.)

The following values of this function are determined for the various chambers considered in this study:

Prototype, $1 \div 16^{0.57} = 0.206$
Smallest scale model, $0.258 \div 1.067^{0.57} = 0.247$
Grand Rapids model, $0.9 \div 6^{0.57} = 0.324$
Dearborn model, $0.9 \div 3^{0.57} = 0.480$

It should be noted that no study has been made of the concentration of sand in the various tests. Without considering such effects these calculations indicate that the foregoing order of enumeration should also be the order of magnitude of the percentage removal of sediment in the basins. There is thus a certain measure of qualitative agreement between the foregoing statement and the observed percentage removal of sand in the model basins—the smallest model removed the greatest percentage of sand even when the sand used was half as large as in the larger models. Whether or not there is any value in this consideration the author can decide.

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[15] "Transportation of Sediment in Suspension," by Lorenz G. Straub, *Civil Engineering*, May, 1936, p. 321.

[16] "Study of Sewage Settling Tank Design," by C. H. Capen, *Engineering News-Record*, November 24, 1927, Vol. 99, p. 833.

[17] *Engineering News-Record*, February 16, 1928, Vol. 100, p. 290.

[18] "A Theory of Silt Transportation," by W. M. Griffith, *Proceedings*, Am. Soc. C. E., May, 1938, p. 859.

[19] "On Sedimentation," by the late Allen Hazen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LIII, December, 1904, p. 45.

The writer would like to inquire if there are any data available showing the possible effect, if any, of the quantity of sand added upon the observed percentage removal for any given basin.

In conclusion, the writer would like to point to the value of running a similar test on the percentage removal of 0.2-mm sand in the prototype when built. Since the two sets of models used were designed on different principles, a final check on the prototype should help to answer such questions as these: Should sedimentation models, designed for identical values of $\frac{a}{t}$, have the same Froude number as the prototypes, or should, instead, the Reynold's number be made as nearly the same as possible as in the prototypes? Should the "transporting power," as defined herein, receive attention in the design of such models?

George E. Hubbell,[20] Assoc. M. Am. Soc. C. E. (by letter).—At this time the writer has little further to offer on the Detroit grit chamber model tests. Only tests of the finished structure itself will suitably close this discussion. However, there appears to have been some misunderstanding of the writer's conclusions. Professor Ridenour's interpretation of the data leads him to believe that shallow grit chambers are more efficient than deep ones. In Table 6, the comparison of removal on the basis of percentage of total sand is questionable since the actual pounds of the various sizes used in the two tests differ widely. Furthermore, it would appear that the critical size at which the $\frac{1}{V_h}$-ratio equals the $\frac{d}{V_s}$-ratio is best indicated by the size of sand that is completely removed. For the Grand Rapids test, this is 0.295 mm to 0.417 mm; and for Dearborn it is 0.2 mm, the Dearborn $\frac{L}{d}$-ratio equaling 29.8. The equivalent length for Grand Rapids, therefore, would be 179 ft, and the tests indicate a critical size of 0.2 mm, as would be expected if the relationship $\frac{1}{V_n} = \frac{d}{V_s}$ is substantially correct. With regard to the average size of sand removed in the equivalent section of the two chambers, the different character of the sands involved should be considered, the Grand Rapids test sand having an over-all average size of 0.57 mm as compared to 0.42 mm for the Dearborn sand. The writer does not conclude that slightly greater removal efficiency can be expected from the deeper tank, but rather that the same degree of efficiency can be obtained with deep chambers properly proportioned.

Professor Camp has developed, in a masterly manner, the theoretical grit-chamber efficiency to be expected, based on the laws governing sedimentation in quiescent water, and has shown that the efficiencies actually obtained in the Grand Rapids, Dearborn, and model tests, and estimated for the Detroit grit chamber, all in flowing water, are about 10% less than those theoretically expected. These results appear to indicate a reasonably approximate measure of the effects of cross currents and unequal distribution of velocities which prevail in grit chambers under the usual operating conditions. It was assumed arbitrarily that a prototype velocity of 5 ft per sec would remove scum and grease, and no conclusion was reached from the model test other than that, if 1.29 ft per sec were adequate in the model, the higher velocity would be adequate in the prototype.

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[20] Cons. Engr. (Hubbell, Roth & Clark, Inc.), Detroit, Mich.

Professor Howland has raised some interesting questions with regard to the relationship between the transporting power of flowing water and grit-chamber design. Considerable information on the transporting power of streams is available, but it is difficult to apply it directly to grit-chamber design. The average velocity in the model was 0.26 ft per sec, and the sand used had a diameter of approximately 0.1 mm. Competent velocity computations would tend to indicate that sand of this size is critical and would be transported. Referring to the prototype, it would appear from similar computations that a sand, for example, of 0.35 mm would not settle at a velocity of 1.0 ft per sec since the competent velocity for such a particle size is variously estimated at from 0.5 to 0.7 ft per sec. Furthermore, the writer believes that any general expression that does not take cognizance of particle size is, of necessity, limited in usefulness, because there can be no question that, with reference to sedimentation, settling velocities, particle sizes, and tank velocities are intimately related. In so far as the writer knows, no data are available on the effect of concentration on removals.

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STRUCTURAL BEHAVIOR OF BATTLE-DECK FLOOR SYSTEMS

By Inge Lyse,[1] M. Am. Soc. C. E., and Ingvald E. Madsen,[2] Jun. Am. Soc. C. E.

With Discussion by Messrs. H. N. Hill and R. L. Moore, Jonathan Jones, and Inge Lyse and Ingvald E. Madsen.

Synopsis

Results of two years of investigating the behavior of battle-deck flooring are reported in this paper. Four one-third sized models and one full-sized floor panel, designed on the basis of the results obtained on the preliminary models, were tested under the action of a concentrated wheel load, such as the specifications recommended for H-20 loading. The battle-deck flooring acted as an integral unit distributing the wheel load over various stringers, with the amount of load taken by the several stringers depending on their spacing. The plate acted with the stringer so as to form a T-beam which, if taken into account in the design, might result in an economy of 10 to 15 per cent. The width of plate contributing to the T-beam action was also found to depend on the stringer spacing. When the stringers were coped in on the floor-beams, partial fixation resulted with further economy in design. The models were loaded with dead weights, and the full-sized floor panel was tested by means of a jack and spring device. The test results gave a basis on which to formulate a rational design method for battle-deck floor systems.

Introduction

Experiments were made on four one-third sized models, based on a prototype bridge floor consisting of two panels 20 ft long and 10 ft wide. The first model represented a floor with a $\frac{3}{8}$-in. plate on stringers, spaced at 24 in. The tests showed that such a floor was not capable of supporting an H-20 loading. Consequently, the second model had stringers welded between those of the first model, making the prototype a floor with a $\frac{3}{8}$-in. plate on stringers, spaced

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Note.—Published in January, 1938, *Proceedings*.

[1] Prof. of Reinforced Concrete and Solid Bridges, Inst. of Technology, Trondheim, Norway.

[2] Associate in Civ. Eng., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

12 in., center to center. This model proved adequate in supporting the load, and the next step was to determine the more economical design. Comparative cost data indicated that although a widening of the stringer spacing might increase the weight of the floor slightly, the decrease in welding cost would more than offset the gain in weight. Thus, for the third floor model, a prototype was selected which consisted of a $\frac{3}{4}$-in. plate laid on stringers spaced at 30 in., and for the fourth model, a $\frac{9}{16}$-in. plate on stringers spaced at 24 in.

The results of the model tests showed that the floors behaved according to certain relationships. Design methods and procedure, as established from these results, served for the construction of the full-sized floor panel. The agreement between the design stresses and the measured stresses in this full-sized floor indicated a check on the design assumptions. The full-sized floor consisted of 12-in. standard I-beams, spaced on 26-in. centers, with a $\frac{11}{16}$-in. plate. The floor was 16 ft 9 in. in span and 9 ft 5 in. wide. This span length was adopted because the full-sized panel was tested as a simple beam with the stringers resting on the floor-beams, representing the distance between the points of contra-flexure of a 20-ft panel length with the stringers in the bridge floor coped into the beams.

THE PROBLEM

The problem of determining the behavior of battle-deck flooring may be divided broadly into two parts: (1) The stresses in the plate; and (2) the action of the stringer; that is, the amount of load carried by each stringer and the interaction of plate and stringers in resisting flexural stresses. The investigation was limited to the study of these conditions in relation to battle-deck flooring for highway bridges subjected to concentrated loads. For an adequate solution of the problem it was necessary to determine:

(*a*) The strength and deflection of battle-deck flooring under concentrated loads;

(*b*) The properties of the floor-plate in distributing the load over various stringers;

(*c*) The width of plate acting with the stringers as the compression flange of a T-beam;

(*d*) The length of plate under concentrated load affected by the load;

(*e*) The effect of changing the distance between the stringers;

(*f*) The degree of fixation at the ends of the stringers; and,

(*g*) The degree of fixation at the ends of the plate.

Little work has been done in this field, although some mathematical studies have been made on the width of plate acting in a T-beam under various loading conditions.[3] Even less is known of the distribution of load among the various stringers of the floor. For uniform loads over a floor, simple relationships obtain, but for concentrated wheel loads such as were used in this investigation, the problem becomes very difficult.

What actually happens to a stringer floor under load is readily visualized, but a strictly mathematical analysis is practically impossible because the floor is statically indeterminate to a high degree. As the load is applied the stringer beneath it deflects and the plate deflects with it, acting as a beam to transmit shear to the next stringer. This second stringer will also deflect and carry on the distribution. Theoretically, the distribution will go to all the stringers, although, practically, the effect may become so small after being distributed over three or four of them that any further distribution may be neglected. This action is quite different from that usually assumed in design, namely, that the load is spread equally over a certain definite number of stringers. Since the deflection of a beam varies as the cube of the span (that is, the spacing between the stringers) and inversely as the cube of its thickness, a floor with equal spacings between the stringers has a constant proportion of the shear transmitted between each stringer. In other words, the load on a floor is distributed throughout the floor in a geometrical ratio.

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[3] "Die Mittragende Breite," by Theodor von Kármán, M. Am. Soc. C. E., in Beiträge zur Technischen Mechanik und Technischen Physik, August Föppl Festschrift, Berlin, 1924.

The action of the plate under a concentrated load cannot be determined readily. The plate may be regarded as a rectangular plate supported along the edges, but the behavior is complicated by the fact that the deflection of the stringers varies along the plate, giving it an elastic support; and, in addition, the rotation of the stringers makes the degree of fixation in the plate an uncertain quantity.

In this investigation strain readings were taken before and after loading to determine the stresses. Slope readings were also taken along the stringers before and after loading. These readings were plotted, and the resulting curves were differentiated once to obtain the moments, and then a second time to obtain the shears. They were also integrated to obtain the deflections, and the deflections were measured directly for a check. From mechanics, the slope curve, differentiated twice, and multiplied by $E\,I$, gives the shear. Since the sums of all the shears of the stringers must be equal to the applied load, the value of the moment of inertia, I, was selected so that this became true for the experimental results. Knowing the value of I, it was easy to determine how much plate was acting with the stringer as a T-beam. The values of the shears on each stringer indicated how much load was carried by the various stringers. Furthermore, when the T-beam action of the stringer was known, the section modulus could be computed, and the stress could be determined from the moment curves which had been computed. These stresses were compared with the measured stresses, and their agreement served as a check on the work. The stresses in the plate were determined by means of Huggenberger tensometers.

Program

The four models were tested with the dead loads in various positions. In what is called a typical run, strain readings were taken along, and transversely between, the stringers, and level-bar readings were taken along the stringers both before and after the load was applied. The difference between the initial and final readings was the effect due to the applied load. The strain, multiplied by the modulus of elasticity, gave the stress at any point in the stringer; and the difference in the level-bar readings gave the slope due to the applied load. Such tests were made on the models with the load in all parts of the floor. However, the load in the middle of the stringer caused the larger stresses and, therefore, governed the design. In the full-sized panel, tests were made only with the load in the middle of the floor.

For these tests, the sum of all the slope readings along a stringer at any point would give the deflection of the stringer at that point. However, this was supplemented by tests in which Ames dials were placed along the stringers to measure the actual deflections. These results agreed quite closely with the integration of the slope readings and served as a check on them.

The plate stresses were obtained by measuring the strains in the plate with the tensometers. The load was placed in various parts of the plate, and tensometers were located at all points where any effect was noticed, thus recording the distribution of the plate stresses. In making some of the models, strain measurements were also taken on the stringers before and after welding to furnish an idea as to the severity and the effect of the welding stresses.

TESTING

The models were made of standard rolled-steel sections and plate. They were constructed on the basis that the model should be one-third size. However, since the prototype could not be reduced in all proportions without expensive machining, the stringers were machined so as to keep the clear spans of the plate in proportion, and they were designed so that the stresses in the model would be approximately the same as in the prototype. The ideal could not quite be attained since the smallest I-beams rolled gave about a 25% excess over the computed section modulus. The panels in the model were 80 in. by 40 in. The first two models consisted of two panels welded together. The last two models consisted of only one panel each, but they were welded in the same frame and their floor-beams consisted of channels. After the

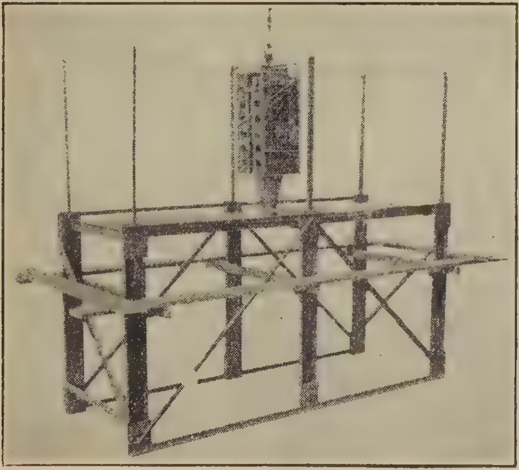

FIG. 1.—FLOOR MODEL AND LOAD

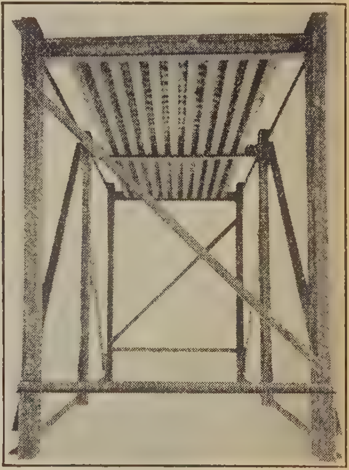

FIG. 2.—BOTTOM VIEW OF SECOND FLOOR MODEL

testing of the last two models was completed, a filler plate was inserted between the two adjacent channels, and weld metal was deposited so as to make an I-beam and cause the panels to be continuous. A view of the first two models is shown in Figs. 1 and 2. All the models had the same type of stringers, but the stringer spacing and the plate thickness varied. The plate in the first two models was steel metal sheeting of a low strength and yield point. This type was selected because the uniformity of thickness which sheeting possesses was an important factor in the $\frac{1}{8}$-in. plate thickness used in these models. The $\frac{1}{4}$-in. and $\frac{3}{16}$-in. plates in the third and fourth models were of the regular structural grade of steel and passed the specifications.

Great care was necessary in welding the models in order to avoid warping, particularly in the first two where the plate was so thin. They were fabricated by first tacking the stringers on to the plate, after which the weld metal was deposited, alternating from one spot to another on the floor so as to minimize the heat and thus decrease the tendency to warp.

The models were loaded by a dead-weight loading rig which is shown in Fig. 1. The Bureau of Public Roads, United States Department of Agriculture, has approved [4] a loading area of 20 in. by 10 in. for the rear wheel of an H-20 loading. This assumes a tire 20 in. wide, with a pressure of 112 lb per sq in., giving 10 in. of longitudinal bearing.

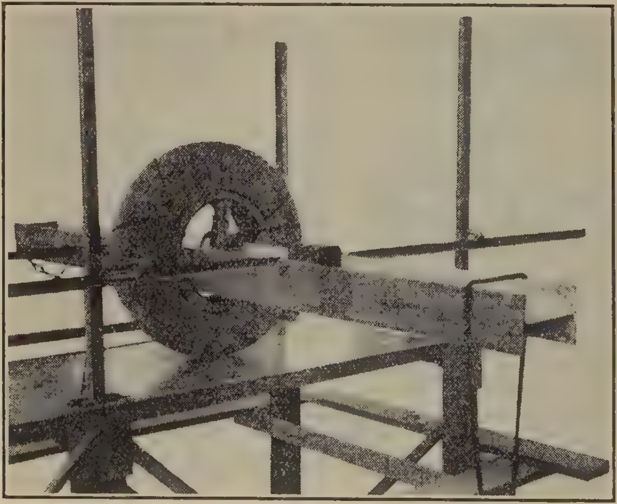

FIG. 3.—TIRE LOADING RIG

The loading rig consisted of a cast-iron block to which a frame was fastened to carry the additional dead weight. The initial weight of the rig was 400 lb. The load was applied, in 50-lb increments, through a steel bearing plate which was one-third size, or $6\frac{2}{3}$ in. by $3\frac{1}{3}$ in. A piece of soft rubber, $1\frac{1}{2}$ in. thick, was placed under the bearing plate to keep the load constantly uniform as the plate deflected, and a piece of cellotex was placed under the rubber to keep the area in contact with the plate constant.

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[4] Specifications for Highway Bridges, Am. Assoc. of State Highway Officials, 1935, p. 173.

A load of 2 489 lb should cause the same stresses in the model as would the rear wheel of an H-20 truck in the full-sized panel. A load of 2 500 lb was used in the model tests.

To show that the loading rig gave a uniform load distribution on the plate, and to compare its action with that of a tire, comparative load tests were made.

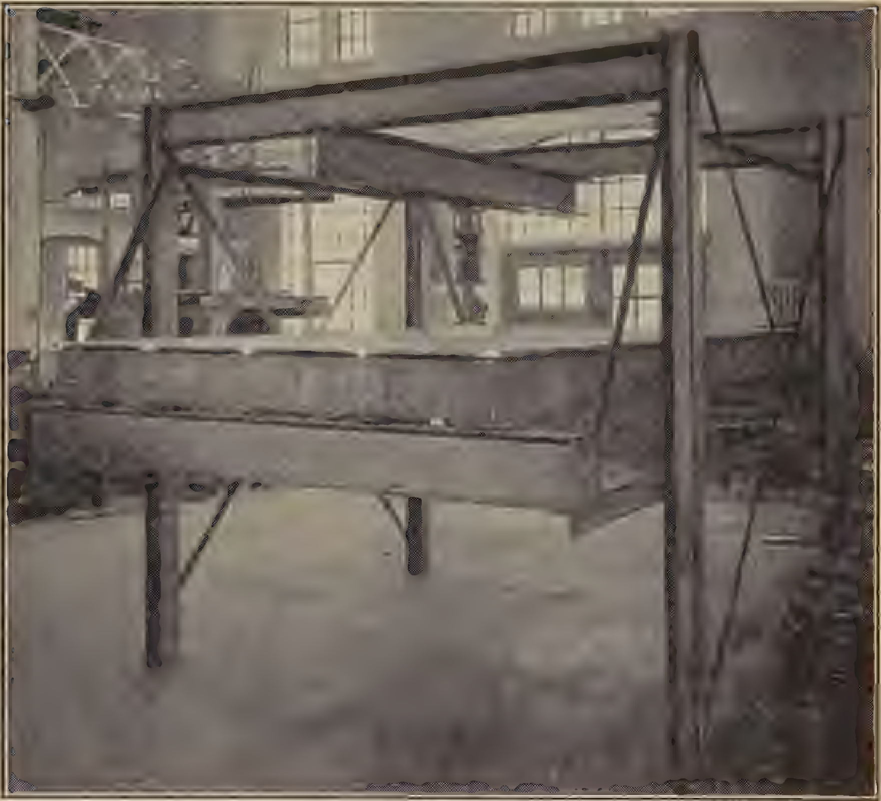

Fig. 4.—Full-Sized Floor

The floor was tested by means of the loading rig, and strain measurements were taken around the load at a number of critical points. Then the floor was tested by means of a tire, as shown in Fig. 3. The results of these tests were compared and found to be equal within the limits of experimental error, showing that the loading rig gave essentially the same results as an actual wheel load.

No trouble was experienced with warping in the full-sized floor panel, due to welding, so that it was unnecessary to adopt any special welding procedure. The plate was so thick that it dissipated the heat rather quickly. The structural grade of steel was used in the floor.

The floor was set in a frame as shown in Fig. 4. It was tested by jacks such as that shown in Fig. 5, the load being measured by the deflection of calibrated springs. It was possible to ascertain the load on the floor by this method to within 1 per cent. In order that the pressure on the floor be kept uniform as the plate deflected, the deflection of the plank on which the spring set-up was placed, was computed, and the plank was made of such a thickness that its deflection would be approximately equal to that of the plate. The deflection of the plank was computed on the basis of a beam on a yielding foundation under uniform pressure.

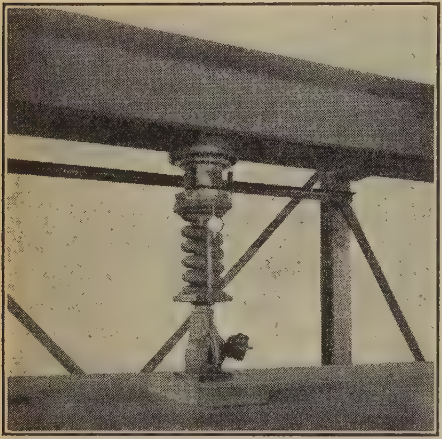

Fig. 5.—Jack Used on Full-Sized Floor

The models were held in a frame consisting of vertical posts made of 8-in. channels braced with angles (see Figs. 1 and 2). The floor-beams of the models were welded to the vertical channels to simulate the connections of beams to hangers in an actual bridge construction. The full-sized panel was held in a truss made of beams and angles, the details of which are shown in Fig. 4.

The strains in the stringers were measured by a fulcrum-type Whittemore strain-gage, equipped with a 0.0001-in. Ames dial. It is accurate to about 600 lb per sq in., as a tolerance of 0.0002 was allowed in repeating a reading. Since temperature changes cause strains in a structure that would be measured by the gage, the observations were made when these changes were at a minimum. Usually, therefore, temperature could be neglected, but when variations occurred, corrections were applied to the strains from observations on mild steel standards.

The tensometers which were used in the plate tests had gage lengths of 1 in. and 0.5 in. They were accurate to within about 500 lb per sq in., depending on the gage length.

Two level-bars were used. The one in the model tests had gage lengths varying from 1 in. to 6 in. However, the 5-in. gage was used almost entirely. It was fitted with a very sensitive bubble so that readings could be repeated to 0.0001 in., if the point hit the same spot on the floor. Hitting the exact spot was practically impossible, and, therefore, the accuracy was limited by the irregularities in the plate surface. For the full-sized model, the bubble was mounted so that the level-bar had a 15-in. gage length. This would make the instrument three times more accurate than it was on the 5-in. gage length, but in this case, again, the accuracy was limited by the irregularities in the plate surface. These irregularities were worse in the full-sized plate than in the models. In all cases the spots where the micrometer point of the level-bar rested were ground smooth and polished with an emery cloth.

TEST DATA AND RELATIONSHIPS

Many tests were made on all the models and on the full-sized panel. It is impossible to describe all these tests, however, and only the results of the significant and important runs will be given.

Tests on First Model of 24-Inch Stringer Spacing.—A series of nine runs was made on the first model. Some of the load positions included the quarter-points of the stringers, the center of the span, and the points adjacent to the floor-beams. As far as stringer stresses are concerned, the load in the center of the span caused the critical stresses.

In Series 16 (see Fig. 6), the load was placed between Stringers E and G, in the middle of the first panel. The results of the tests are shown in Tables 1 and 2. The two adjacent stringers, E and G, received 80% of the load, whereas Stringers C and I received about 10 per cent. The stringers that were away from the direct influence of the load carried over about 25% of their load to the adjacent stringer. The beams acted as if they were partly fixed, the average fixation factor on the left being about 15% and that on the right about 39 per cent. Due to this fact, the shear on the left was less than that on the right. These fixation factors were the ratios of the moments at the supports to those of a fixed-end beam.

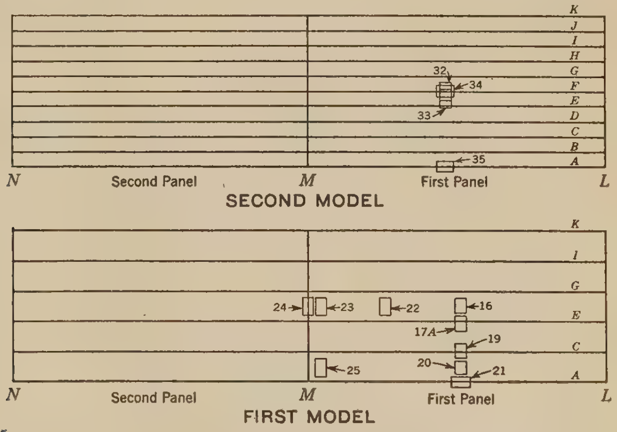

FIG. 6.—LOADING POSITIONS FOR FIRST AND SECOND MODELS

In order that the shears should equal the applied load, the moment of inertia of a stringer had to be 3.54 in.^{4}. This required 5.50 in. of plate acting in the compression flange. With these values the computed center moment was found from the slope curves, and the fiber stresses were computed. The measured stresses did not agree with those computed as well as expected, and the reason for this will be discussed subsequently.

The center deflection was computed from the formula:

$$y = \frac{W L^3}{192 E I}(-4 + 2 F_l + F_r) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

TABLE 1.—TEST RESULTS

Stringer*	SHEAR, IN POUNDS		Load, in pounds	Ratio to next stringer	Percentage of total load	PERCENTAGE FIXATION		Computed center moment, in thousands of inch-pounds‡	Ratio to next stringer	CENTER STRESS, IN THOUSANDS OF POUNDS PER SQUARE INCH		CENTER DEFLECTION, IN INCHES			Ratio to next stringer*
	Left	Right				Left, F_l	Right, F_r			Computed	Measured	Computed	From slope readings	Measured	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(*a*) SERIES 16, FIRST MODEL															
C . .	101	119	220	0.21	8.8	13	42	3.8	0.21	+2.3 −1.3	0 −1.5	0.0185	0.0178	. . .	0.23
E . .	507	555	1 062	1.00	42.6	18	34	18.5	1.00	+10.3 −6.0	+9.6 −2.3	0.0880	0.0769	. . .	1.00
G . .	470	488	958	1.00	38.3	15	26	17.2	1.00	+9.6 −5.6	+10.1 −2.2	0.0828	0.0854	. . .	1.00
I . .	122	138	260	0.27	10.4	11	55	4.3	0.25	+2.4 −1.4	+0.8 −1.8	0.0211	0.0205	. . .	0.24
T†.	1 200	1 300	2 500	. . .	100.1	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
(*b*) SERIES 17A, FIRST MODEL															
A . .	61	75	136	0.23	5.4	27	29	2.3	0.25	+1.1 −1.4	0 −2.5	0.0135	0.0129	. . .	0.30
C . .	265	315	580	0.47	23.2	38	42	9.3	0.43	+5.3 −3.4	+5.2 −2.6	0.0434	0.0426	. . .	0.41
E . .	538	699	1 237	1.00	49.5	11	35	21.6	1.00	+12.2 −7.9	+15.4 −4.0	0.1120	0.1075	. . .	1.00
G . .	201	241	442	0.38	17.7	15	27	7.9	0.37	+4.5 −2.9	+4.5 −3.1	0.0402	0.0386	. . .	0.36
I . .	47	58	105	0.24	4.2	23	43	1.7	0.22	+1.0 −0.6	+0.8 −0.9	0.0087	0.0083	. . .	0.22
T†.	1 112	1 388	2 500	. . .	100.0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
(*c*) SERIES 21, FIRST MODEL															
A . .	717	918	1 635	1.00	65.4	32	55	25.1	1.00	+15.5 −15.5	+20.0 −9.2	0.161	0.149	. . .	1.00
C . .	336	350	686	0.42	27.4	33	36	11.1	0.44	+6.5 −5.4	+5.7 −5.4	0.062	0.061	. . .	0.41
E . .	89	86	175	0.25	7.0	51	57	2.5	0.23	+1.5 −1.2	0 −2.3	0.013	0.011	. . .	0.18
T†.	1 142	1 354	2 496	. . .	99.8	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

in which: W = wheel load; L = span length; E = modulus of elasticity; I = moment of inertia; F_l and F_r are the fixation factors at the left and right ends of the span, respectively. Equation (1) was derived from moment-area theorems and, although it is approximate, it is reasonably accurate, if the fixation factors do not differ by more than 50 per cent.

TABLE 1.—(*Continued*)

Stringer*	SHEAR, IN POUNDS Left	 Right	Load, in pounds	Ratio to next stringer	Percentage of total load	PERCENTAGE FIXATION Left, F_l	 Right, F_r	Computed center moment, in thousands of inch-pounds‡	Ratio to next stringer	CENTER STRESS, IN THOUSANDS OF POUNDS PER SQUARE INCH Computed	 Measured	CENTER DEFLECTION, IN INCHES Computed	 From slope readings	 Measured	Ratio to next stringer*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
							(*d*) SERIES 32, SECOND MODEL								
C..	44	52	96	0.37	3.8	6	31	1.7	0.39	+1.0 −0.7	+0.5 −1.2	0.0092	0.0089	. . .	0.41
D..	129	134	263	0.50	10.5	23	35	4.5	0.45	+2.6 −1.7	+1.3 −2.2	0.0226	0.0218	. . .	0.44
E..	263	258	521	0.68	20.8	7	23	10.1	0.77	+5.7 −3.8	+6.3 −2.4	0.0508	0.0500	. . .	0.76
F..	352	416	768	1.00	30.7	19	37	13.1	1.00	+7.5 −5.0	+9.8 −3.0	0.0672	0.0659	. . .	1.00
G..	233	238	471	0.61	18.8	10	25	8.6	0.65	+4.9 −3.2	+6.4 −2.5	0.0440	0.0440	. . .	0.67
H..	109	124	233	0.50	9.4	18	43	3.9	0.46	+2.2 −1.5	+2.5 −1.5	0.0201	0.0190	. . .	0.43
I ..	49	49	98	0.42	3.9	0	40	1.8	0.45	+1.0 −0.7	+0.3 −1.0	0.0095	0.0090	. . .	0.47
T†.	1 179	1 271	2 450	. . .	97.9	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
							(*e*) SERIES 6, THIRD MODEL								
A..	17	11	28	0.06	1.1	−16	−5	0.6	0.06	+0.3 −0.2	+0.5 −0.9	0.0028	0.0037	0.0045	0.08
B..	220	234	454	0.31	18.5	−38	−27	10.4	0.39	+5.4 −2.3	+5.8 −2.0	0.0458	0.0450	0.0435	0.48
C..	735	722	1 457	1.00	59.4	+27	+24	25.4	1.00	+13.1 −5.5	+14.2 −2.8	0.0944	0.0938	0.0935	1.00
D..	234	240	474	0.32	19.3	−47	−36	11.4	0.45	+5.9 −2.5	+6.6 −2.6	0.0495	0.0497	0.0490	0.53
E..	28	15	43	0.09	1.7	+5	−23	1.2	0.10	+0.6 −0.3	+1.0 −1.2	0.0037	0.0038	0.0045	0.07
T†.	1 234	1 222	2 456	. . .	100.0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

The deflection was obtained by adding the areas under the slope curve. The agreement between the observed and measured deflections served as a check on the accuracy of the differentiation and the computations.

In Table 1(*b*) are shown the results of Series 17A on the first model. In this run, the load was placed directly on Stringer *E* (see Fig. 6), at its center. However, the loading area was so wide that the bearing block overlapped the plate and some of the load was transferred directly through the plate to Stringers *C* and *G*. The ratio between the shear carried from one stringer to another

TABLE 1.—(*Continued*)

Stringer*	Shear, in Pounds		Load, in pounds	Ratio to next stringer	Percentage of total load	Percentage Fixation		Computed center moment, in thousands of inch-pounds‡	Ratio to nex stringer	Center Stress, in Thousands of Pounds per Square Inch		Center Deflection, in Inches			Ratio to next stringer*
	Left	Right				Left, F_l	Right, F_r			Computed	Measured	Computed	From slope readings	Measured	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
							(*f*) Series 19, Fourth Model								
A..	92	82	174	0.30	6.7	+16	+6	3.3	0.26	+1.8 −1.2	0 −2.0	0.016	0.017	0.012	0.30
B..	288	301	589	0.47	22.8	−16	−10	12.6	0.57	+6.7 −3.3	+6.0 −2.8	0.058	0.057	0.054	0.63
C..	625	625	1 250	1.00	48.4	+23	+24	22.1	1.00	+11.9 −5.7	+12.8 −2.8	0.091	0.091	0.091	1.00
D..	235	214	449	0.36	17.4	−4	−13	9.3	0.42	+5.0 −2.4	+4.4 −1.8	0.042	0.042	0.043	0.46
E..	63	63	126	0.28	4.9	+7	+7	2.4	0.26	+1.3 −0.6	0 −0.7	0.011	0.011	0.010	0.26
T†.	1 303	1 285	2 588	...	100.2	...	...	...	...	...	...	...	...	...	...
							(*g*) Series 31, Full-Sized Floor								
A..	...	...	405	0.08	1.6	...	...	16.8	0.07	+0.4 −0.2	0 −0.5	0.006	0.006	0.009	0.09
B..	...	...	4 850	0.37	20.8	...	...	250.0	0.41	+5.6 −1.8	+5.4 −1.6	0.066	0.070	0.075	0.42
C..	...	...	13 120	1.00	56.2	...	...	610.0	1.00	+13.6 −4.4	+13.9 −2.1	0.179	0.165	0.170	1.00
D..	...	...	4 820	0.37	20.6	...	...	252.0	0.41	+5.6 −1.8	+5.2 −1.3	0.066	0.070	0.080	0.42
E..	...	...	199	0.04	0.8	...	...	12.3	0.05	+0.3 −0.1	0 −0.3	0.003	0.004	0.003	0.06
T†.	...	...	23 394	...	100.0	...	...	...	...	...	...	...	...	...	...

* See Fig. 6. † *T* = total. ‡ Values actually computed to nearest 10 lb; recorded as shown to conserve space.

in the direction away from the load is 0.24. The average value of F_l was 23% and that of F_r, 35 per cent. The value required for the moment of inertia of the stringers to make the shear equal to the load was 3.35 in.^{4} for the interior stringers, and 2.84 in.^{4} for the exterior stringer, *A* (see Table 2). These values for *I* were developed as follows: First, the shears were computed using a value of *I* corresponding to full T-beam action; that is, using 8 in. of plate for the top part of the T in the interior stringers, and 4 in. for the exterior stringers. This made the value of the shears too large. Next, the values for the moment of inertia were multiplied by a factor so that the shears would equal the load,

TABLE 2.—PROPERTIES OF SECTIONS USED IN COMPUTATIONS FOR TABLE 1

Description		SERIES 17A, FIRST MODEL		SERIES 21, FIRST MODEL			SERIES 6, THIRD MODEL			SERIES 19, FOURTH MODEL		SERIES 31, FULL-SIZED FLOOR	
	Series 16, first model	Interior stringers	Exterior s ringer, *A*	Interior stringers	Exterior stringer, *A*	Series 32, second model	Stringer *A*	Stringers *B*, *C*, and *D*	Stringer *E*	Stringer *A*	Stringers *B*, *C*, *D*, and *E*	Stringers *A* and *E*	Stringers *B*, *C*, and *D*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
T-beam action*...	5.5	4.25	$1\frac{1}{8}$	2.0	0.5	4.0	3.25	6.0	$4\frac{5}{8}$	3.25	5.75	12.0	17.5
I†.............	3.54	3.35	2.84	2.93	2.53	3.30	3.82	4.43	4.15	3.51	4.02	392.0	427.0
S‡.............	{1.79 {3.07	1.77 2.72	1.67 2.00	1.70 2.06	1.62 1.62	1.76 2.64	1.86 3.20	1.93 4.62	1.89 3.90	1.80 2.82	1.87 3.86	43.6 106.0	44.7 137.0

* Inches of plate in T-beam action. † Moment of inertia = 1 in.^{4} ‡ Section modulus = S in.^{3}

and then the width of plate required for this value of the moment of inertia was computed.

The left reactions were again found to be less than the right, as would be expected, since the left fixation factor, F_l, was the smaller. It is seen that Stringer *E*, directly beneath the load, received fully 50% of the load.

In Series 19, the load was placed on Stringer *C*, which was next to the exterior stringer. Consequently, the load could not be distributed so widely. In this case, the loaded Stringer *C* carried 55% of the load, or 5% more than in the former series.

The worst loading condition for the floor occurred in Series 21, Table 1(*c*), when the load was placed directly over Stringer *A*, which thus received 65% of the load. However, this loading would not be attained in an actual bridge, since, in order to place the wheel directly over an exterior stringer, a considerable part of the wheel would project beyond the floor. When the load was placed as close to the exterior stringer as is practical, it took about 50% of the load, and its design would be about the same as that of an interior stringer.

Referring to the stringer stresses of Series 21, the measured stress gave values that would require much more plate in T-beam action than the total plate width between stringers. This shows that there were other stresses in the stringer besides those due to bending. The observed slopes of Stringer *A* during this test are shown in Fig. 7(*a*), and the corresponding stresses in Fig. 7(*b*).

The results of quarter-point loading on the floor were quite interesting. The length of plate in T-beam action was the same as that in the corresponding center of span loadings, but the shear transferred from one stringer to another was only 0.08 of the shear in the stringer, instead of the 0.25 which occurred in the former cases. This agrees with the fact that the shear transferred from one stringer to another depends on their relative deflection; and since the stringers deflected less at the quarter-point, the difference in deflection between two adjacent stringers was less. Carried to the extreme, when the load is at the end of the stringer, one stringer will take all the load and must be designed for it.

Supplementary tests were also made on this model. In Fig. 8 the deflection of Stringer *E* is plotted against the load, which was placed in a position corresponding to Series 16. It is seen that the load-deflection relation is a straight line (as would be expected), showing that the width of plate acting as a T-beam was constant and did not vary with the load. The center deflection diagram for the plate is also given in Fig. 8. The curve slopes upward to the left showing that the load required for equal increments of deflection increased with the increase in the load on the plate. In other words, catenary action helped to support the plate.

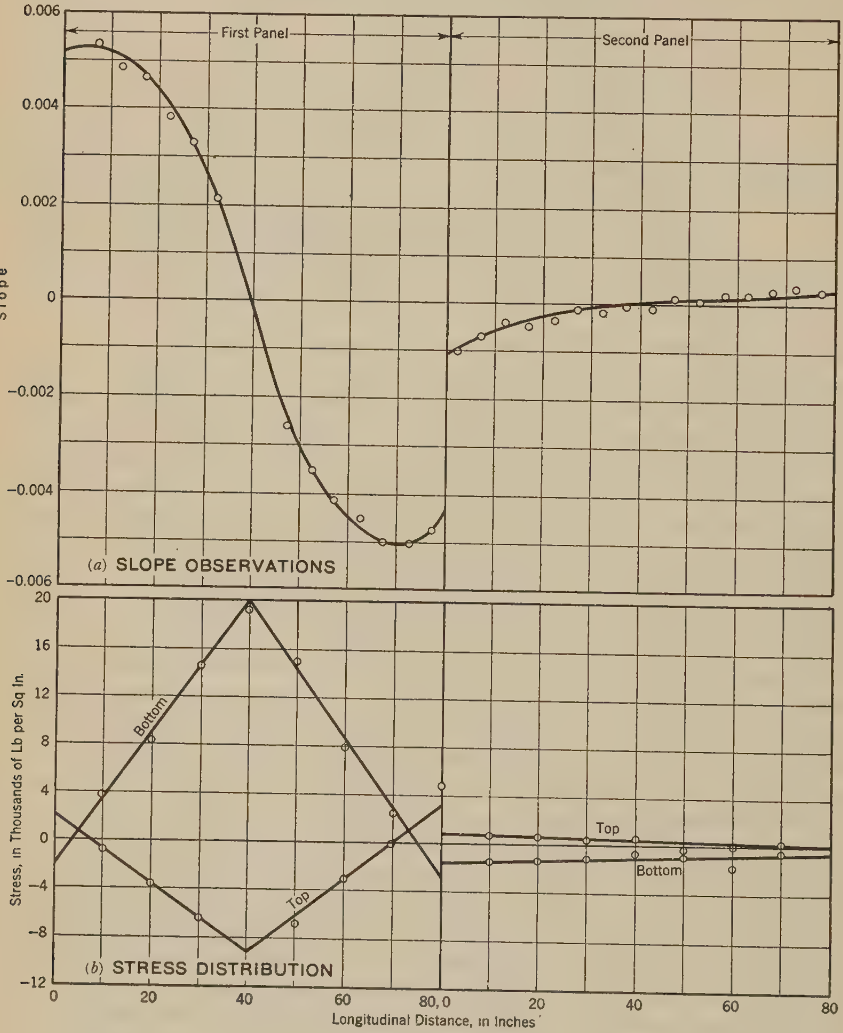

FIG. 7.—TESTS OF STRINGERS *A*

In Fig. 9, the deflections of the plate and Stringer *E* are plotted against the longitudinal axis of the stringer for a 2 500-lb load. The deflection curves for quarter-point loading are also given. In the stringers, there is a slight initial reverse curvature of the deflection curve at the ends of the span. This shows a slight degree of fixation. The curve for the plate, in sharp contrast to that of the stringers, slopes gradually and then changes quite sharply beneath the load.

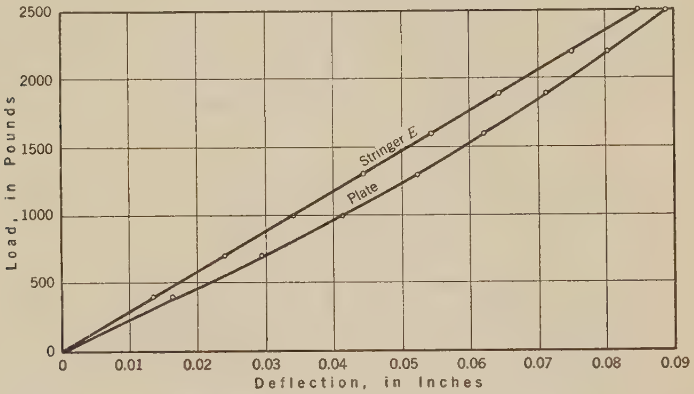

FIG. 8.—LOAD-DEFLECTION RELATIONS FOR CENTER STRINGER AND PLATE

In all these tests the stringers rotated considerably. However, the top of the stringer showed little or no rotation, which indicated that the stringers must have rotated about the plate. This rotation is at least partly due to the deflection of the floor plate. Evidently, the stringers were subjected to torsional forces and thus contributed to the load distribution.

Tests on Second Model of 12-Inch Stringer Spacing.—The second model was the same as the first, except that the stringers were spaced 4 in., center to center, instead of the former 8 in. The results were similar to those of the first test, but due to the decreased stringer spacing, the floor was stiffer so that the load was distributed over more stringers.

The results of Series 32 (see Fig. 6), in which the load was placed directly on top of Stringer *F*, are shown in Table 1(*d*). The width of the loaded area was so great, relatively, to the stringer spacing that when Stringer *F* was loaded, the area overlapped on the two other stringers, *E* and *G*. Full T-beam action was present in this series. Stringer *F* took the largest proportion of the load—about 30 per cent. The ratio of shear carried from one stringer to another, in the direction away from the load, was about 0.43, instead of the 0.25 found in the first model. The average fixation factor was about 12% at the left (F_l) and about 35% at the right (F_r).

The center moment, stress, and deflection were computed for the second model in the same manner as for the first. The slope readings checked the deflection. However, the ratios between the various stringers did not stay quite as constant for the shear, moment, and deflections, as they did in the first model. This shows that the slope curves were not exact second-degree curves, although they were so assumed, and, consequently, a slight error occurred in the differentiation and integration. The load on the other sections of the floor gave results similar in nature to those shown for Series 32.

After the regular runs had been made on the second model an attempt was made to test it to destruction. It was loaded by means of a 20-in. I-beam extended out as a cantilever from an 800 000-lb testing machine. It was first loaded to 15 710 lb, which is more than six times the design load, at which time the deflection under the load was 0.531 in. The load was released to 4 700 lb and the deflection was 0.240 in., of which 0.118 in. was permanent set.

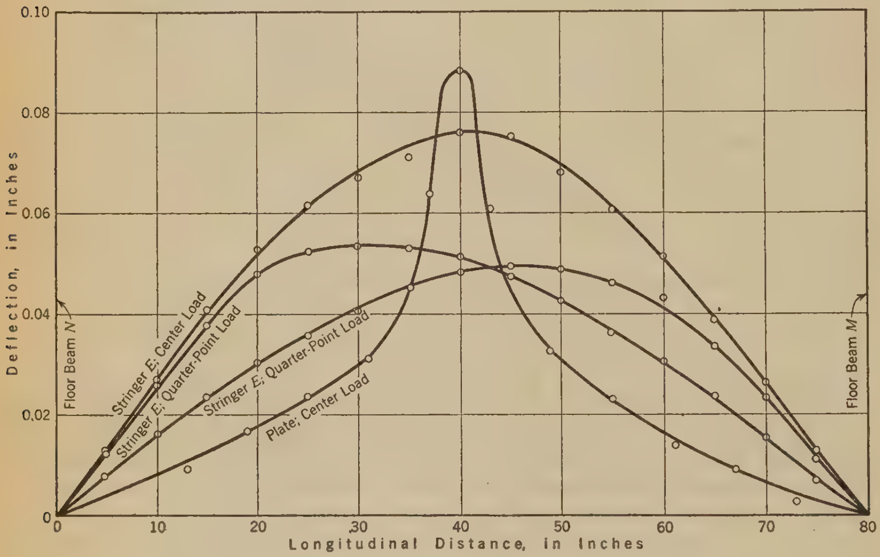

Fig. 9.—Load-Deflection Relations for Stringer *E* and Plate

The model was loaded again to 28 510 lb, when the testing had to be discontinued due to the incipient yielding of the loading beam and the bowing of the vertical legs of the frame holding the floor. At this load, the total deflection was about 1 in. The stringer under the load had yielded and thrown much of the load on to the two adjacent stringers, which were also beginning to yield. Upon removal of the load, a permanent set of about 0.5 in. was observed.

There was no sign of yielding in the plate, and the only sign of failure was a scaling of the whitewash in one of the welds holding the floor-beam in the supporting frame.

Tests on Third Model of 30-Inch Stringer Spacing.—The third model was based on a prototype with a $\frac{3}{4}$-in. plate welded to stringers spaced on 30-in. centers. This model, as well as the fourth, consisted of only one span, and since the stringers were coped into the floor-beams there was a partial degree of fixation. In these models, the plate was extended to the center of the exterior stringer, *A*, whereas it overlapped 2 in. from the center of the exterior stringer, *E*. This was done to see whether the overlapping plate was efficient in T-beam action.

The results of the tests on the third model were very similar to those of the first two. The results of the case in which the load was placed on top of Stringer *C* are given in Table 1(*e*). Stringer *C* took 59.4% of the load, which is a greater proportion than in the first two models. This was due to the greater stringer spacing. For a similar reason the width of plate in T-beam action was not as large as in the first two models. The fixation factor for the loaded stringer was small, as would be expected. For the other stringers the value of *F* is negative, denoting an applied moment at the end of the stringer. The reason it is negative is that, due to the partial fixation of the loaded stringer, the floor-beam rotates and imposes applied moments on the adjacent stringers.

A number of tests were made in a study of the plate stresses in the third and fourth models. Similar tests were made on the full-sized floor and since these were more complete and gave essentially the same results as those on the models, only the results on the full-sized floor plate are given in this paper. Full T-beam action was not present in these models. Accordingly, the compression in the top part of the T-beam would be expected to decrease away from the stringer. This decrease was found to be linear, as shown in Fig. 10, for the variation of the compression in the plate between Stringers *C* and *D* when the load was placed between them.

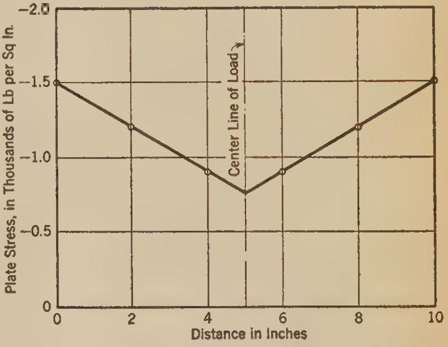

FIG. 10.—STRESS DISTRIBUTION IN FLOOR PLATE WHEN LOAD IS PLACED BETWEEN STRINGERS *C* AND *D*

Cross-bracing was welded between the stringers in one of the bays of this model. It consisted of $\frac{1}{4}$-in. by 2-in. plates welded to the stringers and the plate, at 12-in. intervals. Although the secondary bracing was spaced this closely, the plate stresses were reduced very little.

Tests on Fourth Model of 24-Inch Stringer Spacing.—The fourth model was based on a prototype with a $\frac{9}{16}$-in. plate welded to stringers at 24-in. centers. In this model, an attempt was made to evaluate the welding stresses. Strain readings were taken on the stringers and on the part of the plate over the stringers, before and after the floor was welded together. The welding stresses were greatest in the plate because most of the welding was done there. The longitudinal welding stresses in the plate varied from about 4 000 lb per sq in. over the exterior, to 9 000 lb per sq in. compression over the interior, stringers. The stress along the bottom of the stringers was about 2 000 lb per sq in. in tension. The transverse welding stress in the plate between the stringers varied from 15 000 lb per sq in. to 20 000 lb per sq in. in compression. These stresses did not seem to affect the test results.

The results of the various tests of the fourth model were similar to those of the previous models. Table 1(*f*) gives the results for the case in which Stringer *C* was loaded. This stringer is seen to take 48% of the load.

After the regular runs had been made on the third and fourth models, they were welded together to form a two-panel floor. A run was taken similar to that in the third model for which the results are given in Table 1(*e*), and the results for the two-panel model were very similar to those for the third model except that the fixation factor at the intermediate floor-beam was increased to 50 per cent. The stresses and deflections in the floor-beams were found to be smaller in accordance with the increased fixation factor.

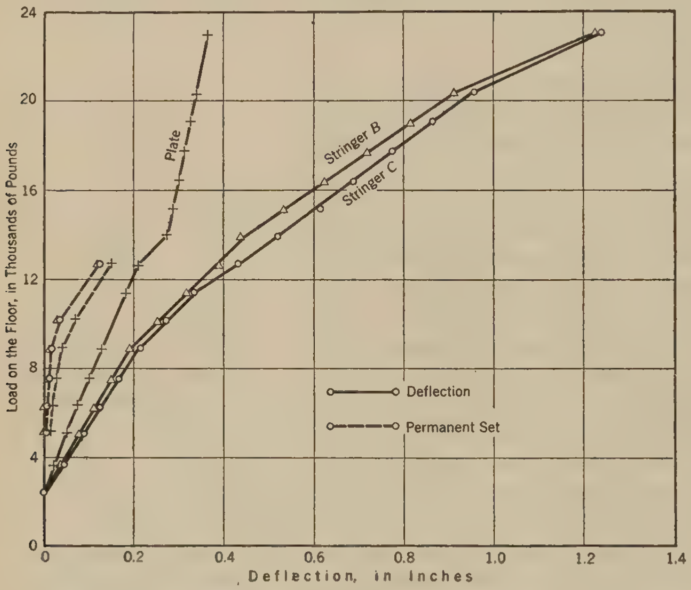

Fig. 11.—Deflection and Permanent Set of Plate and Stringers *B* and *C*. Breaking Test

Both the third and fourth models were tested to destruction. They were loaded in a manner similar to that used for the second model; that is, a cantilever load from the big testing machine. The third model was loaded up to 26 744 lb and the fourth, to 27 911 lb. In each case, the testing had to be discontinued due to incipient yielding of the loading beam. At these maximum loads, which were about eleven times the design load, large deflections were present although nothing broke. The large reserve strength is due to the fact that as one stringer yields, the increase in load is taken by the adjacent stringers while the original stringer still holds its yield-point load. This process continues until all stringers have yielded.

In the third model the load was placed on the plate between Stringers *B* and *C* (Fig. 6). Fig. 11 shows a diagram of the load-deflection and permanent-set curves of the stringers and the plate. At the maximum load, the deflection was 2 in. and the permanent set was 1 in.

The fourth model was tested with the load directly over Stringer *C*. The results were about the same as for the third model. A diagram of the deflection of the centers of the stringers under various load increments is shown in Fig. 12.

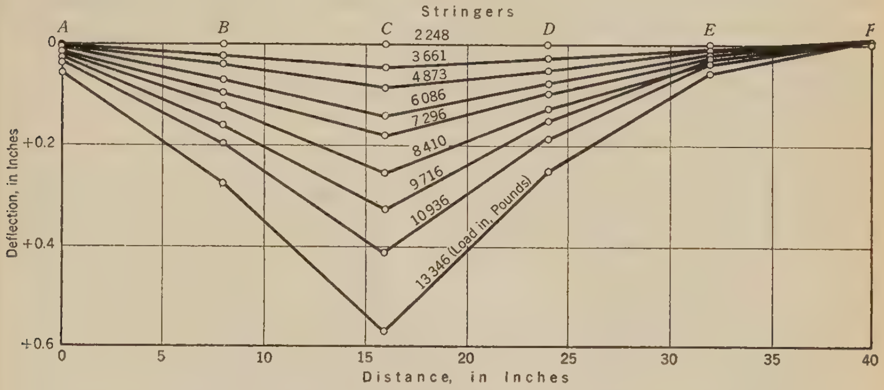

FIG. 12.—DEFLECTION CURVES FOR CENTER OF STRINGERS. BREAKING TEST, MODEL NO. 4

Tests of Full-Sized Floor Panel.—The full-sized floor panel was built according to a design procedure determined by a study of the results of the model tests. The panel was 16.75 ft long, center to center bearing, and 9.42 ft wide. This floor was built so that it would act as a test on the design procedure, and serve as a check on the model tests. How well it did this is illustrated in the comparison of the design values with the measured values of stress and deflection for the critical sections of the floor, shown in Table 3. The check is very good considering that the properties of structural sections may vary as much as 5 per cent. The tests on this full-sized floor were similar to those of the models. Table 3 gives the results when the load was on top of Stringer *C*.

TABLE 3.—TEST OF FULL-SIZED FLOOR

Description (1)	Maximum stress in plate, in pounds per square inch (2)	Maximum deflection of plate, in inches (3)	Maximum stress in loaded stringer, in pounds per square inch (4)	Maximum deflection of loaded stringer, in inches (5)	Percentage of wheel load taken by loaded stringer (6)	Width of plate in T-beam action, in inches (7)	Weight of floor (stringers and plate), in pounds per square foot (8)
Design value.....	27 900	0.113	13 300	0.169	53	17.5	44.0
Measured value...	26 000	0.111	13 900	0.170	56	17.5	...

Because of the large dimensions of the full-sized floor, it was possible to obtain more data concerning the stresses in the plate than had been possible in the models. A large number of tensometer readings were taken in order to obtain the stress distribution in the plate. Fig. 13(*a*) shows the distribution of the transverse stress in the plate along the length of the floor. This stress is the greatest in the plate, since it lies along the short span between the stringers. It is seen to have a peak of 26 000 lb per sq in. at the center of the load, rapidly decreasing asymptotically along the plate. The compression in the top of the plate is seen to be similar to the tension in the bottom. Complete readings could not be obtained for the compression side since the load was in the way. The distribution of the transverse stress in the bottom of the plate, across the width of the floor, is shown in Fig. 13(*b*). In the loaded span, the stress in the plate was zero at the edges of the stringer flanges. The stress changed to compression along the flange, and rapidly dropped to zero at the next stringer.

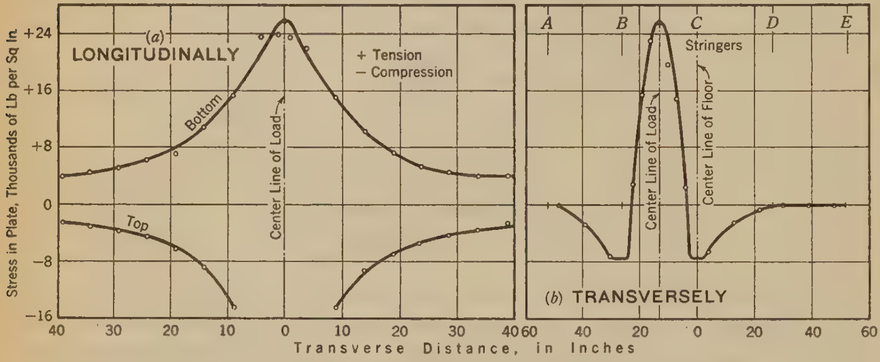

Fig. 13.—Transverse Stress in Plate of Full-Sized Floor

An over-load test was made on the full-sized floor. It was loaded up to 56 000 lb, or two and one-half times the design load. The deflection of the floor increased linearly with the increase in load. At a 50% over-load, there was an over-all permanent set under the plate of 0.005 in. At the maximum load of two and one-half times the design load the permanent set was 0.025 in. No signs of failure or yielding could be determined with the exception of a scaling of the whitewash near one of the welds, between the plate and the stringer. After the load was removed, the floor appeared just as perfect as ever, and one could never have detected by eye that it had been overloaded. The maximum deflection of the plate had been 0.269 in.

Discussion of Results

The results of these tests show that the battle-deck floors acted as an integral unit. The load was distributed from one stringer to another by means of the plate, which acted as a cantilever beam, in proportion to the relative deflection of the stringers. This distribution factor was a constant for a definite stringer spacing and plate thickness. In all cases, as shown by the tables, the carry-over factor was larger for the stringers close to the load than for those away from the load. Considering the case in which the load was directly over one stringer, the adjacent stringers took, not only the load carried over by their relative deflections, but also some of the load itself, as the wheel load was so wide that it overlapped the loaded stringer.

In some cases, the stresses in the stringer did not check the computed stresses, particularly for the smaller stringer spacings, because the computed stresses only included bending stresses. Since the stringers were welded to the floor-beams, direct tension could occur in addition to the bending moments. Four types of stresses are probably added to the simple bending stresses along the tension flange: First, due to the tension in the lower flange, its length is increased, imposing compression on the ends of the beam, and thus tending to reduce the flange tension; second, due to the deflection of the beam, the longitudinal axis shortens and causes tension along that axis; and, third, due to the rotation of the bottom flange, shortening takes place which causes tension along the gage line. Finally, these effects in any one stringer have a reaction on the floor-beam which, in turn, applies a couple and a tensional stress to the other stringers.

The effect of these stresses is greatest on the loaded stringer as it reduces the compression and increases the tension. This stringer has a great effect on the rotation of the floor-beam which, in turn, tends to offset the secondary stresses in the adjacent stringers. In the stringers a distance away from the load, the tension is reduced in some cases to zero, and the compression is increased. Table 1(*b*) illustrates this phenomenon.

The width of plate in T-beam action was found to increase as the stringer spacing decreased, whereas the load taken by a stringer increased with the increase in spacing. Both these results are logical and Fig. 14 presents the relationships obtained. These relationships are useful for the design of battle-deck floors.

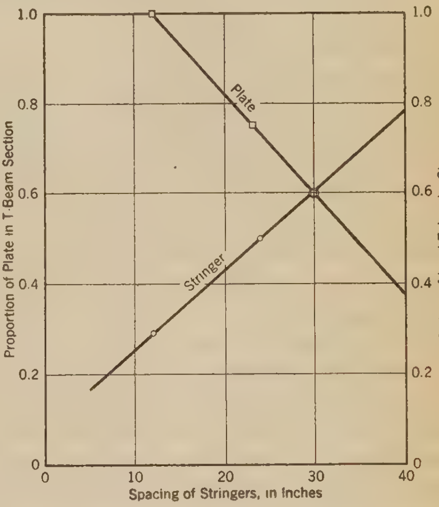

FIG. 14.—AMOUNT OF PLATE IN T-BEAM ACTION AND LOAD TAKEN BY STRINGER

A stringer needs to be designed only for the effect of one rear wheel load since the usual axle spacing on trucks is so large that the effect of one wheel is not carried over to the other. There seems to be no need of making the exterior stringer larger than the interior stringers since it is practically impossible for the center of the wheel to come over the center of the exterior stringers. Usually, the exterior stringer will not be stressed higher than the interior stringers if it is the same size. The stringer next to the exterior will take about 5% more of the load than any of the other interior stringers, because a full wheel load can rest on it and the exterior will not help support it as much as the interior stringers. Thus, it will be overstressed about 5%, if it is the same size as the other interior stringers.

When coped into the floor-beams of a single span, the stringers had a fixation factor of about 25 per cent. When the spans were continuous, the factor was as high as 50 per cent. A substantial saving in material can be effected if this partial fixation is taken into account in design. The foregoing statements apply to web-plate connections; the fixation factors for web-angle connections were slightly smaller.

In designing battle-deck floors the smaller the stringer spacing is made, the lighter will be the resultant floor. However, the increased welding cost of the lighter floors will probably make them uneconomical unless the importance of light weight is particularly great, such as in lift spans.

The over-load tests showed that the battle-deck floor had a large reserve strength and was practically impossible to break. The tests on the full-sized floor panel would indicate that, although the measured stress was fairly high, the plate thickness could be reduced to $\frac{5}{8}$ in., or $\frac{9}{16}$ in., and still be amply strong to take an H-20 load.

Most plates in battle-deck flooring have been designed on the assumption that the plate under the load acts as a fixed-end beam. No account has been taken of the longitudinal distribution of the load. Fig. 13 shows the distribution of the plate stress. The longitudinal distribution is seen to extend over about four times the clear span of the plate, or about 84 in.

The point of contra-flexure of the plate fell close to the edge of the stringer flange. One of two assumptions may explain this: The first is that the plate acts as a simple span between the flange of stringers; and, the second is that the plate acts as a fixed beam with the point of contra-flexure at the edge of the stringer flange. In either case the result is the same, and the first assumption is the easier to use in computation. The second assumption is probably closer to what actually takes place since the plate forms a fixed beam of varying cross-section, the depth between the flanges being the depth of the plate itself, and the depth over the stringer flanges being the thickness of the plate plus the flange. The increased depth of beam at the stringers decreased the stress over the stringer far below what would be expected. The tension stress was larger than the compression stress, probably because a catenary stress of about 1 000 lb per sq in. was present.

The curve in Fig. 13(*b*) shows that the longitudinal distribution of stress varied from a peak at the center of the load, decreasing asymptotically. Neglecting the small curve at the peak, this variation may be assumed to be parabolic. Thus, the computation of the stresses in the plate becomes very simple: First, the total moment which the plate must support is computed on the assumption that the plate is a simple beam between the edges of the stringer flanges. Next, the average stress in the plate is computed over the length of four times the clear span of the plate, since the load is distributed over that distance. For the average stress this gives:

$$s_{\text{ave}} = \frac{M}{4\,S\,L} \qquad \text{(2)}$$

in which M is the moment; L, the clear span; and S is the section modulus of the plate per inch of plate. The maximum stress in the plate will be three times the average since the distribution of stress is parabolic. This semi-empirical method of determining the plate stress was used in computing the stresses in the full-sized floor and was checked by the observed stresses. The values of the stresses in the one-third sized models checked even more closely than those for the full-sized floor. This method of determining plate stresses may be expected to give quite accurate results for one-way slabs under concentrated wheel loads.

Recommended Method of Design

The results of the four models and the full-sized floor panel indicated that battle-deck floors may be designed by the following procedure:

(1) When the stringer spacing is determined, obtain from Fig. 14 the load taken by T-beam action of one stringer and its contributing plate width. With this information, the moment on the stringer is easily found and a trial stringer is selected. The properties of the T-beam section can be determined as soon as the plate thickness is found.

(2) When the trial stringer section has been selected for the T-beam, the clear span between the stringers is known. The trial section can be determined quite closely in the first step since great changes in the top of the T-beam change the value of the section modulus only slightly.

(3) The required plate thickness, t, is determined by the formula:

$$s_{\max} = \frac{3\,M}{4\,S\,L} \qquad \text{(3)}$$

and is found directly by changing Equation (3) to,

$$t = 3\sqrt{\frac{M}{2\,s_{\max}\,L}} \qquad \text{(4)}$$

(4) Knowing the plate thickness, the section of the T-beam is determined, and the stresses in the stringer are computed. If the stresses are not satisfactory another section is selected.

(5) Design the stringer connections for the full load in shear.

(6) Partial fixation may be taken into account and resulting economies effected by using a fixation factor of 25% when the span is simple and 50% when it is continuous.

Wide-flanged beams, in general, will be economical since they reduce the clear span of the plate. However, the lightest wide-flanged beam may not meet the specification that the web must be at least $\frac{3}{8}$ in. in thickness. In general, lateral bracing should not be necessary for the floor when the stringers are coped in on the floor-beams.

When the plate and stringers are selected, the remainder of the floor is designed according to the usual methods. The welds between the plate and the stringers are designed for longitudinal shear. The resulting welds will also be strong enough to take care of the horizontal and catenary stresses in the plate.

SUMMARY

The results of the tests indicated that:

1.—Battle-deck flooring acts as an integral unit.

2.—Inherent welding stresses may be found in the plate but they do little harm under static load. Care in welding will minimize these stresses.

3.—A tire imposes an essentially uniform load over the area upon which it rests.

4.—The plate acts with the stringers to form a T-beam, reducing the stringer stress about 15 per cent.

5.—The load taken by a stringer and the width of plate acting with the stringer vary with the stringer spacing.

6.—The stringers distribute the load in proportion to their relative deflections; thus, the distribution is greatest when the load is in the center of the panel and decreases as the load approaches the floor-beams. The distribution factor varies with the thickness of the plate and the distance between the stringers.

7.—The plate acts as a simple beam between the edges of the stringers. The load is distributed longitudinally over a distance equal to four times the clear span. The distribution is parabolic, with the maximum stress three times the average stress.

ACKNOWLEDGMENTS

The American Institute of Steel Construction, through its Technical Research Committee, sponsored this investigation of the action of concentrated loads on battle-deck flooring, in order to obtain information for design purposes. A research fellowship supported by the Institute was established at the Fritz Engineering Laboratory of Lehigh University, at Bethlehem, Pa., and the investigation was conducted under the guidance of the Committee. Acknowledgment is due all members of this Committee for their active interest in the work and their advice and guidance. Acknowledgment is also due to the members of the Fabricating Division of the Bethlehem Steel Company for constructive criticism and valuable suggestions during the investigation, and to H. J. Bowles, Welding Supervisor, Bethlehem Steel Company, for advice and assistance in all the construction work, and to C. C. Keyser, Instructor at the Laboratory, for his assistance in the construction and testing. The full-sized floor panel was fabricated in the shops of the Bethlehem Steel Company.

DISCUSSION

H. N. Hill,[5] Assoc. M. Am. Soc. C. E., and R. L. Moore,[6] Esq. (by letter). —Having clearly recognized the essential elements of this complicated problem, the authors have proceeded logically toward an experimental study of each element. It seems to the writers, however, that certain phases of the experimental approach and the subsequent analysis of the data are open to question. The writers also feel that a word of caution should be offered against the general use of design methods empirically formulated from a limited number of tests without the benefit of a rational analysis of the problem.

The determination of the load distribution along and between stringers in a panel of battle-deck flooring, subjected to a concentrated load, is indeed a complex problem. Assuming that deflections are produced entirely by bending, the method of approach used by the authors (that of measuring slopes along the stringers and differentiating to obtain moments and shears) is theoretically sound. It is extremely sensitive, however, both to experimental inaccuracies and to slight variations in the manipulation of the slope data in obtaining the derivatives. From the nature of the level-bar readings, it was evidently not possible to obtain the slopes at the extreme ends of the stringers. As a result, the end moments and shears, which were the critical values in determining the end-fixation and load-distribution factors, were obtained from the derivatives of extrapolated curves. Fig. 7(*a*) suggests that the liberties taken in sketching in the slopes at the ends of the panels may have had considerable influence upon the accuracy of the results obtained.

That the bending moments and load-distribution factors determined in this manner are not altogether satisfactory is indicated by certain inconsistencies found between the loads carried by the different stringers and the corresponding center moments, as shown in Table 1. It would seem from the nature of the problem that the load on the stringers not carrying the wheel-load concentration would be distributed along the span according to the relative deflections of adjacent stringers. Table 1 shows, however, that for the full-sized floor the computed center moments for Stringers *B* and *D*, on either side of the loaded stringer, were greater than would have been obtained had the load computed for these stringers been concentrated at the center of the span.

Although the question of load distribution between stringers is of interest from the standpoint of analysis, the designer is usually concerned only with the maximum intensity of load and the corresponding maximum stresses that may be produced. In every case but one, given by the authors, the computed stresses in the bottom flange of the loaded stringers were less than the measured values, the difference in three cases being more than 20 per cent. As long as load-distribution factors are to be determined experimentally, it would seem that an attempt should be made to make them as consistent as possible with the maximum stresses observed.

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[6] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

Table 1 shows that in every case the computed stresses in the top flange of the loaded stringers (plate stresses, considering T-beam action) were about twice the measured values. Although these stress differences are probably not important from the standpoint of design, the writers believe that they may be attributed very definitely to the small widths of plate computed to be effective in T-beam action. Since the end shears were determined from the second derivatives of the slope curves, multiplied by $E\,I$, any errors in the slopes, of course, would be reflected in a greatly magnified manner in the I-values. Any tendency to assume greater negative moments at the ends of the stringers than were actually present, as appears to be the case at the left end of Stringer A in Fig. 7(a), would result in high values for the second derivative of the slope curve and correspondingly low values of $E\,I$.

The authors conclude that "the width of plate acting with the stringer varies with the stringer spacing." For certain conditions of loading this statement is undoubtedly true; but it should be borne in mind that the effective width is dependent to a much greater extent on other variables. Professor S. Timoshenko[7] has shown by mathematical analysis that for certain types of loading on a wide-flanged T-beam, in which the flange thickness is small compared to the depth of the beam, the effective width of flange is a function of the span length alone. In general, however, the effective width depends not only upon the span length and type of loading, but upon the plate thickness and the elements of the stringer section as well. It may be shown from an extension of Professor Timoshenko's analysis that the theoretical effective width of plate acting with a stringer in the full-sized floor, assuming a concentrated load at the center and a span of 16.75 ft, is about 45 in. Such a width is greater than the stringer spacing, but as long as the loads on all the stringers are not of the same magnitude, there seems to be no reason for placing an arbitrary limit on effective width.

TABLE 4.—SECTION ELEMENTS FOR STRINGER C OF FULL-SIZED FLOOR

Description	Width of Plate in T-Beam Action, in Inches		Percentage increase
	17.5*	45.0†	
Moment of inertia, in inches^4	427	509	19
Section modulus, in inches^3:			
Top	137	280	104
Bottom	44.7	46.8	5

* From authors' Table 2. † Computed by writers.

Table 4 gives section elements for Stringer C of the full-sized floor, assuming effective widths of plate in T-beam action of 17.5 in. and 45.0 in., as computed by the authors and writers, respectively. It is clear from the differences shown that a great increase in the effective width of the plate would have little effect on the calculated stresses in the bottom flange of stringers, but would decrease those at the top considerably.

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[7] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill, p. 156.

Using the value for computed bending moment, given by the authors in Table 1, and an effective width of 45 in., the calculated stress for the top of Stringer C of the full-sized panel would be 2 200 lb per sq in., as compared to a measured stress of 2 100 lb per sq in. The authors' calculated top flange stress for this stringer, using an effective width of 17.5 in., was 4 400 lb per sq in. The curve in Fig. 14, pertaining to the length of plate affected by T-beam action, can scarcely be considered accurate since it was based on values for effective width of plate that are so greatly in disagreement with the top flange stringer stresses, the only measured values which are sensitive to effective widths.

In computing the deflections shown in Table 1, the authors assumed the load on each stringer to be concentrated at the center of the span, despite the fact that the actual load distribution was presumably known from the slope measurements and the computed values of center moments given in the same table did not correspond to such a distribution. It is also apparent that shearing deformations were omitted from the values of computed deflections. The effect of shear on the deflections would not be very great, of course, producing an increase of only about 7% for the full-sized floor. The addition of this percentage, however, would produce a greater discrepancy between measured and computed values for the loaded stringer than that shown. The computed deflection of Stringer C, for instance, would be increased from 0.179 to 0.192 in., as compared to a measured value of 0.170 in. If an effective width of plate of 45 in. rather than 17.5 in. had been used, the computed deflection, including shear, would have been 0.163 in.

During the past seven years (1931–38) the writers have had occasion to make tests on several bridge-floor panels of the battle-deck type, and on numerous other panels in which the action was somewhat similar. All the panels tested were fabricated from high-strength aluminum alloy plates (or sheets) and structural shapes.

From measurements of the bearing areas of dual-tired wheels on large trucks, used to provide load in several field tests, the writers arrived at a laboratory loading to simulate H-20 plus 35% impact. In this loading a total load of 21 600 lb is distributed over two parallel bearing pads which are 8 in. wide and 11$\frac{3}{4}$ in. in the direction of traffic. These bearing pads are of felt, 1 in. thick, backed up by steel plates 1 in. thick, and are spaced 12 in., center to center, so that the total width is 20 in. It is believed that this type of loading more nearly simulates the true wheel load than the application of a uniform pressure over a continuous bearing area.

It is questionable whether Equation (3), recommended by the authors, is adequate for the design of battle-deck floor systems of proportions different from those tested. The writers have tested floor panels composed of plates $\frac{1}{2}$ in. and $\frac{7}{16}$ in. thick on 8-in. channel stringers, spaced 11 in. apart, loaded with a truck having dual rear wheels and carrying 35 000 lb on the rear axle. With the truck so placed that the tires on each of the rear wheels straddled a stringer, the plate stresses shown in Fig. 15 were measured. It will be noticed that the transverse stresses found under the inside tires were considerably higher than those measured under the outside tires. This was true to about the same degree for both thicknesses of plate. The test was made so that the panel with the $\frac{1}{2}$-in. plate was loaded by the right rear wheel of the truck, whereas the left rear wheel was placed on the panel with the $\frac{7}{16}$-in. plate. This unequal distribution of the wheel load between the inside and outside tires may have been influenced by a number of factors, such as the deflection of the rear axle under load, the air pressures in the tires, and the extent of wear on the tires. Whatever the reason, the fact remains that this unequal distribution was obtained for a typical case of a truck in service and the existence of such a condition should be recognized.

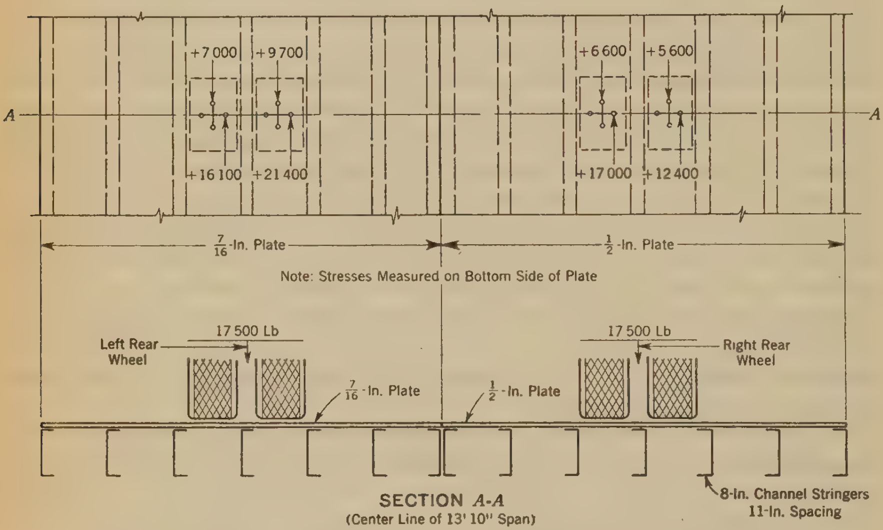
+7 000
+9 700
+6 600
+5 600
A
A
+16 100
+21 400
+17 000
+12 400
$\frac{7}{16}$-In. Plate
$\frac{1}{2}$-In. Plate
Note: Stresses Measured on Bottom Side of Plate
17 500 Lb
Left Rear Wheel
17 500 Lb
Right Rear Wheel
$\frac{7}{16}$-In. Plate
$\frac{1}{2}$-In. Plate
SECTION A-A
(Center Line of 13' 10" Span)
8-In. Channel Stringers
11-In. Spacing

FIG. 15.—MEASURED STRESSES IN PLATES OF BATTLE-DECK BRIDGE FLOOR

If the wheel loads in Fig. 15 are distributed between the tires in the same proportion as the ratio of the measured transverse stresses, it is found that the inside tires carried about 57% of the total, which is equivalent to a load of 10 000 lb on a bearing area of 8 in. by $11\frac{3}{4}$ in. According to Equation (3), this load would produce maximum transverse stresses of 22 800 lb per sq in. in the $\frac{1}{2}$-in. plate and 30 400 lb per sq in. in the $\frac{7}{16}$-in. plate, which values are 35% to 40% higher than those actually measured. As far as the effect of the load on the action of the plate is concerned, the writers' tests differed in two essential details from the tests described by the authors:

(1) In the writers' tests the load was distributed in such a manner that the plate was loaded on both sides of a stringer, whereas in the authors' tests the load was carried entirely between two stringers. It would seem that this difference might produce the effect of a greater degree of fixation in the plate at the stringer.

(2) The length of the bearing area (in the direction of traffic), expressed in terms of clear distance between stringers, was about three times as great in the writers' tests as in those of the authors'. This greater relative length of

bearing area would produce a distribution of longitudinal stress that would be without a sharp peak, such as that shown in Fig. 13(*a*).

The writers do not feel justified in attempting to derive an equation for maximum plate stress that would be generally applicable to this type of construction on the basis of the meager test data available. Because of the small deflections in the writers' tests (as in those of the authors') the load may be assumed to be carried entirely by bending of the plate. In the case of thin plates, however, in which the deflection might be several times the thickness of the plate, a part of the load will be carried by direct stresses, and any equation derived on the basis of bending resistance only would obviously be inadequate.

In discussing the manner in which a load applied over the center stringer is distributed laterally to the other stringers, the authors state (see "Summary," Item 6) that "the distribution factor varies with the thickness of the plate and the distance between the stringers." Nevertheless, in Fig. 14, which is recommended as a basis for design, the percentage of the total load carried by the loaded stringer is plotted against stringer spacing, with no regard to plate thickness. There are at least three other variables which affect the distribution of the load among the stringers: (1) The stiffness of the stringer (or composite plate and stringer section); (2) the width of the panel (or the number of stringers); and (3) the span length of the panel.

Since, as the authors state, the load transferred from one stringer to an adjacent stringer is a function of the difference in the deflections of the two stringers, it follows that if the stiffness of the stringers is increased the deflections will decrease and the condition of very stiff stringers and a relatively thin plate will be approached. In such a case the total applied load would be carried by the loaded stringer, regardless of the stringer spacing.

The effect of the panel width on the part of the load carried by the loaded stringer can be demonstrated by considering a panel with a large number of stringers and of such proportions that the center stringer carries 35% of the applied load, and the two stringers adjacent to the center each carry 15% of the load. The remainder of the load would then be carried by the stringers beyond the center three. If the width of the panel is decreased so that it contains only the center three stringers (which previously carried 65% of the load) they must carry the entire load.

The importance of the span length in determining the manner in which a centrally applied load is distributed among the various stringers can be demonstrated by data obtained by the writers from tests on a bridge-floor panel of the interlocking-channel type. The action of this panel was similar to that of the battle-deck type since the panel was essentially composed of a series of ribs connected by flat plates. The panel (which was 11 ft 6 in. wide) was tested on simple spans of 3-ft, 4-ft, and 5-ft lengths. Load was applied over a bearing area which had a dimension of 12 in. in the direction of the width of the panel. With a rib-spacing of only 7.25 in. and the load centered on the middle rib, it is evident that the loaded area extended almost to the two ribs adjacent to the middle. The proportion of the load carried by each rib for the different span lengths is given in Table 5. These percentages were determined from measured deflections on the basis that the load carried by each of the ribs was proportional to its deflection. Stresses measured on the bottom of each rib were substantially in agreement with such a distribution of the load. It will be noticed in Table 5 that for all spans the outside ribs were unloaded and that the center rib carried about 50% more load with the panel on a 3-ft span than when the panel was tested on a 5-ft span.

TABLE 5.—LOAD DISTRIBUTION IN PANEL OF INTERLOCKING CHANNEL

Span, in feet	PERCENTAGES OF TOTAL LOAD CARRIED BY THE FOLLOWING RIBS, FOR LOAD CONCENTRATED ON CENTER RIB AT MID-SPAN																		
	LEFT OF CENTER									CENTER	RIGHT OF CENTER								
	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9
3..........	0.0	0.0	0.3	0.6	1.3	2.2	4.4	7.9	18.9	31.4	18.6	7.9	3.5	1.6	0.9	0.6	0.0	0.0	0.0
4..........	0.0	0.5	1.0	1.5	2.1	2.6	4.4	9.1	17.8	24.8	18.3	8.3	4.3	2.0	1.5	1.0	0.7	0.2	0.0
5..........	0.0	0.4	1.0	1.6	2.2	2.9	5.0	10.1	17.1	21.6	17.6	8.8	4.7	2.4	1.9	1.3	0.9	0.4	0.0
5*.........	0.1	0.8	1.6	3.2	4.4	6.1	7.3	8.6	11.0	12.2	11.2	9.1	7.8	6.7	4.6	2.7	1.8	0.6	0.1

* Distributor beam at center of span.

In some of the tests made by the writers, one or more "distributor" beams were fastened to the ribs in a direction normal to that of the ribs. The function of these beams was to produce a more favorable distribution of the load among the ribs. The degree in which this was accomplished on the panel of the interlocking channel can be seen from the values in Table 5. With the distributor beam at the middle of the 5-ft span, the center rib carried only about 12% of the total load (as determined from measured deflections), whereas about 22% of the load was carried by the same rib without the distributor beam. Obviously, the effectiveness of the distributor beam will depend on its stiffness.

JONATHAN JONES,[8] M. AM. SOC. C. E. (by letter).—It would be quite futile to undertake, either by reasoning or experiment, a general solution for the case of a floor plate stiffened by ribs. Enough is known of the mathematical complexity of the problem, and of the many variables entering in, to guarantee that any such generalization, if obtainable, would be too formidable to have much practical value.

About all that is practically worth doing is to select, for the purpose in mind, an approximate arrangement of parts and sizes that promises to be successful and economical, and then, by a combination of experiment and reasoning, to develop some general rules of design that may be applied within a narrow field. This process may appear crude, but it may also be highly useful. It may not be rigorous mathematics, but it can be good engineering. This piece of research was conducted in just such a fashion, and it has produced results and recommendations of great value in its intended range, but which should not be extrapolated far therefrom.

It was the desire of the Committee of the American Institute of Steel Construction, that initiated this research, to discover the constructional advantages of the "battle-deck floor," in steel, which it believed had been obscured by the lack of a basis for design. In particular, it believed that plates of excessive thickness were being used, to the great economic disadvantage of this type of floor, simply for lack of any better method of design than a guess at a distribution, and then a continuous beam formula and the usual working stresses.

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[8] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.; Chairman, Committee on Technical Research. Am. Inst. of Steel Constr.

This Committee, therefore, guided the research into just those arrangements of parts in which it believed this type of construction would be most successful. Thus, it opened up the beam spacing at the expense of added weight, to minimize manufacturing costs; it discarded transverse distributors for the same reason. All this it has reported adequately elsewhere.[9] The successive models, and the final full-sized panel, were designed by the Committee, as more data became available, to come closer and closer to an economical and reasonable design.

The results have been extremely satisfactory. Within the purposely narrow range, simple rules of design have been developed by approximate analysis of the test results. Upon these rules the sponsor has compiled a booklet[9] giving recommended practice as to the arrangement of parts, beam spacing, beam sizes, plate thicknesses, etc., which give certainty of success.

Not mentioned by the authors, but important to the designer, is the choice of unit stresses by the Sponsor Institute for its recommended practice. For tension in the bottom flange of its beams, it recommends 20 000 lb per sq in., but in its tables it uses 18 000 in deference to present bridge specifications. For plate stress, it recommends 28 000 lb per sq in.

Thus, in Table 3, for the full-sized panel, it will be noted that 27 900 lb per sq in. was the conscious objective of the final design, although only 26 000 was actually measured. At first sight it may seem to be taking great liberties to propose, for regular design, a unit stress as high as 28 000 lb per sq in. As a matter of fact, it will be seen from Fig. 13 that it is truly conservative. All that this 28 000 means is a "spot" of highly stressed material backed up by a wide area of under-stressed plate; danger is impossible under such conditions. It is perhaps analogous to the, say, 40 000 lb per sq in. that exists at the edge of a hole in a tension member, but in no way is it analogous to the 16 000 or 18 000 lb per sq in., for which that member was designed.

The behavior of these floors under over-load, and their recovery on release (as described by the authors under "Test Data and Relations: Tests of Full-Sized Floor Panels"), convinced all observers that the design rules set up by A. I. S. C. will produce a floor that simply cannot be made to fail under any condition of highway service. The authors are to be congratulated upon the manner in which they fitted their technical skill into a problem which was narrowed not by, but for, them.

It is interesting to observe[10] that German engineers are also experimenting in an endeavor to reduce the weight of their highway bridge floors. As might be expected, however, they are working with designs of great complexity in order to effect a maximum saving of material; their transverse stiffeners, four-sided supports, stiffened buckle-plates, etc., represent a degree of fabrication which, in general, American designers must avoid.

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[9] "The Battledeck Floor for Highway Bridges," Am. Inst. of Steel Construction, New York, N. Y., 1938.

[10] *Die Bautechnik,* June 3, 1938, p. 306 ff.

INGE LYSE,[11] M. AM. SOC. C. E., AND INGVALD E. MADSEN,[12] JUN. AM. SOC. C. E. (by letter).—An experimental solution of a complicated structural problem must of necessity be limited within the available resources. For a problem as complicated as that of the battle-deck floor system the test program had to be greatly limited in scope. Mr. Jones describes and justifies this limitation so well that it is unnecessary for the writers to elaborate on it. Mr. Jones thus answers some of the questions and criticism raised by Messrs. Hill and Moore, and since the writers subscribe in full to the discussion by Mr. Jones they will concern themselves only with other facts of the discussion by Messrs. Hill and Moore.

In discussing the bending moments and the load-distribution factors Messrs. Hill and Moore state that Table 1 gives computed center moments for Stringers *B* and *D* on either side of the loaded stringer of the full-sized floor, which are greater than would have been obtained had the loads computed for these stringers been concentrated at the center of the span. According to Table 1 Stringer *B* carried 4 850 lb; the span length is 16.75 ft; so that the center moment becomes $M_c = \frac{4\,850 \times 16.75 \times 12}{4} = 244\,000$ lb-in. against 250 000 lb-in. given in the table. It seems that this is well within experimental errors.

The width of plate in T-beam action is also questioned and reference is given to theoretical analyses indicating that for the full-sized floor panel an effective width of 45 in. could be assumed when spacing of the stringers was only 24 in. It seems to the writers that the maximum effective width could not very well exceed the stringer spacing. It should also be noted that the T-beam analysis, referred to in the discussion, is based on a single T-beam with very wide flanges, which is not the same case as the series of parallel beams in the battle-deck floor. Furthermore, the observed tension in the stringers agreed very well with the width of plate given in Table 3.

Messrs. Hill and Moore compare the results of the battle-deck steel floor with aluminum plates on channel sections. It seems that this is an extrapolation of the writers' experimental results beyond reasonable limits. The differences obtained, therefore, are what might be expected when materials and sections that differ so widely are used. It cannot be emphasized too often that experimental results of complicated structures must not be generalized to apply far outside the range covered in the experiments.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2022

FLOOD ROUTING

By Edward J. Rutter,[1] and Quintin B. Graves,[2] Assoc. Members, Am. Soc. C. E., and Franklin F. Snyder,[3] Jun. Am. Soc. C. E.

With Discussion by Messrs. Cecil S. Camp, R. D. Goodrich, E. L. Myers, Ralph W. Powell, William T. Collins, George E. Townsend, Waldo E. Smith, and Edward J. Rutter and Quintin B. Graves and Franklin F. Snyder.

Synopsis

A method used for routing floods through the Tennessee River is described in this paper. First, a method for routing floods in the natural condition from Knoxville, Tenn., to the mouth of the Tennessee River was developed, and then the procedure was adapted to routing through the reservoirs of authorized and proposed projects. To analyze flood reduction, the complete river system was considered as a unit for various floods.

Routing was accomplished by dividing the river length into seven reaches and applying the storage equation to each reach. Total inflow into each reach was determined day by day from the routed outflow of the adjacent up-stream reach, published discharges of metered streams flowing into the reach, and an inflow estimated from rainfall on the unmeasured areas draining into the reach. Total flood volume at each dam site was assumed to be known. The storage in the reach for various flows was determined from topographic maps and cross-sections of the river valley in conjunction with flow profiles determined from back-water curves. The outflow was then derived from the relation of storage and discharge for the reach.

The general procedure for all reaches is described, and a detailed example for the Watts Bar-Chickamauga Reach is included. Finally, results are given for the effect of the proposed system of dams on the 1926–1927 flood and on a flood 50% larger.

Introduction

The development of the Tennessee Valley by the Tennessee Valley Authority (TVA) involves the construction of a series of reservoirs for navigation and

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Note.—Published in February, 1938, *Proceedings*.

[1] Hydr. Engr., Tennessee Valley Authority, Knoxville, Tenn.

[2] Instr. in Civ. Eng., Dept. of Civ. Eng., Univ. of Texas, Austin, Tex.

[3] Hydr. Engr., Dept. of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pa.

flood control, with incidental power development. The program that has been authorized by Congress includes storage reservoirs on two tributaries of the Tennessee River and a series of five high dams on the Tennessee River itself. The ultimate plan for the development of the Tennessee Valley includes the construction of seven high dams on the main river, in addition to the existing Wilson Dam at Muscle Shoals, Ala.

The determination of the reduction in flood crests that may be effected by such a system of reservoirs is a complex problem. It may be a comparatively simple matter to determine how a storage reservoir on a tributary modifies a flood flow, if stream-flow data and the storage capacity of the reservoir are known. However, the total volume of water that passes through the main river within a flood period is so great, with respect to the available storage capacity in the river reservoirs, that much study is necessary for an estimate of the reduction which these reservoirs will afford. Since river reservoirs are not level pools, consideration must be given to the water-surface profiles of each reservoir. In some of the river reservoirs the back-water effect of the dam disappears some distance from the upper end of the pools when there is a large flood discharge. The solution of the problem, therefore, requires a knowledge of the total storage in each pool which comes into use by an increase in water level—at the lower end of the pool due to the dam itself, and in the upper end due to the limitation of channel capacities. In comparing flood flows under "natural" conditions with those under conditions after the dams are built, it is necessary to determine the effect of channel or valley storage on the natural flood flows.

A complete knowledge of conditions during past floods would require copious data on channel cross-sections, topography in the valley reached by the flood, the amount of clearing within the flooded areas, and the actual profiles of the river during the high flows. It is possible to obtain much of this information, but a reliable estimate of the effect of some factors requires considerable research. (The study of floods has involved, first, as thorough an analysis of actual floods of the past as existing data permit; and, second, a determination of the effect of reservoir storage, now or soon to be available, upon floods of the past and others that may be expected in the future.) Certain reservations of storage capacity are made for flood control, but it is necessary not only to determine flood-control benefits which may be derived by the reservoir system, but also to provide data which may be necessary or useful for the proper operation of the system in flood periods.

General Information

The Tennessee River (Fig. 1) has a total length of 652 miles from its mouth, near Paducah, Ky., to the confluence of the French Broad and Holston Rivers, at Knoxville, Tenn. The valley is divided into two basins by a constriction near Chattanooga, Tenn., the upper basin being much more mountainous than the lower. The river has a number of relatively large tributaries, and many small tributary streams. The slope of the main river channel varies from rather slight slopes on the lower river to steep slopes on the upper river and at Muscle Shoals, in Alabama. Along parts of the river there are wide over-bank sections, as is the case just below Pickwick Landing Dam, in Tennessee, and in other parts there are gorge sections, as is the case just below Chattanooga. For a general description of the Tennessee River Basin, the reader is referred to the March, 1936, report to Congress of the TVA on "The Unified Development of the Tennessee River System."

Tennessee River Survey maps made under the direction of the United States Engineer Department and of silt ranges made by the TVA are important sources of information for use in determining storage in the various reaches. River-stage records and high-water marks and profiles were obtained from both published and unpublished data of the United States Weather Bureau, the United States Geological Survey, the Mississippi River Commission, and the Tennessee River Survey of the U. S. Engineer Department. Later data have been obtained and compiled directly by the TVA. Rating curves were obtained from the same sources when available; otherwise, they were constructed and extended from gage relationships taken from the river-stage records and from back-water curve computations. (Guntersville Dam, not shown in Fig. 1, is situated about 9 miles down stream from the town of Guntersville.)

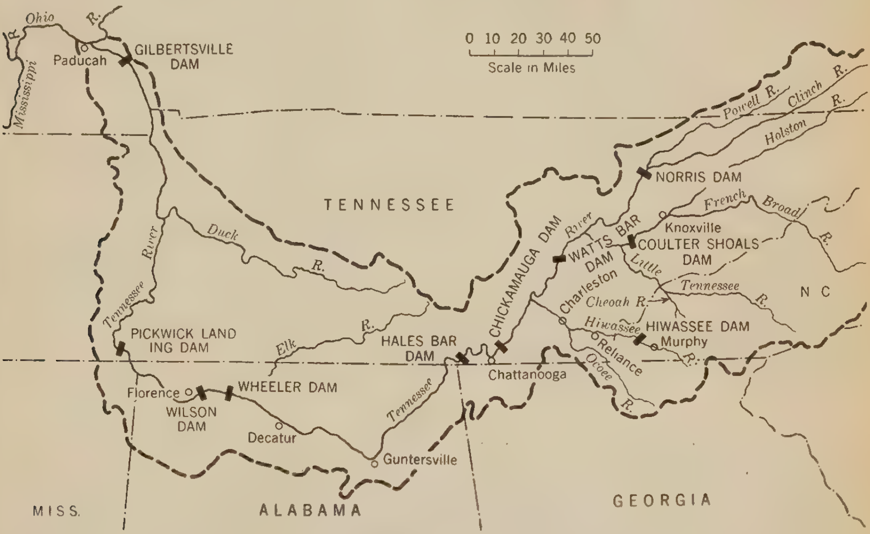
Ohio
R.
R.
Mississippi
Paducah
GILBERTSVILLE DAM
0 10 20 30 40 50
Scale in Miles
Powell R.
Clinch R.
R.
Holston
NORRIS DAM
TENNESSEE
French Broad R.
Knoxville
COULTER SHOALS DAM
River
Duck R.
River
WATTS BAR DAM
CHICKAMAUGA DAM
Little Tennessee R.
Charleston
Cheoah R.
Hiwassee
HIWASSEE DAM
Murphy
R.
Reliance
Ocoee R.
N C
Tennessee
PICKWICK LANDING DAM
Elk R.
HALES BAR DAM
Chattanooga
Florence
WHEELER DAM
WILSON DAM
Decatur
Tennessee
Guntersville
MISS.
ALABAMA
GEORGIA

FIG. 1.—TENNESSEE RIVER AND TRIBUTARIES, SHOWING EXISTING AND PROPOSED DAMS

GENERAL PROCEDURE

The storage equation, "inflow minus outflow equals change in storage," has been made the basis of the routing. There are a number of solutions of this equation adaptable to flood routing, and the one used is that described by R. D. Goodrich, M. Am. Soc. C. E.[4]

In routing a flood through a reach, it is necessary to know: (1) Total inflow into reach: (*a*) Inflow at upper end of reach; and (*b*) local and tributary inflow; (2) profile of water surface at any instant; and (3) storage under the profile.

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[4] "Rapid Calculation for Reservoir Discharge," by R. D. Goodrich, *Civil Engineering*, February, 1931, pp. 417-418.

If these three factors are known, or can be closely approximated, an outflow hydrograph will be obtained by solving the storage equation,

$$\frac{I_1 + I_2}{2}t - \frac{O_1 + O_2}{2}t = S_2 - S_1$$

in which I is the total inflow rate; O, the outflow rate; S, the volume in storage in the reach; t, the length of the period; and the subscripts, 1 and 2, refer to the beginning and the end of the period, respectively. The average inflow and outflow for a period are $\frac{I_1 + I_2}{2}$ and $\frac{O_1 + O_2}{2}$, respectively. However, if the unit of flow is cubic feet per second and the period is 24 hr, $I_1 + I_2$ and $O_1 + O_2$ give the total inflow and outflow in acre-feet, since 1 cu ft per sec for 24 hr is almost exactly 2 acre-ft. With these units the storage equation can then be written as $I_1 + I_2 - O_1 - O_2 = S_2 - S_1$, and transposing, $O_2 + S_2 = I_1 + I_2 + S_1 - O_1$. For other units and lengths of routing periods the equation must be solved accordingly.

The outflow, O_1, for the first period must be estimated, but can be closely approximated as the routing is started a day before the river begins to rise; any error made in this estimate is soon eliminated. With O_1 assumed for the beginning of the first period, I_1, I_2, S_1, and O_1 are known and determine an "outflow-plus-storage" factor, $O_2 + S_2$, for the first period. Then, with the relation between O and S known for various values of their sum, the separate values of O_2 and S_2 are determined and are equivalent to O_1 and S_1 of the following period. The four quantities, I_1, I_2, S_1, and O_1, are then known for the second period and the process is repeated.

REACHES

The first part of this study consists of routing actual floods in the natural river channel, under conditions existing at the time those floods occurred. As a preliminary, the river must be divided into "reaches" of suitable length.

In the present study of the Tennessee River, a time unit of 24 hr is used. A more flashy stream would require a shorter routing period since, in the solution of the storage equation, it is assumed that changes in rates of inflow and outflow are uniform for the period used. The length of reach should be such that the time of travel through the reach is about the same as the time unit used in routing, so that changes in the condition of inflow at the upper end of the reach at the beginning of a period will be reflected at the lower end of the reach by the end of the period, with proportional intermediate changes. The desirable length of reach in the present case is thus seen to be about one day's travel.

The ideal situation would be to have a "routing station" near the mouth of each large tributary, and as many others as necessary; but it is difficult to fix the stage-discharge relation at such points, and the ratings are seldom available. However, it is desirable to know the natural hydrograph at each dam location so that a comparison may be made between the natural peak discharge and the regulated peak discharge after the dam is built. Moreover, local and tributary inflows that have proved satisfactory in the natural condition will be available for use in routing with the dams built. The routing stations were thus fixed by the location of the dams whose potential effects on floods it was desired to study. Fortunately, all the reservoirs except Gilbertsville, in Kentucky, are close to the length corresponding to a time of travel equal to one day. Table 1 gives pertinent data for the reaches and dams considered.

TABLE 1.—GENERAL DATA ASSUMED FOR DAMS AND RESERVOIRS

Name of project (1)	Miles above mouth (2)	Drainage area, in square miles (3)	Surcharge elevation for floods (4)	Pool elevation in advance of floods (5)	Minimum elevation for navigation (6)	Spillway crest elevation (7)	Spillway length, in feet (8)
Knoxville	647.7	8 990	....	....	....	....	....
Coulter Shoals	603.5	9 600	815.0	806.0	804.5	775.0	400
Watts Bar	529.9	17 400	746.0	740.0	740.0	721.0	760
Chickamauga	471.0	20 800	685.0	675.0	673.5	645.0	720
Hales Bar	431.1	22 000	....	....	....	....	....
Guntersville	349.0	24 300	595.0	591.0	590.0	555.0	720
Wheeler	274.9	29 600	558.0	550.0	546.0	541.0	2 400
Wilson	259.4	30 800	....	....	....	....	....
Pickwick Landing	206.7	32 900	418.0	408.0	408.0	378.0	880
Gilbertsville	22.5	40 200	370.0	352.0	352.0	330.0	840

RATING CURVES

Natural Condition at Dam Sites.—Most of the gaging stations on the Tennessee River and its main tributaries from Knoxville to the mouth were used in some way in this work. Where discharge measurements have been made, rating curves based on the latest data have been used. At some gaging stations where river-stage records are available but where no discharge measurements have been made, and at dam sites, rating curves have been constructed by using stage-relation curves between the gaging station or dam site in question and the nearest gaging stations which have been rated. In some cases, particularly at dam sites, it was necessary to construct or extend the rating curves by means of back-water curve computations. Only the rating curves which have been made from actual flow measurements can be taken without reservation. All rating curves constructed by other methods and all extensions of rating curves must be accepted subject to revision when better data are available.

At all routing stations except the Pickwick Landing Dam site and the Gilbertsville Dam site, the rating curves predicated a constant relationship between stage and discharge. At the Pickwick Landing Dam site a "changing-stage" rating curve was constructed to take account of slope and back-water effects. It was based on a normal curve and on two other curves—one for falling stages and one for rising stages—giving the percentage correction for each foot change of stage from the preceding day for any elevation. The normal rating curve was derived from back-water computations and other data, and the correction curves were determined by trial routings of several floods. The changing stage rating curve, then, consisted of a family of curves plotted with elevation against discharge and with the change in stage from the preceding day as the parameter, or third variable.

The method used in obtaining the Gilbertsville rating curve is similar to that developed by the U. S. Geological Survey, in the Missouri (St. Louis) District

Office, in the construction of a rating curve for the Ohio River, at Paducah. The relation between percentage submergence and a submergence factor was determined from the data available. For any percentage submergence and stage at Gilbertsville the actual discharge was obtained by applying the submergence factor to the actual stage to determine an effective stage. Then, with this effective stage, the corresponding discharge was determined from a base rating curve. For convenience, the rating curve was constructed, from the foregoing information, as a family of curves with discharge at Gilbertsville as abscissa, elevation at Paducah as ordinate, and elevation at Gilbertsville as the third variable.

Tail-Water and Head-Water Curves.—The tail-water rating curves at the dams are the same as the rating curves in the natural condition modified, in most cases, by the back-water from the nearest reservoir down stream.

The spillway discharge curves were based on the usual weir formula, $Q = C\,L\,H^{3/2}$, and were computed for unregulated conditions only; that is, all gates open. Values of the coefficient, C, were taken the same as those used by the U. S. Army Engineers.[5] When the spillway was submerged, the coefficient, C', as obtained from the results of the United States Deep Waterways experiments[6] on submerged-weir models, was substituted in the weir formula for the coefficient, C. The resulting spillway discharge curves were single-line curves, except for the Pickwick Landing and Gilbertsville Dams. At these two dams it was necessary to construct a family of curves, each member being based on a fixed tail-water elevation.

INFLOW

Inflow into each reach was divided into two parts: (1) The discharge from the reach immediately above; and (2) the local and tributary inflow, which consisted of the measured flow at U. S. Geological Survey gaging stations on the larger tributaries, together with an estimated flow from the unmeasured area. The sum of the discharge from the upper reach and the local and tributary flow is the total inflow, I, which is used in solving the storage equation. If the distance of a tributary station from the main stream is great, the flows passing the gaging station on any day were assumed to enter the main river storage one or two days later. This time lag, to the nearest day, was determined from a study of the time of travel of one particular flood.

The first step was to obtain the relation between the drainage area and the volume in the flood hydrograph at the various dam sites. This was done by plotting the flood volumes at the regular U. S. Geological Survey gaging stations against the drainage areas. The drainage area at the dam site then determined the volume of the flood at that point. The total volume to come from rainfall on the unmeasured area was determined from the difference between the volumes of the flood hydrograph at the upper and lower ends of each reach. Table 2 shows the total volume for the 1926–1927 flood at each location, the volume from the measured tributaries in each reach, and the volume from the unmeasured area. (In Table 2 the total volume in 1 000 sec-ft days is not necessarily the exact figure as taken from the total volume-drainage area relation. The total volume for the local areas is obtained from this relation, as previously stated, but the total volume shown to any point is the sum of the discharges obtained by routing. Unless the storage stage at the end of the routing period is the same as that at the beginning and unless the outflow is the same as the inflow at the beginning and end of the routing period, the total volume of outflow cannot equal the total volume of inflow. For instance, the sum of the Watts Bar total volume of 2 297 000 and that for the area between Watts Bar and Chickamauga of 544 000 is 2 841 000 and is the volume of inflow into the Chickamauga Reach, but the volume of discharge passing Chickamauga Dam site is 2 838 000 as obtained by routing.) The distribution of the unmeasured local inflow was obtained from rainfall and a distribution graph, the latter being determined from distribution graphs of streams of about the same drainage area and shape as those in each reach herein considered. The procedure for transforming rainfall into run-off by means of a unit graph was developed by Leroy K. Sherman,[7] M. Am. Soc. C. E., and enlarged by Merrill Bernard,[8] M. Am. Soc. C. E.

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[5] H. R. Doc. No. 328, 71st Cong., 2d Session, Appendix 1, Chart CIV.

[6] "Weir Experiments, Coefficients, and Formulas," by Robert E. Horton, M. Am. Soc. C. E., U. S. Geological Survey *Water Supply and Irrigation Paper No. 200.*

TABLE 2.—TOTAL, TRIBUTARY, AND LOCAL VOLUME OF 1926–1927 FLOOD

Location	Drainage area, in square miles	Total volume, 1 000 second-foot days (29 days)	MEASURED TRIBUTARIES		LOCAL AREAS, UNMEASURED					
			Drainage area, in square miles	Volume, 1 000 second-foot days (29 days)	Drainage area, in square miles	Volume, 1 000 second-foot days (29 days)	Base flow, in cubic feet per second	Surface run-off, 1 000 second-foot days (30 days)	Rainfall, in inches	Surface run-off, percentage
Knoxville, Tenn.	8 990	886	..	..	..	..	..	..	..	..
Knoxville to Coulter Shoals	610	83	235	30	375	53	400	44	6.92	63.1
Coulter Shoals, Tenn.	9 600	968	..	..	..	..	..	..	..	..
Coulter Shoals to Watts Bar	7 800	1 345	5 662	1 009	2 138	336	2 800	275	8.25	58.0
Watts Bar, Tenn.	17 400	2 297	..	..	..	..	..	..	..	..
Watts Bar to Chickamauga	3 400	544	2 300	299	1 100	245	2 030	189	7.65	83.4
Chickamauga, Tenn.	20 800	2 838	..	..	..	..	..	..	..	..
Chickamauga to Hales Bar	1 200	259	..	..	1 200	259	..	..	..	..
Hales Bar, Tenn.	22 000	3 100	..	..	..	..	..	..	..	..
Hales Bar to Guntersville	2 300	445	389	85	1 911	360	3 000*	273	6.87	77.4
Guntersville, Ala.	24 300	3 566	..	..	..	..	..	..	..	..
Guntersville to Wheeler	5 300	1 097	1 700	388	3 600	709	5 412	556	8.92	64.4
Wheeler, Ala.	29 600	4 678	..	..	..	..	..	..	..	..
Wheeler to Wilson	1 200	237	355	71	845	167	930*	141	9.36	66.5
Wilson, Ala.	30 800	4 878	..	..	..	..	..	..	..	..
Wilson to Pickwick	2 100	474	621	141	1 479	333	1 850*	284	9.36	76.4
Pickwick, Tenn.	32 900	5 372	..	..	..	..	..	..	..	..
Pickwick to Gilbertsville	7 300	1 588	3 049	684	4 251	904	6 400	742	8.40	78.1
Gilbertsville, Ky.	40 200	7 108	..	..	..	..	..	..	..	..

* Average for the period.

In every reach the unmeasured area was relatively a small part of the total drainage area above that point, the maximum being 12.3%, so that the distribution graph could be somewhat in error without materially affecting the discharge.

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[7] "Stream Flow from Rainfall by the Unit Graph Method," by Leroy K. Sherman, *Engineering News-Record*, Vol. 108, 1932, pp. 501–505.

[8] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

The same local and tributary flows were used in the case with dams as in the natural condition, modified where necessary, of course, in accordance with the assumed operation of tributary storage dams.

The local and tributary inflow can be obtained by working backward from known daily discharges and storage on the main stream, but in every trial of this method the results were found to fluctuate. This fluctuation is due to placing all the error in the local and tributary flow, since a relatively small error in the main river discharge would be a large error in the local and tributary inflow. Then, if on one day, the computed local and tributary inflow is too low, the next day it will be correspondingly higher and the next day that much lower, etc., until negative quantities of local and tributary inflow result. However, if the fluctuating values obtained by working backward are smoothed graphically, reasonable local and tributary flows are obtained. In the example (see heading, "Example of Routing—Chickamauga Reach") given in this paper this method was not used.

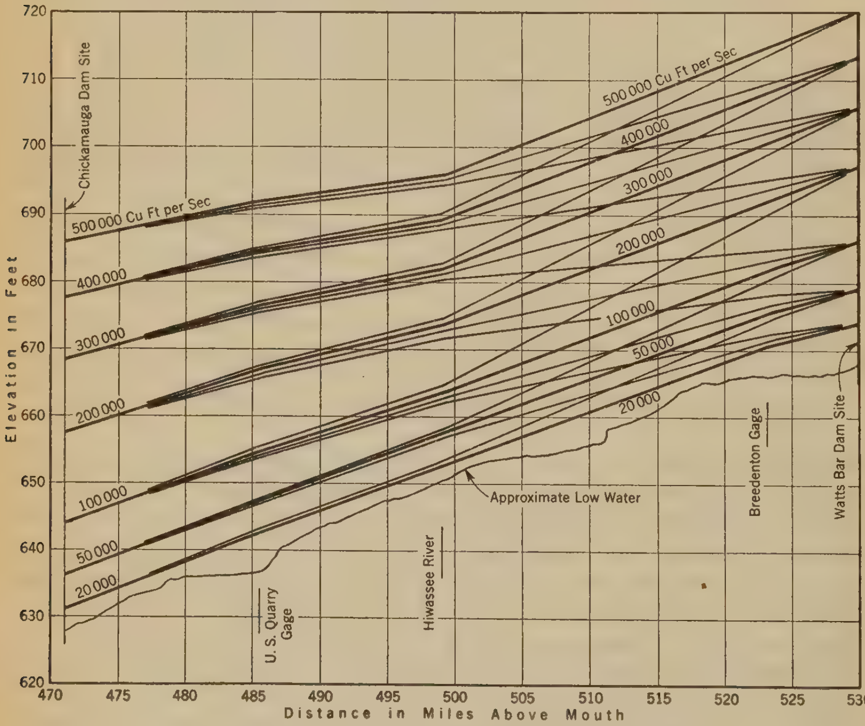

FIG. 2.—PROFILES FOR NATURAL CONDITIONS, WATTS BAR TO CHICKAMAUGA

STORAGE

Each reach was considered as a reservoir, but instead of assuming the reservoir as level, storage was computed from the volume under a sloped profile based on rating curves and back-water studies. In the natural condition, when there was a constant relation between elevation and discharge, profiles were drawn for a series of steady flows, the highest one being at least as large as the largest flood to be routed. In order to obtain a series of storage curves for use in routing that would cover the entire range of conditions, profiles of non-steady flow were drawn between the steady flow profiles. A typical set of these curves is shown in Fig. 2. The manner in which the flow was varied from the upper end of the reach to the lower end differed for each reach. In a reach where there was no large tributary, the flow was varied directly with the length of the reach; for example, with 200 000 cu ft per sec flowing at the upper end, and 300 000 cu ft per sec flowing at the lower end, the flow at the mid-point of the reach was taken to be 250 000 cu ft per sec. The elevation at the mid-point was determined by that flow, applied to a rating curve drawn from the calculated back-water curves. In reaches where a relatively large tributary entered, the flow was varied on the basis of drainage area; for example, if the drainage area from the upper end of the reach to just below the mouth of the tributary is three-fourths of the total drainage area in the reach, then the flow at that point is taken as 275 000 cu ft per sec for the same flow conditions as given previously.

To represent fully the condition after the dams are built, the storage curves for each reach must be in such a form as to satisfy any flow conditions in that reach for any fixed head-water elevation at the dam at its lower end. This is necessary because the discharge from any dam may possibly be regulated to maintain any elevation required. To provide the necessary data, three sets of profiles were drawn—one set for an unregulated condition (that is, all gates open), and the other two sets for regulated conditions of fixed elevations at the dam. The fixed elevations usually selected were the maximum flood surcharge elevation and the minimum navigation elevation. Typical curves are shown in Figs. 3 and 4, and their use is explained under the routing procedure described in the example (see heading, "Example of Routing—Chickamauga Reach").

Storage under each of the profiles was obtained by dividing the reservoirs into smaller reaches varying from 2 miles in length on the upper river to 10 miles in length on the lower river. A volume curve was obtained for each of these smaller reaches, either by planimetering a contour map or by taking cross-sections and assuming them to be average for a certain distance. Where the latter method was used, the final individual volume curves were adjusted so that the total volume from them for a horizontal pool would agree with a level-reservoir volume curve which had been obtained by planimetering the best available maps. Low-water elevation was taken as the point of zero volume for all profiles. The total storage under the profile was obtained by summing up the volumes corresponding to the elevations taken from the profiles at the mid-points of the individual reaches. When outflow-plus-storage curves were required (as in the natural condition), the discharge at the lower end of the reach, for the corresponding profile, was added to the storage. This sum was plotted as abscissa, with discharge as ordinate, and with the discharge at the

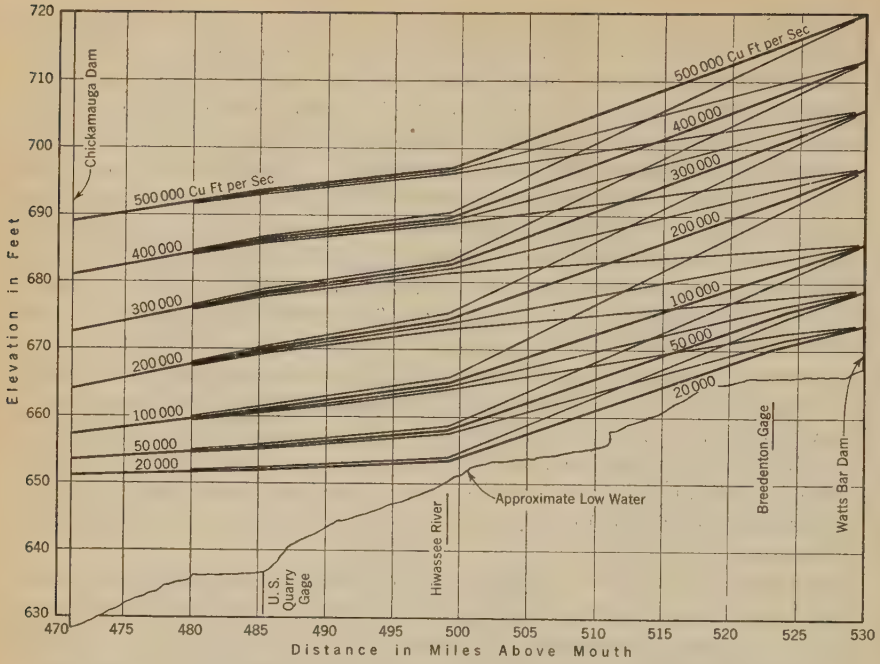

Fig. 3.—Profiles for Unregulated Condition, with Dam, Watts Bar to Chickamauga

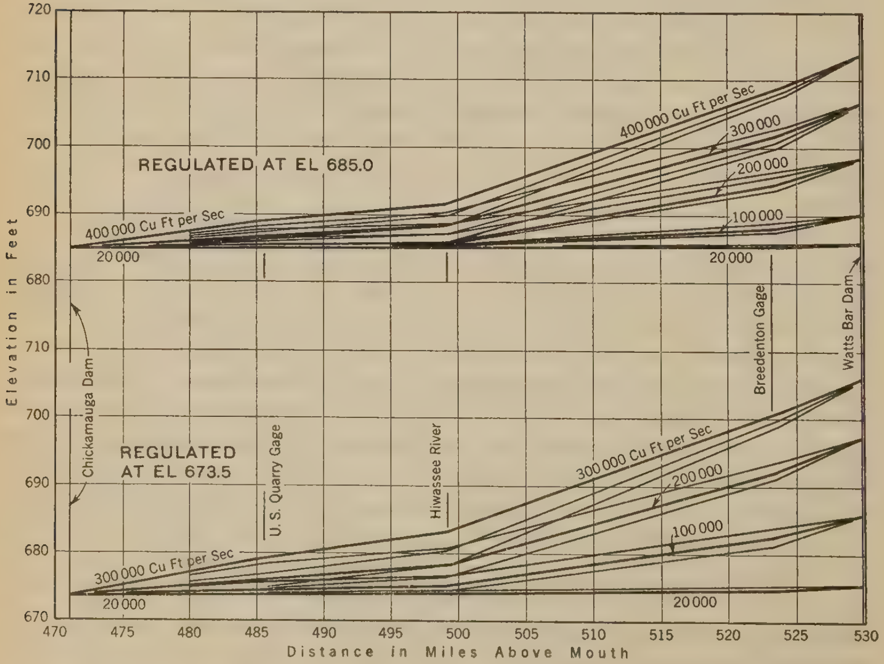

Fig. 4.—Profiles for Regulated Condition, with Dam, Watts Bar to Chickamauga

upper end of the reach as the parameter, thus forming a family of curves (see Fig. 5). For the conditions after the dams are built, storage alone is plotted as abscissa (Fig. 6). For each of the five upper reaches (Coulter Shoals, Watts Bar, Chickamauga, Guntersville, and Wheeler) there is one set of curves

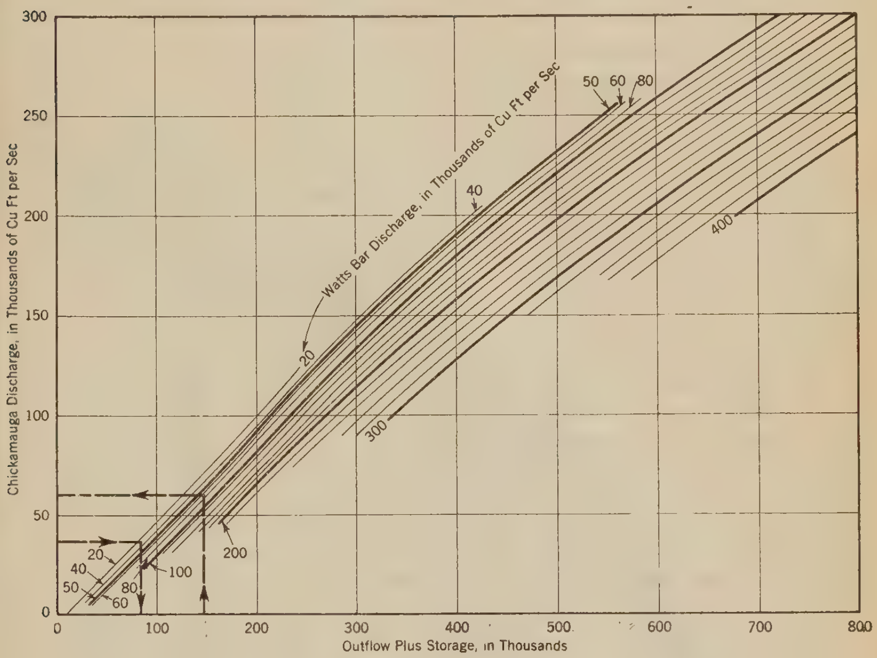

Fig. 5.—Outflow-Plus-Storage Curves for Natural Condition, Chickamauga Reach

of outflow plus storage, for use in the natural condition, and three sets of curves of storage, for use after the dams are built.

For the two lower river reaches (Pickwick Landing and Gilbertsville) a different procedure was used for routing in the natural condition, because of the effect of slope and back-water on the stage-discharge relation. In each of these reaches three sets of outflow-plus-storage curves were constructed, based on three different rating curves for the respective dam sites, in order to allow for different discharges that can occur for any given elevation at the lower end of the reach. At Pickwick Landing, the rating curves selected were the normal curve, the 10-ft rise-in-stage curve, and the 4-ft fall-in-stage curve. The resulting sets of outflow-plus-storage curves, were plotted against the Pickwick elevation, with discharge at Wilson Dam as the third variable. In the routing procedure the true elevation, discharge, and storage are interpolated from the three sets; this process is explained under routing procedure (see heading, "Routing"). At Gilbertsville, the rating curves are based on the elevation of the Ohio River at Paducah, which is assumed to be known in all Gilbertsville routings. The rating curves used in this case were for three different percentage submergences of the Gilbertsville gage, and the resulting outflow-plus-storage curves were plotted against the Paducah elevation, with the Pickwick elevation as the third variable.

For routing after the dams are built, the storage curves for the two lower reaches were drawn in the same manner as those for the upper reaches. At all dams, when an unregulated condition is assumed, the routing procedure is exactly the same as in the natural condition, except that the outflow-plus-storage curves are based on the rating curve for unregulated head-water elevation.

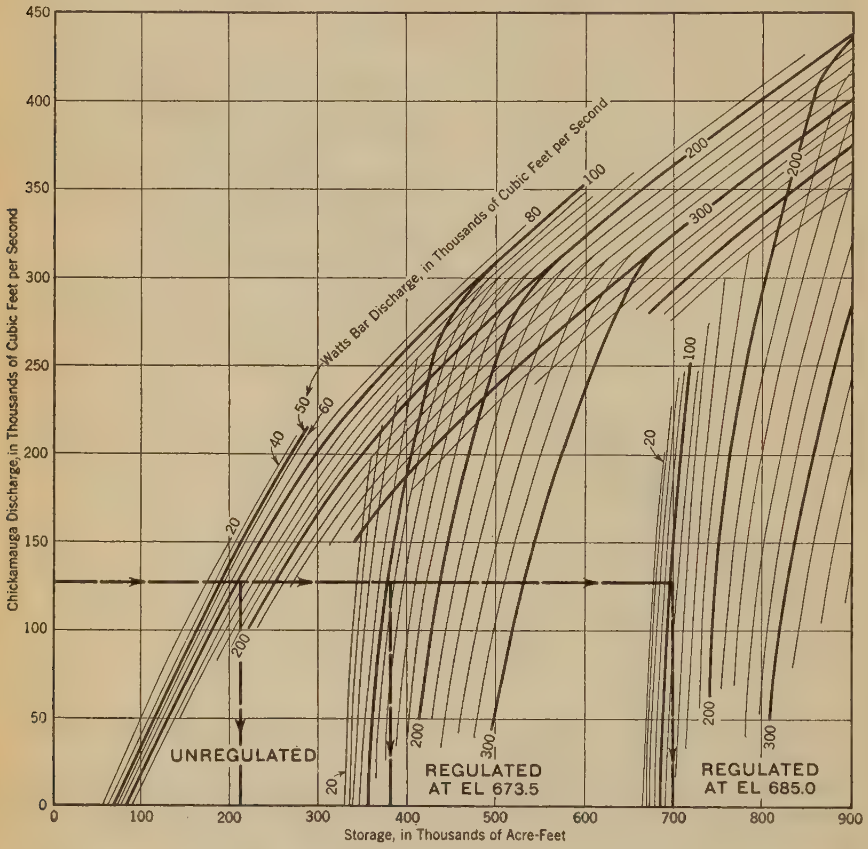

Fig. 6.—Storage Curves for Chickamauga Dam

Other methods of obtaining storage were considered in the course of this study. Throughout the period covered by a flood hydrograph it is possible to determine the storage change on each day if the discharge at the upper and lower ends of the reach and the local and tributary inflow are known. Then, knowing the initial storage, the total storage at the end of each time period can be determined. This method has been described by Harold A. Thomas,[9]

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[9] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, *Engineering Bulletin*, Carnegie Inst. of Technology, p. 65.

M. Am. Soc. C. E. The objections to its use are that the hydrograph at each end of the reach must be known fairly accurately (which was not the case in this study) and that storage curves obtained by this method are not easily adapted to the condition with the dams built. It is also necessary to have a different storage curve for each flood or an average curve based on a number of floods.

If the profiles and the storage under them are correct, it is not necessary to introduce any time element into the routing procedure, provided the length of each reach is such that the true storage within it at any time can be determined by the end elevations. The profiles show the storage at a particular instant, and the storage conditions in the entire river are considered at the same instant. It is believed that if the length of the reach is such that the time of travel is approximately equal to the time period used in the routing, the use of the two end elevations in fixing the profile of the water surface, and, consequently, the storage, will result in reasonable accuracy. The Gilbertsville Reach was the only one in which a correction was necessary to allow for an extreme length, which in this case was 184 miles. Since the time of travel in this reach is several days, a flood wave passing through the reach does not agree with the assumptions used in the shorter reaches as to surface profiles and storage.

One purpose of routing in the natural condition to check an actual hydrograph was to learn whether the routing would give an accurate indication of the time of occurrence of the peak. It was found that it did; there was no marked difference between the observed and routed time. It is an accepted fact that an increase in flow at the upper end of a reach is transmitted to the lower end more rapidly through a reservoir than along the river in its natural condition. That the routing procedure for the condition with dams automatically takes this into account will be clear from inspection of the profiles for the natural and regulated conditions for a typical reach (Figs. 2 and 4) and the corresponding outflow-plus-storage curves or storage curves (Figs. 5 and 6). For example, assume that a steady flow of 100 000 cu ft per sec throughout the reach is increased to 200 000 cu ft per sec at the upper end. The increment of storage that must be filled before an outflow of 200 000 cu ft per sec can occur at the lower end of the reach is: (1) For the natural condition, 174 000 acre-ft; (2) for the dams in place, with regulation at Elevation 673.5, 100 000 acre-ft; and (3) for regulation at Elevation 685.0, 70 000 acre-ft. The routing method thus provides for the faster passage of water through the reservoirs, and its accuracy is limited only by that of the profiles and storage.

EXAMPLE OF ROUTING—CHICKAMAUGA REACH

The Watts Bar-to-Chickamauga Reach is now discussed in detail as an example of the routing procedure.

Chickamauga Inflow.—Inflow into the upper end of this reach was made the same as the routed discharge from the Watts Bar Reach immediately above. (The Watts Bar routing is not shown in this example.)

The increase in drainage area between the upper and lower ends of the Chickamauga Reach, as shown in Table 2, is 3 400 sq miles. The run-off from 2 300 sq miles was measured on the Hiwassee River at Charleston, Tenn., leaving the run-off from 1 100 sq miles to be estimated from an average rainfall of 7.65 in. in 9 days. This average rainfall was obtained from six stations corrected to a calendar-day basis. The unmeasured drainage area is made up of many small streams running directly into the main river and, for this reason, a distribution graph was used which gave a relatively quick run-off. The percentages used were 50, 30, 12, 5, 2, and 1, which means that 50% of the run-off from a day's rain arrived in the reach on that day, 30% on the next day, etc. The total volume of run-off from the unmeasured area was obtained as follows: (1) From the total volume-drainage area relation for a 30-day period, the total flood volume at Watts Bar was 2 346 000 sec-ft days, and that at Chickamauga was 2 899 000 sec-ft days; (2) the difference (553 000 sec-ft days) of these two volumes was the total flood run-off from the area between Watts Bar and Coulter Shoals; (3) the total flood volume of the Hiwassee River, at Charleston, Tenn., was 303 000 sec-ft days, and this volume was subtracted from 553 000 to give 250 000 sec-ft days, the total flood volume from the unmeasured area; (4) the base flow of 2 030 cu ft per sec per day or 61 000 sec-ft days was subtracted from 250 000 to give 189 000 sec-ft days, the total volume from rainfall of 7.65 in. on the unmeasured area. Since there was some rainfall on the day preceding the beginning of the routing period, the quantities given for a 30-day period are somewhat greater than those shown in Table 2 for a 29-day period. The percentage of run-off was assumed to be constant for the entire storm.

The sum of the flow of the Hiwassee River, at Charleston, and the estimated flow from the unmeasured area was added, in the routing, to the inflow at the upper end of the reach to give the total inflow, I, used in the storage equation.

When the Hiwassee Dam was assumed to be in operation, the flow at that point was interpolated from the records of the Hiwassee River at Reliance, Tenn., and Murphy, N. C., and the quantity stored at Hiwassee Dam was subtracted from the Chickamauga local and tributary flow one day later than it occurred at the Hiwassee Dam site.

Chickamauga Storage.—Natural storage in the Chickamauga Reach was obtained as follows: (1) A series of back-water curves for various non-steady flows were computed, based on values of n in Kutter's formula as derived from a study of several known flood profiles and a Chickamauga rating curve determined from gage relations with the Chattanooga curve; (2) from the calculated curves, rating curves were drawn at all required points in the reach; (3) from these rating curves the elevations at the desired flows were taken and flow profiles were constructed (Fig. 2); and (4) storage under each profile was obtained by summing the 2-mile reach volumes which had been planimetered from topographic maps made in 1924 by the U. S. Corps of Engineers. To the storage was added the outflow at Chickamauga, and this sum was plotted against discharge at Chickamauga with the discharge at Watts Bar as the third variable (Fig. 5). In explanation of Step (3), it should be noted that, the non-steady flow was assumed to vary with the drainage area; for example, the drainage area between Chickamauga and the mouth of the Hiwassee River is 380 sq miles; the drainage area of the Hiwassee River is 2 660 sq miles; and the total drainage area between Watts Bar and Chickamauga is 3 400 sq miles. Then, if the flow is 300 000 cu ft per sec at Chickamauga and 100 000 cu ft per sec at Watts Bar, it is assumed to decrease uniformly from 300 000 at Chickamauga to 300 000 $- \frac{380}{3\,400}(300\,000 - 100\,000)$ at the mouth of the Hiwassee River. At that point it is further decreased by $\frac{2\,660}{3\,400}(300\,000 - 100\,000)$, and from there it is decreased uniformly to 100 000 at Watts Bar.

Three sets of profiles for the conditions after the dam is built (Figs. 3 and 4) were constructed in a similar manner. However, for the cases shown in Fig. 4 (which apply when the outflow from the dam is being regulated) the non-steady flow profiles were varied with the distance instead of the drainage area, as it was believed that with regulation the character and shape of a flood is so changed that the tributary does not necessarily contribute inflow in the same proportion as in the natural condition. Storage under the profiles was obtained from the same 2-mile reach volume curves that were used for the natural condition. In this case, however, storage was plotted against discharge at Chickamauga with discharge at Watts Bar as the third variable (Fig. 6).

Chickamauga Routing.—A sample of routing in the natural condition (the latter part of the flood has been omitted) from Watts Bar to Chickamauga is given in Table 3(*a*).

TABLE 3.—EXAMPLES OF ROUTING, CHICKAMAUGA REACH

(Flows and Storages Are in Units of 1 000 Cubic Feet per Second and 1 000 Acre-Feet, Respectively.)

Item	VALUES FOR THE FOLLOWING CALENDAR DAYS IN DECEMBER, 1926										
	21	22	23	24	25	26	27	28	29	30	31
(*a*) NATURAL CONDITIONS											
Watts Bar discharge	29	76	95	138	187	188	200	209	189	179	146
Local and tributary inflow	16	16	24	40	54	52	49	52	47	35	24
Total inflow, I_1	45	92	119	178	241	240	249	261	236	214	170
Total inflow, I_2	92	119	178	241	240	249	261	236	214	170	129
Storage, S_1	48	88	135	187	278	345	370	396	394	355	291
$I_1 + I_2 + S_1$	185	299	432	606	759	834	880	893	844	739	590
Chickamauga discharge, O_1	37	60	104	141	187	227	237	247	252	237	211
$O_2 + S_2$	148	239	328	465	572	607	643	646	592	502	379
Elevation at dam site	635.2	639.1	645.4	650.0	655.8	660.5	661.7	662.9	663.4	661.7	658.7
(*b*) WITH DAMS											
Watts Bar regulated discharge*	70	76	91	112	112	112	112	112	112	112	110
Local and tributary inflow†	14	14	21	37	50	40	35	45	37	21	18
I_1	84	90	112	149	162	152	147	157	149	133	128
I_2	90	112	149	162	152	147	157	149	133	123	....
S_1	387	373	369	395	452	512	557	607	659	687	694
Sum	561	575	630	706	766	811	861	913	941	948	....
Regulated Chickamauga discharge	90	98	108	127	127	127	127	127	127	127	127
Head-water elevation	675.0	674.1	673.5	674.1	676.7	679.1	680.7	682.3	683.8	684.6	684.9
Tail-water elevation	643.4	644.5	646.0	648.3	648.3	648.3	648.3	648.3	648.3	648.3	648.3

* Modified from the natural case, Table 3(*a*), due to storing in Norris Reservoir and regulation at Coulter Shoals and Watts Bar Reservoirs.

† Modified from the natural case, Table 3(*a*), due to storing in Hiwassee Reservoir.

The Watts Bar discharges or inflows at the upper end of the reach are those obtained by routing from up stream. The Chickamauga discharge, O_1, is estimated for the first day to be 37 000 cu ft per sec. From Fig. 5, using the Watts Bar and Chickamauga flows for December 21, 1926, a value of 85 000 is obtained for $O + S$, and subtracting O_1, $S_1 = 48\,000$ acre-ft. (This interchange of units of flow and storage has been explained under the heading, "General Procedure.") The total inflow, I_1, for December 22, 1926, is brought over as I_2 of December 21, and the sum of 185 000 is obtained, from which the Chickamauga O_1 of 37 000 is subtracted, giving an $O_2 + S_2$ of 148 000 for December 21, which becomes the $O_1 + S_1$ for December 22. This value of outflow plus storage, the Watts Bar discharge of 76 000 for December 22, and the $O + S$ curves determine the Chickamauga outflow, O_1, of 60 000, and the storage, S_1, of 88 000 (that is, 148 000 − 60 000) for December 22. The procedure is then repeated for as many periods as desired. Table 3(*a*) shows a peak Chickamauga discharge of 252 000 sec-ft on December 29. The elevations are obtained from the Chickamauga tail-water curve (Fig. 7). The routed hydrograph at Chickamauga is compared with the published U. S. Geological Survey discharges at Chattanooga in Fig. 8. Chickamauga is slightly less than eight miles up stream from Chattanooga.

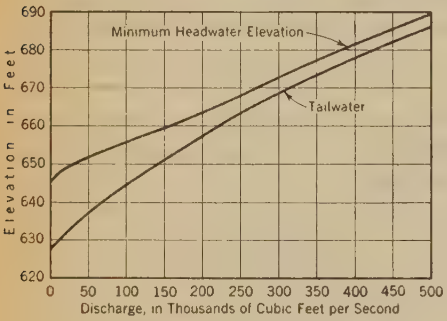

FIG. 7.—HEAD-WATER AND TAIL-WATER RATING CURVES, CHICKAMAUGA DAM

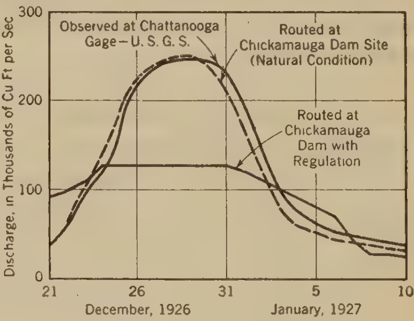

FIG. 8.—COMPARISON OF ROUTED AND OBSERVED HYDROGRAPHS

Table 3(*b*) is an example of Chickamauga routing with the dam regulating the flow. The reservoir at the dam is at Elevation 675.0 at the beginning, with an assumed drawdown to Elevation 673.5 and a flood surcharge to Elevation 685.0 (Table 1).

The constant outflow to which it is possible to regulate a particular flood without violating the assumed elevations is estimated and then checked by routing, and if a greater or less reduction is indicated, the routing is revised.

The outflows were regulated as desired and in the given example made such, from December 21 to December 23, 1926, that the reservoir elevation at the dam would be drawn down to the minimum allowable and held there as long as possible without exceeding the flow to which it was possible to reduce the main peak of the flood. The constant outflow of 127 000 cu ft per sec was made such that the reservoir elevation at the dam would not exceed Elevation 685.0 at any time.

The principal problem of routing with regulated operation of the dams is to determine the correct head-water elevation from day to day. As in all the routing, the storage on the first day must be determined from the river conditions. On succeeding days the storage is then determined from the regulated Watts Bar inflows, the local and tributary inflows, and the regulated Chickamauga outflows.

For a particular day, such as December 24, the three sets of storage curves in Fig. 6 are entered with the Watts Bar inflow of 112 000 cu ft per sec and the Chickamauga discharge of 127 000 cu ft per sec. A storage of 214 000 acre-ft is obtained from the curves for the unregulated condition, 382 000 acre-ft from the set for regulation at Elevation 673.5, and 700 000 acre-ft from the set for regulation at Elevation 685.0. The elevation corresponding to the storage for the unregulated condition is found to be 658.0 from the head-water curve of Fig. 7 for the Chickamauga flow of 127 000 cu ft per sec. The three storage values are plotted against their respective elevations (Fig. 9) and a curve is drawn through the points. Then, entering Fig. 9 with the actual storage for December 24 (395 000 acre-ft), the corresponding elevation of 674.1 is determined for that date. The process is repeated for each day.

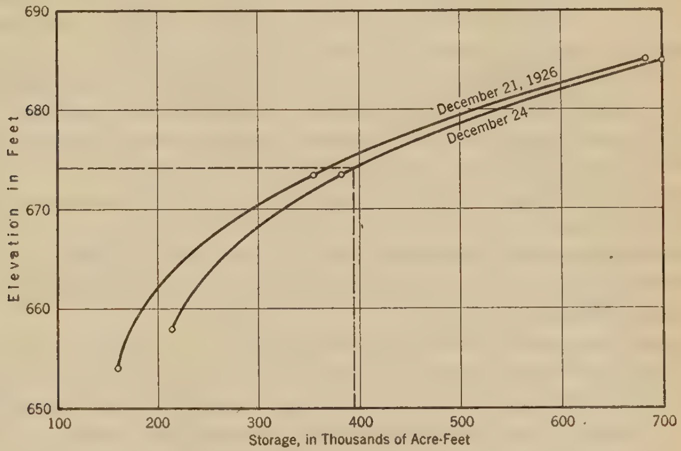

Fig. 9.—Relation Between Elevation and Storage on Typical Days, Chickamauga Reach, with Dams

The tail-water elevations are determined by applying the regulated Chickamauga discharges to the tail-water rating curve of Fig. 7.

Special Problems

In routing from Knoxville to Gilbertsville a number of special problems were encountered. The Knoxville hydrograph must be known to route through the Knoxville-Coulter Shoals Reach, and the regular procedure is then followed in routing through Watts Bar and Chickamauga.

No attempt was made to route through the Hales Bar Pool. In the natural routing the Hales Bar flows were obtained by applying gage heights to the latest rating curve and were used in routing through the Guntersville Reach. All the Guntersville storage curves were prepared on this basis.

The natural routing thus required no local inflow for the area between Chickamauga and Hales Bar. However, with the dams operating a local inflow was necessary, and it was obtained by plotting the Chickamauga and Hales Bar hydrographs on separate sheets to the same scale. The Chickamauga plotting was placed on the other and lagged in time, a distance approximately equal to the time of travel between the two points. The difference between the discharges on each day, using the date on the Hales Bar hydrograph, was taken as the local inflow on that day and was assumed to represent any storage effect of the Hales Bar Pool. These values were added to the Chickamauga regulated outflows to obtain the inflows into the upper end of the Guntersville Reach. The Guntersville and Wheeler Reaches were routed in the regular manner.

The floods were not routed through the Wilson Reservoir. For the natural condition, inflow into the upper end of the Pickwick Landing Reach was made the same as the published U. S. Geological Survey discharges at Florence, Ala. For the condition with the dams built, the Pickwick Landing inflow at the upper end was obtained by adding to the Wheeler outflow the local and tributary flow into Wilson Reservoir, allowing no time lag or reduction for storage.

The Pickwick Landing and Gilbertsville Reaches required special methods because of the types of rating curves necessary at each station; and, in addition, the length of the Gilbertsville Reach presented a difficult situation. As mentioned in the discussion of storage (see heading, "Storage"), three sets of outflow-plus-storage curves were prepared for both the Pickwick Landing and Gilbertsville natural routings, to take care of the additional variable introduced by the effect of slope or back-water.

For rising stages at Pickwick Landing the $O + S$ curves for no change of stage and for a 10-ft rise were entered with the values of $O + S$ and the Wilson flow for a particular day, and two Pickwick elevations were thus obtained. These two elevations were plotted against zero and 10-ft change of stage, respectively, and connected by a straight line. Then, by trial, a change in stage was determined from this plotting such that when added to the previous day's elevation, the new elevation was that given on the curve for the change in stage used. Falling stages were handled similarly except that the discharge plus storage curves for zero change of stage and those for a 4-ft fall were used. Having found the elevation and change of stage, the discharge for that day was known from the rating curve.

At Gilbertsville, the treatment was similar to that at Pickwick Landing, except that the additional variable taken care of by the three sets of outflow-plus-storage curves was percentage submergence. In addition, all three sets were used in the supplementary plotting to obtain the true Gilbertsville elevation.

An additional variation was made in the natural Gilbertsville routing to take care of the length of the reach. For use in entering the outflow-plus-storage curves, the Pickwick Landing elevation was adjusted. The time of travel was assumed to be about three days, and the effect of a given day's change in stage at Pickwick Landing was spread over three days.

The routing with the dams built at Pickwick Landing and Gilbertsville was slightly different from the example for Chickamauga. The difference at Pickwick Landing was in obtaining the reservoir elevation against which to plot the storage figure obtained from the storage curves for the unregulated condition. The Pickwick tail-water elevations were obtained by trial and error from the changing-stage rating curve before the head-water elevation could be obtained from the special head-water curves previously described.

For routing in Gilbertsville, the Pickwick Landing tail-water elevations were adjusted in the same manner as in the natural routing. Here, again, the tail-water elevation was obtained before the head-water elevation could be found. The Paducah elevation, corrected for the regulation of the Tennessee River, the natural rating curve, and the regulated outflow, fixed the tail-water elevation. Knowing the tail-water elevation and the discharge, the head-water elevation was fixed for use with the set of unregulated storage curves.

Conclusions

Table 4 gives the results of routing for two of the cases studied. The dams were operated in accordance with the limiting elevations given in Table 1.

TABLE 4.—Peak Discharge in Thousands of Cubic Feet per Second at Main-River Dam Sites

Location	Flood of 1926–1927		1926–1927 Flood Plus 50 Per Cent	
	River in natural condition	Dams in place*	River in natural condition	Dams in place*
Coulter Shoals	85	64	125	103
Watts Bar	209	112	307	180
Chickamauga	252	127	358	216
Guntersville	286	162	407	276
Wheeler	352	229	504	418
Pickwick Landing	375	270	595	500
Gilbertsville	367	344	545	510
Columbus, Ky., on the Mississippi River	1 320	1 180	1 580	1 448

* Norris, Hiwassee, and all main-river dams.

Elevations at the beginning of the flood period were assumed as those in Column (5) of that table. A drawdown was used where allowed by minimum navigation limits (Column (6)). The flood surcharge elevations used are given in Column (4). It must be kept in mind that this range of operation of the dams is tentative and subject to change. The flood, 50% greater than that of 1926–1927, was built up by increasing the Knoxville hydrograph by 50% and then routing down stream with the local and tributary inflow of each reach increased by 50 per cent.

It is believed that the procedure developed gives results as accurate as can be obtained with the data available. The storage curves and profiles as compiled in this paper are not suitable for low flows or sharp-peaked floods of short duration, but with some modification they could be made applicable.

For floods as great as any that have occurred and for which fairly accurate ratings are available at the various stations, the method seems satisfactory in studying past floods or in forecasting flows for desirable operation of dams as future floods occur.

In forecasting, the beginning point at Knoxville and the local and tributary inflows must be predicted as far into the future as it is desired to forecast the main river. This is best done by consideration of rainfall and extension of tributary records as they become available.

ACKNOWLEDGMENTS

The writers wish to express their appreciation to J. H. Kimball, M. Am. Soc. C. E., Head of the Flood Control Section of the TVA, under whose direction the studies described in this paper were made; and to B. J. Buehler, Jun. Am. Soc. C. E., for reading and criticizing the manuscript; and to other members of the Flood Control Section for preparation of some of the material used in the study.

DISCUSSION

Cecil S. Camp,[10] Assoc. M. Am. Soc. C. E. (by letter).—The method of routing floods through reservoirs, presented in this paper, should be of value to all engineers concerned with analyzing the effect of various dams, or groups of dams, on the control of floods. The method can also be used to advantage in determining the operating schedule of a system of flood-control or navigation dams. Because of the many data on stream cross-sections and profiles required for its use, the method will probably be of more value in studies of large river systems than for single dams.

It was rather surprising to the writer that the routed natural hydrographs agreed so well with the actual stream-flow records when it is considered that there was considerable variation in the extent of the area for which local and tributary inflow was calculated from rainfall for the various floods studied. This variation was due to some stream-gaging stations not being in operation for the earlier floods studied. However, the fact that there was considerable variation in this area serves to emphasize the adaptability and usefulness of the method.

The necessity of obtaining accurate stage-discharge relations, covering the maximum possible range of these values, at the different "routing stations" cannot be emphasized too strongly. The effect of channel storage between the "routing station" and the stream-gaging stations which are used in the construction of the rating curve, must be given careful consideration. Furthermore, in those cases where the submergence factor between a down-stream station and the "routing station" is used, the rating curve must be constructed to cover the extreme values of the submergence factor that may occur when the stage at the down-stream station is affected by excessive run-off from a tributary below the "routing station," or by confluence with another stream. In studies of the 1937 flood on the Lower Tennessee River the writer found that the effect of the extremely high stages on the Ohio River, at Paducah, Ky., was such that it was impossible to check observed stages at the Gilbertsville, Ky., dam site by the use of a Gilbertsville rating curve, which had given very good results when used for several other floods in which the Paducah stages had not been so high.

R. D. Goodrich[11] M. Am. Soc. C. E. (by letter).—The importance of flood routing is becoming greater with every added disaster and a year seldom passes without serious loss and damage in some quarter from this cause. Hence, an accurate and detailed description and record of methods, as given by Messrs. Rutter, Graves, and Snyder, is a valuable contribution to the literature on floods.

The reliability of discharge estimates and flood frequencies based on such computations as those described by the authors may be questioned by some and, therefore, when predictions based on similar studies are confirmed by later experience, one's confidence in these methods seems to be justified.

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[10] Asst. Prof. of Civ. Eng., Coll. of Applied Science, Syracuse Univ., Syracuse, N. Y.

[11] Dean, Coll. of Eng., Univ. of Wyoming, Laramie, Wyo.

A case in point occurred with the floods of December, 1937, on the Sacramento River, in California. It was while engaged on a detailed study of the Sacramento Valley Flood Control Project and the probable increased protection that would be provided by the construction of the Kennet Dam (Shasta Reservoir) that the writer developed the solution of the flow-storage relation used on the Tennessee River investigation. The flood frequencies computed from the results of routing floods on the Sacramento River indicated that the location where flood damage was most probable was at Butte City, Calif., with an average probability that the capacity of the project would be equalled or exceeded about once in twenty-one years.[12] In 1915, the capacity of the project was practically equalled but with little or no damage.

A little less than twenty-five years later a flood from the Upper Sacramento River breached the levees in three places and the entire area of Butte City was inundated.[13] Great damage was also done on the Lower Feather River, in California, which the studies showed to be the next most vulnerable area in the Sacramento Valley.

Modifications and improvements in flood-routing methods are made from time to time, and the writer wishes to suggest one possibility which he utilized recently in studies on the Spokane River, in Washington. The authors used a "family of curves" for conditions involving consideration of changing stage (see heading, "Rating Curves"). To substitute a single curve the changing stage factor may sometimes be introduced into the well-known logarithmic gage-discharge relation, thus:

$$Q = A\left[(G - e) + b\,\Delta G\right]^n \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which A, e, b, and n are constants for any given discharge section and ΔG is the change in stage for the period preceding any observed value of G. The discharge curve is obtained by plotting values of $[(G - e) + b\,\Delta G]$ against corresponding discharges. The constants may be found by trial or by the method of least squares. Evidently, when ΔG is very small it may be neglected and the ordinary equation of the discharge curve results.

With a single discharge curve and a single capacity curve, a very easy and rapid graphical-mechanical solution of the storage-discharge equation was suggested in a personal communication to the writer by C. O. Wisler, M. Am. Soc. C. E., which is an improvement on the method published by the writer.[4] By plotting curves for discharge and storage against corresponding elevations, two additional curves for values of $(S + 0)$ and $(S - 0)$ may also be plotted against the same corresponding elevations, as shown in Fig. 10. Elevation scales for values of $(S + 0)$ and $(S - 0)$ are then plotted below the curves with a distance apart such that an engineer's scale may be placed between them. To use this method, suppose that a value of 34 000 has been computed for $(S_2 + 0_2)$ at the end of a given period. This value will be found to occur at Elevation 2.5 in Fig. 10 and the corresponding value of $(S_1 - 0_1)$ is 15 800. Now if the grid section of Fig. 10 is enlarged to 10 in., the zero of an engineer's scale (with the same number of divisions per inch as the horizontal scale of the curves) can be placed at 2.5 on the $(S_1 - 0_1)$ elevation scale; then add the value of $I_1 + I_2$ by measuring along the $(S_2 + 0_2)$ elevation scale (say, 27 800), and the elevation at the end of the next period is seen to be 3.0, corresponding to a storage of about 43 600. The engineer's scale is then moved to the right until the zero mark comes to 3.0 on the $(S_1 - 0_1)$ scale, and the operator is ready for the value of the inflow for the next period. In this manner two computers can do very rapid routing of floods.

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[12] H. R. Doc. No. 191, 73d Cong., 1st Session, p. 55, Zone 2.

[13] *Engineering News-Record,* December 16, 1937.

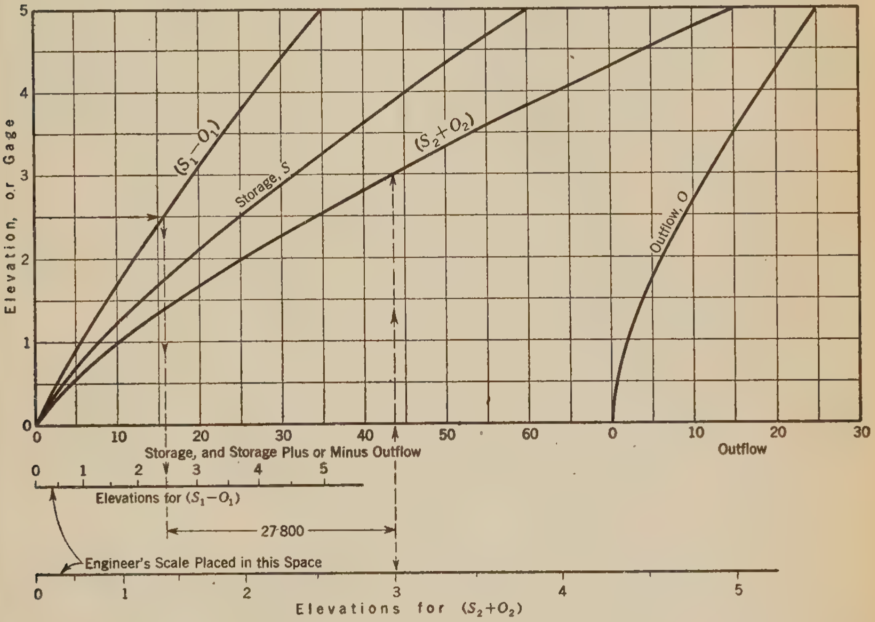

FIG. 10

If actual flood hydrographs are available for the inflow records, almost any desired degree of accuracy may be obtained by using some shorter period than the 24 hr used by the authors. In some cases, when the flood rise or fall is very rapid, the simple solution given for the storage-discharge relation may not produce sufficiently accurate results. Other solutions of the relationship are possible, as mentioned in the paper.[9] A solution for this case, which would permit the use of curves and methods similar to those used by the authors, can be made quite general by using first and second differences of inflow and outflow rates. The following equation gives one solution,

$$\left(I_1 + \frac{1}{2}\Delta I_1 + \frac{2}{3}\Delta^2 I_0\right)\frac{t}{12} + \Delta 0_0 \frac{t}{18} + \left(S_1 + \frac{1}{6} 0_1 \frac{t}{12}\right) = \left(S_2 + \frac{7}{6} 0_2 \frac{t}{12}\right) \quad ..(2)$$

In Equation (2), the symbols have the same significance as in the paper, Δ and Δ^2 referring to the first and second differences and the subscript, 0, referring to a quantity determined by using the flow at the beginning of the period preceding that for which the computation is being made.

The authors should be commended for publishing this record of their exhaustive studies of the effect of reservoirs on Tennessee River floods.

E. L. MYERS,[34] M. AM. SOC. C. E. (by letter).—In a definite and concise manner, this paper describes a method of arriving at probable flood discharges and gage heights in the Tennessee Valley. This is a most constructive piece of work and should prove of great assistance to engineers making flood studies of other streams. Until recently, little information was available as to the manner of arriving at flood stages due to assumed floods. An excellent report of a flood study of the Illinois River was published in 1929 by the Division of Waterways of the State of Illinois. It was used largely by the State Reclamation Department of Texas in 1930 in a study of floods in the Trinity River.

The conditions on the Trinity River and the Illinois River, no doubt, are quite different from those on the Tennessee River; however, the general formulas used are applicable in either case. The study of the Illinois River was complicated by the existence of many levee districts which also applies to the Trinity River in a lesser degree. The effects of reservoir storage entered into the Trinity River study, but not into that of the Illinois River.

A noteworthy feature of the study of the Tennessee River as outlined in the flood-routing report is the small effect that reservoir storage has upon the magnitude of floods in the Lower Tennessee Valley. This is characteristic of most studies of flood control by reservoirs. In the case of the study of the Trinity River, it was found that prevention of floods by reservoirs was impracticable and much more expensive than protection by levees.

RALPH W. POWELL,[35] M. AM. SOC. C. E. (by letter).—Since the subject of flood routing is so important not only for flood forecasting, but also for the rational design of any system of control by reservoirs, the authors should be thanked for placing the methods they have developed at the disposal of the profession. "It is not difficult to point to cases where primary decisions on levee systems or retarding-basin systems of the first magnitude have been [in the past] based on flood-routing computations of doubtful reliability."[36] When the writer compares the elaborate studies summarized in this paper with the crude methods he used when working on the same problem of the Tennessee River, in 1927, it is striking how much progress has been made in ten years.

This is, as far as the writer knows, the first published method for routing floods through reaches affected by back-water from navigation dams, and it seems to solve the problem satisfactorily. The only questions the writer wishes to raise concern reaches not affected by back-water—what the authors call "natural conditions." In this case, cannot the method be simplified further? And cannot satisfactory routings be made without the cross-sections at frequent intervals upon which this method is based?

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[34] Cons. Civ. Engr. (Myers & Noyes), Dallas, Tex.

[35] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[36] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, M. Am. Soc. C. E., *Engineering Bulletin*, Carnegie Inst. of Technology, 1934 (revised 1937), p. 46.

In an attempt to answer these questions the writer secured from the authors the continuation of Table 3 through January 12, 1927, and plotted S against $O + S$ and then against $I + O + S$. As was to be expected, in the first curve, S was definitely higher on the rising stage than on the falling stage for the same value of $O + S$. In the second curve, S was greater on the falling stage than on the rising (for the same value of $I + O + S$), and the difference was larger than before. Trials were then made of $0.5\,I + O + S$; $0.4\,I + O + S$; and, $0.3\,I + O + S$, and it was found that the use of $0.4\,I$ gave the best agreement. The values are shown in Fig. 11. In this case, in determining the storage in the reach, the up-stream flow seems to have a weight of 0.4 compared to 1.0 for the down-stream flow.

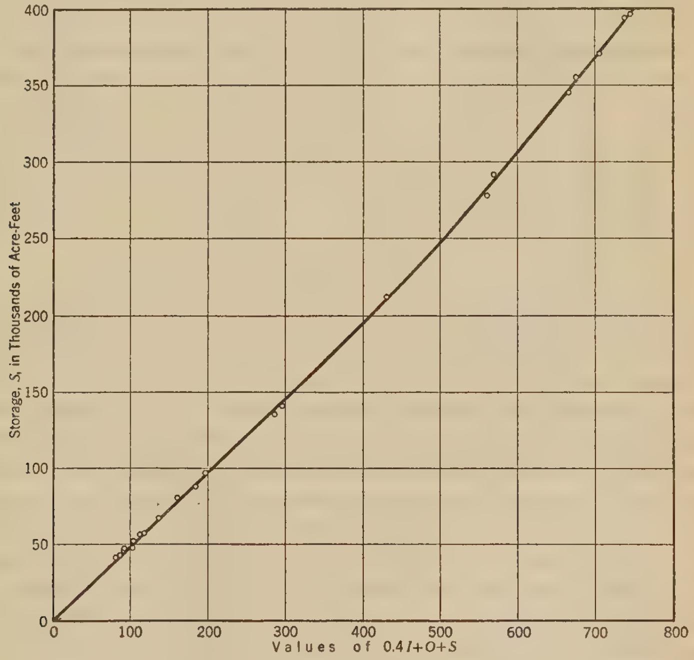

FIG. 11

Using this curve, the values in Table 5 were computed. All the values of I_1 (Columns (1), Table 5) are taken from Table 3 (and its continuation supplied by the authors); O_1 and S_1 for December 21, 1926 (Columns (3) and (4)) are also taken from Table 3. From these, are computed $I_1 + 1.4\,I_2 + S_1 - O_1 = 45 + 129 + 48 - 37 = 185$ (Column (5)) which also equals $0.4\,I_2 + O_2 + S_2$ for December 21, and $0.4\,I_1 + O_1 + S_1$ for December 22, 1926. Subtracting $0.4\,I_1$ for December 22 (= 37) gives $O_1 + S_1$ for December 22 (= 148). Entering Fig. 11 with $0.4\,I + O + S = 185$ gives S_1 (Column (4)) for December 22 (= 89). Subtracting this from $O_1 + S_1 = 148$ (Column (3)) gives $O_1 = 59$. The remainder of Table 5 was computed by a similar process. It should be noted that the values given are not those plotted in Fig. 11, except for the values of I; but they differ from them only slightly. The maximum difference between any value of O and that computed by the authors for the corresponding day was four units on January 2, 1927, which is less than 4% and well within the probable error of the data. In fact, on 15 of the 22 days computed, the difference was one unit or less. It seems, therefore, that if this sample is typical, the channel storage can be computed with sufficient accuracy by this method wherever accurate hydrographs of one large flood have been obtained, without the necessity of a complete flood profile and field surveys of cross-sections. All that is necessary is to make a rough estimate of the storage in the reach before the beginning of the rise (from the known length and assumed average width and average depth). Then, tabulating the known values of I and O, and starting with the assumed S_1, the other values of S can be computed by the formula, $S_2 = S_1 + I_1 + I_2 - O_1 - O_2$, and a curve like Fig. 11 plotted; or, if preferred, one may start with an assumed storage at the end of the period and work backward. In cases where the data are insufficient for making an accurate estimate of the local and unmeasured tributary inflow, this backward order would be preferred, and it is believed that it will be possible to construct a fairly accurate curve of the type shown in Fig. 11 by using the falling stage data only. For this the unmeasured inflow will form such a small part of the whole that even the roughest approximation of it will introduce little error. As has been shown by Robert E. Horton,[37] M. Am. Soc. C. E., beyond the point of inflec-

TABLE 5.—VALUES OF $I_1 + 1.4\,I_2 + S_1 - O_1 = 0.4\,I_2 + O_2 + S_2$, AS COMPUTED* FROM FIG. 11

Day: December, 1926	Total inflow, I_1 (1)	$0.4\,I$ (2)	Chickamauga discharge, O_1 (3)	Storage, S_1 (4)	$I_1 + 1.4\,I_2 + S_1 - O_1 = 0.4\,I_2 + O_2 + S_2$ (5)	$O_1 + S_1$ (6)	Day: January, 1927	Total inflow, I_1 (1)	$0.4\,I$ (2)	Chickamauga discharge, O_1 (3)	Storage, S_1 (4)	$I_1 + 1.4\,I_2 + S_1 - O_1 = 0.4\,I_2 + O_2 + S_2$ (5)	$O_1 + S_1$ (6)
(*a*) COMPUTATIONS FOR DECEMBER, 1926							(*b*) COMPUTATIONS FOR JANUARY, 1927						
21	45	18	37	48	185	...	1	129	52	166	205	294	371
22	92	37	59	89	289	148	2	90	36	117	141	202	258
23	119	48	102	139	405	241	3	63	25	80	97	161	177
24	178	71	142	192	565	334	4	58	23	60	78	136	138
25	241	96	184	285	678	469	5	43	17	54	65	116	119
26	240	96	228	354	715	582	6	44	18	42	56	114	98
27	249	100	238	377	753	615	7	40	16	43	55	102	98
28	261	104	248	401	744	649	8	36	14	39	49	92	88
29	236	94	255	395	676	650	9	33	13	35	44	87	79
30	214	86	238	352	566	590	10	32	13	33	41	82	74
31	170	68	213	285	423	498	11	30	12	30	40	81	70
..	...	..	...	...	...	...	12	29	12	30	39	..	69

* As in Table 3, flows and storages are in units of 1 000 cu ft per sec and 1 000 acre-ft, respectively

tion of the hydrograph the flow is essentially the draw-down from channel storage, so that it should give a fairly accurate measurement of it.

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[37] "Surface Run-Off Phenomena," *Publication 101,* Horton Hydrological Laboratory, 1935, p. 84.

Of course it should be understood that the coefficient 0.4 applies to this particular reach. For other reaches a trial would have to be made to determine whether that value or some other gave the most reasonable results. It should also be understood that no single value of the coefficient can be expected to fit the facts of all floods perfectly, or even all parts of the same flood, even on the same reach. This is only intended to be an approximate method.

As stated by the authors, Professor Thomas [9] has given a method of accomplishing this same result, but it is believed that the method herein given is simpler, and that it partly meets his objection to what he calls the first approximate method, that change of slope is "the very essence of the behavior of a flood wave," [38] and is entirely neglected in the older methods. To meet this objection more fully, the writer would recommend that if the method herein proposed is to be used for a reach in which actual gagings are available at each end, the rating curves should be drawn for a steady stage and then corrected for rising or falling stage. Perhaps a few words on this correction would not be out of order.

Professor Thomas' equation [39] can be written:

$$\frac{dy}{dx} = \frac{S\left(1 - \frac{Q^2}{Q_n{}^2}\right)}{1 - \frac{(u - v)^2}{g\,d}} \qquad \text{(3)}$$

in which S = slope of the water surface at the station; Q = actual flow at the station (in cubic feet per second); Q_n = flow as given by rating curve for steady flow; u = velocity of a flood wave, in feet per second; v = mean velocity of water at the station; g = acceleration of gravity, say, 32.16; and d = mean depth of stream at a station (in feet). Now if j = rate of increase of gage height, in feet per hour (negative for falling stage):

$$\frac{dy}{dx} = \frac{-j}{3\,600\,u} \qquad \text{(4)}$$

and,

$$\left(\frac{Q}{Q_n}\right)^2 = 1 + \frac{j}{3\,600\,S\,u}\left[1 - \frac{(u - v)^2}{g\,d}\right] \qquad \text{(5)}$$

As Professor Thomas states, this applies strictly only to cases in which the velocity is so low that velocity-head effects can be neglected, but this limitation would not forbid the use of Equation (5) at most gaging stations; and, ordinarily, the values of u and v will be close enough so that the last term will be unimportant. The maximum correction for changing stage occurs when $u = v$ and,

$$\frac{Q}{Q_n} = \sqrt{1 + \frac{j}{3\,600\,S\,v}} \qquad \text{(6)}$$

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[38] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, *Engineering Bulletin*, Carnegie Inst. of Technology, p. 53.

[39] *Loc. cit.*, p. 25, Equation (22).

This is the same as the Jones formula used by the U. S. Geological Survey [40] except that v is the average velocity rather than the surface velocity. The use of the latter somewhat reduces the correction and, therefore, may give a better result, but actually the surface velocity depends largely on the wind, and is not a real factor in the problem. The writer is not suggesting that all flood problems require that all gage readings be converted into flow by Equation (6), but that if the data are available, the correction should be applied to the hydrographs used in obtaining the channel storage correction, as outlined herein.

It might be added that the upper part of most rating curves (if uncorrected for changing stage) are generally weighted to give falling rather than rising stage quantities, because gaging parties more often arrive at the station after the peak has passed and get a measurement on falling stage.

William T. Collins,[41] Jun. Am. Soc. C. E. (by letter).—The influence of volume occupied by vegetation and of ground storage on the amount of storage for flood routing does not nullify the usefulness of volumes found from contours. This fact has been demonstrated by the authors. Heretofore, no study using that method to determine storage has been published as far as the writer knows, either because of lack of sufficiently accurate topography and data on profiles, or for lack of the perspicuity to realize that that approach could be used. To obtain the amount of storage from contours probably involves more labor than to build it up from differences of inflow and outflow hydrographs. Establishing the profiles necessary in each reach is a major hydrologic problem in itself. To those engineers who do not have the time or the data necessary to determine storage in this manner, a comparison of these storage volumes with those that might have been determined from differences of hydrographs, would be extremely interesting. Probably storage determined from hydrographs is absolutely accurate if the hydrographs are correct. Because of conditions peculiar to a flood, hydrographs are often not correct unless they are determined from rating curves based on discharge measurements made during that flood. Rating curves are usually based on measurements taken at various times; therefore, hydrographs and storage determined from them are not absolutely correct, and it would be desirable to compare storage determined in this manner with that determined from contours.

The profiles of non-steady flow are an original and logical approach to determining volumes under conditions which prevail most of the time during floods. Until now the best method of computing storage under such conditions was using storage volume as "equal to the length of the reach times the average end area."[42] This is probably all right as long as the stream remains entirely in its channel; but as soon as the flood goes over-bank, it is difficult to imagine that the end, or any sections, is typical. Gaging stations are usually located where the over-bank flow is relatively restricted, which tends to make this relation even less valid. Possibly a solution would be to find the storage from hydrographs and plot it as a function of end area.

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[40] *Water Supply Paper No. 375–E.*

[41] Junior Asst. Engr., Muskingum Watershed Conservancy Dist., New Philadelphia, Ohio.

[42] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, M. Am. Soc. C. E., *Engineering Bulletin*, Carnegie Inst. of Technology, 1934, p. 54.

In any reach where there is a major tributary, the authors assume that distribution of discharge from the head to the foot should vary with drainage area. Consider such a reach where the stages are rising—in it inflow will exceed outflow, and discharge at the head of the reach may also (see, for example, December 22 in Table 3(*a*)). Under this circumstance the authors step discharge down at the mouth of the tributary when actually it would step up, for the tributary will always be adding some discharge. During the falling stages their assumptions closely approximate the actual discharge profile. A different treatment of local and tributary discharges might improve the method.

Tributary and local flows can be routed to the foot of the reach by adjusting time lag and distribution coefficients, or it would be possible to find the total storage and separate it according to whether it affects flow that enters at the head of the reach or that which enters along the reach. Straight-line distribution of discharge in a reach will suit conditions better, provided the stations are close enough together for the sharpness of the flood wave, if tributary and local flows are treated separately. With this plan, the discharge at the end of a reach would be the outflow as routed from the head plus local and tributary discharges. The steps in the discharge profile would come at routing stations where they are easily handled. Total storage would not be obtained under this treatment of tributary flows, but it would be approximately the storage that affects the water entering at the head. Since tributary water enters at some point other than the head, it should be treated differently from that which does enter at the head. Consider the discharge profile of a reach under two conditions: First, when head inflow is greater than outflow during a rising stage and tributaries are not adding materially to the inflow, the profile may be considered a straight line; and second, during a faster rise, when tributaries are adding, the profile will no longer be considered a straight line, but will have an increment, and, in most cases, a change of slope wherever a tributary enters. If the head inflow in the second condition is the same as before, if the outflow of water that entered at the head is less than in the first case, and if the tributaries have added enough so that the total outflow is equal to that in the first case, the authors' method would give the same storage in the two cases. The storage in the second case may be less, equal to, or more than, in the first case, depending on the position of the tributaries and whether or not they have risen or fallen since the time that their discharge influenced the present outflow. These conditions, which are redundant with the present treatment, would not be so if tributary and local flows were routed separately.

It should not be taken as general, that "the distribution graph could be somewhat in error without materially affecting the discharge computed for the main stream." On the Muskingum River, the drainage area at Zanesville, Ohio, is 6 839 sq miles, and the local and tributary area from Dresden, the station next above, is 857 sq miles, 12% of the whole. The crest at Zanesville is in some cases caused by the tributary which is flashy, and at those times occurs when the tributary adds its sharp crest to the relatively flat hydrograph of the main stream. The tributary may add as much as 35% to this crest, so that in this particular case it is necessary to know the tributary accurately.

The Muskingum Watershed Conservancy District uses curves similar in purpose to those of Fig. 9. It is realized that the outflow influences the regulated profiles of Fig. 4. However, it is easily seen that the flow at the head of the reach has more weight in determining the amount of storage. The curves made up by the District are for various uniform flows throughout the reach, and since the effect of variation of outflow from inflow is ignored they are drawn in permanent form. There is one other difference, the District being primarily a flood-control agency its interest is in the storage available for flood reduction, and its curves show what is called net reservoir storage, which is the difference between actual storage for the range of reservoir water-surface elevation at each of the chosen flows, and the storage for the same flows in the natural state without the dam.

The accuracy of the authors' method is stated to be satisfactory, but no comparison with other methods or percentage of this accuracy is given. It might have been advisable to route to the stations at which hydrographs of the floods studied were available, as well as to the stations at the dams where the main interest is. This would allow comparison of computed with actual hydrographs for the same stations.

George E. Townsend,[43] Jun. Am. Soc. C. E. (by letter).—In this flood routing paper, the authors state that the time period is 24 hr, but that in routing on flashy streams a shorter time period is required. When any inflow hydrograph shows a sharp change from rising to falling stage, or when it shows marked differences from day to day, a shorter period should be used in order to give more accurate results. The necessity of a routing having shorter time periods is definitely shown when a greater peak outflow than peak inflow is found when using a longer time period. The peak outflow cannot be greater than the peak inflow because, on the rising side of the hydrographs, the stages are rising and a part of the inflow goes into storage. As long as some of the inflow goes into storage, the outflow will always be less than the inflow, until the peak of the outflow is reached. The condition is unusual and can be eliminated by the use of the shorter time period.

Fig. 12 shows a comparison between routings using the 24-hr and the 6-hr time period. The reservoir projects were operated at normal depressed winter elevations for each of the main Tennessee River dams, and all the flow was stored at the tributary reservoir projects above Knoxville, Tenn. The hydrographs apply to the Chickamauga Reach. The use of the 24-hr period gives an outflow about 11 000 cu ft per sec greater than the inflow. The application of the 6-hr time period results in an outflow peak discharge slightly less than the inflow and occurring 12 to 18 hr before the peak, as determined in the 24-hr routing. In addition, the outflow hydrograph is crossed by the inflow hydrograph nearly at the highest point of the outflow hydrograph, a criterion which usually may be applied as one check upon the results. The differences in the computed outflow hydrographs using the two different time periods may be accounted for by the impossibility of determining any flow profile to fit all flow conditions, and to the difficulty of determining an accurate local inflow. Upon the flow profiles, of course, depend the storage curves which are used in the routing. The method used in obtaining the local inflow gives good average results, but its determination depends on taking into account the water losses, the season of the year, and the condition of the ground during the particular flood. The results, therefore, are not accurate.

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[43] Junior Hydr. Engr., Tennessee Valley Authority, Knoxville, Tenn.

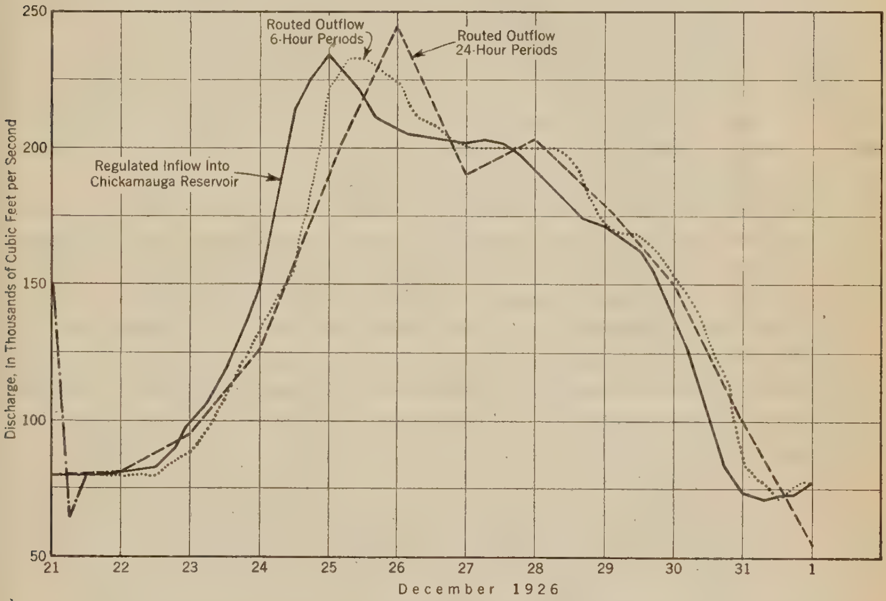

FIG. 12.—COMPARISON OF 1926 FLOOD HYDROGRAPHS AT CHICKAMAUGA DAM SITE USING 24-HOUR AND 6-HOUR PERIODS IN ROUTING

The method of using the shorter time period when it is required does not entail much extra work. It can be accomplished with the same sets of storage curves used in the 24-hr routing. Since available discharge or stage records often indicate the average day's flow or the rate of flow at a certain hour, it is advisable to convert these records into discharge curves or hydrographs. Then, from the curves, or hydrographs, the probable rate at any hour or the total for any period may be taken. The flow into the upper end of the reservoir and the local and tributary flow into the reservoir are adjusted in that manner. The sum of these two flows for each period is the flow at the beginning of each period, termed I_1, in the paper. Then all values of I_1 are divided by 4 if the time period desired is 6 hr. Next, the outflow at the beginning of the period (O_1) is found in the usual manner, but it is converted to $\frac{O_1}{4}$ so as to find the storage at the beginning of the next period (S_1) and to find $O_2 + S_2$. Several other methods may be developed, but this one may be applied to both natural conditions and to those with regulated conditions. If the storage curves for the natural conditions, however, are in the form of "outflow-plus-storage," these curves must be put in the form of "storage" alone before this method may be applied.

Also, in Fig. 12, is shown the use of two different starting flows and their result on the computed outflow hydrograph. The lower starting flow is one in which the outflow is assumed equal to the inflow to start the routing which is usually just about the actual condition at the start. The other starting point, the higher flow, can be seen to dip sharply immediately in the first 6-hr period used; but the satisfactory part about it is the rapidity with which it gets back to the original outflow hydrograph with its 6-hr period. This demonstrates that practically any outflow may be assumed to start a routing and that, regardless of which one is used, the critical peak point is not affected. Furthermore, it shows that small errors in curve reading are compensated for within several periods in the routing procedure.

A simple check on the arithmetic of a routing computation is convenient. First, all the inflows at the beginning of each period are added. Then a "correction" is determined which, when applied to the summation of the inflows, should equal the summation of all the outflows. To find this "correction" the inflow, outflow, and storage are considered for the first and the last periods of the routing computation. The first and last periods are used because the actual routing computation involves the period between the middle of the first and the middle of the last periods. Actually, the first half of the first period and the second half of the last period do not enter into the total volume of the routed hydrograph. The difference between inflow and outflow, in which the unit is in day-second-feet, therefore, is divided by 2. This is done for both the first and the last period. The difference between storages, in which the unit is acre-feet on the first and the last period, is divided by 2 to convert it to day-second-feet. The sign applied to the flows is negative when the inflow exceeds the outflow. If the storage in the last period is larger than in the first period, its sign is negative. Finally, the algebraic sum of these three corrections is applied to the summation of all inflows at the beginning of each period, and this should equal the summation of all the outflows.

Using Table 3(*a*) the correction for December 21 is $37 - 45 = -8$ day-sec-ft; for December 31, it is $211 - 170 = +41$ day-sec-ft; and for the storage correction the difference between December 21 and 31 is $48 - 291 = -243$ acre-ft. Then, $\frac{-8 + 41 - 243}{2} = -105$ day-sec-ft; and,

$$\begin{aligned} \Sigma I_1 &= 2\,045 \text{ day-sec-ft} \\ &\underline{-\ 105} \text{ day-sec-ft} \\ \Sigma O_1 &= 1\,940 \text{ day-sec-ft} \end{aligned}$$

which checks.

Possibly, some improvement may be made in the manner of determining a set of storage curves for the Tennessee River. Another method might take care of bank storage and water losses, where the authors' method does not, because storages were determined through the use of topographic maps. The manner in which the flow is varied from the upper end of the reach to the lower end might be changed so that better routing computations would result. Good results are obtained in reaches where a relatively large tributary enters near the middle of the reach, and the flow is varied on the basis of the drainage area. In reaches where no large tributary enters and the flow is varied directly with the length of the reach, results are only fair. Floods in which there is a sharp peak do not work out as well in reaches where the storage curves were made up by varying the flow with the length of the reach. Floods with a peak of long duration seem to yield better results; and, better results might be obtained if the flow were varied by a curve instead of a straight line between the upper and lower end of a reach in which no large tributary contributes its flow.

WALDO E. SMITH,[44] ASSOC. M. AM. SOC. C. E. (by letter).—The authors are to be commended for their splendid contribution to hydraulic literature pertaining to the routing of floods. The writer is pleased to note that their method abandons completely the old conceptions of prism and wedge storage and likewise circumvents the seemingly insurmountable step from theoretical and approximate formulas for uniform slopes and channels to their application to actual channels of varying section and slope. Furthermore, after the preliminary work is complete and the basic curves are computed and plotted, the method is expeditious and is satisfactory for use for analytical purposes especially, and for limited use in forecasting and in operation for the flood control of a reservoir system applied to the natural channel and channel reservoir reaches.

Flood-routing problems, quite naturally, resolve themselves into two quite distinct although overlapping parts, the first of which, in turn, is further divided into two aspects, as follows:

(1) The routing of floods in natural channel and channel reservoir reaches:
  (*a*) For analytical purposes, such as the critical survey of the operation and routing undertaken during the progress of a flood; and,
  (*b*) For forecasting and flood-control operation of reservoirs.
(2) The routing of floods through comparatively deep, broad, valley reservoirs in which the back-water transition curve is a negligible item in the volume of flood storage available. (The latter may likewise be subdivided but that subdivision is not pertinent to the subject as developed by the authors.)

The work of the authors is especially applicable to Problem 1(*a*). It can be used in forecasting Problem 1(*b*) also, but only to the extent that the upper end of the uppermost reach may be forecast.

The authors mention that the time of travel through the reach should usually be about the interval used in the analysis and that, in most cases, this appeared to be about 24 hr. The Muskingum Watershed Conservancy District has a method under study and in partial use that facilitates forecasting to successively later times as progress is made down stream. Assuming the period of forecasting to be for 24 hr, the technique developed by the District bases the forecasting for the lower station on the inflow into the reach at the present time, the anticipated storage in the reach 12 hr hence, and the local inflow 24 hr hence by the distribution graph method. Upon the completion of a storm, knowing the run-off factor, the forecaster can complete his work for the remainder of the flood period. By the District method it has been found, if the time of travel is 24 hr, that 12, 6, or 4-hr intervals can be used in routing as satisfactorily as 24 hr. The time interval use should never be greater than the so-called time of travel through the reach but it may be less. The development of outflow-plus-storage curves appears to have the effect of limiting the use of these curves to 24-hr periods. In the District work on analytical studies, the stage at the lower station has been based on estimating the outflow at the same instant of time that the inflow is known, similar to the work outlined by the authors.

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[44] Hydr. Engr., Muskingum Watershed Conservancy Dist., New Philadelphia, Ohio.

The writer acknowledges that there is a substantial difference between the problems on the Tennessee Valley and those of the Conservancy District. In the former the upper end of the uppermost forecast reach drains about 9 000 sq miles and the routing is carried through a system of reservoirs that are essentially channel reservoirs. The total drainage area involved in the Conservancy District, on the other hand, is only 8 000 sq miles and the reservoirs are largely head-water reservoirs with storage capacities relatively large in comparison to the water-shed above; and routing and forecasting on the main streams are almost entirely in natural channels. The forecast method used by the District is found not to be uniformly applicable to all reaches, and, in some reaches, in which a large tributary enters, it has not yielded a satisfactory solution. It has been necessary to use a number of variations in the general procedure.

The authors were fortunate indeed to secure contour maps of the channel and valley floor which gave them the opportunity to determine storage in 2-mile reaches by means of contours and the profile of the stream. They were fortunate even to have cross-sections at sufficiently representative and frequent intervals. Such maps and cross-sections are not usually available and one must rely on the hydrographs to find the valley storage at any particular time for a past flood, for the development of the storage curves. This has the disadvantage of involving the local inflow and its distribution with regard to time as well as valley storage and the difference between the two hydrographs at any particular time is thus a function of two independent variables which can only be determined by trial and error. The volume of local inflow is determined by means of the hydrographs as outlined by the authors; but one must realize that this throws the entire cumulative error in rating curves into the local inflow which is usually the smallest part. The procedure on the work of the Muskingum Watershed Conservancy District has been to assume an apparently reasonable distribution graph and determine the valley storage in a reach for a given flood on this basis. A storage curve, usually with a hysteresis loop, is found and smoothed up, and the distribution coefficients are modified accordingly. Analyzing a second flood of similar magnitude, using the same distribution graph, the new storage relationship is determined, which usually differs from the first. The distribution graph is then adjusted to bring the two curves as closely together as possible and the resulting channel storage relationship is used. Local inflow in all cases has been applied to the lower end of the reach, however, instead of at the upper end as has been done by the authors. This has the tendency to change the characteristics of the distribution graph considerably, giving a relatively small percentage of run-off for the first day (or other time interval) and a much larger percentage on the second day. The effect is substantially the same, but the technique used by the District makes it convenient to find the discharge at the lower end of the reach and then add the value for the local inflow to establish the forecast gage height.

One of the difficulties of the old prism and wedge storage idea was that it failed to take into account the fact that the flood crest, or the trough between crests, is sometimes between the limits of the reach. Either condition alters prism and wedge valley storage substantially. The authors seem to offer no means of overcoming this difficulty inasmuch as the method assumes a constant rate of change of inflow, outflow, and, thus, the storage rate for each 24-hr routing period. The same problem has been a stumbling block in some cases with the forecast method used by the District outlined briefly; but by using a shorter interval than the time of travel in the reach it has been possible in some instances to compensate for it satisfactorily.

The back-water transition curves in Figs. 3 and 4 scarcely appear to be typical even of channel reservoirs. If the straight lines indicated for the profile are chords of the actual curve and were used as shown, it appears that a large thin volume of storage is included between the Chickamauga Dam and the Hiwassee River that is not actually available because of the tendency for the profile in the reservoir to be concave upward and to be nearly horizontal at the dam.

There is one question that the writer raises in connection with the speeding up of the flow through the channel reservoir. The authors have indicated that they believe that the fact that it takes less storage to increase the flow a given amount down stream compensates satisfactorily for this. The writer is not sure that this is complete compensation if the time of travel is decreased substantially. The writer would also like to know why, in the first instance, O_1 must be estimated or assumed if there is a gaging station at the lower end of the reach?

EDWARD J. RUTTER,[45] AND QUINTIN B. GRAVES,[46] ASSOC. MEMBERS, AM. SOC. C. E., AND FRANKLIN F. SNYDER,[47] JUN. AM. SOC. C. E. (by letter).—The writers appreciate the interest shown by the discussions of this paper. Several interesting suggestions have been given which add to the treatment of the subject.

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[46] Instr. in Civ. Eng., Dept. of Civ. Eng., Univ. of Texas, Austin, Tex.

[47] Hydr. Engr., Dept. of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pa.

Mr. Camp raises the question as to the accuracy of discharges at a "routing station" which is affected by confluence with another stream. This factor is an important feature in any stage-discharge relation, and the more accurately the relation is known the more accurate will be the discharge and elevation obtained by the routing method described in the paper. It should be noted, in studies of the 1937 flood on the Lower Tennessee River, mentioned by Mr. Camp, that the reason observed stages at the Gilbertsville, Ky., dam site were not checked is that the rating curve had to be extended into a region where no measurements of flow were available and an extremely high submergence obtained.

Mr. Goodrich offers an improvement on his method of routing with a single discharge curve and single capacity curve. It does not seem possible, however, that the graphical-mechanical solution is more rapid than his earlier method, which was readily adapted to the writers' problem of using sloped storage instead of level storage.

Mr. Myers mentions several flood studies which have been made and compares them in a general way with those on the Tennessee River. As the flood wave travels down stream the effect of up-stream tributary reservoirs tends to become less, the percentage of uncontrolled local area becomes larger, and the natural hydrograph becomes broader. All these factors combined tend to reduce the reservoir storage effect, provided there is insufficient main-river reservoir storage to counterbalance that tendency.

Mr. Powell has demonstrated a procedure for routing floods in the "natural condition" through a reach of a river for which the storage quantities were based on a study of past floods and not necessarily on storage obtained from topographic maps or cross-sections. He has, effectively, eliminated the storage loop that is obtained when storage is plotted against outflow alone. This requires accurate stream-flow records to determine the values of storage in the reach for various values of inflow and outflow. Difficulty might be experienced if it were necessary to extend these data to take care of floods considerably larger than any experienced to date, although the curve of Fig. 11 could be extended reasonable amounts.

It is desirable to use the same routing method in the "natural condition" as in the condition with the dam in place. No accurate check can be made on any procedure for routing through a reservoir created by a dam until after the dam is built. The writers believe that by treating the two cases as nearly as possible in the same manner, and by making as accurate a check as possible on the "natural condition" routing, the results obtained for the case with the dam in place will be more reliable.

The writers have applied Mr. Powell's procedure to several different conditions and have found that it will give satisfactory results. It should be noted, in his discussion, that the inflows used in Fig. 11 are also used in the routing shown in Table 5, and that all results depend on the routing method of the writers and are not taken from any really natural flows. This does not constitute an adequate check on the method for use with other floods; hence, the writers used Fig. 11 on flows for the Chickamauga reach for February and March, 1917, and for March and April, 1936, the results (Table 6) of which indicate very clearly that Mr. Powell's procedure will work satisfactorily. His procedure was then tried on three other reaches and the factor, $0.4\ I + O + S$, was found to vary according to the conditions of the reach. Having found the curve for another reach (similar to Fig. 11), other floods were routed using this curve. All results indicate a workable procedure.

TABLE 6.—COMPARISON OF OUTFLOW RATIO O_1, APPLIED TO CHICKAMAUGA (TENN.) REACH

(a) 1936 FLOOD						(b) 1917 FLOOD					
Period (1)	Writers' method (2)	Powell's method (3)	Period (1)	Writers' method (2)	Powell's method (3)	Period (1)	Writers' method (2)	Powell's method (3)	Period (1)	Writers' method (2)	Powell's method (3)
March:			April:			February:			March:		
16	25	25	1	117	118	18	43	43	6	310	310
17	29	29	2	97	93	19	55	54	7	332	336
18	36	36	3	136	129	20	85	82	8	330	335
19	46	48	4	167	173	21	120	120	9	278	285
20	55	54	5	156	159	22	142	144	10	180	177
21	62	65	6	140	137	23	140	141	11	103	98
22	77	76	7	175	173	24	127	127	12	87	89
23	74	74	8	217	220	25	101	100	13	86	85
24	71	72	9	217	222	26	83	88	14	96	96
25	93	91	10	191	190	27	83	81	15	118	118
26	143	141	11	149	148	28	80	80	16	118	117
27	185	187	12	113	113	March:			17	111	110
28	225	223	13	91	93	1	83	81	18	126	126
29	243	246	14	78	79	2	115	110	19	143	144
30	235	239	15	68	69	3	165	167	20	145	147
31	188	180	16	62	62	4	214	211	21	135	135
....	....	....	17	58	56	5	268	265	22	124	120
....	....	....	18	52	52	....	....	....	....	....	....

TABLE 7.—RELATION BETWEEN UP-STREAM FLOW AND LOCAL FLOW

Day	(a) NO LOCAL FLOOD					(b) NO UP-STREAM FLOOD				
	I_1	Powell		Writers		I_1	Powell		Writers	
		O_1	S_1	O_1	S_1		O_1	S_1	O_1	S_1
December:										
21	45	37	48	37	48	45	37	48	37	48
22	92	60	88	60	88	45	45	56	45	56
23	110	99	131	100	130	53	47	62	49	60
24	153	124	171	124	169	68	59	77	61	71
25	201	165	236	160	239	82	72	96	76	84
26	202	195	279	194	288	80	82	104	81	89
27	213	202	297	200	309	76	78	100	78	86
28	222	212	318	211	333	79	77	100	77	86
29	201	216	313	216	329	74	76	100	77	85
30	191	204	285	205	300	62	72	88	67	77
31	157	187	242	187	256	50	58	70	56	66
1*	124	150	186	155	195	42	47	57	46	56

* January 1.

Another condition that affects the value of the coefficient 0.4 is the relation between the up-stream flood and the local flood. A comparison was made between Mr. Powell's method and the writers' method for two cases—(1) assuming no local flood; and (2) assuming no up-stream flood. As expected, the peak storage value was smaller in the first case and larger in the second as compared to that from the curves in the paper. Table 7 shows the comparison for the two cases.

For the "natural condition," Mr. Powell's procedure is simpler than the writers' method, as far as curve reading is concerned, but required calculations are increased; hence, the type of procedure to use depends upon personal preference, or upon original information available.

Values of $0.4\,I + O + S$ were computed for the Chickamauga reach for the case with the dam in place, with regulation to several different elevations and with no regulation. These were plotted against storage, and intermediate curves for actual use were interpolated. With this set of curves, direct routings are possible without resorting to a trial-and-error procedure as in the writers' method. However, additional arithmetical work is involved, as in the "natural condition." There is one other point to consider, namely, the $(0.4\,I + O + S)$-curves for use with the dam in place must be taken from routing curves similar to those developed by the writers when making a study of a dam which is to be constructed.

Mr. Collins' criticism of the assumed profile which gives a decrease in flow at the mouth of a large tributary at the beginning of a rise (December 22) is a valid one. However, the actual value of storage cannot be greatly in error since the large tributary is near the middle of the reach. The assumed flows at the upper end of the reach, just above the tributary, just below the tributary, and at the lower end, are 76 000, 74 000, 62 000, and 60 000 cu ft per sec, respectively. The actual flows at the same points are approximately 76 000, 62 000, 74 000, and 60 000 cu ft per sec, respectively.

Mr. Collins is correct in his statement that, for some reaches, it may be better to treat tributary flows separately and add them to the routed main stream outflow at the lower end of the reach. This is particularly true of a tributary that enters the reach close to the lower end, and, consequently, has a relatively small effect on the storage values. However, by the same reasoning, the flow from a tributary which enters near the upper end of a reach should be routed with the main stream flow. Considerable study is necessary to determine if, and when, separate routing of tributary flows would be required. Each reach must be treated as a separate problem, and the procedure that gives the best results should be used. As stated in the paper (see heading, "Reaches"), the ideal procedure is to have routing stations at the mouth of all large tributaries.

Mr. Townsend presents a comparison of routing with 24-hr and 6-hr time periods, with the same inflow. In the case shown, the inflow has a relatively sharp peak, due to the method of operation used, and hence the reach assumes the characteristics of one in which a shorter time period than 24 hr gives more reasonable results. The method of checking the arithmetic described by Mr. Townsend is convenient and rapid. However, it does not check the reading of curves; these must be read again for a complete check. Mr. Townsend also notes several points in which improvements might be made in the procedure but gives no suggestions as to the method of making these improvements. Possibly more study would reveal better methods for the determination of the non-steady flow profiles in some reaches.

Mr. Smith is partly correct in his statement that, if the time of travel in a reach is 24 hr, 12-hr, 6-hr, or 4-hr intervals can be used as satisfactorily as 24-hr intervals, although this would not be the case for a reach with little local or tributary inflow. Since the interval should be determined by the stream characteristics, the use of shorter intervals in routing adds to the work without materially increasing the accuracy. The same base data used in plotting the outflow-plus-storage curves for 24-hr periods can be used to plot similar curves for any time interval desired. For example, if a 6-hr routing period is required, the outflow O would be plotted against $O + 4\,S$, instead of $O + S$.

The back-water profiles shown in Figs. 3 and 4 have the same general appearance as those in Fig. 2 for the natural condition. At the time these curves were computed, every reasonable attempt was made to eliminate any sharp changes of slope, and the lines shown represent, as nearly as possible, the profiles that might occur. Probably, after the reservoir has been constructed and operated through several floods, a revision could be made in the shape of the profiles of water surface.

Mr. Smith questions the speeding up of flow through the reservoir. The writers have based the storage on the back-water profiles. If the profiles are correct for the flow conditions indicated, the discharge at the dam cannot increase faster than the corresponding inflows, profiles, and storage allow. The change in time of travel is shown in the shape of the profile, and as a result, the value of storage is different.

The value of O_1, in the first instance, need not be assumed nor estimated for an actual flood if there is a gaging station at the routing station. For hypothetical floods and at all routing stations where there has been no gaging station the initial value of O_1 must be estimated.

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PIN-CONNECTED PLATE LINKS

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WITH DISCUSSION BY MESSRS. HAROLD D. HUSSEY, AND BRUCE G. JOHNSTON.

SYNOPSIS

Results of 106 tests of differently proportioned steel plates are presented in this paper. The specimens were loaded in tension by means of 3-in. steel pins which passed through holes in either end of the plate. The effects of plate thickness, side and bottom edge distances, and pin clearance were studied in relation both to the general yield and the ultimate strength of the links. The "dishing" phenomenon, which greatly reduces the ultimate strength of thin plates unsupported laterally, was given particular attention. Empirical formulas are given for predicting the general yield and ultimate strength of the plates, together with formulas for producing a pin-plate connection of balanced design.

INTRODUCTION

The design of steel eye-bars has developed an accepted practice in regard to the relative proportions and distribution of material in the eye-bar head. The present investigation is concerned with pin-connected plates which do not have reduced width in the body as in the case of the forged eye-bar, but which are made simply by boring pinholes near the ends of structural steel plates. Examples of this type of connection are found in steel sheave blocks, derrick tackle, anchorage bars for cables, and various types of erection equipment. Although the plates tested differ from forged eye-bars, their action around the pinhole during the progress of failure is similar to that in the eye-bar head.

The problem of designing such pin-connected plate links was encountered on a large scale in arranging the hangers for raising the suspended span of the Quebec Bridge.[2] Tests were made both on model links and on full-sized hangers with 12-in. pins, and the results show a good correlation with those of the present tests. The general relation between plate thickness and "dishing" of thin plates was noted in the Quebec tests.

The bibliography on the subject of stresses in plates around pinholes is extensive, but most of it is concerned primarily with the study of stress dis-

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[2] "The Quebec Bridge," Vol. 1, pp. 222 to 227, Rept. of Canadian Govt. Board of Engrs., 1919.

tribution in the elastic range. References to various photo-elastic, mathematical, and experimental studies of the stresses in eye-bar heads may be found in the work of Professor S. Timoshenko.[3] Mathematical stress analyses require an assumed distribution of bearing pressure between pin and pinhole as a starting point. The actual bearing stress distribution depends on pin clearance and varies somewhat during the application of load. The maximum stresses are localized and, for ductile materials, the initial yielding is restrained from rapid progress by surrounding low-stressed areas and, in the early stages, has no appreciable effect on the elastic behavior of the link as a whole. Furthermore, these localized stresses have practically no effect on the ultimate strength of the link under static load. Hence, the use of a ductile material such as structural steel is very advantageous.

Local yielding in the plate at the contact point, except in the case of tight-fitting pins, will occur even at working loads. The general, or over-all yield at the end of the plate, occurs at a much later stage than initial local yielding. The term, "general yield point," will be used herein to designate the load or average bearing stress at which the slope of the curve of load plotted against deformation between pin and plate (about 3 pin diameters away) is three times the initial slope.

Because of the early local yielding and associated stress redistribution, mathematical formulas for maximum local stresses based on the assumption of elastic behavior have doubtful utility in the actual design of pin-connected steel plates under steady loads. For non-ductile materials, or for ductile materials designed for repeated stress, the elastic stress distribution is of importance. An approximate mathematical solution by J. Beke[4] has been applied to experimental tests of large pin-connected members in the Hugo-Preuss Bridge, in Berlin, Germany.[5]

Beke's formula is based on the simplest possible assumptions and is applicable in the elastic range. More exact solutions become increasingly complex and, in any case, give little indication as to the actual load carried at the general yield point or ultimate. To have practical value, a mathematical solution should be based on the theory of plasticity, and should include the study of the phenomenon of plastic "dishing" of thin plates. The solution of this problem has not been attempted in the present investigation. Of first importance to the designer are the answers to the questions: "How much load will a structural unit carry without excessive deformation?" and "What is its strength at failure?" In the present instance, the answer is given in the form of empirical equations, validated by a wide range of tests, and concerned with average stresses at general yield and ultimate. The large number of relatively thin plates tested give information on the greatly reduced margin between general yield and ultimate which is caused by the "dishing" type of failure in laterally unrestrained plates.

TEST PROGRAM AND PROCEDURE

The program of 106 tests was designed to cover the critical range of variables. The same size of pin, 3 in. in diameter, was used in all tests. The four variables ranged as follows: (1) Plate thicknesses varying from $\frac{1}{8}$ to $\frac{3}{4}$ in.; (2) plate widths of 6, 8, and 10 in.; (3) edge distance behind pinhole varying from 1 in. to 3.2 in.; and (4) pin clearance varying from a tight fit to 0.2 in.

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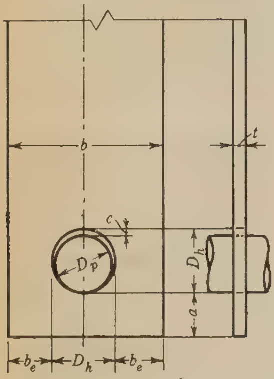
[3] "Theory of Elasticity," p. 120.

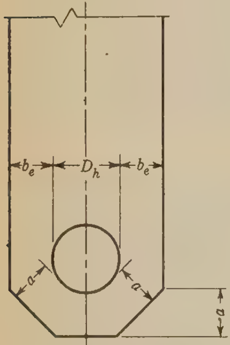
[4] "Der Eisenbau," 1921, p. 233.

[5] *Die Bautechnik*, February 12, 1932, pp. 79–83.

The dimensions of the test specimens were measured to the nearest thousandth of an inch by micrometer calipers, and are indicated in Table 1. This table shows in detail the range of variables covered. Fig. 1 shows the notation adopted in regard to pin dimensions and dimension ratios.


b
t
c
Dp
Dh
a
be
Dh
be

(a) TESTS 1 TO 103



(b) TESTS 104 TO 106

FIG. 1.—DIMENSION NOTATION FOR TEST SPECIMENS (SEE TABLE 1)

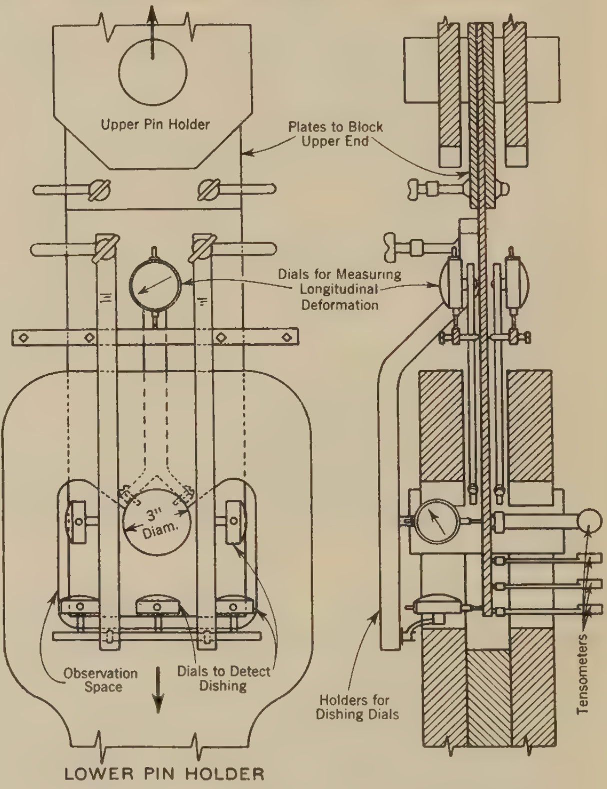

Upper Pin Holder
Plates to Block Upper End
Dials for Measuring Longitudinal Deformation
3" Diam.
Observation Space
Dials to Detect Dishing
Holders for Dishing Dials
Tensometers
LOWER PIN HOLDER

FIG. 2.—APPARATUS FOR TESTING PIN-CONNECTED STEEL PLATE LINKS

The material specified was Universal Mill Plate with rolled edges, a medium grade structural steel free from rust. All the plates of similar width and thickness were cut from the same stock with an additional section cut for tensile test coupons. The ends of the specimens were flame-cut to a length an inch or more longer than required, and were cut at the testing laboratory by hacksaw to the exact length. Two tensile coupons cut parallel with the axis of rolling and two cut transversely were made from each plate series, the results being given in Table 2.

The arrangement of the apparatus used during a typical test is shown diagrammatically in Fig. 2, and by photograph in Fig. 3. The upper pin-holder

TABLE 1.—DIMENSIONS AND TEST DATA; PIN-CONNECTED PLATE LINKS

Test No.	MEASURED DIMENSIONS, IN INCHES				SURFACE SLIP LINES; LOAD, IN KIPS		LOAD, IN KIPS†			
	Diameter of hole, D_h (1)	b_e (2)	t (3)	a (4)	First observed line* (5)	General spread of lines (6)	Local yield* (7)	General yield (8)	Initial "dishing" (9)	Ultimate strength* (10)
1	3.015	2.510	0.128	0.997	6.5 *BS*	8.0	5.6 *B*	8.5	8.0	14.5 *D*
2	3.019	2.510	0.128	1.671	7.0 *BC*	10.0	9.0 *B*	11.6	12.5+	18.0 *D*
3	3.017	2.511	0.128	2.193	10.0 *BC*	14.5	10.8 *B*	14.1	14.0	17.3 *D*
4	3.016	2.510	0.128	2.592	7.0 *BC*	15.0	11.2 *E*	15.2	13.0	17.4 *D*
5	3.015	2.512	0.128	2.881	9.0 *BC*	....	11.5 *E*	15.1	14.0	16.3 *D*
6	3.015	2.515	0.128	3.204	7.0 *BC*	....	12.0 *E*	14.4	14.0	15.0 *D*
7	3.069	2.486	0.127	0.970	6.0 *BS*	....	5.2 *B*	6.3	7.5+	15.2 *D*
8	3.070	2.482	0.128	1.700	7.5 *BC*	9.5	7.2 *B*	9.1	9.8+	15.2 *D*
9	3.069	2.482	0.128	2.185	6.0 *BC*	12.5	6.0 *B*	12.6	10.0	15.0 *D*
10	3.066	2.484	0.128	2.582	5.0 *BC*	12.5	10.5 *E*	12.4	11.0	13.5 *D*
11	3.067	2.479	0.128	2.905	5.0 *BC*	....	10.3 *B*	11.1	11.0	12.0 *D*
12	3.068	2.484	0.127	3.190	4.0 *BC*	....	11.9 *B*	12.6	11.0	13.5 *D*
13	3.017	2.517	0.190	1.018	....	11.5	10.0 *B*	10.8	10.5	25.1 *D*
14	3.014	2.510	0.189	1.700	12.0 *BS*	15.0	13.7 *B*	16.6	14.5	27.7 *D*
15	3.019	2.509	0.190	2.209	12.0 *BS*	18.0	16.4 *B*	17.7	19.0+	26.5 *D*
16	3.018	2.518	0.190	2.584	14.0 *BC*	22.0	20.3 *B*	21.3	22.0	28.9 *D*
17	3.019	2.511	0.190	2.888	16.0 *BC*	22.0	....	19.8	21.0	26.3 *D*
18	3.020	2.513	0.190	3.180	12.0 *BS*	17.0+	19.0 *E*	21.5	20.0	26.0 *D*
19	3.073	2.484	0.190	1.008	4.0 *BS*	7.5	4.2 *B*	9.7	7.0	25.3 *DF*
20	3.074	2.478	0.189	1.889	9.0 *BC*	14.5	12.1 *B*	14.1	14.5	29.7 *D*
21	3.074	2.479	0.189	2.206	5.0 *BC*	16.0	15.8 *B*	18.5	18.5+	27.0 *D*
22	3.074	2.479	0.188	2.606	7.0 *BC*	18.0	20.0 *E*	21.5	....	26.4 *D*
23	3.075	2.473	0.189	2.880	....	20.0	16.0 *E*	18.2	16.0	22.0 *D*
24	3.074	2.480	0.188	3.208	10.0 *BC*	18.0+	....	20.0	20.0	24.4 *D*
25	3.017	2.485	0.254	1.016	13.0 *BS*	20.0	12.3 *B*	16.8	....	38.0 *FB*
26	3.018	2.483	0.254	1.697	21.0 *BS*	23.0	19.1 *B*	23.2	20.0	42.5 *D*
27	3.016	2.486	0.255	2.216	15.0 *BC*	....	24.4 *B*	28.2	27.0	43.4 *D*
28	3.019	2.480	0.254	2.619	13.0 *BC*	31.0	26.2 *E*	29.7	29.0	43.4 *D*
29	3.018	2.483	0.255	2.915	15.0 *BC*	27.0	38.3 *B*	31.0	25.0	43.3 *D*
30	3.019	2.481	0.253	3.219	11.0 *BC*	31.0	29.4 *E*	32.0	34.0	41.0 *D*
31	3.072	2.458	0.256	0.998	7.0 *BC*	12.0+	8.1 *B*	12.4	10.0	36.9 *FB*
32	3.069	2.460	0.258	1.685	10.0 *BC*	20.0	15.2 *B*	19.5	20.0	49.0 *FB*
33	3.069	2.460	0.257	2.211	15.0 *BS*	26.0	21.1 *B*	25.8	25.0	48.2 *D*
34	3.072	2.452	0.256	2.590	9.0 *BC*	24.0+	24.2 *B*	27.8	27.0	43.0 *D*
35	3.073	2.459	0.257	2.908	9.0 *BC*	30.0	28.3 *B*	28.0	31.0	41.5 *D*
36	3.071	2.459	0.257	3.175	9.0 *BC*	27.0	25.8 *B*	29.0	29.0	39.6 *D*
37	3.016	2.493	0.373	0.997	17.0 *BS*	23.0	15.0 *B*	21.6	None	53.7 *FB*
38	3.016	2.495	0.372	1.709	20.0 *BC*	30.0	24.3 *B*	29.4	None	72.0 *FB*
39	3.018	2.489	0.373	2.201	21.0 *BC*	36.0	30.4 *B*	36.5	38.0+	76.5 *D*
40	3.017	2.491	0.374	2.593	21.0 *BC*	40.0	34.5 *E*	40.0	46.0+	76.5 *D*
41	3.019	2.489	0.374	2.896	18.0 *BC*	49.0	37.8 *B*	42.0	48.0+	73.4 *D*
42	3.017	2.492	0.371	3.199	21.0 *BC*	....	39.6 *E*	43.0	42.0	75.7 *D*
43	3.066	2.466	0.373	0.959	13.0 *BC*	16.0	11.6 *B*	17.0	None	58.9 *FB*
44	3.066	2.474	0.371	1.681	11.6 *BC*	26.0	21.7 *B*	24.5	None	78.0 *FB*
45	3.066	2.469	0.371	2.193	25.0 *BC*	32.0	27.4 *B*	32.5	34.0+	78.5 *D*
46	3.069	2.467	0.371	2.594	30.0 *BC*	36.0	31.2 *B*	36.5	39.0+	78.5 *D*
47	3.067	2.465	0.372	2.910	16.0 *BC*	38.0	29.0 *B*	39.5	42.0+	73.00
48	3.066	2.469	0.371	3.200	16.0 *BC*	40.0	35.5 *B*	40.5	40.0+	70.20
49	3.014	2.506	0.496	1.019	....	30.0	22.1 *B*	28.5	None	73.4 *F*
50	3.014	2.514	0.500	1.700	18.0 *BC*	39.0	36.0 *B*	40.0	None	101.8 *F*
51	3.016	2.510	0.495	2.210	35.0 *BS*	42.0	36.0 *B*	47.0	....	....
52	3.014	2.511	0.498	2.580	27.0 *BC*	....	46.4 *B*	54.5	Ult.	133.4 *D*

* *B* = below pin; *C* = crushing; *D* = "dishing"; *E* = side of pin; *F* = fracture; and *S* = shear.
† 1 kip = 1 000 lb.

passed through the upper fixed head of the testing machine where it engaged transverse bars by means of lugs which allowed freedom of motion of the holder. The lower holder passed through the moving head of the testing machine and was open at the side to allow observation of the test. The upper end of the test specimen was blocked by plates on either side to ensure failure at the lower pinhole. The holders were built entirely by flame-cutting and welding steel slabs.

TABLE 1.—(*Continued*)

Test No.	MEASURED DIMENSIONS, IN INCHES				SURFACE SLIP LINES; LOAD, IN KIPS		LOAD, IN KIPS†			
	Diameter of hole, D_h (1)	b_e (2)	t (3)	a (4)	First observed line* (5)	General spread of lines (6)	Local yield* (7)	General yield (8)	Initial "dishing" (9)	Ultimate strength* (10)
53	3.014	2.496	0.492	2.900	15.0 *BC*	46.0	46.0 *E*	53.5	Ult.	128.3 *D*
54	3.014	2.509	0.499	3.215	30.0 *BC*	52.0	49.8 *E*	58.0	"	126.4 *D*
55	3.067	2.479	0.494	0.993	20.0 *BC*	22.0	14.5 *B*	21.0	"	72.6 *D*
56	3.067	2.471	0.494	1.685	22.0 *BC*	31.0	28.5 *B*	33.6	None	101.5 *FB*
57	3.066	2.480	0.494	2.174	25.0 *BC*	44.0	35.3 *B*	45.0	"	118.9 *FB*
58	3.066	2.475	0.495	2.601	20.0 *BC*	48.0	35.0 *B*	47.5	"	138.2 *FB*
59	3.066	2.478	0.497	2.902	25.0 *BC*	51.0	39.5 *B*	50.8	Ult.	130.4 *D*
60	3.069	2.475	0.498	3.183	25.0 *BC*	52.0	42.0 *E*	57.0	"	121.2 *D*
61	3.015	2.494	0.745	0.988	30.0 *BC*	....	....	38.0	None	108.4 *FB*
62	3.016	2.505	0.747	1.702	30.0 *BC*	....	....	58.0	"	150.6 *FB*
63	3.016	2.491	0.746	2.176	24.0 *BC*	....	....	66.8	"	162.0 *FB*
64	3.016	2.492	0.745	2.579	....	....	....	72.6	"	185.9 *FB*
65	3.019	2.497	0.747	2.906	....	....	....	80.5	"	204.0+
66	3.017	2.490	0.745	3.207	42.0 *BC*	....	....	82.0	"	204.0+
67	3.067	2.465	0.745	1.010	....	....	....	31.2	"	111.6 *FB*
68	3.067	2.459	0.742	1.679	....	....	....	43.5	"	147.2 *FB*
69	3.067	2.465	0.745	2.215	25.0 *BC*	....	....	54.6	"	172.8 *FB*
70	3.067	2.473	0.748	2.607	....	....	....	64.0	"	200.0+
71	3.065	2.464	0.745	2.916	30.0 *BC*	....	....	73.0	"	200.0+
72	3.070	2.475	0.746	3.186	42.0 *BC*	....	....	76.0	"	200.0+
73	3.024	1.471	0.188	0.981	11.0 *BS*	15.0	7.4 *B*	11.8	12.5+	24.4 *D*
74	3.019	1.476	0.187	1.687	10.0 *BS*	15.0+	12.8 *E*	16.6	17.0+	28.4 *D*
75	3.018	1.473	0.187	2.213	14.0 *ES*	20.0	18.0 *B*	20.0	20.0+	26.2 *D*
76	3.019	1.475	0.188	2.611	12.0 *BC*	22.0	11.2 *E*	20.8	19.7	24.0 *D*
77	3.018	1.475	0.188	2.909	15.0 *ES*	22.0	13.2 *E*	21.0	18.0	25.4 *D*
78	3.019	1.474	0.187	3.205	12.0 *BC*	22.0	17.3 *E*	22.0	21.0	24.8 *D*
79	3.018	1.510	0.514	0.978	21.0 *BS*	36.0	24.3 *B*	33.0	None	64.6 *FB*
80	3.016	1.509	0.512	1.674	24.0 *BC*	....	39.8 *B*	44.0	"	98.6 *FE*
81	3.017	1.504	0.515	2.201	....	49.0	43.0 *E*	55.0	"	100.5 *FE*
82	3.016	1.506	0.512	2.571	35.0 *BC*	53.0	38.0 *E*	58.0	"	99.8 *FE*
83	3.015	1.508	0.513	2.905	35.0 *BC*	....	46.0 *E*	58.0	"	100.9 *FE*
84	3.015	1.506	0.513	3.210	25.0 *BC*	....	45.0 *E*	58.8	"	100.5 *FE*
85	3.018	3.487	0.192	0.988	8.0 *BS*	....	10.0 *B*	11.3	11.0+	25.5 *D*
86	3.018	3.487	0.190	1.711	12.0 *BC*	14.0	13.9 *B*	15.2	15.0+	29.2 *D*
87	3.020	3.480	0.190	2.216	12.0 *BC*	....	16.2 *B*	19.6	20.0+	28.1 *D*
88	3.020	3.480	0.190	2.587	15.0 *BS*	21.0	17.0 *B*	20.2	22.0+	28.1 *D*
89	3.024	3.488	0.191	2.906	12.0 *BC*	21.0	19.4 *B*	21.8	18.0	27.6 *D*
90	3.023	3.475	0.190	3.198	11.0 *BC*	21.0	18.6 *B*	21.5	20.0	27.0 *D*
91	3.016	3.485	0.508	1.008	20.0 *BC*	....	21.4 *B*	28.2	None	83.8 *FB*
92	3.015	3.487	0.507	1.731	28.0 *BC*	....	35.6 *B*	41.0	"	113.3 *FB*
93	3.015	3.485	0.509	2.191	20.0 *BC*	....	43.5 *B*	50.0	"	129.2 *FB*
94	3.020	3.481	0.507	2.573	20.0 *BC*	....	49.0 *B*	56.0	"	139.8 *FB*
95	3.017	3.482	0.507	2.897	....	....	46.5 *B*	59.0	Ult.	147.5 *D*
96	3.025	3.481	0.507	3.190	....	....	50.0 *B*	62.0	"	149.0 *D*
97	3.001	2.507	0.500	2.888	44.0 *BS*	64.0	....	62.4	"	130.0 *D*
98	3.007	2.502	0.502	2.883	49.0 *BS*	57.0	....	59.0	"	129.9 *D*
99	3.028	2.486	0.492	2.881	33.0 *BC*	57.0	....	58.5	"	127.7 *D*
100	3.058	2.480	0.501	2.895	17.0 *BC*	55.0	....	58.0	"	131.2 *D*
101	3.100	2.447	0.495	2.870	20.0 *BC*	56.0	....	55.0	"	132.5 *D*
102	3.137	2.441	0.496	2.887	20.0 *BC*	56.0	....	55.0	"	128.7 *D*
103	3.200	2.400	0.492	2.935	16.0 *BC*	52.0	....	55.0	"	122.6 *D*
104‡	3.000	2.495	0.492	2.902	49.0 *BS*	61.0	....	58.0	"	122.9 *D*
105‡	3.058	2.480	0.502	2.908	22.0 *BC*	56.0	....	58.0	"	129.7 *D*
106‡	3.200	2.402	0.492	2.897	12.0 *BC*	54.0	....	51.5	None	151.2 *FB*

‡ Corners clipped at 45 degrees.

Four different observations were made at successive loads during each test, as follows:

(1) Longitudinal deformation between pin and plate was measured by two Ames dials $\left(\frac{1}{10\,000}\right)$ attached rigidly to arms which were fixed to the pins by set screws as shown in Fig. 2. The plungers of these dials engaged bars at-

TABLE 2.—PHYSICAL PROPERTIES OF MATERIALS BASED ON THE AVERAGE RESULTS OF TWO TESTS

Plate Nos.	LONGITUDINAL					TRANSVERSE				
	Yield point stress $\frac{}{1\,000}$	Ultimate strength $\frac{}{1\,000}$	Percentage reduction of area	Percentage elongation in 2 inches	Modulus of elasticity, $\frac{E}{10^6}$	Yield point stress $\frac{}{1\,000}$	Ultimate strength $\frac{}{1\,000}$	Percentage reduction of area	Percentage elongation in 2 inches	Modulus of elasticity, $\frac{E}{10^6}$
1–12	41.1	59.6	62.4	40.3	30.8	45.6	62.0	61.1	28.0	31.0
13–24	41.3	57.9	69.0	46.7	30.3	41.0	59.3	61.4	38.2	30.0
25–36	41.6	63.7	62.6	47.5	28.4	40.2	64.4	45.3	34.5	29.5
37–48	39.4	63.4	62.7	50.0	29.1	37.4	63.3	47.5	34.8	30.8
49–60	38.5	63.4	59.2	51.8	29.3	39.0	63.3	30.3	30.0	30.1
61–72	31.3	60.2	63.3	56.8	28.9	31.3	61.5	50.3	41.5	29.1
73–78	42.0	62.6	66.6	42.2	30.6	....	63.8	57.4	32.5	30.0
79–84	39.6	62.6	62.7	49.3	29.7	38.4	63.6	33.9	20.0	29.9
85–90	41.0	59.0	71.8	50.0	29.1	39.3	58.5	57.2	37.5	30.3
91–96	36.5	63.1	60.5	51.3	29.2	36.1	64.3	27.1	38.5	28.8
97–106	41.0	65.5	60.8	49.0	30.2	39.1	65.6	35.3	27.5	29.2
Average	39.2	61.6	64.1	48.6	29.5	38.7	62.4	47.2	33.6	29.9

tached to the test pieces by pointed screws about 8.5 in. from the center of the lower pin. The graph of the longitudinal deformation, plotted as abscissa with the load as ordinate, was used as a basis for determining the general yield point of the links. The point of general yield was taken as the load at which the slope of the load-deformation curve was three times that in the elastic range.

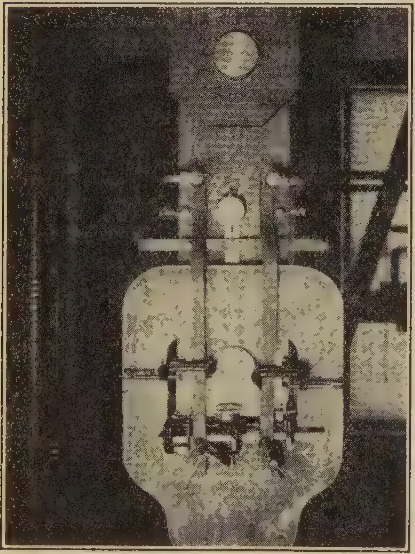


FIG. 3.—GENERAL VIEW OF TEST ARRANGEMENT, WITH DIALS PLACED TO DETECT "DISHING"

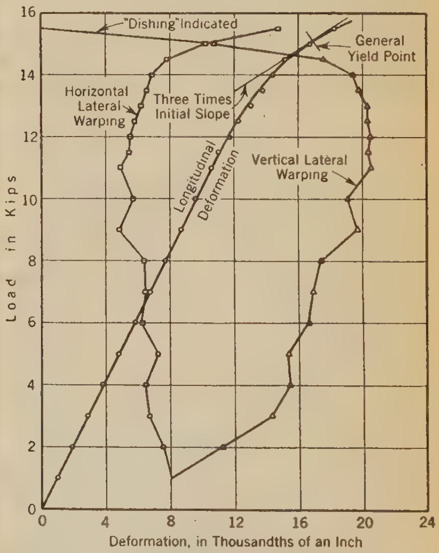


FIG. 4.—LOAD-DEFORMATION GRAPHS FOR TEST NO. 5

(2) Five Ames dials fixed to two main arms and one cross-piece engaged the lower surface of the test piece in order to measure any lateral movement

which would indicate the beginning of "dishing." The arms for holding these dials were fixed rigidly to the center of the test specimen, the arrangement being shown in Fig. 2. There were three points along two vertical lines, therefore, and three across the lower face of the plate. A study of these dial readings during successive load increments indicated any lateral movement along these three lines.

(3) Strains in the pin-plate material were measured along critical sections by means of Huggenberger tensometers, which were calibrated by an interferometer device designed and built by Raymond D. Mindlin, Jun. Am. Soc. C. E. The tensometers served two functions: (1) To obtain the stress distribution across the net section within an elastic range; and (2) to detect stress redistribution as indicated by variations in the slope of the strain curves. This, of course, does not necessarily indicate that the limit of proportionality had been exceeded at the exact point where the strain is measured. The tensometers were also used in determining the modulus of elasticity of the steel in the tensile coupon tests.

(4) The surface slip lines which are visible in the mill scale of structural steel at points of over-strain were accentuated by coating the surfaces of the plate at the lower pinhole with whitewash. The progress of the slip lines was recorded during successive load increments, and photographs were taken of all typical patterns. The whitewash consisted of 35 grams of hydrated lime mixed with 50 cu cm of water.

A similar routine was followed in all the tests, and mimeographed forms were used for recording dimensions, dial and tensometer readings, and the successive development of surface slip-lines.

Test Results

Types of Failure.—The general yield point, arbitrarily determined in every test from the load-deformation curve, was preceded by surface evidence of local failure below the pin in every instance except Tests Nos. 75 and 77 (see Table 1), these being two of the twelve tests of narrow plates 6 in. wide. Considerable surface slip-line development, extending over a relatively large area of the plate, was usually in evidence before the general yield point.

The type of ultimate failure in the pin-connected plate depends on the relative proportions of the plate at the pinhole. Three general and overlapping classifications may be made: (1) Tension failure in the net section at one side of the pin; (2) crushing and shearing failure below the pin, in some cases followed by a tearing fracture in "hoop" tension after considerable deformation; and (3) "dishing" failure of thin plates which are laterally unrestrained.

Typical Tests.—Failure by "dishing" at very low load is typified by Test No. 5, Table 1. The load-deformation curve, to determine the general yield point, and the curves of lateral and vertical warping indicative of "dishing" are shown in Fig. 4. A photograph of the surface slip-line pattern after "dishing" of Test No. 5 is shown in Fig. 5. This plate "dished" at a load of only 1 200 lb above the general yield point, and, in such cases, the slip lines appeared only on the convex surface. The "dishing" itself is not evident in this side view,

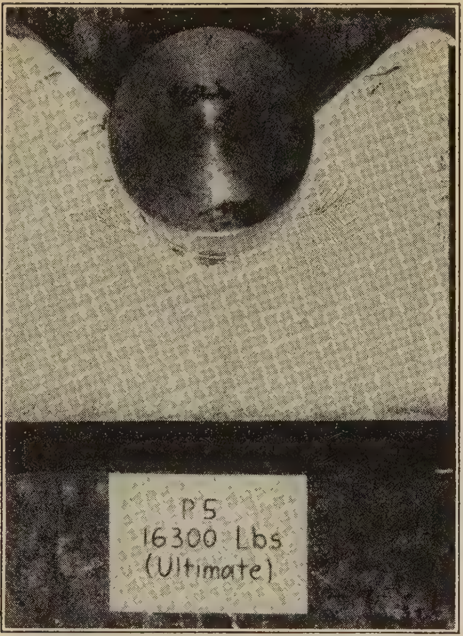


FIG. 5.—TEST NO. 5 AFTER FAILURE BY "DISHING"; SHOWING SLIP-LINES ON CONVEX SURFACE

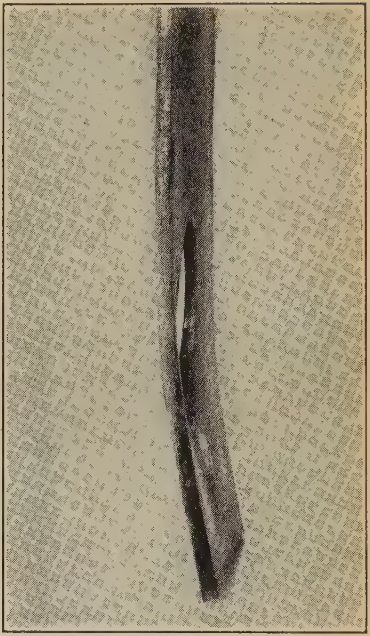


FIG. 6.—SHAPE OF TEST PIECE AFTER FAILURE BY "DISHING"

but is clearly indicated in Fig. 6, showing the shape of the plate after complete failure by "dishing."

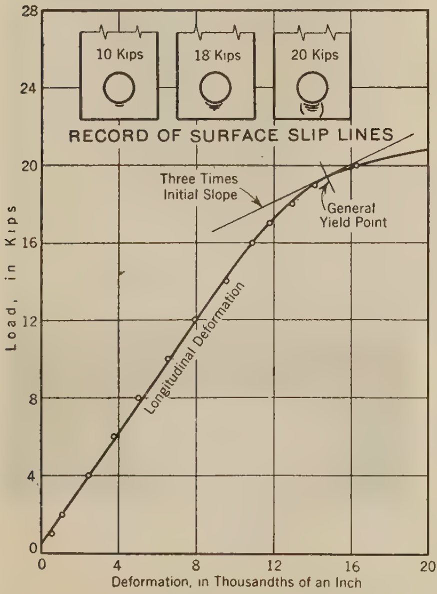


FIG. 7.—TEST NO. 32; LOAD DEFORMATION GRAPH AND SURFACE SLIP-LINE RECORD

The test of Plate No. 32 is typical of failure behind the pinhole, due to insufficient edge distance, *a*. Fig. 7 gives the load-deformation curve for this test and shows sketches similar to those taken in every test to record the progressive growth of surface slip-lines. Fig. 8 illustrates the further spread of the surface slip-lines at a load of 26 kips, and Fig. 9 shows the fracture after failure. The condition of the whitewash in Fig. 9 shows the completely plastic region below the pin prior to failure and the relative absence of surface slip-lines along the sides and above the pinhole. Fig. 10 shows an end fracture in another test of more balanced design and greater plastic elongation. Fig. 11 shows a surface slip-line pattern near the ultimate strength in

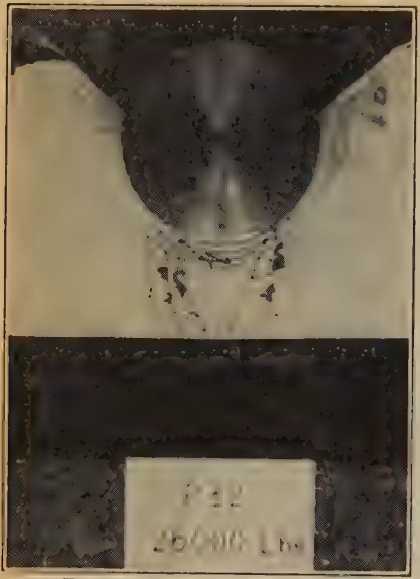

P32
26000 Lbs

Fig. 8—Test No. 32 at 26 Kips

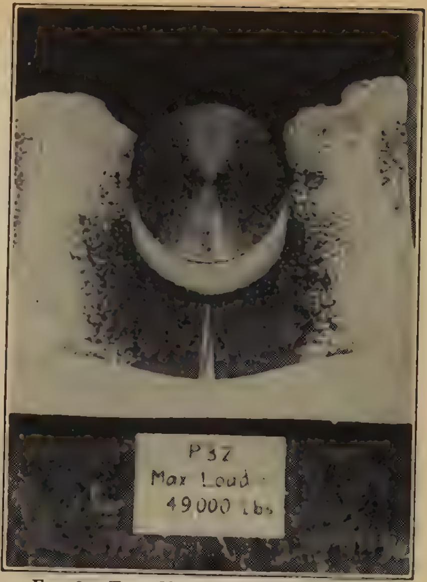

P32
Max Load
49000 lbs

Fig. 9.—Test No. 32 After Fracture Behind the Pin

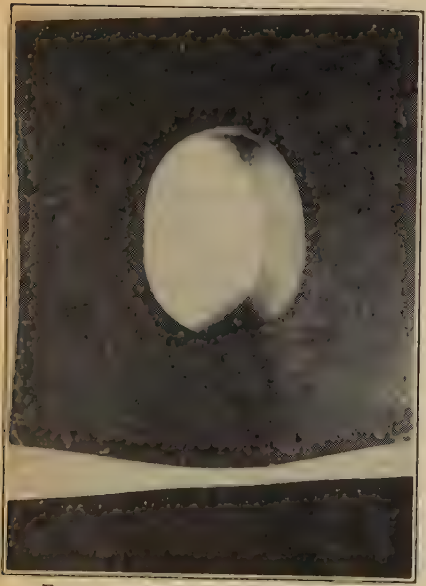


Fig. 10—Test No. 64 After Partial Failure Behind the Pin

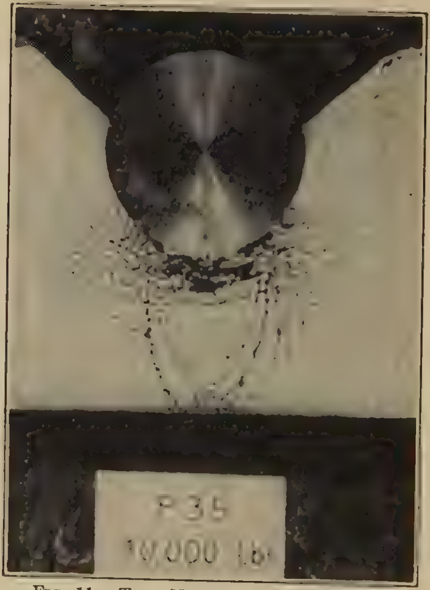

P35
40000 lbs

Fig. 11.—Test No. 35 at 40 Kips and Just Prior to "Dishing"

Plate No. 35, similar to Plate No. 32, but of more balanced design. This plate was the same in every respect as Plate No. 32 with the exception of greater edge distance, *a*, with the result that considerable local crushing below the pin developed before the final spread of the slip-lines. The slip-lines indicated in Fig. 11 were duplicated on both sides of the plate. At a load of 41.5 kips, the plate "dished" causing a slip-line pattern similar to that in Fig. 5 to be superposed on the convex side. Unlike Test No. 5, however, Plate No. 35 was thick enough to develop considerable reserve strength between the general yield stress and ultimate failure by "dishing."

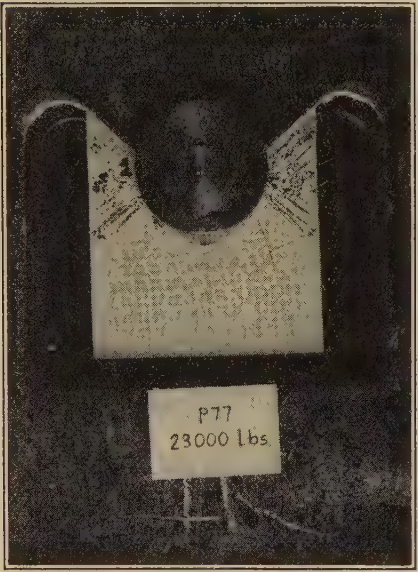


FIG. 12.—TEST NO. 77 AT 23 KIPS, SHOWING YIELDING AT THE SIDE

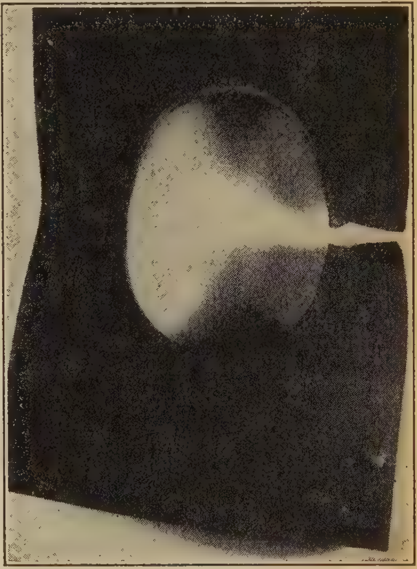


FIG. 13.—TEST NO. 82 AFTER FRACTURE AT SIDE OF PINHOLE

With one exception the series of plates which were 6 in. wide failed either in the net section in tension, or by "dishing." Fig. 12 shows the slip-line pattern soon after the general yield point of a 6-in. by $\frac{3}{16}$-in. plate prior to ultimate failure by "dishing," and Fig. 13 shows the side fracture, at ultimate strength of a 6-in. by $\frac{1}{2}$-in. plate.

Stress Distribution.—The stress distribution in the plate around the pinhole is complex, and may best be studied within the elastic range by means of photo-elastic tests. Complete studies of the stress pattern and distribution in a plate around a pinhole have been made by previous investigators.[6]

The maximum stresses are the localized contact compressive stresses at the point of bearing of pin on plate, and the concentration of tensile stress at each side of the pinhole. Except in the case of tight-fitting pins, the localized contact stresses produce the first surface evidence of local yielding. For a discussion of the subject of contact stresses reference may be made to an in-

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[6] *Journal*, Franklin Inst., Vol. 199, 1925, p. 289; and *Bulletin*, Aero. Research Inst., Tokyo Univ., September 12, 1926.

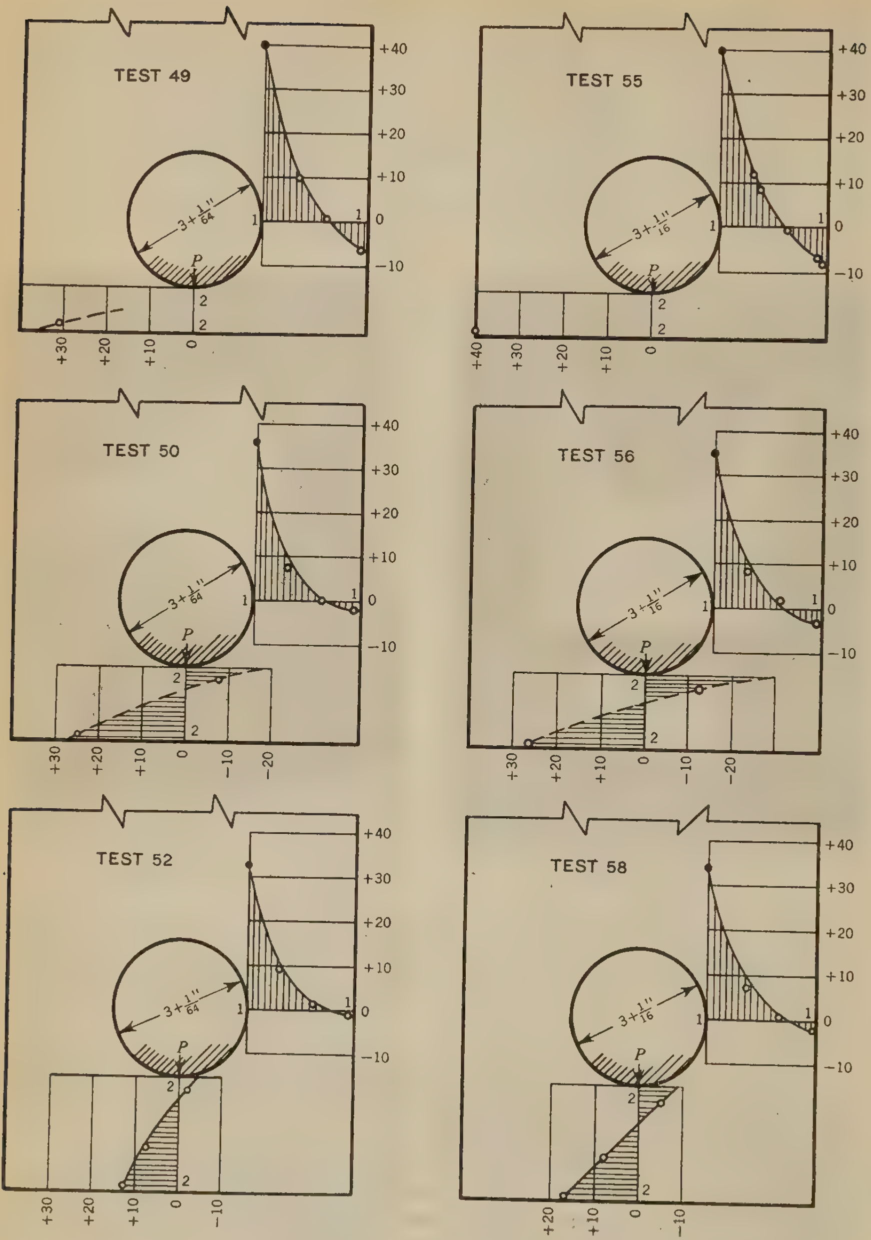

FIG. 14.—STRAIN TIME MODULUS OF ELASTICITY NORMAL TO THE HORIZONTAL AND VERTICAL LINES BY TENSOMETERS ($P = 20$ KIPS)

vestigation of stresses under bridge rollers reported[7] in 1936, by V. P. Jensen, Assoc. M. Am. Soc. C. E.

Insufficient clearance around the pin made it impossible to place tensometers for measuring strains at the exact locations of maximum stress. The tensometers were useful in all tests, however, in detecting initial stress redistribution which indirectly determined loads at which local yielding had occurred at points of maximum stress. Additional tensometers were used in some of the tests to determine the strain distribution along vertical and horizontal lines (see Fig. 2). The results are indicated in Fig. 14 for six tests on 8-in. by 0.5-in. plates with a varying distance, *a*, and two different pin clearances. The values indicated in Fig. 14 are the products of the strain in one direction multiplied by the modulus of elasticity, and are not true stresses, although very nearly so along Line 1–1, where the normal horizontal stresses are negligible. The maximum value of the vertical strain times the modulus of elasticity, *E*, normal to Section 1–1 adjacent to the pin was determined indirectly by extending from the measured values a stress area that would be in equilibrium with the known total load on the section. The contact stresses below the pin vary rapidly and could not have been measured satisfactorily over the half-inch gage length available in a Huggenberger tensometer, even if clearance had been available. The strains normal to the vertical section, 2–2, were measured at as many points as possible and the products of these strains and the modulus of elasticity, *E*, are also indicated in Fig. 14. The true stresses normal to Section 2–2 in the vicinity of the pin are considerable larger than the values of strain times the modulus of elasticity, *E*, because of the unmeasured contact stresses.

It is important to note that considerable local yielding, either below or to the side of the pinhole, was always evidenced before the general yield point of the plate was noted, and that stress concentrations had little effect upon the ultimate strength.

Pin Clearance.—A special series of tests was made in which clearance between pin and pinhole was the only variable. The load-deformation curves were extended far beyond the general yield point in these tests and three typical curves are shown in Fig. 15. The results of this series of tests are discussed in the general "Summary of Test Results" which follows.

Summary of Test Results.—Table 1 presents an abbreviated summary of the results of all the tests. A detailed explanation of the various columns in this table follows: Columns (1), (2), (3), and (4) give measured dimensions of the specimens before test, at the end near the observed pinhole, based on the average value of micrometer readings made to the nearest 0.001 in.

Column (5) indicates the load at which the first surface slip-line was noted, together with its location and type. The first letter, *B* or *E*, indicates whether the line was below or to one side of the pin. The second letter (*S* or *C*) indicates whether the flaking was a definite line, indicating slip or shear along a certain path, or general flaking below the pin, indicative of compression yield by crushing (see Fig. 11 for illustrations of both types). In this column, of which only a qualitative interpretation may be made, the trend of the lowest noted loads is most nearly correct, since the whitewash at times had enough cohesion to remain in place after yielding had commenced.

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[7] *Bulletin*, Iowa Eng. Soc., Vol. XI, No. 4, October, 1936, p. 9.

Column (6), Table 1, gives the load at which surface indication of failure had become general. By this is meant either the presence of several lines extending from the pinhole through to the end or side of the plate, or a general network of lines developed in the vicinity of the pinhole. The same qualifications must be made regarding interpretation as in the case of Column (5). Column (7) gives the first indication of general stress redistribution due to local yielding, either to one side (E) or below (B) the pinhole. The noted load in each case is the limit of proportionality of the tensometer readings plotted against load.

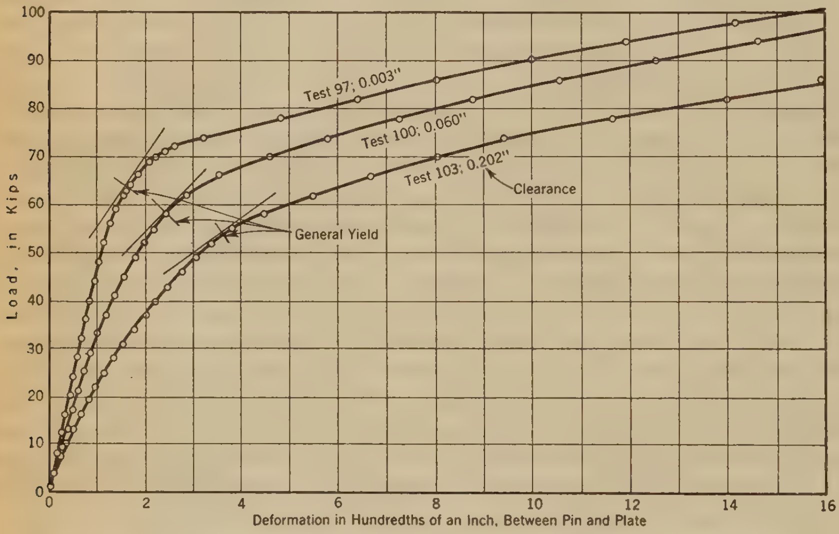
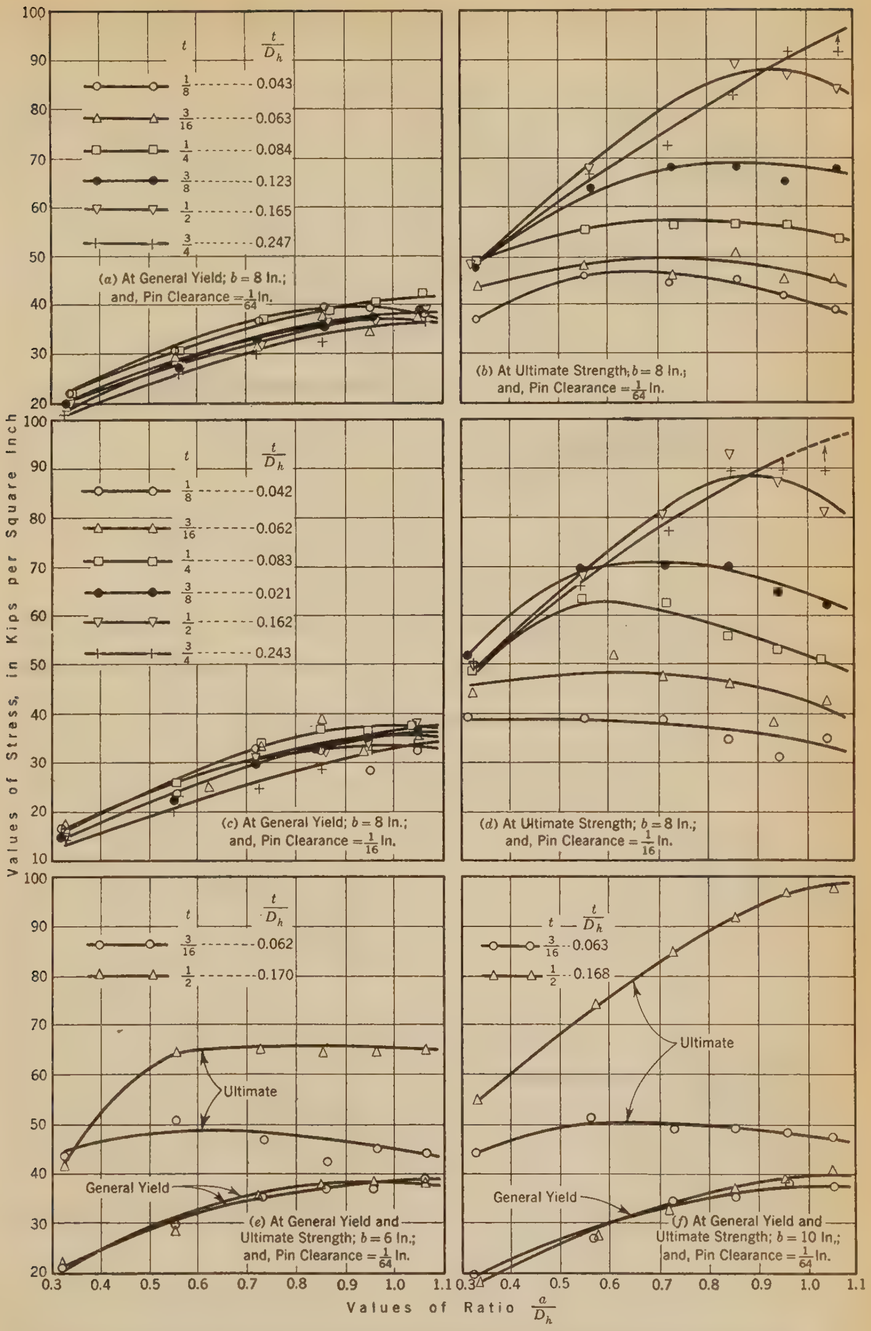


FIG. 15.—LOAD-DEFORMATION CURVES WITH PIN CLEARANCE AS THE ONLY VARIABLE

Column (8), Table 1, gives the load at the arbitrarily determined "general yield" strength at which point the longitudinal deformation between pin and plate was progressing at three times the initial rate; Column (9) indicates load at the start of "dishing," as detected by the limit of proportionality of the differences between the readings of the lateral dials, plotted against load; Column (10) gives the maximum load either at fracture or "dishing," in the latter case, just prior to large falling off of load during the progress of "dishing."

In Tests Nos. 49 to 106, the set readings were taken at five or more different load intervals to give a complete curve of set plotted against load. These data are omitted to conserve space but the permanent set in all tests followed closely the curve indicated in Fig. 19(b).

Graphical Summary of Test Results.—Average bearing stress at the general yield point, and the ultimate strength for the plates 8 in. wide (Tests Nos. 1 to 72) are plotted in Figs. 16(a) to 16(d), with the ratio, $\frac{a}{D_h}$, as abscissa. Figs.



(*a*) At General Yield; $b = 8$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

(*b*) At Ultimate Strength; $b = 8$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

(*c*) At General Yield; $b = 8$ In.; and, Pin Clearance $= \frac{1}{16}$ In.

(*d*) At Ultimate Strength; $b = 8$ In.; and, Pin Clearance $= \frac{1}{16}$ In.

(*e*) At General Yield and Ultimate Strength; $b = 6$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

(*f*) At General Yield and Ultimate Strength; $b = 10$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

FIG. 16.—AVERAGE BEARING STRESSES CORRESPONDING TO VALUES OF THE RATIO, $\frac{a}{D_h}$.

16(*e*) and 16(*f*) give the same results for the 6-in. and 10-in. plates, Tests Nos. 73 to 96, inclusive.

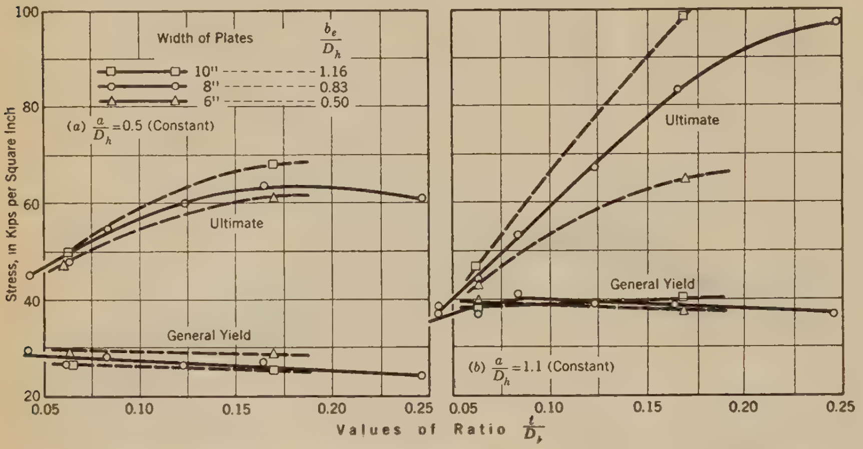


FIG. 17.—AVERAGE BEARING STRESSES AT THE GENERAL YIELD POINT AND ULTIMATE STRENGTH, WITH VARYING RATIO, $\frac{t}{D_h}$

Fig. 17 gives average bearing stresses plotted against the variable thickness ratio, $\frac{t}{D_h}$, for Tests Nos. 1 to 96, Table 1, and for two different ratios, $\frac{a}{D_h}$, of 0.5 and 1.1. The values plotted were taken from the curves in Fig. 16.

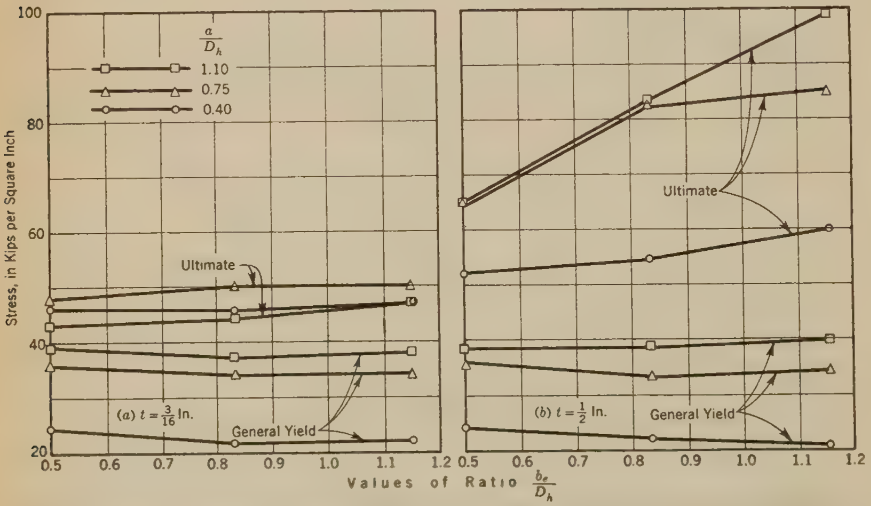


FIG. 18.—AVERAGE BEARING STRESSES AT THE GENERAL YIELD POINT AND ULTIMATE STRENGTH, WITH VARYING RATIO, $\frac{b_e}{D_h}$

In Fig. 18 the average bearing stress is plotted against the ratio, $\frac{b_e}{D_h}$, as abscissa for all the tests of $\frac{3}{16}$-in. and $\frac{1}{2}$-in. plates.

Fig. 19(*a*) gives the average bearing stress at the general yield point and ultimate strengths for Tests Nos. 97 to 106 (Table 1), in which the pin clearances are the variable, and are plotted as abscissa. Fig. 19(*b*) shows the relation between permanent set and variable pin clearance for the same series at different bearing stresses and at the general yield.

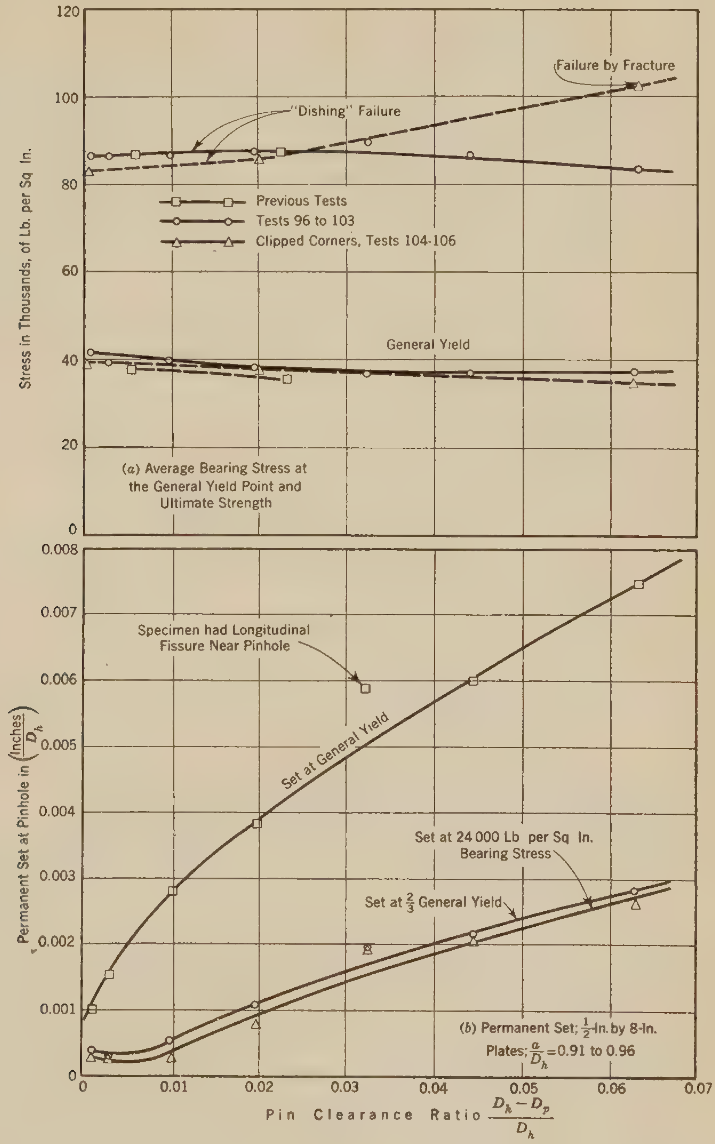


FIG. 19.—EFFECT OF VARIATIONS IN THE PIN-CLEARANCE RATIO, $\frac{c}{D_h} = \frac{D_h - D_p}{D_h}$

RELATION OF TEST RESULTS TO DESIGN

This paper includes no recommendations as to proper working stresses, factors of safety, or allowable permanent sets, which are properly the subject of specifications set up by engineering groups. In the following text, empirical formulas are presented for predicting the average bearing stress at the general yield and ultimate strength of pin-connected plate links similar to those tested. In order to study the four variable dimension ratios which affect the strength of the plates in the regions of greatest variation, many of the test pieces have dimensions outside the usual range of design. As possible design proportions are approached, however, the test results show a uniformity of trend which warrants the application of the formulas to all usual design proportions. Definite limits within which the formulas are applicable are noted in connection with each. The formulas agree closely with the average test results; and, in case of variation of more than a small percentage, they are adjusted to agree with the lowest trend of test results. In applying these formulas to design, these further restrictions are emphasized:

(1) Material should be structural grade steel for bridge work as covered by standard specifications.[8]

(2) Loads are static and pins do not rotate. For repeated stress conditions or for rotating pins appropriate allowances must be made.

(3) No rivet or bolt-holes may be drilled or punched in the vicinity of the pinhole. Previous tests[2] have noted the weakening effect of such small holes, either below or above the pinhole.

(4) The body of the plate shall have the same dimensions as the gross section at the pinhole.

Notation.—Reference should be made to Fig. 1 for the notation in regard to dimensions of plate. Ratios to which reference has already been made are:

$$\frac{t}{D_h} = \frac{\text{Plate thickness}}{\text{Pinhole diameter}};\ \frac{b_e}{D_h} = \frac{\text{Edge distance to side of pinhole}}{\text{Pinhole diameter}};$$

$$\frac{a}{D_h} = \frac{\text{Edge distance below pinhole}}{\text{Pinhole diameter}};\ \frac{c}{D_h} = \frac{D_h - D_p}{D_h} = \frac{\text{Pin clearance}}{\text{Pinhole diameter}}.$$

The unit stresses are: s_{by} = average bearing stress between pin and plate at general yield of plate; s_{bt} = average bearing stress between pin and plate at ultimate strength of plate; s_y = tensile yield point of material as determined by standard tension test; and s_t = ultimate tensile strength of material as determined by standard tension test.

General Yield Point.—The average bearing stress at the general yield point is not appreciably influenced by plate thickness ratios, $\frac{t}{D_h}$ greater than $\frac{1}{20}$, nor by the edge distance ratio, $\frac{b_e}{D_h}$, for values greater than $\frac{1}{2}$. The test data do not give results for $\frac{b_e}{D_h}$ greater than 1.2, but the bearing stress in such cases at the general yield point is probably not influenced greatly for values of $\frac{b_e}{D_h}$ within the usual range of design. The clearance between pin and pinhole, as defined by the ratio, $\frac{c}{D_h}$, has a minor effect on the general yield point, as indicated in Fig. 19(*a*). The general yield point of the plate is induced principally by a yielding of the material behind the pin. This yielding is preceded by a crushing failure below the pin which progresses until all the metal behind it starts to give way in a plastic state. In very thin plates the general yield is followed quickly by "dishing" and ultimate failure, whereas in thicker plates there is a large plastic reserve before ultimate failure occurs.

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[8] Specification A7-34, Am. Soc. for Testing Materials.

The following formula gives the average bearing stress at the general yield point:

$$s_{by} = \frac{s_y}{2}\left[3\left(\frac{a}{D_h}\right) - \left(\frac{a}{D_h}\right)^2 - 2\left(\frac{c}{D_h}\right)\right] \quad \ldots\ldots\ldots\ldots (1)$$

Equation (1) applies within the following limits:

$\frac{a}{D_h}$ between 0.3 and 1.2

$\frac{c}{D_h}$ between 0 and 0.07

$\frac{b_e}{D_h}$ greater than 0.5 (otherwise general yield will be entirely in net section)

$\frac{t}{D_h}$ greater than 0.05 (otherwise ultimate failure by "dishing" may precede any yielding)

Equation (1) is based on the test average of s_y equal to 39 kips per sq in. Based on the minimum specification requirement of 33 kips per sq in. for structural steel for plates, shapes, and bars in bridges (A. S. T. M. A7–34) and using ratios of $\frac{a}{D_h} = 1.2$ and $\frac{c}{D_h} = \frac{1}{250}\left(\frac{1}{50}\text{-in. clearance for a pin 5 in. in diameter}\right)$, by Equation (1), s_y is 35.4 kips per sq in. If a factor of safety of 1.5 were suitable with respect to the general yield point, the allowable working stress would be 23.6 kips per sq in., which corresponds closely with the frequently specified allowable stress of 24 kips per sq in. If such a working stress is suitable, it should only be used when the plate is thick enough to prevent ultimate failure at a low load by "dishing." The tests will be examined later to develop criteria of proportions which will insure against "dishing" failure and at the same time give a balanced design.

Ultimate Failure.—The three general types of ultimate failure have been described previously (see heading, "Test Results: Types of Failure"). Three empirical formulas (in close agreement with test results) will be presented to serve as indices of the average bearing stress at the ultimate strength in the case of failure by any of the three types. Applied to a plate of any proportions, the lowest value obtained from the three equations indicates the expected ultimate strength and the type of failure. All the equations are written in terms of average bearing stress between pin and plate, as follows:

Failure by fracture at side of pinhole,

$$s_{bt} = 2s_t\left(\frac{b_e}{D_h}\right) \quad \ldots\ldots (2)$$

Failure by fracture below pinhole,

$$s_{bt} = s_t\left[1.13\left(\frac{a}{D_h}\right) + \frac{0.92\left(\frac{b_e}{D_h}\right)}{1 + \left(\frac{b_e}{D_h}\right)}\right] \quad \ldots\ldots (3)$$

Failure by "dishing,"

$$s_{bt} = 20 + 315\left(\frac{t}{D_h}\right) + 75\left(\frac{tb_e}{D_h^2}\right) + 20\left(\frac{a}{D_h}\right) - 20\left(\frac{a}{D_h}\right)^2 \quad \ldots (4)$$

In correlating Equations (2) and (3) with the test results, the test coupon average of $s_t = 62$ kips per sq in. applies. In applications to design the minimum specification requirement for s_t should be used. The following limitations as to dimensions are made: $\frac{a}{D_h}$ between 0.3 and 1.2; and, $\frac{c}{D_h}$ between 0 and 0.07.

The first limitation automatically limits the other dimensions (whenever they are determining factors) to values close to the test range. The second limitation indicates the test range but does not enter the equations because the clearance was not found to influence the ultimate strength appreciably. Although $\frac{a}{D_h}$ varied from 0.325 to 1.05 in the tests, the extension to 1.2 is based on the fact that the test results approached a uniform trend in the upper limit of $\frac{a}{D_h}$. Further remarks on each type of failure follow.

Failure by Fracture at Side of Pinhole.—This type of failure occurred only in the 6-in. plates. The test results showed that the material in the net section did not fail until the average stress in that section reached the ultimate tensile strength of the material as determined by a standard tension test. This indicates that the stress concentrations near a circular hole, which exist in the elastic range, have no effect on the ultimate strength of structural steel under static loads.

Equation (2) simply states that if failure is by tension in the net section, the average stress through this section will be the same at failure as in the case of a standard tension test. The stress actually developed, based on original areas and a test average of $s_t = 62$ kips per sq in., was slightly higher than that determined by Equation (2), due probably to the fact that the lower stressed material above and below the minimum section did not allow so much "necking down" and reduction of area as in the case of the straight tensile coupons.

Ultimate Failure Below Pinhole.—A study of the plates that failed in this manner shows that the ultimate strength of the plate is dependent both on the material below and to the side of the pinhole, the results agreeing with Equation (3) for $s_t = 62$ kips per sq in.

Ultimate Failure by "Dishing."—Equation (4) is for plates unsupported laterally at the ends. If the plate is blocked securely against "dishing" at the end, either Equation (2) or Equation (3) may determine the ultimate strength. (Although no blocked plates were tested in the present investigation, this statement is verified by tests made on the pin-plates of the Quebec Bridge (see Table 3)). Equation (4) is not written in terms of s_t, as are Equations (2) and (3), because the "dishing" is primarily a function of stability rather than strength.

TABLE 3.—COMPARISON OF TEST DATA, MAUMEE BRIDGE AND QUEBEC BRIDGE

DIMENSIONS, IN INCHES						RATIOS			BEARING STRESSES, IN KIPS PER SQUARE INCH			
Diameter of hole, D_h	Width of Plate b		Thickness, t	Edge Clearance		$\frac{b_e}{D_h}$	$\frac{a}{D_h}$	$\frac{t}{D_h}$	At failure*	Predicted Average Ultimate Strengths		
	At pin	Body		At bottom, a	At sides, b_e					Side	End	"Dishing"
						(*a*) MAUMEE BRIDGE						
8.00	22.0	10.0	1.375	7.00	7.00	0.875	0.875	0.172	68.6 *D*, *FB*	107.7	*87.8*	*87.8*
8.00	22.0	10.0	1.375	7.00	7.00	0.875	0.875	0.172	61.4 *D*, *FB*	107.7	*87.8*	*87.8*
8.00	20.0	10.0	1.375	7.00	6.00	0.750	0.875	0.172	69.2 *D*, *FB*	92.3	*85.6*	86.1
8.00	20.0	10.0	1.375	7.00	6.00	0.750	0.875	0.172	67.1 *D*, *FB*	92.3	*85.6*	86.1
8.00	22.0	10.0	1.375	8.00	7.00	0.875	1.000	0.172	75.0 *D*, *FB*	107.7	96.6	*85.5*
8.00	22.0	10.0	1.375	8.00	7.00	0.875	1.000	0.172	77.3 *D*, *FB*	107.7	96.6	*85.5*
8.00	22.0	14.0	1.375	8.00	7.00	0.875	1.000	0.172	80.5 *D*, *FB*	107.7	96.6	*85.5*
8.00	22.0	14.0	1.375	8.00	7.00	0.875	1.000	0.172	79.5 *D*, *FB*	107.7	96.6	*85.5*
						(*b*) QUEBEC BRIDGE						
5.03	13.0	13.0	0.375	3.75	4.00	0.790	0.745	0.075	57.9 *D*	97.2	77.4	*51.9*
5.03	13.0	13.0	0.375	3.75	4.00	0.790	0.745	†	77.4 *FB*	97.2	*77.4*	†
5.03	11.0	11.0	0.375	3.75	3.00	0.597	0.745	0.075	52.8 *D*	73.5	73.6	*50.8*
5.03	11.0	11.0	0.375	3.75	3.00	0.597	0.745	0.075	55.6 *D*	73.5	73.6	*50.8*
5.03	9.0	9.0	0.375	3.75	2.00	0.398	0.745	0.075	52.3 *D*	*49.0*	68.2	49.7
5.03	9.0	9.0	0.375	3.75	2.00	0.398	0.745	0.075	55.0 *FE*	*49.0*	68.2	49.7
12.14	28.3	28.3	2.000	12.1	8.08	0.668	1.000	0.165	77.2 *FE*	82.2	92.8	*80.3*
12.14	28.3	28.3	2.000	12.1	8.08	0.668	1.000	0.165	78.4 *FB*	82.2	92.8	*80.3*
12.14	26.3	26.3	1.500	10.5	7.10	0.585	0.865	0.123	70.5 *D*, *FE*	72.0	82.7	*66.6*
12.14	26.3	26.3	1.500	10.5	7.10	0.585	0.865	0.123	72.4 *D*, *FB*	72.0	82.7	*66.6*

* D = "dishing"; F = fracture; B = below pin; and, E = side of pin.

† End of plate blocked to prevent "dishing."

Balanced Design.—Assuming that the plate is thick enough, or that it is blocked laterally, so as to prevent "dishing," a comparison of Equations (2) and (3) yields the following criteria for equal strength or balanced design with respect to the edge distances to the side and below the pin:

$$\frac{a}{b_e} = \frac{1 + 1.75\left(\frac{b_e}{D_h}\right)}{1 + \left(\frac{b_e}{D_h}\right)} \qquad (5)$$

Equation (5) gives a slight excess of a distance.

What plate thickness is required to prevent "dishing"? Assuming that the design is balanced in other respects according to Equation (5), the following thickness ratio, obtained from a comparison of Equations (2) and (4), gives the required minimum allowable thickness ratio, $\frac{t}{D_h}$:

$$\frac{t}{D_h} = \frac{-20 + 123\left(\frac{b_e}{D_h}\right) - 20\left(\frac{a}{D_h}\right) + 20\left(\frac{a}{D_h}\right)^2}{315 + 75\left(\frac{b_e}{D_h}\right)} \quad \ldots\ldots\ldots (6)$$

Although Equation (6) gives sufficient thickness to prevent "dishing" in an otherwise balanced design, it also gives more than sufficient thickness for plates which are not balanced according to Equation (5). The "dishing" is more nearly a direct function of the ratio, $\frac{a}{t}$, than $\left(\frac{t}{D_h}\right)$, and Fig. 20 shows the maximum allowable ratio, $\frac{a}{t}$, to prevent "dishing" for plates balanced in design according to Equations (5) and (6).

In general, the tests indicate that the most efficiently designed pin-plate connectors are those narrow in width, thick enough to prevent "dishing," and having material below the pinhole in the proportions specified by Equation (5).

Comparison with Previous Tests.—The results of two previous series of tests on pin-connected plates of larger dimensions than those of the present investigation are presented in Table 3. The second group of ten tests is taken from the report[2] on the Quebec Bridge. Six of these tests were models, using pins 5 in. in diameter, and the remaining four were full-sized hangers with 12-in. pins, and plate widths of 26 in. or 28 in. The results of these tests check well with the strengths predicted by Equations (2) to (4), only two of them falling below, with a maximum difference of about 4%, and the remainder above, by as much as 10% in some instances. The material used in these pin-plates had slightly higher ultimate strength than that used in the present study.

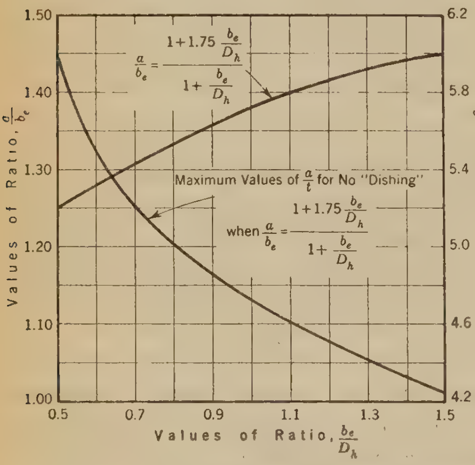

FIG. 20.—BALANCED DESIGN OF ULTIMATE STRENGTH[1]

The results of tests of eight pin-connected plates to determine the best design for anchorage bars for the Maumee River Suspension Bridge are also presented in Table 3. These bars had a reduced width in the main section between pinholes. All the test strengths of these bars fall below the values given by Equation (2), Equation (3), or Equation (4). Apparently, the reduced width in the body of the bar increased the tendency toward "dishing," and reduced their strength, although this may not necessarily be the cause. It is to be noted that the two tests with largest plate width in the body are only 7% less than the strengths indicated by Equation (5). Fracture failures below the pin were reported in these tests but this was combined at the same time, with "dishing." Since the exact dimensions of the specimens are not known and their number is limited, no final conclusion can be given in connection with these tests.

Illustrative Example.—Assume that clearance requirements make it necessary to use a pin-plate 1 in. thick to carry a load of 200 kips at a working stress in bearing of 24 kips per sq in. (A. S. T. M. Specification A7–34). The required pin diameter at 24 kips per sq in. $= \dfrac{200}{1 \times 24} = 8.33$ in., which dictates the use of a pin with a diameter of 8.5 in. The required net edge-distance at the pinhole, at 18 kips per sq in., is $b_e = \dfrac{200}{2 \times 1 \times 18} = 5.55$ in. With $b_e = 6$ in., this makes a total width of 20.5 in. required, and the ratio, $\dfrac{b_e}{D_h} = \dfrac{6}{8.5} = 0.706$. From Fig. 20, for balanced design: $\dfrac{a}{D_h} = 0.75 \times 1.31 = 0.985$, from which $a = 0.985 \times 8.5 = 8.4$ in. (use $a = 9$ in.). By actual measurement, $\dfrac{a}{D_h} = 1.06$; $\dfrac{t}{D_h} = \dfrac{1}{8.5} = 0.118$; and, $\dfrac{a}{t} = \dfrac{1.06}{0.118} = 9.0$, which is more than the 5.27 allowable maximum for no "dishing," for the ratio, $\dfrac{a}{D_h} = 1.06$, from Fig. 20. The plate, therefore, will fail by "dishing." By Equation (4), the average bearing stress at ultimate strength is, $s_{bt} = 20 + 315 \times 0.118 + 75 \times 0.118 \times 0.75 + (1.06 - 1.06^2)\,20 = 62.63$ kips per sq in. The ultimate strength is $62.63 \times 8.5 \times 1 = 532$ kips; and, the safety factor with respect to the ultimate $= \dfrac{532}{200} = 2.66$.

As an alternative, suppose that the plate is securely blocked to prevent "dishing." The ultimate strength will then be given either by Equation (2) for side failure, or by Equation (3) for end failure For $s_t = 60$, the minimum by A. S. T. M. Specification A7–34, Equation (2) gives $s_{bt} = 120 \times 0.75 = 90.0$ kips per sq in.; and, by Equation (3), $s_{bt} = 60\left[1.13 \times 1.06 + \dfrac{0.92 \times 0.75}{1 + 0.75}\right] = 96$ kips per sq in. The ultimate strength, then, if the plate is blocked $= 90 \times 8.5 \times 1 = 765$ kips; and the factor of safety with respect to the ultimate $= \dfrac{765}{200} = 3.82$.

General Yield Point of Plate.—If a pin clearance is $\dfrac{1}{32}$ in., the pin-clearance ratio, $\dfrac{b_e}{D_h} = \dfrac{1}{32 \times 8.5} = 0.00367$. By Equation (1), the average bearing stress at the yield point will be: $s_{by} = 33 \left[\frac{3 \times 1.06 - 1.06^2 - 2 \times 0.00367}{2} \right] = 33.8$ kips per sq in.; the total strength at the yield point = 33.8 × 8.5 × 1 = 288 kips; the safety factor with respect to the yield point = 1.44; and the permanent set for a stress of 24 kips per sq in. (from Fig. 19(*b*)) = 0.00025 × 8.5 = 0.002 in.

It should be noted that the general yield point may be raised by arbitrarily increasing the end ratio, $\frac{b_e}{D_h}$, but this will also increase the tendency to "dish." In cases where "dishing" is eliminated by suitable plate thickness or by side blocking, a low factor of safety with respect to the general yield point may be justified, in the opinion of the engineer, due to the high plastic reserve strength prior to ultimate failure.

Clipped Corners.—As indicated by previous tests and a very limited number of the present tests (three in all), the corners of the plate below the pin may be clipped with only slight loss of strength. It is recommended that 20% more net section be maintained on a 45° line through the corner than the minimum at the side or below.

SUMMARY

Within certain well-defined limits as to material and shape, this paper has presented empirical equations for predicting the general yield point, ultimate strength, type of failure, and balanced proportions of pin-connected steel plates. These equations agree closely with the 106 tests reported, in which the pin diameter was 3 in., and are further validated by tests on similar pin-connected links used in the erection of the Quebec Bridge in which the pin diameter was as much as 12 in. Although the results do not check well with a number of tests of pin-connected anchorage bars for the Maumee River Suspension Bridge, this may possibly be accounted for by reason of the reduced width in the body of these bars. Hence, the results of this paper are not intended to apply directly to pin-connected plates having the shape of standard eye-bars unless tests on such eye-bars show agreement with the present results.

ACKNOWLEDGMENT

The tests were conducted as a research project of the Department of Civil Engineering, of which James K. Finch, M. Am. Soc. C. E., is Executive Officer. The author is indebted for the close co-operation of the late Albin H. Beyer, M. Am. Soc. C. E., who was Director of the Testing Laboratories, and to the present Director, W. J. Krefeld, M. Am. Soc. C. E. Leon Krantz assisted in setting up the apparatus and operating the tests. The test program was originally suggested by Sterling Johnston, M. Am. Soc. C. E., and was conducted in co-operation with the American Institute of Steel Construction. Engineers of the American Bridge Company, the Bethlehem Steel Company, and the Chicago Bridge and Iron Works, contributed valuable criticism both as to the proper test program and the conclusions to be drawn from the results.

DISCUSSION

Harold D. Hussey,[9] M. Am. Soc. C. E. (by letter).—This paper is of much value in its field of structural design and especially in giving an empirical solution to a very complicated problem in buckling of plates; that is, the buckling of plates behind the pin in a pin-connected tension member. The tests which are reported were made on plain plates of uniform width and the author does not claim that his conclusions will apply to members of non-uniform width, such as eye-bars. The writer wishes to call attention, however, to some tests on eye-bars that agree very closely with the results obtained by Mr. Johnston.

When the width of the plate is given, the thickness required to prevent "dishing" can be determined from Equations (5) and (6) as follows: Let $\frac{a}{b_e} = M$; and, $\frac{a}{t} = N$. Solving for t,

$$t = \frac{M}{N} b_e \dots\dots\dots\dots (61)$$

Dividing both sides of Equation (61) by D_h and letting $\frac{t}{D_h} = t_D$; and, $\frac{b_e}{D_h} = b_D$:

$$t_D = \frac{M}{N} b_D \dots\dots\dots\dots (62)$$

Calculations show that the factor, $\frac{M}{N}$, has an approximate value of $0.3 \sqrt{b_D}$; therefore $t_D = 0.3\, b_D \sqrt{b_D}$; and,

$$t = 0.3\, b_e \sqrt{\frac{b_e}{D_h}} \dots\dots\dots\dots (63a)$$

Solving Equation (63*a*) for b_e,

$$b_e = 2.23\, t \sqrt[3]{\frac{D_h}{t}} \dots\dots\dots\dots (63b)$$

The value of a is determined from Equation (5).

The use of Equations (63*a*), (63*b*), and (5) will result in a design which has equal resistance to failure by fracture at the side of a pin-hole, fracture below a pin-hole, and "dishing." Specimens that failed by fracture at the side of a pin-hole did so at an average stress in that section equal to, or slightly higher than, the ultimate tensile strength of the material as determined by standard tensile tests. A common specification is that the net section through a pin-hole shall be at least 40% greater than the net section of the member.[10] This shows that standard design requirements are on the side of safety.

In the "Summary," the author states, "* * * the results of this paper are not intended to apply directly to pin-connected plates having the shape of standard eye-bars unless tests on such eye-bars show agreement with the present results." Records of tests of standard eye-bars which broke in the head are extremely rare because of the fact that eye-bars are usually made to certain minimum requirements, one of which is that the diameter of the pin shall be not less than seven-eighths the width of the bar.

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[9] Designing Engr., Am. Bridge Co., New York, N. Y.

[10] A. R. E. A. Specification for Steel Railway Bridges, 1935.

Occasionally, however, eye-bars are made, which vary from standard practice, and they sometimes fail in the head. The latest case of this kind, to the writer's knowledge, occurred in 1931, when a 7-in. by 1-in. eye-bar was tested to destruction. The two heads were 16.5 in. in diameter and the pin-holes were 7 in. and 5.25 in. in diameter, respectively. The eye-bar failed by "dishing" and fracture in the head having the smaller pin-hole.

An analysis of this eye-bar head according to Equations (2), (3), and (4) indicates that failure should take place by "dishing." Solving Equation (4), $S_{bt} = 93.8$ kips.

This means that a 16.5-in. rectangular plate with the pin-holes in the two ends should have failed by "dishing" back of the 5.25-in. pin when the bearing stress between the pin and plate reached the foregoing value. The total tension on the plate would then be 492 450 lb. The eye-bar in question failed by "dishing" and fracture back of the 5.25-in. pin under a load of 492 000 lb (reading to the nearest kip), indicating agreement between this eye-bar test and the author's work. This is further verified by the fact that another eye-bar, which was a duplicate of the one noted, had been tested a few months earlier, and the failure had occurred in the body of the bar. This appears contradictory, but when it is noted that this eye-bar failed under a tension of 470 kips it is seen that the material failed just before the head of the eye-bar was about to fail by "dishing." These tests indicate that the author's results will apply to standard eye-bars.

If standard practice had been followed when making these eye-bars the minimum size of the pin would have been $6\frac{1}{8}$ in. in diameter (seven-eighths of 7 in.). A re-test of two more 7-in. by 1-in. eye-bars was made, but instead of using a 5.25-in. pin at one end the pin was made 6.25 in. in diameter in order to determine whether this increase would have any influence on the result. Both these eye-bars broke in the body of the bar, as would be expected from the author's results (a solution of Equation (4), using a 6.25-in. pin, shows that it would require a total tension of 520 kips to cause this head to fail by "dishing," whereas the eye-bars failed under tensions of 474 kips and 470 kips, respectively).

The author refers (in the "Introduction") to "forged" eye-bars. The writer wishes to emphasize that, although the two heads are produced by upsetting (or forging), the main body of the eye-bar is a plain rolled plate, or bar. For example, an eye-bar 65 ft from center to center of end pins has nearly 60 ft of rolled material with 4 ft to 5 ft at each end that has been forged. When a "forging" is specified, the material is manufactured to A.S.T.M. Specification A18, whereas A.S.T.M. Specification A7 gives complete requirements for annealed eye-bars. There is a considerable difference between these two materials. The latter specification is generally used for eye-bars, whereas the former never is; and when the term, "forged," is introduced, it leads to confusion.

Bruce G. Johnston,[11] Assoc. M. Am. Soc. C. E. (by letter).—The discussion presented by Mr. Hussey is entirely constructive in nature. The close agreement between the tests of two eye-bars and the strength and type of failure predicted by Equations (2), (3), and (4) is very gratifying. It is unfortunate that these are the only two tests of eye-bars brought to light in discussion. Equations (63*a*) and (63*b*), used with Equation (5), will give a well-balanced design entirely safe with respect to failure by dishing. Equations (2), (3), and (4) should be used when close clearances or other special requirements make a balanced design impossible.

In connection with the application of this paper to the design of eye-bars it has been brought to the writer's attention that Section 411 of the A. R. E. A. Specification for Steel Railway Bridges has been changed to recommend a maximum net width-to-thickness ratio at the pin-hole of 8 instead of 12.

The writer wishes to express his appreciation to Mr. Hussey for his courtesy in discussing the paper.

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[11] Asst. Prof. of Civ. Eng., and Asst. Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

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Paper No. 2024

WATER-HAMMER PRESSURES IN COMPOUND AND BRANCHED PIPES

By Robert W. Angus,[1] Esq.

With Discussion by Messrs. K. J. DeJuhasz, Harold A. Thomas, F. Knapp, Martin A. Mason and Pierre F. Danel and Antoine Craya, A. A. Kalinske, and Robert W. Angus.

Synopsis

In two previous papers,[2, 3] the writer has derived the principal equations for water hammer and has given the theory in some detail, so that only the barest outline of the general theory is given herein. Beginning with the two general equations derived by L. Allievi,[4] a graphical construction has been explained in some detail and illustrated in two simple problems of uniform pipes.

For the application of the method to simple pipes, the reader should consult the earlier papers, as this paper deals with other problems, and particularly those on compound and branched pipes. Illustrations are given of the application to parallel pipes, pipes with dead ends, surge tanks in systems, pipes leading from reservoirs and having two branches, each discharging water, the effect of gate closure on turbines and draft-tubes, and the case of a pumping system in which the pressure falls so low as to cause the column to separate. In all these cases the variation of pressure and velocity with time is found for various points in the system.

All these problems, and many more, may be solved by a careful worker, both accurately and quickly, on the drafting-board, although their solution by analytical means is almost impossible.

Notation.—The letter symbols used in this paper are defined where they are first mentioned and, for convenience of reference, a complete list is given in the Appendix.

Note.—Published in January, 1938, *Proceedings*.

[1] Prof. of Mech. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[2] "Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines," by Robert W. Angus, *Journal*, Eng. Inst. of Canada, 1935, Vol. 18, pp. 72, 264.

[3] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps: Graphical Treatment," by R. W. Angus, *Proceedings*, Inst. M. E., London, England, Vol. 136 (1937).

[4] "Theory of Water Hammer, Notes I–V," by L. Allievi; Tr. by E. E. Halmos, M. Am. Soc. C. E.; Distributed by A. S. M. E., 1925.

Outline of General Theory

When a closed pipe is filled with moving liquid (water will be referred to hereafter), the laws governing the changes of pressure and discharge will depend upon the conditions under which flow occurs. If the motion is steady (by which is meant that the volume per second passing through any section of the pipe remains constant as time goes on), the Bernoulli equation may be applied.

When the motion is unsteady (that is, when the discharge at each section is varying from one instant to the next), the Bernoulli equation no longer serves, and the pressures and velocities are not connected by this equation. Sometimes during unsteady motion there is a mass flow of the entire column of water in the pipe, such as occurs when the water surges back and forth between two reservoirs, or between a reservoir and a tank; such cases are common and there must be an open tank or reservoir, or an air chamber, in the line.

On the other hand, variable motion may occur in a fully closed system provided there is anything to start it, the motion being then due to the elasticity of the water and pipe. Thus, variable motion may occur in a section of a pipe system with a "dead end," due to some change in the system, such as the closing of a valve somewhere, although there may be no delivery of water through the pipe.

The three cases are distinct and important: The first is used, for example, in finding the size of a pipe for given service; the second case is common in water-power plants where surge tanks are necessary to store and restore the water during load changes; and, the third case is of common occurrence in all systems. Quite frequently, the latter two cases occur together; there is a mass surge of the water in the system, but at the same time the elasticity of the water and pipes produces independent action, often building up high pressures at different points and not infrequently producing pressures at the bottom of an open tank many times greater than that corresponding to the depth of water in the tank. Under certain conditions, too, the elasticity of the water and pipe walls induces extremely low or collapsing pressures, and may even cause separation of the water column itself.

This paper refers to the latter case of unsteady motion which produces the phenomenon of water hammer. It will be best to start with the simple case of a horizontal uniform pipe connected at one end to a reservoir that has a fixed level and terminating in a nozzle, gate, or valve (the three terms are used in the paper for the same type of device) which may be used to control the flow. The equation of continuity is used and will be specifically applied to branch pipes, but it is to be noted that at a given instant the elasticity of the water and of the pipe walls enables the discharges at different points on the same pipe to differ at the same time.

In many problems, velocity head is relatively small, as is also friction loss, and the cases under consideration are simplified if the two terms are omitted; the pressures computed when friction loss is omitted are nearly always greater than the actual pressures and, therefore, the results are on the safe side. Where pipes burst, or in other unusual conditions, both friction and velocity may have to be taken into account, and the method of doing this is explained subsequently. For convenience, the reservoir level is assumed constant, although variations in it are easily allowed for; these variations are very slow, however, in comparison with the pressure wave velocity.

The case described is illustrated by Pipe $A\,B\,C$ of length L in Fig. 1, in which Point A is at the discharge end, Point C, at the reservoir end, and Point B, at the center. Under conditions of steady flow with the gate fully open, H_0 will represent the pressure on the gate as well as the elevation of the water surface above the gate, and if the pipe area is A_a, the steady velocity, V_0, in the pipe with the discharge, Q_0, is given by,

$$Q_0 = A_a\,V_0 \dots\dots\dots\dots\dots\dots (1)$$

in which the units are in feet and seconds. Suppose, now, that a closing movement of the gate is begun which would effect complete closure in T sec; any law of closure may be used, but if it is such that the "effective gate area" (by which is meant the actual gate area multiplied by the proper coefficient of discharge) decreases at a uniform rate with time, the motion is referred to as "straight-line" or "linear," a nomenclature used in the remainder of this paper.

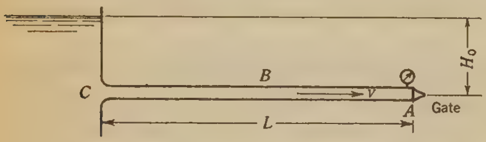

FIG. 1

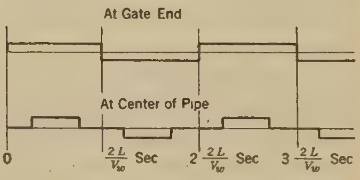

FIG. 2

The actual gate movement may be assumed made in small steps, each one of which causes a small change in the pipe velocity with a corresponding pressure change along the pipe. The mechanics of the phenomena are fairly well understood and need not be repeated, except to state that the first movement causes a direct pressure wave to travel up the pipe with velocity V_w, the round trip of this wave requiring $\frac{2\,L}{V_w}$ sec, and it returns a second time up the pipe from the gate as an indirect pressure wave unchanged in magnitude, etc. These waves continue to follow one another, each small gate movement producing corresponding waves, the ultimate pressure at any point being the resultant of those due to the separate waves. Fig. 2 illustrates what is meant by each wave, and its effect at the end and center of the pipe.

The magnitude of the pressure waves at the gate may be determined from the following considerations: After a small closing movement of the gate, a pressure wave travels up toward the reservoir with velocity, V_w, and in the time, δT, it will have traveled a distance (see Fig. 3),

$$L_w = V_w\,\delta T \dots\dots\dots\dots\dots\dots (2)$$

the pressure rise for such change being indicated by δh. Denoting the part of the velocity extinguished in this movement by δV, Newton's second law gives,

$$-w A_a \delta h = \frac{m\,\delta V}{\delta T} = \frac{A_a\,w\,V_w\,\delta T}{g}\frac{\delta V}{\delta T} \ldots\ldots (3)$$

in which m is the mass of the water changed in velocity; and w is the weight of a cubic foot of water. The minus sign appears in Equation (3) because an increase in δh corresponds to a decrease in δV. The formula reduces to,

$$-\delta h = \frac{V_w}{g}\delta V \ldots\ldots (4)$$

Integration of Equation (4) gives, for the first interval:

$$h = H - H_0 = \frac{V_w}{g}(V_0 - V) \ldots\ldots (5)$$

in which H is the new reading of the gage at the gate. The maximum value of h results when $V = 0$; in other words, when the entire velocity is extinguished in the first interval, in which case the pressure rise is,[5]

$$h = H - H_0 = \frac{V_w}{g}V_0 \ldots\ldots (6)$$

and this shows the interesting fact that the maximum pressure rise is independent of the dimensions of the pipe and of the head, and depends only on the velocity extinguished.

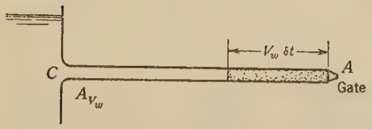

FIG. 3

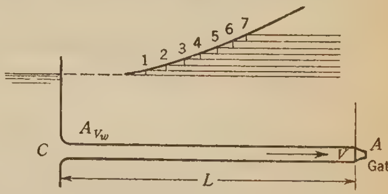

FIG. 4

Since Equation (6) refers to the direct waves and takes no account of the reflected ones, it can only be applied to find the direct change in pressure corresponding to an individual gate movement and not to find the pressure rise in the pipe, directly, unless closure is effected in one interval, in which case no reflection occurs. The equation applies naturally within any interval provided the proper velocities are used. The magnitude of the direct wave in the nth movement of the gate will be indicated by F_n and is given by,

$$F_n = H_n - H_{n-1} = \frac{V_w}{g}(V_n - V_{n-1}) \ldots\ldots (7)$$

Having traced the course of events following the first gate movement, it is now possible to see what happens during subsequent movements. If the closing time, T, is less than $\frac{L}{V_w}$, the events are as shown in Fig. 4, where the numbers indicate the pressures computed from Equation (5) corresponding to the small movements of the gate; the combined series of movements produces a wave of sloping (curved) front traveling first from the gate, and this wave will reach its maximum height before its "toe" reaches Point C, as represented in Fig. 5(a). When the closure requires $\frac{L}{V_w}$ sec, the wave front is shown in Fig. 5(b), the wave front covering the entire length of the pipe; but, in Case 5(c) in which the closure time exceeds $\frac{L}{V_w}$, the returning opposition wave neutralizes some of the positive pressures and the maximum pressure is exerted on the pipe until Point g is above Point j. In Case 5(d) the maximum pressure is reached only, but does not remain, as the opposition wave front cancels it, whereas, in Case 5(e), the pressure is never as high as in the former cases. The maximum pressure reached is the same in Cases (a) to (d) where $T \leqq \frac{2L}{V_w}$ (provided the same velocity is extinguished at each step); but the quicker the closure the longer the time will be during which this maximum pressure continues.

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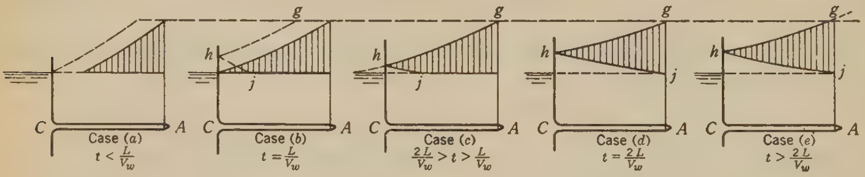
[5] A summary of the work done by N. Joukowsky, of Moscow, in 1898, has been given in a paper entitled "Water Hammer" by Miss O. Simin, *Proceedings*, Am. Water Works Assoc., 1904, p. 341.



Case (a) $t < \frac{L}{V_w}$ Case (b) $t = \frac{L}{V_w}$ Case (c) $\frac{2L}{V_w} > t > \frac{L}{V_w}$ Case (d) $t = \frac{2L}{V_w}$ Case (e) $t > \frac{2L}{V_w}$

FIG. 5

For the case in Fig. 5(e) the same maximum pressure is not reached. Furthermore, it may be shown that for linear gate movement the curvature of the wave front is not great; it is convex to the horizontal axis as drawn, so that in such cases as those in Figs. 5(b), 5(c), 5(d), and 5(e), the total pressure rise has roughly a straight-line variation from zero at the reservoir to a maximum value at the gate, a fact also confirmed by many computations.

The general proposition may then be stated that the maximum pressure rise is produced for closing times equal to, or less than, $\frac{2L}{V_w}$; this is commonly referred to as sudden closure. For short pipes the closure may easily be too slow to produce the most serious results, but it is quite possible in the longer pipes, and, for this reason, it is frequently stated that the danger of damage from water hammer is greater in long pipes than in short ones; this statement, however, is true only in the sense that the time of closure to produce the worst results is often within the range of operation of the valves on long pipes.

FUNDAMENTAL EQUATIONS FOR VARIABLE FLOW IN CLOSED PIPES

In order to establish the fundamental relations, the variation in flow is considered to be due to closing a valve in a pipe line, thereby causing a decrease in the flow. The equations thus established apply equally well to a gate-opening.[4] The pipe is assumed to be of a uniform diameter, D, and a thickness, t; and the problem frequently occurs in one or the other of the two forms shown in Fig. 6. The former (Fig. 6(*a*), with the enlarged section in Fig. 6(*b*)) corresponds to closing a nozzle on the end of a pipe, whereas Fig. 6(*c*) represents a pumping line with a control valve, decreased flow being caused by closing the valve or by cutting off power to the pump. Elsewhere,[3] the writer has demonstrated that, at any point, situated a distance, x, from the gate (Case (*a*), Fig. 5) and at a time, T, after closure begins:

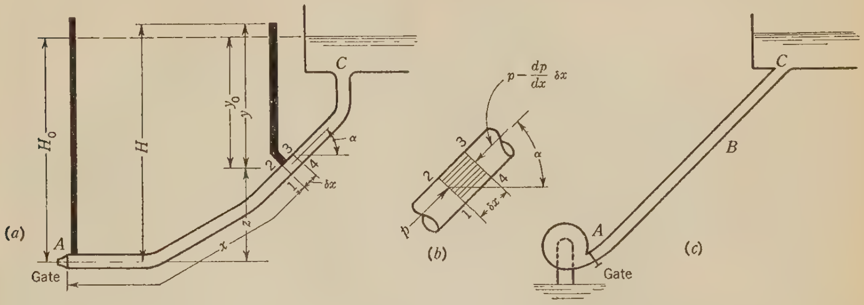


FIG. 6

$$\frac{\delta V}{\delta T} = g\,\frac{\delta H}{\delta x} \qquad (8a)$$

and,

$$\frac{\delta V}{\delta x} = \frac{g\,\delta H}{V_w^2\,\delta T} \qquad (8b)$$

In Case (*c*), Fig. 5, both left-hand members have a negative sign and, therefore, Equations (8) apply, with proper correction to the sign. Furthermore,

$$V_w = \frac{\sqrt{\dfrac{g\,K}{w}}}{\sqrt{1 + \dfrac{K\,D}{E\,t}}} = \frac{4\,665}{\sqrt{1 + \dfrac{K\,D}{E\,t}}} \qquad (9)$$

in which K is the bulk modulus of elasticity for water; and, E is Young's modulus for the material in the pipe. Some approximate values of the ratio, $\frac{K}{E}$, are:

Steel plate	0.0097 (usually assumed as 0.01)
Cast iron	0.02 to 0.022
Concrete	0.10
Wood	0.20

In Equations (8) and (9), $\frac{\delta V}{\delta T}$, $\frac{\delta H}{\delta x}$, $\frac{\delta V}{\delta x}$, and $\frac{\delta H}{\delta T}$, refer to changes in a small cylinder (1 2 3 4, Fig. 6(*b*)) situated a distance, x, from the gate. The general integrals of Equations (8) were first obtained by Allievi as follows:

$$H - H_0 = F\left(T - \frac{x}{V_w}\right) + f\left(T + \frac{x}{V_w}\right) \ldots\ldots\ldots\ldots\ldots (10a)$$

and,

$$V_0 - V = \frac{g}{V_w}\left[F\left(T - \frac{x}{V_w}\right) - f\left(T + \frac{x}{V_w}\right)\right] \ldots\ldots\ldots (10b)$$

and the corresponding integrals for Case (*c*), Fig. 6, are similar except for the change in sign of the left-hand member in each case.

One point deserves special emphasis: The pressure head, H, is measured above the horizontal plane of the nozzle, so that at Section 1–2, Fig. 6(*a*), the actual pressure in the pipe is only y and is connected with H by the relation,

$$H = y + z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The pressure rise at this section is $H - H_0 = y - y_0$; but the ratio of the pressures after, and before, gate movement is,

$$\frac{y}{y_0} = \frac{H - z}{H_0 - z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

In Equations (10) (which will be referred to as Allievi's equations), H represents the pressure head, in feet, and V the velocity, in feet per second, at a point distant x from the gate, at a time, T, after closure begins; $F\left(T - \frac{x}{V_w}\right)$ denotes the sum of all direct pressure waves (positive, in closing), each persisting for a time, $T = \frac{2L}{V_w}$, at the gate; and $f\left(T + \frac{x}{V_w}\right)$ denotes the sum of all reflected pressure waves (negative, in closing), each of which quantities also persists for a time, $T = \frac{2L}{V_w}$ sec, at the gate. A little consideration will show that the direct wave in any interval is equal in magnitude, but of opposite sign, to the reflected wave, $\frac{2L}{V_w}$ sec later, if the reflected and direct waves each correspond to the same gate movement. This has been shown in the general discussion of the pressure waves in the pipe.

Allievi's equations (Equations (10)) may be used to find the conditions at any point on the pipe, but if the pressure at a point, $x = 0.75\,L$, is desired, the steps of the valve motion must be not more than $\frac{1}{4}\left(\frac{2L}{V_w}\right) = \frac{L}{2V_w}$ sec apart, and a similar statement applies to other points; for the gate the movements may be $\frac{2L}{V_w}$ sec apart. Similar equations may be written[2] where friction and velocity head are considered if the following quantity, H_T, is substituted for H:

$$H_T = H + \frac{V^2}{2g} + h_r \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which h_r is the friction loss, in feet, up to the point considered. The equations applying to Fig. 6(a) would then be:

$$H_T - H_0 = F\left(T - \frac{x}{V_w}\right) + f\left(T + \frac{x}{V_w}\right) \ldots\ldots\ldots\ldots (14a)$$

and,

$$V_0 - V = \frac{g}{V_w}\left[F\left(T - \frac{x}{V_w}\right) - f\left(T + \frac{x}{V_w}\right)\right] \ldots\ldots (14b)$$

and for the case of Fig. 6(c) they are easily written. The solution of the problem is virtually the same whether H_T or H is used, and hence the graphs will be constructed using Equations (10).

The analytical method of dealing with water hammer has been used to some extent in practice. For example, Dr. Charles Jaeger[6] has described the analytical solution in considerable detail and has illustrated it by some examples. It must be evident, however, that: (a) Much care is to be exercised in its application; (b) some of the calculations are long and somewhat involved; and, (c) there is a possibility of error in using it unless one is doing it very frequently. It is quite difficult to apply in case of branched and complicated pipes, but has proved of great value in establishing basic relationships.

In order to simplify and hasten the answers to problems in water hammer, a number of graphical methods have been devised of which probably that of Allievi[4] was the first; but his method was only applied to uniform pipes, without friction, and usually with straight-line valve movement. Furthermore, the method is not practicable on account of the large scale to which the drawings should be made. Allievi used it to develop his useful series of charts. In 1926, Dr. Hans Kreitner[7] described a method of merit which was translated and summarized by the writer in 1935.[8] This method has been used by Hruschka.[9]

The construction described and applied in this paper is based on that of earlier writers, but its development is due very largely to Dr. O. Schnyder,[10] of Klus, Switzerland, and to Professor L. Bergeron,[11] of Paris, France, who have shown that the method is general and may be very simply used in solving the most complicated problems quickly.

GRAPHICAL METHOD

The graphical method proposed herein proceeds from Equations (10) and as these formulas are also used in the analytical development the same results will be obtained by the graphical as by the analytical solution. By addition and subtraction, they reduce to:

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[6] "Theorie General du Coup de Belier," by Charles Jaeger, Dunod, Paris, 1933.

[7] "Druckschwankungen in Turbinenrohrleitungen," *Die Wasserwirtschaft*, 1926, No. 10; a translation by Professor Angus has been filed for reference in Engineering Societies Library, 33 West 39th St., New York, N. Y.

[8] *Mechanical Engineering*, December, 1935.

[9] "Druckrohrleitungen der Wasserkraftwerke," by A. Hruschka, Julius Springer, Berlin, 1929.

[10] "Über Druckstösse in Rohrleitungen," by Dr. O. Schnyder, *Wasserkraft und Wasserwirtschaft* (Berlin), 1932, Vol. 5/6; also, "Über Druckstösse in verzweigten Leitungen mit besonderer Berücksichtigung von Wasserschlossanlagen," by O. Schnyder, *Wasserkraft und Wasserwirtschaft* (Berlin), 1935, Vol. 12.

[11] "Variations de régime dans les conduites d'eau," by Prof. L. Bergeron, *Comptes Rendus* des Travaux de la Société Hydro-technique de France, May, 1932, p. 605H; also, "Etude des variations de régime dans les conduites d'eau: Solution graphique generale," by Prof. L. Bergeron, *Revue Generale de l'Hydraulique* (Paris), 1935, p. 12, etc.

$$H - H_0 = -\frac{V_w}{g}(V_0 - V) + 2F\left(T - \frac{x}{V_w}\right) \dots\dots\dots (15a)$$

and,

$$H - H_0 = +\frac{V_w}{g}(V_0 - V) + 2f\left(T + \frac{x}{V_w}\right) \dots\dots\dots (15b)$$

If Equations (15) are applied to the gate end of the pipe, where $x = 0$, they will be written:

$$H - H_0 = -\frac{V_w}{g}(V_0 - V) + 2F(T) \dots\dots\dots\dots\dots (16a)$$

and,

$$H - H_0 = +\frac{V_w}{g}(V_0 - V) + 2f(T) \dots\dots\dots\dots\dots (16b)$$

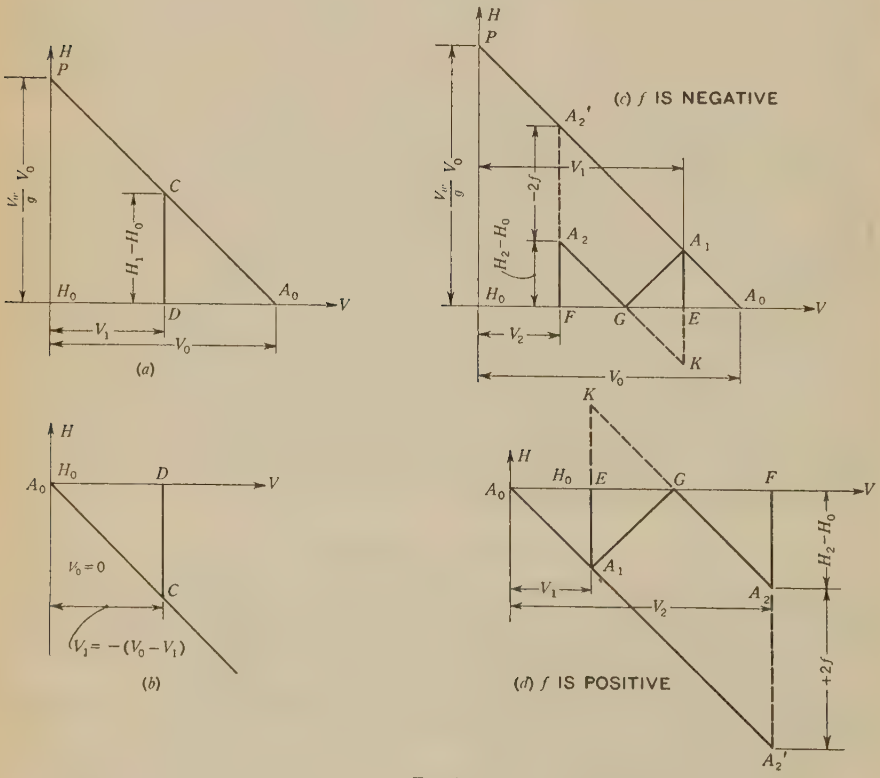


FIG. 7

In Fig. 7 the axes of H and V have been drawn as shown, and Point A_0 represents the initial steady velocity, V_0, and static head, H_0. In the first interval there is no reflected wave, or $2f(T) = 0$, and Equation (16b) represents a straight line through A_0 with a tangent $-\frac{V_w}{g}$ as represented at $A_0\,P$ in Fig. 7(a). For a closing motion of the gate, reducing the velocity to V_1 in $T \leqq \frac{2\,L}{V_w}$, the rise in pressure is given by Line $C\,D$ which corresponds to $-\frac{V_w}{g}(V_0 - V_1) = H_1 - H_0$, as shown in Fig. 7($a$), and Equation (16$a$) shows that the direct wave, $C\,D$, equals $F_1\,(T)$ (see also Equation (7)), since,

$$2\,F_1\,(T) = H_1 - H_0 + \frac{V_w}{g}(V_0 - V_1) \quad \text{.............} (17)$$

When the opening occurs from a closed gate, $V_0 = 0$, at $H = H_0$, and the line, $A_0\,C$, is the locus of the values of H as before. Both $V_0 - V_1$ and $H_1 - H_0$ are negative in this case (see Fig. 7(b)). If, however, the velocity, $V = V_2$, is not reached until an interval beyond the first—for example, in the time $2\left(\frac{2\,L}{V_w}\right) > T > \frac{2\,L}{V_w}$—then the pressure reached is modified by the reflected wave. When the gate is being closed, the previous discussion shows that the reflected wave produces a negative pressure equal in magnitude, but opposite in sign, to the direct or F-wave produced in the previous interval. In Fig. 7(c) let V_1 be the velocity reached at the end of the first interval, then the pressure corresponding to this velocity will be represented at Point A_1 and Line $A_1\,E$ is the pressure rise, or the value of $F_1\,(T)$ in the first phase. It is also the numerical value of the reflected pressure or f-wave in the second interval; that is,

$$\overline{A_1\,E} = -f_2\,(T) \quad \text{......................} (18)$$

Equation (16b) with Fig. 7(c) shows that,

$$A_2'\,F = +\frac{V_w}{g}(V_0 - V_2) \quad \text{..................} (19a)$$

and,

$$2\,A_1\,E = A_1\,K = -\,2\,F_1\,(T) = 2\,f_2\,(T) \quad \text{...........} (19b)$$

Hence, $H_2 - H_0 = A_2\,F$; and, A_2 represents the conditions corresponding to the change of velocity, $V_0 - V_2$, since $A_2'\,A_2 = A'\,K$. Point A_2 is determined graphically either by making Line $E\,K$ = Line $A_1\,E$ and drawing Line $K\,A_2$ parallel to Line $A_0\,P$; or, by drawing Line $A_1\,G$ at the slope $+\frac{V_w}{g}$ (that is, $\overline{A_0\,E} = \overline{E\,G}$) and drawing Line $G\,A_2$ parallel to Line $A_0\,P$.

When the gate is being opened, the result is quite similar. Suppose the gate is opened from a closed position with $V_0 = 0$ and the velocity, V_1, is reached in the first interval, it is evident that Line $C\,D$, Fig. 7(b), represents $H_0 - H_1$. If the velocity, V_2, is reached in the second interval, the construction would be as shown in Fig. 7(d), in which both $V_0 - V_2$ and $H_2 - H_0$ are negative, the slope of the line has the same tangent, $-\frac{V_w}{g}$, as before, and

A_1 represents the pressure at the end of the first interval. Therefore, $F_1(T)$ is negative and $f_2(T)$ is positive and equal to $A_1 E$, and the point, A_2, representing H_2 lies at A_2 above A_2'. Equations (16), therefore, are easily represented by this graphical construction.

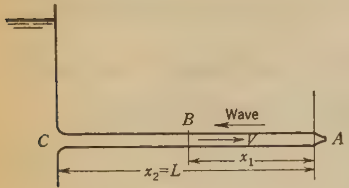


FIG. 8

These two formulas, Equations (15), then, define the laws of the graphical method, and it only remains to re-arrange them into a more useful form. Applying them to the pipe shown in Fig. 8 (in which case T_0, T_1, and T_2, denote the times that the same direct wave passes, respectively, Point A on the pipe at the origin, Point B at a distance, x, and Point C at the reservoir, which is a distance, $x_2 = L$ from Point A), it is to be noted that,

$$T_1 = T_0 + \frac{x_1}{V_w} \qquad \text{(20a)}$$

and,

$$T_2 = T_0 + \frac{x_2}{V_w} \qquad \text{(20b)}$$

Therefore, while the direct wave is proceeding from Point A on the pipe to Point C, the following equations apply (using H_{A0}, etc., to denote the steady condition):

$$H_{AT0} - H_{A0} = -\frac{V_w}{g}(V_{A0} - V_{AT0}) + 2F(T_0) \qquad \text{(21a)}$$

$$H_{BT1} - H_{B0} = -\frac{V_w}{g}(V_{B0} - V_{BT1}) + 2F\left(T_1 - \frac{x_1}{V_w}\right)$$

$$= -\frac{V_w}{g}(V_{B0} - V_{BT1}) + 2F(T_0) \qquad \text{(21b)}$$

and,

$$H_{CT2} - H_{C0} = -\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2F\left(T_2 - \frac{x_2}{V_w}\right)$$

$$= -\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2F(T_0) \qquad \text{(21c)}$$

Furthermore, while the reflected wave is proceeding toward the gate, the reverse of Fig. 8, and reaches Point B at the time, T_3, and Point A at the time, T_4, the following equations apply:

$$H_{CT2} - H_{C0} = +\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2f\left(T_2 + \frac{x_2}{V_w}\right)$$

$$= +\frac{V_w}{g}(V_{C0} - V_{CT2}) + 2f(T_4) \qquad \text{(22a)}$$

$$H_{BT3} - H_{B0} = +\frac{V_w}{g}(V_{B0} - V_{BT3}) + 2f\left(T_3 + \frac{x_1}{V_w}\right)$$

$$= +\frac{V_w}{g}(V_{B0} - V_{BT3}) + 2f(T_4) \qquad \text{(22b)}$$

and,

$$H_{AT4} - H_{A0} = + \frac{V_w}{g}(V_{A0} - V_{AT4}) + 2f(T_4) \quad \text{(22c)}$$

Generally, at any point, such as Point B, and at any instant, two waves are proceeding: The direct wave toward Point C; and the reflected wave toward Point A. For the direct wave the pressure rise above the initial steady pressure, H_{B0}, at Point B is given by Equation (21b), and for the reflected wave it is given by Equation (22b). If friction is not being considered, then $H_{A0} = H_{B0} = H_{C0}$ as the water level will be the same in piezometers attached to the pipe at Points A, B, and C; and subtractions of Equations (21) and (22), made in suitable order, give:

$$H_{AT0} - H_{BT1} = + \frac{V_w}{g}(V_{AT0} - V_{BT1}) \quad \text{(23a)}$$

$$H_{BT1} - H_{CT2} = + \frac{V_w}{g}(V_{BT1} - V_{CT2}) \quad \text{(23b)}$$

$$H_{CT2} - H_{BT3} = - \frac{V_w}{g}(V_{CT2} - V_{BT3}) \quad \text{(23c)}$$

and,

$$H_{BT3} - H_{AT4} = - \frac{V_w}{g}(V_{BT3} - V_{AT4}) \quad \text{(23d)}$$

The case illustrated is that of a pipe discharging from a reservoir through a closing gate. A very common case is also that of a pump discharging through a pipe into a reservoir, as shown in Fig. 6(*c*). The flow is controlled by a gate near the pump, the movement of the gate causing pressure disturbances. It has already been shown that the algebraic signs of the fundamental differential equations are the opposite of those in the case of Fig. 8. This only means that all terms on the right-hand side containing V will change sign, and, hence, for the case of the pump with controlling discharge valve near it, the Equations (23) would be written:

$$H_{AT0} - H_{BT1} = - \frac{V_w}{g}(V_{AT0} - V_{BT1}) \quad \text{(24a)}$$

and,

$$H_{BT1} - H_{CT2} = - \frac{V_w}{g}(V_{BT1} - V_{CT2}) \quad \text{(24b)}$$

etc., the letters, A and C, referring to the valve and reservoir ends, respectively, of the pipe, as in the previous case. In Fig. 8, let $x_2 = 10\,000$ ft, in which the length, $x_1 = 4\,500$ ft, and $V_w = 4\,000$ ft per sec. Then, reckoning time from the beginning of closure, it follows that $T_0 = 0$; $T_1 = 0 + \frac{4\,500}{4\,000} = 1.125$ sec; $T_2 = 0 + \frac{10\,000}{4\,000} = 2.5$ sec; $T_3 - T_2 = \frac{10\,000 - 4\,500}{4\,000} = 1.375$ sec; and, $T_4 - T_3 = \frac{5\,000}{4\,000} = 1.125$ sec.

For many problems the series of equations is best left in the form shown in Equations (23) or Equations (24), and some illustrations will be given subsequently in which this method is adopted; but it often happens that the engineer is more interested in the proportional rise in pressure than in the actual pressure and wishes to know the ratio, $\frac{H}{H_0}$, directly. The results are the same whether one obtains H, $H - H_0$, or $\frac{H}{H_0}$; it is purely a matter of convenience for the particular application. If the equations are modified to yield the value of $\frac{H}{H_0}$, Equation (23*a*) may be written:

$$\frac{H_{AT0}}{H_0} - \frac{H_{BT1}}{H_0} = 2\,\frac{V_w\,V_{A0}}{2\,g\,H_0}\left(\frac{V_{AT0}}{V_{A0}} - \frac{V_{BT1}}{V_{A0}}\right) \qquad (25)$$

Ordinarily, small letters are used to designate the ratios; thus: $h_{BT1} = \frac{H_{BT1}}{H_0}$; $v_{BT1} = \frac{V_{BT1}}{V_{B0}}$, etc. Furthermore, the relation, $\frac{V_w\,V_{A0}}{2\,g\,H_0} = \rho$, will be referred to as the pipe-line characteristic, as defined by Allievi. Similar relationships are found for the other equations and Equations (23) then take the form:

$$h_{AT0} - h_{BT1} = +\,2\,\rho\,(v_{AT0} - v_{BT1}) \qquad (26a)$$

$$h_{BT1} - h_{CT2} = +\,2\,\rho\,(v_{BT1} - v_{CT2}) \qquad (26b)$$

$$h_{CT2} - h_{BT3} = -\,2\,\rho\,(v_{CT2} - v_{BT3}) \qquad (26c)$$

and,

$$h_{BT3} - h_{AT4} = -\,2\,\rho\,(v_{BT3} - v_{AT4}) \qquad (26d)$$

which is the form in which they are mostly commonly used in the graphical construction. For the pumping system (see Equations (10)), the algebraic signs of the second terms would be reversed.

The derivation of Equations (23), (24), and (26) has been made for closing movements of the gate only, but they will apply in exactly the same form to opening movements, because in this case, the change of pressure has the opposite sign to the change of velocity as in the case of closure; therefore, the two series (Equations (23) and (26) and the corresponding formulas for the pumping system) apply also to opening movements.

To represent Equations (26) graphically, axes of h and v, Fig. 9, are assumed, the origin having the value, $h = 0$, $v = 0$. The values of h should preferably be plotted on the vertical axis. Continuing still with the problem of closure, as in Fig. 6(*a*), the values at Point A during steady flow before closure begins are $H = H_0$ and $V = V_0$; hence, at the start, $h = 1$ and $v = 1$, corresponding to the time, T_0, in Equations (26). Furthermore, the pressure wave does not reach Point B until the time, T_1, and does not reach Point C until the time, T_2, therefore, up to these respective times the conditions at Points B and C remain unchanged, or $h_{BT1} = h_{CT2} = 1$, and $v_{BT1} = v_{CT2} = 1$. In Fig. 9(*a*), a point A_{T0}, will represent the conditions at Point A at the time,

T_0; similarly, B_{T_1} will represent those at Point B at the time, T_1, etc. Therefore, A_{T_0}, B_{T_1}, and C_{T_2} are all at Point $h = 1$, $v = 1$.

For this problem, if it is assumed that the reservoir level is constant, then all values of $h_C = 1$, and hence all points, C, must lie at $h = 1$ on the axis

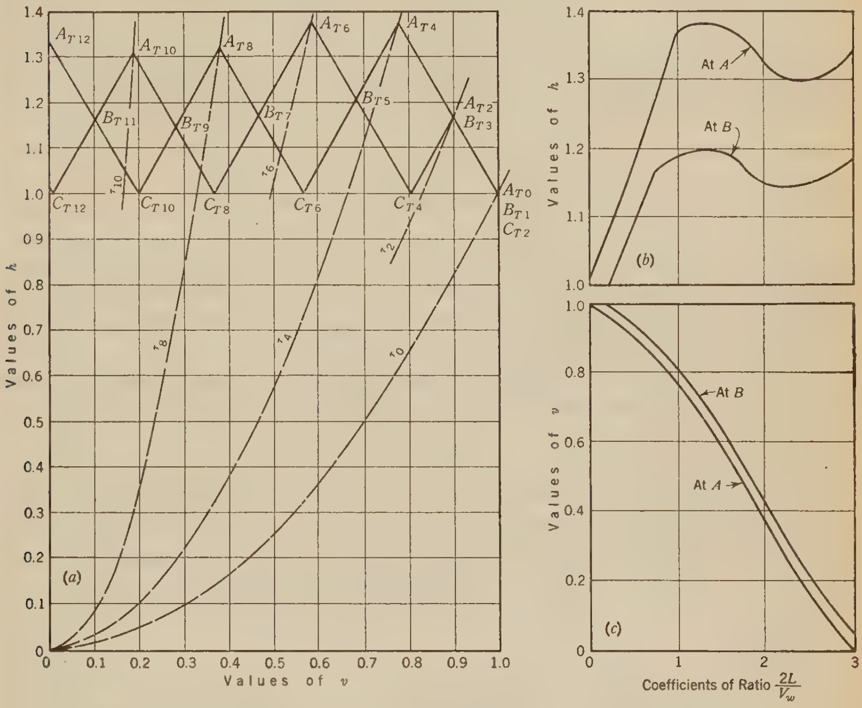


FIG. 9

of v; Equations (26*a*) and (26*b*) are then represented by a point. It may be simplest for the moment to proceed at once from Points A to C, thus combining Equations (26*c*) and (26*d*) into:

$$h_{CT_2} - h_{AT_4} = -\,2\,\rho\,(v_{CT_2} - v_{AT_4}) \ldots\ldots\ldots\ldots\ldots (27)$$

This is evidently a line through C_{T_2} sloping at the angle, $\tan^{-1} - 2\,\rho$, which defines one line on which A_{T_4} lies. It is now necessary to establish the law connecting the head, H_A, and the velocity, V_A, at the gate. No matter what form the gate has, the velocity of discharge from it is,

$$V_A = c_d\,\sqrt{2\,g\,H_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which c_d is the coefficient of discharge, and in which the discharge is found by multiplying this velocity by A_g, the area of the gate-opening. Hence, the pipe velocity is,

$$V_A = \frac{A_g \times c_d\,\sqrt{2\,g\,H_A}}{A_a} \doteq B\,\sqrt{H_A} \ldots\ldots\ldots\ldots\ldots (29)$$

in which B is a function of the effective gate area, $(c_d\,A_g)$; and, therefore, B is a function of the time. For the initial steady condition:

$$V_{A0} = B_0\,\sqrt{H_0} \qquad (30a)$$

and, therefore:

$$\frac{V_A}{V_{A0}} = \frac{B}{B_0}\sqrt{\frac{H_A}{H_0}} = \tau\sqrt{\frac{H_A}{H_0}} \qquad (30b)$$

in which τ is also a function of the time and has a maximum value, $\tau = \tau_0 = 1$, for full gate-opening. Expressing Equation (30b) in the notation adopted:

$$v_A = \tau\,\sqrt{h_A} \qquad (31a)$$

or, at the time, T_4, at Point A on the pipe:

$$v_{AT4} = \tau_4\,\sqrt{h_{AT4}} \qquad (31b)$$

Equation (31b) is evidently the formula for a parabola with its vertical axis coinciding with that of h and with its vertex at $h = 0, v = 0$. Since $\tau = \tau_0 = 1$ for the full open gate, the corresponding curve will pass through $h = 1$, $v = 1$, and is easily plotted. Similarly, the parabola for τ_4 is readily drawn if the law of gate closure is known, because for each instant during gate motion the value of $\frac{B}{B_0} = \tau$ is known.

If a parabola corresponding to τ_4 is drawn in Fig. 9(a), it will be the locus of the points, A_{T4}, and where it intersects the other line containing A_{T4}, it will locate the desired point. It is now possible to locate B_{T3} and, for simplicity, B will be assumed at the center of the pipe. Equations (26) show that it lies on the line, $C_{T2} - A_{T4}$, and following the method already adopted, one may write:

$$h_{AT2} - h_{BT3} = +\,2\,\rho\,(v_{AT2} - v_{BT3}) \qquad (32)$$

but A_{T2} lies on the parabola, with the value, τ_2, found as in the case of that for A_{T4} (for linear closure, $\tau_2 = \frac{1}{2}(1 + \tau_4)$), and, therefore, Equation (32) represents a point and B_{T3} coincides with A_{T2}. If one desired to know the conditions at Point A one interval later, it would be necessary to find the point, A_{T8}, for which the following equations would be written:

$$h_{AT4} - h_{CT6} = 2\,\rho\,(v_{AT4} - v_{CT6}) \qquad (33a)$$

and,

$$h_{CT6} - h_{AT8} = -\,2\,\rho\,(v_{CT6} - v_{AT8}) \qquad (33b)$$

Passing through A_{T4}, Equation (33a) locates C_{T6} (for which $h = 1$), and Equation (33b), passing through C_{T6}, gives A_{T8} on the parabola, τ_8, drawn similarly to τ_4. Fig. 9(a) has been drawn for $\rho = 0.8$ and for closure in three intervals, conditions at Point A and Point B being shown. The corresponding values of v and h have been plotted on a time base in Fig. 9(b) and Fig. 9(c). Evidently, in solving such a problem it is necessary to know: (a) Complete data on the conditions in the pipe for initial steady flow; (b) the law of variation of gate-opening with time; and (c) further information about the conditions at some other point such as C. When the pipe is connected to a large open reservoir, or to a large surge tank, the variations of water level in the latter during the water-hammer period are generally negligible, because the water-hammer intervals are very short compared with those required for the mass movement of the water. Practically, however, there is no difficulty in dealing with variations of level at Point C, or with any other condition that may exist there. With water-turbine gates the linear law of movement is used approximately, but for ordinary hand-controlled gates the movement may be quite irregular; once the effective areas are known, however, on a time base, the values of τ for each interval or fraction thereof are known and the problem is easily solved. Of course, the spacing of the τ-lines will not be uniform in the latter case.

GATE-OPENING

For gate-openings, precisely the same set of equations is used, but the value of V to be chosen in computing ρ requires some thought. If the gate is opened from a closed position, the initial pipe velocity is zero and, of course, cannot be used; and if it is moved from one open position to another position with larger area, there are two steady velocities, that at the beginning and that at the end of the movement. An examination of the theory will show that either of these velocities may be used, and v will then be solved for relative to the selected reference velocity. It is always most convenient to use the steady velocity corresponding to the final position of the gate within the limits of the problem, because this gives values of v less than unity on the whole, and such values are easier to work with than when all values of v exceed unity. For that reason this method has been adopted herein, although the same results will be obtained for the other values of V, provided the meanings of ρ and v are kept in mind.

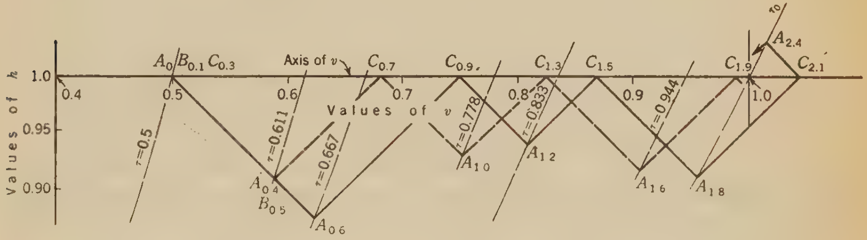
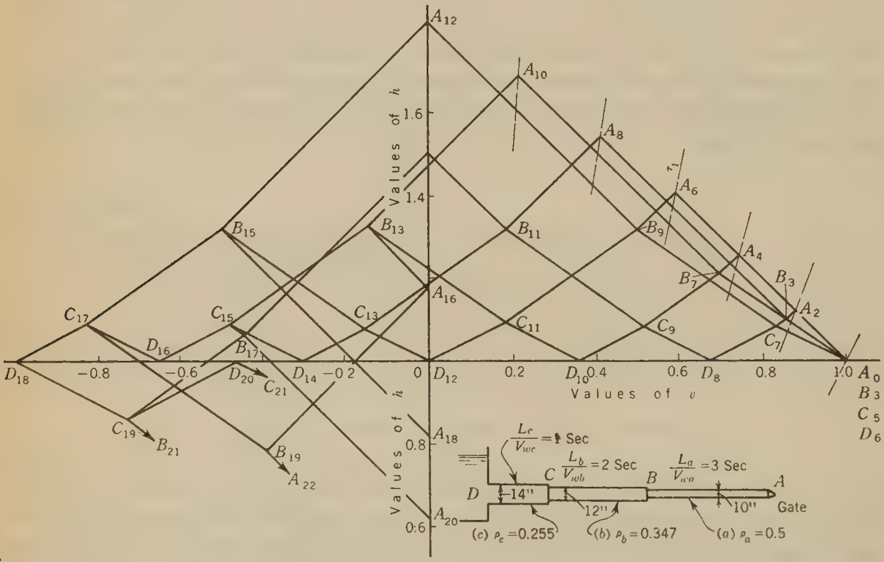


FIG. 10.—OPENING FROM HALF TO FULL GATE IN $T = 3\left(\frac{2L}{V_w}\right) = 18$ SEC

The example shown in Fig. 10 is for a pipe working under a head, $H_0 = 800$ ft, and with a steady velocity, $V_1 = 4$ ft per sec; and the changes of pressure have been determined at Point A and also at Point B ($x_1 = \frac{1}{3}L$). The valve is opened in 1.8 sec to such a position that the final steady velocity will be $V_2 = 8$ ft per sec; and, $V_w = 3\,500$ ft per sec has been assumed. If the pipe is 1 050 ft long, then $\frac{2L}{V_w} = 0.6$ sec; and, hence, $T = 1.8$ sec $= 3\left(\frac{2L}{V_w}\right)$ sec. In other words, opening occupies three intervals. Furthermore, $\rho = \frac{3\,500 \times 8}{2 \times 32.2 \times 800} = 0.543$, so that the lines in Fig. 10 have a slope such that $\tan^{-1} \pm 2 \times 0.543 = \tan^{-1} \pm 1.086$. The equations for the solution of this problem are written from Equations (26). The parabolas, $\tau = 0.5$, $\tau_1 = 0.667$, $\tau_2 = 0.833$, and $\tau_3 = \tau_0 = 1$, are drawn in light lines, but since Point B is at $x_1 = \frac{1}{3} L$, the parabolas one-third the way between the foregoing are also drawn dotted; for example: $\tau = 0.611$, $\tau = 0.778$, and $\tau = 0.944$. The starting point is evidently at Point A_0, Fig. 10, where $v_{A0} = 0.5$; and $h_{A0} = 1$. This point is also $B_{0.1}$ and $C_{0.3}$, and the construction should be followed without difficulty. (In all values such as $B_{0.1}$, etc., the subscript following each letter gives the time, in seconds, at which the values of h and v are represented by the point.) The actual head at the point at the time marked, of course, will be the indicated value of h multiplied by $H_0 = 800$ ft, and the actual velocity is the value of v multiplied by the reference velocity, $V_2 = 8$ ft per sec. All points, $A_{1.8}$ and beyond, are plotted on $\tau_0 = 1$. Since the gate remains in a fixed position from that time on, the pressures along the pipe reach H_0 quickly.

COMPOUND PIPES

The analytical solution for the pressures in compound and branched pipes becomes extremely difficult, if not actually impossible, in many cases; and yet the graphical construction furnishes a very simple method of solving the same problems. The first illustration is the pipe of Fig. 11, consisting of three parts of different lengths and diameters, all necessary data being shown on the inset. In order to clarify the illustration, linear gate closure and convenient ratios between the values of $\frac{L}{V_w}$ have been assumed. Any other data



A_{12}
A_{10}
A_8
A_6
A_4
A_2
B_3
B_{15}
B_{13}
B_{11}
B_9
B_7
A_{16}
C_{17}
C_{15}
C_{13}
C_{11}
C_9
C_7
D_{16}
B_{17}
D_{18}
−0.8
−0.6
D_{20}
C_{21}
D_{14}
−0.2
0
D_{12}
0.2
D_{10}
0.4
0.6
D_8
0.8
1.0
A_0
B_3
C_5
D_6
Values of v
Values of h
1.6
1.4
0.8
0.6
C_{19}
B_{21}
B_{19}
A_{22}
A_{18}
A_{20}
$\frac{L_c}{V_{wc}} = 1$ Sec
$\frac{L_b}{V_{wb}} = 2$ Sec
$\frac{L_a}{V_{wa}} = 3$ Sec
D
C
B
A
14"
12"
10"
Gate
(c) $\rho_c = 0.255$
(b) $\rho_b = 0.347$
(a) $\rho_a = 0.5$

FIG. 11.—LINEAR GATE CLOSURE IN $T = 2\left(\frac{2L}{V_w}\right) = 12$ SEC

may be treated just as easily although the drawing might have more lines on it and, therefore, might become larger. Only the first few equations will be written:

$$h_{A0} - h_{B3} = +\ 2\ \rho_a\ (v_{A0} - v_{B3}) \qquad (34a)$$

$$h_{B3} - h_{C5} = +\ 2\ \rho_b\ (v_{B3} - v_{C5}) \qquad (34b)$$

$$h_{C5} - h_{D6} = +\ 2\ \rho_c\ (v_{C5} - v_{D6}) \qquad (34c)$$

$$h_{B3} - h_{A6} = -\ 2\ \rho_a\ (v_{B3} - v_{A6}) \qquad (34d)$$

$$h_{A4} - h_{B7} = +\ 2\ \rho_a\ (v_{A4} - v_{B7}) \qquad (34e)$$

$$h_{C5} - h_{B7} = -\ 2\ \rho_b\ (v_{C5} - v_{B7}) \qquad (34f)$$

$$h_{A2} - h_{B5} = +\ 2\ \rho_a\ (v_{A2} - v_{B5}) \qquad (34g)$$

$$h_{B5} - h_{C7} = +\ 2\ \rho_b\ (v_{B5} - v_{C7}) \qquad (34h)$$

and,

$$h_{D6} - h_{C7} = -\ 2\ \rho_c\ (v_{D6} - v_{C7}) \qquad (34i)$$

Equations (34*a*), (34*b*), and (34*c*) correspond to the single points, A_0, B_3 C_5, D_6, at $h = 1$, $v = 1$; Equation (34*d*) locates the line, $B_3\ A_6$, on $A_0\ A_6$, since A_6 is fixed by the value corresponding to the time and consequent gate-opening at the end of one interval for Section $A\ B$, Fig. 11. Having determined the values of A_0 and A_6, the points, A_2 and A_4, are found by the method described for the simple pipe. Equation (34*e*) with Equation (34*f*) locates Point B_7 and Equation (34*g*) then gives Point B_5; from these, Equations (34*h*) and (34*i*) serve to locate Point C_7. By writing out the successive equations, the points, D_8, C_9, D_{10}, B_9, C_{11}, etc., may be found. All A–B lines have the slope, $\pm\ 2\ \rho_a$, all B–C lines, the slope, $\pm\ 2\ \rho_b$, and all C–D lines, the slope, $\pm\ 2\ \rho_c$. All points, D, are on the line, $h = 1$, since the reservoir level is assumed constant.

Whereas the pressures at Point A rise continually until closure and then fall rapidly, the pressures at Point B rise quickly during the first 9 sec; then they remain nearly constant to the 15th sec, after which they fall. On the other hand, the pressures at Point C remain nearly constant for the first 17 sec, after which they become less than H_0. The velocities may also be read easily, but usually are not desired. A pipe with any number of sections may be investigated just as easily, although those with fractional values of $\frac{L}{V_w}$ may require a lengthy but simple construction, similar to Fig. 11. It is to be noted further that all the values of ρ are connected since they are all referred to the same values of H_0 and, therefore, must be related in the same manner as the pipe areas.

BRANCHED PIPES; THE SURGE TANK

Pipes with branches form a particularly useful application of the principles under discussion, and these include penstocks with surge tanks, water pipes with a branch parallel to parts of the main line, but connected to the line at both ends of the branch, branches with dead ends, etc. Perhaps the easiest case to begin with is the water-power system with an open-top surge tank (*b*) between the conduit (*c*) and the penstock (*a*), as illustrated in Fig. 12, in which Point A

is the gate (it would be a turbine in this case), Point B, the junction point of the penstock, conduit, and tank, Point D, the water surface in the tank, and Point C is at the conduit entrance. The case of load rejection with non-linear gate closure will be considered.

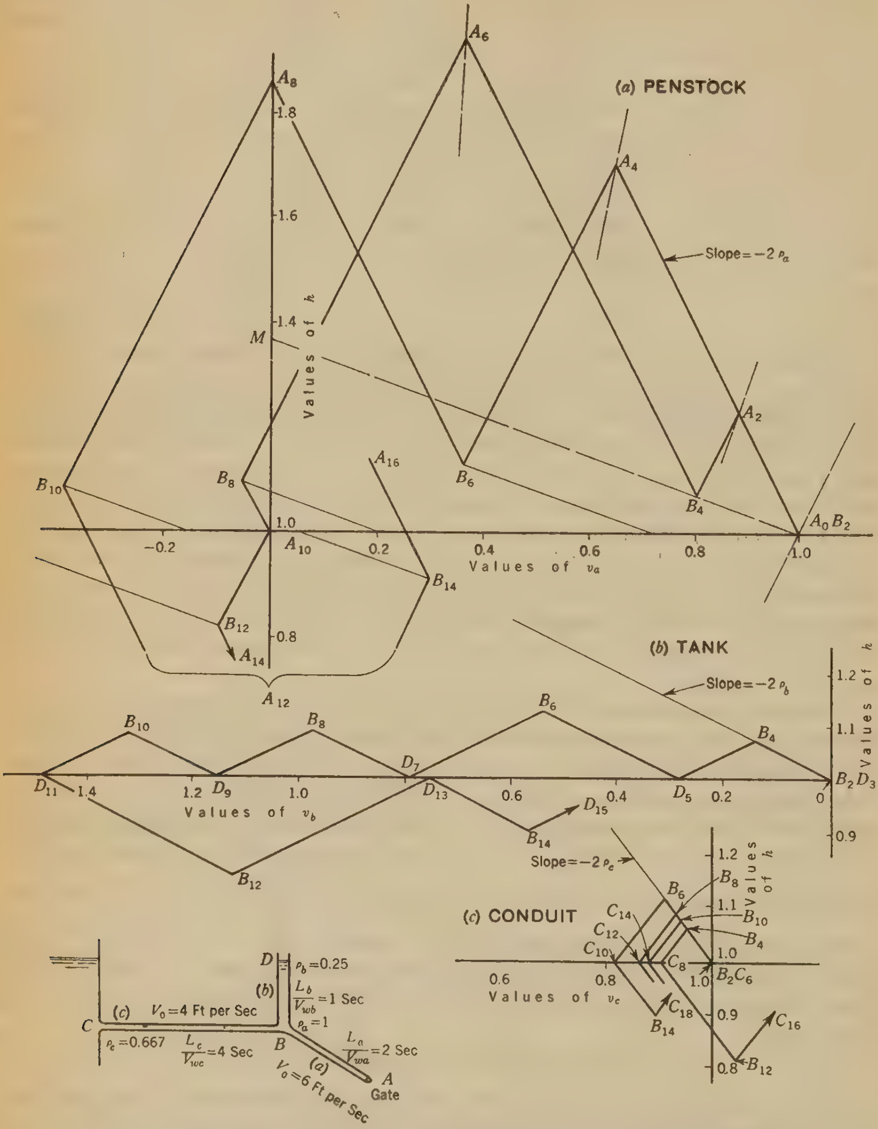


FIG. 12.—NON-LINEAR GATE CLOSURE IN $T = 2\left(\frac{2L}{V_w}\right) = 8$ SEC

When the load is rejected, a mass movement of the water takes place up into the tank; but compared to the interval, $2\frac{L}{V_w}$, for the penstock or tank, this movement is quite slow, and with tanks of usual proportions the change of level of the tank, say, for five intervals of $\frac{2L}{V_w}$ sec of either the tank or the penstock would be negligible as far as it affected the pressure at Point B. On the other hand, the gate movement produces pressure or water-hammer waves, which are very rapid and which, at times, do produce serious pressure changes at Point B; but in the first instance, at least, changes of level at Point C and Point D are not considered. It then follows that in the graphical representation all Points C and D lie on the line, $h = 1$.

The method of dealing with this case is the same as the others. Friction and velocity head being neglected, the pressure at a given instant at Point B is the same whether it is considered as a point in (a), (b), or in (c). Hence,

$$h_{aB} = h_{bB} = h_{cB} = h_B \qquad (35)$$

and during steady flow, $h_{B0} = 1$. In addition, if the positive direction of V_c is toward Point B, while those of V_a and V_b are away from Point B, the equation of continuity for that point will be written, $Q_c = Q_a + Q_b$; or, $Q_a = Q_c - Q_b$; that is,

$$A_a V_a = A_c V_c - A_b V_b \qquad (36a)$$

For steady flow Equation (36a) becomes,

$$A_a V_{a0} = A_c V_{c0} - A_b V_{b0} \qquad (36b)$$

Dividing Equation (36a) by $A_a V_{a0}$ gives $\frac{A_a V_a}{A_a V_{a0}} = \frac{A_c V_c}{A_a V_{a0}} - \frac{A_b V_b}{A_a V_{a0}}$; or,

$$\frac{A_a V_a}{A_a V_{a0}} = \frac{A_c V_{c0}}{A_a V_{a0}} \frac{V_c}{V_{c0}} - \frac{A_b}{A_a} \frac{V_{b0}}{V_{a0}} \frac{V_b}{V_{b0}} \qquad (37)$$

In the notation already adopted the left-hand term is written v_a and each of the two right-hand terms has the same form as the left-hand one, thus, the term, $\frac{A_c V_{c0} V_c}{A_a V_{a0} V_{c0}}$, contains a fixed constant multiplied by the velocity ratio, $\frac{V_c}{V_{c0}}$, and the entire term may be written, v_c, by arbitrarily defining V_{c0} so that $A_c V_{c0} = A_a V_{a0}$. A similar statement applies to the second right-hand term, and, therefore, Equation (37) may be written,

$$v_a = v_c - v_b \qquad (38)$$

This method of defining V_{c0} and V_{b0} at once gives a value to the velocities used in finding ρ for the pipes, (a), (b), and (c), and, therefore, these values of ρ will be written:

$$\rho_a = \frac{V_{wa} V_{a0}}{2 g H_0} \qquad (39a)$$

$$\rho_b = \frac{V_{wb} V_{b0}}{2 g H_0} = \frac{A_a}{A_b} \rho_a \frac{V_{wb}}{V_{wa}} \qquad (39b)$$

and,

$$\rho_c = \frac{V_{wc} V_{c0}}{2 g H_0} = \frac{A_a}{A_c} \rho_a \frac{V_{wc}}{V_{wa}} \qquad (39c)$$

Using Equations (38) and (39) this problem presents no more difficulty than the simple pipe. It is possible to follow the methods described at the beginning of this paper and represent the conditions in the three pipes, (*a*), (*b*), and (*c*), Fig. 12, on a single diagram, but the latter then becomes very complicated and it will be better to draw them separately. The data for the problem are given in Fig. 12, the tank area being assumed uniform and equal to four times the penstock area and the head, $H_0 = 300$ ft. The data assumed give $\rho_a = 1.00$, $\rho_b = 0.25$, and $\rho_c = 0.667$, and closure is assumed in two intervals of the conduit; that is, 8 sec. The lengths of the pipe and tank have been distorted deliberately so that the problem may be solved on a small sheet and still show the principles. In attacking these problems it is best and quickest to have the three diagrams side by side so that all axes of v are in the same straight line; to save space, the figures have been moved from their original positions used when solving the problem.

The diagram for A_2 and A_4 for the penstock, (*a*), should require no explanation, the points, $A_0\,B_2$, $B_2\,C_6$, $B_2\,D_3$, evidently are placed correctly at each starting point and, although v_a and v_c are measured to the right, it is necessary to measure v_b to the left, as it has a negative sign. Having found A_2 and A_4, the formula, $h_{A2} - h_{B4} = +\,2\,\rho_a\,(v_{A2} - v_{B4})$ (from Equations (26)), gives one line on which B_4 lies. The point, B_4, however, must be at the same height in the diagrams for Pipes (*a*), (*b*), and (*c*) from Equation (35) and also, from Equation (38),

$$v_{aB4} = v_{cB4} - v_{bB4} \dots\dots\dots\dots (40)$$

The notation in Equation (40) is complicated but v_{aB4} is written for the velocity in Pipe (*a*) at the point, B, and 4 sec after gate motion begins. Equation (40) shows that Point B_4 lies on a line, $A_0\,M$, located as follows: $v_a = 0$ in Equation (40) if $v_b = v_c$, all being taken at the same pressure. This is easily seen to be the pressure at which a line (not shown for lack of space) through the point, $v_c = 0$, $h = 1$ at slope $+\,2\,\rho_b$ on the (*c*)-diagram, intersects a line through the point, $v_c = 1$, $h = 1$, and at the slope, $-\,2\,\rho_c$ on the same diagram. This point, so found on Conduit (*c*), gives the height of the Point M so that M represents the same pressure; B_4 then lies on $A_0\,M$. This also locates B_4 on the (*b*) and (*c*)-diagrams since,

$$h_{D3} - h_{B4} = -\,2\,\rho_b\,(v_{D3} - v_{B4}) \dots\dots\dots\dots (41a)$$

and,

$$h_{C6} - h_{B10} = -\,2\,\rho_c\,(v_{C6} - v_{B10}) \dots\dots\dots\dots (41b)$$

and B_4 clearly lies on $B_2 - B_{10}$, or this line produced.

Next, A_6 is found since it lies on $B_4\,A_6$ from the equation,

$$h_{B4} - h_{A6} = -\,2\,\rho_a\,(v_{B4} - v_{A6}) \dots\dots\dots\dots (42)$$

It also lies on the parabola corresponding to 6 sec after closure begins. The next point is D_5 which lies on Line $B_4\,D_5$ at Slope $+\,2\,\rho_b$ and at $h = 1$. Next, B_6 lies on a line through D_5 since,

$$h_{D5} - h_{B6} = -\,2\,\rho_c\,(v_{D5} - v_{B6}) \dots\dots\dots\dots (43)$$

and it also lies on $A_4 B_6$ of the slope, $+ 2 \rho_a$ and, furthermore, it must be located so that $v_{aB6} = v_{cB6} - v_{bB6}$. A little consideration will show that it must then lie on a line parallel to $A_0 M$, but horizontally displaced to the left by a distance, $D_5 D_3$.

The remainder of the construction is similar. To locate such a point as B_{14}, the line, $A_0 M$, is displaced horizontally by the distance, $(D_3 - D_{13}) + (C_6 - C_{10})$, and, similarly, for other points.

It is well to check the method frequently by scaling the drawings and remembering that the continuity equation holds at Point B at any instant and pressure. Thus, the point, B_8, represents the conditions at Point B at 8 sec after closure begins, and scaling from the drawing at this point for each section gives $v_a = -0.05$; $v_b = 0.97$; and $v_c = 0.92$, so that $v_a = v_c - v_b = 0.92 - 0.97 = -0.05$, as it should. At this instant, all the water flowing through the conduit, (*c*) Fig. 12, together with some water from the penstock, (*a*), is flowing into the tank. However, at 10 sec after closure begins the water is coming from the penstock at a high rate, as shown at B_{10} on the diagram for Penstock (*a*). It must not be thought, however, that water is actually flowing into the nozzle from without, for no Point A ever gets to the left of the axis of h, although all points, from A_8 on, lie on this line because the gate is closed.

A little care is needed in the interpretation of the results; for example, at 6 sec, the point, B_6, corresponds to $h = 1.13$ and, therefore, to a rise in pressure at Point B of $0.13 \times H_0 = 39$ ft, and, similarly, for other points.

Although the left-hand scale for Tank (*b*), Fig. 12, is confusing at first, it soon ceases giving any trouble. In dealing with all such problems the points, A, must be chosen close enough together to enable the solution to be made. In most cases the ratio of $\frac{2L}{V_w}$ has the smallest value for the tank, but whether it has or has not, the time unit is selected at least as small as the lowest value of $\frac{2L}{V_w}$ in the system and parabolas are drawn with this time spacing. In this case, the smallest value of this term is 2 sec in the tank and on Fig. 12, therefore, values of A are found for each 2 sec. When the periods of the pipes are not multiples of one another, a sufficient number of values of A must be found to complete the problem, and sometimes the work is prolonged, but gives no trouble of any kind.

The volume of water flowing into the tank due to water hammer is easily computed. At 4 sec, $v_{bB} = 0.14$, and the volume passing into the tank in the first 4 sec, therefore, is:

$$A_a V_{bB} \times \frac{2 L_b}{V_{wb}} = v_{bB} v_{a0} A_a \frac{2 L_b}{V_{wb}} = 0.14 Q_0 \frac{2 L_b}{V_{wb}} \quad \text{.........(44)}$$

in which by definition of V_{b0}, $Q_0 = A_a V_{a0} = A_b B_{b0}$ is the initial steady flow. At the end of 6 sec, the total volume that has passed in is $(v_{bB4} + v_{bB6}) Q_0 \times \frac{2 L_b}{V_{wb}}$ cu ft. The velocities, of course, are all measured on the diagram for Tank (*b*) and, for accuracy, D_5 and D_7 should be raised above $h = 1$ to suit this increased volume; but usually the change of level is so small as to have no practical effect.

WATER MAIN WITH BRANCH; EFFECT OF DEAD END

The next example is the case of the water pipe illustrated in Fig. 13 and Fig. 14. It represents a 15-in. pipe supplied by a reservoir with constant level and discharging through a nozzle or gate, for which it is assumed the law of closure is known. This pipe has an 18-in. branch connected to the main at Points B and C in Fig. 13, but only at Point B in Fig. 14, the free end of the 18-in. pipe being plugged in this latter case. The pressures in the pipe are sought in the two cases, and, in that way, a comparison is possible between the closed circuit and the one with "dead end" under exactly similar conditions.

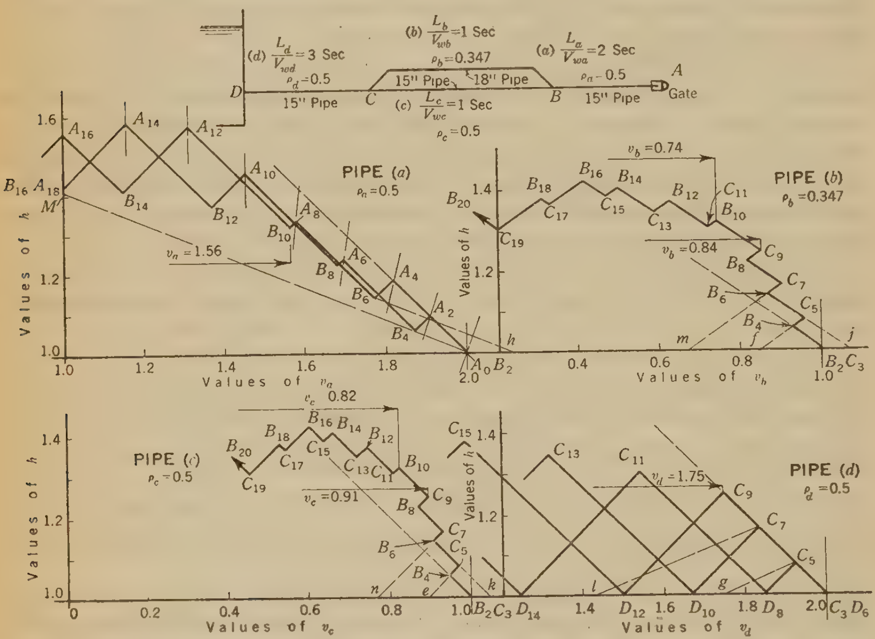

(d) $\frac{L_d}{V_{wd}}$=3 Sec
ρ_d=0.5
(b) $\frac{L_b}{V_{wb}}$=1 Sec
ρ_b=0.347
(a) $\frac{L_a}{V_{wa}}$=2 Sec
ρ_a=0.5
(c) $\frac{L_c}{V_{wc}}$=1 Sec
ρ_c=0.5
15" Pipe
18" Pipe
D
C
B
A
Gate
PIPE (a)
ρ_a=0.5
PIPE (b)
ρ_b=0.347
PIPE (c)
ρ_c=0.5
PIPE (d)
ρ_d=0.5
v_a=1.56
v_b=0.74
v_b=0.84
v_c 0.82
v_c=0.91
v_d=1.75
Values of h
Values of v_a
Values of v_b
Values of v_c
Values of v_d
A_{16} A_{14} A_{12} A_{10} A_8 A_6 A_4 A_2 A_0 A_{18}
B_{16} B_{14} B_{12} B_{10} B_8 B_6 B_4 B_2 B_{18} B_{20}
C_{19} C_{17} C_{15} C_{13} C_{11} C_9 C_7 C_5 C_3
D_{14} D_{12} D_{10} D_8 D_6
M
h
m
f
j
n
e
k
l
g
1.0
1.2
1.4
1.6
1.8
2.0
0
0.2
0.4
0.6
0.8

FIG. 13.—LINEAR GATE CLOSURE IN 4 $\left(\frac{L_a}{V_{wa}}\right)$ = 16 SEC; 18-INCH BRANCH CONNECTED TO MAIN AT TWO POINTS

Starting with the first case (Fig. 13), separate diagrams will be made for the four parts, (a), (b), (c), and (d), designated in the drawing. The data are selected to give values of $\frac{L}{V_w}$ in the four parts as 2 sec, 1 sec, 1 sec, and 3 sec, respectively, and, also, $\rho_a = 0.5$, from which the previous discussion shows that this is also the value of ρ_c and ρ_d, and that $\rho_b = \frac{A_c}{A_b} \rho_a = 0.347$.

Furthermore, assuming that all velocities have their positive directions toward Point A, the continuity equation at Point B is $v_a = v_b + v_c$, and at Point C, it is $v_d = v_b + v_c$. Therefore, the point, A_0, in Fig. 13(a), corresponds to $v_{a0} = 2$; in Fig. 13(b) and Fig. 13(c) both v_{b0} and v_{c0} are unity, whereas, in Fig. 13(d), the value is $v_{d0} = 2$. The initial starting points on the four diagrams are designated $A_0 B_2$, $B_2 C_3$, $B_2 C_3$, and $C_3 D_6$, in accordance with the notation already adopted, and the slopes of the principal lines on these diagrams are $\pm 2 \rho_a$ etc.; furthermore, in Fig. 13(*a*), parabolas for 2 sec, 4 sec, 6 sec, etc., after closure begins, are drawn and the points, A_2 and A_4, are found in the usual manner. Point B_4 is located at the same elevation in Fig. 13(*a*), Fig. 13(*b*), and Fig. 13(*c*), and also, for this point, $v_a = v_b + v_c$; and in order to carry out this condition, an auxiliary line, $A_0 M$, is drawn, exactly as in Fig. 12, such that at any pressure the velocity shown by $A_0 M$ is the sum of the velocities in Fig. 13(*b*) and Fig. 13(*c*) at the same pressure. Then,

$$h_{A2} - h_{B4} = + 2 \rho_a (v_{A2} - v_{B4}) \dots\dots\dots\dots (45)$$

gives the line, A_2-B_4, and, consequently, the point, B_4, and this point is then projected over to Fig. 13(*b*) and Fig. 13(*c*).

Next Point C_5 is to be found and the equations by which it is located are, for Fig. 13(*b*):

$$h_{B4} - h_{C5} = + 2 \rho_b (v_{B4} - v_{C5}) \dots\dots\dots\dots (46a)$$

and for Fig. 13(*c*):

$$h_{B4} - h_{C5} = + 2 \rho_c (v_{B4} - c_{C5}) \dots\dots\dots\dots (46b)$$

so that lines through Point B_4 representing these equations are drawn on the two diagrams. The continuity equation also gives $v_b + v_c = v_d$ so that Point C_5 will also lie on a line, $g\,C_5$ (Pipe *d*), having the same inclination as $A_0 M$, but with its tangent of opposite sign, the point, g, being located so that $B_2 e + B_2 f = D_6 g$. (This is quite easy to demonstrate by geometry or by actual measurement, as the reader desires.) Again, the equation,

$$h_{D6} - h_{C9} = - 2 \rho_d (v_{D6} - v_{C9}) \dots\dots\dots\dots (47)$$

shows that Point C_5 is located at a point on the line through Point D_6 and at a slope, $- 2 \rho_d$, so that the location of Point C_5 is determined in Fig. 13(*d*), and then projected horizontally to Fig. 13(*b*) and Fig. 13(*c*) as indicated. The next point, B_6, is similarly found and, in this case, $A_0 h = B_2 j + B_2 k$; and the line, $h\,B_6$, is parallel to $A_0 M$. Point B_6 is also located on $A_4 B_6$ at the slope, $+ 2 \rho_a$. Another point, C_7, is obtained by making Line $D_6 l = \overline{B_2 n} + \overline{B_2 m}$, and drawing Line $l\,C_7$ parallel to Line $g\,C_5$.

The process is then carried on to the final solution, it being remembered that all *A–B* lines have a slope, $\pm 2 \rho_a$, all *C–D* lines, a slope, $\pm 2 \rho_d$, and all *B–C* lines, a slope, $\pm 2 \rho_b$, or $\pm 2 \rho_c$, depending on whether they belong to Pipe (*b*) or to Pipe (*c*). The dimensions of one point, *B*, and one point, *C*, are shown, and the drawing made by the method described gives, for B_{10}: $v_{bB10} = 0.74$, $v_{cB10} = 0.82$, and $v_{aB10} = 0.74 + 0.82 = 1.56$; and, for C_9: $v_{bC9} = 0.84$, $v_{cC9} = 0.91$, and $v_{dC9} = 0.84 + 0.91 = 1.75$. In constructing the diagram it is well to check the results frequently by actually measuring the values of v and adding, as in Fig. 13.

Although the pressures at Points *A*, *B*, and *C*, have been found (Points *D*, of course, lie on the line, $h = 1$), for the first 20 sec, only curves of pressures at Points *A* and *C* have been plotted on the separate pressure diagram, Fig. 14, and it shows that Point *A* reaches the highest pressure at about 12 sec, and the pressure at Point *C* its maximum value at about 15 sec.

Fig. 14 represents precisely the same piping system as Fig. 13, with the valve at Point A closed in the same manner and time as the former case, but the connection at Point C has been cut off and Pipe (b) plugged. As this case is quite similar to the surge-tank problem, very little explanation will be necessary. In the diagram for Pipe (b) all points, C, must lie on the vertical line, $v_b = 0$, and the pressures at Point C, therefore, rise rapidly; furthermore, the velocities at Point B in Fig. 14(b) change only as much as the elasticity of the water and pipe walls permit, which naturally is not very much. It is like a very small surge tank. In this case, as with the surge tank, $v_{a0} = 1$, $v_{b0} = 0$, and $v_{c0} = 1$. A comparison with the previous case shows that the pressure rise at Points A and C, plotted on Fig. 14, is considerably more in the latter case than in the former, and this agrees with Joukowsky's statement that dead ends aggravate the surge pressures and show the exact amount of such pressures. Fig. 14 is easy to allow for pressure variations at Point D, provided the law of such variation is known, but in that case, of course, the several points, D, would not lie on the one horizontal line. Pressures at any intermediate point in the system are found as with the uniform pipe.

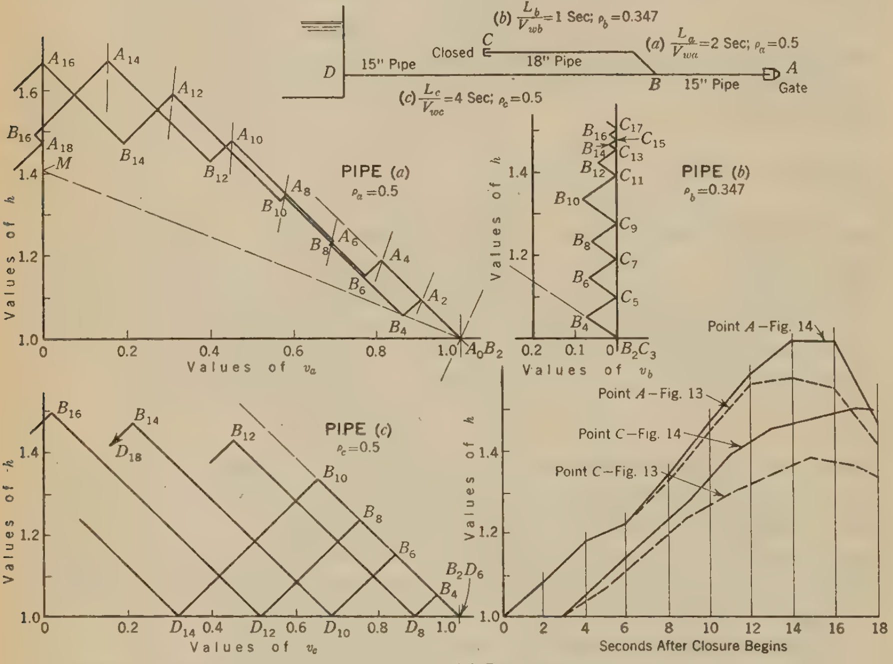


Fig. 14.—Linear Gate Closure in 4 $\left(\frac{2\,L_a}{V_{wa}}\right)$ = 16 Sec; 18-Inch Branch Connected to Main at Only One Pont

Pipe with Two Branches Each Discharging Water

Another interesting case is shown in Fig. 15 in which a pipe, $C\,B$, designated by (c) issuing from a fixed level reservoir, has two branches: $B\,A$, marked (a); and, $B\,D$, denoted by (b), and discharge is occurring at both Points A and D. Without going into the details of the pipe dimensions, values have been selected to give a clear drawing, and $\frac{L}{V_w}$ is taken at 1.5 sec for Branch (a), 1 sec for Branch (b), and 0.5 sec for Line (c), whereas the selected values of ρ are 0.5 for Branch (a), 0.4 for Branch (b), and 0.6 for Line (c). It is assumed that the gate at Point A is closed by a known law in 6 sec, or two intervals for Pipe (a); also that the law of variation between pressure and discharge at Point D is known. For example, a turbine may be connected to Point D in which the power produced is constant; or some other specified condition may be imposed. In the problem, the law of variation has been assumed as represented by the dotted line on the drawing for Branch (b).

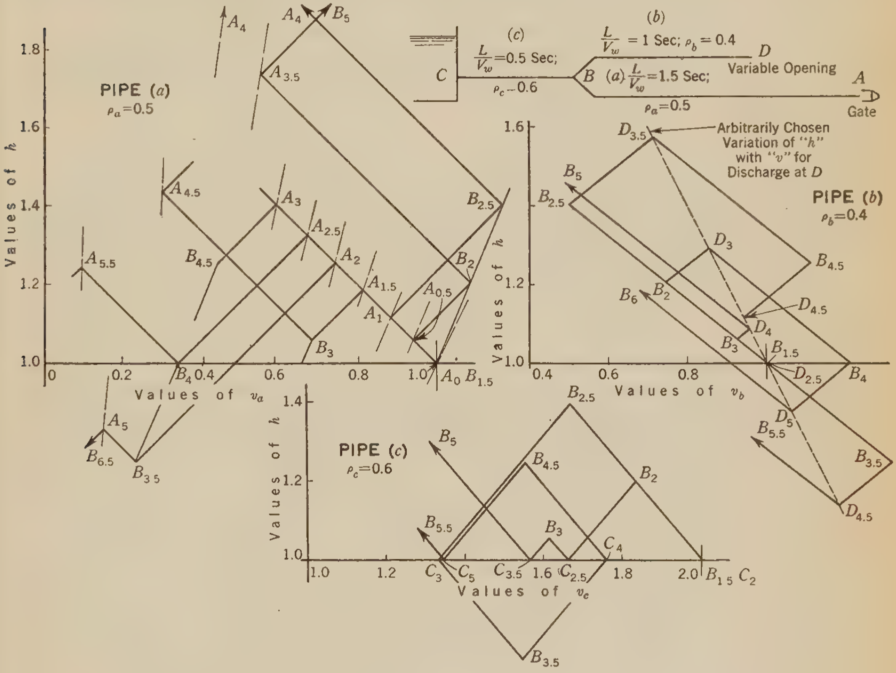

PIPE (a)
ρ_a=0.5
Values of h
Values of v_a
1.8
1.6
1.4
1.2
1.0
0
0.2
0.4
0.6
0.8
1.0
A_4
B_5
$A_{3.5}$
$A_{4.5}$
A_3
$A_{2.5}$
A_2
$A_{1.5}$
A_1
$A_{0.5}$
$A_{5.5}$
$B_{4.5}$
$B_{2.5}$
B_2
B_3
B_4
A_0 $B_{1.5}$
A_5
$B_{6.5}$
$B_{3.5}$
(c)
$\frac{L}{V_w}$=0.5 Sec;
ρ_c=0.6
C
(b)
$\frac{L}{V_w}$ = 1 Sec; ρ_b = 0.4
D
Variable Opening
B
(a) $\frac{L}{V_w}$=1.5 Sec;
ρ_a=0.5
A
Gate
PIPE (b)
ρ_b=0.4
$D_{3.5}$
Arbitrarily Chosen Variation of "h" with "v" for Discharge at D
B_5
$B_{2.5}$
D_3
$B_{4.5}$
B_6
B_2
$D_{4.5}$
D_4
B_3
$B_{1.5}$
$D_{2.5}$
B_4
D_5
$B_{5.5}$
$B_{3.5}$
$D_{4.5}$
Values of h
Values of v_b
1.6
1.2
1.0
0.4
0.6
0.8
PIPE (c)
ρ_c=0.6
Values of h
Values of v_c
1.4
1.2
1.0
1.0
1.2
1.6
1.8
2.0
$B_{2.5}$
B_5
$B_{4.5}$
B_2
$B_{5.5}$
B_3
C_4
C_3
C_5
$C_{3.5}$
$C_{2.5}$
$B_{1.5}$ C_2
$B_{3.5}$

Fig. 15.—Gate A Has Linear Closure in $2\left(\frac{2\,L_a}{V_{wa}}\right) = 6$ Sec

This case is dealt with in a manner similar to the others discussed, and, here, $v_{a0} = 1$, $v_{b0} = 1$, and $v_{c0} = 2$. All points, D, fall on the dotted line, and the construction need not be explained. The line through A_0 locating Point B_2 has a positive tangent with the data used. The pressure variations at Points A, B, and D are so large and so erratic that it is evidently impossible to use the system in practice. A second case has been developed and illustrated in Fig. 16 for the same system and rate and time of closure of A, and the same control at Point D, but with the addition of a surge tank at Junction B.

This, of course, introduces an extra branch, making four pipes in all, and if the surge tank is denoted by (*d*), then,

$$v_c = v_a + v_b + v_d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

and $v_{c0} = 2$; $v_{a0} = 1$; $v_{b0} = 1$; and, $v_{d0} = 0$. Using Equation (48) in a method exactly similar to the foregoing, the line, through A_0, is found on the penstock drawing in such a way as to locate Point B_2 from Point $A_{0.5}$, and all points, B, on this penstock drawing will lie on lines parallel to Line $A_0 B_2$. Point B_2 is then transferred to the drawings for Pipes (*b*), (*c*), and (*d*) by making them all represent the same pressure and, therefore, Point B_2 is given on each diagram. From Point B_2, Point D_3 is located on Fig. 16(*b*), Point $C_{2.5}$ on Fig. 16(*c*), and Point $E_{2.5}$ on Fig. 16(*d*). It is also evident that Point $B_{2.5}$ lies in each diagram on the same line as Point B_2. Furthermore, at any time (such as 4.5 sec after closure begins) the value of v_c indicated by Point $B_{4.5}$ on Fig. 16(*c*) is the sum of v_a indicated by Point $B_{4.5}$ on Fig. 16(*a*) plus v_b, as shown at Point $B_{4.5}$ on Fig. 16(*b*) plus v_d as given by Point $B_{4.5}$ on Fig. 16(*d*); this is easily checked by measurement.

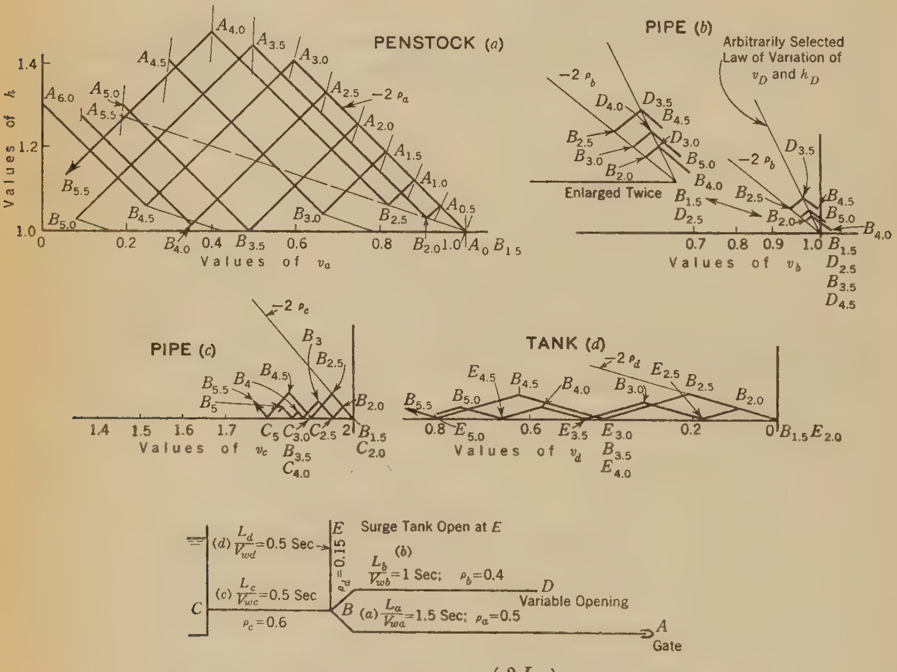

PENSTOCK (a)
Values of h
1.4
1.2
1.0
Values of v_a
0
0.2
0.4
0.6
0.8
1.0
$A_{4.0}$
$A_{3.5}$
$A_{3.0}$
$A_{4.5}$
$A_{6.0}$
$A_{5.0}$
$A_{5.5}$
$A_{2.5}$
$-2\rho_a$
$A_{2.0}$
$A_{1.5}$
$A_{1.0}$
$A_{0.5}$
A_0
$B_{5.5}$
$B_{5.0}$
$B_{4.5}$
$B_{4.0}$
$B_{3.5}$
$B_{3.0}$
$B_{2.5}$
$B_{2.0}$
$B_{1.5}$
PIPE (b)
Arbitrarily Selected Law of Variation of v_D and h_D
$-2\rho_b$
$D_{4.0}$
$D_{3.5}$
$B_{4.5}$
$B_{2.5}$
$B_{3.0}$
$D_{3.0}$
$B_{2.0}$
$B_{5.0}$
$B_{4.0}$
Enlarged Twice
$B_{1.5}$
$D_{2.5}$
$D_{3.5}$
$B_{4.5}$
$B_{5.0}$
$B_{4.0}$
Values of v_b
0.7
0.8
0.9
1.0
$B_{1.5}$
$D_{2.5}$
$B_{3.5}$
$D_{4.5}$
PIPE (c)
$-2\rho_c$
B_3
$B_{2.5}$
$B_{4.5}$
$B_{5.5}$
B_4
B_5
$B_{2.0}$
1.4
1.5
1.6
1.7
C_5
$C_{3.0}$
$C_{2.5}$
2
$B_{1.5}$
$C_{2.0}$
Values of v_c
$B_{3.5}$
$C_{4.0}$
TANK (d)
$-2\rho_d$
$E_{4.5}$
$B_{4.5}$
$B_{4.0}$
$B_{3.0}$
$E_{2.5}$
$B_{2.5}$
$B_{2.0}$
$B_{5.5}$
$B_{5.0}$
0.8
$E_{5.0}$
0.6
$E_{3.5}$
$E_{3.0}$
0.2
0
$B_{1.5}$
$E_{2.0}$
Values of v_d
$B_{3.5}$
$E_{4.0}$
(d) $\frac{L_d}{V_{wd}}$=0.5 Sec
E
Surge Tank Open at E
ρ_d=0.15
(b)
$\frac{L_b}{V_{wb}}$=1 Sec; ρ_b=0.4
D
(c) $\frac{L_c}{V_{wc}}$=0.5 Sec
Variable Opening
C
B
(a) $\frac{L_a}{V_{wa}}$=1.5 Sec; ρ_a=0.5
ρ_c=0.6
A
Gate

FIG. 16.—GATE A HAS LINEAR CLOSURE IN $2\left(\frac{2L_a}{V_{wa}}\right) = 6$ SEC WITH SURGE TANK

Fig. 16 affords an interesting illustration of the effect of the surge tank. In this case the tank has a very small area, being only $\frac{\rho_c}{\rho_a}$, or four times that of Pipe (*c*), and yet it renders the system of Fig. 15 quite practicable as far as the investigation has gone. With no tank, the pressure at Point B after 5 sec is 2.2 times the original pressure, whereas with the tank it is only about 3% above the original level. With no surge tank the fluctuations at Point D up to 5 sec vary from 64% to 158% of normal, a condition that would render any governing mechanism at Point D useless; on the other hand, the surge tank keeps the pressure at Point D within 9% of normal during the first 5 sec. At Point A, the variation in the same time with no surge tank is from 83% to 210% of H_0, whereas the surge tank reduces this variation to 30% of H_0, although at 4 sec it was 48%; and, of course, a larger tank will reduce the variations still further.

This is only a further example of how these difficult problems may be solved simply and accurately, on the drawing board. Although the proportions chosen in the example are most unusual, they serve to make the reduced scale drawings clear, and simplify the method of attack.

PENSTOCK, TURBINE, AND DRAFT-TUBE

A reaction turbine installation, for any but very low heads, consists of a penstock, a turbine with a distributor, and a draft-tube. Water hammer will be produced for every load change, because each change is accompanied by a movement of the distributor gates causing, in turn, an increase or decrease in the volume of water per second being delivered to the turbine; so that the velocities in the penstock and draft-tube change very frequently. It is quite usual to have the gates complete the full opening or closing movement in 2 sec, which corresponds roughly to two intervals in which the penstock is about 1 600 ft long. As a matter of fact, it would correspond to one interval if the turbine happened to be operating at half load, which it suddenly rejected. Governing of such turbines, therefore, may cause high pressures, and the study of this problem will be solved by the graphical method.

Usually, both penstock and draft-tube have a taper, and the method of dealing with compound pipes has already been discussed; but in order to make the diagram simpler, both the penstock and the tube will be assumed to be uniform in size, the draft-tube being the larger and also the shorter. (A method used by Allievi to avoid working with tapering pipes is to assume an equivalent uniform value of V_w from $L = V_w \sum \frac{l_x}{V_{wx}}$. Dr. O. Schnyder has also proposed that an equivalent area may be determined from $A\,L = \sum A_x\,l_x$. In both formulas, l_x, A_x, and V_{wx} refer to the data for the uniform pieces of pipe composing the entire line.[10, 4] Friction and velocity head are not taken into account. The turbine distributor gates serve to cause the varying velocities according to the load demand. In the problem assumed herewith, the dimensions have had to be distorted for clarity, but the diagram to suit any case is easily constructed.

The system is shown on Fig. 17 in which the head, $H_0 = 450$ ft; Penstock (b) is joined to the forebay at Point C and to the turbine at Point B''; and Draft-Tube (a) carries the water from Turbine B' to the tail-water at Point A; the two points, B' and B'', are assumed close enough together so that the volume of water between them has no effect on the water-hammer pressures. Without more definite data the value of V_w is taken as 3 220 ft per sec in both Draft-Tube (a) and Penstock (b); the steady velocity, V_0, in the penstock is 4.5 ft per sec; and that in the draft-tube is 3.4 ft per sec. These data give $\rho_a = 0.375$ and $\rho_b = 0.5$, and if the penstock length is 1 290 ft the value of $\frac{L_b}{V_{wb}} = 0.4$ sec. Furthermore, for the purpose of clarity $\frac{L_a}{V_{wa}} = 0.2$ sec (a length of draft-tube which could not be tolerated in practice).

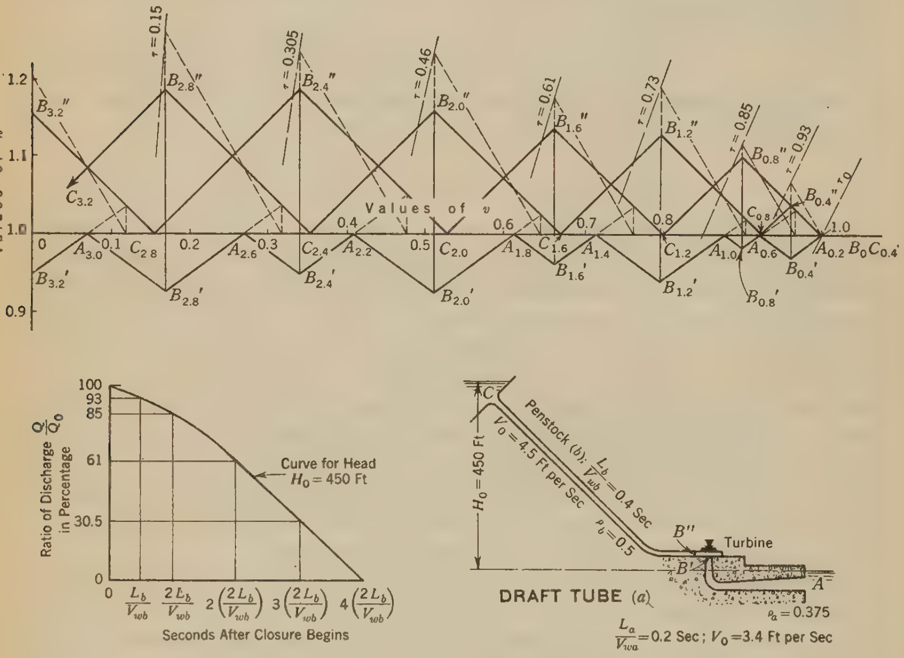

Values of h
1.2
1.1
1.0
0.9
τ = 0.15
τ = 0.305
τ = 0.46
τ = 0.61
τ = 0.73
τ = 0.85
τ = 0.93
τ = 1.0
B3.2″
B2.8″
B2.4″
B2.0″
B1.6″
B1.2″
B0.8″
B0.4″
C3.2
C2.8
C2.4
C2.0
C1.6
C1.2
C0.8
Values of v
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.0
A3.0
A2.6
A2.2
A1.8
A1.4
A1.0
A0.6
A0.2 B0C0.4
B3.2′
B2.8′
B2.4′
B2.0′
B1.6′
B1.2′
B0.8′
B0.4′
Ratio of Discharge Q/Q0 in Percentage
100
93
85
61
30.5
0
Curve for Head H0 = 450 Ft
0
Lb/Vwb
2Lb/Vwb
2(2Lb/Vwb)
3(2Lb/Vwb)
4(2Lb/Vwb)
Seconds After Closure Begins
C
H0 = 450 Ft
Penstock (b), Lb/Vwb = 0.4 Sec
V0 = 4.5 Ft per Sec
ρb = 0.5
B″
B′
Turbine
A
DRAFT TUBE (a)
ρa = 0.375
La/Vwa = 0.2 Sec; V0 = 3.4 Ft per Sec

Fig. 17.—Gate Closure in $4\left(\frac{2\,L_b}{V_{wb}}\right) = 3.2$ Sec

From tests on a large turbine, the curve plotted in Fig. 17, showing the relation between gate position and discharge at normal constant head, has been obtained and there can be little error in assuming that the gate position is a linear function of the time, which has been done in this case; the curve of discharge on a gate-position base, therefore, is also taken to be a curve of discharge on a time base. It is further assumed that closure is effected in 3.2 sec, or in $4\left(\frac{2\,L_b}{V_{wb}}\right)$ sec, which is certainly not unduly short. The time interval in this case must be 0.4 sec, corresponding to the draft-tube value, $\frac{2\,L_a}{V_{wa}} = 0.4$ sec.

Fig. 17 shows the axes of h and v with the point, $h = 1$, $v = 1$, marked $B_0\,A_{0.2}\,C_{0.4}$, since in this case the controlling mechanism is at Point B. Only a few of the equations to be applied will be written (see Equations (26)):

$$h_{C0.4} - h_{B''0.8} = -2\,\rho_b\,(v_{C0.4} - v_{B''0.8}) \quad \ldots\ldots\ldots\ldots (49a)$$

$$h_{B''0.8} - h_{C1.2} = +2\,\rho_b\,(v_{B''0.8} - v_{C1.2}) \quad \ldots\ldots\ldots\ldots (49b)$$

and,

$$h_{B''0.4} - h_{C0.8} = +2\,\rho_b\,(v_{B''0.4} - v_{C0.8}) \quad \ldots\ldots\ldots\ldots (49c)$$

Furthermore, from the corresponding formulas for the pump series:

$$h_{A0.20} - h_{B'0.4} = +2\,\rho_a\,(v_{A0.20} - v_{B'0.4}) \quad \ldots\ldots\ldots\ldots (50a)$$

and,

$$h_{B'0.4} - h_{A0.6} = -2\,\rho_a\,(v_{B'0.4} - v_{A0.6}) \quad \ldots\ldots\ldots\ldots (50b)$$

etc., so that the lines on which Points $B'_{0.4}$ and $B''_{0.4}$ lie are readily found, but their position is not determined. The discharge curve for the turbine, however, shows that at 0.4 sec $= \frac{L_b}{V_{wb}}$, the proportional discharge at the head, H_0, is 0.93, and experience with the action of turbines proves that the flow at a given gate-setting varies with $\sqrt{H_{B''} - H_{B'}}$; that is, with $\sqrt{h_{B''} - h_{B'}}$. A parabola, therefore, is drawn in Fig. 17 with the value, $\tau = 0.93$, and the problem then is to locate $B'_{0.4}$ and $B''_{0.4}$ on the same vertical line (since the value of v is the same at both points at this instant), and having a vertical spacing equal to the height of the parabola, above the axis of v at this same velocity. The reason for this is that $h_{B''0.4} - h_{B'0.4}$ will be the same as the pressure rise to which this velocity corresponds. Schnyder[10] has devised a simple graphical solution to a similar problem by drawing through Point B_0 the dotted line such that its tangent is $2\,(\rho_a + \rho_b) = 1.75$, and where it intersects the parabola, $\tau = 0.93$ is directly above Point $B'_{0.4}$ and Point $B''_{0.4}$. In this manner, these two points are determined and Point $A_{0.6}$ and Point $C_{0.8}$ are found without difficulty (accidentally, they nearly coincide in Fig. 17) since all points, A and C, lie on the line, $h = 1$, if the forebay and tail-water levels do not vary.

The construction for the points, $B'_{0.8}$ and $B''_{0.8}$, is arrived at precisely as before by the aid of the parabola, $\tau = 0.85$, corresponding to the time, $\frac{2\,L_b}{V_{wb}}$, the Schnyder construction being shown by short dots.

Although no definite conclusion can be drawn from the numerical results obtained in this problem (because of the distortion of the dimensions), some comment on the results will be of value. Taking as an example the condition at the turbine 2 sec after closure begins, the values are $h_{B''2} = 1.165$ and $h_{B'2} = 0.93$, which mean that the pressure at the lower end of the penstock has risen $0.165\,H_0$, or 74 ft above normal, whereas at the top of the draft-tube it has fallen $(1 - 0.93) = 0.07 \times H_0$, or 31.5 ft below normal. Of course, with ordinary plants, the column in the draft-tube would separate with a slightly greater pressure drop than this shows, and the condition in the tube could then be investigated as in one of the problems solved in this paper.

BREACH OF WATER COLUMN DUE TO LOW PRESSURE

A case that has recently come to the writer's attention is illustrated on Figs. 18 and 19, in which a pump discharges through a long pipe line laid with the profile shown. The first part of the pipe slopes somewhat steeply from Valve A at the pump to a point, B, whereas the latter part has a more gentle slope up to the reservoir. The characteristic curve for the pump at its normal speed of $n = 750$ rpm is shown, and the efficiency curve for the machine is also known, as well as the $W R^2$-value of the rotating parts ($W =$ the equivalent weight of the rotating parts and $R =$ its radius). The pump is assumed to be delivering 30 000 000 U. S. gal per day in a pipe in which it produces a velocity of 4 ft per sec. Friction is not considered.

Suppose, now, that the power is suddenly cut off; then the pump begins to slow down, its deceleration depending on the value of $W R^2$ for the rotating parts and on its output; and δn, the decrease in speed, in revolutions per minute in the time, δT, is given by the easily derived formula,[12]

$$\delta n = 183\,540 \frac{Q\,H\,\delta T}{W\,R^2\,e\,n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

in which feet and pounds are the units, and e is the efficiency of the pump.

The method of plotting the characteristic of a pump at any speed, from the curve obtained at some other speed, is well known and need not be explained, and, therefore, the speed corresponding to the characteristic curve passing through any point is readily found. The speed of the pump at any time after disconnecting the power supply can be computed by Equation (51).

The slowing down of the pump causes the pressure to decrease all along the line, and at Point A its value is fixed by the characteristic curve at the reduced pump speed. At very slow speeds, the pressure at Point A is nearly atmospheric unless the pump reverses; the valve at A is assumed to close at the instant the flow reverses. At Point B the low pressure at Point A may produce a pressure much less than the atmosphere, but in this problem a vacuum valve is installed at Point B in such a manner as to prevent the pressure from falling to that extent; that is, the hydraulic gradient at Point B cannot fall below 160 ft above the pump.

Referring to Fig. 18, which has axes of H and V, the starting point is $A_0\,B_1\,C_6$, as marked, and according to Equations (24) the water-hammer line will be drawn through this point, and with a slope, $+\frac{V_w}{g}$. The pump characteristics passing through the point where the water-hammer line intersects the pressure, $H = 160$ ft, corresponds to a speed of 590 rpm and Equation (51) shows that if average values of Q, H, and e, for the period are used, this speed will be reached 0.60 sec after the power is cut off; therefore, Point $A_{0.60}$, which is also Point $B_{1.60}$, is located. In 1.60 sec air begins to enter at Point B and the column parts at that point.

The admission of air and its subsequent release at Point B maintains atmospheric pressure there until the columns re-unite; the original column separates into the two parts, Line $A\ B'$ and Line $B''\ C$, the values of $\frac{L}{V_w}$ for which are, respectively, 1 sec and 5 sec, and it follows, therefore, that Point $B_{1.60}$ also represents the conditions of the upper column for 5 sec after the point, C_6; that is, the velocity at B'' of the upper column remains constant

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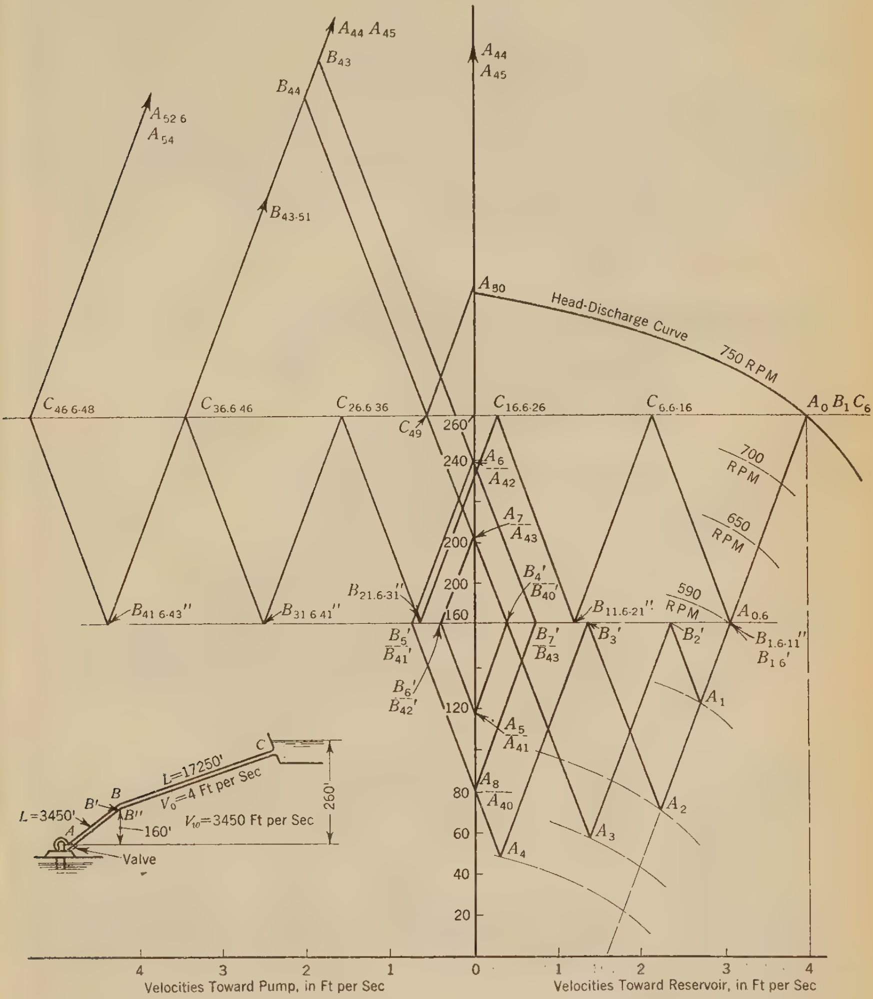
[12] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps," by R. W. Angus, *Proceedings*, Inst. Mech. Engrs., 1937.



$A_{44}\ A_{45}$
B_{43}
B_{44}
A_{44}
A_{45}
$A_{52\ 6}$
A_{54}
$B_{43\text{-}51}$
A_{50}
Head-Discharge Curve
750 RPM
$A_0\ B_1\ C_6$
$C_{46\ 6\text{-}48}$
$C_{36.6\ 46}$
$C_{26.6\ 36}$
C_{49}
$C_{16.6\text{-}26}$
$C_{6.6\text{-}16}$
260
240
A_6
A_{42}
700 RPM
A_7
A_{43}
200
650 RPM
B_4'
B_{40}'
$B_{21.6\text{-}31}''$
200
160
$B_{11.6\text{-}21}''$
590 RPM
$A_{0.6}$
$B_{41\ 6\text{-}43}''$
$B_{31\ 6\ 41}''$
B_5'
B_{41}'
B_7'
B_{43}
B_3'
B_2'
$B_{1.6\text{-}11}''$
$B_{1\ 6}'$
B_6'
B_{42}'
120
A_1
A_5
A_{41}
C
$L=17250'$
$V_0=4$ Ft per Sec
B
B'
$L=3450'$
B''
A
160'
$V_w=3450$ Ft per Sec
260'
Valve
80
A_8
A_{40}
A_2
60
A_3
A_4
40
20
4
3
2
1
0
1
2
3
4
Velocities Toward Pump, in Ft per Sec
Velocities Toward Reservoir, in Ft per Sec

FIG. 18

from 1.6 sec to 11 sec at $V_{B1.6}$. This is readily seen by applying Equations (24) to this case, which give $H_{C6} - H_{B''11} = V_g\,(V_{C6} - V_{B''11})$, so that Point $B''11$ lies on Line $C_6\ A_{0.6}$, and as its pressure is atmospheric, it is fixed at $B'_{1.6} = B''_{1.6}$. The construction already adopted enables one to find Points $C_{6.6-16}$, $B''_{11.6-21}$, etc., and these points are marked on Fig. 18; the location of Points B_2', B_3', etc., give no trouble.

If the valve at Point A is closed in slightly less than 5 sec, the flow through the pump will not reverse and all subsequent values of Point A will lie on the axis of H. On Fig. 19, the velocities at Point B (that is, for Point B' and Point B'') are plotted against time, and it is evident that the hatched area, M, representing an integral of a velocity time diagram gives the linear distance between Point B' and Point B'' at any instant; the columns reach a maximum separation at the time, 21.6 sec, where the two curves cross. After that they approach one another and finally come together again at the time where Area N = Area M, which in this case is at 43 sec. At the time of re-uniting, the velocity of the upper column is 4.4 ft per sec toward the pump, and that of the lower column is about 0.9 ft per sec in the opposite direction, so that the pressure rise will be caused by the sum of these two velocities, and may be determined as shown. It is assumed that the air admitted to the pipe is released before the columns meet again.

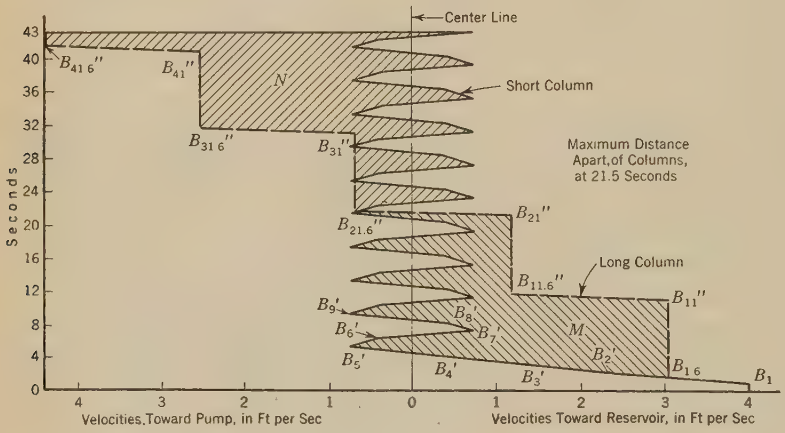


FIG. 19

Evidently, the study may be made with equal ease for any other time of closure of the valve at Point A, or for any control desired at Point B. The only error arises from the fact that the values of $\frac{L}{V_w}$ are assumed to remain constant during the study, whereas the columns actually decrease in length after separation; but in most cases this change produces a very small effect. In this particular problem the maximum distance between the columns is approximately 37 ft.

The odd form of the velocity curve for Point B' is interesting but, of course, the average velocity there could be taken as zero after closure, without affecting the answer. The pressure rise at Point A would be very high and that at Point B_{43} is easily scaled.

In computing values of δn the time intervals must be taken very short and Fig. 19 shows only a few of the points used in the calculation, as the complete set of lines would complicate a small scale drawing.

Effect of Friction and Velocity Head

The equations necessary to allow for friction and velocity head have been already given and, although the results obtained from them are not exact, they enable closely approximate answers to be obtained in the few cases where these factors are important. In general, it may be said that the friction decreases the bad effects due to water hammer, and computations made by neglecting it are usually on the safe side. In dealing with the problem, it has been frequently assumed that friction loss varies directly with the square of the velocity and, although the calculation may readily be made without this modification, the work is somewhat simplified by it. With this assumption the two quantities, velocity head and friction loss, may be combined into a single term, thus:

$$\text{Velocity head} + \text{friction loss} = \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} = V^2\left(\frac{1}{2g}\right) + \frac{f}{2g}\frac{L}{D} = b\,V^2 \quad (52)$$

in which the coefficient, b, varies slightly, as f is the only variable in it. To apply this to the h, v-diagram, it must be divided by H_0 and is then written:

$$\frac{b\,V^2}{H_0} = \frac{b\,V_0^2}{H_0}\left(\frac{V}{V_0}\right)^2 = \frac{b\,V_0^2}{H_0}v^2 \quad (53)$$

which gives the proportional effect on h for each value of v. Thus far, the only known method of allowing for friction loss has been to assume that the total loss is concentrated at certain points along the pipe. In Fig. 20, for example, it has been assumed to be concentrated at two points, B and C; that is, if the friction loss at Velocity V is 12 ft, it is assumed that in each of the obstructions

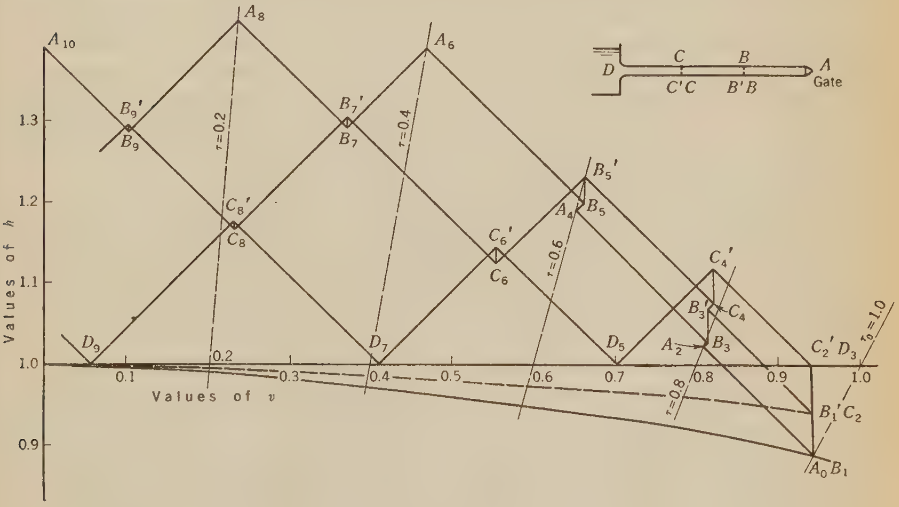


Fig. 20

at Point B and Point C, the loss of head is 6 ft and there is no loss in Pipes $A\ B$, $B'\ C$, and $C'\ D$, each having a length of $\frac{L}{3}$ ft.

With this assumption, each of the parts may be dealt with exactly as in the earlier discussion, but, at any given velocity, Points B and B' must be separated vertically by $b\,\frac{V_0^2}{H_0}\,v^2$, and they must be at the same velocity at the same instant. In Fig. 20 the values $\frac{L}{V_w}$ for each section are the same and are taken as 1 sec $\left(\text{for the entire pipe, } \frac{L}{V_w} = 3 \text{ sec}\right)$; $\rho = 0.5$; and closure is effected in 10 sec, or five intervals of $\frac{2\,L}{V_w}$ sec for the part, $A\ B$. The construction is shown well enough to need little explanation; the curve through $v = 0$, $h = 1$, and A_0 is separated from the axis of v by $\frac{b\,V_0^2\,v^2}{H_0}$; and the dotted line, which represents the loss at each obstruction, is half-way between the axis of v and the curve. Each pair of points, $B\ B'$, $C\ C'$, is separated vertically by the distance from the axis of v to the dotted curve corresponding to the velocity, v.

Conclusion

The variety of the illustrations shows that the method is not only quick, but is easily applied to problems met in practice. Knowing the starting point on the diagram and the law of operation of the valve or other device causing the pressure disturbance, one can solve the problems easily. A variable reservoir level introduces no trouble and merely means that such points as D (Fig. 11) no longer lie on the line $h = 1$ or H_0, but at distances above and below this line, corresponding to the variation from the original level. When the friction head is large compared with H_0 it is best to use axes of H and V rather than h and v, and to choose scales that make distinct intersections of the lines; but a little practice gives one much guidance in the plotting.

The method avoids the tedious tracing of the various pressure waves, and the complicated study of various reflection factors, as these have all been included automatically in the construction. Such experimental studies as have been made available, check the accuracy of the construction, but it is hoped that many more experiments will be conducted by those who are able to do so, and that the results of these will be available for comparison. The writer expresses his obligation to Dr. Schnyder,[10] and Professor Bergeron,[11] for their papers on this work and for the illustrations they have given. He is also indebted to many other authors and to the excellent paper on "High Head Penstock Design," by Messrs. A. W. K. Billings, O. H. Dodkin, F. Knapp, and Adolpho Santos, presented on June 30, 1933, under the joint auspices of the Hydraulic Division, A. S. M. E., and the Power Division, Am. Soc. C. E.[13]

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[13] First Symposium on Waterhammer, 1933; Limited Special Edition; distributed by A. S. M. E.

APPENDIX

NOTATION

The following symbols, as defined in the paper where they first appear, conform essentially with "Symbols for Hydraulics" compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929:[14]

A = area of a pipe, a subscript designating the pipe referred to; A_g = area of a gate-opening; $A_{4.5}$ = a point on the diagram representing both the pressure and velocity at Point A on the pipe at 4.5 sec after water hammer begins;

$B = \dfrac{c_d A_g \sqrt{2g}}{A_a}$ and is a function of effective gate area $= c_d A_g$; B_0 = the value of B for the initial, steady flow condition;

b = a coefficient (see Equation (52));

c_d = coefficient of discharge;

D = inside diameter of pipe;

E = Young's modulus for the material in the pipe;

e = efficiency of pump;

F = short form for $F\left(T - \dfrac{x}{V_w}\right)$, is the magnitude of a direct wave at time, T, and at x ft from gate; F_n = magnitude of a wave in its nth movement; F_1 = magnitude of a wave at the end of the first movement;

f = short form for $f\left(T + \dfrac{x}{V_w}\right)$, is the magnitude of a reflected wave at time, T, and at x ft from the gate;

g = acceleration due to gravity;

H = general term for pressure head at a given time and place on the pipe; H_0 = initial pressure head at the gate under steady flow conditions; H_{BT_2}, etc. = head at Point B on the pipe (Fig. 8) at the time, T_2 sec after gate movement begins; H_A = head at Point A, usually at the point of disturbance;

h = pressure head change; δh = small pressure head change; also, h_{BT_2} etc. = ratio $\dfrac{H_{BT_2}}{H_0}$, etc.; h_r = friction loss in feet;

K = bulk modulus of elasticity of the liquid;

L = length of pipe, or the distance traveled by a wave;

l = short length of uniform diameter in a tapering pipe;

m = mass of the water changed in velocity;

n = speed of the rotating part of a centrifugal pump and motor, in revolutions per minute; δn = a decrease in speed;

p = pressure intensity at any point in a pipe line, due to water;

Q = discharge; Q_0 = a uniform steady discharge;

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[14] A. S. A.—Z10b—1929.

R = effective radius of the rotating parts of a centrifugal pump and motor;

T = time; δT = short interval of time; also, δT = time required for a decrease in speed of δn;

t = thickness of a pipe;

V = average velocity; V_0 = steady, uniform velocity; V_w = velocity of pressure wave; δV = velocity extinguished; V_1 = velocity reached at the end of the first interval; V_2 = velocity reached at the end of the second interval; V_{BT2} = velocity at the Point B on the pipe at the time, T_2, after disturbance begins; V_A = velocity at Point A; v_{BT2} = a ratio, $\frac{V_{BT2}}{V_{B0}}$, etc.;

W = equivalent weight of the rotating parts of a centrifugal pump and motor;

w = weight of 1 cu ft of water;

x = variable distance along a pipe measured from the gate;

y = piezometer pressure reading at Point x during water hammer; y_0 = piezometer pressure reading at Point x under steady flow conditions;

z = height of any point, x, above the gate;

α = slope angle of a pipe;

ρ = pipe line characteristic $= \frac{V_w V_{A0}}{2 g H_0}$;

τ = a function of time (see Equation (30*b*)) $= \frac{v_A}{\sqrt{h_A}} = \frac{B}{B_0}$.

DISCUSSION

K. J. DeJuhasz,[15] Esq. (by letter).—For the purpose of analyzing complicated inter-relationships, the graphical method is far superior to numerical computations. The writer's experience with water-hammer is based not on water conduits, but on fuel-injection systems of Diesel engines. These types also involve complicated water-hammer phenomena for which numerical computations are scarcely practicable, although they can be analyzed readily by graphical methods. Therefore, the writer fully endorses the author's preference for the graphical method.

The examples given in this paper treat important practical cases and are highly instructive. In the writer's opinion, however, their clarity would be enhanced by a supplementary diagram, with time as the abscissa and distance as the ordinate, showing when and where the v and h-flow conditions exist. Indexing the v and h-points, as was done by the author, is helpful, but does not tell the entire story. Such distance *versus* time diagrams are easily constructed if the acoustic velocity for the various pipe subdivisions are known. It is admitted, however, that for knowledge of the absolute magnitude of the surge pressure (and this is the most important question), the v, h-chart alone is sufficient.

In dealing with fuel-injection phenomena the writer has also been confronted by the question as to how to represent the pipe friction in the diagram. The author's approach appears workable, but it is open to the objection that the friction is assumed to be localized at definite points in the pipe, whereas actually it is distributed all along the pipe. As yet the writer cannot suggest any definite procedure that would more closely approach the actual conditions. It seems, however, that the friction could be taken into account by varying the slopes of the directrix, that is, using a directrix of greater slope when pressure is converted into velocity (because the frictional resistance absorbs part of the pressure energy), and using a directrix of smaller slope when velocity is converted into pressure (because the frictional resistance absorbs part of the velocity energy). Admittedly, such a method would be handicapped by some complications that the author's simplified procedure does not involve, and it is offered solely as another line of attack along which the solution of this question might be attempted.

Transient phenomena in elastic columns constitute a wide group, including, in addition to water-hammer, spring surges, surges in gas columns, and electric transients, all having important applications in engineering. The graphical analysis appears to be a highly useful tool. It is to be hoped that Professor Angus' scholarly paper will help to direct attention to the advantages of the graphical method and serve as an incentive to its application in related engineering fields.

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[15] Associate Prof., Eng. Research, The Pennsylvania State Coll., State College, Pa.

HAROLD A. THOMAS,[16] M. AM. SOC. C. E. (by letter).—American hydraulic engineers have been rendered a notable service by the publication of this clean-cut and workable graphical method for solving problems of water-hammer in branching and compound pipes. While studying this method the writer has noted that Professor Angus is not correct in stating that the solution of these problems by analytical means is almost impossible (see "Synopsis"). The fundamental equations underlying the method lead to a convenient analytical solution which gives the same results as the graphical one.

Since numerical methods of computation have certain advantages over graphical ones, it seems desirable to present a brief outline of the analytical method for solving problems of the type under consideration. Among the advantages of the analytical method may be mentioned the possibility of obtaining high accuracy by the use of computing machines, the convenience of checking by another person, the keeping of the work on sheets of convenient size for filing and incorporation in reports, freedom from troubles due to paper shrinkage and wrinkling, and freedom from trouble due to having the work extend beyond the boundaries of the paper. In important cases, each method should give a valuable check on the other.

In general, the analysis is applicable to problems involving a valve or gate of variable opening, connected to a group of uniform pipes of various lengths and diameters, these pipes being joined together by two-way or three-way junctions. Before starting the computation, a sketch of the system should be made, each end of each uniform pipe designated by a letter, and an arrow marked on each uniform pipe to indicate the assumed positive direction of flow in that pipe. This direction may be assumed arbitrarily, except that in the case of two pipes in series both arrows must point in the same direction, and in the case of a three-way junction all three arrows must not point either toward or away from the intersection. In certain pipes of the system the flow will have a known direction under a condition of steady discharge with the gate or valve open, and it is convenient to make this the positive direction of flow for those pipes.

Notation Pertaining to a Frictionless Uniform Pipe.—The notation to be used herein will follow that of the author, with some necessary amplifications. The flow is assumed to be frictionless except where otherwise mentioned. The analytical work is materially simplified by presenting the equations in terms of discharge rather than velocity. A = area of cross-section; V_w = velocity of acoustic wave; T = time to any given instant; $T' = T - \frac{L}{V_w}$ = time to an instant, $\frac{L}{V_w}$ sec earlier; X designates that end of the pipe at which the head and discharge are unknown at a given instant, T; X' designates the other end of the pipe where the head and discharge are known at the earlier instant, T'.

In the following notation for head and discharge the simpler or abbreviated symbols can be used in cases where no ambiguity results:

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Symbol	Definition
H, H_X, or H_{XT}	= head at Section X, at time, T.
Q, Q_X, or Q_{XT}	= discharge at Section X, at time, T.
H', H'_X, or $H_{X'T'}$	= head at Section X', at time, T'.
Q', Q'_X, or $Q_{X'T'}$	= discharge at Section X', at time, T'.

The term, "head," at any section of a pipe is used herein to designate the elevation of the center of the pipe above the datum, plus the pressure head at the center of the pipe.

Finally, N = a factor whose value is $+1$ or -1 according to whether the positive direction of flow in the given uniform pipe is toward X or away from X; $S = \frac{N\,V_w}{g\,A}$ = slope of a line in graphical construction (see Fig. 23); and, C = a water-hammer constant, such that,

$$C = H + S\,Q = H' + S\,Q' \qquad (54)$$

Notation Pertaining to a Valve or Gate.—If the gate discharges into the open air, take the datum plane through the *vena contracta* of the jet, and let H and Q be the head and discharge, respectively, just above the gate, whose coefficient of discharge is C_d and whose area of opening is A_g. Then, $Q = C_d\,A_g\,\sqrt{2\,g\,H} = k\,\sqrt{H}$, or,

$$H = \frac{Q^2}{k^2} \qquad (55)$$

in which k is a known function of the time, T.

If the gate discharges under water, assume the datum plane arbitrarily, and let H_X and H_Y be the heads just up stream and down stream from the gate, respectively. In cases where $H_X - H_Y$ is positive, Equation (55) then becomes,

$$Q = k\,\sqrt{H_X - H_Y} \qquad (56)$$

In order to provide for cases in which the flow through the gate may be reversed so that Q and $H_X - H_Y$ become negative, the relation may be generalized to read: $Q = \frac{k\,(H_X - H_Y)}{\sqrt{|\,H_X - H_Y\,|}}$ or,

$$H_X - H_Y = \frac{Q\,|\,Q\,|}{k^2} \qquad (57)$$

in which the symbol, $|\;|$, enclosing a given quantity designates the absolute (positive) numerical value of that quantity.

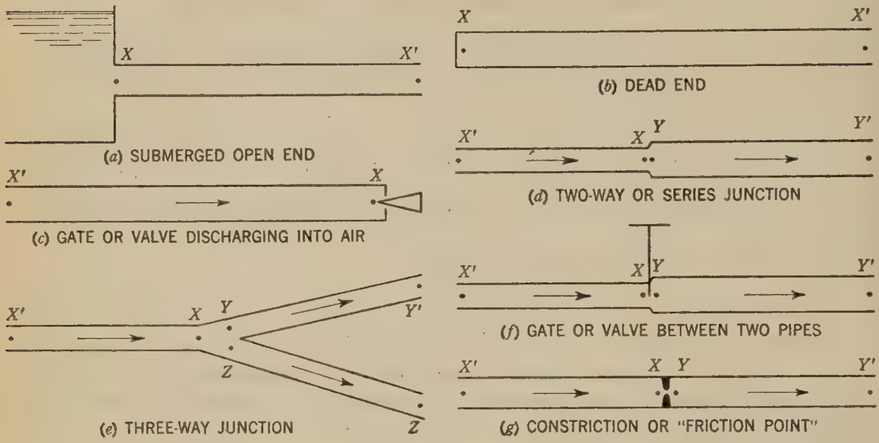
General Relation Between Head and Discharge in a Frictionless Pipe in Which Waves of Pressure Change Are Present.—According to the principles derived in Professor Angus' paper the head, H_{XT}, and the discharge, Q_{XT}, are related as follows to each other and to the head, $H_{X'T'}$, and discharge, $Q_{X'T'}$:

$$H_{XT} + \frac{N\,V_w}{g\,A}\,Q_{XT} = H_{X'T'} + \frac{N\,V_w}{g\,A}\,Q_{X'T'} = \text{constant} \qquad (58)$$

In the foregoing abbreviated notation, this relation may be written:

$$H + S\,Q = H' + S\,Q' = C \quad \text{.................. (59)}$$

The following paragraphs give the application of this general formula to a variety of special cases.



(a) SUBMERGED OPEN END

(b) DEAD END

(c) GATE OR VALVE DISCHARGING INTO AIR

(d) TWO-WAY OR SERIES JUNCTION

(e) THREE-WAY JUNCTION

(f) GATE OR VALVE BETWEEN TWO PIPES

(g) CONSTRICTION OR "FRICTION POINT"

FIG. 21

Submerged Open End; Fig. 21(*a*).—In this case since the head in the open water of the tank or reservoir is always equal to H_0 the head at X at any instant, T, is given by $H = H_0$. Moreover, by Equation (59), $H_0 + S\,Q = C$. Therefore, the discharge at Section X at any instant, T, is given by,

$$Q = \frac{C - H_0}{S} \quad \text{.......................... (60)}$$

Dead End; Fig. 21(*b*).—In this case the discharge past the dead end is zero at all times. Therefore, the discharge at Section X at any instant, T, is given by $Q = 0$; also, by Equation (59), the head at X at the instant, T, is given by:

$$H = C \quad \text{.......................... (61)}$$

Valve or Gate Discharging into Air; Fig. 21(*c*).—The datum plane is taken through the *vena contracta* of the jet. In this case by Equation (59) and Equation (55):

$$H + S\,Q = C \quad \text{...................... (62a)}$$

and,

$$H = \frac{Q^2}{k^2} \quad \text{.......................... (62b)}$$

Therefore, the discharge at Section X at any given instant, T, is given by:

$$Q = \frac{S\,k^2}{2}\left(\sqrt{1 + \frac{4\,C}{S^2\,k^2}} - 1\right) \quad \text{................ (63a)}$$

and the corresponding head is given by,

$$H = C - S\,Q \qquad \text{(63}b\text{)}$$

Two-Way or Series Junction; Fig. 21(*d*).—Neglecting the effects of velocity head and energy losses due to sudden contraction or expansion, the head at the junction is the same whether the junction is considered a part of Pipe $X'\,X$, or of Pipe $Y\,Y'$. Therefore, let $H = H_X = H_Y$. Assuming that the positive direction of flow has been chosen the same in both pipes, the law of continuity of flow may be written: $Q = Q_X = Q_Y$. Also, by Equation (59),

$$H + S_X\,Q = C_X \qquad \text{(64}a\text{)}$$

and,

$$H + S_Y\,Q = C_Y \qquad \text{(64}b\text{)}$$

Therefore, the discharge and head at the junction at any given instant, T, are given by:

$$Q = \frac{C_X - C_Y}{S_X - S_Y} \qquad \text{(65}a\text{)}$$

and,

$$H = C_X - S_X\,Q \qquad \text{(65}b\text{)}$$

Three-Way Junction; Fig. 21(*e*).—Neglecting the effects of velocity head and energy losses due to expansion or contraction, the head at the junction is the same whether the junction is considered a part of Pipe $X'\,X$, Pipe $Y\,Y'$, or Pipe $Z\,Z'$. Therefore, let $H = H_X = H_Y = H_Z$. Assuming that all three arrows do not point either toward or away from the junction, and that the points, X, Y, and Z, are chosen so that branches Y and Z are the two in which the positive directions of flow with respect to the junction are the same, the law of continuity of flow takes the form: $Q_X = Q_Y + Q_Z$. By Equation (59), $H + S_X\,Q_X = C_X$; $H + S_Y\,Q_Y = C_Y$; and, $H + S_Z\,Q_Z = C_Z$. Therefore, at any given instant the head at the junction is given by,

$$H = \frac{\dfrac{C_X}{S_X} - \dfrac{C_Y}{S_Y} - \dfrac{C_Z}{S_Z}}{\dfrac{1}{S_X} - \dfrac{1}{S_Y} - \dfrac{1}{S_Z}} \qquad \text{(66)}$$

and the discharges in the three branches are given by,

$$Q_X = \frac{C_X - H}{S_X} \qquad \text{(67}a\text{)}$$

$$Q_Y = \frac{C_Y - H}{S_Y} \qquad \text{(67}b\text{)}$$

and,

$$Q_Z = \frac{C_Z - H}{S_Z} \qquad \text{(67}c\text{)}$$

Gate or Valve Between Two Pipes; Fig. 21(*f*).—In this case it is considered that the positive direction of flow has been assumed the same in both pipes, and that X and Y are so chosen that the positive direction of flow is from X to Y. Then, at any instant, T, the discharge through the gate is $Q = Q_X = Q_Y$. It follows from Equations (57) and (64) that, at any instant, T, the discharge through the gate is given by:

$$Q = \frac{k^2 (S_X - S_Y)}{2} \frac{(C_X - C_Y)}{|C_X - C_Y|} \left(\sqrt{1 + \frac{4|C_X - C_Y|}{k^2 (S_X - S_Y)}} - 1 \right) \ldots\ldots (68)$$

and the heads just above and below the gate are given by,

$$H_X = C_X - S_X Q \ldots\ldots (69a)$$

and,

$$H_Y = C_Y - S_Y Q \ldots\ldots (69b)$$

Friction Point; Fig. 21(g).—The approximate friction loss, H_f, in any uniform pipe is considered to be $j\,Q^2$, in which Q is the discharge at the middle of the pipe at the given instant and j is the coefficient of Q^2 in the ordinary pipe-friction formula,

$$H_f = f \frac{L}{D} \frac{V^2}{2\,g} = f \frac{L}{D} \frac{Q^2}{2\,g\,A^2} \ldots\ldots (70)$$

For purposes of computation the loss is assumed to be concentrated in a very short distance, $X\,Y$, at the mid-section of the pipe, the letters being so chosen that the positive direction of flow is from X to Y. Therefore, $Q = Q_X = Q_Y$ and $S = S_X = -S_Y$. In order to provide for possible reversal of the direction of flow in the pipe, the relation between lost head and discharge is written in the generalized form, Equation (57), $H_X - H_Y = j\,Q|Q|$. By Equation (64), $H_X + S\,Q = C_X$ and $H_Y - S\,Q = C_Y$. Therefore,

$$Q = \frac{S\,(C_X - C_Y)}{j\,|C_X - C_Y|} \left(\sqrt{1 + \frac{j\,|C_X - C_Y|}{S^2}} - 1 \right) \ldots\ldots (71)$$

$$H_X = C_X - S\,Q \ldots\ldots (72a)$$

and,

$$H_Y = C_Y + S\,Q \ldots\ldots (72b)$$

It is to be noted that the effect of a "friction point" is exactly the same as that of a constriction at the mid-section of the pipe. If it is desired to compute the friction loss in a uniform pipe with more refinement than this, the pipe may be regarded as divided into several segments and a "friction point" located at the middle of each. The value of j for each segment will then be based on the length of that segment. If two segments of equal length are used, the two constrictions producing the friction loss will thus be located at the quarter-points of the original pipe. Professor Angus' method of finding the friction head for this case, as shown in Fig. 20, is slightly different from the foregoing, his two constrictions being at the one-third points instead of at the quarter-points. Where several segments are used, the method outlined herein is more accurate than that used by Professor Angus, since it makes the stepped hydraulic grade line conform more closely to the true linear hydraulic grade line. However, in many problems, the difference of the results obtained by the two methods would be inappreciable.

Solution of Problems.—The foregoing equations are sufficient for the solution of all ordinary problems of water-hammer in compound and branching pipes. The work is greatly facilitated by tabulating the numerical quantities in a systematic manner. A suitable arrangement for the computations is suggested in the following illustrative problem. In general, a separate table must be arranged for each of the following kinds of points occurring in the given pipe system: submerged open end, dead end, valve or gate, series junction, and three-way junction. If the effects of friction are to be considered, one or more "friction points" are placed in each uniform pipe, and a table is included for each of these points.

When the blank tables have been prepared and the initial values of head and discharge have been written into the tables in their proper places, the remainder of the computation becomes automatic. The values of head, H, and discharge, Q, computed in one table for a given instant, T, at one end of one of the uniform pipes, can always be inserted in another table as the values of H' and Q' for the abutting end of an adjacent uniform pipe. Repetition of this process solves the problem.

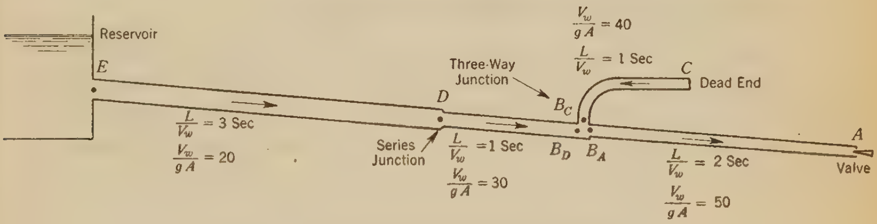


FIG. 22.—DATA FOR ILLUSTRATIVE PROBLEM

Illustrative Problem.—For this problem (see Fig. 22) $H_0 = 100$ ft; and $Q_0 = 10$ cu ft per sec; therefore, $k_0 = \frac{Q_0}{\sqrt{H_0}} = 1$. The valve is closed at a uniform rate in 10 sec; and, therefore, $k = 1 - \frac{T}{10}$. Values of head and discharge are to be computed at 2-sec intervals at the various sections designated by letters in Fig. 22. Friction is neglected. For this problem the foregoing general formulas reduce to the following special ones:

For Section A, a valve discharging into air, $X = A$; $X' = B_A'$; $N = 1$; $S = 50$; $C = H' + 50\,Q'$; and $k = 0.1\,(10 - T)$:

$$Q = 25\,k^2\left(\sqrt{1 + \frac{C}{625\,k^2}} - 1\right) \quad \ldots\ldots (73a)$$

and (see Equations (63)),

$$H = C - 50\,Q \quad \ldots\ldots (73b)$$

Equations (73) are solved in Table 1.

TABLE 1.—SOLUTION OF EQUATIONS (73); SECTION A, FIG. 22; A VALVE DISCHARGING INTO AIR

Item No.	Section-instant $X'\ T'$	B_A 0	B_A 2	B_A 4	B_A 6	B_A 8
1	Discharge, Q'	10	10	8.97	7.53	5.28
2	$50\ Q'$	500	500	448.5	376.5	264.0
3	Head, H'	100	100	117.5	153.3	198.4
4	C (Item No. 3 plus Item No. 2)	600	600	566.0	529.8	462.4
5	$k = 0.1\ (10 - T)$	0.8	0.6	0.4	0.2	0
6	$25\ k^2$ (see Equations (73))	16	9	4	1	0
7	$625\ k^2$	400	225	100	25	0
8	$C \div 625\ k^2$	1.500	2.667	5.66	21.19	....
9	$\sqrt{1 + \text{Item No. 8}} - 1$	0.582	0.915	1.581	3.70	....
10	Q = Item No. 9 × Item No. 6 (see Equation (73a))	9.31	8.23	6.32	3.70	0
11	$50\ Q = 50$ × Item No. 10	465.5	411.5	316.0	185.0	0
12	$H = C - 50\ Q$ = Item No. 4 minus Item No. 11	134.5	188.5	250.0	344.8	462.4
Item No.	Section-instant $X\ T$	A 2	A 4	A 6	A 8	A 10

For Section B, a three-way junction (see Fig. 22):

$X = B_A$; $Y = B_C$; $Z = B_D$; $C_X = H_X' - 50\ Q'_X$;
$X' = A$; $Y' = C$; $Z' = D$; $C_Y = H_Y' + 40\ Q'_Y$;
$N_X = -1$; $N_Y = 1$; $N_Z = 1$; $C_Z = H_Z' + 30\ Q'_Z$;
$S_X = -50$; $S_Y = 40$; $S_Z = 30$; and (see Equations (66) and (67)):

$$H = \frac{C_X}{3.916} + \frac{C_Y}{3.132} + \frac{C_Z}{2.349};$$

$$Q_X = \frac{C_X - H}{-50}; \quad Q_Y = \frac{C_Y - H}{40}; \quad \text{and} \quad Q_Z = \frac{C_Z - H}{30}.$$

The foregoing equations are solved in Table 2.

TABLE 2.—SOLUTION OF EQUATIONS FOR THREE-WAY JUNCTION

Item No. (1)	Section-instant (2)	$X'\ T'_X$ (3)	$Y'\ T'_Y$ (4)	$Z'\ T'_Z$ (5)	A 2 (6)
1	Discharge, Q'	Q'_X	Q'_Y	Q'_Z	9.31
2	S	-50	40	30	-50
3	$S\ Q'$ (Item No. 2 × Item No. 1)	$-50\ Q'_X$	$40\ Q'_Y$	$30\ Q'_Z$	-465.5
4	Head, H'	H'_X	H'_Y	H'_Z	134.5
5	C (Item No. 4 + Item No. 3)	C_X	C_Y	C_Z	-331.0
6	....	3.916	3.132	2.349	3.916
7	Item No. 5 ÷ Item No. 6	....	....	....	-84.6
8	Head, H	H	H	H	117.5
9	Item No. 5 minus Item No. 8	$C_X - H$	$C_Y - H$	$C_Z - H$	-448.5
10	Discharge, Q (Item No. 9 ÷ Item No. 2)	Q_X	Q_Y	Q_Z	8.97
Item No.	Section-instant	$X\ T$	$Y\ T$	$Z\ T$	B_A 4

TABLE 2.—(*Continued*)

Item No.	$C\ 3$	$D\ 3$	$A\ 4$	$C\ 5$	$D\ 5$	$A\ 6$	$C\ 7$	$D\ 7$
(1)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	0	10	8.23	0	9.30	6.32	0	7.72
2	40	30	−50	40	30	−50	40	30
3	0	300	−411.5	0	279.0	−316.0	0	231.6
4	100	100	188.5	135.1	114.0	250.0	171.3	145.6
5	100	400	−223.0	135.1	393.0	−66.0	171.3	377.2
6	3.132	2.349	3.916	3.132	2.349	3.916	3.132	2.349
7	31.9	170.2	−57.0	43.2	167.4	−16.9	54.7	160.6
8	117 5	117.5	153.3	153.3	153.3	198.4	198.4	198.4
9	−17.5	282.5	376.8	−18.2	239.7	264.4	−27.1	178.8
10	−0.44	9.41	7.53	−0.45	7.98	5.28	−0.68	5.96
Item No.	$B_C\ 4$	$B_D\ 4$	$B_A\ 6$	$B_C\ 6$	$B_D\ 6$	$B_A\ 8$	$B_C\ 8$	$B_D\ 8$

For Section C, a dead end (see Equation (61)): $X = C$; $X' = B$; $N = -1$; $S = -40$; $Q = 0$; and $H = C = H' - 40\,Q$. The solution is arranged for convenience in Table 3.

TABLE 3.—SOLUTION OF EQUATIONS FOR DEAD END

Item No.	Section-instant $X'\ T'$	$B_C\ 4$	$B_C\ 6$	$B_C\ 8$
1	Discharge, Q'	−0.44	−0.45	−0.68
2	$-\ 40\ Q'$	17.6	18.0	27.2
3	Head, H'	117.5	153.3	198.4
4	Head, H (Item (3) plus Item (2))	135.1	171.3	225.6
5	Discharge, Q	0	0	0
Item No.	Section-instant $X\ T$	$C\ 5$	$C\ 7$	$C\ 9$

For Section D, a series junction:

$X = D_E$; $Y = D_B$; $C_X = H'_X + 20\,Q'_X$;
$X' = E$; $Y' = B_D$; $C_Y = H'_Y - 30\,Q'_Y$;
$N_X = 1$; $N_Y = -1$;
$S_X = 20$; $S_Y = -30$; and (see Equations (65)):

$$Q = \frac{C_X - C_Y}{50}; \quad \text{and,} \quad H = C_X - 20\,Q.$$

The solution of these formulas is given in Table 4.

For Section E, a submerged open end, at reservoir: $X = E$; $X' = D$; $N = 1$; $S = -20$; $C = H' - 20\,Q'$; $H = 100$; and (see Equation (60)), $Q = \dfrac{C - 100}{-20}$. The solution is arranged as in Table 5.

Tables 1 to 5 are arranged so that the known values, Q' and H', appear at the top of each column and the unknown values, Q and H, appear at the bottom. The term, "section-instant," is used to denote a given place and time. Thus, in Table 5, at the section-instant D 5, the discharge, Q', is 9.30 cu ft per sec. The letter at the bottom of the first numerical column in each table designates the given section, and the numeral designates the instant 2 sec after the

TABLE 4.—SOLUTION OF EQUATIONS FOR SERIES JUNCTION

Item No.	Section-instant...	$X'T'_X$	$Y'T'_Y$	E 2	B_D 4	E 4	B_D 6	E 6	B_D 8
1	Discharge, Q'....	Q'_X	Q'_Y	10	9.41	10	7.98	10	5.96
2	S..............	20	−30	20	−30	20	−30	20	−30
3	SQ' (Item No. 2 × Item No. 1)..	$20Q'_X$	$-30Q'_Y$	200	−282.3	200	−239.4	200	−178.8
4	Head, H'........	H'_X	H'_Y	100	117.5	100	153.3	100	198.4
5	C (Item No. 4 plus Item No. 3).....	C_X	C_Y	300	−164.8	300	−86.1	300	19.6
6	$C_X - C_Y$ (difference in Item No. 5).............	$C_X - C_Y$	....	464.8	....	386.1	....	280.4	....
7	Discharge, Q (Item No. 6 ÷ 50)....	Q	....	9.30	....	7.72	....	5.61	....
8	$20Q$ (20 × Item No. 7).........	$20Q$	....	186.0	....	154.4	....	112.2	....
9	Head, H (C_X, Item No. 5 − Item No. 8).........	H	....	114.0	....	145.6	....	187.8	....
Item No.	Section-instant...	XT	....	D 5	....	D 7	....	D 9	....

first impulse from the valve movement reaches the given section. For example, at Section E the initial conditions remain unchanged for 6 sec after the valve movement begins. Therefore, in the table for Section E (Table 5) the symbol, E 8, is written at the bottom of the first numerical column. The numerals at the bottom of the remaining columns (or groups of columns) of each table increase by 2-sec increments. The letters and numerals at the top

TABLE 5.—SOLUTION OF EQUATIONS FOR A SUBMERGED OPEN END, AT RESERVOIR

Item No.	Section-instant $X'T'$....................	D 5	D 7	D 9
1	Discharge, Q'..........................	9.30	7.72	5.61
2	$-20Q'$..............................	−186.0	−154.4	112.2
3	Head, H'............................	114.0	145.6	187.8
4	C (Item No. 3 plus Item No. 2)............	−72.0	−8.8	75.6
5	C (Item No. 4 − 100).....................	−172.0	−108.8	−24.4
6	Q (Item No. 5 ÷ − 20)....................	8.60	5.44	1.22
7	Head, H..............................	100.0	100.0	100.0
Item No.	Section-instant XT......................	E 8	E 10	E 12

of each column are determined from those at the bottom. Thus, in Table 5, the symbol, D 5, is written at the top of the first numerical column, because Section D is at the far end of the uniform pipe terminating in Section E, and an impulse requires 3 sec to travel from D to E.

The computation is started by writing into the tables the known initial head of 100 ft for section-instants, B_A 0, B_A 2, C 3, D 3, E 2, E 4, E 6, E 8, E 10, and E 12; the known initial discharge of 10 cu ft per sec for section-instants, B_A 0, B_A 2, D 3, E 4, and E 6; and the known initial discharge of 0 cu ft per sec for section-instants, C 3, C 5, C 7, and A 10.

The values of head and discharge are now computed for all the remaining section-instants in the following order:

A 2
A 4
A 6
B_A 4
B_A 6
B_A 8
B_C 4
B_C 6
B_C 8
B_D 4
B_D 6
B_D 8
C 5
C 7
C 9
D 5
D 7
D 9
E 8
E 10
E 12, etc.

A schedule for each individual computation is indicated in the tables. The numerical work in this example has been extended to the tenth second at Section A, and by means of the same process it can be carried to any desired subsequent even second.

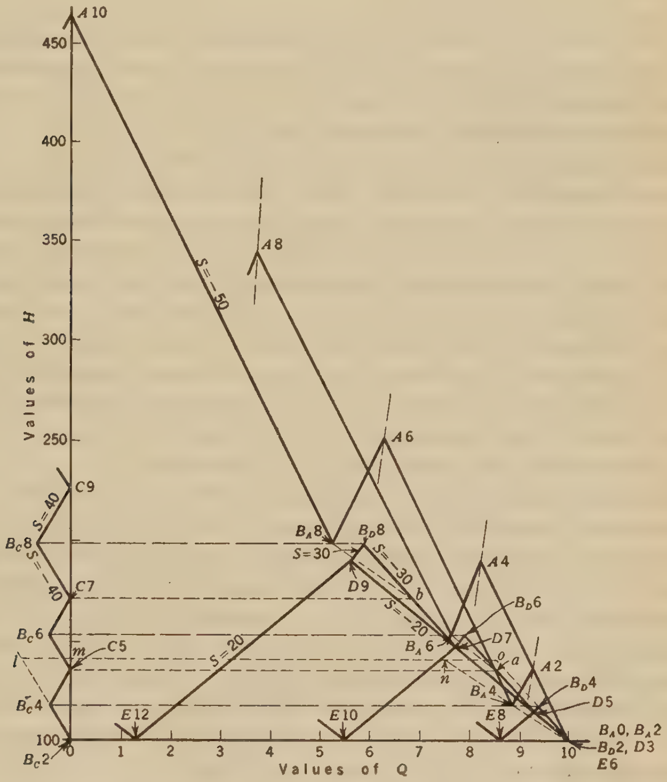


FIG. 23

Comparison with Graphical Solution.—A solution of the foregoing problem by Professor Angus' graphical method, slightly modified, is shown in Fig. 23. It is seen that the analytical and graphical methods give identical results. In this diagram the abscissas are plotted in terms of discharge rather than of velocity, in conformity with the principle given in Equation (59). This device of expressing the flow in terms of discharge rather than velocity is very helpful in clarifying the general analysis of water-hammer problems in compound and branching pipes, since it eliminates the use of complicated velocity relationships such as those described by the author in connection with Equation (37). However, it does not change the shape of the final graphical figure.

Professor Angus is to be complimented especially on the elegance of his graphical solution for the problem of the three-way junction. Practice in the use of this device suggests modifications for cases in which the construction falls beyond the boundaries of the paper. Such a modification is shown in Fig. 23. On the arbitrarily located horizontal line, *ol*, lay off *on* equal to *ml* and draw a line from D 3 to n. The direction of the latter line defines the direction of the lines which solve the three-way junction problem by joining D 3 to B_A 4, a to B_A 6, and b to B_A 8.

F. KNAPP,[17] ESQ. (by letter).—All engineering theory is based upon certain fundamental hypotheses, and the degree of agreement between the theory and the observed behavior depends on the approximation with which the assumptions made describe the actual facts.

The analytical theory of water-hammer and the graphical method derived from it do not escape this necessity. Although the basic assumptions are of such a nature as to give excellent agreement between test and computation for the more common cases occurring in practice (and, as a matter of fact, this agreement is better than is actually required and hoped for in most hydraulic computations), it is well to remember, for exceptional cases, that the elastic surge theory assumes proportionality between stress and strain; that is, the validity of Hooke's law. Furthermore, without a more complete theory than has been published to the present, it is not possible to compute the surge conditions affected by cavitation of the water-column; that is, the subnormal pressures approximating the vapor pressure of the water.

The theory, furthermore, cannot account for the frictional losses as they occur, distributed along the pipes. Several approximate (and graphical) methods, however, have been devised in order to overcome this difficulty.

As far as the first mentioned hypothesis is concerned (the proportionality between stress and strain) the usual theory considers only cases with surges of such a magnitude as to remain well below the yield point of the pipes. There is no difficulty in completing the theory by replacing the observed stress-strain relation above the yield point of the pipe by a series of tangents or secants. The theory thus completed furnishes several important indications for the hydraulic and mechanical design of pipe lines.

The author's example of the "breach of the water column due to low pressures" is based on several assumptions that are neither corroborated by experimental verification nor by a careful examination of the subnormal pressure conditions in the pipe line. As stated by the author, the graphical method avoids the tedious tracing of the various pressure waves and the complicated study of various reflection factors, as these have all been included automatically in the construction. It is just this facility of the construction which may lead to considerable errors and for this reason the writer[13] treated, at length, the surge conditions in a pipe line by means of reflection and transmission factors combined with the "wave-plan." Although tedious, such a study represents an excellent basis for the complete understanding of the graphical method of surge computations.

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[17] With the São Paulo Tramway Light & Power Co., São Paulo, Brazil; Assoc. Member, Waterhammer Committee, A. S. M. E., representing Brazil.

Referring to Fig. 18 of the paper, the line connecting the points, $A_{0.6}$ and $C_{6.6-16}$, indicates a negative pressure wave with a magnitude of 100 ft; the author assumes this wave to travel up the flatter part of the pipe line without causing disturbances of any kind. He also assumes this wave to be reflected totally, but with opposite sign, at the intake. According to Fig. 18, the pressure head at the intake is only a small fraction of the magnitude of the negative surge wave and this means that the water column between Sections *B* and *C* is subject to cavitation conditions. It follows, also, that the maximum subnormal pressure in the upper flatter section of the pipe line for any section cannot be greater than the difference between the static water level and the elevation head of the section considered and an additional amount given by the prevailing barometric pressure minus the vapor pressure of the water. The wave thus travels up the line with diminishing magnitude in accordance with the profile of the pipe line. As a consequence, the reflection at the intake occurs with only a fraction of the original wave.

The column of water, subject to subnormal pressure conditions, releases, partly, the air contained in it. When the wave returns, the elastic properties are found to have changed to such an extent that the surge velocity is considerably less than before. Again, reaching Section *B*, the original pressure conditions are being restored, and it should be noted that during this process the atmospheric pressure plays an important part.

The author assumes that Section *B* represents a point of total reflection and, consequently, treats the two columns, *A B′* and *B″ C*, separately. This assumption is not justified; nor has it been corroborated by actual experimental verification. The conditions in the pipe line occur in accordance with simple physical considerations as described herein. Without anticipating the results of a probable future publication regarding this special aspect of the theory of water-hammer, the writer would remark that it is not surprising to find that an approach to a difficult problem such as that presented by the author (under assumptions that are fundamentally incorrect) gives results in accordance with these assumptions.

In 1935, the writer called attention[18] to the incorrect solution of the problem under discussion, first proposed by O. Schnyder,[19] later, by M. L. Bergeron,[20] and now reviewed by the author.[21]

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[18] "Ueber eine allgemeine graphische Berechnungsmethode der Druckstoesse in Rohrleitungen," by F. Knapp, *Wasserkraft und Wasserwirtschaft,* 1935, p. 279; also, "Operation of Emergency Shut-Off Valves in Pipelines," by F. Knapp, *Transactions,* A. S. M. E., November, 1937.

[19] "Ueber Druckstoesse in Rohrleitungen," by O. Schnyder, *Wasserkraft und Wasserwirtschaft,* 1932, p. 49.

[20] "Pompes centrifuges e usines elevatoires," by M. L. Bergeron, *La Technique Moderne,* 1935, No. 5.

[21] The same criticism applies to the author's paper, "Air Chambers and Valves in Relation to Waterhammer," *Transactions,* A. S. M. E., November, 1937.

Referring to the text following Equation (40), it may be interesting to indicate the solution of branch-pipe problems in a different manner. Simultaneous equations for Branches (*a*) and (*c*) may be written as follows:

$$h_{At-2} - h_{aBt} = 2\,\rho_a\,(v_{At-2} - v_{aBt}) \tag{74a}$$

and,

$$h_{Ct-4} - h_{cBt} = -\,2\,\rho_c\,(v_{Ct-4} - v_{cBt}) \tag{74b}$$

From Equation (74*a*) it follows that:

$$v_{aBt} = v_{At-2} - \frac{1}{2\,\rho_a}\,(h_{At-2} - h_{aBt}) \tag{75a}$$

and, from Equation (74*b*):

$$v_{cBt} = v_{Ct-4} + \frac{1}{2\,\rho_c}\,(h_{Ct-4} - h_{cBt}) \tag{75b}$$

Introducing Equations (75) into the equation of continuity and considering, also, the relation expressed by Equation (35), the following is obtained:

$$\left(\frac{\rho}{\rho_a}\,h_{At-2} + \frac{\rho}{\rho_c}\,h_{Ct-4}\right) - h_{Bt} = -\,2\,\rho\left[(v_{At-2} + v_{Ct-4}) - v_{bBt}\right] \tag{76}$$

in which,

$$\rho = \frac{\rho_1\,\rho_2}{\rho_1 + \rho_2} \tag{77}$$

Equation (76) has exactly the same structure as the simultaneous Equations (26). The slope of the line, defined by $-\,2\,\rho$, is constant and equal to that of the line, $A_0\,M$, in Fig. 12. As a matter of fact, Equation (76) represents a momentary "end condition," replacing the two branches, (*a*) and (*c*), temporarily, by a single line. In practice, and in order to speed up the application, it is more convenient to start the line with the slope, $-\,2\,\rho$, at a point corresponding to the velocity in the surge tank at the wye with the pressure-ordinate, $h = 1.0$, instead of computing the starting point of this line by means of Equation (76). This relation, of course, may be extended so as to be applicable to any number of branches, meeting in one single point.

The treatment of the draft-tube problem, given by the author, is most interesting. The writer would only remark that, in practice, the change of the speed of the unit would also have to be considered in the computation.

As far as the "large and erratic" pressure variations at Points *A*, *B*, and *D* of Fig. 15 are concerned, it becomes evident from a closer study that this unsatisfactory behavior is due to the unstable characteristic of the valve at Section *D*, assumed by Professor Angus. Certain valves exist with such a head-velocity relation, but in practice the effect is not as pronounced as is assumed in Fig. 15. It is certainly not desirable, as proposed by the author, to make such an installation stable by the addition of a stand-pipe or surge tank, because the engineer who designs such a layout would choose a valve with a satisfactory characteristic.

In spite of the fact that he treats the basic relations in considerable detail, the author still uses the old-fashioned method of developing Equations (10) by means of differential equations. Their development, by simple physical considerations, was demonstrated by the writer in 1937.[22]

In spite of the writer's criticism of the paper, the author deserves the thanks of the profession for having brought to the attention of a greater circle of engineers a subject of lively interest, heretofore limited to a small number of specialists.

MARTIN A. MASON,[23] JUN. AM. SOC. C. E., PIERRE F. DANEL,[24] ASSOC. M. AM. SOC. C. E., AND ANTOINE CRAYA,[25] ESQ. (by letter).—There are a few points of importance omitted from Professor Angus' excellent paper which should be discussed as concerning the accuracy of the graphical method and as showing a further liaison between it and the analytical method of studying water-hammer phenomena.

Consider, first, the case of a compound pipe consisting of a number of dicontinuities provided by changes in section, as that studied by Professor Angus in Fig. 11. At each of these discontinuities there is a loss of head due to kinetic energy losses, depending upon the sense and magnitude of the flow, and due directly to the change in character of the conduit. This loss may be evaluated[26] as $\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \frac{V_1^2}{2\,g}$ in the case of an enlargement of section, and as $\left(\frac{1}{c} - 1\right)^2 \frac{V_2^2}{2\,g}$ in the case of a contraction. The effect of these losses, whether due to contraction or enlargement, is always the same in that the pressure indicated by the graphical method is not the true pressure at the point being studied for both sections of the conduit. It is the true pressure, plus or minus the kinetic energy correction, for one of the sections; for instance, in the case treated by Professor Angus, with flow from Points D to A (that is, from the reservoir to the gate), there is a change in pressure at Points C and B due to the change in section, which is not shown in Fig. 11. The true pressure may be found in the following manner: Consider Section $A\,B$ of the conduit, with a change in régime introduced by the gate closure. Then, the pressure-velocity relations in Section $A\,B$ at some time, i, later are shown by the régime points, A_i and B_i. In Section $B\,C$ of the conduit, however, the true pressure at Point B is not that shown by Point B_i since the loss of kinetic energy due to the sudden contraction at Point B has been considered. Therefore, to find the true pressure at Point B, Section $B\,C$, add to the pressure shown by Point B_i the head lost in contraction. In Fig. 24 this may be accomplished by extending Point B_i vertically upward a distance equal to $\left(\frac{1}{c} - 1\right)^2 \frac{V_{AB}^2}{2\,g}$ times the pressure scale of the graph. Then the characteristic lines representing the pressure-velocity relations in Section $B\,C$ should start, not from Point B_i but from the corrected Point B_i. The same correction is necessary for each change in section.

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[22] "O golpe de ariete: Theoria, confirmação experimental e applicaçœs praticas," by F. Knapp, *Boletin* da Inspectoria de Serviços Publicos, São Paulo, November, 1937.

[23] Asst. Hydr. Engr., National Hydr. Laboratory, National Bureau of Standards, Washington, D. C.

[24] Research Engr., Ateliers Neyret-Beylier et Piccard Pictet; Director of Hydr. Laboratory, Ecole des Ingénieurs Hydrauliciens, Univ. of Grenoble, Grenoble, France.

[25] Research Engr., Ateliers Neyret-Beylier et Piccard Pictet, Grenoble, France.

[26] "Fluid Mechanics," by R. A. Dodge and M. J. Thompson, McGraw-Hill Co., 1937.

As noted by Professor Angus, and as more fully discussed elsewhere,[27] these terms may usually be neglected. If they are neglected, however, it should be remembered (particularly in regard to the pressure conditions at intermediate points in the conduit) that the results are not exact. Admittedly, the error in the final result is often very small. It is obvious that as the velocity head term, $\frac{V^2}{2\,g}$, increases in ratio to the pressure head the error becomes larger.

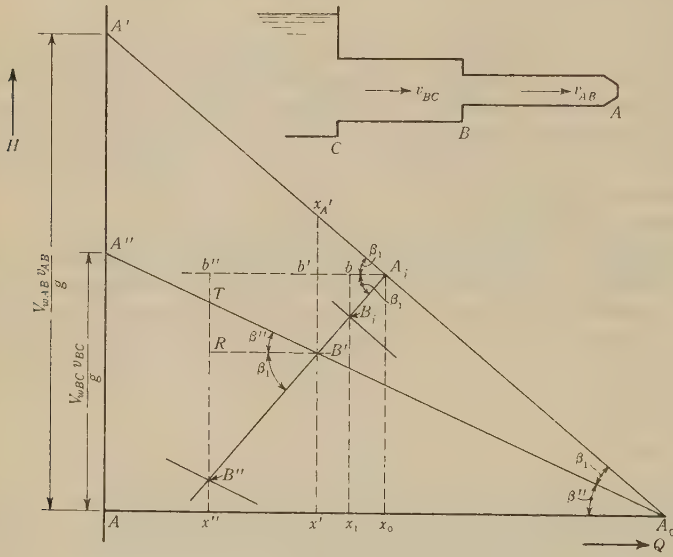

FIG. 24

In connection with corrections of this kind it should be noted that conditions frequently occur in practice which demand care in their interpretation as to the necessity for applying the foregoing corrections. Professor L. Bergeron has shown[27] that sometimes a correction need not be applied for flow in one sense, but for flow in the other sense a correction is required. For instance, he discusses the case of a nozzle opening into an air chamber, and the case of a simple pipe entrance. In the first case, the correction to be applied for reversed flow is four times that for normal flow; whereas, in the second case, no correction is required for flow in the normal sense (there being a negligible loss at the entrance); but, for flow in the reverse sense, correction must be made for the exit losses.

The study of water-hammer conditions in compound pipes by the graphical method offers a great advantage over the analytical method in that many of the factors that must be calculated in the latter method are automatically considered in the graphical method. Among these terms may be mentioned the so-called "coefficients" of reflection and transmission of the pressure waves, and in case the problem is being studied by the theory developed by Jaeger,[6] a third coefficient, α_i.

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[27] Contributions to discussion of the papers by Messrs. L. Allievi, J. N. LeConte, and R. T. Knapp, at the Water Hammer Committee Meeting, A. S. M. E., 1937, by Prof. L. Bergeron and Martin A. Mason, Assoc. M. Am. Soc. C. E. (Publication pending.)

These terms may be easily found from the graphs drawn for a particular case, although their calculation is not at all necessary to the use of the graphical method. Fig. 24 is the graph for a simple case of a compound pipe consisting of two sections; the line, $A_0\,A'$, represents the pressure-velocity relation in Section $A\,B$, and Line $A_0\,A''$, the same relation as for Section $B\,C$. For simplicity, a pipe of only two sections has been chosen as the same proof applies equally well to compound pipes of any number of sections.

The construction of the graph by the graphical method shows the magnitude, F_i, of the direct pressure wave in Section $A\,B$, leaving End A at Time i, to be the length, $\overline{X_0\,A_i}$, in Fig. 24. Likewise, the magnitude of the reflected wave, f_i, from Point B, Section $A\,B$, is equal to the length, $\overline{-B'\,b'}$; and, the magnitude of the direct pressure wave, F', in Section $B\,C$, is equal to the length $\overline{X'\,B'}$, assuming that there are no reflected waves as yet in Section $B\,C$. Prolonging $X'\,B'$ to its intersection, $X_A{}'$, with the line, $A_0\,A'$, $\overline{b'\,X_A{}'} = \overline{B'\,b'} = -f_i$; from which,

$$\overline{X'\,B'} = \overline{b\,X_1} - \overline{b'\,B'} = F_i - (-f_i) = F_i + f_i \quad\text{.........}(78a)$$

and,

$$\overline{X'\,X_A{}'} = \overline{b\,X_1} + \overline{b'\,X_A{}'} = F_i + (-f_i) = F_i - f_i \quad\text{........}(78b)$$

Considering, however, the triangle, $A_0\,A'\,A''\,A$, cut by the parallel lines, $A\,A'$ and $X'\,X_A{}'$:

$$\frac{\overline{X'\,B'}}{\overline{X'\,X_A{}'}} = \frac{\overline{A\,A''}}{\overline{A\,A'}} = \frac{V_{wBC}\,v_{BC} \div \dfrac{1}{g}}{V_{wAB}\,v_{AB} \div \dfrac{1}{g}} = \frac{\dfrac{V_{wBC}}{A_{BC}}}{\dfrac{V_{wAB}}{A_{AB}}} \quad\text{..............}(79)$$

in which A represents the cross-sectional area of the conduit considered. Substituting the values of Equations (78) into Equation (79):

$$\frac{F_i + f_i}{F_i - f_i} = \frac{\dfrac{A_{AB}}{V_{wAB}}}{\dfrac{A_{BC}}{V_{wBC}}} = \frac{1 + \dfrac{f_i}{F_i}}{1 - \dfrac{f_i}{F_i}} \quad\text{....................}(80a)$$

and solving for $\dfrac{f_i}{F_i}$,

$$\frac{f_i}{F_i} = \frac{\dfrac{A_{AB}}{V_{wAB}} - \dfrac{A_{BC}}{V_{wBC}}}{\dfrac{A_{AB}}{V_{wAB}} + \dfrac{A_{BC}}{V_{wBC}}} = r \quad\text{......................}(80b)$$

which is recognized as the coefficient of reflection used by Jaeger, Loewy, Comte de Sparre, and others.

Furthermore, since $\overline{X' B'} = F' = \overline{X_1 b} - \overline{b' B'} = F_i + f_i$:

$$\frac{F'}{F_i} = 1 + \frac{f_i}{F_i} = \frac{2\,\dfrac{A_{AB}}{V_{wAB}}}{\dfrac{A_{AB}}{V_{wAB}} + \dfrac{A_{BC}}{V_{wBC}}} = s \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

which is the coefficient of transmission of Section $A\,B$ into Section $B\,C$, applied in a manner similar to Equation (80*b*).

Consider, now, that there is a reflected wave, f', in Section $B\,C$ (Fig. 24). For this case it is known by construction that Line $A_0\,A''$ has been displaced parallel to itself a vertical distance equal to twice the magnitude of the reflected wave; that is, $2\,f'$. Therefore,

$$\overline{B''\,b''} = \overline{B''\,R} + \overline{R\,b''} = \overline{B''\,R} + \overline{B''\,b'} = \overline{B''\,R} - f_i \ldots\ldots (82a)$$

and since $\overline{B''\,T} = -\,2\,f'$:

$$\overline{B''\,R} + \overline{R\,T} = -\,2\,f' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82b)$$

From the similar triangles, $T\,R\,B'$ and $A''\,A\,A_0$, $B''\,R\,B'$ and $A'\,A\,A_0$, one finds,

$$\frac{\overline{R\,T}}{\overline{B''\,R}} = \frac{\overline{A_0\,A''}}{\overline{A_0\,A'}} = \frac{\dfrac{A_{AB}}{V_{wAB}}}{\dfrac{A_{BC}}{V_{wBC}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83a)$$

and substituting Equation (83*a*) into Equation (82*b*):

$$\overline{B''\,R} = -\,f'\left(\frac{2\,\dfrac{A_{BC}}{V_{wBC}}}{\dfrac{A_{AB}}{V_{wAB}} + \dfrac{A_{BC}}{V_{wBC}}}\right) = -\,f'\,s' \ldots\ldots\ldots\ldots (83b)$$

in which s' is the coefficient of transmission of Section $B\,C$ into Section $A\,B$.

To find Jaeger's coefficient, α_i, it is necessary only to consider Equations (83*b*) and (80*b*) with respect to Equation (82*a*), or,

$$\overline{B''\,b''} = r\,F_i + s'\,f' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84a)$$

and, dividing by F_i,

$$\frac{\overline{B''\,b''}}{F_i} = r + s'\,\frac{f'}{F_i} = \alpha_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84b)$$

in which α_i, Jaeger's coefficient, represents the ratio of $\overline{B''\,b''}$ (the reflected wave in Section $A\,B$ due to reflection in Section $B\,C$) to the direct wave in Section $A\,B$.

Professor Bergeron, who with Dr. O. Schnyder, is mainly responsible for the rapid development of the graphical method of study of water-hammer phenomena, has shown[28] that the method may be applied successfully in fields other than that of hydraulics. In fact, theoretically, this method of study of pressure-velocity relationships may be adapted to any case in which the disturbing waves are propagated as plane waves. Thus, it can be used to study the propagation of plane waves in a stretched chord, longitudinal waves in a metal bar, torsional waves in a rotating cylinder, and the study of transients in electric lines.

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[28] Mémoires de la Société des Ingénieurs Civils de France, *Bulletin*, July–August, 1937.

However, in many of these cases occurring in domains other than hydraulics (as well as in the field of hydraulics itself) difficulties are frequently introduced in the use of the method by the physical conditions under which the disturbance occurs. The principal of these difficulties is that of determining, at some time after the beginning of the disturbance, the characteristic curve of the pressure-velocity relations imposed by the disturbing device. In most cases this curve must be calculated by analytical means, as in the case of a surge tank consisting of an air reservoir, where, by means of Boyle's law, the pressure-velocity relations at any instant may be calculated. Unfortunately, this computation introduces, into the use of the graphical method for such cases, considerable additional work which it would be desirable to avoid. This phase of the problem of graphical solution of water-hammer problems is worthy of considerable study by those engineers who frequently use the graphical solution method.

A. A. KALINSKE,[29] ESQ. (by letter).—This paper, with previous writings by the author (which he lists in footnotes), provide, for American engineers, complete information on the solution of water-hammer problems by the graphical method. The writer suggests that any one wanting to become thoroughly familiar with this method of analysis should also consult the papers that are cited.

The presentation of a graphical method of analysis in a technical paper is extremely difficult as the author usually cannot get the reader to follow through, completely, the details of the graphical construction. Unless the reader does this, patiently and diligently, he will not grasp the principles of the graphical method. Furthermore, he is likely to use this method of analysis just as a tool, without any knowledge of the physical phenomena with which he is dealing.

The writer believes that the paper would be more lucid if Professor Angus had devoted additional space to showing what the graphical method of analysis really is; namely, a method for solving simultaneous equations. There is nothing mysterious about it; it is simply one method of solving, step by step, simultaneous equations that relate various phenomena which occur together. In fact, these simultaneous equations can be solved analytically just as well as graphically. Furthermore, only after solving a few problems analytically by a step-by-step numerical solution will the physical phenomena be entirely clear to any one who is just beginning the study of water-hammer. For example, when using the graphical method outlined by the author for compound pipes, the physical phenomena occurring at pipe junctions are completely obscured. The graphical method is a useful tool and is very convenient for some problems; it is unnecessary and cumbersome for certain others, and it is practically never indispensable. Some problems may be solved most rapidly by a combination of the numerical and graphical methods. An engineer interested in this problem should be able to use either method.

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If an engineer knows how to write the different simultaneous equations that relate and control the various phenomena occurring in water-hammer, he will have no difficulty solving the problem. He may use a numerical or a graphical method, but, in either case, he should first have a correct understanding of what he is doing. The writer believes that Professor Angus might well have explained in more detail how to set up the various simultaneous equations needed to solve the more complicated piping problems, and thus have generalized the method of attack to be used in solving any particular problem.

In general, the solution of any water-hammer problem involves the formulation of the following: (1) Relation between the head and the velocity at the control gate; (2) relation between the head and velocity at one end of a section of uniform conduit, and the head and velocity at the other end of the same section taken an instant earlier than the first (this interval of time being equal to the time required for the pressure wave to travel the length of the conduit section); and, (3) discharge relationships at the junction of pipe branches.

As Professor Angus states, if friction loss is not neglected, the solution of any problem is considerably complicated. However, he minimizes the number of problems in which friction must be considered. In most water-supply pipe systems and pump discharge lines, the head lost due to friction is far from negligible. In many such problems the flow head may be only a small fraction of the static head. This fact is of great importance in the design of relief devices. By considering the friction loss as concentrated at a single point or points, an approximate solution can be obtained. However, such a procedure is far from satisfactory.

Probably the best physical pictures of the effect of friction on the form of the pressure wave for instantaneous closures are those given by W. F. Durand[30] and H. A. Gibson.[31] A complete, satisfactory explanation of the effect of friction for gradual gate closures does not seem to exist. In a paper published in 1937, Allievi[32] states:

> "During the perturbed regimen, moreover, the velocity of each single liquid filament is varying with time, and, at the same instant, is different for each filament. Therefore, the effect of liquid friction is transmitted to the lowest filament which we are considering, according to very complex laws in which it is legitimate to assume that the velocity, V_i, of the lowest identical filament plays a predominant part."

Probably the only way to obtain a correct idea as to the part that friction plays in altering the results obtained when it is neglected, or when it is considered as localized at various points, is through experiments. In fact, the writer believes that any further knowledge that is to be gained regarding water-hammer will have to be obtained from carefully conducted experiments. Mathematical and graphical analyses have solved most of the general features of the water-hammer problem; it is now time to investigate some of the minor assumptions that have been made. Some other special features of the problem, which merit experimental investigation, are the effects of sharp bends in pipes, gradual and sudden pipe transitions, partly closed valves, and the characteristics of various relief devices, including those that store up energy and those that dissipate it partly.

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[30] "Hydraulics of Pipe Lines," by W. F. Durand, D. Van Nostrand, New York, N. Y., 1921, p. 89.

[31] "Mechanical Properties of Fluids," by H. A. Gibson, Blackie & Sons, London, England, 1925, p. 218.

[32] "Air-Chambers for Discharge Lines," by Lorenzo Allievi, *Transactions*, A. S. M. E., Vol. 59, 1937, p. 655.

ROBERT W. ANGUS,[33] ESQ. (by letter).—The discussion by Professor DeJuhasz shows that the matter dealt with is of broad application. The writer agrees with the suggestion that the results become much clearer if represented by a solid instead of a plane diagram, and the elegant models made by Professor DeJuhasz[34] illustrate the meaning of the diagrams well. For problems met with in water-works and hydraulic systems, however, it is only necessary to determine the magnitude and location of the most dangerous pressure by the use of the diagrams shown.

It must be admitted that the method of dealing with friction is open to some criticism, but as the number of points of concentration of the friction is increased, the assumed pipe approaches the actual case more and more closely. If the number of points was infinite, the solution would be an exact one from the friction standpoint. Therefore, it becomes largely a question as to the number of points to concentration that must be used to avoid serious errors in the results, and judgment thus comes into the question. As the diagrams may be drawn very quickly, even the inexperienced person can try the same problem with one, two, three, or more, points of concentration, and in that way the minimum number is soon found; but the writer has observed that it is rarely necessary to assume more than two points, and very often one is sufficient.

In Fig. 20 the solution is offered for the case of two points of concentration where the resistance, $A_0 D_3$, is a little greater than 10% of the static head; but it would have been just as easy to solve the problem had $A_0 D_3$ been taken, say, at 95% of the static head, as would be the case in many water-works problems. Usually, sufficient accuracy is obtainable by assuming all the loss concentrated at the pipe entry and the corresponding diagram is easily deduced (see Fig. 25).[35]

Several of the discussers have dealt with friction, suggesting different ways of allowing for it, and some of these solutions may approach closer to the correct results than any yet suggested. There was no attempt in the paper to minimize the effect of friction, but it makes the diagrams more difficult to follow than if it were omitted; and the very small size of the diagrams suggested to the writer that it was best to omit it in most cases. The writer includes it in dealing with actual problems, and believes that, on the whole, the errors due to this phase of the work are much less than those due to uncertainties in the value of V_w and the exact effects caused by the turbine governor or other source of disturbance.

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[33] Prof. of Mech. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[34] "Hydraulic Phenomena in Fuel Injection Systems for Diesel Engines," by K. J. DeJuhasz, *Transactions*, A. S. M. E., November, 1937, p. 669.

[35] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps: Graphical Treatment," by R. W. Angus, *Proceedings*, Inst. M. E., London, England, Vol. 136 (1937), Fig. 23*a*, p. 288. (Slightly revised.)

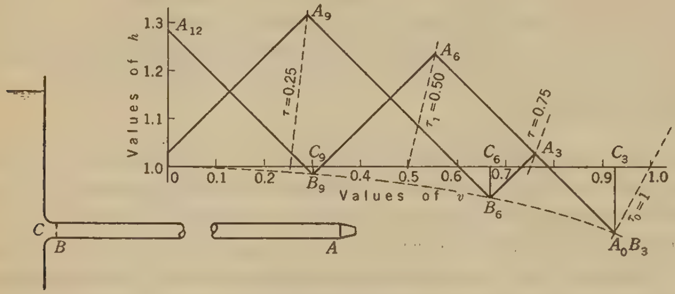

FIG. 25.—EFFECT OF FRICTION FOR STRAIGHT-LINE CLOSURE IN 12 SECONDS; $\rho' = 0.5$; $\frac{L}{V_w} = 3$ SECONDS

The analytical solution given by Professor Thomas is most instructive, and may also be derived from the writer's diagrams by writing down in succession the equations to the various lines on the diagrams. Since the slope of each line is known, and also its starting point, it is not difficult to solve for each point without the use of the drafting-board. The analytical solution forms a valuable check on the graphical one. There is more danger of error in the former than in the latter method, however, and it appears easier to detect mistakes on the diagrams than on the tabulated set of data. Mistakes may easily be made on the diagrams by drawing lines at the wrong slope, but once the correct slopes are established they are transferred very easily from place to place.

The writer has found that it is essential to be very careful with the slopes of the lines; otherwise they may become mixed when they are nearly parallel, and for this reason he used a separate diagram for each pipe, as in Fig. 14, etc., of the paper, in preference to the combined diagram, Fig. 23, shown by Professor Thomas. With the particular data of Fig. 23, that diagram is quite clear, but in many cases the lines become very much entangled and then the separate diagrams are best. The writer always puts all axes of v or V in the same horizontal line when working problems, so as to reduce the labor somewhat. Many geometrical shortcuts will suggest themselves, of which the one shown by Professor Thomas is ingenious. Scales must be arranged so that intersections are not too acute, and frequently it is desirable to use two vertical scales on the same drawing; in one case the writer found it convenient to use a scale for H of 10 ft = 1 in. up to H_0 = 100 ft, and 50 ft = 1 in. for values of H_0 greater than 100 ft; the engineer quickly adjusts himself to these solutions.

The points raised by Mr. Mason, Professor Danel, and Mr. Craya touch on several important matters. Undoubtedly, in principle, velocity head should be taken into account, as the writer has stated, and as is further emphasized by this discussion. It may be allowed for in various ways; thus, in Fig. 20, it has been treated similarly to friction by adding the velocity head to the friction loss, which has lengthened such lines as $B_3 B_3'$. In general, velocity head is relatively small and has very little influence. The modification shown in Fig. 24 is neat and ingenious and does not seriously lengthen the work.

Those who read the analytical method of computing water-hammer will appreciate the demonstration given by Messrs. Mason, Danel, and Craya of the derivation of the formulas for the transmission and reflection coefficients, which are difficult to understand and are well illustrated in the diagrams. The graphical construction includes them without appearing to do so. The derivation given shows the connection between the two methods of treatment and is a distinct addition to the literature on the subject.

The writer is pleased to find the desire, on the part of so many workers in the subject, to compare the analytical and the graphical methods. Jaeger[6] has been one of the exponents of the former method and Equation (84*b*) shows the connection of one of his important coefficients with the corresponding lines on the graphical diagram. Undoubtedly, the two methods must go side by side, recourse being had to parts of one and parts of the other where necessary; and the writer realizes that his own statement that in some cases analytical solutions were almost impossible, is capable of a much wider interpretation than was intended.

The three writers have called attention to a practical difficulty in solving water-hammer problems by any method due to the lack of exact knowledge of the pressure-velocity-time relations for the disturbing valve or gate; the solution cannot be more accurate than the data in this respect.

The discussion written by Mr. Knapp calls attention to the solution of the turbine and draft-tube problem illustrated by Fig. 17. Naturally, the turbine would vary somewhat in speed during gate closure and this would add to the disturbances entering the problem, but if all these features were discussed there would be ample material for a complete paper on that one problem. The writer was trying to illustrate the many different types of problems that could be solved, and Fig. 17 shows one in which he had been interested for some time.

The writer cannot conceive of a competent engineer deliberately designing a plant with the characteristics at Point *D* of Fig. 15, and then correcting it with a surge tank as in Fig. 16; nor has he suggested making the plant stable by means of the surge tank. Fig. 15 represents a type of plant that is common —that is, one in which a single supply pipe, *C B*, delivers water to two penstocks terminating in gates or nozzles used for turbines. In Fig. 15(*b*) the writer selected an arbitrary variation of *h* with *v* for the outlet at Point *D* and then investigated what happened. The inference from the investigation, clearly, is that such an arbitrary variation at Point *D* is incorrect; there may be other ways of determining this fact but the construction shown in the paper is a simple one.

The writer then added one more accessory to the plant by examining a surge tank located at Point *B* with the results shown in Fig. 16; but whether or not that is a desirable method has not been discussed. The tank certainly reduces the pressure swings and makes the system practicable "as far as the investigation has gone," and is "a further example of how these difficult problems may be solved simply and accurately, on the drafting board."

In regard to the derivation of Equations (10) the writer feels that he could scarcely have been expected to read the paper to which Mr. Knapp refers.[22] There are many methods of deriving these equations, the one adopted being sound and as reliable as any. This method appeals to some, but others may prefer a different one; and it makes little difference in the final outcome so long as there is confidence in the correctness of the equations.

The statements by Mr. Knapp that the writer worked "under assumptions that are fundamentally incorrect" and has obtained "results in accordance with these assumptions" is rather broad and, with a single qualification, to be discussed subsequently, is tantamount to a denial of Newton's Second Law. Before studying the paper in which Mr. Knapp calls "attention[18] to the incorrect solution of the problem under discussion," the reader would do well to analyze, carefully, the conditions imposed by the writer in solving the problem of Fig. 18 and the similar problem[21] reported by the writer in 1937. In referring to Fig. 18, the present paper states that "a vacuum valve is installed at Point *B* in such a manner as to prevent the pressure falling to that extent" (namely, below atmospheric pressure), and further, "the admission of air and its subsequent release at Point *B* maintains atmospheric pressure there until the columns re-unite." A similar statement is made for the 1937 paper to which Mr. Knapp refers[21] and in both cases it is assumed (although only stated in the latter), that Pipe *C B* rises somewhat in the vicinity of Point *B* so that the surface of the water must be horizontal while the columns are divided.

Under the first assumption, there could be no connection whatever between Columns *A B′* and *B″ C* from the time they part until they re-unite; and, both may be treated as distinct columns, as has been done, the air at Point *B* being always at atmospheric pressure. Mr. Knapp's first paper[18] refers to the case in which air is not admitted at Point *B* and where cavitation may occur with somewhat uncertain effects, in the present knowledge of this phenomenon. It is unfortunate that he has not given full details of his experiments[18] so that those interested might form an independent judgment on this subject.

The writer agrees with the criticism of the profile of Pipe *B C* which should be always at least 66 ft below Surface *C*. The actual case suggesting this problem was of a pipe where the total rise in Line *B C* occurred very close to the reservoir, in which case the solution is correct. It is only to this point that Mr. Knapp's sweeping criticism applies.

In his discussion, Mr. Kalinske thinks that the writer should have emphasized the fact that the method described is one for solving a set of simultaneous equations; but as Equations (23) and (26) make this quite clear, he had not thought it necessary to enlarge further on the fact. Instead of depending on an undue number of equations, a better grasp of the subject can be obtained from the broad variety of illustrations shown.

The writer agrees with Mr. Kalinske that with most engineers a clear conception of what happens in water-hammer is best obtained by first working a number of examples in a step-by-step process, and he uses this method with students as an introduction to the subject. He analyzed the "wave plan" described by R. Löwy,[36] and illustrated by Mr. Knapp,[13] but felt that it was not helpful enough to justify the time spent on it.

The writer expresses his thanks to all those who have been kind enough to contribute to this paper.

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[36] "Druckschwankungen in Druckrohrleitungen," by R. Löwy, pub. by Julius Springer, Berlin, 1928.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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NATURAL PERIODS OF UNIFORM CANTILEVER BEAMS

By Lydik S. Jacobsen,[1] Esq.

With Discussion by Messrs. K. Bert Hirashima, Merit P. White, and Lydik S. Jacobsen

Synopsis

An attempt has been made to give a quantitative answer to the question of the relative influence of different types of distortions on the natural periods of uniform cantilever beams elastically fixed at their bases. Very few buildings approximate the uniform and symmetrical beam dealt with in the theory, so that close agreement with actuality cannot be expected if the theory is applied to buildings. However, it is felt that the study enables the designer of earthquake resistant structures to put his finger on the most important factors entering into a determination of periods. The order or magnitude of errors committed by neglecting either flexure or shear in the building and by neglecting elastic yielding of the ground may be inferred from the tables, curves, and approximate formulas of the paper. Quantitative data relating to ground rigidities are as yet rather speculative, but it is hoped that the theory presented will help investigators to form reasonable estimates of the magnitudes of the ground rigidities from field data of natural vibrational periods of buildings.

Introduction

Transverse vibration frequencies of uniform beams have usually been computed from a consideration of the dynamic, flexural distortions. (The term "flexural distortion" refers to distortions produced by bending moments only.) When the beams are long and slender, experimental evidence is in fair agreement with the theoretical results, especially for the fundamental mode, but if higher modes are considered, or if the beams are relatively stubby, the deviations from the simple theory increase with the order of the mode. Retaining Lord Rayleigh's correction for the effect of rotatory inertia of the beams, S. Timoshenko[2] introduced a second correction for shear and solved the problem of

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Note.—Published in March, 1938, *Proceedings*.

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[2] "Vibration Problems in Engineering," by S. Timoshenko, 1928, p. 231.

frequencies for a uniformly loaded beam resting simply on rigid supports. In the case of a beam of rectangular cross-section, he found that the shear correction is four times larger than the one due to rotatory inertia. Using methods of successive approximations, V. I. Novotorzev[3] studied the effect of shear, rotatory inertia, and longitudinal elasticity on the fundamental frequency of beams and of simple bents. He found that for a rigidly built-in cantilever beam of rectangular cross-section, with a length to depth ratio of 4.56, the effect of shear distortion is to lower the frequency 3.3% from that obtained by considering flexural distortions alone, and that the combined effect of flexure, shear, and rotatory inertia amounted to a 4.2% lowering of the frequency. (The term, "shear distortion," refers to distortions produced by shear only.)

In regard to the higher mode frequencies the correction occasioned by shear will increase with the order of the mode until very soon a much better approximation of the frequencies results from a consideration of shear alone rather than from flexure alone. Quantitative data for the higher modes of cantilever beams, to the writer's knowledge, have not been previously published.

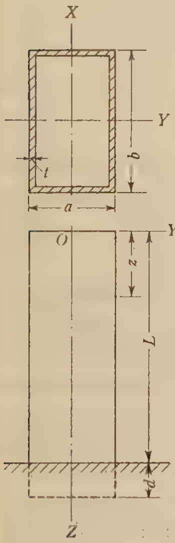
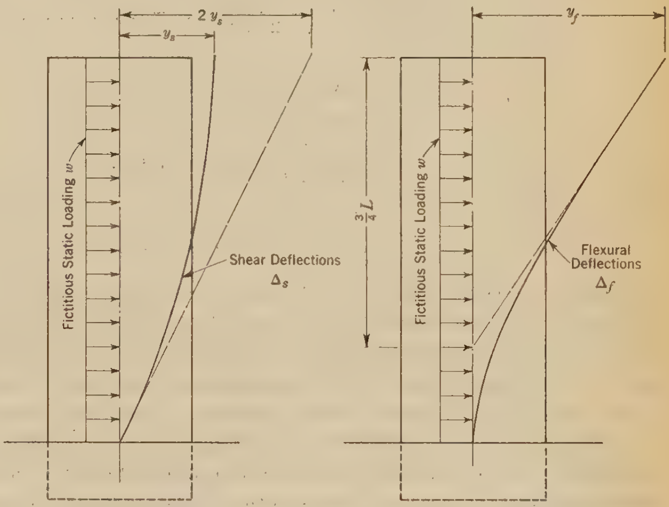


FIG. 1.—CO-ORDINATE SYSTEM AND DIMENSIONS



(a) DUE TO SHEAR (b) DUE TO FLEXURE

FIG. 2.—STATIC DEFLECTIONS OF A UNIFORMLY LOADED CANTILEVER

In vibration studies of beams it has been customary to assume simple end conditions as, for instance: (1) Entirely free; (2) rigid for translation and free for rotation; (3) rigid for rotation and free for translation; and (4) rigidly built-in. Frequency studies of cantilever beams with elastic constraints at the built-in end have undoubtedly been made for cases of flexure alone and shear alone,[4] but it is believed that the cantilever frequencies due to flexure, shear, translatory and rotatory elastic yielding of the built-in end have not been published before.

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[3] *Bulletin No. 70*, Academie des Sciences de l'Union des Republiques Sovietiques Socialistes, 1935.

[4] *Zeitschrift für technische Physik*, No. 7, 1927, p. 269. (E. Schwerin considers elastic restraints at the end of a non-uniform beam subjected to flexure only.)

Consider a vertical, uniform cantilever beam of rectangular cross-section and of a free length L. The beam is "planted" to a depth, d, in an elastic medium (see Fig. 1).

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in Appendix I.

Static Distortions of the Beam Itself

Since the elastic properties of the cantilever beam have something to do with its vibrations, the first step is to examine some of the static distortions that would occur if a hypothetical, horizontal, uniform loading of the same intensity as the uniform weight of the beam were to act in the Y direction. The beam will then deflect horizontally so that shear deflections, Δ_s, and flexural deflections, Δ_f, become established as shown in Fig. 2. If the origin of the co-ordinate system is at the top of the beam and z is measured positive downward, the deflections of a cantilever beam with a rigidly built-in end are:

$$\Delta_s = \frac{Y_s\, w}{2\, A\, G}\,(L^2 - z^2) \qquad \text{(1a)}$$

$$(\Delta_s)_{\max} = y_s = \frac{Y_s\, w\, L^2}{2\, A\, G} \qquad \text{(1b)}$$

$$\Delta_f = \frac{w}{24\, E\, I_x}\,(3\, L^4 - 4\, L^3\, z + z^4) \qquad \text{(1c)}$$

$$(\Delta_f)_{\max} = y_f = \frac{w\, L^4}{8\, E\, I_x} \qquad \text{(1d)}$$

in which, w is the weight per unit height of beam, in pounds per inch; A is the cross-sectional area of beam, in square inches; I_x is the moment of inertia of beam section about the X-axis, in inches^4; E is the modulus of elasticity, in pounds per square inch; G is the modulus of elasticity in shear, in pounds per square inch; and, Y_s is the shear deflection coefficient, which is dimensionless.

The value of Y_s is generally not the same as the shearing stress distribution coefficient of 1.5 resulting from the simplified theory of a parabolic shearing stress distribution over the cross-section. According to Theodor von Kármán, M. Am. Soc. C. E., and F. Seewald[5] the influences of the non-linear distribution of longitudinal stresses as well as the transverse stresses due to the lateral deformation depend on the value of Poisson's ratio, $\frac{1}{m}$, and for a solid rectangular cross-section of a narrow width must be taken into account as follows:

$$Y_s = \frac{3}{2} - \frac{3\, m}{10\,(m + 1)} - \frac{3}{4\,(m + 1)} \qquad \text{(2)}$$

The first member on the right-hand side of Equation (2) is the usual shear distribution coefficient first evaluated by Grasshof.[6] The second member gives the correction due to the longitudinal stresses, and the third represents the correction necessitated by the lateral deformation of the beam. Thus, if $\frac{1}{m}$ is one-third, $Y_s = 1.500 - 0.225 - 0.188 = 1.087$; and, if $\frac{1}{m}$ is one-fourth, $Y_s = 1.50 - 0.24 - 0.15 = 1.11$.

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[5] Abhandlungen aus dem Aerodynamischen Institut der Techn. Hochschule, Aachen, Vol. 7, pp. 3, 11, Berlin, 1927.

[6] "Der Stahlbau," by R. Gran Olsson, 1934, Vol. 2.

It should be mentioned that for any built-in beam, such as a cantilever, several possible but different assumptions relating to the type of distortional constraint, as amplified by warping of the cross-section at the built-in fixed end, lead to different expressions for the effect of shearing force on the deflection.[7]

If the cross-section of the beam is different from the rectangular (as, for instance, those shown in Fig. 3) the foregoing considerations in regard to the value of Y_s do not apply quantitatively, and the designer is forced to assign somewhat arbitrary values to the shear deflection coefficients. In view of the fact that deflection experiments on H-beams in the Y-direction, Fig. 3(c), are in fair agreement with the simple theory when Y_s is taken equal to the shearing stress distribution coefficient, this assumption may be accepted until better approximations become available. The simple static moment formula of elementary strength of materials, therefore, is used,

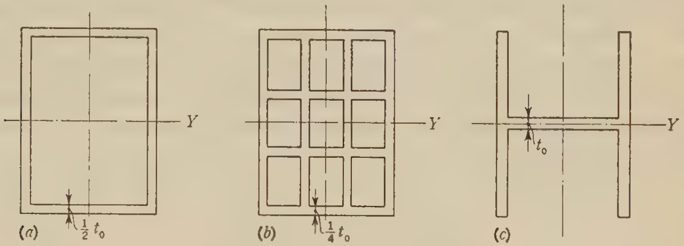

FIG. 3.—EXAMPLES OF CROSS-SECTIONS

$$Y_s = \frac{A \int_0^{a/2} y \, dA}{t_0 I_x} \quad \text{(3)}$$

Example 1.—If t_0 (the thickness of the section at the neutral axis) is small in comparison with a and b, and if $a = b$, the value of Y_s for the box section is 2.25; for the cellular section, 2.58; and for the H-section, 3.11. These values are two to three times greater than for the rectangular solid cross-section; in other words, shear is of relatively great importance in a study of the sections shown in Fig. 3.

In addition to the fictitious horizontal loadings producing the lateral deflections in the Y and in the X-directions, the beam may be twisted about its vertical axis by a fictitious torsional twisting moment of $w\,k$ lb-in. per unit height. Under these circumstances the angle of twist will be given by:

$$\phi = \frac{w\,k}{2\,G\,R_t}(L^2 - z^2) \quad \text{(4a)}$$

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[7] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill, 1934, pp. 37, 150.

and,

$$(\phi)_{\max} = \phi_R = \frac{w\,k\,L^2}{2\,G\,R_t} \quad \ldots\ldots (4b)$$

in which k = radius of gyration of the cross-section, in inches; and $G\,R_t$ = torsional rigidity, in pound-inches². The value of R_t for the beam of rectangular cross-section is found from the St. Venant table printed in most textbooks.[8] If the cross-section of the beam is of the types shown in Fig. 3, R_t is best evaluated by use of the Prandtl membrane analogy, also found in textbooks.[8] However, if the box section or the cellular section has transverse horizontal diaphragms spaced at regular vertical intervals, as in the case of buildings with floors, the torsional shear distribution is influenced by these diaphragms, and somewhat arbitrary assumptions must be made in regard to the torsional properties of the beam. It is suggested that in this case R_t be defined as a summation of the products of the individual areas by the squares of their normal distances from the center of twist. For example, in the case of the rectangular box section of thickness, t,

$$R_t = \frac{1}{2}\,(a\,b^2 + a^2\,b)\,t \quad \ldots\ldots (5a)$$

If the membrane analogy had been used:

$$R_t = 2\,\frac{a^2\,b^2}{a + b}\,t \quad \ldots\ldots (5b)$$

Equations (5*a*) and (5*b*) give identical results when $b = a$, but when $b = 2\,a$, the first, and recommended, expression gives a value of 12% greater rigidity than the second. The arbitrary assumptions are especially necessary when sections of the cellular type are considered.

HYPOTHETICALLY ISOLATED VIBRATION FREQUENCIES OF A RIGIDLY BUILT-IN CANTILEVER BEAM

The transverse vibrations resulting from a consideration of shear alone are found from the well-known equation of motion:[9]

$$\frac{\partial^2 y}{\partial z^2} = \alpha^2\,\frac{\partial^2 y}{\partial t^2} \quad \ldots\ldots (6)$$

in which $\alpha^2 = \frac{Y_s\,w}{g\,A\,G}$, giving the frequency equation:

$$\cos \alpha\,p\,L = 0 \quad \ldots\ldots (7)$$

which means that the hypothetically isolated periods due to shear only may be expressed by:

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[8] See, for instance, "Strength of Materials," by S. Timoshenko, Van Nostrand, 1930, Vol. I, p. 77, and Vol. II, p. 623.

[9] "Vibration Problems in Engineering," by S. Timoshenko, p. 216.

$$(T_s)_1 = 0.288 \sqrt{y_s} \qquad (8a)$$

$$(T_s)_2 = \frac{1}{3} (T_s)_1 \qquad (8b)$$

$$(T_s)_3 = \frac{1}{5} (T_s)_1 \qquad (8c)$$

$$(T_s)_4 = \frac{1}{7} (T_s)_1 \qquad (8d)$$

in which the maximum shear deflection, y_s, is given in inches. A consideration of flexural distortions only leads to the equally well-known equation of motion[10]:

$$\frac{\partial^4 y}{\partial z^4} = -\beta^2 \frac{\partial^2 y}{\partial t^2} \qquad (9)$$

in which $\beta^2 = \dfrac{w}{g\, I_x\, E}$, giving a frequency equation:

$$\cos \sqrt{\beta\, p\, L^2} \cosh \sqrt{\beta\, p\, L^2} = -1 \qquad (10)$$

which means that the hypothetically isolated periods due to flexure only may be expressed by:

$$(T_f)_1 = 0.258 \sqrt{y_f} \qquad (11a)$$

$$(T_f)_2 = \frac{1}{6.27} (T_f)_1 \qquad (11b)$$

$$(T_f)_3 = \frac{1}{17.6} (T_f)_1 \qquad (11c)$$

$$(T_f)_4 = \frac{1}{34.4} (T_f)_1 \qquad (11d)$$

in which the maximum flexural deflection, y_f, is given in inches.

The torsional vibrations are not influenced by flexure since they involve shear or torsional distortions only. They may be obtained from Equations (6), (7), and (8), therefore, by replacing the coefficient, α, with α_ϕ, giving:

$$\frac{\partial^2 \phi}{\partial z^2} = \alpha_\phi^2 \frac{\partial^2 \phi}{\partial t^2} \qquad (12)$$

in which $\alpha_\phi^2 = \dfrac{w\, k^2}{g\, R_t\, G}$, giving a frequency equation:

$$\cos \alpha_\phi\, p\, L = 0 \qquad (13)$$

Therefore, the actual torsional periods are expressed as follows:

$$(T_R)_1 = 0.288 \sqrt{k\, \phi_R} \qquad (14a)$$

$$(T_R)_2 = \frac{1}{3} (T_R)_1 \qquad (14b)$$

$$(T_R)_3 = \frac{1}{5} (T_R)_1 \qquad (14c)$$

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[10] "Vibration Problems in Engineering," by S. Timoshenko, p. 222.

and,

$$(T_R)_4 = \frac{1}{7}(T_R)_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14d)$$

in which the maximum torsional angle at the top of the beam is ϕ_R.

TRANSVERSE PERIODS OF A RIGIDLY BUILT-IN CANTILEVER BEAM

Before considering the elastically yielding end fastening of the beam, it is interesting to inquire as to the extent to which flexural and shear distortions are of importance in a study of the transverse periods of the rigidly built-in beam. For very stubby beams Equations (6), (7), and (8) give good results, and for very slender beams Equations (9), (10), and (11) apply. If the beams are neither stubby nor slender, Equation (45*a*) of Appendix II must be used. Table 3(*a*) (Appendix II), gives solutions of this equation in the form of period ratios, $\frac{T}{T_s}$ and $\frac{T}{T_f}$, in terms of the flexure-shear ratio, $\mu = \frac{y_f}{y_s}$. Thus, if the maximum static deflections from Equations (1) of the fictitiously loaded beam are equal, the ratio, μ, is unity, and the true periods will be:

For the fundamental mode,

$$T_1 = (1.32)\,0.288\sqrt{y_s}, \quad \text{or} \quad T_1 = (1.47)\,0.258\sqrt{y_f};$$

for the second mode,

$$T_2 = (1.20)\,\frac{0.288}{3}\sqrt{y_s}, \quad \text{or} \quad T_2 = (2.80)\,\frac{0.258}{6.27}\sqrt{y_f};$$

and for the fourth-mode period,

$$T_4 = (1.03)\,\frac{0.288}{7}\sqrt{y_s}, \quad \text{or} \quad T_4 = (5.64)\,\frac{0.258}{34.4}\sqrt{y_f}.$$

A comparison of the ratios, $\frac{T}{T_s}$ and $\frac{T}{T_f}$, of Table 3(*a*) (Appendix II) for any specific value of μ and for a specific mode of vibration gives an idea of the relative importance of shearing and of the flexural distortions of the beam in regard to period calculations. Thus, for the fundamental mode of the beam with $\mu = 1$, the ratios, $\frac{T}{T_s} = 1.32$, and $\frac{T}{T_f} = 1.47$, signify that if flexure is neglected entirely and if only shear distortions are considered, the true period is 32% longer than the computed period. On the other hand, if shear is neglected the true period is 47% longer than the period resulting from a consideration of flexure only. Therefore, in this case, the error due to neglecting shear is greater than that resulting from a neglect of flexure. When $\mu = 1.25$, the errors in the period due to neglecting either shear or flexure are equal and amount to 40 per cent. For values of μ greater than 1.25 a better approximation of the fundamental period results from a consideration of flexure only than from one of shear only.

For the second-mode periods the errors due to neglecting either shear or flexure are not equal until the ratio, μ, is 5.4. At this value the errors are 56 per cent. In the third-mode and the fourth-mode periods equalities of errors for the two methods of computation do not occur within the limits of extrapolation of Table 3(*a*) (Appendix II). In general, one may conclude that for cantilever beams, rigidly built-in, a consideration of shearing distortions rather than one of flexural distortions is of importance for the higher modes of vibration unless the beam is very slender so that the ratio, μ, becomes large.

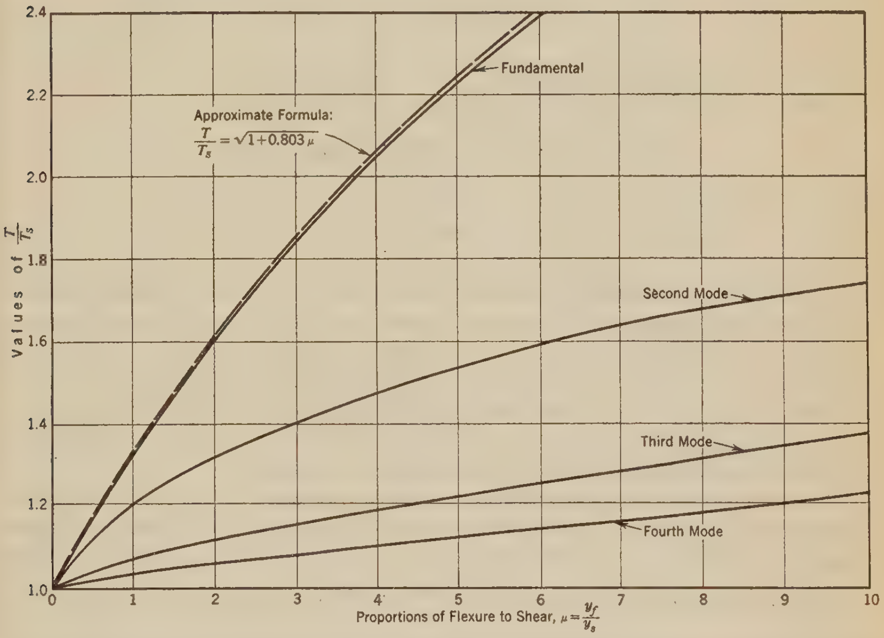

FIG. 4.—GENERAL CURVES FOR FIRST FOUR TRANSVERSE MODE PERIODS OF UNIFORM CANTILEVER BEAMS SUBJECTED TO DYNAMIC SHEAR AS WELL AS TO DYNAMIC FLEXURE; RIGID GROUND

In order to emphasize the significance of Table 3(*a*) (Appendix II), Fig. 4 and Tables 3(*b*), 3(*c*), and 3(*d*) have been included. Table 3(*b*) (Appendix II) refers to a cantilever beam with a solid square cross-section. The shear deflection coefficient, Y_s, is taken equal to 1.2, and the modulus of elasticity, E, is taken equal to 2.5 G. In this case equalities of errors in period of the fundamental mode occur when the height-to-depth ratio, $\frac{L}{a}$, is 1.1; for the second mode when $\frac{L}{a}$ is 2.3; for the third mode when $\frac{L}{a}$ is 3.9; and for the fourth mode when $\frac{L}{a}$ is 5.3. This means that in most beams of solid square cross-section, only relatively stubby ones require that both types of distortions be considered for computations of periods. Shear distortions may be neglected except for the higher modes of vibration.

It is interesting to note that for $\frac{L}{a} = 4$, Ratio $\frac{T}{T_f}$ is 1.04, which means that the lengthening of the period due to shear is 4 per cent. This is in good agreement with the result obtained by Novotorzev[3] of 3.3% for a beam with $\frac{L}{a} = 4.56$.

In 1894, S. Dunkerley[11] showed that the resultant period, T_r, of a complicated system may be expressed empirically to a fair degree of approximation in terms of the "isolated component" periods, T_a, T_b, $\cdots$ T_n, as follows:

$$T_r^2 = T_a^2 + T_b^2 + \cdots T_n^2 \qquad (15)$$

In the present case, in which the two "isolated component" periods are the one due to shear alone, and the one due to flexure alone,

$$T_{sf}^2 = T_s^2 + T_f^2 = 0.288^2\, y_s + 0.258^2\, y_f \qquad (16)$$

and,

$$\frac{T_{sf}}{T_s} = \sqrt{1 + 0.803\frac{y_f}{y_s}} = \sqrt{1 + 0.803\,\mu} \qquad (17)$$

The empirical formula, Equation (17), is in good agreement with the exact theory as may be seen from the dotted curve on Fig. 4. In this case the error is positive, but in other cases it may be either positive or negative (see Fig. 6, for instance). However, it should be emphasized that the approximate formula does not have the slightest application to the period ratios of the higher modes, and that it is precisely for these modes that considerations of flexure alone give erroneous results unless the beams are very slender indeed.

Table 3(*c*) (Appendix II) relates to a cantilever beam of a square box cross-section with Y_s equal to 2.25 and with E equal to 2.5 G. In this case equality of errors in periods for computations involving shear alone and flexure alone occurs for the fundamental mode when $\frac{L}{a}$ is 2.2; for the second mode when $\frac{L}{a}$ is 4.5; and for the third mode when $\frac{L}{a}$ is 7.6. Table 3(*c*) does not extend far enough to give this ratio for the fourth mode. It is seen that in the case of the cantilever beam with a square box cross-section shearing as well as flexural distortions must be considered even if the values of $\frac{L}{a}$ are appreciable.

The period ratios for the beam with the rectangular box cross-section of a width-to-depth ratio of 3.1 are given in Table 3(*d*) (Appendix II). In this case Y_s is 4.45 so that a relatively large shear effect is present. From Table 3(*d*) it is found that equalities of errors in periods for the two types of distortion occur when $\frac{L}{a}$ is 3.4 for the fundamental mode, when $\frac{L}{a}$ is 7.2 for the second mode, and when $\frac{L}{a}$ is between 12 and 13 for the third mode.

If the beam with rectangular box cross-section is considered to be somewhat akin to a very much idealized building, it is clear that the influence of shearing distortions on its natural periods is far from negligible; indeed, for buildings of ordinary proportions a consideration of shear alone will give better approximations of the true natural periods, including the second mode and those above it, than a consideration of flexure alone. Of course, for the fundamental-mode period both types of distortion are of importance.

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[11] *Philosophical Transactions*, Royal Soc., Lond., Vol. 185 (1894), p. 311.

RATIOS OF NATURAL PERIODS

Table 3(*d*) (Appendix II) also gives the ratios of the true fundamental periods to the true higher mode periods for the rigidly built-in cantilever beam with rectangular box cross-section of $\frac{b}{a} = 3.1$. The ratios of 3, 5, and 7 for the case of shear alone (Equations (8)) are obtained when $\frac{L}{a}$ approaches zero, and the ratios of 6.27, 17.6, and 34.4 for the case of flexure alone are theoretically approached as $\frac{L}{a}$ becomes infinite. When $\frac{L}{a}$ is 10—a very large ratio for a building—the period ratios, 5.28, 11.1, and 17.5, are quite far from either of the two sets of limiting ratios. Finally, it may be concluded that even in the case of a rigidly built-in, uniformly loaded, cantilever beam the ratios of natural periods are not very simple.

END CONSTRAINTS OF THE CANTILEVER BEAM

In proceeding to a study of the effect, on the periods, of the elastic constraint at the built-in end, it will be assumed that the medium in which the cantilever beam is "planted" is capable of exerting elastic restoring forces and moments on the end of the beam. The inertia of the medium is neglected and, for the time being, the restoring force properties of the medium and "planting" may be thought of as resulting from static experiments. Appendix III includes a description of a crude, rational method of relating the restoring properties of the ground and "planting" to ground deformation moduli.

The translatory rigidities offered by the ground to the "planted" beam in the X-direction and in the Y-direction are defined by R_x and R_y. The units are pounds per inch of translation. If the fictitious horizontal loading of the weight, $w\,L$, of the beam itself is assumed to act in the Y-direction, the corresponding horizontal translation of the base of the beam in this direction, in inches, is given by:

$$y_t = \frac{w\,L}{R_y} \quad \text{(18)}$$

In regard to the rotatory or tilting rigidity of the ground, it is assumed that the horizontal axis about which rotation or tilting occurs lies in the plane of the surface of the ground. The rotatory restraints of the ground upon the beam about the X- and Y-axes in the plane of the ground are then given by $R_{x\theta}$ and $R_{y\theta}$. The units are pound-inches per radian. If the fictitious horizontal loading of the weight of the beam itself, $w\,L$, is assumed to act in the Y-direction, the vertical axis of the beam will be inclined in the $Z\,Y$ plane and the maximum linear displacement at the top of the beam, in inches, will be given by,

$$y_r = \frac{w\,L^3}{2\,R_{y\theta}} \quad \text{(19)}$$

The fifth rigidity of the ground relates to the torsional constraint of the ground acting through the "planted" part of the beam. It will be assumed again that this rigidity is measured about the vertical axis of the beam in the plane of the ground. It is denoted by R_ϕ and its dimensions are pound-inches per radian. The effect of the fictitious torsional moment of $w\,L\,k$ lb-in. about the vertical axis of the beam will be an angle of rotation of the base of the beam, in radians, with respect to the ground of the magnitude,

$$\phi_g = \frac{w\,L\,k}{R_\phi} \qquad (20)$$

It is important to note that in the case of a homogeneous elastic ground the five rigidities, R_x, R_y, $R_{x\theta}$, $R_{y\theta}$, and R_ϕ are functions of the properties of the ground as well as of the dimensions, a, b, and d of the "planting."

INFLUENCE OF THE GROUND ON THE TORSIONAL PERIODS

Since flexure in the building as a whole does not affect its torsional properties, the torsional-mode periods of the elastically built-in cantilever beam can be found from a general solution of Equation (47*a*), of Appendix II, if the shear coefficients, α and τ, are replaced by the torsional coefficients, α_ϕ (in radians per inch per second) and $\tau\phi$ (in second² per inch per radian), defined as follows:

$$\alpha_\phi = \sqrt{\frac{w\,k^2}{g\,R_t\,G}} = \sqrt{\frac{2\,k\,\phi_R}{g\,L^2}} = 0.072\sqrt{\frac{k\,\phi_R}{L^2}} \qquad (21a)$$

and,

$$\tau_\phi = \frac{w\,k^2}{g\,R_\phi} = \frac{k\,\phi_g}{g\,L} \qquad (21b)$$

With these substitutions, Equation (47*a*) (Appendix II) may be written thus:

$$\frac{0.454\,\sqrt{k\,\phi_R}}{T}\tan\left(\frac{0.454\,\sqrt{k\,\phi_R}}{T}\right) = 2\,\frac{\phi_R}{\phi_g} \qquad (22)$$

Solutions of this transcendental equation yield the torsional periods for any cantilever beam with elastic, torsional constraint at the built-in end for assumed values of the torsional rigidity ratio, $\psi = \frac{\phi_g}{\phi_R}$. The torsional periods of the nth mode of vibration are expressed by,

$$T_n = \left(\frac{T_{Rg}}{T_R}\right)_n \frac{0.288}{2\,n-1}\sqrt{k\,\phi_R} \qquad (23)$$

Fig. 5 shows the values of the period ratios that must be used in Equation (23). Furthermore, it shows that as elastic torsional yielding of the ground increases, the fundamental torsional period becomes longer and longer, so that the second-mode torsional period approaches the fundamental mode torsional period for the beam with both ends free; similarly, the third-mode period approaches the second-mode period for the free beam, etc. The ratios of the fundamental torsional periods to those of the higher modes are close to the values 3, 5, 7, and 9 for small values of the ratio, ψ.

If Dunkerley's empirical formula (see Equation (15)) is applied to the fundamental mode of torsion,

$$T_{Rg} = \sqrt{T_R^2 + T_g^2} = \sqrt{(0.0830\,\phi_R + 0.1024\,\phi_g)\,k} \quad\ldots\ldots\ldots(24)$$

and,

$$\frac{T_{Rg}}{T_R} = \sqrt{1 + 1.23\,\frac{\phi_g}{\phi_R}} = \sqrt{1 + 1.23\,\psi} \quad\ldots\ldots\ldots\ldots\ldots(25)$$

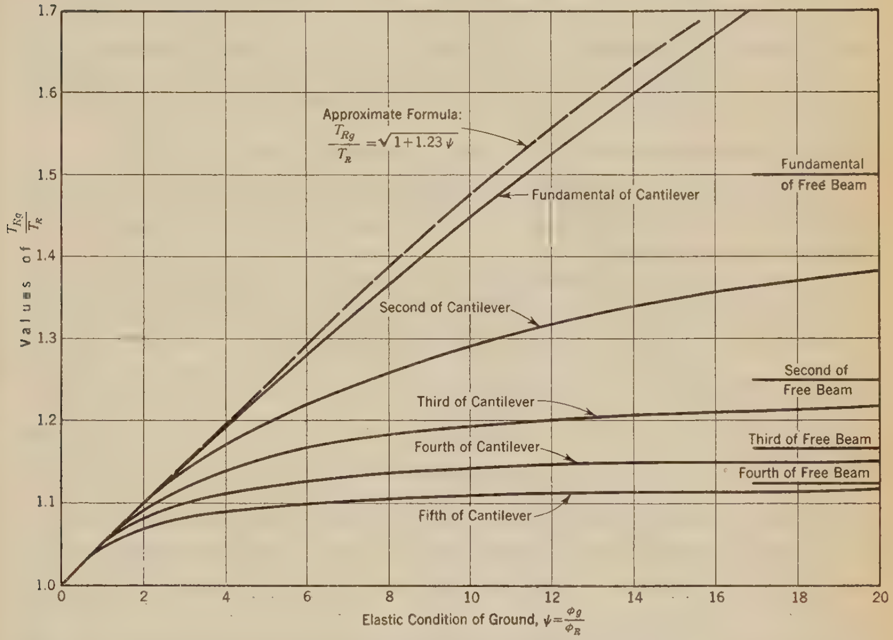

FIG. 5.—GENERAL CURVES FOR FIRST FIVE TORSIONAL MODE PERIODS OF UNIFORM CANTILEVER BEAMS "PLANTED" IN A TORSIONALLY ELASTIC GROUND

The results of Equation (25) are plotted on Fig. 5 as a broken-line curve. It is suggested that the approximate formula is accurate enough for practical purposes.

INFLUENCE OF THE GROUND ON THE PERIODS OF THE TRANSVERSE VIBRATIONS

In Appendix II the general problem of the free vibration periods of cantilever beams has been solved by considering shear and flexure of the beams as well as translatory and rotatory elastic yielding of the ground. Since the writer's primary interest in the application of the results centers on idealized buildings, a specific but fictitious building has been chosen to illustrate the significance of the theoretical deductions in Appendix II.

Example 2.—A cantilever beam of rectangular box cross-section, Fig. 3(*a*), is defined as follows: (1) Period due to shear alone, 1 sec = $(T_s)_1$; (2) height, 200 ft, or 2 400 in. = L; (3) depth, 52.5 ft, or 630 in. = a; (4) width, 170.0 ft, or 2 040 in. = b; (5) radius of gyration, 35 ft, or 420 in. = k; and (6) depth of "planting," 20 ft, or 240 in. = d.

The ground in which the cantilever beam is planted is defined by: (7) Horizontal deformation modulus, ϵ_h, and shear deformation modulus, ϵ_s, are equal (see Appendix III); and, (8) vertical deformation modulus, ϵ_v, is twenty-five times larger than ϵ_h; or $\frac{\epsilon_v}{\epsilon_h} = \gamma = 25$.

The yielding of the ground in translation, in rotation, and in shear may then be expressed, according to Appendix III, by the ground constants, ϵ_h and γ, taken in the proper combination with the physical dimensions of the building. The ratio of the translatory ground deflection, y_t, to the maximum shear deflection, y_s, of the beam is then sufficient for characterizing uniquely the yielding of the ground. This ratio is called ν $\left(\text{that is, } \nu = \frac{y_t}{y_s}\right)$.

The discussion of the chosen example is confined to a study of the transverse translatory vibration periods in the Y-direction and to the torsional vibration periods about the Z-axis.

By considering the specifications of the beam and some of the given equations it is found that: $y_s = 12.1$ in. (Equation (8a)); $Y_{sz} = 4.45$ and $Y_{sy} = 1.70$ (Equation (3)); $A = 2\,(a + b)\,t = 5\,340\,t$ sq in.; $I_x = (\frac{1}{2}\,a^2\,b + \frac{1}{6}\,a^3)\,t = 444 \times 10^6\,t$ in.^{4}; $\frac{w}{G} = 5.04 \times 10^{-3}\,t$ in. (Equation (1b)); $\frac{w}{E} = 2.02 \times 10^{-3}\,t$ in. (if $E = 2.5\,G$); $y_f = 18.6$ in. (Equation (1d)); $(T_f)_1 = 1.11$ sec (Equation (11a)); $\mu = \frac{y_f}{y_s} = 1.54$; $R_t = 1.71 \times 10^9\,t$ in.^{4} (Equations (5)); $\phi_R = 3.57 \times 10^{-3}$ radians (Equation (4b)); and, $(T_R)_1 = 0.353$ sec (Equation (14a)).

From the specifications of the ground and from Equation (53a) (Appendix III): $\frac{y_t}{y_s} = \nu$ (different values of ν to be assumed); $\frac{y_r}{y_t} = \frac{(12\,d + 6\,b)\,L^2}{\gamma\,a^3 + 8\,d^3} = 13.8$; $\lambda = \frac{y_r}{y_s} = \frac{y_r}{y_t}\frac{y_t}{y_s} = 13.8\,\nu$; and, $\psi = 3.86\,\nu$ from Equation (54) (Appendix III).

Torsional Periods in Example 2.—The torsional coefficients, $\frac{T_{Rg}}{T_R}$, of Equation (23) are easily found from Fig. 5 for the values of ψ corresponding to the assumed values of ν. From the results, as given in Table 1, it is seen that the fundamental torsional period increases indefinitely as ν increases. This means that a cantilever loses its fundamental mode as ν approaches infinity, or as its constraint at the built-in end approaches zero. The second-mode period of the motion of the cantilever with built-in end changes into the fundamental-mode period of the beam with free ends as ν approaches infinity. The third-mode period becomes the second-mode period, etc.

Transverse Periods in Example 2.—The solution of the frequency Equation (44) from Appendix III for the structure described by the foregoing constants and for assumed values of the ratio, ν, is a long and laborious task. The results are plotted in Figs. 6, 7, and 8.

TABLE 1.—TORSIONAL PERIODS, IN SECONDS, FOR THE FIRST FIVE MODES OF VIBRATION OF THE CANTILEVER BEAM REPRESENTING THE IDEALIZED BUILDING CHOSEN AS AN ILLUSTRATIVE EXAMPLE OF THE THEORY

(The Elastic Condition of the End Fastening Is Measured Either by ν or by ψ. The Ratios of the Fundamental to the Higher Mode Periods Are Also Given.)

SYMBOLS	VALUES OF ν AND ψ									
ν ψ	0 0	0.05 0.193	0.10 0.386	0.20 0.770	0.30 1.16	0.40 1.54	0.50 1.93	1.0 3.86	2.0 7.72	∞ ∞
(*a*) FUNDAMENTAL TORSIONAL MODE										
T_1	0.351	0.386	0.417	0.474	0.530	0.579	0.628	0.806	1.17	∞
(*b*) SECOND TORSIONAL MODE										
T_2	0.117	0.128	0.136	0.146	0.153	0.158	0.162	0.166	0.170	0.176
$\frac{T_1}{T_2}$	3.00	3.02	3.07	3.29	3.47	3.66	3.88	4.86	6.88	....
(*c*) THIRD TORSIONAL MODE										
T_3	0.0702	0.0766	0.0795	0.0830	0.0844	0.0850	0.0857	0.0865	0.0870	0.0878
$\frac{T_1}{T_3}$	5.00	5.05	5.25	5.71	6.28	6.81	7.35	9.33	13.5	....
(*d*) FOURTH TORSIONAL MODE										
T_4	0.0502	0.0543	0.0558	0.0568	0.0573	0.0578	0.0583	0.0584	0.0585	0.0586
$\frac{T_1}{T_4}$	7.00	7.11	7.46	8.34	9.25	10.0	10.8	13.8	20.0	....
(*e*) FIFTH TORSIONAL MODE										
T_5	0.0390	0.0414	0.0425	0.0429	0.0432	0.0434	0.0436	0.0437	0.0437	0.0438
$\frac{T_1}{T_5}$	9.00	9.33	9.87	11.04	12.3	13.3	14.4	18.5	26.8	....

Referring to Table 2(*a*), Item No. 1 gives the transverse periods of the structure on a rigid ground, computed on the assumption that shear alone governs the periods; thus, for instance, the fourth-mode period becomes one-seventh of the fundamental period, or 0.143 sec. The periods recorded as Item No. 2 involve the assumption that flexure alone governs the vibration. Item No. 3, still relating to the condition of a rigid ground, gives the so-called true periods of the structure. Items Nos. 4 to 12, inclusive, give true periods of the structure on a yielding ground; the degree of yielding is expressed by the value of ν. Item No. 13, for $\nu = \infty$ (that is, for a beam free at both ends), is a limiting case, and, consequently, when applied to a building is a physical impossibility. Its value lies in the fact that it indicates how closely the results for $\nu = 2$ approach those for $\nu = \infty$. It should be noted that for the higher mode periods the agreement between Items Nos. 12 and 13 is fairly close. The fundamental-mode period, however, increases without limit, which means that instability of motion results. Item No. 14 gives the periods of the structure with both ends free, computed from shear distortions only, whereas Item No. 15 gives the periods resulting from considering flexure alone. The third column of Table 2(*a*), relating to the fundamental period, has been plotted in Fig. 6.

TABLE 2.—PERIODS AND PERIOD RATIOS FOR THE FIRST SIX MODES OF TRANSVERSE VIBRATION OF THE CANTILEVER BEAM REPRESENTING THE IDEALIZED BUILDING CHOSEN AS AN ILLUSTRATIVE EXAMPLE OF THE THEORY

Item No.*	Values of the ratio, ν	(*a*) PERIODS, IN SECONDS, FOR THE FOLLOWING MODES:						(*b*) PERIOD RATIOS				
		Fundamental	Second	Third	Fourth	Fifth	Sixth	$\frac{T_1}{T_2}$	$\frac{T_1}{T_3}$	$\frac{T_1}{T_4}$	$\frac{T_1}{T_5}$	$\frac{T_1}{T_6}$
1†	Rigid¶	1.00	0.333	0.200	0.143	0.111	0.0910	3.0	5.0	7.0	9.0	11.0
2‡	Rigid¶	1.11	0.176	0.0634	0.0323	0.0195	0.0130	6.27	17.6	34.4	56.9	85.0
3§	Rigid¶	1.48	0.425	0.216	0.149	0.114	0.0925	3.48	6.86	9.92	13.0	16.0
4	0.05	1.70	0.458	0.226	0.154	0.117	0.0957	3.71	7.52	11.1	14.5	17.8
5	0.10	1.90	0.472	0.233	0.157	0.119	0.0959	4.02	8.16	12.1	16.0	19.8
6	0.20	2.18	0.480	0.236	0.156	0.117	0.0888	5.22	9.23	14.0	18.6	24.6
7	0.30	2.56	0.472	0.232	0.140	0.108	0.0872	5.42	11.0	18.3	23.7	29.4
8	0.40	2.88	0.465	0.223	0.132	0.105	0.0867	6.20	12.9	21.8	27.4	33.2
9	0.50	3.11	0.452	0.210	0.131	0.104	0.0862	6.89	14.8	23.7	29.9	36.1
10	0.80	3.76	0.400	0.200	0.130	0.103	0.0857	10.3	18.8	28.9	36.5	43.9
11	1.0	4.10	0.367	0.196	0.130	0.103	0.0857	11.2	20.9	31.7	39.4	47.9
12	2.0	5.80	0.322	0.192	0.130	0.103	0.0857	18.0	30.2	44.6	56.3	67.7
13	∞	∞	0.320	0.192	0.130	0.103	0.0857	. . .	1.67**	2.46**	3.20**	3.73**
14†	∞	∞	0.250	0.167	0.125	0.100	0.0833	. . .	1.50**	2.00**	2.50**	3.00**
15‡	∞	∞	0.173	0.0628	0.0320	0.0194	0.0130	. . .	2.76**	5.50**	8.92**	13.3**

* With the exceptions noted, the types of distortion are those due to shear, flexure, translation, and rotation, combined.

† Shear distortion only.

‡ Flexure distortion only.

§ Shear distortion and flexure distortion.

¶ The elastic condition of the ground; the elastic condition of the end fastening is measured by the ratio, ν.

** In these items the numerator of the ratio is T_2 instead of T_1.

If Dunkerley's empirical formula (Equation (15)) is applied to the fundamental-mode period of the cantilever:

$$T_{sftr}^2 = 0.0830\, y_s + 0.0666\, y_f + 0.1024\, y_t + 0.0681\, y_r \ldots\ldots (26)$$

For the specific cantilever under consideration, in which: $\frac{y_f}{y_s} = 1.54$; $\frac{y_r}{y_s} = 13.8$; and, $\frac{y_t}{y_s} = \nu$: $T_{sftr}^2 = (1.855 + 1.042\,\nu)\, y_s$: and,

$$\frac{T_{sftr}}{T_s} \doteq \sqrt{2.24 + 12.55\,\nu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The results of this empirical formula are given by the dotted curve on Fig. 6. It is seen that the error is negative in this case. However, it is suggested that for practical problems the Dunkerley formula, as expressed in the general form by Equations (8), is satisfactory.

Fig. 7 shows a plot of the higher mode periods as functions of ν, the yielding of the ground. The dotted, straight horizontal lines on the left and on the right of the individual graphs relate to the periods for zero and for infinite yielding of the ground. The second-mode period increases rather rapidly for a small yielding of the ground, reaches a maximum when ν is about 0.2, and then decreases along a smooth curve. When ν is 0.6 the period of the mode is the same as for the second mode with $\nu = 0$. As ν increases further, the period drops, and as ν approaches infinity the period approaches asymptotically the fundamental-mode period of 0.320 sec for the beam or structure with both ends free. A similar variation in period resulting from a yielding of the ground occurs for all the higher mode vibrations.

It has now been shown that the fundamental-mode period of the cantilever increases indefinitely as the yielding of the ground is increased; moreover, that the second-mode period of the cantilever on a rigid ground transforms into the fundamental-mode period of the beam with free ends as ν increases. The gradual transformations from the second mode to the fundamental, from the third to the second, from the fourth to the third, etc., are shown in Fig. 7. It is to be noted that the higher the mode, the smaller the value of ν necessary for bringing about the transformation.

On Fig. 8, the ratios of the fundamental to the higher mode periods are plotted as functions of ν. It is seen that these ratios are far from simple; in general, they increase with the yielding of the ground. If the curve for $\frac{T_1}{T_2}$ is considered, it is seen that the ratio of 6.3, due to flexure alone and for a rigid ground, really corresponds to a yielding ground of which the coefficient, ν, is more than 0.4. On the other hand, the ratio of 3 due to shear alone does not exist, since the curve for $\frac{T_1}{T_2}$ starts at 3.4 for a rigid ground.

The practical value of the tendencies indicated by these curves may be twofold: First, from observations of periods on existing buildings (the United States Coast and Geodetic Survey has done much observational work in California) the ground factors may be estimated. These factors must then be related to the geologic characteristics of the ground as well as to soil tests of foundation mechanics. Second, if enough types of cases have been studied, it becomes possible to predetermine the periods of proposed buildings with a fair degree of accuracy.

It will undoubtedly be realized that the significance of such analyses is not restricted to the somewhat narrow, as well as hypothetical, value of periods for purposes of aseismic design. The periods of the building proper are indices of its mass and elastic properties, but they cannot be evaluated correctly until the ground effect on the periods becomes separable. Finally, from the point of view of foundation mechanics the periods of existing buildings, when analyzed, will give information about the ground itself.

Applications of the Theory

The theory applies to any symmetrical cantilever beam provided that its weight per unit height, its shearing rigidity, and its flexural rigidity are constant. Whether or not the ground rigidities are given by the equations of Appendix II makes no difference as far as the application of the theory is

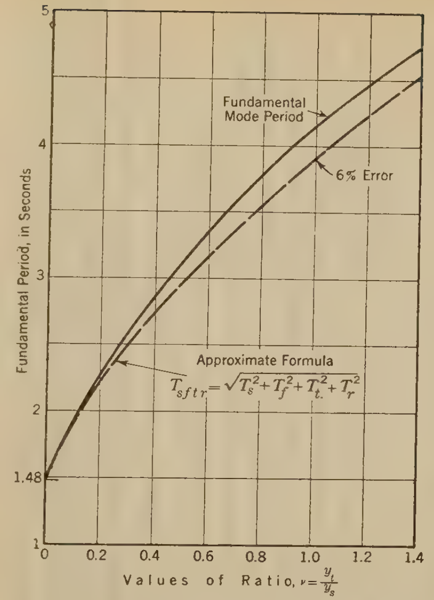

FIG. 6.—EFFECT OF AN ELASTICALLY YIELDING GROUND ON FUNDAMENTAL TRANSVERSE MODE PERIOD OF SPECIFIC CANTILEVER BEAM SUBJECTED TO SHEAR, FLEXURE, GROUND TRANSLATIONAL YIELDING AND GROUND ROTATIONAL YIELDING

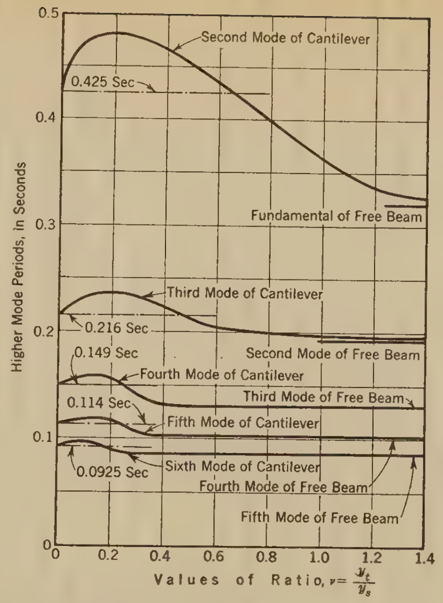

FIG. 7.—EFFECT OF ELASTIC GROUND ON HIGHER TRANSVERSE MODE PERIODS OF THE SPECIFIC CANTILEVER SUBJECTED TO THE FOUR TYPES OF YIELDING

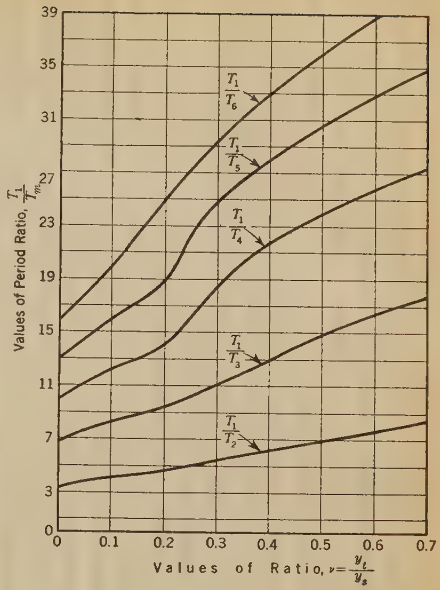

FIG. 8.—RATIOS OF FUNDAMENTAL TRANSVERSE MODE PERIOD TO HIGHER TRANSVERSE MODE PERIODS OF SPECIFIC CANTILEVER BEAM AS FUNCTIONS OF GROUND YIELDING

concerned. The necessary and sufficient condition for the ground is that its behavior must be elastic and symmetrical so that the ground coefficients, τ and ζ, Appendix II, can be evaluated.

In regard to buildings the theory will not apply with a satisfactory degree of quantitative accuracy on account of the fact that actual buildings, with a few exceptions, are far from satisfying the demands of constancy of mass and elastic properties with height; neither do they satisfy the condition of symmetry. It is thought, however, that qualitatively the theory explains the seemingly erratic data obtained from wind vibration records, or from shaking-machine vibration records of buildings. The theory indicates that for most buildings shear as well as flexure must be considered, and that the elastic yielding of the ground will influence the periods greatly.

If the data from period tests of actual symmetrical buildings are to be analyzed correctly, it becomes necessary to use graphical or mechanical methods for finding the component deflections due to shear, flexure, translatory, and rotatory rigidities of the ground.

A rough indication of the relative importance of the four factors influencing the periods of buildings may be obtained statistically from the observed data of the U. S. Coast and Geodetic Survey. The most important part of such an analysis is to choose carefully from the mass of data (about 200 to 300 buildings) the ones that satisfy most nearly the demands of twofold symmetry as well as isolation from other structures. Such a selection of data is found to decimate the number of buildings available.

Since it is known that the Dunkerley formula gives sufficiently accurate results for the fundamental-mode periods of uniform bars, and since, in the case of actual buildings, it may be supposed to apply, at least dimensionally, one may write:

$$T_{sftr} = \sqrt{c_1 l + c_2 l^2 + c_3 l^3 + c_4 l^4} \ldots\ldots\ldots\ldots\ldots (28)$$

in which l is a characteristic length-to-depth ratio of a building $\left(\text{that is, } \frac{L}{a} \text{ or } \frac{L}{b}\right)$, and the coefficients, c_1, c_2, c_3, and c_4, are to be determined from the available data. Thus, if four transverse fundamental periods of, say, two buildings of four different values of λ are known, four items of experimental information are available and may then determine uniquely the four coefficients. The significance of the four terms may then be explained as the component periods due to: Elastic ground translation, $\sqrt{c_1 l}$; shear in the building, $\sqrt{c_2 l^2}$; elastic ground tilting, $\sqrt{c_3 l^3}$; and flexure in the building, $\sqrt{c_4 l^4}$.

Of course, if more than four independent observations are available from which to find the four coefficients, the information will be redundant, and the method of least squares suggests itself for a statistical determination of the four coefficients.

Graphical and Mechanical Method of Computing the Periods

In its simplest form the problem of finding the period of one of the natural modes of vibration of an elastic system may be stated as the problem of finding a dynamic loading curve that will produce a geometrically similar, dynamic deflection curve. Although the two curves refer to dynamic conditions, they may be thought of as being static, so that a system subjected to an assumed dynamic loading curve may be treated by the well-known methods of statics in order to find its component deflections due to shear, flexure, and elastic ground yielding. The methods in question may be purely graphical, or graphical and numerical, or they may be a combination of static experimentation upon a mechanical system made up to represent the elastic properties of the system in shear, in flexure, and in end-fastening rigidity. It is possible, by means of simple strips, to build a static, elastic model of a building. The problem of finding the dynamic (that is, really, the static) loading curve that will produce an elastic deflection curve geometrically similar to the loading curve is merely a matter of sufficient experimentation by trial and error.

APPENDIX I

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[12] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

x = horizontal co-ordinate;
y = horizontal co-ordinate; direction of transverse vibration;
z = vertical co-ordinate, origin at top of beam;
b = dimension of beam in horizontal direction;
a = dimension of beam in horizontal direction; direction of vibration;
L = dimension of beam in vertical direction;
A = cross-sectional area of beam;
Y_s = shear deflection coefficient for Y-direction;
I_x = moment of inertia of cross-section for vibrational direction;
w = weight per unit height of beam;
g = acceleration due to gravity;
E = modulus of elasticity for bending;
G = modulus of elasticity for shear;
m = Poisson's number;
k = radius of gyration of cross-section;
$G\,R_t$ = torsional rigidity of beam about vertical axis;
Δ_s = shear deflection due to fictitious gravity load in horizontal direction;
Δ_f = flexural deflection due to fictitious gravity load in horizontal direction;
ϕ = torsional angle due to fictitious moment, $w\,k$, per unit height;
y_s = shear deflection at top of beam; horizontal gravity loading;
y_f = flexural deflection at top of beam; horizontal gravity loading;

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[12] A.S.A.—*Z* 10*a*—1932.

μ = ratio, $\frac{y_f}{y_s}$;
ϕ_R = torsional angle at top of beam due to moment, $w\,k$, per unit height;
t = time in derivative notation; also, thickness of cellular sections;
t_0 = thickness of cross-section at neutral axis;
l = characteristic length ratio of beam or building;
c = coefficient;
d = depth of "planting" or embedment of beam in ground;
R_x = translatory rigidity of ground in X-direction;
R_y = translatory rigidity of ground in Y-direction;
$R_{x\theta}$ = rotatory or tilting rigidity of ground in $x\,z$-plane;
$R_{y\theta}$ = rotatory or tilting rigidity of ground in $y\,z$-plane;
R_g = torsional rigidity of ground for twist of beam about vertical axis;
y_t and x_t = displacements of top of building due to ground translational yielding;
y_r and x_r = displacements of top of building due to ground rotational yielding;
ϕ_g = angle of twist due to torsional yielding of the ground;
ϵ_v = deformation modulus of ground in vertical direction;
ϵ_h = deformation modulus of ground in horizontal direction;
ϵ_s = deformation modulus of ground in shear;
ν = ratio of $\frac{y_t}{y_s}$;
λ = ratio of $\frac{y_r}{y_s}$;
ψ = ratio of $\frac{\phi_g}{\phi_R}$;
α^2 = dynamic shearing constant of beam $= \frac{Y_s\,w}{g\,G\,A}\left(\frac{\text{seconds}^2}{\text{length}^2}\right)$;
β^2 = dynamic flexural constant of beam $= \frac{w}{g\,E\,I_x}\left(\frac{\text{seconds}^2}{\text{length}^4}\right)$;
p = time-frequency $\left(\frac{\text{radians}}{\text{second}}\right)$;
q = trigonometric space-frequency $\left(\frac{\text{radians}}{\text{length}}\right)$;
r = hyperbolic space-frequency $\left(\frac{\text{radians}}{\text{length}}\right)$;
u = space-frequency for case of flexure only $= \sqrt{\beta\,p}\left(\frac{\text{radians}}{\text{length}}\right)$;
ρ = beam parameter $= \frac{\beta^2}{\alpha^4}$ (seconds^{-2});
$f(z)$ = transverse loading intensity a function of z;
V = shear in beam;
M = moment in beam;
Z = normal space-function;
$f(t)$ = trigonometric function of time;
$A\ B\ C\ D$ = temporary constants of integration;
T = natural period of vibration (seconds);

T_1 = fundamental natural period;
T_n = nth mode natural period;
T_s = isolated natural period due to shear distortions only;
T_f = isolated natural period due to flexural distortions only;
T_{sf} = natural period due to shear and flexure, rigid ground;
T_{sftr} = natural period due to shear, flexure, and elastically yielding ground;
T_R = natural torsional period of beam or building; rigid ground;
T_{Rg} = natural torsional period of beam or building; yielding ground;
$T_a\ T_b \cdots$ = isolated component periods; general case; and,
T_r = resultant period of several isolated component periods.

APPENDIX II

NATURAL FREQUENCIES OF A CANTILEVER BEAM SUBJECTED TO SHEAR AS WELL AS TO FLEXURE AND WITH ELASTIC CONSTRAINTS AT THE BUILT-IN END

It will be assumed that the principle of superposition of distortions is valid so that the simple theories of beam deflections may be expressed as follows:

Intensity of loading,

$$f(z) \qquad (29a)$$

Gradient of shear,

$$\frac{\partial V}{\partial z} = -f(z) \qquad (29b)$$

Variation of gradient of moment,

$$\frac{\partial^2 M}{\partial z^2} = -f(z) \qquad (29c)$$

Curvature of beam,[13]

$$\frac{\partial^2 y}{\partial z^2} = -\frac{M}{E\,I_x} - \frac{Y_s}{G\,A} f(z) \qquad (29d)$$

Variation of curvature of beam,

$$\frac{\partial^3 y}{\partial z^3} = -\frac{V}{E\,I_x} - \frac{Y_s}{G\,A}\frac{\partial f(z)}{\partial z} \qquad (29e)$$

Second variation of curvature,

$$\frac{\partial^4 y}{\partial z^4} = \frac{f(z)}{E\,I_x} - \frac{Y_s}{G\,A}\frac{\partial^2 f(z)}{\partial z^2} \qquad (29f)$$

The investigation will be limited to a consideration of the case in which the shearing rigidity, $\frac{G\,A}{Y_s}$, the flexural rigidity, $E\,I_x$, and the weight per unit length, w, are constants for the beam. If the beam is set into vibration, the loading intensity is dynamic, and may be expressed as follows:

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[13] "Strength of Materials," by S. Timoshenko, Pt. I, p. 187.

$$f(z) = -\frac{w}{g}\frac{\partial^2 y}{\partial t^2} \qquad (30)$$

Substituting Equation (30) into Equation (29*f*), the general differential equation for the motion of the beam is obtained, as follows:

$$\frac{\partial^4 y}{\partial z^4} = -\frac{w}{g\,E\,I_x}\frac{\partial^2 y}{\partial t^2} + \frac{Y_s\,w}{g\,G\,A}\frac{\partial^4 y}{\partial z^2\,\partial t^2} = -\beta^2\frac{\partial^2 y}{\partial t^2} + \alpha^2\frac{\partial^4 y}{\partial z^2\,\partial t^2} \qquad (31)$$

Equation (31) takes into account the effect of shear as well as of bending moment on the elastic distortions, but it neglects the effect of rotatory inertia on the dynamic loading. The dimensions of β are seconds per unit length², whereas those of α are seconds per unit length.

If one assumes that the solution of Equation (31) is given by a normal function, Z, so that:

$$y = Z\,f(t) = Z\,(E\cos p\,t + F\sin p\,t) \qquad (32)$$

then Equation (32) may be substituted in Equation (31) with the result that the differential equation for Z becomes:

$$\frac{\partial^4 Z}{\partial z^4} + \alpha^2 p^2\frac{\partial^2 Z}{\partial z^2} - \beta^2 p^2 Z = 0 \qquad (33)$$

Equation (33) is a linear differential equation, and yields the following solution:

$$y = (A\cos q\,z + B\sin q\,z + C\cosh r\,z + D\sinh r\,z)\,f(t) \qquad (34)$$

subjected to the restriction that q^2 and r^2 must satisfy the following quadratic equations,

$$q^4 - \alpha^2 p^2 q^2 - \beta^2 p^2 = 0 \qquad (35a)$$

and,

$$r^4 + \alpha^2 p^2 r^2 - \beta^2 p^2 = 0 \qquad (35b)$$

This means that:

$$q = \frac{\alpha\,p}{\sqrt{2}}\sqrt{\sqrt{1 + \frac{4\,\rho}{p^2}} + 1} \qquad (36a)$$

and,

$$r = \frac{\alpha\,p}{\sqrt{2}}\sqrt{\sqrt{1 + \frac{4\,\rho}{p^2}} - 1} \qquad (36b)$$

in which the units are radians per unit length. The parameter, ρ (in radians per second)² is defined as follows:

$$\rho = \frac{\beta^2}{\alpha^4} = \frac{g\,G^2 A^2}{Y_s^2\,w\,E\,I_x} \qquad (37)$$

Thus, for the case in which the beam distortions are due to shear only, $\rho = 0$; $r = 0$; and $q = \alpha\,p$, in radians per unit length. If the distortions of the beam were due to flexure only, the value of ρ would approach infinity; and, $r = q = \sqrt[4]{\rho\,\alpha^4 p^2} = \sqrt{\beta\,p} = u$.

If the origin of the co-ordinate axes is placed at the free end of the beam, (see Fig. 1) the following end conditions prevail: At the free end where $z = 0$, the moment must be zero, hence from Equations (29*d*) and (30):

$$\left(\frac{\partial^2 y}{\partial z^2}\right)_{z=0} = \alpha^2 \left(\frac{\partial^2 y}{\partial t^2}\right)_{z=0} \qquad (38a)$$

Moreover, at the free end the shear must be zero, hence from Equations (29*e*) and (30):

$$\left(\frac{\partial^3 y}{\partial z^3}\right)_{z=0} = \alpha^2 \left(\frac{\partial^3 y}{\partial t^2\, \partial z}\right)_{z=0} \qquad (38b)$$

At the built-in end where $z = L$ the shear, $V_{z=L}$, acting upon the ground of translatory rigidity, R_y, will produce a displacement,

$$(y)_{z=L} = \frac{V_{z=L}}{R_y} = -\frac{1}{R_y}\int_0^L f(z)\, dz = \frac{w}{g\, R_y}\int_0^L \frac{\partial^2 y}{\partial t^2}\, dz$$
$$= \tau \int_0^L \frac{\partial^2 y}{\partial t^2}\, dz \qquad (39)$$

Moreover, the slope of the beam at the built-in end is due to the shear in the beam itself as well as to the rotation of the ground. This rotation depends on the moment exerted by the beam upon the ground and on the rotatory rigidity, $R_{y\theta}$, of the ground itself. The shear contribution to the slope at $z = 0$ is:

$$\left(\frac{\partial y_s}{\partial z}\right)_{z=L} = \frac{Y_s\, V_{z=L}}{G\, A} = -\frac{Y_s}{G\, A}\int_0^L f(z)\, dz$$
$$= \frac{Y_s\, w}{g\, G\, A}\int_0^L \frac{\partial^2 y}{\partial t^2}\, dz = \alpha^2 \int_0^L \frac{\partial^2 y}{\partial t^2}\, dz \qquad (40a)$$

Whereas the contribution due to the rotation of the ground is:

$$\left(\frac{\partial y_r}{\partial z}\right)_{z=L} = \frac{M_{z=L}}{R_{y\theta}} = -\frac{1}{R_{y\theta}}\int_0^L \int f(z)\, dz\, dz$$
$$= +\frac{w}{g\, R_{y\theta}}\int_0^L \int \frac{\partial^2 y}{\partial t^2}\, dz\, dz = +\zeta \int_0^L \int \frac{\partial^2 y}{\partial t^2}\, dz\, dz \qquad (40b)$$

Consequently:

$$\left(\frac{\partial y}{\partial z}\right)_{z=L} = \left(\frac{\partial y_s}{\partial z}\right)_{z=L} + \left(\frac{\partial y_r}{\partial z}\right)_{z=L} = \alpha^2 \int_0^L \frac{\partial^2 y}{\partial t^2}\, dz$$
$$+ \zeta \int_0^L \int \frac{\partial^2 y}{\partial t^2}\, dz\, dz \qquad (41)$$

From the first two end conditions given by Equations (38),

$$C = \frac{r^2}{q^2} A \qquad (42a)$$

and,

$$D = \frac{r}{q} B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42b)$$

If Equations (42) are introduced into Equation (34), the following expression for y will be obtained:

$$y = \left[A \left(\cos q\, z + \frac{r^2}{q^2} \cosh r\, z \right) + B \left(\sin q\, z + \frac{r}{q} \sinh r\, z \right) \right] f\,(t) \,.\,.\, (43)$$

The constants, A and B, depending on end conditions, are complicated and will not be evaluated here, but the frequency equation, resulting from substituting Equation (43) into the expressions for the end conditions at the built-in end (that is, into Equations (39) and (41)), assumes the form:

$$\begin{aligned} & 2\, \alpha^4 \rho + 2\, \tau\, \zeta\, p^2 \\ & + (2\, \alpha^4 \rho - 2\, \tau\, \zeta\, p^2 + \alpha^4\, p^2) \cos q\, L \cosh r\, L \\ & + \left(+ \frac{\zeta}{r} - \tau\, r \right) \alpha^2\, p^2 \sqrt{1 + \frac{4\, \rho}{p^2}} \cos q\, L \sinh r\, L \\ & + \left(- \frac{\zeta}{q} - \tau\, q \right) \alpha^2\, p^2 \sqrt{1 + \frac{4\, \rho}{p^2}} \sin q\, L \cosh r\, L \\ & + \left(- \frac{\tau\, \zeta}{\sqrt{\rho}} p^2 - \alpha^4 \sqrt{\rho} \right) p \sin q\, L \sinh r\, L = 0 \ldots\ldots\ldots (44) \end{aligned}$$

The frequency equation, of course, is rather involved, containing either explicitly or implicitly the four constants: α and β for the beam itself, and τ and ζ for the ground.

Discussion of Frequency Equation

It will now be shown that a number of particular cases for some of which the frequency equations are well known, can be derived from the general frequency formula, Equation (44).

Case a.—If the ground is assumed to be rigid in translation as well as in rotation, τ and ζ are zero and the general equation reduces to:

$$\cos q\, L \cosh r\, L - \frac{p \sqrt{\rho}}{p^2 + 2\, \rho} \sin q\, L \sinh r\, L = - \frac{2\, \rho}{p^2 + 2\, \rho} \ldots\ldots (45)$$

Therefore, Equation (45) will give the frequencies for the transverse modes of vibration of a cantilever beam rigidly built-in when shear distortions as well as flexural distortions are present (see Table 3(a)).

Case a_1.—If the problem is simplified further and the shear deformations of the beam are assumed to be negligible in comparison with the flexural deformations, Equations (37) and (36) show that $\rho = \infty$, $q = r$, and $r = \sqrt{\beta\, p} = u$. Equation (45) then reduces to the well known frequency equation for a slender cantilever beam, namely:

$$\cos u\, L \cosh u\, L = - 1 \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

TABLE 3.—RATIOS OF TRUE PERIODS, T, TO PERIODS COMPUTED ON ASSUMPTIONS OF SHEAR ONLY (T_s) AND ON ASSUMPTIONS OF FLEXURE ONLY (T_f); CANTILEVER BEAM RIGIDLY BUILT-IN

EQUATIONS TO EXPRESS MODES OF VIBRATION:

(*a*) Fundamental Mode of Vibration,

$$T_1 = \left(\frac{T}{T_s}\right)_1 0.288\sqrt{y_s};\quad \text{or,}\quad T_1 = \left(\frac{T}{T_f}\right)_1 0.258\sqrt{y_f}$$

(*b*) Second Mode of Vibration,

$$T_2 = \left(\frac{T}{T_s}\right)_2 0.0960\sqrt{y_s};\quad \text{or,}\quad T_2 = \left(\frac{T}{T_f}\right)_2 0.0412\sqrt{y_f}$$

(*c*) Third Mode of Vibration,

$$T_3 = \left(\frac{T}{T_s}\right)_3 0.0576\sqrt{y_s};\quad \text{or,}\quad T_3 = \left(\frac{T}{T_f}\right)_3 0.0147\sqrt{y_f}$$

(*d*) Fourth Mode of Vibration,

$$T_4 = \left(\frac{T}{T_s}\right)_4 0.0411\sqrt{y_s};\quad \text{or,}\quad T_4 = \left(\frac{T}{T_f}\right)_4 0.0075\sqrt{y_f}$$

Variable (see sub-caption)	VALUES OF PERIOD RATIOS FOR THE FOLLOWING MODES OF VIBRATION										
	Fundamental (Equations (*a*))		Second Mode (Equations (*b*))			Third Mode (Equations (*c*))			Fourth Mode (Equations (*d*))		
	$\left(\frac{T}{T_s}\right)_1$	$\left(\frac{T}{T_f}\right)_1$	$\left(\frac{T}{T_s}\right)_2$	$\left(\frac{T}{T_f}\right)_2$	$\frac{T_1}{T_2}$	$\left(\frac{T}{T_s}\right)_3$	$\left(\frac{T}{T_f}\right)_3$	$\frac{T_1}{T_3}$	$\left(\frac{T}{T_s}\right)_4$	$\left(\frac{T}{T_f}\right)_4$	$\frac{T_1}{T_4}$
(*a*) PERIOD RATIOS FOR DIFFERENT VALUES OF $\frac{y_f}{y_s}$, OR μ (IN FIRST COLUMN)											
0	1.00	....	1.00	....	....	1.00	....	....	1.00	....	....
0.25	1.09	2.43	1.06	4.94	....	1.02	8.08	....	1.01	12.00	....
0.5	1.17	1.72	1.11	3.62	....	1.04	5.78	....	1.02	7.84	....
1	1.32	1.47	1.20	2.80	....	1.07	4.23	....	1.03	5.64	....
2	1.60	1.26	1.32	2.18	....	1.11	3.09	....	1.06	4.11	....
3	1.84	1.19	1.41	1.90	....	1.15	2.61	....	1.08	3.44	....
4	2.04	1.14	1.48	1.73	....	1.19	2.32	....	1.11	3.03	....
5	2.23	1.11	1.54	1.60	....	1.22	2.16	....	1.13	2.76	....
6	2.39	1.09	1.59	1.51	....	1.25	2.00	....	1.15	2.56	....
7	2.54	1.07	1.64	1.44	....	1.28	1.90	....	1.17	2.41	....
8	2.69	1.06	1.68	1.39	....	1.31	1.82	....	1.18	2.29	....
9	2.83	1.05	1.71	1.34	....	1.34	1.75	....	1.20	2.23	....
10	2.97	1.05	1.74	1.28	....	1.36	1.71	....	1.22	2.11	....
∞	....	1.00	....	1.00	....	....	1.00	....	....	1.00	....
(*b*) PERIOD RATIOS FOR DIFFERENT VALUES OF THE HEIGHT-TO-DEPTH RATIO, $\frac{L}{a}$ (IN FIRST COLUMN); SOLID SQUARE CROSS-SECTION $\left(\mu = \frac{L^2}{a^2}\right)$											
1	1.32	1.47	1.20	2.80	3.30	1.07	4.23	6.17	1.03	5.64	8.97
2	2.04	1.14	1.48	1.73	4.13	1.19	2.32	8.41	1.10	3.03	13.0
3	2.83	1.05	1.71	1.34	4.96	1.34	1.75	10.6	1.20	2.23	16.5
4	3.70	1.04	1.94	1.11	5.77	1.50	1.47	12.4	1.33	1.83	19.4
(*c*) PERIOD RATIOS FOR DIFFERENT VALUES OF HEIGHT-TO-DEPTH RATIO, $\frac{L}{a}$ (IN FIRST COLUMN); SQUARE BOX CROSS-SECTION $\left(\mu = 0.267\frac{L^2}{a^2}\right)$											
1	1.10	2.38	1.07	4.81	3.08	1.03	7.83	5.34	1.01	10.7	7.6
2	1.34	1.43	1.21	2.72	3.33	1.08	4.09	6.20	1.04	5.49	9.02
3	1.70	1.22	1.36	2.03	3.75	1.13	2.86	7.52	1.07	3.73	11.1
4	2.09	1.13	1.50	1.68	4.19	1.20	2.28	8.72	1.10	2.91	13.3
5	2.49	1.07	1.62	1.46	4.61	1.27	1.93	9.80	1.15	2.45	15.1
6	2.92	1.05	1.73	1.30	5.06	1.35	1.71	10.8	1.21	2.14	16.9
7	3.41	1.04	1.82	1.20	5.62	1.44	1.57	11.8	1.26	1.91	18.9
∞	....	1.00	....	1.00	6.27	....	1.00	17.6	....	1.00	34.4
(*d*) PERIOD RATIOS FOR DIFFERENT VALUES OF THE HEIGHT-TO-DEPTH RATIO, $\frac{L}{a}$ (IN FIRST COLUMN); RECTANGULAR BOX CROSS-SECTION WITH $\frac{b}{a} = 3.1$ AND $\mu = 0.107\frac{L^2}{a^2}$											
1	1.04	4.56	1.03	7.41	3.03	1.01	12.1	5.15	1.01	16.9	7.22
2	1.14	1.95	1.10	3.96	3.11	1.04	6.20	5.48	1.02	8.52	7.83
3	1.31	1.48	1.20	2.88	3.28	1.07	4.27	6.12	1.04	5.78	8.82
4	1.53	1.31	1.29	2.31	3.56	1.10	3.29	6.96	1.06	4.44	10.1
5	1.77	1.31	1.38	1.98	3.85	1.14	2.72	7.76	1.08	3.63	11.5
6	2.02	1.16	1.47	1.76	4.12	1.18	2.35	8.56	1.09	3.07	13.0
7	2.27	1.11	1.55	1.59	4.39	1.22	2.10	9.31	1.12	2.69	14.2
8	2.52	1.08	1.63	1.46	4.64	1.27	1.90	9.92	1.16	2.44	15.2
9	2.79	1.06	1.70	1.35	4.92	1.33	1.77	10.5	1.19	2.24	16.4
10	3.08	1.05	1.75	1.25	5.28	1.39	1.67	11.1	1.23	2.07	17.5

Case a_2.—If the problem had been simplified by assuming that flexural deformations of the beam were negligible in comparison with shear deformations, Equations (37) and (36) would have given $\rho = 0$, $r = 0$, and $q = \alpha\, p$, and the frequency Equation (45) would have reduced to:

$$\cos \alpha\, p\, L = 0 \qquad (46b)$$

This is the frequency equation for a stubby beam when shear alone is of importance. If the constant, α, had been made to refer to torsional vibrations instead of to translational shear vibrations, the same frequency equation would hold.

Case b.—Going to the other extreme in regard to the character of the end fastening and assuming that the cantilever beam becomes a free beam, the ground constants, τ and ζ, must approach infinity, and the general frequency equation reduces to:

$$\cos q\, L \cosh r\, L + \frac{p}{2\sqrt{\rho}} \sin q\, L \sinh r\, L = 1 \qquad (46c)$$

This equation applies to the frequencies of free beams as, for instance, the bars used in xylophones. Flexure as well as shear is considered.

Case b_1.—Simplifying Equation (46*c*) further by assuming that flexure alone is of consequence:

$$\cos u\, L \cosh u\, L = 1 \qquad (46d)$$

another well-known result.

Case b_2.—If the free beam is very stubby, shear alone governs the frequencies and Equation (46*c*) reduces to:

$$\cos \alpha\, p\, L = 1 \qquad (46e)$$

This type of frequency equation also applies to the torsional or to the longitudinal vibrations of a free bar.

Case c_1.—If the hypothetical situation of a ground, yielding elastically in translation but infinitely rigid in rotation is to be considered, $\zeta = 0$, and $\tau \neq 0$. Then for the case in which shear alone is of importance the general frequency equation (44) would reduce to:

$$\tan \alpha\, p\, L = \frac{\alpha}{p\, \tau} \qquad (47a)$$

This type of equation also applies to the torsional vibrations of cantilever beams with elastic torsional end constraint (see Fig. 5).

Case c_2.—If the translatory rigidity vanishes, τ approaches infinity while the rotatory ground rigidity remains infinite (that is, $\zeta = 0$); the result for flexure alone then becomes:

$$\tan u\, L = -\tanh u\, L \qquad (47b)$$

Case c_3.—The result for shear alone assumes the form:

$$\tan \alpha\, p\, L = 0 = \sin \alpha\, p\, L \qquad (47c)$$

Case d_1.—An equally hypothetical case of a ground yielding elastically in rotation, but being infinitely rigid in translation would give for shear alone,

$$\tan \alpha\, p\, L = \frac{\alpha^3\, p}{\zeta} \qquad (47d)$$

Case d_2.—Again, if the rotatory rigidity were to vanish completely, ζ must approach infinity and the result is a beam with a hinge at one end. Under these circumstances flexural considerations alone would give:

$$\tan u\, L = \tanh u\, L \qquad (47e)$$

Case d_3.—Shear alone gives:

$$\tan \alpha\, p\, L = 0 \qquad (47f)$$

APPENDIX III

A CRUDE, RATIONAL ANALYSIS OF THE ELASTIC CONSTRAINING ACTION OF THE GROUND OF A CANTILEVER BEAM OR IDEALIZED BUILDING

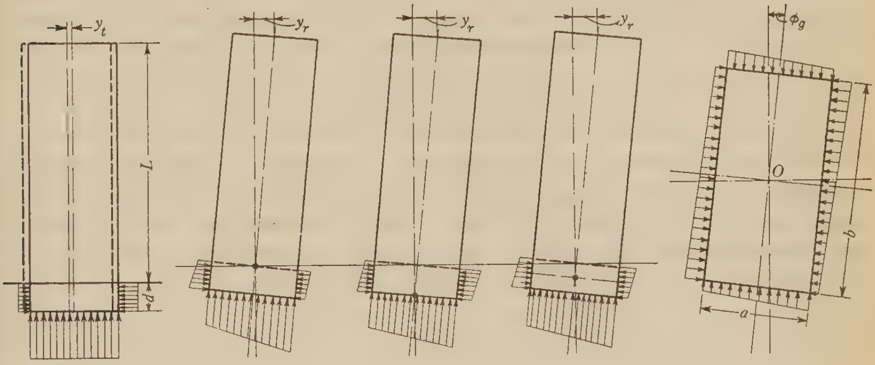
In regard to the "fixed"-end conditions of the cantilever beam, it was assumed that the medium in which it is "planted" is a semi-infinite elastic continuum; further assumptions can now be made, namely, that the inertia properties of the continuum may be neglected, and that the interaction pressures between the medium and the cantilever beam are linearly distributed over the contact surfaces involving ignorable discontinuities at edges and at corners. It is true that these assumptions are very crude, and that some doubt exists in regard to their validity even as first approximations to reality.

For purposes of analysis one may recognize three ground deformation moduli, a vertical ϵ_v, a horizontal ϵ_h, and over the base of the cantilever a shear modulus ϵ_s. These moduli differ from moduli of elasticity in being expressed in stress per total deformation; that is, in pounds per cubic inch instead of in pounds per square inch. The translatory rigidity of the ground in the Y-direction (in pounds per inch) is then given by:

$$R_y = 2\, \epsilon_h\, b\, d + \epsilon_s\, a\, b \qquad (48)$$

The corresponding horizontal translation, y_t, due to a hypothetical horizontal loading of the cantilever will be constant for the entire height of the beam (see Fig. 9(a)) and will be as expressed by Equation (18).

Directing attention to the rotation of the center line of the cantilever resulting from the same hypothetical loading as before, the investigator is confronted with some uncertainty in regard to the location of the center of rotation. Figs. 9(b), 9(c), and 9(d) indicate three possible locations of this center with the resulting pressure distributions over the contact surfaces. If the elastic restoring moment of the ground per unit angular rotation is evaluated for the three positions of the center of rotation, they will be found to increase as the center moves downward, but the resulting linear translations of the top of the cantilever, corresponding to the ground influence on a fundamental mode, will not differ as much as the angles. For this reason the question of the exact location of the rotation center may be dismissed, assuming that it lies in the plane of the ground surface as shown in Fig. 9(*b*). The rotatory rigidity of the ground for this location is given, in pound-inches per radian, by:



(*a*) ELASTIC TRANSLATION, y_t

(*b*) ELASTIC TILTING, y_r Rotation Center at Ground Surface

(*c*) ELASTIC TILTING, y_r Rotation Center at at End of Beam

(*d*) ELASTIC TILTING, y_r Rotation Center Between (*b*) and (*c*)

(*e*) ELASTIC TORSION, ϕ_g

FIG. 9.—SIMPLIFIED DIAGRAMS OF GROUND PRESSURE DISTRIBUTIONS AT END OF CANTILEVER BEAM SUBJECTED TO ELASTIC DEFORMATION OF GROUND

$$R_{y\theta} = \epsilon_v \frac{a^3\, b}{12} + \epsilon_h \frac{2\, b\, d^3}{3} \quad \text{(49}a\text{)}$$

and,

$$R_{x\theta} = \epsilon_v \frac{a\, b^3}{12} + \epsilon_h \frac{2\, a\, d^3}{3} \quad \text{(49}b\text{)}$$

to which the corresponding horizontal translations of the top of the beam (y_r and x_r, in inches), are expressed by Equation (19).

Finally, the torsional rigidity of the ground is considered for a hypothetical twisting moment of $w\,k$ lb-in. per unit height of the beam. Fig. 9(*e*) shows the resulting pressure distributions on the vertical surfaces or basement walls of the beam, and the torsional rigidity coefficient of the ground (in pound-inches per radian) is easily shown to be:

$$R_\phi = \epsilon_h \frac{1}{6} (a^3 + b^3)\, d + \epsilon_s \frac{1}{12} (a\, b^3 + a^3\, b) \quad \text{(50)}$$

which corresponds to an angle of twist of the beam (ϕ_g), in the ground itself which is expressed by Equation (20).

A few words in regard to the relative values of the deformation moduli are appropriate if the theory must be simplified.

If the ground medium is homogeneous and isotropic, the values of ϵ_h, ϵ_v, and ϵ_s are related to: (1) The two elastic constants, E and $\frac{1}{m}$; (2) the dimensions of the cross-section; and (3) the depth of "planting." On the other hand, if the ground medium is ordinary soil it becomes very difficult to make any theoretical predictions of the values of the ground deformation moduli.

Since the medium, or soil under the beam or building, is usually highly consolidated compared to the soil adjacent to the basement walls, it is reasonable to expect that ϵ_v is much greater than ϵ_h; the order of magnitude of the ratio, $\gamma = \frac{\epsilon_v}{\epsilon_h}$, may well lie between the values 10 and 100. The relative value of ϵ_s is perhaps equally problematic since it depends to a certain extent on the frictional adhesion of the soil to the base. However, it would be reasonable to expect that ϵ_s is of the same order of magnitude as ϵ_h. With the object of simplification in mind, and for want of better information, the investigator may choose to make ϵ_s equal to ϵ_h. The simplified expressions for the rigidities (compared with Equations (48), (49), and (50)) are then:

$$R_y = \epsilon_h\,(2\,b\,d + a\,b) \quad \text{(51a)}$$

$$R_x = \epsilon_h\,(2\,a\,d + a\,b) \quad \text{(51b)}$$

$$R_{y\theta} = \epsilon_h\,\frac{1}{12}\,b\,(\gamma\,a^3 + 8\,d^3) \quad \text{(51c)}$$

$$R_{x\theta} = \epsilon_h\,\frac{1}{12}\,a\,(\gamma\,b^3 + 8\,d^3) \quad \text{(51d)}$$

$$R_\phi = \epsilon_h\,\frac{1}{6}\left[(a^3 + b^3)\,d + \frac{1}{2}\,(a\,b^3 + a^3\,b)\right] \quad \text{(51e)}$$

The chief value of this analysis is to recognize that since there are five rigidities R_x, R_y, $R_{x\theta}$, $R_{y\theta}$, and R_ϕ expressed by the ground deformation constant, ϵ_h, and the ratio, γ, plus the dimensions of the basement, a, b, and d, the following relations are determined for the rigidities:

$$R_y = \frac{b\,(24\,d + 12\,a)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,R_\phi \quad \text{(52a)}$$

$$R_x = \frac{a\,(24\,d + 12\,b)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,R_\phi \quad \text{(52b)}$$

$$R_y = \frac{b\,(2\,d + a)}{a\,(2\,d + b)}\,R_x \quad \text{(52c)}$$

$$R_{y\theta} = \frac{\gamma\,a^3 + 8\,d^3}{24\,d + 12\,a}\,R_y \quad \text{(52d)}$$

$$R_{x\theta} = \frac{\gamma\,b^3 + 8\,d^3}{24\,d + 12\,b}\,R_x \quad \text{(52e)}$$

$$R_{y\phi} = \frac{b\,(\gamma\,a^3 + 8\,d^3)}{a\,(\gamma\,b^3 + 8\,d^3)}\,R_{x\phi} \quad \text{(52f)}$$

The hypothetical static displacements are then related as follows:

$$y_r = \frac{(12\,d + 6\,a)\,L^2}{\gamma\,a^3 + 8\,d^3}\,y_t \qquad (53a)$$

$$x_r = \frac{(12\,d + 6\,b)\,L^2}{\gamma\,b^3 + 8\,d^3}\,x_t \qquad (53b)$$

$$\phi_g = \frac{12\,R\,(2\,b\,d + a\,b)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,y_t \qquad (53c)$$

and,

$$\phi_a = \frac{12\,R\,(2\,a\,d + a\,b)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,x_t \qquad (53d)$$

If two independent rigidities can be found from a study of observed periods, the remaining three follow. It must be remembered, however, that only in rare instances are the two ground deformation constants, ϵ_h and γ, sufficient for expressing the conditions of the soil around a building, because not only geological features, but also man-made constructions, as exemplified by the proximity of other buildings, will generally call for different values of ϵ_h on the different basement walls; in addition to this complication, ϵ_v may vary over the foundation, especially if a spread footing foundation is adopted.

INFLUENCE OF THE GROUND ON THE TORSIONAL PERIODS

It was mentioned previously that flexure does not complicate the problem of determining the periods of torsion. In regard to the effect of the ground on these periods a similar simplification exists since the vertical deformation constant, ϵ_v, and, consequently, the ground rotation, do not enter into the problem. Equations (1*b*), (4*b*), and (21) enable one to define the ratio of the torsional yielding of the ground to that of the beam as follows:

$$\psi = \frac{\phi_g}{\phi_R} = \frac{12\,b\,(2\,d + a)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,\frac{Y_s\,R_t}{A}\,\frac{y_t}{y_s} \qquad (54)$$

DISCUSSION

K. Bert Hirashima,[14] Esq. (by letter).—This interesting paper by Professor Jacobsen should prove an important addition to the literature of the subject treated. Especially valuable are the tables and graphs illustrating the theory.

The normal modes of transverse vibrations of a rigidly built-in cantilever, resulting from a consideration of shear only and flexure only, are well known. Taking the frequency equation for each of these two elementary cases, the vibrations that occur in the Y-direction are given by Equations (7) and (10). It will seem that Equation (7) (shear only considered) is comparatively simple and easy to solve. The solution of Equation (10) (flexure only) is more complicated; but it can be solved by a method of successive approximations.[15]

However, as shown in the paper, neither of these comparatively simple equations gives the correct result for the general case in which neither shear nor flexure is negligible. Thanks to Table 3, however, the periods of the various modes for this general case can readily be obtained by simple multiplication from the solution of either equation. Equation (7) will probably be preferable for reasons of simplicity.

The author's analysis of the effect on the vibration periods of the elastic yielding of the ground is highly interesting; but the solution of the frequency equation for this case is very involved. However, Table 2 and the graphs in Figs. 6, 7, and 8, give a clear indication of what can be expected when the ground yields elastically. The ground deformation moduli, ϵ_v, ϵ_h, and ϵ_s, of course, can be determined by experimental methods. The ground rigidities, R_x, R_y, $R_{x\theta}$, $R_{y\theta}$, and R_ϕ, can then be determined in the manner described in the paper. Finally, with numerical values of the ground constants thus determined the frequency equation can be solved for the values of the various periods. The disadvantage of the foregoing procedure is that it is long and tedious with the likelihood that errors will be introduced.

In practical engineering work, as differentiated from theoretical investigations, what is most desired is not 100% theoretical accuracy, but an accuracy sufficient for the nature and needs of the problem. With this standard limitation in mind it seems that a table similar to Table 3 could be constructed giving conversion factors for various types of soils and foundations. The period of the various modes for a cantilever "planted" in sandy soil, for example, can then be determined by simple multiplication from the solution of either Equation (7) or Equation (10); or, by reference to Tables 2 and 3 and Figs. 6, 7, and 8, the engineer could derive his own factors for converting the periods as obtained from a solution of Equation (7) or Equation (10) into the estimated true periods for the case of an elastically yielding ground. Such procedure requires sound judgment on the part of the engineer. The errors incident thereto are probably not more serious than the uncertainties inherent in the nature of the problem. On important work, investigation along the lines indicated in the paper is justified.

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[14] Asst. Highway Planning Engr., Territorial Highway Dept., Honolulu, Hawaii.

[15] "Applied Elasticity," by J. Prescott, p. 206.

It may be remarked that the various frequency equations, involving as they do either trigonometric or hyperbolic functions, have an infinite number of roots. When a rod (or, in the present case, a cantilever) is set in motion by being bent to one side and then released, it is not likely to begin vibrating in any one of the normal modes, unless the curve into which the cantilever is first bent was the same curve as the cantilever assumes in one of the extreme positions of that particular normal mode. However, it is known that the subsequent motion is composed of a number of superposed normal modes of which the lower modes are the more important.

The further discussion of the superposition of normal modes and the development in terms of Fourier's series is beyond the scope of the paper. The interested reader is referred to the various special treatises such as Byerly's "Fourier's Series and Spherical Harmonies"; Lord Rayleigh's "Theory of Sound"; Prescott's "Applied Elasticity"; and Love's "Mathematical Theory of Elasticity."

Merit P. White,[16] Jun. Am. Soc. C. E. (by letter).—Since this paper is intended primarily for the designer of earthquake-resistant buildings, a question arises as to the errors involved in treating an ordinary building, even one of constant cross-section, as a uniform cantilever beam with definite shear and bending flexibility. On the other hand, even a very simple building is a tremendously complicated elastic-dynamic system. The interaction of frame, walls, and partitions, the variations in stiffness of these elements due to doors and windows, the concentrations of mass at the floor levels—these and similar factors make impossible any mathematically exact analysis. Obviously, in order to secure any information at all on the dynamical characteristics of a building many simplifying assumptions are necessary.

In his "Synopsis" the author states that close agreement with actuality is not to be expected if the theory is applied to buildings. The writer agrees with this statement to the following extent: Certainly, any one possessing only a complete set of plans for a given building, including data on the deformation characteristics of the site, and with no empirical information obtained by observations on similar buildings on similar sites, would be fortunate indeed to be able to predict within 20% the fundamental period of the building in question. On the other hand, possession of the empirical information just mentioned would permit a much closer prediction of the building period. Furthermore, there are certain other, very important quantities, such as the shapes of the several modes of vibration, the relation between the fundamental period and the periods of higher modes, and the change in building period due to foundation yielding, all of which, the writer believes, can be determined with considerable accuracy from the theory presented by Professor Jacobsen.

Although it is quite true that an ordinary building is so complicated that any attempt to make a mathematically exact analysis in an endeavor to test the validity of the author's simplifying assumptions would be an endless task, nevertheless it may be possible to check these assumptions by making others, quite different, and performing the calculation which they make possible.

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The author simplified his problem by replacing discontinuous distributions of weight and stiffness by continuous functions. Actually, these characteristics were considered constant. Another possible simplification would be the following: Consider all weights to be concentrated at the floors, and replace the stiffness of frame, walls, and partitions by fixed-ended columns. It must not be thought that the fixed-ended columns in the proposed model are identical with the columns of the real building. Actually, they are much stiffer because the greatest part of the lateral stiffness of an ordinary building is due to its walls and partitions. This can be shown by comparing measured building periods with those computed, neglecting the stiffening of the walls. These hypothetical columns are purely schematic and are intended to duplicate the stiffness, and not the structure, of a building.

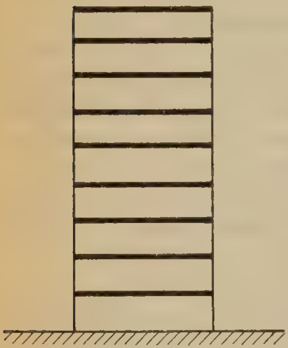

FIG. 10

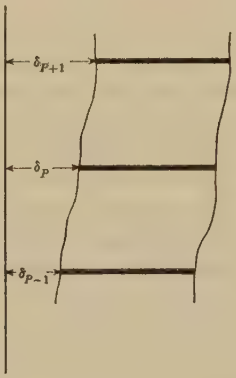

FIG. 11

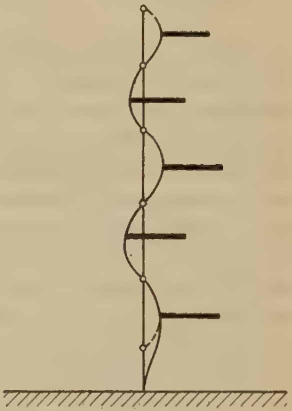

FIG. 12

Although this idealized building is no more real than that of the author, nevertheless, a comparison of the results of both assumptions will give an idea of their reliability, particularly since an actual building is somewhat intermediate in nature.

To simplify matters as much as possible, it will be assumed that: (1) Bending of the building as a whole (extension and compression of the hypothetical columns) is negligible; (2) foundation yielding is also negligible; (3) the weights of all floors are equal; and (4) all stories are equally stiff. Although these limitations detract much from the reality of the model, if the agreement in this very particular case is satisfactory, it may safely be assumed that the author's assumptions are generally permissible.

The following problem is to be solved: To find expressions for the frequencies of the several modes of vibration of the structure shown in Fig. 10, consisting of a given number (N) of floors, each of Mass m, supported by columns giving a stiffness, K, to each story (K is the force, in pounds, which will produce unit relative deflection between two adjacent floors). Since N quantities (the deflections of the several masses, for example) are necessary to describe the configuration of the system completely at any instant, the system is said to have N degrees of freedom. Thus, there will be N different modes and frequencies of vibration which can be calculated by solving a system of N simultaneous linear equations.

Except for the boundary condition at the roof, this problem is identical with that of finding the modes and frequencies of vibration of a series of identical, equally spaced beads on a stretched string, first solved by Lagrange.[17] In fact the solutions to the present problem are identical with half the solutions of Lagrange, those possessing an even number of nodes or an odd number of half waves. This follows from the fact that the boundary condition at the top of a vibrating building—namely, that at this point the shear is zero—is satisfied at the center of a vibrating string of beads provided this point is not a node. The computation is quite simple.

Assume that the system shown in Fig. 10 is vibrating in any one of its normal modes (fundamental, second harmonic, etc.). In such a vibration each point moves with simple harmonic motion, of frequency, f_n, this being the natural frequency of the mode in question (the nth mode). Furthermore, in any normal vibration all points move in phase; that is, they pass through their extreme positions simultaneously. If δ_p is the amplitude of motion of the pth mass, then its displacement at any instant is,

$$\Delta = \delta_p \sin (\lambda_n t - \xi_n) \ldots\ldots\ldots\ldots (55)$$

in which ξ_n is the phase angle for the nth mode of vibration and is the same for all points of the structure; and, λ_n is equal to $2\pi f_n$. By differentiating the expression for displacement, it can be shown that the acceleration of the pth mass is: $-\lambda_n^2 \delta_p \sin (\lambda_n t - \xi_n)$. This acceleration is always equal to λ_n^2 times the displacement and is in the opposite direction.

The present problem is to determine f_n—in other words, to calculate the frequency of the nth mode of vibration. Fig. 11 shows the masses at the points, $p - 1$, p, and $p + 1$, in their extreme positions. Consider the equilibrium of the pth mass on which the following forces act: (1) A D'Alembert force equal to mass times acceleration; (2) a force due to bending of the columns above; and (3) the same due to the columns below.

For the pth mass,

$$m \lambda_n^2 \delta_p + K (\delta_{p+1} - 2 \delta_p + \delta_{p-1}) = 0 \ldots\ldots\ldots (56a)$$

For the mass at the top (the Nth),

$$m \lambda_n^2 \delta_N + K (- \delta_N + \delta_{N-1}) = 0 \ldots\ldots\ldots (56b)$$

and, for the lowest mass (the first),

$$m \lambda_n^2 \delta_1 + K (\delta_2 - 2 \delta_1) = 0 \ldots\ldots\ldots (56c)$$

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[17] "Mechanique Analytique," by Joseph L. Lagrange, Tome I, p. 390.

Making $2 - \frac{m}{K}\lambda_n^2 = C$, the complete set of N-equations can be written:

$$\left.\begin{array}{rcl}
C\,\delta_1 - \delta_2 \quad \cdots & & = 0 \\
-\delta_1 + C\,\delta_2 - \delta_3 \quad \cdots & & = 0 \\
-\delta_2 + C\,\delta_3 - \delta_4 & & = 0 \\
\cdots\cdots\cdots\cdots\cdots\cdots & & \\
& -\delta_{N-2} + C\,\delta_{N-1} - \delta_N & = 0 \\
& -\delta_{N-1} + (C-1)\,\delta_N & = 0
\end{array}\right\}\ldots(57)$$

The homogeneity of Equations (57) arises from the fact that all terms contain δ. Thus, the shape, but not the absolute dimensions, of the mode of vibration can be found. One of the δ-ratios will remain arbitrary. The remaining δ-ratios and C are the N unknowns to be determined.

Several methods exist for finding the natural frequencies of the given system from Equations (57). The usual procedure in such a case is based on the fact that the determinant of the coefficients of the unknown amplitudes must be zero in order that the δ-amplitudes may be different from zero. Since the coefficients of the δ-ratios involve only C and unity, it is clear that a single algebraic equation (identical with the determinant of the coefficients) can be obtained. This will be an Nth degree equation in C, the N roots (or solutions) of which will correspond to the N frequencies of the system. By making certain trigonometric substitutions, expressions for the frequencies in the general case (that is, in terms of N) can be found.

A somewhat simpler method, described by Professor A. G. Webster,[18] is the following: The quantity, $C\left(= 2 - \frac{m}{K}\lambda_n^2\right)$, is always less than $+\,2$. This is obvious since m, K and λ_n^2 are all positive. The least value of C occurs with maximum λ_n $(= \lambda_N)$. This corresponds to a mode (deflection curve) in which alternate masses are deflected in opposite directions as shown in Fig. 12. The frequency in this case is slightly less than it would be if all columns were pinned at their centers (including a half-column added at the top) as indicated in Fig. 12. Due to the pins, which allow rotation but not displacement of the center point of each column, certain constraints would be imposed on the system, thus raising its frequency. In the case of the pinned columns it is seen that the system is equivalent to N one-degree-of-freedom systems, each of frequency, $\lambda' = 2\sqrt{\frac{K}{m}}$. Since $\lambda_N < \lambda'$, it follows that $-2 < C < +2$.

On examining Equations (57) it is seen that each equation except the last resembles the trigonometric equation relating the sines of three consecutive multiples of an angle, θ:

$$-\sin(r-1)\theta + 2\cos\theta\sin r\theta - \sin(r+1)\theta = 0 \ldots\ldots\ldots(58)$$

Resemblance becomes identity if: $\delta_r = A\sin r\theta$; and $C = 2\cos\theta$. This is permissible since $-2 < C < +2$. All equations except the Nth are satisfied by any value of θ. To satisfy the Nth equation, however, it is clear that only certain values of θ are possible. Rewriting the last of Equations (57):

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[18] "Dynamics," by A. G. Webster, Second Edition, p. 167.

$$A\,[-\sin (N-1)\,\theta + 2\cos\theta \sin N\,\theta - \sin N\,\theta] = 0 \qquad (59)$$

since $-\sin (N-1)\,\theta + 2\cos\theta\sin N\,\theta - \sin (N+1)\,\theta = 0$,

$$\sin N\,\theta - \sin (N+1)\,\theta = 0 \qquad (60)$$

is the equation that determines possible values of θ. Therefore, $(2N+1)\,\theta = \pi, 3\pi, 5\pi \cdots (2n-1)\,\pi \cdots (2N-1)\,\pi$; and,

$$\theta = \frac{2n-1}{2N+1}\times\pi; \qquad n = 1, 2, 3 \cdots N \qquad (61)$$

Since $C = 2 - \frac{m}{K}\lambda_n^2 = 2\cos\theta = 2\cos\frac{2n-1}{2N+1}\pi$,

$$\lambda_n^2\frac{m}{K} = 2\left(1 - \cos\left(\frac{2n-1}{2N+1}\right)\frac{\pi}{2}\right) = 4\sin^2\left(\frac{2n-1}{2N+1}\right)\frac{\pi}{2} \qquad (62a)$$

and,

$$\lambda_n = 2\sqrt{\frac{K}{m}}\sin\left(\frac{2n-1}{2N+1}\right)\frac{\pi}{2}\,(n = 1, 2, 3, \cdots N) \qquad (62b)$$

For a system of N-masses there will be N-frequencies and modes of vibration, the former obtained by making $n = 1, 2, 3, \cdots N$, in Equation (62*b*).

If $\frac{n}{N}$ is small, Equation (62*b*) becomes approximately:

$$\lambda_n = \sqrt{\frac{K}{m}}\frac{2n-1}{2N}\pi\,(n = 1, 2 \cdots \ll N) \qquad (63)$$

TABLE 4.—VALUES OF $\frac{T}{\sqrt{y_s}}$ FOR DIFFERENT VALUES OF N (NUMBER OF STORIES) AND n (ORDER OF HARMONIC)

Values of of n	VALUES OF N						
	1	2	3	5	10	20	∞
1	0.320	0.299	0.294	0.290	0.289	0.288	0.288
2	....	0.115	0.105	0.100	0.097	0.096	0.096
3	....	....	0.073	0.063	0.059	0.058	0.058
4	....	....	....	0.049	0.043	0.042	0.041
5	....	....	....	0.043	0.035	0.033	0.032

Equation (63) may be compared with the author's Equations (8). In the present case:

$$y_s = \frac{m\,g}{K}(1 + 2 + 3 + \cdots N) = \frac{m\,g}{2\,K}(N^2 + N) \qquad (64)$$

in which y_s, as defined by the author, is the static deflection, in inches, due to a horizontal acceleration equal to g. If $N \ll N^2$ this gives, when substituted in Equation (63):

$$T_n = \frac{0.288}{2\,n - 1}\,\sqrt{y_s} \quad \text{(65)}$$

which is equivalent to Equation (8). In terms of y_s, Equation (62b) becomes,

$$\lambda_n = \frac{2.772}{\sqrt{y_s}}\,\sqrt{N^2 + N}\,\sin\left(\frac{2\,n - 1}{2\,N + 1}\right)\frac{\pi}{2} \quad \text{(66)}$$

and,

$$\frac{T_n}{\sqrt{y_s}} = \frac{0.226}{\sqrt{N^2 + N}\,\sin\frac{2n - 1}{2\,N + 1}\,\frac{\pi}{2}} \quad \text{(67)}$$

Table 4 gives, for comparison, values of $\frac{T}{\sqrt{y_s}}$ for various values of n (order of harmonic) and N (number of floors in the structure). The last column ($N = \infty$) is from Equation (65) and corresponds to the values obtained by the author. It appears that even for structures of one or two stories the agreement is fairly close, while the lower frequencies in structures of several stories agree almost exactly with those given by the author's equations.

Lydik S. Jacobsen,[19] Esq. (by letter).—As Mr. Hirashima states, tables with corrections for periods of buildings due to various types of foundations might be computed so as to make the designer's job of predetermining natural periods easier. However, for most of the practical designs, engineering judgment, fortified by qualitative knowledge of the relative importance of the different factors entering into the problem, will lead to results that are sufficiently accurate for design purposes. It must not be forgotten that the practical question relates to stresses in the building, and that it is only indirectly that natural periods of a structure merit study. Since the ground periods likely to be present in an earthquake are not definitely known, it is not necessary to predetermine the natural periods of the structure very accurately. The study presented in this paper attempts only to give quantitative backing for a sound evaluation of general tendencies; extensions from the specific examples given in the paper should rely chiefly on engineering judgment.

Mr. White's discussion of the paper introduces the question of the magnitude of error arising from considering continuous, rather than discontinuous, mass and rigidity distributions. He shows that for a highly simplified case satisfactory agreement is obtained if the structure consists of two or more stories. It should be pointed out, however, that the method used by Mr. White is definitely limited to a consideration of shear distortions and ground translations.

In the writer's opinion the effect of flexure of the building as a whole, as well as the effect due to ground tilting, may be estimated for most practical cases if one makes use of existing experimental data and is guided by a rational theory. In this case shear alone, or shear with estimated ground translation, becomes the only calculation that can reasonably be demanded by practice.

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[19] Prof. of Mech. and Civ. Eng., Stanford Univ., Stanford University, Calif.

For this purpose a schedule form of calculation involving successive approximations, and first given by H. Holzer, enables the designer to calculate the natural periods of non-uniform buildings with considerable accuracy and ease. As an example of Holzer's method a torsional vibration problem has been solved by Professor Timoshenko.[20] The writer has used a slight modification of this method for dealing with non-uniform buildings, and it can be said that the amount of work is very small, even for a building of many stories, when it is compared to the customary static calculations for the idealized framework of a building.

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[20] "Vibration Problems in Engineering," by S. Timoshenko (Second Edition), p. 264.

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Paper No. 2026

INCREASING THE TRAFFIC CAPACITY AND SAFETY OF THOROUGHFARES

A SYMPOSIUM

Page

Causes of Failure in Handling Traffic.
By Lewis W. McIntyre, M. Am. Soc. C. E. 441

Remedies to Relieve Traffic Congestion.
By Donald M. McNeil, Assoc. M. Am. Soc. C. E. 461

Improvements to Reduce Traffic Accidents
By Arnold H. Vey, Esq. 474

Economics of Alignment, Grade, and Width.
By E. L. Schmidt, Esq. 485

With Discussion by Messrs. John A. Oakey, L. W. Mahone, Fred Lavis, Hugh A. Kelly, Sidney J. Williams, Harold Nevin Carey, W. W. Crosby, Roger L. Morrison, W. R. Flack, Hawley S. Simpson, Charles M. Noble, Lewis W. McIntyre, and Arnold H. Vey.

Note.—The Symposium on Increasing the Traffic Capacity and Safety of Thoroughfares was presented at the meeting of the City Planning Division, Pittsburgh, Pa., October 15, 1936, and published in November, 1937, *Proceedings*.

CAUSES OF FAILURE IN HANDLING TRAFFIC

By Lewis W. McIntyre,[1] M. Am. Soc. C. E.

Synopsis

The Bureau of Traffic Planning of the City of Pittsburgh, Pa., has conducted observations on down-town intersections to determine the space between moving automobiles, and also the time taken by automobiles to clear the intersection. Interference was at a minimum during these observations. The theoretical maximum traffic flow of a single lane of traffic was computed from these data. It is the purpose of this paper to show how various factors reduce the theoretical maximum traffic flow to the much lower values that occur in actual practice, in the usual congested district, and as far as possible to evaluate, quantitatively, the effect of each factor.

Definitions

The following definitions, essentially as proposed in 1934 by a Committee of the Institute of Traffic Engineers,[2] have been adopted for use in this paper:

(1) Traffic consists of pedestrians, mounted or herded animals, vehicles, street cars, and other conveyances together with their load, either singly or as a whole, while using any "way" for the purpose of transportation or travel.

(2) Traffic flow is the number of elemental components of traffic that pass a given point in a specified direction during a specified period of time (usually 1 hr). The traffic flow in an individual lane of traffic is determined by the effective headway and the speed.

(3) Traffic volume is the total number of elemental components of traffic that pass through a given intersection or way in any specified period of time (usually 8 hr, or more).

(4) Traffic capacity is the total number of elemental components of traffic that can pass through a given intersection or way in any specified period of time (usually 1 hr). The traffic capacity of any street depends upon the number of lanes of moving traffic on that street and the traffic flow in each individual lane.

(5) Traffic delay is the time spent while the elemental components of traffic that are being studied cannot move because of interference.

(6) Traffic speed is the rate of movement with which specified elemental components of traffic move over a specified course or section of way during any specified period.

(7) Over-all speed is the total distance traversed, divided by the total time required (usually expressed in miles per hour and includes all delays).

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[1] Traffic consultant, Pittsburgh, Pa.

[2] Rept. of Committee on Terminology and Nomenclature, Inst. of Traffic Engrs., T. M. Matson, *Chairman*. Not published.

(8) Weaving is the movement of vehicular traffic in such manner that the elemental components cross each other's paths at low angles of approach or in nearly the same direction of movement.

(9) A point of conflict is the point or limited area where elemental component parts of traffic intersect; the apex of the angle of approach.

(10) A lane of traffic movement is a series of elemental components of traffic moving, or standing ready to move, so as to use a lane of roadway effectively.

(11) A critical lane is a line of traffic which, for inherent reasons, requires more time to pass a given point than other parallel lanes.

(12) The lane width of a roadway is the measure of the number of possible free moving lanes of traffic which the roadway can pass in both directions: (*a*) A lane is a width of roadway sufficient to accommodate the free movement of an average vehicle, customarily considered to be 10 ft.

(13) Headway is the time space between the front ends of vehicles or street cars when moving past a specified point in a given direction and lane.

(14) Minimum headway is the time space (see Definition (13)) obtained when the elemental components of traffic are moving as closely together as practicable.

(15) Effective headway is the time space (see Definition (13)) obtained for elemental components of traffic moving under minimum headway in one or more lanes.

(16) A way, as applied to traffic, is that upon which traffic passes. The term, "way," shall be considered generic to all places along which traffic may move. It includes highway, parkway, limited way, freeway, road, path, route, street, avenue, boulevard, lane, alley, arcade, drive, etc.

(17) A major street is a way that carries traffic between sections of an urban district. The major streets of a city are the main traveled ways. They may form important connecting links in the traffic facilities of the State. (A major street is sometimes referred to as an arterial street.)

(18) A minor street is a way that connects individual property with a major street or other minor streets.

(19) An alley is a narrow inner street, either without sidewalks, or with very narrow ones, or a street on which the rears of buildings adjoin. It serves as a connection with other minor streets.

(20) A radial street is a major street that runs from the central business district to the outlying districts of a city. A radial street signifies a way that carries traffic from surrounding territory to a main focal point. The term is also applied to rural highways.

(21) A distribution street is a major street that collects and passes traffic from the other major streets and from which traffic distributes itself to advantageous points of entry into a district.

(22) A by-pass street is a major street that carries traffic around congested districts (sometimes referred to as a belt line or loop).

(23) A cross-town street is a major "way" that carries traffic so as to intersect radial streets at approximately right angles. (A cross-town street is also referred to as circumferential street.)

(24) A roadway is that part of a way particularly devoted to vehicular traffic; ordinarily, it lies between the regularly established curb lines.

(25) The pavement is that part of a roadway having a constructed surface for the facilitation of vehicular traffic.

(26) An intersection is the area of a junction included within lines joining the meeting points of property lines (together with any sections of property lines necessary to enclose the area) at the junction of two or more ways at the same elevation.

(27) A congested district is any area in which most of the streets or ways carry such a volume of traffic during a considerable part of the day as materially to reduce the headway and cause traffic delay (usually a business district).

TABLE 1.—Space Between Moving Automobiles at Intersections

No. of intersections	No. of observations	Average space, in feet	Remarks
3	574	17.82	Free movement
1	139	17.42	Following street car
1	71	12.8	Especially dense traffic

The spacing between vehicles (Table 1) was obtained by marking, on the street pavement, various distances from the property line. An observer recorded the distance of the rear end of each vehicle from the property line at the instant when the front end of a following vehicle passed this line. Under ordinary circumstances of fairly congested traffic, the average spacing between vehicles was 17.82 ft. When these vehicles followed street cars the space was 17.42 ft. In a number of cases where traffic was especially dense and was moving under obvious pressure, the spacing was as low as 12.8 ft. The data in Table 2 indicate that the first car of a group moved across the 36-ft roadway and one 12-ft sidewalk at an average speed of 10.8 or 10.9 miles per hr, depending on whether the vehicle was a passenger car or a truck. The data also indicate that following cars, because of their greater opportunity to get started, actually proceeded through the intersection at a speed of 15.3

TABLE 2.—Time Taken by Automobiles to Clear Intersection (12-Foot Sidewalk Plus 36-Foot Roadway)

Type of car	No. of observations	Time, in seconds	Speed, in miles per hour	Effective Headway		Lane Capacity (Theoretical)	
				Free movement, in seconds	Dense traffic, in seconds	Dense traffic	Free movement
Passenger:							
First car group.........	93	3.92	10.8	2.01	1.70	2 100	1 790
	161	2.77	15.3	1.43	1.20	3 000	2 500
Truck:							
First car group.........	21	4.61	10.9	2.78	2.44	1 475	1 300
	10	4.06	12.4	2.42	2.14	1 680	1 480
Mixed group...........	199	2.95	15.3	1.60	1.38	2 600	2 250

miles per hr in the case of passenger cars or a mixed group, and 12.4 miles per hr in the case of trucks. Using these speeds and the average spacing shown in Table 1, effective headways were computed under the conditions of reasonably free movement and of especially dense traffic. In these computations the length of a passenger automobile was taken as 14 ft and the length of a truck as 26 ft.

These data indicate that a single lane of a street in a congested district has a capacity of from 1 800 to 2 500 passenger cars per hr at an intersection, with interferences eliminated, and that reasonable speeds range from 11 to 15 miles per hr. The data also indicate that such a street has a capacity for trucks of from 1 300 to 1 500 vehicles per hr at speeds from 11 to 12.5 miles per hr. Where the traffic is mixed, as is usually the case, the capacity of a single lane is indicated to be approximately 2 250 vehicles per hr. Naturally, this capacity will depend upon the proportion of trucks and other vehicles present. The data further indicate that where traffic is especially dense, even higher values than the foregoing may be obtained. It is not likely, however, that these especially dense conditions could be maintained over a considerable period of time.

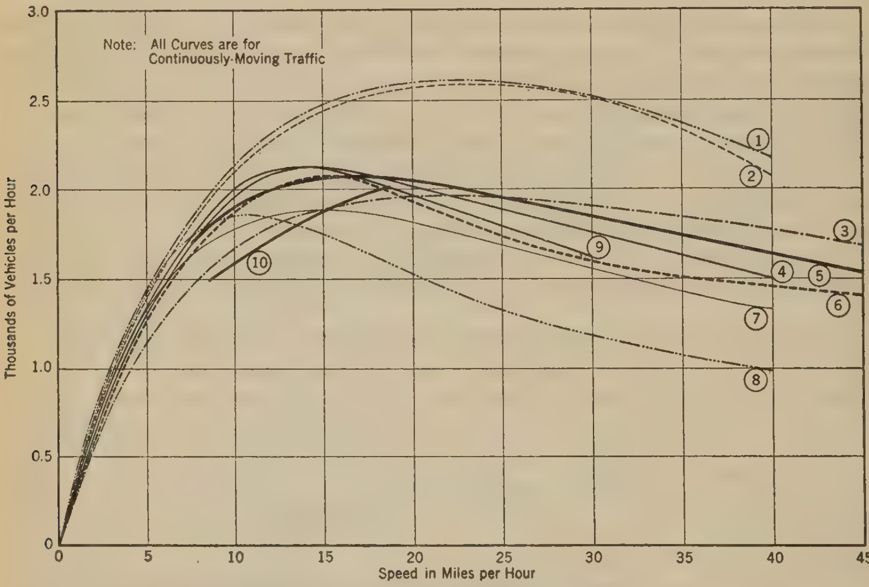

Fig. 1.—Maximum Hourly Capacity of a Single Traffic Lane at Different Speeds

Some verification of these data was obtained from a group of curves (Fig. 1) obtained from various sources, which represent maximum hourly capacity of a single traffic lane at varying speeds. These curves show a range of hourly capacity of from 1 700 to 2 300 vehicles at 11 miles per hr, and from 1 900 to 2 500 vehicles at 15 miles per hr. Referring to Fig. 1, Curves 1, 4, and 9, respectively, are for Los Angeles, Calif., Detroit, Mich., and Norristown, Pa.[3]

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[3] "A Study of the Traffic Problem," by William S. Canning, 1928, p. 16. The curve for Los Angeles (Curve 1) is shown in more detail in *Traffic Bulletin No. 143*, Am. Gas Accumulator Co., Elizabeth, N. J.

Curve 2, originally, was plotted from computations made on the basis of field data referring to the spacing of vehicles at various speeds both on crowded and on high-speed thoroughfares.[4] It applies to a continuously moving lane of traffic, on the assumption that the average length of vehicle is 14 ft. Curve 3 was plotted from data observed by the Pennsylvania State Highway Department. This curve is intended to show the "theoretical maximum capacity of one traffic lane under ideal conditions."[5] The sources of the data, study conditions, assumptions, etc., from which the computations were made, are not available.

Curve 5, Fig. 1, was derived and plotted by the Bureau of Traffic Planning, City of Pittsburgh, and represents the mean average of the nine remaining curves in this drawing. A blueprint of Curve 6, with its formula, supplied by N. W. Dougherty, M. Am. Soc. C. E., was the only item of reference material available for this curve. Evidently, it applies to continuously moving traffic, assuming all cars with four-wheel brakes, and, being derived from an equation, is theoretical.

Curve 7 is intended for an "uninterrupted traffic lane."[6] It was derived from a formula, the factors of which were obtained from actual field data and observations. Mr. H. S. Swan,[7] who is the authority for Curve 8, makes no mention of study conditions, how data were obtained, etc., except that "safe braking distance" is assumed and that no time is allowed for the application of brakes. Two other curves presented by Mr. Swan (not included in Fig. 1) are computed on different assumptions. Finally, Curve 10 was plotted by the Pittsburgh Bureau of Traffic Planning, based on studies of down-town Pittsburgh. This curve is theoretical, representing the flow of traffic, for a full hour, moving in a flexible, progressive, signal system.

Further light on the accuracy of Fig. 1 was obtained from studies of the vehicle flow on the Wilmot Street Bridge, in Pittsburgh (see Table 3). This is a two-lane bridge—one lane in each direction. It has a large reservoir at one end for entering vehicles to accumulate, and at the other end for leaving vehicles to spread out promptly, so as not to delay those on the single lane of movement on the bridge. Table 3 shows an average rate of 1 710 vehicles per hr at a speed of 20 miles per hr, which rate was sustained over a period of 5 min at a time. A rate of 1 680 vehicles per hr was sustained for as long as 25 min.

Further supporting data (Curve 10 of Fig. 1) were obtained based upon a number of studies in down-town Pittsburgh, principally on Second Avenue, where dense groups of traffic were observed moving through the progressive traffic signal system. These data indicate a movement of 1 700 vehicles per hr at a speed of 11 miles per hr, and 1 900 vehicles per hr at a speed of 15 miles per hr. Fig. 2 is a comparison of Curves 7 and 10 in Fig. 1. Both curves are on the basis of a cycle of 53 sec and a lane-moving, or "Go" time of 45.3% of the cycle, or 24 sec. It is to be noted that, at the points of greatest divergence of the two, Curve 10 differs from Curve 7 by only 15% at 12 miles per hr and 19% at 17 miles per hr. These data give substantial ground for the traffic capacity of a single lane of a street in a congested district as 2 000 to 2 500 passenger cars per hr with speeds of movement of from 11 to 15 miles per hr.

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[4] "Roadway Capacity," by W. D. Hudson, July, 1924. Not published.

[5] "Facts and Figures of the Automobile Industry," National Automobile Chamber of Commerce, 1928 Edition, p. 85.

[6] "Regional Survey of New York and Its Environs," 1927 Rept., Vol. III, "Highway Traffic," p. 86, Fig. 45.

[7] "Theoretical Capacity of Roadways," by H. S. Swan, *Traffic Bulletin No. 142*, Am. Gas Accumulator Co., Elizabeth, N. J.

TABLE 3.—VEHICLE FLOW ON WILMOT STREET BRIDGE, PITTSBURGH, PA. (ONE LANE)

Date (1933)	No. of vehicles in 25 minutes	No. of vehicles in a maximum 5 minutes	Rate of flow, in vehicles per hour	Speed, in miles per hour
October 5	701	145	1 740	20
October 6	619	142	1 680	20
Average	143.5	143.5	1 710	20

Of course, these capacities are not actually obtained for all lanes, or even for the critical lane. Counts of actual traffic volumes and the hourly rate per lane for several typical congested Pittsburgh streets (Table 4), show a range of from 132 cars per hr to 754 cars per hr per lane. The highest rate (754 cars per hr per lane) was obtained on the Boulevard of the Allies at Grant Street, which is a viaduct approach to the congested district. The next highest rate of 508 was obtained on a bridge approach. At both these locations many of the usual interferences were not present, and there was constant pressure on the intersection from a long reservoir of automobiles waiting to get through.

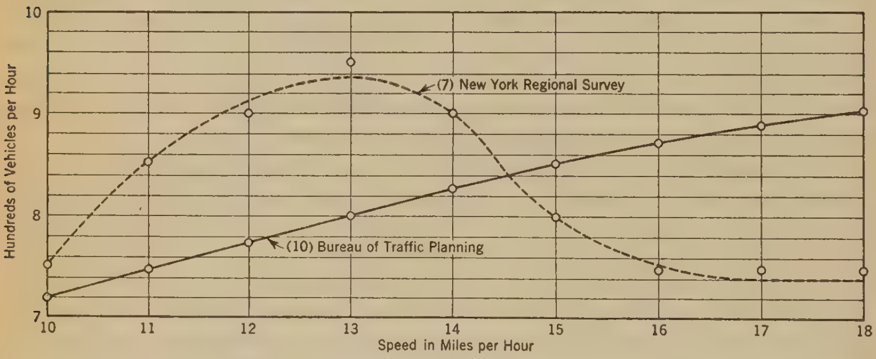

FIG. 2.—COMPARISON OF LANE CAPACITY DATA

The reason for this startling difference between capacities and actual flow is interference of one kind or another. Experience has shown that considerable interference can be eliminated, or at least reduced. Four types are recognized: (1) Cross-interference; (2) marginal interference; (3) internal interference; and (4) medial or center-line interference.

Interference by cross-traffic is the most obvious. It is evident that traffic from the main streets must stop while traffic on the cross-streets is moving.

If both the main street and the cross-street are given an equal proportion of the available time, it is obvious that the flow on each cannot be more than 50% of the capacity. Although this interference by cross-traffic can only be eliminated entirely by complete grade separation, it can be reduced by accurate proportioning of signal timing so that each street is given its proper share, but no more and no less than its proper share, of the available time. This proportion will vary throughout the day, but study indicates that the variation occurs with reasonable regularity, both as to time of the day, day of the week, and season of the year. Naturally, the facts concerning these variations must be obtained if the street is to be used at its maximum efficiency.

TABLE 4.—TRAFFIC VOLUME PER LANE, TYPICAL PITTSBURGH INTERSECTIONS

Street	At:	Traffic from:	Volume maximum, in 15 minutes	No. of lanes	Traffic per lane	Hourly rate per lane	Remarks
Fifth Avenue	Smithfield Street	East	124	2	62	248	Street cars
Fifth Avenue	Smithfield Street	West	195	2	98	392	Street cars
Ninth Street	Liberty Street	North	173	2	86	346	Street cars
Sixth Avenue	Grant Street	East	261	2	130	520	Street cars
Sixth Street	Penn Avenue	North	135	2	67	270	Street cars
Allies*	Grant Street	East	377	2	188	754	No street cars
Allies*	Grant Street	West	305	3	102	408	No street cars
Grant Street	Allies*	North	217	3	72	288	Street cars
Grant Street	Allies*	South	169	3	56	224	Street cars
Grant Street	Seventh Street	North	207	3	69	276	No street cars
Grant Street	Seventh Street	South	252	3	84	336	Street cars
Seventh Avenue	Grant Street	East	233	3	78	312	No street cars
Seventh Avenue	Grant Street	West	241	3	80	320	Street cars
Fourth Avenue	Smithfield Street	West	100	3	33	132	One-way†
Smithfield Street	Water Street	South	254	2	127	508	Bridge
Penn Avenue	Sixth Street	East	185	4	46	185	One-way
Smithfield Street	Fourth Avenue	South	196	4	49	196	One-way
Smithfield Street	Fifth Avenue	South	205	4	51	205	One-way

* Boulevard of the Allies. † Three lanes.

Cross-interference shows further restrictive effect in the delay that follows the stopping of a vehicle. Some data showing the time taken to start automobiles at an intersection, after the "Go" signal has been given, have been obtained as follows:

Position of car in line	No. of observations	Average time, in seconds
First car	224	1.6
Second car	224	2.9
Third car	202	4.1
Fourth car	185	5.1
Fifth car	101	7.4
Sixth car	81	7.6

The average time required to start equals approximately 1.41 times the position of the car in line. Naturally, the more stops that are caused by intersections which are too close together, the more frequently will this delay occur and the greater will be its effect in reducing street capacity. One of the fundamentals of good traffic regulation is to keep the traffic moving. The following simple illustration, involving the sixth and last car of a line stopped at an intersection, will show the importance of effective headway.

Assume the six cars to be stopped with a distance of 3 ft between each car, and each car to be 14 ft long. The rear of the sixth car will be 99 ft from the intersection, and must travel 147 ft for the rear of the car to clear the far curb of a 36-ft roadway. The data in Table 2 demonstrate that the first passenger car of a group will travel 48 ft in 3.92 sec and that the following cars of this group will travel 48 ft in 2.77 sec. The speed of the first car in traveling 48 ft was 10.8 miles per hr and that of following cars, as they cleared the intersection, 15.3 miles per hr. Assuming the average of these two speeds as the speed which the sixth car made in the remaining 51 ft which it was required to travel, gives a total of 17 sec before this sixth car clears the intersection, or an effective headway for these 6 cars of 2.82 sec, and a rate of flow of 1 275 cars per hr per lane. This is a reduction in lane capacity of 35 to 50 per cent.

Expressed in tabular form:

Description	Time required, in seconds
For sixth car to start	7.6
To travel 48 ft, at 10.8 miles per hr, after starting	3.92
To travel 48 ft, at 15.3 miles per hr, through the intersection	2.77
To travel another 51 ft, at 13.0 miles per hr	2.67
Total time	16.96
Effective headway	2.82

If intersections are close together there is no opportunity for vehicles to have sufficient free running time at higher speeds to overcome this delay, and the capacity of the street is permanently reduced.

A second cross-interference of considerable magnitude is caused by pedestrians who cross against the traffic. The degree of this interference depends upon the degree to which pedestrians obey the signals. Some data taken at typical intersections in the down-town district in Pittsburgh show that 56% of the pedestrians confronted by a "red" signal, failed to heed it, and crossed the street. The best obedience in the down-town district was at Smithfield Street and Sixth Avenue where 34% of the pedestrians confronted with a "red" signal crossed in the face of it. The corresponding value for Penn Avenue and Sixth Street was 71 per cent. An attempt was made to establish the relationship between pedestrian volumes, vehicle volumes, or turning volumes, and the degree of pedestrian observance of the traffic signals; but although a large volume of data was obtained, no consistent relationship could be determined.

In considering this pedestrian interference, it should be noted that no real effort to secure pedestrian control, except through educational means, has been attempted in Pittsburgh. Such interference, as well as that caused by pedestrians crossing in the middle of the block (commonly called "jaywalking"), has been reduced materially in a number of cities where real efforts have been made to enforce pedestrian regulations in congested districts. The value of such enforcement in increasing street capacity can be seen when it is realized that observations covering 4 hr for each of twelve intersections in down-town Pittsburgh showed that 22.9% of all pedestrians at the intersection crossed with interference of one sort or another. Naturally, the degree to which pedestrian interference reduces the street capacity depends on the volume of pedestrians present. Where such a large proportion crosses against the traffic the conclusion is inevitable that the street capacity has been materially reduced.

TABLE 5.—DELAYS CAUSED BY TURNS

Type of Turn		Type of vehicle	Number of observations	Time Required, in Seconds:		Delay, in Seconds, Due to Interference with Oncoming:			Delay, in Seconds, in Its Own Lane, at Straight-Ahead Speeds (in Miles) per Hour) of:			
From Lane No.:*	To Lane No.:*			To clear its own lane	To turn	Lane No. 3*	Lane No. 2*	Lane No. 1*	6	12	18	Actual
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	1	Passenger	18	2.5	4.5	...	...	...	0.0	1.0	1.5	0.9
		Trucks	10	2.8	5.0	...	...	...	0.0	0.6	1.4	0.6
		Horse-drawn	1	6.0	8.8	...	...	...	2.4	4.2	4.8	1.2
1	3	Passenger	76	3.2	5.2	...	...	...	0.0	1.3	1.9	1.1
		Trucks	16	4.1	6.5	...	...	...	0.0	1.5	2.4	1.4
1	4	Passenger	22	3.0	5.0	...	...	...	0.0	0.0	0.9	0.0
		Trucks	16	3.2	6.3	...	...		0.0	0.0	0.7	0.0
		Horse-drawn	2	3.3	6.5	...	...	‡	0.0	0.0	1.0	0.0
2	2	Passenger	65	2.8	5.7	...	...	3.1	0.0	1.3	1.8	1.2
		Trucks	18	4.2	6.6	...	...	4.2	0.0	2.0	2.8	2.0
		Horse-drawn	3	4.9	7.6	...	...	5.4	1.3	3.1	3.7	0.1
2	3	Passenger	103	3.3	5.6	...	...	3.2	0.3	1.8	2.3	1.7
		Trucks	26	3.9	6.8	...	...	4.2	0.0	1.7	2.5	1.7
		Single street car	51	7.0	10.2	...	...	7.7	0.5	3.8	4.8	2.3
		Street-car train	14	12.7	22.6	...	...	13.5	0.9	6.8	8.8	4.2
		Horse-drawn	2	5.5	10.9	...	...	6.8	1.9	3.7	4.3	0.7
2	4	Passenger	45	3.4	5.5	...	...	3.1	0.4	1.9	2.4	1.8
		Trucks	29	3.3	6.8	...	...	3.2	0.0	1.1	1.9	1.1
		Horse-drawn	4	8.9	16.2	...	...	8.2	5.3	7.1	7.7	4.1
3	2	Passenger	167	4.0	6.9	...	2.7	2.7	1.0	2.5	3.0	2.4
		Trucks	30	5.6	9.1	...	4.3	4.5	1.3	3.4	4.2	3.4
		Single street car	46	8.2	12.0	...	8.4	8.7	1.7	5.0	6.0	3.5
		Street-car train	30	12.4	16.4	...	12.2	11.9	0.6	6.5	8.5	3.0
		Horse-drawn	1	5.2	9.5	...	5.4	5.2	1.6	3.4	4.0	0.4
3	3	Passenger	17	3.2	5.7	...	5.0	1.8	0.2	1.7	2.2	1.6
		Trucks	17	4.9	6.2	...	4.5	2.1	0.6	2.7	5.5	2.7
		Horse-drawn	1	7.0	12.2	...	5.2	4.8	3.4	5.2	5.8	2.2
4	2	Passenger	21	8.2	11.1	1.2	3.0	2.5	4.3	6.3	6.9	6.1
		Trucks	16	8.7	11.6	3.7	5.3	4.5	3.5	6.1	7.0	6.0
		Horse-drawn	1	4.2	9.4	...	4.4	5.2	0.0	1.9	2.7	0.0
1	1†	Passenger	20	2.3	3.8	...	...	...	0.0	0.8	1.3	0.7
		Trucks	7	2.6	4.5	...	...	...	0.0	0.0	1.2	0.4
2	2†	Passenger	90	2.4	4.3	...	...	2.8	0.0	0.9	1.4	0.8
		Trucks	29	2.7	4.9	...	...	3.2	...	0.5	1.3	0.5
		Single street car	21	8.2	11.4	...	...	11.1	0.1	4.2	5.5	2.4
		Street-car train	31	14.7	16.5	...	...	15.3	1.3	8.0	10.2	5.0
3	2†	Trucks	18	5.9	9.7	...	4.4	4.9	1.6	3.7	4.5	3.7

* All left turns, except as indicated in Column (2). † Right turn. ‡ Actual speed.

Turns are a further type of cross-interference, particularly left turns. Factual data regarding the magnitude of such interference have been obtained.

These data, in Table 5, show: (*a*) The time required by vehicles of various types to clear their own lanes; (*b*) the delay to following traffic; (*c*) the time required to complete the turn; and, (*d*) the delay to oncoming traffic where the turn crosses such traffic.

Turns are designated by the number of the lane from which, and to which, the turn is made. Lanes are numbered across both streets starting from the corner about which the turn is made. Average delays to following traffic range from zero to 6.1 sec, depending on the type of turn, whether the streets are one-way or two-way, the type of vehicle, the volume of oncoming traffic, etc. Interference to oncoming traffic ranges to as high as 15.3 sec, and time to complete the turn to as much as 22.6 sec in the case of a street-car train making the ordinary left turn from one two-way, four-lane street to another. Table 5 shows that one passenger automobile, turning left from the center lane of a two-way, six-lane street, may delay following traffic in its own lane an amount equal to the time required for four such vehicles to go straight through; and that the street-car train making the turn previously described, occupies the time required for twenty-one passenger cars to move through one of the lanes in the opposite direction.

Of course, the actual reduction in capacity caused by these movements at any given intersection depends on the number of such turns per hour and the actual traffic volumes moving in both directions. The data can only be used in a practical manner when these volume data are known from actual traffic counts at the points being studied.

In considering these and other data presented in this paper, it should be borne in mind that they are presented merely with the idea of evaluating the various interferences in terms of their reduction of street capacity and so that the value of providing remedies for these interferences can be measured. The economic phase has not been discussed; for instance, it might be stressed, with considerable validity, that even if the left-turning street-car train mentioned did take as much time as is required for twenty-one passenger automobiles to move straight through, the street-car train actually carried twice as many passengers as all twenty-one of the passenger cars.

The time taken to make turns is further influenced by the volume of pedestrian traffic moving parallel with the traffic stream from which the turn has been made. Such pedestrian data should be available when studies are being made in connection with the gains to be expected from prospective turn eliminations.

In making turn elimination studies, the alternate routes, with the interferences likely to develop, must receive careful consideration. It should be accepted as a fundamental principle in such studies that two alternate routes must be available for each turn eliminated: One to the motorist before he reaches the point where the turn is to be eliminated; the other, after he has reached this point, in case he has forgotten to take the first alternate route. As far as possible turn eliminations should follow some reasonably regular pattern. It is extremely disconcerting and it adds to the confusion, for a motorist, especially a stranger, to find turns eliminated at random. This, also, is a factor in reducing street capacity. Properly located and legible signs are essential. An origin and destination study of the traffic involved in the turn is also extremely helpful in determining a proper course of action.

The second type of delay is caused by marginal interference, which is due to: (*a*) Vehicles entering and leaving parking lots and garages; (*b*) vehicles parked along the curb; (*c*) vehicles pulling out from the curb and backing in; and (*d*) the natural slowing down of moving cars in the lane adjacent to a line of parked cars, etc. Loading and unloading at the curb has a similar effect in reducing street capacity to parked cars. This interference is more difficult to correct because facilities for off-street loading and unloading have not yet been provided in most cities.

The reduction in traffic capacity due to parked cars pulling into, and out from, the curb is naturally dependent on the degree of turn-over in the parked cars. This depends on the type of district, the parking regulation, and the degree of enforcement secured.

In down-town Pittsburgh the average stopping time of all cars parked or loading on the streets is 21.5 min; 48% are loading and unloading, requiring 8.5 min; 24% are parked while the occupant is shopping, requiring 24.5 min; 28% are parked for 40.5 min while the occupant has gone to his work or is engaged in recreation. On the 1-hr parking streets, the average parking time is 37 min.

The effect of some of these factors can be illustrated by examining the ratio of lane use of two typical Pittsburgh thoroughfares, shown in Table 6. The first of these thoroughfares is Sixth Avenue, a four-lane, two-way street with a line of street cars moving in each direction. The data were obtained at the near side of each intersection.

TABLE 6.—RATIO OF LANE USE, TYPICAL STREET

Block	Intersection	Lane No. 1	Lane No. 2	Lane No. 3	Lane No. 4	Remarks
	Sixth Avenue at:					
1	Fifth Avenue	0.077	0.923	0.762	0.238	Parking garage next block
2	Wylie Avenue	0.499	0.501	0.715	0.285	No parking or stopping
3	Webster Avenue	0.499	0.501	0.715	0.285	No parking or stopping
4	Grant Street	0.506	0.494	0.650	0.350	No parking or stopping
5	Smithfield Street	0.239	0.761	.....	.....	Partial parking
6	Wood Street	0.070	0.930	0.798	0.202	Considerable parking
7	Liberty Avenue	0.580	0.420	.....	.....	No parking or stopping
	Fifth Avenue at:					
	Grant Street	0.133	0.867	0.728	0.272	....
	Smithfield Street	0.052	0.948	0.855	0.145	Parking in Lane No. 1
	Market Street	0.237	0.763	0.687	0.313	....

At Fifth Avenue, which may be called the entrance to Block 1, 0.077 of the traffic was in the curb lane whereas 0.923 was in the adjacent lane. The reason is plain when it is known that in the next block there is a large parking garage with a capacity of about 600 cars. Motorists simply will not use the curb lane when they know they will be blocked by cars entering and leaving this parking garage. In other words the capacity of the street has been reduced nearly one-half. In the next three blocks, where there is no such interference and where the no-parking regulation is strictly enforced, there is a practically even distribution of traffic; in fact, in the third block the curb lane even carries slightly more traffic than the middle lane. The character of the business in the next block (Block 5, Table 6) is such that a certain amount of parking, or at least short-time stopping is permitted. The curb-lane use drops to 0.239. In the next block (Block 6) even more parking and stopping is permitted, and the curb-lane use drops to 0.070. The next block (Block 7) is a short one with parking again absolutely banned; the curb-lane use increases to 0.580. In the latter block, however, some influence is felt from the fact that considerable of the traffic turns to the right. As far as the law is concerned, a no-parking regulation is "on the books" for this entire side of Sixth Avenue, but the business use and degree of enforcement are different.

The second street that will be examined—Fifth Avenue—is one of the main streets in the down-town district. The parking ordinance calls for "no parking at any time" for all the blocks in question, on both sides of the street. In spite of this regulation, the fact remains that considerably more parking is permitted on one side than the other. On the north side, where this parking exists, the ratio of curb-lane use is 0.052 whereas, on the other side, it is 0.272.

Some factual data regarding the actual delays to traffic in the adjacent lane caused by cars pulling into, and out from, the curb have been obtained. Each car which pulls out from the curb interrupts the adjacent lane for an average of 11.25 sec, whereas each car backing into a parking space interrupts the adjacent lane 16 sec. The total number of observations for cars leaving as well as entering was sixteen. The more frequent the turn-over the more serious is this effect in reducing street capacity. It must be remembered, however, that since the time necessarily allowed for cross-traffic to move has already had its effect in reducing capacity, some time is available between blocks for delays such as these, without their complete effect showing on the over-all capacity of the street. There is also a speed-reducing effect on the lane immediately adjacent to the parking lane. Its capacity is reduced as compared to a lane not immediately adjacent to a parking lane.

The third type of delay is caused by internal interference, such as the weaving of cars from one lane to another, the conflicts that occur when cars pass each other in adjacent lanes when the lanes themselves are too narrow, the effect of vehicles of different characteristics, etc.

Formerly, the width of roadways was determined by the old three-fifths rule. This rule determined the width as three-fifths of the distance between property lines, which distance itself had been somewhat arbitrarily determined —usually being some multiple of ten. Thus, for a 30-ft street, the roadway was 18 ft wide; for a 40-ft street, 22 ft, etc. Such widths permitted no similarity in the actual lane widths available for moving traffic. In few cities has any one attempted to make the actual lane widths available for traffic movement, suitable for such movement, by adjusting the sidewalk width. Furthermore, in the establishment of parking regulations scant consideration has been given to the width of the roadway, so as to provide for a given number of lanes of adequate width, for moving traffic. In some cases an 8-ft lane is available, in others, 9 ft, and, in others, 10 ft. A lane that is too wide is economically wasteful from the standpoint of pavement cost; a lane that is too narrow inevitably decreases the capacity of the street.

The traffic flow on a city street is not directly proportional to the number of lanes. Arthur N. Johnson,[8] M. Am. Soc. C. E., has demonstrated that, for rural highways, three lanes had twice the capacity, and four lanes, three times the capacity, of two-lane roads, etc. The same effect is not so evident on city streets. In down-town Pittsburgh there are a number of four-lane streets that are actually carrying more traffic than other six-lane streets with about the same apparent congestion. The difference is in the interference factors, which are variable from street to street.

Rough pavements limit street capacity, as do steep grades. Grades of less than 5% have little effect on modern traffic, however. Poor street illumination has its effect on traffic speeds, with a consequent effect in reducing capacity in districts where street use is heavy after dark.

Where routings are such that traffic must weave from one lane to another in order to move into the proper position for a turn, another factor which reduces traffic capacity is introduced, especially where the weaving must take place across several lanes. Some brief studies on this subject, applicable to downtown Pittsburgh conditions, were made on one down-town street (Second Avenue). These studies cover the weaving from one lane to a second lane (not multiple-lane weaving across several lanes). The studies were made in the evening peak period.

The method of the study may be briefly stated as follows: A measured course was laid out on Second Avenue, between Smithfield and Grant Streets, on the side for eastbound traffic. Several automobiles were hired, and drivers were instructed to weave from the lane nearest the center of the street to the next lane to the right, starting always at the first line of the measured stretch. The distance in which the weave was made and the speeds of the weaving vehicle were recorded. The speeds of following vehicles on both lanes were also recorded, as well as the speed of traffic when there was no weaving. Other observers checked the volume of traffic per lane in given time periods, so that the density of the traffic through which the test automobiles did their weaving was determined (see Table 7).

The composite average of these data shows a reduction of approximately 40% in the speed of traffic in the lane from which the weaving started, and a reduction of more than 30% in the speed of traffic in the lane to which the weaving vehicle moved. The average delay to vehicles in the lane from which the weaving started was 0.75 sec, and 2.6 sec in the lane to which the weaving vehicle moved. The range of weaving delays was from a negligible time to 9 sec. Translated into terms of effective headway, each weaving vehicle displaced from one to six vehicles moving straight through, the over-all reduction in capacity of the street depending on the number of weaving vehicles. Weaving, therefore, is another interference factor to be avoided if a street is to attain its maximum capacity.

Other elements that reduce the effective width of traffic lanes are structures such as loading zones and safety islands for street cars. Wherever possible the width taken out of traffic lanes by such structures should be compensated by cutting back the sidewalk opposite them a similar or greater distance. Even so, the necessary diversion of the traffic from a straight line is certain to reduce the over-all capacity of the street. Pillars of elevated railways are similar obstructions for which compensation should be made. On the other hand, structures such as loading zones for street cars introduce a helpful factor in that they permit traffic to move past street cars which are stopped to take on passengers.

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[8] *Proceedings*, Highway Research Board, 1930, pp. 218–224.

TABLE 7.—DELAYS DUE TO WEAVING FROM LANE NO. 3 TO LANE NO. 2*

Period No.:	Fifteen-minute period (afternoon), ending:	SPEED, IN MILES PER HOUR					Average distance required, in feet	TRAFFIC FLOW (NUMBER OF VEHICLES)				DELAY, IN SECONDS, CAUSED TO TRAFFIC BY WEAVING VEHICLES		Time, in seconds, required to weave
		With no weaving in:		With weaving in:		Of the weaving vehicle		Actual flow in vehicles per 15-minute period		Equivalent flow, in vehicles per hour				
		Lane No. 3	Lane No. 2	Lane No. 3	Lane No. 2			Lane No. 3	Lane No. 2	Lane No. 3	Lane No. 2	Lane No. 3	Lane No. 2	
	(1)	(2)	(3)	(4)	(5)	(9)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(*a*) FRIDAY, AUGUST 9, 1929														
1	4:30	17.7	17.3	6.2	...	9.3	65	115	142	460	568	9.0	...	4.75
2	4:45	17.2	17.3	11.7	10.2	7.0	52	107	162	428	648	2.3	3.5	5.07
3	5:00	18.0	16.1	...	...	...	...	177	115	708	460	...	...	2.2
4	5:15	14.4	13.7	9.9	6.8	6.9	55	171	215	684	860	2.7	6.3	5.4
5	5:30	11.7	11.8	8.1	10.3	10.3	65	199	205	796	820	3.2	1.1	4.3
6	5:45	14.2	13.5	...	13.7	6.1	73.5	149	165	596	660	...	1.9	8.2
Average.....	.....	15.8	14.7	8.7	9.9	7.4	60	153	167	696	700	3.8	2.5	4.8
(*b*) FRIDAY, AUGUST 16, 1929														
7	4:30	19.0	14.6	12.2	8.8	10.2	74	117	169	468	676	2.8	3.8	4.9
8	4:45	15.6	13.6	14.1	10.9	8.8	66	158	156	632	624	0.8	1.0	5.4
9	5:00	16.2	12.8	10.2	19.2	6.4	82	150	186	600	744	3.0	2.6	8.4
10	5:15	17.7	14.3	...	12.0	6.0	80	136	227	544	908	...	2.2	9.0
11	5:30	13.1	12.4	...	9.0	9.0	58	199	272	796	1 088	...	2.6	4.1
12	5:45	15.1	14.1	9.5	12.8	7.0	75	168	221	672	884	3.2	1.0	7.2
13	6:00	17.6	14.7	10.7	13.1	9.8	90	151	155	604	620	3.1	0.8	6.3
Average..........		16.8	14.7	10.9	10.1	8.3	70	153	198	644	906	2.6	2.2	6.0
Composite average..		16.1	14.7	9.8	10.2	8.0	66.4	153	184	696	906	3.4	2.6	5.6

* Lane No. 3 is next to the middle of the street.

Another factor in internal interference is based on the character of the vehicles themselves. Taxicabs are a delaying factor, not so much because the vehicles are incapable of moving with the traffic stream, but because the drivers lag in their effort to pick up passengers, or to get into desirable taxi stands. If cruising is prevalent, the extra volume of traffic added to the street displaces vehicles of other types. For fifteen intersections selected at random in downtown Pittsburgh, taxicabs comprised more than 8.5% of the total motor-vehicle traffic and in one block (Fifth Avenue, between Smithfield and Wood Streets) taxicabs comprised 25.8% of the total motor-vehicle traffic. In four other blocks they exceeded 10% of the total. Of the 4 591 cabs counted in this study, 76.7% were empty.

In another study (Table 8) covering 856 cabs and three intersections, 84% of the cabs were empty. The first appearance of a cruising taxicab was considered legitimate and, therefore, this vehicle was not included in Column (6), which includes only the second and subsequent appearance of each cab. During a 30-min period cabs were observed to enter an intersection two or more times as follows:

TABLE 8.—STUDY OF TAXICAB CRUISING

Time of observation	Location	No. of observations	Percentage of Observations:		
			Loaded	Empty	Cruising
(1)	(2)	(3)	(4)	(5)	(6)
8:00 A.M. to 10:00 A.M.	William Penn Way and Oliver Avenue	335	16	84	23
4:00 P.M. to 6:00 P.M.	Grant Street and Diamond Street	340	19	81	31
February 8, 20, and 27, 1933.	Fifth Avenue and Wood Street	181	11	89	25

No. of appearances	No. of cabs	No. of appearances	No. of cabs
2	80	6	2
3	32	7	1
4	6	9	1
5	6		

Even granting that the first appearance of an empty cab was legitimate on the assumption that it was going to a call or a stand, 25% of the empty cabs were cruising; that is, they appeared, empty, two or more times at the same location within 30 min. Taxicab stands themselves furnish the same type of interference as parked cars. Such stands, of course, should never be permitted, except on streets where other vehicles are permitted to park.

Buses are also a delaying influence, in that they usually stop at the curb to pick up passengers and as a consequence delay the following traffic. One item receiving considerable attention is whether the bus stop should be on the near side, or on the far side of the intersection. Practice varies in different cities, although the prevailing practice seems to be to have the stop on the near side of the intersection so that stops necessary on account of the red light can be utilized for picking up passengers. On the other hand, when the bus arrives on the green light, such stops delay following traffic, which otherwise would go on through. Physical characteristics, such as the width of the street, number of lanes for moving traffic, whether parking is permitted or not, whether the street is flowing nearly to capacity, the number of right turns made into the street, etc., are determining factors in any individual case, although the practice must be made uniform throughout the congested district.

The effect of trucks and other commercial vehicles has been indicated previously. Due to their slower speeds they reduce the capacity of any thoroughfare. For instance, a traffic load of one-third trucks reduces the capacity about 20% as compared to a flow consisting entirely of passenger cars. Wherever possible, trucks should be segregated on certain streets. Trucks following regular routes to established terminals should be carefully considered and their route established so as to cause the least possible interference.

Because of their fixed use of the street-car lane, street cars limit following traffic in congested districts to the speed that these vehicles themselves are able to maintain, and thus lower street capacity. Many cities are using antiquated types of cars with not only slow speeds, but also slow accelerations.

The modern, high-speed street car developed by the President's Conference of the American Transit Association has an acceleration and speed enabling it to hold its own with motor traffic in the traffic stream, with the exception of the necessary stops to pick up and discharge passengers. This new car is being introduced in a number of cities. Of necessity, when the car stops to take on passengers, it blocks its own lane. Careful signal planning and study of the

TABLE 9.—LOADING TIME OF STREET CARS, ON "RED" AND "GREEN" TRAFFIC SIGNALS (OBSERVATIONS TAKEN BETWEEN 4:30 P.M., AND 6:00 P.M.; 104 SIGNAL CYCLES AT EACH LOCATION; TOTAL, 416 CYCLES)

Description	No Loading Platform		Loading Platform		Total
	Traffic from north on:				
	Wood Street at Fifth Avenue	Sixth Street at Penn Avenue	Grant Street at Fifth Avenue	Traffic from west on Liberty Avenue at Sixth Avenue	
(1)	(2)	(3)	(4)	(5)	(6)
Total number of observations	87	60	78	56	281
Percentage of total time used for loading	23.6	21.9	14.9	16.8	19.2
Percentage of total time that is stop time	55.8	48.0	46.1	48.0	49.5
Percentage of Time Used for Loading:					
"Stop" time	34.7	27.2	14.7	20.0	24.6
"Go" time	9.4	17.0	15.0	13.9	14.0
Percentage of Cycles with Loading on:					
"Stop" time	74.0	44.2	41.3	46.1	51.4
"Go" time	43.3	26.9	50.0	33.6	38.4
Percentage of Cars Loading, Exclusively, on:					
"Stop" time	47.2	43.4	28.2	35.7	38.8
"Go" time	0.0	5.0	44.9	12.5	16.0
Percentage of Loading Time That Occurs on:					
"Stop" time	82.3	59.8	45.5	57.2	63.3
"Go" time	17.7	40.2	54.5	42.8	36.7
Percentage of Cars That Arrive on:					
"Stop" time	56.3	95.0	50.0	60.7	63.7
"Go" time	43.7	5.0	50.0	39.3	36.3
Percentage of Time (in Cycles in Which Cars Are Actually Loaded), Used for Loading:					
"Stop" time	47.0	61.5	35.4	43.3	47.0
"Go" time	21.6	56.3	30.1	41.2	36.5
Ratio of the Loading Time on "Stop" Signals to Loading Time on "Go" Signals	4.64	1.49	0.84	1.33	1.73

characteristics of individual stops will enable the stopping and loading of street cars on the "red" signal, to a surprisingly large degree, thus eliminating much of this delay. Loading zones permitting the by-passing of motor traffic are a further help.

Some observations of results actually obtained in this respect at four rather heavy loading points selected at random in down-town Pittsburgh are presented in Table 9. At two of these points, loading platforms which permit the by-passing of other traffic, have been installed; at the other two, all automobile traffic must stop while the street car is loading. The readings were made during the evening peak from 4:30 P.M. to 6:00 P.M., and a total of 281 street cars was observed. The average "stop" time for the four points was 49.5% of the total time.

These data show that (see Column (6), Table 9): (1) 63.3% of the total loading time occurred on "stop" time; (2) 38.8% of the cars loaded exclusively on the "stop" time, and only 16.0% loaded exclusively on the "go" time; (3) some loading on the "go" time occurred in only 38.4% of the signal cycles; (4) 36.5% of the "go" time in the cycles in which cars loaded, was actually used for loading, and 47% of the "stop" time in the cycles in which cars loaded, was actually used for loading; and (5) 1.73 times as many seconds were used in loading on the "stop" time as on the "go" time.

A distinct difference was observed between the locations where loading platforms were installed, and those where patrons were required to board street cars without this protection. This difference illustrates not only the value of the loading platforms themselves, but the fact that the design of the signal system had actually taken this difference in street-car loading procedure into consideration. At the two locations without loading platforms (Columns (2) and (3), Table 9), the percentage of total loading time occurring on "stop" was 82.3 and 59.8 respectively; at the two locations with such platforms (Columns (4) and (5), Table 9) the corresponding percentages were 45.5 and 57.2, because it was not so important to have cars load on "stop" in the latter case. At the locations without the platforms, street cars loading exclusively on the green light were zero and 5%; at the locations with platforms these values were 44.9 and 12.5 per cent. Furthermore, the ratio of the seconds of loading on "stop," to the seconds of loading on "go," was 4.64 and 1.49 where there were no loading platforms, but 0.84 and 1.33 where there were loading platforms.

In discussing the effect of taxicabs on street capacity, the writer mentioned the fact that, in so far as considerable of this traffic was probably not serving a useful purpose, it was occupying the space that could be occupied by other vehicles, thereby reducing the street capacity to useful traffic.

In most congested districts there is another type of traffic, present in even greater volume, that is reducing street capacity in the same manner. This is through traffic, as contrasted to the strictly local, business-producing traffic.

In down-town Pittsburgh 49.7% of the vehicles entering between the hours of 6:00 A.M. and 12:00 midnight remained in the district less than 20 min and left by a different route from that by which they entered. The volume of this through traffic is illustrated by Fig. 3. In the evening peak period from 4:00 P.M. to 6:00 P.M., it amounted to 53.3 per cent. Progress through restriction

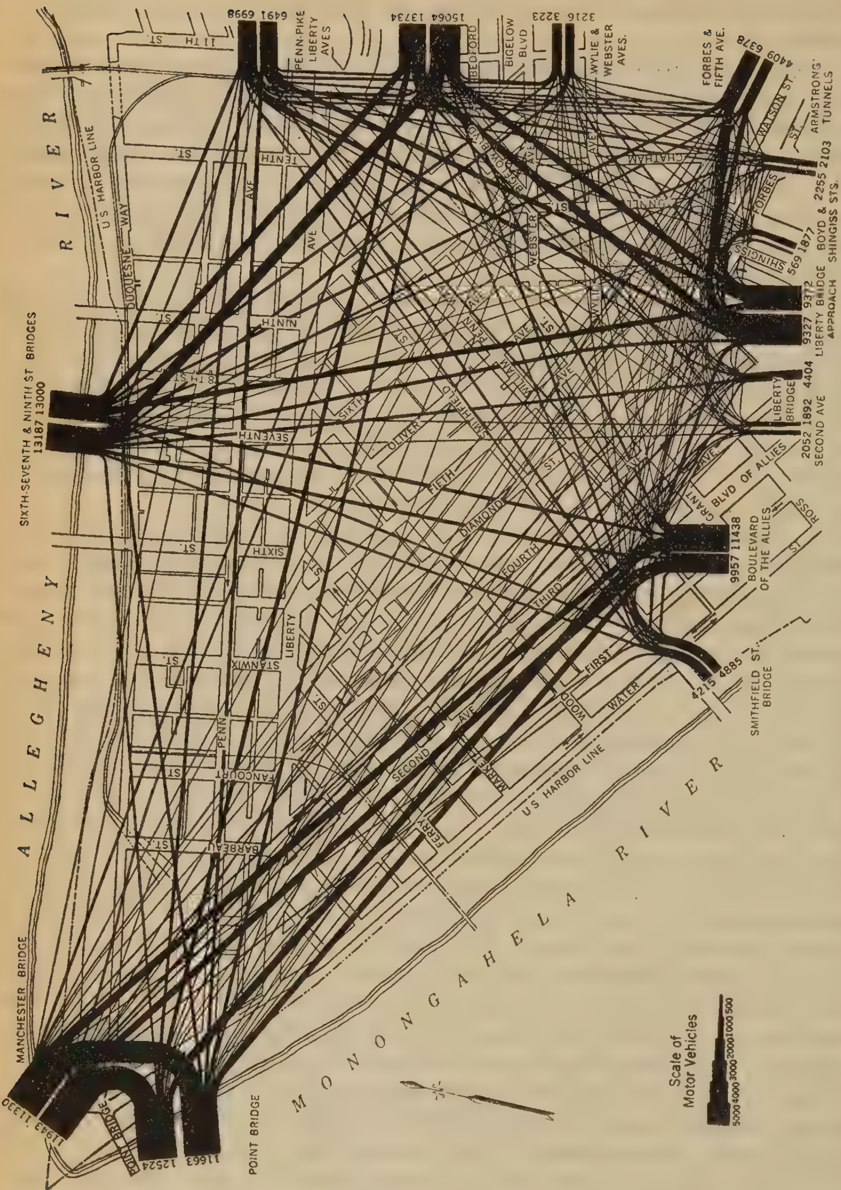
ALLEGHENY RIVER
MONONGAHELA RIVER
SIXTH-SEVENTH & NINTH ST BRIDGES
13187 13000
MANCHESTER BRIDGE
11943 11330
POINT BRIDGE
12524
11663
6998 6491
PENN-PIKE LIBERTY AVES
13734 15064
BIGELOW BLVD
3223 3216
WYLIE & WEBSTER AVES.
FORBES & FIFTH AVE.
6378 4409
ARMSTRONG TUNNELS
2103 2255
BOYD & SHINGISS STS.
1877 569
9372 9327
LIBERTY BRIDGE APPROACH
4404
LIBERTY BRIDGE
1892 2052
SECOND AVE
11438 9957
BOULEVARD OF THE ALLIES
4885 4215
SMITHFIELD ST. BRIDGE
US HARBOR LINE
DUQUESNE WAY
11TH ST.
TENTH ST.
NINTH ST.
8TH ST.
SEVENTH ST.
SIXTH ST.
STANWIX ST.
FANCOURT ST.
BARBEAU ST.
PENN AVE
LIBERTY AVE
BEDFORD
BIGELOW BLVD
WEBSTER AVE
WYLIE AVE
CHATHAM
FORBES
SHINGISS
WATSON ST.
GRANT
BLVD OF ALLIES
ROSS ST
SIXTH
OLIVER
FIFTH
DIAMOND
FOURTH
THIRD
FIRST
SECOND AVE
WATER
WOOD
MARKET
FERRY
SMITHFIELD
WILLIAM
PENN AVE
Scale of Motor Vehicles
5000 4000 3000 2000 1000 500

FIG. 3.—Through Traffic Flow, Central Business District, Pittsburgh, Pa.

of parking, elimination of turns, the creation of one-way streets, more efficient traffic signal systems, etc., cannot be continued indefinitely.

Traffic that merely passes through congested districts on its journey from one outlying district to another, adds nothing to the prosperity of a congested area. The tremendous relief from congestion which would result from the elimination of 40% to 50% of the present traffic is evident. What a rejuvenation business organizations could experience through the removal of that 40% to 50% of non-business producing traffic and its replacement by an equal volume of business-producing traffic! Such a change would not entail any greater burden on the streets of Pittsburgh than at present, and could be accomplished by the construction of a close in by-pass system around the congested district. It is essential if permanent traffic relief is to be secured. In the meantime some improvement could be accomplished by establishing, and thoroughly marking, preferred through routes over existing streets. Normally, through traffic filters to a considerable extent from one street to another in down-town areas, producing unnecessary confusion. A preferred route will often shunt this traffic flow to an adequate street hitherto neglected.

The fourth type of delay is due to medial or center-line interference. This is the interference that occurs between traffic moving in opposite directions. On radial, by-pass, and distribution streets, as well as on the open highway, where such highways approach large centers of population, it can best be eliminated by the use of a divided roadway with a center strip 15 to 30 ft wide separating the roadways used by traffic moving in opposite directions. Such a construction is largely impractical in congested districts. It is fortunate, therefore, that in such districts this type of interference is at a minimum and may be neglected, in view of the large amount of interference of other types.

Conclusions

On the basis of data presented herein, the following conclusions can be advanced:

(1) The traffic capacity of a single lane of a street in a congested city district, with interferences eliminated, is from 2 000 to 2 500 passenger automobiles per hr.

(2) Interference by cross-traffic at grade inevitably reduces the capacity of any street approximately 50 per cent.

(3) The stopping of a lane of traffic reduces the capacity of that lane 35% to 50% when the stops occur close together.

(4) In the absence of pedestrian control, as much as 23% of the pedestrian traffic crossing the street will interfere with vehicles, thus reducing the street capacity. As many as 56% of the pedestrians crossing the street in congested districts cross against the traffic. Crossing in the middle of the block or "jay-walking" is also prevalent. These practices, which reduce street capacity, can be greatly reduced with adequate pedestrian control.

(5) On the average, a passenger automobile turning left may delay following traffic a length of time equal to that required for four such vehicles to go straight through; a street car and trailer turning left may delay following traffic a length

of time equal to that required for twenty-one passenger automobiles to go straight through.

(6) A large parking lot or garage with the entrance on a main thoroughfare may have the effect of reducing the capacity of that street from 20% to 40% of the traffic which would otherwise use it. The extent of the reduction depends on the number of lanes of moving traffic on the street.

(7) The effect of curb parking is to reduce the traffic capacity of the street in question to 50% of the traffic that could otherwise use it.

(8) The traffic flow of city streets is not proportional to the number of lanes. City streets in congested districts do not follow the same rules in this regard as rural highways because of considerable variation in interference factors from street to street.

(9) Where street-car loading platforms, safety islands, elevated railway pillars, etc., obstruct the roadway, the curb should be "cut back" a distance at least equal to the width taken out of the roadway by such structures, to avoid any reduction in lane widths.

(10) One automobile weaving from one lane to another causes a change in effective headway equivalent to six vehicles moving straight through.

(11) Empty taxicabs, to a considerable extent cruising, may comprise as much as 20% of the motor-vehicle traffic on a street. To the extent that they are not serving a useful purpose and are occupying the space that could be occupied by other motor vehicles, they are reducing street capacity.

(12) Trucks reduce the capacity of a street 25% to 40% as compared to passenger automobiles. With trucks equal to approximately one-third of the traffic, the reduction in capacity is approximately 20% as compared to all passenger automobiles.

(13) Antiquated types of street cars reduce street capacity through slow speeds and slow accelerations. Modern street cars are available which can hold their own in the traffic stream. Street cars delay following traffic when they stop to take on passengers. Through the use of loading platforms and careful traffic signal system design considerable of this apparent reduction in capacity can be eliminated.

(14) As much as 50% of the traffic in congested districts is non-business producing, through traffic. This must be largely eliminated by the construction of convenient by-passes, if permanent traffic relief is to be secured. Improvement in this direction can usually be accomplished by establishing and thoroughly marking preferred through routes over existing streets.

Acknowledgment

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REMEDIES TO RELIEVE TRAFFIC CONGESTION

By Donald M. McNeil,[9] Assoc. M. Am. Soc. C. E.

Synopsis

Growth of the automobile has resulted in congestion in business districts where the street system has not been expanded to compensate for this growth. Traffic interference, both pedestrian and vehicular, causing delays is discussed, and remedies are suggested. From an engineering standpoint most delays can be eliminated, but all improvements should be weighed from the standpoint of cost *versus* benefit to business. Inexpensive suggestions are given in the paper for relieving cross, marginal, internal, and medial interference.

Introduction

With rapid development of the automobile, practically every city has been confronted with serious congestion in its business section. This congestion, which can be defined as an over-crowded condition causing a decided reduction in the rate of speed of the movement of traffic, unquestionably reflects on the volume of business in the area. The development of the automobile has been so rapid that municipalities have been unable to devise regulations and develop improvements fast enough to cope with the situation. Re-arrangement of the existing street systems is decidedly expensive and, in many cases, from a financial and economic standpoint it is impossible; and the engineer is faced with the problem of examining the existing street systems to determine whether the ultimate capacity is now being obtained, and if not, to develop methods for obtaining greater capacity.

When the area of the main business section of Pittsburgh, Pa., is considered as 0.54 sq mile, comprising 14.9 miles of streets, on which, at present (1937), 75 000 vehicles enter daily; and when one considers the necessity of moving 19 127 vehicles, plus 541 street cars, in the peak hours, during which period approximately 100 000 persons use the various modes of transportation, a series of delays becomes extremely costly. The time required for automobiles to depart from the down-town area is five times greater, and, for street cars, three times greater, than on a thoroughfare on which there are no delays. The present average time required to depart from the down-town area is approximately 20 min. This average is three times greater than that which would be required on a street system with no delays. Reducing this to man-hour delays at the current rate of day labor (40 cts per hr), the cost of delays in Pittsburgh during only the peak hours of one day equals $10 000, not to mention the cost of the accidents which always accompany such delays. Therefore, it is easy to appreciate the exorbitant loss with which each municipality is faced under present

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[9] Traffic Engr., City of Pittsburgh, Pittsburgh, Pa.

street systems. The analysis for the peak period in Pittsburgh is more or less general in all cities.

It would be unreasonable to expect a street system improved so as to avoid all delays, because of the tremendous cost involved. However, many of the delays that are now causing congestion in cities of the United States can be eliminated, and the capacity of their street systems can be increased; but it is essential that, in each case, the benefits to be gained shall be weighed carefully against the cost of such improvements (which improvements can be decidedly expensive), and careful decision must be made as to which solution is warranted.

Where the expression, "speed," is used in this paper, it must be kept in mind that the prevailing average gross speed in a business section varies between approximately 5 and 10 miles per hr. From studies made on the relation between the capacity of a lane and speed, it was shown that capacity varied approximately in direct proportion to a speed of about 23 miles per hr, but for greater speeds, the capacity decreased as the speed increased. This condition is due to the difference in headway because of the distance required to stop a vehicle in case of an emergency. Therefore, when reference is made to speed in this paper, unless otherwise noted, it relates to a value of less than 23 miles per hr—that critical speed of the volume graph. The various delays possible may be classified in four groups as follows:

(1) Cross-Interference: That interference to a stream of traffic moving in one direction crossing the path of another stream of traffic on the same plane.

(2) Marginal Interference: That type of interference which occurs between moving vehicles and other vehicles, persons, or objects alongside the outer edge of the roadway.

(3) Internal Interference: That type of interference which occurs between traffic units moving in the same direction in the same traffic area.

(4) Medial Interference: That interference between vehicles moving in opposite directions at, or near, the center of the roadway.

REMEDIES FOR DELAYS DUE TO CROSS-INTERFERENCE

To minimize cross-interference, the necessity of traffic in one direction to cross-traffic in another direction must either be eliminated, reduced, or regulated so that the traffic involved does not arrive at its point of conflict at the same time. In cross-interference, the delay encountered is the time lost due to the necessity of slowing down and, in many cases, stopping, and re-starting. At an intersection it can be eliminated completely by the construction of grade separations on the intersecting streets or, at a series of consecutive intersections, by an elevated roadway through that section of the town. Such grade separations are decidedly expensive, frequently requiring the acquisition of more land area. Theoretically, the average typical intersection reduces the capacity of the intersecting streets to 50%, not considering the delay due to slowing down, stopping, and re-starting, because when traffic is moving on one street, the traffic on the other street must wait.

The grade-separation intersection does not require a change in the speed of traffic traveling straight through it. However, a slightly reduced speed is required by that traffic traveling from one street to the other street, provided the streets have ramp connections. Such ramp connections should be designed so that drivers wishing to travel from one street to the other may do so by a right turn. This type of movement provides less delay to traffic using the street on which the vehicle is traveling. The ramps that connect the roadway to which the vehicle is traveling should preferably be designed so that only a right turn can be made. The "clover leaf" design is an example.

Cross-interference can be greatly reduced by signal control or other automatic devices, so that cross-traffic does not arrive at points of conflict at the same time. This is illustrated by the results of the flexible progressive traffic signal system installed in Pittsburgh in 1928. Prior to this time the streets of the city were seriously congested. Since the installation of the system (in which time there has been practically no physical change in the street system) the traffic volume has increased approximately 40%; yet the average speed of traffic in this section was practically the same in 1936 as it was in 1928. This success is due largely to the frequent modification of the signal timings at individual intersections and the co-ordination pattern that has been developed to provide progressive movement.

A study in 1934 of the action of approximately 1 000 000 pedestrians revealed the fact that 56% of the total number approached the intersection on the red signal, and one-half, or 28%, showed no regard for it. Of the pedestrians who disregarded the red signal, 22.9% interfered with traffic that had the right of way on the street the pedestrian was crossing. These data show that a pedestrian interferes with the movement of street traffic on an average of one out of every fifteen times he crosses an intersection. This value was obtained on fixed time signals having a signal cycle of 50 sec. Further studies were made at these same intersections when the signal cycle was 60 sec, and the results were practically the same as those reported when the signals were operating at a 50-sec cycle. Delays due to this interference by pedestrians can be eliminated only by rigid enforcement, requiring the strict observance of the law prohibiting pedestrian movement during the red-signal period. In determining the proper length of the signal cycle, consideration should be given to the length of time that pedestrians are required to wait to cross the street. Studies made by Mr. J. Rowland Bibbins[10] indicate that the total violations of pedestrians crossing against traffic on a 20-sec red period would be perhaps only one-half that on a 30-sec red period, and the latter only about one-third that on a 60-sec red period. Although no data are available of the number of pedestrians crossing in the middle of the block, thus interfering with the movement of traffic, it is evident that delays from this cause are considerable. Records indicate that this jaywalking of pedestrians is decidedly dangerous and frequently causes accidents. The remedy for this type of delay is a rigid enforcement of the laws eliminating jaywalking and confining pedestrian crossings to the intersections.

When turning right or left at an intersection, a vehicle is required to cross through the pedestrian traffic crossing on the cross-walk of the throat of the street into which the vehicle is turning. No data are available to show the number of times such vehicles interfere with pedestrians. However, from studies of turn delays made in Pittsburgh at typical business intersections, it was found that each right-turning vehicle delayed through traffic, traveling in the lane from which the vehicle turned, an average of 0.7 sec; and that each left-turning vehicle delayed not only through traffic in the lane from which the vehicle turned, an average of 1.7 sec, but also delayed the traffic in the opposite direction an average of 2.6 sec. This excessive delay of 2.6 sec is due largely to the interference of pedestrians legally crossing the cross-walk of the throat into which the vehicle is turning. Some of the remedies for these turning delays are as follows:

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[10] *Transaction*, National Safety Council, Annual Safety Congress, 1930, p. 126.

(*a*) If there are many turning vehicles, in a heavy pedestrian traffic, it is possible that the remedy may be the provision of a pedestrian interval in the signal sequence, during which period no vehicles shall move.

(*b*) If it is possible to sacrifice the flexibility of the intersection, the remedy could be the restriction of all turns, or of certain turns.

(*c*) If property can be obtained reasonably, it is possible that the remedy would be the construction of a circular arrangement, so that traffic desiring to turn left would proceed slightly beyond the intersection, turning right around a circle and waiting for the red light before completing the movement through the intersection. Such an arrangement would likewise interfere materially where there was heavy pedestrian movement, and would require the handling of the traffic twice through the intersection.

(*d*) It is possible that the remedy would be the construction of a circular roadway requiring all traffic to move in a rotary manner. This circle should be of ample diameter, preferably 250 ft, or more. Although it would eliminate cross-interference, it would create a weaving interference—to be described subsequently.

(*e*) Possibly, at least a partial remedy would be to construct connecting tunnels between basements of large stores located at the intersections.

(*f*) A possible remedy would be the provision of pedestrian under-passes where traffic would not permit turn eliminations, or the provision of a pedestrian interval. The experience with pedestrian under-passes on outlying main arteries in Pittsburgh has been extremely unsatisfactory, because of their little use. If pedestrian under-passes are provided, they should be extremely sanitary, well-lighted, designed to require the least possible exertion on the part of pedestrians walking through them, and preferably arranged to attract their use, such as with show windows and connections to buildings. Pedestrians should be educated as to their advantage.

In summarizing, the remedies for delays encountered by cross-interference are likely to be found in one or more of the following:

I.—Elimination of cross-interference due to pedestrians:

- (*A*) Rigid enforcement of pedestrian regulations: (1) No crossing on the red signal light; (2) no jaywalking.
- (*B*) Provision of a pedestrian interval, during which period no vehicles shall move, and on the other periods, no pedestrians shall move.

- (*C*) Connecting the basements of large buildings by pedestrian tunnels. (Partial remedy.)
- (*D*) Construction of pedestrian under-pass tunnels.

II.—Elimination of cross-interference due to vehicles:

- (*A*) Provision of grade separation intersections or elevated roadways.
- (*B*) Provision of a progressive co-ordinated signal system, or its equivalent.
- (*C*) Elimination of turning by vehicles.
- (*D*) Provision of a far-side, right-turn, circle roadway to replace left-turning movement.
- (*E*) Provision of a circular roadway.
- (*F*) "Channelize" wide uneven intersections.

Remedies for Delays Due to Marginal Interference

Marginal interference is usually created by the granting of conveniences to a few at the expense of a majority. Its remedy in general is inexpensive and involves limiting these conveniences. The greater part of marginal interference is caused by street and off-street parking, and the loading and unloading of vehicles. Its entire elimination is impossible, as the Courts have ruled that a municipality cannot restrict a vehicle from stopping for that period of time necessary to load or unload a vehicle expeditiously. It is an admitted fact, substantiated by the Courts, that streets are primarily for the purpose of the moving vehicle. However, such a vehicle is decidedly limited if it cannot stop to discharge any of its occupants or materials. The necessity of permitting vehicles to stop for loading or unloading has resulted in a considerable restriction of the use of the curb lane. From lane-ratio data available, the curb-lane usage of a street on which there is a moderate loading or unloading traffic is less than one-half that of the center lane, whereas on other streets where there is considerable loading, the curb-lane usage becomes practically zero. Such delays due to loading and unloading can be eliminated by restricting all stopping of vehicles (at least during the rush periods), in which case, unless other interference is encountered the lane distribution ratio will be approximately equal, as indicated by other available data. This elimination of stopping has the possibility of providing an increase of 40% in the capacity of a four-lane street. Where parking is permitted on a street, it is found that there is generally no usage obtained from the curb lane. If this parking were prohibited, and if reasonable loading were permitted in the curb lane, it would be possible in some cases to increase the capacity of the street 50%; and, with the restriction of loading during the peak periods, it would be possible to increase the capacity of the street 100 per cent. This statement, of course, is made with the provision that the curb lane has ample width for moving vehicles and that the intersections are capable of handling this additional traffic.

In addition to the obstruction of the curb lane, the parked or loading vehicle interferes with free-moving traffic in the second lane by pulling in and out of the curb lane. The seriousness of this delay depends upon the number of vehicles loading and unloading, and the rate of turn-over of parked cars. It has been observed, particularly in the business section, that the traffic in the lane adjacent to parked vehicles proceeds at a lower rate of speed than it would have traveled had there been no parked vehicles present. This is due to the preparedness of the operator to be able to stop his vehicle in the event that a parked car suddenly pulls out from the curb lane, or a jaywalking pedestrian steps from between parked cars into the path of his vehicle. If parking were eliminated, there would be no cars to pull out from the curb lane, and the distance in advance that the operator would be able to see the jaywalking pedestrian would permit the vehicle to progress at a higher rate of speed. In January, 1928, the City of Chicago, Ill., put into effect a blanket "No Parking" ordinance in the Loop District. In comparing the data of 1926 with that of several years subsequent to the restriction of parking, it is found that, in spite of 18.3% increase in traffic, street-car speeds increased from 18% to 33%, and speeds of all traffic increased 15% to 40%; in addition, total accidents were reduced 10 per cent.

The delays caused by vehicles entering and leaving parking lots and garages is another form of marginal interference. Off-street parking facilities are essential in a business section, and as it becomes necessary to restrict more and more street parking, the off-street parking facilities will be increased. Their success depends largely upon their location and their entrances, and it is for this reason that proprietors endeavor to locate them centrally and with entrances on main thoroughfares. The location of entrances to parking lots or garages on a main artery is similar to the addition of an intersecting street and, unfortunately, these lots and garages receive and discharge their tenants during the peak periods of traffic movement. Efforts should be made to have entrances to parking lots and garages open on to streets having the least congestion, and preferably on to streets that do not have through-artery characteristics. These lots and garages should be designed with sufficient reservoir space and operated with a sufficient number of attendants, so that vehicles desiring to enter them will not be required to wait on the street or the sidewalk before admittance. The exits should be located so that vehicles can be discharged on to a secondary street and, by way of connecting streets, distribute themselves to their respective main arteries.

Remedies for Delays Due to Internal Interference

Internal interference, being the type that occurs between traffic units moving in the same direction in the same traffic area, is due in general to the difference in speeds of the various vehicles, the tendency of certain vehicles to foul the lane or lanes adjacent to that in which they are traveling, and any physical condition of the roadway that may change the speed at which the vehicle is traveling.

Many of the streets in the business sections were planned during those periods when traffic consisted largely of horse-drawn wagons, bicycles, and other slow moving vehicles, with the result that in many cases the lane widths are not adequate for the free movement of present-day traffic. Formerly, it was the practice in designing streets, to make the roadway itself three-fifths of the street width; thus, when the street was 30 ft wide, the roadway was made 18 ft wide; for a 40-ft street the roadway was made 24 ft wide, etc. From an analysis of such roadway widths it will be noted that there is no similarity in the actual lane widths for moving traffic. The current design of new roadways in a business section provides lane widths for moving traffic of 9 and 10 ft, preferably 10 ft, and sufficient sidewalk area for the future development of that section. Where the lane widths are not of proper width (at least 9 ft) the speed of traffic is reduced materially, and thus the capacity of the street is curtailed.

Pedestrian traffic has increased materially in business sections, and in many cases sidewalks are not adequate to accommodate pedestrians properly. Some cities have attempted to correct the roadway deficiency by encroaching on the sidewalk areas, with the result that in many cases pedestrian traffic has been penalized. One possible remedy for an over-crowded condition of both vehicles and pedestrians would be to elevate the sidewalks to provide more roadway area. This type of improvement, while decidedly less expensive than the construction of arcades in existing buildings, or the razing of buildings, places an inconvenience on pedestrian traffic, unless the plan is adopted for an entire area, as it requires the constant walking up and down to, or from, the upper sidewalk.

Many of the old street designs provided for such small curb radii at the intersections as 6, 8, and 10 ft, which required traffic that was turning right from and on to relatively narrow streets, to proceed at a slow rate of speed. Frequently, such traffic is obliged to encroach on the wrong side of the roadway in making the turn. In remedying this condition by enlarging the curb radii, the engineer must be careful that the curb is not made so inviting as to encourage an excessive rate of speed, thus subjecting pedestrians to a hazard. Where pedestrian traffic is relatively light and the speed of traffic is approximately 25 miles per hr, curb radii of 35 ft are preferable. Where pedestrian traffic is heavy, it is suggested that the curb radii be modified to 20 ft, which will permit a comfortable speed for passenger cars of 10 miles per hr, and in some cases as much as 15 miles per hr, and a somewhat slower speed for commercial vehicles having large turning requirements. Where the street paving is in poor condition, or where it is decidedly slippery for driving, the speed of traffic is considerably slower than traffic traveling on improved non-skid pavements. Street car rails also have a tendency to reduce the speed of traffic, because of the possibility of skidding during slippery periods. At night, the speed of traffic in busy areas that are congested, is somewhat governed by the degree of visibility afforded the driver. Where there is sufficient street lighting, whether by store windows or street lights, or a combination of both, so that the driver can readily see any object that may appear suddenly in his path, he is inclined to drive at a faster speed than at places where he must rely largely on the headlights of his vehicle.

Another form of internal interference is that caused by vehicles weaving from one lane to another. In order to permit flexibility of a street system, this weaving is essential in many cases. Frequently, however, vehicles weave from one lane to another, due either to the inattention of the driver, or to his desire to travel faster than the vehicle preceding him. In many cases, this unnecessary weaving can be eliminated by "channelizing" the roadway (that is, by establishing closed lanes), and requiring persons traveling in a given lane to confine themselves to it. Frequently, the marking of the street will help materially to guide motorists. In a preceding part of this paper, a suggested solution for cross-interference is to provide a rotary circle of proper diameter. The success of this type of a roadway is due to the elimination of cross-interference at the sacrifice of weaving interference. The number of arteries connecting the rotary circle and the movement of the traffic using the circle determines its capacity. Unfortunately, sufficient data are not available to determine the exact capacity of a circle; nor are there sufficient developments of a rotary circle from which to obtain data, due to the fact that of the circles in use to-day, their traffic movements, roadway design, diameters, etc., are different. From such data as the writer has been able to obtain, a circle having an island 250 ft in diameter and a roadway 50 ft wide, has a capacity of approximately 5 000 vehicles per hr. This capacity is considerably less than that of a grade separation of the "clover leaf" design, but considerably less expensive.

The success of any business area depends upon the facilities available to the business houses to receive and deliver their merchandise, which often requires the use of large trucks. In general, these trucks travel at a decidedly slow rate of speed, compared to passenger automobiles and street cars, and require more lane area when turning either right or left. In addition to the delays caused by the difference in speed, the trucks (unless properly provided for) again create serious delays when in the act of loading or unloading. The municipality should encourage all new buildings to provide all-street loading, as well as off-street parking facilities. It should provide places such as loading zones where these vehicles can pull into the curb to discharge or to pick up their loads, without stopping in the second lane. Establishments that are already developed and have not provided off-street loading facilities should be encouraged to confine their loading, if possible, to their alley or their secondary street exits. The municipality should reserve these alleys for such use by the restriction of parking because, not only do they provide the merchants with loading facilities, but they are important fire-protection factors for the buildings in the congested areas. It would not be practical to prohibit the use of streets in the congested area to commercial vehicles, because of the hardship it would inflict on the business establishments. It is possible, however (at least, at certain periods of the day) to restrict certain types of deliveries, such as coal, produce, etc.; and if such arrangement can be made with the express companies, that they endeavor to do most of their delivering prior to noon. From studies made by the Bureau of Traffic Planning Survey, in December, 1934, in Pittsburgh, 48% of the vehicles that stop at the curb are being loaded or unloaded with either passengers or merchandise, which indicates the demand for the aforementioned facilities.

The operation of taxicabs in the congested area creates another form of internal interference during the peak periods of the day. The taxicab, unless otherwise regulated, will travel at a slow rate of speed, because the driver hopes to have a pedestrian hail him for service. This is encouraged by the practice of taxicab companies that compute the wages of their drivers on a percentage basis of their day's receipts. Were the driver proceeding at the same rate of speed as the other vehicles, he is liable to lose this business, as he could not stop suddenly without the possibility of an accident to the vehicle traveling behind him; or he might not see the pedestrian signaling for him. A study of the operation of taxicabs in Pittsburgh showed that 26% were cruising (that is, they approached the same location empty twice or oftener within a 30-min period). The municipality may remedy this interference caused by cruising and slow moving taxicabs by requiring taxicab companies to provide adequate off-street parking facilities, and to see that ordinances be enforced rigidly. However, taxicabs are not the only vehicles that "cruise." Studies made at many intersections in the business district of Pittsburgh showed that in some cases 17% of the traffic on certain streets was cruising traffic (that is, traveling around the block either looking for a place to park, or looking for a person they were to meet, who happened to be late, etc.). To remedy the delay caused by the cruising passenger car is decidedly difficult. This problem has been solved in one or two cases in Pittsburgh by restricting the parking, which eliminates those vehicles looking for a place to park, or by restricting turns at the intersections where cruising is prevalent. These restrictions of turns and parking have made it so inconvenient for the person cruising that many have been discouraged in this practice, and have arranged to meet people in less congested areas.

At wide, uneven intersections having a large street area, where many of the vehicles weave and cross each other, the construction of islands and the "channelization" of the roadway confines the various movements to their proper lanes, and frequently eliminates many of the weaving and crossing delays that formerly existed at the old intersection, in addition to creating a safer condition for both vehicular and pedestrian traffic.

The co-ordination of signals enters into internal interference in that the time for starting and stopping the various types of vehicles differs materially. If the signals are co-ordinated so that the necessity for stopping and re-starting is minimized, many of these delays will be eliminated.

Probably the most serious type of internal interference is due to the loading and unloading of public carriers—the street car and the bus. With the modern improvement of these mass transportation vehicles and the recent development of the model street car, the delay due to different traveling speeds has been greatly reduced. The bus, when stopping to pick up or to discharge passengers, pulls over to the curb and obstructs the curb lane in the same manner as a parked car. The street car does not have the flexibility of being able to pull to the curb to pick up or to discharge its passengers, so that, in many cases, when in the act of loading or unloading, it obstructs not only the curb lane, but also the free lane in which the street car travels. If the adjacent area is controlled by a co-ordinated signal system, signals should be timed so that most of these mass transportation vehicles will arrive at their stops at that period of the signal sequence when traffic in their direction is stopped. That this is practicable is illustrated by the example in Pittsburgh, where it is possible, due to correct signal timing, to do 63.3% of the loading on the stop signals. Another remedy for delays in both lanes by street cars is the construction of street-car loading platforms, which permit vehicles between the loading platform and the curb to move during the time the street car is loading. There has been considerable agitation to pass legislation such that, if a common carrier arrives at an intersection during the green signal, it should not be permitted to stop to pick up or to discharge passengers, but must proceed to the next intersection. This is a drastic remedy and would create a decided hardship for the people who rely on the mass-transportation vehicles.

As previously stated, considerable internal interference has been due to the difference in the speeds of the various types of vehicles and the necessity for the mass transportation vehicles to stop frequently to pick up or to discharge passengers. A possible remedy for this interference is illustrated by the experience in Pittsburgh where two main adjacent thoroughfares, Smithfield Street and Wood Street, were restricted to one-way traffic, northbound and southbound, respectively. The effect was to restrict the parking of automobiles to the right-hand curb lane, and the slow-moving and mass-transportation vehicles to the lane adjacent to the parked vehicles. The success of this plan is illustrated by the lane-ratio data recorded at every intersection along these two main streets, which show that the extreme right lane has a usage of 2.5%, the lane adjacent to the parked vehicles, a usage of 24.5%, the next or third lane from the right, a usage of 60.5%, and the extreme left curb lane has a usage of 12.5 per cent. This indicates that the left half of the street carries 73% of the traffic, whereas the right half carries 27 per cent. If two-way traffic were permitted on these streets, there would be considerable request for permission to park on both sides of the street, and it is probable that street cars would operate in both directions on both streets. These data indicate that making these streets one-way has permitted an increase in capacity of 80% more than if they were operated on a two-way basis. This accomplishment was due to the provision of lanes for the faster moving vehicles; and it does not restrict the capacity of the streets to that of the slower moving vehicles. It is not practicable, however, to confine any street to one-way traffic. Study should be made of the origin and destination of the traffic using the streets, and the possibility of ready access to and from the various destinations. Unless such consideration is given, one-way streets are likely to cause an unnecessary distance of travel and the addition of turns to offset the acknowledged advantages.

Remedies for Delays Due to Medial Interference

The degree of medial interference is largely dependent on the lane widths of the roadways. Where the lanes of a street have a width of not less than 9 or 10 ft, there is little or no medial interference. As the lane width is reduced, vehicles traveling in opposite directions will proceed at slower rates of speed. Medial interference is created when faster moving vehicles desire to overtake and pass other vehicles by pulling around to the wrong side of the roadway. This type of interference can be eliminated readily by the construction of center strip islands, which have yet another advantage in that they can be designed to provide a "reservoir" off the roadway proper permitting vehicles to turn left without obstructing free-moving traffic. The effect of the center strip island is more to eliminate the hazard of accidents than to minimize delays of traffic. It is felt, in general, that such constructions are impracticable in congested districts, because of the additional valuable street area required—an area already limited in many cases. The delays due to medial interference are practically negligible where proper lane widths are available, and it is suggested that concentration be given to the other types of delays because of their seriousness.

OTHER REMEDIES FOR DELAYS

The preceding parts of this paper have dealt with the various types of interference that cause delays to motor traffic. As to the exact seriousness of these delays, little has been published except to evaluate it in terms of units. The relative seriousness is dependent upon the volume of traffic that is involved during certain periods; as this volume increases the congestion will decrease, and as it decreases, congestion will decrease correspondingly. The engineer is faced with the additional problem of analyzing the street system to determine, at no hardship to the inhabitants of the municipality, methods of decreasing the volume of traffic that must travel on the various streets and thus reduce existing congestion.

A possible remedy for reducing the volume of traffic during the peak period is to stagger the business hours. This solution has been discussed for many years and, in some cases, has been found impracticable because of the interlocking of the various types of businesses. It is doubted whether any sincere experiment has been made of the staggering of business hours, because of the habits of the American people and their desire to associate at common hours. For example, when production is necessary, factories and mills operate 24 hr per day; milk companies deliver milk in the early hours of the morning because the public demands fresh milk for breakfast; and meat packers, in order that their products shall not spoil, must deliver their loads before the sun is high. There is no difficulty in the operation of the milk companies or the meat packers, except that there has been a demand that has required the staggering of their business hours. As conditions become more critical and as the various industries realize their loss because of the congestion of traffic, they will undoubtedly favor changing their transportation schedules to more profitable hours. Such a solution of a whole-hearted staggering of business hours is possibly "far-fetched." However, it may be the ultimate solution that the American people must expect, in order to remedy their traffic "snarl." If such a plan were followed, the critical peak period, because of being distributed over several periods, would be materially reduced, and congestion would be correspondingly decreased.

Another possible means of decreasing the volume of traffic is to make a careful analysis and survey of the existing mass transportation system. During the ten years, 1927 to 1936, the private automobile (which carries merely an average of 1.8 persons) has transported about 50% more passengers into a business district than formerly, whereas the total number of passengers carried by mass-transportation vehicles has decreased. This has been due to a change in the mode of transportation of the general public from that of the original mass-transportation vehicle to the private automobile. This change has probably been due to the additional convenience afforded by the private automobile. The engineer of a municipality, whose duty it is to facilitate the movement of traffic safely, should strive to handle the largest number of people in the least number of vehicles. There is little question but that an individual, when driving his automobile to and from work and parking it in the congested area, spends several times the money that he would in using a mass-transportation vehicle. As the mass-transportation facilities are improved, there is little question but that the number of private automobiles in the congested area will be reduced, which will relieve the congestion and also decrease the demand for street and off-street parking spaces.

A survey should be made of the various routes of the surface common carriers: Their speeds, their points of loading and unloading, and the type of their service. There has been considerable controversy over the proper type of routing for surface carriers in a congested area. Three types have been advanced: "Short-looping," "through routing," and general routing. "Short-looping" consists in bringing the common carriers to the outskirts of the business area, discharging and picking up their passengers, and proceeding away from that section. In "through routing," the common carriers proceed straight through the business section, stopping to discharge and pick up passengers as required, and proceeding in the opposite direction from which they originated, to the end of the line, where they turn around and proceed back to town. General routing is the type most commonly practiced to-day. This is the routing of the common carriers through the business section, past the various large department stores, into the center of the section, and turning and proceeding back through the town. This type requires the carriers to make many turns, cross one another frequently, and travel practically a double distance in the congested area. There is no "hard and fast" solution for this condition of the private automobile vying with the surface common carrier. Some of the solutions advanced to the present day involve the construction of elevated lines and subways; and yet the city that does not have these facilities, and must rely entirely on surface common carriers, has a more critical problem. The engineer of a municipality should strive to improve mass transportation facilities, which in the end will decrease the use of the private automobile in the busy areas, and thus reduce congestion.

A third possible solution for reducing traffic volumes is to study and analyze street systems carefully to determine whether adequate by-pass facilities are available to avoid congested areas. From an origin and destination count made in Pittsburgh, indications are that 49.7% of the vehicles entering the business section between the hours of 6:00 A.M. and midnight, left by a different throat from that by which they entered, within a period of 20 min, which time was practically equal to that required to travel from the entry throat to the exit throat. This indicates that the vehicles did not stop in the business section, but, because of not having adequate by-pass facilities, were obliged to use the connection through the down-town section. From further analysis of these entering and leaving data, it is evident that not only does this through traffic hold consistently through the day, but that during the peak period from 4:00 P.M. to 6:00 P.M., 53.3% of the flow in the business section was through traffic, which, if eliminated, would materially relieve the congestion. This is a startling example to illustrate the relief of congestion that would be afforded the City of Pittsburgh if adequate by-passes were provided around the business district.

One should not be misled by this statement, to conclude that facilities are not available in Pittsburgh to enable the avoidance of the congested area, as there are several entry routes that can be used. However, these routes do not have the characteristics that would encourage motorists to use them in preference to the route through the business section. By-pass routes provided should be of a characteristic high-speed design, so as to minimize delays, and should be connected properly to all the entering and leaving arteries. Unless a by-pass has a high-speed characteristic and is attractive to motorists, it will be of little value, and will fail to serve the purpose for which it was created.

Prior to making the "origin and destination" count an engineer making the statement that 53.3% of the flow was through traffic would have been criticized, as it was felt to be an absurd value. Consequently, it is probable that this condition is present in other cities, even if highway officials feel that they have adequate routes for avoiding the business sections; but from a study such as the one cited, the engineer will determine whether these facilities are being used for the purpose intended, and if they are not being so used, an analysis of these by-pass routes should be made to eliminate the delays that are diverting traffic to the congested areas.

Another means of reducing delays falls within the jurisdiction of the police authorities. In preceding parts of this paper, there was discussion of possible restriction of parking, possible restriction of turns, regulation of pedestrian traffic, etc. After a careful study has been made and regulations have been recommended and enacted, it is necessary that they be enforced rigidly, and this is the province of the police. The engineer should be assured of the full co-operation of the police authorities on any regulations established. To enact a regulation does not fulfill the remedy for the delay in traffic; the removal of the vehicle causing the conflict is also required, and it is that conflicting vehicle that must be removed by the police officials.

A municipality should have highly trained police officers, deserving, and having, the respect of the motoring public. Such a staff will facilitate materially the movement of traffic and will be prepared to reduce congestion in the event of an emergency.

Conclusion

To relieve traffic congestion and increase the capacity of a street system is possible, as illustrated in several ways in this paper. Responsibility for the particular type of remedy to be devised is placed upon the traffic and city planning engineers. Their decisions must take many phases into account, such as the habits of the people, the demands of merchants, the cost of the proposed improvements, and the extent of the relief that is to be expected. In many cases, it has been found advantageous, in making such decisions, to temper the recommendations of the engineers by the opinions and suggestions of a group of citizens representing all classes of society, so that by providing an opportunity for every phase of business life to consider the problem, the best possible solution may be obtained. However, the engineers know the causes of congestion and the remedies for increasing the capacity of existing street systems, and any final decisions should be made with their guidance and advice.

IMPROVEMENTS TO REDUCE TRAFFIC ACCIDENTS

By Arnold H. Vey,[11] Esq.

Synopsis

Any detailed analysis of motor-vehicle accident reports discloses numerous conditions, circumstances, and situations involved in the problem of highway accidents. Consequently, hundreds of causes must be considered and, to complicate the problem further, these causes may operate either singly or in combination. Therefore, it is difficult to determine, with any degree of accuracy, the exact number of accidents that may be charged to the car, the highway, or the highway user. All three elements are involved, and they contribute in one way or another, and in varying degrees, to highway accidents; but the highway user (the driver and pedestrian) is responsible for the greatest number. In other words, most motor-vehicle accidents are caused by some improper action on the part of the person or persons involved, regardless of whether such action was wilful, or due to negligence or ignorance.

It seems reasonable, therefore, to suppose that if all highway users knew, and would voluntarily observe, safe highway practices at all times, the present-day accident problem would be materially lessened. Actually, however, the solution is not so simple. Because of human frailties and the impossibility of supervising and governing the actions of all persons at all times, safety on the highway is made extremely difficult. Educational endeavors and increased enforcement activities are not only essential but invariably prove helpful. However, such activities, despite their importance, are not in themselves a complete solution. It is necessary that roadway facilities be designed and applied which are inherently safe, as far as possible, in order to render the failures of human beings of lesser or little importance.

Roadway facilities, of course, are of many component parts, a number of which may be classed as major highway improvements. Time and space will not permit a discussion of all such improvements, and it is the purpose of the writer, therefore, to discuss only those about which accident facts are immediately available. As most major highway improvements during recent years have concerned State highways and, primarily, in rural or less densely populated areas, the scope of this paper is confined largely to improvements on the State Highway System of New Jersey.

Note.—Presented at the meeting of the Highway Division, New York, N. Y., January 21, 1937.

[11] State Traffic Engr., Div. of Traffic Control and Regulation, State Dept. of Motor Vehicles, Trenton, N. J.

In this connection, it is important and also of interest to have some knowledge of the accident problem as it exists in New Jersey and as it concerns New Jersey State Highways.

Accident Experience in 1934, New Jersey State Highway System

The data included herein are taken from reports of accidents filed by the motorists involved, together with supplemental reports from police and other officials concerned with motor-vehicle traffic and highway accidents. The reports include accidents involving death and personal injury, as well as those resulting in property damage to the extent of $25 or more.

Total Accidents.—Table 10(*a*) represents the number and severity of accidents in New Jersey during 1934. Of the total accidents, 3.5% involved fatalities, 55.9% involved personal injury, and the remaining 40.6% concerned property damage only; 7 217, or 21.4%, occurred on the State Highway System.

TABLE 10.—Number and Severity of Accidents in New Jersey During 1934

	(*a*) Classified by Type of Roadway						(*b*)* Classified by Number of Lanes		
Severity of accidents	State Highways		All Other Roadways in the State		Total		Two lanes	Three lanes	Four lanes
	Number	Percentage of total	Number	Percentage of total	Number	Percentage of total			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Fatal	400	5.5	742	2.8	1 142	3.5	5.05	6.24	6.27
Non-fatal	3 768	52.3	15 069	56.9	18 837	55.9	50.23	54.51	52.59
Causing property damage	3 049	42.2	10 625	40.3	13 674	40.6	44.72	39.25	41.14
Total	7 217	100.0	26 436	100.0	33 653	100.0	100.00	100.00	100.00

* Section (*b*) pertains to State Highways only.

It is interesting to compare this percentage of State highway accidents to total accidents with the percentage of State highway mileage to total miles of roads in New Jersey; for example, of the more than 35 000 miles of improved and unimproved roadways in the State, only slightly more than 1 500 miles, or nearly 6%, represent roadways under the jurisdiction of the State Highway Department. In other words, about 6% of the State roads are the scene of more than 21% of the accidents. Such a comparison, however, is not necessarily a measure of the hazards of State highways, as consideration must be given to the use made of them. Unfortunately, vehicle-mile data, segregated by the jurisdictions of the different classes of official bodies, are not available.

When comparing the severity of accidents on State highways with those on all other roads, it is found that, of the total State highway accidents, those involving fatalities, in percentages, equalled nearly twice the percentage of fatal accidents to total accidents for the remainder of the State, whereas a similar comparison for injury accidents showed a percentage slightly less than that for the remainder of the State. However, if pedestrian accidents are excluded from the results (as most accidents of this class occur in urban or built-up districts), it is found that the percentage of fatal and non-fatal accidents to total accidents is greater on State highways, in each instance, than on the remaining roadways of the State. This is the result of higher average speeds on State highways, of course, and a lesser enforcement of laws and supervision of activities in rural areas than in urban and built-up areas.

A comparison of pedestrian accidents occurring on State roads with those on the remainder of the State, indicates that, of the total pedestrian accidents on State highways, 25% concerned fatalities, whereas for all other roadways only 6% resulted in fatalities. This, too, is the result not only of greater speeds, but also of the lack of pedestrian sidewalks or pathways along most State highways, as well as lack of proper and sufficient illumination.

Accident Location and Time of Occurrence.—Of the total State highway accidents in 1934, 40.4% occurred at street intersections and the remaining 59.6% at intermediate points. It may be seen, therefore, that the sections of State highway having little or no cross-movements of traffic are the more hazardous. Accidents that occurred at night equalled 46.3% of the total, whereas those occurring during hours of daylight represented the remaining 53.7 per cent. Despite the fact that approximately one-fifth of the 24-hr traffic represents night-time travel, this lesser volume of traffic produces nearly one-half the accidents. In other words, it is more than three times as hazardous to drive at night as during the day.

Accidents per Million Vehicle-Miles.—To afford a comparison of accident experience and hazard on State highways of different widths, an analysis was made of accident rates on those roadways or parts of roadways having two, three, and four improved, hard-surfaced lanes used for the purpose of vehicular travel. Table 11 gives the results of this study.

TABLE 11.—ACCIDENTS PER MILLION VEHICLE-MILES, NEW JERSEY STATE HIGHWAYS, IN 1934

Number of lanes	Million vehicle-miles	Number of accidents	Number of accidents per million vehicle-miles
Two	1 178.78	3 241	2.75
Three	390.85	1 378	3.53
Four	483.21*	1 743	3.61
Total	2 052.84	6 362	3.10

* Includes about thirty miles of divided highway.

From this analysis it is determined that, for the State Highway System studied, the average accident rate equals 3.10 per million vehicle-miles, whereas on two-lane roadways the rate is 2.75, on three-lane roadways, 3.53, and on four-lane roadways, 3.61. In other words, an increased accident rate occurs with an increased width of highways; or (contrary to common belief) the wider the road the more hazardous it is. It is interesting, too, to note that, contrary to common belief, four-lane highways of the undivided type are more hazardous than three-lane roadways.

The writer does not mean by this statement that he supports or endorses the construction of three-lane roadways. However, the facts do prove (at least in so far as New Jersey is concerned) that the rate on four-lane roads is greater by 0.08 accident per million vehicle-miles than on three-lane roadways. There are many possible explanations for this condition, the primary one being that, the wider the road is, the more flexible will be the movement of traffic from one side to the other, unless congestion or enforcement prevents such movements.

A comparison of the severity of accidents on two-lane, three-lane, and four-lane roadways is shown in Table 10(*b*). It is seen that fatal accidents increased slightly with the width of the highway, whereas non-fatal and property-damage accidents fluctuated without regard to highway width. Segregating the accident experience of two-lane, three-lane, and four-lane roadways by accident types, it is found that the percentage of right-angle collisions decreased with the width, as did head-on collisions and fixed-object collisions. Same-direction accidents, however, increased with the width, and opposite-direction accidents (involving a left turn), and pedestrian accidents, fluctuated without regard to highway width. In other words, despite the increase in accident rates with increased width of highways, certain types of accidents conversely decreased, whereas other types increased, and still others fluctuated without regard to width.

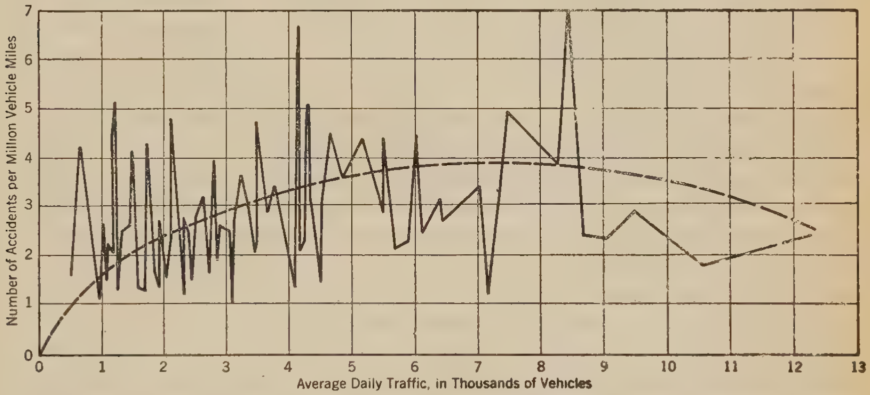


FIG. 4.—RELATIONSHIP BETWEEN DAILY TRAFFIC AND ACCIDENTS PER MILLION VEHICLE MILES FOR TWO-LANE ROADS

Relationship Between Daily Traffic and Accident Rates.—To determine the relationship between daily traffic and accident experience, a survey was made on two-lane roads, or on sections of two-lane roads, in the State Highway System, the results of which are shown in Fig. 4. It is interesting to note the rather definite relationship between daily traffic volume and the number of accidents per million vehicle-miles. In other words, as the average daily traffic increases, accident experience, on a million vehicle-mile basis, likewise increases to approximately 7 000 vehicles per day, which is considered by many authorities to be the capacity of a two-lane highway. Beyond that point, primarily because of congestion and the resultant decrease in speed and flexibility of movement during heavy traffic hours, there is a gradual decrease in the accident rate, despite the increase in traffic.

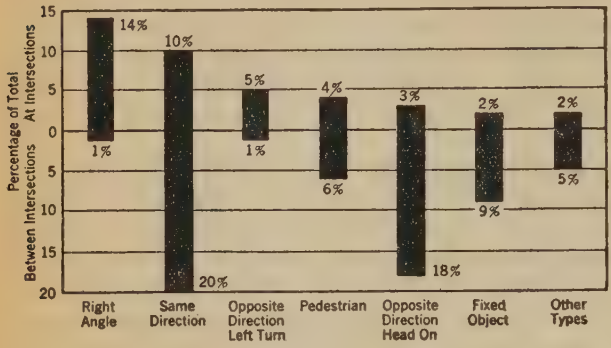


Fig. 5.—Types of Accidents on State Highways

Types of Accidents.—An analysis of the extent of accidents of various types which have occurred on the New Jersey State Highway System will give an indication as to the probable degree of usefulness of some of the major highway improvements of to-day. Fig. 5 illustrates the relationship of various types of accidents to total accidents, arranged so as to compare those occurring at intersections and between intersections.

Assuming that it is physically and economically possible to reconstruct the New Jersey State Highway System to the extent of applying major highway improvements now familiar to the highway engineer, including controlled rights of way, in addition to physical separation or segregation of conflicting flows of traffic, adequate shoulders for stopping, pedestrian sidewalks or pathways, proper and sufficient highway lighting, plus other possible advanced features, 75% or more of the accidents that occur to-day could be avoided. In other words, the use of such highway safeguards would make it physically impossible to experience many of the types of accidents indicated in Fig. 5.

For example, a controlled right of way, prohibiting ingress to, and egress from, the highway, except at specific points where there are grade separations, would almost entirely correct the right-angle collision type of accident, as well as collisions between vehicles going in opposite directions with one making a left turn. A center island would prevent most head-on collisions and possibly, too, it might have some effect on same-direction, as well as other, accidents. Pedestrian sidewalks or pathways along major highways would have a materially favorable effect in reducing rural pedestrian accidents.

Furthermore, proper and adequate highway lighting, not only at roadway crossings, but also along sections of heavily traveled highways between crossings, will enable a reduction of all accidents, many of which are of the same-direction type, fixed-object type, as well as the pedestrian type, and which are chargeable to inadequate vision and illumination.

It is impossible, of course, to reconstruct the State Highway System in such a manner without prohibitive expenditures of money. However, much can now be done toward applying some of these so-called highway safeguards or major improvements. Such action will enable a reduction in accidents sufficient to make the necessary expenditures economically proper. Consider therefore, the accident experience as a result of the application of some of these improvements.

Accident Experience: "Before" and "After" Improvements.—Unfortunately, in most instances, major highway improvements on the State highways of New Jersey were constructed as a part of the initial system and, therefore, reliable "before" and "after" records are not available. In the case of divided roadways, however, it is possible to obtain some rather interesting and reliable "before" and "after" facts for parts of highways that have been converted from an undivided highway to that of a divided type. In the case of other major improvements, and where it has not been possible to obtain "before" and "after" records, interesting comparisons can be made indicating the usefulness of the improvement.

Divisional Islands.—Divisional or center islands that separate opposing flows of traffic, when properly constructed, make it practically impossible for motorists traveling in opposite directions to collide head-on. This type of highway construction has been used in New Jersey for the past several years but, until recently, proper and adequate "before" and "after" studies to determine the usefulness of such construction have not been possible. As a result of converting parts of State Highway Route No. 26 from a four-lane undivided highway to a four-lane divided highway (by sliding one lane of concrete to the side for a sufficient distance to permit the construction of a 12-ft center island, and adding an additional lane to make available a two-lane roadway on either side of the island), it has been possible to obtain "before" and "after" records for a rather limited period. Table 12(*a*) represents a "before" and "after" study on 2.1 miles of this highway in the Township of North Brunswick.

A review of Table 12(*a*) indicates a general reduction of 36% in accidents of all types, after the conversion. It will be noted, too, that not all types of accidents have decreased. Some indicate no change, whereas one type shows an increase. It is important to note, however, that this study—representing not only a rather short section of road but also a limited period of time (not quite five months)—is inadequate to determine with any degree of certainty that the conversion of this roadway to one of the divided type will eventually bring about such a reduction as is now indicated. Nevertheless, it is the writer's opinion that a reduction somewhat comparable to this will be shown as time goes on. Incidentally, all characteristics of the converted highway remained the same as before, with the exception of the construction of the center island as mentioned.

Another example of accident experience after the construction of a center island, but where the roadway was converted from a two-lane highway to a four-lane divided highway, is State Highway Route No. 25 in Cinnaminson and Delran Townships. Table 12(*b*) shows the results of a "before" and "after" analysis of this stretch of highway, representing approximately seven miles of road.

The study indicates a total reduction of more than 46% in all types of accidents, and whereas some types of accidents are reduced, others are increased, and still others remain the same. In this case, however, the reduction

TABLE 12.—ACCIDENT EXPERIENCE, NEW JERSEY STATE HIGHWAYS

Type of accident	EXPERIENCE BEFORE AND AFTER IMPROVEMENT						COLLISION RECORD						
	At Intersection		Between Intersections		Total		Daylight				Night		
	Before	After	Before	After	Before	After	Fatal	Not fatal	Property damage	Total	Not fatal	Property damage	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	(a) CONSTRUCTION OF CENTER ISLAND, ROUTE NO. 26, NORTH BRUNSWICK TOWNSHIP						(d) CLOVERLEAF AT INTERSECTION OF ROUTES NOS. 4 AND 25, AT WOODBRIDGE						
Approaching at right angle	..	..	1	..	1	..	..	..	..	..	..	..	..
Approaching in same direction	..	1	3	3	3	4	..	3	2	5	..	..	..
Approaching in opposite directions:													
Left turn	..	1	1	..	1	1	..	..	..	..	..	..	..
Head-on	..	..	2	..	2	..	..	..	..	..	..	..	..
Fixed object accidents	..	..	2	..	2	..	..	..	..	..	..	..	..
Pedestrian accidents	..	..	2	2	2	2	..	..	..	..	..	..	..
Other types of accidents	..	..	..	..	..	..	..	..	1	1	..	..	..
Total	..	2	11	5	11	7	..	3	3	6	..	..	..
	(b) CONVERSION OF ROUTE 25, IN CINNAMINSON AND DELRAN TOWNSHIPS, FROM TWO-LANE TO FOUR-LANE DIVIDED PARKWAY						(e) INTERSECTION OF ROUTE NO. 25 AND AVENEL STREET; IN WOODBRIDGE TOWNSHIP						
Approaching at right angle	3	3	..	..	3	3	..	1	..	1	..	..	..
Approaching in same direction	1	1	5	4	6	5	..	3	1	4	2	2	4
Approaching in opposite directions:													
Left turn	..	1	..	3	..	4	..	1	1	2	..	..	..
Head-on	1	1	12	..	13	1							
Fixed object accidents	1	..	5	1	6	1	..	2	..	2	..	1	1
Pedestrian accidents	1	..	1	2	2	2	..	..	..	..	..	..	..
Other types of accidents	..	..	..	..	..	..	..	1	..	1	..	..	..
Total	7	6	23	10	30	16	..	8	2	10	2	3	5
	(c) CONSTRUCTION OF CENTER ISLAND, ROUTE NO. 25, IN THE CITY OF ELIZABETH						(f) INTERSECTION OF GEORGES ROAD WITH ROUTES NOS. 25 AND 26, IN NORTH BRUNSWICK TOWNSHIP						
Approaching at right angle	7	6	..	..	7	6	..	1	..	1	1	..	1
Approaching in same direction	21	47	4	3	25	50	..	3	3	6	3	3	6
Approaching in opposite directions:													
Left turn	7	7	..	..	7	7	1	4	3	8	1	2	3
Head-on	..	..	2	1	2	1							
Fixed object accidents	3	1	..	1	3	2	..	..	1	1	..	..	..
Pedestrian accidents	10	1	1	..	11	1	..	..	..	..	..	..	..
Other types of accidents	2	6	1	..	3	5	..	..	..	..	..	..	..
Total	50	68	8	5	58	72	1	8	7	16	5	5	10

cannot be entirely credited to the conversion of this highway into a divided roadway as, in addition to the construction of the 10-ft to 12-ft center island, the roadway width was increased from two lanes to four lanes plus a favorable straightening of the alignment in many instances. This study concerns a period considerably longer than that for Route No. 26 as can be noted from Table 12(*b*) and is, therefore, somewhat more indicative of the ultimate results of the improvement.

A further example of the effect of the construction of a center island, but where the improvement differed slightly from that of either of the two previous cases, and where the district is substantially urban rather than rural, is that of Route No. 25 in the City of Elizabeth. Table 12(*c*) shows a comparison of accident experience along 1.3 miles of this highway before and after the improvement. This thoroughfare was initially constructed with narrow islands on each side, approximately 4 ft wide and 20 ft from the curb lines, to segregate local traffic from through traffic. As the roadway width between curb lines was 96 ft, there remained approximately a 48-ft roadway in the center between islands for through traffic.

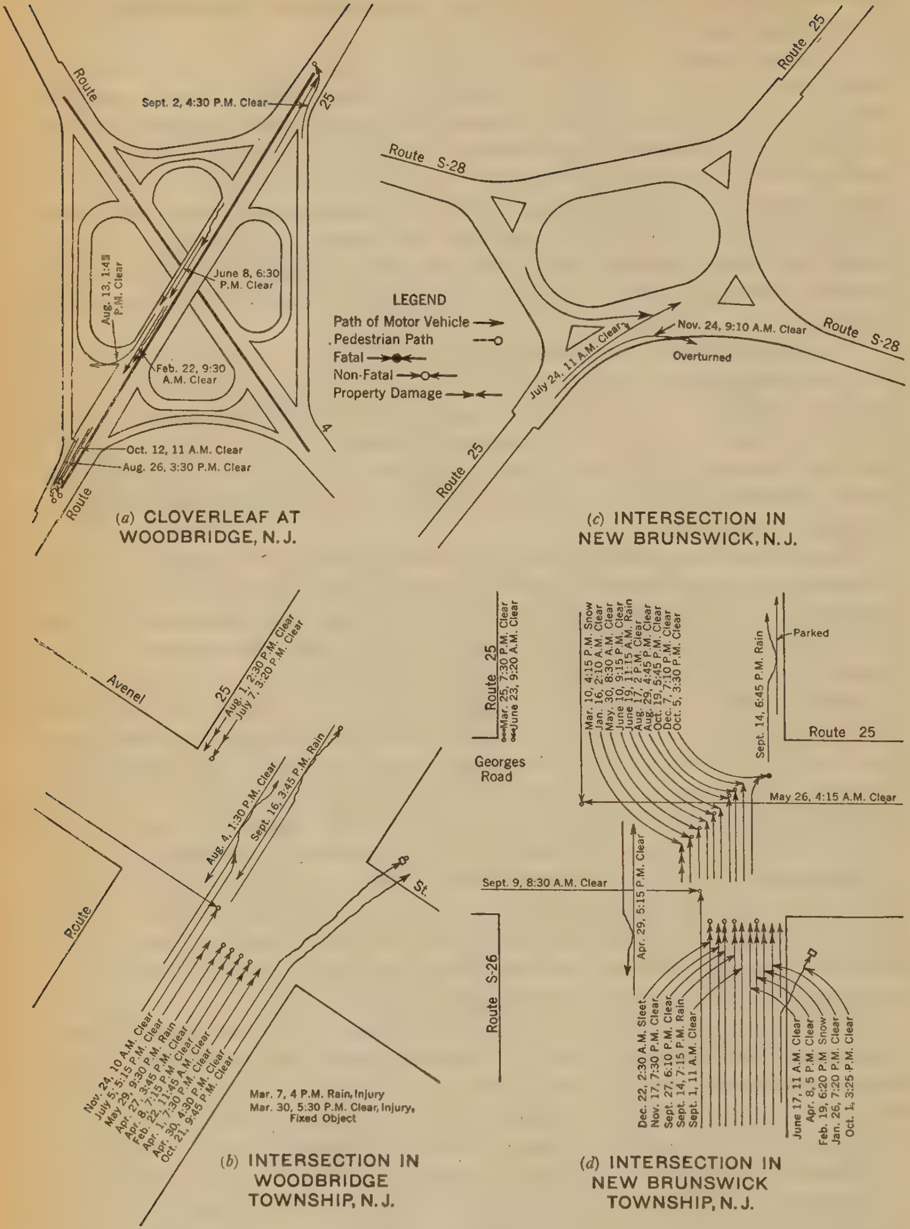
The improvement concerned the removal of the so-called local islands and the construction of a center island, 16 ft wide, down the center of the highway, leaving, therefore, a 40-ft roadway on either side of the center island. A review of Table 12(*c*) shows an increase of about 24% in total accidents after the improvement. Only two types of accidents have increased, however, and one of these types—same-direction accidents, primarily rear-end collisions—increased 100 per cent.

The increase in same-direction accidents is probably the result of improper re-location of signal units at the ten signalized intersections along this stretch of highway after the improvement, in a manner such that motorists traversing the highway can unmistakably receive a signal command sufficiently in advance of the crossing to govern their actions accordingly. It is planned to re-locate signal units properly along this highway, and to improve their operation; and the writer firmly believes that after this further improvement there will be a material decrease in this type of accident. Assuming that a decrease can be accomplished, it will be seen that the improvement of this stretch of highway—eliminating the so-called local islands and constructing a center island—will have a favorable effect on accident experience.

With regard to the improvements at all three of the aforementioned locations, it is interesting to compare the severity of accidents before and after the improvement. For example, on Route No. 26, in North Brunswick, fatal accidents decreased from 2 to none; injury accidents increased from 3 to 5; and property damage accidents decreased from 6 to 2.

On Route No. 25, in Cinnaminson and Delran Townships, fatal accidents decreased from 2 to none; injury accidents decreased from 14 to 5; and property damage accidents decreased from 14 to 11; and on Route No. 25, in Elizabeth, fatal accidents decreased from 2 to 1, injury accidents decreased from 48 to 19, and property damage accidents increased from 8 to 52.

Grade Separations and Traffic Circles.—Unfortunately, it has been impossible to obtain "before" and "after" records for intersections of highways separated by grade, and at traffic circles. However, it is interesting to make a comparison of accident experience at one of the first grade separations constructed in New Jersey with that of a signalized highway intersection along the same roadway only a short distance to the north; and also at a traffic


Route
25
Route
Sept. 2, 4:30 P.M. Clear
June 8, 6:30 P.M. Clear
Aug. 13, 1:45 P.M. Clear
Feb. 22, 9:30 A.M. Clear
Oct. 12, 11 A.M. Clear
Aug. 26, 3:30 P.M. Clear
4
Route 25
Route S-28
Route S-28
Route 25
LEGEND
Path of Motor Vehicle
Pedestrian Path
Fatal
Non-Fatal
Property Damage
July 24, 11 A.M. Clear
Nov. 24, 9:10 A.M. Clear
Overturned
Avenel
25
Aug. 1, 2:30 P.M. Clear
July 7, 3:20 P.M. Clear
Aug. 4, 1:30 P.M. Clear
Sept. 16, 3:45 P.M. Rain
Route
St.
Nov. 24, 10 A.M. Clear
July 5, 5:15 P.M. Clear
May 29, 9:30 P.M. Rain
Apr. 27, 3:45 P.M. Clear
Apr. 8, 7:15 P.M. Clear
Feb. 22, 11:45 A.M. Clear
Apr. 1, 7:30 P.M. Clear
Apr. 30, 4:30 P.M. Clear
Oct. 21, 9:45 P.M. Clear
Mar. 7, 4 P.M. Rain, Injury
Mar. 30, 5:30 P.M. Clear, Injury, Fixed Object
Route 25
Mar. 25, 7:30 P.M. Clear
June 23, 9:20 A.M. Clear
Georges Road
Mar. 10, 4:15 P.M. Snow
Jan. 16, 2:10 A.M. Clear
May. 30, 8:30 A.M. Clear
June 10, 9:15 P.M. Clear
June 19, 11:15 A.M. Rain
Aug. 17, 2 P.M. Clear
Aug. 29, 4:45 P.M. Clear
Oct. 19, 5:45 P.M. Clear
Dec. 7, 7:10 P.M. Clear
Oct. 5, 3:30 P.M. Clear
Sept. 14, 6:45 P.M. Rain
Parked
Route 25
May 26, 4:15 A.M. Clear
Sept. 9, 8:30 A.M. Clear
Apr. 29, 5:15 P.M. Clear
Route S-26
Dec. 22, 2:30 A.M. Sleet
Nov. 17, 7:30 P.M. Clear
Sept. 27, 6:10 P.M. Clear
Sept. 14, 7:15 P.M. Rain
Sept. 1, 11 A.M. Clear
June 17, 11 A.M. Clear
Apr. 8, 5 P.M. Clear
Feb. 19, 6:20 P.M. Snow
Jan. 26, 7:20 P.M. Clear
Oct. 1, 3:25 P.M. Clear

(*a*) CLOVERLEAF AT WOODBRIDGE, N.J.

(*c*) INTERSECTION IN NEW BRUNSWICK, N.J.

(*b*) INTERSECTION IN WOODBRIDGE TOWNSHIP, N.J.

(*d*) INTERSECTION IN NEW BRUNSWICK TOWNSHIP, N.J.

FIG. 6.—COLLISION DIAGRAMS (SEE CORRESPONDING DATA IN TABLES 10, 11, AND 12)

circle along this same roadway with that of a signalized intersection a short distance south of this circle.

Fig. 6(*a*), with Table 12(*d*), indicates the number and type of accidents that occurred on the Woodbridge Cloverleaf, where State Highway Routes Nos. 4 and 25 intersect; and Fig. 6(*b*), with Table 12(*e*), indicates accident experience at the signalized intersection of State Highway Route No. 25 and Avenel Street, a short distance to the north of the Cloverleaf. In this vicinity Route No. 25 averages approximately 25 000 vehicles daily, whereas State Highway Route No. 4 carries about 15 000 vehicles daily, or considerably more traffic than the local roadway intersecting with Route No. 25, shown in Fig. 6(*b*). In other words, despite considerably greater volume of traffic using the Cloverleaf grade separation there were nine less accidents than at the Avenel Street grade crossing for the same period. Of the six accidents at the Woodbridge Cloverleaf three were caused by motorists who, apparently in error, took the wrong roadway or proceeded beyond their turn, and then, in an effort to correct themselves (and in violation of traffic regulations), attempted to make a left turn where such a turn was prohibited.

Figs. 6(*c*) and 6(*d*), with Table 12(*f*), show a similar comparison but between a traffic circle and a signalized intersection. The collision data corresponding to Fig. 6(*c*) are:

Accidents due to approaching at right angles	1
Other accidents	1
Total	2

both cases being property damage only. In this case, the signalized crossing (Fig. 6(*d*)) is on the same highway only a short distance to the south of the traffic circle (Fig. 6(*c*)). The volume of traffic on Route No. 25 in this section equals approximately 13 000 to 15 000 vehicles per day. Despite the constant operation of traffic-control signals at the Georges Road intersection 26 accidents occurred there during 1934. Most of these accidents, however, were the result, either directly or indirectly, of the numerous left turns from the north to the east. Without doubt, a properly designed and constructed traffic circle at this intersection would greatly curtail these accidents.

Highway Lighting.—Unfortunately, reliable "before" and "after" studies enabling a determination of accident experience after the installation of highway lighting, are not available in New Jersey. As a result of a survey made along parts of Routes Nos. 25 and 26 in New Jersey (the first of which is properly lighted with up-to-date equipment, whereas the part along Route No. 26 has no lighting whatever, except at a few isolated points or intersections) some rather interesting facts were developed. These facts were reported in a paper entitled, "Relation of Highway Lighting to Highway Accidents," presented before the New York Section of the Illuminating Engineering Society, in 1934.[12] The conclusions arrived at as a result of this study were, in part, as follows:

> "* * * there is a definite relationship between highway lighting and highway accidents and * * *, in general, where highway lighting is adequately pro-

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[12] Not published.

vided, there is a substantial reduction in night accidents. This reduction in night accidents results in an economic saving in all cases in excess of the cost of providing street lighting, not mentioning, of course, the suffering and sorrow which inevitably accompany motor vehicle accidents."

Other Improvements.—There are many other highway improvements, of course, which may be classed as major improvements and which have a favorable effect on accident experience. Such improvements might include better sight distance at curves and hills, as well as at street intersections; flatter crowns and curves; shoulders wide enough to permit vehicles to park off the normal traveled part of the highway; and the proper use of standard signs, signals, and markings (and these only at locations where they will perform useful functions).

CONCLUSIONS

Safety on the streets and highways depends upon the application of proper "safeguards" as well as the use of proper "safe practices"; that is, unless and until the highways as well as motor vehicles are provided with proper and adequate "safeguards" and the highway user performs "safe practices" when making use of these facilities, the motor-vehicle accident problem will not be solved.

Properly designed and applied, major highway improvements or "highway safeguards" will generally produce a favorable effect upon traffic accidents. In other words, such improvements designed for safety make it less possible for highway users to perform improper practices that cause accidents. Even these improvements, however, are not cure-alls and, with streets and highways as they generally exist to-day, and the impracticability of their immediate re-construction, it is necessary through educational endeavors and enforcement activities to cause drivers and pedestrians alike to practice safe highway habits, as well as to maintain the proper attitude toward the other fellow when on the highways. Such campaigns will do much to curtail the horror and suffering now caused by motor-vehicle accidents. A most fruitful field of educational endeavor to-day is that of teaching the safe use of the highway to young and prospective drivers.

ECONOMICS OF ALIGNMENT, GRADE, AND WIDTH

By E. L. Schmidt,[13] Esq.

Synopsis

The preceding papers of this Symposium have dealt with causes of traffic congestion and accidents, and have opened a discussion as to necessary remedies. No adequate discussion of this subject would be complete without summarizing the factors relating to cost, and that is the burden of this paper.

Introduction

The function of a major highway is to connect two or more centers of population and commerce by a roadway and its appurtenances. In the old days, before and during the early stages of the automobile, the guiding principle in considering the relative merits of two or more locations for proposed highway projects was the question of direct costs, and usually the best route was deemed to be that upon which the improvement could be constructed at the least outlay of funds. On the basis of the then existing light volumes of vehicular traffic this probably was not bad economics. As a result of this application of concept, however, most highways at that time followed the "lines of least resistance" to construction problems and rode upon the back of Mother Nature's surface, following contours, hills, and valleys. The consequence, in territory where topography was not kindly subdued, was roads of irregular alignment, heavy grades, and narrow widths.

With the rapid development of the automobile gradually came the realization that costs other than the direct payment of highway construction were involved. The riding public was asked to pay these bills, and rightly so; but if the design of the highway for which this public paid was such as to make him travel longer distances, require greater time, and combat more accident hazards than would be necessary over other possible alternate routes, then this public obviously would be paying proportionately higher costs for automobile operation and probably a greater total price for highway use than would be required with improved and more costly location and development. This condition prompted highway designers to inquire into the question of operation costs. Considering it from this angle alone, the ideal highway would be the one that followed a straight line, was of practically uniform grade throughout, and, of course, provided sufficient traffic-carrying capacity. Such a highway often would not justify the expenditures that would be required to overcome the obstacles of Nature and property developments. Thus, for the conditions of to-day in general, neither extreme is proper. The problem now usually resolves itself into the determination of the most economical highway—the one which will connect the points of termini in the shortest possible traveling distance consistent with reasonable convenience and safety of travel and the economics involved.

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[13] Pittsburgh, Pa.

HIGHWAY ECONOMICS

The costs involved in highway construction have been designated "direct" and "indirect." Direct costs include all expenses for the actual physical construction of the highway improvement. They consist of the following: (1) Right of way and damages; (2) construction; (3) overhead; and (4) depreciation and maintenance. The indirect costs are: (5) Automobile operation; and (6) automobile accidents.

These costs are not all the factors of value that should be considered in highway improvement problems. Among others, two very important factors to be considered are the values of scenic and beautification advantages, and the values that may accrue from the appreciation of property. The former is an intangible consideration and difficult to evaluate; the latter is more easily measured; but this paper is confined to considerations of highway economics from the point of view of traffic.

Traffic Considerations.—When considering two or more locations and types of development for a proposed highway, the most economical route will be that which requires the least annual expenditure for fixed charges, automobile operation costs, depreciation, and maintenance.

In order to make this determination, it becomes apparent that the starting point in the analysis of all highway problems is traffic. The first question to be answered is "What are the volume and characteristics of traffic likely to be during the life period of the highway improvement?" In the past, this has been difficult to foretell because of the almost unbelievable advance of the automobile. However, it seems that future forecasts will be somewhat easier because primarily the volume of traffic bears a proportional relation to population gain, which is decreasing and probably will continue to decrease to a point of practical stability. Moreover, the ownership of automobiles, now at slightly less than one per five persons, surely must be approaching a "point of saturation." Although the average daily use of cars probably will continue to increase, a practical limit cannot be far remote. Certainly, automobile manufacturers cannot continue forever to install more and more speed in their new models. Their appeal to the wants of car owners, of necessity, will have to be in other directions, such as greater safety and comfort, and in simplifying operation technique so that practically every one can drive with ease. The necessary analysis of traffic demands should be made with these and other considerations in mind.

No general statements can be formulated as to what the average conditions may be because the volume and characteristics of highway traffic are changing continuously from section to section. The only way to make a reasonably accurate determination is by means of a careful and painstaking study of the local circumstances of the problem. In general, however, traffic characteristics may be classified somewhat in accordance with the type of community development through which the highway is traversing. For example, in the commercial districts, practically always, the traffic volume is relatively heavy and speed is restricted. In the urban residential sections, traffic volume is less; but usually it is still high, and the average speeds generally are advanced. In rural residential and further outlying rural sections, traffic volumes have diminished to a relatively small value but, on the other hand, speeds have been materially increased.

Standards of Width, Alignment, and Grades.—It is obviously impossible to establish "hard and fast" general standards in regard to desirable highway designs for these different types of development districts. Every single highway problem must be studied and treated in the light of its own peculiarities, and it can be stated without danger of dispute that no two sets of conditions will ever be found to be exactly the same. Nevertheless, there is usually a general similarity between problems that may arise in similar community sections and an attempt will be made herein to define standards that will fit the rule.

Commercial Districts

It is easy to define desirable minimum standards of width, alignment, and grade of major highways that traverse commercial districts, but it is quite another problem to secure the general adoption of such standards. The simplest rule would be to have no major thoroughfares routed directly through commercial sections of any magnitude or importance. Quite often, of course, this rule is not practicable, although there is a decided definite trend toward the by-passing of major thoroughfares around congested business sections. In Pittsburgh, Pa., there is an excellent example of the unnecessary burden of traffic which is superimposed upon city streets because the major thoroughfares leading from outlying districts converge and become a part of the downtown street system. Almost 50% of the local traffic in the business section of Pittsburgh is there, not because of a definite destination in the section, but because it is forced into the "Triangle" due to the absence of adequate by-pass routes. In order to correct this situation, it is proposed to construct a boulevard type of highway along the Monongahela and Allegheny River fronts which, in turn, will connect with a proposed cross-town thoroughfare in the vicinity of the Pennsylvania Railroad Station in Pittsburgh. The cost of this river-front improvement probably will be approximately $5 000 000. No doubt it will be justified time after time, however, by the value of accommodation which it will afford traffic.

In a commercial district, where there may be no choice but to utilize the existing street system for major highway use, attention should be directed toward selecting those streets that are best adapted for the purpose. Wherever possible, streets should be chosen that will afford convenient movement for through traffic with the least interference to local traffic. Of course, no rule can be made for this determination. It can only be solved by a study of the conditions at hand. However, in a commercial district, the hourly traffic almost always will exceed the working capacity of a two-lane roadway for moving vehicles and, therefore, wherever possible, such streets should be selected as will provide capacity for four moving lanes—two in each direction—as well as adequate provision for parking spaces. Generally, six lanes for moving vehicles on one street within the limits of a single roadway, is not advisable, even if there may be evidence of congestion on the four-lane highway. Where it appears that there is need for additional traffic capacity, it is probably better to devise methods by which part of the traffic may be diverted into other streets.

In commercial sections, because of interference caused by traffic on intersecting streets, parked vehicles, pedestrian movement, street cars, and the other usual factors, automobile velocity generally is quite low, probably varying from 10 to 25 miles per hr and averaging about 15 miles per hr. At this range of speeds, the question of alignment does not present any troublesome problems. As long as there are no awkward effects, movement of traffic would not be retarded nor accident hazards increased because of short radius curves. At street intersections, it is desirable, if possible, to provide minimum radii of approximately 50 ft in order that large buses and trucks may turn without causing interference in other lanes of traffic.

The maximum grade in commercial districts, where justified economically, should not exceed 4 per cent. In old established and concentrated business sections, highway improvements, such as providing better alignment, reduced grades, or increased widths of roadways, usually will require enormous direct costs due to the high values of property along the right of way. Ordinarily, in order to justify these expenditures, the advantages that will accrue from the improvements must be of considerable magnitude.

Urban Residential and Other Outlying Districts

Outside the commercial districts, traffic characteristics will vary considerably, and desirable minimum standards of alignment, grade, and width of highways will depend upon the conditions peculiar to the problem in question. In general, it may be stated that the desirable curvature, gradient, and width of roadway, depend upon the volume and speed of the traffic that probably will use the highway, as well as the general topography of the territory in which the improvement is to be made.

Physical Characteristics

Widths of Roadways.—The determination of roadway widths is fundamentally a study of roadway lane capacity. Ordinarily, the definition of lane capacity has been interpreted as the maximum number of vehicles that can pass a given point on the highway, in a given time, at specified speeds. This measure of capacity, an arithmetical computation, has been determined frequently and it has been found possible, without interferences, to pass 2 000 cars per lane per hr, at a speed of 20 miles per hr. This finding, however, has no real practical value since there is no consideration given to the speeds that drivers would naturally be inclined to follow in the absence of limitations imposed by traffic, and no clue as to the point where congestion may be said to begin. The latter is an extremely important consideration since it determines, more or less, the time when attention should be given to the question of highway capacity improvement.

A better definition of capacity was given by the Highway Research Board, National Research Council, at its Eighth Annual Meeting in 1931, as follows:

"The traffic capacity of a roadway is reached when any further increase in traffic volume, all other factors remaining constant, results in a marked decrease in traffic speed." This interpretation of traffic capacity is significant because it indicates the point at which any additional traffic will begin to result in retardation of traffic movement. Bruce D. Greenshields, Assoc. M. Am. Soc. C. E., appropriately designates such capacity as the "working capacity" of the highway. As the result of experiments conducted by him in 1934, on Ohio State highways, by which traffic volumes and speeds are recorded and correlated, he reaches the conclusion that the average automobile driver (if not most drivers) will operate his car at a speed of about 44 miles per hr, whenever traffic conditions permit him to drive as he sees fit. Professor Greenshields designates this velocity as "free speed" and finds that it obtains a density of approximately 400 cars per hr on a two-lane highway, with a varying percentage of truck traffic to 10%, and with traffic about equally divided in both directions. Whenever traffic exceeds this density, velocity begins to decrease. Apparently, however, the decrease in velocity could not be called marked until a density of approximately 600 to 700 cars per hr is reached.

These experiments "dovetail" with the arithmetical computations made by Sigvald Johannesson, M. Am. Soc. C. E., who computes "working capacity" of a two-lane roadway as approximately 200 vehicles per hr per lane. At this density, vehicles are likely to be spaced so as to permit passing in both directions with reasonable safety.

The writer substantiates these findings by his local experience and observations. It is his conclusion that the working capacity of a two-lane roadway approximates 600 vehicles per hr, and he would recommend that whenever it is found or estimated that the average daily density of traffic exceeds this value for 3 hr or more of the day, consideration should be given to the economics of widening.

In the Pittsburgh area, a maximum hourly density of 600 cars on a two-lane highway (as determined by a comprehensive traffic survey conducted in 1934 under the direction of John M. Rice, M. Am. Soc. C. E.) is equivalent to a total volume of approximately 5 000 cars per day. It is interesting to note the comparison between this volume and the standard of practice for road design established by the American Association of State Highway Officials. This group recommends a roadway capacity of more than two traffic lanes in cases where the average daily volume exceeds 4 000 vehicles.

Two lanes for moving vehicles, of course, is the minimum for any highway. Whenever traffic conditions indicate the need of widening a road, or the construction of a new highway of greater capacity than two moving lanes the change should be from a two-lane to a four-lane roadway. Only in exceptional cases should the widening from two to three lanes be considered. Although the addition of one lane will increase capacity very materially, it usually will convert the highway into a most dangerous thoroughfare.

A highway with four lanes for moving traffic will carry a large volume, probably (under favorable conditions) as great as 50 000 cars per day without appreciable congestion. Where more than four traffic lanes seem to be needed to take care of the traffic, most likely it will be found better to construct an additional thoroughfare.

Because of the seriousness of head-on collisions, the present tendency is to advocate the construction of a divided roadway with two or three traffic lanes for each direction instead of a four-lane pavement. This type of development, certainly, is desirable and, wherever economically justified, should be provided. The width of separation between roadways, where provided, will depend upon the character of local conditions and may vary from a few feet to as many as several hundred feet or more. Wherever possible the minimum distance between directional roadways, at least where cross-overs are provided, should be 40 ft. This will permit vehicles to turn from the inside lane of one roadway to the inside lane of the other without obstructing the traffic movement in other lanes of travel. Where there is considerable truck or slow-moving traffic, consideration should be given to the desirability of providing, physically or by regulation, separate lanes of travel for this traffic.

Curvature.—The most important reasons for curvature and variations in alignment are: (*a*) To make the highway serve intermediate points between the principal termini; (*b*) to decrease damage, right-of-way, and construction costs by the utilization of existing locations, or by taking advantage of favorable topography or other natural conditions; and (*c*) to reduce the rate of grades by lengthening the distance of travel. Except as it affects distances, curvature does not appreciably affect the cost of motor-vehicle operation, provided proper radii and length, super-elevation, sight distances, and roadway widening are made part of such curvature. The desirable magnitudes of these factors depend mostly upon the rate of speeds at which traffic will move normally. On a two-lane roadway, the standards should be more liberal than those for a four-lane highway where directional traffic is confined to its own lanes.

The desirable minimum standards of curvature and sight distances are as follows:

Item:	Desirable minimum standard:
Sight distance, in feet	800
Horizontal curvature; radius, in feet	1 000
Vertical curvature; sight distance, in feet	500

In each case these are desirable minimums and should be exceeded wherever practically possible. However, in territory such as Western Pennsylvania, because of the rugged terrain and topography, it sometimes becomes expensive, and not economically justified.

Current practice in Allegheny County is to superelevate all curves on a two-lane roadway which are in excess of 1° curvature. As a rule, the superelevation is fixed at $\frac{1}{8}$ in. per ft of width per degree of curvature, with a maximum superelevation of 1 in. per ft. On four-lane highways, superelevation usually is provided, but the extent and design ordinarily are determined by special study of each individual case rather than by the results of fixed formulas.

Grades.—The rugged topography in the Pittsburgh territory does not lend itself to the construction of grades as low as those in many other sections of the United States. Engineers in the region, of course, attempt to keep grades as low as practicable and try never to exceed a maximum of 8 per cent.

Costs and Savings

Direct Costs.—The sums that are expended for the direct cost items in the improvement of highways vary materially, and it is difficult to determine the exact effect which alignment, grade, and width have on these items. Costs of rights of way and damages are the most variable, and in the writer's experience have ranged from $15 000 to $100 000 per mile. In commercial districts, obviously, this item is usually high and may constitute the major part of total direct costs; or, in fact, in many instances, it may cause the cost of a proposed improvement to be prohibitive, making it necessary to construct a highway along an alternate route. In sections outside commercial districts, this item of cost decreases with the intensity of development and the extent of property values.

In Allegheny County, a fair average cost for 60-ft rights of way and damages in districts outside of commercial and highly developed residential areas, is about $25 000 per mile. For a 100-ft right of way, the corresponding cost probably would average about $40 000 per mile, provided no material building developments were encountered. These averages give a reasonable measure of the probable difference in cost of rights of way for a highway in Allegheny County with two lanes and four lanes for moving vehicles.

Cost of construction does not cover the wide range of variation per mile of highway as rights of way and damages; and, unless influenced by peculiar or unusual construction problems, such cost is practically the same in the different types of community districts. For the type of development customary in connection with two moving vehicular traffic lanes, construction costs in Allegheny County average about $75 000 per mile. Where four lanes for moving vehicles are provided, construction costs average about $140 000 per mile. Thus, on the average, the cost of constructing a four-lane highway is about 1.86 times that for a highway with two lanes for moving traffic. This is a measure of the effect of width on the direct costs of major thoroughfares. This additional expense, of course, is offset by reduced automobile operation costs.

As already stated, the effect of introducing curvature and grades into a highway usually is to reduce construction costs. However, where it becomes desirable, or necessary, or economically justifiable, to reduce grades or improve alignment on existing highways, the effect is to provide a highway of greater direct cost than the existing one that is being replaced.

Cost of overhead (being composed of expenses for contingencies, engineering, organization, interest, office, legal, and similar items which bear a relation to both rights-of-way and construction costs) is affected, therefore, by the range of variation in these two cost items. Overhead costs usually are expressed as percentages of the total estimated cost of the improvement. For conditions in Allegheny County, overhead costs average about 15% of the total estimated costs of rights of way and construction, and on the basis of the average values given for these items, would equal about $15 000 to $27 000 per mile for highways with two and four traffic lanes, respectively.

Cost of maintenance is a continuously recurring charge in contrast to the other direct cost items, which are non-recurring. Maintenance varies principally with the construction cost item and reflects expenditures for keeping the highway in proper condition for traffic use. Obviously, it varies mostly with width of pavement, but alignment and grades also affect this cost item since there is more "wear and tear" and need for conditioning pavements as the sharpness of curvature and grades increases. Experience with high-type road construction in Allegheny County indicates that the annual cost of maintenance, covering all items for a two-lane roadway, averages about $1 000 per mile, and about $1 200 per mile for four-lane highways.

Indirect Costs.—Various attempts have been made to evaluate the benefits to traffic that accrue by the reduction of travel distances, traffic delays, and traffic danger hazards. Except for the question of traffic hazards, the bases of such estimates are automobile operation costs, and include the items of gasoline, lubrication, tires and tubes, maintenance, depreciation, license, garage, interest on investment in car, insurance, taxes, and sometimes driver's wages. In a study made by Thomas R. Agg and Harold S. Carter, Members, Am. Soc. C. E., in 1928,[14] it was found that the cost of operation of the average passenger automobile which traveled 11 000 miles per yr, was 5.44 cts per mile when operating on high-type road pavements. Mr. Johannessen concludes[15] that the average operation cost of passenger cars used for private purposes at that time amounted to 7.5 cts per mile, based on an annual 10 000-mile use; that the cost for passenger cars used for business purposes on the same annual mileage basis was 12 cts per mile; and that about two-thirds of the passenger cars are used for pleasure and one-third for business. In a similar manner, other estimates have been made for the cost of truck operation and the results have been found to vary from approximately 20 to 70 cts per mile, depending upon the size and tonnage capacity.

In Allegheny County, as the result of various traffic surveys, the average conditions seem to be as follows:

Passenger traffic (percentage)	80
Pleasure purposes (percentage)	70
Business purposes (percentage)	30
Truck traffic (percentage)	20
Average truck capacity, in tons	1.5
Average annual mileage	12 000

Based upon these averages the writer's estimates of automobile operation costs are as shown in Table 13. These unit costs are the bases for the following evaluation of benefits arising from highway improvements which affect reductions in travel distance, traffic delays, and danger hazards.

Saving by Reduction in Distance.—Distance affects both operating costs and time, but consideration is given herein only to the economics of the former, as the value of time will be considered separately under the heading, "Reduction of Traffic Delays." The values in Columns (2) to (5), Table 13, will be affected directly by any reduction of distance, assuming that ample roadway capacity is provided for traffic, but the total annual sums spent for the other values will not be changed to any significant extent.

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[14] *Bulletin 91*, Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

[15] "Highway Economics," by Sigvald Johannessen, M. Am. Soc. C. E., 1931.

These operation costs become 3.75 cts for passenger automobiles (pleasure or business purposes), and 9.83 cts for trucks. Based on a distribution of 80% passenger cars and 20% trucks, the average cost per vehicle amounts to approximately 5 cts per mile or about 1 ct per 1 000 ft. At this rate the annual savings due to a reduced traveling distance of 1 mile for 5 000 cars per average day amounts to $91 250. This is equivalent to a capitalized cost (at 6% interest) of $1 520 000, and is a measure of the sum of money that can be expended justifiably to reduce the traveling distance for an average daily traffic of 5 000 vehicles by 1 mile. Expressed in other terms, these results indicate that under the conditions assumed, the annual expenditure of about $57 500 will be justified to reduce the travel distance 1 000 ft per 1 000 vehicles per day. Among other methods, such reduction of distances can be accomplished by improving the alignment and reducing the grades.

TABLE 13.—ESTIMATE OF AUTOMOBILE OPERATION COSTS, IN CENTS PER MILE (12 000 MILES ANNUALLY)

Vehicle (1)	Gasoline (2)	Lubrication (3)	Tires and tubes (4)	Maintenance (5)	Depreciation (6)	License (7)	Garage (8)	Interest (9)	Driver's wages (10)	INSURANCE		Total (13)
										Fire and theft (11)	All accidents (12)	
Passenger:												
Pleasure	1.33	0.36	0.40	0.83	1.66	0.13	0.42	0.50	....	0.10	0.83	6.56
Business	1.33	0.36	0.40	0.83	1.66	0.13	0.42	0.50	5.00	0.10	0.83	11.56
Trucks	3.00	0.63	1.10	3.00	4.20	0.40	0.50	1.50	7.50	0.25	1.25	23.33

Savings by Reduction of Delays.—The value of savings in traffic delays has been estimated as from 1 ct to 2 cts per vehicle-minute. Under the conditions that prevail in Allegheny County a fair and conservative value is 1.25 cts. This may be supported upon an analytical basis either by placing a reasonable value upon the rider's time or by expressing the fixed charges on a minute-of-time basis. However, an actual practical basis has greater meaning and, therefore, the writer refers to such an example in the Pittsburgh District. For a part of the suburban South Hills Section two routes are available for travel to downtown Pittsburgh, one of which, between common points, is 5.7 miles long and the other is only 4.0 miles. However, there is considerable traffic congestion on the shorter route and the average travel time for the 4 miles is about 13.5 min, whereas the longer route can be traversed in 9.5 min. Because of the greater convenience of travel and the shorter time required for those to whom both routes are available, drivers of passenger cars invariably select the longer one. This means that in so doing, preference is given to spending about 6.5 cts more for operation costs in order to save 4.0 min. On this basis the value of a vehicle-minute would be 1.6 cts. Considering all the circumstances involved in this example, and the theoretical computations used, 1.25 cts is a conservative value. Its significance is that wherever a highway improvement—either by providing greater width, better alignment, reduced grades, by-passing of congested areas, or by any other means—results in a reduction of travel time, the value of this reduced time is equivalent to 1.25 cts per vehicle-minute under the average conditions that prevail in Allegheny County. Using the example of 5 000 vehicles per average day, the annual savings of 1 min per vehicle would equal $22 800, or an equivalent 6% capitalized cost of $380 000. On the basis of 1 000 vehicles per day this would be $76 000.

Savings in Reduction of Accident Hazard.—The economic loss caused by traffic accidents in the United States has been estimated at about $3 000 000 000 per yr. On the basis of 10 000 miles per vehicle, this is equivalent to an average accident-hazard value of about 1 ct per vehicle-mile. This is a general measure of a value that may be applied where reduction of accident hazards is accomplished by highway improvement. Of course, a better method is to study the accident proneness and the costs of the particular highway or section of highway under consideration and, thereby, derive a value that will be more applicable to the conditions at hand.

In a general presentation of this kind, it is practically impossible to evaluate savings due to reduction of traffic accidents and, therefore, no consideration is given to such values in the following specific examples.

Examples

The effects of alignment, grade, and width on the direct and indirect costs of major highways cannot be equated, varying as they do for each individual highway improvement. In general, however, curvature is introduced into a highway to reduce rights-of-way, damage, or construction costs. The introduction of such curvature does not materially affect vehicular operation costs in so far as the reduction of traffic speeds are concerned, provided such curvature is of proper radii, length, and superelevation, and if it provides good sight distance. If it increases traveling distances, curvature obviously increases vehicular operation expenses.

The economics of grades is difficult to evaluate. Where grades are introduced in order to fit topography, the effect is to reduce construction cost. Provided such grades do not exceed a rate of about 6%, and if there is sufficient width of roadway and sight distances, the cost of vehicular operation does not become a particularly significant factor. On the other hand, where heavy cuts and fills are made in order to reduce grades to desirable standards, or to decrease the distance of travel or the need for curvature, the effect is to increase construction costs materially but, compensatingly, to decrease vehicular operation costs. The provision of additional width causes a slight increase of rights-of-way and damage costs and a heavy increase in construction costs. On the other hand, however, such improvement increases vehicular speeds and, consequently, lessens vehicular operation costs.

Case 1.—The application of the economics of highway improvement can best be illustrated by reference to specific examples. In 1936, a study was made of a proposed major highway extension in the Pittsburgh area. The proposed improvement was about 3 miles long and it was estimated that the average annual traffic during the next 20 yr would approximate 2 000 000 cars per yr. The problem resolved itself into two alternatives, one of which would adapt the location of the highway to topography, involving curvature and grades, and would make the line about 3.6 miles long. The other location was more direct (about 3.0 miles long) but would involve considerable cut and fill and the construction would cost about twice as much as the longer route. The economics of this problem were as shown in Table 14. Project B, the shorter but more costly project in so far as direct costs apply, is the more economical.

TABLE 14.—ECONOMICAL COMPARISON, ROUTES A AND B

Project	Length, in miles	Total physical cost, in thousands of dollars	Annual Costs, in Thousands of Dollars					
			Interest at 4%	Depreciation	Maintenance	Vehicular operation: For distance, at 5 cts per mile	Vehicular operation: For time, at 40 miles per hour	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	3.6	500.0	20.0	12.5	3.6	360.0	135.0	531.0
B	3.0	1 000.0	40.0	25.0	3.0	300.0	112.5	480.5

Another excellent example is the Liberty Tubes. These are twin vehicular roadways, about 1⅛ miles long, constructed by tunneling through a steep hill known as Mount Washington, so as to connect down-town Pittsburgh with the South Hills (residential) District. Although they are exceptional in character, the tubes are nothing more than a four-lane highway. However, the construction cost of this highway was more than $5 000 000 per mile. Nevertheless, in overcoming obstacles of topography to eliminate excessive curvature, grades, and distances, this expenditure has been justified over and over again by the value of the accommodation to traffic. It has resulted in a daily savings of both time and distance to about 25 000 cars, which is equivalent to more than 9 000 000 vehicles per yr. Without recourse to any computations, it is apparent that this convenience is easily worth 10 cts per vehicle. At this rate, the annual savings to traffic is $900 000. Capitalized at 6%, the worth of this savings is $15 000 000. On the other hand, if this improvement had only affected, in the same manner, 1 000 vehicles per day, the annual savings would be only $36 000 which, capitalized, would represent approximately $600 000. Under the latter condition, it would be obviously impossible to justify the $5 500 000 construction cost. Incidentally, this particular improvement is an outstanding example of one which has resulted in a tremendous appreciation of property values. In one township alone, in the South Hills area, affected by the construction of this project, the assessed value of property has increased from $8 000 000 to $29 000 000 within 10 yr after its opening in 1924.

Case 2.—One other splendid example of the economics involved by improvement of alignment, grade, and width, is the so-called Westinghouse Cut-Off. This is a relocation improvement of the Lincoln Highway in an area immediately outside the city limits of Pittsburgh. Between common points, the old route, 3.5 miles long, provided capacity for two moving lanes. Of the total 3.5 miles approximately one-third followed an indirect route through a heavily congested commercial district and the remaining two-thirds was routed over a hillside, resulting in an average rate of grade of about 10%, and several extremely sharp curves. The average time for traffic to travel this old course was 14 min.

The new highway, following an entirely new location, is 2.5 miles long, has a capacity for four lanes of moving vehicles and provides a practically uniform grade of 4 per cent. To accomplish these improvements it was necessary to make very heavy cuts and fills, the maximum of each exceeding 120 ft. The total physical costs amounted to almost $3 000 000, of which 42.5% was for rights of way and damages. The average travel time over the new route is 4.3 min.

The annual charges for interest at 6%, depreciation, and maintenance, amount to $333 000. Offsetting these charges, the annual costs of automobile operation have been reduced at least $425 000 due to the value of the savings in time and distance to the 2 500 000 vehicles at present utilizing the highway.

Conclusion

It is impossible to present general formulas of the effect of alignment, grade, and width on the direct and indirect costs of major highways. This applies particularly to direct costs because of the extreme variations of physical conditions throughout the United States. This paper has outlined the procedure followed and the experience gained in Allegheny County. Because of the ruggedness of the terrain, the problems are much more difficult, and the direct costs heavier, than in most other sections. This results in greater expenditures for value received. In practically all cases, however, the engineers concerned have been able to justify the expenses for major highway improvements.

DISCUSSION

JOHN A. OAKEY,[16] ASSOC. M. AM. SOC. C. E. (by letter).—Work such as that described in this Symposium does much toward establishing traffic engineering as a practicable science. However, there is one item in the section dealing with the "Economics of Alignment, Grade, and Width," as presented by Mr. Schmidt which, in the writer's estimation, requires further consideration. That item is the cost of operating a motor vehicle.

Ever since Agg and Carter[14] published the results of their investigation on the cost of operating a passenger car, these results have been accepted with little or no consideration of the fact that motoring costs have undergone radical changes since 1928. These findings, based upon an annual travel of 11 000 miles, are presented in Table 15, Columns (2), (3), and (4).

The results of studies[15] by Mr. Johannesson on the cost of operating a passenger car, are shown in Table 15, Column (5). These values are based upon an annual travel of 10 000 miles and show a total somewhat in excess of those of Agg and Carter. In other words, according to these authorities, the annual vehicle cost for the average motorist lies between $700 and $750. A momentary consideration of the fact that the average motorist has an annual income surely not exceeding $2 400 seems ample proof of the fact that motoring costs cannot be this high. If they were, there would not be 25 000 000 passenger cars in operation in the United States.

The writer has no experimental data to support his contention that these costs are excessive other than his own experience and that of his acquaintances, and certain statistics published by the Automobile Manufacturers' Association.[16a] It is interesting, nevertheless, to study each item in Table 15, Column (1), from a common-sense viewpoint and with full consideration of the habits of the average motorist of to-day.

(*1*) *Depreciation.*—In 1936, the average passenger car had an f.o.b. price of $603. The average retail price, therefore, was approximately $750. This car travels about 9 000 miles per yr and has a life slightly greater than 8 yr. It is fair, therefore, to assume that the life is 75 000 miles. On this basis, the depreciation is 1.0 cts per mile (see Table 15, Column (6)).

(*2*) *Gasoline.*—The average cost per gallon of gasoline was 18.84 cts in 1936. Assuming that the average car makes 15 miles to the gallon, the fuel cost is 1.2 cts per mile. The values in Table 15, Columns (3) and (5), based upon the same mileage per gallon but upon a fuel cost of 20 cts per gal, are, therefore, quite reasonable. The annual gasoline consumption of the average passenger car is 600 gal, giving a total annual travel of about 9 000 miles.

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[16] Asst. Prof., Civ. Eng., Villanova Coll., Villanova, Pa.

[16a] "Automobile Facts and Figures," Automobile Manufacturers' Assoc., 1936 Edition.

(*3*) *Lubrication.*—The average motorist changes the oil in his car between the 1 000-mile and the 2 000-mile mark. Assume that he changes the oil at the 1 500-mile mark, that the change requires 6 qt of oil costing 25 cts per qt, and that he finds that he requires an additional quart between changes. Assume further that he has his car greased every 3 000 miles at a cost of 50 cts. These assumptions place the total cost of lubrication at $2.00 per 1 500 miles, or 0.13 ct per mile. However, since no consideration has been made of the fact that many thousands of motorists service their own cars with chain-store products at a considerable saving, this item may reasonably be reduced to 0.1 ct per mile.

TABLE 15.—COMPARISON OF OPERATING COSTS, IN CENTS PER MILE

Description	AGG AND CARTER			Johannesson	Writer
	High type of roads	Intermediate type of roads	Low type of roads		
(1)	(2)	(3)	(4)	(5)	(6)
Fuel	1.09	1.31	1.61	1.39	1.2
Lubricants	0.22	0.22	0.22	0.20	0.1
Tires and tubes	0.29	0.64	0.84	0.75	0.3
Maintenance	1.43	1.72	2.11	2.03	0.5
Depreciation	1.26	1.39	1.57	1.73	1.0
License	0.14	0.14	0.14	0.19	...*
Garage	0.44	0.44	0.44	0.44	0.5
Interest	0.36	0.36	0.36	0.53	0.1
Insurance	0.21	0.21	0.21	0.23	0.1
Taxes	....	....	....	0.01	0.2†
Total	5.44	6.43	7.50	7.50	4.0

* Included in taxes. † Includes license fees.

(*4*) *Tires and Tubes.*—The modern light-weight average car can be equipped with tires and tubes at a cost not exceeding $50. These tires will have a life between 15 000 and 20 000 miles. On this basis, the tire cost will be slightly less than 0.3 ct per mile. However, the slight additional cost of repairs will bring this item up to 0.3 ct per mile.

(*5*) *Maintenance.*—The writer has driven his present car almost 100 000 miles at a total maintenance cost of slightly less than $200. The resulting maintenance cost of 0.2 ct per mile, while simply the result from one car, appears as reasonable as the other values shown in Table 15. The latter represent annual total maintenance costs of $189 and $203, respectively, and are obviously much too high. Acquaintances of the writer have reported maintenance costs from $5 to $50 per 10 000 miles. In any case, an arbitrarily assumed annual maintenance charge of $45, or 0.5 ct per mile should be sufficient to properly maintain the modern average car.

(*6*) *Interest.*—For some unknown reason, whenever an engineer is confronted with a problem involving economics, he writes down as his first item of annual cost "6% on the investment." To the average motorist, his car is no more of an investment than are the clothes he wears. The argument is frequently raised that, if he had not purchased his car, the motorist could have put his money out at interest. The apparent logic of this argument is "exploded" when one considers that the motorist would get, at most, 3% from a savings bank and that 60% of all new cars and 65% of all used cars are purchased on the instalment basis. Apparently, a large percentage of the motoring public in this country lacks either the ability or the inclination to save. The dialogue composed of: "Now that you own a car, you'll be broke all the time," and "I'm broke all the time anyway so I might as well have the car," is indicative of the national habit of owning a car at all costs. For these reasons, a nominal charge of 1% on the investment, or 0.1 ct per mile, seems all that can be justified on this item.

(*7*) *Taxes.*—Passenger car and truck owners to the number of 26 200 000 paid $396 000 000 for registration fees and personal property, city, and county taxes in 1935. The average cost, with no attempt to segregate the two types of vehicles, is $15. This figure results in a cost of somewhat less than 0.2 ct per mile.

(*8*) *Garages.*—Storage charges vary from $15 per month in thickly populated urban areas to nothing in rural areas. The commonly accepted charge of $4.00 per month, resulting in a cost of 0.5 ct per mile, is reasonable.

(*9*) *Insurance.*—In 1935, the owners of the 26 200 000 passenger cars and trucks in the United States paid a total of $446 000 000 for all forms of automobile insurance, $242 000 000 of which were returned in losses paid. The net charge of $204 000 000 results in a cost of $8.00 per vehicle, or 0.1 ct per mile.

(*10*) *Miscellaneous.*—The writer is loath to venture an opinion as to whether such items as "time" and "driver's wages" can be given a definite value. The arguments from authorities on either side seem to be equally balanced. Abstract time, to the average motorist, has little value, except as a convenience. When weighing the respective values of time and mileage, he usually considers the cost of gasoline as his only mileage charge. Whether engineers have the right to make him pay for a saving he does not appreciate is a question which may be argued at great length.

From a consideration of the tangible items comprising the cost per mile, it would appear that this cost is about 4.0 cts per mile for the average passenger car. Items (6), (7), (8), and (9) and, to a certain extent, Item (1), are relatively unimportant since these items vary with time and are not affected by any saving in distance or betterment in type of surface. If one-third of the depreciation may be charged to time and two-thirds to usage, the saving resulting from a reduction of distance is 2.8 cts per mile. This is considerably lower than the commonly accepted values.

There is another aspect to the reduction of distance which is seldom considered. This is the effect of the relative length of the reduction. In railroad work, a sliding percentage is applied depending upon the length of the reduction. Unfortunately, it is impossible to break down motor-vehicle costs in a manner comparable to that successfully used by the railroad companies. Yet the effect of relative length of the reduction of highway mileage may be of sufficient importance to warrant some additional study. It requires no great imagination to see that the saving of 1 000 ft on a 25-mile trip would have less effect on motoring costs than the vagaries of wind and weather. As every one knows, a stiff head wind increases gasoline consumption considerably; but as yet, no one has suggested that highways be enclosed to prevent these head winds even although it may be proved, on the soundest of economic bases, that such a step would be justified in many cases.

In the foregoing, the writer has attempted to "iron out" one of the difficulties confronting the present-day highway economist. These difficulties are legion, but they cannot be solved by over-estimating the costs of vehicle operation. Other important and still uncertain basic elements in economic analysis also come to mind. What is the probable life of this particular piece of pavement? What will the maintenance charges be? What will the average traffic volume be? With what savings to the motorists may one credit the pavement during its life? These are some of the questions that must be settled. In time, engineers may be able to accumulate sufficient data to permit their successful solution. It is doubtful, however, if highway economics will ever become sufficiently exact to permit its application in anything but the broader aspects of highway planning. Within certain broad limits, highway economics will be useful in justifying certain projects. These justifications, however, will continue to be swayed by the dictates of politics, safety, preservation of natural beauty, and improvement of recreational or educational facilities.

L. W. MAHONE,[17] ASSOC. M. AM. SOC. C. E. (by letter).—Quoting Mr. McNeil (see heading, "Remedies for Delays Due to Marginal Interference"): "It is an admitted fact, substantiated by the Courts, that streets are primarily for the purpose of the moving vehicle." It is encouraging to know that the Courts realize this, as certainly the unthinking "public" does not seem to. It would be interesting to know how many actual studies have been made of the relative cost of providing streets for parking, as against storage lots or garages for the same number of cars.

Project after project of street widening has been undertaken, presumably to provide more traffic lanes, but actually to permit Mr. Public (to say nothing of Mrs. Public) to continue his "squatter's rights" on a section of public street, and thus maintain the usual two lanes of parked cars.

Using words of Mr. McNeil: "Marginal interference is actually created by the granting of conveniences to a few at the expense of a majority." That is exactly what happens when parking is permitted on a heavily traveled street. In the future, engineers should consider the economy of providing off-street parking for the relatively few cars that can park on the street; and thus gain two traffic lanes without widening expense. Admittedly, each case will be peculiar unto itself.

Traffic engineers realize that in heavy traffic one parked car blocks a traffic lane just as thoroughly as several. For this reason a car stopping only to unload is undesirable. As a remedy for this could not loading and unloading be taken "around the corner" and off the main thoroughfare?

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[17] Asst. Prof., Eng. Extension Service, Iowa State Coll., Ames, Iowa.

In view of the various causes of traffic delay it would seem that the traffic engineer's ideal would be a street carrying no street cars or buses, with no parking, and with "around-the-corner" loading zones. Possibly, some one may be able to "put it across." The extra width of street due to the elimination of two parking lanes would in itself stop most "jay-walking."

As regards the loading and unloading of trucks in congested traffic areas, there would seem to be only two alternatives; either handle this during the night, or provide suitable space on private property. Such regulations would probably be no more drastic than many others.

A very pertinent sentence in the paper by Mr. McNeil (see heading, "Other Remedies for Delays") is: "The engineer of a municipality should strive to improve mass transportation facilities, which in the end will decrease the use of the private automobile in the busy areas, and thus reduce congestion." Much could be done to show that there is economy and time saving in using a good system of mass transportation. By placing further limitations on parking this worthy cause could be helped. The down-town merchant should be shown that he has no cause for complaint, because many are already turning to suburban stores because of traffic difficulties. He had better face the facts and "get on the band wagon" to promote the use of mass transportation.

Fred Lavis,[18] M. Am. Soc. C. E. (by letter).—The several papers included in this Symposium are all valuable additions to current information on this vitally important subject.

Referring to highways carrying large volumes of traffic, it is evident from the data presented that speeds of about 50 miles per hr are the maximum which should govern highway design, this being indicated both on account of safety and by economics. This also agrees with the conclusions of the writer as set forth in a paper[19] presented in 1937 at a meeting of the Highway Research Board, in Washington, D. C.

Mr. Vey's general statement that highway improvements might do away with 75% of the accidents, which now occur, does not seem to be borne out by the facts presented. Specific observations of the actions of drivers approaching certain grade crossings in Michigan, where danger signals were displayed, confirm the observations at Pittsburgh as recorded by Mr. McIntyre in the text following Table 4 and by Mr. McNeil under the heading "Remedies for Delays Due to Cross-Interference." The human element is the most important factor in the problem of highway safety, not the highway itself.

Mr. Vey's records which confirm other observations indicate that speed is an important factor both in the number and severity of accidents and, while the elimination of some of the points at which accidents occur may do away with these particular types of accidents, it seems likely that the human element, being what it is, coupled with the inducement to high speed due to the removal of some obstructions, will introduce other and perhaps worse hazards.

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[18] Cons. Engr., New York, N. Y.

[19] "Safety and Speeds as Affecting Highway Design," Washington, December 3, 1937.

The improvements mentioned by Mr. Vey are desirable, because they will tend to increase traffic capacity and also because they will eliminate some causes of accidents. It seems very doubtful, however, even if the money was available to do all these things, that the accident toll would be reduced by anywhere near 75 per cent.

This latter statement has been widely quoted but to the writer it seems a very dangerous one to make in such an apparently authoritative manner as it tends to divert attention from the real cause of most accidents, the careless and incompetent driver. Mr. Vey himself states that the highway user (the driver and pedestrian) is responsible for the greatest number of accidents and admits that the money is not available to make all highways meet the highest engineering standards.

Mr. Vey's records of "before and after" results of improvements are, as he states, hardly sufficient to warrant anything like definite conclusions, but they do at least indicate that his optimism as to reduction of accidents by reason of such improvements, if the money were available, is not quite justified.

In regard to the economics of location, Mr. Schmidt's conclusions do not vary greatly from those stated by the writer in 1930.[20]

The writer cannot altogether agree with the definition of "traffic capacity" to which reference is made by Mr. Schmidt under the heading, "Physical Characteristics: Widths of Roadways." The validity of this definition depends on the cost of providing additional facilities for the critical speed and the value of the time saved. As in the case of most highway traffic problems, because of the very great variety of vehicles and circumstances, this problem cannot usually be disposed of so easily.

As there is little chance of sufficient money being provided to meet and overcome present and anticipated congestion, probably some sacrifice of speed will have to be made in order to provide facilities for the passage of the largest number of vehicles at such corresponding speed.

The direct assertion by Mr. Schmidt that if increased capacity is required over that provided by a two-lane highway, the change should be to a four-lane highway, is not borne out by considerable practical experience with the satisfactory operation of three-lane highways.

The increase of distance by the use of curvature does not "obviously" (see heading, "Examples"), or necessarily increase operating expenses; the alternative might be increased rise and fall. In certain types of rolling country a judicious use of light curvature may not only decrease costs of construction, but may tend to provide a more sightly highway and one which may be more pleasing and perhaps easier to drive over. Long tangents slashing across rolling to hilly country may not provide the best location if all proper factors are taken into consideration.

Hugh A. Kelly,[21] Assoc. M. Am. Soc. C. E. (by letter).—In any paper of the same length, the writer has never seen the entire subject so tersely, yet so thoroughly, covered as in that by Mr. Vey.

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[20] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

[21] Civ. Engr.-Archt.-Appraiser, Jersey City, N. J.

In 1936, 36 000 souls perished by automobiles! why this annual slaughter?

In 1937, 40 000 souls perished by automobiles! why this annual increase in slaughter?

Any one who has ever stood for hours watching a parade swing by to the tunes of many blaring brass bands, will have some conception of the vastness of these numbers of deaths; 40 000 living men could not pass a given point, except "at the double," and in the very closest of formations, in less than 4 hr.

Why This Annual Slaughter?—The Engineering Profession has done much to make highway travel safe in the way of providing "fool-proof" roads and safeguards of all kinds. Daily, week-end, and holiday head lines serve to prove that the last word in safe road construction is not the solution of the problem, confronting the nation, of curbing death and injury by automobile.

Mr. Vey correctly concludes: "Because of human frailties * * *, safety on the highway is extremely difficult." However, with his other conclusion—"the impossibility of supervising and governing the actions of all persons at all times"—the writer is forced to disagree, for the following reasons:

Many years ago, he attended a meeting of the Society for the purpose of discussing the proposed Holland Tunnel. During the discussion he suggested that a physical barrier be erected in each tube of the tunnel to provide separate traffic lanes and prevent cars from crossing from one lane to the other. After thorough discussion and exhaustive research, the late Clifford M. Holland, M. Am. Soc. C. E., decided that it would not be practical to use a physical barrier; that the same measure of safety could probably be obtained by installing a painted white line down the center of each tube of the tunnel and by thoroughly patrolling each tube to see that drivers kept in line and did not cross the white line.

That Mr. Holland accomplished the results by adequate patrol (the writer had this in mind when he suggested a physical barrier), is amply borne out by ten years of safe operation, during which time, and in the face of the heaviest and most concentrated traffic in the world, there have been only two accidents involving patrons, causing three deaths, the more serious of the two accidents being caused by the illegal crossing of the white line.

Why This Annual Slaughter?—When the Pulaski Skyway, the main high-speed westward traffic artery to and from Jersey City, N. J., was first opened to traffic, trucks as well as pleasure cars were allowed to use it, with the result that the Skyway, erected at enormous cost to relieve congestion and speed up traffic in the Metropolitan Zone, became known as "Death Highway." Fatalities resulting from the careless use of the Skyway, mounted to enormous proportions. Like the Holland Tunnel traffic problems, the hazards of the Pulaski Skyway, too, were solved. The Mayor of Jersey City and the Governor of New Jersey joined forces to prohibit trucks from using the Pulaski Skyway; and Jersey City furnished adequate police officers and equipment to patrol this vital stretch of costly highway properly. Fatalities ceased, and minor accidents were reduced to a minimum.

Why This Annual Slaughter?—Is it not plain, therefore, that one of the major factors in reducing deaths and accidents on the highways—one tried and found to be successful—is proper and adequate patrol of highways?

This apparently simple solution of such a grave problem has a sound psychological basis; that is, the vast majority of drivers will, to the very best of their ability, do what is right and proper, and the malicious or careless minority will fall in line when they are positively certain, as they will be under a patrol plan, that they are being watched carefully, and will positively feel the heavy hand of the law for failure to comply strictly with reasonable traffic rules.

Therefore, the writer proposes that a highway patrol be organized, that the highways be patroled 24 hr per day, that a man be assigned to such areas as he can control—which in congested districts may be not more than a half mile. A distinctive uniform to be worn at all times while on duty, and a flood-lighted car or motorcycle at night, should be provided, so that law-abiding citizens and others shall be under no misapprehension as to the penalty for failure to observe the rules of the road.

Another matter that has not been given the attention it should receive, is one that the writer presented to the State Highway Department of New Jersey many years ago; that is, to eliminate the great hazards of night driving by properly illuminating the highways.

Why This Annual Slaughter?—Motorists, of all citizens, are the taxpayers par excellence, and the public should be selfish enough to keep them alive and free of injury for their continued ability to pay taxes, if for no other reason. The writer feels that no increase in taxation is needed to give effect to this proposed plan of proper and adequate highway patrol. Ample funds are now provided by the car-owning and driving taxpayers *via* gasoline taxes and license fees to take care of the increased personnel necessary to do away with this wanton destruction of life and property.

This is the only manner in which this blight may be removed forever from the highways of the United States and this is a solution far simpler than the sending of careless drivers to morgues, and the reading or memorizing of "horror" tales based on destruction of human life by automobiles, and other sincere but vain efforts to bring about the same result.

Why This Annual Slaughter?

SIDNEY J. WILLIAMS,[22] M. AM. SOC. C. E. (by letter).—In commenting on this Symposium, in general, the writer can only commend the thorough and sound analyses of the various papers presented. More States and cities should make these accurate detailed studies of how traffic moves and how accidents occur. Such data are the only sound bases for efforts, both national and local, to reduce accidents and congestion.

One subject, briefly alluded to in these papers, deserves further discussion, namely, pedestrian hazards, pedestrian protection, and pedestrian control. The streets and highways, traffic control equipment, and traffic control by police have been devised almost entirely to expedite and protect vehicular traffic. About 40% of the persons killed in motor vehicle accidents in 1937 were pedestrians. In cities alone the percentage is about 67, and, in large cities, it is even greater. Pedestrian deaths in cities have been reduced slightly since 1930, but have increased 64% in the rural districts.

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[22] Director, Public Safety Div., National Safety Council, Chicago, Ill.

How is this problem being met? Many persons advocate laws requiring the pedestrian to observe traffic signals, although, with a few notable exceptions, the signals were not designed for that purpose and take no account of the speed limitations of the pedestrian. They try to "educate" pedestrians chiefly by telling them to keep out of the way of vehicles, to cross at cross-walks, and with the green light, but with little corresponding "education" of the drivers to give the pedestrian "a break" when he does try to cross at the corner and with the green light.

The City of Pittsburgh has done far more for the pedestrian than most cities, yet even Mr. McNeil, in referring strictly to his topic of relieving traffic concentration, cites the considerable delays to vehicles caused by pedestrians, but mentions nothing of the enormous delays to pedestrians caused by vehicles. He suggests only "rigid enforcement of the laws eliminating jaywalking and confining pedestrian crossings to the intersections." This proposal is entirely correct, but with it must go equally rigid enforcement of the laws that give the pedestrian the right of way over the vehicle if the former is crossing at an intersection where there is no traffic control, or if he is crossing or has started to cross with a green light.

Actually, most vehicles turning on a green light do not yield any right of way to the pedestrian crossing on the same light, and most vehicles start instantly on the green light (if not before) with no regard for pedestrians who have started to cross and are thus left stranded in the middle of the street. Is it any wonder that under these circumstances many pedestrians feel they can place no reliance on the "protection" of the law or the courtesy of the driver and are thus tempted to depend entirely on their agility and dash across wherever and whenever they can see an opening in the traffic?

The writer does not intend at all to deny that a great many pedestrians are inexcusably careless and that all efforts to educate them out of this habit must be continued. He maintains only that, along with this, the engineer and the police officer must give much greater consideration to the pedestrian than most of them have done in the past. The engineer can help the pedestrian by providing raised loading platforms at street-car stops, and other safety islands in the center of wide or very busy thoroughfares, adequate sidewalks along rural highways traveled by pedestrians, and special "walk" signals where the volume justifies. Police departments can give more attention to requiring drivers to observe the legal and moral rights of pedestrians at crossings, as well as dissuading the latter from careless practices and in extreme cases arresting them. Educators, especially of the adult public, can study the entire problem more closely and can apply "selective education," which means determining the social and geographic groups that suffer the most pedestrian accidents and reaching them through appropriate channels.

The problem is well summarized by the following quotation:[23]

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[23] Rept. of the Committee on Pedestrian Control and Protection, National Safety Council, 1937.

"In approaching the pedestrian problem it ought to be recognized that, except in a few cases of suicide, pedestrians do not want to be injured or killed in traffic. It should also be admitted that drivers do not want to be injured or killed in traffic. It should also be admitted that drivers do not want to injure pedestrians. Accidents occur where unexpected conditions arise, or traffic becomes so complex that normal walking habits and ordinary precautions do not supply the necessary protection."

HAROLD NEVIN CAREY,[24] ESQ. (by letter).—Some questions concerning the economic justification for expenditures for highway improvements are raised by Mr. Schmidt. He emphasizes the commonly accepted proposition that a shortening of highway length results in a savings to the individual motorist, and hence to the public in general.

Without raising the question of the proper data to use for the cost of operating an automobile, or whether any value can be claimed for savings in time, one should consider the effect on the expenses of one automobile driver of a reduction in highway length of a quarter of a mile.

The items of operating cost that may be said to be affected to any extent by the distance traveled are those for fuel, lubrication, tires and tubes, maintenance and, possibly, depreciation (see Table 13). What is the effect on these factors of shortening the distance traveled by the quarter of a mile?

Fuel.—The saving in fuel would be the sum necessary to drive a quarter of a mile, perhaps (on a basis of 15 miles per gal) 1/60 gal, or a little more than 2 oz. At 20 cts per gal this would represent 1/3 ct. It is doubtful, however, if the driver has saved that 1/3 ct when there are so many other factors affecting gasoline consumption—the adjustment and condition of the motor, the operation of the choke, and the speed of driving (a change of driving speed from 25 to 50 miles per hr may make as much difference as 4 miles per gal with a light car), whether the driving is continuous or stop and go, etc. It scarcely seems that a savings of that 1/3 ct can be claimed for the driver, inasmuch as the next month or the next day a varying combination of conditions may waste more than the "saved" 2 oz of gasoline.

Lubrication.—Regardless of what data are used for miles per gallon of oil or per pound of grease, there would seem to be no monetary savings to the driver due to the quarter of a mile saved. The distance traveled has little influence on these values in comparison with such factors as whether the driver remembered to change oil at 1 500 miles or at 1 800 miles, or whether he took the car down to be greased three, four, or five times in 10 000 miles of driving.

Tires and Tubes.—In this case, again, there will be no monetary savings to the driver due to the quarter of a mile saved when his tire cost is almost entirely governed by the time when he decides to replace his tires, because tires are not used until they are absolutely worn out. A decision to buy new tires this Monday instead of next Friday has such an influence on costs as to overshadow, entirely, a saving in driving distance of a quarter of a mile.

Maintenance.—On the record, the fact that the driver did not cover the quarter of a mile could scarcely save him any money on maintenance expense.

Depreciation.—Here, again, the distance traveled has a relatively small bearing on the final depreciation cost to the driver because that is determined mainly by the time when he decides to turn in the car. Therefore, the saving in distance results in no monetary savings.

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[24] Civ. Engr., Ayres, Lewis, Norris & May, Ann Arbor, Mich.

From the foregoing it would seem that it cannot be shown that, as far as the one driver is concerned, there has been any actual monetary savings due to the shortening of the distance traveled by a quarter of a mile.

The economic justification for highway expenditures based on a shortening of the distance is customarily computed by multiplying the mileage saved by the assumed affected operation costs per mile for one car and, taking this figure as the savings per car, multiplying by the annual number of cars to give the total annual savings; but if it cannot, definitely, be shown that one driver actually saves anything by the shortening of the distance, how can he be multiplied by a legion to obtain anything? Zero times *n* is still zero.

W. W. Crosby,[25] M. Am. Soc. C. E. (by letter).—The writer's interest in the paper by Mr. McIntyre prompts the following questions:

What degree of betterment to the speed of through traffic would result from "rotary" instead of right-angled intersections?

What reduction of speed would result from "weaving" through a rotary intersection of rational diameter?

What progress has been made, and with what results on the congestion in down-town Pittsburgh, through the diversion of through traffic over the "Belt Roads"[26] around the city?

Referring to Mr. McNeil's paper the writer offers the following: His experience convinces him that many present traffic difficulties come from too narrowly weighing an improvement "from the standpoint of cost *versus* benefit to business" (see "Synopsis"). Genuine economy is a high ideal, but "benefit to business" is too mean a "yardstick." Elasticity of results, that is, the capacity for adjustment to changes in demands, convenience, and even human pleasure, is also part of the functions of highways. Mr. Schmidt refers to this under the heading, "Highway Economics."

"* * * grade separations are decidedly expensive * * *" (see heading, "Remedies for Delays Due to Cross-Interference," in Mr. McNeil's paper), and as Mr. McNeil suggests, many could be avoided (by rotary intersections, for instance) had the "elbow room,"[27] that is, wide right of way, been acquired in advance. It must be remembered constantly that the road of to-day is the street of to-morrow.

Pedestrian under-passes, unless unavoidable, will not prove efficient when they are built with steps. To encourage their use to the maximum benefits, ramps are obligatory.

Good results have been obtained in many places by requiring pedestrians, as well as vehicles, to obey traffic signals. One difficulty sometimes yet remains (referred to by Mr. McIntyre following Table 5) that of vehicular turns through "pedestrian traffic moving parallel with the traffic stream." It has been met, to some benefit, by permitting vehicular turns to the right on the red light.

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[25] Col., U. S. Army (*Retired*); Cons. Engr., Coronado, Calif.

[26] "Highway Location," by W. W. Crosby, Gillette Pub. Co., 1928, p. 127.

[27] *Roads and Streets,* October, 1934, p. 362 *et seq.*

Mr. McNeil states (in the paragraph preceding "Remedies for Delays Due to Medial Interference"): "It is not practicable, however, to confine any street to one-way traffic." One wonders if, for instance, the authorities of New York City would agree. He states further (see heading "Other Remedies for Delays") in effect, that an individual using his private car "spends several times the money that he would in using a mass-transportation vehicle." This situation is one to be faced squarely by highway engineers, even if deplored. He also states that unless a by-pass has certain characteristics it will fail. The writer would enlarge the list of characteristics by adding that it must have the width, or other means of protecting its flanks, sufficient to prevent an early demand for by-passing the by-pass.

Mr. Schmidt (see heading, "Highway Economics: Traffic Considerations") thinks the ownership of automobiles "surely must be approaching a 'point of saturation'." It might be well to consider the suggestions made by such an eminent authority as Frederic Law Olmstead who, in 1926, implied, that "The Saturation Point (of automobiles on roads) might depend greatly on the time available to people for using them."[28] If working hours per week become less, the writer sees no prospect for a saturation point to be reached in the near future, especially when other factors, such as the "more abundant life" and "higher standards of living," etc., are considered.

The writer wishes to commend many of the statements of the various authors in this Symposium, particularly those as to divided roadways, service ways, the elimination of parking on through-ways, the by-passing of through traffic, the use of ample diameter for circles or rotary intersections, the location of entrances to parking spaces, larger curb radii,[29] loading lanes (such as that of Stern Brothers' Department Store on 43d St., in New York, N. Y.), the "channelization" of roadways, and the desirability of a sufficient width for the "islands" or divisional strip so that turning-traffic dangers may be minimized.

Roger L. Morrison,[30] M. Am. Soc. C. E. (by letter).—Of the many items discussed in the Symposium the most controversial are probably those relating to economics.

In estimating the cost of motor-car operation Mr. Schmidt uses data which were determined a number of years ago and these costs are now probably too high. A friend, who drives approximately 35 000 miles each year, kindly gave the writer detailed operating cost data on five cars of the same make which he used between January, 1927, and November, 1935. The records show a constant decrease in cost from 5.53 cts per mile on the first car to 2.80 cts per mile on the last one.

Another point, apparently not considered in Mr. Schmidt's paper, is that if the average cost per mile for gasoline, lubrication, tires, maintenance, and depreciation is, say, 3.1 cts per mile, that does not mean that a saving of 1 mile in distance gives a saving of 3.1 cts per car. The average per-mile cost of gasoline includes that used in starting and stopping, standing with motor running, and driving many short distances with a cold motor, none of which is involved, as a rule, in driving an extra mile on a trip. Furthermore, the average per-mile cost of tires includes deterioration from time, weather, injuries, and excessive use of brakes, which are not appreciable in driving an extra mile. Of course, no one knows just how much should be deducted to cover these items, but 2.5 cts per mile for passenger cars would probably be as large a value as should be used in computing savings in operating costs due to decreased mileage.

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[28] Rept. No. 39, to 5th Congress (Milan, 1926), Permanent International Assoc. of Road Congresses.

[29] "Improving Street Corners," *American City*, November, 1930, p. 140.

[30] Prof. of Highway Eng. and Highway Transport, Univ. of Michigan, Ann Arbor, Mich.

Many laymen, and some engineers, are inclined to adopt a rather contemptuous attitude toward the small per-mile savings used in the computations of highway economists. The writer has often heard it stated that such savings are too trifling to be worth considering; but in this connection it should be remembered that a gasoline tax of 1 ct per gal amounts to only about 1/15 ct per mile, and the writer has neither seen nor heard any statements that such a tax is too small to be considered. On the contrary, many a bitter fight has been waged over that particular 1/15 ct per mile.

W. R. Flack,[31] Assoc. M. Am. Soc. C. E. (by letter).—The most important and often the most neglected aspect of highway design is that treated in the valuable and interesting paper by Mr. Schmidt. Although the subject matter is somewhat removed from the general theme of the Symposium, the writer is most particularly concerned with the section on "Costs and Savings," which analyzes the financial investment in highway construction. One of the governing factors in such an analysis is the operating cost per mile assigned to the typical vehicle. Believing the study of this important problem might be advanced by the collection of more data, the writer desires to submit some original findings on the costs of owning and operating passenger automobiles. In Table 16, Car 1 was a 1931, two-door sedan; Car 2 was a 1931 coach; and Car 3 was a 1936 model of Car 1. Mr. Schmidt cites the studies made by Dean Agg and Professor Carter,[14] in 1928, and by Mr. Johannesson,[15] in 1931. The writer's data are based on a record of costs that began in March, 1932, and is continued to January, 1938. The values are so at variance with Mr. Schmidt's estimate that a detailed itemization and description are in order. The values shown in Table 16 are cents per mile.

These cars were owned and operated in Medina County, Ohio. The average annual mileage was 11 900. About 95% of these miles were on pavement. The values for depreciation are actual for the first two cars, and those on the last car are an appraisal made from advertised prices of used cars. The garage rent is a somewhat arbitrary estimate, and is open to the question of whether it should have been included at all; nearly all houses in the vicinity had garages, and it is difficult to find a tenant for an empty garage; it is necessary for a house tenant to pay rent on a garage whether or not he has a car. The license includes tax; there are no other property taxes levied on cars in

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[31] With The Babcock & Wilcox Co., Barberton, Ohio.

Ohio except that contained in the cost of the license; this item also includes the driver's license. The insurance covers fire, theft, property damage, personal liability, and, in 1937, collision; there was no insurance in 1932. Gasoline (various brands) was purchased retail. Oil was usually bought in 5-gal lots and added as needed; changes were made about every 4 000 miles.

TABLE 16.—COMPARATIVE COSTS OF OWNING AND OPERATING PASSENGER AUTOMOBILES, IN CENTS PER MILE

Description	Car 1, March, 1932, to March, 1934	Car 2, March, 1934, to August, 1936	Car 3, August, 1936, to January, 1938	All cars entire period	Description	Car 1, March, 1932, to March, 1934	Car 2, March, 1934, to August, 1936	Car 3, August, 1936, to January, 1938	All cars entire period
Depreciation.....	0.559	0.701	1.186	0.801	Greasing........	0.033	0.024	0.056	0.036
Garage rent......	0.365	0.364	0.187	0.309	Tires...........	0.090	0.103	0.004	0.068
License..........	0.060	0.063	0.086	0.069	Batteries.......	0.047	0.041	....	0.031
Insurance.......	0.133	0.326	0.239	0.231	Parking........	0.045	0.068	0.037	0.050
Tools...........	0.019	0.005	....	0.009	Alcohol.........	0.015	0.013	0.017	0.015
Equipment......	0.022	0.084	0.064	0.056	Bridge tolls	....	....	0.014	0.004
Gasoline.........	0.998	1.373	1.180	1.176	Fine...........	0.033	....	....	0.012
Repairs.........	0.295	0.152	0.003	0.156	....	....	....	....	....
Oil.............	0.116	0.058	0.081	0.086	....	....	....	....	....
Total.......	....	....	....	....	....	2.830	3.375	3.154	3.109
Miles.......	....	....	....	....	....	24 158	23 118	21 090	68 366

No item is included for interest. These cars were regarded as necessities, and over some periods were absolutely required for the owner to reach his employment. The existence of a question as to whether or not to own a car was not recognized. A corporation may sell bonds and assume the expense of the interest on them when raising the capital for an improvement or addition to its property. It is difficult to understand how this procedure can apply to an individual in the purchase from income of what he regards as a necessity. Furthermore, as Harry J. Engel, Assoc. M. Am. Soc. C. E., has stated:[32] The theory about capital that saved money somehow automatically deserves to earn interest compounded at 6% is not well supported by the facts.

These costs are subject to adjustments to eliminate peculiarities of the conditions and to conform to more generally accepted accounting methods. The record of out-of-pocket spendings is complete and accurate.

The net operating cost per mile should include the following items from the costs given in Table 16:

Item	Cost, in cents per mile
Gasoline........................	1.11
Repairs........................	0.16
Oil............................	0.09
Greasing.......................	0.04
Tires..........................	0.07
Batteries......................	0.03
Total........................	1.50

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[32] "Social Horizons for the Engineer," by Harry J. Engel, *Civil Engineering*, November, 1937, p. 763.

These are the only items that will be affected directly by any reduction of distance. The gasoline cost has been reduced; the sum given in the record included considerable "stop-and-go" winter driving of the kind in which a mother takes the car out on especially cold mornings to drive the children a mile to school. This service is performed at a cost of gasoline the economist has no right to consider in examining highway relocations. Data on gas consumption, considered as a function of the distance driven per month, showed a remarkable consistency; the miles per gallon equalled about $12.00 + 2.66 \left(\frac{\text{miles driven in month}}{1\,000}\right)$. The months of low mileage per gallon and few miles per month were in the winter when starting required more than the average quantity of gas. The fact remains that the road designer is concerned with a car—or a million cars—rolling on the pavement, not as it moves, choked, in reverse gear from the garage to the street. The cost of repairs might also have been reduced, as some of the breakages had no relation to distance traveled. However, this owner was probably more fortunate than the average.

It seems to the writer that an analysis of total cost should be based upon a minimum value for the expense of vehicle operation; one should not attempt to make an economic justification of highway expenditures based on operating costs of inefficient, luxurious, or carelessly operated vehicles. As to estimating future savings, it is likely that advances in drainage, soil mechanics, design of Portland cement and bituminous concretes, and the performance of automobiles will make anticipating economies beyond a period of, say, fifteen years, imprudent. The designer might be on better ground to describe his relocation as one that would be paid for in savings to drivers over a term of fifteen years than to claim that the cost of the proposal is an equivalent 6% capitalized cost. The results of the two methods might be the same, but the vehicle costs can be estimated much more accurately than the interest the money might have earned.

The cost of vehicle operation may be used in estimating the expense to the public of detouring a project during construction. The writer knows of one badly timed project during which the additional operating cost of the traffic making the detour amounted to at least 30% of the cost of the construction. If this factor had been recognized some other design would certainly have been used.

Hawley S. Simpson,[33] M. Am. Soc. C. E. (by letter).—The clear analysis of the highway accident situation, presented by Mr. Vey, throws needed light on a subject of major importance. Of particular interest was his discussion of the accident experience on two-lane and four-lane highways in which he casts considerable doubt on the often reiterated theory that "congestion breeds accidents."

His data indicate that the reverse may be true when he shows that two-lane roads carrying more than "capacity" traffic have less accidents on a usage basis than when travel is lighter. Probably, however, the limited number of opportunities to make observations on the more heavily traveled two-lane roads makes data for them of somewhat less reliability than where more samples were obtained.

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[33] Research Engr., Am. Transit Assoc., New York, N. Y.

Furthermore, four-lane roads are shown to have a greater accident expectancy than narrower pavements. This is probably due partly to the fact that the per lane usage of the four-lane roads is less than that of the two-lane roads, so that, as Mr. Vey states, the individual motorist is allowed greater flexibility of movement, which always leads to higher accident rates. It is quite probable that if New Jersey's four-lane roads were carrying as much traffic per lane as the two-lane roads, the accident rates per vehicle-mile on the two types would not differ greatly. If this conclusion is even approximately correct, it is obvious that there is little to be gained, from the standpoint of highway safety, from the customary types of highway construction, regardless of width.

Consequently, the natural conclusion is that, unless substantial changes can be made in the aptitudes and attitudes of the driving population, which is most unlikely, safety can only be promoted by the construction of highways that will make it impossible for many types of accidents to occur.

That the expected reduction from this higher type construction will reach 75% seems to be unwarranted on the basis of available data. Although it is true that the highway which physically segregates the more important conflicting traffic flows would undoubtedly largely reduce certain types of accidents, there is as yet insufficient experience to indicate the extent to which other types of accidents may be increased in volume and perhaps even in seriousness. By removing all, or substantially all, apparent hazards, motorists may, on these new roads, be lulled into an entirely false sense of security and, therefore, attempt to obtain from their vehicles more nearly the maximum speed built into them. Thus, drivers may unconsciously be led into accident-provoking situations, which they cannot, or do not, recognize until it is too late. In the higher range of speeds the ability of an individual to control an automobile decreases as the speed increases, resulting in a lessened margin between safety and accidents. If this is carried far enough, the accident toll, instead of being appreciably reduced, may be only slightly affected. On the narrow, winding, and congested roads on the other hand, the many apparent hazards induce drivers to exercise an extreme degree of caution and so to compensate, or even over-compensate, for these hazards by extra careful and prudent operation.

There are certain comparatively inexpensive measures, however, which could, if more widely used, create appreciable improvement in accident experience without such great expenditures of money as would be required to produce the "fool-proof" highway. Two of these are particularly important—highway lighting and rural sidewalks.

At speeds of the order common on rural highways, automotive headlights provide wholly inadequate pavement lighting. There are numerous limitations that seem to lie in the way of looking to the vehicle, itself, to provide sufficient light for night operation at speeds to which drivers are accustomed during the day. For a relatively small percentage of the cost of highway construction, adequate lighting can be furnished. Mr. Vey has already made studies of the effect of highway lighting on accidents and has found an economic saving in every instance. Although highway lighting is not as spectacular as the construction of high-speed, grade-separated highways, and, therefore, does not attract as much attention, economic considerations suggest that a greater proportion of highway funds be put into lighting than in the past.

Highway sidewalks, now practically non-existent, should be constructed along every mile of heavily traveled rural highway. They should be so attractive that the pedestrian will wish to use them in preference to the paved roadway. Pedestrians, as a group, have suffered greatly from the automobile; yet little attempt has been made to provide facilities that will enable them to move about with dignity and safety.

More analyses of the kind conducted by Mr. Vey, in New Jersey, should point the way to the development of a highway system with a balanced consideration of all the numerous factors involved.

Charles M. Noble,[34] M. Am. Soc. C. E. (by letter).—Highway engineers should welcome Mr. Vey's excellent paper since it presents an analysis of the value of highway improvements based on actual accident records in a State small in area but with a heavy concentrated population. The area lies between the largest city and the third largest city in the country, and forms part of a cluster of States containing less than 4% of the area and more than 25% of the population of the United States. Indeed, New Jersey is known as the "Highway of the Nation." Certainly the accident experience in such a State should include a sizable proportion of the total vehicle-miles operated in the United States and should be representative of present-day congested traffic conditions as well as indicative of future trends in the less congested regions.

It is somewhat strange that highway engineers should have neglected making an appraisal of the value of each type of highway improvement from the standpoint of the accident experience of the user of the highway, particularly since it is an essential function of the engineer to deal with well substantiated facts in the design of engineering structures. It would appear axiomatic that the designer should provide a structure as ideally suited to the use that will be made of it as is practicable. The paper by Mr. Vey is a start in this direction which should prove an incentive to highway organizations to continue this form of statistical analysis in order that the use value of each type of design may be more clearly defined, and to establish its economic value. The inherent safety and "accident-proof" qualities of a design should be thoroughly established by actual tests and records before the engineer expends large sums on its construction. A design that develops a bad accident record will not reflect any credit on the designer and, in addition, it will create the impression among laymen that the engineer is either not fully alive to his responsibilities or that the problem is incapable of an engineering solution. Either conclusion would be disastrous to the highway engineer and the highway transportation industry.

The business depression years have produced a feeling among many classes of people that the highway system of the nation has been completed to as large an extent as required and that the sums collected from the motorist in the form of gasoline and oil imposts should be diverted to unemployment relief or for other governmental purposes. Indeed, large sums have already been diverted from highway funds, and some of the philosophy behind such diversion may be traced to this idea. Even among engineers the measurement of adequate highway transportation service has been computed, in the past, on the mass basis of mileage and the condition of the roadway surface rather than from the quality standpoint of the safety, comfort, and convenience of the user. Design which provides inherent safety for the motorist is distinctly a new trend. Mr. Vey's paper presents accident facts to support this trend and to substantiate the engineer's conviction that henceforth the "measuring stick" must be quality and not quantity; and, furthermore, that the country is not overbuilt but, on the contrary, that it is in urgent need of all the funds collected from motorists to modernize the existing highway system as well as to construct new trunk routes and farm-to-market roads.

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[34] Engr., Pennsylvania Turnpike Comm., Harrisburg, Pa.

It is stated that the data in the paper were derived from statements filed by the participants in accidents, with supplementary notes, in some cases, by the State Police. It is recognized, of course, that this type of data will reveal only the most rudimentary causes of accidents and will merely serve to classify the general type of accident. Mr. Vey recognizes this and rightly only attempts to utilize the latter information. The more subtle causes cannot be brought forth by participants, since it is scarcely possible for them to recognize such causes. All too often a particular set of conditions in or surrounding the highway combine to cause the motorist to commit an error in judgment which with lightning-like rapidity climaxes in an accident; and thus it is quite possible that more accidents are caused by defects in highway design than are indicated in the paper. Nevertheless, it is interesting to analyze the information in Fig. 5 and list the percentages which certainly can be almost wholly prevented by perfected highway design; and to speculate on the approximate percentage of reduction of those types which one cannot state with absolute certainty will be wholly eliminated.

Referring to Fig. 5, it is noted that 14% of all accidents are of the right-angle type at intersections, and that 1% occur between intersections. If grade separations are constructed at all crossings with proper acceleration and deceleration lanes this first type of accident should be completely eliminated—namely, 14% of all accidents. In the case of opposite direction left-turn accidents, those occurring at intersections will also be eliminated by grade separation, namely 5 per cent. In the case of pedestrians, if their presence is removed from the motor areas and provision is made for them elsewhere, this type of accident can be almost entirely prevented; and it is safe to state that all but 2% of the total can be eliminated, namely 8 per cent. The opposite-direction accident can be eliminated by the provision of a wide space separating opposite-direction traffic, accounting for a total of 21 per cent. If all fixed objects are removed for a reasonable distance from the pavement, the fixed-object accident can be largely prevented; but in order to be conservative it may be assumed that surely one-half these accidents can be eliminated, namely 6 per cent. Many types of same-direction accidents are caused by the influence of oncoming opposite-direction traffic, and if the divided highway principle is utilized, this type of causation is eliminated. Furthermore, the use of curbs that hamper lateral freedom has an effect on this type of accident because the motorist does not have the opportunity to dodge sidewise on to the shoulder to avoid a crash in case of an emergency. Wide, smooth, stabilized shoulders, level with the pavement, permit lateral freedom and will have a favorable influence on same-direction accidents. If divided design and wide shoulders are utilized, and if it is assumed that one-half the same-direction accidents will be eliminated between intersections because of such design, then it is probable that 10% of all accidents will be prevented. The same-direction accident at an intersection is caused by the stopping of vehicles at traffic lights or slowing down or stopping to make a turn. If grade separation is utilized with acceleration and deceleration lanes these accidents will be eliminated, namely, 10% of all accidents.

To summarize, the use of known design technique will surely eliminate the following types and percentages of accidents: Right angle, 14%; opposite direction left turn, 5%; pedestrian, 8%; opposite direction, 21%; same direction at intersections, 10%; giving a total of 58 per cent. If the probable reduction of fixed object accidents (6%) and same direction between intersections (10%) is added the total becomes 74%, which is a very close check on Mr. Vey's statement of 75 per cent. Thus, it can be seen that improved engineering technique and design will have a pronounced favorable effect on accidents.

In addition to the savings resulting from the foregoing general features of design, it is very possible to reduce the accident toll still further by close attention to highway details in order that all features will conform to the requirements of the user and may be as accident proof as possible. Mr. Vey recognizes the entire problem very clearly, summing it up concisely in his paper as follows (see heading "Synopsis"): "It is necessary that roadway facilities be designed and applied, which are inherently safe, as far as possible, in order to render the failures of human beings of lesser or little importance."

The vehicle-mile basis of comparing accident experience presents a rather accurate method of measuring comparative conditions between various general types of highways, and it is hoped that properly designed automatic recording stations will be established generally in order that such comparisons may be more widespread and that accident statistics may be compared on a more accurate basis than at present. In comparing various individual design features it may prove more illustrative, however, if some kind of rating could be established whereby each different design of the same type would be classified as being a certain percentage of a perfect, or 100%, score. In this manner, definitely bad design would very quickly score a low rating in comparison with other types of design and, being based on actual accident statistics, such a method would be of immense value in establishing correct design as well as furnishing unassailable proof to governing authorities of such good design.

The statement made in the paper that 59.6% of State highway accidents occur between intersections and 40.4% at intersections, and (see heading "Accident Location and Time of Occurrence") "that the sections of State highway having little or no cross-movements of traffic are the more hazardous" may cause some misconception of relative accident exposure. The statistics are undoubtedly correct, but it must be remembered that the actual space occupied by intersections on State highways is much less than the space traversed by a vehicle between intersections. Thus, the unit of exposure to accidents based on distance traversed, on time, or per vehicle-mile, or on any other basis of actual comparison, is much greater at intersections than on the comparatively long stretches between. As a crude illustration: If it is assumed that each intersection is 100 ft wide and that intersections are 1 mile apart, it will be thirty-six times more hazardous to traverse each foot of intersection than each foot of open highway between intersections.

It is frequently stated that each year only 15% of the drivers are the cause of 90% of all accidents, and the inference is that if these 15% were eliminated from the highways the accident problem would be solved without having to make any further improvements in the present highway system. It is true that 15% of the drivers cause a large percentage of the accidents each year, but, unfortunately, it is not the same 15% every year, except for a small percentage of accident-prone drivers who are known as "accident repeaters." If it were true that the same drivers were involved in the majority of accidents year after year they would soon destroy themselves and the accident problem would have solved itself long since, without any exertion on the part of the engineer. This suggests that the average driver is involved in the greater number of accidents—motorists who are sober and substantial citizens. In addition, the experience to date indicates that the behavior of the driver is the most difficult element in the situation to control, whereas the design of the highway can unquestionably be controlled with any degree of accuracy required. It would appear, therefore, that full use should be made of this fact, and that all the known safeguards should be incorporated in the design to as great an extent as is economically justified in order to overcome the weaknesses and mistakes of the user.

The situation is similar to the conditions existing in industry some years ago when workmen were not protected from moving machinery and other hazards by proper safeguards, and, consequently, the industrial accident toll was quite high. At that time accidents were assumed to be an unavoidable evil and part of the risks of manufacturing, little effort being made to improve the situation. Then, as to-day in the motor field, the operator of the machine was assumed to be at fault in case of injury, the assumption being that he was inept or careless. The tremendous reduction in industrial accidents by the use of proper safeguards is an indication of the results possible when the problem is attacked with the proper mental attitude and with vision and courage. Thousands of men and women are alive and well to-day who would be dead or permanently disabled had it not been for that effort. A similar opportunity exists for humanitarian service in the motor accident field; and since, in contrast with the industrial accident problem, the highway user will pay the cost of such safeguards, he is entitled to the most perfect product which knowledge and skill can produce.

The facts and figures given by Mr. Vey form a powerful tool which highway engineers should be quick to seize in order to prove conclusively to laymen and officials that highway improvements are a vital and effective means for combating the accident problem, that the need for such improvements is imperative if the accident disease is to be checked, and, furthermore, that diversion of gasoline and oil imposts is causing the death and injury of thousands of human beings yearly. Therefore, Mr. Vey is to be congratulated on his valuable paper, and it is hoped that it will create an incentive to highway authorities to continue and extend this form of analysis in order to establish authoritatively the theory that improved highway design is a fundamental need.

Lewis W. McIntyre,[35] M. Am. Soc. C. E. (by letter).—The primary purpose of this paper was to record, for the use of other engineers, certain factual data concerning the actual movement of city traffic. Although these data were obtained possibly as long ago as 1928, for the purpose of establishing whether a signal system could be used in down-town Pittsburgh, they have been found useful in comparing the traffic capacity of various plans for separation of grades, for bridge plazas, traffic by-passes, etc. In a number of cases, the decision as to which of a number of suggested plans should be selected has depended on an analysis of traffic capacity based on these data.

To the best of the writer's knowledge, no similar data have been published. The technique for determining traffic capacity, which has been developed from these data, has been found extremely useful. It is hoped that other engineers may also find the data of service.

Arnold H. Vey,[36] Esq. (by letter).—In this closing discussion, it is the writer's intention to discuss the comments expressed by Messrs. Lavis, Kelly, and Simpson, all of whom more or less took the writer to task for the optimism he expressed concerning the favorable reduction in accidents which might be expected following major highway improvements.

Mr. Lavis states that the statement to the effect that highway improvements might do away with 75% of the accidents which now occur "does not seem to be borne out by the facts presented." Mr. Simpson makes the same general statement and uses the words "seems to be unwarranted on the basis of available data." Colonel Kelly is of the opinion that safe road construction is not the solution of the problem and disagrees with the conclusion, contained in the original paper, as to "the impossibility of supervising and governing the actions of all persons at all times." Mr. Lavis considers the real cause of traffic accidents to be the careless and incompetent driver.

The driver, of course, is the cause, and upon him can be placed the largest share of responsibility. After all, of the three elements concerned—the highway, the car, and the highway user—he is in control of the movement of the vehicle; and, too, he shapes the path of the vehicle along the public highways. Therefore, if all highway users were perfect and at all times drove in a manner having regard for the conditions of the vehicle as well as highway conditions, all would agree that there would be no traffic accident problem. However, when indicting the highway user as the element of greatest responsibility, it is necessary to consider all highway users and not any one particular class or group as Mr. Lavis advocates.

It is true that in any one year, a rather small proportion of the licensed drivers (ranging from 10% to 15%) are involved in all the accidents of that year. Unfortunately, however, with few exceptions, the citizens identified in this small group are not responsible for the highway ills year after year. If this were the case, the solution would be rather simple as it would not be difficult to identify these accident breeders quickly and either correct their habits or remove them from the roadway entirely by revoking their driving privileges.

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Those involved in motor-vehicle accidents come from all walks of life. For the most part, the accident problem concerns average, law-abiding citizens, many of whom have driven for a number of years and have never before had an accident and, of course, who did not intentionally drive in a manner to cause an accident.

The solution of the motor-vehicle accident problem, despite the complication of causes, can be summed up in two words—"safeguards" and "safe practices"; that is, the application of adequate and appropriate safeguards and the performance of safe practices on the part of highway users.

Safeguards include the construction of highways that are inherently safe—highways that render the failures of human beings of lesser or little importance; and, highways that make it less possible for drivers to perform improper practices, causing accidents. Unfortunately, there are only a few examples of such types of construction in the United States. However, there are many examples of parts of a highway embodying all of the principles and features in one form or another but not confined to, and along, the same section of roadway.

An indication may be had, however, of the probable reduction in accidents through such highway improvements by reviewing accident experience along present-day roadways and determining, from such an analysis, the percentage of accidents that might be eliminated had the roadway system embodied all of the present-day safeguards known to the highway engineer.

For example, the accident experience along the New Jersey State Highway System over a period of one year was segregated into the various types of accidents and further separated by their place of occurrence—whether at or between intersections—and the following distribution was shown:

Type of collision	Percentage of accidents at intersections	Percentage of accidents between intersections
Right angle	14	1
Same direction	10	20
Opposite direction (head-on)	3	18
Fixed object	2	9
Pedestrian	4	6
Opposite direction (left turn)	5	1
Other types	2	5

Reviewing the foregoing tabulation, it is apparent that by controlling the right of way (prohibiting acess to abutting property as well as eliminating all cross traffic at grade), mostly all right-angle and opposite-direction (left-turn) types, as well as pedestrian accidents—all of which aggregate 31% of the total—would be eliminated.

The center island, segregating opposing flows of traffic, would prevent nearly all head-on collisions which comprise 21% of the total; fixed-object accidents, equaling 11% of the total, would also be curtailed through the elimination of fixed objects within the travelable part of the highway, together with the elimination of fixed objects adjacent to, and along, the traveled way.

The classification "same direction" concerns three more or less common types of collisions involving vehicles traveling in the same direction—"rear-end," "cutting-in," and "side-swipes." Of these three, the first is by far the most common. Although there has not yet been developed a practical physical means of dealing effectively with accidents of this type, particularly those occurring at between-intersection locations, nevertheless it is the opinion of the writer that the type of highway improvement with which this discussion is concerned would not only correct most of the same-direction accidents that now occur at street intersections but would also affect, favorably, the same-direction accidents that occur between intersections. Furthermore, the application of proper and adequate highway lighting would curtail night-time accidents effectively, particularly those of the same-direction type, many of which occur during hours of darkness, primarily because of insufficient visibility.

It was through a detailed analysis such as this that the writer arrived at the probable 75% reduction in accidents experienced to-day, through proper highway improvements. Neither Mr. Lavis nor Mr. Simpson offers any factual data to support their contentions.

Further indication of the favorable effect of roadway design on present-day accidents may be shown from a comparative study of the accident rate on a million-vehicle-mile basis along a section of an important highway in New Jersey that contains most of the advanced features herein concerned (with the exception of a center island dividing opposing flows of traffic) with sections of a surface roadway not embodying any of these characteristics.

The accident experience for a period of one year along State Highway Route No. 25 (U. S. Route No. 1, including the 12th Street viaduct, the depressed highway and the Pulaski Skyway" which represents the main approach to the Holland Tunnel, equaled 5.71 accidents per million vehicle miles, whereas for three sections of a surface route—State Highway Route No. 26 (U. S. Route No. 1) in South Brunswick, Plainsboro, and West Windsor Townships—the accident experience during the same year equaled 10.12, 10.88, and 10.34, respectively. A center island along the viaduct, the depressed roadway and the Skyway, would produce further reductions in accident experience.

Another indication of the benefits derived from a controlled highway is the accident experience along the Merritt Parkway in Connecticut, opened to vehicular traffic in June, 1938. Accident experience for the first few months of operation indicates a rate of 0.13 accidents per million vehicle miles of travel as compared with an average rate of 3.1 for the entire highway system of New Jersey.

Both Mr. Lavis and Mr. Simpson are of the opinion that, with all hazards removed from the roadways, motorists will be (as Mr. Simpson states), "lulled into an entirely false sense of security," and that the average speed of the vehicles will increase materially.

Speed surveys made along sections of improved highways, in New Jersey, such as four-lane divided roadways, do not support this contention. It is true that these roadways do not contain all the features of a controlled highway but intersection frequency is such that there is little, if any, border interference. The average speeds of vehicles over these roadways are not substantially different from the speeds along other roadways of the State, other conditions being equal.

In support of his contention, Colonel Kelly cites the experience in the Holland Tunnel as an example of effective enforcement. Since its opening, the Holland Tunnel has had an admirable accident record—a record for which the tunnel authorities can justly be proud. However, in the humble opinion of the writer, this record is due not alone to extraordinary enforcement activities plus the "no passing" regulation but, also, in no small measure, to the elimination of usual conflicts normally experienced or possible along ordinary present-day highways. In other words, the Holland Tunnel is, in a practical sense, a controlled highway—a highway designed for safety.

Colonel Kelly also states that since the passage of an ordinance prohibiting commercial vehicles from using the Pulaski Skyway, and since an increase in enforcement activities along the Pulaski Skyway, fatalities have ceased. The accident records along the Skyway for the five years, 1934 to 1938, inclusive, do not bear out this statement. As a matter of fact, in this 5-yr period, twelve persons were killed as a result of motor-vehicle accidents within the boundary lines of Jersey City and Kearny.

The history of industrial safety is somewhat parallel to the present highway accident situation. When industrial accidents became sufficiently extensive to warrant attention, industrialists first attempted prevention through a program based entirely on worker-safe practices and completely ignored machine safeguards.

The meager results soon determined that the cart was before the horse, and it was not until proper and adequate safeguards were applied that industrial accidents declined appreciably. In 1938 industrial accidents were less than half of the number twenty-five years ago.

As stated previously, the solution of the motor-vehicle accident problem is dependent upon the application of proper and adequate highway safeguards plus the use of safe practices on the part of highway users. Admittedly, the reconstruction of streets and highways in the manner herein discussed (that is, the rebuilding of highways to eliminate all possible conflicting or interfering flows of traffic by physical separation or segregation) would be economically impossible. However, if the public is to avoid the catastrophes occurring daily along roadways of the United States, it is essential that roadways of the future include design features which automatically lessen the importance of human failures.

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RELATIVE FLEXURE FACTORS FOR ANALYZING CONTINUOUS STRUCTURES

By Ralph W. Stewart,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Frederick Shapiro, Dean F. Peterson, Jr., George W. Housner, Homer M. Hadley, Adolphus Mitchell, John B. Wilbur, Leon Blog, A. Floris, D. B. Hall, Ralph W. Hutchinson, Arthur B. McGee, E. Neil W. Lane, and Ralph W. Stewart.

Synopsis

An effective method of analyzing the moments in continuous structures, by using the geometrical properties of elastic curves, is offered in this paper.

The area of any $\frac{M}{E\,I}$-diagram for a beam or part of a beam represents the curvature of the beam or the part to which the $\frac{M}{E\,I}$-area is related. This curvature is measured by the angle between the tangents to the elastic curve at the ends of the beam or of the part considered. It may be treated as a rotation of the tangent at one end of the curve with reference to the tangent at the other end, the center of rotation being the point where the center of gravity of the $\frac{M}{E\,I}$-area is projected upon one of these tangents to the elastic curve. In using the method, a graph consisting of a series of lines connecting these centers of rotation is substituted for a graph consisting of a series of moment diagrams. Each angle between the lines represents a moment area, both as to its magnitude and also as to the location of its center of gravity. It is not necessary that the graph be drawn to scale as the work is analytical. The method also involves use of the familiar mechanical requirements that the moments about a joint must balance, for equilibrium, and that the shear in a member must equal its combined end moments divided by its length. The series of lines referred to is designated as a "traverse of the elastic curves," because the lines which pass through points of support or joints in the structure will be, in fact, tangents to the curves formed by the flexed structure.

Note.—Published in January, 1938, *Proceedings*.

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Although the method of analysis used in the paper is new, the construction of the type of graph used was described by the writer in a paper published in 1934.[2]

True Traverse and Equivalent Traverse of a Series of Elastic Curve Units

Fig. 1(*a*) represents a column with one end deflected laterally; Fig. 1(*b*) and Fig. 1(*c*) represent the induced bending moments and the true traverse of the elastic curves; and, Fig. 1(*d*) and Fig. 1(*e*) represent equivalent moment diagrams and an equivalent traverse. The moment diagrams (Fig. 1(*d*)) are considered as running the full length of the member, thereby fixing their centers of gravity at known fractions of the lengths. If the member is of uniform section, these points will be at the third points. If intermediate loading is applied, the simple moment diagram due to the loading, and its corresponding angle in the traverse, are added, as in Fig. 1(*f*) and Fig. 1(*g*). This assembly of moment diagrams is the same as that used in treatises on slope deflection and other forms of analysis.[3] The traverse diagrams express the same data as the moment area constants, but in a much more workable form.

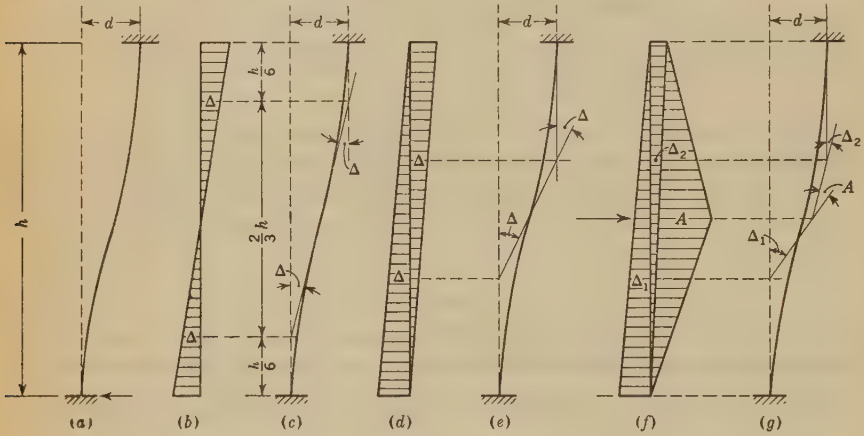

Fig. 1

In the following solutions, the symbol, Δ, represents the relative curvature in the entire length of a member due to an end moment, the different Δ-angles being identified by numerical subscripts. The symbol, θ, represents the relative angular rotation of a joint, the location being identified by a subscript. Symbols for fixed-end moments will follow the practice standardized for slope deflection.[3] Simple moment areas or their corresponding traverse angles will not be used, as solutions will be based on the fixed-end moments produced by any loading.

The stiffness of a member, as used in this paper, is measured by the moment, applied to one end of the member, necessary to produce a unit Δ-angle when the other end is hinged; or, briefly, is the ratio of an end moment to its Δ-angle. Under this definition, stiffnesses of a group of non-tapering members will have the same relative values as the stiffnesses used in the end-moment distribution method.[4] For tapering members, however, the relative stiffnesses will be different from those used in the end-moment distribution method, and will be easier to compute.

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[2] "Analysis of Continuous Structures by Traversing the Elastic Curves," by Ralph W. Stewart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

[3] *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois; also "Moments in Restrained and Continuous Beams," by F. E. Richart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 51.

The values of the separate items of curvature in the members, which are the angles in the traverse of the elastic curves, constitute the flexure factors mentioned in the title.

APPLICATION TO A CONTINUOUS BEAM OF NON-TAPERING SPANS

In Fig. 2 it is desired to determine influence lines for a load moving along Span *B C*. The relative stiffnesses of the spans are indicated by the numerals in circles.

Apply a moment at Joint *B* as shown in Fig. 2(*b*). The next step is to draw the traverse for the entire structure. This is easily done by the following procedure: (1) Draw Line 31-51 representing the rotated position of the tangent to the elastic curve at Joint *B*; (2) from Point 51 (which is under the third point of the span) draw Line 51-9 which, if produced, would pass over Joint *C*, and under the third point from the right end of Span *B C* (directing Line 51-9 to pass over Joint *C* is done because there is restraint at Joint *C*, and passing it under the third point mentioned indicates that the restraint is less than fixture); (3) from Point 9 draw Line 9-18 representing the tangent to the elastic curve passing through Joint *C*, and continue the traverse to Point 6, all angles being opposite the third points of the beam; and (4), since the hinged joint at End *E* offers no restraint, the traverse is finished by drawing a straight line from Point 6 to Point 2.

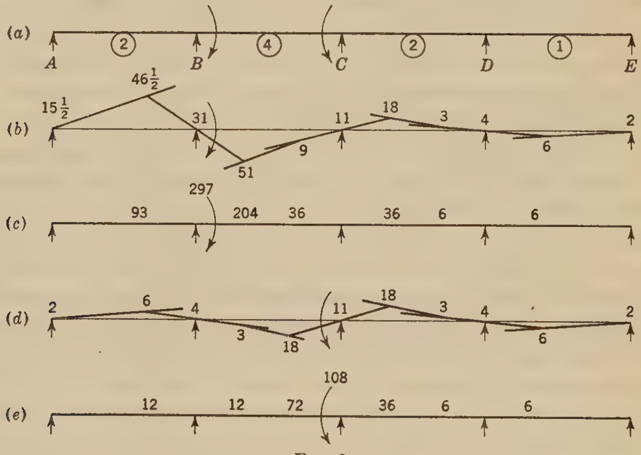

FIG. 2

Now, assign the arbitrary values of 2, 6, and 4, to the angles of the triangle in the traverse of the right span, these values being proportional to the opposite sides of the triangle. Since $M_{DC} = M_{DE}$ and since Span *C D* is twice as stiff as

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[4] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

Span $D\ E$, the angle due to M_{DC} will be one-half the angle due to M_{DE} giving it the value, 3, as shown. (The reason for assigning the even number, 2, to the rotation at End E was to avoid fractional values in Span $C\ D$, this division by 2 being foreseen). The next Δ-angle to the left of 3 is quickly obtained by the following deflection computation: Using one-third the length of the span as the unit of length, $(4 \times 3) + (3 \times 2) - \Delta \times 1 = 0$. Therefore, $\Delta = 18$. The rotation of Joint C is $4 + 3 - 18 = -11$. Again, by stiffness ratios, the angle at the left of Joint C is 9 and, proceeding as described, the rotation at Joint B is found to be 31. The two remaining angles in Span $A\ B$, being proportional to the opposite sides, are, respectively, 1.5 and 0.5 of 31, or 46.5 and 15.5.

Fig. 2(*b*) now shows all relative flexure factors for the beam due to a moment at Joint B. To reduce the relative flexure factors to relative moment factors, multiply them by the stiffness factors of their respective members. All the moments thus obtained resist the clockwise rotation of Joint B. The applied moment that produces these resisting moments is the numerical sum of the resisting moments acting at Joint B, and is equal to 297, as shown in Fig. 2(*c*). Note that if this moment, 297, acts through Member $B\ C$ as a medium, the actual moment, M_{BC}, will be the applied moment, 297, less the resisting moment, 204, leaving an actual moment of 93, which balances $M_{BA} = 93$. For any fixed-end moment, M_{BC}, the moments throughout the beam can now be computed by one setting of the slide-rule, as each will have the same ratio to its corresponding relative moment in Fig. 2(*c*) that the fixed-end moment has to 297.

Fig. 2(*d*) and Fig. 2(*e*) represent the same procedure for a moment at Joint C. In this case, another traverse beginning with an arbitrary angle assumption at End A, is run to Joint C. Owing to the accidental sequence of stiffness factors, each traverse yields the same end slope, 11, at Joint C. Ordinarily, the computed slope at Joint C, due to the traverse starting with an arbitrary angle assumption at the right end would not be the same as the computed slope at C resulting from the traverse starting with an arbitrary angle assumption at the left end; and it would be necessary to eliminate the difference in slopes at C with one slide-rule setting by multiplying all flexure factors on one side of the joint by the ratio necessary to equalize the end slopes at this joint. With the data in Fig. 2(*c*) and Fig. 2(*e*), all joint moments can now be determined quickly for any set of fixed-end moments in Span $B\ C$.

This beam was selected because it has heretofore been used as a demonstration problem and this solution may be compared with slope deflection solutions heretofore published.[5] The superiority of the solution by flexure factors will become greater if more spans are added. The extension of the traverse computation across another span is a small item, whereas, with slope deflection, and also with methods based on virtual work, another simultaneous equation is added to an already cumbersome group.

For a complete solution of this structure, for influence lines involving any fixed-end moments in any span, it is only necessary to run one traverse computation in each direction the entire length of the beam and then obtain all other values by simple ratios requiring only a single setting of the slide-rule for any series of moments. These right and left traverse computations have an automatic check by Maxwell's theorem of reciprocal deflections since the final moment at Joint *D* for an angle assumption at End *B* will be the same as the final moment at End *B* due to the same angle assumption at End *D*. As far as the writer knows, this elegant check, which avoids duplication of computations, is not available in automatic form with any other method.

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[5] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 347, 356, and 1048.

For influence-line computations, involving side-sway of an unsymmetrical frame, as illustrated by Fig. 3, it is necessary to determine a set of relative flexure factors for each moving joint and the unbalanced shear resulting from them. The remainder of the work is by direct proportion and summation.

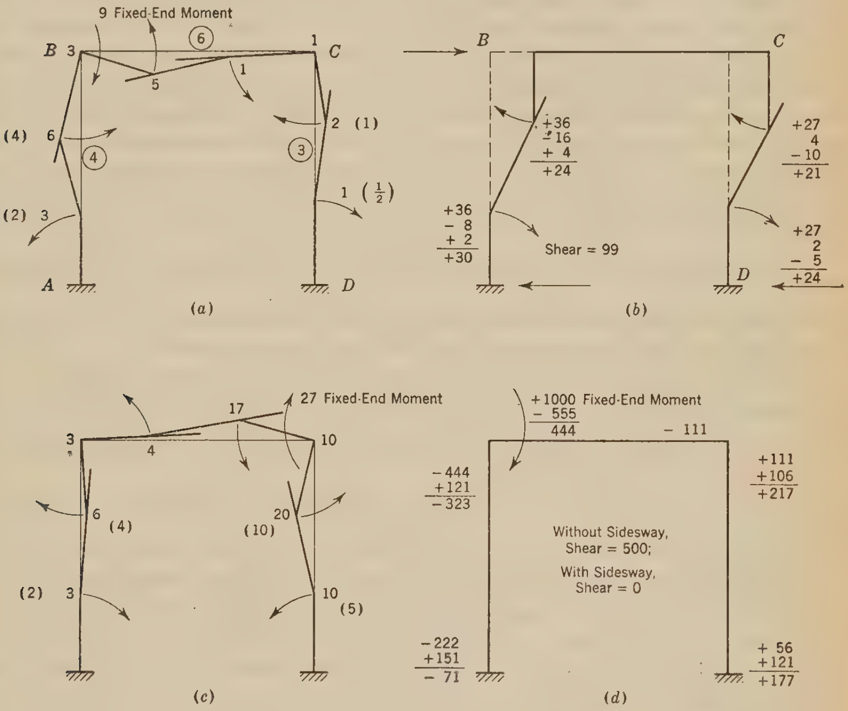
9 Fixed-End Moment
B 3
⑥
1 C
5
1
(4) 6
④
③
2 (1)
1 (½)
(2) 3
A
D
(a)
B
C
+36
−16
+ 4
+24
+27
4
−10
+21
+36
− 8
+ 2
+30
Shear = 99
+27
2
− 5
+24
D
(b)
27 Fixed-End Moment
17
3
4
10
6
(4)
20
(10)
(2) 3
10
(5)
(c)
+1000 Fixed-End Moment
− 555
444
− 111
−444
+121
−323
+111
+106
+217
Without Sidesway,
Shear = 500;
With Sidesway,
Shear = 0
−222
+151
− 71
+ 56
+121
+177
(d)

FIG. 3

To illustrate: Fig. 3(*a*) shows a set of flexure factors, computed easily by mental arithmetic, beginning with the relative value of 1 for the lower angle in the right column and progressing around the frame in the direction, *D C B A*. The numerals in parentheses opposite the column flexure factors are the corresponding moment factors computed on the basis of accepting the beam as representing the standard of stiffness, so that its flexure factors and moment factors are equal. The column moment factors are then equal to their flexure factors multiplied by the ratio of column stiffness to beam stiffness. Adding the moment factors for M_{BA} and M_{BC} it is found that the fixed-end moment acting on Joint *B* to produce these relative moment factors is 9. If this fixed-end moment acts through the beam as a medium, the actual moment in the beam is equal to the fixed-end moment, 9, minus the resisting moment, 5 = 4, which balances the moment, 4, at the top of the column. The upper numbers of each group beside the columns in Fig. 3(*d*) (namely, 222, 444, 111, and 56) are the moments without side-sway for a fixed-end moment of 1 000, being $\frac{1\,000}{9}$ times the moment factors in Fig. 3(*a*).

For side-sway correction the unbalanced shear due to these moments is computed assuming the height of the frame as equal to 1. This unbalanced shear of 500 is illustrated by the arrow at Joint B, in Fig. 3(*b*), and the structure must deflect to the right until resisting moments sufficient in magnitude to hold the structure in equilibrium are developed. To obtain these moments, first consider the frame deflected as illustrated by Fig. 3(*b*), without rotation at Joints B or C, causing moments of as yet unknown, and of unassumed, magnitude but with directions illustrated by the curved arrows. In order to transform this into a true picture of flexure by releasing Joints B and C so they can rotate, it is necessary to obtain a series of flexure and moment factors for Joint C, as illustrated by Fig. 3(*c*). This is done by the same procedure as for Fig. 3(*a*), beginning with the angles in the left column. These assumed column flexure factors were selected so that in crossing Joint C from the column to the beam, the multiplication by the stiffness ratio of $\frac{2}{3}$ would give a whole number. It is found by easy mental arithmetic that the flexure and moment factors shown in Fig. 3(*c*), are produced by a fixed-end moment of 27.

The flexure of Fig. 3(*b*), may now be evaluated. The method usually given in existing treatises is to distribute the unbalanced shear between the columns in proportion to their $\frac{I}{l^3}$-values, or to distribute the moments due to the shear in proportion to $\frac{I}{l^2}$-values. The writer considers this method somewhat undesirable because it requires remembering formulas and also because it is not a universal method applicable to tapering members. The following procedure is more easily remembered, equally easy to use, and is applicable to all types of members. Note that the equal deflections at Joints B and C in Fig. 3(*b*), are governed entirely by the inclined courses in the traverse along the centers of the columns. This will also be true if tapering members are used. For the case in hand, since each column has a constant value of I, the Δ-angles will be at the third points, and since the columns are of equal length all Δ-angles will be equal and the distribution of moments will be in proportion to the stiffness factors. For the general case, the distribution of moments is obtained by taking relative Δ-values directly from the geometry of the figure and reducing them to relative moment values by multiplying them by the respective stiffness factors. For the case in hand the most convenient set of relative moment values is obtained by adopting the trial moment of 27 in Fig. 3(*c*), as the relative end moment for Column $C\,D$ in Fig. 3(*b*). For M_{BA} and M_{AB} in Fig. 3(*b*), then, $\frac{4}{3} \times 27 = 36$, which is four times the trial moment in Fig. 3(*a*). Joints B and C are now held in their deflected position, but are made free to rotate. They will then turn until the moment at the top of each column is in balance with its adjoining beam moment and in accomplishing this adjustment the flexure at the bottoms of the columns will also receive an adjustment. The adjustments of the column moments due to the rotation of Joint A are obtained by multiplying the moments in Fig. 3(a), by $\frac{36}{9} = 4$, and are written on Fig. 3(b). The moments due to the adjustment of Joint C are copied directly from Fig. 3(c).

It may be noted that the adjustment of moments at the bottoms of the columns is one-half that at the tops of the columns, corresponding to a carry-over factor of one-half. However, this factor of one-half is for the special case of non-tapering members and the method of this paper does not use or advocate carry-over factors.

By summation, the final relative moments for side-sway influence are 30, 24, 21, and 24, corresponding to a side-sway force, or the shear of their sum = 99. The side-sway moments due to any shear can now be determined by direct proportion.

For a fixed-end moment, $M_{BC} = 1\,000$, the column moments without side-sway are $\frac{1\,000}{9}$ times the column moments in Fig. 3(a), giving a computed shear of 500. The correction moments for side-sway will be $\frac{500}{99}$ times the moments in Fig. 3(b). The column moments without side-sway, the side-sway correction moments, and the final column moments, are shown on Fig. 3(d).

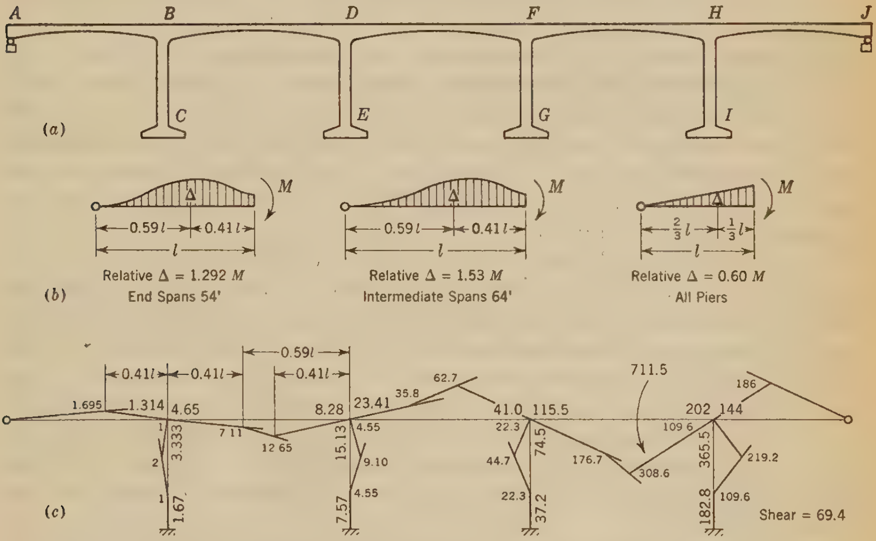
A
B
D
F
H
J
C
E
G
I
(a)
M
0.59l
0.41l
l
Relative Δ = 1.292 M
End Spans 54'
M
0.59l
0.41l
l
Relative Δ = 1.53 M
Intermediate Spans 64'
M
⅔l
⅓l
l
Relative Δ = 0.60 M
All Piers
(b)
0.41l
0.41l
0.59l
0.41l
1.695
1.314
4.65
7 11
12 65
8.28
23.41
35.8
62.7
1
3.333
2
1
1.67
15.13
4.55
9.10
4.55
7.57
41.0
115.5
22.3
44.7
22.3
74.5
37.2
176.7
711.5
308.6
109 6
202
144
186
365.5
219.2
182.8
109.6
Shear = 69.4
(c)

FIG. 4

Multi-Span Rigid Frame with Variable Moment of Inertia.—Fig. 4(a) represents a five-span rigid-frame bridge, the deck spans having parabolic soffits. The moment-area constants for each member may be obtained by the method described by George E. Large, Assoc. M. Am. Soc. C. E.,[6] or by tables based on integrations. In this structure all crowns are 27 in. deep, the haunches over the centers of piers are 54 in. deep, the piers are 40 in. thick, and d^3 is assumed to represent relative I-values. Young's modulus, E, being constant throughout, is omitted from the computations.

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[6] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 104, Fig. 48.

It is desired to obtain influence lines for any loading. Rotate Joint H as shown in Fig. 4(*c*) and draw the traverse diagram. Designate the relative values of the angles in Column $B\ C$ as 1, 2, and 1, as shown. For Span $A\ B$, the angles in the traverse triangle, being proportional to the opposite sides are computed, from the assumed relative joint rotation of 1, to be 0.69 and 1.69. The Δ-angle of 1.69 corresponds to a moment of 1.31 and the Δ-angle of 2 for the column corresponds to a moment of 3.33, these values being entered in their respective places on the traverse diagram.

These two moments, which act contra-clockwise, must be balanced by a clockwise moment, M_{BD}, equal to their sum, or 4.65. Enter this in Fig. 4(*c*) and compute and enter its corresponding Δ-angle of 7.11. The regular form of traverse equation from Joint B to Joint D now gives $1 + 7.11 \times 0.59 - 0.41\,\Delta = 0$, from which Δ for $M_{DB} = 12.65$ and its moment $= 8.28$ as shown. Enter these in Fig. 4(*c*) and by summation of angles obtain $\theta_D = 4.55$. The traverse angles for Column $D\ E$ are now added to the figure, the top moment in the column $= 15.13$ computed, entered on the figure and combined with the 8.28 deck moment to get $M_{DF} = 23.41$. This process is then repeated across the structure, and it is found that M_{HF} must equal 711.5 to produce the relative flexure factors and relative moments shown in Fig. 4(*c*). Since the structure is symmetrical the same relative factors may be reversed to compute the effect of a moment at Joint B. If the structure were not symmetrical a right to left traverse would be computed beginning with an angle of 1 to Joint H and ending, if correctly computed, with an external moment of 711.5 at Joint B. The distribution factors for fixed-end moments at Joints D and F are computed as already explained for Fig. 2. The effect of any fixed-end moment is now computed by simple ratios, and the effect of any combination of fixed-end moments can easily be tabulated.

Application to Closed Box-Frame.—In a closed box-frame each flexure factor is governed by stress flow along two separate routes. The solution is illustrated by Fig. 5, which shows such a structure, subjected to unsymmetrical loading. consisting of a single fixed-end moment at Joint B. Designate the rotation at Joint D by χ and traverse from Joint D to Joint B by Routes $D\ C\ B$ and $D\ A\ B$. The first traverse gives $\theta_B = 5.5\,\chi + 3$, and the second traverse gives $\theta_B = 7\,\chi - 4$. Equating these two values of θ: $\chi = 4.667$; from which value all flexure factors may be evaluated and the moments determined. The solution can then be extended to include side-sway corrections by the procedure applied to Fig. 3.

Application to Partial Fixture.—An easily understood and easily used method for treating partial fixation is available. In Fig. 6, let it be desired to analyze the effect of a partly fixed-column base. Add to the figure an imaginary underground member, as shown in dotted lines. Any desired degree of fixation can now be taken into account in the design by a proper adjustment of the stiffness factor, K_0, of the imaginary member, with reference to the stiffness factor, K, of the column. For 50% fixture, $K_0 = K$. For any ratio of fixation, designated as r, $r = \frac{K_0}{K_0 + K}$; and, $\theta = \frac{1 - r}{2r} \Delta_1$. For 50% fixture $\theta = \frac{1}{2} \Delta$.

Application to Large Buildings.—Buildings like the Empire State Building, in New York City, and even those of much lesser size, have so many members

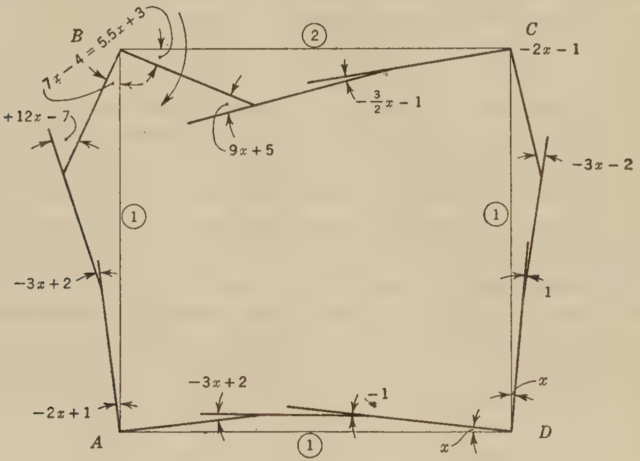
B
$5.5x+3$
$7x-4=$
$+12x-7$
②
C
$-2x-1$
$-\frac{3}{2}x-1$
$9x+5$
$-3x-2$
①
①
$-3x+2$
1
$-3x+2$
-1
x
$-2x+1$
A
①
x
D

FIG. 5

that it is impracticable to analyze them by extending moment distribution over their entire width and height. This is especially the case when slow convergence of the successive approximations is encountered. For such cases a plausible method is to assume a degree of partial fixation at the various bound-

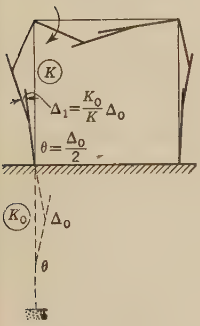
K
$\Delta_1 = \frac{K_0}{K} \Delta_0$
$\theta = \frac{\Delta_0}{2}$
K_0
Δ_0
θ

FIG. 6

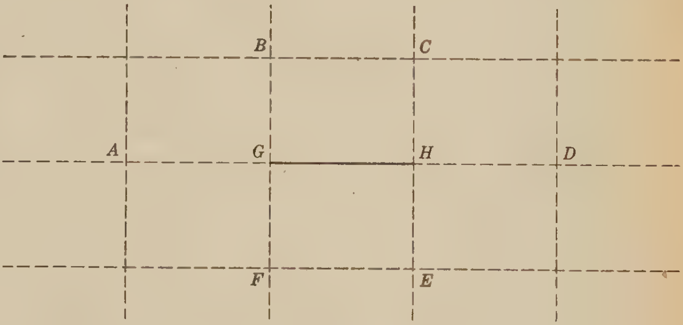

FIG. 7

ary joints around the part that is to be analyzed. For example, in Fig. 7, the moments in Girder GH can be determined accurately with only ordinary judgment as to the degrees of fixation at Joints A, B, C, D, E, and F. This is because a given percentage of error in assuming the moment at Joints A, B, C, D, E, or F will produce a smaller percentage of error in M_{GH} or M_{HG}.

SUMMARY AND CONCLUSIONS

It has been noted that the essential feature of the method set forth herein is to establish, as centers of rotation, the points of intersection of the tangents to elastic curves. Slope deflection is based on rotations at the joints where, due to continuity, the governing tangents to elastic curves do not establish angles. No simultaneous equations were used in this paper, whereas with slope deflection and also with the Maxwell-Mohr method of virtual work, very annoying groups of simultaneous equations would be needed to solve some of the problems presented.

A distinction between the method of relative flexure factors and end-moment distribution (which also avoids simultaneous equations) is that the latter begins the computation at the member affected by the load and works outward toward unloaded members, whereas the former begins the computation at a point remote from the loaded member and works toward it. The relative flexure-factor method does in a "once-over" computation what end-moment distribution accomplishes by a series of computations.

The writer feels quite confident that, for the class of structures illustrated in this paper, the method of treatment set forth will prove to be superior, not only in reduced arithmetical work but also in other respects, to the other methods mentioned.

DISCUSSION

FREDERICK SHAPIRO,[7] JUN. AM. SOC. C. E. (by letter).—In the conclusion to his excellent paper, Mr. Stewart seems too enthusiastic over the advantages of his method over that of end-moment distribution.

The writer, who has made an extensive study of the various methods of rigid-frame analysis, feels that for every problem there is at least one system of solution which is superior to others for speed in computation, or for accuracy, or for both. For example, in problems in which an approximate answer is desired quickly for a given loading, the Hardy Cross method[4] is excellent; for a short series of continuous beams, the theorem of three moments is superb, especially when modified by certain short-cuts; and for a rigid-frame bridge with curved beam, the method[8] used by A. G. Hayden, M. Am. Soc. C. E., is very convenient. For the problem of the multi-span rigid-frame bridge with variable moment of inertia which is solved in the paper, the method of transmission coefficients introduced by R. G. Brumfield,[9] M. Am. Soc. C. E., is probably a better one. For those who prefer graphical methods, conjugate points[10] offer an excellent solution to the continuous beam problems. The Beggs' deformeter[11] is ideal for a mechanical solution.

An objection might be raised to some of these methods because they involve the use of formulas. Formulas are intended as short-cuts; they express in one brief mathematical sentence what might occupy several paragraphs of written thought. The devices generally used to avoid them may be likened to going around the block to reach next door. It might be claimed that some of these methods do not give a picture of what is happening to the structure as computation progresses. Is it not easy and simple to put a coin in a slot-machine, pull a lever, and take what is desired from the proper compartment? There is a great advantage in applying this slot-machine principle to the solution of rigid frames in that mistakes are limited to arithmetical ones. The method of transmission coefficients,[12] in the writer's opinion, is one of the best for general application; not only does it give a good picture of the structure at all times, if the computer will pause to look, but it also "grinds" out the solution, if the computer cannot take time to stop.

If only one system of analysis is to be used, it is a matter of personal opinion what method should be selected; but actually different methods handle certain problems more advantageously than others, depending on the purpose in view.

DEAN F. PETERSON, JR.,[13] JUN. AM. SOC. C. E. (by letter).—The extension of the moment area principle, by the use of flexure factors, seems to the writer to have widespread application to continuous structures. The criterion for its application, as with the other so-called "special methods," is: (1) That the members have essentially straight center axes; and (2) that the deformations are essentially flexural.

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[7] Designer, Madigan-Hyland, Inc., Long Island City, N. Y.

[8] "The Rigid Frame Bridge," by Arthur G. Hayden, John Wiley & Sons, 1931.

[9] "Moving Loads on Beams with Restrained Ends," by R. G. Brumfield. (Complete manuscript filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.)

[10] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[11] *Loc. cit.*, Vol. 88 (1925), p. 1208.

[12] *Proceedings*, Am. Soc. C. E., March, 1937, p. 638.

[13] Troy, New York.

Although actual influence lines for moment or shear at any section are not generally necessary to analysis, when cases occur in which they are desirable, they may be constructed directly from the method of flexure factors and the use of the Müller-Breslau relation between influence lines and deflection curves.[14] For a moment at Point B (see Fig. 8(*b*)), a unit rotation of the axes is inserted and the flexure factors are written. Span $A\,B$ is solved since $M_{BA} = M_{BC}$. The unit rotation is represented by the number, $31 + 68 = 99$. The actual end rotations of the various members may be found by dividing through by 99. The ordinate of the influence line may be found graphically, with the aid of the funicular polygon, or analytically, by a convenient, self-checking table. It is only necessary to construct the deflection curve of each member separately. To do this the length of the member may be divided into a finite number of elementary lengths, Δl. Then, the rotation in the length, Δl, is $\frac{M\,\Delta l}{E\,I}\cdot$ This rotation, with sufficient accuracy, may be applied at the center of the length, Δl. Then the designer, knowing the end slopes of each member, traverses these rotations to form the deflection curve. Any accuracy desired may be attained by choosing a sufficiently small value of Δl. If one radian is taken as the initial rotation, the scale of the influence ordinate, in foot-pounds, will be the same as the scale of the abscissa, in feet. If, to begin with, some other rotation is used at the section, the scale of the ordinate will be equal to the initial rotation, in radians. In the graphical method the rotations, $\frac{M\,\Delta l}{E\,I}$, are plotted as vertical forces, $A\,B$, $B\,C$, etc., on the equivalent force diagram. It is not necessary that the true value of $\frac{M\,\Delta l}{E\,I}$, or that the actual moments for the known end slopes, be computed, because the rotations, $A\,B$, $B\,C$, etc., may be plotted on the equivalent force diagram to any scale desired in any one span and, therefore, need only be relative. All that is required is that the rays on the equivalent force polygon, through the points, A and G, be parallel to the end tangents of the beam (see Fig. 8(*d*)).

The influence line for moment at the center of Span $B\,C$ may be drawn (see Fig. 8(*e*)) almost as readily. The flexure factors are written from each direction up to 4.5 on the right and $3a$ on the left, in which a is a proportionality factor to balance the rotations from each direction. This factor may be evaluated since $\Delta_1 = \Delta_2$ (because the moments producing them are equal) and $3 \times 8a + 2 \times 3a + \Delta_1 = 3 \times 11 + 4.5 \times 2 + \Delta_2$ (since the deflection is the same, approaching from the right or from the left). Thus a is found to be $\frac{7}{5}$ and it is possible to express the rotations on each side as comparative units; Δ_1 and Δ_2 may be found from statical considerations of Beam $B\,C$ because, disregarding

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[14] For discussion on Müller-Breslau relation see "Structural Theory," by Hale Sutherland, and Harry Lake Bowman, Members, Am. Soc. C. E., Article 10–4.

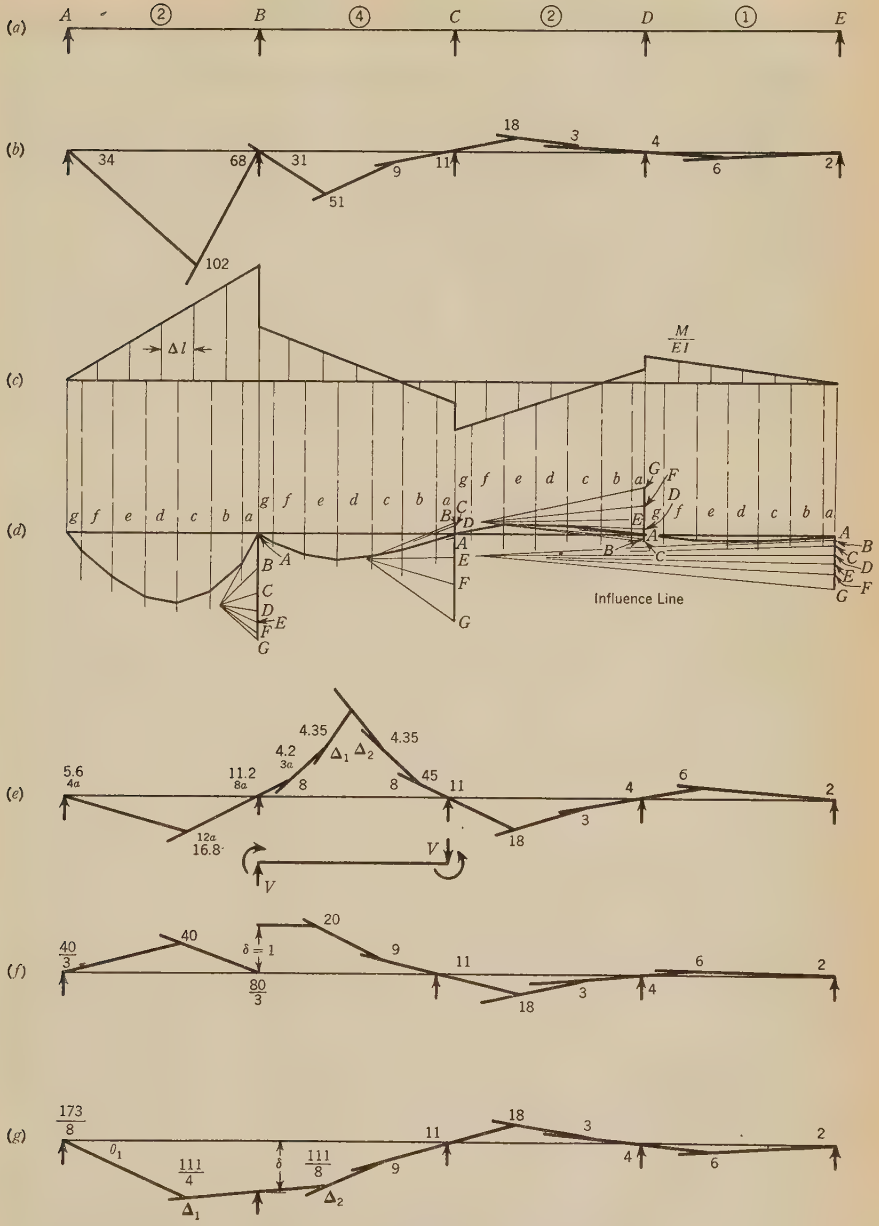
(a)
A
②
B
④
C
②
D
①
E
(b)
34
68
102
31
51
9
11
18
3
4
6
2
(c)
Δl
M/EI
(d)
g f e d c b a
A B C D E F G
Influence Line
(e)
5.6
4a
11.2
8a
12a
16.8
4.2
3a
4.35
Δ1
Δ2
4.35
8
8
45
11
18
3
4
6
2
V
V
(f)
40/3
40
δ = 1
80/3
20
9
11
18
3
4
6
2
(g)
173/8
θ1
111/4
Δ1
δ
111/8
Δ2
9
11
18
3
4
6
2

FIG. 8

the common elastic factor necessary to change the rotations into end moments, the end shear, V, equals $\frac{M_{CB} - M_{BC}}{l} = \frac{4.5 - 4.2}{l} = \frac{0.3}{l}$, and the moment at the center is $M = 4.2 + \frac{0.3}{l} \times \frac{l}{2} = 4.35$ which is the rotation, Δ_1. To reduce to exact rotations, the flexure factors are divided by $11.2 + 4.2 + 4.35 + 4.35 + 4.5 + 11 = 39.6$. It is to be noted that, for rotations in Span $B\,C$, the stiffness is doubled since the member is cut in the center. The use of the funicular polygon, although not shown in Fig. 8, may be extended to this case and to the case for a shear influence line.

For shear at Point $B\,C$ (Fig. 8(*f*)), deflect Span $B\,C$ from Span $B\,A$ without rotation and write the flexure factors. They are determined in Span $A\,B$ since $M_{BA} = M_{BC}$. The unit deflection is represented by the quantity, $\frac{2}{3}l \times 11 + \frac{1}{3}l \times 9 = \frac{31}{3}l$. To reduce to exact scale, all the rotations must be divided by $\frac{31}{3}l$.

For a settlement, δ, of Support B (Fig. 8(*g*)) the problem is likewise easily solved. The flexure factors are written from the right end, and since $M_{BA} = M_{BC}$, $\Delta_1 = 2\Delta_2$. Equating the deflections and rotations at Point B calculated from the right side to those calculated from the left side, $\theta_1 - \frac{1}{3}\Delta_1 = 11 + \frac{2}{3} \times 9 - \frac{1}{3}\Delta_2$ and $-(\theta_1 - \Delta_1) = 11 + 9 - \Delta_2$. Solving these three equations simultaneously, $\Delta_2 = \frac{111}{8}$, $\theta_1 = \frac{173}{8}$, and $\Delta_1 = \frac{111}{4}$. Since δ is represented by $\frac{173}{8}l - \frac{1}{3}l \times \frac{222}{8} = \frac{99\,l}{8}$, the actual rotations are obtained by dividing by $\frac{99\,l}{8\,\delta}$. Although simultaneous equations were used in the last solution they were very simple and solvable almost by inspection.

The foregoing two problems presented are solved for a very simple case. The solution may be extended, without difficulty, to frames of greater complexity and to members with varying moment of inertia. In general, the directness of the attack made and the inevitable final check furnished by the method of flexure factors will undoubtedly cause it to be viewed with favor as a means of analyzing many problems in continuous structures. The writer does not know of a more direct method for the solution of the complete influence line for a continuous frame.

George W. Housner,[15] Jun. Am. Soc. C. E. (by letter).—The method of presenting this paper can be criticized in that Mr. Stewart professes to derive his solution from the theory of moment-areas and from geometrical principles. Upon examination, however, it is seen that the method of traversing continuous structures is identical with the conjugate beam theory, except that the author chooses to represent the $\frac{M}{E\,I}$-areas graphically as angle changes.

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[15] Pasadena, Calif.

Although the author sketches a traverse of the deflected structure he reasons from the conjugate beam theory. For example, under the heading "Application To a Continuous Beam of Non-Tapering Spans," he states "Now, assign arbitrary values of 2, 6, and 4, to the angles of the triangle in the traverse of the right span, these values being proportional to the opposite sides of the triangle." This statement is objectionable, because a triangle with sides proportional to 2, 6, and 4 is an impossibility. These proportions define a straight line and not a triangle. They are perfectly proper in any application of the conjugate beam theory as distinct from the principles of geometry.

To demonstrate that the method of traversing continuous structures is the same as the conjugate beam theory, the author's first example is reproduced in part. For any real beam, construct a fictitious beam of equal length loaded with the $\frac{M}{E\,I}$-diagram. Then the shear at any point on the fictitious beam is equal to the rotation at that point on the real beam, and the bending moment at any point on the fictitious beam is equal to the deflection at that point on the real beam. Therefore, given the continuous beam shown in Fig. 9(*a*), the approximate bending moment diagram can be sketched as shown in Fig. 9(*b*).

Assigning an arbitrary value of 2 to the reaction at End *E*, it follows from statics that the triangular load on the fictitious beam, *D E*, equals 6 and the left reaction of Beam *D E* equals 4. These beam reactions represent the rotation of the real beam at the supports. Therefore, the right reaction of Span *C D* is also 4 since there is continuity over the supports. Then, since M_{de} equals M_{dc} the value of the triangle load (measured by M_{dc}) is 6 × 1 ÷ 2 = 3. The deflection at Reaction *C* is zero, so that, taking moments about Point *C*, with the length of beam equal to 3, Area × 1 − 3 × 2 − 4 × 3 = 0 and the area equals 18. From statics the reaction at Point *C* equals 18 − 3 − 4 which is 11.

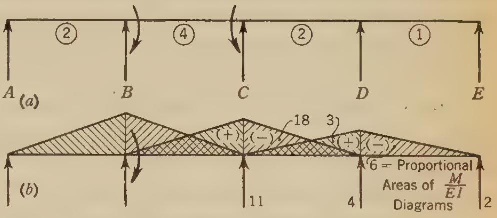

FIG. 9

This reasoning continued will reproduce the author's solution. The other examples in the paper can be solved similarly.

Although it may be simpler to sketch the traverse of a beam than to sketch the bending moment diagrams, the use of the traverse is less obvious. This is to be expected as it is a secondary form of the conjugate beam theory. In view of this it is doubtful whether the use of the traverse is justified as it somewhat obscures the reasoning, thus making the solution more difficult to comprehend.

HOMER M. HADLEY,[16] ASSOC. M. AM. SOC. C. E. (by letter).—When the tangled complexity of the Gordian knot was laid before Alexander that he should untie it, he drew his sword and cut it. This was a wholly new approach to the problem but it effectively disposed of difficulties.

Mr. Stewart has devised a somewhat similar method of dealing with stress distribution in continuous structures, at least in all their common varieties. Essentially, it is recognition of the fact that the flexed form of a beam is a direct expression of the stresses that are in it. However, his method proceeds a step further in recognizing that the direction of the tangents to the elastic curve at the ends of a member and the direction of an intermediate course connecting critical points upon the end tangents tell all that needs to be known —the direction and magnitude of the end moments. Therefore, in lieu of much involved clutter there is presented a method of analysis which, by means of the most elementary geometry and a fundamental or two of mechanics, yields an exact solution by a single operation.

It is not the least important part of Mr. Stewart's method that its first step is to draw a traverse diagram, the shape of which reflects the deformations in the structure it represents. This diagram requires no scale, and it can be exaggerated and distorted so long as it shows, correctly, the inclination of its courses with respect to one another. It is a framework upon which the relative angular changes are calculated and recorded. It furnishes, at all times, a valuable image of the structure under consideration, its various inclined courses indicating clearly the deformations and stress tendencies in the original subject.

However, the essential feature of the method is its determination of the relative magnitude of the moments at the ends of a member by expressing these moments as functions of angles developed at certain critical points in a traverse and then developing the respective magnitudes of the angles from the geometrical necessities of the situation. It is because the flexural effects of a moment applied at one end of a member can be represented graphically by the proper intersection of two lines at a point offset from the center of gravity of the $\frac{M}{E\,I}$-diagram for this moment and member that this method has the elegant nicety which characterizes it.

Beginning at the member in the system most remote from the one subjected to applied stress, convenient relative values are assigned to the angles developed in it under the conditions of end restraint which prevail at its extremity. Thence, one proceeds to the next member and the relative values of the angles in its traverse are precisely determined from the relationship which that member's stiffness bears to the first member's stiffness; and so on, progressively obtaining angle values which are accurate and exact in their relationship to one another. Either a final or concomitant conversion of these angles gives a set of end moments likewise accurately related to one another and to the applied moment that produces them. With this series of moments established, multiplying it by the ratio which any other applied or causal moment bears to the applied moment of the established series gives correct values for the new series. It is simplicity itself.

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[16] Regional Structural Engr., Portland Cement Assoc., Seattle, Wash.

The writer believes that Mr. Stewart could have improved his paper if, at the outset of his illustrative problems, he had stated in definite categorical detail the procedure to be followed: Do this—then do this—then do this, etc. After all, it is in the use and application of a method that its value resides. The reasons for the various steps can be elaborated in the discussion of the several examples.

Fig. 4(*b*) would be improved if "stiffnesses" were substituted for "Relative Δ" in each case. When every one is familiar with stiffnesses, why introduce another term for another aspect of the same thing?

ADOLPHUS MITCHELL,[17] JUN. AM. SOC. C. E. (by letter).—The writer has used the traverse method since its first presentation.[18] The ensuing years have given the opportunity to test the method in the design office and classroom. It is an attribute that engineers familiar with the area-moment method experience no difficulty in analyzing continuous structures by traversing the elastic curves. The present paper does much to simplify further the traverse method and is a distinct contribution to engineering literature.

The subject of closed box-frames has scarcely been touched. The true shape of the traverse is seldom known to the designer at the beginning of an analysis. This point is illustrated by Figs. 10 and 11 showing two variations of Fig. 5. The problem becomes more difficult in multiple box-frame design especially in extending existing culverts many of which utilized 12-in. interior walls. The writer would like to see a general solution of this difficulty that would require little added work for the designer.

Many problems show great advantages in the traverse solution. The following problem was solved by the slope-deflection method by Mr. A. W. Fischer, in 1936.[19] Fig. 12 shows the required dimensions and values of K, and Fig. 13 shows the traverse. To determine D_v, Fig. 14 will be used. Applying trigonometry,

$$\delta_1 = h - h_s \cos(\theta + \alpha) = h - h_s(\cos\theta\cos\alpha - \sin\theta\sin\alpha) \ldots\ldots (1)$$

Since α is very small, $\sin\alpha = \alpha$; $\cos\alpha = 1$; $h_s\cos\theta = h$; $h_s\sin\theta = l$, and $\alpha = \frac{D}{h_s}$. Hence,

$$\delta_1 = h - h + l\alpha = l\alpha = \frac{lD}{h_s} \ldots\ldots (2)$$

Likewise,

$$\delta_2 = h_s\cos(\theta - \alpha) - h = h_s(\cos\theta\cos\alpha + \sin\theta\sin\alpha) - h$$

$$= h + l\alpha - h = \frac{lD}{h_s} \ldots\ldots (3)$$

and,

$$D_v = \delta_1 + \delta_2 = 2\frac{lD}{h_s} = 0.329\,D \ldots\ldots (4)$$

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[17] Designer, Bridge Dept., State Highway and Public Works Comm., Raleigh, N. C.

[18] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105; its first publication, without discussion, was in *Proceedings*, Am. Soc. C. E., October, 1934, p. 1125.

[19] "Successive Elimination of Unknowns in the Slope Deflection Method," by John B. Wilbur, Assoc. M. Am. Soc. C. E.; discussion by A. W. Fischer, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Fig. 7, p. 362.

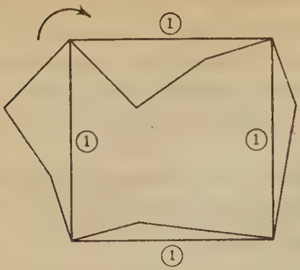

FIG. 10

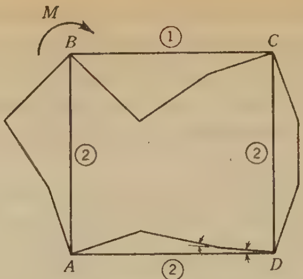

FIG. 11

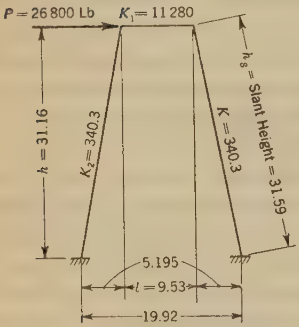
P = 26 800 Lb
K_1 = 11 280
h = 31.16
K_2 = 340.3
K = 340.3
h_s = Slant Height = 31.59
5.195
l = 9.53
19.92

FIG. 12

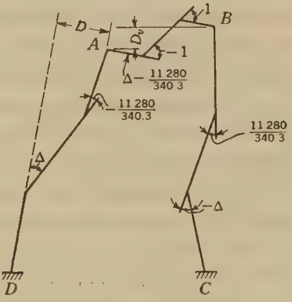
D
D_v
1
B
A
–1
Δ – 11 280 / 340.3
11 280 / 340.3
11 280 / 340.3
Δ
–Δ
D
C

FIG. 13

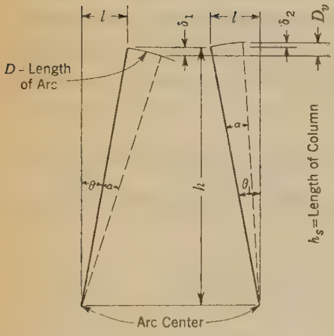
l
δ_1
l
δ_2
D_v
D - Length of Arc
α
θ
α
h
θ
h_s = Length of Column
Arc Center

FIG. 14

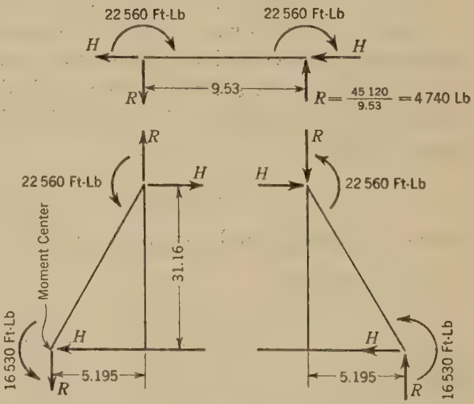
22 560 Ft-Lb
22 560 Ft-Lb
H
H
R
9.53
R = 45 120 / 9.53 = 4 740 Lb
R
R
22 560 Ft-Lb
H
H
22 560 Ft-Lb
Moment Center
31.16
16 530 Ft-Lb
H
H
5.195
5.195
R
R
16 530 Ft-Lb

FIG. 15

Keeping in mind that breaks in the traverse occur at the third points of members of constant moment of inertia and using the traverse of the beam, the offsets at End *B*, Fig. 11, may be added to obtain,

$$0.329\,D + \left(\Delta - \frac{11\,280}{340.3}\right)9.53 - \frac{2}{3}(9.53) + \frac{1}{3}(9.53) = 0 \ldots\ldots (5a)$$

Similarly, for the column,

$$\frac{2}{3}(31.59)\,\Delta - \frac{1}{3}(31.59)\,\frac{11\,280}{340.3} - D = 0 \ldots\ldots\ldots\ldots\ldots (5b)$$

Solving Equations (5) simultaneously, $\Delta = 26.3$; and:

$$M_{AB} = 2\,K_1\left(\text{area of }\frac{M}{I}\text{-diagram}\right) = 2(11\,280)\,1 = 22\,560\text{ ft-lb};$$

$$M_{AD} = 2\,K_2\left(\text{area of }\frac{M}{I}\text{-diagram}\right) = 2(304.3)\,\frac{11\,280}{340.3} = 22\,560\text{ ft-lb};$$

and,

$$M_{DA} = 2\,K_2\left(\text{area of }\frac{M}{I}\text{-diagram}\right) = 2(340.3)(26.3) = 17\,930\text{ ft-lb}.$$

These values are based on an assumed area of the $\frac{M_{AB}}{I}$-diagram equal to unity. The corresponding value of P is obtained by taking moments as shown in Fig. 15, giving $P = 2\,H = 4\,180$ ft-lb.

The given value of P is 26 800 lb and the correct moments are obtained by direct proportion, resulting in $M_{AB} = 22\,560\left(\frac{26\,800}{4\,180}\right) = 144\,600$ ft-lb; $M_{AD} = 22\,560\left(\frac{26\,800}{4\,180}\right) = 144\,600$ ft-lb; and,

$$M_{DA} = 17\,930\left(\frac{26\,800}{4\,180}\right) = 115\,000\text{ ft-lb}.$$

All computations have been made with a slide-rule and the explanation assumes familiarity with the author's paper.

For continuous beams the three-moment theorem and moment distribution cannot compare with the author's method. This is especially true when members of variable cross-section are involved because of the directness of its application. Slopes and deflections at any point are easily found by using the load, shear, moment, slope, and deflection series of diagrams as repeatedly brought out in discussions of the shear-area method.[20] Simpson's rule may be used to advantage when great accuracy is desirable in handling areas bound by complex curves.

No method is best for handling all problems, but any designer would be well repaid for time spent in learning when to use the Stewart traverse method.

JOHN B. WILBUR,[21] ASSOC. M. AM. SOC. C. E. (by letter).—In presenting a new expedient for analyzing continuous structures, Mr. Stewart has contributed a paper of real interest. Just what arrangement of the details of a solution leads to the best results is a difficult matter to determine, but certainly Mr. Stewart's procedure for analyzing continuous structures has the merit of simplified computation under some circumstances.

In simplifying analysis procedure, there is always danger that the reasoning involved will become obscure. The writer is of the opinion that simplicity of thought is of at least equal importance with simplicity of procedure. With this in mind, it might be questioned as to whether the concept of flexure factors does not pay too great a price for the advantage it may gain.

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[20] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 947.

[21] Associate Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

Although the process of traversing the elastic curve by assuming the slopes at certain joints, and progressively computing slopes at adjacent joints, is excellent under some circumstances, the same results can be accomplished with precisely the same mathematical steps by working directly from the moment-area theorems. Treating the $\frac{M}{EI}$ -curve as an elastic load, it may be shown easily that the following theorem holds: If a section, A–B, of a member which is initially straight, has its slope changed due to bending only, the slope at one end of the section, referred to the chord, A–B, of the elastic curve, is equal to the reaction at that end of an imaginary beam, A–B, which is carrying a distributed elastic load of intensity, $\frac{M}{E\,I}$.

For the case of an end-supported beam, if the entire beam is considered as the Section A–B, this theorem leads directly to the computation of true slopes referred to the original position of the axis of the beam, inasmuch as this original position of the axis coincides with the chord, A–B.

In applying this theorem, the following sign convention may be observed: (*a*) Positive values of $\frac{M}{E\,I}$ correspond to a downward elastic load; (*b*) negative values of $\frac{M}{E\,I}$ correspond to an upward elastic load; (*c*) an elastic reaction that acts in a direction to cause positive shear in the adjacent section of the beam considered, corresponds to a positive or clockwise slope; and (*d*) an elastic reaction that acts in a direction to cause negative shear in the adjacent section of the beam considered, corresponds to negative or counter-clockwise slope.

To determine the slopes obtained by Mr. Stewart in Fig. 2(*b*), by means of this theorem, one may proceed as follows:

(1) Sketch the resultant moment curve for the case under consideration as shown in Fig. 16(*a*), and sub-divide this moment curve into its component parts as shown by dotted lines. Note that all moments may be assumed as positive.

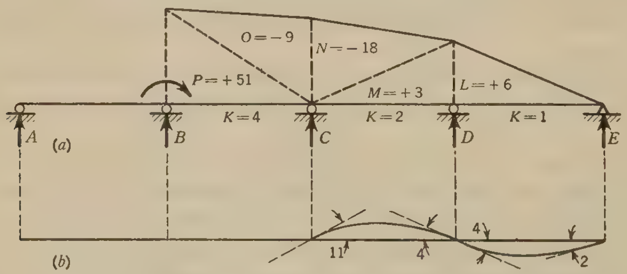

FIG. 16

(2) Assume that the slope at End E is -2. Note that this assumption corresponds to a counter-clockwise slope, as will actually occur at this point. It also corresponds to an upward reaction due to the $\frac{M}{E\,I}$ loading, since this produces negative shear to the left of End E.

(3) In order that the triangular elastic load shown by Area L may produce an upward reaction of 2 at End E, when acting on the end-supported beam, D–E, Area L must equal $+ 6$; the elastic load, $\frac{M}{E\,I}$, must act downward, corresponding to positive moment.

(4) The elastic load, $L = + 6$, causes an upward reaction of 4 at Point D. This reaction causes positive shear to the right of Point D; and, the slope at this point, therefore, is equal to 4, and is clockwise.

(5) The moment area, L, is given by:

$$\frac{M_{DE}}{E\,I_{DE}} \times \frac{L_{DE}}{2} = \frac{1}{2\,E} \times \frac{M_{DE}}{K_{DE}} \dotfill (6)$$

Similarly, the moment area, M, is given by:

$$M = \frac{1}{2\,E} \times \frac{M_{DC}}{K_{DC}} \dotfill (7)$$

Since $M_{DE} = M_{DC}$, the following relation holds:

$$\frac{M}{L} = \frac{K_{DE}}{K_{DC}} \dotfill (8)$$

For the case under consideration, $\frac{K_{DE}}{K_{DC}} = \frac{1}{2}$, so that $M = \frac{1}{2} (+ 6) = + 3$.

(6) Clockwise slope at Point D corresponds to positive elastic shear to the left of D in the beam, $C\,D$; hence, for purposes of considering the equilibrium of the elastic loads on Beam $C\,D$, the elastic reaction at Point D is downward, and equals 4. This reaction is due to the elastic loads, N and M, acting on Beam $C\,D$; whence, $N \times 1 + (M = + 3) \times 2 + 4 \times 3 = 0$; and $N = - 18$. This negative value shows that the elastic load, N, acts upward, corresponding to negative moment.

(7) The slope at Point C is now obtained by evaluating the elastic reaction at that point. The upward value of this reaction equals $\frac{1}{3}(3) - \frac{2}{3}(18) = - 11$; the reaction, therefore, is down; this causes negative shear to the right of Point C; and the slope at that point, therefore, is counter-clockwise.

In a similar manner, relative slopes may be determined at other joints. From the relative slopes, relative moments are easily determined.

Mr. Stewart refers to a solution for a continuous beam given by the writer[5] and suggests a comparison of the labor involved by the two methods. As explained in the closing discussion of the writer's paper, this example was included as a pedagogical step in leading the reader gradually into the more complicated problem of a building frame, rather than because it was believed to illustrate a desirable method of analyzing continuous beams.

Although the technique of Mr. Stewart's solution differs considerably from that suggested by the writer, the two methods have certain features in common.

Both are based fundamentally on the moment-area theorems, and both work, successively, from joint to joint.

LEON BLOG,[22] Assoc. M. AM. Soc. C. E. (by letter).—Eight years have transpired since, in May, 1930, Hardy Cross, M. Am. Soc. C. E., published his valuable method of analysis.[23] As has been the case with the original analyses for continuous beams and for single span and continuous span arches, such publication has stimulated the presentation of improvements and of alternative methods of obtaining the same results as by the prototype. Of these additions to the subject, the one offered by T. Y. Lin, Jun. Am. Soc. C. E., in 1934,[24] was a variation of the Cross method. The writer has shown that the Lin method was applicable to one-story structures and offered quicker solutions than the Cross method when such structures had numerous spans. It was followed by one presented by the author[25] which was a combination of the area-moment method and a set of simultaneous equations for each member, based upon the concept that for every part of a bent beam a triangle may be conceived which has, for its three sides, the line joining the ends of the bent part and the two end tangents. The tangents form an angle which is equal to the sum of the two interior angles of the triangle and is measured by $\frac{M}{E\,I}$, which is Moment-Area Principle 1.[26] The series of tangents, when taken continuously along a beam or a structure, may be considered as a surveyor's traverse in conjunction with the attendant angles, a concept which gives Mr. Stewart's method its name.

The traverse method was followed by L. E. Grinter, Assoc. M. Am. Soc. C. E., in devising his method of balancing angle change,[27] and it, in turn, is now followed by a restatement of the Stewart traverse method which eliminates the cumbersome equations of the original presentation.

The purpose of this discussion is to register the advantages and disadvantages of each of these developments based upon the following criteria:

(1) Is the method a general tool, or is it a special tool of analysis?
(2) Must approximations be made as to the degree of end restraint in the terminal members in the highest story?
(3) Does the method yield approximate or accurate results?
(4) Is the method faster than its competitors?
(5) Is it easy to remember?

The Cross method is a general tool. It applies to all true continuous frames whether they are single or multi-storied. Distribution can be begun without approximations as to restraint in either the beams or the columns. This method yields approximate results which are accurate enough for the purpose. For certain problems it is not as fast as some of its competitors. It is the easiest method to remember.

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[23] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[24] *Loc. cit.*, Vol. 102 (1937), p. 61.

[25] *Loc. cit.*, Vol. 101 (1936), p. 105.

[26] "Mechanics of Engineering," by I. P. Church, pp. 490–492 incl.

[27] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1020.

The direct method of moment distribution, by Mr. Lin, is not a general tool. It is not applicable to multi-storied structures without approximation as to end restraint in the terminal members in the highest story. The method yields approximate results which are as accurate as the Cross results. For one-story structures (such as bridges) for which influence lines are required, it is faster than the Cross method despite the extra preliminary computations required for the modified stiffness and carry-over factors which lead to its rapid convergence. Like the Stewart method it is a one-direction method. It is not as easy to remember as the Cross method.

The Grinter method of analyzing continuous frames by balancing angle changes is a general tool. It is not moment distribution in the strict sense of the term, although some of the moment distribution technique, such as balancing around a joint and carrying over angles instead of moments, is used. Approximations at the upper stories need not be made. The answers are approximate, but accurate enough for most purposes. It is distinctly not faster than its competitors and is the most difficult of all the methods proposed after the Cross method to remember. It is really slope deflection in reverse.

The restatement of the Stewart method is a special tool. It is not applicable to multi-storied structures inasmuch as assumptions as to restraint of the terminal members of the highest story must be made. For one-story, or closed, frames it is the most accurate of all the methods under discussion because it is distinctly not a method of approximation. This is because no convergence is involved in the solution of problems. As a special tool, it is faster than all the methods being discussed. This statement is made with the reservation that the user is not under the necessity of familiarizing himself with the method every time he has to use it. It is of use to those who are continually analyzing one-story and closed frames. It ranks second in ease of remembrance.

The Stewart method of analyzing continuous frames by flexure factors is neither a moment distribution, nor a slope deflection, method. For the end spans it makes use of Moment-Area Principle 1 and for interior spans it applies Moment-Area Principles 1 and 2. It involves less computation preliminary to obtaining the desired moment, but aside from that superiority it is not simpler than the Lin method.

Much is made by authors and discussers of the Grinter and Stewart methods of the point that the deflection of the structure can be visualized; but this is true of all the methods when used by competent engineers. Under dead load, live load, temperature, shrinkage, plastic flow, movement of the footings —laterally or vertically—or under earthquake movements, there is no difficulty in ascertaining the deflected shape of the structure. When, however, some, or all, of the distortion agents act simultaneously, the resultant shape is ascertainable only from the moment envelope, which may or may not be the result of the addition of moments of opposite signs. Under such load and natural phenomena combinations, the visualization advantage claimed for the Grinter and Stewart methods is not impressive and should not be a determinant in the selection of a method of analysis.

For those engineers who prefer not to tax their memories with new concepts of stiffness and the adaptation of long-known principles such as slope deflection and area moments in the alternatives to the Cross method, the latter, despite some additional calculation, will serve best for all continuous frames. For one-story frames, requiring influence lines, the modified Cross-Lin method is the fastest method of approximating moments by distribution; but review will be required before its use.

For closed frames and one-story structures, the Stewart method of traversing flexure factors will involve less computation. It will yield accurate results because it is not a method of approximation.

A. Floris, Esq.[28] (by letter).—In the proposed method for analyzing continuous beams and rigid frames by means of the relative flexure factors, only the influence of the bending moments upon the deformation of the structure is taken into account. The influence of the shearing forces is neglected. This is the usual procedure in the analysis of continuous structures. However, cases may arise in practice, in which a knowledge of the influence of the shearing deformation is not only desirable but necessary. This is especially true in short bars of considerable depth.

In the following discussion the author's analysis will be modified in such a manner as to include the shearing deformation of structures composed of bars with constant section. This is done very easily, because all the necessary data for this purpose are already available.[29]

In developing his theory the author uses the distances of the centers of gravity of the moment areas from the ends of the bars, the stiffness factors of these bars, and the fixed-end moments. By considering the shearing deformation of the structure the distances of the centers of gravity from the ends of the bars (Fig. 17) are: $x_1 = 2\,\lambda$; and, $x_2 = 2\,\zeta$, in which,

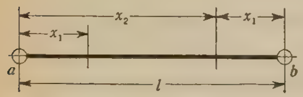

Fig. 17.—Centroids of Joint Moment Areas with Shears

$$\lambda = \frac{l}{6} - \nu\frac{\gamma}{l} \qquad (9a)$$

and,

$$\zeta = \frac{l}{3} + \nu\frac{\gamma}{l} \qquad (9b)$$

In Equations (9), $\gamma = \frac{I}{A}$; I = moment of inertia; A = sectional area; and $\nu = 3.0$ for steel and, approximately, 2.8 for concrete.

The stiffness factors in this case are for unyielding supports,

$$K = I\frac{\zeta}{\zeta^2 - \lambda^2} \qquad (10a)$$

and, for yielding supports,

$$K = \frac{I}{(\lambda - \zeta)\,l} \qquad (10b)$$

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[28] Dipl.-Ing., Los Angeles, Calif.

[29] "Shearing Deformation in Continuous Beams and Rigid Frames," by A. Floris, *Journal*, Am. Concrete Inst., November–December 1937, p. 165.

In both cases the carry-over factor will be,

$$\beta = \frac{\lambda}{\zeta} \qquad (11)$$

By considering the shearing deformation of the structure, the fixed-end moments are expressed by,

$$S_{ab} = \frac{Q_r \zeta - Q_l \lambda}{(\zeta^2 - \lambda^2)\, l} \qquad (12a)$$

and,

$$S_{ba} = \frac{Q_l \zeta - Q_r \lambda}{(\zeta^2 - \lambda^2)\, l} \qquad (12b)$$

in which Q_l and Q_r are the static moments of the moment area of the freely supported bar relative to the left and right supports, respectively. For symmetrical loadings the fixed-end moments in Equations (12) reduce to the ordinary fixed-end moments, found in almost all handbooks on structural engineering.

For a concentrated load, P, at a distance, l_1, from the left and l_2, from the right supports, for instance, these moments are,

$$Q_l = \frac{P\, l_1}{6} (l^2 - l_1^2) \qquad (13a)$$

and,

$$Q_r = \frac{P\, l_2}{6} (l^2 - l_2^2) \qquad (13b)$$

If several concentrated loads, unsymmetrically spaced, are acting on the bar, the statical moments are found by substituting the values in Equations (13). It should be noted in passing that, if the structure is composed of different materials, the stiffness factors should be multiplied by the modulus of elasticity of the material, everything else remaining the same.

In all the foregoing expressions, λ and ζ are taken from Equations (9). By neglecting the last terms of this equation containing ν and γ and showing the influence of the shearing deformation, the expressions used by the author are obtained immediately.

The further analysis of the structure follows the steps that were so ably outlined by Mr. Stewart. For this reason a repetition of this procedure is not necessary.

In conclusion, it should be emphasized that the proposed method, in addition to its theoretical interest, has an intrinsic practical value. Dispensing with the inconvenient simultaneous equations and by the aid of few physical concepts and pure geometrical relationships, the author succeeds in solving, in a natural, speedy, and elegant manner, problems of marked complexity. In certain cases, the method is almost without rival.

D. B. HALL,[30] ASSOC. M. AM. SOC. C. E. (by letter).—In this paper the author further demonstrates, by new applications, the usefulness of his conception of the elastic curve as a traverse.[2] The first requisite in applying the methods of this paper, or in developing and applying still different methods, is a thorough grasp of this principle. The property of the traverse used in the paper (which makes it at once so simple and yet effective for solving continuous structures, and which has been extensively used but not explicitly stated by the author), is this: In a traverse such as Fig. 18, Angles *a* and *d* measure the rotations at the ends of the member; and, Angles *b* and *c* measure the moments at the ends of the member. Thus, the simple geometrical relationship among these four angles fully defines the elastic properties of the beam as it acts in a structure.

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[30] Designer, Ash-Howard-Needles & Tammen, New York, N. Y.

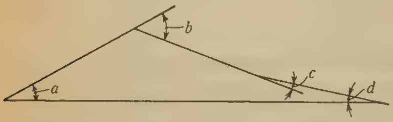

FIG. 18

The author has mentioned the fact that many complicated structures may be analyzed by considering only the members in the vicinity of any particular point, but he has devoted his paper mainly to exact solutions, as was appropriate for purposes of demonstration. From a practical standpoint the approximate solutions may be the more important.

Consider the closed box in Fig. 5. If End *B*, of Member *BC*, is given a rotation of 1, regarding it as the free end of a four-span structure, and then the traverse is traced all the way around to Point *B* in Member *AB*, the rotation at that point will be 142, and the moment, 1 × 246. Although a theoretical analysis of the structure should take into account the net rotation of Member *BA* with respect to Member *BC*, it is evident that, for all practical purposes, the stiffness of Member *BA* is $\frac{246}{142}$. The stiffness of Member *BC* found similarly by starting with a unit rotation at Point *B* in Member *BA*, is $\frac{2 \times 123}{74.5}$. It may be of academic interest, but it is scarcely worth remembering, that the true relative stiffness factors are $\frac{246}{141}$ and $\frac{2 \times 123}{73.5}$, since the rotation of one member with respect to the other is 1 less than its absolute rotation. Using one less span each way (that is, going around clockwise from *C* to *B*, or counter-clockwise from *A* to *B*) gives, for Member *B A*, $\frac{45}{26}$; and, for Member *BC*, $\frac{2 \times 33}{20}$, values which differ from the preceding by about 1 per cent. Thus, although a theoretically exact solution for the closed box required quite a little ingenuity, a practically exact solution presents no special difficulty.

RALPH W. HUTCHINSON,[31] ASSOC. M. AM. SOC. C. E. (by letter).—The relative flexure method of analyzing continuous structures presents a direct solution of these structures which may be applied efficiently to continuous-span bridges. Since haunched and tapered beams are used freely in continuous bridges, the determination of the properties of the members becomes an important detail of the analysis. Many computers use prepared charts or tables to obtain the properties of these beams for use with the familiar methods of analysis. The lack of such charts prepared for the relative flexure method need not discourage the use of the method as a tool because, by simple interrelations, the position of the points of inflection and the stiffness factors of haunched and tapered beams may be obtained quickly from the tables and charts in common use for obtaining the properties of these beams, which are used in connection with the method of end-moment distribution.

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[31] Associate Bridge Engr., State of California, Sacramento, Calif.

When a moment is applied to one end of a beam which is hinged at the other end, the ratio of the rotation at the opposite ends of the beam is the carry-over factor, γ. Furthermore, $\frac{x\,l}{l - x\,l} = \gamma$; or, $x = \frac{\gamma}{1 + \gamma}$, which locates the point of inflection. For the same beam the angle of flexure, Δ (Fig. 19), is the sum of the rotations of the two ends of the beam and is equal to $(1 + \gamma)\,\theta_L$ in which θ_L is the rotation at the applied moment.

Inversely, the stiffness for flexure, $K\,F = \frac{K\,C}{1 + \gamma}$, in which the product, $K\,C$, is the stiffness of a free-ended beam as used in end-moment distribution. Furthermore, for symmetrical beams the stiffness, $K\,F = K\,(1 - \gamma)$, in which K is the stiffness of the fixed beam as used in end-moment distribution. For tapering beams there will be two carry-over factors and, therefore, two stiffness factors and the points of inflection will not be symmetrical.

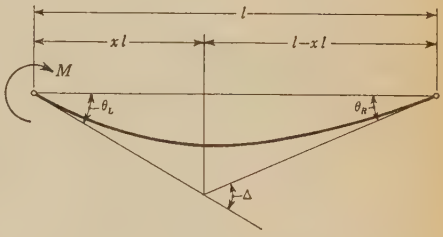

FIG. 19

Using the charts prepared by Thor Germundsson, M. Am. Soc. C. E.,[32] in 1932, the properties of the haunched beams from the author's illustrated example, Fig. 4(*a*), can be readily obtained. From the charts $\gamma = 0.71$; and, $K\,C = 6.2$. The point of inflection is at $x = \frac{0.71\,l}{1.71} = 0.415\,l$, which checks the value of $0.41\,l$ given by the author. The stiffness factor, $K\,F = \frac{6.2}{1.71} = 3.62$. The actual stiffness is $\frac{3.62 \times 2.25^3}{54} = 0.765$ or, as stated by the author, the relative $\Delta = \frac{M}{0.765} = 1.308\,M$, which checks the value of $1.292\,M$ given by the author.

The relative flexure method is most efficient when applied to a series of continuous spans on flexible piers which are subject to moving loads. For analyzing a structure of this type the method appears to be faster than any other. For continuous spans on stiff piers it requires the introduction of another flexure system, that of the pier, and thus becomes inferior to the end-moment distribution method which gives moments in the piers from the beam moments without additional effort or complications. Hinged beams can also be solved readily by end-moment distribution with no more effort than that required for a solid beam whereas the solution by the relative flexure method appears to be excessively cumbersome.

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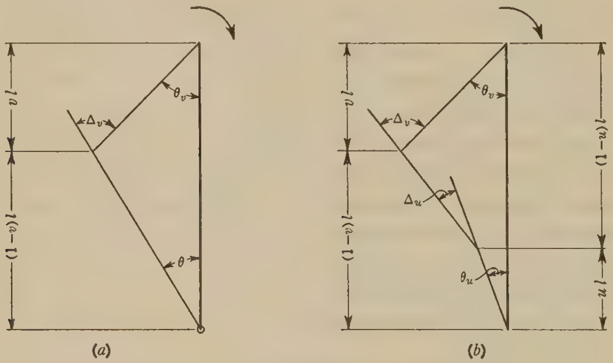
[32] *Civil Engineering*, October, 1932, p. 647.

The case of continuous spans on flexible piers appears to the writer to be the only one in which the relative flexure method has any advantages over the common methods of solution. It is the writer's opinion that only those computers who frequently handle this type of structure should attempt to master the method. Those who work with this type of structure infrequently can best solve it by one of the methods with which they are already familiar.

Arthur B. McGee,[33] Esq. (by letter).—The analysis of partial fixture, in this paper, is very brief. Partial fixture of a member with variable moment of inertia, and treatment of a moment applied to the frame at a point of partial fixture, are not included.



(a) (b)

Fig. 20

The writer prefers to omit the idea of an imaginary member and deal directly with the angle of rotation of the partially fixed end. Fig. 20(*a*) shows a "traverse of the elastic curve" of a member with variable moment of inertia, hinged at the lower end. Fig. 20(*b*) shows a traverse of the same member with the lower end partially restrained. The terms, u and v, are identical with those same terms in the tables by Walter Ruppel, Assoc. M. Am. Soc. C. E.[34] The fixture factor, r, as used herein, is defined by:

$$r = 1 - \frac{\theta_u}{\theta} \qquad (14)$$

in which θ is the angle of rotation of the hinged end, and θ_u is the angle of rotation of the restrained end (see Fig. 20). When the end is hinged, θ_u equals θ and $r = 0$. When the end is fixed, $\theta_u = 0$ and $r = 1$. The fixity thus varies from 0 (hinged) to 1 (fixed); 75% fixture means that $r = 0.75$ and the partially fixed end rotates through one-fourth the angle of a hinged condition.

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[33] Structural Engr. in Bridge and Structural Design Div., Bureau of Eng., City of Los Angeles, Los Angeles, Calif.

[34] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

To draw a traverse involving partial fixture and to evaluate the angles, it is only necessary to find the relation of θ_u to Δ_u (Fig. 20(*b*)). From Fig. 20(*a*):

$$\theta = \frac{v\,l}{(1 - v)\,l}\,\theta_v = \frac{v}{1 - v}\,\theta_v \qquad \text{(15a)}$$

from Fig. 20(*b*):

$$\theta_v \times v\,l = \theta_u\,(1 - v)\,l + \Delta_u\,[(1 - v)\,l - u\,l] \qquad \text{(15b)}$$

and,

$$\theta_v = \frac{1 - v}{v}\,\theta_u + \frac{1 - v - u}{v}\,\Delta_u \qquad \text{(15c)}$$

Substituting θ_v in Equation (15*a*):

$$\theta = \theta_u + \frac{1 - v - u}{1 - v}\,\Delta_u \qquad \text{(15d)}$$

and, substituting θ in Equation (14):

$$\frac{\theta_u}{\Delta_u} = \frac{(1 - u - v)\,(1 - r)}{(1 - v)\,r} \qquad \text{(16)}$$

Equation (16) is the general formula of the relation between the angles, θ_u and Δ_u. A member with a uniform moment of inertia has values of u and v equal to $\frac{1}{3}$, and Equation (16) is simplified to,

$$\frac{\theta_u}{\Delta_u} = \frac{1 - r}{2\,r} \qquad \text{(17)}$$

which is the same formula as that given in Mr. Stewart's paper. When a moment is applied at a partially fixed end, a part of the moment is taken directly by the support and may be treated as a fixed-end moment not affecting any other joint in the frame. The part of the moment that is not taken by the support goes back into the frame. It is not necessary to make a traverse starting from the point of partial fixture to distribute this moment. Merely transfer this moment to the far end of the member by multiplying it by $\frac{v}{1 - v}$, the usual carry-over factor, and subtracting or adding the result to the existing moment at the far end, depending on whether there is an odd number or an even number (including zero) of points of inflection between the ends.

The problem shown in Fig. 21(*a*) will now be worked in detail. Fig. 21(*b*), (*c*), (*d*), (*e*), shows the properties of the members. Each Δ equals the area of an $\frac{M}{E\,I}$-diagram. Thus, in the traverse an angle represents an area. The formulas for Δ may be found in a discussion by Fang-Yin Tsai, Assoc. M. Am. Soc. C. E.[35] The values of u, v, p, and q are taken from Ruppel's tables.[34] In Fig. 21(*a*) the length of Member *C D* is 20; and in Fig. 21(*d*) and Fig. 21(*e*): $u = 0.250$; $v = 0.427$; $p = 0.292$; and $q = 0.171$. In Fig. 22(*a*) apply an unknown moment at Joint *B* as shown by the arrow. Rotate Joint *B* and start drawing the traverse from that point, progressing both ways to Points A and D. Angle points occur at each corner and opposite each center of gravity of the $\frac{M}{E\,I}$-diagrams. The lines of the traverse are tangent to the elastic curve at the joints, but the middle section on each side is not tangent to the elastic curve. The numbers in parentheses are moments; the other numbers are angles; and, ratios of Δ-angles to moments are given in Fig. 21(b), (c), (d), (e).

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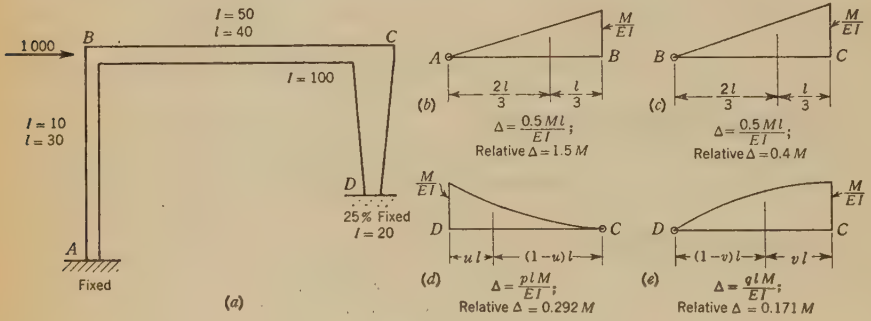
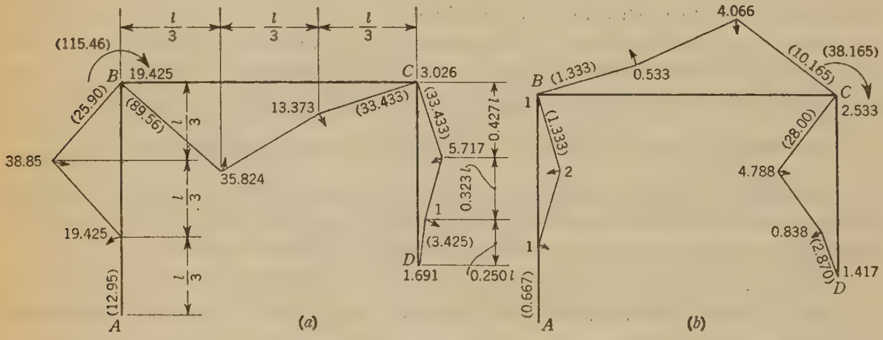
[35] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 129.



1000
B
C
I = 50
l = 40
I = 100
I = 10
l = 30
D
25% Fixed
I = 20
A
Fixed
(a)
(b)
A
B
M/EI
2l/3
l/3
Δ = 0.5Ml/EI;
Relative Δ = 1.5 M
(c)
B
C
M/EI
2l/3
l/3
Δ = 0.5Ml/EI;
Relative Δ = 0.4 M
(d)
M/EI
D
C
ul
(1 − u)l
Δ = plM/EI;
Relative Δ = 0.292 M
(e)
D
C
M/EI
(1 − v)l
vl
Δ = qlM/EI;
Relative Δ = 0.171 M

FIG. 21

Solving Equation (16) gives $\theta_u = 1.691\ \Delta_u$. Now, start at Point D letting θ_u, the angle of rotation, equal 1.691 and the next angle, 1. Then, $1 \div 0.292 = (3.425)$; $[1.691 \times l + 1 \times (0.427 + 0.323) \times l] \div 0.427 \times l = 5.717$; $5.717 - 1 - 1.691 = 3.026$; $5.717 \div 0.171 = (33.433)$; $33.433 \times 0.4 = 13.373$; $13.373 \times 1 + 3.026 \times 2 = 19.425$; $19.425 + 13.373 + 3.026 = 35.824$; $35.824 \div 0.4 = (89.56)$; $19.425 \times 2 = 38.85$; $38.85 \div 1.5 = (25.90)$; $19.425 \div 1.5 = (12.95)$; and, $25.90 + 89.56 = (115.46)$.

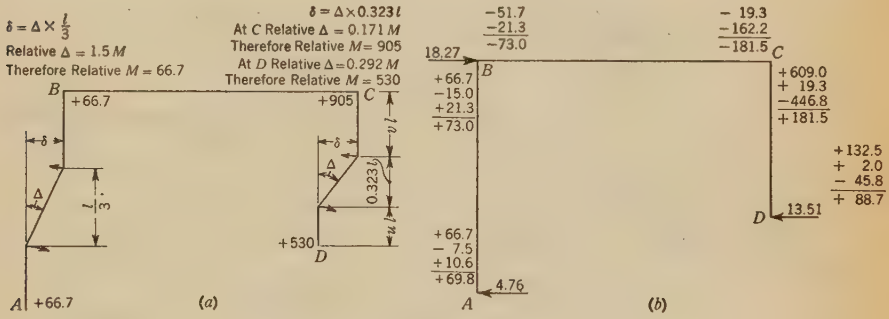


l/3
l/3
l/3
(115.46)
B
19.425
C
3.026
(25.90)
(89.56)
l/3
13.373
(33.433)
(33.433)
0.427l
38.85
35.824
5.717
l/3
0.323l
1
19.425
(3.425)
l/3
D
1.691
0.250l
(12.95)
A
(a)
4.066
B
(1.333)
0.533
(10.165)
(38.165)
C
2.533
1
(1.333)
2
(28.00)
4.788
0.838
(2.870)
1
1.417
D
(0.667)
A
(b)

FIG. 22

The traverse in Fig. 22(b) is drawn in a similar manner. Begin assigning angle values at End A and progress to Corner C whose angle of rotation is found to be 2.533. The angles from Point C to Point D must be proportional to those in Fig. 22(a) since $\frac{\theta_u}{\Delta_u} = 1.691$ as before. Therefore, the angles will be $\frac{2.533}{3.026}$ times the values in Fig. 22(a).

Fig. 23(*a*) shows the frame deflected sidewise with the top member straight and both legs fixed at the bottom. The deflection, δ, is the same in each leg so that the relative moments may be found for this condition. End *D*, however, being only 25% fixed will have a value of $0.25 \times 530 = 132.5$; $0.75 \times 530 \times \frac{v}{1 - v} = 296$, which is carried over to End *C*, and, $905 - 296 = 609$, the moment at End *C* when End *D* is 25% fixed. These moments are shown on the frame in Fig. 23(*b*). Now, Member *B C* is allowed to deflect; and, the moment at Joint *B* is distributed by using $\frac{66.7}{115.46}$ of the moment values in Fig. 22(*a*) and the moment at Joint *C* is distributed by using $\frac{609}{38.165}$ of the moment values in Fig. 22(*b*). Signs of the moments are readily seen by referring to the arrows on the traverses, clockwise being positive. The arrows are always placed so that they tend to straighten the elastic curves. Shears in the legs of Fig. 23(*b*) now show the moments to be due to a horizontal force of 18.27. Therefore, multiply all moments by $\frac{1\,000}{18.27}$ and find the moment at End $A = 3\,822$; Joint $B = 3\,994$; Joint $C = 9\,934$; and End $D = 4\,855$.



δ = Δ × l/3
Relative Δ = 1.5 M
Therefore Relative M = 66.7
δ = Δ × 0.323 l
At C Relative Δ = 0.171 M
Therefore Relative M = 905
At D Relative Δ = 0.292 M
Therefore Relative M = 530
B
+66.7
+905
C
vl
δ
Δ
l/3
0.323 l
ul
+530
D
A
+66.7
(a)
−51.7
−21.3
−73.0
18.27
B
+66.7
−15.0
+21.3
+73.0
− 19.3
−162.2
−181.5
C
+609.0
+ 19.3
−446.8
+181.5
+132.5
+ 2.0
− 45.8
+ 88.7
D
13.51
+66.7
− 7.5
+10.6
+69.8
4.76
A
(b)

FIG. 23

From preference, the writer has been using the Stewart traverse method since about 1936, for continuous and rigid frame bridges.

E. NEIL W. LANE,[36] JUN. AM. SOC. C. E. (by letter).—The use of relative flexure factors in the analysis of continuous structures, as described by Mr. Stewart, is simple and fundamental in theory. It is a general method for attacking the solution of moments in rigid frameworks that is particularly valuable for use in conjunction with moving and variable load systems. Combining bending moments by superimposing influence diagrams aids in visualizing the elastic behavior of a structure. Moreover, the process is quite time-saving.

The author's method somewhat parallels the steps in the slope deflection method in that the process is indirect. Angles are found first, and then substituted in the original equations to obtain the desired moments. Similarly, the author borrows from moment distribution in his procedure of correcting for side-sway. Trial moments are found first, to which corrections are added to satisfy statics.

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[36] Junior Engr., U. S. Dept. of Interior, National Park Service, Eng. Div., Omaha, Nebr.

In solving for moments under a single fixed condition of loading, the writer prefers the method known as traversing the elastic diagram that was published

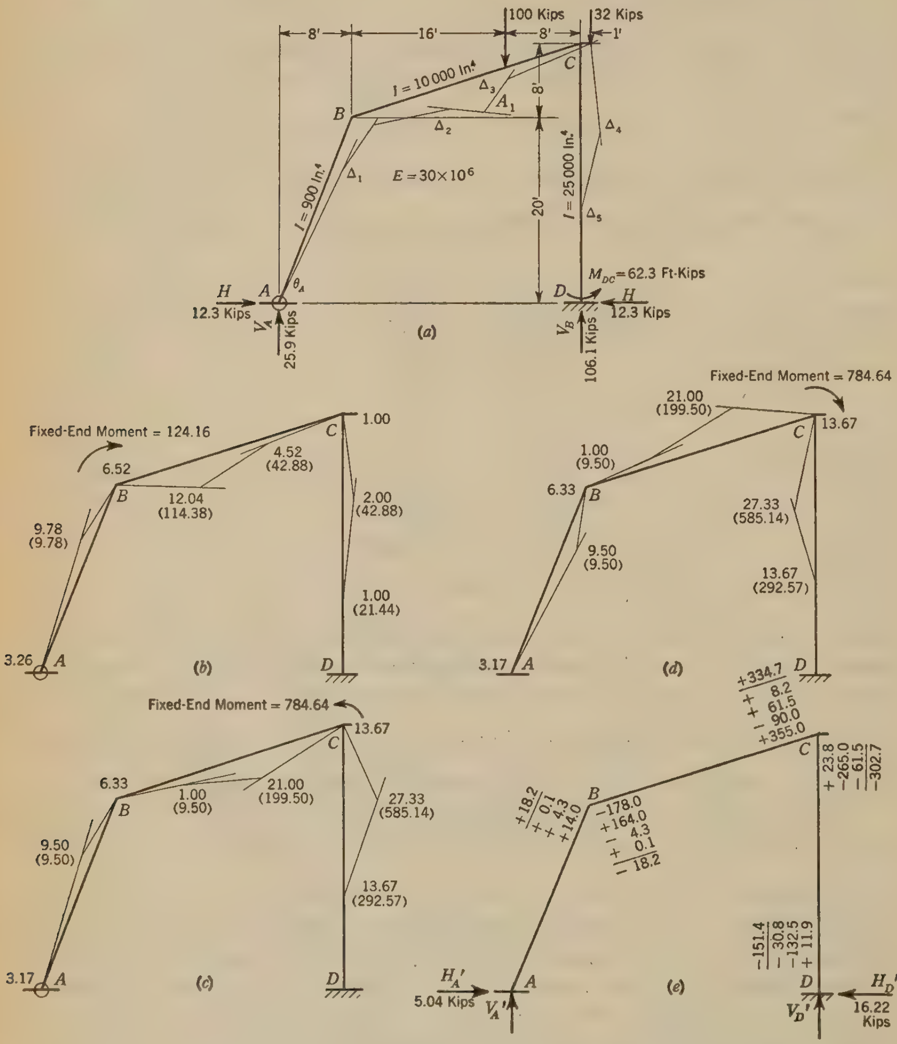
100 Kips
32 Kips
8'
16'
8'
1'
I = 10 000 In.⁴
C
Δ3
A1
8'
B
Δ2
Δ4
Δ1
E = 30×10⁶
I = 900 In.⁴
20'
I = 25 000 In.⁴
Δ5
θA
M_DC = 62.3 Ft-Kips
H
A
12.3 Kips
D
H
12.3 Kips
V_A
25.9 Kips
(a)
V_B
106.1 Kips
Fixed-End Moment = 124.16
6.52
B
4.52 (42.88)
C
1.00
12.04 (114.38)
9.78 (9.78)
2.00 (42.88)
1.00 (21.44)
3.26
A
(b)
D
Fixed-End Moment = 784.64
21.00 (199.50)
13.67
C
1.00 (9.50)
6.33
B
27.33 (585.14)
9.50 (9.50)
13.67 (292.57)
3.17
A
(d)
D
Fixed-End Moment = 784.64
13.67
C
6.33
1.00 (9.50)
21.00 (199.50)
B
27.33 (585.14)
9.50 (9.50)
13.67 (292.57)
3.17
A
(c)
D
+334.7
+ 8.2
+ 61.5
− 90.0
+355.0
C
+ 23.8
−265.0
− 61.5
−302.7
+18.2
0.1
4.3
+14.0
B
−178.0
+164.0
− 4.3
+ 0.1
− 18.2
−151.4
− 30.8
−132.5
+ 11.9
H_A'
5.04 Kips
V_A'
A
(e)
D
H_D'
16.22 Kips
V_D'

FIG. 24

by the author in 1934.[37] The method of relative flexure factors is an outgrowth and adaptation from the previous paper. It is believed that the evolution of the new method will be of interest. As a means of widening the scope of the author's present work, an example has been chosen to show the solution for moments in a frame with sloping members (see Fig. 24(*a*)).

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[37] "Analysis of Continuous Structures by Traversing the Elastic Curves," by Ralph W. Stewart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

The solution by the original method requires eight equations, the first of which traverses the elastic diagram:

$$\theta_A + \Delta_1 + \Delta_2 - A_1 + \Delta_3 + \Delta_4 - \Delta_5 = 0 \dots\dots\dots\dots (18)$$

in which, $A_1 = 0.00307$ radians. Equation (18) is analogous to others frequently used in elastic energy theory.[38] The next two equations are statements of relative stiffness:

$$\Delta_1 \left(\frac{900}{258.5}\right) = \Delta_2 \left(\frac{10\,000}{304.6}\right) \dots\dots\dots\dots\dots\dots (19a)$$

and, $M_{CB} = 384\,000 + M_{CD}$; or,

$$\Delta_3 = 0.000194 + 2.26\, \Delta_4 \dots\dots\dots\dots\dots\dots (19b)$$

The vertical deflection of Point *A* relative to Point *C* is zero:

$$32\, \theta_A + 26\tfrac{2}{3}\, \Delta_1 + 16\, \Delta_2 - 10\tfrac{2}{3}\, A_1 + 8\, \Delta_3 = 0 \dots\dots\dots\dots (19c)$$

The horizontal deflection of Point *A* relative to Point *D* is also zero:

$$-13\tfrac{1}{3}\, \Delta_1 - 22\tfrac{2}{3}\, \Delta_2 + 24\tfrac{4}{9}\, A_1 - 25\tfrac{1}{3}\, \Delta_3 - 18\tfrac{2}{3}\, \Delta_4 + 9\tfrac{1}{3}\, \Delta_5 = 0 \dots (19d)$$

Now, to obtain the sixth equation, three member equilibrium equations must be solved simultaneously. These are: $M_{CD} + M_{DC} - 336\, H = 0$, or,

$$10^6\, (\Delta_4 + \Delta_5) - 0.0752\, H = 0 \dots\dots\dots\dots\dots\dots (19e)$$

$M_{BC} \div M_{CB} + 96\, H - 288\, V_A + 96 \times 10^5 = 0$; or,

$$10^6\, (\Delta_2 - \Delta_3) + 0.0485\, H - 0.1455\, V_A + 4\,850 = 0 \dots\dots\dots (19f)$$

and, $M_{BA} - 240\, H + 96\, V_A = 0$; or,

$$10^6\, \Delta_1 - 1.153\, H + 0.461\, V_A = 0 \dots\dots\dots\dots\dots\dots (19g)$$

Slide-rule values give $H = 12.3$ kips; $V_A = 25.9$ kips; and, $M_{DC} = -62.3$ ft-kips.

By using relative flexure factors the moment coefficients, as shown in Figs. 24(*b*), 24(*c*), and 24(*d*), are obtained. These are relative moments, immediately adaptable and easily checked for any system of loading. The moments, uncorrected for side-sway, are shown in Fig. 24(*e*). By the foregoing procedure, Equations (18), (19*a*), and (19*b*) have been eliminated and the problem simplified. The true moment at the end of each member will consist of the uncorrected moment, plus or minus some correction. Let these corrections be: *X* at Point *B*; *Y* at Point *C*; and, *Z* at Point *D*. Then, the solution of the remaining five equations will furnish the desired results. Equations (19*e*), (19*f*) and (19*g*) then become,

$$16\,220 - 0.00298\, Y - 0.00298\, Z - H = 0 \dots\dots\dots\dots (20e)$$

$$60\,440 - 0.01042\, X = 0.01042\, Y + H - 3.0\, V_A = 0 \dots\dots\ (20f)$$

---

[38] "Elastic Energy Theory," by J A. Van den Broek, M. Am. Soc. C. E. (1931), p. 81, Equation III.

and,

$$910 - 0.00417\,X - H + 0.40\,V_A = 0 \qquad \text{(20}g\text{)}$$

For convenience, Equations (19*c*) and (19*d*) were replaced. The fourth equation is a statement of a statement of moments about Point *D*: $M_{DC} + 9\,216\,000 - 384\,V_A = 0$; or,

$$28\,730 - 0.00261\,Z - V_A = 0 \qquad \text{(21}a\text{)}$$

The fifth formula is a statement equating the vertical deflection of Point *A* when released to the opposite deflection caused by the redundant reaction:

$$1.372 + 0.00484\,X + 0.00278\,Y - 0.00133\,Z = 0 \qquad \text{(21}b\text{)}$$

This is a means of correcting for side-sway in any frame.

It is worth while to note that the elastic energy theory probably furnishes the most simple and direct solution for this type of problem. When used with area moments, Equation (21*b*) becomes:

$$0.212\,V_A - 0.385\,H - 0.00103\,M_{DC} - 2.19 = 0 \qquad \text{(22}a\text{)}$$

Using a statement of horizontal deflections similar to Equation (22*a*):

$$0.288\,V_A - 0.594\,H - 0.00024\,M_{DC} - 1.19 = 0 \qquad \text{(22}b\text{)}$$

The use of Equations (21*a*), (22*a*), and (22*b*) makes a much more desirable form of solution of problems of this nature than the method proposed by the author. The writer does believe the method of relative flexure factors to be an improvement over the procedure published by Mr. Stewart in 1934 for use in analyzing continuous beams and frameworks not subject to side-sway.

RALPH W. STEWART,[39] M. AM. SOC. C. E. (by letter).—In presenting a new approach to the analysis of continuous structures it cannot be expected that the profession will adopt it quickly. The writer feels gratified, therefore, that of those contributing discussion several engineers have endorsed the method for the class of structures treated.

Mr. Shapiro thinks that the unpublished method of "transmission coefficients"[9] is the best for all-round use. By private correspondence with Mr. Shapiro the writer obtained a solution of Fig. 25,[40] by the method of transmission coefficients and found that the number of arithmetical operations necessary to solve it by that method was greater than by the method of relative flexure factors; and that the former method lacked the automatic checks and valuable by-products offered by the latter method.

The direct construction by Mr. Peterson of influence lines for both moments and shear, and also for yielding supports by using a traverse of the elastic curves in connection with the Müller-Breslau relationship between influence lines and elastic curves, is a valuable addition to the paper. Any computer who wishes to avoid the graphical construction of funicular polygons to obtain the ordinates of the final elastic curves can use the following formula which, for beams of constant section, expresses the ordinates to the elastic curves in terms of the end slopes:

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[40] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 1, p. 5.

$$\text{Ordinate} = (\theta_1\, k_1\, k_2^2 + \theta_2\, k_1^2\, k_2)\, l \ldots\ldots\ldots\ldots\ldots (23)$$

in which θ_1 and θ_2 are the left and right end slopes respectively, and k_1 and k_2 the fractions of the beam length from the left and right ends respectively.

The computation and tabulation of ordinates from this formula will be less laborious to most computers than the graphical construction.

Mr. Housner objects to considering the length of the base of a traverse triangle as equal to the lengths of the two inclined sides. If the validity of the traverse method of treating elastic structures is rejected on this account, it will be necessary also to reject the classic theory of flexure which neglects the difference between the abscissa of a point in a beam and the diagonal distance to the same point after the beam is deflected. Textbooks on flexure are based on the radius of curvature of a flexed beam being taken as

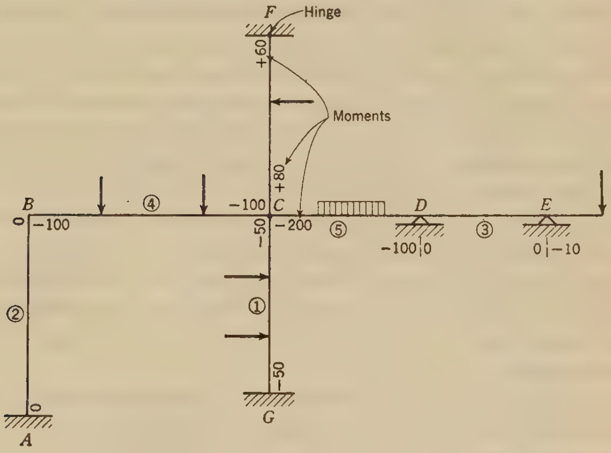

FIG. 25

$$r = \frac{(dl)^3}{dx\, d^2y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Treatises on the calculus show that the radius of curvature is

$$r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

An attempt to reduce Equation (25) to Equation (24) will disclose a point in the treatment of flexure used in all texts to which Mr. Housner's objections would be applicable. The value of an infinitesimal, or nearly infinitesimal, angle may be taken as equal to the value of either its sine or its tangent. In stress computations for bridges and buildings it is entirely proper and customary to consider the angles due to elastic deformation as conforming to the law of infinitesimal quantities, as is done in this paper.

The illustration by Professor Wilbur that for a continuous beam the same set of arithmetical computations used in the writer's paper can be applied in connection with the moment-area method is correct. However, the extension of the moment-area delineation to more complex problems will give rise to difficulties, whereas the traverse delineation will make it easy for the computer to keep account of what he is doing. Mr. Hadley's complimentary statements about the flexure-factor method are very pleasing. Mr. Blog gives interesting general comments and opinions on various methods of analyzing continuous structures. As time passes and new literature is published the rating of these various methods will become determined more accurately. Messrs. Mitchell, Floris, Hall, and McGee endorse the flexure-factor method for one-story structures, and all have done constructive work to expand the scope of the paper. The statement of Mr. Mitchell that for continuous beams the three-moment theorem and end-moment distribution cannot compare with the flexure-factor method, and the statement of Mr. McGee that he has adopted the method for rigid frames, are very gratifying.

The solution by Mr. McGee of a laterally loaded unsymmetrical frame with one partly fixed pier base supporting a tapering pier is an interesting illustration of the utility of the flexure-factor method. This solution is worth saving for reference, not only on account of the problem but also because of the complete exposition of the method of adopting the Ruppell tables[34] to an elastic curve traverse.

Mr. Hutchinson expresses an opinion that the relative flexure-factor method is advantageous only for continuous spans on "flexible piers," and that in cases in which the elastic properties of the piers must be considered, or in which a hinge occurs in a span, the method is less convenient than other methods. According to studies by the writer and others, the inclusion of elastic piers adds no more to the work of computation than if methods other than the flexure-factor method are used. If side-sway can be neglected it adds less work to include piers in a flexure-factor analysis than to include them in a moment distribution or slope deflection analysis. It is also easy to traverse across a hinged span as the following solution will demonstrate. Fig. 26 represents the same continuous beam as Fig. 2 except that a hinge has been introduced at the third-point of Span $B\ C$. The original $\frac{I}{l}$-value for this span was 4. After inserting the hinge the $\frac{I}{l}$-values of the two fractional parts will become 12 and 6, as shown.

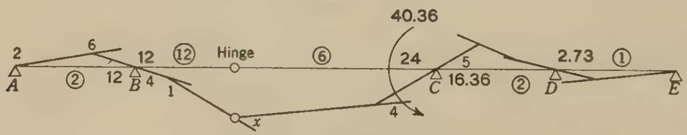

FIG. 26

By statics, the relationship of M_{BC} to M_{CB} must be in proportion to the distances of Points B and C from the hinge, or as 1 : 2. Dividing by the stiffnesses, the ratio of the corresponding Δ-angles is as 1 to 4. Write these relative Δ-values on the figure; multiply them by the stiffnesses to get the relative moment values; designate the angle at the hinge by x; balance the moment of 12 with M_{BA} also equal to 12; and write the angle values appurtenant to $M_{BA} = 12$, in Span A B. For convenience, consider Span B C as divided into nine equal units of length. The usual type of traverse equation then gives $(4 \times 9) + (1 \times 8) - 6\,x - (4 \times 2) = 0$; and $x = 6$. Combining angles from Points B to C gives $\theta_c = 5$.

The resisting moments in Spans C D and D E will now be $\frac{5}{11}$ times those in Fig. 2. Write these in the figure, add the two resisting moments at Point C to get the activating (fixed-end) moment of 40.36 at Point C, and the picture of the flexure is complete with all moments and rotations given. If the hinge is placed at one-seventh of the distance from Point B to Point C (which is the point of contraflexure in Fig. 2(e)) the angle x will be found by the foregoing procedure to be zero, and the hinge will be without effect on the moments in the structure. This form of analysis will enable the computer to determine the desirable location for a hinge more quickly than by other forms.

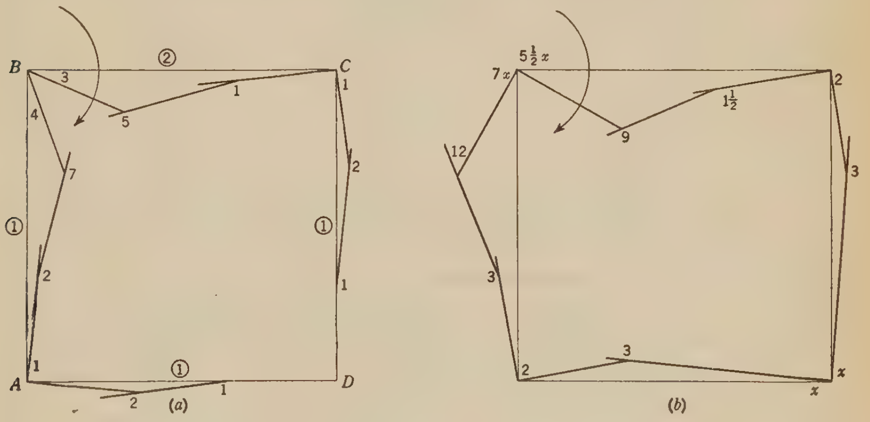

FIG. 27

Mr. Hall's solution of the closed box illustrates very well how an exceedingly accurate approximation of the bending moments can be obtained by a simplified application of the method. Although it is as exact as the common theory of flexure, the solution given in the paper involves the somewhat troublesome mental effort of carrying computations along the figure with each angle expressed as a two-term quantity. After further study the writer has concluded that most computers will find it easier to make two separate computations for this structure, using a single term for each angle. This is done as follows:

In Fig. 27(a), consider rotation as zero at Joint D and draw a traverse D C B due to the moment M_{BC} as shown, ending with a joint rotation of 3 at Point B. Now, with the moments about Joint D in balance, begin at D and draw the traverse *D A B*, as shown, ending with a joint rotation of 4 at Point *B*. To equalize these joint rotations at Point *B* it is obvious that Joint *B*, and consequently Joint *D*, must rotate in clockwise direction. Therefore, draw Fig. 27(*b*), in which Joint *D* is without restraint and has clockwise rotation equal to x. Numerals in this figure are coefficients of x. Now, combining Figs. 27(*a*) and 27(*b*), since Rotation *B A* must equal Rotation *B C*: $7x - 4 = 5\frac{1}{2}x + 3$; and $x \doteq 4.667$, which checks Fig. 5.

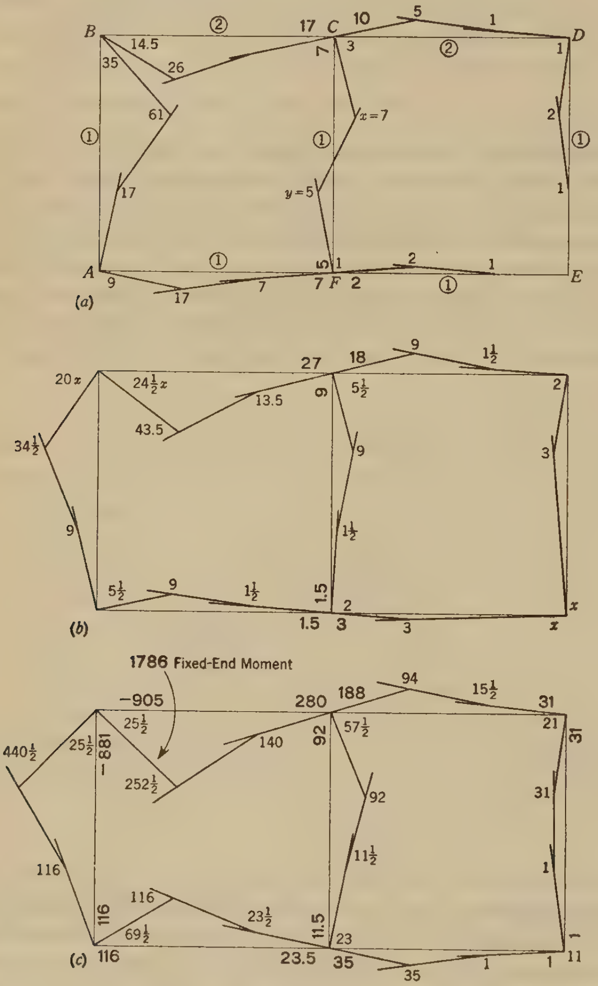
B
14.5
35
26
②
17 C
10
5
1
D
7
3
②
1
61
$x = 7$
2
①
①
①
17
$y = 5$
1
A
9
17
①
5
1
2
1
7
7 F 2
①
E
(a)
20x
$24\frac{1}{2}x$
27
18
9
$1\frac{1}{2}$
9
13.5
$5\frac{1}{2}$
2
43.5
$34\frac{1}{2}$
9
3
$1\frac{1}{2}$
9
$5\frac{1}{2}$
9
$1\frac{1}{2}$
1.5
2
x
1.5
3
3
x
(b)
1786 Fixed-End Moment
−905
280
188
94
$15\frac{1}{2}$
31
$25\frac{1}{2}$
92
$57\frac{1}{2}$
21
31
$440\frac{1}{2}$
$25\frac{1}{2}$
−188
140
$252\frac{1}{2}$
92
31
$11\frac{1}{2}$
116
1
116
$23\frac{1}{2}$
116
11.5
$69\frac{1}{2}$
23
−1
(c)
116
23.5
35
35
1
1
11

FIG. 28

This treatment can readily be extended to multiple box openings, the solution involving one pair of simultaneous equations at each intermediate vertical member. To illustrate this case the traverse in Fig. 28(*a*) is drawn from Point *E* to Point *C* and from Point *E* to Point *F*, letting Point *E* be fixed and balancing the moments about Point E. To close the traverse along Member CF: $(3 \times 3) - 2x + y = 0$; and $-3 + 2y - x = 0$. Solving, $x = 7$; and $y = 5$.

The remainder of the solution involves nothing that has not already been explained. Bending moments not necessary for the computation are omitted from Figs. 28(*a*) and 28(*b*), for simplicity. Any omitted moment may be added to the figure at once by multiplying its Δ-angle by the stiffness factor of the member.

Mr. Lane's solution of an unsymmetrical frame with a sloping column is of interest. It avoids the use of trigonometric functions which were applied to the solution of the same type of problem by Mr. Mitchell. His statement that the flexure-factor method is time saving is appreciated.

In conclusion it may be stated that the method of flexure factors based on a traverse of the elastic curves is not as restricted in its scope as some of the discussion indicated. It can be extended to curved beams and arch analysis. For arch analysis the Δ-angles are not in the unsprung axes of the curved members but are at the intersections of those end slope tangents which are selected to control the design. The ordinates of these Δ-angles can be computed, and by using the Δ-angle points as centers of rotation, the usual arch formula can be computed. Some of this extension work might be suitable for college theses.

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Paper No. 2028

DEOXYGENATION AND REOXYGENATION

By C. J. Velz,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Earle B. Phelps and C. J. Velz.

Synopsis

Too frequently, sewage disposal is thought of as synonymous with discharge into the nearest body of water. Such is not the case. Ultimate disposal (that is, final oxygen equilibrium) is inescapable and if it does not occur, at least partly, before the effluent is discharged, the sewage becomes a stream liability. The public is fast becoming impressed with the fact that it must be concerned with this problem beyond the sewer outlets. There is a general awakening and a demand that something be done about the menace and nuisance of stream pollution.

With advances in the knowledge and art of treatment permitting the reduction of almost any waste burden, the remedial problem is largely one of procedure and financing. If economy in efforts and funds were of no consequence, the problem could be solved simply by the invariable application of the highest and most elaborate treatment known. A sound and economical solution, however, requires an evaluation of pollution liabilities and stream assets, in advance of building, to permit the determination of the effect upon the stream of any proposed remedial program.

In design it is highly important to give careful consideration to plant details and mechanical efficiency, but in the enthusiasm over the development of the "means" one should not lose sight of the "end." First and foremost in importance is the effect of the design on the stream.

A technique for stream analysis is presented in this paper which is easy and rapid to apply and should serve as an aid in determining where, when, and how much to build, and what degree of treatment to provide.

Notation.—The introduction of some change in letter symbols is due to an effort to avoid conflict with standard symbols in allied fields of study.

Note.—Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in April, 1938, *Proceedings*.

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INTRODUCTION

Nuisance in streams is a combination of noxious conditions. Although no single measure fully reflects all these conditions, oxygen content of the stream is the most inclusive and best single index of the degree of pollution. Deoxygenation is a complex bio-chemical process influenced radically by time, temperature, and the nature of waste. Reoxygenation, no less complex, is primarily a chemical process, influenced radically by concentration of dissolved oxygen, time, temperature, depth, and turnover. Professor Earle B. Phelps and H. W. Streeter, M. Am. Soc. C. E., have integrated these factors into a single expression for resultant oxygen, as defined by the well-known formula:[2]

$$O_t = \frac{k\,O_{0d}}{K - k}\left(e^{-kt} - e^{-Kt}\right) + O_0\,e^{-Kt} \ldots\ldots\ldots\ldots\ldots (1)$$

in which: O_0 = initial dissolved oxygen saturation deficit of the water, in parts per million; O_t = saturation deficit, in parts per million, after time, t; O_{0d} = initial oxygen demand of the organic matter of the water, in parts per million; k = coefficient defining rate of deoxygenation; K = coefficient defining rate of reoxygenation; t = elapse of time, in days; and e = Naperian base.

An integrated formula such as Equation (1) has many advantages but, owing to its complexity and the many factors involved, is subject to the dangers of misunderstanding and misapplication. In certain instances, a better understanding and wider application of the fundamentals can be obtained by breaking down these complexities and considering deoxygenation and reoxygenation separately.

REOXYGENATION

The most important asset of a stream is the natural process of reoxygenation —that is, its absorption of oxygen from the atmosphere. Without it the entire world would long ago have been faced with complete treatment, or sewage everywhere would have to be disposed in deep sea water. Fortunately, reoxygenation is a gift of Nature. It is important in the pollution problem that this natural asset be evaluated because economics demands that it be used wisely.

As early as 1910, in his studies of the pollution of New York Harbor, Professor Phelps[3] established a fundamental basis for an evaluation. He considered reoxygenation as a separate phenomenon governed by two fundamental laws: (1) The law of rate of solution; and (2) the law of diffusion. The rate of absorption is inversely proportional to the concentration of dissolved oxygen; and the rate of diffusion through water and between two proximate points is proportional to the difference in concentration at these two points. With these two fundamental laws and his experimental determination of the coefficient of diffusion in quiescent water, Professor Phelps developed the formula for absorption of oxygen:

$$O_t' = 100 - \left(1 - \frac{O_{0d}'}{100}\right) \times 81.06\left(0.779^6 + \frac{0.105^c}{9} + \frac{0.0019^c}{25} \cdots\right) \ldots (2)$$

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[2] "A Study of the Pollution and Natural Purification of the Ohio River: III, Factors Concerned in the Phenomena of Oxidation and Reaeration," *Public Health Bulletin No. 146*, U. S. Public Health Service, 1925.

[3] "Location of Sewer Outlets and Discharge of Sewage into New York Harbor," Rept. by the late Maj.-Gen. William Black, U. S. Army (*Retired*), and Prof. Phelps, 1910.

in which O_{0d}' = initial dissolved oxygen expressed as the average content in percentage of the saturation value; O_t' = final dissolved oxygen expressed as the average content in percentage of the saturation value; and c = a constant for each set of conditions and equal to,

$$c = \frac{\pi^2 a t}{d^2} \qquad (3)$$

in which t = time of exposure, in hours; d = depth of water, in centimeters; and, a = a diffusion coefficient which, in turn, is a function of temperature.

From an analysis of Professor Phelps' experimental determination of diffusion coefficient, the writer develops the relationship between the coefficient and temperature represented by the formulas:

$$\log a = 0.04125\, T - 0.672 \qquad (4)$$

or,

$$a = 1.42 \times 1.1^{(T-20)} \qquad (5)$$

in which T = temperature, in degrees Centigrade. It is apparent from the graphical representation shown in Fig 1 that there is linear relationship between the log of the coefficient of diffusion and temperature.

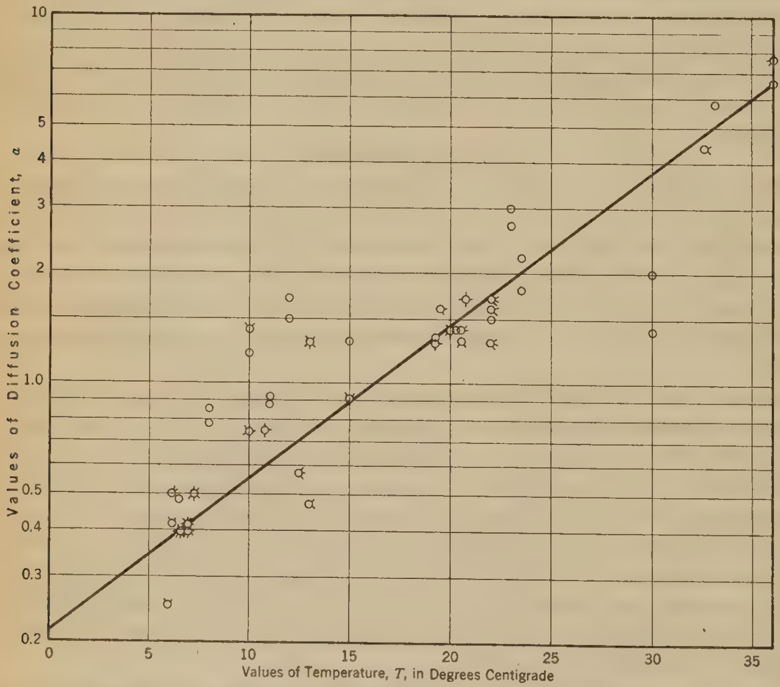

Fig. 1.—Relationship Between Diffusion Coefficient and Temperature (Degrees Centigrade), Based on Phelps' Experimental Data

The fundamental relationships established mathematically and experimentally are applicable to actual stream conditions. For example, consider a flowing stream: The integration of the average oxygen content of a stretch of the stream is analogous to mixing the entire volume prior to exposure to the atmosphere. To maintain the integrated average oxygen content at a constant value (that is, along given profiles and contours) by the equilibrium established between deoxygenation and reoxygenation, is analogous to considering the body of water quiescent in so far as movement on a horizontal plane is concerned. As long as that equilibrium is maintained the initial oxygen deficit remains constant, deoxygenation factors utilizing oxygen just as fast as, but no faster than, the water absorbs oxygen from the atmosphere. Under such conditions the quantity of oxygen absorbed in any given time is dependent only upon the frequency of vertical mixing—the stream turnover. Thus, by substituting the factors in Equation (2), using time of exposure, t, as the interval between vertical mixes, the oxygen absorbed by the river stretch is obtained. Under the conditions of equilibrium maintained, the same quantity will be absorbed in each succeeding mixing interval. Hence, Professor Phelps' integrated equation expressing total quantity, in actual application becomes a differential equation expressing rate of absorption. The only factor not readily measurable in the stream is frequency of vertical mixing, which, for the most streams, falls between relatively narrow limits.

Equation (2) permits direct evaluation of the influence of depth, which, as is apparent from Equation (3), is a dominating factor. In Equation (1) all reoxygenation influences are combined into a single constant, K. The comprehensiveness of K is likely to lead to error when it is applied to streams other than the one for which it was evaluated, whereas, in Equation (2) cognizance is taken of the particular physical characteristics of each stream. Furthermore, Equation (1) is limited in application to inland waters. The complexities of tide-water flow which preclude the application of Equation (1) to brackish water, have been indicated in a paper on the pollution of New York Harbor written in 1932.[4] In practical application, the disadvantage of Equation (2) is the mathematics necessary to solve it. A large number of terms of the converging series, involving Exponent c must be used in order to obtain accuracy (40 to 60 terms, when the value of c is small). The writer has developed a graphical means of solution from which oxygen absorption, under any set of conditions, can be obtained accurately at a glance. By mathematics the process would require much painstaking effort.

Standard Reoxygenation Curve

From Equation (2) it is apparent that oxygen absorption is inversely proportional to initial concentration, or directly proportional to oxygen deficit $\left(1 - \frac{O_{0d}'}{100}\right)$. Furthermore, it is a function of the summation of a converging series involving time, depth, and a specific diffusion coefficient as an exponent, c, in each term of the series. By making O_{0d}' equal to zero (beginning with water devoid of oxygen), the summation of the series,

$$\Sigma s = 81.06\left(0.779^c + \frac{0.105^c}{9} + \frac{0.0019^c}{25} \cdots\right) \ldots\ldots\ldots\ldots (6)$$

represents the deficit at any time; or the deficit becomes a function of c.

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[4] "Report on the Pollution of New York Harbor," by Earle B. Phelps and C. J. Velz, *Sewage Works ournal*, January, 1933.

A solution of the series for various arbitrary values of c provides a set of control points indicating the relationship between c and the deficit which, when plotted on natural scales, produces Fig. 2. The section of the curve most frequently applicable to actual stream conditions lies between values of c from 0.000001 to 0.001, for which accurate interpolation is not permitted by Fig. 2. In order to emphasize the range of the curve and particularly to provide a convenient means of evaluating c in terms of the factors by which it is influenced, the writer has developed the graphical representation shown in Fig. 3. It is at once apparent from this chart that for the most useful range of the curve there is linear relationship between the dissolved oxygen and the log of c, a very convenient relationship for interpolation and for conversion.

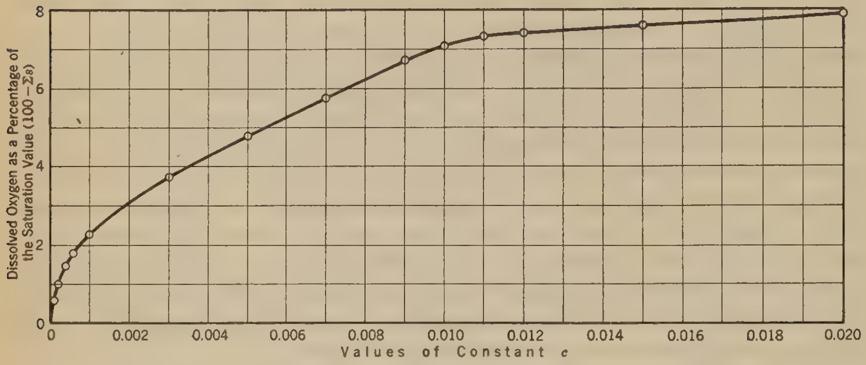

FIG. 2

In the evaluation of c in Equation (3), in terms of time, depth, and diffusion coefficient, including temperature, the writer has used the characteristics of a logarithmic scale adaptable in the principle of a slide-rule: Constant c is inversely proportional to the square of the depth, d, and directly proportional to time, t, and the diffusion coefficient, a. The diffusion coefficient, a, is a function of temperature as defined by Equation (5); and, since c is directly proportional to a, for each unit change in temperature, T, a fixed rate of change occurs in c. The use of the standard reoxygenation curve is illustrated by examples.

Example 1.—Given a stretch of stream in which the average depth, d, is 10 ft; the average temperature, T, is 24° C; the dissolved oxygen profile of the stretch is maintained constant at an integrated average of 40% of the saturation value; and the flow conditions were such as to produce a time of mix, t, of 15 min.

Referring to Fig. 3: (1) Opposite 10 ft on Scale (4) read 0.000151 on Scale (3); (2) slide the control point of Scale (8) to coincide with 0.000151 on Scale (3); (3) opposite $\frac{1}{4}$ hr on Scale (8) read 0.000038 on Scale (3); (4) slide the control point of Scale (7) to coincide with 0.000038 on Scale (3); (5) opposite 24° on

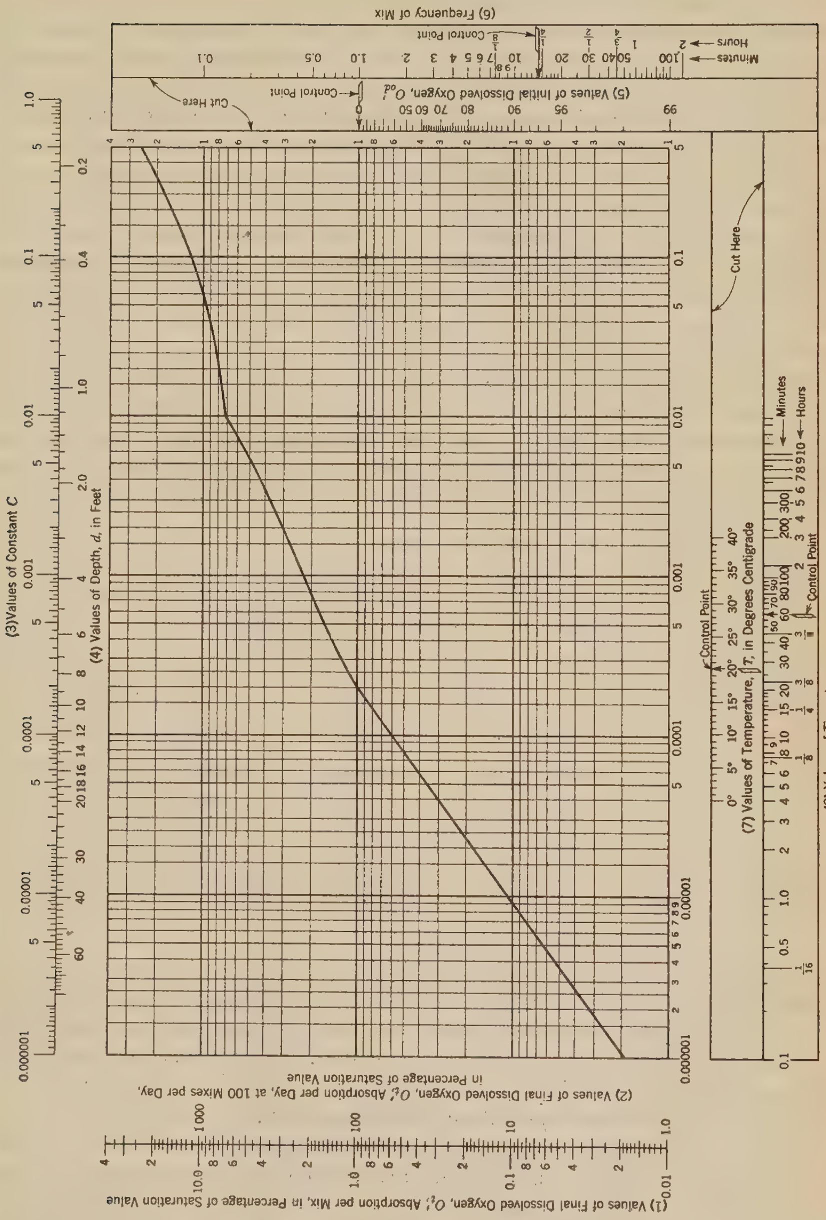

FIG. 3.—STANDARD REOXYGENATION CURVE, BASED ON BLACK AND PHELPS' FORMULA

Scale (7) read 0.000055 on Scale (3) (this gives the value of the constant, c, at a depth of 10 ft, a mixing time of $\frac{1}{4}$ hr, and 24° C); (6) descend vertically from 0.000055 on Scale (3) to an intersection with the standard reoxygenation curve; (7) extend horizontally to Scale (1) and read 0.38% (the quantity absorbed under the conditions in one mix period if water were initially devoid of oxygen); (8), slide the control point of Scale (5) to coincide with 0.38% on Scale (1), and opposite 40% on Scale (5) read 0.228% on Scale (1) (this represents the percentage of oxygen absorbed in each $\frac{1}{4}$-hr interval between mixes under the given conditions); and (9) to convert the result of Step (8) to a percentage of saturation absorbed per day, slide the control point of Scale (6) to coincide with 0.228% on Scale (1), and opposite $\frac{1}{4}$ hr on Scale (6) read 22.0% per day on Scale (2). Thus, under the conditions given, and as long as the oxygen balance remains constant at an average of 40% of saturation, this body of water will absorb oxygen each day equal in amount to 22% of its saturation value.

Some workers prefer to express reoxygenation in terms of pounds of oxygen absorbed per million gallons per day. A convenient graphical conversion from the "percentage of saturation" value absorbed per day to "pounds per million gallons per day" is readily effected by utilizing Fig. 4, which the writer constructed by adapting the principles that govern the development of Fig. 3.

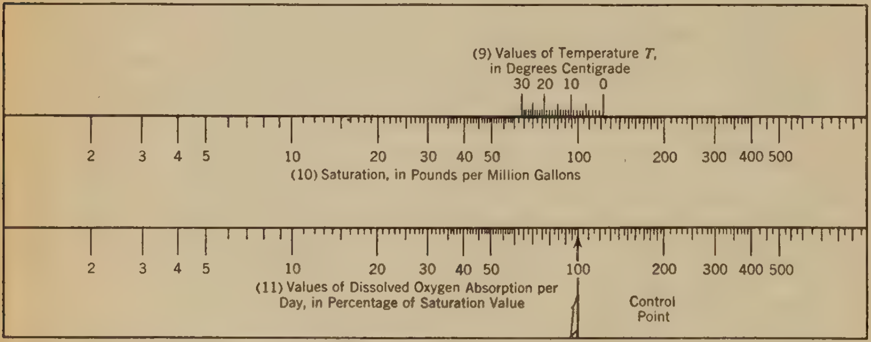
(9) Values of Temperature T, in Degrees Centigrade
30 20 10 0
2 3 4 5 10 20 30 40 50 100 200 300 400 500
(10) Saturation, in Pounds per Million Gallons
2 3 4 5 10 20 30 40 50 100 200 300 400 500
(11) Values of Dissolved Oxygen Absorption per Day, in Percentage of Saturation Value
Control Point

FIG. 4.—CONVERSION OF PERCENTAGE OF SATURATION VALUE PER DAY TO POUNDS PER MILLION GALLONS PER DAY

Example 2.—Assume the same conditions as in Example 1. Opposite 24° C on Scale (9) read 71.1 lb per million gallons (the saturation value at 24° C) on Scale (10); then slide the "control point" of Scale (11) to coincide with 71.1 lb per million gallons, on Scale (10); then opposite 22.0% per day, on Scale (11), read 15.6 on Scale (10), which represents the conversion from 22% of saturation value per day into pounds per million gallons per day.

Reoxygenation temperature has a double influence, first in the rate of diffusion and again in the limit of the saturation value, with opposite effects which, however, do not necessarily balance each other. Increase in temperature increases the rate of diffusion and, consequently, the rate of absorption, but the limit of quantity absorbed (namely, the saturation), decreases with increase in temperature.

VERIFICATION OF EQUATION (2)

An excellent check on Equation (2) and Fig. 3 is afforded by the experiments reported by H. W. Streeter and Chilton Wright, Members, Am. Soc. C. E., and Mr. R. W. Kehr in 1936.[5] By applying depth, time, and temperature values from these experiments to Fig. 3, a close agreement is noted between the dissolved oxygen computed and that observed in the experiments. Furthermore, if curves in the foregoing report[6] are replotted on a basis of percentage of saturation, beginning with initial oxygen devoidance, the time of turn-over, or frequency of mix, is obtained at the point where the dissolved oxygen coincides with that obtained by Fig. 3 at the same time interval.

The frequencies of mix computed by this method, at depths of 0.318 ft and 1.0 ft, indicate a relationship with the velocity shown in Fig. 5. It will be noted that the logarithm of the frequency of mix bears a linear relationship to velocity and, logically, as depth increases, the frequency of mix diminishes. Further controlled experiments extended to depths normally encountered in streams will doubtless lead to a set of curves similar to Fig. 5. In the absence of such a set of curves the writer has obtained satisfactory checks against observed dissolved oxygen profiles by using an average frequency of $\frac{1}{4}$ hr per mix, ranging from $\frac{1}{8}$ hr for shallow, rapidly flowing streams to $\frac{1}{2}$ hr for deep, slowly flowing streams.

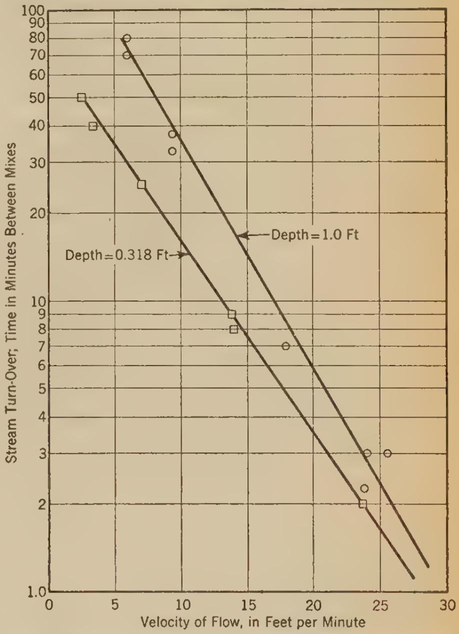

FIG. 5.—RELATIONSHIP BETWEEN STREAM TURN-OVER, VELOCITY, AND DEPTH

The experimental confirmation of Equation (2) and Fig. 3 under controlled conditions of flow and the close agreement between computed and observed dissolved oxygen values when applied to actual stream conditions, substantiate Professor Phelps' original fundamental assumptions. It is hoped that the advantages of simple, direct evaluation of reoxygenation for both inland and tidal waters will assist in routine stream-analysis technique as applied in the design of pollution remedial work.

DEOXYGENATION

The major pollution liability in streams is deoxygenation resulting from the satisfaction of the natural oxygen demand of organic waste matter. In the expression of his law, namely, that "the rate of biochemical oxidation of organic matter is proportional to the remaining concentration of unoxidized substance measured in terms of oxidizability," Professor Phelps changed the concept of pollution from that of a simple, static, chemical dilution phenomenon to one of changing, bio-chemical complexity. The law has been checked repeatedly by experiment and has been confirmed in actual observation of stream pollution studies; and can be incorporated, safely and advantageously, as routine technique in the design of remedial works. It is recognized that there are modifying elements, such as immediate demand, nitrification stage demand, sedimentation, and scour, which are highly important in a theoretical scientific analysis; but, in so far as routine technique for the design of remedial works is concerned, until such time as these refinements are firmly established, the writer prefers to include them in the factor of safety used in design.

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[5] "Measures of Natural Oxidation in Polluted Streams: III, An Experimental Study of Atmospheric Reaeration Under Stream Flow Conditions," by H. W. Streeter, C. T. Wright, and R. W. Kehr, *Sewage Works Journal*, Vol. 8, No. 2, p. 282, March, 1936.

[6] *Loc. cit.*, p. 298.

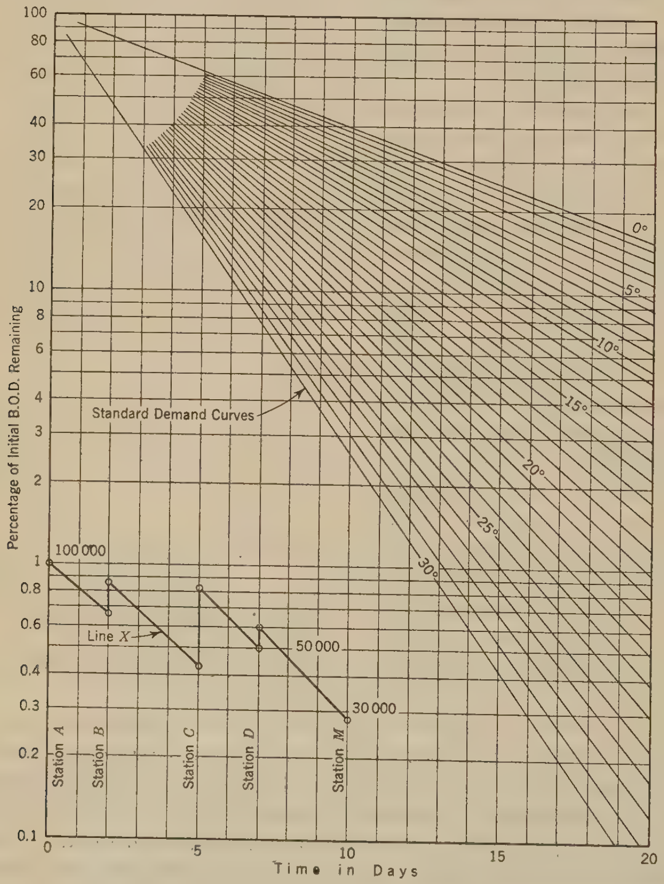

Fig. 6.—Relationship Between Bio-Chemical Oxygen Demand and Time at Temperatures, 0°C to 30°C

The technical literature of the ten years, 1927–1937, is replete with discussion of the fundamentals of bio-chemical oxidation, but these fundamentals are not so widely used in actual practice. As a stimulus to their further application to practical problems, the writer presents a convenient graphical representation of the well-known time-temperature functions. By plotting the logarithm of oxygen demand expressed as a percentage of the total initial demand against time, for rates at temperatures from 0° C to 30° C, the nest of Standard Demand Curves shown on Fig. 6 is obtained. These curves signify that, at a given temperature in successive intervals of time, the proportion of the demand satisfied is always a constant percentage of that remaining at the end of the previous interval—a very convenient relationship. The influence of temperature upon the rate of oxidation (by coincidence) is similar to that determining the rate of diffusion in reoxygenation; namely, the log of the rate is directly proportional to temperature. With the logarithmic slopes of the standard demand curves known, and with the distance between stations expressed in time of passage, the temperature of the water and the quantities of pollution contributed (expressed in any convenient term such as population equivalents), at each station, known, it is a simple graphical operation to integrate the resultant demand and determine the relative responsibility of polluters at any place in the course of the stream.

Line *X*, Fig. 6, represents such a resultant demand curve for the hypothetical conditions given in Table 1. This technique for integrating resultant demand affords a rapid, direct means of evaluating deoxygenation, which is the "load factor" in the design of projects to remedy pollution problems.

TABLE 1

Station	Mean temperature of water between stations, in degrees Centigrade	Time of Passage in Days		Pollution added at station, expressed in population equivalents
		Mean time between stations	Total time from Station A	
(1)	(2)	(3)	(4)	(5)
A	18	2.0	0.0	100 000
B	20	3.0	2.0	20 000
C	22	2.0	5.0	40 000
D	22	3.0	7.0	10 000
M*	....	....	10.0	0

* M = Mouth.

Oxygen Balance

Oxygen balance is the algebraic summation of satisfied demands of pollution and stream assets. A scientific guide in planning measures to correct stream pollution is afforded in a series of oxygen-balance profiles computed for variations in the three major factors: Predicted pollution loads, probable stream flows, and proposed combinations of remedial plans. Evaluation of the relative effect of these factors is essential to safeguard natural resources and to insure the greatest return from remedial works with the least expenditure of funds. For the convenience of the computer the writer has indicated an oxygen balance procedure in Tables 2, 3, and 4.

Variation in pollutional load is a function of population and, except for unusual concentrations of industrial waste, is directly proportional to it. How much to build now, how much excess capitalization to provide for growth, when to add units, or to increase degree of treatment—these questions depend to a large extent upon the prediction of future loads. A well co-ordinated long-range program permits a more conservative allowance for these factors in the basis of design. Restriction of immigration increase in the average age of the population, a declining birth rate, and redistribution of population, are

TABLE 2.—FORM FOR ANALYZING THE PHYSICAL CHARACTERISTICS OF A STREAM

(At —% Chance Flow, — Cubic Feet per Second per Square Mile)

Station (location of major sources of pollution and dilution) (1)	Drainage area tributary to station, in square miles (2)	Run-off, at the station, in cubic feet per second (3)	Mean depth between stations, in feet (4)	River volume between stations, in cubic feet (5)	Time of passage, in days, between stations (computed from volume displacement) (6)	Estimated frequency of stream turnover; in hours per mix (7)	Mean temperature of water between stations, in degrees Centigrade (8)

TABLE 3.—FORM FOR ANALYZING DEOXYGENATION

(At —% Chance Flow, — Cubic Feet per Second per Square Mile and Predicted Load as of ——— (Year))

			BIO-CHEMICAL OXYGEN DEMAND, IN POPULATION EQUIVALENTS*:						
Station (1)	Mean temperature of water, in degrees Centigrade (2)	Time of passage, in days from initial station (3)	Tributary to station (no treatment) (4)	Removed by treatment prior to discharge (5)	Discharged into stream at station (6)	Residual from stations up stream (7)	Total at station (tributary residual plus residual from up stream) (8)	Satisfied in stream between stations (9)	Total satisfied above station (10)

* Bio-chemical oxygen demand of industrial and municipal waste expressed as population equivalents (0.22 lb per day per person).

retarding influences tending toward stabilization in numbers, particularly in urban centers where the disposal problem is most acute.

Variation in stream flow is a paramount factor in oxygen balance by virtue of its influence on volume and, more important, on the element of time in deoxygenation and reoxygenation. Uncontrollable disturbance of equilibrium in Nature's oxygen balance is almost entirely associated with variations in the rate of stream flow. Of run-off so little is known that it is considered a chance phenomenon, and, consequently, it is evaluated by standard statistical methods, a determination of probability of occurrence based upon recorded past behavior. With the possible exception of winter when an ice cover closes the stream to reoxygenation, the coincidence in summer of high temperature and low run-off provides the critical stream condition. For most problems, therefore, an analysis of the summer monthly average run-off records is sufficiently accurate for purposes of design.

TABLE 4.—Form for Analyzing Reoxygenation and Oxygen Balance (At —% Chance Flow, — Cubic Feet per Second per Square Mile and Predicted Load as of ———)
(Year)

							Dissolved Oxygen, in Population Equivalents*								
Station (1)	Run-off at station, in cubic feet per second (2)	Mean temperature, in degrees Centigrade (3)	Mean depth, in feet (4)	Frequency of turnover, in hours per mix (5)	River volume, in cubic feet (6)	Total demand satisfied above station, in population equivalents (7)	Added by run-off between stations (8)	Total added by run-off above station (9)	Total run-off at station at saturation (10)	Net, at station (Column (9) minus Column (7)) (11)	Added by reoxygenation between stations per million gallons† 12)	Total, added by reoxygenation between stations (13)	Total, added by reoxygenation above station (14)	Net oxygen balance at station (Column (11) plus Column (14)) (15)	Percentage saturation at station (Column (15) divided by Column (10)) (16)

* Dissolved oxygen expressed in population equivalents (0.22 lb per day per person).

† Per million gallons per day from Fig. 2 and Fig. 3 converted to population equivalents.

A convenient system for establishing the run-off probability curve is afforded by the graphical method developed by the late Allen Hazen, M. Am. Soc. C. E.[7] If a duration flow curve is made of monthly average run-offs, showing the relative frequencies of flows exceeding any given value, a curve of the logarithms of these flow values against percentage frequencies of occurrence plotted on probability paper approaches a straight line. Fig. 7 represents a typical probability curve. From such a curve the most probable minimum flow for any desired percentage chance of occurrence is readily obtained.

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[7] "Flood Flows," by Allen Hazen, John Wiley & Sons, Inc., 1930.

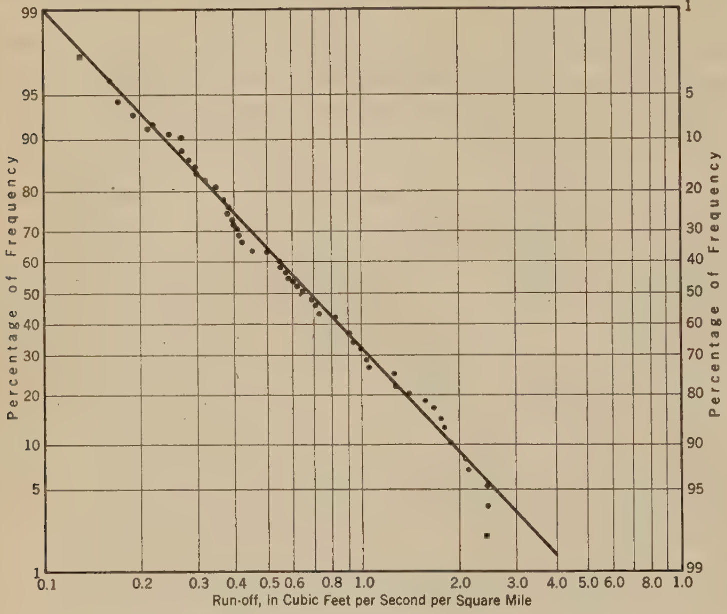

FIG. 7.—RUN-OFF PROBABILITY CURVE

Although any particular oxygen-balance profile is possible of reasonably exact determination from the fundamental laws governing deoxygenation and reoxygenation, load factors and run-off probabilities are not definitely predictable. However, by computing a series of oxygen-balance profiles over a range of loads and run-off, accurate limits of stream condition are defined. Obviously, these defined limits afford a foundation work from which to measure the necessity and effectiveness of any arrangement of treatment plants designed to relieve the stream of portions of the pollution load. The subjection of various remedial combinations to such evaluation insures the proper protection to the stream and to the taxpayer.

DISCUSSION

EARLE B. PHELPS, ESQ.[8] (by letter).—In two ways, this paper is a distinct contribution to the practical art of stream pollution control. By an ingenious development of graphic procedures, Mr. Velz has simplified the application of the complicated formulas of re-aeration and de-aeration and has rendered them directly applicable to any given situation. It is especially important that this simplification has been introduced without altering the form of the basic equations. The latter are rational in form and their constants are capable of experimental determination. Their conversion to simpler empirical forms leads too readily to loss of significance and to faulty applications. This, the author has happily avoided.

Of equal importance is the author's insistence upon the study of stream loadings as a factor in design. If a highway engineer were to build a dirt road and utilize it until traffic became impossible because of stalled cars, and then replace it with a six-lane concrete highway when two lanes would suffice, he would be subject to legitimate criticism; and yet, raw sewage is often discharged in increasing loads until the resulting stream conditions become intolerable. Then treatment is undertaken upon the basis of some successful procedure elsewhere, or under some general specifications of a State authority, and wholly apart from any but the most casual reference to stream capacity.

In his paper entitled "Economics of Sewage Treatment,"[9] George J. Schroepfer, Assoc. M. Am. Soc. C. E., has emphasized the fact that the first point of attack is a determination of the required degree of treatment. Mr. Velz' contribution fits exactly into this vacant niche and, in a sense, "completes the picture." It may quite reasonably be stated, that, in sewage treatment of any kind, the economic determination of the extent of treatment required is at least of correlative importance with the complementary discussion of the economics of design.

Perhaps the first systematic attempt at establishing stream capacities was that made by the late Frederic P. Stearns, Past-President, Am. Soc. C. E., who, in 1890, studied Massachusetts streams carrying varying sewage loadings upon the basis of dilution per contributing capita. He suggested,[10] as tentative limits, that a dilution of less than 2.5 cu ft per sec per 1 000 persons leads to inadmissible pollution and that a corresponding dilution of 7 cu ft leads to results that are clearly inoffensive. The late Allen Hazen, M. Am. Soc. C. E., in his 1898 report on the Ohio River,[11] set the limits of necessary dilution at 1.5 to 4.0 cu ft, noting, however, that in the case of sluggish streams the dilution might have to be increased even to 10 cu ft. In 1902, the late X. H. Goodnough, M. Am. Soc. C. E., on the basis of more extensive Massachusetts data agreed[12] essentially with Mr. Stearns but set the upper safe limit at 6 cu ft, and the lower limit of necessary dilution at 3.5 cu ft. The late Rudolph Hering, M. Am. Soc. C. E., based his Chicago Drainage Canal design upon a dilution ratio of 3.33 cu ft. It will be particularly noticed that throughout all this period only Mr. Hazen made any allowance for the influence of velocity of flow although this influence is traditional. With present-day knowledge, a simple computation will readily show that, in any of these minimum dilution volumes that have been set up, the stream must rely largely upon re-aeration to preserve any semblance of purity.

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[8] Prof. of San. Science, Coll. of Physicians and Surgeons, Columbia Univ., New York, N. Y.

[9] *Proceedings*, Am. Soc. C. E., April, 1938, pp. 737–765.

[10] "Examination of Water Supplies," Special Rept., Mass. State Board of Health, 1890.

[11] "Investigation of Rivers," Special Rept., Ohio State Board of Health, 1897.

[12] Rept., Mass. State Board of Health, 1902.

The British Royal Sewerage Commission, in its 8th Report (1912), recognized re-aeration as a factor in stream pollution standards and made an attempt to evaluate it.

The Engineering Board of Review, of the Sanitary District of Chicago (1925), clearly recognized the essential relation existing between dilution and the oxygen demand of sewage but the most recent textbooks either omit all reference to re-aeration as a stream asset or merely note the lack of quantitative knowledge concerning it.

In 1915 the writer made a report to the International Joint Commission in which the principles of permissible loading and of allocation of stream assets among the users of the boundary waters of the Great Lakes system were laid down as bases for a remedial program.[13]

Under the direction of Mr. C. A. Holmquist, Director of the Division of Sanitation, New York State Department of Health, a similar study was made in 1932 of the capacity of the Hudson River below Albany, N. Y. Mr. Velz was associated with the writer in that study. The oxygen profile was determined under summer conditions, and estimates were made of the rate of de-oxygenation and re-aeration, and of the relation of the contribution of each community to the total pollution of the stream. A similar treatment of the pollution and self-purification of New York Harbor, as influenced by the ebb and flow of the tidal prism as well as by the flow of the Hudson, has been published.[14]

In making plain the necessity for such considerations and in developing and presenting a simplified and workable method for their application the author has rendered the profession a distinct service.

Briefly stated, de-aeration is a function of concentration of oxidizable organic matter, temperature, and time. The form of this function, and its time and temperature constants, have been adequately investigated and are comparable in their accuracy and applicability to weir formulas and others of similar nature upon which engineers rely in many of their design problems. Re-aeration is a function of the initial state of de-aeration or saturation deficit, of time and temperature, and of stream turbulence. The basic constants of this relation are also known with sufficient accuracy, except that, for the present, the item of turbulence must be determined in a more or less empirical manner. As shown by H. W. Streeter, M. Am. Soc. C. E., and the writer[15] it may be developed in streams as a somewhat complicated function of depth, velocity, and wetted perimeter. In larger areas of more nearly quiescent water, the late William Murray Black, M. Am. Soc. C. E., and the writer have shown that it may likewise be developed as a function of depth, wind velocity, agitation by passing boats, and the movement of tidal flow through constricted sections.[16] In each case the turbulence factor appears in the basic formula as a time relation expressed in the form "time between mixings." The author's experience, indicating that under many conditions occurring in actual practice on small streams this time may be taken at from $\frac{1}{8}$ hr to $\frac{1}{2}$ hr for streams ranging from shallow, rapid-flowing, to deep slow-flowing, probably represents satisfactory approximations. The British Royal Sewerage Commission, on the basis of direct observations with dye solutions, estimated for some of the larger streams times of mixing ranging from 1 hr in the case of moderately rapid flows to 6 hr in the case of very sluggish flows. The results, computed on this basis, were found to be in practical agreement with those of field investigation and of laboratory experiments in tanks.[17]

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[13] Rept. of the Cons. Engr. on Pollution of Boundary Waters, International Joint Comm., Washington, 1918.

[14] "Pollution of New York Harbor," by Earle B. Phelps and C. J. Velz, *Sewage Works Journal*, Vol. 5, 117, 1933.

[15] "Factors Concerned in the Phenomena of Oxidation and Re-aeration," by H. W. Streeter and Earle B. Phelps, *Public Health Bulletin 146*, Washington, D. C., 1925.

It is believed that the procedures advocated by the author are worthy of much wider use on the part of those charged with the responsibility of stream control and that such use will result in economy of effort and conservation of resources.

C. J. VELZ,[18] ASSOC. M. AM. SOC. C. E. (by letter).—A technique for the evaluation of stream assets and pollution liabilities, in relation to remedial programs, is essential to a sound and economical solution of the pollution problem. Professor Phelps has drawn an apt parallel with highway construction and has shown that the functions and constants in deoxygenation and reoxygenation are established, and that they are comparable in accuracy and applicability to those used by engineers in other fields. When he is designing for specific loads and conditions, in order to be reasonably certain that the structure will fulfill, safely and economically, the functions intended, the bridge engineer subjects his design to a rigorous analysis. Although the sanitary engineer deals with constantly changing, living, forces rather than inanimate, static, forces, knowledge concerning the rates of change are sufficiently established to permit evaluations similar to those made by the bridge engineer. If sanitary engineering is to be placed on a plane equal to that of other branches of engineering, arbitrary selection in design, based on practical experience alone, must become secondary to a systematic technique of analysis based upon a knowledge of the fundamentals involved.

In discussing the rate of stream turnover, Professor Phelps mentions the work of the British Royal Commission[17] in which it is estimated that the rate of mixing ranges from 1 hr to 6 hr. Analysis of the experimental work of the Stream Pollution Laboratory of the United States Public Health Service and comparisons of computed and observed dissolved oxygen profiles of streams indicate shorter periods—$\frac{1}{8}$ hr to $\frac{1}{2}$ hr. Experimental data that are suitable in the analysis of rates of mix are meager, and it is hoped that further flume studies on reoxygenation, similar to those undertaken by the United States Public Health Service (particularly with greater depths and with more definite control of mix intervals) will bring forth more exact knowledge concerning this factor. Confirmation of the shorter mix period is evidenced from a comparison of computed and observed conditions on the Ohio and the Hudson rivers. H. R. Crohurst[19] has presented rates of reoxygenation, in pounds of oxygen per thousand square feet of water surface per day between sampling stations. They are based upon the assumption that the difference in oxygen balance between April and other months represents reoxygenation that occurred during those months. In a strict sense, such a method of determining rates of reoxygenation is not exact since all discrepancies and errors are "lumped" in the residual oxygen balance and, therefore, are reflected in the rates determined from them. In order to minimize any discrepancies thus included, a stretch of river, and months of the year that show a decided regeneration in dissolved oxygen, were selected by the writer for comparison with rates obtained from the Standard Reoxygenation Curve (Fig. 3).

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[16] Rept. on Discharge of Sewage into New York Harbor, by William Murray Black and Earle B. Phelps, Board of Estimate and Apportionment, New York City, March, 1911.

[17] Royal Comm. on Sewage Disposal (Great Britain), 8th Rept., Vol. 1, p. 10, 1912.

[18] Prof., San. Eng. and Head, Civ. Eng. Dept., Manhattan Coll., New York, N. Y.

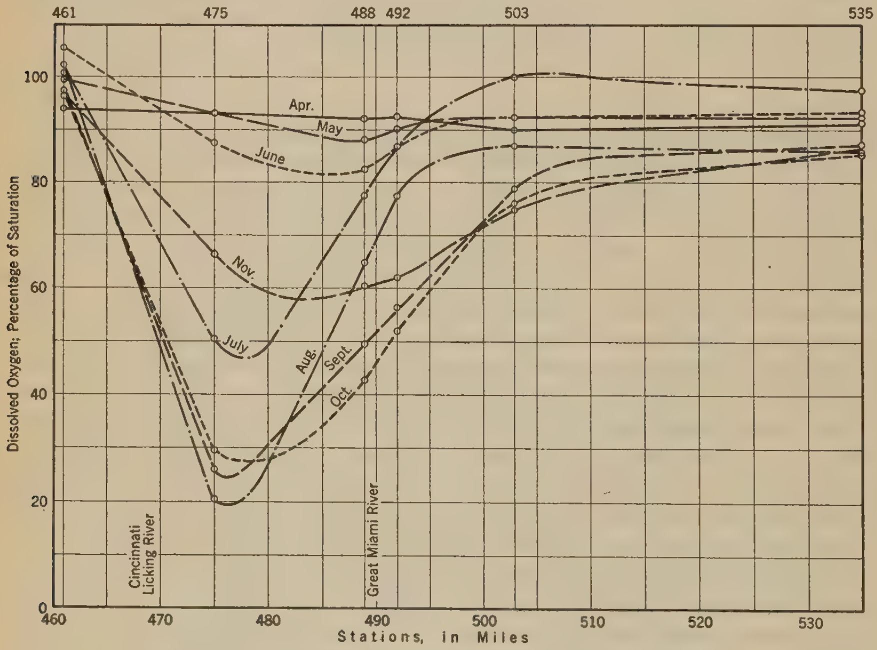

FIG. 8.—DISSOLVED OXYGEN PROFILE; OHIO RIVER, 1930

The dissolved-oxygen profiles of the Ohio River (see Fig. 8) show[19] that, during the months of July, August, September, and October, 1930, the stretch of the river below Cincinnati, Ohio, and between the junctions of the Licking and the Great Miami rivers was subject to a very pronounced regeneration of oxygen content. Utilizing the velocities of flow reported for these months and the corresponding mean rates of stream turnover determined by the author from the experimental data reported by the U. S. Public Health Service,[20] the writer has computed rates of reoxygenation in this stretch of the river by means of the Standard Reoxygenation Curve. These computed values agree closely with the rates based upon observed dissolved oxygen at Stations 475 and 488 as shown in Table 5. Under Item 4, Table 5, the computations are: Pounds per 1 000 sq ft times 4.9, divided by depth of river, in meters, times mean observed oxygen-saturation deficit, expressed as a decimal ratio. The variation in the rates are within the probable errors of observation and under these conditions the experimental rates of mix are identical with the actual rates of turnover in the river; namely, between 5 min and 25 min.

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[19] "A Study of the Pollution and Natural Purification of the Ohio River," by H. R. Crohurst, *Public Health Bulletin No. 204*.

TABLE 5.—COMPARISON OF COMPUTED AND OBSERVED RATES OF REOXYGENATION; OHIO RIVER

Item No.	Description	July (1)	August (2)	September (3)	October (4)
1	Velocity, in feet per minute	18.5	11.7	14.1	10.0
2	Stream turnover; time in minutes between mixes, corresponding to velocities in Item 1 (mean of both curves in Fig. 5)	5	22	14	25
	Reoxygenation (Percentage of Saturation per Day; the Maximum Rate with the River at 0% of Saturation):				
3	Computed from Fig. 3; depth = 16.1 ft and temperature = mean of stations 475 and 488*	30.0	18.5	17.9	8.7
4	Observed*; pounds per 1 000 sq ft per day, converted to percentage of saturation value absorbed per day, with the river at 0% of saturation	38.8	19.6	17.4	8.4

* "A Study of the Pollution and Natural Purification of the Ohio River," by H. R. Crohurst, U. S. Public Health Service, *Public Health Bulletin No. 204*.

A further verification of the shorter period of stream turnover is indicated by a comparison of the computed and observed dissolved oxygen values on the Hudson River. In these computations the writer used rates of mix ranging from $\frac{1}{8}$ hr for shallow reaches to $\frac{1}{2}$ hr for deep reaches. The agreement between the observed and the computed profiles is shown in Fig. 9.

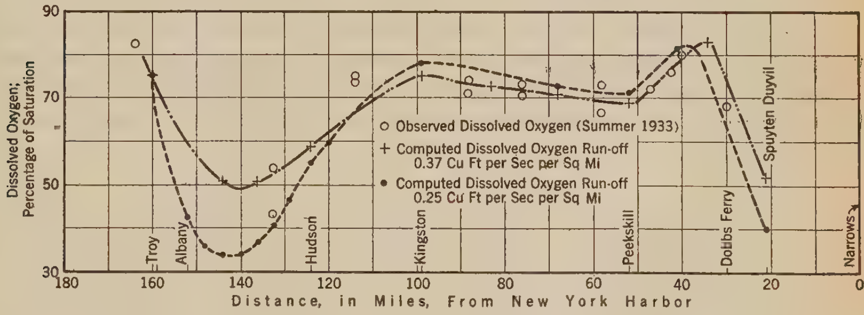

FIG. 9.—DISSOLVED OXYGEN PROFILE, HUDSON RIVER

In addition to verification of the shorter period of stream turnover, the Hudson River data emphasize, strikingly, the importance of the element of depth in reoxygenation.

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[20] *Sewage Works Journal*, March, 1936.

In the upper reaches from Albany, N. Y., to Hudson, N. Y., the river is relatively shallow and here both observed and computed values of reoxygenation are shown to be pronounced. In passing through this stretch, the river satisfies 90% of a heavy pollution debt and at the same time rebuilds, by reoxygenation, to about 75% of saturation. From Kingston, N. Y., to Peekskill, N. Y., the river passes through a deep gorge section (60 ft to 100 ft); and, with only 10% residual pollution load from up stream, and very small additional pollution loads added in the stretch, reoxygenation through the deep section is unable to satisfy the relatively light demand evidenced by the change from positive to negative slope of the dissolved oxygen profile. Upon entering the shallow reaches of Tappan Zee below Peekskill, reoxygenation is again pronounced, as indicated by a decided increase in computed, and also observed, dissolved oxygen. The sudden drop in oxygen below the Zee is caused by the heavy concentration of pollution from the Metropolitan area.

Quite apart from the verification of the rate of turnover, the important fact to be observed from the Hudson River data is the dominating influence that depth plays in the reoxygenation phenomenon. The only rational approach to computation of reoxygenation which directly employs this important factor of depth is that laid down by Professor Phelps in 1908 in his reoxygenation equation.[3] This formula is the basis for the writer's Standard Reoxygenation Curve (Fig. 3). The importance of depth is frequently obscured by other factors, which has led some workers, particularly Europeans, to express rates of reoxygenation in terms of oxygen absorbed per unit of surface area without designation of specific depth. Such values are applicable only to streams of the same depth; use of these surface rates at other depths may lead to error.

Technique is no substitute for judgment; in the last analysis judgment is the deciding factor in any given situation. It is hoped, however, that the method of analyzing stream assets and pollution liabilities, offered in this paper, will be a useful tool, and will be an aid to sounder judgment in the solution of pollution problems.

The writer wishes to thank Prof. Gordon M. Fair and H. W. Streeter, Members, Am. Soc. C. E., and Prof. Edward W. Moore, for their valuable criticism. In particular he wishes, respectfully, to acknowledge indebtedness to Professor Phelps, whose earlier writings were primarily the foundation for this paper.

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Paper No. 2029

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

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With Discussion by Messrs. A. W. Fischer, Jacob Karol, Glenn B. Woodruff and Norman C. Raab, Hardy Cross, A. A. Eremin, A. Fraser Rose and William A. Rose, Leon S. Moisseiff, O. H. Ammann, Leon Blog, and Shortridge Hardesty and Harold E. Wessman.

Synopsis

A rapid and accurate method of analysis for the preliminary design of the stiffening trusses of suspension bridges is presented in this paper. The method demonstrates clearly the related functions of the cable and the stiffening truss, emphasizing the fact that the former is the major structural element and that the latter is added to reduce undesirable grade changes. It is useful in determining quickly the effect of variations in the proportions, chord sections, and materials of the stiffening trusses, of differences in loadings, or of changes in major dimensions of the bridge. The proposed method gives results at the center and quarter-points which agree closely with those obtained by the deflection theory. Typical computations for the Triborough Suspension Bridge, at New York, N. Y., accompanied by detailed explanations, illustrate the procedure.

Introduction

Description of Method.—Certain types of statically indeterminate structures are hybrid[3] in their action. Every time a change is made in dimensions or design sections, a change occurs in the moments at various sections of such structures. The stiffening trusses of a suspension bridge act in this manner. Regardless of arbitrary limits imposed upon deflections or grade changes, the designer has a choice of many different stiffening trusses; and in some cases, with large dead load and small live load, he may omit them entirely, as was

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Note.—Published in January, 1938, *Proceedings*.

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[3] "The Relation of Analysis to Structural Design," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

done in the George Washington Bridge over the Hudson River at New York, N. Y. There is a definite need for a rapid method of analysis for preliminary design studies of stiffening trusses, so that the engineer may form some intelligent conclusions as to what happens when he varies the depth or changes the chord sections, selects a new sag ratio, changes the weights of flooring, or moves main towers and anchorages to different relative positions. Heretofore, three methods of analysis have been available, but none of them is satisfactory for making rapid studies of a variety of designs.

The so-called "elastic theory" used in earlier days is obsolete. Its equations are misleading and erroneous. True enough, when the structure is very stiff, the elastic theory gives results which are not much in error; but, as the stiffening truss becomes more and more flexible, the error becomes very large. One need only to point to the George Washington Bridge and ask the question, "How much error would have resulted if this structure had been designed by the elastic theory?"

The "deflection theory" is usually termed the exact method because it accounts for the change in shape of the cable under live load and temperature variations and the resulting changes in moments and shears in the stiffening truss. The term, "exact," is relative, however, because, as ordinarily applied, the method involves assumptions which result in some error. On the other hand, it is much more scientific and accurate than the elastic theory. Unfortunately, the exponential formulas of the deflection theory require long and involved computations. It is necessary to use a computing machine or a table of logarithms in order to avoid possibilities of serious error.

The "trigonometric series" method is the third method of analysis. The equations of this method, which are expressed in terms of trigonometric series that converge very rapidly, also account for changes in the shape of the cable. Although it is not commonly used, the trigonometric series method has distinct advantages over the deflection theory. Non-uniform suspender pull is accounted for in the determination of the horizontal component of the cable stress. What is more important, a slide-rule may be used for practically all computations without sacrificing accuracy. This simplifies procedure and saves time, but the computations are still lengthy and involved. Consequently, like the deflection theory, it is not satisfactory for studies of a variety of preliminary designs, although it is an excellent tool for the analysis of the final design. The trigonometric series method has been extended by the writers to provide a basis for precise computations made to support the derivation of certain formulas developed in this paper; but this treatment is not included.

In the proposed preliminary design method, the maximum moments at the quarter-point and at the center of the main span are computed in two major steps: First, the deflections of an unstiffened cable under partial live load, for various ratios of live load to dead load, are computed on the assumption that the cable length is unchanged and the tower tops do not move. An average value of load length is used, which value was determined from a study of the load lengths giving maximum moments at the quarter-point and at the center of the stiffening trusses of several actual bridges.

The effect of adding the stiffening truss is then considered. Evidently the truss will attempt to conform to the curve of deflection of the cable. It will reduce the deflection, however, and assume some compromise position depending upon its stiffness. The bending moment taken by the truss in its attempt to conform to the deflected cable will be a function of the maximum deflection of the cable at certain points and the truss stiffness. A trial moment of inertia is used, and corrected later if found necessary. A single numerical coefficient accounts for non-uniform suspender loading. The moment is evaluated primarily from the important consideration that, if the cable is kept from assuming its natural position as an unstiffened cable, the stiffening truss must provide the bending moment required to hold the cable in its restrained position.

Finally, the changes in the length of the cable due to live load or to rise and fall of temperature, and the sag changes caused by movement of the tower tops or by the interaction between the side spans and the main span, are combined into one change in the center sag. This lowers or raises the truss, causing a moment which is added algebraically to the moment obtained in the first step.

The changes in center sag of the side span are necessarily found in order to evaluate loading conditions that govern the main span. From these sag changes, it is a simple matter to compute the maximum positive and negative moments at the center of the side span trusses.

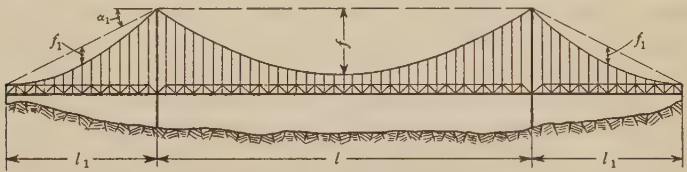

FIG. 1.—SUSPENSION BRIDGE WITH TWO-HINGED STIFFENING TRUSS AND SUSPENDED SIDE SPANS

Fig. 1 is a line diagram of a two-hinged suspension bridge of the type discussed in this paper. The method applies equally well to suspension bridges with straight back-stays.

Notation.—The meanings of the symbols used in this paper are given where they first appear and are assembled, for convenience of reference, in the Appendix.

DEFLECTIONS OF AN UNSTIFFENED CABLE

Cable Ordinates for Dead Load and Mean Temperature.—The cable of a suspension bridge is a flexible member having practically no resistance to bending. Its shape depends upon the initial sag and the loading; in fact, the cable curve is an equilibrium polygon for the loads suspended from it. Consequently, for any condition of loading, the unstiffened cable assumes a curve that represents the moment diagram for the same loading on a simple span. The horizontal component of the cable stress, multiplied by the ordinate at any section, must equal the bending moment at the same section due to the loads on the cable span, considered as a simple beam. With only dead load on the span,

$$H_w\, y = \frac{w}{2}(l\, x - x^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

in which w = total dead load per unit horizontal length of cable; H_w = horizontal component of cable stress due to dead load and mean temperature; l = length of main span; and x and y = co-ordinates of any point of cable curve, measured from an origin at the top of the left tower, under dead load at mean temperature. If the center sag is f,

$$H_w = \frac{w\,l^2}{8\,f} \quad \text{.........................(1b)}$$

Substituting Equation (1b) in Equation (1a):

$$y = \frac{4\,f}{l^2}\,(l\,x - x^2) \quad \text{.........................(2)}$$

which is the familiar parabolic formula for the cable ordinates under dead load at mean temperature.

The deflections of an unstiffened cable for partial live loads and varying ratios of live load to dead load are first to be found, with cable stretch omitted. Consequently, these deflections are based on an unchanged cable length L, and no relative movement of the tower tops. The equations for the cable ordinates under various live load conditions must first be derived. The differences between the resulting ordinates and those computed from Equation (2) will then give the desired deflections.

Live Load on One End of Span.—When live load moves upon the structure and extends a distance $k\,l$ from the left tower, the cable changes shape. The new curve, which represents the moment diagram for the combined live and dead load on a simple span, consists of two parabolas that have a common tangent at the head of the live load. The new ordinates are given by the following equations, which are simply expanded statements of the law of statics, $\Sigma\,M = 0$. It should be noted that H_w has been increased by a value H_a due to live load. When $x < k\,l$ (that is, for the part of the span under live load), there results the following equation, in which w_l = live load on roadway, per unit length of truss,

$$(H_w + H_a)\,y = \frac{w}{2}\,(l\,x - x^2) + w_l\,k\,l\left(1 - \frac{k}{2}\right)x - \frac{w_l\,x^2}{2} \quad \text{......(3a)}$$

or,

$$y = \frac{(w + w_l)\,(l\,x - x^2) - w_l\,l\,(1 - k)^2\,x}{2\,(H_w + H_a)} \quad \text{.........(3b)}$$

When $x > k\,l$ (that is, for that part without live load), there is found:

$$(H_w + H_a)\,y = \frac{w}{2}\,(l\,x - x^2) + w_l\,k\,l\left(1 - \frac{k}{2}\right)x - w_l\,k\,l\left(x - \frac{k\,l}{2}\right) \quad \text{...(4a)}$$

or,

$$y = \frac{(w\,x + w_l\,k^2\,l)\,(l - x)}{2\,(H_w + H_a)} \quad \text{...................(4b)}$$

With only dead load on the span, the horizontal component, H_w, of the cable stress is easily determined from Equation (1b), which involves the initial center sag, f. When live load moves on to the span, the center sag changes. As a consequence, the determination of the new horizontal component, $H_w + H_a$, is not quite so simple. The quantity $H_w + H_a$ will be determined in this step on the assumption that the cable length l does not change. The effect of changes in cable length will be discussed subsequently. If there is no change in length, the external work performed in deflecting the cable must equal zero; hence,

$$\int_0^{kl}\left(w+\frac{w_l}{2}\right)\eta\,dx+\int_{kl}^{l} w\,\eta\,dx=0 \ldots\ldots\ldots\ldots(5)$$

in which η = the deflection of truss and cable, at any section, from their initial position under dead load at mean temperature. For values of $x < k\,l$, the deflection, η, equals Equation (3*b*) minus Equation (2); and for values of $x > k\,l$, it equals Equation (4*b*) minus Equation (2). Making these substitutions in Equation (5), performing the integrations, and combining terms, there results:

$$H_w + H_a = \frac{w\,l^2}{8f}\left[\frac{2+3\dfrac{w_l}{w}k^2(3-2k)+\dfrac{w_l^2}{w^2}k^3(4-3k)}{2+\dfrac{w_l}{w}k^2(3-2k)}\right]=\frac{w\,l^2}{8f}C_1 \ldots(6)$$

Substituting Equation (6) in Equations (3*b*) and (4*b*), the following important equations are obtained:

For $x < k\,l$,

$$y=\frac{4f}{l^2}\,\frac{\left(1+\dfrac{w_l}{w}\right)(l\,x-x^2)-\dfrac{w_l}{w}(1-k)^2\,l\,x}{C_1}\ldots\ldots\ldots\ldots(7a)$$

and, for $x > k\,l$,

$$y=\frac{4f}{l^2}\,\frac{\left(x+\dfrac{w_l}{w}k^2\,l\right)(l-x)}{C_1}\ldots\ldots\ldots\ldots\ldots\ldots(7b)$$

in which C_1 is the quantity within the brackets in Equation (6).

Live Load in Middle of Span.—When the live load occupies the middle of the span, symmetrically placed about the center line with a total length of $k\,l$, the cable curve consists of three parabolas with common tangents at the ends of the live load. The two end parabolas are identical. The derivation of the equations is similar to the preceding case and will not be repeated. The final equations are as follows:

$$H_w + H_a = \frac{w\,l^2}{8f}\left[\frac{2+\dfrac{3\,w_l}{2\,w}(3\,k-k^3)+\dfrac{w_l^2}{w^2}(3\,k^2-2\,k^3)}{2+\dfrac{1\,w_l}{2\,w}(3\,k-k^3)}\right]=\frac{w\,l^2}{8f}C_2 \ldots(8)$$

For the end sections under dead load only,

$$y=\frac{4f}{l^2}\,\frac{(l\,x-x^2)+\dfrac{w_l}{w}k\,l\,x}{C_2}\ldots\ldots\ldots\ldots\ldots\ldots(9a)$$

(in which C_2 is the quantity within the brackets in Equation (8)); and for the middle part subject to both live and dead load,

$$y = \frac{4\,f}{l^2} \frac{\left(1 + \frac{w_l}{w}\right)(l\,x - x^2) - \frac{w_l}{w}\frac{l^2}{4}(1 - k)^2}{C_2} \ldots\ldots\ldots\ldots (9b)$$

Live Load on Both Ends of Span.—When live load is placed on both ends, covering a length $k\,l$ at each end, the new cable curve consists of three parabolas, the two end ones again being identical. The final equations are:

$$H_w + H_a = \frac{w\,l^2}{8\,f}\left[\frac{2 + 3\frac{w_l}{w}(6\,k^2 - 4\,k^3) + 8\frac{w_l}{w^2}k^3}{2 + \frac{w_l}{w}(6\,k^2 - 4\,k^3)}\right] = \frac{w\,l^2}{8\,f}C_3 \ldots (10)$$

For the end parts, subject to both live and dead load,

$$y = \frac{4\,f}{l^2}\frac{(l\,x - x^2) + \frac{w_l}{w}(2\,k\,l\,x - x^2)}{C_3} \ldots\ldots\ldots\ldots (11a)$$

in which C_3 is the quantity within the brackets in Equation (10). For the middle part under dead load only,

$$y = \frac{4\,f}{l^2}\frac{(l\,x - x^2) + \frac{w_l}{w}k^2\,l^2}{C_3} \ldots\ldots\ldots\ldots (11b)$$

Application of Equations (1) to (11).—Equations (1) to (11) are employed only in computing the basic deflections, and need not be used in applying the approximate method. They are stated in order to demonstrate how the proposed method has been developed.

It will be noted, by examining the governing equations for cable ordinates, that the span of the cable has no effect whatever upon the deflections. When x in the numerator is considered as a proportional part of l, it is evident that the term l^2 in the denominator is cancelled. The deflections of the cable depend only upon the initial sag f, the ratio of live load to dead load $\frac{w_l}{w}$, and the load length factor k. Now k can be made a constant in securing maximum effects at definite points in a span, such as the quarter-point or the center point, regardless of the span length. Hence, curves of deflection for the cable may be plotted for different ratios of live load to dead load, but with a constant proportional load length, $k\,l$. The deflections may be expressed as percentages of the initial sag, f. Then, if the deflection of a cable is wanted for a definite $\frac{w_l}{w}$ ratio, one needs only to select the percentage of deflection corresponding to the proper ratio and multiply the value by the cable sag.

An investigation of various load lengths, $k\,l$, indicated that changes in the $\frac{w_l}{w}$ ratio had practically no effect on the loaded length that would produce maximum deflections at the quarter-point and at the center of the span.

Moreover, it was found that the load lengths which produced maximum downward and upward deflections were about the same as those which caused maximum positive and negative moments in the stiffening trusses. The following values of $k\,l$ were found to be accurate enough for preliminary design purposes:

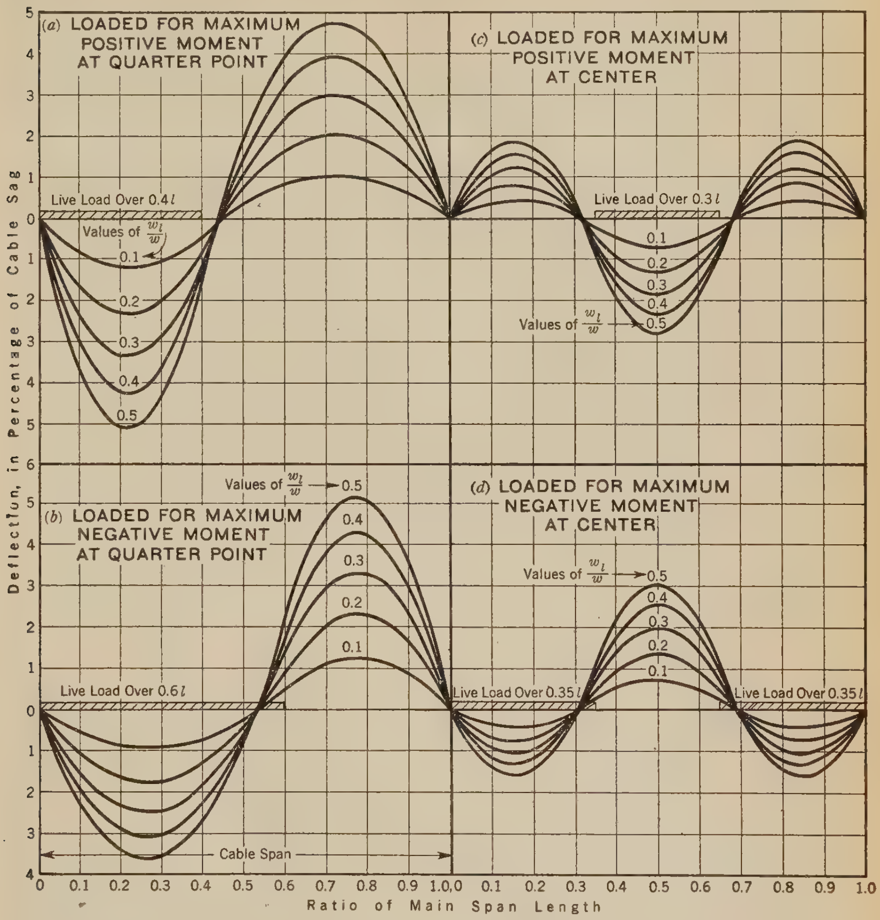

FIG. 2.—DEFLECTIONS OF UNSTIFFENED CABLE DUE TO PARTIAL LIVE LOADS PRODUCING MAXIMUM MOMENTS AT QUARTER-POINT AND AT CENTER. DEAD LOAD OVER ENTIRE SPAN

Condition	Load length
Maximum positive moment at the quarter-point,	0.4 l at the same end of span
Maximum negative moment at the quarter-point,	0.6 l at far end of span
Maximum positive moment at the center	0.3 l at center of span
Maximum negative moment at the center	0.35 l at both ends of span

These load lengths were used to compute the deflections of the unstiffened cable for ratios of live load to dead load ranging from 0.1 to 1.0. Some of the

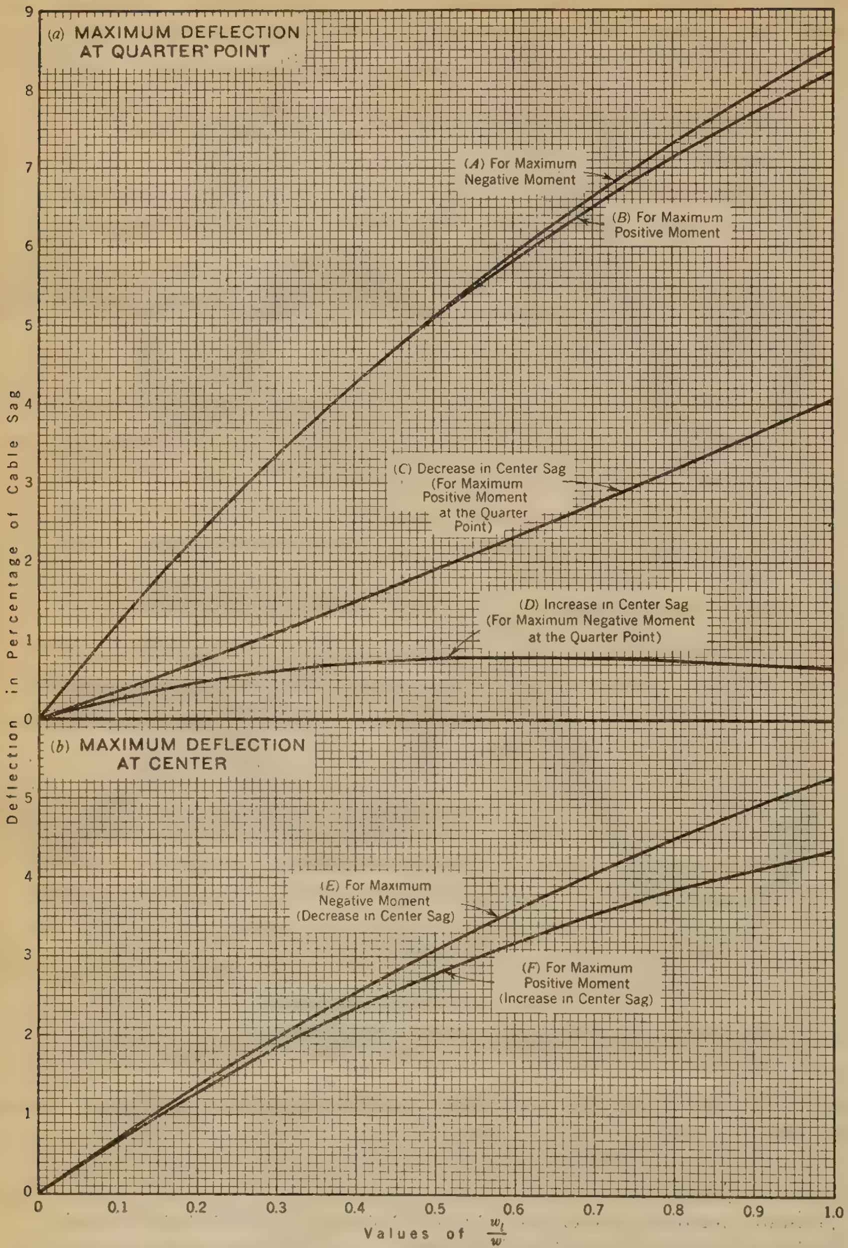

FIG. 3.—MAXIMUM DEFLECTIONS OF UNSTIFFENED CABLE AT QUARTER-POINT AND CENTER OF MAIN SPAN DUE TO PARTIAL LIVE LOADS

curves showing the procedure at the quarter-point and the center are drawn in Fig. 2. The maximum downward deflections in Fig. 2(a) were then plotted against the $\frac{w_l}{w}$ ratio, as Curve B in Fig. 3(a). This curve obviates interpolation for intermediate $\frac{w_l}{w}$ ratios. The decreases in center sag corresponding to the loading giving maximum positive moment at the quarter-point are also plotted in Fig. 3(a) (see Curve C). The other curves were obtained in a similar manner.[4]

Figs. 3(a) and 3(b) are the only curves used in applying the proposed method.

EFFECT OF ADDING STIFFENING TRUSS TO UNSTIFFENED CABLE

Bending Moment, M, in Stiffening Truss.—When a stiffening truss is attached to the cable and the live load is moved on to the span, the cable tends to force the stiffening truss to conform to the deflection curve of the unstiffened cable. If the truss were very flexible, it would practically conform to that curve; whereas, if it were infinitely stiff, it would not deflect at all, and the cable would be forced to remain in its original position. Since the cable offers resistance to anything tending to force it back to its initial position, and since the truss is neither very flexible nor infinitely stiff, the final position of truss and cable will evidently be a compromise between the original dead load position of the cable and the deflection curve of the unstiffened cable, as indicated in Fig. 4(a).

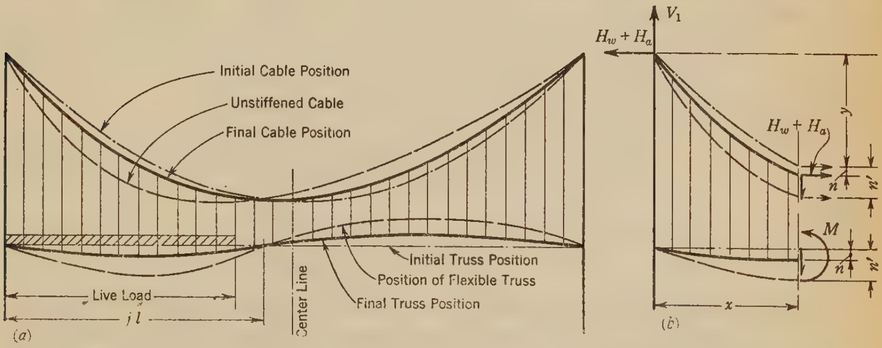

FIG. 4.—DEFLECTION OF CABLE AND STIFFENING TRUSS UNDER PARTIAL LIVE LOAD

In reaching this compromise position, the laws of statics will necessarily be satisfied (note Section x in Fig. 4(b)). The external bending moment due to dead load at mean temperature is balanced by $H_w\, y$. When live load comes upon the roadway, there is an additional external moment acting at Section x. If there were no stiffening truss, or if the truss were so flexible that it could resist no moment, the cable would deflect through a distance η'; and the total external moment would be balanced by $(H_w + H_a)\, y + (H_w + H_a)\, \eta'$. If, on the other hand, the truss had infinite stiffness, the cable deflection would be reduced from η' to 0; in other words, the truss would force the cable back through the entire distance η' to its original dead load position. The change η' in the lever arm of the force, $H_w + H_a$, would upset the balance of moments at Section x, in so far as the cable is concerned; and this balance would be restored by the stiffening truss taking a moment equal to $(H_w + H_a)\,\eta'$, in order to satisfy the equation of statics, $\Sigma M = 0$.

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[4] The extensive computations for these curves are not included in the paper, but have been filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.

Since the truss is neither infinitely flexible nor infinitely stiff, the cable will be forced back only a part of the distance η'. Let the final deflection, shown in Fig. 4(*b*), be η. The change in lever arm is then $\eta' - \eta$; and the moment resisted by the stiffening truss is,

$$M = (H_w + H_a)\,(\eta' - \eta)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

From one point of view, this moment resisted or developed by the stiffening truss may be considered as moment taken away from the cable when its position is changed from that of an unstiffened cable. This statement is not intended to convey the idea that the cable itself resists bending moment. The cable is flexible, but the force in the cable, multiplied by a lever arm, most certainly constitutes a moment at a section of the structure.

The deflection η of Equation (12) is found in the following manner. Let M_t be the bending moment at Section x that would be induced in the stiffening truss if it were bent to the deflection η' of the unstiffened cable. Its value will be determined later in terms of the deflection η', the physical properties of the truss, and a loading which accounts for variation in suspender pull. Assuming the final stiffening truss moment, M, to bear the same ratio to M_t as η does to η', there is found:

$$M = M_t \frac{\eta}{\eta'}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Equating the right-hand sides of Equations (12) and (13):

$$\eta = \eta' \frac{(H_w + H_a)\,\eta'}{M_t + (H_w + H_a)\,\eta'}\ldots\ldots\ldots\ldots\ldots\ldots(14a)$$

Hence,

$$M = M_t \frac{(H_w + H_a)\,\eta'}{M_t + (H_w + H_a)\,\eta'}\ldots\ldots\ldots\ldots\ldots\ldots(14b)$$

Equation 14(*b*) plays an important part in the proposed method of analysis. From an inspection of this formula, it will be noted that the final moment M in the stiffening truss is less than the moment $(H_w + H_a)\,\eta'$ that would be induced by forcing the cable back to its original position. One may consider either that the ratio $\frac{(H_w + H_a)\,\eta'}{M_t + (H_w + H_a)\,\eta'}$ determines the proportion of the moment M_t taken by the truss, or that the ratio $\frac{M_t}{M_t + (H_w + H_a)\,\eta'}$ represents the proportion of the moment $(H_w + H_a)\,\eta'$ taken by it. The latter ratio shows also the proportion of the deflection η' that is eliminated by the stiffening truss in this step of the process.

Horizontal Component of Cable Stress.—By an inspection of the terms of Equation 14(*b*), it will be noted that the deflection η' is determined from

Fig. 3(*a*) and Fig. 3(*b*), which leaves two other quantities, M_t and $(H_w + H_a)$, to be evaluated. The latter quantity is the horizontal component of the cable stress due only to dead load and live load on the unstiffened cable. This horizontal component is found by computing the moment M_c' of the dead load and live load at the center of the span, just as if the span were a simple beam, and then dividing by the center sag. In equation form:

$$H_w + H_a = \frac{M_c'}{(f + \eta_c')} \dots\dots\dots\dots\dots\dots\dots (15a)$$

in which $(f + \eta_c')$ = the center sag, and η_c' = increase or decrease in the center sag taken from the curves of Fig. 3(*a*) and Fig. 3(*b*). With only the dead load, w, acting:

$$H_w = \frac{w\, l^2}{8f} = 0.125\, \frac{w\, l^2}{f} \dots\dots\dots\dots\dots\dots\dots (15b)$$

Evidently, in this case, f is the center sag of the cable in its original position. When live load comes on, however, the center ordinate f changes by an amount η_c'. It should be kept in mind that the effect of cable stretch and temperature is to be considered subsequently. For the time being, deflections which are caused by partial live load are used, with the length of the cable l kept constant.

For the cases of loading summarized in Fig. 2(*c*) and Fig. 2(*d*), the values of $H_w + H_a$ are as follows:

Maximum positive moment at the quarter-point, with live load at the same end, over a length of 0.4 l,

$$H_w + H_a = \frac{1}{f + \eta_c'} (0.125\, w\, l^2 + 0.040\, w_l\, l^2) \dots\dots (16a)$$

Maximum negative moment at the quarter-point, with live load at the opposite end over a length of 0.6 l,

$$H_w + H_a = \frac{1}{f + \eta_c'} (0.125\, w\, l^2 + 0.085\, w_l\, l^2) \dots\dots (16b)$$

Maximum positive moment at the center, with live load at the center over a length of 0.3 l,

$$H_w + H_a = \frac{1}{f + \eta_c'} (0.125\, w\, l^2 + 0.0638\, w_l\, l^2) \dots\dots (16c)$$

Maximum negative moment at the center, with live load over a length of 0.35 l at each end,

$$H_w + H_a = \frac{1}{f + \eta_c'} (0.125\, w\, l^2 + 0.0613\, w_l\, l^2) \dots\dots (16d)$$

In each of the preceding equations, the quantity within the parenthesis is the bending moment at the center of a simple span due to dead load over the entire span and live load over a part of the span. The deflection η_c' is positive when downward and negative when upward.

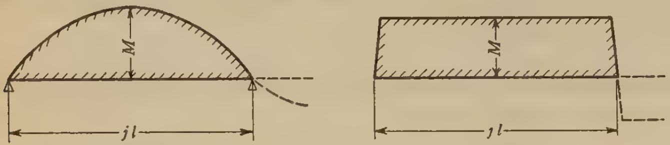
The horizontal component of cable stress may be found in a similar manner for other conditions of loading. Equations (16) are sufficient, however, for preliminary design purposes.

Bending Moment, M_t, in Stiffening Truss.—The bending moment M_t in Equation (14*b*), denoting the moment that would be induced in the stiffening truss if it were bent to the deflection curve of the unstiffened cable, will now be evaluated. For convenience, consider the case of maximum positive moment at the quarter-point. Deflection curves for this case are plotted in Fig. 2(*a*) and Fig. 2(*b*). The stiffening truss will be bent into the two loops shown, one downward and one upward. This is also indicated in Fig. 4. If η' is the maximum deflection of the downward loop and $j\,l$ is the length of the base of the loop, the bending moment may be stated thus, in general terms:

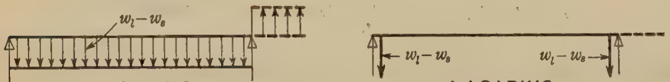
$$M_t = C\,\frac{E\,I\,\eta'}{(j\,l)^2} \quad \text{(17)}$$

in which C is a numerical coefficient depending upon the shape of the moment diagram; or, in other words, the type of loading on the truss. The loading involves both the live load on the roadway and the live load suspender pulls.

It will be well to consider approximate moment diagrams first, in order to show the relationship between M_t and η'.



(*a*) MOMENT (*c*) MOMENT



(*b*) LOADING (*d*) LOADING

Fig. 5.—Approximate Moment and Load Diagrams

If the moment diagram on one loop of the truss is a parabola, as shown in Fig. 5(*a*), the deflection at the center of the loop is,

$$\eta' = \frac{5}{48}\,\frac{M_t\,(j\,l)^2}{E\,I} \quad \text{(18a)}$$

and, the maximum moment is,

$$M_t = 9.6\,\frac{E\,I\,\eta'}{(j\,l)^2} \quad \text{(18b)}$$

The loading creating such a parabolic moment diagram is uniform over the length, $j\,l$, and is designated as $w_l - w_s$ in Fig. 5(*b*). The deflection curve would evidently be a fourth degree curve in this case.

Next suppose that the moment diagram is almost rectangular, as in Fig. 5(*c*), rather than parabolic. Then the center deflection would be,

$$\eta' = \frac{1}{8}\,\frac{M_t\,(j\,l)^2}{E\,I} \quad \text{(19a)}$$

and the maximum moment would be,

$$M_t = 8.0 \frac{E I \eta'}{(j l)^2} \qquad (19b)$$

The loading creating such a moment diagram would consist of two equal concentrated loads near the ends of the span, as in Fig. 5(*d*), or two equal end couples. The deflection curve in this case would be a second degree curve or a parabola.

The loadings in the two cases shown are radically different, but the maximum moments, M_t, for the same deflection differ by about 17%; and the final M-values, determined from Equation (14*b*) for these two values of M_t, would differ from one another by less than 8% in most cases.

The actual loading diagram for each loop of the stiffening truss falls somewhere between the two cases shown in Fig. 5. In an approximate method, one might assume arbitrarily that the suspender loading is uniform; but the final moments computed by the deflection theory are not determined from a uniform suspender pull, even if that assumption is used in finding the value of H_a. Hence, to make the proposed preliminary method as accurate as possible, actual moment and load patterns for partial live loads should be studied. Some mathematical relations must first be established to facilitate such a study.

Relationship Between Suspender Pull and Truss Moment.—The initial curve of the cable under dead load and mean temperature is,

$$y = \frac{4 f x}{l^2} (l - x) = \frac{w x}{2 H_w} (l - x) \qquad (20a)$$

The familiar differential equation of this curve is,

$$H_w \frac{d^2 y}{dx^2} = - w \qquad (20b)$$

When an additional load, w_s, due to live load or temperature change, acts through the suspenders upon the cable, the cable ordinates become $y + \eta$, and the differential equation becomes,

$$(H_a + H_w) \frac{d^2(y + \eta)}{dx^2} = - (w + w_s) \qquad (21a)$$

that is,

$$- \frac{H_a}{H_w} w - w + (H_a + H_w) \frac{d^2 \eta}{dx^2} = - (w + w_s) \qquad (21b)$$

or,

$$w_s = \beta w - (H_a + H_w) \frac{d^2 \eta}{dx^2} \qquad (21c)$$

in which β is the ratio $\frac{H_a}{H_w}$. Equation (21*c*) gives the value of the suspender pull w_s.

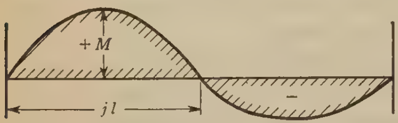
The differential equation of curvature for the stiffening truss is,

$$\frac{d^2 \eta}{dx^2} = - \frac{M}{E I} \qquad (22)$$

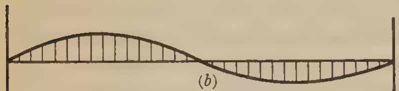
The curvature of the stiffening truss is the same as that of the curve of deflection of the cable. The value in Equation (22) when substituted in Equation (21*c*) gives the following important relationship:

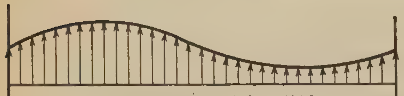
$$w_s - \beta\, w = \frac{H_a + H_w}{E\, I} M \qquad (23)$$

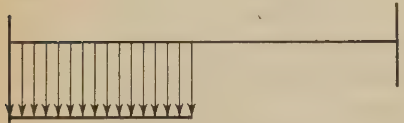
Since $\frac{H_a + H_w}{E\, I}$ is a constant for one loading condition, the final bending-moment diagram on the truss must have the same shape as the diagram of live load pull, w_s, on the suspenders, minus a constant proportion of the dead load pull, w. The diagram may be determined quantitatively from Equation (23), but the major significance of this is qualitative. It is evident that a parabolic moment diagram such as that in Fig. 5(*a*) and a load diagram such as that in Fig. 5(*b*) are not possible.

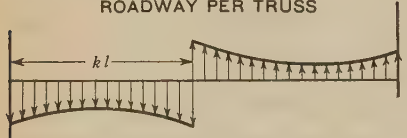


(*a*) MOMENT DIAGRAM


(*b*)
$(w_s - \beta w)$ DIAGRAM


(*c*) SUSPENDER LOADING, w_s


(*d*) LIVE LOAD, w_l, ON ROADWAY PER TRUSS



(*e*) FINAL TRUSS LOADING, $w_t = w_l + w_s$

FIG. 6.—MOMENT AND LOAD DIAGRAMS FOR STIFFENING TRUSS

For a live load extending over a length of 0.4 l, the moment diagram on the truss, with no stretch effects included, is a curve of two loops, approximately like that in Fig. 6(*a*). The (w_s − β w) diagram must be similar, as in Fig. 6(*b*). Adding β w to this curve, the hanger-pull diagram in Fig. 6(*c*), for live load only, is the result. Combining Fig. 6(*c*) with the live load w_l on the roadway (Fig. 6(*d*)) gives the final load diagram, Fig. 6(*e*), for the truss. This is the loading that is associated with the shears and bending moments in the stiffening truss. It should be kept in mind that the dead load, w, has no effect on the stiffening truss at mean temperature, and is not considered, therefore, as a load on the truss. Even if $k\, l$ should be somewhat less than $j\, l$, it is evident that the loading for each loop falls somewhere between the two cases shown in Fig. 5. Consequently, the coefficient C in Equation (17) must lie between 9.6 and 8.0. To verify this, several deflection, moment, and load diagrams, computed precisely by the trigonometric series method, have been studied, omitting the effects of cable stretch due to live load and temperature change, and the effect of side-span interaction. These studies have been presented in detail elsewhere.[5]

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[5] "Related Functions of Cable and Stiffening Truss in Suspension Bridges," by Harold E. Wessman, Assoc. M. Am. Soc. C. E.; Thesis presented to the Univ. of Illinois in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

As a result of these studies and comparisons of results obtained by the preliminary design method with final values for a number of bridges computed by the deflection theory, the following values of the coefficient C, to be used in Equation (17), have been adopted for the preliminary design method:

Maximum positive or negative moment at:	Value of C
Quarter-point	9.1
Center	9.0

Specific Values of M_t.—Definite values of $j\,l$ for Equation (17) have been determined by using the average lengths of the bases of the deflection loops of the unstiffened cable. For positive moment at the quarter-point, $j\,l = 0.44\,l$. For negative moment at the same point, $j\,l = 0.46\,l$. For positive moment at the center, $j\,l = 0.37\,l$, and for negative moment at the center, $j\,l = 0.39\,l$. When these values and the preceding values of C are inserted in Equation (17), the following specific values of M_t are obtained:

Maximum positive moment at quarter-point:

$$M_t = 9.1 \frac{E\,I\,\eta'}{(0.44\,l)^2} = 47.0 \frac{E\,I\,\eta'}{l^2} \ldots\ldots\ldots\ldots (24a)$$

Maximum negative moment at quarter-point:

$$M_t = 9.1 \frac{E\,I\,\eta'}{(0.46\,l)^2} = 43.0 \frac{E\,I\,\eta'}{l^2} \ldots\ldots\ldots\ldots (24b)$$

Maximum positive moment at center:

$$M_t = 9.0 \frac{E\,I\,\eta'}{(0.37\,l)^2} = 65.8 \frac{E\,I\,\eta'}{l^2} \ldots\ldots\ldots\ldots (24c)$$

Maximum negative moment at center:

$$M_t = 9.0 \frac{E\,I\,\eta'}{(0.39\,l)^2} = 59.2 \frac{E\,I\,\eta'}{l^2} \ldots\ldots\ldots\ldots (24d)$$

Equations (24) are based upon a constant length, $j\,l$, regardless of the stiffness of the truss attached to the cable. In other words, they imply that the points of zero deflection for partial live loads (with cable stretch and side-span effects omitted) remain practically fixed in position. An inspection of the deflection loops in Fig. 2(a) and Fig. 2(b) indicates that the points of zero deflection do not move much for different ratios of live load to dead load. Consideration was also given to the possible effect of stiffness of truss upon the point of zero deflection, as it was thought that a very stiff truss might possibly cause the point to travel appreciably toward the unloaded end of the truss. To test this hypothesis, the main truss of the Manhattan Suspension Bridge, in New York, N. Y., was replaced by a truss ten times as stiff, and the deflections were then computed for a live load from 0 to $0.4\,l$, with cable stretch and side-span effects omitted. It was found that the point of zero deflection remained practically fixed in position. This check, combined with a number of comparisons of values obtained in actual designs, gave assurance relative to the use of the preliminary design method for a wide range in truss stiffness.

All the values for use in Equation (14*b*) are now established; η' is determined from Fig. 3(*a*) and Fig. 3(*b*); values of $H_w + H_a$ are found from Equations (16); and values of M_t are computed by using Equations (24). This completes the first and major step of the approximate method. It is now necessary to evaluate the effects of cable stretch, temperature change, and side-span interaction.

EFFECT OF CABLE STRETCH, TEMPERATURE CHANGE, AND SIDE-SPAN INTERACTION

Introduction.—When live load moves upon the bridge, or when a temperature change occurs, stress is induced in the cable. This stress causes a change in the normal length of the cable; that is, the length corresponding to full dead load and mean temperature. When a temperature change takes place, the cable lengthens or shortens a distance which is in addition to any length change produced by the stress from live load or temperature change. These changes affect the shape of the cable, increasing or decreasing the center sag and producing positive or negative bending moments in the stiffening truss.

Any change in cable stress or length of the cable also causes a change in the side-span sag. This, in turn, causes a movement of the tower tops which, finally, affects the center-span sag. A change in center sag is accompanied by a change in the cable stress, which will again modify the side-span sag. Evidently, there is an interaction between the side span and the main span, which occurs simultaneously with any change in loading or temperature. When the bending moments in the stiffening truss due to changes in cable length and side-span interaction have been found, the results are to be added algebraically to the moments from live load found by Equation (14*b*).

Change in Length of Cable Due to Stress.—The strain in any section of the cable of differential length, due to stress from live load or temperature, is:

$$\delta = \frac{H_a\,dL}{A\,E_c\,dx}\,dL \qquad (25)$$

The total strain or change in length of the main-span cable may then be expressed as follows:

$$\Delta L_s = \int_0^l \frac{H_a}{A\,E_c}\frac{dL^2}{dx} \qquad (26)$$

When evaluated, this gives the following approximate equation to be used for the main span in the preliminary design method:

$$\Delta L_s = \frac{H_a\,L}{A\,E_c\,l}\,L = \frac{H_a\,L^2}{A\,E_c\,l} \qquad (27)$$

This simplified equation was used by Allston Dana, M. Am. Soc. C. E., and G. M. Rapp, Assoc. M. Am. Soc. C. E., in the cable calculations for the Delaware River Bridge, from Philadelphia, Pa., to Camden, N. J.[6] It should be noted that l is the horizontal length of the main span and L is the actual length of the cable; L is given quite accurately by Equation (28), as follows:

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[6] "The Delaware River Bridge Connecting Philadelphia, Pa., and Camden, N. J.," Final Rept. of the Board of Engrs., p. 107.

$$L = l\left(1 + \frac{8}{3} n^2\right) \qquad (28)$$

in which n = ratio $\frac{f}{l}$. (D. B. Steinman, M. Am. Soc. C. E., has published[7] a comparison of exact and approximate formulas for cable length.) Likewise, the change in length of the cable over each side span may be expressed as,

$$\Delta L_{1s} = \frac{H_a \, L_1^2}{A \, E_c \, l_1} \qquad (29a)$$

in which l_1 is the horizontal length of the side span, and L_1 is the actual length of the side-span cable, or (see Fig. 1):

$$L_1 = l_1 \left(\sec \alpha_1 + \frac{8}{3} \frac{n_1^2}{\sec^3 \alpha_1} \right) \qquad (29b)$$

There are other short sections of the cable over the tower saddles and at the anchorages, which also change length and contribute to the effects upon the stiffening truss. For one-half the bridge, these sections will be designated by the length term, L'. Then,

$$\Delta L_s' = \frac{H_a \, (L')^2}{A \, E_c \, l'} \qquad (30)$$

in which l' is the horizontal length of the section of cable corresponding to L'. At the tower saddle, l' is virtually equal to L'.

Change in Length of Cable Due to Temperature.—The change in length of the cable over the main span due to temperature variation is,

$$\Delta L_t = \omega \, \Delta t \, L \qquad (31a)$$

The corresponding change over the side span is,

$$\Delta L_{1t} = \omega \, \Delta t \, L_1 \qquad (31b)$$

The change in length of other sections of length, L', for one-half the bridge is,

$$\Delta L_t' = \omega \, \Delta t \, L' \qquad (31c)$$

It should be noted that the changes of length indicated by Equations (27), (29), (30), and (31) are separated into units corresponding to the main span, the side span, and incidental sections of the cable. This separation is convenient in order to evaluate the changes in cable sag due to cable strains.

Relations Between Cable Strain, Cable Sag, and Span Length.—Any change in the length of the cable is accompanied either by a change in the sag or by a change in the horizontal span length. The relation between a small change in sag and a small change in the length of cable, with the span length kept constant, is expressed approximately as follows:[8]

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[7] "Suspension Bridges," by D. B. Steinman, p. 6.

$$df = \frac{15}{16\,n\,(5 - 24\,n^2)}\,dL \qquad (32a)$$

The relation between a small change in sag and a small change in span length with the cable length kept constant is:

$$df = -\frac{(15 - 40\,n^2 + 288\,n^4)}{16\,n\,(5 - 24\,n^2)}\,dl \qquad (32b)$$

The relation between a small change in span length and a small change in cable length, with the sag kept constant, is:

$$dl = \frac{15}{(15 - 40\,n^2 + 288\,n^4)}\,dL \qquad (32c)$$

Equations (32) apply to the main span, and, corresponding equations may be developed for the side span. Usually, the side-span sag ratio is so small that the terms involving n_1^2 and n_1^4 may be omitted without sacrificing reasonable accuracy in a preliminary design method. Then Equations (32) take the following form for the side span:

$$df_1 = \frac{3}{16\,n_1}\sec^3\alpha_1\,dL_1 \qquad (33a)$$

$$df_1 = -\frac{3}{16\,n_1}\sec^2\alpha_1\,dl_1 \qquad (33b)$$

and,

$$dl_1 = dL_1 \sec\alpha_1 \qquad (33c)$$

To find the change in the cable sag at the center of the main span due to cable stretch in the main span from stress, or due to change in length of the cable caused by temperature, the values of ΔL_s or ΔL_t from Equations (27) or (31*a*) are substituted for dL in Equation (32*a*); but changes of length, ΔL_1, of the cable in the side spans cause the tower tops to move, thus changing the horizontal length, l, of the main span by a length, dl. The horizontal movement of each tower top is approximately equal to $\Delta L_1 \sec\alpha_1$. If the side spans are symmetrical, the total change is as follows:

$$dl = -2\,\Delta L_1 \sec\alpha_1 \qquad (34)$$

Any change in length of the part of the cable from the end of the side span to the anchorage must be included in ΔL_1 in Equation (34). Moreover, the change in length of the short section of the cable over the tower saddles causes the tower tops to move horizontally a distance equal to the change in length. This is doubled to account for the other side span and then added to the increment, dl, in Equation (34). When all the horizontal movements of the tower top have been combined, the differential change in sag, df, is found from Equation (32*b*).

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[8] "Modern Framed Structures," by Johnson, Bryan, and Turneaure, Pt. II, Ninth Edition, p. 199.

Effect of a Change in Sag Upon the Main-Span Stiffening Truss.—Whenever the center sag of the main-span cable changes by an amount, df, the stiffening truss is raised or lowered by the same amount at the center. Forces are transmitted to the truss through the suspenders or hangers. If the hanger forces are uniformly distributed, the moment in the truss at the center will be:

$$M = 9.6 \frac{E\,I}{l^2}\,df \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Moments at other points would vary as the ordinates of a parabola. The hanger forces are not uniformly distributed, however. An inspection of the $(w_s - \beta\, w)$ diagram, which must look like the moment diagram, will verify this. An investigation of exact load diagrams indicated that the stiffening truss is subject to a non-uniform hanger load of greatest intensity at the ends and decreasing toward the middle. The moment diagrams are slightly elliptic in shape rather than parabolic. Consequently, the following equations are proposed in the preliminary design method for the computation of the center moment in the stiffening trusses caused by changes in sag:

For the main span at the center,

$$M = 9.4 \frac{E\,I}{l^2}\,df \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36a)$$

and, for the side span at the center,

$$M = 9.4 \frac{E\,I_1}{l_1^2}\,df_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36b)$$

For exact results, the value of M given by Equation (36a) should be adjusted, such adjustment being carried out by means of equations analogous to Equations (14a) and (14b); but the correction would be small, and not worth while.

The moment diagrams, as already indicated, are elliptic in shape. For a parabola, the moment ordinate at the quarter-point would be 0.75 of the center ordinate. For a true ellipse, the ordinate at the quarter-point is 0.866 of the center ordinate. A study of calculated moment diagrams for cable-stretch effects indicates that the quarter-point moment should be approximately 0.81 of the center moment for preliminary design purposes. Hence, the following equations are proposed for the computation of the moments at the quarter-points caused by changes in center sag:

For main span at the quarter-point,

$$M = 0.81 \times 9.4 \frac{E\,I}{l^2}\,df = 7.6 \frac{E\,I}{l^2}\,df \ldots\ldots\ldots\ldots (37a)$$

and, for the side span at the quarter-point,

$$M = 7.6 \frac{E\,I_1}{l_1^2}\,df_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37b)$$

Side-Span Interaction.—When the side spans of a suspension bridge are suspended from the cable, the side-span sag, f_1, is made such a value that the horizontal component of the cable stress under dead load and mean temperature has the same value for both the side span and the main-span; then:

$$H_w = \frac{w\,l^2}{8f} = \frac{w_1\,l_1^2}{8\,f_1} \qquad (38a)$$

or,

$$f_1 = \frac{w_1\,l_1^2}{w\,l^2}f \qquad (38b)$$

What happens to the side-span sag when live load is placed on the main span, or when there is a change in temperature? First, consider the case with no stiffening truss in the side span. The horizontal component of cable stress has increased to the value $H_w + H_a$. If the tower saddle is free to move, or if the tower has very little resistance to horizontal forces at the top, the sag, f_1, in the side span must decrease by an amount df_1, in order to preserve equilibrium; then:

$$H_w + H_a = \frac{w_1\,l_1^2}{8\,(f_1 + df_1)} \qquad (39a)$$

or,

$$(H_w + H_a)\,(f_1 + df_1) = \frac{w_1\,l_1^2}{8} = H_w\,f_1$$

and,

$$df_1 = -\frac{H_a}{H_a + H_w}f_1 \qquad (39b)$$

In Equation (39*b*) the minus sign indicates a decrease in sag for positive values of H_a. When a stiffening truss is added to the side span, a decrease, df_1, in side-span sag creates negative bending moments in the stiffening truss, as indicated in Equations (36*b*) and (37*b*). At the center of the span, the equation of statics, $\Sigma\,M = 0$, is satisfied when,

$$(H_w + H_a)\,(f_1 + df_1) + 9.4\,\frac{E\,I_1}{l_1^2}\,df_1 = \frac{w_1\,l_1^2}{8} = H_w\,f_1 \qquad (40a)$$

and,

$$df_1 = -\frac{H_a}{(H_w + H_a) + 9.4\,\dfrac{E\,I_1}{l_1^2}}f_1 \qquad (40b)$$

Due to the stiffening truss, the change in sag in Equation (40*b*) has a smaller value than the change in sag given by Equation (39*b*). If there is a uniform live load, w_{1l}, over the side span, in addition to the dead load, the equation of equilibrium becomes,

$$(H_w + H_a)\,(f_1 + df_1) + 9.4\,\frac{E\,I_1}{l_1^2}\,df_1 = \frac{w_1\,l_1^2}{8} + \frac{w_{1l}\,l_1^2}{8} \qquad (41)$$

but, since $\frac{w_1 l_1^2}{8} = H_w f_1$, and $\frac{w_{1l} l_1^2}{8} = \frac{w_{1l}}{w_1} H_w f_1$, Equation (41) becomes:

$$df_1 = \frac{-H_a + \frac{w_{1l}}{w_1} H_w}{(H_w + H_a) + 9.4 \frac{E I_1}{l_1^2}} f_1 \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

It is often convenient to find cable-stretch effects and sag changes in terms of $H_a = 1\,000\,000$ lb, or $H_a = 100\,000$ lb, and then compute the effects for the final value of H_a by direct proportion. This is accurate enough when H_a is small relative to H_w. A trial value of H_a based on Equations (16) is first used. Sag changes are then computed for temperature effect and cable stretch from stress, using this trial value of H_a. From the new sag, a new value of H_a is determined, and the process is repeated until the trial H_a and the final H_a agree, or differ by a very small amount.

The total change in the sag f at the center of the main span due to temperature, cable stretch, and side-span interaction is then consistent with the final value of H_a. From the total sag change, df, the moments in the stiffening truss at the quarter-point and at the center are found from Equations (36*a*) and (37*a*). These moments are added algebraically to the live load moments found from Equation (14*b*), in order to find the final values at the quarter-point and the center to be used in the preliminary design of the stiffening truss.

Since the sag change, df_1, in the side span must be found in order to compute maximum positive or negative moments in the main span, and since the side span is either unloaded or fully loaded with live load to find these main-span moments, it requires little additional computation to find the maximum positive and negative moments at the center of the side span for preliminary design purposes. The value of df_1, when found, is inserted in Equation (36*b*), and the moment is then evaluated.

COMPUTATIONS ILLUSTRATING THE PRELIMINARY DESIGN METHOD

Triborough Suspension Bridge.—A definite structure, the Triborough Suspension Bridge, has been used to demonstrate the typical preliminary design computations rather than some fictitious span, in order that the results obtained by the preliminary analysis may be compared with final values for an actual design.

The procedure outlined in Table 2 will be exactly the same as that for the investigation of an entirely new layout, except for the choice of the moments of inertia of stiffening trusses. Preliminary design data and constants are given in Table 1(*a*), Column (2). In the following computations (Tables 1 and 2) final design values for the moments of inertia of the stiffening trusses are used. In a new layout, it is advisable to use an arbitrarily selected moment of inertia, based on a tentative chord section and truss depth, for the initial determination of maximum moments and deflections. Then the procedure should be repeated with other values of moments of inertia, say, one-half as much and two times as much as those used in the first analysis. By doing this, a fairly accurate idea is obtained of the manner in which maximum bending moments vary with truss stiffness. Moreover, a good measure of the reduction in the deflections of an unstiffened cable, caused by each trial truss, is afforded by this process.

Maximum positive and negative moments at the quarter-point and center of the main span, and at the center of the side span, are computed. Considerable space is devoted to the explanations accompanying each step of the method; as a result, the computations may seem to be more extensive than they actually are. Ordinarily, the numerical computations, which are slide-rule results, require very little space and relatively little time when compared

TABLE 1.—COMPARISON OF DESIGN CHARACTERISTICS OF SUSPENSION BRIDGES

Description (1)	Triborough Bridge, at New York, N. Y. (2)	San Francisco-Oakland Bay Bridge, in California (3)	Manhattan Bridge, at New York, N. Y. (4)	Maumee River Bridge (5)	Mount Hope Bridge, in Rhode Island (6)	Moline-Bettendorf Bridge, at Moline, Ill. (7)
(*a*) DESIGN CONSTANTS						
Live load, w_l, in pounds per foot of truss	2 000	3 000	4 000	2 000	750	1 120
Total dead load, w, in pounds per foot of truss	9 580	9 200	5 820	6 500	2 650	2 185
Ratio, $w_l \div w$, of live load to dead load	0.209	0.326	0.687	0.308	0.283	0.512
Dead load, w_1, in pounds per foot of side span truss	9 670	9 700	6 130	6 500	2 650	2 230
Lengths, in Feet:						
Main span, l	1 358	2 294	1 447	777.6	1 188	735
Side span, l_1	660.7	1 152	713.5	230.3	498.3	366.7
Main span cable, L	1 394	2 354	1 485	811.1	1 220	753.9
West side-span cable, L_1	685	1 192	740.0	259.5	521.4	377.6
East side-span cable, L_1		1 184				
Short cable section, L', at tower	22	16.3	31.8	9.7	12.5	5.2
Short cable section, L', at anchorage	108	913.4*	119.1	266.6	341.3	193.4
Sag of cable, f, at center of main span	133.7	228.7	145.3	96.0	118.8	73.0
Sag of cable, f_1, at center of side span	31.6	60.8	37.2	8.41	20.9	18.42
Ratio, $n = f \div l$	0.0985	0.0997	0.1004	0.1234	0.10	0.0994
Ratio, $n_1 = f_1 \div l_1$	0.0478	0.0528	0.0522	0.0365	0.0419	0.0502
Moment of Inertia (Unit, Inches^2 — Feet^2):						
Stiffening truss of main span	15 000	69 500	43 900	2 600	4 259	1 835
Stiffening truss of side span	17 500	99 100	50 860	2 200	4 152	2 115
Horizontal component, H_w, of cable pull, in kips	16 450	25 810	10 480	5 120	3 940	2 030
Modulus of Elasticity, in Kips per Square Inch:						
Stiffening truss and towers, E	29 000	29 000	29 000	29 000	29 000	29 000
Cable, E_c	28 000	28 000	29 000	27 000	29 000	24 000
Area of cable, A, in square inches	276.8	500	275	106.6	73.9	44.9
Temperature change, Δt, in degrees Fahrenheit	±55	±30	±55	±55	±60	±60
Coefficient of thermal expansion, in ten-millionths	65	65	66	65	65	65
Secant of Slope Angle, α:						
West span	1.03	1.03	1.03	1.12	1.04	1.03
East span		1.02				
(*b*) COMPARISON OF PRELIMINARY DESIGN METHOD WITH DEFLECTION THEORY (UNITS, IN FOOT-KIPS)						
Main Span; Maximum Positive Moment at:						
Quarter-point, by preliminary design	28 500	111 800	113 300	11 300	10 300	8 400
Quarter-point, by deflection theory	28 000	111 000	119 100	11 580	10 390	9 095
Center, by preliminary design	23 600	90 400	91 300	8 450	8 350	7 060
Center, by deflection theory	22 100	88 830	89 710	7 900	7 954	6 665
Main span; deflection at quarter-point, in feet	4.58	10.1	7.4	3.0	4.5	3.7
Side Span; Maximum Positive Moment at:						
Center, by preliminary design	43 400	209 000	181 000	9 400	12 940	12 750
Center, by deflection theory	42 300	208 230	181 000	9 550	12 970	13 000

* West anchorage, 913.4 ft; and east anchorage, 56.6 ft.

TABLE 2.—PRELIMINARY DESIGN OF TRIBOROUGH SUSPENSION BRIDGE

Step No.	Explanation	Computation	Result	Units
	(*a*) MAXIMUM POSITIVE MOMENT AT THE QUARTER-POINT OF THE MAIN SPAN			
1	From Fig. 3(*a*), for $w_l \div w = 0.209$ and $f = 133.7$:			
	η' at the quarter-point..........	0.0242×133.7	3.24	Feet
	η_c' at the center................	-0.0075×133.7	−1.00	Feet
	New center sag, $f + \eta_c'$............	$133.7 - 1.0$	132.7	Feet
2	H_w, by Equation (15*b*)..............	$2\,200\,000 \div 133.7$	16 450	Kips‡
	$H_w + H_a$, by Equation (16*a*)........	$0.04 \times 2\,000 \times 1\,358^2 = \dfrac{148\,000}{2\,348\,000 \div 132.7}$	17 680	Kips‡
3	Trial value of $H_a = (H_w + H_a) - H_w$	....	1 230	Kips‡
4	M_t from Equation (24*a*)............	$(47.0 \times 29 \times 10^6 \times 15\,000 \times 3.24) \div 1\,358^2$	35 900	Foot-kips‡
	Moment induced by forcing cable back through the distance, $\eta' = (H_w + H_a)\eta'$	$17\,680 \times 3.24$	57 300	Foot-kips
	Adjusted moment in truss, from Equation (14*b*).....................	$(35\,900 \times 57\,300) \div 93\,200$	22 100	Foot-kips
5	Adjusted deflection, η..............	$3.24 \times 57\,300 \div 93\,200$	1.99	Feet
6	Change in Cable Length, ΔL Due to a Temperature Change, $\Delta t = +55°$ F (see Equation (31)):			
	(*a*) In the main-span cable......	$0.0000065 \times 55 \times 1\,394$	0.498	Feet
	(*b*) In the side-span cables......	$0.0000065 \times 55 \times 685$	0.245	Feet
	(*c*) In the cable at the tower saddle	$0.0000065 \times 55 \times 22$	0.008	Feet
	(*d*) In the cable at the anchorages	$0.0000065 \times 55 \times 108$	0.039	Feet
7	Change in Cable, ΔL, Due to $H_a = 1\,000$ Kips:			
	(*a*) In main-span cable, by Equation (27).................	$\dfrac{1\,000\,000 \times 1\,394^2}{277 \times 28 \times 10^6 \times 1\,358} = 0.000129\,\dfrac{1\,394^2}{1\,358}$	0.185	Feet
	(*b*) In side-span cable, by Equation (29*a*)................	$0.000129 \times 685^2 \div 661$	0.092	Feet
	(*c*) At the tower saddle, by Equation (30).................	0.000129×22	0.003	Feet
	(*d*) At the anchorage, by Equation (30).................	$0.000129 \times 108^2 \div 105$	0.014	Feet
8	Change in Center Sag, df, Due to a Temperature Change, $\Delta t = +55°$ F:			
	(*a*) From ΔL_t in main-span cable (Equation 32*a*)............	$\dfrac{15 \times 0.498^*}{16 \times 0.0985(5 - 24 \times 0.0097)} = 2.0 \times 0.498^*$	1.00	Feet
	(*b*) From Equation (34), for $\Delta L_{l t}$ in side-span, $dl =$	$-2 \times 0.245^† \times 1.03 = -0.50$	....	....
	and, from Equation (32*b*), $df=$	$-\dfrac{15 - 0.39 + 0.027}{7.51}(-0.50) = 1.95 \times 0.50$	0.97	Feet
	(*c*) From Equation (32*b*) for $\Delta L_t'$ at tower saddle; $df =$......	$1.95 \times 2 \times 0.008^a$	0.03	Feet
	(*d*) From Equation (34), for $\Delta L_t'$ at anchorage; $dl =$.........	$-2 \times 0.039^b \times 1.03 = -0.081$	....	....
	and, from Equation (32*b*), $df=$	1.95×0.081	0.16	Feet
	(*e*) Total, Steps 8(*a*), 8(*b*), 8(*c*), and 8(*d*)..................	....	+2.16	Feet

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*a*) Maximum Positive Moment at the Quarter-Point of the Main Span—(*Continued*)			
9	*df* Due to $H_a = 1\,000$ Kips:			
	(*a*) From Equation (32*a*), for ΔL_s in main-span cable	2.0×0.185^{c}	0.37	Feet
	(*b*) From Equation (34) for ΔL_s in side-span cable, dl = ...	$-2 \times 0.092^{d} \times 1.03 = -0.19$		
	and from Equation (32*b*), df =	1.95×0.19	0.37	Feet
	(*c*) From Equation (32*b*), for ΔL_s at tower saddle	$1.95 \times 2 \times 0.003^{e}$	0.01	Feet
	(*d*) From Equations (32*b*) and (34), for $\Delta L_e'$ at the anchorages: dl = ...	$-2 \times 0.014^{f} \times 1.03 = -0.029$		
	df = ...	1.95×0.029	0.06	Feet
	(*e*) Total, Steps 9(*a*), 9(*b*), 9(*c*), and 9(*d*)	. . . .	+0.81	Feet
10	*df* Due to Side-Span Interaction, for $H_a = 1\,000$ Kips:			
	(*a*) From Equation (40*b*), df_1 = . .	$-\dfrac{1\,000 \times 31.6}{17\,450 + 10\,900}$	−1.11	Feet
	(*b*) From Equation (33*b*), dl_1 = . .	$\dfrac{16 \times 0.0478}{3 \times 1.03^2} \times 1.11$	0.267	Feet
	(*c*) $dl = -2\,dl_1$ = ...	-2×0.267	−0.534	Feet
	(*d*) From (32*b*), df = ...	1.95×0.534	+1.04	Feet
	Total change, df (Step 9(*e*) + Step 10(*d*))	. . . .	+1.85	Feet
11	(*a*) Trial value of H_a (see Step 3)	. . . .	1 230	Kips‡
	(*b*) df due to $\Delta t = +55°$ F, and $H_a = 1\,230$ kips	$2.16 + 1.23 \times 1.85$	4.43	Feet
	(*c*) Corrected center sag	$132.7^{g} + 4.43$	137.1	Feet
	(*d*) $H_w + H_a$	$2\,348\,000^{h} \div 137.1$	17 130	Kips‡
	(*e*) $H_a = (H_w + H_a) - H_w$	$17\,130 - 16\,450$	680	Kips‡
	(*f*) New trial, H_a	Between 680 and 1 230; assume, H_a =	780	Kips‡
	(*g*) df due to $\Delta t = +55°$ F, and $H_a = 780$ kips	$2.16 + 0.78 \times 1.85$	3.60	Feet
	(*h*) Revised center sag, f = ...	$132.7 + 3.60$	136.3	Feet
	(*i*) $H_w + H_a$ = ...	. . . .	17 230	Kips‡
	(*j*) H_a (this value checks Step 11(*f*)) . .	. . . .	780	Kips‡
	(*k*) Therefore, the final change, df, and the deflection of the truss, is	$2.16 + 0.78 \times 1.85$	+3.60	Feet
12	Moment at quarter-point of truss (Equation 37(*a*))	$7.6 \times 29 \times 10^6 \times 15\,000 \times 3.60 \div 1\,358^2$	+6 450	Foot-kips‡
13	Final moment, from Steps 4 and 12 . .	$22\,100 + 6\,450$	+28 500	Foot-kips‡
14	Final moment, by the deflection theory	. . . .	+28 000	Foot-kips‡
15	Total deflection, η	$1.99^{i} + 0.73^{j} \times 3.60^{k}$	+4.62	Feet
	(*b*) Maximum Positive Moment at the Center of the Main Span			
1	From Fig. 3(*b*), for $w_l \div w = 0.209$ and $f = 133.7$:			
	η' at the center = ...	0.0133×133.7	1.78	Feet
	New center sag, $f + \eta'$ = ...	$133.7 + 1.78$	135.5	Feet
2	H_w, by Equation (15*b*) = ...	$2\,200\,000 \div 133.7$	16 450	Kips‡
	$H_w + H_a$, by Equation (16*c*) = ...	$0.0638 \times 2\,000 \times 1\,358^2 = 235\,000$		Kips‡
		$2\,435\,000 \div 135.5$	17 950	Kips‡
3	Trial value of $H_a = (H_w + H_a) - H_w$	. . . .	1 500	Kips‡
4	M_t, from Equation (24*c*) = ...	$(65.8 \times 29 \times 10^6 \times 15\,000 \times 1.78) \div 1\,358^2$	27 600	Foot-kips‡
	Moment induced by forcing cable back through the distance, $\eta' = (H_w + H_a)\eta'$	$17\,950 \times 1.78$	32 100	Foot-kips‡
	Adjusted moment in truss, by Equation (14*b*)	$27\,600 \times 32\,100 \div 59\,700$	+14 800	Foot-kips‡

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*b*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE MAIN SPAN—(*Continued*)			
5	Adjusted deflection, η	1.78 × 32 100 ÷ 59 700	0.95	Feet
6–10	Same as Table 2(*a*)	....	....	....
11	(*a*) Trial value of H_a (see Step 3)	....	1 500	Kips‡
	(*b*) *df* due to Δt = + 55° F, and H_a = 1 500 kips	2.16 + 1.5 × 1.85	4.93	Feet
	(*c*) Corrected center sag	135.5 + 4.93	140.4	Feet
	(*d*) $H_w + H_a$	2 435 000^h ÷ 140.4	17 330	Kips‡
	(*e*) $H_a = (H_w + H_a) - H_w$	....	880	Kips‡
	(*f*) New trial, H_a =	Between 880 and 1 500; assume H_a =	1 000	Kips‡
	(*g*) *df* due to Δt = + 55° F and H_a = 1 000 kips	2.16 + 1 × 1.85	4.01	Feet
	(*h*) Revised center sag, *f* =	135.5 + 4.01	139.5	Feet
	(*i*) $H_w + H_a$	....	17 450	Kips‡
	(*j*) H_a (this value checks Step 11(*f*))	....	1 000	Kips‡
	(*k*) Therefore, the final change, *df*, and the deflection of the truss, is	....	4.01	Feet
12	Moment at center of truss (Equation (36*a*))	$9.4 \times 29 \times 10^6 \times 15\,000 \times 4.01 \div 1\,358^2$	+8 890	Foot-kips‡
13	Final moment, from Steps 4 and 12	....	+23 700	Foot-kips‡
14	Final moment, by the deflection theory	....	+22 100	Foot-kips‡
15	Total deflection, η	0.95 + 4.01	4.96	Feet
	(*c*) MAXIMUM NEGATIVE MOMENT AT THE QUARTER-POINT			
1	From Fig. 3(*a*), for $w_l \div w$ = 0.209 and *f* = 133.7:			
	η' at the quarter-point =	−0.0242 × 133.7	−3.24	Feet
	η_c' at the center =	0.0048 × 133.7	0.64	Feet
	New center sag, $f + \eta_c'$ =	133.7 + 0.64	134.3	Feet
2	H_w by Equation (15*b*) =	2 200 000 ÷ 133.7	16 450	Kips‡
	$H_w + H_a$, by Equation (16*b*) =	$0.085 \times 2\,000 \times 1\,358^2$ = 314 000 2 514 000 ÷ 134.3	18 700	Kips‡
3	Trial value of $H_a = (H_w + H_a) - H_w$	....	2 250	Kips‡
4	M_t, from Equation (24*b*)	$(-43.0 \times 29 \times 10^6 \times 15\,000 \times 3.24) \div 1\,358^2$	−32 800	Foot-kips‡
	Moment induced by forcing cable back through the distance, η' = $(H_w + H_a)\eta'$	18 700 × 3.24	−60 600	Foot-kips‡
	Adjusted moment in truss, by Equation (14*b*)	− 32 800 × 60 600 ÷ 93 400	−21 300	Foot-kips‡
5	Adjusted deflection, η	− 3.24 × 60 600 ÷ 93 400	−2.10	Feet
6	ΔL when Δt = − 55° F	(Same as Step 6, Table 2(*a*), with all negative values)	....	....
7	ΔL due to H_a = 1 000 kips	(Same as Step 7, Table 2(*a*))	....	....
8	*df* when Δt = − 55° F	(Same as Step 8, Table 2(*a*), with opposite signs) Σ *df* =	−2.16	Feet
9	*df* due to H_a = 1 000 kips	(Same as Step 9, Table 2(*a*)); Σ *df*	+0.81	Feet
10	*df* Due to Side-Span Interaction, for H_a = 2 250 kips:			
	(*a*) From Equation (42), df_1 =	$\frac{-2\,250^m + (2\,000 \div 9\,670)16\,450}{18\,700 + 10\,900} \times 31.6$	1.23	Feet
	(*b*) dl_1 = from Equation (33*b*)	$-\frac{16 \times 0.0478 \times 1.23}{3 \times 1.03^2}$	−0.30	Feet
	(*c*) $dl = -2\,dl_1$ =	2 × 0.30	0.60	Feet
	(*d*) From Equation (32*b*), *df* =	− 1.95 × 0.60	−1.17	Feet

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*c*) MAXIMUM NEGATIVE MOMENT AT THE QUARTER-POINT—(*Continued*)			
11	(*a*) Trial value of H_a (see Step 3) .	....	2 250	Kips‡
	(*b*) df due to $\Delta t = -55°$ F, and $H_a = 2\,250$ kips...........	$-2.16 + 2.25 \times 0.81 - 1.17$	−1.50	Feet
	(*c*) Corrected center sag.........	$134.3^{g} - 1.50$	132.8	Feet
	(*d*) $(H_w + H_a)$................	$2\,514\,000^{h} \div 132.8$	18 900	Kips‡
	(*e*) $H_a = (H_w + H_a) - H_w$.....	....	2 450	Kips‡
	(*f*)^{n} Repeat Steps 10 and 11 to find H_a..................	....	2 400	Kips‡
	(*k*) $\Sigma\, df$ for $\Delta t = -55°$ F and $H_a = 2\,400$ kips............	$-2.16 + 2.4 \times 0.81 - 0.99$	−1.20	Feet
12	Moment at quarter-point of truss (Equation (37*a*))................	$7.6 \times 29 \times 10^6 \times 15\,000 \times (-1.20) \div 1\,358^2$	−2 100	Foot-kips‡
13	Final moments from Steps 4 and 12..	....	−23 400	Foot-kips‡
14	Final moments, by deflection theory .	....	−22 700	Foot-kips‡
15	Total deflection, η, from Steps 5 and 11.	$-2.10 + 0.73^{j}(-1.20)$	−2.97	Feet
	(*d*) MAXIMUM NEGATIVE MOMENT AT THE CENTER OF THE MAIN SPAN			
13	Final moment from Steps 1 to 12....	$-15\,000 - 3\,300$	−18 300	Foot-kips‡
14	Final moment by the deflection theory	....	−16 500	Foot-kips‡
15	Total deflection, η.................	$-1.08 - 1.52$	−2.60	Feet
	(*e*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE SIDE SPAN^{o}			
1	H_w, by Equation (15*b*) =..........	$2\,200\,000 \div 133.7$	16 450	Kips‡
2	df due to $\Delta t = +55°$ F............	....	2.16	Feet
3	df due to $H_a = 100$ kips............	....	0.08	Feet
4	df Due to Interaction of Loaded Side Span, for $H_a = +100\,000$ lb:			
	(*a*) From Equation (42), df_1 =...	$\dfrac{-100 + 3\,400}{16\,550 + 10\,900}\,31.6 =$	+3.80	Feet
	(*b*) From Equation (33*b*), dl_1 = ..	$-0.240 \times 3.80 =$	−0.910	Feet
	(*c*) From one side span, $dl = -dl_1$..............	....	+0.910	Feet
	(*d*) From Equation (32*b*), df =...	$-1.95 \times 0.910 =$	−1.77	Feet
	df Due to Interaction of Unloaded Side Span:			
	(*e*) From Equation (40*b*), df_1 = ..	$\dfrac{-100 \times 31.6}{16\,550 + 10\,900}$	−0.115	Feet
	(*f*) By proportion from Step 4(*d*), df =......................	....	0.05	Feet
	Total change, $\Sigma\, df$, from both side spans (Step 4(*d*) + Step 4(*f*))......	....	−1.72	Feet
5	(*b*) df due to $\Delta t = +55°$ F, and $H_a = +100$ kips..............	$2.16 + 0.08 - 1.72$	0.52	Feet
	(*c*) Revised center sag, f	$133.7 + 0.52$	134.2	Feet
	(*d*) $H_w + H_a$ =....................	$2\,200\,000 \div 134.2$	16 360	Kips‡
	(*e*) H_a...........................	....	−90	Kips‡
	(*f*) New trial H_a = ...	Between − 90 and + 100; assume, H_a =	−50	Kips‡
	(*g*) Repeat Steps 4 and 5; H_a = (practically)......................	....	−40	Kips‡
6	df_1, due to $H_a = -40$ kips = (Equation (42))......................	$\dfrac{+40 + 3\,400}{16\,410 + 10\,900}\,31.6$	+3.98	Feet

TABLE 2.—(*Continued*)

Step No.	Explanation	Computation	Result	Units
	(*e*) MAXIMUM POSITIVE MOMENT AT THE CENTER OF THE SIDE SPAN[o]—(*Continued*)			
7	Moment at center of side span (Equation (36*b*))	$(9.4 \times 29 \times 10^6 \times 17\,500 \times 3.98) \div 661^2$	+43 400	Foot-kips‡
8	Moment from deflection theory	....	+42 250	Foot-kips‡
	(*f*) MAXIMUM NEGATIVE MOMENT AT THE CENTER OF THE SIDE SPAN[p]			
6	Final change, *df*, in sag of unloaded side span	....	−3.30	Feet
7	Moment at center	....	−36 000	Foot-kips‡
8	Moment from deflection theory =	....	−34 200	Foot-kips‡

* See Step 6(*a*). † See Step 6(*b*). ‡ The result is given in kips, or thousands of pounds to conserve space. [a] See Step 6(*c*). [b] See Step 6(*d*). [c] See Step 7(*a*). [d] See Step 7(*b*). [e] See Step 7(*c*). [f] See Step 7(*d*). [g] See Step 1. [h] See Step 2. [i] See Step 5. [j] At the quarter-point, η is about 0.73 times its value at the center. This was determined from a study of precise deflections due to cable stretching. [k] See Step 11(*g*). [m] In Equation (42) use a trial value of $H_a = 2\,250\,000$ lb from Step 3, Table 1(*c*), because this formula is not a linear function of H_a. [n] Steps 10(*f*), 10(*g*), 10(*h*), 10(*i*), and 10(*j*). [o] The side span under consideration is fully loaded with live load; the main spans and the other side span are not loaded; and the value, Δt, is + 55° F. [p] The span under consideration is not loaded; the remainder of the bridge is fully loaded; and the value of Δt is − 55° F.

to the procedure necessary in using the deflection theory or the trigonometric series method. A study of the computations emphasizes the importance of Fig. 3(*a*) and Fig. 3(*b*).

Preliminary Design Values for Other Suspension Bridges.—Maximum moments and corresponding deflections have been computed by the preliminary design method for five additional suspension bridges, ranging from a relatively short and light span to a long and heavy span. Detail computations are omitted from the paper; but the constants for each bridge are given in full in Table 1(*a*). The final results are tabulated in Table 1(*b*), together with the corresponding moments computed by the deflection theory.

Effect of Variation in Truss Stiffness.—In preliminary design studies of suspension bridges, it is important to know approximately what happens to moments and deflections when the truss stiffness is varied. The results in Table 1(*b*) for the San Francisco-Oakland Bay Suspension Bridge are based upon the equivalent uniform moments of inertia of the main span and side-span trusses as actually designed. If an unstiffened cable had been used, the downward deflection at the quarter-point would have been approximately 14.2 ft. Consequently, the truss as designed has reduced the deflection of the cable at the quarter-point from about 14.2 ft to about 10.1 ft.

If the moments of inertia of the main-span and side-span trusses are doubled, the moments become larger and the deflections decrease. If the moments of inertia are decreased, the moments decrease, but the deflections become greater. Fig. 7 shows theoretical values plotted for moments of inertia, I_t, four times, twice, one-half, and one-quarter as much as the moment of inertia I_a used in the actual design. The curves are not exact; nevertheless they give the designer a fairly accurate picture of what happens.

Fig. 7(*a*) indicates that the most economical stiffening truss is no stiffening truss whatever; but Fig. 7(*b*) indicates that the deflections of an unstiffened cable may be too great, and that, therefore, a stiffening truss may be needed. The maximum grade change is probably of more importance than the maximum deflection; even so, Fig. 7(*b*) gives a clue as to what may be expected with trusses of different stiffness, since the deflections and grade changes will vary in about the same ratio. What constitutes the limiting grade change, however, is still a subject for research. Present tendencies are to make stiffening trusses more flexible. Studies like those embodied in Fig. 7 may then be of great help in selecting a proper design. As stated[9] by Hardy Cross, M. Am. Soc. C. E.: "Where the action is hybrid there are many possible structures; the designer must make his choice." The problem becomes one of intelligent selection.

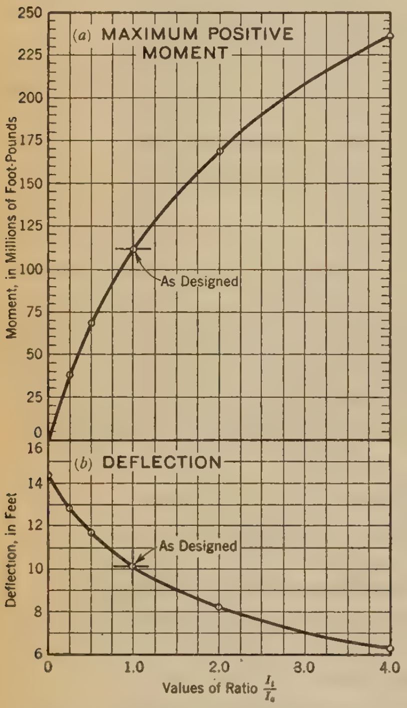

FIG. 7.—EFFECTS AT QUARTER-POINT OF MAIN SPAN OF SAN FRANCISCO-OAKLAND BAY BRIDGE DUE TO CHANGES IN TRUSS STIFFNESS

CONCLUSION

A rapid and accurate method of analysis for the preliminary design of suspension bridges is presented in this paper. The method begins logically with an unstiffened cable and then shows what happens when a stiffening truss is added. The effects of cable stretch from stress and from temperature, and the effect of side-span interaction, are obtained separately, thus giving the designer a good idea of the contributions of the various factors to the final moment and deflection in the stiffening truss. The method demonstrates clearly the related functions of cable and stiffening trusses in suspension bridges.

The method emphasizes the fact that there is no single design for a stiffening truss. There are any number, depending upon what limits are established for deflection and change of grade. The method will also prove a valuable aid in making additional studies which are needed to "throw light" upon suspension bridge design. These studies are indicated in the following questions: When may the stiffening truss be omitted? What is the maximum permissible grade change? Is it more economical to obtain stiffness by increasing the chord section or by increasing the depth of the truss? May one count on the floor system to help the stiffening truss in reducing cable deflections? If so, how must the floor system be connected to insure participation? Will participation "hurt" the floor system more than it will "help" the truss? When should one use alloy steels with higher working stresses than those for ordinary structural steel? Is there a place for alloys with different moduli of elasticity? What are the economics of using light-weight floors?

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[9] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1369.

ACKNOWLEDGMENTS

The writers greatly appreciate the valuable suggestions made during the course of this study by Professor Cross. They also acknowledge the generous co-operation of the following engineers, who supplied final design data for various suspension bridges: Allston Dana, Leon S. Moisseiff, D. B. Steinman, Montgomery B. Case, and C. E. Paine, Members, Am. Soc. C. E., and Ralph A. Tudor, Assoc. M. Am. Soc. C. E.

APPENDIX

NOTATION

The following symbols, introduced where they first appear in the paper, conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials" compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:[10]

A = area; cross-sectional area of cable;
C = constants;
E = modulus of elasticity of the stiffening truss and towers; E_c = modulus of elasticity of the cable;
f = sag of cable at the center of the main span; f_1 = sag of cable at the center of the side span;
H = horizontal component of force; H_w = component of cable stress due to dead load and mean temperature; H_a = additional component due to any change from the condition of dead load at mean temperature;
I = moment of inertia of the stiffening truss of the main span; I_1 = moment of inertia of the stiffening truss of the side span;
k = a ratio defining loaded length of stiffening truss;
L = length of cable in main span; L_1 = length of cable in side span;
l = horizontal length of main span; l_1 = horizontal length of side span;
M = moment; actual bending moment in the stiffening truss; M_t = bending moment induced in the stiffening truss if it were bent to the deflection η' of the unstiffened cable; M' = bending moment in the stiffening truss, assuming it to be a simply supported beam;
t = temperature; Δt = temperature change, or variation;

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[10] A. S. A.—Z10a—1932.

V = actual shear in stiffening truss at any section; V' = shear at any section in the stiffening truss, assuming it be a simply supported beam;

w = total dead load per unit horizontal length of cable; w_l = live load on the roadway per unit length of truss; w_s = suspender pull expressed as a load per unit of horizontal length of cable due to live load or temperature, or both; w_t = final load acting on truss per unit length.

x = co-ordinate horizontal distance of any point of the cable curve, measured from an origin at the top of the left tower, under dead load at mean temperature;

y = co-ordinate vertical distance corresponding to x;

α = slope of the straight line joining the tower tops; α_1 = slope of the chord of the side-span cable;

β = a ratio, $\frac{H_a}{H_w}$;

δ = unit elongation; strain;

η = deflection of truss and main cable, at any section, from its initial position, under dead load at mean temperature; η' = corresponding deflection of unstiffened main cable; and

ω = coefficient of thermal expansion.

DISCUSSION

A. W. Fischer,[11] Esq. (by letter).—In their "Introduction" the authors state that the so-called "elastic theory" used in earlier days is obsolete. The writer thinks, however, that for short spans of, say, 750 ft, or less, the "elastic theory" can be used. (Generally, for spans of 750 ft and less, other types of bridges are more economical.) Referring to the "deflection theory," the authors state that it is quite long and that a computing machine or a table of logarithms must be used in order to avoid possibilities of serious error. The writer agrees that the deflection theory is rather long and he would not use it, for that reason especially, for calculating the value of H_a. After H_a is known, however, the equations for moment and shear given by the "deflection theory," derived by using hyperbolic functions, are as short as any of the exact methods.

The authors state that the "trigonometric series" is not satisfactory for studies of a variety of preliminary designs. The writer much prefers the trigonometric series for the solution of H_a because a slide-rule can be used which gives results that are accurate enough for practical purposes. If the effect of the change of curvature is neglected, then the value of H_a can be calculated in a very short time, and the results will be amply accurate for preliminary design.

The authors then advance their method as being more rapid than the deflection theory or the trigonometric series method. After studying the proposed new method the writer is not convinced that any time is saved by it. It is true that the moments at the quarter-points and the center points of the main span and the moment at the center of the side span agree remarkably with the deflection theory, but even so the method requires considerable computation. The writer did not check the moment of 113 300 ft-kips as given for the quarter-point by the authors' preliminary design for the Manhattan Bridge, but it seems that the results are somewhat too small. By solving for H_a, using the trigonometric series,[12] with the live load covering 0.425 of the main span, neglecting the effect of the change of curvature, and assuming that the ratio of H_a to H_w is 0.2, the writer obtains an answer of 2 028 kips for H_a, and substituting this in an equation involving hyperbolic functions,[13] the value of the moment at the quarter-point is + 118 870 ft-kips. If the effect of the change of curvature is considered, $H_a = 2\ 040$ kips and the moment at the quarter-point is + 118 650 ft-kips.

For the Manhattan Bridge, by solving for H_a, using the trigonometric series with the live load covering 0.3 to 0.7 of the main span, considering the effect of the change of curvature, and assuming that the ratio, H_a to H_w, is 0.3, the writer finds 3 166 kips for H_a; and substituting this value in an equation involving hyperbolic functions the value of the moment at the center of the main span is + 88 300 ft-kips. In their preliminary design the authors give 91 300 ft-kips.

In their entire paper the authors do not mention how they propose to compute shear, and for a preliminary design it is also important that the shear is known. The value of H_a can also be readily solved by the trigonometric series, and then substituting in a general equation containing hyperbolic functions,[14] the value of shear can be ascertained.

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[11] Care, Constr. Service, Veterans Administration, Washington, D. C.

[12] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 387.

[13] *Loc. cit.*, Vol. 100 (1935), p. 1199, Equation (80); see, also, p. 1200.

For a preliminary design for moment at the quarter-points and the center of the main span, etc., it also seems to the writer that such a method as given by Arvid H. Baker, Assoc. M. Am. Soc. C. E.,[15] is considerably shorter than the authors' method, and the results are such that, for a preliminary design, they are satisfactory. The moment is first solved by reading coefficients from a chart introduced by D. B. Steinman, M. Am. Soc. C. E.,[16] and then applying a correction factor derived by Mr. Baker.[15]

Throughout the entire paper, the writer fails to grasp the meaning of just what is meant by "preliminary design of stiffening trusses of suspension bridges." If it means a preliminary design for estimating purposes, it seems to the writer that too much time will be required when compared with the chart presented in 1926 by the late J. A. L. Waddell, Hon. M. Am. Soc. C. E.,[17] from which the steel in the stiffening trusses must be read by an experienced engineer familiar with suspension bridge designs. Any one who knows the cost of steel in place can make a very close estimate in a short time, and for that reason the writer really sees no particular need for a preliminary design such as that given in the paper.

In their closing discussion the authors should show just how they propose to apply their method to the computation of shear in the main span and in the side spans and, furthermore, they should present a general method for calculating the moments at points other than the quarter-points and center.

JACOB KAROL,[18] ESQ. (by letter).—The principal merit of this valuable paper lies in the "picture" it presents of the related action of the cable and the stiffening truss. The writer wishes to discuss this pictorial aspect in greater detail.

It has been emphasized, in the literature on the deflection theory of suspension bridges, that stresses for a combined loading cannot be found by a summation of the stresses due to the component loads. Although this statement is strictly true, it is also true, and much more important, to state that superposition of component effects under certain conditions gives a result that is approximately correct.

The authors' method of calculation permits a direct determination of these component effects. In Professor Wessman's original thesis[5] the effect due to the addition of the stiffening truss to the unstiffened cable was designated as Step I, and the combined effect of cable stretch, temperature change, and side-

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[14] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), Equation (81); and, also, p. 1200.

[15] "Suspension Bridge Analysis by the Exact Method Simplified by Knowledge of Its Relations to the Approximate Method," by Arvid H. Baker, Eng. and Science Series No. 24, Pl. I and II, Rensselaer Polytechnic Inst., June, 1928, Troy, N. Y.

[16] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, Charts I, II, and III.

[17] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 894, Fig. 5; see, also, p. 942, Table 5.

[18] Graduate Fellow in Civ. Eng., Univ. of Illinois, Urbana, Ill.

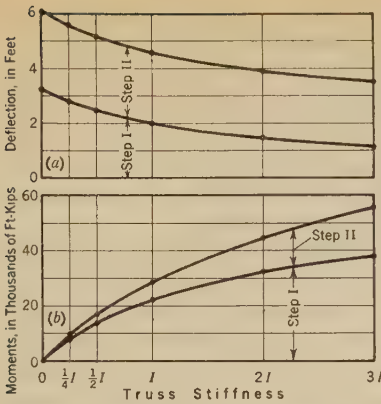

FIG. 8.—EFFECTS AT QUARTER-POINT OF MAIN SPAN OF TRIBOROUGH BRIDGE DUE TO CHANGES IN TRUSS STIFFNESS (LOADING CONDITION, DEAD LOAD + LIVE LOAD + 55° CHANGE OF TEMPERATURE)

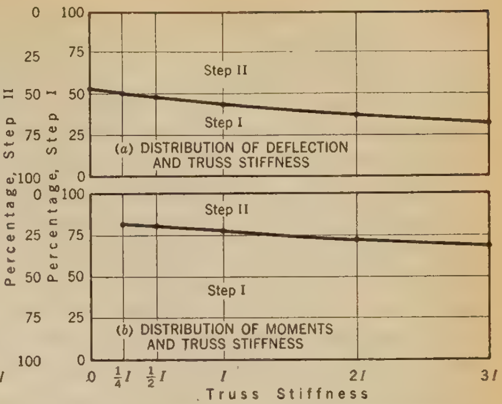

FIG. 9.—EFFECTS AT QUARTER-POINT OF MAIN SPAN OF TRIBOBOUGH BRIDGE DUE TO CHANGES IN TRUSS STIFFNESS (LOADING CONDITION, DEAD LOAD + LIVE LOAD + 55° TEMPERATURE)

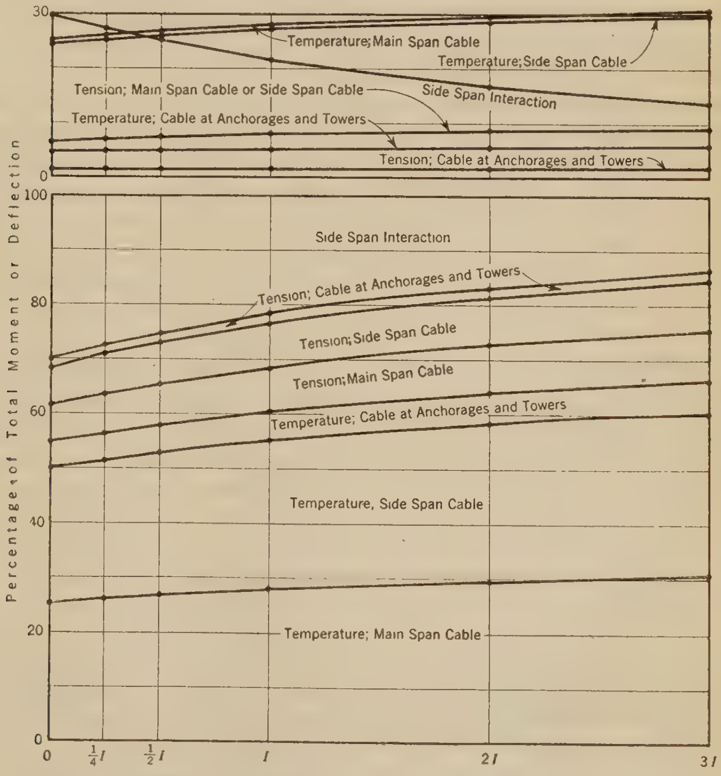

FIG. 10.—EFFECTS AT QUARTER-POINT OF MAIN SPAN OF TRIBOROUGH BRIDGE ON MOMENTS AND DEFLECTIONS FROM CABLE SAG, DUE TO CHANGES IN TRUSS STIFFNESS (LOADING CONDITION, DEAD LOAD + LIVE LOAD + 55° TEMPERATURE)

span interaction was designated as Step II. It may be noted, in passing, that Equation (14*b*), which gives the moment due to Step I, is based on Equation (13), which is an assumption. The writer found that the moment due to Step I, as calculated by the deflection theory, was slightly less than that calculated by the authors' method, whereas the reverse was true of the deflection. However, the effect of Step II as computed by the authors' method is compensating, so that the final moment and deflection are in substantial agreement with the values obtained by the deflection theory.

From the preceding discussion it is obvious that the separation of the total moment and deflection into their component effects is quantitatively approximate. It may be added that this procedure is applicable to the main span only.

Economical design implies an understanding of the relations among the various factors entering into the design. Hence, only by studying the changes produced by the variation of the several factors can one obtain the necessary understanding.

Consider, as an illustration, the positive moment and deflection at the quarter-point of the main span of the Triborough Suspension Bridge, in New York, N. Y., as affected by a variation in truss stiffness, with the ratio of the main span truss stiffness to that of the side span held constant. Fig. 8(*a*) indicates the relation between the deflection and the truss stiffness, and Fig. 8(*b*), the relation between the moment and the truss stiffness. These same data may be plotted in another manner as proportions of the total moment or deflection. Fig. 9(*a*) indicates that the proportion of the total deflection due to Step I is 53% for the unstiffened cable and 31% for a truss stiffness of 3 *I* (*I* is the truss stiffness as designed), and Fig. 9(*b*) shows that the proportion of the total moment due to Step I is 82% for 0.25 *I* and 68% for 3 *I*.

Fig. 10 indicates the proportions of the total moment or deflection from Step II, due to its component factors, in relation to the truss stiffness. It shows that the effects of cable stretch due to temperature and tension are sensibly constant, and that side-span interaction causes 30% of the total for an unstiffened cable and 14% of the total for a truss stiffness of 3 *I*. By considering both Figs. 9 and 10, it seems that about 50% of the total deflection and about 20% of the total moment is due to the unvarying cable stretch effects of Step II.

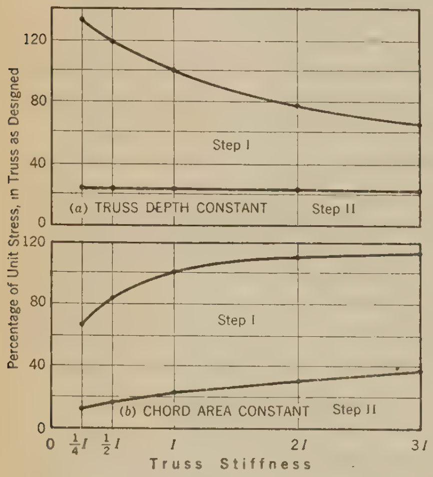

FIG. 11.—EFFECTS AT QUARTER-POINTS OF MAIN SPAN OF TRIBOROUGH BRIDGE ON UNIT STRESSES IN STIFFENING TRUSS DUE TO CHANGES IN TRUSS STIFFNESS (LOADING CONDITION, DEAD LOAD + LIVE LOAD + 55° TEMPERATURE)

Another interesting study is the effect of the truss proportions on the stresses in the chords. Assuming that the truss as designed has a stress of 100%, the preceding data were plotted in Fig. 11. For a constant truss depth, Fig. 11(*a*) indicates that the stress due to Step II is approximately constant, whereas that from Step I decreases with an increase in I. Fig. 11(*b*) shows that for a constant chord area the stresses due to Step I and Step II both increase with an increase in I. Hence, if a structure has been designed with a truss of stiffness, I, depth, d, and chord area, A, the following relations obtain:

Decreasing the value of I, with depth, d, constant, increases the unit stress.
Increasing the value of I, with depth, d, constant, decreases the unit stress.
Decreasing the value of I, with Chord A constant, decreases the unit stress.
Increasing the value of I, with Chord A constant, increases the unit stress.

If it is desired to maintain a constant unit stress, the required relationships can be determined as follows: For the designed truss, let I represent the stiffness; M, the moment; A, the chord area; d, the depth; and s, the unit stress. For a truss of stiffness, $I_n = n\,I$, the corresponding terms are denoted with a subscript, n. Letting $I = \frac{A\,d^2}{2}$,

$$s = \frac{\frac{M\,d}{2\,A\,d^2}}{2} = \frac{M}{A\,d} \qquad (43)$$

Similarly,

$$s_n = \frac{M_n}{A_n\,d_n} \qquad (44)$$

From Equations (43) and (44),

$$\frac{M}{A\,d} = \frac{M_n}{A_n\,d_n} \qquad (45)$$

Also, by definition:

$$A\,d^2 = \frac{1}{n}\,A_n\,d_n^2 \qquad (46)$$

From Equations (45) and (46),

$$\frac{A_n}{A} = n\left(\frac{d}{d_n}\right)^2 = \frac{M_n}{M} \times \frac{d}{d_n} \qquad (47)$$

or,

$$\frac{d_n}{d} = n\,\frac{M}{M_n} \qquad (48)$$

and,

$$\frac{A_n}{A} = \frac{1}{n}\left(\frac{M_n}{M}\right)^2 \qquad (49)$$

Applying Equations (48) and (49) to the preceding data gives the results indicated in Fig. 12.

The writer made some further studies to determine the effect of the side-span stiffness on the positive moment and deflection at the quarter-point of the main span, the stiffness of the latter being kept constant and equal to the design value. Fig. 13 indicates a reduction of about 4% in the moment and 10% in the deflection for the extreme variation in side-span stiffness from 0 to 3 I_1.

This, and other, studies by the writer seem to indicate that for a given cable and span layout, the moments and deflections in any span are primarily a function of the stiffness of that span. This statement does not apply unreservedly to the case of continuous stiffening trusses. It should be evident from the foregoing that the authors' method of analysis is of undoubted value in giving a clear and simple picture of the action of a suspension bridge with hinged stiffening trusses. This picture is admittedly approximate quantitatively, but not so approximate as to be misleading. The writer has extended the authors' method to include the determination of the maximum shears in the stiffening trusses. From 80% to 90% of the total shear is due to Step I.

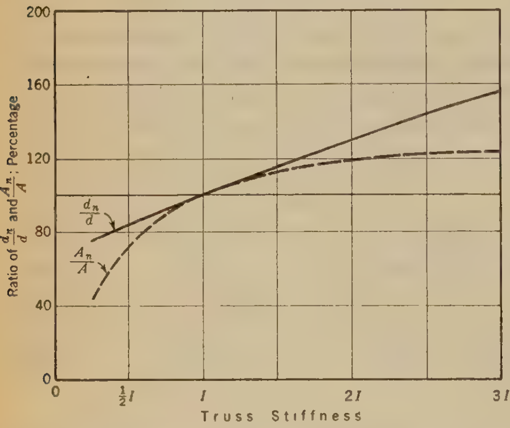

FIG. 12.—EFFECT AT QUARTER-POINT OF MAIN SPAN OF TRIBOROUGH BRIDGE; REQUIRED VARIATION IN TRUSS DEPTH AND CHORD AREA FOR CONSTANT UNIT STRESS WITH VARYING TRUSS STIFFNESS

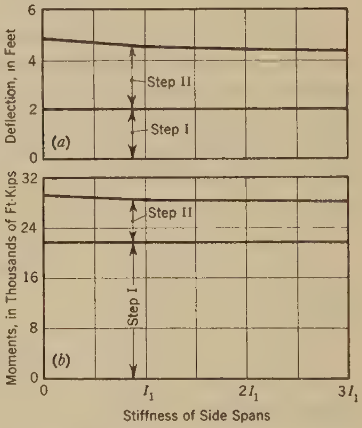

FIG. 13.—TRIBOROUGH BRIDGE; MAXIMUM EFFECTS AT QUARTER-POINT OF MAIN SPAN FOR DESIGN STIFFNESS I IN MAIN SPAN AND VARIOUS VALUES OF I IN SIDE SPANS

In closing, the writer wishes to take exception to the authors' criticism of the deflection theory on the ground that it requires long and involved computations. He has developed an influence line procedure for the generalized deflection theory whereby slide-rule computations are accurate enough for all practical purposes.

Acknowledgment.—These studies of suspension bridges were made under the direction first of Hardy Cross, and, later, of T. C. Shedd, Members, Am. Soc. C. E., as part of the writer's graduate study, and their helpful comments are hereby acknowledged.

GLENN B. WOODRUFF[19] AND NORMAN C. RAAB,[20] MEMBERS, AM. SOC. C. E. (by letter).—Even to the experienced designer, the deflection theory for suspension bridges has heretofore been largely a matter of following formulas without a complete conception of their meanings. The authors are to be congratulated upon their success in presenting a clear picture of the interaction of the various parts of the suspension system. The paper is so complete and the results so convincing that, within its scope, there remains but little to be said. In their "Conclusion," the authors have emphasized several subjects for further investigation. The following is submitted, not as an answer to these problems, but rather as a suggested basis for these investigations.

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[19] Engr. of Design, San Francisco-Oakland Bay Bridge, San Francisco, Calif.

[20] Senior Designing Engr., San Francisco-Oakland Bay Bridge, San Francisco, Calif.

Before commenting on these problems, reference is made to Fig. 7 which, it is believed, neglects the fact that unless other changes in design are made, changing the truss stiffness would also affect the dead load, which, in turn, would affect the deflections. Assuming that the designers of the San Francisco-Oakland Bay Bridge considered an increase in rigidity necessary, the question arises as to the most economical means of securing such increase. For the present, only the methods of increasing the dead load of the suspended structure, increasing the moment of inertia of the stiffening trusses, or a combination of the two, are considered. To keep the dead load of the structure as low as practicable, light-weight aggregates were used for the 6-in. upper-deck paving slab, at an increased cost for the light-weight as compared to standard aggregates. The cost of the paving, as well as its supporting floor steel, could have been reduced by using standard aggregate. Still further savings in the floor construction could have been made by using a 9-in. paving slab with standard aggregate.

Assuming that the quarter-point deflection is a satisfactory index of rigidity, calculations have been made for the following cases: (I) Original design; (II) 50% increased I (no increase in dead load); (III) 50% increased I (additional weight of trusses and cables included); (IV) changing pavement slab to standard concrete (no increase in I); (V) changing paving slab to standard concrete (30% increased I); and (VI) changing paving slab to 9-in. standard concrete (no increase in I).

TABLE 3.—EFFECT OF MOMENT OF INERTIA ON RIGIDITY

Case	Dead load, in pounds per linear foot of cable	Area of cable, in square inches	Quarter-point moment, in foot-kips	Deflection, in feet	Decrease in deflection	Cost change	Cost per foot, decrease
I	9 200	500	110 000	10.10	....	....	....
II	9 200	500	142 500	8.92	1.18	....	....
III	10 000	530	137 600	8.52	1.58	$1 150 000	$730 000
IV	9 800	525	107 900	9.68	0.42	294 000	700 000
V	10 200	540	124 900	8.94	1.16	976 000	840 000
VI	11 100	570	100 000	9.03	1.07	978 000	910 000

The results for these cases, shown in Table 3, indicate that, for this particular structure, increasing the moment of inertia of the stiffening trusses is, in general, the most economical means of increasing rigidity. They do indicate, however, that there would be no economy in extra expenditures for an extremely light-weight floor and adding truss metal to regain the rigidity lost by decreasing the dead load.

Certain of the additional studies proposed by the authors lead to a specification for the design of stiffening trusses which, in the present stage of suspension-bridge design, is desirable. For this purpose, it is necessary to specify the intensity and distribution of live load, a measure of the required rigidity of the stiffening truss, and the permissible unit stresses. Considering only vehicular loadings, the probabilities of several of the loading conditions assumed in the calculations of maximum stresses in stiffening trusses are so remote that they may be safely discounted to a large extent. The most probable loading, for a bridge subjected to heavy traffic, is to have the roadway filled with passenger cars interspersed with comparatively few trucks and buses. With traffic having an unusually high percentage of trucks and buses, and assuming that the roadway is completely filled, bumper to bumper, the results per 100 vehicles would be approximately as follows:

80 passenger cars	@ 3 200 lb	= 256 000 lb	@ 16 ft	= 1 280 ft
5 buses	@ 16 000 lb	= 80 000 lb	@ 26 ft	= 130 ft
15 trucks	@ 20 000 lb	= 300 000 lb	@ 26 ft	= 390 ft
100 vehicles	@ 6 360 lb	= 636 000 lb	@ 18 ft	= 1 800 ft

With traffic moving slowly, the average space between cars would be about 12 ft, or a total of 30 ft per vehicle. With traffic at a standstill, a load of 35 lb per sq ft is possible. With traffic moving at slow speed, this reduces to 25 lb per sq ft.

It is recognized that traffic is not distributed uniformly and that there may be a sequence of heavy trucks. The probability of such a sequence on one part of the bridge with no loading on the other parts is small, but should be considered a possibility. The probability of such a critical (as used in stiffening truss analysis) loading on more than one lane is more remote. Unless deliberate efforts are made to achieve such a result, the writers do not believe there is any chance of a loading heavier than that of an H-20 truck in the lane adjacent to the stiffening truss and 250 lb per lin ft in the other lanes. As applied to customary practice, the foregoing rule would have the effect of increasing the design loads for two-lane bridges and decreasing those for wider bridges. For the Triborough Bridge, it would produce a live load of 1 500 lb per ft of truss in comparison with the 2 000-lb design load.

In the writers' opinion, the question of omitting the stiffening truss (see "Conclusion") should be reframed to read, "How flexible may the stiffening trusses safely be made?" A certain degree of vertical stiffening is always desirable to spread concentrations over several hangers; otherwise, severe distortion may occur at the stringer connections and at the cable bands. The case of a damaged hanger should be considered. Chords are always required for the wind truss and, as a minimum, the material required therefore should be disposed so as to afford as much vertical stiffening as practicable. Vertical stiffness is also desirable to dampen the vertical waves induced by wind gusts.

A suggestion toward a specification for the degree of vertical stiffening has been made by Sterling Johnston, M. Am. Soc. C. E.[21] The writers, however, suggest that the angular change at any panel or suspender point, rather than the change of grade, be considered the criterion for the required stiffness of the system. Further data are required as to the effects of angular changes on the cable.

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[21] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1210.

If a definite specification based on rigidity is established, there will be so many variables in any specific case (such as economical panel arrangement, back-stay conditions, dead loads, etc.), that each structure will be a separate study and no definite rules as to economic truss depths will cover the large range of possibilities.

The writers believe that efforts should be made to reduce the participation of the floor system in stiffening truss action as much as practicable. Usually, it is not economical to combine direct and bending stresses in any member. In most designs, floor expansion joints are introduced to reduce the floor participation. Such joints interrupt the continuity of the pavement and generally involve considerable maintenance. Efforts should be made to reduce the number of such joints to a minimum. Participation from vertical loads may be eliminated by keeping the plane of the stringers at the neutral axis of the truss.

On the basis of the usual calculations, stresses in stiffening trusses are complete reversals. Under these conditions, W. M. Wilson, M. Am. Soc. C. E., has shown[22] that the usual alloy steels have strengths no greater than carbon steel; however, the loadings that produce these stresses are of rare occurrence. Silicon steel costs only slightly more than carbon steel, and for occasional over-stresses, should have an increased strength in the ratio of their yield points. It is believed, therefore, that silicon steel should be generally used for the chords of stiffening trusses.

The authors have furnished a method whereby comparative studies of any variable can be quickly made. It is better engineering to make these comparisons than to give any direct statement as to the economies of light-weight floors or other detail problems. As long as the designer considers the rigidity furnished in both vertical and horizontal planes by the dead load, the solution can be reached quickly.

The writers trust that this paper will become the basis for a specification for the design of stiffening trusses for vehicular suspension bridges.

HARDY CROSS,[23] M. AM. SOC. C. E. (by letter).—This paper is an important contribution both to the literature of suspension bridges and to the general theory of structural design. That it makes possible an immediate picture of the action of the truss and of the effect of variation of proportions of the structure is a rare virtue.

A case having important points of similarity to that discussed in the paper is presented by two parallel beams so connected that they deflect together. Assume that the dimensions of one of the beams (represented by the cable) is fixed and that its strength is of no further interest. Assume, also, that the second beam (stiffening truss) is added merely to stiffen the primary beam, whatever that may mean, and that structural failure of the secondary beam is to be feared only because it will then fail to stiffen properly. The shape of the deflection curve for a loaded beam and for a loaded cable are not alike; this does not destroy the usefulness of the comparison for present purposes.

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[22] Rept. on Fatigue Tests of Riveted Joints, by Wilbur M. Wilson, presented at the meeting of the Structural Division, New York, N. Y., January 20, 1938. (Not published.)

[23] Prof., Civ. Eng., Yale Univ., New Haven, Conn.

The secondary beam may produce two effects: It may change the shape of the deflection curve; or, it may change the amount of the deflection. The authors present some evidence that the first effect is not important.

If the deflection of the primary beam is fixed, the angular distortions of the secondary beam are also fixed. Linear strains are then a function of depth and stresses are related to strains by the elastic modulus. Such a condition is represented in suspension bridges by cable stretch, expansion of cable, and movement of tower connections of cable due to the deformation of shore spans. These "deformation" stresses have no useful effect; they merely threaten the structural integrity of the secondary beam.

If the load on the combination is fixed, the presence of the secondary beam affects the moments on the primary beam and so affects the deflections. If the secondary beam is very flexible compared with the primary beam, this change in deflection is small, and the stresses vary as do the deformation stresses. Such a case is represented by such heavy bridges as the George Washington Bridge.

If the secondary beam is very stiff it will carry practically all the moment, and the deflection of the system under load will be primarily a function of the stiffness of the secondary beam. This is true of light suspension bridges; for these the elastic theory gives close approximations.

In most bridges the stress in the stiffening truss due to live load does not vary inversely as its strength. The truss is relatively inefficient as a stiffening member. The structure is "hybrid," acting partly as indicated by the elastic theory of suspension bridges where the moment to be resisted is fixed and partly as if its stresses due to load were of the nature of deformation stresses where the deflection produced is fixed.

One of the most important results of the paper may prove to be that more attention will be given to the question of the function of the stiffening truss. Stiffness may be discussed in terms of angular deformation, of deflection, of grade change. It may also be discussed in dynamic terms, and discussion of angular deformation, or of deflection, or of grade change, is not a completely adequate basis for estimating vibration characteristics. Probably stiffness needs discussion in all these terms. Engineers know that they want to stiffen the bridge, but they have not yet satisfactorily defined stiffness.

Valuable studies in the design of stiffening trusses, some of which are correlated to the thesis presented by the paper, have been made by Jacob Karol.[24]

A. A. EREMIN,[25] ASSOC. M. AM. SOC. C. E. (by letter).—In this paper the authors have developed a simple method of computing stresses in the preliminary design of suspension bridges. However, it has various limitations which should be considered by designers.

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[24] "A Partial Influence Line Procedure for Suspension Bridge Analysis by the Deflection Theory," by Jacob Karol. Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering in the Graduate School of the University of Illinois, 1938.

[25] Assoc. Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

The error in computing stresses in stiffening trusses increases rapidly with a decrease in the span length of the bridge. From Table 1 it is evident that the error in computing the bending moment in the stiffening truss of the San Francisco-Oakland Bay Bridge by the preliminary method is about 1.77 per cent. The span length of the Bay Bridge is 2 294 ft. The error in computing the bending moment in the stiffening truss of the Maumee River Bridge, with its span length of 777.6 ft, is 7 per cent. The increased error in computing stresses in short-span suspension bridges is caused by the greater influence of approximate assumptions used in computing deformations of stiffening trusses in short-span bridges.

Equations (3) to (9), involving expressions for ordinates of suspension cables loaded with uniformly distributed dead and live loadings, were developed by the authors from equations in statics. Similar equations were developed in a slightly different manner in 1917.[26] It is to be regretted that, in computing the deformation of suspension cables, the authors considered only uniformly distributed loading. However, the maximum stresses in stiffening trusses of suspension bridges with moderate span length occur for a combined loading uniformly distributed, and concentrated forces at various points along the span length. Evidently, the computation of deflections and the deflection loops of suspension cables sustaining the combined loading is complicated and impracticable considering the simplicity of other available approximate methods of designing suspension bridges. The authors already have demonstrated the simplicity and other advantages of the design methods based on trigonometric functions. Approximate analyses of stresses in suspension bridges with moderate span lengths may also easily be made with model test, as shown by Mr. E. Rothenberg.[27] Likewise, an interesting preliminary study of the Bay Bridge was performed by the use of a model.[28] In the model tests various combinations of uniformly distributed and concentrated loading may be considered. Furthermore, the stiffening truss may be considered either with uniform or variable moment of inertia along the span length.

A. FRASER ROSE,[29] M. AM. SOC. C. E., AND WILLIAM A. ROSE,[30] ASSOC. M. AM. SOC. C. E. (by letter).—This timely paper is quite in line with the modern trend in structural analysis and design—simplification through successive approximations—and is a worthy addition in this field to the method of moment distribution.[31] The proposed method is developed in such a manner that it possesses the added advantage of affording the designer a clear mental picture of the contributions to the final results made by the various factors.

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[26] "Modern Framed Structures," by the late J. B. Johnson, and C. W. Bryan, Members, Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Pt. 2, 1917 Edition, pp. 203–208.

[27] *Bautechnik,* Heft 53, 1929, p. 844.

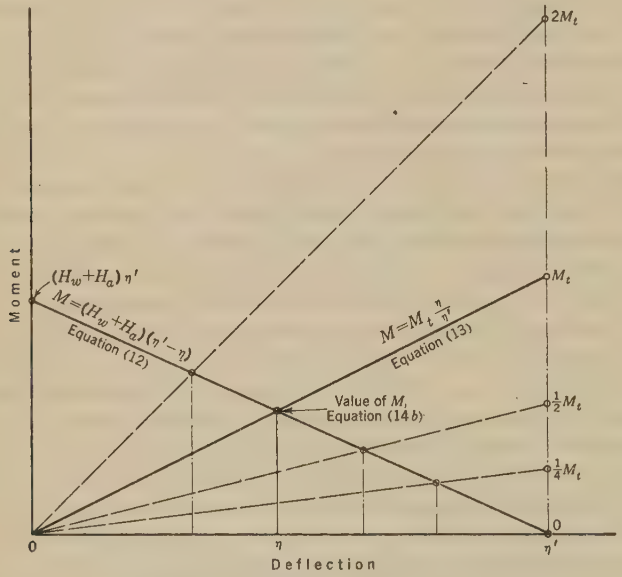
[28] "Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge," by G. E. Beggs and R. E. Davis, Members, Am. Soc. C. E., and H. E. Davis, Jun. Am. Soc. C. E., *Publications in Engineering,* Univ. of California, Vol. 3, No. 2, pp. 59–166, 1933.

[29] Cons. Engr., New York, N. Y.

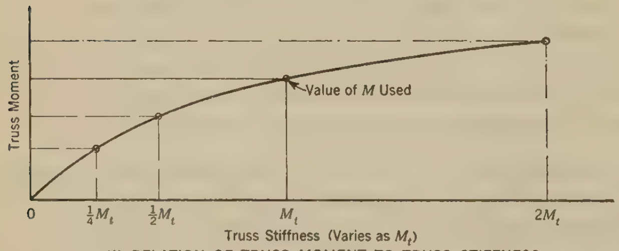
[30] Asst. Prof. of Structural Eng., New York Univ., New York, N. Y.

[31] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 96 (1932), p. 1.

The great difficulty in analyzing a structure of the nature of the stiffened suspension bridge is that superposition of results is not permissible, inasmuch as they are inter-related. The authors have overcome this obstacle by the clever device of considering the contributing factors in their relative order of importance, and subsequently correcting for the errors introduced by trial superpositions.



(*a*) RELATION OF MOMENT AND DEFLECTION FOR VARIOUS TRUSS STIFFNESSES



(*b*) RELATION OF TRUSS MOMENT TO TRUSS STIFFNESS

FIG. 14.—EFFECT OF TRUSS STIFFNESS ON MOMENT, M, AND DEFLECTION, η (NEGLECTING CABLE STRETCH, ETC.)

As cited in the paper, Hardy Cross, M. Am. Soc. C. E., has proposed a logical classification of structures[4] which tends to clarify the relation of analysis to design. Four types of structural actions are emphasized: (1) The action which produces deformation stresses; (2) that which produces participation stresses; (3) normal structural action; and (4) hybrid structural action. The proposed method clearly indicates that the stiffened suspension bridge exhibits characteristics of all four of these types of action, but that it is primarily a hybrid structure. As the authors point out, there are any number of possible designs for the stiffening truss in a given instance. Any change in truss stiffness is reflected in a change in the moment resisted by the truss, a typical hybrid action. Where hybrid action exists, preliminary designs for comparative study are profitable and enlightening.

The use of exact, or relatively exact, methods of analysis for preliminary studies is not justified. The small added degree of accuracy attained consumes an unwarranted length of time and serves no useful purpose. The importance of developing, on rational bases, approximate methods for rapid analysis in such cases cannot be over-estimated. Their availability is conducive to a proper study of the effects on the analysis of the variation of the several factors which enter into the design. In this way, the engineer is enabled to secure the maximum economy consistent with stability.

The writers agree with the obvious conclusion of the authors that the determination of the maximum shears is not essential for preliminary design purposes. One finds in designing the web members of the stiffening trusses that minimum section rather than stress governs the areas in many cases. Even in the final design, maximum shears are needed at only a few points; for the preliminary work, sufficient information may be obtained from the maximum moments at critical sections.

A number of interesting conclusions may be reached after careful inspection of the equations that enter into the calculation of the moments at a critical section. For example, consider Equations (14*b*), (24*a*), (37*a*), (40*b*), and (42) in their relation to the determination of maximum moments at the quarter-point of the main span.

The final total moment at the quarter-point is the sum of the values obtained from Equations (14*b*) and (37*a*). In the first of these values, it is evident that an increase in M_t, which from Equation (24*a*) is proportional to I, results in a less than proportional increase in M, due to the fact that M_t appears in the denominator as well as in the numerator of the expression on the right-hand side. This is clearly shown in Fig. 14, in which the final moment is the ordinate of the point of intersection of the lines, $M = (H_w + H_a)(n' - n)$, and $M = M_t \frac{n}{n'}$ (Equations (12) and (13)).

Fig. 14 and Equation (14*b*) emphasize the existence of hybrid action. Hybrid action is further shown by inspection of the factors that contribute to the change in sag, df. From Equation (37*a*), it might hastily be concluded that df should vary inversely with I. This conclusion tacitly assumes that M would remain fairly constant. Careful study reveals that M, due to this cause, increases almost as rapidly as I, so that the decrease in df is considerably less than proportional.

Three factors contribute to df. Temperature change contributes a constant deformation in the cable and a constant change, df, in sag, regardless of truss stiffness, except for a small correction due to the additional stress in the cable caused by the temperature change. This action by itself then comes under the first of Professor Cross' classification—action which produces deformation stresses.

The second factor is the change in cable stress from that under dead load at mean temperature, caused by the addition of live load; this contributes almost a constant amount to *df* except for small changes due to its own action.

The third factor is the element termed side-span interaction. The contribution to *df* from this cause is by no means constant, but is affected by any change in side-span stiffness. This is brought out in Equations (40*b*) and (42), and in Table 2. Side-span interaction comes under the second of Professor Cross' classifications—action which produces participation stresses. It should be noted that maximum negative moment in the side span itself is primarily participation moment.

The preceding discussion on the change in sag is necessarily somewhat general because the three factors really merge into one another in a simultaneous action. Nevertheless, the proposed preliminary design method does give an idea of the related action of the various factors, a picture which is lost in mathematical formulas in the more exact methods.

The authors are to be congratulated upon their choice of subject and their manner of presentation. Considerable effort has apparently been made to keep the percentage error in the various approximations consistent, so that needless computation is avoided. The illustrative material has been carefully chosen to cover a wide range of cases. Obviously, the method may be extended for the computation of maximum moments at sections other than those given, should such extension be necessary or desirable in a given case.

LEON S. MOISSEIFF,[32] M. AM. SOC. C. E. (by letter).—The literature on suspension bridges has received an interesting contribution in this paper. A clear understanding of the distortional behavior of suspension bridges, and of the functioning of stiffening trusses, is not as common among engineers as it is generally assumed to be. The facts that, with the exception of suspension bridges, all other bridges may be treated as rigid structures and that suspension bridges are relatively small in number, rightly lead to directing the attention of engineering students to rigid bridges. The distortions of rigid bridges under live load, wind, and temperature are small enough not to affect the primary stresses of their frames. This does not hold true for suspension bridges. In these bridges the distortion of the structure under live load and other outside causes exerts a marked and determining effect on the stresses in the various members of the combined structure. Suspension bridges, therefore, require a different analytical approach than rigid bridges and, as a consequence, the considerations covering their design are also different.

The authors' approach to the problem, through a study of the behavior of an unstiffened cable, is efficient for the purpose of a qualitative appreciation of the functioning of suspension bridges. Such studies were made a century ago by Navier, Rebhan, and others. That these studies dealt with the unstiffened cable was an outcome of the fact that, in those days, suspension bridges were not stiffened systematically. For the light wagon loads of those times a braced railing was considered sufficient. These studies contained much informative material, and offered a good insight into the general behavior of suspension bridges.

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[32] Cons. Engr., New York, N. Y.

After experience had shown that light, unstiffened suspension bridges were neither comfortable for travel nor safe against wind, engineers began to stiffen them. For the short-span bridges built in the early times the attachment of any sizable stiffening girder supplied more than sufficient stiffness. In fact, the girders were so stiff that it appeared to be logical to assume the beam as perfectly rigid. A simple static solution then became possible and was readily established. It formed the basis of the well-known Rankine-Ritter theory. This was the suspension bridge theory taught in engineering schools fifty years ago, which assumed a rigid truss suspended from the cables. The assumption of absolute rigidity of the stiffening truss tacitly involved the unchanged form of the original curve of the cable, and precluded the recognition of the deformations of the structure. The effect of the "dead," or original, load on a suspension bridge could not be considered. The Rankine-Ritter theory ignores it. To this statement should be added that at that period of engineering the elastic behavior of bridges had not as yet come into practical application. In general, bridges were small, relatively stiff, and the elasticity of the members could equally be ignored.

With the greater use of metallic bridges the effect of temperature variation was recognized in the course of time, and a correction was introduced in the stiffening-truss formulas to allow for the change in length of cable due to change in temperature. Soon, thereafter, the elongation of the cables due to change of load also received recognition, and the correction was extended to provide for it.

By that time the study of the elastic behavior of metallic structures had attained a high development and was extended to suspension bridges. The elastic theory, which was then developed, took account of the elasticity of the various parts forming a suspension bridge and forgot the effect of the dead or original load of the bridge. The deflection theory takes this effect into consideration as its basic idea. It has proved that the effect of the dead load or, more precisely, the pull in the cable due to it, is of serious importance and sometimes completely controls the design and behavior of the bridge. The elastic theory has outlived its usefulness as a tool for designing suspension bridges.

The authors are aware of these developments, and they approach the problem with an educated understanding. Step by step, consistently adhering to the function of the stiffening truss and its conditioned behavior in the bridge, the authors approach an intelligent approximation of the more accurate solution. Taking into account the results of the computations of some modern suspension bridges, they make empirical approximations adjusted to these results and arrive at a procedure for a preliminary design.

The comparisons, given in Table 1, of the results obtained by the preliminary method as against those obtained by the deflection theory, are close and satisfactory for the purpose. The range of the data, however, has too narrow a compass.

In the Golden Gate Bridge, in San Francisco, Calif., with a main span of 4 200 ft, dead load of 21 000 lb per lin ft, and live load of 4 000 lb, the greatest positive moment is found at 0.126 of the span length from the tower, and it is 10% greater than at the quarter-point. Attention is also called to the fact that the stress due to live load in the Golden Gate Bridge is only 30% of the total stress and that the remaining 70% is due to the transverse wind.

In the Whitestone Bridge, in New York, N. Y., with a span of 2 300 ft, dead load of 11 000 lb per lin ft, and live load of 3 000 lb, the moment at the quarter-point, by the authors' approximate method, is 5 172 kip-ft, and by the deflection theory, it is 5 000 kip-ft, which is very close; but, again, this moment is not the maximum moment. The maximum moment occurs at one-tenth the span length from the tower, and amounts to 6 360 kip-ft. This is 23% greater than at the quarter-point. Evidently, the assumption that the maximum moment is at the quarter-point in all bridges is incorrect, and the method proposed should be handled with care.

The writer disagrees with the authors that the steps involved at arriving at results by the approximate method will take less time than by using the deflection theory procedure directly. After all, it depends entirely on one's familiarity with this procedure. That a computation machine should be used for rapid and accurate results is true; but an engineering office engaged in designing suspension bridges is supposed to be so equipped.

O. H. AMMANN,[33] M. AM. SOC. C. E. (by letter).—The authors have made a useful contribution to the design of stiffened suspension bridges. The principal merit of their method of analysis lies in the fact that it aids the student more clearly to visualize the behavior of the unstiffened cables and the effect thereon of the stiffening trusses and other factors, and to evaluate the economic value of stiffening trusses of various degrees of rigidity.

The paper does not attempt to go beyond the mathematical analysis as an aid to preliminary design studies; but in their conclusions the authors indicate that there are a number of problems in the design of stiffening trusses which require clarification. They may be summarized in the two important questions: What is the appropriate degree of rigidity? and how is it to be obtained structurally?

Judging from the widely varying degrees of rigidity of stiffening trusses, even in recently built suspension bridges, it would seem, indeed, that these questions offer a fruitful field for further study with a view to establishing a more uniform, and possibly improved, practice. However, a comprehensive study of the problem leads to the conclusion that both the criteria for rigidity and the factors involved in the design of an adequate and economical stiffening system are so complex, and to some extent necessarily matters of individual judgment, that an attempt to devise formulas or rules of design on a theoretical basis appears fruitless.

The problem is peculiar to suspension bridges because, contrary to its effect in truss types, rigidity is secured by increased expense and must be kept to a minimum. This, in fact, becomes one of the major economic factors in suspension bridges of long span.

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[33] Director of Eng., The Port of New York Authority, and Chf. Engr., Triborough Bridge Authority, New York, N. Y.

The very object of rigidity in a bridge is complex. In smaller and lighter structures the principal object is rigidity against vibrations or oscillations which may be detrimental to the structure as well as objectionable to traffic. No valid formula has ever been devised on theoretical grounds as a measure for the required degree of rigidity in this respect. Experience has set up very crude and purely empirical criteria in the form of limiting deflections.

As the bridge increases in span and weight this object becomes less and less important, except in the design of the floor system. In fact, flexibility of the structure as a whole becomes advantageous, because it tends to reduce local impact effects and vibrations. The principal purpose of limiting deflections and other deformations of the main carrying system in long spans is to avoid excessive gradients, and to prevent deformations that may result in excessive local bending, distortions, or transverse tilting of the floor structure.

Limiting grades may be important in bridges carrying rail traffic, but for modern highway traffic even that object loses significance, because the grade changes that may be actually produced in a well designed structure are not sufficient to form a serious impediment to traffic.

The effect of the deflections upon the floor depends so much upon the width, depth, and general arrangement of the floor structure as to require careful study in each individual case.

The authors state that present tendencies are to make stiffening trusses more flexible. This is undoubtedly due to the recognition of the fact that with increasing magnitude of bridges, and particularly those serving modern highway traffic, rigidity, as measured by deflections of the structure as a whole, ceases to be a virtue, and that flexibility offers material economic advantages, besides aiding the designer to produce more graceful structures.

This recognition is due partly to the abandonment of the so-called elastic theory which ignored the influence of the dead weight of the structure upon the proportioning of the stiffening system and thus, with increasing magnitude of span, led to more erroneous and wasteful design. The general rule, found in many former treatises and textbooks on suspension bridges—that the most appropriate depth of stiffening trusses, irrespective of span length, character of moving load, or dead weight of suspended structure, was between $\frac{1}{40}$ and $\frac{1}{60}$ of the span—was based upon that erroneous theory.

The writer became aware of the economic significance of flexible trusses in his preliminary studies for the George Washington Bridge. In the attempt to determine the proper proportions of the stiffening trusses for a span of such unprecedented length, he resorted to a careful study by analysis, as well as by model, of the behavior of the unstiffened cables, in respect to deflections, grade changes, and angular distortions under various load conditions.

These studies convinced him that this bridge, which was to be designed principally for highway traffic with the possibility of later addition of suburban transit trains, would have ample rigidity to accommodate highway traffic without any stiffening system whatsoever—that is, without any restraint upon grade changes and deflections of the unstiffened cables—but that, even for rapid transit trains, although the unstiffened system as a whole would be amply rigid in so far as deflections and grade changes are concerned, comparatively flexible trusses between the two decks, with a depth ratio of only $\frac{1}{120}$, were advisable to secure local rigidity and avoid excessive local distortions of the floor.

The bridge was actually designed in accordance with these conclusions, and to date (1938) it has only a single completely unstiffened deck. This was a radical departure from conventional practice,[34] but experience has corroborated the validity of the conclusions.

The Golden Gate Bridge, at San Francisco, Calif., which has since been completed with a span of 4 200 ft and carries only highway traffic on a single deck, was designed with trusses 25 ft deep, which is only $\frac{1}{168}$ of the center span. Thus far, it also has proved amply rigid. It is the writer's present opinion that even shallower trusses or solid plate girders would be just as adequate for this great span.

A further radical step in the direction of greater flexibility of stiffening trusses was taken in the design of the Bronx-Whitestone Bridge across the East River in New York, N. Y. It has a center span of 2 300 ft, side spans of 735 ft, and is designed to carry only highway traffic on a single deck 75 ft wide. It is stiffened by plate girders of a depth of 11 ft, which is only $\frac{1}{210}$ of the span length. The moment of inertia of the girders is only 2 600 in.^{2}–ft^2 in the center span, and 2 140 in the side spans. These values are practically the same as the corresponding ones of the stiffening girders (10 ft deep) used in the Maumee River Bridge in Toledo, Ohio, which has a center span of only 778 ft, and side spans of 230 ft.

The Bronx-Whitestone Bridge still requires the test of actual operation. If it proves to be amply rigid, as the writer does not doubt, it should set a precedent for spans of that magnitude and similar proportions. It should also indicate that some of the bridges of shorter span might well have been designed with shallower, more economical, stiffening trusses or girders, particularly such structures in which the deflections of the unstiffened cables are reasonably restrained by the compound effect of other influencing factors, such as relatively wide and heavy floor structure, shallow cable sag, and short side spans.

Suspension bridges for heavy modern highway traffic are a comparatively recent development. They will never be common structures and each case will call for and merit most careful study. In view of the complexity of the factors which influence the design of the stiffening system, the trial method of approaching a final design, together with judgment based upon experience, will be found as serviceable as and less likely to lead to erroneous practice than "cut and dried" formulas or rules of design.

LEON BLOG,[35] ASSOC. M. AM. SOC. C. E. (by letter).—A method for the preliminary design of two-hinged stiffening trusses of cable suspension bridges is presented in this paper. The object of developing a preliminary method for proportioning any structure is the rapid consummation of the final design. The use of the method must assure the designer that the proportions selected for the application of the preliminary method will result in the ideal design when used in a more accurate method. The preliminary method presented by the authors enables the designer to arrive, rapidly and understandingly, at deflections and moments, thereby yielding an idea as to the combined stiffness of the cable and stiffening truss, the effect of the live load on the total cable tension, and the unit stresses in the truss selected.

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[34] *Transactions*, Am. Soc. C. E., Vol. 97 (1933), p. 43.

[35] Asst. Chf. Structural Engr., Div. of Bridges and Structures, City of Los Angeles, Los Angeles, Calif.

The writer has investigated the possibility of further reducing the total time for the preliminary design required by ignoring the width of the top of the tower. This tower width is a factor in the authors' method and, of necessity, considerable time must be spent in investigating for a rational width of tower top in order to arrive at L', the short cable section over the saddle. The horizontal component of the live load tension as well as that of temperature is a function of $I \div I_1$. The horizontal components are first converted into tensions along the cable and these tensions, in turn, into vertical components acting on the tower. The tower can then be designed. Thus, the final tower design cannot proceed without satisfactory stiffening trusses. If a suitable method of determining such trusses can be developed independently of a predetermined tower-top width, considerable time will be saved.

Accordingly, the writer has explored the degree of accuracy to be obtained by using the authors' method with all steps involving L' deleted from their solutions. To do this, the cable curves were produced backward to the center lines of the towers, new sags, cable lengths, and horizontal spans were computed and then used in the authors' solutions with Operations 6(*c*), 7(*c*), and 8(*c*), in Table 2(*a*), eliminated.

Design constants in Table 1(*a*) are compared with the results of the writer's simplification in Table 4.

TABLE 4.—COMPARISON OF DESIGN CONSTANTS

Method	Length of Span, in Feet		Length of Cable, in Feet		Sag at Center of Span, in Feet	
	Main span, l	Side span, l_1	Main span, L	Side span, L_1	Main span, f	Side span, f_1
Table 1(*a*), Column (2).......	1 358	660.7	1 394.00	685	133.70	31.600
Writer's simplification........	1 380	671.7	1 416.82	696	138.07	32.337

The authors have observed that the horizontal projection of L' practically equals the true value of L' which is supported by noting that L = 1 416.82 ft = 1 394 + 22.82 ft, the additional 0.82 ft arising from the fact that the main and side-span cables intersect at the center line of the tower in a cusp, the sum of whose sides is greater than L' = 22 ft. Note that the side-span cable has been increased 11 ft to 696 ft, from 685 ft. In short, the total length of the bridge cable is practically unchanged. Dropping the 0.82 ft and using

1 416.00 ft did not affect the writer's solutions. The writer's values in Table 4 were used in solving Tables 2(*a*), 2(*b*), 2(*c*), and 2(*e*). The answers obtained by the authors and by the writer's simplification of their method are compared in Table 5.

The writer did not solve the cases of Tables 2(*d*) and 2(*f*) because no step-by-step check was available. However, the nature of the computations in the authors' method indicates that results close to those of the authors for these unsolved cases would result by using the writer's simplification.

Table 5 shows that the maximum deviation for moment for any case studied was about 2% less than the authors' result and occurs in Table 2(*b*). Assuming that the stiffening trusses being investigated are actually working up to a maximum allowable fiber stress of 18 000 lb per sq in., the writer's result yields 0.02 × 18 000 = 360 lb per sq in. less, or the stress in the chords would be 17 640 lb per sq in. Inspection of Table 5 shows that the variations for other cases in Table 2 are not large. Since the essence of the simplification being demonstrated is the use of longer stiffening trusses than those proposed, with the incidental greater loads, there is never any danger of adopting trusses that will be too weak.

TABLE 5.—COMPARISON OF MAXIMUM MOMENTS AND CORRESPONDING DEFLECTIONS

Item No.	Method	QUARTER-POINT, MAIN SPAN* (TABLE 2(*a*))		CENTER, MAIN SPAN* (TABLE 2(*b*))		QUARTER-POINT, MAIN SPAN† (TABLE 2(*c*))		CENTER, SIDE SPAN* (TABLE 2(*e*))	
		Moments, in foot-kips	Deflection, in feet	Moments, in foot-kips	Deflection, in feet	Moments, in foot-kips	Deflection, in feet	Moments, in foot-kips	Deflection, in feet
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Authors'.....	28 500	4.58	23 600	4.90	23 300	2.93	43 400	3.98
2	Writer's......	28 545	4.68	23 200	4.98	23 160	3.08	43 700	4.13
3	Ratio‡.......	1.001	1.021	0.983	1.017	0.995	1.050	1.007	1.037

* Maximum positive moments; deflection is down. † Maximum negative moment; deflection is up. ‡ $\frac{\text{Item No. 2}}{\text{Item No. 1}}$.

Table 5, Column (9), further shows that the maximum deviation between the two methods for any downward deflection is plus 4% and equals about 2 in.; and, Table 5, Column (7), shows the deviation for the upward deflection to be 5% and also equals about 2 in. Even for so stiff a structure as the modern Triborough Bridge (New York City), a 2-in. variation in accuracy between two preliminary methods is not serious.

The results obtained by the writer's simplification are evidently similar to those of the authors. No preliminary tower design is required. Time is saved. One more trenchant tool has been added to the engineer's kit by the authors. Once the underlying formulas are understood and accepted, only the simplest mathematics is required. Visualization of the otherwise intricate interaction of all parts of the hybrid assembly has been made easy.

"Preliminary Design of Suspension Bridges" will take its place beside "Analysis of Continuous Frames by Distributing Fixed-End Moments"[36] as a drudgery reducer in structural analysis.

SHORTRIDGE HARDESTY,[37] AND HAROLD E. WESSMAN,[38] MEMBERS, AM. SOC. C. E. (by letter).—The writers greatly appreciate the comments made by those who have discussed this paper. Mr. Ammann emphasizes the major significance of the method when he notes, first, its value as an aid in visualizing the behavior of the unstiffened cables and the effect thereon of the stiffening trusses and other factors; and, secondly, its merit in appraising the economic value of stiffening trusses of various degrees of rigidity. In fact, its value in a variety of economic studies is demonstrated clearly by Messrs. Woodruff and Raab in their interesting discussion.

In the preliminary studies which are necessary for any suspension bridge, it is not only important to have a method which will give approximate maximum moments and deflections quickly at one or two points, so that trusses of different rigidity may be compared, or that other variables may be investigated; but it is even more important that the method permit and facilitate an understanding of the action of all parts of the structure. Here is where the deflection theory, championed by Mr. Moisseiff, displays its shortcomings, for the very nature of the exponential formulas, as is evidenced by even a casual inspection, is such that the related functioning of truss and cable, side span and main span, is completely disguised. In the application of these formulas to the determination of stiffening truss moments, no scale is obtained regarding the relative contributions of each factor to the total moment. In preliminary studies, at least, it is important to know how variations in one factor or another will affect final results, in order to obtain an economical design. In commenting about the preliminary design method, Mr. Blog states: "Visualization of the otherwise intricate interaction of all parts of the hybrid assembly has been made easy."

The important studies made by Mr. Karol again emphasize this. Figs. 8, 9, and 10 give an excellent break-down of the contributions of various factors to the moment and deflection at the quarter-point of the main span of the Triborough Bridge.

It is important to note that the writers emphasize the method in its relation to preliminary design studies. It is the preliminary study of any project which calls for the highest degree of engineering judgment, talent, and experience. The final design and its analysis are usually of a routine nature. They demand technical skill, it is true, for their consummation, but they rarely result in major changes in conclusions already reached as a result of thorough preliminary studies. Mr. Fischer, apparently, does not recognize a distinction between, nor the relative importance of, preliminary and final design. Which is more important—making the decision that the stiffening truss of the Bronx-Whitestone Bridge shall be an unprecedented shallow plate girder only 11 ft deep $\left(\frac{1}{210}\text{th of the span length}\right)$—or computing the final curve of maximum moments for the girder as selected?

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[36] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[37] Cons. Engr. (Waddell & Hardesty), New York, N. Y.

[38] Prof., Structural Eng., Coll. of Eng., New York Univ., New York, N. Y.

In making the analysis of the final design, the deflection theory in one form or another should be used. Mr. Moisseiff prefers the deflection theory expressed in exponential formulas, which require a computing machine for accurate results. The writers prefer the deflection theory expressed in terms of a trigonometric series, which gives just as accurate results with an ordinary slide-rule. That is a matter of personal preference.

Mr. Fischer believes that the elastic theory can be used for spans less than 750 ft long. The writers see no merit in using a theory of analysis which is definitely erroneous. They heartily agree with Mr. Moisseiff when he states, "The elastic theory has outlived its usefulness as a tool for designing suspension bridges." Moreover, any preliminary design method based on the elastic theory may lead to distinctly erroneous conclusions, when one realizes that there are a thousand possible stiffening trusses of various rigidities for any suspension bridge. The elastic theory, which ignores cable deflections and dead-load effects, if applied to the Bronx-Whitestone stiffening girders as designed, is 1 300% in error. Other stiffening girders or trusses might have been used in which the error, by using the elastic theory, would be less; but they would be stiffer—and more expensive. A preliminary design method that does not disclose the true structural action is of little value to the engineer who must make his choice primarily in terms of rigidity and economy.

In making economic studies, diagrams, such as Figs. 11 and 12 plotted by Mr. Karol in applying the writers' method, are a distinct help in getting the answer to the following question raised in the paper:

"Is it more economical to obtain stiffness by increasing the chord section or by increasing the depth of the truss?"

In a simple truss, increasing the chord area lowers unit stress. Increasing the depth of truss also lowers stress. On the other hand, if the depth of a stiffening truss is increased, the unit stress is actually increased. If unit stresses are too high, decreasing the depth and increasing the chord area will bring them down; but, at the same time, the effect upon rigidity must be kept in mind.

Messrs. Woodruff and Raab have presented an interesting summary in Table 3 showing the effect of moment of inertia on rigidity, associated with changes in dead load of floor slab and dead load of trusses and cable. It is desirable to keep dead load down in order to save on cable and tower steel. Using a light-weight deck will help to do this, but the light-weight deck and the supporting floor steel may cost more than a standard concrete slab and its framing. Will the saving in one item offset the increase in another item?

Moreover, if a definite rigidity (whatever that may mean) is to be attained, higher dead load is of help in reducing deflections, but it may be cheaper to use a stiffer and more costly stiffening truss to get the same result, in order to save on cables and towers.

Mr. Eremin states that there are various limitations to the writers' method which should be considered by designers, but he fails to point out any limitations of significance.

The writers are indebted to Mr. Moisseiff for calling their attention to the fact that the point of maximum positive moment in the Bronx-Whitestone Bridge is at a point one-tenth of the span length from the tower. The moment at that point (6 360 ft-kips) has been verified by the writers, using the trigonometric series method. On the Golden Gate Bridge, the point of maximum moment is near the eighth-point. It is an interesting fact that, on these two large spans with unusually flexible stiffening units, the point of maximum moment does not occur at or near the quarter-point. That does not nullify the value of the writers' method as an aid in preliminary design studies. As stated by Messrs. Rose and Rose, the method may be extended for the computation of maximum moment at sections other than those given. It is questionable, however, whether this is worth doing for preliminary studies, although the point is one that warrants additional investigation. In making initial designs, it is advisable to correlate stiffening truss moment at some section with deflections at the same section, in order to get a scale on rigidity, as indicated by Fig. 7.

Fig. 15 has been plotted to show how the unstiffened cable deflects under an advancing uniform live load with the cable length kept constant. Note that the maximum deflection occurs almost at the quarter-point with live load extending over a distance of $0.4\,l$ into the main span. A load extending about $0.2\,l$ into the main span causes a loop with maximum deflection near the eighth-point, but the deflection is considerably less than the maximum at the quarter-point. When cable-stretch effects are included, the difference between the two becomes even greater. It was because of this, and also because maximum positive moments on a large number of spans already designed occurred at or near the quarter-point, that the writers developed the preliminary design method primarily in terms of quarter-point effects.

Fig. 15 indicates the possibility of further interesting studies on suspension bridges. It is not the deflection, but the curvature of the curve of deflection of the cable which is important in evaluating stiffening truss moments. The stiffening truss must have the same curvature as the curve of deflection of the cable, neglecting suspender deformations.

The differential equation of curvature for the cable given in Equation (21c) may be restated, as follows:

$$\frac{d^2\eta}{dx^2} = -\frac{w_s - \beta\,w}{H_a + H_w} \qquad (50)$$

The differential equation of curvature for the stiffening truss is:

$$\frac{d^2\eta}{dx^2} = -\frac{M}{E\,I} \qquad (22)$$

In the unstiffened cable, an inspection of Equation (50) indicates that the curvature of the curve of deflection of the cable, with cable-stretch effects omitted, may be slightly greater at the eighth-point than at the quarter-point. It is dependent somewhat on the ratio of live to dead load. The denominator decreases if H_a (live load horizontal component of cable stress) becomes less. The numerator will increase, because $\beta\,w$ decreases.

When a flexible stiffening truss, such as that of the Bronx-Whitestone Bridge, is added to the cable, the stiffening effect is somewhat localized in nature. The moment corresponding to Step I of the preliminary design method is then greater at the eighth-point than at the quarter-point. The quantity $w_s - \beta w$ (which is the suspender pull minus a fractional part of dead load) is directly related to the moment, as evidenced by Equations (50) and (22). A study of Fig. 6 clarifies this relationship.

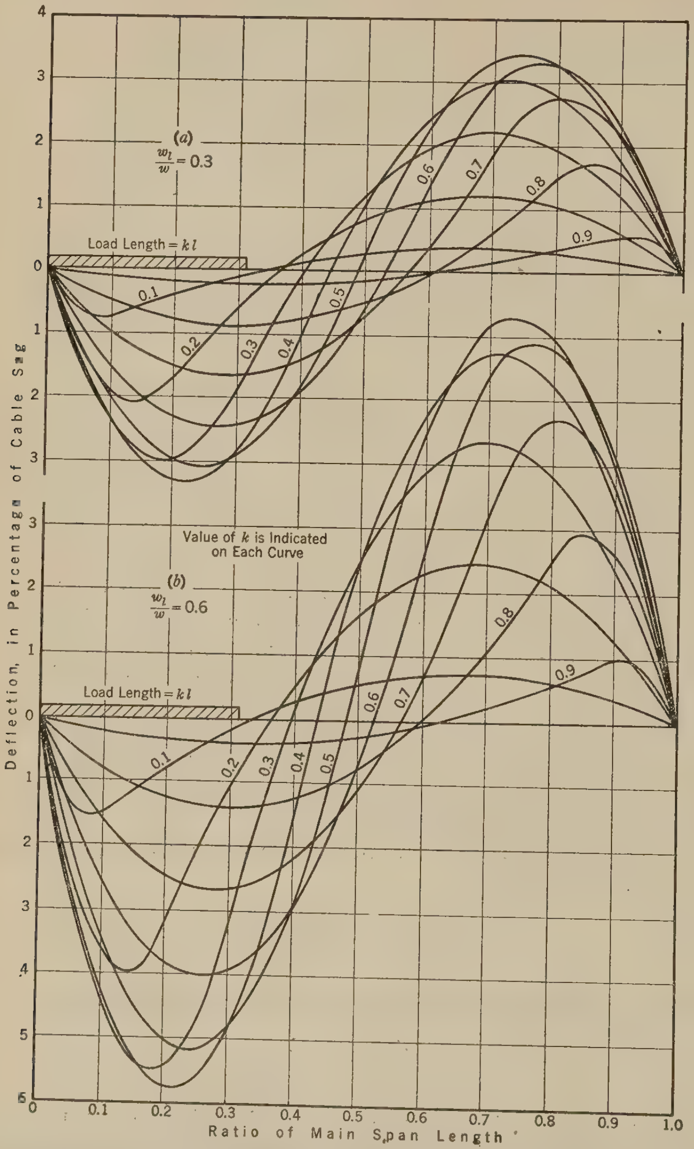

FIG. 15.—DEFLECTIONS OF UNSTIFFENED CABLE DUE TO PARTIAL LIVE LOAD OF VARYING LENGTH EXTENDING FROM LEFT END OF MAIN SPAN. DEAD LOAD OVER ENTIRE SPAN

The moment from Step II, however, involving cable-stretch effects, is definitely greater at the quarter-point than at the eighth-point. As noted under the heading, "Effect of Cable Stretch, Temperature Change, and Side-Span Interaction: Effect of a Change in Sag Upon the Main-Span Stiffening Truss," the moment diagram is elliptic in shape rather than parabolic, with the maximum moment at the center of the span. As flexibility increases, however, the moment due to Step II becomes a smaller part of the total, as shown in Fig. 9. Consequently, even if the moment from Step II is greater at the quarter-point than at the eighth-point, the difference is not as great as the difference in Step I where the effect at the eighth-point predominates. Hence, the point of absolute maximum positive moment may have a considerable range depending upon the "stiffness" of the stiffening truss. A detailed study of characteristics of maximum moment curves would be of some value, in view of the tendency toward more flexible stiffening units. It is quite possible that the maximum moments in some cases may be almost constant over that part of the span, say between the tenth- and the fifth-point. This is an added argument supporting the tendency to use virtually the same section throughout for the stiffening girder or truss, a practice which, of course, appeals to the fabricator.

Mr. Moisseiff undoubtedly recognizes that, as one gets closer to the tower, the effect of suspender elongation, and the effect due to the location of the end pin of the stiffening girder not being in the same vertical line as the end of the cable above, upset considerably the moments and shears derived by the usual deflection theory formulas.

Messrs. Rose and Rose have submitted a valuable discussion, linking the suspension bridge to the classification of structures proposed by Professor Cross,[39] a classification which is of tremendous importance to the profession. When its significance becomes more widely recognized, practicing engineers will design structures even more intelligently than they do now; teachers of indeterminate structures will be more careful in relating procedures of analysis to design.

The writers are also indebted to Messrs. Rose and Rose for calling attention to the relatively unimportant part played by shears in preliminary design studies. Incidentally, Mr. Karol notes that he has extended the method to account for maximum shears. It is important to note again that maximum end shear cannot be determined accurately by any theory that ignores the location of the end pins of stiffening trusses relative to cable.

Mr. Blog submits a simplification of the method which is quite acceptable. It will undoubtedly save time to eliminate the small elements of length denoted by L', and to work with all initial dimensions referred to the center line of the tower. Table 5 indicates that the results obtained by this simplification are accurate enough for preliminary studies.

Professor Cross draws upon the structural action of the flitched beam in clarifying the picture of the interaction of cable and stiffening truss. It is also important to keep Fig. 6 in mind in picturing the load distribution between the two elements. He also expresses the hope that more attention will be given to the function of the stiffening truss, with a view to defining stiffness adequately. There is undoubtedly a definite need for further research in this field, a need that challenges the resourceful investigator. The problem is not a simple one.

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[39] "The Relation of Analysis to Structural Design," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

This is further emphasized by Mr. Ammann in his enlightening discussion concerning rigidity and recent trends toward more flexible trusses. The writers are indebted to him for noting that the factors which influence the design of the stiffening system are so complex that the trial method of approaching a final design, combined with judgment based upon experience, is less likely to lead to erroneous practice than "cut and dried" formulas or rules of design.

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ENGINEERING ECONOMICS AND PUBLIC WORKS
A SYMPOSIUM

PAGE
Advantages of Orderly Planning.
By FREDERIC H. FAY, M. AM. SOC. C. E. 636

Influence of Public Opinion.
By DANIEL W. MEAD, PAST-PRESIDENT AND HON. M. AM. SOC. C. E.. 652

Hazards of Uneconomical Construction.
By HENRY EARLE RIGGS, PAST-PRESIDENT, AM. SOC. C. E....... 668

An Appeal to Reason.
By WILLIAM J. WILGUS, HON. M. AM. SOC. C. E............... 690

WITH DISCUSSION BY MESSRS. J. K. FINCH, ELLIOTT J. DENT, C. FRANK ALLEN, BRADLEY G. SEITZ, ALFRED ALLEN STUART, PIERCE P. FURBER, R. F. BESSEY, DONALD M. BAKER, PHILIP W. HENRY, H. K. BARROWS, HARRY A. WIERSEMA, J. D. GALLOWAY, E. S. MARTIN, K. BERT HIRASHIMA, LOUIS E. AYRES, JOHN H. MEURSINGE, SAMUEL B. FOLK, DANIEL W. MEAD, AND WILLIAM J. WILGUS.

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ADVANTAGES OF ORDERLY PLANNING

By Frederic H. Fay,[1] M. Am. Soc. C. E.

Synopsis

Attention has been focused upon public works as a possible stabilizing factor on employment, by the vast sums spent by the Federal Government in this field in the past few years for unemployment relief. The paper presents facts as to the construction industry as a whole, its relation to national income, and the share of public works in the industry, for the years 1925 to 1935, inclusive, the period for which detailed statistics were available. For public works themselves, analysis is made of expenditures in this period by the Federal Government, by the States, and by local governmental units.

During the depression expenditures in the construction industry as a whole (which in the pre-depression period accounted for one-sixth of the national income) dropped far more rapidly than the national income, primarily because of the great slump in private construction. Public works declined considerably after 1930, notwithstanding the greatly increased expenditures of the Federal Government, since the excess of Federal expenditures was nullified and much more than offset by the sharp drop in expenditures by counties and municipalities. Contrary to the general impression that the past few years has been a period of substantially increased activity in public works, the facts are that the country's public works program was a full year behind schedule by the end of 1935.

The theory of a controlled public works policy, by which construction will be retarded to some extent in prosperous times and expanded to relieve unemployment in periods of depression, is noted. This principle has not yet been given a fair trial since what has been done thus far has been done under pressure of an emergency and without the benefit of advance planning. The recent depression has taught its lessons, however, which may serve to guide future policy. Some of these lessons are stated. The importance of advance planning of public works and long-term budgeting, whether or not a policy of controlled public works as a stabilizing factor is to be attempted, is reviewed. Suggestions are offered whereby orderly advance planning may be made effective, thereby promoting efficiency in administration and economy in government.

Introduction

The efforts of the Federal Government during the past five years or more to stimulate public works as a means of relieving unemployment and lessening the severity of the depression throughout the United States, has forcibly impressed upon engineers engaged in carrying out this program, and to some slight extent has awakened the public to, the need of orderly advance planning of public works. Although long advocated by a few economists, engineers, planners, and others of vision, it is only during the recent depression that an attempt has been made in this country on a large scale to stimulate business and alleviate unemployment by accelerating and expanding public works construction.

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[1] Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

Although it has been measurably successful, this attempt has failed to produce the full degree of stimulation anticipated. However, the experience of the five years (1932 to 1937) affords no adequate criteria for passing judgment on the merits and defects of the principle of planned flexibility in public works, for the reason that (with certain exceptions as regards Federal programs) the element of advance planning has been absent. In State, county, and municipal public works particularly, which constitute by far the greater part of the public works field, carefully prepared plans were almost non-existent. Instead, in a desperate effort to meet an unexpected emergency, project after project was authorized and constructed by means of hastily conceived, unstudied plans which all too often had to be improvised almost over night. The many obstacles in the way of initiating public works at short notice in the absence of carefully prepared plans presented almost insuperable difficulties.

On the engineering side, this lack of thorough preliminary investigation and planning has frequently resulted in uneconomical expenditure through modifications of plans necessary to meet unforeseen conditions arising during construction, the resulting extra cost of which might have been avoided had it been possible to exercise careful engineering foresight based on adequate data as to physical conditions to be met. On the economic side (and this often was of even greater importance) insistence on immediate action left no time for deliberate investigation of the economic worth of many projects undertaken, nor a wise selection between projects according to their comparative merits and need.

The orderly planning of public works in advance of public requirements, both as to their engineering and their economic aspects, is at all times a sound governmental policy. This is so fundamental as to require no argument. That such sensible foresight has not been exercised in greater degree in the past is due largely to lack of understanding by the public generally as to the nature and importance of planning. On the physical side there is little comprehension by the public of the time, effort, and money necessarily required for securing adequate knowledge of site conditions and for the preparation of designs which shall meet conditions in a proper and economical manner. Such engineering investigation, instead of being needless expense, may save far more than its cost by economies effected in construction. On the economic side, there is too often but little attempt, not only to weigh projects as to their need and comparative worth, but also from the standpoint of ability to finance. Contrary to popular impression, there are distinct limits to the funds that can be drawn from the public purse.

Whether or not one holds that there should be some control over public works as a stabilizing factor on employment, the fact remains that advance planning of public works and long-range budgeting are necessary elements of wise governmental economy. They are but the application of the foresight and the common-sense practices of private business, to the business of Government.

On the Engineering Profession, more than on any other group, falls the important duty of guiding and educating the public to the need of orderly planning of public works. It is a highly important part of engineering-economics.

In his address on "The Engineering Profession," delivered at the Commencement Day exercises of the Massachusetts Institute of Technology, on June 8, 1937, Gano Dunn, M. Am. Soc. C. E., laid emphasis upon the responsibilities of the engineer in the field of economics. In discussing science and engineering he stated:

> "The essential difference between the scientist and the engineer lies in the economic involvement of the engineer's work. It is in this hot crucible of the economic test that all an engineer does must be tried * * * . Economics is not a physical science. It is a social science, bringing in problems of the human spirit and the behavior of man as man, and it is because the engineer's art deals with dollars and economic relations that he is bound into the great business structure of society in a way that the scientist is not. Being bound into this structure, he must be a man among men. He must be able to make his views prevail. He must be able to persuade and contend. And he must be able to give blows and to take them."

THE CONSTRUCTION INDUSTRY OF THE UNITED STATES

Statistics of the construction industry of the United States have not been available in comprehensive detailed form until recently. The information at hand is that of estimates prepared after long and arduous analysis of statistics of many territorial divisions of the construction field and of many classes of construction. Where these statistics do not cover the entire country estimates must be made of the construction in the territory omitted. A chief source of information is the F. W. Dodge Corporation, of New York, N. Y., which, since 1924, has reported construction contracts awarded in 37 States east of the Rocky Mountains. Another is the volume of building permits issued in more than 800 cities throughout the country, compiled by the United States Department of Labor. Analysis must be made of the construction work done by the many Federal departments, bureaus, and relief and other agencies, supplemented by analysis of construction done by State, county, and municipal agencies. Allowance must be made for construction and maintenance work, by force account labor and not by contract. Duplications must be eliminated. Then, too, it should be recognized that the aggregate totals of contracts awarded and of building permits issued may generally be less than the ultimate cost of the projects which they represent. Innumerable difficulties beset the path of the statistician who seeks to determine the true financial picture of the construction industry.

From its establishment in 1931, the Federal Employment Stabilization Board did notable work in statistical analysis of the national construction industry; this having been undertaken by Corrington Gill, Chief Economist, in co-operation with the Board's executive head, Donald H. Sawyer, M. Am. Soc. C. E. The reader interested in the subject may well consult the writings of Arthur D. Gayer.[2] Estimates of total construction in the United States, based chiefly on the volume of construction materials produced each year, have been prepared by Simon Kuznets, of the National Bureau of Economic Research.[3] The latest publication on this subject that has come to the writer's attention is a *Bulletin* entitled, "The Construction Industry," issued by the Bureau of Foreign and Domestic Commerce of the U. S. Department of Commerce, in April, 1936.

Total Construction, Public and Private.—A comparison of several principal estimates of the total volume of construction in the United States is presented in Table 1(*a*).

TABLE 1.—COMPARISON OF CONSTRUCTION WITH NATIONAL INCOME, IN MILLIONS

Year	(*a*) COMPARATIVE ESTIMATES, TOTAL CONSTRUCTION IN THE UNITED STATES					(*b*) ESTIMATES OF NATIONAL INCOME PRODUCED		
	Gill (original estimate) (1)	Gill (revised by Gayer) (2)	Gayer (3)	Kuznets (4)	U. S. Department of Commerce (5)	Brookmire (6)	Brookings Institution (7)	U. S. Department of Commerce (8)
1925	$10 805	$11 768	$12 295	$14 032	$11 550	$67 952	$75 918	*
1926	10 912	11 884	12 578	14 343	12 040	71 899	77 177	*
1927	11 153	12 147	12 924	14 876	12 050	71 703	77 003	*
1928	11 339	12 349	13 019	15 919	12 230	76 414	79 679	*
1929	10 492	11 427	12 279	14 381	11 630	79 886	81 940	$80 757
1930	10 108	10 108	10 208	11 921	10 180	71 208	....	67 969
1931	7 586	7 586	7 592	8 920	7 580	59 186	....	53 584
1932	4 361	4 361	4 068	5 458	4 350	45 750	....	39 545
1933	....	....	....	....	3 580	44 681	....	41 813
1934	....	....	....	....	4 440	51 667	....	49 575
1935	....	....	....	....	....	54 826	....	54 955
1936	....	....	....	....	....	....	....	63 799

* No estimate.

Estimates of the Federal Employment Stabilization Board, as compiled by Gill,[4] are listed in Column (1), Table 1. For the years, 1925 to 1929, values for pipe-line, gas, telegraph, and water-works companies are omitted; but they are included in the years, 1930 to 1932. In Column (2), Table 1, Gayer [5] has corrected the values in Column (1) to include the estimates for pipe-line, gas, telegraph, and water-works construction in 1925 to 1929, by an average ratio, 8.91%, which these items bore to the other construction in 1930 to 1932. Estimates prepared by Gayer, based on data released by the F. W. Dodge Corporation, the Federal Employment Stabilization Board, and the U. S.

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[2] "Public Works in Prosperity and Depression," by Arthur D. Gayer, Rept. to the National Planning Board, National Bureau of Economic Research, 1935.

[3] "Gross Capital Formation," 1919–33, by Simon Kuznets (November 15, 1934), *Bulletin No. 52*, National Bureau of Economic Research.

[4] "Public Works in Prosperity and Depression," by A. D. Gayer, p. 65, 1935.

[5] *Loc. cit.*, pp. 65, 69, and 70.

Bureau of Public Roads, are given in Column (3).[6] Kuznets (see Column (4), Table 1) bases his estimates mostly on the volume of construction materials produced each year.[3] These estimates are distinctly higher than the others and are believed by the writer to be in excess of actual amounts. The estimates of the Department of Commerce (Column (5), Table 1) are the most recent, and presumably have been presented with previous estimates in mind. It will be noted that in the 5-yr period, 1925 to 1929, inclusive, the total volume of construction, public and private, averaged approximately $12 000 000 000 annually.

Total Construction in Relation to National Income.—The importance of the construction industry in the national economic structure may be gaged by comparing its amount with that of national income. For this purpose estimates of national income produced are given in Table 1(*b*). In the case of national income, as in total construction, authorities differ in their estimates. Comparison of Table 1(*a*) and Table 1(*b*), however, indicates that in the predepression years, 1925 to 1929, inclusive, the total volume of expenditures for construction was roughly one-sixth of the national income. In 1933, however, when construction had reached its lowest level, total construction as estimated by the Department of Commerce (3.5 billion dollars) was only about one-twelfth of the national income produced that year.

Analysis of Estimates of Total Construction.—Beginning with 1925, estimates of total construction in the United States from three sources are available in such detail as to permit a comparative analysis under the classifications of: (*a*) Private construction; (*b*) public utilities (including railroads); and (*c*) public works. Such analysis is given in Table 2. Of the three groups of estimates in Table 2, the first in point of time is that of the Federal Employment Stabilization Board made by Gill. Next followed the estimates of Gayer prepared in 1934 for the National Planning Board and, after some revision, published in 1935 by the National Bureau of Economic Research under the title, "Public Works in Prosperity and Depression." Data for both the Gill and Gayer estimates in Table 2 have been taken from this book. The latest available estimates are those of the Bureau of Foreign and Domestic Commerce of the U. S. Department of Commerce and published in 1936 in its *Bulletin*, "The Construction Industry," from which the data of the third group of estimates in Table 2 are derived. Attention is called particularly to the fact that the estimates in Table 2(*a*) for the years, 1925 to 1929, inclusive, do not include, in the utilities group, expenditures for pipe-line, gas, telegraph, and waterworks construction; hence, for these years, the totals for both utilities and total construction are below their true values. The estimates in Table 2(*b*) and Table 2(*c*) include allowances for construction by these several public utilities in the years named. Beginning with 1930 the construction expenditures of these utilities have been reported.

It will be noted that the public works estimates in Table 2(*b*) are generally higher than those given in Table 2(*a*). From reading the report by Gayer[2] one gains the impression that he makes a good case for the authenticity of his own estimates, and, at the same time, one is impressed by the difficulties and complexities which confront the statistician working in this field. The U. S. Department of Commerce, in its later publication, appears to have favored the estimates by Gill rather than those by Gayer.

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[6] "Public Works in Prosperity and Depression," by A. D. Gayer, p. 62.

TABLE 2.—COMPARATIVE ANALYSIS OF ESTIMATES OF TOTAL CONSTRUCTION IN THE UNITED STATES, IN MILLIONS OF DOLLARS

	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935
(*a*) ESTIMATES BY CORRINGTON GILL, FEDERAL EMPLOYMENT STABILIZATION BOARD*											
Private	5 237	5 365	5 175	5 416	4 451	2 746	1 761	667	....	....	....
Utilities†	2 851	2 935	2 933	2 900	3 265	4 062	2 898	1 629	....	....	....
Public works	2 717	2 612	3 045	3 023	2 776	3 300	2 927	2 065	....	....	....
Total†	10 805	10 912	11 153	11 339	10 492	10 108	7 586	4 361	....	....	....
(*b*) ESTIMATES BY ARTHUR D. GAYER‡											
Private	5 919	5 935	5 552	5 763	4 643	2 558	1 652	450	....	....	....
Utilities	3 564	3 669	3 666	3 625	4 081	4 018	2 873	1 614	....	....	....
Public works	2 812	2 974	3 706	3 631	3 555	3 632	3 067	2 004	....	....	....
Total	12 295	12 578	12 924	13 019	12 279	10 208	7 592	4 068	....	....	....
(*c*) ESTIMATES BY THE U. S. DEPARTMENT OF COMMERCE§											
Private	5 240	5 630	5 240	5 490	4 610	2 800	1 810	680	750	870	‖
Utilities	3 600	3 780	3 770	3 740	4 260	4 150	2 950	1 660	1 240	1 440	1 480
Public works	2 710	2 630	3 040	3 000	2 760	3 230	2 820	2 010	1 590	2 130	2 030
Total	11 550	12 040	12 050	12 230	11 630	10 180	7 580	4 350	3 580	4 440	‖

* "Public Works in Prosperity and Depression."

† Estimates incomplete for years, 1925 to 1929, inclusive, due to omission of values for pipe-line, gas, telegraph, and water-works companies.

‡ Prepared for National Planning Board, published by National Bureau of Economic Research in "Public Works in Prosperity and Depression," 1935. (Compilation by writer.)

§ "The Construction Industry," pub. by Bureau of Foreign and Domestic Commerce, U. S. Department of Commerce, 1936.

‖ Estimate for 1935 is preliminary and incomplete.

PUBLIC WORKS

The distribution of public works expenditures between the Federal Government, the States, and political subdivisions of States, has been investigated by the Federal Employment Stabilization Board and subsequently by the U. S. Department of Commerce. The estimates of these two agencies are in close agreement from 1925 to 1929, after which there is some variation in the item of Federal expenditures. Although in both cases the estimated total volume of public works in each year prior to 1932 is less than that in the Gayer estimates prepared for the National Planning Board (as will be noted from Table 2), the estimates of the Department of Commerce are used herein as the latest and most complete breakdown of expenditures for public works. These data of the distribution of public works expenditures between the Federal Government, States, counties, and cities, for the years, 1925 to 1935, inclusive, are given in Table 3. In this table, Federal aid is included in Federal expenditures and excluded from those of States and their political subdivisions.

TABLE 3.—Distribution of Public Works Expenditures

(Estimated* by Department of Commerce; Federal Aid Is Included in Federal Expenditures and Excluded from Other Divisions)

Year	(a) Dollar Distribution, in Millions					(b) Percentage Distribution				
	Cities	Counties	States	Federal	Total	Cities	Counties	States	Federal	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1925	1 280	780	410	240	2 710	47	29	15	9	100
1926	1 300	680	400	250	2 630	49	26	15	10	100
1927	1 480	880	440	240	3 040	49	29	14	8	100
1928	1 420	830	500	250	3 000	47	28	17	8	100
1929	1 340	560	580	280	2 760	49	20	21	10	100
1930	1 500	710	710	310	3 230	46	22	22	10	100
1931	1 300	330	780	410	2 820	46	12	28	14	100
1932	800	140	550	520	2 010	40	7	27	26	100
1933	400	100	500	590	1 590	25	6	32	37	100
1934	500	130	400	1 100	2 130	23	6	19	52	100
1935	550	130	350	1 000	2 030	27	6	17	50	100

* "The Construction Industry," 1936, Bureau of Foreign and Domestic Commerce, U. S. Dept. of Commerce.

In the pre-depression years, 1925 to 1929, inclusive, the average public works expenditure in each of these four divisions was as follows:

Division	Expenditure	Percentage of total
Cities	$1 364 000 000	48.2
Counties	746 000 000	26.4
States	466 000 000	16.5
Federal	252 000 000	8.9
Total	$2 828 000 000	100.0

Thus, in this pre-depression period, local governmental units accounted for three-quarters of the total expenditures for public works, and the State Governments for one-sixth, whereas the expenditures of the Federal Government were only about 9% of the total. Analysis of Table 3 reveals some striking facts as to what actually happened to the public works program during the depression.

Federal expenditures which, prior to the depression had averaged about 250 million dollars annually, rose steadily during the Hoover Administration and reached 590 million dollars in 1933. The Roosevelt Administration program inaugurated in 1933, increased them to one billion or more in 1934 and 1935. The States themselves expanded their construction activities from 1929 and carried forward enlarged programs until 1933, after which year State expenditures dropped somewhat below the pre-depression average. City and county expenditures, however (which together averaged 2 110 million dollars prior to 1930 and amounted to 2 210 millions in the latter year), dropped sharply to 500 millions, or less than one-quarter of the former values, in 1933, and increased but slightly in the two years following.

Considering the 6-yr depression period, 1930 to 1935, as a whole (6 yr being taken as 1935 is the latest year for which data are available), and contrasting it with the pre-depression period, 1925 to 1929, it is found that, in the aggregate, city and county expenditures were more than 6 billion dollars less than they would have been had the pre-depression average been maintained. On the other hand, the aggregate of State expenditures was nearly half a billion, and of Federal expenditures 2.4 billion, more than the maintained pre-depression figures. The net result is an aggregate shrinkage of more than 3 billion dollars in public works expenditures from 1930 to 1935, inclusive. In other words, contrary to the general impression that these depression years were years of stimulated activity in public works, the facts are that the country's public works program was a full year behind schedule at the end of 1935.

Federal vs. Local Public Works.—A paper entitled "Federal *vs.* Local Public Works," was presented in 1937 by William Stanley Parker,[7] based on the data reproduced in Table 3. Mr. Parker's conclusions as to the "depression relationship" between Federal public works and those of the combined State, county, and city groups (which he classifies, collectively, as "local" public works) are so pertinent that all interested students of the subject will wish to read it at the source.

According to Mr. Parker, the 1929 commitments for "local" public works formed the peak in 1930. The beginning of the slump began to show in 1931, and the decline in local expenditures than continued sharply to a low point in 1933. Federal expenditures, on the other hand, began to increase in 1929; in 1932 they had doubled, and had increased still more in 1933. Quoting Mr. Parker: "These increases represent the effort of the Hoover administration to offset the mounting unemployment in the construction industry." The comparison of the cumulative totals for the 3-yr period, 1931, 1932, and 1933, is as follows:

Decrease in "local" expenditures (below the 1930 peak)	$3 800 000 000
Increase in Federal expenditures (including the 1929 and 1930 increases)	$ 860 000 000

An accumulated shortage of about 3 billion dollars is indicated by the foregoing data, or the equivalent of an entire "normal" year's work. The efforts of the Federal Government are thus seen to be relatively ineffective.

The new Federal relief program was initiated in 1933 and developed in subsequent years. Expenditures on Federal projects were increased, "grants were made to local and state authorities and loans at low interest rates sought further to stimulate local expenditures." At the same time, Mr. Parker states that the Federal Government was spending large sums for relief, a substantial proportion being on minor construction projects which were not included in his basic statistics. Quoting:

> "Federal expenditures on public works, including grants, mounted to $1 100 000 000 in 1934 and to an estimated billion in 1935, four times their 'normal' amount. What happened to local expenditures in these two years? The expenditures of states dropped steadily, from the 1933 total of 500 million to 400 million in 1934 and 350 million in 1935. County expenditures increased about 30 million and cities increased theirs about 150 million, the net result being that in 1934 and 1935, in spite of the urgent need of employment, especi-

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[7] *The Architectural Record*, May 1937, p. 12.

ally in the construction industry, in spite of the efforts of the Federal government to incite local governments to develop their programs, and in spite of the local help through Federal relief expenditures, the combined expenditures of states, counties and cities for public works remained practically the same through 1934 and '35 as they were in the low peak year of 1933, being only little more than one-third of their 'normal' amount."

Furthermore, Mr. Parker stresses the fact that statistics for 1935 reveal that the total bonded indebtedness of States, counties, cities, and towns was 350 million dollars less than in 1932, indicating "a strange failure of local governments to cooperate with the Federal program which had come to their rescue."

His comparison of cumulative totals for 1934 and 1935, similar to the 3-yr period, 1931, 1932, and 1933, is as follows:

Decrease in local expenditures (1934 and 1935)	$3 780 000 000
Increase in Federal expenditures (1934 and 1935)	1 600 000 000

Comparing the new annual totals in these two categories:

Federal expenditures	$1 000 000 000
All local expenditures	1 000 000 000

and, as a final summary, for the years since 1930:

Cumulative decrease in local expenditures	$7 580 000 000
Cumulative increase in Federal expenditures	2 460 000 000

In other words (to quote Mr. Parker), "for every dollar the Federal government increased its expenditures in order to prime the pump the local governments reduced their expenditures by three dollars, and even after the 1933 relief program developed the local reduction was more than twice the Federal increase."

Planning and Control of Public Works

Several of the States, notably New Jersey and Idaho in 1915, Pennsylvania in 1917, California in 1921, and Wisconsin in 1923, passed legislation providing for expansion of public works construction as a remedy for unemployment, but these measures either remained inoperative or produced negligible results. A few city and county governments have undertaken to prepare long-term public works programs. Thus far but little has been accomplished in this direction by the States and their political subdivisions.

For a dozen years following the World War, attempts were made repeatedly to secure Federal legislation for the orderly planning and control of Federal public works. These efforts proved fruitless until 1931 when Congress passed legislation creating the Federal Employment Stabilization Board. The purpose of the Act was to provide for "the advance planning and regulated construction of public works, for the stabilization of industry, and for aiding in the prevention of unemployment during periods of business depression."

By the Employment Stabilization Act of 1931 the Federal Government established a policy which, if followed consistently by State and municipal governments throughout a period of years, may not only assist in stabilizing employment, but at the same time should result in wiser and more economical expenditures for public works. An important feature is the requirement that Federal departments and other agencies having charge of construction shall "prepare a six-year advance plan with estimates showing projects allotted to each year," with the further provision that the 6-yr program be kept up to date by annual revision and extension. Thus, the principle of advance planning has been established by law. A second important feature is recognition of the need of co-operation in advance planning between the Federal Government and State and municipal governments if control over public works is to be in any degree effective. The Federal Employment Stabilization Board was directed to watch, constantly, the pulse of the construction industry, to keep the President informed regarding business activity and the trend of employment, and to give warning of the existence or approach of periods of business depression and unemployment so that at such times the President might advise the Congress regarding needed expansion of the public works program.

In the Federal Employment Stabilization Board the theory of planned public works was put into large-scale operation for the first time in the United States. The Board compiled the first reliable data on construction expenditures in the post-war period. Its work was of great assistance to the Public Works Administration in determining the allocation of funds appropriated under the National Industrial Recovery Act of 1933. The latter Act created the Federal Emergency Administration of Public Works which, in turn, set up the National Planning Board as its advisory body. Both these agencies took over some of the functions of the Stabilization Board which then continued to operate as the Federal Employment Stabilization Office of the Department of Commerce.

Basic Principles.—A controlled public works policy is usually looked upon as a flexible policy which will exert some degree of stabilizing effect on changing business cycles—a policy by which public works construction will be retarded to some extent in prosperous periods and expanded to relieve unemployment in periods of depression. The principle of controlled public works has not yet been given a fair trial since what has been done thus far has been done under the pressure of an emergency and without careful advance planning. It has not yet been put into large-scale operation in the United States in prosperous times and the extent to which it may lessen the severity of a depression has not been demonstrated. The recent depression has taught its lessons, however, which may well serve as a guide to future policy. Among them may be noted the following:

(1) Long-range advance planning and budgeting are absolutely indispensable for the success of any plan of controlled public works.

(2) The Federal Government cannot undertake the task alone. The excess of Federal expenditures for public works since 1930 has been nullified and much more than offset by the sharp drop in expenditures by counties and cities. To be successful, State and local governments must do their share. It is a nation-wide problem.

(3) The financial program, or long-range budgeting, is an important element. In prosperous times some cities have wisely followed, to a greater or less extent,

a "pay-as-you-go" policy, paying for capital expenditures out of current income, thus accumulating reserves of borrowing capacity and having strong credit with which to face a depression. This has been true of certain State Governments as well. Recently, the suggestion of creating reserve funds in boom periods to be drawn upon for public works construction in periods of depression has been given publicity. In general, the financing of public works during depressions, especially if construction programs are then to be expanded, may be accomplished in one of two ways; either by drawing upon previously accumulated reserves, or by long-term or short-term borrowing. Of these two methods, borrowing, preferably for short terms, is generally favored. Federal and State credit is usually such that borrowing presents no serious difficulties. This may not be the case with local units on which rigid limits may have been placed as to borrowing capacity. Such restrictions have doubtless accounted, in part, for the sharp drop in city and county public works expenditures since 1930. Long-term budgeting must concern itself with the problem of conserving the borrowing capacity of these local units or of finding means (as has recently been done by legislation in several of the States) whereby restrictive limits on borrowing by local units are raised somewhat during an emergency caused by a depression.

(4) A controlled public works policy should not be looked upon as a means of relieving, directly, unemployment among workers outside the construction industry by giving them employment on construction projects. In any important depression period private construction will surely decline, public utilities may be forced to curtail their construction activities somewhat, and inevitably there will be considerable unemployment among workers normally employed in these branches of the industry. Controlled public works construction, however much it be expanded during a depression, should give direct employment only to those workers within the construction field.

(5) The unemployed must be supported in any event out of public or private funds. The real net cost of public works during a depression, apart from any stimulating effect upon the business cycle, is actually much less than it seems because, in public works, the community gets something in return for its expenditure.

Some may ask, "Can a sufficient volume of genuinely needed public works be found in depression periods to give substantial employment?" The answer is "yes," if construction programs are carefully planned in advance and retarded in periods of prosperity. Roads and bridges are examples of construction suitable for a flexible program.

The construction industry, as a whole (in which, in normal times, expenditures are roughly one-sixth of the national income) is a highly important part of the national economic structure. Public works normally constitute about one-fourth of this industry. Public works alone give primary and secondary employment to a substantial number. In primary employment it has been estimated that normally about 800 000 workers are directly employed in the field, and for every worker directly employed one or two others are employed indirectly in the manufacture and transportation of construction materials. Thus, the normal primary employment due to public works may be, roughly, 2 000 000 wage earners. These workers, in turn, spend a very large proportion of their wages for consumer goods and services thus creating a "secondary" employment in other fields perhaps, roughly, as large again.

If it were possible to maintain the construction industry as a whole at a fairly uniform level, much would be accomplished in "flattening" the business cycle curve and in effecting a marked stabilizing influence on national income. The volume of private construction, however, is not susceptible to public control and is certain to fluctuate widely. The control of public works construction such that its volume remained fairly constant, or was expanded somewhat in depressions, would have appreciable effect in lessening the severity of a depression in the construction industry and, at the same time, would stimulate employment, to some extent, outside that field. If public utilities should join with public works in stabilizing their construction programs, these two (which, together, account for more than one-half of all construction activity of the country, because of the increased primary and secondary employment thus created), would measurably improve the business of the country in depression periods. Such co-operation is not possible at present, however, under existing taxation laws.

Federal Aid in Non-Federal Construction Work

During the recent depression, the Federal Government has made a radical departure from principles to which it had adhered for a century and a half by embarking upon the new policy of lending its credit, and of giving grants outright, to aid the construction of projects which it does not own. That this is an emergency policy, of temporary nature, which will shortly be brought to an end, is generally understood. The precedent having been established, however, undoubtedly there will be an insistent demand for the resumption of this policy when the next severe depression comes. If that is the case, long-range advance planning becomes vitally necessary.

It was in 1932 that the United States Government inaugurated the policy of using its credit to aid borrowing by local governments for their public works. By an Act passed in July of that year, Congress authorized the Reconstruction Finance Corporation to make loans, up to the aggregate amount of $1 500 000 000, to States and their political subdivisions and agencies, and in certain instances to private corporations, for "self-liquidating" construction projects. Although this was the first time that Federal credit had been used in this country for construction of other than Federal public works, the idea was not new. For forty-five years this policy had been at work in England where, through the agency of the Public Works Loans Board, most of the funds for housing and similar projects undertaken by the smaller municipalities had been borrowed from the National Treasury.

This 1932 Act was a demonstration of the growing belief in the value of expanded public works programs as a means of affording unemployment relief. Because of the stipulation of the Act that loans by the Reconstruction Finance Corporation must be restricted to revenue-producing or "self-liquidating" projects, and of the policy of the RFC in charging relatively high interest rates, authorizations were made very slowly and disbursements more slowly still. Only a minute fraction of the one and one-half billion dollars authorized had been put to work for the relief of unemployment before this Act was superseded by that of 1933.

The National Industrial Recovery Act of June, 1933, appropriated $3 300 000 000 for public works; authorized grants as well as loans to States, municipalities, and other public bodies; created the Federal Emergency Administration of Public Works (PWA) to administer the Act; and required the PWA to "prepare a comprehensive program of public works," Federal, State, and local.

Engineers throughout the country, perhaps better than any other group, realize the chaotic conditions as regards public works planning precipitated by the 1933 Act. In the feverish haste to put the vast Federal appropriation to work for the relief of unemployment there was no time to prepare orderly programs or even adequate preliminary plans. Almost over night as it were, engineers everywhere were called upon to prepare applications for public works projects in the form required by the PWA, which all too often involved guesses as to site conditions, estimates which were scarcely more than guesses of probable cost, and plans which were the merest sketches. There was no time for economic planning as to the need and comparative worth of various projects. Thus, at the outset and for a long time after the 1933 Act became operative, orderly planning was not possible and scant attention was paid to a "comprehensive program of public works."

Under this Act, however, the Administration committed itself to the principle that orderly planning and control was necessary to secure efficient and socially desirable expenditure of public monies in the public works field. The Act contained no statement concerning the possible utilization of public works as a stabilizing factor, but the prominence, in the Act, of the stipulation regarding the formulation of a comprehensive, nation-wide program seemed to imply that in the future such a program could be utilized toward that end.

Within the framework of its initial organization the PWA set up in 1933 the National Planning Board, an advisory body, to which was assigned the preparation of the comprehensive program required by the 1933 Act. The functions of the National Planning Board, as stated, were to engage in research concerning the potential development of public works in relation to social and economic needs and to draw up a comprehensive plan for regional areas in co-operation with National, State, and local agencies. As for planning agencies throughout the country, in the summer of 1933 there existed at the top of the pyramid the newly created National Planning Board and at its base about 700 municipal planning boards in the principal cities and in certain of the larger towns. Between the top and the base, State and regional planning agencies were almost wholly non-existent.

In that year the President addressed communiciations to the Governors of the several States urging the creation of State planning boards to assist in the orderly planning and programming of public works in their respective States, particularly as an aid to the PWA in the allotment of grants and loans. Applicants to the PWA for such Federal aid were required to state whether the projects in question formed parts of comprehensive plans and had received the approval of State and local planning agencies.

New Hampshire was the first State to comply with the President's request and immediately Governor Winant appointed an unofficial State Planning Board. The example set by New Hampshire was followed by a large proportion of the States. In most cases these boards were necessarily unofficial bodies, at first, created by executive order so as to avoid the delay of waiting for State legislation. At present (1937) there are State Planning Boards in forty-five of the forty-eight States, and of these, thirty-six Boards are now permanent official bodies established by legislative act.

The National Planning Board has fostered the creation of interstate regional planning commissions when groups of States have problems of special regional interest. The New England and Pacific Northwest Regional Planning Commissions are examples of such bodies.

The experience of the four years, 1933 to 1937, in attempting to promote useful and needed public works projects as a part of the Federal aid program for unemployment relief, has demonstrated conclusively to those concerned with these public works, if not to the public generally, both the need of advance planning and that of fitting public works plans into a logical pattern. If any lesson is to be learned from the experience of the last few years, it is that public works should no longer be planned and constructed in a "hit-or-miss" fashion but that henceforth, the need for them should be studied with proper foresight. The Roosevelt Administration deserves commendation for its stand regarding the necessity for advance planning, and for its efforts toward co-ordinating public works projects, Federal, State, and local, into something like an orderly program which will promote wise and economical use of public funds in this field.

Advance Planning

Whatever differences of opinion may exist as to the wisdom of some control over public works as a stabilizing factor on employment, all must agree that advance planning and long-range budgeting are common-sense business practices. They are absolutely essential in private business. They are practiced likewise by leading public utilities of which the nation-wide telephone company is a notable example. It is in the business of Government, particularly in the case of public works, that the necessity for foresight and economy is all too often ignored.

Advance physical planning of public works requires some modification of the practices usually obtaining in governmental departments, particularly those of the State and local governments. Rarely do these departments take a long-range view and give thorough and detailed study to their requirements for some years in advance. Usually, the departments themselves are not primarily at fault. Mistaken governmental economy in far too many cases allows such meager appropriations for technical service that thorough engineering investigation of public works projects well in advance of their authorization is not possible. Frequently such projects are authorized on hastily prepared plans and estimates, without careful advance study based on adequate knowledge of conditions to be met. Only when funds for construction become available is it possible, in many cases, to start a general engineering investigation and by that time pressure to get construction "under way" leaves little time for thorough engineering planning.

The departments themselves may often be subject to criticism, however, for failure to impress upon appropriating agencies the wisdom and ultimate economy of adequate appropriations for engineering service which will provide a competent organization sufficient to handle current work and, at the same time, give proper consideration to work foreseen as needed in the future. Too many governmental departments live on a "hand-to-mouth" or day-by-day policy. At most, it is from administration to administration. A new governor or a mayor is elected; new department heads are appointed. There is usually a sincere intention on the part of the Chief Executive to administer the affairs of the State or city wisely and well, but no carefully prepared and formally accepted long-term program or pattern awaits his coming. There is all too often, therefore, a perfectly understandable human tendency to discount the efforts, the efficiency, and perhaps even the sincerity, of earlier administrations; to abandon projects already begun; to divert—with consequent loss—millions of dollars from one channel of public activity to another; and to venture into new and unexplored fields without the benefit of proper advance technical planning. In the absence of advance planning, under the present system of Government, and in the light of human nature, this cannot be otherwise. No one can change human nature. Americans do not want to depart from their democracy in Government; they can and should prepare well in advance, and within reason should adhere to, carefully considered programs for public works activities.

The public thinks engineering inconsequential and something that can be done in a hurry when required. There is great need of public education as to the importance of engineering investigation before proceeding with construction. The serious lack of adequate advance physical planning by departments of State and local governments has been glaringly evident in connection with the current program of Federal aid in non-Federal public works.

Advance financial planning or long-range budgeting of the public works program is of equal importance with advance physical planning. A few American cities have already led the way in preparing long-time bond or financial programs for public improvements. If the example which they have set were general practice in State and local governments, greater economy and more uniformity in expenditure would result. The danger of rushing ahead with poorly considered and less necessary projects, and with projects which the community can ill afford, would be largely reduced.

Timing of public improvements is an important part of advance planning. Proper timing will result if there is co-ordination of advance physical planning and advance financial planning; in fact, all three are factors of a single problem. Timing so as to retard public works programs somewhat in periods of prosperity and to advance them in depressions is necessary if public works are to be utilized as one phase of an economic stabilization program.

Co-ordinated, long-range public works programs are made up of many diverse elements. They are desirable and economically necessary for city, county, State, and Federal Governments. How then is proper co-ordination to be secured? It is not a matter that can be handled adequately by individual governmental departments alone. Each department, it is true, is best qualified to forecast and plan for the needs of its own particular field, but its vision is limited, it may know little or nothing of the needs of another department, it is naturally biased to a greater or less extent by its conception of the importance of its own problems, and it is not an impartial judge. In no governmental unit, State or local, will a well-considered and well-co-ordinated long-range program of public improvements, best suited to the needs of that unit as a whole and within its capacity to finance, result from a round-table discussion between representatives of the departments directly concerned with carrying out these improvements. A broader, more detached, and more impartial viewpoint is needed.

Co-ordination is planning in its broadest terms. It can well be secured through some form of non-political planning agency, acting independently of other departments of the Government, free from all administrative responsibility for the construction of projects, and one in which a substantial proportion of its members, at least, shall be appointed in rotation for fairly long terms in order to secure continuity of policy. Such planning agency should work in closest co-operation and harmony with other governmental departments responsible for public improvements and for the Government's financial administration. Heads of certain of these departments may well be members, *ex-officio*, of the central planning agency in order to secure closer co-operation and a better understanding of public works problems and their relation to the Government's financial policy. It would seem best, however, that the majority of the members of the planning body should be free from other governmental duties and responsibilities in order that they may exercise unbiased judgment. Such membership presents an opportunity for engineers, architects, planners, economists, business men, and civic leaders of highest standing to render service of great value to their community.

If the challenge is fairly met, and public officials work in co-operation with the planning group, then an orderly progress may be expected in promoting a long-term program of public works, in establishing efficiency in administration, and in effecting real economy in government.

INFLUENCE OF PUBLIC OPINION

By Daniel W. Mead,[8] Past-President and Hon. M.
Am. Soc. C. E.

Synopsis

A well informed public can and will raise such objection to projects which they know to be unsound that no Federal, State, or Municipal Administration will dare to undertake such projects.

Public works are, in general, so complicated as to be little understood by the general public. In recent years and in the past, many uneconomical public works have been undertaken, for reasons of political expediency, and have gone unchanged because the public has not understood their unsound character. This lack of understanding by the public is largely due to the fact that the Government seldom furnishes the information necessary for a basis of sound judgment, but misleads and misinforms the public by extravagant statements and propaganda favorable to such projects.

Engineers, from their training and experience, can and should study these projects and inform the public, in simple language that can be readily understood, the facts concerning such projects that will give the best possible knowledge from which the economic feasibility and desirability of such ventures may be judged. Some of the Government projects of the past, present, and probable future, are discussed briefly, herein, and these objectionable features pointed out.

The Record of Government in Business

There is no profession more affected by depressions than that of Engineering. In consequence, there are no men who are more interested, and none better qualified to assist, in the re-establishment of normal conditions than engineers.

After seven years of waiting (1930 to 1937) which includes five years of Federal Administration experiments, the depression is still present. Whether partial recovery, as far as recovery has occurred, is due to, or in spite of, administrative activities will be a moot question for years to come. There are still under consideration many suggestions for further radical changes in social and business relations.

The engineers of the United States will do less than their duty if they hesitate to discuss these matters frankly, honestly, intelligently, and fearlessly, and bring to bear on the solution of these vital problems the best thoughts of their profession.

Many ill-conceived and unwarranted public and semi-public works have been undertaken by the Federal Administration since the beginning of the depression. The more these ill-considered and extravagant projects are discussed and their defects made plain to the public, the more likely it is that they will be prevented in the future by the development of a sound public opinion. To the mind of the writer, a well informed public can raise such objections to unsound projects that no administration—municipal, State, or Federal—will dare attempt projects that are not based on thorough investigation, sound judgment, and sound economies.

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The writer believes that there are many matters at the present time that are even more important to the engineer than the very important matter of ill-advised public works. On account of the introduction of labor-saving machinery and other mechanical devices that have saved labor, increased production, and reduced cost, the Administration has charged the Engineering Profession with the production of unemployment. The facts are diametrically opposite to this theory.

The National Resources Committee reports:[9]

> "* * * in the period from 1870 to 1930, the population of the country multiplied by three while the number gainfully employed multiplied by nearly four. * * * The eighteen major industries of today which employ directly and indirectly about 25 per cent of those gainfully employed in this country were absolutely unknown in 1870. While mechanical power has been instrumental in separating the worker from his tools and agriculture from industry, it has definitely created more callings and more employment than it eliminated."

The Administration has also urged the engineering schools to teach their students to consider not only the design and construction of engineering works but also to study and determine their ultimate effects on society.

Can the Administration believe that the recent construction of vast public works, many of which will compete with projects financed through the sale of securities to the public under the supervision of the various State commissions, is fair to such investors, or that the construction of such works will effect recovery, or be a real benefit to the nation? Are not all honest investors in the securities of public utilities entitled to the consideration of the Administration in the planning of public benefits to the citizens of the United States? This question has been raised frequently but the writer has heard no answer.

The Administration's suggestion for careful consideration of the effects of improvements would have been received more sympathetically if the plans for the construction of these great public works undertaken for the alleged purpose of recovery, could stand sound economic analysis.

The entrance of Government into business is being carried so far by the present Administration that it seems to threaten the ultimate socialization of business and industry and the regimentation of all professional men and workers into Government service.

The record of Government in industry, whenever it has been tried, is not a satisfactory one and warrants no further extension of Government into business. The fiascos of the Government control of the railroads during the World War, of the Government-owned railroad in Alaska, the Government ownership of railroads in Canada, seem to have little weight on public or political opinion.

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[9] "Report on Technological Trends and National Policy," June, 1937, p. 254.

The economical records of Federal irrigation projects since 1902 and of the Inland Waterways Corporation since its organization, are thoroughly unsatisfactory. The recent records of the Resettlement Administration and of the Federal Housing Projects are still later exhibits of Federal economic ideas. Can the public and its representatives in Congress learn nothing from experience? or is the trouble due to lack of information? The unfortunate results of these disastrous experiments should be brought to the attention of the public so that they will realize the danger of further experimentation in Government ownership and operation.

An immediate danger that confronts the nation is the further entrance of the Government into the hydro-electric power field. Already the Government is committed to vast expenditure for power development, partly by Acts of Congress, but largely by administration orders. Among such ventures may be named the Tennessee Valley Authority (TVA) involving an expenditure of about a half billion dollars, the Grand Coulee Project which will involve a similar sum, and diverse other Federal projects, including the Passamaquoddy Project, now sleeping, but not forgotten.

The bill before Congress (S. 2555), if it becomes a law, will divide the United States into seven districts, in each of which an authority will be established, essentially similar to the TVA. This will practically place all future hydraulic power development under Federal ownership and operation. It will probably involve the Government in the installation and operation of steam or other auxiliary power plants as in the case of the original Muscle Shoals Project. It may also involve not only Federal navigation and flood protection, but other industries, and ultimately may eliminate local, State, and probably all private industries on a theory of "unified" developments.

It seems to the writer that this bill, if passed, and these policies if continued, will involve "Federal engineering," "Federal domination," and will prove a serious detriment to the future, of not only the Engineering Profession, but of all other professions, trades, and industries.

The writer is opposed to these policies as they involve a direct attack on the liberty of the American people, on individual freedom, and on industrial and professional independence.

Destruction of Private Property by Government Ownership

In the early development of the United States water transportation and water power had a great influence on settlement, and on community development. The eastern seaboard was the section of the country first available to settlement, and local water powers first became the centers of industrial development. The extension of settlement developed, normally and naturally, along the navigable waters. In early days, and quite up to the time of the Civil War, traffic was largely water-borne. Many of the early canals and the early river improvements, even while in themselves largely unprofitable, were warranted by the development of the country which resulted therefrom. As the ox-team and the canal-boat were displaced by the railroad and other more rapid means of transportation, internal navigation on rivers and canals has gradually assumed less and less importance; yet the tradition of the great value of internal navigation has caused the Federal Government to waste hundreds of millions of dollars on navigation projects, such as the Fort Peck Reservoir and the 9-ft channel in the Upper Mississippi River, of little present or prospective economic value.

In the same way the early use of water power has given it a value in the public mind, and more especially to the politician, which in many cases it does not possess. The fact that flowing water is "energy going to waste" is overemphasized and the false value attributed to water power is causing the Federal Government to undertake, and to encourage others to undertake, many developments that are entirely unwarranted, and some of which will probably be utter failures. Such success as is attained will be largely at the expense of private investments already made and will probably involve very great losses to many investors. It required many and severe losses in investments to prove the necessity of limiting competition in the matter of public utilities. In the past, competition was permitted which encouraged unintelligent expenditures and which resulted in heavy losses to competitors, and in entirely unsatisfactory and unduly expensive public service. Every loss to an individual is a loss to the nation. The destruction of private property makes the entire country poorer. It was only after much unfortunate experience that competition was eliminated and regulation substituted with much more satisfactory results.

The Administration, however, has again initiated the uneconomic principle of unrestricted competition, not only by the duplication of plants but also by paralleling transmission lines. Private enterprises suffer in competition with Government ownership because of the unfair advantages the latter has over the former. With capital furnished from taxation and not directly charged against the development, and by omission of taxes, certain Government projects can be made to appear successful and can destroy any private competition.

Indeed many private competitors are being taxed to effect their own destruction. "The right to tax is the right to destroy." The writer believes that intelligent public opinion is in favor of a "square deal" and the securing of satisfactory public service at reasonable prices through intelligent regulation and without uneconomical or unfair Government competition. If Government ownership and operation is other than politically desirable, it should be introduced in a frank, honest, and honorable way.

Government Function in Business Recessions and Recovery

Depressions are not uncommon in this or other countries. Many now living will recall some or all of the depressions of 1884, 1893, 1907, and 1921; and the depressions of 1817, 1827, 1837, 1847, 1858, and 1873 are matters of history.

In all previous depressions, the responsibility for recovery has been met largely by the business interests of the country, and recovery has been brought about within a reasonable length of time. The present is the first time that the Federal Administration has assumed the responsibility of recovery.

Administrative action is apparently based on the theory that recovery can be effected only by an entire re-adjustment of business and social relations under strict Federal laws and regulations, and that radical changes are necessary in the basic principles of the American Government, which principles have been highly satisfactory during the last one hundred and fifty years.

Long experience has demonstrated that legislation passed to improve social conditions frequently results in effects quite different from what were expected, and usually must repeatedly be changed and modified to accomplish the results desired even approximately. It seems strange, therefore, that apparently both public and political opinions still adhere to the belief that almost any evil in social or business relations can be remedied immediately by "passing a law." Satisfactory social changes are not accomplished so simply.

In spite of the warnings of experience, the Federal Government has been, and still is, passing radical legislation which can but delay recovery on account of the experimental character of the changes involved and the effect of the uncertainties on public confidence. C. F. Roos, Director of Research of the defunct National Recovery Administration (NRA), stated:

> "* * * although I was invited to join the NRA staff as a friend of the new administration and went to my work with a sympathetic attitude, the data accumulated by our research workers forced upon me the conviction that their type of legislation was a mistake. * * * These studies also show that NRA regulations of hours and wages kept business recovery at a standstill for a two year period."

To care for the unemployed, the Public Works Administration (PWA) was organized and has been the most satisfactory and effective activity attempted by the Administration. It has been, as far as it has been allowed to act, fully as satisfactory as could be expected from any Federal emergency agency. Securities issued by various municipal organizations and not otherwise salable under the conditions of the depression, were taken by the Federal Government, and the avails plus a bonus of 30% (which was afterward increased to 45%) were furnished under Federal supervision of projects.

The writer believes that the principle of Federal grants was a serious mistake. Such grants must ultimately be paid by the taxpayers of the entire nation and could not be made with any degree of equality throughout the country. The grant encourages extravagance in local public works, not immediately necessary, in order to obtain the subsidy which the locality must pay in part in any event. Works of real necessity would have been assured by Government loans at low rates, which the community benefited would ultimately have been obliged to pay. Such a plan, therefore, would not have resulted in an unfair distribution of benefits and taxes.

The increase in grant from 30% to 45% has resulted in some communities receiving 50% greater grants than other adjoining communities that were more prompt in the efforts to adopt relief work. Other communities that had not sufficient influence to secure such grants were independent enough to pay the entire costs for needed works and will now, by the payment of taxes, unfairly be obliged to help other communities that received Federal assistance.

In the main, the PWA seems to have been well organized and the works well selected and well conducted, when all conditions are considered. In some cases, however, monies were appropriated for the purpose of constructing municipal plants where cities were already served by private corporations.

Power districts were also organized and subsidized in locations adequately served at reasonable rates. In some cases not only were the projects subsidized by furnishing 45% of the cost, but the Administration purchased the bonds issued as no other market for such bonds was available, and thus such questionable projects have been or are being constructed at Federal risk with no other responsible organization behind them.

In all these power projects subsidized by the PWA as well as in the case of larger projects which have been undertaken by the Administration directly, private power companies already in the field have been seriously threatened, and thousands of stockholders who through their faith in fair treatment by Federal and State utility commissions have invested their limited savings in such enterprises, are threatened with serious loss through Federal competition, often in a market already adequately supplied with electrical facilities.

To start the work of the PWA in an intelligent and effective manner required time and, temporarily at least, it did not respond to the immediate need for work relief. To meet this need the Civil Works Administration (CWA) was organized. Under this Administration the municipalities paid in part for materials used, and the remainder of the cost and labor costs were met by the Federal subsidies. This work was not let by contract but was done by day work under Federal supervision. Thus began the diversion of work from its legitimate channels of the construction industry, and the spending of Federal funds for more or less unnecessary and questionable projects.

The PWA was somewhat free from political and local influence, and most projects undertaken were valuable and worth while. The men employed were largely, if not entirely, from the relief rolls, but they were required to perform a proper amount of work for the time spent and the pay received. Under the CWA the work frequently was performed under inexperienced supervision, and was done, in general, at a cost greatly in excess of what could have been paid under the corresponding industry. Even where other work was available, the men generally refused to accept private service which frequently involved less pay for more work as well as the loss of their positions on the relief rolls. Then, too, the selection of projects was far more susceptible to political and local influence, and the insistence of States and municipalities that they should receive their share of Federal funds, often regardless of the necessity or desirability of the work to be done. In this way, Federal funds could be spread more uniformly over the country, meeting more readily the necessity of local employment but without much regard to the real necessity or desirability of the work to be done.

The CWA was succeeded by the Federal Emergency Relief Administration (FERA) and, later, by the Works Progress Administration (WPA) by means of which relief was widespread and many questionable projects were undertaken, often at an enormously large expense. In this manner, billions of dollars were diverted from the construction industry on the revival of which full recovery and the permanent reduction of unemployment so largely depend.

The first relief appropriation of 1933 was for $3 300 000 000. It was commonly expected that all this money would be spent on a great public works program that would revive the construction industry and furnish needed public works as well as needed employment; but about 25% of this sum or approximately $825 000 000 was used for CWA and the other day-labor agencies. Of the still greater appropriation of 1934 of $4 880 000 000, about $2 500 000 000 was spent by day-labor agencies; and of the 1936 appropriation, about $2 200 000 000 went for day labor. In the last appropriation of $1 500 000 000, it is understood this will largely be spent in the same manner. Thus, the Government apparently has spent and will spend for day-labor enterprises about $7 000 000 000 to the great detriment of the construction industry which under normal conditions and when running at normal rates would employ, in addition to the number now engaged in construction work, an additional force fully equal to the number now on the relief rolls. To the writer it seems doubtful whether the Government has accomplished, by its day's-work program, 20% of the volume of the really valuable work which could have been accomplished had it been done under the PWA and by the construction industry.

Although every right-thinking man must recognize the desirability and the necessity of aiding the needy and taking care of the afflicted, few will agree with Harry L. Hopkins, Administrator, Works Progress Administration, that it is desirable to give the recipients of Government charity a "dignified status" and thus augment the feeling now all too prevalent among those who have received Government aid that the Government owes them a living and that it is a rank imposition that they be required to work at all for what they receive.

Although much has been said concerning intelligent planning and careful consideration of the ultimate results of engineering developments, it is evident that the public works program under construction, so far as its economic aspects are concerned, is the result, not of sound study and thorough investigation, but either of political expedience or irrational inspiration. It seems to be assumed that the ordinary intellect cannot hope to understand the objectives but must be impressed and satisfied by the fact that these projects are the grandest and greatest that the world has ever seen. The fact that some of them are the greatest mistakes that any Government has ever undertaken since the building of the Pyramids, and about as useful, may dawn slowly on the public mind, unless public opinion can be enlightened and freed from false propaganda and political misinformation.

During the decade in which the United States entered the World War many unfortunate enterprises were undertaken. The building of hundreds of ships that were never used; the construction of the Wilson Dam, of an auxiliary steam plant, and of nitrate plants that never were able to function; and the taking over and mismanagement of the railways—all are examples of Government failures which cost the country unnecessary billions of dollars and which should have served as a warning to the people of the necessity of greater intelligence in Congress and of limitations in the authority of the Executive so that vast public expenditures should be based only upon thorough investigation, careful consideration, and intelligent action.

It must be recognized, of course, that emergencies of war and depressions entail conditions which warp the judgment and which confuse intelligent thinking; that under such conditions the people of the nation seem always ready to follow a leader who has positive ideas and who can express those ideas in a manner that seems to make them rational. The idea seems to prevail that not knowing what to do, people should do something, regardless of consequences. The mistakes of the past have left a heritage of evil. The Shipping Board long added an annual deficit to the burden of the taxpayer, and the Wilson Dam and Power Plant furnished the basic excuse for the establishment of the Tennessee Valley Authority.

Government Interference as a Permanent Policy

Power.—The Tennessee Valley Authority is an undertaking of the Federal Government of an idealistic "unified" development of the valley of the Tennessee River for navigation, flood relief, power development, and various altruistic ideals. Power development as an objective is not a legal function of the Federal Government. It can be undertaken only if incidental to navigation. Power development is undoubtedly the primary objective of the TVA. It is also to constitute a "yardstick" for measuring the cost of generating power for comparison with privately owned power utilities.

The TVA is an interesting project which would not be warranted at Federal expense during prosperous times on account of its purely local character and benefits. Under conditions of the depression, and as an aid to national recovery, it is utterly ill-advised and unwarranted, except as a political gesture of undertaking something big and novel under the excuse of aiding recovery. Like most of the Federal works now in progress, exact and detailed information concerning this project is impossible to obtain, and its detailed ultimate objectives are probably as indefinite in the minds of the Administration as they are to the taxpayers.

As a "yardstick" for measuring the cost of power, this and other Federal hydro-electric power projects are complicated by the fact that "navigation," "flood protection," and other "objectives" are also involved, and that both first cost and cost of development, operation, and maintenance must be assigned to these different objectives. Can it be hoped that the TVA or other Federal Authorities, the leaders of which are responsible for the success of these power projects, will be able to assign fair and equitable parts of the great investments to each of the so-called objectives of the projects, or will power unfairly be found to be a mere by-product of navigation and flood protection?

Arthur E. Morgan, M. Am. Soc. C. E., Chairman of the Tennessee Valley Authority, stated:[10]

> "In case public power is used as a 'yardstick' or as a measure of what the private power industry should charge for its services, then it is imperative that records and accounts be honest and fair and open and that there be no hidden element of subsidy. The very fundamental element of such comparison is honesty, fairness, and openness in measurement. Take away those characteristics and the supposed comparison may becloud the issue rather than clarify it."

All fair-minded engineers will certainly agree with the foregoing specifications for a Federal "yardstick"; but it is fair to ask how it is to be established in practise.

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[10] "Public Ownership of Power," by Arthur E. Morgan, *Atlantic Monthly*, September, 1937, p. 342.

On account of the absence of exact data, the TVA project is very difficult to analyze. The best information the writer has been able to obtain is the testimony before the House Appropriations Committee.[11] At this hearing the inquiries of the members of the Committee brought out certain estimates which, so far as the writer is aware, are otherwise unavailable.

The pertinent data given in this evidence are as follows:

Continuous capacity of integrated system, in kilowatts		666 000
Average annual power output, in kilowatt-hours		5 780 000 000
Estimated annual income		$23 120 000
Less annual cost of power	$12 200 000	
Less other expenses	1 170 000	13 370 000
Net revenue		$ 9 750 000
Estimated cost of dams, locks, and reservoirs		$343 700 000
Cost of power-house and equipment		135 450 000
Total estimate for navigation, floods, and power		$479 150 000

In these estimates no allowance is made for interest during construction or for carrying charges while the market is being developed. An estimate of $20 000 000 for contingencies is also omitted as possibly unnecessary (a deficiency appropriation from the Federal Treasury always can be anticipated).

All these expenses are just as much a part of the actual cost of a power plant development, Federal or private, as the cost of buildings or machinery. Taxes during the development period can be ignored in this case, because, in lieu of taxes, Congress has provided that the States shall receive 5% of the gross income—no sales, no taxes; a very generous provision for the TVA.

Based on the estimated income the TVA finds it can pay the entire cost of the project in fifty years, without interest. Interest, however, is as real an expense as capital cost, cost of operation, or cost of maintenance. The estimated net return (without interest), even if it could ever be realized, will not pay interest on any reasonable basis even on the fair cost of the power project alone.

Even if the TVA estimates are correct, the real net income will be negative and the actual cost of this project to the taxpayer will probably be two or more times the total estimated cost before the Federal bonds, issued to meet the cost of this project, are paid.

It is assumed for this analysis that the estimates of continuous power are correct. Such estimates are of necessity always doubtful on account of great variations in annual stream flow, and in this case, are still more doubtful, on account of necessary regulations for navigation and flood control. It should also be noted that the annual output is based on the daily use of the continuous capacity for 365 days per yr.

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[11] First Deficiency Appropriations Bill, March, 1936.

It has been assumed in one critical article written on this subject,[12] that, on account of navigation and the necessary load factor (which can only approximate 50%), not more than one-half this power will be available. The writer believes, however, that with the proposed installation of machinery contemplated it may be possible to generate the full daily output of kilowatt-hours within 24 hr at any reasonable load factor without serious effect on navigation. The effect of systematic efforts for flood control will probably be more serious. The income estimate is calculated on the basis of 4 mills per kw-hr on 100% of the estimated continuous capacity.

From the same source of information the wholesale rates established by the TVA on October 22, 1934, are:

Demand charge, in dollars per kilowatt	0.90
First 100 000 kw-hr, in mills	4
Next 200 000 kw-hr, in mills	3
Next 700 000 kw-hr, in mills	2.5
All more than 1 000 000 kw-hr, in mills	2.0

From these rates it is evident that the annual returns, if and when markets are found for the entire output, cannot average as much as 3 mills per kw-hr and that the returns, therefore, would be more than $5 780 000 less than estimated by the Authority. On this basis the project would require about 125 yr, without interest, to pay for itself. (Press dispatches of August 20, 1937, state that a chemical company has contracted with TVA for 12 000 kw of "firm power" at 2.74 mills per kw-hr and 12 000 kw of "run-of-river power," at 2.27 mills per kw-hr.)

The estimated output is based on continuous efficiencies of more than 80%, which is scarcely possible to obtain under all conditions of head and load. There are other losses due to plant use, transmission losses, transformer losses, and losses due to regulation for navigation and flood prevention. Basing his judgment on a somewhat extended experience the writer believes that if, with a complete market, the Authority realizes 80% of this estimate, it will be accomplishing more than should normally be expected.

The writer recognizes that there is available, in addition to the firm power estimate, a large block of irregular seasonal power that will be available during periods of more than average flow, which the TVA calls "run-of-river power," and which is more commonly termed "dump power." He is also aware that contracts have already been made to supply a considerable quantity of such power. This power can be made firm where auxiliary steam power is installed. No such auxiliary power is proposed by the TVA at present. Whether or not this power project can be made to pay on a fictitious cost estimate is of little moment. The real factor of importance is that the TVA seems to be deliberately attempting to "becloud the issue" and to set up a "yardstick" that is clearly dishonest instead of "honest, fair, and open."

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[12] "Black and White Coal," by C. W. Kellogg, *Bulletin*, Edison Electric Inst., May, 1937, p. 163.

In an attempt to show honesty of purpose the cost of power is estimated on the following basis which on its face would seem to be fair: Interest, 3.5%; depreciation, 3%; taxes, 1%; operation, 1.5%; the total being 9% of the cost of power development. The estimate of power house and equipment ($135 450 000 at 9%) equals $12 191 500, which is essentially the same as the annual cost of power previously quoted. This shows conclusively that the cost of the power "yardstick" is estimated at $135 450 000, and does not include any part of the cost of dams and reservoirs. Under such conditions, are these estimates of cost, income, and profits, free from "hidden elements of subsidy"?

The system of high dams is believed to have been built almost exclusively on account of power development, and any honest estimate of the cost of such development should include a large portion of this cost.

The Army Engineers have estimated[13] the average seasonal flood loss in the Tennessee Valley and its tributaries ($1 784 061). What reasonable sum should be expended to prevent this annual flood loss? A fund of about $51 000 000, if set aside at 3.5%, would pay such annual flood damage. The works proposed, however, will not furnish 100% flood protection. Chattanooga, Tenn.,[14] will have to expend $15 000 000 to secure effective relief; and probably other localities may require extensive local works. In any event, an expenditure of $36 000 000 would seem to be about the limit for this purpose, although indirect damages might warrant some additional expense.

The Army Engineers have estimated[15] the cost of navigation works with low locks and dams at $74 709 000. Although the writer doubts the economic desirability of this investment, he agrees that the higher dams will possibly improve navigation facilities and that an increase to, say, $90 000 000 for this purpose might be warranted.

The estimate of cost for the eleven dams and power plants (as of March, 1936) was $479 150 000; the difference in cost between this sum and a fair estimate for navigation and flood protection will indicate the real cost of power development:

Total estimated cost		$479 150 000
Cost of navigation	$ 90 000 000	
Cost for flood protection	36 000 000	
	$126 000 000	126 000 000
Total cost of power installations		$353 150 000

This is more than 2.5 times the estimate of the Authority, and to it should be added interest during construction, carrying charges during development of the power market, taxes (which would be paid by a private company), and presumably cost of transmission, etc.

As a "yardstick" the writer believes that "the supposed comparison may becloud the issue rather than clarify it." It is quite evident that power, the real primary object of these Government ventures, is to be regarded financially by the Administration as a by-product of "navigation" and "flood protection."

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[13] H. R. Doc. No. 328, 71st Cong., 2d Session, p. 730 *et seq.*

[14] "Flood Control at Chattanooga, Tenn.," by E. L. Chandler, *Civil Engineering*, June, 1937, p. 405 *et seq.*

[15] H. R. Doc. No. 328, 71st Cong., 2d Session, p. 5.

The Chairman of the TVA has expressed the opinion[16] that the project is worth all it will cost for navigation and flood protection alone; also that the value of the flood-protection feature will be worth $100 000 000 to flood protection on the Lower Mississippi River. As to the first opinion, it is so extreme that it seems useless to discuss it. As to flood protection value, the writer understands that one primary objective of the TVA is flood protection in the Tennessee Valley. This may or may not co-ordinate with the interest of flood protection on the Mississippi River. The Miami District of Ohio, with automatic reservoir discharge, adds to the floods in the Ohio River about as often as it reduces such floods, depending on the direction of movement, and the length and intensity of storms. Can the TVA accomplish better results?

On the unwarranted basis of the TVA estimates, it is apparent that it estimates the cost of power at about 2.11 mills per kw-hr. On a more reasonable basis of cost of the plants and of the power that can be sold, the probable cost will be from 5 to 7 mills per kw-hr.

Steam power can certainly be generated in the Tennessee Valley for not to exceed 4 mills per kw-hr. As about 65% of the cost of coal is due to labor, it would seem that if recovery was the true purpose of these power developments it would be of greater advantage to use steam power at a much less capital cost as well as a greatly reduced operating expense.

That the power to be generated by this development will never warrant the great expense involved, is further indicated by the findings of the National Resources Committee, which, in its report[17] to the President on June 18, 1937, states that "the high efficiency and the low fixed charges now possible in large fuel burning plants, places hydro-electric developments at a disadvantage in most sections of the United States if low power cost is the objective."[18] (By a majority vote the Tennessee Power Commission has granted permission for the construction of a $2 000 000 steam generating plant in Nashville.)

If there is any honest and intelligent defense for the expenditure of approximately half a billion dollars which is apparently now being largely wasted in the Tennessee Valley, it should certainly be made manifest. This is especially true if the Administration ever expects to secure the backing of public opinion for repeating this seemingly irrational experiment in six other sections of the country.

It seems apparent that the TVA is only slightly less absurd than the Passamaquoddy Project which, however, it greatly exceeds in cost. It is quite equal to the Grand Coulee Project by which it is proposed to develop not only an enormous amount of power for which there can be no market for years to come, but also to irrigate more than a million acres of land, largely in private ownership, for which there will be no demand within a still longer period.

Reclamation.—Almost the entire record of irrigation under Federal laws and Federal control in the United States has been a series of economic failures.

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[16] Testimony before Appropriations Committee, 1936, *Proceedings*, Am. Soc. C. E., February 1938, p. 253.

[17] *Electrical World*, August 23, 1937, p. 260.

[18] *Loc. cit.*, p. 52.

The Carey Act of 1894 resulted, by 1925, in the irrigation of only 1 158 926 acres under private and corporate efforts. In general, the cost of bringing water to the land was so great that success was practically impossible. This was due (in part, at least) to promoters' profits. Such lands were partly colonized through misrepresentations, and, in many cases, the settlers lost everything in their attempt to cultivate the land.

In 1902, reclamation was started by the Government as a self-sustaining and self-liquidating venture. These projects were to be financed from a revolving fund. The time of payment was first planned at 10 yr, then 20 yr, and, finally, at 40 yr, but the funds refused to revolve.

The following description[19] of the situation that followed can but remind the reader of recent history. Such results are, with few exceptions, the indispensable condition of all Federal ventures into business:

> "Once irrigation work got under way, every state west of the 100th meridian besieged Congress to approve one or more pet projects. When government money was to be had for the asking, it put a new zest into politics. Here was something tangible to work for—pie to bring the faithful back home. * * * In consequence every patch of brown sagebrush became a prospective Garden of Allah in the mind of some one. All it needed was a Moses with his staff to make the water gush forth, and lo, the desert would burst into bloom.
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> "Those were grand days for the promoters, the real estaters, the speculators, the chambers of commerce, the business men and the politicians. Of the latter every one who carried a staff aspired to be a Moses, and he was the most honored and the most loved who was able to bring forth the biggest fountain.
>
> "Every project had a political father and mother and a host of godfathers, sons, uncles, and other close relatives. * * *"

Flood Control.—The space available will not permit further consideration of Federal public works now under construction. The writer desires, however, to consider briefly the matter of Federal flood-control works that apparently is soon to become important. An appropriation of $34 177 000 chiefly for flood control on the Ohio River has already been passed and approved. This is only a start on the vast expenditure that may be involved in this important problem.

The public has many erroneous ideas concerning the cause of floods. When the writer was one of the commission called to China, in 1914, to determine what could be done concerning the control of the Huai River which frequently inundated its lower valley, it was found that the people of China believed that these floods were produced by the Hydra, a mythical dragon, which they believed dug into the hills and let out the flood waters. Even the average citizen of America smiles at such a crude idea but, nevertheless, believes in the no lesser superstition that floods in America are caused by "deforestation." It is apparently not generally known that floods, quite equal to any that have recently happened, occurred long before the forests of the United States were cut for timber. Neither does it seem to be understood that devastating floods occur only in valleys and lowlands which have been built up by the rivers in the past for the purpose of passing their flood waters. When mankind builds cities, factories, or farms on such lands, troubles from the occasional high floods must necessarily be expected. Flood control is, in fact, an attempt to save the people from the results of their own folly and ignorance, not in the cutting of timber, but in attempting to usurp the lands which the rivers occasionally need to discharge their maximum floods. In most cases, these lands, convenient to the river for the purposes of navigation, water supply, and pleasure, are the most accessible for railroads, highways, and buildings during normal river flow, and the most attractive sites for cities. The farm lands of the valleys and flood-plains, often greatly enriched by silt deposits, were naturally sought for farms. The great floods usually occur only at long intervals of time. The damages due to occasional floods were not great when the country was young and the people few in number. With the great growth of the centers of population and manufacturing often located in the exposed positions, flood losses have become enormous.

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[19] Editorial in the *Country Gentleman*, January, 1927, p. 26.

Some communities that have experienced heavy losses in property and life, such as the Miami River Valley, have had sufficient energy and foresight to accomplish flood control at their own expense. Under present conditions of Federal spending, it is only to be expected that such communities are now calling on the Federal Government to save them from their own shortsightedness. The Federal Government has already committed itself to power development and flood control under the guise of navigation. Almost everywhere that floods are experienced, navigation either at the point where the damage occurs, or on the main river below, may be used as an excuse for Federal expenditures.

The writer believes that, in the future, with the precedent of the TVA and other recent Federal projects where flood control has been named as one of the objectives, the Federal expenditures for flood control will be one of the great items of Government expense. It is to be hoped that both public and political opinions may be so informed that their great and important works will be handled in an intelligent and economical manner.

In the last few years so much has been heard of "little waters," "up stream engineering," and "holding each drop of water when it falls," that at times it has looked as if flood control would be attempted by means of a few small reservoirs at the head-waters of streams, by a few small dams to prevent erosion, and by the planting of forests.

In his message to Congress, on floods, President Roosevelt has stated, however, that floods cannot be prevented by forests alone or by dams alone, but that such control demands the consideration of many factors. He has deferred action on the plans of the Army Engineers for certain flood-control projects until other interested Departments of the Government could be consulted. It is quite certain that the theoretical values of forest, improved farming methods, and erosion control will not be overlooked when these projects are finally considered.

The writer fully appreciates the importance of forestry, erosion control, and improved farming methods. All are important and should receive due consideration in their time and place. There is no question but that the creation of forests on the hills and mountains, the grassing of sloping fields, and the construction of check dams for preventing erosion, have an effect on run-off during moderate rains and will distinctly reduce ordinary run-off and even light floods. With frozen or saturated soils, however, and the downpour of maximum flood-producing rainfalls, such preventives of flood run-off are only as effective as footprints in a field, which may hold back water to a small extent, but are so utterly insignificant as to be unworthy of any consideration under these extreme conditions.

There are no men more interested in every means of reducing maximum floods than engineers who have studied the subject of flood control. The effect of forests on stream flow has been carefully investigated and thoroughly studied many times, but the writer has yet to be convinced from such studies that deforestation or reforestation has ever had the slightest practical effect on the maximum flood of any stream.

Forests for pleasure and profit, and for the prevention of erosion, have real value; and reforestation for its real intrinsic value has the earnest support of the writer.

The study of cause and effect is a somewhat difficult and intricate matter. The writer was in China at the time of a lunar eclipse. The natives thought that a dragon was attacking the moon. Fire crackers were exploded, gongs were beaten, and pandemonium broke loose. The results were apparently fully satisfactory. In a short time the trouble was over. The moon came forth serenely, even the part that had apparently been bitten out by the dragon was replaced. The moon was saved to the satisfaction of the Chinese people as it has been many times in the 4 000 yr of Chinese history and as it probably will be for hundreds of years to come. Such ideas are not confined to the Chinese. The writer found in the *Proceedings* of the Society not long ago the statement that the author knew that forests produced rainfall because he had frequently stood on the sunlit lowlands of Central America and had seen heavy rainfalls over the forests of the foot-hills and mountain slopes. Did the forests produce this rainfall or do trees normally grow where rain naturally falls? The people of the United States may, perhaps, receive a somewhat expensive answer to this question when they have the final results of the shelter belt, which, the writer understands, is still being planted on the Great Plains in spite of the refusal of Congress to appropriate money for that purpose.

Responsibility of the Engineering Profession.—The welfare of the future depends much on the Engineering Profession. The engineer, through his training and through his experience in connection with past enterprises, is best fitted to make a truthful analysis of the results which must be anticipated from most of these great projects. If he will assume the responsibility of informing the public concerning those matters, he can assist materially in creating and arousing an intelligent public opinion that will demand sane and intelligent consideration of future governmental activities.

The members of various Engineering Societies in Chicago, Ill., recently voted to subject Government public works projects to critical examination by competent members and to publish the findings. Similar action should be taken by every Engineering Society in America. Much good can be accomplished by such action. Will the Engineers and Engineering Societies of America rise to this important duty?

The engineering features of the TVA, of navigation, and other works done under the Army Engineers, of the Reclamation projects, and of such of the present projects, as the writer has seen, are, in general, well designed and well constructed and he has nothing to criticize in this regard. Fortunately to this extent politics has been eliminated; but sound engineering cannot make an uneconomical project an economic success.

Government projects, both in the present and in the past, have been based most largely on political expediency, especially when the Government has ventured into business enterprises. This has been true regardless of the party in power, whether Federalist, Whig, Democrat, Republican, or Progressive. The principle of spreading the spoils of Federal activities over the entire country in order to satisfy the electorate, regardless of any real justification of the individual projects involved, has become too well established and, in the opinion of the writer, is the only real explanation of many Federal activities of both the present and the past.

The Grand Coulee Project, as an immediate objective, was so utterly condemned by official reports of the Army Engineers, the Reclamation Service, and the Department of Agriculture before it was undertaken under executive order, that it seems hopeless to anticipate that Government undertakings can ever be based on sound economic principles, unless an intelligent public opinion can be both created and aroused to active influence in municipal, State, and Federal activities. The conditions prevailing at present are certainly a sad reflection on the present so-called civilization. Unless the thinking men of this country, the men who are willing to stand for a square deal, for sanity, for truthfulness, honesty, and intelligence, aside from party and aside from personal interests, soon begin to take an active part in public affairs, the writer sees little future hope for the success of democracy or of the American nation.

HAZARDS OF UNECONOMICAL CONSTRUCTION

By Henry Earle Riggs,[20] Past-President, Am. Soc. C. E.

Synopsis

The magnitude of the unemployment problem which faced the present Administration when it came into power, the country-wide extent of the relief program, the fact that billions of dollars have been paid out since 1933 either in the form of direct relief or for many hundreds of projects of work relief, and the multitude of different agencies for the distribution of relief funds—all of these influences have tended to divert the attention of the general public from the vast program of major public works which has been entered upon as part of the relief program by various Federal agencies.

Many of these great undertakings are wholly unknown to the public. Where they are located; what they are for; the great magnitude of some of them; the cost of them running into tens or hundreds of millions of dollars; the annual costs of interest, maintenance, and operation that will be incurred after they are completed; the assurance that they have of success or failure—all of these phases of the problem are a closed book to the great majority of taxpayers. The non-technical visitor to the construction work is awed by the immensity of it and is furnished with printed information, some of it in the nature of propaganda, which emphasizes the great size but is silent on all matters of economics.

What kind of projects should be adopted for the purpose of furnishing work relief? Is the nation justified, in a time of great stress, and of rapidly rising national debt, in engaging in business ventures that are new to Government in the United States? Is it economically sound to spend vast sums of money in building works for the far distant future, which are distinctly not needed now? Is it wise to build without long and careful planning when the building involves tens of millions of dollars? Is a project economically sound when it may be seriously questioned whether there is a public need for it, a demand for its services, or a doubt as to whether it can be self-liquidating?

These and other similar questions which are suggested by the present program of public works should be proper matters of discussion by the Engineering Profession. This paper is an attempt to present the available evidence not only as to a few of the projects approved in the four years, 1933 to 1937, but also of some projects of earlier administrations which undoubtedly can be classified as failures of publicly owned State and Federal business undertakings.

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It is written for the purpose of directing attention to the need for long-time planning, by Congress, of Federal public works and of the necessity for establishing definite requirements as to accounting, publicity of statistics, and the elimination of subsidy in connection with the operation of any publicly owned business or public-service enterprise.

No criticism is made or is intended to be implied of the design or construction methods of the projects discussed. The work of the engineers on the design and the handling of the field work of construction on all the major projects which the writer has visited, is excellent. Bonneville Dam and the dams on the Tennessee Valley Project are fine examples of modern engineering skill. Grand Coulee Dam and Fort Peck Dam will be the greatest structures of their kind in the world. They will continue as great monuments in the desert to the ability of American engineers to overcome well-nigh unsurmountable difficulties.

The construction plant that is used on these major projects and the construction methods and administration come in for equal praise. The most modern types of labor-saving equipment are in use. The impression one gains on a visit to the work, whether it is done by contract or by force account under the Corps of Engineers of the U. S. Army or other Government agency, is one of utmost efficiency of men and machines. Whatever there is of future credit on account of these undertakings will belong to the designers and the builders.

The question is whether such projects are economically justifiable as relief undertakings. In many cases, projects have not been selected by the Congress and, in some cases, they have been adopted in the face of adverse recommendations by competent engineers. The fact that the construction of many of the projects constitutes an entry of the Government into direct competition with private enterprise in the electric power or transportation industries has no bearing on any of the criticisms made herein. The writer is a thorough believer in the regulation of public service corporations and in the public ownership of utilities whenever the public authority can give better service, or lower rates, or can safeguard health or other public interests, more effectively than private corporations. He is by no means convinced, however, that Government ownership and operation of great and complex utilities is likely to be as successful or as efficient as fully regulated private ownership. A study of the results of Federal and State ownership does not tend to increase confidence in the success of many of the businesses thus involved.

This paper is an attempt to present exact facts regarding certain selected projects, both old and new; to present analyses and comparisons which throw light on the economic phases of the projects; and to indicate the financial hazards attendant on the hasty adoption of projects for great public works for which no complete surveys have been made, for which final plans and estimates are not available, and for which not only have no contracts for the sale of the product been made but, often, no present market for the product is in sight.

Passamaquoddy Tidal Power Project

The first example of present-day projects selected is the Passamaquoddy Tidal Power scheme, situated within two miles of the Canadian International Boundary on the Maine coast. This is an electric power project. As originally adopted by the Public Works Administration (PWA), it was a modification of a promotor's scheme which had been under discussion for about twenty years. The plan was to develop power from the high tides at the mouth of the Bay of Fundy by creating a reservoir out of Cobscook Bay, in Maine, which should always be held near high-tide level. Power would be generated by releasing water from this reservoir through turbines after the tide had fallen 4.5 ft, and continuing through low tide and until the rising water had returned to a level of 4.5 ft below that in the reservoir. This would permit power development for about 7 hr out of every 12. Because of the constant change of head during the 7-hr period and the fact that in each month the range of tides varies from about 10.5 ft to 24 ft, there can be no possible uniformity of power output, and, consequently, a small amount of firm power when the total installed capacity is considered.

To supply power during the 5 hr of each 12 when tidal power is not available it was proposed to build an artificial reservoir about 120 ft above sea level, to use the secondary power developed during the 7 hr of tidal operation to pump sea water into this high-level reservoir, and to permit it to run back into the ocean and operate turbines during the 5-hr period and thus furnish the power required for the stand-by plant.

The original plan for an international project was rejected in 1925 and 1926 by responsible private interests. The modified plan was submitted to the PWA in 1933 and twice rejected, and again turned down as unjustifiable after hearings by the Federal Power Commission. The original request to the PWA was for $47 000 000.

On May 16, 1935, Harold Ickes, Secretary of the Interior, announced that his Allotment Board had approved an allotment of $10 000 000 initially on an estimated total cost of $36 284 000.

The task of building the project was assigned to the Corps of Engineers, U. S. Army. Preliminary construction work was begun at once, and at the same time extensive surveys, explorations, and research studies were carried on and an estimate of cost was made. That estimate, which has never been made public, is known to approximate $62 000 000. Shortly after it was submitted, a commission of three nationally known and thoroughly competent hydraulic engineers (Charles H. Paul and Joseph Jacobs, Members, Am. Soc. C. E., with

Professor William F. Durand, of Stanford University) was appointed and directed to review and reconcile all estimates of cost. That report likewise has been filed and not made public. It approved in general the plans proposed by the Corps of Engineers, and the estimate of the cost of construction was somewhat higher than was estimated by the Army Engineers.

It does not seem out of place to emphasize here the fact that these are but two of a large number of engineering reports which should be available to members of the Congress, the public, and the Engineering Profession but which repose in the files at Washington, D. C.

The estimate of $62 000 000 was based on the original plan which included the high-level pumped storage reservoir and was for the same work covered by the estimate of $36 284 000 given out by the Secretary of the Interior at the time the project was approved. The Army Engineers proposed a revised plan which eliminated this high-level reservoir and substituted therefor a Diesel or steam plant as an auxiliary source of power to be used when the tidal power plant was non-operative. The estimated cost of this revised plan was approximately $37 000 000.

A paper by Captain Hugh J. Casey, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., presented before the Boston (Mass.) Section of the American Institute of Electrical Engineers,[21] stated that the tidal plant will be capable of producing "a continuous output of 30 000 kw on a 100% load factor" and that "the combined plants will be capable of an annual net output of 260 000 000 kw-hr of prime energy."

It is obvious that either $62 000 000 or $37 000 000 for a plant with a firm power capacity of only 30 000 kw is many times the cost of a modern steam plant. It is equally obvious that if a steam or Diesel plant is built for stand-by use during 5 hr out of every 12 there is no reason why it should not operate all the time. If this stand-by plant is built it reduces the tidal power part of the project to an absurdity. It is evident that none of the engineering boards that studied this project for private interests or for the PWA between 1924 and 1933 could find any market that would warrant such an expenditure. The data published by the Federal Power Commission indicate beyond any question that there is no present market for this power in Maine.

In July, 1935, the Hon. Louis J. Brann, Governor of Maine, appointed a commission composed of Kenneth C. M. Sills, President of Bowdoin College; and Messrs. W. K. Campbell, W. F. Cram, H. B. Crawford, and E. S. French. The report[22] of the Commission which finally secured favorable action by the Administration made no attempt to justify the project as economically sound. This report stated:

> "We would argue for federal support for this project largely on the grounds of social desirability, that is, of benefit in the long run and in many ways without definite assurance that the project would be within a definite number of years self-liquidating. * * * In the present economic situation private capital would not be available and the government has already denied a loan to a private corporation on the ground that a sufficiently sure market for the power generated has not been shown."

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[21] Not published.

[22] *Portland (Me.) News.*

Approximately $12 000 000 was allocated to this project. Work on housing, surveys and explorations, research studies, and some minor construction work—all costing slightly more than $6 000 000—was done before Congress refused to make any further appropriations.

IMPROVEMENTS FOR NAVIGATION

The second illustration that comes to mind is a navigation project. Navigation on the Missouri River by stern-wheel steamers commenced prior to the Civil War. It was of major importance to the West in the days before the railroad and, although the tonnage moved was not very large, it continued during the period 1860 to 1885, when the railroad network had fully covered the district between the Mississippi River and the Rocky Mountains.

The first appropriations for improvement work on the Missouri River were made under the River and Harbor Act of August 14, 1876,[23] and the original plan for comprehensive improvement of the river was adopted August 2, 1882.[24] The earlier Republican Administration initiated these expenditures on the Missouri River; and the present projects were adopted or modified in 1912, 1917, and 1927. Only the Fort Peck Dam Project was adopted under the present Administration. Therefore, prior Administrations must share any criticism as to the propriety of the great expenditures on this river. Over the years from 1876 to June 30, 1933, the total expenditures for new work and maintenance was a little less than $95 000 000, and during those years it was termed by the critics, "pork barrel legislation."

On October 3, 1933, "it was recommended to Congress that the project for navigation on the main stem of the Missouri River as heretofore authorized, namely from the mouth to Sioux City, be vigorously pressed to completion, and that, in addition the reservoir at the site of Fort Peck be built to the maximum practical capacity, and be operated primarily for navigation."[25]

During the first three years of the present Administration $47 000 000 was spent on this river improvement, and more than $60 000 000 of an estimated $109 000 000 was spent on the construction of Fort Peck Dam,[26] so that the net total of Federal money spent for navigation on the Missouri River to June 30, 1936, was $202 495 633, and the total appropriations amounted to $248 232 000, leaving $45 736 000 of available funds for work in the fiscal year 1937. To complete Fort Peck Dam another $18 500 000 will have to be appropriated, bringing the total for navigation on the Missouri River to $267 000 000, or more.

At least 90% of the reported traffic on the Missouri River was material for Government construction, and has been for the past eight or ten years. In 1929, when total traffic was reported as 1 372 000 tons, all except 6 155 tons was noted as "used in river improvement and moved by owners." Since that year specific items used in Government construction are not indicated but the general footnote appears in each yearly report, as follows, "a large percentage of the traffic reported was for river improvement work."

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[23] Rept. of Chf. of Engrs., U. S. Army, 1933, Pt. I, p. 712.

[24] *Loc. cit.*, p. 707.

[25] *Loc. cit.*, p. 708.

[26] Rept. of Chf. of Engrs., U. S. Army, 1936.

There can be no possible criticism of reasonable Federal expenditures for keeping navigable and navigated waterways and harbors in condition for use. Comparatively few such waterways can be self-liquidating and the value of any of them to the public is incapable of exact determination, such value being principally in the form of modified rail rates over large areas due to the competition of water transportation. One yardstick of the economic justification of these expenditures on the Missouri River can be had by comparison with like expenditures on the Great Lakes, and by a comparison of the tonnage and value of freight moved on the two waterways.

The total cost of all capital and maintenance expenditures on the Great Lakes between 1824 and 1936, including all work on the harbors of the Great Lakes and the navigable rivers entering them, all connecting rivers such as the Detroit, St. Clair, and the St. Marys, with the locks at the "Soo," in short, all Government expense of every kind on this great system of internal waterways, was $248 277 000, or almost exactly the same as the appropriations on the Missouri River to the same date ($248 232 000).

In 1935 gross total traffic at all Great Lakes ports totaled 186 879 000 tons, or more than 101 times as much as the 1 841 000 tons reported on the Missouri River, nearly all of which was construction material, and therefore, temporary traffic. The total value of Great Lakes freight was $2 913 070 000 which is 236 times as much as the $11 941 000 total freight value on the Missouri River. If the expenditure of $95 000 000 for navigation on the Missouri in a period of 57 yr was "pork barrel" is not the appropriation of $153 000 000 in three years "pork barrel supreme"?

FORT PECK DAM

Fort Peck Dam has been considered as part of the Missouri River navigation project. Further brief reference to it appears to be necessary. This great structure is about 1 600 miles northwest of Kansas City, Mo., in North Central Montana. It will be the world's largest earth dam, requiring more than 100 000 000 cu yd of earth and gravel in its construction, with spillway, tunnels, and auxiliary structures of equally imposing magnitude. It began with an estimated cost of $86 000 000.[27] The revised estimate of 1936 places the cost at $108 600 000.[26] It is expected that all major structures will be finished by June 30, 1938, and that the dam itself will be finished and placed in operation in the autumn of 1939. It is a magnificent piece of engineering design and construction, and no possible criticism can attach to the builders of it.

Although the plans provide for a possible future power plant there is small likelihood of one being built except for irrigation pumping as the territory is sparsely settled and there are not more than three or four towns with more than 5 000 population within a 150-mile radius. Ample power is now developed in the western part of the State to meet every requirement for years to come. No present irrigation development is proposed, although, like power, this is a possible future by-product of the dam.

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[27] Rept. of Chf. of Engrs., U. S. Army, 1934.

The Division Engineer of the Corps of Engineers, U. S. Army, is quoted[28] as placing $3 000 000 as the justifiable expenditure at Fort Peck for flood protection in the river below St. Joseph, Mo., and he states, further, that "it would have little if any effect upon Mississippi floods."

This clearly is a navigation project. It will insure the maintenance of a minimum flow of 30 000 cu ft per sec at Yankton, S. Dak., and at points down stream. Power, irrigation, and flood control are practically "out of the picture."

This dam will be completed within a year or two. Water is now (1938) flowing through the tunnels, all the auxiliary structures are nearing final completion, and there will be left after this year only the completion of the earth dam.

POWER IN THE NORTHWEST

In the summer of 1933, in the first wild rush to find projects for putting the unemployed to work at once, $63 000 000 was allocated by the PWA for the construction of a low dam, which later could be increased in height, across the Columbia River at the head of the Grand Coulee, in Northern Washington, and the building of a power "yardstick" for the Northwestern States. This project had been estimated by the Army Engineers to cost about $108 000 000.[29]

Politics "got to work" as soon as this project was approved. The State of Oregon had a power and navigation project for which it demanded approval, and within a few weeks $31 000 000 was allocated by the PWA to the Bonneville Dam and Power Plant, which was close to the power market, which alone was capable of furnishing as much power as will be needed in the area, and which extends deep-water navigation about fifty miles to The Dalles, Ore.

Construction at Grand Coulee was put in charge of the Bureau of Reclamation, a contract for the dam and power plant was entered into on July 16, 1934, and work on the dam was actively begun.

The fact that the Government had commenced work on two power projects, either of which was capable of meeting all reasonable present or anticipated needs for additional power in Washington, Oregon, and Idaho, the three States having the cheapest power in the country and the greatest per capita consumption of power, indicated the necessity for a reconsideration of the original plans for Grand Coulee Dam. Therefore, extended studies were made by Engineers of the U. S. Bureau of Reclamation, and on June 5, 1935, an official order was issued changing the contract from one for the low dam and power house to a contract for the foundations of an ultimate high dam[30] which would back water to the Canadian Boundary and constitute the first step in a gigantic project to irrigate more than a million acres of land known as the Columbia Basin.

The plan adopted by the Bureau was one which promotors had been urging for a number of years, in which irrigation was the final objective, but irrigation which could only be justified if it was subsidized by the sale of vast quantities of power. The order made most revolutionary changes in the contract, substituting foundation work on a much larger dam, for the originally planned complete low dam and power plant, all in the bed of a swift river with a mean flow of more than 100 000 cu ft per sec, and a maximum recorded flow of 460 000 cu ft per sec,[31] involving increases in quantities of more than 3 000 000 cu yd of common excavation and more than 850 000 cu yd of mass concrete. The contractor was to do this work for the same price as that named in the contract for the original project "subject to adjustment."[30]

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[28] *Engineering News-Record*, November 29, 1934.

[29] House Doc. No. 103, 73d Cong., 1st Session, p. 735.

[30] Paper by Kenneth B. Keener, Senior Engr., U. S. Bureau of Reclamation, *Engineering News-Record*, August 1, 1935.

The Columbia Basin Reclamation Project, substantially as now contemplated by the Bureau of Reclamation, was estimated by the Corps of Engineers, U. S. Army, in its study of power, irrigation, and flood-control possibilities on the Columbia River, as follows:[32]

Total for power and navigation.............	$215 483 453
Total for irrigation........................	221 721 880
Total power, navigation and irrigation.......	437 205 333

This was a preliminary estimate and, in view of the unprecedented difficulties of construction, may conceivably be substantially exceeded. A statistical abstract of the "United States Under Government Irrigation Projects" shows the charge to irrigation of $11 419 070 of the cost of Grand Coulee Dam to June 30, 1935. Inasmuch as the entire cost of the river dam is charged to "Power" in this estimate the final distribution of cost will be very different from these data.

The present work under contract includes only the foundations of the Columbia River Dam, of which Kenneth B. Keener, Senior Engineer of the Bureau of Reclamation, states:[30]

> "* * * completion of the Grand Coulee Dam is a problem for the future and one that need not be given serious consideration until the work now under contract is nearing completion. Conditions and requirements at that time will no doubt govern the next step to be taken in the development of the Columbia Basin project."

Within a year or two at the most Congress will be asked for more money to continue this project. That body may very properly ask just what it is proposed to do, why it is needed, and whether the project will be self-liquidating. The dam is advertised in literature given out to visitors to be "the world's largest masonry structure, larger than the great pyramid, four times as long as Boulder Dam." It will be 500 ft high, 4 300 ft long; it will contain 11 250 000 cu yd of concrete, and will involve 16 000 000 cu yd of excavation. According to a tabulation of the Bureau of Reclamation the power plant will have a rated capacity of 2 500 000 hp, and will be able to develop 7 000 000 000 kw-hr of firm power and more than 4 000 000 000 kw-hr of secondary power per yr. This rated horse-power capacity is almost 16% of the rated capacity of all the water-wheels in the United States,[33] and more than the 2 358 000 hp of the California turbines. It is made perfectly clear in the report of the U. S. Corps of Engineers that "the feasibility of the pumping plan [this project] is dependent on the possibility of selling practically the entire output of primary power from this plant at a remunerative rate."[34] The late Elwood Mead, M. Am. Soc. C. E., has stated:[35] "This development to be solvent must be based on revenues from power and these revenues must contribute to the cost of the irrigation works to avoid injurious burdens on irrigation farmers." Therefore, the first consideration as to this plant is the question of market for power.

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[31] Rept. of Div. Engr., H. R. Doc. No. 103, 73d Cong., 1st Session, p. 58.

[32] H. R. Doc. No. 103, 73d Cong., 2d Session, p. 15.

[33] Given as 16 179 475 hp as of January 1, 1937, in the Dept. of the Interior Press Memorandum, dated February 16, 1936.

In considering the market it must be remembered that Bonneville, in Northern Oregon, is in the same territory. The Bureau of Reclamation states that 7 000 000 000 kw-hr of firm power will be generated at Grand Coulee. The statement has been made[36] that the two dams on the Columbia—Bonneville and Grand Coulee—will have a combined final installed capacity of 2 322 000 kw and can produce 10 220 000 000 kw-hr.

The combined capacity of the two plants is greater than that of all classes of generating equipment of any State in the Union in 1935, except only New York and California. It is 840 000 kw in excess of the total equipment in Oregon, Washington, and Idaho, in 1935.[37]

The total power generated in that year in the eight States of Oregon, Washington, Idaho, Montana, Wyoming, Colorado, Utah, and Nevada was 6 772 000 000 kw-hr or less than the firm power capacity of Grand Coulee alone. The total power generated in Oregon, Washington, and Idaho, which is where these plants must find their market, was only 4 570 281 000 kw-hr.[37]

Need anything more be said? Any study of the power market in the Northwest indicates that there would be difficulty in finding a market at present for 1 000 000 000 kw-hr, and the market for any such amount as would justify Grand Coulee as a power project is more than a generation in the future.

The Columbia Basin Project

This Grand Coulee Project cannot be dismissed without emphasizing the fact that, in 1927, six years before the emergency approval of Fort Peck, Grand Coulee, and Bonneville, the Congress attacked the problem of the wise and proper development of the Columbia River in a highly proper manner. In that year an Act was passed providing for an exhaustive study of certain rivers, including the Missouri and Columbia, with the view to determining the possibilities for the development of navigation, flood control, power, and irrigation, on these great rivers.[38] One of the so-called "308 reports" dealing with the Columbia River is dated July 31, 1931, and was available to the Federal officials in 1933 when Grand Coulee was approved as a project.[39] This report discussed the building of either eight or ten dams which would develop more than 92% of the total head of the Columbia between the International Boundary and Tidewater. Of these dams, one at Warrendale, Ore. (or Bonneville as an alternate site), was the lowest, at Tide-water, and the one at Grand Coulee was the last one before reaching the International Boundary. The Corps of

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[34] H. R. Doc. No. 103, 73d Cong. 1st Session, p. 8.

[35] *Loc. cit.* p. 5.

[36] *Electrical West*, February, 1935.

[37] *Statistical Bulletin No. 4*, Edison Electric Inst., January, 1937.

[38] H. R. Doc. No. 308, 69th Cong., 1st Session.

[39] H. R. Doc. No. 103, 73d Cong., 1st Session.

Engineers of the Army went at length in this report into the consideration of building Grand Coulee in connection with the Columbia Basin irrigation. Apparently, the fact that this was a promoter's project was the reason for the exhaustive study of it.

A careful analysis of this report disclosed strong adverse recommendations to the project now under construction; for example, Maj. Gen. Lytle Brown, U. S. Army, M. Am. Soc. C. E., then Chief of Engineers, U. S. Army, stated:[35] "In fact the local reports demonstrate that the irrigation of land as pertains to the Columbia River area under consideration is not an economical proposition at this time and should await the future."

The Board of Engineers for Rivers and Harbors stated:[34]

> "One large area of upland known as the Columbia Basin could be irrigated by gravity from the Clark Fork and Spokane River or by pumping from the Columbia River by means of secondary power developed at the plant near Grand Coulee. The feasibility of the pumping plan is dependent on selling practically the entire output of primary power from this plant at a remunerative rate."

Subsequently, the Board discusses this subject at length, as follows:[40]

> "The economic feasibility of this plan is largely based on subsidizing irrigation by profits from the sale of power. * * * The Board * * * is of the opinion that the estimates of the reporting officers that the growth of power demand will be such that the entire prime output would be absorbed in a period of 15 years after 1940 * * * is unduly optimistic. * * * The Board * * * is unable to recommend the adoption of this project at this time."

This report, dated March 29, 1932, was available when the project was decided upon and is full of sound argument why Grand Coulee should not be built. A "Report on the Columbia Basin Project," dated February, 1925, two years prior to the action of Congress ordering the Corps of Engineers, U. S. Army, to make its study, was addressed to the U. S. Bureau of Reclamation by Louis C. Hill, Past-President, Am. Soc. C. E., Joseph Jacobs, Charles H. Locher, Richard R. Lyman, and A. J. Turner, Members, Am. Soc. C. E., and Mr. O. L. Waller. This Board found that economic conditions definitely favored the adoption of a gravity project by diverting water from Clark Fork at Albany Falls, which would cost $158 per acre to irrigate 1 224 000 acres, and did not deem it necessary to make an elaborate analysis of the Grand Coulee scheme.

Discussing Grand Coulee on March 19, 1912, Elwood Mead stated:[35] "It will require at least ten years after the works are authorized, to build the dam and power plant, and another ten or fifteen years to absorb the power thus made available. These things must precede the large expenditure to build the works required for irrigation."

It is impossible in such a paper as this to brief all the evidence fully. To the writer the essential points appear to be as follows:

(1) In 1925, a study of the irrigation project was made by the Hill Commission which recommended a gravity system at a cost of $158 per acre. This report has not been made public, but apparently the total cost of the gravity plan approximated $195 000 000.

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[40] H. R. Doc. No. 103, 73d Cong., 1st Session, p. 12.

(2) In 1927, the Congress ordered the Corps of Engineers of the U. S. Army to make a study of the entire Columbia River and to report on the possibilities of navigation, irrigation, power, and flood control. This was done and an 1 845-page report was submitted which considered this specific project at length, and which did not approve it.

(3) It appears from various sources that the larger part of the land in question is in private ownership, much of it in the hands of bankers and speculators.

(4) The project as first approved by PWA was for an entirely different plan involving only power but with a low dam which, subsequently, could be raised in height. Later, this plan was abandoned and the change to the large power development to support the irrigation scheme was made by the Bureau of Reclamation.

(5) It does not appear that the project was presented to the Congress before its adoption or discussed by Congress in the light of the report to it by the Corps of Engineers, and with full understanding of the magnitude of the undertaking.

(6) The Corps of Engineers' estimate of cost of the present project was $437 000 000. The Statistical Abstract for 1936 shows the total cost to the Government of all the forty-five other reclamation projects of the Federal Government, excluding all charges to the Grand Coulee project, to have been less than $237 500 000. Based on charges to date to irrigation on this project, it is evident that this one irrigation project alone will cost more than all previous Federal reclamation projects, and the entire project will exceed the cost of the Panama Canal by many millions of dollars.

(7) A study of existing power development in Oregon, Washington, and Idaho, a large percentage of which is publicly owned by the Cities of Seattle and Tacoma, Wash., and of the production and sale of power in these States, seems to indicate that there is no possibility of the sale of all the power of both the present Bonneville installation and Grand Coulee; but all the power of Grand Coulee must be sold at a profit to subsidize irrigation in order to justify the scheme.

It is to be hoped that the Congress will not take hasty action in approving further appropriations for this project without the most complete expert study. The request for more money, which will inevitably be made by 1938, should be cause for an exhaustive study and a report which will be available to the public by a commission of nationally known, highly qualified, and wholly disinterested engineers. If there is no market for practically all the energy (and that appears to be the fact) the entire project will fall.

Freely granting the urgent need that existed in 1933 to find public works to relieve unemployment, the writer is of the opinion that such a project as this one, which will require years to complete, and which may involve half a billion dollars, does not fall into the class of emergency relief works. It should never have been adopted without the full discussion and consideration of Congress.

"COUNTING THE COST"

The nation will do well to count the cost of the great program of public works which has been commenced. The sums of money published as the estimated cost of construction are imposing enough in themselves, and would total somewhere between $1 000 000 000 and $2 000 000 000, if all the major projects were listed (such as the ones already named, and others like the Tennessee Valley Authority, the Florida Canal, Central Valley, California, and other Federal projects, and such Federal financed State undertakings as the Central Nebraska Power Project, the South Carolina Public Service Authority, and the Lower Colorado Authority in Texas).

In "counting the cost" it is not the intention herein to consider anything except actual cost which sooner or later must be paid in the form of taxes. Possible losses of private capital invested in power properties or transportation agencies whose business may be injured and whose credit may be wrecked by competing Government agencies, or the possible destruction of industries such as that of the salmon fisheries by the building of dams, may be substantial, but they are controversial matters which will not be discussed.

Too much emphasis cannot be placed on the fact that every dollar that is invested by the Government in river improvement in aid of navigation, in great dams to regulate the flow of streams in aid of navigation, in power plants for the manufacture of electric energy, in irrigation, or in any other public service, is a dollar added to the public debt. Whether the project is wise or foolish, every dollar that it costs adds to the national debt and contributes in proportionate amount to the annual burden of interest.

Exactly the same sound business principles should control the selection of public construction projects which are to render a service that the public must pay for as would prevail in the case of a successful private undertaking of the same kind. Many of the plants now being built appear to be foredoomed to failure because of the adoption of the project without adequate study of the site, or complete and accurate plans, or without a proper economic study to determine whether they could be self-liquidating.

The ultimate cost to the taxpayers is the final cost of the project with all of the auxiliary construction made necessary for its complete use, plus all interest and other fixed charges as long as any part of the cost is not amortised, plus all deficits in operation and maintenance less than total revenues.

This may be made more definite by a consideration of actual cases to show that:

(*a*) Imperfect explorations and preliminary and partly digested plans result in misleading and inaccurate estimates, in large contingent expense due to unforeseen difficulties, or to the need for changes in plan after the beginning of work.

(*b*) The adoption of plans for a single project which may take years to build (such as a great dam and power house), involves the building of auxiliary structures to permit its complete use, such, for example, as the transmission lines and sub-stations needed to permit the power plant to serve the market.

These structures must inevitably be built, but no estimates of their cost have been made public by Federal agencies.

(*c*) Where the economic studies of the market and of probable future growth of business were over-optimistic or just plain "bad," the resulting business losses extending over many years, plus fixed charges, constitute an intolerable burden.

These conditions have occurred in the past on numerous occasions when the Congress was misled or was at fault by reason of political pressure. A large part of the public works now in progress (1938) were hastily approved by temporary emergency boards and commissions, with a changing personnel, acting under a pressure not unlike that during the World War when hundreds of millions of dollars were wasted in building nitrate plants that never operated and ships that never sailed. The magnitude of these projects far exceeds anything ever undertaken in a like period of time in the United States.

These hazards of uneconomic public works, construction cost that cannot be repaid, great expense of auxiliary plant not now disclosed, fixed charges that cannot be carried by the project and that may never be reported in its operation, and possible operating losses, cannot be cured by the "writing off" of part or all of the capital account. This has been done in the past, and on the books the account of the particular project looks better. That, however, retires no bonds; nor does it reduce the taxpayers' annual burden of interest. It merely transfers the item to other general accounts. Every project which involves Government in business and which is built as a self-liquidating enterprise, should be compelled by the Congress to keep its accounting records so as to show every transaction, and to carry as its share of fixed charges every dollar that the nation is obligated for by reason of its construction. Failure to treat interest as a deduction from gross revenue is by no means rare in the accounting of publicly owned plants, especially those that are not successful.

SIGNIFICANCE OF CONTINGENT EXPENSE

A study of the annual reports of the Chief of Engineers of the U. S. Army discloses the fact that Bonneville Dam affords an excellent example of contingent expense due to the hasty adoption of sketch plans based on only a preliminary investigation of the site.

Under the National Industrial Recovery Act the sum of \$20 250 000 was allotted to the project on September 30, 1933. Actual construction began in October, 1933. Confident that the preliminary plans were sound, construction work was in progress at the same time that surveys and explorations for the final plans were being made. The details of the preliminary estimate of the dam and power house with two units, totaling \$31 000 000, were published in 1934.[41] The PWA allotted an additional \$11 000 000 to this project in August, 1934, making a total allotment of \$31 250 000.

Detailed study of the site as work progressed indicated that the spillway section of the dam (north of the island) should be moved down stream about

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[41] *Engineering News-Record*, November 1, 1934.

3 000 ft. This was done and contracts for the dam and power house were let. In the report of the Chief of Engineers for 1934 the estimated cost was placed at $40 000 000.

Salmon fishing is a most important industry in the area, and public clamor was such that most elaborate fishways and fish locks were installed. The total cost of these provisions for the passage of fish in the river approximates $4 000 000. The Chief of Engineers' report for 1935 gives "the estimated cost of the dam, lock, two power units, and fishways as $42 415 700." The navigation lock was changed from 350 ft to 500 ft in length, and from a depth of 15 ft over the sill to 27 ft. This change cost approximately $1 200 000.

The Chief of Engineers' report for 1935 stated[42] that the project was "adopted by the River and Harbor Act of Aug. 30, 1935." In other words, Congressional approval was not secured until two years after the work was begun. The Annual Report for 1936 states that the estimated cost revised in that year is $51 000 000.

These facts are not presented in any spirit of criticism of the Corps of Engineers, which is doing a magnificent piece of construction; nor is it implied that Bonneville is not economically justified. The writer believes that if there is any warrant for this class of Federal expenditures, Bonneville can be classed as a self-liquidating project. The subject is discussed to illustrate the fact that when such a difficult project as one of these Columbia River dams is selected without the benefit of complete explorations and preliminary plans, and construction is begun immediately, the inevitable result is a great contingent expense the cost of which must be provided for by a further appropriation of funds raised by taxation. In this case the Administrators of the National Recovery Act and the PWA allocated $31 000 000. Two years later, the orphan was "adopted" and Rivers and Harbors appropriations have had to take care of the additional $20 000 000 which was not anticipated.

Estimates made by the U. S. Corps of Engineers include liberal allowances for engineering, interest not only during the period of actual construction but up to the time when the plant is on an earning basis, and all legitimate over-head costs. Contingent costs due to changes of plan, difficult foundations not disclosed by the preliminary survey, weather, flood, or bad soil conditions may increase the final cost of the work from 10% to 50%, or more, beyond the estimate.

Here are two Army built projects so nearly completed that the difference between the 1934 estimate and the revised estimate of 1936 may be accepted as the cost of contingencies:

Dam	1934 Estimate	Revised Estimate of Final Cost
Fort Peck	$86 000 000[43]	$108 600 000[26]
Bonneville	31 250 000[44]	51 000 000[45]
Total	$117 250 000	$159 600 000

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[42] Rept. of Chf. of Engrs., U. S. Army, 1935, p. 1315.

[43] *Loc. cit.*, 1934, pp. 641, 742.

[44] *Loc. cit.*, pp. 1335, 1337.

[45] *Loc. cit.*, 1936, p. 1517.

The excess cost over the originally stated estimate is $42 350 000, or an over-run of 36 per cent. If the final cost of Grand Coulee should exceed the estimate of the Corps of Engineers, made under exactly the same conditions as those for Bonneville and Fort Peck, by the same percentage, it means the addition of $157 394 000 of unforeseen cost, or almost as much as the total final cost of Fort Peck and Bonneville. Any one who has visited the site of Grand Coulee in the remote desert, with a wide, deep, and swift river to contend with, whose maximum flood is 300 000 cu ft per sec in excess of the mean flow, can realize that such a contingent expense is entirely possible. These data are based on estimates of the Corps of Engineers, a body capable of making fully as accurate estimates as any other governmental agency or, indeed, as any engineers. The reports of the Chief of Engineers give facts in such full detail as to inspire absolute confidence.

The fact seems to be that the data issued at Washington as the estimates of cost of these major public works projects do not represent the final cost, and that tens of millions of dollars more will have to be provided to complete the work now under way.

SUPPLEMENTARY COSTS

Auxiliary construction made necessary by the completion of Federal emergency projects, of which no mention was made at the time of approval of the work, constitutes another great source of future expense.

On December 10, 1935, a Committee on Power of the Oregon State Planning Commission, of which J. C. Stevens, M. Am. Soc. C. E., was Chairman, submitted a report on the "Cost of Bonneville Power." This Committee estimated the capital cost of a transmission system, including receiving sub-stations, for the adequate distribution of power from the finally completed Bonneville plant with ten generating units to Oregon communities, at $43 270 000.

The report of the Division Engineer of the War Department, North Pacific Division, on the study of the Columbia River,[46] refers to this subject of power transmission. No attempt is made to submit any final plan, or total estimate of cost of transmission lines and transformer stations needed to carry the Grand Coulee power to market.

Appendix II of this report includes a study of this problem by Dean Edgar A. Loew, of Washington University.[47] Data in this estimate give the cost of one double-circuit, 220 000-volt line, with its terminal equipment estimated for several circuit lengths, using Government construction costs with interest at 4%, as shown in Table 4.

What has been stated regarding the capacity of these two plants makes it very obvious that the Federal Government has an extremely serious marketing problem, and a very costly auxiliary construction project ahead of it to get the more than 7 000 000 000 kw-hr from Grand Coulee Dam to any possible market.

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[46] H. R. Doc. No. 103, 73d Cong., 1st Session, pp. 48–49.

[47] *Loc. cit.*, pp. 456–479.

This same problem of transmission of power will be found at every one of the Federal power projects and will call for tens of millions of dollars in nearly every case. Not only are there no definite figures as to the cost of the dams and power plants, but it is impossible to determine for some years the number of millions, or hundreds of millions, of dollars that must be incurred in transmission costs to which the projects are committed.

TABLE 4.—COST OF ONE DOUBLE-CIRCUIT, 220 000-VOLT TRANSMISSION LINE

Length of line, in miles	Peak capacity, receiver end	Investment	Annual expense of operation and maintenance	Annual fixed charges*
100	325 000	$8 078 200	$30 000	$551 700
150	271 000	8 495 700	45 000	580 300
200	216 000	9 144 400	60 000	624 600
250	162 000	9 933 600	75 000	678 500

* Fixed charges include interest at 4%; depreciation reserve at 1.78%; and amortization at 1.05%; total, 6.83 per cent.

OPERATING LOSSES UNDER GOVERNMENT OWNERSHIP

The third hazard which has been referred to is that of operating losses on unsuccessful projects. This can best be illustrated with certainty by turning to the history of operating projects.

The experience of the Federal Government in business does not lead one to be optimistic about many of the projects now under construction. Several of the States have engaged in enterprises that have proved disastrous. One or two notable exceptions tend to emphasize the fact that the more far flung the enterprise and the more centralized the administration of it the greater is the certainty of failure.

Municipal ownership, being nearer home, where the taxpayers are neighbors of the officials, has done better. There are scores of examples of well managed and profitable publicly owned water-works. There are several notable examples of highly successful and well operated municipal electric plants in large cities, giving good service at low rates and full and fine publicity to statistics based on accounting that is done in accordance with the most approved classifications. There are many such plants in smaller cities that were built as publicly owned properties in the early days of the industry, which have been, for many years, in charge of non-political commissions and skilled managers, and which compare favorably in service and rates with privately owned utilities of comparable size. On the other hand, there are apparently more than a thousand small-town electric plants that are miserable failures—many more failures than there are good plants.[48]

Early State expenditures for canals were great and there were many projects none of which paid expenses for more than a brief period and none of which ever paid much, if any, interest, to say nothing of any return of capital. The principal surviving canal, the New York Barge Canal, successor to the Erie Canal, had cost the State of New York nearly $300 000 000 from 1903 to 1933. At the time that the Barge Canal System was authorized in 1903, State funds of New York had already been drawn upon to the amount of $73 000 000 in excess of canal revenues for the support of the old Erie Canal. In 1933, a total of 4 000 000 tons of freight was moved, and the annual maintenance and operating expense was $2 788 000.

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[48] "Do Municipal Electric Plants Really Pay?" by H. E. Riggs, *Public Utilities Fortnightly*, September 24, 1936, pp. 363–392.

Prior to the Civil War and extending into the 1880's several of the States expended large sums of money on railroad construction, nearly all of which was lost. Only the State of Georgia with its Western and Atlantic Railroad can report a successful investment. This road, still owned by the State, has been leased for more than fifty years at a rental which pays a fair return on its cost.

The Alaska Railroad cost the Government $70 110 000 to June 30, 1930.[49] Not in any one year since it began operating in 1916 has it paid its operating expenses. Added capital investment and interest on the capital have all been met by the taxpayers, in addition to making up the deficits. This property is 540 miles long, serving a population in 1930 of less than 8 400 in a tributary area of 50 000 sq miles, with very little export freight. In the fiscal year 1930 it moved 915 freight car loads.[50] The Government has spent more than $6 000 000 building a highway from Valdez to Fairbanks. In the report of the Senate Committee, the late Hon. Benjamin F. Howell, U. S. Senator from New Jersey, stated:[51]

> "These facts afford a notion of the competitive potentialities of the government built and maintained highway in juxtaposition with the also government built and maintained Alaska Railroad, and it well might be urged that these facts also denote, at least, an apparent lack of coordination in the practical promotion of government activities."

In this same report, the following statement appears: "A factor, other than inadequate passenger and freight rates, that contributes to the annual deficits is looseness and inefficiency in the conduct of railroad business and affairs and of the activities incidental thereto."

An examination of the accounts of the Alaska Railroad was reported[52] to the Interstate Commerce Commission on August 31, 1930, by Examiner Leroy S. Price, which is of interest. He reveals the fact that there had never been an audit of the accounts of the Alaska Railroad other than "intermittent and irregular audits of cash in the hands of the disbursing clerk at Anchorage," and finally concludes, "not until the accounts are stated at their true value can the government know the cash outlay in the Alaska Railroad."

The losses assumed by the Government on account of the Federal operation of the railroads during the World War are estimated at $1 120 000 000. Losses due to the Emergency Fleet Corporation appear to have exceeded $2 000 000 000. The operating losses of the United States Shipping Board for the eight years, 1922 to 1929, were in excess of $220 000 000.

The Muscle Shoals Plant on the Tennessee River cost more than $127 000 000, of which approximately $40 000 000 was spent on electric power development.

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[49] Senate Rept. 1230, 71st Cong., 3d Session.

[50] *Loc. cit.*, p. 3, Second paragraph.

[51] *Loc. cit.*, 71st Cong., 3d Session, p. 6.

[52] *Loc. cit.*, p. 20.

The project was built primarily for the manufacture of nitrates. The nitrate plants were not completed until the war was ended. The first plant proved impractical; the second was given a trial run of two weeks in 1919 after which it was closed and held as a stand-by plant. Obsolescence has probably been very great on a plant built for such a special purpose as this one, in the eighteen years in which it has stood idle.

The Inland Waterways Corporation is owned and controlled by the United States through the Secretary of the Treasury in whose name all the stock was subscribed. This Corporation operates the Federal Barge Lines on the Mississippi, Illinois, and Missouri Rivers, and from New Orleans, La., to Bingamport, Ala., *via* Mississippi Sound and Black and Warrior Rivers, and through stock ownership controls a railroad known as the Warrior River Terminal Company.

From 1924 to 1935 the system as a whole shows an average tonnage moved of approximately 1 500 000 per year. During the twelve years the system net revenue from operations was only $1 003 000, and the net income, $307 865. The total investment in real property and equipment used exclusively by this Government agency in transportation is $23 500 000. There appear to have been adequate allowances for depreciation of floating and harbor equipment in all years. It will be noted that there is not enough return reported over the twelve-year period to pay even a low rate of interest for a single year on plant actually used in transportation, and nothing at all in the way of recompense to the Government for work done on river and harbor improvement. The property pays practically no taxes and nothing appears as a deduction from earnings for interest on the investment in property and plant used. Had such a deduction been made to represent the actual interest paid by the Government and passed on to the taxpayers, net income would be shown between $6 000 000 and $7 000 000 "in the red." The story of this enterprise, in detail, furnishes material for an interesting paper.

THE TEST OF ENGINEERING ECONOMICS

The foregoing partial roll call of Federal and State projects in which Government has entered into business especially in times of emergency, causes one who has studied the statistical history of the projects to be decidedly pessimistic as to the outcome of the many and large projects which the Government has undertaken during the past four years.

It will probably be argued that private ownership has had many failures. That is true. There have been hundreds of cases of ill-advised, poorly planned, and badly or dishonestly managed private business undertakings that have resulted in complete failure and the loss of the entire investment. There is a long and unenviable record of privately owned railroads and utilities in which all thought of service was lost, and financial gain alone was the objective of promoters. There are chapters in the history of the railroads, of irrigation, or water-works, and of the electric industry that are scandalous and that gave ample cause for the development of regulation. The answer is that all the money that was lost in the failure of privately owned projects was the money of the individual investors, and the evils of management can be, and should be, cured by Government regulation. Just as soon as uniformity of accounting and full publicity of accounting and statistics were required of the railroads and real power was given to the Commissions, all the abuses and evils of a generation ago were abolished.

Regulation of private enterprise to prevent improper financing, discrimination of all kinds, rebating, or giving of subsidies, and excessive charges or deficient service is a true and legitimate function of Government that has proved its worth in the last fifty years.

One issue that is clearly raised by the great construction program of the present day is that of national policy. Should the people of the United States continue to favor individual initiative, enterprise, and opportunity regulated by law, or should they adopt a policy of nationalization of utilities and ultimately bring all industry and labor under the control of Government? That is for the nation to decide.

Another question that must be answered deals with the character of public work which should properly be undertaken in an emergency, such as that of 1933 which amounted almost to national paralysis. Millions of men were out of work. One of the great problems was to find work which, promptly and quickly, would furnish labor for the greatest number. Does the adoption of such projects as Bonneville, Grand Coulee, Tennessee Valley, and the many other long-time projects (which require from four to eight or ten years to complete) solve the emergency employment requirement? Or would not just as much employment have been furnished, and just as much steel, cement, and other structural material have been used had there been adopted some such a project as the building of safe and adequate bridges and railroad grade separations on all Federal highways.

In one case there is the planning of a single great structure and the organization of a single force of men for long-time employment. In the other case there is the planning of many structures in each State and the organization of many different construction forces in all parts of the country under the supervision of established highway commissions.

In one case the taxpayers of the nation pay in the end for the entire cost of some of the projects. In the other case, they pay it all, but they would get the lasting benefit of work, which in some of the States would be delayed for years, but which is a direct benefit to public transportation.

The writer believes that in passing upon whether or not any of these great projects are economically wise each one must be judged on its own merits, and all argument that it was an emergency undertaking must be disregarded.

Passamaquoddy certainly cannot qualify as an economic project. There is no demand for power at that location. If there were such a demand the economic answer would be to build a steam plant for $6 000 000 or $7 000 000 instead of the proposed tidal plant for ten times as much.

The extensive evidence that was secured by the U. S. Corps of Engineers in its study of the Columbia River seems to show that the bringing into cultivation of more than a million acres of additional farm land is not a present urgent need, that it could only be justified by the sale of vastly more power than can be absorbed by the territory for several decades in order to subsidize irrigation, and that there exists an engineering report to the effect that the area in question can be irrigated by gravity for less money.

Bonneville, were it alone being built, appears to be a reasonable project with good prospects of being self-liquidating. The one great question involved in that case is that of national policy; and the precedent of the Hoover Dam, designed and commenced during a Republican Administration, cannot be overlooked in considering it.

The policy of Federal improvement of waterways is of long standing. No one can question the propriety of spending money for navigation on streams such as the Ohio and Lower Mississippi which have been main arteries of traffic over the life of the nation; but to spend a quarter of a billion dollars on such a river as the Missouri which has no traffic, and has never had any worth the name, raises another question.

Can the nation afford to increase its burden of national debt at a time when every effort should be made to eliminate unnecessary expense and balance the budget, and in addition increase its annual expenditure for fixed charges, maintenance, and operation by more than $10 000 000 per yr, in the hope that a future water-borne traffic can be developed?

The writer thinks that the answer is "No," but wishes to remind readers that this problem was created by the Republican party.

The entire discussion comes down to a question of engineering economics. Putting to one side all argument as to whether Government ownership is wise or unsound, it would appear to the writer that the general policy that should control the adoption of any program of public works which involves the entry of the Government into any form of business or public service, whether in time of emergency, or not, should be determined by the Congress only, and not delegated to any bureau or commission for final decision.

No such project can be economically sound unless it can earn all operating expenses, including: (1) All cost of maintenance and upkeep of plant; (2) a reasonable allowance for a reserve to replace structures and equipment as they wear out; (3) interest on every dollar of the national debt incurred by the construction of the plant; and (4) a reasonable allowance for amortizing that debt over a period of forty to sixty years. The determination of these facts is an engineering problem. Therefore, no project should be undertaken without an exhaustive study of construction features, and market for the output, by qualified engineers reporting to the Congress. No better example of such a study can be found than that on the Columbia River made by the Corps of Engineers after four years of study.

Every such report should be a matter of record available to the public. There must be scores of reports of highly qualified engineering commissions, such as, for example, the PWA reports on Passamaquoddy, the estimate and recommendation of the Corps of Engineers, and the later report of the Special Engineering Board on the same project, on file in Washington, none of which has been made available to the public.

Allocation of funds was made for the Grand Coulee project in the face of adverse reports by the Chief of Engineers of the U. S. Army, the Board of Engineers of Rivers and Harbors, the Division Engineer, and the bitterly adverse argument of the Secretary of Agriculture. This was done without any guaranty of revenue notwithstanding the fact that the Chief Engineer of the Bureau of Reclamation, under date of January 7, 1932, submitted to the Commissioner of Reclamation the following perfectly definite recommendation:[53]

> "9. No construction on the Columbia River Dam and power plant should be undertaken until the contracts are executed for the sale of power which will insure sufficient revenue for annual expenses and the repayment of the investment in the dam and power plant with interest at 4 per cent within fifty years.
>
> "10. No construction on the irrigation development should be undertaken until the power revenues are assured and a suitable contract for the repayment of the investment in irrigation works within 40 years had been executed."

Every railroad, and practically every privately owned public utility, is required to keep all its accounts under rigidly prescribed classifications issued by Federal or State commissions, and to keep accurately and make public all pertinent statistics. This is as it should be. This requirement has bared many bad practices and made their abolition possible. Unfortunately, no such requirement applies to publicly owned utilities, either Federal or municipal.

In a recent article Arthur E. Morgan, M. Am. Soc. C. E., states:[54]

> "Under any method there are certain proprieties and decencies of government which should be observed. Where the public has invited private capital to supply an essential public service, there should be no capricious arbitrariness in destruction of duplication of facilities to the loss of honest, necessary and useful investment. Where bad government or lax administration has allowed inflated securities to be sold to innocent investors the public is not without responsibility. In case public power is used as a 'yardstick' or as a measure of what the private power industry should charge for its services, then it is imperative that records and accounts be honest and fair and open, and that there be no hidden element of subsidy. The very fundamental element of such comparison is honesty, fairness and openness in measurement. Take away those characteristics, and the supposed comparison may becloud the issue rather than clarify it."

It does not appear unreasonable to insist that every publicly owned utility be required to use exactly the same accounting rules and statistical data that are required of similar privately owned utilities. This is of as much importance to the Congress and the Administration as it is to the public.

Conclusions

Twenty-four hundred years ago Pericles made the statement that "acts are foredoomed to failure that are undertaken without discussion." That is just as true to-day as it was before the Christian era.

Who should do the discussing? The Congress, in American Government, is vested with the authority to represent the taxpayers and decide which expenditures are wise and which are not.

Disregarding all the earlier discussions of the Panama Canal and Nicaragua Canal routes between 1870 and 1875 prior to the French activities, the Congress began active discussion of the Panama Canal in 1899, and in March,

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[53] Rept. of U. S. Bureau of Reclamation, H. R. Doc. No. 103, 73d Cong., 2d Session, pp. 484–485.

[54] "Public Ownership of Power," by Arthur E. Morgan, *Atlantic Monthly*, September, 1937, p. 342.

1903, passed the Act which authorized the President to negotiate with the Republic of Panama. Not until December, 1903, were negotiations completed. Then came the long discussion as to whether a lock canal or a sea-level canal should be built, ending with the adoption by Congress in 1906 of the lock canal. The actual construction cost of this project was $336 400 000. When the $40 000 000 paid to the French Company, and the $10 000 000 paid to the Republic of Panama are added, the total cost reached $386 400 000, practically $30 000 000 less than the estimate of the Grand Coulee project.

This great canal had the benefit of years of study by a number of the greatest engineers of the time. There was no undue haste. Great questions were not settled by individuals or bureau heads. The records of the Congress show that all questions of general policy, even to deciding the engineering question as to whether the canal should be a lock canal or a sea-level canal, were submitted to and decided by the Congress. The decision was based on the recommendation of the engineers.

If policy has changed; if individuals or bureaus are to authorize expenditures of many hundreds of millions of dollars without Congressional approval, or are allowed to change contracts by the issuance of departmental orders, thereby changing the entire character and objective of the project, is it not time for members of the Engineering Profession to break silence?

The writer believes that those members of the profession whose hands are not tied, who are free to tell the facts, should do so freely and should discuss the economics involved as well as the technical features of public works.

Conditions in the United States to-day call for "hard-headed" economy, for the careful analysis of every proposal to spend money raised by taxation, and for the firm rejection of every one that is not economically sound.

Members of the Engineering Profession, better perhaps than any one else, can advise the people of the United States as to the economic soundness of proposals to spend vast sums on public works, especially those designed to render public service of a kind that should make the project self-liquidating.

They will fail in their duty if they keep silence.

AN APPEAL TO REASON

By William J. Wilgus,[55] Hon. M. Am. Soc. C. E.

Synopsis

In this paper, the writer wishes to avoid the impression that he disbelieves in the creation of worth-while public works of the type considered in this Symposium. In the words of the Hon. George D. Aiken,[56] Governor of Vermont:

> "I hold no brief for any private utility corporation. I even hold that through their manipulations in pyramiding values [too often fraudulent] and the acquiring of unearned and unwarranted profits at the consumer's expense, they are largely, if not principally, responsible for the present state of affairs."

The bracketed comment is by the writer. Neither does the writer hold with those who would maintain the unemployed in corrosive idleness, destructive of all that is best in American life, rather than at self-respecting work, at a somewhat greater expense, from which the public gains material and spiritual returns. The work, in fact, must be worth while in the public interest and not uneconomic and wasteful, involving a tragic loss of public monies for ever-continuing interest charges and upkeep.

Worth-while public works need not necessarily suffer from the evils of corrupt, incompetent political management. There are notable examples of admirably run publicly owned water-works, highly successful and well operated municipal electric plants, toll tunnels and bridges and the other commercial activities, such as those of the Port of New York Authority, the revenue-producing enterprises operated under the auspices of the New York City Park Department, and various toll-bridge authorities in New York City, and the publicly owned transit facilities in the same municipality which eventually are to be expanded by the addition en masse of its privately owned subways and elevated lines. There are, too, many city managerships to be looked to as outstanding examples of successful administration; this can likewise be said of many works under the jurisdiction of the U. S. Army Engineers. Under proper safeguards and conditions, it is possible to run public enterprises with honesty, competence, and resourcefulness, coupled with a strict observance of proper accounting at all times open to the public. The quicker the people realize this and prepare for it, the better off this country will be. To drift blindly, without a chart or compass for guidance, will mean the continuation of the ills of public management, vastly to be multiplied, as private utilities, whether one likes it or not, come more and more under public control. With the public ownership of railways and other utilities "in the offing," it behooves the civil engineer to abandon a King Canute-like attitude of negation and to favor a constructive course leading to a reasonably satisfactory conclusion.

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[55] Weathersfield, Vt.

[56] Address at Plymouth, Vt., August 8, 1937, reported in *Rutland* (*Vt.*) *Herald*, August 9, 1937.

It is deeply to be regretted, therefore, that the gargantuan projects analyzed in this Symposium have been promoted by the Government without at the same time offering evidence that they hold promise, within a reasonable time, of becoming socially and economically justifiable under efficient management.

MAJOR PROJECTS

In the Passamaquoddy Tidal Power Project, the estimates of cost have ranged widely from $37 000 000 to $62 000 000, to which must be added unknown sums of magnitude for distribution systems and for interest, and other carrying charges, until the project, if ever, shall become self-supporting. It would also appear that the intended dual methods of operation—by water power and by coal or oil—will result in an absurdly high cost per unit of capacity; and there is no market for the power in sight. Furthermore, it is said that certain engineering reports on the project have been withheld from the public. As so far revealed, it is not a "pretty" story, especially in view of the threat that workers in the coal and oil fields, and in the transportation industry, are to be made to suffer to the extent that water power thus is to be wastefully used.

As to the Fort Peck Dam, the estimate of cost has risen from $86 000 000 to $108 000 000, for which the resulting improvement in the low-water flow in the Missouri River will bring a grossly inadequate return to the public. It may be that in some unexplained manner the improved river channel is expected, within a reasonable time, to afford a water outlet for the products of the Northwest at savings in the cost of transportation sufficient to offset the added interest charges, the expense of upkeep to be borne by the people of the country at large; or, it may be that other benefits worth the price are to be expected by the public in the way of flood control, improved sanitation, cheap power, and the reclamation of waste lands. The evidence, however, appears to be lacking.

Likewise, the estimated cost of the Grand Coulee project in the Columbia River Basin has risen from $108 000 000 to upward of $437 000 000 for the combined purposes of navigation, power, and irrigation, to which must be added the first cost of needed distributing systems and auxiliary structures of magnitude, as well as the annual costs of depreciation, maintenance, operation, and interest on the capital cost, from the time of completion of the project until it shall perchance become fully self-supporting in the uncertain future. In this, as well as in the case of the Fort Peck Project, the claim is to be borne in mind that already there is a superabundance of irrigated lands in the West needing competent settlers in large numbers to make them an economic success.

JUSTIFICATION OF FORESIGHT

In total, the present estimated cost of these three outstanding projects is upward of $600 000 000, plus the unfigured enormous additional costs due to contingent expense and operation. In judging their merits or demerits from the economic angle it seems necessary to use caution. Vision—or, perhaps, prophecy—has been the moving cause from which some of the world's most notable products of the brain and the hand of Man have come forth. Witness, for instance, the Canadian Pacific Railway, built by men of vision across plains and mountains then unpeopled and undeveloped, with buffalo bones at first as the only freight in sight. The electrified Grand Central Terminal, in New York, N. Y., may also be pointed to as an example of imagination come true, with its rich by-products gained through the utilization of air rights, initiated in the face of forebodings that offices in that region could not be rented in competition with the established down-town Wall Street district; that guests would not patronize hotels and apartments over or neighboring a smoky, dingy, noisy railroad station; that cabs, then horsedrawn, with their attendant odors, could not be parked underground; and that vehicular travel could not be diverted from Fifth Avenue to Madison and Park Avenues, nor to the intended elevated driveways around the station. A vision of the future, bearing directly on this subject, may be quoted from a report issued,[57] in 1937, by the National Resources Committee:

> "Another material (than manganese) which is essential to the production of alloy steels, as well as for a coating metal, is chromium. It also is largely produced from foreign ores. There is a recently developed Bureau of Mines process for producing ferrochrome electrolytically from chromite ore. There is still considerable developmental work to be done on this, but if it proves feasible it will very likely be a practical outlet for the power to be produced at the Grand Coulee Dam. A new ore deposit of very significant size has recently been opened up in the Philippines and contains about 35 per cent of chromium oxide. It would be feasible to ship this material to the Columbia River water-head for electrolytic recovery of ferrochrome. There are also considerable deposits of this same type of ore in Montana and one county in Wyoming, which would be available in case of need:"

Other possible developments in electro-metallurgy in the field of such metals as aluminum, magnesium, manganese, and iron, are pointed to in the same report, as well as the possible creation of new manufacturing centers convenient to sources of raw materials and a consuming public, and the reclamation of waste lands for the purposes of agriculture.

Wishful Thinking versus Foresight

These are only hints of what may "come out of the blue" when new conditions are created, such as improved channels for transportation, changes of motive power, and pools of cheap power of huge proportions accessible to ocean vessels. Wishful thinking or idle guessing should carry no weight in prophesying what new developments may cause the Passamaquoddy, Fort Peck, and Grand Coulee Projects to be economically justifiable. Civil engineers have a right to ask just what line of reasoning in this respect has guided their Government, so that an intelligent conclusion may be reached as to its soundness, and the total carrying charges—interest and upkeep—that necessarily must be borne by the people of the United States before the project in each instance shall, perhaps, become self-supporting.

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[57] Rept. on Technological Trends and National Policy, by the National Resources Committee, 1937, p. 357.

In a word, then, there is reason to demand: (1) A defense, if one is possible, of the original committal of the people of the United States to projects based on the promoters' "half-baked" plans in connection with which the estimated costs later have been found to be far too low; (2) an explanation of the excessively high cost per unit of capacity or development that characterizes many of the projects; (3) reasonably accurate estimates of cost inclusive of all elements of every nature; (4) an outline of the reasoning, with data, in support of the alleged economic justification of each of the projects; and (5) a disclosure of the engineering and other reports and opinions hitherto held *en camera* pertaining to some present Federal projects.

Florida Ship Canal

It is not alone to the three major projects at Passamaquoddy Bay, Fort Peck, and Grand Coulee (for knowledge of which the writer is indebted to Professor Riggs' paper) that this paper is confined. The writer has also had occasion to familiarize himself with the Government's Florida Ship Canal project, which, in many of its aspects, is even more forbidding.

Basic Considerations.—The Peninsula of Florida, about 95 miles wide at its narrowest point, projects (with its terminating arc of islands) about 440 miles south and southwest to Key West and beyond, bounded by the Atlantic Ocean on its east, and the Gulf of Mexico on its west. The Straits of Florida, on its south and southeast, extend about 250 miles to the Island of Cuba and 40 miles to the Bahama Banks, with a minimum depth of 1 000 ft. Generally low in altitude, the backbone or central axis of Florida, for a width of 40 to 50 miles, rises to elevations of from 100 to 200 ft above sea level with a few heights or knobs rising to 300 ft and one to an elevation of 325 ft.

Underlying the Peninsula is the porous Ocala limestone formation, honeycombed with caverns and intricate channels of unknown location, number, and size, and dipping from north to south to levels far below sea level. On the flanks of this rock formation, to a greater or less degree, a comparatively impervious material is superimposed except at two eroded areas on the backbone of the State which act as catchments for the rain that there descends into and permeates the limestone cavities and, as fresh water, flows upward under pressure through artesian wells and at breaks at lakes and springs, and in the open sea. The northerly of these eroded areas, roughly 50 miles wide and 120 miles long, is separated from the one on the south by a low-lying saddle, about 100 ft above the sea, centering at Ocala, Fla., about 75 miles below the latitude of Jacksonville, Fla., on the Saint Johns River. Draining eastward from this saddle are the waters of the Oklawaha River emptying into the Saint Johns River near Palatka, Fla., flowing northward and eastward to the Atlantic Ocean, and westward the waters of the Withlacoochee River, emptying into the Gulf about 94 miles north of Tampa Bay. The fresh water in profusion thus under pressure beneath the Peninsula is vital to the culture of citrus fruits and vegetables, and the attraction of visitors from the North, on which the people of the State depend for their livelihood. This fact, as will be seen, is first to be borne in mind in giving study to the Florida Ship Canal problem.

Along with this is to be viewed the effect of the Peninsula on the time consumed by water-borne commerce in passing between the Gulf of Mexico and the Atlantic Coast, and foreign countries, through the Straits of Florida. Since the number has steadily increased in recent years, it is estimated that in 1935 as many as 12 000 vessels carrying more than 60 000 000 short tons were engaged in this service, principally in the movement of petroleum products, of which about 80% or 90%, it is variously claimed, is to be considered in weighing the pros and cons of a short-cut across the State.

History of Engineering Analyses.—In the 1920's the growing desire to shorten the journey between the Gulf of Mexico and the Atlantic led to various surveys by the Corps of Engineers, U. S. Army, for a barge canal connecting the Atlantic and Gulf intra-coastal protected waterways, and in the early years of the present decade for that as well as an alternate ship canal. A series of reports then ensued, eight in all, in which the 9-ft barge canal project was ignored or disapproved, and the ship canal of differing designs and estimates was in some instances approved and in others disapproved.

In all these reports the adopted general course of the proposed ship canal has been the same, running, as it does, westerly and southerly in the Saint Johns River from deep water off its mouth in the Atlantic Ocean for about 78 miles, past Jacksonville, to Palatka; thence, southwesterly and westerly in the valleys of the Oklawaha and Withlacoochee Rivers to the Gulf coast, in a 95-mile artificial cut of earth and rock having a maximum depth of 141 ft or more as recommended in the later reports; and thence, for from 25 to 27 miles in the rock-underlaid shoal waters of the Gulf to deep water, a total length of nearly 200 miles. Thus would be provided, it is claimed,[58] "a route between the Gulf and the Atlantic seaboard from 300 to 350 nautical miles shorter than the present route around its [Florida's] southern end," and a saving of "somewhat less than one day's steaming time."

It is the notching of the lip of the ground-water reservoir by this deep cut at the edge of the northerly catchment area to which reference has been made, that has given rise to the fear that the resultant lowering of the head in artesian wells and springs in many parts of the State will permit salt water to rise and cause damage to fruit and vegetable culture and seriously affect lake levels and municipal water supplies.

The first of these reports, a preliminary one made at the instance of the President by a Special Board of Army Engineers, June 3, 1933, was intended for the use of the engineers of the Reconstruction Finance Corporation and was not made public. The recommendation in a second report, made by the engineers of the Public Works Administration (PWA), October 19, 1933 (in connection with an application of the Ship Canal Authority of the State of Florida for a public loan of $115 000 000 to cover the estimated cost of a toll 30-ft lock canal), was not approved by the PWA Administrator, and failed of action.

The third report, made by a Special Board of Army Engineers, December 30, 1933, placed the cost of a 35-ft lock canal, with a minimum channel width of 250 ft, at $199 481 000 plus $8 562 000 for land and damages, a total of $208 043 000, exclusive of interest charges during construction. This report, at the request of local interests favorable to the enterprise, was not made public until three years later, after work had been started on the canal, when it developed that its verdict had been adverse to the economic justification of the project at this time, as had been the findings of the Shipping Industry contained in a report made by the Department of Commerce on February 1, 1934.

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[58] Committee Print on "Atlantic-Gulf Ship Canal, Fla.," 75th Cong., 1st Session, pp. 2, 3, and 28, respectively.

The fourth and fifth reports were made by the President's Interdepartmental Board of Review on June 28 and September 15, 1934, respectively, which estimated the cost of a 30-ft sea-level canal, with a minimum channel width of 250 ft, at $142 700 000, exclusive of interest charges, and $159 834 000, inclusive of interest, plus $3 000 000 for rights of way to be borne by local interests. The project was declared to be not self-liquidating by tolls collected from shipping, the annual charges being placed at $7 250 000 and the estimated benefits at $7 500 000, which was too close to finance the project on that basis. In the following January the PWA reached the same adverse conclusion as had been the verdict contained in the report of the Department of Commerce a year before. Nevertheless, the President, on August 30, 1935, allocated $5 400 000 from the Emergency Relief Fund for starting the work, in addition to which money for acquiring the necessary rights of way was provided by the Florida Ship Canal Navigation District, created by the State Legislature. Apparently, from then on the project was to be considered as toll free to all users, foreign and domestic.

The sixth report was made by the Revisory Board of Army Engineers, on November 1, 1936, after the relief allotment had been exhausted, in which the estimated cost of a 33-ft sea-level canal, with a minimum channel width of 250 ft, was placed at $162 985 000, exclusive of interest charges, or $185 471 000, inclusive of interest charges, and of $3 000 000 to be borne by local interests for rights of way, damages, etc. The project was declared to be economically justifiable, a verdict arrived at by calculating annual savings in ship-operating expenses, interest on ship cargo, freight rates, and navigation hazards plus recreational value to pleasure craft at $9 553 244, and annual net carrying charges at $8 242 969, leaving an alleged net annual gain of $1 310 275, which was held to show that the project was economically justifiable on a toll-free basis. The latter figure, however, would not in its entirety accrue to American interests. In the estimated savings ($9 553 244) is included the value of savings in ship time and interest on cargo ($7 155 720) of which 20% (amounting to $1 431 144) should be enjoyed by vessels flying foreign flags. This would leave annual savings of only $8 122 100 for the benefit of American citizens, in contrast with the annual carrying charges amounting to $8 242 969. On this showing the project is not economically justifiable, even granting that the benefits flowing to private interests in some manner really would be passed on to the public called upon to pay the bill.

Then came the long deferred hearings before the Board of Engineers for Rivers and Harbors late in December, 1936, without which in the usual course the project could not be considered by Congress for approval or disapproval.

This resulted in the report of the Board on February 24, 1937—the seventh in this series—fixing the estimated cost of a 35-ft sea-level canal, in preference to a shallower one, with a minimum channel width of 400 ft, at the sum of $263 838 000, exclusive of interest charges (computed at $25 005 000), and exclusive of $3 000 000 for right of way to be paid for by local interests. The total cost to all interests, therefore, was placed at $291 843 000, in which provision is made for the removal of nearly a billion cubic yards of earth and rock. In this estimate, as in all the others, nothing was included for consequential damages resulting from disturbances of the underground water supply. As bearing on this the Board stated:

> "In view of the undeterminable possibility that the excavation of a sea-level canal might open underground channels in the Ocala limestone of such size and extent as would drain a wide area, with consequent extensive damage to ground-water supplies, any authorization for the construction of the canal should empower the Secretary of War, on the recommendation of the Chief of Engineers, to modify the plans to provide for the construction of a lock canal if at any time during the progress of the work such a modification should be found warranted in the interest of protecting the ground-water supply."

No estimate, in such an event, was named for the additional cost of making the change, nor for claims for indirect damages which, according to the Board, the Federal Government under the law would face the onus of meeting. The Board declared that the benefits to be reaped from such a canal "do not establish economic justification for the large expenditures necessary for its construction," this conclusion having been predicated on estimated annual charges of $14 930 000 and benefits or savings of $8 543 000, leaving an annual net loss of $6 387 000 apart from diversions of benefits to vessels flying foreign flags.

Then came an eighth report—that of the Chief of Engineers on April 1, 1937—disagreeing in large part with the recommendations of the Board of Engineers for Rivers and Harbors. The Chief of Engineers estimated the cost of a 33-ft sea-level canal, with a 400-ft minimum width of channel, at $197 921 000, apparently exclusive of interest charges (or $215 735 000 including them), plus $3 000 000 for right of way to be borne by local interests, a total of $218 735 000 in contrast with the aforementioned Board's estimate of $291 843 600 for corresponding items applicable to a canal 2 ft deeper required, in its opinion, for safety. The Chief of Engineers does not "share the apprehension expressed in the report of the Board of Engineers, as to the possible adverse effect [of a sea-level canal] on ground-water supplies." He places the annual charges against the United States and Florida at $8 905 000 and the benefits to shipping at $8 741 000, or practically a stand-off of what the writer considers private gains at public expense. Making allowance for the portion of these benefits that would flow to vessels flying foreign flags, the result would be even more painful to the people of the United States.

Conclusions from Available Data.—At this point it should be remarked that, for some mysterious reason, the item of interest charges during construction, in the Government estimates, is treated as a thing apart, to be ignored in giving the public an impression of the total cost of the project. Hire of money, termed interest, is, of course, just as much an element of cost to be borne by the taxpayer, as hire of labor or costs of material. To omit it from the total estimates is in this respect to misguide the public.

It will be seen that the estimated costs of the project, to the Federal Government and Florida "interests," have varied from a low of \$115 000 000 to a recent high of nearly \$300 000 000, equal in amount per ship-mile saved to many times that incurred at the Panama and Suez Canals, to which is to be added the cost of damage arising from "possible" injury to the ground-waters of Florida and the added cost incident to the changing of the canal from sea-level to a lock design in case the later discovered certainty of such injury would make that course necessary.

Whether or not the net benefits to be gained from this vast and uncertain expenditure make the enterprise justifiable from a self-liquidating or economic standpoint has been open to a variety of opinions. The answer is "yes" on the part of the Revisory Board in 1936 and the Chief of Engineers in 1937. It is "no" on the part of the Special Board of Army Engineers in 1933, the shipping industry as reported by the Department of Commerce in 1934, the President's Interdepartmental Board of Review in 1934, the Administrator of PWA in 1935, and the Board of Engineers for Rivers and Harbors in 1937.

Such a jug-handled arrangement whereby the beneficiaries of the canal, domestic and foreign, are to be free to keep their savings while the taxpayers —that is the people—"foot the bills" of undefined magnitude, has an even more repugnant look when it is understood that by the time the canal should be completed in six or more years, it would run the risk of being already obsolete and unused.

In viewing this aspect of the situation, it is well to bear in mind that the margin of saving in steaming distance *via* the proposed Florida Canal is very slight in comparison with that enjoyed *via* the world's existing great ship canals. In the case of the Suez Canal, the saving over its Cape of Good Hope rival is, say, 40 times its length of 100 miles; and in the case of the Panama Canal the saving over its Cape Horn and Cape of Good Hope rivals is, say, 100 to 200 times its length of 50 miles. Both are in sharp contrast with the Florida Canal's saving over its Florida Straits rival of less than two times its length of 200 miles. The two cape routes are hopelessly out-distanced by the interoceanic cut-offs with which they have to cope. The Florida Straits route, on the other hand, is not so handicapped and through improvements in vessel design, and a less hazardous course, has every prospect of "holding its own," for reasons on which it may be well to touch.

The estimated saving in ship time through the use of the proposed Florida Canal as against the Straits of Florida (22 hr) is based on a difference of vessel speeds *via* the two routes which will become less and less as progress is made in the art of shipbuilding. The Chairman of the U. S. Maritime Commission recently has stated[59] that "the United States is third in world tonnage (mostly obsolete), fifth in speed of ships, and eighth in ranking of modern ships in operation." With an increase of 50% in the speed capacity of vessels, already said to have been realized in the mercantile fleets of foreign countries with which the United States must compete, there will be no appreciable saving in time to be made through the use of the proposed shorter canal route in which advantage may not be taken of modern open-sea speeds. Even under present conditions many of the steamship companies now navigating the Straits of Florida assert that, despite its intended absence of tolls, they will not use the canal, if built, because of higher insurance premiums on hulls and cargo passing through the canal, and[58] "the hazard of damage by collision or grounding in the constricted waters of the canal" which the Board of Engineers for Rivers and Harbors considers "would greatly exceed the hazard of damage in the open waters of the Florida Straits including damage and delays due to hurricanes in the latter."

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[59] Reported in *New York Times* in dispatch from Washington, D. C., August 16, 1937.

In a word, unlike the Panama and Suez Canals and unlike other artificial waterways having no alternative routes to threaten their use, the proposed Florida Canal would ever face the prospect of the continued use of a competitor having lesser handicaps and the superior advantages of open-sea navigation. The margin of saving in time *via* the new route is too slight to warrant an expenditure upon it of $200 000 000 or $300 000 000, or more, which the people never can "get back," to quote the words of the President who is reported to have named this as a condition.[58]

From what has been stated herein[58] in respect to the toll-free Florida Ship Canal project the situation may be summarized as follows:

(1) The estimated cost of the project has risen in the four years, 1933 to 1937, from $115 000 000 to as high as $291 843 000, with the assurance of a marked increase, of unknown size, should a change be found necessary from a sea-level to a lock design, and should consequential damages be incurred through injury to the underground water supply of Florida;

(2) The estimated carrying charges to be borne in this connection by the people of the United States have risen in the same period from $7 250 000 to as high as $14 930 000 per yr, also with the assurance of a marked increase should the aforementioned conditions prevail;

(3) The estimated benefits to be enjoyed by the users of the canal and its other beneficiaries—foreign and domestic—have ranged from a low of $7 500 000 to a high of $9 553 244 per yr, constituting in effect a public subsidy to private interests from which the people of the United States as a whole would gain nothing whatever, other than the possibility of some lowering of freight charges of unknown extent, of which there is, to say the least, a doubtful chance when competition rather than cost of service is what governs the fixing of rates;

(4) In thus failing so signally to show benefits in excess of carrying charges, the canal project falls far short of meeting the condition laid down by the President that, to have his approval, it should in effect bring to the people of the United States the assurance that they would get their money back;

(5) The possibility of grave injury to the underground water supply of Florida, through the building of a sea-level canal, is one to be viewed with concern by the people of the State and nation. It is comparable to a community, informed that an important bridge on which the people depended for access to the world might fail, with resulting death or injury to their families

—a case in which to doubt is to condemn; where there are no compensating advantages to offset the risks;

(6) The disruption of Florida's highway system along the canal, involving the creation of needed future crossings at the expense of the State, also to be burdened with the inconvenience and cost of upkeep and operation of highway ferries and drawbridges as well as the costs of rights of way and other lands for the canal; the cost of appropriate terminal and transfer facilities; and, the cost of maintenance and operation of the canal in excess of a fixed annual sum to be borne by the Federal Government, should the recommendation of the Revisory Board in this respect be endorsed by Congress, are all matters of great moment to the taxpayers of that State in which, too, sight is not to be lost of threatened injury to its wild life; and,

(7) The modernizing of the country's merchant marine, about to be undertaken, will result in the employment of speedier vessels in the Gulf-Atlantic trade and, in consequence, the continued use of the safer present open-sea route around the tip of Florida as against the canal route then offering no substantial saving in time and greater dangers of navigation, thereby threatening the canal, if built, with early obsolescence, even if the exhaustion of oil fields tributary to the Gulf is long postponed.

Recommendations.—In view of the foregoing conclusions, therefore, the Florida Ship Canal project, as a substitute for, or rival of, the existing open-sea route through the Straits of Florida, is without a single sound argument in its favor, and should not receive the stamp of approval by Congress, accompanied by appropriations for its continuance. It is better thus to abandon what was started as a relief measure at a cost of $5 400 000 of Federal money and an unknown sum thus far expended by local interests for right of way, than to throw good money after bad.

In taking this stand against the financing of useless public works it is not meant, of course, that Federal funds should not be devoted to the construction of really worth-while projects in order to give work to the unemployed. On the contrary, the money saved from loss on uneconomic enterprises like the Florida Ship Canal may, and should, be devoted to the supplying of such crying needs as added public thoroughfares for a rapidly growing automobile traffic. For instance, the building of an International Parkway along the Appalachian Uplift, in co-operation with Canada, connecting many national and State parks and forests and other recreational areas, reaching from Montreal, and Quebec, Que., and the Gaspé Peninsula on the Gulf of St. Lawrence, to Key West, Fla., and other attractions on the Gulf of Mexico, would cost no more in the United States than a Florida Ship Canal; it would give employment to skilled and unskilled labor in seventeen States of the Union instead of one; it would serve to protect the mountain forests and streams from private spoliation and restore many of them to their original charm; and, from a social viewpoint, it would provide the public with a safe, convenient, and inspiring means of movement between the widely varying climates, scenes, customs, and fauna and flora of the North and South, free from commercial usage, billboards, and other objectionable features, in regions neighboring great industrial centers where utility and pleasure thereby would be sanely served in these days of increased leisure.

A CHALLENGE

The papers of the Symposium constitute a terrific arraignment of those who are entrusted by the people of the United States with the handling of their affairs. Great public works are declared on the record, to have been conceived in error. Can it be that the Engineering Profession is not in possession of all the facts with which to reach a just conclusion? If so, the Government's side should be presented without reserve. As citizens of a democracy, and as engineers qualified for the purpose by education and experience, the Civil Engineering Profession should, in the interest of the public, approach this question in a non-partisan spirit, searching only for the truth. In the words of Professor Riggs, "They will fail in their duty if they keep silence."

In the case of the Florida Ship Canal, as in the instances referred to by other authors of this Symposium, involving in all a possible expenditure of more than a billion dollars, it is to be hoped that their proponents—members of the Society and others—will draw attention to any errors of fact which, unintentionally, may have been introduced herein, and to any reasoning that they may care to offer in rebuttal. To arrive at the truth is the important objective, so that engineers may properly advise their fellow citizens who are not experienced in such matters. Thus will there be ground for hope that the hazards of uneconomic public works construction in the future will be avoided, or at least lessened. In that cause there can be no higher call to public duty.

ACKNOWLEDGMENT

In the preparation of this paper, in addition to other sources cited by footnote references, the writer has made liberal use of the factual data presented by Professor Riggs in this Symposium.

DISCUSSION

J. K. Finch,[60] M. Am. Soc. C. E. (by letter).—Although few engineers would question the soundness and force of Mr. Fay's arguments for the future planning of public works, it is quite obvious that very great practical difficulties will be encountered in putting them into effect. There are clearly two major phases of this planning movement.

On the one hand, there is the necessity for foresight in planning public works—a need for engineers to encourage comprehensive planning rather than a hand-to-mouth practice in public construction if such works are to be better adapted to future public needs and requirements. In this respect they must recognize that many existing public works agencies are not set up to undertake such comprehensive planning effectively, and it is doubtful whether the fundamental changes in organization, tenure of office, etc., necessary to permit or to encourage the development of a long-range policy, can be effected. Apparently, the practical approach to the problem is not through reforms in existing public works agencies, but through setting up separate planning boards or commissions, which will be independent of changes in political party and will give particular attention to the longer range viewpoint. This movement is growing rapidly. City and regional planning boards are increasing in number. It was reported in 1935, for example, that there were forty-five State planning agencies, and many municipalities have city planning boards. The attack on this problem, therefore, has been through this more or less indirect plan—far from ideal, perhaps, and still, in many cases, lacking in implementation to make the movement fully effective—but apparently the most promising and practical line of attack yet developed.

On the other hand, the further proposal to use public works as a means of reducing the fluctuations in the construction industry, while fine in theory, seems discouragingly remote in the possibilities of achievement. It is dependent, of course, on the aforementioned future planning movement but, in addition, unless the United States abandons its democratic form of government, public opinion must, as it always has, continue to play an important part in the initiation of actual construction. There may be little difficulty in speeding up public construction in periods of depression as has been done in recent years, but the idea that the Government can retard public spending when prosperity is at its crest sounds well in theory but impossibly impracticable in practice. It might be hoped that the Government could have in hand some well planned and useful projects which can be undertaken in periods of depression, but, in all probability, the really pressing, and, therefore, most important, works will have been built when times were good and their construction was demanded by the public.

Turning to the papers by Messrs. Mead and Riggs, many readers would accept their statements of fact as to the false business economy, or, too frequently, total lack of such considerations, in the construction of the works described. Many engineers would also condemn in an equally forceful way the apparently deliberate suppression of the engineers' reports on these projects and the later attempt to place the onus for the initiation of these economically useless and imposingly wasteful expenditures on the engineer. Many engineers would also agree with Mr. Mead that the TVA "yardstick" notion is a political humbug, and a child of wishful thinking and bad bookkeeping—that, in fact, the entire recent Federal viewpoint on the power problem is unreasonable, economically destructive, and eminently unjust. At the same time, many engineers would not agree with the conclusion implied, at least, in these papers, to the effect that the Government should have nothing to do with any project involving power production and should build no works that are not fully justified from the standpoint of business economics.

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[60] Renwick Prof. of Civ. Eng., and Director, Summer School of Surveying, Columbia Univ., New York, N. Y.

It seems quite obvious, for example, that the Federal Government is already involved in many projects in which the development of power is incidental (is a "contributory benefit," as the Supreme Court puts it) to works which are clearly within the constitutional scope and recognized function of modern government. The time is long since past when Government can limit its activities to the simple functions which constituted its entire scope of duties in Colonial days. The time has passed when the American people will turn over to private interests the remaining public property of the nation for exploitation and development by such interests and for private gain. It seems obvious that this nation has embarked, for better or for worse, on a policy involving the governmental development of its remaining public properties and resources in what it is hoped will be a wider public interest.

It would appear to the writer, therefore, to be futile and impracticable and thus hopelessly reactionary and completely unconstructive, to damn the present entirely and argue for a return of the "good old days." It is all very well to argue that one should not compromise with thieves and murderers, and it is apparently on this ground that the extremists of the New Deal refuse to listen to the power utilities and the power utilities refuse to compromise with the New Deal. To face the facts, however, one must admit that there are two interests both legitimately involved in the power field and, as in the case of capital and labor, these two interests, private and public, must get together, must reach some fair and just basis for co-operation, rather than continue a wasteful and unnecessary battle at the expense of the public. The present Federal viewpoint, if persisted in, will lead to economic suicide. The present public utility attitude often seems planned to alienate rather than woo public support. The engineer is generally considered to be a man of common sense, and he certainly cannot support the extremists on either side. While admitting much which both sides say, he will insist that a compromise is the only practical solution of the problem.

Much can be said, of course, on both sides of this question, and it will probably be necessary, in order to find a sane solution of the problem, to ignore rather than to attempt to reconcile some of the extremists if a compromise is to be effected. It is unquestionably true, for example, that a radical modification of the recent viewpoint of the Federal Administration is the first necessary move toward peace and co-operation. With the Federal Government going ahead with power developments in fields which private companies already serve, with public funds being used to construct municipal plants which put existing plants out of business, it is small wonder that the private utilities are up in arms and that billions in utility constructions are held up. This policy is destructive of economic values; it results in the waste of public funds, depletes the national income, and retards recovery. No engineer whose viewpoint is based on engineering principles rather than on blind bias and emotion can support such a policy.

On the other hand, few thoughtful engineers would attempt to defend the marking up of capitalization and other abuses in some of the power utilities which characterized the pre-depression years. These companies are granted a practical monopoly of an essential service and enjoy a relatively constant and steadily growing market. It seems clear that the lowest possible rates consistent with adequate service should be the goal of their operations. A low interest rate on a fair valuation is all that should be expected. Yet in the halcyon days before the depression the "merchants of debt" argued that capitalization should be marked up to the top figure which income could justify and, through holding companies and similar means, proceeded to burden some utilities with unnecessary and unjustified capital expense. The engineer must condemn with equal warmth this narrow, unsocial, and, in the long run, economically fatal and unconstructive policy. Clearly, if the public utilities take no steps to put their houses in order, and persist in fighting regulation by fair means or foul, the patience of the American people will ultimately be exhausted and the power business, like the water supply business, will be taken over by Government. In the long run, business must be conducted so as to advance the economic welfare of the nation and serve the public interest, if it is to survive.

The writer knows that Messrs. Mead and Riggs would condemn such utility practices and policy, but they apparently feel that this "public utility attitude" reflects the viewpoint of but a small part of the industry, whereas the present policies which have been advanced and encouraged by the New Deal forces are far more fundamental and dangerous to our national life. Although the writer would admit the force of this argument, when advanced by these experienced and careful analysts, it seems clear that the suggestions made in their papers, while directed toward the correction of an existing evil, cannot form the basis for a constructive and progressive general power policy. Co-operation seems essential. After all, it should be the purpose of both Government and business to advance and strengthen the national economic structure, because it is on this structure that our social and national development must rest. Clearly, the basic policy of both Government and business must be based on this principle and must reflect not a narrow, short-sighted opportunism but a reasonable and forward-looking viewpoint. Engineers are so accustomed to planning not for to-day but for to-morrow, of attempting to predict and to plan for future needs, that they assume that this is a common human trait. They evaluate a policy not solely on its immediate advantages but attempt also to picture its future effects. Unfortunately, this is not a common human trait—the world says "take the cash and let the credit go." A little more foresight in both business and political thinking and in the development of business and public works policies is an essential to the continued growth of American institutions. Accepting as a principle the basic idea that mutual understanding and co-operation on a sound economic basis are essential to the future usefulness and sane development of the power business, the writer is confident that engineers are capable of developing, with fairness and impartiality, the necessary details for such a working agreement.

Finally, those engineers who would adopt as a principle the idea that Government should not undertake to build any works except those which can be shown to be economically justified on a business basis, have apparently forgotten or ignored a basic principle of democratic Government. There are many public works which cannot be thus justified and, furthermore, the decision as to whether they should or should not be built is not a simple matter of engineering opinion. It is a question of public opinion. In short, what shall or shall not be done (within the wide limits of constitutional and other law) is determined solely by the state of public opinion at a particular time and place. The result may be the use of public funds for works which are economically justified, or may be believed to be socially desirable, or even for constructions which represent pure waste and extravagance.

If the individual engineer is opposed to all public works which cannot be shown to be economically sound, he is free, as is every other citizen, to express this as a personal view, but it is an impossible principle for the profession as a whole to bring forward. There are certain basic principles, however, on which the Engineering Profession can agree.

Any work, for example, which does not offer opportunities for adequate financial return to the public treasury through direct service charges, through assessment for benefits, or through the creation of additional values which can be taxed, must be classed from the pure economic standpoint, as a luxury. Presumably, such works can be afforded only when the surplus of public funds permits such construction, or when the public is willing to accept such an additional tax burden as their construction entails. This is not a matter of opinion but a statement of fact.

Now, the engineer is an expert in the investment of funds, public or private, in engineering enterprise. It is his duty not only to design such works to meet the service desired with the utmost skill and economy, but also to advise his client, public or private, as to the probable cost and returns from the investment. The Engineering Profession, therefore, is quite within its scope of duty if it insists that the public shall be advised as to the engineering values of any proposed investment of public funds in a public work of an engineering nature. As far as the writer can see, the engineer's duties as a professional man go no further than this. In fact, if he reads these papers correctly, one of the chief objections which the authors offer to recent Government practice rests on the suppression of adverse engineering reports or the assumption that engineering values can be appraised adequately by a layman. The writer, therefore, feels that a great majority of engineers would support the following statements:

(1) That any proposed investment of public funds in any enterprise of an engineering nature should be investigated by (*a*) competent engineers, not directly concerned with, or interested in, the enterprise, who should be asked to review the project as to (*b*) its feasibility and cost, and (*c*) its economic values, reducing these as far as possible to quantitative terms and stating clearly the limits of precision and certainty of the conclusions; and,

(2) That all such reports be given full publicity.

ELLIOTT J. DENT,[61] M. AM. SOC. C. E. (by letter).—The brief discussion of flood control by Professor Mead is timely, and it is to be regretted that it was not possible to make a more extended statement. There is an urgent need that the general public be informed as to the facts as popular ideas do not include a recognition of the basic causes of flood disaster or the difficulties that must be overcome in order to effect a cure.

The occupancy of the flood-plain does not prove folly on the part of those who have selected sites subject to over-flow for their homes or their business activities. The fact is that the inherent advantages of the bottom-lands are such that they will far outweigh the disadvantage due to an over-flow that may occur once in a generation. A settler moving into a new country may have been wise in deciding that the annual gain due to the selection of a flood-plain site will enable him to set aside a reserve that will more than compensate for the loss due to infrequent flooding. A second settler may build near the first for the same sound reason, a third moves in, and eventually a town or city comes into being. As the town was expanding each individual found it to his advantage to build in the flood-plain rather than on higher ground and each individual takes the measures to guard against flood loss that he considers advisable. In spite of the object lessons afforded by the great floods of recent years, large numbers of residences and business establishments are being currently built in areas that were under water only a year or two ago.

The cost of flood protection for a single building may be far more than is warranted by the infrequent losses due to the incidence of the great floods; the time, however, may arrive when the joint losses of all the occupants of a community may warrant the prosecution of a joint project for their joint protection. In some cases the losses in a number of municipalities in a single water-shed may warrant joint action with a view to the construction of reservoirs for the abatement of the floods as contrasted with the local protection works aimed at their exclusion from selected areas. When a community project can be shown to be economically justified it should be carried forward, but, pending the completion of such a project, a new comer may well find it to his advantage to establish himself in the flood-plain, reap the advantages of the site selected, and stand the loss due to an occasional over-flow. Such choice on his part may be good business, not folly.

Wherever there is a tendency for a municipality to expand in an area subject to occasional over-flow, it may be well for the Public Works Planning Agency to require that each new sub-division make provision for eventual flood protection. Such advance planning may materially reduce the ultimate cost and may thereby make it possible to execute the project at an earlier date than would otherwise be possible.

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The people of the United States are accustomed to carry fire insurance and the contrast between fire and flood losses may be worth referring to in the educational campaign so badly needed at this time. The ordinary fire loss is confined to a small area; it may be limited to part of a single building; it may include the whole of a single building, or a number of adjacent buildings; a conflagration covering one or more city blocks is most exceptional. A fire loss is ordinarily very heavy for the property actually involved, and of relatively small magnitude when the wealth of the entire community is taken into consideration.

Flood losses, on the other hand, rarely involve the complete destruction of a piece of property, the average loss per premise flooded is probably less than $500. A few weeks after the greatest flood the visible evidence may have almost completely disappeared. From their very nature, however, flood losses cover all developments within a given area which may include a considerable part of a city or of city after city along the valley of the flooded stream. A great flood may last for many days and the multitude of small losses adds up to very large figures, the numbers of persons driven out of their homes create a dramatic situation that appeals to the public. The scare head-lines about loss of life and outbreaks of disease help to raise relief funds but have little factual justification.

Fire losses are intensive and localized, flood losses are less intensive but have some of the characteristics of a widespread conflagration. Flood losses might be made the subject of reasonable insurance rates, but the insuring companies would be facing huge losses at infrequent intervals instead of the relatively well distributed losses due to the incidence of numerous small fires.

An individual planning to invest his own funds in a home or a business must decide for himself whether it is best to locate in the flood-plain, take his profits during the many years of security, and take his losses when the floods come. In the natural course of events those who elect to occupy the lowlands must accept the consequences of their choice. In due time, they may become numerous enough and wealthy enough to make a co-operative flood-control project an economically sound public enterprise. A judicial engineering study will go far toward showing when the proper time for the execution of the project arrives. Advance planning may also result in the reservation of rights of way for use when the project is undertaken and by reducing the cost may advance the date when the work may become economically warranted.

C. FRANK ALLEN,[62] M. AM. SOC. C. E. (by letter).—In Massachusetts, there is a law that no new railroad enterprise shall be authorized without a certificate of exigency from the proper board. There seems to be no board for that purpose in Washington, D. C., and in the projects that have been undertaken, apparently there has been no regard for what might be termed a certificate of exigency. Instead there is a certificate of emergency. Such a certificate does not reach the emergency of the project itself; it is not a reason, but an excuse, for the expenditure of huge sums of money.

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[62] Prof. Emeritus, Mass. Inst. of Tech., West Roxbury, Mass.

Many of these projects have the element of emergency in the matter of flood control, but, in some cases, flood control has been used as a means for the encouragement of other lines of activity; for instance, power. In the creation of reservoirs for flood control, power is an antagonistic element. For power, a full reservoir is requisite; for flood control, an empty one is requisite; and it is an interesting fact that in the Dayton Flood Control System, a tablet on each of these reservoirs states that it has been designed for flood control and not for storage—in the general sense, not for power.

Engineers should be careful in accepting power accessories as coming within the exigency, or emergency, classes that are established in connection with these huge projects which, apparently, have no economic value.

Most of them have an unfortunate effect in disturbing existing agencies, and there is at present great need for stability. Nearly all of them tend to disturb existing industries, and, to that extent, in many ways, they are unfortunate. For example, why develop irrigation, when it will disturb so many of the established agricultural interests, which have already been restricted by the Government? An important feature of some of the projects is irrigation.

BRADLEY G. SEITZ,[63] JUN. AM. SOC. C. E. (by letter).—In his paper Professor Mead maintains that the Federal public works activities of the past several years have been, in general, economically unjustified, and of little benefit to the nation. Professor Mead further contends that radical legislation has been introduced, and that the broadening of governmental activity threatens the income of investors.

The sociol-economic implications of this paper are open to question. The expansion of governmental functions is not a temporary national phenomenon to be deplored; it is the resultant of all the complex forces acting upon modern society, a force to be directed. In England, France, Russia, Sweden, Switzerland, Finland, and Norway, for example, the "Record of the Government in Business" has been written several chapters beyond that in the United States. Most of the legislation referred to as radical has long been accepted by the conservatives of England.

The fact that some public works projects may be uneconomic does not warrant so general a condemnation. It is to be remembered, also, that conventionally sound economic analysis often leads to conclusions which are socially absurd. It cannot be applied, for example, to public education, or garbage collection. During the past few years, it has been economically unsound for industry to employ about 10 000 000 men; and yet the resulting curtailment of production in the face of a need for products, the waste of man-power and machine power, are both technically and socially appalling.

Although increased governmental activity may well result in decreased profits to the investor group, it is obvious that the condition of maximum profit to investors is frequently in sharp conflict with the general welfare. When this conflict occurs, the general welfare should prevail.

Professor Mead cites the inefficiency of the Government in business. From a social, rather than a profit, standpoint the efficiency of business itself is somewhat marred by over-expansion, duplication of effort, the apathy of mechanized workers, the production of unsound or worthless goods, the interruption of production by strikes and lock-outs, wasteful exploitation of natural resources, over-capitalization, excessive sales, advertising and distribution costs, and periodic depressions.

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These defects are mentioned, not to excuse inefficiency in Government, but rather to suggest that the failure of public enterprise to satisfy the formulas of private enterprise is not, in itself, a valid criticism, since their aims are different.

ALFRED ALLEN STUART,[64] M. AM. SOC. C. E. (by letter).—The papers by Professor Riggs and Colonel Wilgus, especially, have great value, not only because of their able analyses of the subject-matter but also because they expose a great waste of public funds for projects having no appreciable value to the people at large, who must bear the cost. For these several reasons the profession owes a debt of gratitude to each of these authors for bringing to public attention such unwarranted waste of public funds. However, it must be said that the publications of the Society have such a limited circulation among all the people as to preclude the great benefit from being realized.

It is to be regretted that very many of the large works undertaken by the Federal Government are brought to the attention of Congress by incompetent individuals, and are often acted upon favorably by Congress, despite the fact that they may have no merit whatever. It is unfortunate that, as now constituted, the Society has no means of being invited by that body to advise Congress as to the merits, or demerits, of all matters presented to it which, in their very nature, requires a highly trained and experienced civil engineer to comprehend and learn the merits, and demerits, before Congress takes any action on them.

In April, 1906, the writer was asked to go to Portland, Ore., to advise a banking house of New York City as to the merits of an undertaking that would require a large investment of capital. The local newspaper announced his arrival, stating that he represented large financial interests in New York. Immediately following, and continuing as long as he remained in Portland, the writer was importuned by many individuals to interest capital in numerous new developments. Many of these had much merit and could, upon completion, be made very profitable had there been a sufficient number of people to have availed of the benefits to be derived. In view of the fact that as few people were available to utilize the proposed improvements, the writer was compelled to decline even presenting them to the banking interests which he represented.

Congress is constantly having meritorious enterprises presented to it, but as now constituted that body has no means of analyzing them to discover whether they have any merit in fact.

PIERCE P. FURBER,[65] M. AM. SOC. C. E. (by letter).—One aspect of the public works question has not been stressed in the papers of this Symposium.

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[64] Cons. Engr., Winter Park, Fla.

[65] Wayne, Pa.

Several references were made to the uneconomic character of Government projects, although none of the authors offered to show why this happens, or what fundamental principle is involved. The remarks were largely confined to specific criticism of individual projects as samples of Government inefficiency, and frequently to the relation between good engineering and bad economics displayed in most of them.

Often it has been said that the Government cannot operate any business successfully. Experience seems to confirm that opinion. A primary reason may be: Success in business depends upon management more than any other factor within Man's control. Finance, engineering, construction, operation—able management furnishes the impetus that drives them all. The economic success of a construction project depends primarily not on the soundness of the engineering, but upon whether the total actual cost of the completed work will bear a proper relation to the income which may be derived from its operation. This will follow if the judgment of the promoters is sound. Good management is the basis. Engineers may as well acknowledge this situation; it will do no good to cite excellent engineering design as a justification for any project. Engineering work is a means to an end, not the reason for spending money. Often, engineers themselves furnish the good management so essential to success.

Why cannot the Government match that efficient management which may be discovered within the executive branch of private enterprise? Mainly, because private business is undertaken for profit. Men who risk their capital, and their time, in a venture, naturally possess the necessary selfish interest in it to act as a spur to their wits; energy must be applied constantly to avoid loss of their own money. This applies as well to corporations, whose active stockholders exercise constant vigilance over the acts of directors and managers. Some of the direct urge may be lessened in the case of large companies with enormous numbers of small stockholders This condition serves to emphasize the general principle as it concerns Government business, because it has often been noticed that the efficiency of large corporations which have been "orphaned" by wide dissemination of stock holdings has deteriorated in some degree due to the management no longer having a genuine stake in the company.

There is a difference between occupying an executive position of power which carries small financial responsibility, and holding a large proprietary interest in a business. The proprietor will invariably prove a better manager than his hired assistant, granting that he be not simply a financial "angel."

The chief fault in these Government projects—in any Government venture into the field of business or construction—is that no one responsible for the conduct of affairs is in any way responsible for the financial success. None of the managers, or directors, or administrators, has any stake in the enterprise. They are always spending other people's money—tax money—of which the supply always appears to be inexhaustible. The latter presumption accelerates the tendency to uneconomic operations. If only a certain sum were available, and the director knew it, there might be an incentive to try to make that sum do the work. It is fair to credit every public official with the intention to perform his task satisfactorily and efficiently. Likely enough, very few of them are consciously reckless or extravagant; they want their plans to succeed.

However, they may not possess the quality of judgment required to perceive the defects. Experience of large affairs is nearly imperative for any one by whom large sums of money are to be disposed effectively. Judgments may be hampered, furthermore, by the pressure of interests (political, for instance) which should have no voice in determining policies or formulating plans for a business. Public officers are always plagued by geographical considerations which affect practically every dollar spent upon public works.

Notwithstanding the foregoing comments, the entrance of politics, local or sectional interest, or any other influences that burden the management of Government business, actually does not provide sufficient excuse for the failure of public works to pay their way. Neither is it sufficient to assert that they might be constructed or operated economically if their mangagement could be "taken out of politics." To remove all influence of that kind from the directing head would simply leave him more or less free to do as he pleased, without control from any source except public opinion or his own conscience; lack of financial responsibility would be altered in no sense. There would still be no "pocket-book" urge to caution or prudence, no risk of loss staring him in the face before every important decision.

Direct financial interest makes a difference! There may be altruistic souls who believe that a strong man may put forth the same effort on behalf of others—or the public—that he exerts in his own favor. Examples of such deserving virtue may be found in the field of pure Government or civic service, where success is not measured in money. When the cruel criterion of profit and loss enters, the situation changes remarkably. Detachment from direct financial concern cannot fail to alter any person's view of any proposition from that he would hold if his own welfare were involved. Consider trustees, for example. Honest and capable trustees for others' funds are notoriously conservative; risks are feared, sometimes to the actual detriment of the beneficiary. Weaker men sometimes succumb to the temptation to use their positions for personal gain.

Public officials are human, as much so as trustees. They cannot possibly endow themselves with a disinterested quality, or ability to handle large sums of money in widespread business ventures which will produce results commensurate with the achievements of private persons whose self-interest and ability are all centered on one objective: To make a commercial or economic success of whatever they undertake.

Concerning particular suggestions made in the Symposium, a few comments are offered.

Advance planning is well enough to contemplate, as a theory. The prospect of arranging some program of public improvements to be undertaken in the indefinite future, when the need for work is expected to exceed the amount available, may probably now be considered good social theory. It could scarcely be considered good economic theory at any time. The only natural, or rational, economic justification for any building project is either that the construction is needed, or that the improved property will produce income. Governmental projects may be justified by the former; private, or business projects, by the latter consideration.

"Build if, and when (and only if and when) additional space is needed" is a good rule for any business. Successful and growing business applies this rule quite generally, regardless of outside conditions, much the same as large investors, such as insurance companies, handle investment funds; they buy securities when they have idle capital to invest. Waiting for a more favorable market may mean missing the market entirely, or incurring a loss of income. Postponing a building operation may mean loss of time more valuable than the possible increased cost at current prices. Advancing a building project may mean a total failure if the plant is found unnecessary when completed.

Therefore, from an economic viewpoint, success depends upon the decision of some one that the work is needed and should be undertaken at a particular time. Able management is thus the guide to proper timing, also.

Improvements included now in a program for later development during some future depression might be found inadequate, or even undesirable when the time arrives for their execution. No one can foresee the material necessities of even a few years hence. There are many examples of structures planned (at considerable expense) for future extensions, which have not been made and may never be.

The best plans are not always those upon which much time has been spent. A more important factor than time is involved in good design, or planning, namely, brains. That keen sense of discernment, which can be neither explained nor defined, which some possess and others lack, which leads some men directly to practical solutions of difficult problems, outweighs every other qualification of a prospective manager or designer. Obviously, reasonable time is necessary to any work. More time than that spent on reviewing possible methods, especially in conferences, may be the cause of inferior plans. Advance planning, especially for long deferred projects, might easily produce just that condition.

The criticism of public works presented in this Symposium indicates not so much that dissatisfaction is expressed because the projects were hastily planned, as that most of them should never have been planned at all. All the large projects discussed are extremely questionable ventures. Perhaps none of them would have been undertaken had a serious and honest balance been drawn between economy and expediency. The least competent engineer may readily understand, without deep research, that the Florida Canal, Passamaquoddy, Grand Coulee, or TVA, will never be worth their cost if measured by the same standard ordinarily established for lesser operations. Why should not they be so judged?

Whoever attempts to justify this huge construction program as economically feasible is invited to answer or consider a hypothetical question: Could these projects ever be financed if the Government had to attract private capital instead of compelling it through taxation?

True enough, Government bonds have a market; but not on account of the security afforded by public property. They are in no sense a lien upon it. They may be regarded as a satisfactory investment for one reason only: The Government has power to tax the people to pay its debts. Failing this (if, for instance, an extravagant Government should borrow, and spend, more than it could possibly collect in taxes), only one course would be open—repudiation; because, whereas bankrupt corporations liquidate or reorganize, Governments repudiate, directly or indirectly. A Government may issue so-called money (paper notes) to retire its so-called bonds, or it may simply neglect to pay them at all, or it may refuse to pay them. Then, the bondholder has no recourse. No legal security exists for the holder of Government bonds. A bankrupt Government may not be called into Court for liquidation. The creditor's rights are determined by the debtor's pleasure, or good faith. Witness the World War loans.

This situation forces the conclusion that, ultimately, the Government itself is as irresponsible, financially, as any of its agents or officers, when considering the economic aspects of public works. It is liable only to the extent it chooses to be; it may cancel or abridge its agreements with impunity, wherever the executive has discretionary power; otherwise, by Act of Congress. As no one concerned with borrowing or spending these fabulous sums has any direct responsibility for their wise administration or expenditure, neither has the Government, as an entity, any responsibility. Executive discretion may be controlled, or legislative aims directed (after a fashion) only at the ballot box, and not there, even, in case of many appointive officers. Past official action is subject to scarcely any redress, although it may have caused serious damage.

Government extravagance differs from private waste in one important respect: The former wastes public money, the latter wastes its own. The former, therefore, is far more reprehensible, contrary to the cynical notion many people entertain that the Treasury is a huge grab-bag, a bottomless mine to be worked by clever people for their own benefit.

Any unnecessary public works are a drain on the national wealth and welfare. Business projects conceived in false economy, and executed extravagantly, will lead a Government as surely into bankruptcy as similar mismanagement will wreck a private business. The principle is the same, the distinction being one of degree. The evil day may be longer deferred by a Government's using powers denied to private persons.

Whether the Government programs will constitute a national policy, or portend increasing public ownership and operation of business probably depends on how long the people may take to realize the extent of the uneconomic works already under way, and that they are to pay the cost themselves. When they have reached the inevitable conclusion that the management of public business is irresponsible, under all circumstances, they should understand the futility of entrusting economic problems to Government. The necessary functions of pure government (national defense, protection of public safety, etc.) fall into an entirely different category.

The Engineering Profession may serve the country by hastening to develop the national understanding of this momentous question. Its members should view the subject patriotically, and express their deliberate opinions with a single purpose for the best interest of the entire country. It is a subject fit to challenge one's devotion to the truth; a test for one's claim to high ideals. Let engineers hold to the course of reason and common sense, no matter what ignorance or sophistry may argue to the contrary.

Colonel Wilgus cites the vision of certain builders, cautioning that vision is an attribute of unknown value. It is one element of the sense, or judgment, emphasized herein. Vision, however, varies with the point of view. Another hypothetical question is submitted: Would any sensible, level-headed manager, or board of directors—any that ever lived—use their own funds to build these public works if they, themselves, had to assume the risk of possible failure with the attendant losses? (Assume, of course, that they would be entitled to whatever profits might accrue.)

Is it not a question of whose vision to trust? Unless vision is accompanied by substantial financial capacity it seldom inspires confidence. The Canadian Pacific Railroad and the Grand Central Terminal illustrate the point admirably. In these projects, vision, backed by capital, carried responsibility. The vision that promulgated the Florida Canal was devoid of it.

Apparently, some projects have been ordered built in spite of adverse reports by competent engineers. Such proceedings confirm the belief that lack of financial responsibility leads to recklessness, inefficiency, and waste. A responsible executive would hesitate before committing himself or his company to a policy involving large expenditures after receiving not only one, but perhaps several, very discouraging expert reports upon the economic feasibility of a proposition.

The inception of these public works brings to mind the figure of Peer Gynt, in Act IV, Scene V, of Henrik Ibsen's play. Stranded in Morocco, Peter Gynt has wandered inland to a place overlooking the Sahara Desert. After a soliloquy as to the "enormous boundless waste" stretching out before him, he is inspired by an idea:

> "A dam! Then I might ——! The hills are low.
> A dam! Then a cutting—a canal—
> And through the gap the rushing waters
> Would fill the desert with a life-flood."

Then, his thoughts rushing out with a frenzy of idealistic zeal, he visualizes town after town growing up in the Sahara Desert—new industries, new trade, new discoveries; and, finally, with settled conviction, he springs up:

> "I only need
> Some capital, and the thing is done—
> A golden key, and the ocean's gate
> Is open! * * *"

R. F. Bessey,[66] M. Am. Soc. C. E. (by letter).—This Symposium is disappointing because, essentially, the papers (except that by Mr. Fay) represent only one viewpoint of the subject. With Mr. Fay's suggestions for orderly investigation and planning of public works the profession should be in hearty accord. He also suggests, in developing his thesis, some of the fundamental national economic considerations relating to public works, namely, unemployment, the business cycle, national income, the construction industry, public finance, etc.

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As to the remainder of the Symposium, the dominant note is one of criticism and the general inference is that current programs for large public works are uneconomic and unsound. With the constructive suggestions, that public works economics should be studied and discussed by the Engineering Profession, there should be general agreement.

It is not the purpose of the writer to undertake a detailed defense of the projects questioned. Rather, it is his intention to urge a deeper and more general consideration of the economics of public works by all engineers, and briefly to indicate some of the broader economic considerations relating to the public works of the Pacific Northwest.

It is hoped that the Symposium will bring more professional thought to the fundamental economic and social factors—in addition to the purely physical and financial factors—in the design of public works. If engineers are to be of optimum value to society, and are to take the valuable place in public affairs that their basic training warrants, they should acquire a certain knowledge and appreciation of economic, social, and governmental factors. It should be understood, however, that public works is not the field of the engineer exclusively; the engineer must learn to work co-operatively with the economists, the social and political scientists, and the legislators and administrators, who are also vitally concerned.

It is obvious that the economic factors relating to public works are somewhat different from those of private engineering works; also that the factors for the large public projects may be somewhat different from those of the smaller ones. Essentially, the test of the economic feasibility of the private work is profit or loss. Essentially, the test of public work is the extent of public benefit. This is public profit or loss, but all the items are not necessarily measurable in dollars and cents. The smaller public works are quite often designed for a single purpose and the tests of economic merit may be relatively simple: Does the project provide the particular improvement or service needed? Does it fit into a comprehensive plan? Is it physically feasible? Is it legally and administratively feasible? Can it be financed? Will the value of service cover the cost, including capital, interest and amortization, maintenance and operating expenses?

The larger projects involve these tests also; but, in addition, other factors (many of them of a more imponderable nature) must also be taken into consideration. For example, the most important test of the need of a water supply system may be whether a great metropolitan area can continue to grow, or even survive, without such a system. With respect to a slum clearance and housing development, the tests are not solely whether rentals will pay the fixed and operating charges, but whether a community can afford to continue to pay the estimably greater upkeep cost of the blighted area or the inestimably great social costs of the slums.

Public works should be considered as one of the available means of economic adjustment—not only with respect to unemployment of labor, industry, and capital, and the swings of the business cycle, but in correction of, or compensation for, community or regional deficiencies, or unfavorable economic conditions or balances.

Without attempting to elaborate, criteria for larger public works would involve, in addition to comparison of cost and income, or cost and direct benefits, the weighing of various socio-economic factors, including their effect on: (*a*) The stability or movement of population; (*b*) the flow of wealth; (*c*) the economic activities of the community; (*d*) patterns and standards of life; (*e*) public welfare—health, education, safety, recreation, etc.; (*f*) other programs and projects; and (*g*) the conservation and beneficial use of resources, including those of land, soil, water, minerals, etc. Some of these effects might be illustrated by brief discussion of one or two classes of public programs and public projects.

Irrigation development permits a more intensive and stable use of land. In some areas of the United States, very large and important segments of rural and urban civilization would wane and many communities would disappear without it. Irrigation development also involves such economic considerations as movements of population, changes in the pattern of agricultural production, new agricultural industries, and new rural markets. Therefore, the tests of the economic merit for an irrigation project are not merely whether the water users pay their fees and amortize the cost on time; the more indirect effects, the community wealth created, the taxes paid, etc., must also be considered. Reference to a study of the Washington State Planning Council on this subject[67] should broaden the view of some of those who may look at irrigation economics only in terms of project cost and direct repayment. The report of the Council points out that the Yakima Valley, developed through irrigation, produces as much as $50 000 000 annually in farm products, livestock, and value added to raw materials through manufacture; also that, of the new wealth, produced annually, more than one-half is expended for the products of Eastern industries.

Projects for water supply, water conservation, etc., have high importance in any region, but in the Western United States civilization cannot advance, or even be maintained at present levels, without them. Conditions in the high plains[68] point to the great importance of such projects, and also emphasize the principle that in the planning, design, and financing of public works, the fundamental social and economic needs of the community should be determined and such resources as are available for public works should be devoted to meeting these needs in proportion to their importance and urgency, rather than on the basis of custom or precedent.

The economics of hydro-electric power development involve much more than a comparison of costs of fuel-generated and water power; such factors as the conservation of a national fuel and chemical supply (the dwindling oil reserves, for example), and the auxiliary effects of river control (in navigation or irrigation, for example), must also be weighed.

Pointing this discussion more specifically toward the economics of large Pacific Northwest projects concerning which questions are raised in the Symposium: The large projects in the Pacific Northwest are essential to this relatively young region, which cannot meet some of the present needs and cannot advance materially without measures of this kind.

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[67] "Reclamation—A Sound National Policy, as Demonstrated by the Yakima Valley and Other Irrigated Areas in Washington," Washington State Planning Council, 1936.

[68] "The Future of the Great Plains," Rept. of the Great Plains Committee, December, 1936.

Land is a fundamental factor in the Pacific Northwest, as elsewhere. The Pacific Northwest[69] includes 300 000 to 350 000 sq miles, depending on how it is defined. The four Pacific Northwest States of Washington, Oregon, Idaho, and Montana, contain nearly 400 000 sq miles, about 13% of the area of the United States, and less than 3% of the national population. The crop land in these States is limited in extent; it is only about 26 000 000 acres out of a total of 254 000 000, and its productive capacity is generally limited without water.

Lands at present under irrigation total about 5 000 000 acres. There are feasibly irrigable as many as 6 000 000 acres more. Of this area, about 1 200 000 acres lie in the Columbia River Basin, under the Grand Coulee Project. The net additional total of crop land feasibly reclaimable by other means—drainage, diking, stump clearing—is not definitely known, but perhaps would total an additional 2 000 000 to 4 000 000 acres.

Reclamation in the Pacific Northwest also bears an important relationship to national land needs and programs.[70] For example, it should be considered in relation to the indicated need of retirement, nationally, of about 25 000 000 acres from cropping as a soil conservation measure and to 30 000 000 acres, or more, which will be required during the next generation to meet the needs of new population. In addition, or perhaps tending to offset technological advances in land use, there undoubtedly will be a large land requirement during the next generation to meet the growing need of production of industrial raw materials. Mr. J. W. Haw discussed some of the national economic factors in land reclamation before the Annual Convention of the Society in 1936.[71]

In the Pacific West there is a land problem of particular urgency. In 1936 alone, conservative checks and estimates indicate that 10 000 additional families sought settlement on the land in Washington, Oregon, Idaho, and Western Montana. Similar checks and estimates for the first half of 1937 indicate that the totals for that year will be even larger than in 1936. Various factors, such as climatic and economic conditions and age composition of populations elsewhere as well as past trends, indicate that such westward migrations are likely to continue. Of course, all these rural migrants will not settle on irrigated land, although the majority undoubtedly would do so if opportunity offered; but in two years, allowing only 40 acres per family, there were enough migrants to settle the equivalent of about one-seventh of all the feasibly irrigable land in the region to which they have come. In other words, there was a sufficient number of families in two years to settle the equivalent of at least two-thirds of the land of the Grand Coulee reclamation project, which may require twenty years for completion. It might be added that, lacking a comprehensive program of land reclamation, less than 20% of the immigrant farm families in the Pacific Northwest in 1936 were able to settle permanently on economic farm units.

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[69] "Regional Planning," Pt. 1, "The Pacific Northwest" (Columbia Basin Rept.), National Resources Committee, 1936.

[70] Rept. of National Resources Board, December, 1934; also, Supplementary Rept. of the Land Planning Committee.

[71] "Irrigation and the Land-Use Program," by John W. Haw, *Civil Engineering*, October, 1936, p. 663.

Additional factors affecting the economics of Western public works lie in the national ownership of a large part of the land resources. (More than one-half the land area of the eleven Western States is in Federal ownership.)

Hydro-electric power is one of the outstanding resources of the Pacific Northwest, which contains about 40% of the potential hydro-electric power in the United States. The wise and the earliest practicable economic utilization of this non-exhausting natural resource is naturally of great importance to the people of this region and to the nation. In round figures, the potential hydro-electric power capacity of the Pacific Northwest is about 16 000 000 kw. The developed power capacity is about 1 800 000 kw for four States—Washington, Oregon, Idaho, and Montana. The initial capacity at Bonneville (two units) will add 86 400 kw, and the ultimate added at Bonneville will be approximately 500 000 kw. The initial capacity at Grand Coulee will probably be 315 000 kw, and the ultimate planned is 1 890 000 kw. In round figures, the production (four States), in kilowatt-hours, increased from a negligible amount in 1909 to 900 000 000 in 1910, to 3 000 000 000 in 1920; and to 6 000 000 000 in 1929. Following a drop below 5 000 000 000 in 1932, production again advanced to nearly 7 500 000 000 in 1937. The probable production, on the completion of Bonneville, Grand Coulee, and other plants, now under way or planned (1938), will be about 18 000 000 000 kw-hr. Curves of power consumption may be projected into the future on one's own optimistic, pessimistic, or average basis; neither future compounding rate of increase, rate of diminution of the compounding rate, nor the ceiling level can be estimated with any certainty. So many factors are involved that projection of any one curve for more than two decades or so is scarcely warranted; it would be more realistic to try and estimate curves marking upper and lower limits for such long-range use. In any case, however, it should be apparent that there are still many potentialities for expanded and new uses of electric energy, that saturation is not yet in sight, and that if the country advances at all—that is, barring economic ineptitude on the part of the nation and its people—the power will be used, and probably during a time not inconsistent with the probable rate of completion of the current Columbia River power projects.

With reference to Bonneville, the Oregon State Planning Board has conservatively estimated that the assumed Oregon half of Bonneville's ultimate capacity will be absorbed by its Oregon market area, and that 200 000 kw more will be required in sections of the State outside the Columbia Basin area by 1946.[72]

The power projects in the Pacific Northwest should be considered also as potential keys to a needed broader industrial development and a better-balanced regional economy. Complete self-sufficiency is not thought of as a desirable or practical ideal, but studies indicate, for the Pacific Northwest, a marked deficiency in manufacturing activity and a tendency to "live off" of capital through primary economic dependence upon extraction of regional resources which are not readily replaceable—primarily those of soil, forests, and minerals. The fundamental condition, which indicates a flow of wealth to the industrial and financial centers of the East, may be seen at a glance in Fig. 1 which shows the net values of various classes of imports to, and exports from, the region. For a secure region and a secure nation there must be an adequate return flow of wealth. Public works is one of the means of bringing about such a flow. If these public works tend to provide for improvement of the economy of the region, as well as for an immediate return of wealth, so much the better. Abundant and low-cost power, tending to increase industrial activity, payrolls, and dividends, should be highly important to the Northwest economy.

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[72] "Use of Electricity in Oregon with Forecasts of Future Demands," Oregon State Planning Board, September, 1936.

Incidentally, another form of return flow of wealth is tourist travel, which, it will be noted, is also somewhat related to public works through highway construction, national park, and national forest programs.

The Pacific Northwest is one of the regions in which inland navigation is of high present and potential importance. The regional freight rates are among the highest in the United States. Further economic transportation development will be essential to the general development of the region. Modern trunk inland navigation is an important corollary of the land, power, and industrial developments. Both East-and-West and North-and-South trunk lines are feasible from the engineering standpoint. Providing efficient transportation for bulk materials in particular, inland waterway improvements, as found economically feasible, will aid in the further utilization of regional resources. The juxtaposition of economical power and ocean navigation in the Lower and Middle Columbia River is particularly significant. Furthermore, navigation is one of the multiple uses of the water resources that, together, make large-scale development economically feasible.

In connection with multiple water use it should be noted that the Grand Coulee Project, with its more than 5 000 000 acre-ft of useful storage, will provide, in addition to reclamation and power, general benefits in river regulation—primarily, in increasing ultimate efficiency of all down-stream power plants and, secondarily, in flood control and river navigation.

There are other large projects[73] in the Pacific Northwest in various stages of development; for example: Various water conservation and irrigation projects in Washington, Oregon, Idaho, and Montana; further projects in the comprehensive plan for the Lower and Middle Columbia and Snake Rivers; and the Willamette Valley project in Oregon. (Comprehensive studies and reports on this last-mentioned multiple-use project have been made by the Corps of Engineers, U. S. Army, and by the Oregon State Planning Board.)

It is strongly believed that there must be a place in any long-term public works program for the large projects of this character. It is also believed that matters of law and appropriation, rather than demonstration of sound economics, will in most cases be the last hurdle to be topped in getting them under way. Properly conceived, such projects have far greater and more lasting economic effects than small, unrelated public works projects. Ranking in economic importance with the large projects are well co-ordinated programs of smaller public works (for example, such programs as that of the Montana Water Con-

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[73] "Drainage Basin Problems and Programs," National Resources Committee, 1936–1937.

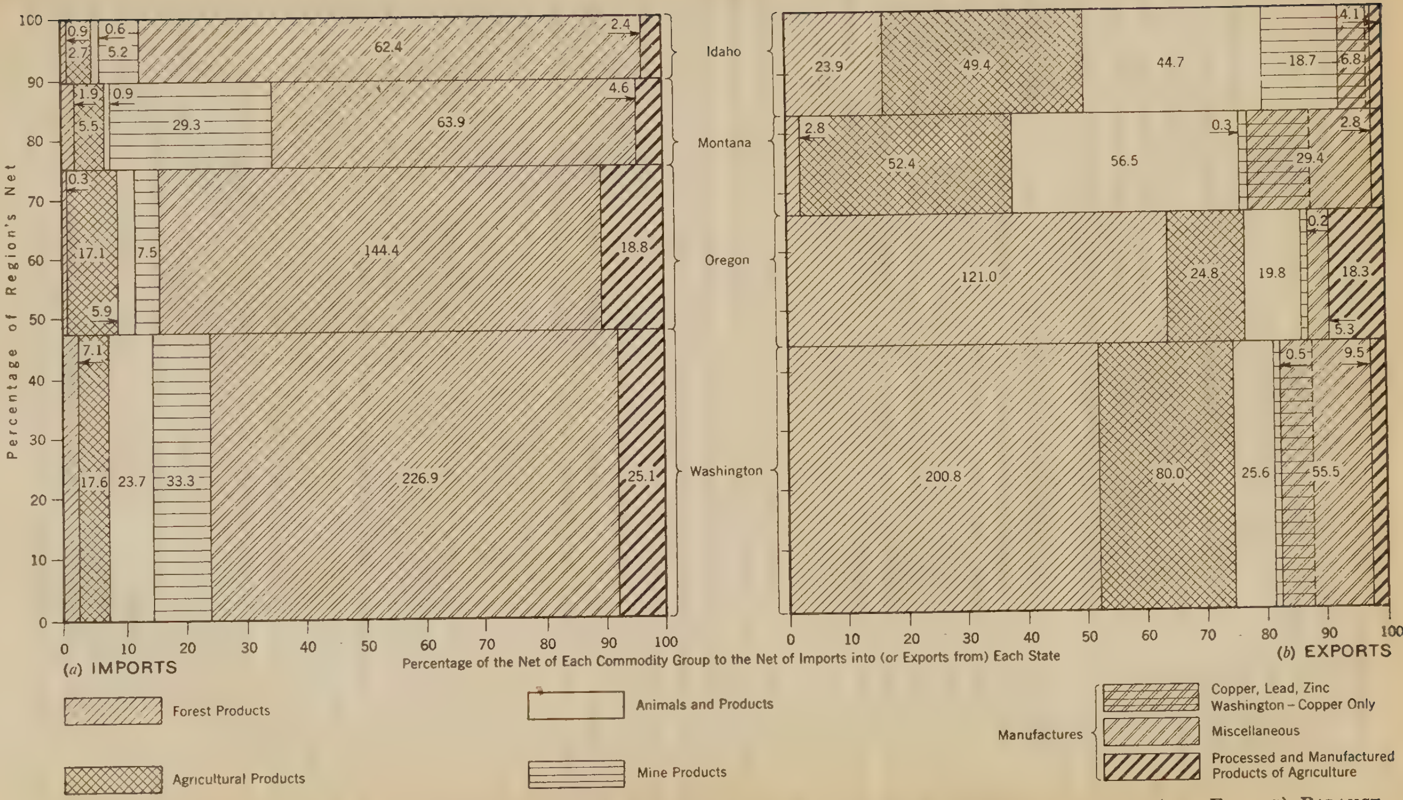

FIG. 1.—ESTIMATED VALUE OF APPARENT NET IMPORTS AND EXPORTS OF THOSE COMMODITIES SHOWING AN IMPORT (AND EXPORT) BALANCE IN THE EXTERNAL TRADES OF THE PACIFIC NORTHWEST STATES, 1929

servation Board, being carried on in co-operation with Federal Public Works and Works Progress Administrations). Such projects and programs are important for their regenerative as well as direct effects—through the foundations provided for an improved or more stabilized community economy or through the stimulus provided for other economic activities.

The programs of the Pacific Northwest have been effective in the employment of labor and of the construction industry. The larger projects have also been particularly effective in providing employment of men and industry in the East in the manufacture and shipment of equipment and supplies. It is believed that few engineers of the Pacific Northwest would doubt the vital importance of these projects to the members of the Engineering Profession in the region.

In connection with the economics of unemployment relief through public works, there must be considered such factors as beneficial effects in industrial activity, national income, national wealth, etc. It should also be borne in mind, that "write-off" to unemployment, where there is any, is generally less in the larger projects and programs than in the smaller works programs and projects, in spite of the fact that these large projects are also effective in emergency relief. Certainly, the savings which they have permitted in direct and indirect relief should be included among the economic factors relating to the large projects.

In conclusion, the writer would like to add a little to the challenge of the Symposium on public works economics. It is hoped that the Engineering Profession at large will be challenged to give a greater consideration to the economic and social bases and consequences of its work. It is hoped that the present and future leaders and educators of the profession will have a constantly broadening concept of engineering economics in relation to public works, and of the responsibility of the profession in this field. It is believed that Mr. Fay's constructive paper presents some of the logical avenues to such consideration by the profession.

DONALD M. BAKER,[74] M. AM. SOC. C. E. (by letter).—Two lessons may be drawn from the four papers comprising this Symposium—the importance of the Construction Industry in the national economy, and the necessity of advance planning and programming of public works projects. When an industry that accounts for an annual expenditure equal to 15% of the annual income (as did the Construction Industry during the five pre-depression years, 1925–1929) evidences such an abrupt decline as occurred after 1930, the resulting effect upon economic conditions is certain to be felt keenly.

Data as to expenditures for construction work presented by Mr. Fay have been re-arranged in Table 5 for the five pre-depression years, 1925–1929, inclusive, and the four depression years, 1932–1935, inclusive, in order to afford a comparison between pre-depression and depression construction activities. This analysis emphasizes the fact that private and public utility construction was the "backbone" of the industry prior to the depression, accounting for more than three-fourths of the average annual expenditures during that period.

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[74] Cons. Engr., Los Angeles, Calif.

In spite of largely increased public expenditures during the four depression years, public utilities accounted for one-third of the expenditures, and private construction for one-fifth.

TABLE 5.—ANALYSIS OF CONSTRUCTION EXPENDITURES, 1925–1934

Agency	Average Annual Expenditures, 1925–1929			Average Annual Expenditures, 1932–1935			Per Capita Expenditures, in Dollars		Ratio, Column (4) / Column (1)
	Millions of dollars (1)	Percentage of totals (2)	Percentage of public works (3)	Millions of dollars (4)	Percentage of totals (5)	Percentage of public works (6)	1925–1929 (7)	1932–1935 (8)	(9)
Public Works:									
Cities	1 364	11.5	4 .2	563	13.3	29.0	11.54	4.47	0.413
Counties	746	6.3	26.4	125	3.0	6.4	6.31	0.99	0.168
State	466	3.9	16.5	450	10.7	23.2	3.94	3.57	0.966
Federal	252	2.1	8.9	802	19.0	41.4	2.13	6.36	3.185
Totals:									
Public works	2 828	23.8	100.0	1 940	46.0	100.0	23.92	15.39	0.686
Private	5 242	44.0	185.1	825	19.5	42.6	44.30	6.55	0.158
Public utilities	3 830	32.2	135.3	1 455	34.5	75.0	32.40	11.55	0.380
Grand total	11 900	100.0	420.0	4 220	100.0	217.5	100.62	33.49	0.355
Total, private plus public utilities	9 072	76.2	320.8	2 280	54.0	117.5	76.70	18.10	0.251
Average, annual national income	78 343	....	....	46 472	....	....	....	....	0.592
Average annual per capita income	663	....	....	368	....	....	....	....	0.552

The greatest proportionate reduction in expenditures occurred in those made by private agencies and by counties, where they fell to about one-sixth of pre-depression rates. Activities of cities and public utilities were reduced three-fifths, those of States showed a slight decrease, and those of the Federal Government increased threefold. In spite of the increased Federal expenditures, public works decreased one-third, and total construction activities were reduced to about one-third of the pre-depression rate.

Data in Table 6, taken from 257 identical cities, indicate that the great reduction in private construction was probably due in large measure to the almost complete cessation of residential building, which was caused by widespread unemployment, reduced incomes, and uncertainty as to the future of those still employed. Counties usually carry on their construction programs from current revenues received from taxation. Deflation of assessed values during the early years of the depression no doubt had a telling effect upon

TABLE 6.—BUILDING CONSTRUCTION IN 257 IDENTICAL CITIES

Description	1925–1929	1932–1935	Ratio, 1932–1935 / 1925–1929
New residential construction per year	$1 983 217 000	$120 777 000	0.0608
Families provided for per year	398 521	32 784	0.0822
Expenditures per family	$4 975	$3 692	0.7420
Total building construction, including additions, alterations, repairs, and other buildings per year	$3 516 282 000	$473 922 000	0.1347

revenues, and the necessity of caring for the local relief load as well as maintaining other functions of county government left little surplus for public works projects. A considerable proportion of the construction expenditures of both cities and public utilities usually goes for expansion of facilities to care for increase in population. The "back to the land" movement, which was very active during the early years of the depression, slowed down urban population increase to a great degree, and both city and public utility construction was probably gaged closely to that necessary for operation and maintenance.

A very large proportion of State expenditures is for highways, financed by gasoline and motor vehicle taxes, and bond issues. Since people still operated their automobiles during the depression, and a large State bond issue takes considerable time to spend, it could be expected that State expenditures for construction would be maintained. In the light of what occurred there remained no agency other than the Federal Government financially able to make an attempt to support the Construction Industry. It was as unprepared to embark upon a large program of public works as it was to enter the World War in 1917; but so was nearly every other agency engaged in construction, except possibly the various States, which, in general, have developed the practice of advance planning and programming of highways, because of the certainty of future income. City planning has become a recognized function of many city governments since 1920, but most city planning boards or commissions have either become involved in the complexities of zoning, or if they have developed anything approaching a comprehensive city plan, have given little attention or thought to the financial or timing aspects of achieving it.

Considerable criticism is offered in the papers of the Symposium to the economic aspects of certain large public works projects embraced in the Federal program. Although not attempting to defend any of the projects referred to, the writer would like to call attention to several facts which must be borne in mind when they are being discussed. These facts are:

(*a*) Lack of preparedness and of planning always causes waste, in public works programs, war, or other activities.

(*b*) Public works projects are seldom initiated or promoted by politicians or public officials. They are usually conceived, and in their early stages promoted, by local interests that expect, individually, to benefit by their construction or operation. This, in itself, is not at all reprehensible in the case of sound projects. Were this not the practice, there would be a far less number of worth while public works projects. These local interests include engineers, architects, material and supply firms, contractors, land-owners, industrialists, financiers, etc., acting individually or through Chambers of Commerce, Boards of Trade, and professional or trade associations. The project is then "sold" to the local public, and becomes a local issue. Once it attains this status, politicians and public officials, under the influence of pressure groups, adopt and support it. Let them question the need or economic feasibility of the project, and other aspirants for public office will be found in large numbers ready to come forward with offers of support if given an opportunity to occupy public office. The writer agrees with the opening statement of Professor Mead that "a well informed public can and will raise such objection to projects which they know to be unsound that no Federal, State, or Municipal Administration will dare to undertake such projects," but wonders how the public is to be informed. He knows of instances where individual engineers and engineering organizations—even Sections of the Society—have expressed disapproval of the economic feasibility of public works projects in serious and carefully prepared reports, only to have their voices drowned out by the loud acclaim of the local public (not the politicians and public officials), this acclaim, in many cases, being based upon other favorable reports made by engineers. There may be some satisfaction in keeping the record clean, even if it is buried, but until engineers learn to adopt the technique of the promoters in gaining a public audience for their conclusions upon uneconomic public works projects, their influence in heading off such projects will be little felt.

(*c*) The amount of attention paid to the economic feasibility or need of any public works project by the general public of a community benefited by its construction or operation is usually in the inverse ratio to the proportionate cost of the project, or of its operation paid by the community. Projects financed by local taxation or bond issues primarily are likely to be widely discussed, pro and con, and criticisms of unwise projects usually find an audience. However, where costs are borne indirectly to a large extent, or are met from a distant source, the customary tendency is not to "look a gift horse in the mouth." This points to the advisability of local participation to the greatest degree possible in the costs of public works projects.

Fig. 2 shows the cumulative expenditures for construction for the 11-yr period, 1925 to 1935, inclusive, according to various classifications as given by Mr. Fay in Tables 1 to 3, and indicates a deficiency in expenditures by 1935 of $36 760 000 000 based upon the pre-depression rate of $11 900 000 000 per yr; or $100.62 per capita per yr. This deficit is classified as follows:

Public works...........	$ 3 150 000 000
Private...............	23 480 000 000
Public utilities.........	10 130 000 000
Total..............	$36 760 000 000

and would indicate annual expenditures of the order of $16 000 000 000 for the 10 yr following 1935 if the deficit is to be overcome and the pre-depression rate resumed. The writer is inclined to believe, however, that the pre-depression rate of $100.62 per capita was artificially stimulated by rapid increase in urban population and an urgent need to supply the population of the nation with a greatly increased standard of living. On the basis of a national population increase of 7 224 000 during that 5-yr period, it meant an expenditure of $8 240 for each person added to the population. However future construction expenditures are computed, they will be large, and the great probability is that those allocated to public works will embrace a continually increasing proportion. This trend was well established prior to the depression and has been greatly fostered during depression years, and such trends are persistent.

Since all construction work is undertaken to serve one or more of the following purposes: (*a*) To replace existing facilities which have worn out or become obsolete; (*b*) to provide new facilities to care for increases in population; and (*c*) to provide new facilities necessary to increase the standards and conveniences of living, it is probable that an analysis of construction expenditures made during the five pre-depression years would show that many of them were made for Purposes (*b*) and (*c*). Indications are to the effect that the trend toward further urbanization, so strong during the 1920–1930 decade, may slacken in the future. This trend, with an approach to stationary national population and a probable reduction of westward migration, would indicate that future construction will be made primarily to serve Purposes (*a*) and (*c*).

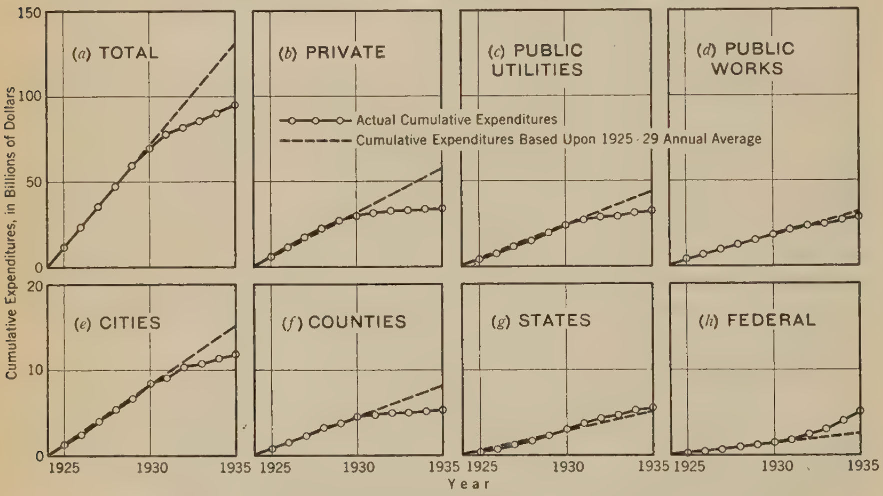

FIG. 2.—EXPENDITURES FOR CONSTRUCTION, 1925–1935

If, as it would appear, a large program of public works construction is indicated in the future, the matter of planning and programming in connection with such program takes on added importance. Such activities must be conducted not only on a national basis, but by State and local agencies. Strictly speaking, they fall into two phases:

(1) The development of physical plans for public works projects, which will result in an effective and efficient utilization and control of natural resources for the public benefit, or will serve the convenience and necessity of the general public; and,

(2) The development of a program for the initiation and completion of these projects which will bring them into existence or operation at such times that increasing public needs and requirements will be satisfied, will keep costs of the projects within the reasonable financial ability of those benefited by and paying for them, and will provide for the allocation of such costs equitably among those benefited.

The first phase is essentially within the province of the engineer, and he has, when engaged in it, acquitted himself with great credit, often under difficult circumstances. The second phase, that of programming, which involves forecasting future needs and requirements, is far more difficult. Although it involves engineering ability and judgment of the highest order, it also requires knowledge and judgment in other fields, including economics, sociology, politics, industry, human nature and habits, geography, and even history. The development of leadership in this phase by engineers affords an opportunity to the Engineering Profession second to none that has occurred heretofore.

In illustration of what is meant by the foregoing two phases of activity in public works planning, consider the case of a large drainage basin which offers possibilities of development for hydro-electric power, irrigation, domestic water supply, and recreation. It is one thing for an engineer to prepare plans and make designs for the various dams, conduits, power-houses, transmission lines, etc., and to build them; but it is another thing for him to plan these structures of proper sizes and to recommend the date for their construction at such times that, when they are placed in operation, fixed charges for unused capacity will not be burdensome, that demands for their services can be met, and that costs will be allocated equitably among those benefited or served. The writer has noticed a tendency on the part of too many engineers to devote 90% of the contents of their reports to the subject of physical plans, and only 10% to the program of development, allocation of costs, etc.

Because of changes that occurred during the 1920–1930 decade, and which will occur because of the depression, it will be more difficult to make predictions of future needs and requirements than in the past. Indications are that many trend curves developed a point of discontinuity about the year 1930.

The problem of national population is an excellent example. Forecasts of probable national population increase[75] indicate a numerical growth between 1930 and 1960 (30 years) but slightly in excess of that which occurred during the 10-yr period, 1920 to 1930. In 1960, it is estimated that 42% of the population will be more than 40 yr of age, as against only 28% in 1930. The effect of this upon habits, needs, and methods of living must be considered. During the 1920–1930 decade the population of urban places increased 27%, whereas it is estimated that during the three decades, 1930–1960, even maintaining the rate of migration to urban areas that occurred during 1920–1930, the total increase in population will be only 19.5%, or 6.5% per decade. If these forecasts are correct, the effect upon such construction activities as residential building, public utilities, etc., will be pronounced. Migration of people has always been a far more important factor in population increase in the Western States than natural increase. This migration follows a law similar to that of Ohm's law in electricity—the rate of migration varies directly with differences in economic pressure at its origin and its destination, and inversely with the resistance to the movement of people. People move away from places where opportunities are few and where it is difficult to make a living, to places where opportunities are greater and living easier, provided they can get away. During the depression the "difference in potential" between the East and the West has been greatly reduced, and the difficulty of getting away and starting to move has increased. Whether these conditions are permanent or temporary is something that is still not clear, but it should be considered seriously by those making long-range programs of public works.

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[75] Rept. of National Resources Committee, 1934–1935, pp. 95–97.

Another factor that must be appreciated and considered in making programs is the fact that curves of per capita production and consumption of many of those things which go to the improvement of standards and conveniences of living are showing tendencies to flatten out, indicating a condition of approaching saturation and a probable future production and consumption, at a rate of increase more closely paralleling that of increase in population. The data in Table 7 are illustrative of this point.

TABLE 7.—RELATION BETWEEN CONSUMPTION AND POPULATION INCREASE

Description	1920	1925	1930	1935
Electric power consumption, in kilowatt-hour per capita	408	573	778	....
Motor vehicles per 1 000 population	87	174	215	....
Telephones (Bell System) per 1 000 population	111	139	163	....
Radios per 1 000 population	....	....	109	169

These factors have had a most important effect upon the thinking and living habits of the American people, and were no doubt responsible to a considerable extent for the pre-depression construction activities.

There is no reason why the engineer should not assume leadership not only in planning but in programming public works projects; in fact, there is every reason why he should, but he must realize, when he attempts to do so, that technical engineering is only a part of his work, and if he enters this phase, he must be ready to assume the blame when the projects for which he is responsible develop into economic failures, when judged by proper standards, just as he now assumes the blame for physical failures.

PHILIP W. HENRY,[76] M. AM. SOC. C. E. (by letter).—Among the "notable examples of admirably run publicly-owned" revenue-producing enterprises, Colonel Wilgus cites the municipal subways owned and operated by the City of New York, N. Y., but he does not mention that they are not earning interest charges. The original rapid transit systems (elevated railways and subways) were built by private capital on which they made a reasonable return; but no capitalist would have under-written the municipal subways which have not only failed to earn interest, but, through competition, have greatly decreased the earnings of privately-owned subways. Had the City of New York left the transit field to private capital, it would not now be levying a 2% sales tax and looking around for other methods of taxation in order to balance its budget. In building subways the City Government was no more interested in the economic aspects of the case than was the Federal Government when it authorized the construction of Bonneville, Fort Peck, Grand Coulee, and other projects.

The capitalist asks of a given project: "Will it make a return in dividends?" The politician asks: "Will it make a return in votes?" Under the capitalistic system Americans have attained the highest standard of living known to history. Under the political system, if adopted generally, they will see this standard stationary or even lowered.

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[76] Cons. Engr., New York, N. Y.

H. K. Barrows,[77] M. Am. Soc. C. E. (by letter).—The papers by Messrs. Riggs and Wilgus contain much that is of interest. The various public works projects, including Passamaquoddy, Fort Peck, Bonneville, Grand Coulee, and the Florida Ship Canal, have all been given thoughtul attention and a clear picture presented of their nature, cost, and economic status.

From the taxpayers' viewpoint, only one of them (Bonneville) appears to have any chance of being classified as a project justifiable in character and as a wise expenditure of public funds.

As has been emphasized, it is the function and duty of Congress to see that proper use is made of Federal funds. In considering these projects the situation was somewhat involved because of their association with relief work. Such large and expensive projects, with their usefulness far in the future, if at all, are obviously not a proper class of undertaking for relief work.

The authors of the Symposium papers have brought the situation to the attention of engineers, and, in so doing, have also reached the public to some extent. It is the latter who should understand the situation, so that the people may express to their Congressmen their disapproval of such unworthy and undesirable projects. This work of education must begin at home, and engineers, who understand what these projects are and mean, have an individual duty in acquainting the layman with the facts, so that such unwise projects may be avoided if possible in the future. As Professor Riggs, in closing, states: "They will fail in their duty if they keep silence."

Harry A. Wiersema,[78] M. Am. Soc. C. E. (by letter).—In his paper in this Symposium, Mr. Mead has given a brief analysis of several Government projects, including the Tennessee Valley Authority (TVA), and has drawn a conclusion regarding the latter which is far from complimentary. He states (see heading "Government Interference as a Permanent Policy: Responsibility of the Engineering Profession") that he has nothing to criticize with regard to the engineering features of the TVA, but gives the principle that "sound engineering cannot make an uneconomical project an economic success." The writer is entirely in accord with this principle, but he is so at variance with the conclusions regarding the economic feasibility of the TVA project that some discussion of this subject seems desirable.

The highest standard of ethical conduct should actuate all engineers in Government service, and to the extent that it does not such engineers are not true to their profession. It is not enough that an engineer render a high quality of technical service, but he also has a responsibility to the public for rendering the same kind of service that a consulting engineer would render to a private client in determining the economic feasibility of the project entrusted to his care. An engineer entering the service of the TVA would be confronted by a situation which might be compared to the relationship between a client and his engineer. In the following theoretical conversation, the United States Government has been personified to represent the client, who is setting out his plans and purposes to his engineer:

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[77] Prof., Hydr. Eng., Mass. Inst. Tech.; and Cons. Engr., Boston, Mass.

[78] Gen. Office Engr., TVA, Knoxville, Tenn.

"Many years ago, spurred on by the necessity for the manufacture of munitions during the great war, I made an investment in a hydro-electric development at Muscle Shoals which has been a financial failure. In the first place, the actual sum of money expended in the construction of this project was much greater than it should have been, due to many factors, including the abnormally high price levels obtaining during the initial construction period and due to the prolonged and interrupted construction period. In accordance with the direction[79] of the Congress, the Tennessee Valley Authority submitted a valuation of Wilson Dam of $31 300 000 which was approved by the President on May 6, 1937. In the second place, due to the fact that this is an isolated plant and that there are no facilities for marketing the power, the income has been insignificant, represented only by sales to a single public utility company at a figure of 2 mills per kw-hr, which was the best I could do in such a monopoly market. The amounts of power thus sold were entirely at the discretion of the buyer, and only about one-fourth of the possible output was actually disposed of. I did receive several offers for the purchase of this plant, none of which was acceptable.

"I believe that the only way to save any considerable part of this investment is to develop further the resources of the Tennessee River. Congress has already recognized the potential value of the Tennessee River as an inland waterway by approving a plan for providing a 9-ft navigable channel from Knoxville to the mouth of the river, a distance of 650 miles. Wilson Dam forms an integral part of this plan. The Tennessee River and its tributaries have a considerable flow at time of flood, and it is desirable to provide flood-control projects on this river to assist in protection against floods, both locally on the Tennessee River and on the Lower Mississippi River. I believe that it is possible to develop a series of projects on this river system which will provide the waterway desired by Congress, together with a substantial degree of flood control, and at the same time develop considerable of the potential water power of the stream. In an integrated system of dams, the surplus power at each lower river project is increased by the released storage from plants above. If this water power can be converted into electricity and sold, the income from such sales will go far toward returning the cost of the entire development.

"The Army Engineers have already constructed the lock for a dam at Wheeler, the next location above the existing Wilson plant, so that the navigation program is already under way. Now is a most favorable time for public works construction; Government credit is at its peak, and long-term money can be obtained at very low interest rates. There is a distinct advantage in constructing public works during times such as these when there is considerable unemployment. Such a development will not only effect an immediate improvement of economic conditions in a large section of the country but will also result in permanent additions to the nation's wealth. In addition to these considerations, we will have in this project a welcome experiment in integrated control of an entire water-shed, involving conservation of natural resources, broad scale regional planning, and an opportunity to demonstrate the advantages of a wider and better use of electricity."

There are several points of view which might be adopted with regard to the validity of the Authority's projects. For example, that part of the work which is justified for flood control and navigation could be considered as non-income producing public works; or the entire project could be considered as semi-revenue producing public works, and allocation of costs made to the different benefits. However, in order not to confuse the issue as to the economic justification of the Authority's projects by entering into a controversy regarding "yardsticks" and "allocations," it seems desirable to make an analysis similar to that made by Mr. Mead to determine how much of the total investment can be liquidated by the sale of surplus power. The following study is such an analysis of the economic feasibility of the project, based on conservative estimates of revenues from the sale of hydro-electric power, and actual project estimates which are fairly definitely determined. It is based on the system that will be developed when dams now (1938) under construction will be finished, which will be by January 1, 1942, rather than on the 11-dam system used by Mr. Mead. The 11-dam system has not yet been approved by Congress and the estimates are, therefore, approximate; for the 7-dam project, on the other hand, Congressional authorization has been fully obtained and estimates of cost and power can be much more accurately determined. The estimated cost of these seven dams[80] is shown in Table 8.

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[79] TVA Act of 1933, Section 14.

TABLE 8.—ESTIMATED COST OF SEVEN PROJECTS, TENNESSEE VALLEY AUTHORITY

Item No.	Project (1)	Initial power installation, in thousands of kilowatts (2)	Estimated cost of initial stage (3)	Item No.	Project (1)	Initial power installation, in thousands of kilowatts (2)	Estimated cost of initial stage (3)
1	Wilson	184	$31 300 000	5	Guntersville	72	$36 334 720
2	Norris	100	36 361 738	6	Chickamauga	81	43 127 903
3	Wheeler	128	36 849 403	7	Hiwassee	60	21 529 579
4	Pickwick Landing	72	33 219 279				
				8	Total	697	$238 722 622

Except for the Wilson Dam (see Item No. 1, Table 8), which is a valuation determined by Congress, these estimates are carefully prepared and contain conservative sums for construction contingencies. Two of the projects, Norris and Wheeler, were finished slightly under these estimates in 1936 and in none of them is there any question of returning to Congress for additional funds to make up deficits, even in the face of increased labor and material costs.

The actual sums spent for transmission lines and sub-stations through the fiscal year 1937, together with the budget estimates for the fiscal years 1938 and 1939, are as follows:

Period	Expenditure
Prior to July 1, 1937	$11 323 182
Budget, fiscal year 1938	5 740 000
Budget, fiscal year 1939	5 560 000
Estimated additions to January 1, 1942	12 500 000
Total (say, $35 000 000)	$35 123 182

An amount has been added for additional investment up to January 1, 1942, which is approximately proportionate to the existing rate of expenditure and which, it is felt, under existing conditions, is adequate to provide a transmission system capable of disposing of the entire power output that will be available as of that date.

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[80] Rept. of Hearings on the Independent Offices Appropriation Bill for 1939, 75th Cong., 2d Session, p. 919.

When the estimated cost of transmission facilities is added to that of the hydro-electric projects, the total investment as of January 1, 1942, becomes \$273 722 622.

It is well to note that the estimate of the total project cost is based on the actual system being constructed. This system is the most desirable from the standpoint of a multi-purpose development, but it is not the most desirable from the standpoint of hydro-electric power alone. If the TVA had planned its projects for power as a single purpose and for the maximum dollar net income, there are other more economical power projects in the Valley which would have been substituted.

The fixed charges constitute by far the largest portion of the total annual cost of such projects, and these charges would reasonably be computed on the basis of the average interest paid by the Government on long-term bonds. Over a long period of years the average interest rate on borrowings by the Federal Government has been less than 3%, and in the period from 1933 to 1938 Government interest rates have been at an especially low level. The Federal Power Commission has determined the weighted average rate on all borrowings of the United States Government from November 1, 1933, to December 31, 1937, to be 1.54271%; but it would be difficult to justify so low a rate of interest in measuring the feasibility of the TVA program. The "Baby Bonds" that are sold to the general public are computed to yield an interest rate of 2.9% over a long period of years, and it appears to the writer that this constitutes a fair basis for amortizing the project.

The life expectancy of the various classes of property included in this project varies widely, ranging from less than 20 yr in the case of certain parts of the transmission system to an indefinite life in the case of reservoir lands. Giving consideration to the portion of the total investment represented by reservoir lands and long-life concrete structures (which will be more than one-half the total cost of the project) it is felt that a service life of as much as 75 or 80 yr is not an unreasonable expectancy. However, in order to adopt a conservative value for the purpose of the present discussion, a 50-yr over-all expectancy has been used. Based on a 50-yr life and an interest rate of 2.9%, the sinking-fund method of calculating annual depreciation charges results in a depreciation rate of 0.9%, or a total amortization rate of 3.8 per cent. Applying this to the total investment previously obtained, of \$273 722 622, results in an annual cost for fixed expense of \$10 401 450.

In order to determine the income from the sale of hydro-electric power, the best estimates available are those contained in the report given to Congress,[81] in which it is estimated that the demand for firm power in the year 1942 will be 480 000 kw, and for secondary power, 98 000 kw. The installed generating capacity in that year in the seven dams under consideration will be more than sufficient to provide for a total demand of this magnitude. A reasonable estimate of the energy sold with this demand would be about 2 600 000 000 kw-hr of primary and 650 000 000 kw-hr of secondary. Analysis of the capability of these seven projects shows that there is energy in excess of the total of the foregoing amounts after allowing for leakage, evaporation, utilization, and transmission losses, as well as efficiency losses in the structures and equipment. It should be noted that, were these projects to be operated for power only, instead of primarily in the interest of flood control and navigation, the potential available energy in these seven plants would be considerably greater than these amounts.

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[81] Independent Offices Appropriation Bill for 1939, 75th Congress, 2d Session, p. 1000.

In estimating the annual revenue accruing from the sale of surplus power, the Authority's standard wholesale rate schedule may be used for firm power. Mr. Mead has quoted this schedule correctly with one exception. The schedule provides for a monthly reduction for energy used in excess of 360 times the billing demand of 0.5 mill per kw-hr from the otherwise applicable rate for such excess. The average rate under this schedule, estimated on a basis of weighted averages of the different sizes and types of load from existing customers, would be at least 4 mills per kw-hr, of which about one-half is represented by the demand charge and one-half by the energy charge.

The average rate for secondary power, as determined from the Authority's existing contracts, is somewhat in excess of 2 mills per kw-hr. The total operating revenue based on the sale of 2 600 000 000 kw-hr of primary energy at the minimum of 4 mills per kw-hr and 650 000 000 kw-hr of secondary energy at the minimum of 2 mills per kw-hr, is $11 700 000.

The Act creating the TVA provides for the payment to the respective States of 5% of the gross revenue received from the sale of power generated within their borders. This payment in 1942 would amount to $585 000.

The operating costs for this 7-dam system will consist of the cost of generation and transmission of electricity, cost of operating the locks, and certain expenses in connection with operating the reservoirs for flood control. The operation of the locks will be in the hands of the Army Engineers, and the estimate as to the cost of this operation is based on their experience. For the purpose of this analysis it appears desirable to make a comparison on two different bases: (1) Using only the cost of generating and transmitting power; and (2) using all the cost of operating the entire system.

Mr. Philip Sporn has given a table[82] of operating costs of hydro-electric plants, in which he shows that for plants of 50 000 to 100 000 kw of installed capacity the operating costs vary from a high of $1.50 to a low of 60 cents per kw, or a median of 75 cents, whereas, for larger plants the median is still lower. It is estimated that an average of 85 cents per kw would be a fair value. To this should be added the cost of operating the transmission system, and also an allowance for the cost of malaria control on the reservoirs (which, on account of the rather complete control that the Authority is instituting, will be a fairly large proportional sum). Adding these estimates, a value of $3.00 per kw of installed capacity appears to be conservative and reasonable, and this sum has been used in setting up the following tabulated estimate of revenue and expenses, as of January 1, 1942 (the estimates for additional expense for operating the system for navigation and flood control, including operation of the locks and certain required dredging, are also given):

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[82] *Proceedings*, Am. Soc. C. E., December, 1937, p. 1927, Table 4.

Revenue and Expense	Per Annum	
Gross Revenue:		
Primary—2 600 000 000 kw-hr at $0.004 =	$10 400 000	
Secondary—650 000 000 kw-hr at 0.002 =	1 300 000	
Total gross revenue	$11 700 000	
Less payment to States at 5%	585 000	
Total revenue		$11 115 000
Expense of Operating Hydro-Electric Facilities:		
697 000 kw at $3.00 per kw		$ 2 091 000
Net revenue from power sales alone		$ 9 024 000
Additional Expense of Operation:		
Navigation facilities	$ 400 000	
Water control and land management	500 000	
Total		900 000
Total net revenue		$ 8 124 000

This tabulation shows that, based on operation of the power system alone, the net revenue would be $9 024 000, whereas based on the entire operation of the project the net revenue would be $8 124 000. Applying this revenue against the fixed costs, the revenue produced from the sale of power is 87% of the total fixed cost of $10 401 500 in the first instance, and even when all costs for operation for navigation and flood control are considered, the revenue is still 78% of the total fixed cost.

Reverting now to interest during construction, there are considerations that make this project unusual in this regard. For instance, the actual interest rates during the construction period have been extremely low and not much more than one-half the long-time average interest rate assumed, of 2.9%, so that the interest during construction will be a relatively small percentage of the total annual interest after the project is completed. As each dam is completed, it contributes to the power available for sale in an amount greater than necessary to carry its own interest, with a surplus to contribute to the interest on uncompleted dams. The income from the plants as they are completed prior to 1942 will so closely meet the total expense that it is certainly fair to assume that it will approximately offset the interest during construction; so that neither the factors of income nor interest during construction need be considered in this analysis.

As stated previously, the actual cost of capital to the Federal Government during the construction period of the TVA projects is not as high as 2.9% and, in fact, may be considered as low as 1.55 per cent. Accordingly, it would seem a safe conclusion that the TVA will liquidate practically all the Government's investment in the project during the life time of the physical structures through the sale of its surplus hydro-electric power.

This is a far different "picture" from that presented by Mr. Mead and justifies the faith of the Engineering Staff of the Authority in the economic feasibility of the project as a whole.

It might not be amiss to discuss the benefits from navigation and flood protection, intangible though they may be. The opinion expressed by Mr. Morgan,[16] that the project is worth all it will cost for navigation and flood protection alone, is shared by many engineers; and although it is impossible to analyze this benefit in the same way that the benefit from the sale of hydro-electric power can be analyzed, it is scarcely "so extreme that it seems useless to discuss it" (see heading, "Government Interference as a Permanent Policy: Power"). The Miami District of Ohio was designed and built for local protection only and could not be expected to contribute to flood protection on the Ohio River. The TVA System, on the other hand, is designed and built not only for flood protection on the Tennessee River but also for flood protection on the Ohio and Mississippi Rivers; and the Gilbertsville Reservoir, the farthest down-stream project of the TVA System, is especially valuable for this latter purpose. The U. S. Engineer Corps has allocated[83] to the Tennessee Basin a total flood storage of 10 589 100 acre-ft, of which the Gilbertsville Reservoir will provide 4 600 000 acre-ft with a possible addition of 1 450 000 acre-ft for extreme floods. The storage of this reservoir will reduce the discharge at Cairo by from 184 000 to 220 000 cu ft per sec, and Gen. J. L. Schley, U. S. Corps of Engineers, M. Am. Soc. C. E., has reported that this reservoir will reduce stages on the Mississippi River by 2 ft.[84] The value of this amount of reduction in providing protection to lands and improvements in the upper parts of the Mississippi River Basin has been estimated by Carl A. Bock,[85] M. Am. Soc. C. E., to be approximately $90 000 000. The estimate of $100 000 000 benefit for flood control of the entire TVA System, as made by Mr. Morgan,[16] would seem, therefore, to be fairly well justified.

The benefits from navigation are also intangible, but there is reason to believe that these benefits may be more than double the amount that Mr. Mead has estimated as justifying the expenditure of about $90 000 000.

From the foregoing analysis it is shown that the direct revenue from sale of hydro-electric power will liquidate all but 13% of the cost of the total investment. Applying this percentage to a total project cost of $500 000 000 would leave only $65 000 000 which is only one-half the sum that Mr. Mead himself estimates for the benefits of navigation and flood control, and only a small fraction of what may be the actual value of these benefits.

Whether or not the navigation and flood-control benefits are as great as Mr. Morgan has stated, and even if they are as low as Mr. Mead estimates, there can be no question that they are worth very much more than the actual unliquidated cost. Surely this is a thorough justification of the TVA program, and one of which its engineers may well be proud.

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[83] House Doc. No. 259, 74th Cong., 1st Session.

[84] *Waterways Journal*, November 27, 1937.

[85] *Engineering News-Record*, April 7, 1937.

J. D. Galloway,[86] M. Am. Soc. C. E. (by letter).—Although it would be possible to bring additional facts to support the data given in the able papers of this Symposium, it is believed that sufficient information is included in the four papers to enable conclusions to be drawn. In the words of Col. Wilgus (see heading, "A Challenge") they "constitute a terrific arraignment of those who are intrusted by the people of the United States with the handling of their affairs." However, the question naturally arises, why has such a condition come about and who is responsible? Many uneconomic projects have been undertaken, money by the hundreds of millions of dollars has been expended, and economic conditions are as bad as, or worse than, before. What is wrong with the program? The writer believes that the underlying causes are inherent in the nature of such programs and will examine the subject from that standpoint.

Advance Planning.—This theory has a superficial appearance of merit and Mr. Fay has outlined the best aspects of the subject. The fundamental defect of such planning lies in the fact that it is impossible to determine in advance what the needs of the future will be, or to anticipate the changes that only a few years will bring about. A few examples will make this point clear.

In the early years of the Nineteenth Century, a system of canals was projected and partly built in the Eastern States. Politics, "log-rolling," and much dishonesty characterized the work. Then railroads were invented and practically the entire system of canals was abandoned. The canals were a complete loss and the one remaining in use—the Erie—has been an excellent example of the losses resulting from Government in business. Of what value would have been a plan of canals over the country, when in a few years the entire system as built was abandoned? The writer has seen, in Indiana, remains of one of these canals that never even contained water.

The railroads of the country are another example. Fifty years ago they were the principal means of transport, and branch lines were built all over the land. The automobile was invented and in the past few years one of the jobs of the engineers of railroads has been the tearing out of abandoned branch lines. Of all the railroads, the interurban lines, generally operated by electric power, furnish an example of the type largely rendered obsolete by the automobile.

Modern highways furnish another example of the inability of engineering works to last. Due to the rapid development of automobile traffic, roads that were built as late as five years ago (1933, say) are now inadequate. An able engineer of the California State Highway Commission recently stated that an expenditure of at least $100 000 000 was required to make over roads built in the past few years.

Examples of this phase of the subject could be cited without number. It is a matter of common knowledge among engineers that change is the order of Nature and that what is built to-day will be found inadequate tomorrow. It is the price paid for progress. When Americans are satisfied with the works that their fathers built, the nation will have reached a state of stagnation that the writer, for one, does not desire to contemplate.

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[86] Cons. Engr., San Francisco, Calif.

It will be contended by the advocates of advanced planning that general plans only should be laid out by the planners of to-day. The answer to this contention is that the men of tomorrow will not follow the plans made to-day. In any single piece of work, plans made by one set of men are usually ignored by their successors. The writer has in mind electric transmission lines in California. Lines built under one set of conditions become obsolete by changes in voltage, changes in the location of load center, and various other factors. The amalgamation of systems has altered the entire program. As long as growth continues, the changes that come about preclude any advance planning that would be of value in an emergency.

The most important factor in the case of advanced planning is that the best of plans will not be followed because the decision to say what program shall be followed in the assumed emergency does not rest with engineers. That decision is made by politicians. The United States has been governed in the past by politicians, is now in their hands, and as long as human nature remains as it is, the country will be governed by politicians. It matters little by what label they are designated, they run true to form. The Columbia River Basin is an excellent illustration of this fact. In reality, advanced plans had been made on that large river. For eight years prior to 1933 the river was made the subject of studies and reports and they were of the general tenor that economic development was not feasible at the present time. The entire river was covered. What was the result of all this study? The politicians gave into the hands of the President enormous sums of money to be expended at will and immediately became active to divert as much of the money as possible to the regions they represented. That was the determining factor in causing the adoption of Bonneville and Grand Coulee. All the advance planning availed as nothing against the chances to obtain slices of "pork."

Electric Power.—Closely connected with such decisions as are mentioned is the plan of the present Administration to place the National Government in the business of developing and selling electric energy. Whenever a dam of any size is projected, it becomes (except in special cases) an economic necessity to make the energy from the falling water available for human use. As a rule the energy in the form of electricity is a by-product of the development. However, in the cases mentioned in the Symposium and in another project on the Tennessee River, power has become the main objective of the installation. The President has stated in no uncertain terms that four projects in different sections of the country were being built as "yardsticks" to determine the cost of power. In addition, funds have been, and are being, donated by the National Treasury to various municipalities for the purpose of installing municipal plants that will drive privately owned plants out of business. At first, the sums advanced were 30% of the cost of the works, which sum was later increased to 45 per cent. In addition, the National Government purchased, or will purchase, the securities of the municipality. The result is an outright gift or subsidy offered by the Government for the purpose of wrecking the private industry and driving the owners out of business.

The same effect is produced by the construction of power plants at the various dams recently completed or nearing completion. In complete disregard as to whether there is a market for the power, expensive hydro-electric installations have been made. This is especially true in the Tennessee Valley where the numerous dams serve practically no purpose except as units in a power development. Some benefit will accrue in flood control but the operation of the system will determine the extent to which the reservoirs will be used for the latter purpose. There is an inherent opposition between the requirements for flood control and those of power generation. Flood control demands that reservoirs be largely or wholly emptied in anticipation of a possible flood. Power demands that the reservoirs be kept full so that a maximum fall of the water be obtained. It may be claimed that flood-control space is left in the upper part of the reservoir, but this again emphasizes the fact that power development is the major element in the project.

There is no deeper seated economic fallacy embedded in the public mind than that power costs as developed by private companies are excessively high. The corollary follows that in every power development there is an unlimited source of revenue, which will carry the burden of all the charges on any project, however lacking in merit it may be. Demagogic newspaper writers, and politicians, have "harped" upon the subject until the public has been completely bewildered by their claims. The development of systems of power generation and distribution either from fuel or by water, in the past fifty years, has had an effect upon modern life in America second in benefits only to the great transportation systems. A cheap source of energy that could be distributed in large or small quantities to all parts of the land has been brought into being by the American genius for invention and for organization. The power thus placed at the service of every one can only be measured by that of billions of men.

The great enterprises have been developed by private initiative operating under the laws of the country. There are numerous instances where some of the enterprises have been misused, and corporation piled on top of corporation, for which the public has been asked to pay. No engineer should defend such practices and few do. On the other hand, there are many States where governmental supervision has prevented such practices and where, as in California, an able commission has worked with the power corporations, to protect the public and also the corporations. As a whole, the power corporations have acted wisely and have reduced rates in accordance with increased business and increased efficiency of larger systems.

As, throughout the country, power systems generally extend over two or more States, it becomes the function of the National Government to provide the necessary regulation. That is the proper function of Government. On the contrary, it is not the function of Government to use its power in building power plants and by use of Federal subsidies, drive the private corporation out of business. There was no need for any "yardsticks" to determine the cost of electric energy. One honest engineer and one honest accountant could determine this cost at any location.

In the Pacific Northwest, where the Bonneville and Grand Coulee power developments are situated, the private and public plants already in existence have brought about a per capita use of electric energy greater than elsewhere in the country. The existing systems have been built to supply immediate needs and are capable of extensions which can supply all the electric energy now required or that may be needed in the future. The Government plants were unnecessary. There has recently appeared[87] a statement that at Bonneville the Government investment on June 30, 1938, amounted to $53 188 800 and that power would be charged with 32.5% of this sum. When the project is completed, the estimated cost will be $74 144 600, of which power will be charged 57%, or $42 191 175. The remainder of the costs (two-thirds in one case and nearly one-half in the other) is charged to navigation. It is with such juggling of costs that the "yardstick" will determine the costs of power. It will be noted that the 1934 estimate of $31 250 000 for this project, stated by Professor Riggs as $51 000 000 in 1936 (see "Significance of Contingent Expense"), has now (1938) grown to an ultimate of $74 144 600.

He also states (see "Power in the Northwest") that the Grand Coulee project now has an estimated cost of $437 000 000. There will be water to irrigate about 1 000 000 acres of land. The experience with the Reclamation Service projects, with few exceptions, shows that farmers cannot, and will not, pay the costs of such projects. They must compete with other lands not subject to the handicap of costly irrigation works and, furthermore, there is a widespread opinion that the Government should furnish the land and water free. The result is that no political management can make them pay. The favorite method has been to write off large sections of the cost of the project, to reduce the interest rates, and to extend the time of payment to a date far in the future. In California, irrigation districts having debts ranging from $15 to $50 per acre have gone insolvent because the farmers could not, or would not, pay their taxes. It is idle to expect that farmers under the Grand Coulee project will act otherwise, especially when Government gifts of all kind are being scattered broadcast. As a possible source of revenue to meet the charges on the Grand Coulee project, the farm lands may be dismissed from consideration. It is doubtful whether the lands could stand the cost of the required canal system. If any revenue is obtained, it will be in the distant future as the country is relatively undeveloped and now amply supplied with power.

Tendencies of Government.—It is recognized that there is a widespread belief throughout the United States in the principles of socialism, the fundamental basis of which is that all means of production should be owned and operated by Government. Parallel to this idea is the one that private ownership of property is an evil and Government should extinguish private rights by confiscation. Since the present Administration came into power it is the belief of the writer that a consistent course has been pursued by the Administration tending to destroy private property and private initiative. The most violent assault has been made upon the various power companies. Some of the means to this end are the several projects discussed in the papers of this Symposium. By apportioning a large proportion of the cost to such items as flood control or navigation, the power plant costs are reduced far below what they should be. This arrangement, together with outright gifts to municipalities written off by the Federal Treasury, may produce low-cost power, but as a measure of the correct costs of electric energy, the so-called "yardsticks" are designedly incorrect and without value as such. In the event that the policy of State socialism is made permanent, it would seem advisable to have the correct costs of any project determined, and not have the projects undertaken with the expressed purpose of altering the true costs by subsidies, direct or indirect, from the National Treasury.

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[87] *Western Construction News,* March, 1938.

The most glaring examples of uneconomic power projects among those cited in the Symposium, is Passamaquoddy. In this case there was lacking the usual excuse of navigation, flood control, or irrigation. All the costs, if properly made in an honest system of accounting, would have had to be charged to the project. The fallacy that energy under modern conditions of demand can be obtained from the varying systems of tides was also present. Hence, a fuel power plant was required of equal capacity to the tidal power plant. The fact that the location was such that there was practically no market for the energy made the entire project a monumental failure from the beginning. However, the politicians at Washington succeeded in spending several millions of money before it fell of its own weight.

In the case of the Wilson Dam and power project, started during the World War, there is another example of unwise Government planning and execution. The country was engaged in a desperate war that required all its energies; yet some one of the politicians secured money from the Treasury to begin the Wilson Dam on the Tennessee River, the power from the plant to be used during the war for the manufacture of nitrates. Years after the war ended, work on the dam was still going forward. When the project was started it was recognized that on account of the variable flow of the river, a steam plant of a capacity about equal to the water-power plant was required, and it was built. It was merely another example of the governmental inaptitude and mismanagement in an emergency. To any one except a politician, it should have been apparent that in a time of a great war, to start a dam and hydroelectric power plant that could not be completed, that could not function, and that required a steam plant of practically equal capacity, was opposed to all ideas of the right thing to do; yet tens of millions of dollars were poured into the project that was obsolete from the start as a producer of nitrates for war purposes.

Opposed to the prevailing tendency toward socialism, there is the individualism of the past. Under whatever term of collectivism it may appear; socialism, communism, Government or municipal ownership, the present tendency is in direct and irreconcilable conflict with the individualism that has built the nation. It is admitted that all over the world at the present time, the doctrine of collectivism is in the ascendency. The moves of the present Administration at Washington are in line with the world tendency. The glaring examples of the adoption by Government of unsound, costly, and uneconomic projects, as set forth in the Symposium, merely express the retrogression into which this country is sliding. In the past, in spite of obvious errors, individualism and personal liberty have accomplished the greatest good for the human race. Compared to the blundering and hampering action of Government, the contrast is a vivid one; and yet, with the other nations of the world, the United States has parted from the system that has made it great, and with the others, is imitating the actions of a certain group of animals of old, which, possessed of devils, rushed down a steep place into the sea. The country is in danger and, in the words of Professor Riggs (see "Conclusions"), "engineers will fail in their duty if they keep silence."

E. S. MARTIN,[88] Assoc. M. Am. Soc. C. E. (by letter).—This Symposium directed to the economic phase of the recently enlarged Federal activity in public works plainly pertains to the exigent character of this activity, and to the competence and propriety of Government in business. The abrupt cessation of the flow of investment funds into private works following 1930 forced the Government unpreparedly into the breach or, failing that, forced a complete moratorium upon the accumulation of investment funds. Professor Riggs' intimation that, as with the Panama Canal, Congress should have deliberated for three years before taking action on the great projects now under way, seems, to the writer, without force in the circumstances. It is certainly a debatable point that Congress should decide engineering questions in any circumstances. It is for Congress to consider and determine the sum, time, and purpose, but not generally to consider and determine technical details which its members are not usually trained to comprehend. This point is the origin and strength of the manager type of municipal government.

The economic aspects of any undertaking mean the trading relation of the undertaking to its economic environment. Everything in the business world is a link in a chain. To study the link alone and to presage how and which way it will jerk the chain along is somewhat bizarre. The condition of the chain demands consideration in determining the safety and service of the link. The writer is offering some considerations herein which the proponents of this Symposium have neglected regarding the will of the people to use new works or even those they have. He pleads that the authors take entirely too narrow a view of the problem. They view public works from the banker's and the investor's standpoint; whereas the purpose of this activity is to use some of the available capacity for service, otherwise idle, to serve the nation, and thereby to command service in return—in the language of the street, to make a living.

Professor Riggs again states (see heading, "Counting the Cost"), "the same sound business principles should control the selection of public construction projects which are to render a service that the public must pay for as would prevail in the case of a successful private undertaking of the same kind." The writer denies the very essence of this statement; public works and public finance are fundamentally different from private works and private finance. Private works are for the purpose of "making money" out of the public presumably by serving it, whereas public works are for use—for service only. The private investor often—but not always—aims to liquidate his investment; but there is no reason whatever for public works to be self-liquidating and there is no need for them to be financed by bonds; in fact, it is a crime against the public to do so. The Government is the one economic entity that has complete control of its income; it can balance its budget without injury to any one in war or peace.

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[88] Secy.-Treas., James A. Wickett, Ltd., Toronto, Ont., Canada.

About 150 yr ago, pioneers migrated from the Atlantic Coast into the wilderness of Western New York and settled and developed that district. They did not prepare a prospectus to show (an investor) that the roads and works they built would pay or be self-liquidating. They were the investors and the users. Making a profession of investing, separate from enterprise, from material capital in modern finance, is not healthy. Mr. Henry Ford has taken pains that that did not happen to him; the same is true of thousands of other firms and of much of agriculture. From the ethical standpoint public works, by and for the public, should be free from private investors; that is, it should be paid for when built. The investor's field is private enterprise attended with risk, where invention and enterprise are essential—a field not suited to Government management.

In so far as this Symposium contributes to prevent waste of labor and material on developments that would be of no service, or of but little service, to Man, it is constructive and meritorious. One must bear in mind that with private enterprise as well as with Government managed enterprise the public provides the labor and materials and the money, too, through its patronage. Money is never consumed or wasted or saved or made. It is only the catalytic agent of trade; so that misapplication of effort is equally a public loss in either type of enterprise; in fact, the only difference between public and private enterprise is the question of who provides the management and gets the income, if any. When Government management excels private management in vision, enterprise, efficiency, and integrity, it will not take long to displace it. It is evident that Government enterprise is rapidly displacing private enterprise quite generally throughout the world, largely alas, because of the demoralization of private enterprise—more precisely, private investment. Professor Mead's reference to Canadian "fiascos" is not a happy one for his contention. Canadian trials at public management are uniformly outstanding successes; it is rather some private promotions in Canada, as in the United States, that are not so aromatic. Although the Canadian Pacific Railway was capably managed, the constituent private railways now forming the Canadian National Railway were in such bad physical and financial condition in 1915 as to threaten the solvency of Canadian banks and, possibly, the nation. Canada assumed and carries this indebtedness (which continues to grow), but the railway management is remarkably improved and compares with the best of American railroads. The same is true of the Toronto Transportation Commission and the provincial telephone systems in the West. The Ontario Hydro-Electric Commission outdoes, by a wide margin, the accomplishments of private systems of the United States. The writer has resided in Toronto for eighteen years following eight in New York, N. Y., and can warn his fellow members in "The States" not to refer to Canada to support an aversion to public ownership. The 8 cents to 10 cents per kw-hr that he paid in New York for domestic current as against the 1 to 1.5 cents in Toronto indicates the extent to which the consumer serves the investor in New York instead of the investor serving the consumer—which is his reason for being.

Now, the very extent, in area and time, of the Tennessee Valley development is such that no private enterprise could undertake this work in order to "make money"; nor could it be tolerated in any event; and, likewise the Columbia River projects. It is impossible for one now living to establish beyond question the tolerably exact value to Man of this kind of development, which depends for its use upon continued development in many directions. Such development is similar to that of the settlement of new domain; it cannot be reduced to a simple financial statement. From the writer's viewpoint, these are very promising, brilliant conceptions, as sure as most human undertakings. Some of the other projects, like the Passamaquoddy, do not offer the same promise, at least, they have not been shown to the writer's satisfaction to do so. The economist and the engineer must not overlook the inevitable risks always attending any enterprise; no one can see the future with perfect vision.

Public work has its pitfalls, as does private work. The selfish local interest which seeks public expenditures that it may benefit from it, whether or not the public does, has its counterpart in the investment broker who markets securities without conscientious solicitude for their soundness, or which do not represent equivalent increase in real material wealth. Mr. G. C. McDonald has stated[89] that total public debts in Canada increased about $1 800 000 000 during the six years, 1932 to 1938, with little or no increase of material wealth. The purpose of money savings and investment is to put and keep people at work creating fixed wealth; turning from this duty is exactly what causes public and private deficits and unemployment. There has been a forced attempt by the Government to discharge this duty in part.

The purpose of public and private improvements is to serve Man; in short to maintain and to raise standards of living, now or in the future. If the people do not live more fully these works are not used; and, conversely, if the nation's capacity for service is not employed, it is because its people are not spending their money. How secure, and how promising can any development be as long as the people of several rich nations decide deliberately not to spend (use) their money? Economics was formerly called Political Economy, and so it is.

K. Bert Hirashima,[90] Esq. (by letter).—The papers of this Symposium should prove of great value to all engineers whether or not they are engaged in planning work. The papers by Messrs Riggs, Mead, and Wilgus reveal some of the colossal mistakes that have resulted from hasty uneconomic planning. Through careful advance planning, it is to be hoped that past errors will not be repeated in the future.

One of the principal objects of long-range advance planning is to relieve (at least to some extent) the large volume of unemployment resulting from the fluctuations in the business cycle; but, as Mr. Fay states "a controlled public works policy should not be looked upon as a means of relieving, directly, unemployment among workers outside the construction industry by giving them employment on construction projects." To do so would only tend to upset the normal relation between the number of workers engaged in the construction industry and the number of workers engaged in other activities. The resulting unbalance may eventually prove to be as great a problem as the original one of unemployment when, with the return of better times, public works projects are "tapered off." Public works expenditures, therefore, should not be regarded as a panacea but as a kind of balance wheel to regulate the activities of one phase of the national economic picture, namely, the construction and allied industries.

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[89] "The Plight of the Taxpayer," by George C. McDonald, *Canadian Business*, Vol. 11, No. 2 (February, 1938), pp. 16–18.

[90] Asst. Highway Planning Engr., Territorial Highway Dept., Honolulu, Hawaii.

As to the financing of a long-range public works program, Mr. Fay mentions two methods: The accumulation of a reserve fund during boom periods, and long-term or short-term borrowing.

The question of just how the reserve fund should be accumulated is a difficult one. In the accumulation of any reserve fund, the assumption is made that assets can be liquidated at approximately the cost price; but the greatest demands upon a public works reserve fund will fall during periods of depression when security prices are at their lowest.

The issuance of bonds for unemployment relief is unjust in that it is nothing less than asking future generations to pay for present unemployment, unless the relief projects are of a self-liquidating character. When the bonds mature, money in the form of taxes must be diverted from the channels of trade to pay for them. This means that some worker, somewhere, is deprived of a chance to work.

In boom times, money is freely exchanged for goods and services; it is then performing its legitimate function as a medium of exchange. During periods of depression, the demand for money is greater than the demand for goods. Capital, unable to earn a profit and fearing taxation, "goes into hiding" in the form of bank deposits or tax-free bonds.

The fact that Government bonds can be sold to finance unemployment relief proves that there is an abundance of idle funds—money that has ceased to perform its function as a medium of exchange. It is not asking too much that such funds be taxed accordingly. Any long-range planning, therefore, should take these factors into account; otherwise, the issuance of public works unemployment bonds will only provide opportunity for additional capital to be removed from circulation. In this connection, President Roosevelt's suggestion that future issues of Government bonds be taxed, is a step in the right direction.

Lastly, the problem of public works planning is not a static but a dynamic one; that is, the factors involved are changing continuously.

Louis E. Ayres,[91] M. Am. Soc. C. E. (by letter).—It is unfortunate that no definite pronouncement, allocating expenditures for power purposes, has ever been made by the Tennessee Valley Authority (TVA). Lacking such pronouncement, Professor Mead calls attention to certain estimates in "testimony before the House Appropriations Committee"[11] (see heading, "Government Interference as a Permanent Policy: Power") in 1936 wherein the "estimated annual power expense" of \$12 191 000 was arrived at by taking 9% of \$135 450 000, the investment in "power houses and power facilities" only. Obviously, such an assumption will not generally be accepted as a basis for determining a power production "yardstick."

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[91] Cons. Engr. (Ayres, Lewis, Norris & May), Ann Arbor, Mich.

[11] First Deficiency Appropriations Bill, March, 1936.

An engineer would normally analyze the basic data in quite a different manner. The usual method to obtain the investment base which may be supported by a given gross income is to divide the gross income by the percentage of annual charges. Those acquainted with water power costs will generally agree that a total of 10%, with interest at 6%, is usually adequate for annual charges in the case of large, privately owned water powers; and if interest is assumed at 3.5%, the annual charges become about 7.5 per cent. In the case, however, of such projects as those constructed by TVA, with possibly as much as one-half the project cost invested in reservoirs (including highway and railroad relocations, which involve very little maintenance and operation and no depreciation), one may use a lower percentage for annual charges than in projects involving relatively larger proportions of the investment in structures and equipment. A value as low as 6.5% was used by H. K. Barrows, M. Am. Soc. C. E.,[92] in computing fixed charges on Boulder Dam, Bonneville, Grand Coulee, and the TVA projects. In the writer's opinion 7% is sufficient, under TVA conditions, to cover all charges, including interest, depreciation, taxes (as assessed under private ownership), insurance, operation, and maintenance. Furthermore, if the "estimated gross annual wholesale power revenue" of $23 120 000 is divided by 0.07, the investment base, or the production "yardstick" of TVA becomes $330 000 000.

However, Professor Mead questions the TVA estimates of annual income, first as regards the amount of output that may be utilized, and second as regards the average unit revenue that may be obtained therefrom, based on the rate schedules now in effect.

As to saleable output, Professor Mead states (see heading, "Government Interference as a Permanent Policy: Power") that the "estimated output is based on continuous efficiencies of more than 80%," and infers that "other losses due to plant use, transmission losses, transformer losses, and losses due to regulation for navigation and flood prevention" have been overlooked. This is not in accordance with the writer's understanding. In the TVA report to Congress, March, 1936, entitled "The Unified Development of the Tennessee River System," the estimate of 660 000 kw of continuous capacity was qualified by the sentence: "By continuous power is meant the dependable power which would be available 24 hours of the day and 365 days of the year after suitable deductions for expected and unavoidable water and energy losses." The writer understands that these deductions included restrictive assumptions as to flood control, allowances for navigation, evaporation, leakage, and seepage, and a final utilization factor of 90 per cent. In effect, all deductions, together with normal allowances for unit efficiencies, approximated an assumption of utilization at the switchboard of less than two-thirds of the theoretical power of all firm water, which is a somewhat more conservative basis than Professor Mead's 80% of 80% or 64%, including transformer and transmission losses.

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[92] *Proceedings*, Am. Soc. C. E., April, 1938, p. 698.

As to average annual unit revenue, Professor Mead presents, incompletely, the TVA wholesale rate schedule and concludes: "From these rates it is evident that the annual returns, if and when markets are found for the entire output, cannot average as much as 3 mills per kw-hr." Such a conclusion can hardly be drawn, from the rate schedule considered, as the demand charge alone of 90 cents per kw per month amounts to 2.46 mills per kw-hr, at 50% load factor, and the lowest step in the energy charge is 2.0 mills per kw-hr. Hence, wholesale power, purchased in large blocks under this schedule, would cost more than 4.46 mills per kw-hr, if a load factor of 50% obtains, as assumed by Professor Mead. One must assume 100% load factor to obtain a value approximating 3 mills, and then only because energy purchased in any month in excess of 360 times the demand is subject to a reduction of 0.5 mill per kw-hr. Also, energy purchased at the Authority's switchboard is subject to a further reduction of 10 per cent. These conditions of the schedule could account for the press dispatches of sales to "a chemical company," referred to by Professor Mead, of "firm power" at 2.74 mills per kw-hr. However, no one will assume that any considerable portion of TVA output will be sold at 100% load factor. If it is used to the best advantage eventually much of the output will be produced at 40% load factor or less, and will return a unit income of 5 mills or more. That firm power can be produced at 40% load factor or less may be judged from the fact that the total installed machinery contemplated is 1 922 000 kw, or slightly less than three times the estimated 660 000 kw of 24-hr firm power. Hence, it would seem that the TVA estimate based on 4 mills per kw-hr for firm output is conservative, having in mind the best ultimate use of this water power.

In addition to the firm output, however, there will be a considerable production of secondary power. Professor Mead is "aware that contracts have already been made to supply a considerable quantity of such power." Why should the income from secondary power be neglected? It would not be overlooked in a private undertaking. Professor Mead refers to press dispatches quoting sales of "12 000 kw of 'run-of-river power,' at 2.27 mills per kw-hr." Although there seem to be no published estimates of the amount of this secondary power, it would probably be not less than 2 000 000 000 kw-hr annually and would have a potential value of from 1 mill to 2 mills per kw-hr.

Assuming an average return of 5 mills per kw-hr for firm power and 1.5 mills per kw-hr for secondary power, the gross annual income amounts to about $32 000 000, which sum, capitalized at 7%, results in an investment base of $457 000 000; or, assuming 4 mills per kw-hr only for firm power and 1 mill only for secondary power, the annual income is computed to be about $25 000 000 and the investment base, $360 000 000, which is still in excess of Professor Mead's estimate of $353 150 000 as the proper "total cost of power installation."

An important hurdle in the path of any general agreement as to a proper power production "yardstick" is the allocation of total expenditures to the several purposes of navigation, flood control, and power. In determining his allowance for navigation, Professor Mead refers to the Army Engineers' estimate[15] of $74 709 000 for "navigation works with low locks and dams," agrees "that the higher dams will possibly improve navigation facilities," and concludes that "say, $90 000 000 for this purpose might be warranted." More recent estimates of the cost of a 9-ft channel, from Paducah, Ky., to Knoxville, Tenn., to the same standards as those used on the Upper Mississippi and Ohio Rivers, are much higher than the older estimates. In January, 1938, C. T. Barker, Assoc. M. Am. Soc. C. E., Head of the Navigation Section of TVA, testified in the constitutionality case in Chattanooga, Tenn., that to secure a navigable channel meeting present standards and requirements, by means of a series of low dams, would cost $144 072 700. If one assumes, therefore, that navigation may properly be charged with an amount substantially equal to what it would cost to provide the equivalent facilities by works constructed solely for that purpose, Professor Mead's allowance should be increased by at least 60 per cent. Obviously, the low-dam scheme would involve greater operating, maintenance, and dredging costs than the high-dam scheme which is being built, and would be less desirable for this and other reasons.

In the matter of flood protection, it may be recalled that the Army Engineers estimated that about $45 000 000 would prevent the maximum possible damage on the Tennessee River. Professor Mead allows $36 000 000 only, although he adds that "indirect damages might warrant some additional expense." In the "Comprehensive Report on Reservoirs in Mississippi River Basin,"[93] there is a list of 21 reservoir sites in the Tennessee Valley which would provide 10 188 100 acre-ft of flood storage at a cost of $167 282 000, or $16.40 per acre-ft. This is part of a list of 81 reservoirs in the Ohio Valley with a total storage capacity of 25 285 400 acre-ft at $20.40 per acre-ft; part of a total of 106 reservoirs up stream from Cairo, Ill., with 52 853 400 acre-ft, at $15.70 per acre-ft; and part of a grand total of 151 reservoirs, above the latitude of Red River Landing, La., with a total capacity of 98 677 900 acre-ft, at $11.40 per acre-ft. If one then assumes a value as low as $11.40 per acre-ft for the approximately 9 000 000 acre-ft of flood-control storage, provided by the TVA projects, the total is about $100 000 000.

Professor Mead quotes Mr. Morgan as testifying[16] in 1936 that the "value of the flood protection feature will be worth $100 000 000 to flood protection on the Lower Mississippi River." This value was based on the cost of raising Mississippi levees 3 to 4 ft, or the estimated lowering effect of TVA dams on a maximum Mississippi River flood. Carl A. Bock, M. Am. Soc. C. E., Assistant Chief Engineer, TVA, has stated[94] that the reservoir value of Gilbertsville alone "in protection to lands and improvements in the upper portion of the alluvial valley of the Mississippi River has been conservatively estimated to be approximately $90 000 000." Professor Mead, however, discards these benefits as problematical and outside the "primary objective of TVA," although under the TVA Act, as amended, the Authority is empowered "to construct such dams and reservoirs as * * * will best serve to * * * control destructive flood waters in the Tennessee and Mississippi River drainage basins."

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[93] H. R. Doc. No. 259, 74th Cong., 1st Session, p. 46.

[94] *Engineering News-Record,* April 7, 1938, p. 500.

On the basis of the foregoing assumptions, one might evaluate the multiple-purpose project as follows:

Purpose		Total	Percentage of total
Power		$360 000 000	56.2
Navigation		144 000 000	22.5
Flood control		136 000 000	21.3
Tennessee Valley (according to Professor Mead)	$36 000 000	....	....
Mississippi Valley (according to Mr. Morgan)	100 000 000	....	....
Total		$640 000 000	100.0

The increase in the size of the Gilbertsville and Fontana Dams, since 1936, and added generating units at several plants have raised the estimated total cost of the eleven projects, exclusive of transmission lines, according to testimony before the House Appropriations Committee (Independent Offices Appropriations Bill for 1939), to about $505 000 000; and, if 56.2% of this sum is chargeable to power production, the power "yardstick" becomes about $285 000 000. In the writer's opinion, a proper allocation to power, exclusive of transmission line costs, should not exceed $300 000 000. Such an allocation would be $60 000 000 less than the investment base, indicated herein, upon which the gross estimated income, from a fully developed market, would pay all charges; and, hence, $60 000 000 might be assumed as available for interest deficits over a period of market development. In other words, on this basis, if the market could be fully developed by, say, 1950, the projects would pay all charges on a fair power "yardstick," assuming 7% for annual charges and the sale of all firm output at 4 mills and all secondary output at 1 mill per kw-hr.

Professor Mead states that: "Steam power can certainly be generated in the Tennessee Valley for not to exceed 4 mills per kw-hr." If this statement is premised on a load factor of 50%, as one may infer from the context, it is believed by the writer to be too low if all costs are considered. Units have been added to plants where the incremental cost was 4 mills per kw-hr at 50% load factor, and plants containing one or two large units, without reserve, may be as low as 4 mills per kw-hr; but a more representative cost of steam power in this area, with all charges considered and adequate reserve provided, can be shown to be 5 mills or more for energy at 50% load factor, construction costs at $85 per kw of installed capacity, and coal at 12 cents per million Btu.

Thus, the engineer may conclude that the Tennessee Valley projects are sound, based on the usual analyses. The construction work done is certainly excellent, as one might expect from the competent engineering personnel that has designed and constructed the projects. The cost of producing power is certainly comparable with, if not less than by, steam substitution; and, the value of the energy will be enhanced as it is utilized in a more fully developed market. The principal economic difficulty involved at present would appear to be one of market, and that in turn is largely dependent upon the settlement of a controversy. One may not condemn the TVA on the basis of switchboard costs. The real TVA controversy is not at the switchboard but at the consumer's meter. One deals in mills at the switchboard, but in cents at the consumer's meter. Most of the "yardstick" is beyond the switchboard. It would seem to be a safe assumption that if the TVA output were allowed to be sold at the switchboard to existing utilities, on the basis of existing TVA rate schedules and without "strings attached" to its use, substantially the entire output would soon be disposed of, to the mutual advantage of both the Government and the purchasers. However, TVA has not seen fit to stop at the switchboard. It insists on lower rates to the ultimate consumers, and its policy in this regard is the real cause of all the acrimonious controversy.

JOHN H. MEURSINGE,[95] ASSOC. M. AM. SOC. C. E. (by letter).—For a period of 300 000 yr human beings have tried to make a living by exploiting the natural resources of the earth to the extent that such exploitation did not tax the human intelligence beyond capacity. While this knowledge was growing in power, Man learned to develop new resources; but it was not until rather recently that he awakened to the fact that each newly developed contrivance meant another change in the social organization in which he happened to live.

Man has paid dearly for his insufficient knowledge by disastrous warfares and expensive social experiments. It was by the method of trial and error, and by reshaping his previous efforts that he was carried along the road of progress. The public works policy of the present Administration, for instance, adds another adventure to the long list of hazardous occurrences which mankind has encountered during his eternal journey. This policy will reflect upon the society of to-day.

Although this may seem very important to contemporaries, nevertheless the present public works policy should be considered as nothing but an infinitely small link in an inexhaustible chain of events which has carried technics through five phases (there will be more) to its present peak in the United States (see Table 9).

Phase 1.—According to the best available knowledge, Man's first ancestors lived in the tropics (the Java Man). Seeds (grains), roots (vegetables) and fruits were his food. Grasses, leaves, and leaf wood made up the materials for construction. The social organization consisted of "individuals." The women took care of the children. This type of social organization is rarely found any more. Even in the most remote parts of the jungle, newer phases have shown their reflection upon this kind of society.

Phase 2.—The hunter's period comes next. Man's technical ingenuity had a tremendous influence upon the society in which he lived. He invented tools with which to kill, and the social result was marriage, or the establishment of the "family."[96] To his vegetarian diet the jungle man began to add the meat of the game. Soon he was not confined to the tropics; Man migrated on to the plains where meat, quite often, became his exclusive food. Fire kept him warm and manure, skins, and bones were added to the construction materials. The women were physically unfit for the hunt, and, consequently, they became dependent upon the males. Technics had accomplished its first change in society.

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[95] Huntington Park, Calif.

TABLE 9.—THE RELATION BETWEEN THE EVOLUTION OF TECHNICS AND OF SOCIETY

Phase	Power resource	Material resource†	Social unit and center of production and distribution	Typical occupation	Civilizations
1	Seeds, roots, and fruits	Grasses, leaves, and leaf wood	Individual	Jungle man	Ancient tropical cultures
2	Men, animals, and fire	Manure, skin, and bones	Family	Hunter	Indians of North American plains; Magdaleniun cultures in Europe
3	Slaves, agricultural products, and domestic animals	Clays, bricks, and natural stone	City	Farmer	Aztecs, Mayas, Incas; Chinese, Babylonians, Assyrians, Arabs; Egyptians, Phœnicians; and, Greeks and Romans
4	Stone, water, and wind*	Cotton, fir wood, and glass	Nation	Business man	Portugal, Spain, Northern Italy, Holland, France, and England
5	Coal, oil, and natural gas*	Metals	World	Industrial worker	Austria, Germany, United States, and Russia

* Electricity is a power created from the resources of Phases 4 and 5.
† Material resources of Phase 6 are rubber, aluminum, and magnesium.

The Indians of the North American plains, and the Magdaleniun cultures which appeared in Europe about 25 000 yr ago are representatives of Phase 2.[97]

Phase 3.—Technical accomplishment kept pace with the ever-growing population. Improved technics made possible the exploitation of plants, animals, and slaves. The new materials—clay, brick, and natural stones—were used for roads, bridges, and houses. An entirely new type of society, which revolved about the farmer, came into existence. Specialization in farming and the instinct of self-preservation gave birth to the "city." Mankind had stopped roaming. The Aztecs, Mayas, and Incas in the Americas; the Chinese, Babylonians, Assyrians, and the Arabs in Asia; the Egyptians and Phoenicians, in Africa; Greeks and Romans in Europe, were the great civilizations of this era (5 000 B.C. to 1 000 A.D.).

Phase 4.—The next step in this evolutionary process was the exploitation of wind and water for power resources; cotton, fir wood, and glass for construction materials. Portugal, Spain, Northern Italy, with Florence and Venice, Holland, France, and England, which countries happened to have the resources of this phase, carried civilization to new peaks. This era has been named eotechnics by Lewis Mumford.[98] Technical accomplishments could prove another change in society; the city had grown into a "nation"; and, as all the countries of this phase had acquired colonies, a new type of occupation had arisen; the trader or business man was added to the jungle man, hunter, and farmer.

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[96] "Kulturgeschichte der Menschheit in ihrem organischen Aufbau," by Julius Lippert, Verlag Ferdinand Enke, Stuttgart, 1886. Partly tr. by Peter Murdock, Assistant Professor of the Science of Society in Yale Univ., under the title, "The Evolution of Culture," Macmillan Co., New York, 1931.

[97] "The New Stone Age in Northern Europe," by John M. Tyler, N. Y., Charles Scribner's Sons, 1921.

Phase 5.—France and England had both shown the signs of a new phase. The underground resources were ready for extensive exploitation. A new type of technical complex, called paleotechnics by the late Professor Patrick Geddes, used coal, oil, and natural gas for power, zinc, copper, nickel, cobalt, iron, manganese, chromium, vanadium, and titanium for construction materials. Judging by contemporary developments, Austria, Germany, the United States, and Russia will make the most of these resources. Their technical accomplishments created the "industrial worker" as a new type of worker, and his international relations point to the entire world, instead of to the nation, as his social unit.

Technics developed Man from a jungle man into an industrial worker; it developed his individualism into internationalism. The relation between technics and social organization cannot be denied. Lewis Mumford[98] expresses this relation as follows:

> "While each of the phases roughly represents a period of human history, it is characterized even more significantly by the fact that it forms a technological complex. Each phase, that is, has its origin in certain definite regions and tends to employ certain special resources and raw material. Each phase has its specific means of utilizing and generating energy, and its special form of production. Finally, each phase brings into existence particular types of workers, trains them in particular ways, develops certain aptitudes and discourages others, and drains upon and further develops certain aspects of the social heritage."

Although each new phase has always strongly reflected upon the civilizations in existence, the old civilizations were never able to regain their positions as world powers by adopting the methods of the newcomers. Most interesting, however, is the fact that they always reluctantly adopted the new ways, and hence always hastened their own decline. Another interesting phenomenon is the fact that the process has been evolutionary. No sharp lines can be drawn between the phases as they penetrate each other.

The typical worker of each phase believes in the economics of his own phase only. The man of the jungle claims that it is uneconomical to be hampered by women and children in his forays; the hunter claims that it is uneconomical to build the city; the farmer objects to the national policies of taxing imports; and the business man cannot understand why the worker should spend a part of his wages on international causes.

The five phases have not only developed five different societies and five different occupations; they have developed five kinds of economics. Although eotechnics (Phase 4) developed the economics of investment, interests, and profits, the jungle man (Phase 1), the Eskimo (Phase 2), the farmer who wrings his entire living from the soil (Phase 3), and the industrial worker (Phase 5), the world over, are rather indifferent to the economics of dollars and cents.

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[98] "Technics and Civilization," by Lewis Mumford, Harcourt, N. Y., 1934.

Here the conclusion may be drawn that these economics are limited to Phase 4. Economics is as much subject to the rules of evolution as anything else. Each of the authors of the Symposium overlooked this phenomenon.

The Symposium resembles the skillful work of a photographer. All the details of the subject have been shown with painstaking accuracy. The exposure has been well timed. Unfortunately, however, the camera has been placed so closely to the subject that only a limited field could be covered. It has been taken from the "eotechnical point" of view (Phase 4).

The science of economics is concerned with the production and distribution of the necessities of life. How has this been done since Man has been on earth? The jungle man searched the vicinity of his abode. The hunter could not leave his home for fear the fire might go out. Hence, the family assisted in production and distribution. In the next phase the specialization in farming made the city the center from which the goods produced were distributed. In the fourth and fifth phases, the city was replaced, respectively by the nation and the world.

The production and distribution center could not have grown in size so continually, had it not been for the aid of technics. Engineering accomplishments made possible better and faster production and transportation. Hence, engineering economics should embrace the problem of making "necessary" additions to the production and distribution system. These additions should keep pace with the evolutionary growth of the system.

Unfortunately, the science of the evolution of economics is still so much in its infancy, and it is looked upon with so much suspicion, that it is rather useless to discuss engineering economics until some kind of an idea has been formulated as to what kind of economics and technics Phases 5 and 6 will eventually create. Hence, it is impossible to say whether or not the present public works policy (addition to the production system) is economically warranted. Those who criticize and those who acclaim it are simply giving vent to their emotions. It behooves the engineer to approach this problem from a scientific angle only.

Conclusion.—The civilizations of each phase have been dominated by the workers created by that phase; but this does not mean that the occupations of the preceding phases had no place in their society. Their place was determined by the age of the phase to which they belonged.

To-day, for instance, in the United States the jungle man (Phase 1) and the hunter (Phase 2) have disappeared. The farmer (Phase 3), the business man (Phase 4), and the industrial worker (Phase 5) are still in existence, and, whereas the farmer's influence upon society is on the down grade, that of the industrial worker is on the up grade. The same can be said about the centers of production and distribution. The old evolutionary law is to-day as keenly felt as it was 300 000 yr ago.

During all this time the civil engineer has kept the road of progress in fairly good condition. Whereas the jungle man (Phase 1) was not in need of transportation, the hunter (Phase 2) needed trails; and the farmer (Phase 3) who traveled by wagon or on horseback, needed roads. The eotechnic (Phase 4) people who used sailing ships for hundreds of years needed the civil engineer to build and maintain canals and harbor works.

To-day, the paleotechnical (Phase 5) people are going forward in gasoline propelled engines, and the road of progress is in need of improvement. Grades and curves must be brought up to modern standards to eliminate obstructed views. Increased width should give ample room for the farmer (Phase 3), the business man (Phase 4), and the industrial worker (Phase 5) alike. Danger and speed-limit signs should keep the evolutionary traffic at a moderate speed; and swift justice should be dealt to those dangerous individuals who insist on driving in reverse "back to normal." They endanger the lives of millions of their fellow men.

If the civil engineer will build this "road" properly it will lead to a society in which the social rights of each worker are determined by the importance of the relative natural resources to society as a whole.

Samuel B. Folk,[99] Assoc. M. Am. Soc. C. E. (by letter).—In succinct manner Mr. Fay has shown the advantages of orderly planning. The next logical step is to note what can be planned. The Administration's budget, as released to newspapers on January 5, 1938, lists:

General Public Works:	
TVA	$ 46 000 000
Commerce	1 850 000
Interior	55 973 100
Justice	2 041 200
State	3 400 000
Treasury	58 107 000
Public Works:	
Highways	$104 640 100
Rivers and harbors	36 168 000
Rural electrification	39 514 800
Other public works	200 049 300
Total	$545 743 500

No amount of orderly planning of $600 000 000 will keep the pendulum of prosperity swinging when the total construction in the United States drops from $12 000 000 000 in 1928 to $4 000 000 000 in 1934 (see Table 1). However, it can help it keep a small oscillation, when the essential factor is to keep it from stopping entirely. Hence, engineers should analyze the types of public works that can be planned in an orderly manner and they should propose a program designed to help keep the wheels moving to elevate the standard of living. Among the numerous public works projects that (*a*) can be planned in advance; (*b*) can be at least partly liquidated; (*c*) will add to the well-being of the nation; (*d*) will reduce the cost of crime; and incidentally (*e*) will assist in returning the engineer to a respectable place in the mind of society—is public housing. Since this activity does not compete with services already offered by private corporations it is not in the category of some of the other projects mentioned in papers of this Symposium. Early in 1933 the idea that private initiative could satisfy all housing needs was abandoned. A small beginning has been made on sound principles, which the authors of these papers must surely endorse:[100]

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[99] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

> "The PWA housing program produced fifty-one sound, workable projects which afforded a total of approximately 22 000 living units. Base rents averaging $5.65 per room per month had been established on 23 of them when the program was transferred to the United States Housing Authority. This figure was the 'shelter' rent and an additional charge of $1.82 per room per month provided heat, hot and cold water, electricity for lighting and cooking, or gas for cooking if that fuel was to be used. At these low figures, former slum dwellers obtained good shelter, ample light and air, modern equipment and generally salubrious surroundings.
>
> "The average income of the families in the PWA projects is low, and every family admitted was taken from substandard housing. In one of the Montgomery, Alabama, projects, the average income of tenants is $12.50 per week. In Techwood, Atlanta, it is $22.11 per week; in Atlantic City, $26.46, Miami, $19.15."

When it is realized that this has been done in spite of the advance in materials prices, high costs, and high taxes, it is even more remarkable. In reference to the general property tax on urban improvements, Harold M. Groves, Professor of Public Finance, at the University of Wisconsin, has remarked:[101] "It seems odd that we should have singled out for especially heavy taxation under the property tax, an interest which we now seek to subsidize as 'low-cost housing'."

Regarding the high prices, the U. S. Bureau of Labor Statistics gives the relative wholesale prices of important building materials cited in Table 10.[102]

TABLE 10.—RELATIVE WHOLESALE PRICES OF BUILDING MATERIALS (1926 EQUALS 100)

Average for year	Brick and tile	Cement	Lumber	Paint, etc.	Plumbing and heating	Structural steel	Other building materials	All building materials
1929	94.3	91.8	93.8	94.9	95.0	98.1	97.7	95.4
1932	77.3	77.2	58.5	71.7	66.8	80.9	79.5	71.4
1937	93.5	95.5	99.0	83.4	78.8	113.2	99.1	95.2

It will be observed that, although the national income in 1937 was only a little more than two-thirds of that in 1929, the prices of all building materials were almost exactly those of 1929 and land costs were still held at exorbitant values by real estate speculators who bought before the Government could get possession of the land for housing projects. These data are affirmed by reports[103] that construction costs are up 9% in 1938, as compared with 1937. Private initiative can do little against these odds to provide housing at the pocket-book level of the lowest quarter of the population. "No industrial organization is equipped to produce the kind and amount of houses we need, while conditions within our cities are such as to discourage rather than encourage industry in so equipping itself."[104]

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[100] "In Defense of the PWA," by Harold L. Ickes, *New Republic*, March 30, 1938, p. 215.

[101] "What to Do About Taxes," *New Republic*, December 29, 1937.

[102] *Peoples Lobby Bulletin*, March, 1938.

To indicate further the complexity of the private building industry, a study of the Knickerbocker Village project in New York City accommodating about 1 600 "white collar" workers and their families, has been made.[105] The materials cost $2 636 000; labor, $2 020 000; overhead, $1 561 000; and land, $3 250 000. To reduce labor costs 25% (labor received an average of $1.15 per hr), as has so often been suggested, would have reduced the cost 5% and would have reduced the rentals only 3.3 per cent. This does not indicate that any one factor is blameless, but that modern methods, reduction in land costs, materials, and overhead, Government subsidy, or outright Federal housing might be the best solution.

From these data it is observed that the engineer's attack on public works as a solution to prosperity must start much deeper than the Symposium indicates. Even after reducing the problem to fundamentals, pressure will be discovered from unexpected sources. Architecture is still very conservative, as it relates to public construction, and financial interests oppose any radical change of types of buildings, as may be noted in the following:[106]

> "When recently the mechanized industries, particularly in metal, entered the housing field with the production of 'Prefabricated Houses,' they were met by the resistance of property holders, especially of the banks, who hold mortgages on about 58% of 1933 value of all urban real estate, and who fear that an influx of cheap modern dwelling would subtract substantially from the market value of existing structures."

Professor Mead has mentioned the interference of Government in business (see "The Record of Government in Business") but has neglected the interference of business in Government. Much legislation is lobbied by trade associations that want, as John T. Flynn, the Economist, has stated:[107] "Less Government in business for me; more Government in business for the other fellow." All want tariffs, subsidies, and laws. Contrary to Professor Mead's opinion, Government in industry has not always been so unsatisfactory. In a brief discussion, data cannot be cited of the success of public water-works, electric plants, highways, bridges, and ferries. There are projects more extensive in scope and area, however, than any one State could undertake and expect success. On the other hand, to be fair, one should note that many private ventures are failures. The railroads have been built and maintained with constant subsidies from the Public Treasury. The story of the financing of the systems is not entirely creditable. Private initiative could not undertake a comprehensive policy of regional development of the Tennessee Valley. Even if it could, many think it is better to have it developed by Federal authority in order that the social shocks can be absorbed.

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[103] *Business Week*, May, 1938.

[104] "Can America Build Houses," *Pamphlet No. 19*, Public Affairs Committee, New York, N. Y., p. 4.

[105] *Fortune*, June, 1938.

[106] "Technological Trends and National Policy," National Resources Committee, June, 1937, p. 59.

[107] Syndicated column in *The Columbus Citizen* and other newspapers, February 5, 1936.

The social effect may be as important as that of many past technical inventions. History has recorded the opposition to nearly every milestone in progress. Note the law sought to prevent the use of coaches in Hungary in 1523,[108] the opposition of the turnpike companies who profited by tolls, and the objections of canal owners to the railroads. The list reads like a catalog of inventions—air-brakes opposed; objections to electric locomotives on account of capital equipment loss, to subways, interurbans, and trucks. Steamships were opposed by the British Navy. The telegraph companies had a chance to buy the telephone, but chose to fight it. Bath-tubs were opposed, legislated against, and taxed. The newspapers fight the radio news. Just now History is writing the opposition to TVA, predicated on the idea that honest people have invested in public utility securities. Fate, however, is not cognizant of the honesty of men when new inventions cause obsolescence of old ideas and old methods or when financial jugglers out-manipulate the investor by pyramiding holding companies. Progress cannot be halted even if bankruptcy is in order for some industries. "In all previous depressions," states Professor Mead, "the responsibility for recovery has been met largely by the business interests of the country" (see "Government Function in Business Recessions and Recovery"). He should further note, as all business leaders did, that in all previous depressions there was not the large debt structure that burdened the country as in 1929, and that business beat a path to Washington demanding that Government "do something" about the sorry state of affairs. Previous depressions were ended when debts were written off, and agriculture and industry started anew to spend for consumptive products, including new inventions which employed a large amount of help. All efforts in this depression were in the direction of saving the debts and creating new ones, thus ear-marking nearly all income for interest. This is neither bankruptcy nor progress.

With his further statement that "The Administration has also urged the engineering schools to teach their students to consider not only the design and construction of engineering works but also to study and determine their ultimate effects on society" the writer, as an educator, agrees wholeheartedly. Most graduates leave college ignorant of the effect of science on culture, and new developments "hatch" so rapidly that the graduate never catches up, but remains saturated with the narrowness of his specialty. One required reading of the "Social Effects of Inventions" and "Resistances to the Adoption of Technological Innovations"[109] might open the doors to a wider knowledge of the effect of their own engineering work on society.

With regard to his opinion that "power development as an objective is not a legal function of the Federal Government" (see "Government Interference as a Permanent Policy: Power"), reference should be made to the Ashwander *v.*

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[108] "Technological Trends and National Policy," National Resources Committee, June, 1937, p. 39 *et seq.*

[109] "Technological Trends and National Policy," Sections III and IV, National Resources Committee, June, 1937, pp. 24–66.

Tennessee Valley Authority case (56 U.S. 466) in 1936 when the Supreme Court held that, "the Wilson Dam and its power plant must be taken to have been constructed in the exercise of constitutional functions of the Federal Government." Further, in Tennessee Electric Power Co. *v.* Tennessee Valley Authority (Fed. Supp. 947 District Court Tennessee E.D.) the Court held:

> "The Court is of the opinion that the relative value of these various plans is immaterial, since it has been established that the TVA project is reasonably adapted to use for combined flood control, navigation, power, and national defense, and that in actual operation the creation of energy is subordinated to the needs of navigation and flood control. * * * The dams and their power equipment, both constructed, under construction, and authorized, must be taken to have been authorized, constructed and planned in the exercise of the constitutional functions of the Government. * * *"
>
> "While the Government, in selling property of the United States, performs many functions, that would be performed in the operation of a private business trading in similar property, inasmuch as the energy sold is created at dams lawfully erected within the Federal power, the Government in performing these functions is not entering into private business. It is merely using an appropriate method of disposing of its property. The Government may sell land belonging to the United States in competition with a real estate agency, carry parcels in competition with express companies, and manage and control its thousands of square miles of national parks even as a private company. The Government has an equal right to sell hydroelectric power, lawfully created, in competition with a private utility. * * * These complainants have no immunity from lawful competition, even if their business be curtailed or destroyed."

However, the essential problem is to develop a modern and progressive viewpoint by engineers, especially civil engineers, who by training and employment are more closely affiliated with public problems than others whose work is allied with large industries not so closely tuned to the common weal. The civil engineer ought to be the first to recognize that "control of power is a social as well as an engineering and economic problem."[110] This being true, perhaps some new methods of analyzing public projects must be devised. The price criteria set up by Mr. Morris L. Cooke might be used as a start. He suggests[111] that the primary principle underlying a national power policy, both as to wholesale and retail prices, should be service at the lowest reasonable cost to consumers, with the benefit of any favorable conditions attached to Federal ownership and operation. Factors involved in establishing the "lowest reasonable cost" are: (1) Rational allocation of the costs of multiple-purpose projects; (2) amortization of the allocated cost of construction during the life of the property; (3) interest; (4) taxes, as under the TVA, where a percentage of gross is paid to the States; (5) cost of maintenance; (6) cost of operation (administrative, commercial, technical); and (7), cost of direct promotion of more intensive use.

Quoting Mr. Cooke:

> "Exceptions to this governing principle, in the form of sale at less than cost as computed by conventional accounting methods, should be allowed only on

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[110] Vice-Chairman Basil Manly, of the Federal Power Commission, quoted report of the National Resources Committee, June, 1937, p. 278.

[111] "Design for National Power Policy," *New Republic*, March 2, 1938.

special occasion in a particular area as a means toward achievement of some objective important to the national welfare, to gain which would effect a proper relation between costs and benefits in terms of social accounting. * * * In place of maximum money profits accruing to private ownership, the objective of public enterprise is maximum social benefits widely and equitably distributed."

Regarding the system of high dams, which Professor Mead states "is believed to have been built almost exclusively on account of power development," the testimony presented in the various engineering reports led the District Court, E.D. Tennessee, to state[112] its judgment that the improvement of the Tennessee River, from its mouth to Knoxville, by a series of low movable dams without power development, would have practically no effect on floods.

Referring to low-lift dams, it also declared that:

> "Such a waterway would be inferior to the high-dam developments and would not permit the economical development of power. * * * The board of engineers pointed out that in addition to having no value whatever for flood control, the 32 low dams, although less expensive to construct than the high dams, provided a navigation channel inferior to the high dam plan. * * * It concluded that 'it is evident that the full utilization of the resources of this river for the public benefit requires its development of power and navigation.'"

No engineer should object to a yardstick. It may not be "clearly dishonest," as Professor Mead states, but only a question of the type of yardstick required to measure the objectives sought. Mr. Cooke has defined it in clear terms:[113]

> "The advantages of public enterprise as a yardstick cannot be realized by attempting to set up a public enterprise in such a manner as to duplicate a private enterprise in every detail, and then comparing the two. They are essentially different kinds of institutions, with different objectives. Comparison of the costs of identical component parts of technical operations by a uniform system of accounting can indeed be useful; but comparison of the costs of a public enterprise in its entirety is neither realistic nor practicable because of the variables involved. For instance, public agencies obtain their money at low rates and almost without exception amortize their investments. The private utilities, by their failure to amortize capital-debt, laid themselves open to the recent collapse of equity values.
>
> "It is a recognition of, and experiment with, the variables that is important. A public enterprise can make experiments and comparisons on many fronts that private enterprise cannot or will not make. It can explore such things as the advantages and disadvantages of mass consumption at low rates, the extent to which electricity can be used advantageously on farms, and the influence of productivity and the costs of good wages and superior working conditions. It can explore the part to be played by electricity as the 'coordinating agent' in the promotion of a comprehensive conservation program."

DANIEL W. MEAD,[114] PAST-PRESIDENT AND HON. M. AM. SOC. C. E. (by letter).—The writer has read with much interest the discussion that followed the publication of his paper. He feels that little can be gained by continuing a discussion on public works, the facts concerning which, in any exact or detailed way, have been so successfully and completely withheld from the public. The important question is not whether "this" or "that" will be accomplished by any of the various projects which have been thrust upon the American people without their desire or consent, but whether or not a new theory of economic procedure is to be inaugurated in this country and whether business relations shall be changed entirely by the destruction of utility and other business investments through Federal competition. There is, and can be, no doubt that any private business can be ruined and destroyed by Federal competition or by the competition of a municipal or district project which is subsidized by the Federal Government to the extent of almost 50%, and in which taxes can be eliminated and interest reduced to a mere fraction of that which must be paid by private investors. If this is the basis of determining the fair cost of a project or the basis of a so-called "yardstick," then there can be no doubt of the inability of the private corporations to compete. In all such public investments the cost of production is an unknown quantity which can be so manipulated that losses can be covered by taxes, and the "yardsticks" so limited that they will assure ruin to all those the Government (Federal, State, or municipal) would destroy.

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[112] 21 Fed. Supp. 947, District Court, E.D. Tennessee.

[113] "Design for National Power Policy," *New Republic*, March 2, 1938.

[114] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

The Administration can, if it will, force this country toward Communism, Socialism, Fascism, or any other such "ism" as its unscrupulous politicians may desire, under the plea of emergency or the excuse of furnishing work to the unemployed. This can be done as long as it can maintain its authority and power by subsidizing selfish individuals and communities who prefer personal profit to the public good and are influenced through public works paid for by tax money and public loans imposed upon the taxpayers on a basis that will not stand sound economic analysis.

The writer stands for a fair deal. He has no defense to offer for private dishonesty in either public utilities or private business; neither can he satisfy his ideals of honesty and integrity by approving unfair and dishonest procedure in Federal or other Government affairs because some private businesses may also have been unfair and dishonest. What is needed at this time is intelligence, honesty, and decency in both business and Government.

The writer does not believe that the country can spend itself out of this depression; neither does he believe that business and confidence can be restored and the depression vanquished by Government competition and the destruction of the investments of its citizens.

WILLIAM J. WILGUS,[115] HON. M. AM. SOC. C. E. (by letter).—A careful reading of the discussions of the Symposium has failed to change the views of the writer that the billion dollar quartet of public works, comprising the Passamaquoddy Tidal Power scheme, the Grand Coulee project, the Fort Peck Dam, and the Florida Ship Canal, are economically unjustifiable. In the case of the Florida Ship Canal the writer finds in the discussions not one single word in its defense. On the record it stands without merit and is unworthy of the further expenditure of the savings of the people of the United States.

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[115] Weathersfield, Vt.

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DESIGN OF PILE FOUNDATIONS

By C. P. Vetter,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Hibbert M. Hill, Odd Albert, August E. Niederhoff, A. A. Eremin, Jacob Feld, John M. Coan, Jr., A. Agatz, D. P. Krynine, and C. P. Vetter.

Synopsis

It is intended, in this paper, to present rational methods for the determination of stresses in the individual piles of a pile foundation which supports a rigid structure such as a bridge pier or a hydraulic gate pier.

Athough it may be formulated in different ways, the purpose of the piles is obviously to minimize the settlement of the pier when it is acted upon by certain exterior forces. For the purpose of this investigation, the weight of the pier and other mass forces are considered exterior forces.

It is evident that the function of the piles is merely one of transfer. The statement is frequently made that the piles "carry" the structure; it would be more logical to state that the piles transfer the load from the structure to such strata of the foundation as are capable of carrying the load with a resultant settlement less than that prescribed for the structure. It is the subsoil that "carries" the load, and consolidation of soft material below the piles will inevitably contribute its share to the settlement of the structure whether or not piles are used.

If all external loads are vertical and the piles are also vertical, the piles transfer the load to deep layers in the subsoil. These layers may be more solid than the surface layers and may be so composed that they will support the load without material settlement. However, they may also be of essentially the same composition as the surface layers, but be more consolidated due to the superimposed load of the upper layers, so that the increased load of the structure may not cause objectionable settlements. Thus, the function of the piles is to transfer the load to the subsoil in such a manner that the compressibility of the upper layers of the foundation material affects the settlement of the structure only to a minor extent, if at all. Again, it may happen that the upper layers are of such composition that they will consolidate only slightly under load, whereas

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Note.—Published in February, 1938, *Proceedings*.

[1] Engr., U. S. Bureau of Reclamation, Denver, Colo.

the lower layers are softer. Under such circumstances, there is obviously no reason for using piles; in fact, the driving of piles may so disturb the upper layers that their original compactness is destroyed with a resultant greater settlement of the structure than would have occurred had no piles been used.

If the external loads are horizontal and the piles are vertical, conditions are entirely different. There is, now, no transfer of load to deeper foundation layers, except such as is caused by the stiffness of the piles, which, especially in the case of wooden piles, is relatively small. The horizontal load is transferred mainly to the upper foundation layers by means of direct pressures between the sides of the piles and the soil. Although the upper layers may be solid enough to withstand these horizontal pressures without material deformation, in their original state, this may not be the case after driving or jetting adjacent piles in the often confined space under a pier. It is believed that, in the case of solid upper layers, the transfer of horizontal forces may better be aided by other methods, such as projections below the pier base extending into excavated trenches in the foundation. In the case of soft upper layers, it is apparent that no great reliance can be placed on the resistance to horizontal forces except such as is furnished by the stiffness of the piles themselves.

It is the purpose of this paper to present, and encourage the use of, methods of design permitting the construction of pile foundations that transfer loads in any direction, including horizontal loads, to deep-lying foundation strata.

It may be argued, in favor of the common practice of relying on vertical piles to resist horizontal loads, that single test piles often show a remarkable resistance to horizontal pulls. However, it often happens, and may be expected, that the deflection of large pile foundations will materially exceed that of a single test pile with a comparable load. This is believed to be due to two main causes: One is the fact that the ground around the single test pile has not been disturbed by the driving or jetting of adjacent piles; the other is the fact, readily verified mathematically or experimentally, that a group consisting of many piles connected to a rigid pier will deflect far more if subjected to a horizontal loading than a single pile acted upon by a horizontal force equal to the average load on an individual pile in the group. In the investigation described in this paper, needless to say, it has been necessary to make certain simplifying assumptions. It may be argued, in general, that the assumptions are so many that the results of the investigation have little, if anything, to do with the actual conditions and, in particular, that considerations from the point of view of soil mechanics have been neglected in favor of those of purely structural mechanics. This argument cannot be refuted in a general manner. There may be cases in which the results are valueless because of the assumptions made. It is believed, however, that in the great majority of cases the simplifying assumptions will have only little effect on the final result. As has been the case in other fields of engineering investigations, assumptions which are now thought necessary may gradually be eliminated by further investigation, so that ultimately a method of design will evolve that gives the entirely correct answer in all cases. Engineering progress is made in this manner.

NOTATION

The letter symbols adopted for this paper are defined where they first appear and are assembled for convenient reference in the Appendix.

GENERAL CONSIDERATIONS

Rational methods for the design of pile foundations that take into account the elastic deformations of the piles have been available for several years. That they are rarely used for practical design may be due to the fact that the general theory upon which they are based is somewhat difficult to follow and because the application to definite problems is often cumbersome. However, in nearly all practical cases, the conditions are simplified to such an extent that neither the necessary theory nor application offers any particular difficulties. The investigation presented herein is confined to such simplified conditions. Those who are interested in the theory in its general form, or who have special problems to solve are referred to the original texts to which references are given under the heading, "Acknowledgments." For the sake of clarity, the presentation, methods, and formulas differ somewhat from those of previous treatments of the subject. The basic principles, however, are not new and have been taken from the works cited.

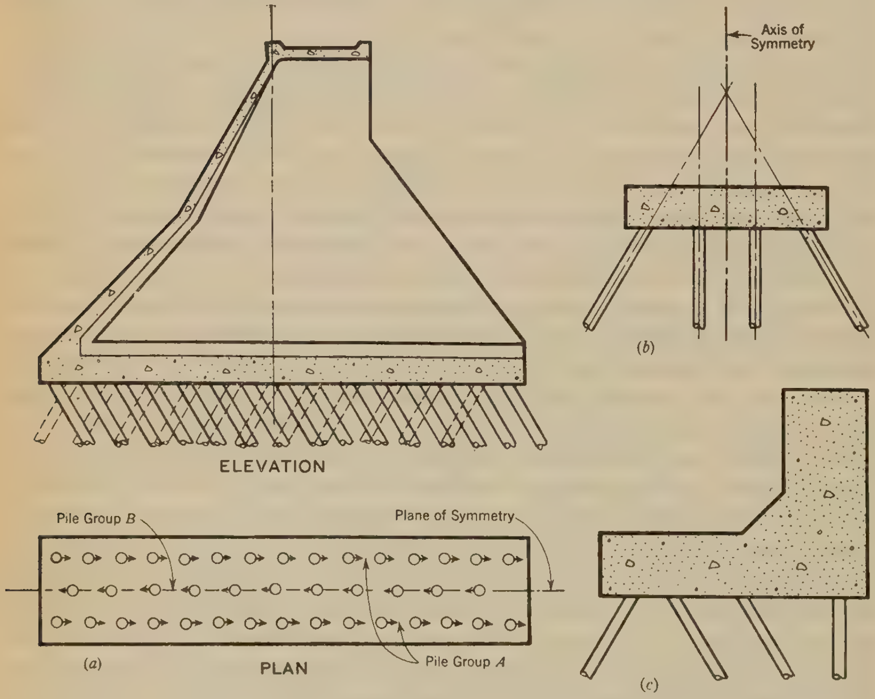


FIG. 1.—TYPES OF PILE FOUNDATIONS

The investigation is confined to two-dimensional systems; that is, to pile foundations having all piles parallel with, and symmetrical with respect to, a vertical plane of symmetry, and to structures in which the resultant of all outside forces, including dead loads, is located in the plane of symmetry. Although all piles are assumed parallel with this plane, they are not assumed to be parallel with each other. Three types of pile foundations are investigated: (*a*) Foundations consisting of two-pile groups, all piles in each group being parallel with each other (see Fig. 1(*a*)); (*b*) foundations having all piles symmetrical about a vertical line (pile bents) (see Fig. 1(*b*)); and (*c*) two-dimensional systems in general (piles in more than two directions and non-symmetrical) (see Fig. 1(*c*)).

It is assumed for all types that the structure on the piles (the pier) is infinitely stiff; that is, the internal deformation of the structure itself is disregarded.

Methods of procedure and formulas are first developed for the simple case in which each pile is considered as a column connected to the pier by means of a frictionless hinge, transmitting its load to the foundation material through a frictionless hinge located at some point of the axis of the pile. In many practical cases, such as bridge piers and gate piers in hydraulic structures, the error introduced by this assumption is insignificant. However, in some applications, such as pile bents for trestles, the error becomes of considerable magnitude. Therefore, methods are developed whereby the restraint of the pile heads in the pier and of the pile tips in the foundation material may be taken into account.

In the case of hinged piles the question arises as to the location of the lower hinge in the axis of the pile. Its location obviously depends on the material into which the pile is driven. If the pile is driven through loose material to firm ground the theoretical point of support would be approximately at the end point, and the so-called effective length of the pile would equal the actual length. If the pile is driven through uniform material so that its bearing capacity depends exclusively on friction, the theoretical point of support would be at some point above the pile tip. If it is assumed that the frictional resistance per linear foot of pile increases rectilinearly from zero at the top of the pile to a maximum value at the tip, the elastic deformation of the pile due to a load, P, applied axially at the top is determined by:

$$\Delta = \frac{1}{EA}\int_0^L \left(P - \frac{1}{2} \times f_x\right) dx \quad \text{.......................(1)}$$

in which x is a distance measured from the base of the pier to some arbitrary depth along the pile; f_x is the frictional resistance per linear foot at that point; E is the elastic modulus of the pile material; and A is the cross-section area of the pile. However, due to the assumed rectilinear variation,

$$f = \frac{x}{L} f_t \quad \text{.............................(2)}$$

in which f_t is the frictional resistance per linear foot at the tip of the pile.

Furthermore,

$$\frac{1}{2} f_t L = P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

since the total frictional resistance must equal the pile load. Hence,

$$\Delta = \frac{1}{E A} \int_0^L \left(P - \frac{x^2}{L^2} P \right) dx = \frac{1}{E A} \frac{2}{3} L P \ldots\ldots\ldots\ldots (4)$$

The deformation of a friction pile is thus the same as that of a pile of two-thirds the length, driven to firm ground, and its effective length is two-thirds of the actual length.

It is apparent that in the case of a pile, hinged both at its connection to the pier and at the point where it transfers its load to the foundation, the lateral resistance of the ground to movements perpendicular to the pile axis has been disregarded. In this case the exterior load on the pier is resisted solely by forces along the axes of the piles. On the other hand, there must be lateral resistance if the pile is considered restrained at one end or at both ends. The resistance must depend on the magnitude of the lateral displacement, on the depth below the surface, and, possibly, on several other factors, and it must be distributed in some manner along the entire length of the pile. For the sake of the investigation presented herein it is assumed that the total effect on the pier of the lateral resistance on the pile is represented by a lateral force (perpendicular to the pile axis) and a restraining moment, both acting at some point along the pile axis. Until the results of further research on the subject are available, the location of this point, the centroid of the lateral forces, can only be fixed by the judgment of the designer.

First, let it be assumed that all piles supporting the pier are hinged, and that the pier is acted upon by an exterior force in the plane of symmetry. The pier will then be displaced from its no-load position, due to the elasticity of the piles. The displacement may take the form of a translation or a rotation, or a combination of both. Since a translation may be defined as a rotation about some point at infinity, any small displacement may be considered a rotation. If the exterior force takes the form of a rotating moment, M (a force couple), the pier will rotate about a particular point, O, which is of special interest and is usually referred to as the elastic center of the pile group. If, on the other hand, the pier is acted upon by a resultant force going through Point O, the pier will rotate about a point at infinity; that is, the displacement will be a translation. This follows directly from Maxwell's general theorem of reciprocity.

Any arbitrary loading on the pier may be resolved into a force through the elastic center and a rotating moment; and, as has been shown, a force through this particular point will produce a translation and a moment will produce a rotation about the same point. Therefore, if the location of the elastic center, O, is known, and if the direction of the translation due to a given force through O has been determined, the problem of individual pile stresses is obviously reduced to finding the pile loads caused by a rotation about O and a translation in a given direction. In the investigation that follows the resultant of all stresses in a pile at its junction with the pier is designated the pile load or the pile reaction. The method followed to determine these quantities differs somewhat according to the type of foundation and will be dealt with for each individual type.

FOUNDATIONS CONSISTING OF TWO-PILE GROUPS

Hinged Piles.—Let the resultant of all outside forces, including dead load, be R and let the two pile groups be A and B, in Fig. 1(a). All piles in each group are parallel with each other and are assumed to be of the same length. Piles of Group A need not necessarily be of the same length as the piles of Group B.

In order to determine the location of the elastic center, give the pier an arbitrary small translation in a direction perpendicular to the direction of the piles in Group B. How this translation is brought about is, for the present, immaterial. As a result of the translation no loads will be induced in the piles in Group B, and all the piles in Group A will have the same load. The resultant of all pile loads will be a force parallel to the piles in Group A and going through the center of gravity of this group. Furthermore, a force, equal in magnitude and location but opposite in direction to this force, acting on the pier will produce a translation perpendicular to the direction of the piles in Group B. Next, give the pier a translation perpendicular to the direction of the piles in Group A. The resultant of all pile loads induced by the translation will be a force parallel to the piles in Group B and going through the center of gravity of this group; and a force, acting on the pier, equal in magnitude and location, but opposite in direction to the resultant, will cause a translation perpendicular to the direction of the piles in Group A. Two lines of force have thus been determined. An outside force acting on the pier along either one of these lines will produce a translation perpendicular to the other. An outside force going through the point of intersection of the two lines may be resolved into components along the two lines. Since each of the components produces translation without rotation, the force itself must also produce translation without rotation and, by definition, the point of intersection must be the elastic center.

The following rule may be established:

> Rule 1.—In a pile foundation consisting of piles in only two directions the elastic center is the point of intersection between lines drawn through the centers of gravity of the two-pile groups and parallel with the directions of the piles in the respective groups.

It is further evident that:

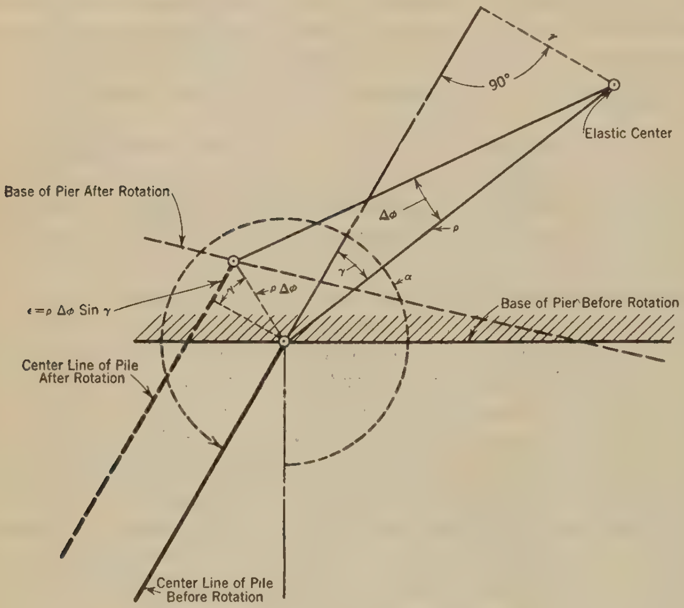
> Rule 2.—An outside force through the elastic center will result in loads on the piles which may be determined by resolving the force into components parallel with the directions of the piles in the two groups and dividing the magnitude of the components by the number of piles in the respective groups.

Pile loads due to a rotating moment may be found as follows: The moment will cause a rotation designated $\Delta\phi$ about the elastic center (Fig. 2). As a result, the length of each pile will be changed by an amount,

$$\epsilon = \rho\,\Delta\phi \sin\gamma = r\,\Delta\phi \qquad (5)$$

in which ρ is the radius vector to the top of the pile and r, the perpendicular distance from the elastic center to the pile. The reaction exerted by the pile due to the change in length is:

$$P_M = \frac{E\,A}{L}\epsilon = \frac{E\,A}{L} r\,\Delta\phi \qquad (6)$$



90°
r
Elastic Center
Base of Pier After Rotation
Δφ
ρ
γ
α
ρ Δφ
ε = ρ Δφ Sin γ
Base of Pier Before Rotation
Center Line of Pile After Rotation
Center Line of Pile Before Rotation

FIG. 2

The moment of all the pile reactions with respect to Point O must equal the rotating moment: $M = \sum \frac{E\,A}{L} r^2\,\Delta\phi$, or,

$$\Delta\phi = \frac{M}{\sum \frac{E\,A}{L} r^2} \qquad (7)$$

The load on an individual pile is then determined by:

$$P_M = \frac{E\,A\,r}{L \sum \frac{E\,A}{L} r^2} M \qquad (8)$$

If M is positive in the clockwise direction and compression is considered positive, r is positive if it extends from the point of rotation toward the right and negative if it extends from the point of rotation toward the left. In Fig. 2, r is negative.

If all piles have the same length, the same cross-sectional area, and are of the same material:

$$P_M = \frac{r}{\Sigma\, r^2} M \qquad (9)$$

Let the number of piles in Group A be n_A and in Group B, n_B, and let the component of the resultant in the direction of the A-piles be R_A and in the direction of the B-piles, R_B. The load on any pile may then be found from,

$$P_A = \frac{R_A}{n_A} + \frac{r}{\Sigma\, r^2} M \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

and,

$$P_B = \frac{R_B}{n_B} + \frac{r}{\Sigma\, r^2} M \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10b)$$

The summation, $\Sigma\, r^2$, should be extended over both the A-piles and the B-piles and the components, R_A and R_B, should be taken as positive when they cause compression in the corresponding pile group.

For design purposes the pile foundations can often be arranged so that the moment, M, becomes zero for two principal loadings. In a hydraulic structure, for instance, the point of intersection between the resultant for maximum water load and the resultant for dead load only should be made the elastic center of the foundation. This is done by drawing two lines through the point of intersection of the resultants parallel to the piles in each group and arranging the piles in each group so that the centers of gravity fall on these lines. The loads for resultants for intermediate water levels may be determined from Equations (10). For preliminary designs the load determination for intermediate resultants is rarely necessary.

Restrained Piles.—If the number of piles in the foundation is great and the rotating moment small, the quantity, $\Delta\phi$ (Equation (7)), becomes small, and the effect of restraint may be safely neglected. However, if the quantity, $\Sigma\, r^2$, is small the rotation may cause considerable bending in the piles with resultant changes in the direct pile loads. It is entirely possible to calculate the pile stresses, both due to axial load and bending, under these conditions. It is done by adding "dummy" piles perpendicular to each of the real piles, the location and length of the dummy piles to be such that the effect on the pier of an unrestrained real pile plus one or two unrestrained dummy piles is the same as that of the restrained real pile. The dummy pile was first introduced by Dr. C. Nökkentved.[5] After the locations and lengths of the dummy piles have been determined, the procedure of load determination is exactly as if all piles were real and unrestrained. The dummy piles are thus purely imaginary piles added for the purpose of facilitating the computations. Certain loads will be found on the dummy piles, and these loads represent the actual shears in the real piles. The addition of the dummy piles is the method whereby the effect of the restraint of the real piles is introduced in the computations.

The restraint of the piles may be classified as follows:

1.—Piles Restrained in the Foundation Material at Some Point of Their Axis and Connected to the Pier Through a Hinge.—It can be shown that the total effect of the pile foundation on the pier is not changed by considering the real piles hinged at both ends, providing a hinged dummy pile is added perpendicular to each real pile, intersecting it at the base of the pier. The dummy pile should be given an area equal to that of the real pile and an effective length,

$$L_1 = \frac{1}{3}\,\frac{A}{I}\,L_r^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where A and I represent the area and moment of inertia, respectively, of the real pile, and L_r, the distance from the base of the pier measured along the axis of the real pile to the point where it is considered restrained.

In Fig. 3 is indicated a pier and one pile, AE, of a pile group. Give the pier a small rotation, $\Delta\phi$, about the arbitrary point, O. As shown in Fig. 3, the shortening of the pile is the distance $A\,D$, or,

$$\epsilon = \rho\,\Delta\phi \sin\gamma = r\,\Delta\phi \quad (12)$$

Furthermore, the lateral deflection of the pile is $D\,B$, or,

$$\delta = \rho\,\Delta\phi \cos\gamma = a\,\Delta\phi \quad (13)$$

in which a = distance from the top of the pile, measured along the axis of the pile to a perpendicular to the axis through the elastic center. The deflection, δ, of the pile will cause a lateral reaction on the pier,

$$H = \frac{3\,E\,I}{L_r^3}\,\delta \quad (14)$$

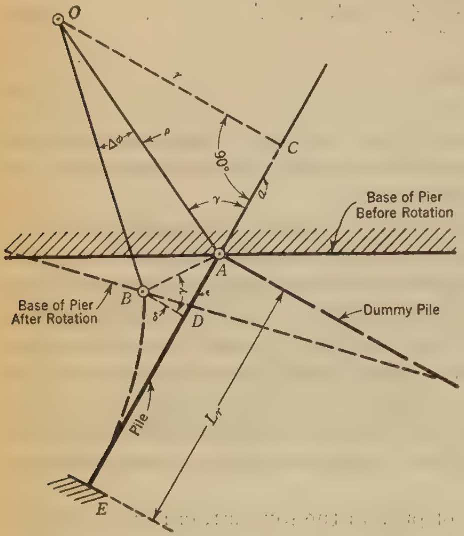


FIG. 3

in addition to the axial reaction:

$$V = \frac{E\,A}{L}\,\epsilon \quad (15)$$

Inserting the value of δ from Equation (13):

$$H = \frac{3\,E\,I}{L_r^3}\,a\,\Delta\phi \quad (16)$$

However, the lateral reaction, H, may also be obtained by adding a dummy pile perpendicular to AE and intersecting the pier base at Point A, provided the length of the dummy pile is,

$$L_1 = \frac{1}{3}\,\frac{A}{I}\,L_r^3 \quad (17)$$

The load on this pile due to the rotation, $\Delta\phi$, would be:

$$P_1 = \frac{E\,A}{L_1}\,\delta = \frac{E\,A}{L_1}\,a\,\Delta\phi \quad (18)$$

Inserting the value of L_1 from Equation (17):

$$P_1 = \frac{3\,E\,I}{L_r^3}\,a\,\Delta\phi = H \quad (19)$$

Since the point, O, is chosen in an entirely arbitrary manner, the substitution of the dummy pile for the restraining action of the real pile is obviously permissible for any displacement of the pier. The stress in the real pile, due to the restraint, is equal to that of a cantilever restrained at Point E and acted upon by a force, P_1, the axial load on the dummy pile, acting perpendicularly to the real pile at Point A.

2.—Piles Restrained at the Pier and not Restrained in the Foundation Material.—The real piles may be considered hinged at both ends if a dummy pile is added to each real pile. The dummy pile should be perpendicular to the real pile and should intersect it at the centroid of the lateral reactions exerted by the foundation material on the real pile. The cross-section area of the dummy pile should equal that of the real pile, and its effective length should be:

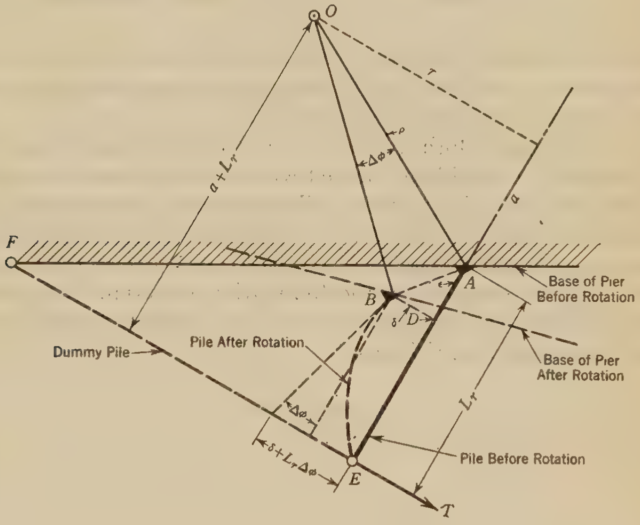


FIG. 4

$$L_2 = \frac{1}{3}\frac{A}{I} L_r^3 \qquad (20)$$

in which L_r is the distance from the base of the pier, measured along the axis of the pile to the centroid of the lateral resistance in the ground. In Fig. 4 is indicated a pier and one pile, $A\,E$, of a pile group. Give the pier a small rotation, $\Delta\phi$, about the arbitrary Point O. Then, as before, the shortening of the pile is $A\,D$, or, $\epsilon = r\,\Delta\phi$; and the deflection is $D\,B$, or, $\delta = a\,\Delta\phi$. In addition thereto, the pile head receives a rotation equal to $\Delta\phi$.

The reaction on the pier due to the restraint is equal to that which, if applied at Point E, would deflect a cantilever, $A\,E$, restrained at Point A, a distance,

$$\delta_2 = \delta + L_r\,\Delta\phi \qquad (21)$$

The force required to produce a deflection, δ_2, is:

$$H = \frac{3\,E\,I}{L_r^3}\,\delta_2 = \frac{3\,E\,I}{L_r^3}\,(\delta + L_r\,\Delta\phi) = \frac{3\,E\,I}{L_r^3}\,\Delta\phi\,(a + L_r) \qquad (22)$$

The lateral reaction, H, may also be obtained by adding a hinged dummy pile through Point E and perpendicular to $A\ E$, intersecting the pier base at Point F and having a length:

$$L_2 = \frac{1}{3}\frac{A}{I} L_r^3 \qquad \text{(23)}$$

The load on this pile, due to the rotation, $\Delta\phi$, would be:

$$P_2 = \frac{E\ A}{L_2}\Delta\phi\ (a + L_r) = \frac{3\ E\ I}{L_r^3}\Delta\phi\ (a + L_r) = H \qquad \text{(24)}$$

The stress in the real pile, due to the restraint, is equal to that of a cantilever restrained at Point A and acted upon by a force, P_2, the axial load on the dummy pile, perpendicular to the real pile at Point E.

3.—Piles Restrained Both at the Pier and in the Foundation Material.—The real pile may be considered hinged at both ends if two dummy piles are added to each real pile. The dummy piles should be perpendicular to the real pile and should intersect it, respectively, at the base of the pier and at a distance below the base of the pier equal to two-thirds of the total distance from the base to the point of restraint in the foundation material. The areas of the dummy piles should equal the area of the real pile and the effective length of the dummy pile intersecting at the base:

$$L_1 = \frac{1}{3}\frac{A}{I} L_r^3 \qquad \text{(25)}$$

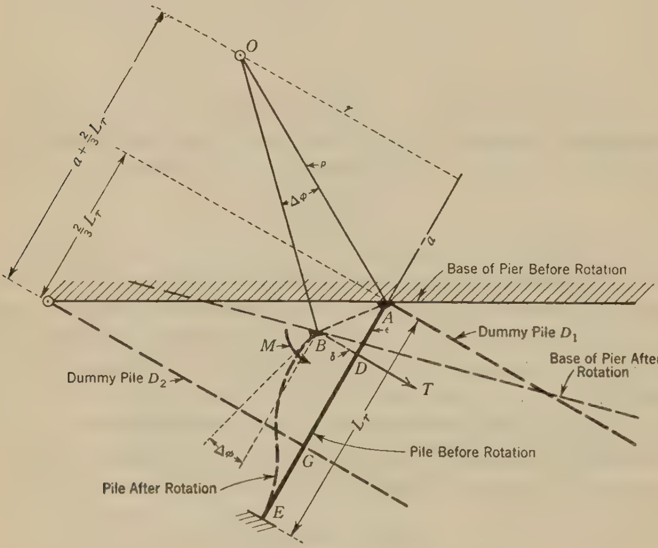

O
r
p
Δφ
$a + \frac{2}{3}L_r$
$\frac{2}{3}L_r$
a
Base of Pier Before Rotation
A
ε
Dummy Pile D_1
M
B
δ
D
T
Base of Pier After Rotation
Dummy Pile D_2
L_r
Δφ
Pile Before Rotation
G
Pile After Rotation
E

Fig. 5

and the effective length of the dummy pile intersecting at the two-thirds point:

$$L_2 = \frac{1}{9}\frac{A}{I} L_r^3 \qquad \text{(26)}$$

Since the two dummy piles are parallel, they will receive the same deformation for any translation and, for determining the elastic center only, may be combined into a single dummy pile with the same cross-section area as the real pile and an effective length:

$$L' = \frac{1}{12}\frac{A}{I}L_r^3 \quad \text{(27)}$$

The combined dummy pile should intersect the real pile at a distance below the base of the pier equal to one-half the total distance from the base to the point of restraint. After the elastic center has been located, the two individual dummy piles should be used for the determination of pile loads. In Fig. 5 is indicated a pier and one pile, $A\ E$, of a pile group. The pile is restrained at both ends. Give the pier a small rotation, $\Delta\phi$, about the arbitrary point, O. Then the shortening of the pile, $A\ D$, is $\epsilon = r\ \Delta\phi$; and the deflection, $D\ B$, is $\delta = a\,\Delta\phi$. In addition, the pile head received a rotation equal to $\Delta\phi$.

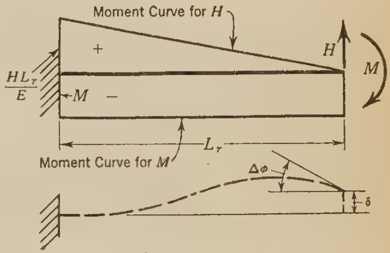


FIG. 6

The effect of the restraint on the pier would be a force, H, and moment, M, equal and opposite to a force and a moment which would produce, in a beam, $A\ E$, restrained at E, a deflection, δ, and a rotation, $\Delta\phi$, at Point A.

Considering Fig. 6 it will be seen that,

$$\delta = \frac{1}{E\,I}\left(\frac{1}{3}H\,L_r^3 - \frac{1}{2}M\,L_r^2\right) \quad \text{(28a)}$$

and,

$$\Delta\phi = \frac{1}{E\,I}\left(-\frac{1}{2}H\,L_r^2 + M\,L_r\right) \quad \text{(28b)}$$

Solving for H and M:

$$H = \frac{6\,E\,I}{L_r^2}\left(\frac{2\,\delta}{L_r} + \Delta\phi\right) = \frac{12\,E\,I}{L_r^3}\Delta\phi\left(a + \frac{L_r}{2}\right) \quad \text{(29a)}$$

and,

$$M = \frac{12\,E\,I}{L_r}\left(\frac{\delta}{2\,L_r} + \frac{\Delta\phi}{3}\right) = \frac{6\,E\,I}{L_r^2}\Delta\phi\left(a + \frac{2}{3}L_r\right) \quad \text{(29b)}$$

The moment, M, and lateral force, H, may also be obtained by adding a hinged dummy pile, D_1, through Point A perpendicular to $A\ E$ and having a length: $L_1 = \frac{1}{3}\frac{A}{I}L_r^3$; and a second hinged dummy pile, D_2, through Point G, perpendicular to $A\ E$, intersecting the pier at Point F, and having the length: $L_2 = \frac{1}{9}\frac{A}{I}L_r^3$. The load on Pile D_1 due to the rotation would be,

$$P_1 = \frac{E\,A}{L_1}\Delta\phi\,a = \frac{3\,E\,I}{L_r^3}\,a\,\Delta\phi \quad \text{(30a)}$$

and the load on Pile D_2:

$$P_2 = \frac{E\,A}{L_1}\Delta\phi\left(a + \frac{2}{3}L_r\right) = \frac{9\,E\,I}{L_r^3}\Delta\phi\left(a + \frac{2}{3}L_r\right) \ldots\ldots (30b)$$

The sum of Equations (30*a*) and (30*b*) is:

$$P_1 + P_2 = \frac{12\,E\,I}{L_r^3}\Delta\phi\left(a + \frac{1}{2}L_r\right) = H \ldots\ldots (31)$$

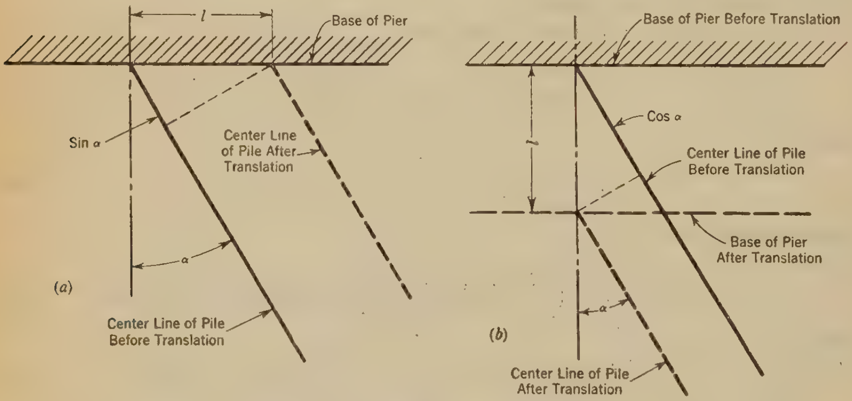
The moment of P_1 with respect to Point A is zero and the moment of P_2 becomes:

$$P_2\frac{2}{3}L_r = \frac{6\,E\,I}{L_r^2}\Delta\phi\left(a + \frac{2}{3}L_r\right) = M \ldots\ldots (32)$$

The stresses in the real pile, due to the restraint, may readily be determined by referring to Fig. 6 and utilizing the relationships, $H = P_1 + P_2$; and, $M = P_2\frac{2}{3}L_r$. Since the dummy piles constitute a pile group in a third direction, the pile loads cannot ordinarily be found by the method outlined for hinged piles, in only the two directions. Other methods are developed elsewhere in this paper which will permit solutions for this case.

PILE FOUNDATIONS WHICH ARE SYMMETRICAL ABOUT A VERTICAL LINE

Hinged Piles.—To determine the elastic center, give the pier a small horizontal translation equal to unity and toward the right (Fig. 7(*a*)). As a result the length of each pile will be changed an amount $\epsilon = \sin\alpha$; in which α is the angle of the pile with the vertical. The reaction exerted by the pile due to the change in length is:



l
Base of Pier
Sin α
Center Line of Pile After Translation
α
(a)
Center Line of Pile Before Translation
Base of Pier Before Translation
Cos α
Center Line of Pile Before Translation
1
Base of Pier After Translation
α
(b)
Center Line of Pile After Translation

FIG. 7

$$P_H' = \frac{E\,A\sin\alpha}{L} \ldots\ldots (33a)$$

Due to symmetry, the resultant of the pile reactions must be a horizontal force:

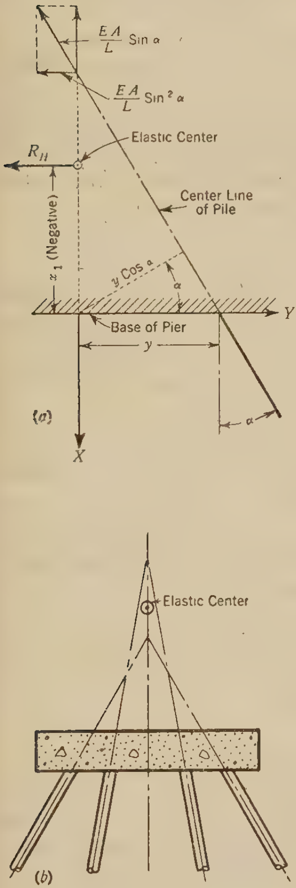
$$R_H = \sum\frac{E\,A}{L}\sin^2\alpha \ldots\ldots (33b)$$

Next give the pier a vertical translation, unity, downward (Fig. 7(*b*)). The change of length of each pile will be $\epsilon = \cos\alpha$; and the reaction:

$$P_V' = \frac{E\,A\cos\alpha}{L} \quad \text{(34a)}$$

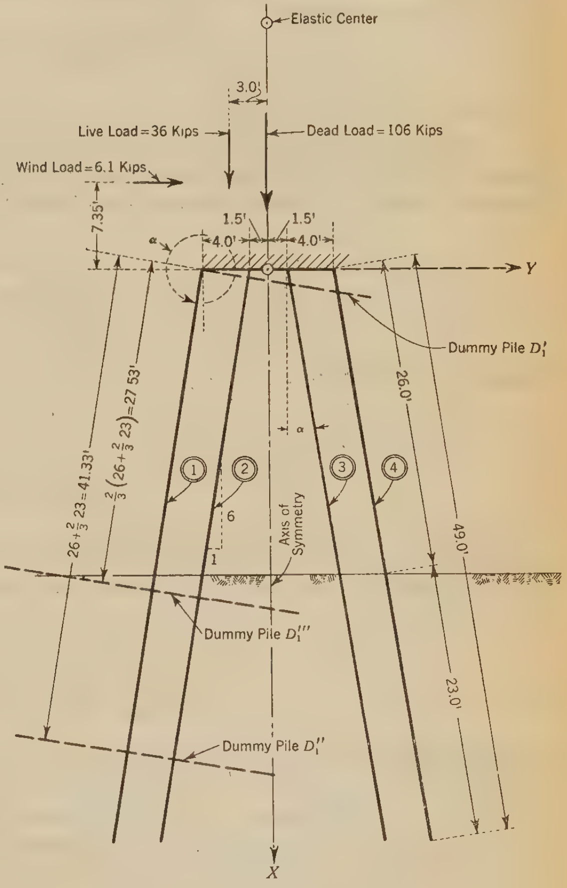
Due to the symmetry, the resultant of the pile reactions must be a vertical force:

$$R_V = \sum \frac{E\,A}{L}\cos^2\alpha \quad \text{(34b)}$$



$\frac{EA}{L}$ Sin α
$\frac{EA}{L}$ Sin² α
Elastic Center
R_H
Center Line of Pile
x_1 (Negative)
y Cos α
α
Y
Base of Pier
y
α
(a)
X
Elastic Center
(b)

FIG. 8



Elastic Center
3.0'
Live Load = 36 Kips
Dead Load = 106 Kips
Wind Load = 6.1 Kips
7.35'
1.5'
1.5'
α
4.0'
4.0'
Y
Dummy Pile D_1'
26.0'
$\frac{2}{3}(26+\frac{2}{3}23) = 27.53'$
$26+\frac{2}{3}23 = 41.33'$
α
①
②
③
④
6
1
Axis of Symmetry
49.0'
Dummy Pile D_1'''
23.0'
Dummy Pile D_1''
X

FIG. 9

located in the axis of symmetry. The location of the resultant, R_H, may be found graphically by drawing a force polygon and a string polygon to the forces expressed by Equation (33*a*).

It may also be found analytically as follows: Lay a co-ordinate system with a *X*-axis vertical along the axis of symmetry with positive direction downward, and a *Y*-axis horizontal, at the base of the pier, if this is horizontal, with positive direction to the right. Let the distance from the point of origin of the co-ordinate system to the intersection of the piles with the Y-axis be y (Fig. 8(a)); then the X-co-ordinate to the elastic center is:

$$x_1 = -\frac{\sum \frac{E\,A}{L} y \sin\alpha \cos\alpha}{\sum \frac{E\,A}{L} \sin^2\alpha} \qquad (35a)$$

If all piles have the same effective length and the same cross-section area, and are of the same material:

$$x_1 = -\frac{\Sigma\, y \sin\alpha \cos\alpha}{\Sigma \sin^2\alpha} \qquad (35b)$$

The elastic center is thus a point on the symmetry axis having an X-co-ordinate of x_1. If a horizontal force, equal in magnitude and location to R_H but in the opposite direction, will cause a translation of unity and a pile load, P_H', then a horizontal force, H', through the elastic center will cause a pile load:

$$P_H = \frac{H'}{R_H} P_H' = \frac{\frac{E\,A}{L} \sin\alpha}{\sum \frac{E\,A}{L} \sin^2\alpha} H' \qquad (36a)$$

Similarly, a vertical force, V', through the elastic center will cause a pile load:

$$P_V = \frac{V'}{R_V} P_V' = \frac{\frac{E\,A}{L} \cos\alpha}{\sum \frac{E\,A}{L} \cos^2\alpha} V' \qquad (36b)$$

By the same reasoning as was used for pile foundations having piles in only two directions, the pile load due to a rotating moment is:

$$P_M = \frac{\frac{E\,A}{L} r}{\sum \frac{E\,A}{L} r^2} M \qquad (37)$$

The total pile load is then $P = P_H + P_V + P_M$; or,

$$P = \frac{\frac{E\,A}{L} \sin\alpha}{\sum \frac{E\,A}{L} \sin^2\alpha} H' + \frac{\frac{E\,A}{L} \cos\alpha}{\sum \frac{E\,A}{L} \cos^2\alpha} V' + \frac{\frac{E\,A}{L} r}{\sum \frac{E\,A}{L} r^2} M \qquad (38a)$$

If all piles have the same length and the same cross-section area and are of the same material,

$$P = \frac{\sin\alpha}{\Sigma \sin^2\alpha} H' + \frac{\cos\alpha}{\Sigma \cos^2\alpha} V' + \frac{r}{\Sigma r^2} M \qquad (38b)$$

The positive direction of M has been given previously. The positive direction of H' is toward the right; and the positive direction of V' is downward. The positive direction of α is from the downward direction of the vertical, counterclockwise to the downward direction of the pile axis.

Comment.—Some interesting conclusions may be drawn from Equations (25) and (26). Since vertical piles do not help to resist a horizontal external force ($\alpha = 0$), structures subject to large horizontal forces should have as many battered piles as possible and the angle, α, should be made as great as possible. In structures where the rotating moment, M, is large, the quantity, Σr^2, should be as large as possible. If Σr^2 approaches zero the pile loads obviously approach infinity unless M is also zero. From Equation (7) it is seen that the rotation due to a moment increases as the quantity, Σr^2, decreases; if, therefore, the piles, approximately, go through the same point, the rotation for a given moment would be large. The assumption that the effect of restraint may safely be neglected does not hold for this case and the stability of the pier depends almost entirely on the restraint and the stiffness of the piles.

It is worth noting that in one of the most commonly used pile bents (Fig. 8(*b*)) the center lines of the piles come very close to intersection at one point. Large displacements may be expected in a structure of this construction if it is subject to loads that do not pass through the elastic center.

Restrained Piles.—The restraint of the piles may be taken into account by the methods outlined previously. It should be noted that the addition of the dummy piles does not destroy the original symmetry so that after they have been added, the computations are exactly as given for hinged piles.

TWO-DIMENSIONAL SYSTEMS IN GENERAL

The procedure is very similar to that used for symmetrical pile groups. Give the pier an arbitrary, small horizontal translation, unity, toward the right. The reactions exerted by the piles due to the translation is given by Equation (33*a*). The location of the resultant of all the pile reactions may be found graphically by drawing a force polygon and a string polygon to the forces, P_H'. Let the resultant of these reactions due to a unit translation be R_H. If the pier is acted upon by an outside force, H', in the same location as R_H, but acting in the opposite direction, the resultant pile loads would be,

$$P_H = \frac{H'}{R_H} P_H' = \frac{E\,A}{L} \sin \alpha \frac{H'}{R_H} \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Since R_H is determined by graphical construction, P_H can be found for any value of H'. Next, give the pier an arbitrary, small vertical translation, unity, downward. The reactions exerted by the piles due to the translation is then given by Equation (34*a*). The location of the resultant of all the pile reactions may be found graphically by drawing a force polygon and a string polygon to the forces, P_V'. Let the resultant of these reactions, due to a unit translation be R_V. As a check it should be noted that the vertical projection of R_H equals the horizontal projection of R_V.

If the pier is acted upon by an outside force, V', in the same location as R_V, but acting in the opposite direction, the resultant pile reactions will be,

$$P_V = \frac{V'}{R_V} P_V' = \frac{E\,A}{L} \cos \alpha \frac{V'}{R_V} \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Since R_V is determined by graphical construction, P_V can be found for any value of V'. It follows from the determination of the forces, R_H, and R_V, that the point of intersection between them is the elastic center for the pile foundation. A moment, therefore, will produce rotation about this point and loads on the individual piles determined by Equation (37). The total load on a pile due to an outside force, with components, H' and V', in the directions opposite to R_H and R_V, respectively, and with a moment, M, about the elastic center is:

$$P = P_H + P_V + P_M = \frac{E\,A}{L}\left[\frac{H'}{R_H}\sin\alpha + \frac{V'}{R_V}\cos\alpha + \frac{r}{\sum \frac{E\,A}{L} r^2} M\right] \quad \ldots (41)$$

The positive directions for M, α, and r have been defined previously. The positive directions for H' and V' are opposite to the directions of R_H and R_V.

If all piles are of the same length and the same material, the string and force polygons should be drawn to the forces: $P_H'' = \sin\alpha$; and, $P_V'' = \cos\alpha$, corresponding to the horizontal or vertical translations, $\Delta = \frac{L}{E\,A}$. Let the resultants of the pile reactions due to these translations be R_H' and R_V', respectively. If the pier is acted upon by outside forces, H' and V', in the same location but acting in the opposite direction of R_H' and R_V', the pile loads would be: $P_H = \frac{H'}{R_H'}\sin\alpha$; and, $P_V = \frac{V'}{R_V'}\cos\alpha$. The total pile load, in case all piles are of the same length and same material, is then,

$$P = \frac{H'}{R_H'}\sin\alpha + \frac{V'}{R_V'}\cos\alpha + \frac{r}{\Sigma r^2} M \quad \ldots\ldots\ldots\ldots\ldots (42)$$

It should be noted that in this general case the forces, R_H, R_H', R_V, and R_V', are not horizontal and vertical forces. They are the resultants of pile reactions due to horizontal and vertical translations. Only in the case of symmetrical pile foundations do they become horizontal and vertical forces.

NUMERICAL EXAMPLE

On Fig. 9 is shown a symmetrical pile bent consisting of four piles driven at a batter of 1 : 6 and spaced as shown. The piles are of reinforced concrete, 49 ft long, and driven 23 ft into fine sand. The cross-section areas of all piles are the same, and equal 331 sq in.; the moments of inertia of the cross-sections are 8 800 in.^{4} The effective lengths are $49 - 23 + \frac{2}{3} 23 = 41.33$ ft. The pile bent is subjected to three loads: A dead load of 106 kips acting vertically in the axis of symmetry; a live load of 36 kips acting vertically 3.0 ft from the axis; and a wind load of 6.1 kips acting horizontally 7.35 ft above the base of the pier.

The pile loads are determined under the assumptions that all piles are: (1) Hinged both at the pier and in the foundation; (2) hinged at the pier and restrained in the foundation material; (3) restrained at the pier and hinged in the foundation material; or, (4) restrained both at the pier and in the foundation material.

The piles are numbered 1 to 4, inclusive, from left to right; the dummy piles for Assumption (2) are designated D_1' to D_4'; for Assumption (3), D_1'' to D_4''; and for Assumption (4), D_1' to D_4' at the base of the pier and D_1''' to D_4''' at the two-thirds point.

TABLE 1.—NUMERICAL VALUES USED IN ILLUSTRATIVE EXAMPLE

Pile No.	Distance, y, in feet (1)	Angle, α (2)	$\sin \alpha$ (3)	$\cos \alpha$ (4)	$\sin^2 \alpha$ (5)	$\cos^2 \alpha$ (6)	$y \sin \alpha \cos \alpha$ (7)
1	−5.50	−9°28′	−0.164	0.986	0.0269	0.972	+0.890
2	−1.50	−9°28′	−0.164	0.986	0.0269	0.972	+0.243
3	+1.50	+9°28′	+0.164	0.986	0.0269	0.972	+0.243
4	+5.50	+9°28′	+0.164	0.986	0.0269	0.972	+0.890
D_1'	−5.50	+80°32′	+0.986	0.164	0.972	0.0269	−0.890
D_2'	−1.50	+80°32′	+0.986	0.164	0.972	0.0269	−0.243
D_3'	+1.50	−80°32′	−0.986	0.164	0.972	0.0269	−0.243
D_4'	+5.50	−80°32′	−0.986	0.164	0.972	0.0269	−0.890
D_1''	−256.9	+80°32′	+0.986	0.164	0.972	0.0269	−41.54
D_2''	−252.9	+80°32′	+0.986	0.164	0.972	0.0269	−40.89
D_3''	+252.9	−80°32′	−0.986	0.164	0.972	0.0269	−40.89
D_4''	+256.9	−80°32′	−0.986	0.164	0.972	0.0269	−41.54
D_1'''	−173.1	+80°32′	+0.986	0.164	0.972	0.0269	−27.99
D_2'''	−169.1	+80°32′	+0.986	0.164	0.972	0.0269	−27.34
D_3'''	+169.1	−80°32′	−0.986	0.164	0.972	0.0269	−27.34
D_4'''	+173.1	−80°32′	−0.986	0.164	0.972	0.0269	−27.99

TABLE 1.—(*Continued*)

Pile No.	L (8)	$\frac{10^4 \sin \alpha}{L}$ (9)	$\frac{10^4 \cos \alpha}{L}$ (10)	$\frac{10^4 \sin^2 \alpha}{L}$ (11)	$\frac{10^4 \cos^2 \alpha}{L}$ (12)	$\frac{10^4 y \sin \alpha \cos \alpha}{L}$ (13)
1	41.33	−39.68	238.57	6.51	235.18	+215.34
2	41.33	−39.68	238.57	6.51	235.18	+58.80
3	41.33	+39.68	238.57	6.51	235.18	+58.80
4	41.33	+39.68	238.57	6.51	235.18	+215.34
D_1'	128 100	+0.08	0.01	0.08	0.00	−0.07
D_2'	128 100	+0.08	0.01	0.08	0.00	−0.02
D_3'	128 100	−0.08	0.01	0.08	0.00	−0.02
D_4'	128 100	−0.08	0.01	0.08	0.00	−0.07
D_1''	128 100	+0.08	0.01	0.08	0.00	−3.24
D_2''	128 100	+0.08	0.01	0.08	0.00	−3.19
D_3''	128 100	−0.08	0.01	0.08	0.00	−3.19
D_4''	128 100	−0.08	0.01	0.08	0.00	−3.24
D_1'''	42 700	+0.23	0.04	0.23	0.01	−6.55
D_2'''	42 700	+0.23	0.04	0.23	0.01	−6.40
D_3'''	42 700	−0.23	0.04	0.23	0.01	−6.40
D_4'''	42 700	−0.23	0.04	0.23	0.01	−6.55

Table 1 contains all numerical values used in the computations, including the lengths of the dummy piles calculated from Equations (11), (20), (25), and (26). Table 2(*a*) gives the computations of pile loads under Assumption (1). The location of the elastic center is found from Equation (35*b*) and the pile loads from Equation (38*b*). Table 2(*b*) gives the computations of pile loads under Assumption (2). The location of the elastic center is found from Equation (35*a*), and the pile loads from Equation (38*a*). Since all piles have the same cross-section and are of the same material, the quantity, EA, cancels out in these two equations. Tables 2(*c*) and 2(*d*) give the corresponding computations under Assumptions (3) and (4), respectively. Zero load on a dummy pile indicates that the load is negligible.

TABLE 2.—Illustrative Example (Loads are in kips)

Pile No.	Distance, r, in feet (1)	Values of r^2 (2)	$10^4 \frac{r}{L}$ (3)	$10^4 \frac{r^2}{L}$ (4)	Live Loads			Dead load,	Wind Loads			Total,
					P_V (5)	P_M (6)	P (7)	$P_V = P$ (8)	P_H (9)	P_M (10)	P (11)	P (12)
(a) Computations Under Assumption (1), $x_1 = -21.00$												
1	−1.97	3.89	..	..	9.1	+13.7	+22.8	+26.9	−9.3	+10.5	+1.2	+50.9
2	+1.97	3.89	..	..	9.1	−13.7	−4.6	+26.9	−9.3	−10.5	−19.8	+2.5
3	−1.97	3.89	..	..	9.1	+13.7	+22.8	+26.9	+9.3	+10.5	+19.8	+69.5
4	+1.97	3.89	..	..	9.1	−13.7	−4.6	+26.9	+9.3	−10.5	−1.2	+21.1
(b) Computations Under Assumption (2), $x_1 = -20.81$												
1	−2.00	4.00	−483.9	967.8	+9.1	+13.4	+22.5	+26.9	−9.2	+10.2	+1.0	+50.4
2	+1.94	3.77	+469.4	912.2	+9.1	−13.0	−3.9	+26.9	−9.2	−9.9	−19.1	+3.9
3	−1.94	3.77	−469.4	912.2	+9.1	+13.0	+22.1	+26.9	+9.2	+9.9	+19.1	+68.1
4	+2.00	4.00	+483.9	967.8	+9.1	−13.4	−4.3	+26.9	+9.2	−10.2	−1.0	+21.6
D_1'	−21.42	458.82	−1.7	35.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_2'	−20.77	431.39	−1.6	33.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_3'	+20.77	431.39	+1.6	33.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_4'	+21.42	458.82	+1.7	35.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(c) Computations Under Assumption (3), $x_1 = -20.31$												
1	−2.08	4.33	−503.3	1 047.7	+9.1	+10.9	+20.0	+26.9	−9.2	+8.0	−1.2	+45.7
2	+1.86	3.47	+450.0	839.6	+9.1	−9.8	−0.7	+26.9	−9.2	−7.2	−16.4	+9.8
3	−1.86	3.47	−450.0	839.6	+9.1	+9.8	+18.9	+26.9	+9.2	+7.2	+16.4	+62.2
4	+2.08	4.33	+503.3	1 047.7	+9.1	−10.9	−1.8	+26.9	+9.2	−8.0	+1.2	+26.3
D_1''	−62.26	3 876	−4.9	302.6	0.0	+0.1	+0.1	0.0	0.0	+0.1	+0.1	+0.2
D_2''	−61.61	3 796	−4.8	296.3	0.0	+0.1	+0.1	0.0	0.0	+0.1	+0.1	+0.2
D_3''	+61.61	3 796	+4.8	296.3	0.0	−0.1	−0.1	0.0	0.0	−0.1	−0.1	−0.2
D_4''	+62.26	3 876	+4.9	302.6	0.0	−0.1	−0.1	0.0	0.0	−0.1	−0.1	−0.2
(d) Computations Under Assumption (4), $x_1 = -19.14$												
1	−2.28	5.20	−551.7	1 258.2	9.1	+9.9	+19.0	+26.9	−8.9	+6.6	−2.3	+43.6
2	+1.66	2.76	+401.6	667.8	9.1	−7.2	+1.9	+26.9	−8.9	−4.8	−13.7	+15.1
3	−1.66	2.76	−401.6	667.8	9.1	+7.2	+16.3	+26.9	+8.9	+4.8	+13.7	+56.9
4	+2.28	5.20	+551.7	1 258.2	9.1	−9.9	−0.8	+26.9	+8.9	−6.6	+2.3	+28.4
D_1'	−19.79	391.6	−1.5	30.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_2'	−19.13	366.0	−1.5	28.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_3'	+19.13	366.0	+1.5	28.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_4'	+19.79	391.6	+1.5	30.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D_1'''	−47.32	2 239	−11.1	524.4	0.0	+0.2	+0.2	0.0	+0.1	+0.1	+0.2	+0.4
D_2'''	−46.67	2 178	−10.9	510.1	0.0	+0.2	+0.2	0.0	+0.1	+0.1	+0.2	+0.4
D_3'''	+46.67	2 178	+10.9	510.1	0.0	−0.2	−0.2	0.0	−0.1	−0.1	−0.2	−0.4
D_4'''	+47.32	2 239	+11.1	524.9	0.0	−0.2	−0.2	0.0	−0.1	−0.1	−0.2	−0.4

A study of the pile loads due to the different external forces will reveal several interesting facts. Under all assumptions, Pile No. 2 gets tension for live load only; it is doubtful whether this result would have been anticipated had other, less rational, methods of load determination been used. Restraint in the foundation material only, has little effect on the pile loads. The effect of restraint in the pier is more pronounced and the effect of fully restrained piles is considerable. Had the horizontal distances between the piles been greater, the effect of all restraints would have been decreased.

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of the United States Bureau of Reclamation in connection with the preparation of this paper. The investigation was conducted in connection with the design of the Imperial Dam, of the Boulder Canyon Project.

He has also consulted, freely, the fundamental work done in this field of study by Messrs. T. Hultin,[2] P. Gullander,[3] H. M. Westergaard,[4] M. Am. Soc. C. E., and C. Nökkentved.[5] In addition the following investigators have treated various phases of the subject: A. Ostenteld,[6] H. Wünch,[7] T. Schultze,[8] A. Labutin,[9] P. Hedde,[10] and Franx.[11]

APPENDIX

NOTATION

The following symbols, defined where first introduced in the paper, are rearranged herein for convenience of reference. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Material,"[12] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = cross-section area of a pile;
a = distance from the top of a pile, measured along the axis of the pile, to a perpendicular to the axis through the elastic center.
E = modulus of elasticity of the pile material;
f = a function factor; the frictional resistance per linear foot of pile; f_x = the resistance at distance, x; and, f_t = the resistance at the tip of the pile;
H = a lateral reaction; H' = a horizontal force;
I = moment of inertia;
L = effective length of a pile; L_r = distance from base of pier measured along axis of pile to point where it is considered restrained;
M = a moment; a force couple;
n = a number; n_A = a number of piles in Group A;
P = a concentrated load; P_M = an additional load imposed by a change in pile length;
R = resultant of all external forces; R_H = a horizontal force, the resultant of pile reactions; R_V = a vertical force, the resultant of pile reactions;

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[2] "Om beräkning ov Grundpålninger," by T. Hultin, Industritidningen, *Norden*, 1911.

[3] "Teori för Grundpålninger," by P. Gullander, Stockholm, 1914.

[4] "The Resistance of a Group of Piles," by H. M. Westergaard, *Journal*, Western Soc. of Engrs., 1917.

[5] "Beregning af Pälevärker," by C. Nökkentved, Copenhagen, 1924.

[6] *Teknisk Tidskrift*, Copenhagen, 1921, No. 1.

[7] "Statische Berechnung der Pfalsysteme" Stuttgart, 1927.

[8] *Zentralblatt der Bauverwaltung*, 1926, p. 469.

[9] "Die graphische Berechnung von Pfalrosten, etc." Riga, 1933.

[10] *Der Bauingenieur*, 1929, p. 1.

[11] *De Ingenieur*, 1928, p. B-189.

[12] A.S.A.—Z10*a*—1932.

r = distance from the elastic center to the axis of the pile;
V = axial reaction; V' = a vertical reaction;
x = a depth from the base of the pier to an arbitrary point along the pile axis;
y = the distance from the point of origin of the co-ordinate system to the intersection of the piles with the Y-axis;
α = an angle (see Fig. 2) of the pile with the vertical;
γ = an angle (see Fig. 2);
Δ = the elastic deformation of a pile due to a load, P, applied axially at the top;
ϵ = the change in length of pile;
ρ = a radius vector from the elastic center to the top of a pile;
ϕ = a rotation; $\Delta\phi$ = a small rotation;
δ = deflection of a pile.

DISCUSSION

Hibbert M. Hill,[13] Assoc. M. Am. Soc. C. E. (by letter).—A few years ago the writer had need of a method of analyzing batter-pile foundations and was impressed, in the search for a method, by the little attention writers in English have given this subject. In this respect the author has performed a very worth-while work. It is to be hoped also that the paper will give an impetus to the use of battered piles to resist lateral loads. The writer is convinced that, in general, it is unsafe practice to rely upon vertical piles to resist lateral loads. Never should any reliance be placed upon assumed frictional resistance between the base of a structure and the earth between the piles.

The writer, some years ago, adopted Krey's[14] method of analysis. This method approaches the subject in a manner sufficiently different from the author's approach to make a brief statement of it worth while.

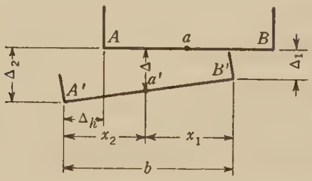

Fig. 10

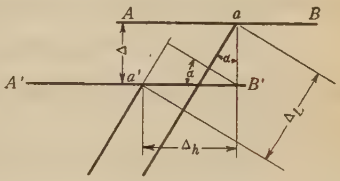

Fig. 11

The assumption is made that the axial deflection of an axially loaded pile is proportional to the applied load, and it is further assumed that the supported structure is rigid, so that the plane of the foundation remains plane after deflection. Consider a foundation, *A-B*, Fig. 10, which, under the action of a given loading, deflects to the position, *A'-B'*. Let a be a point on the foundation, then, Δ = vertical deflection of Point a; Δ_h = horizontal deflection of Point a; and, from the geometry of the figure,

$$\Delta = \Delta_1 \frac{x_2}{b} + \Delta_2 \frac{x_1}{b} \qquad (43)$$

Consider, further, a batter pile in a deflected foundation, Fig. 11: If Δ_h, Δ_1, and Δ_2 are small, and the length of the pile considerable, α, the angle of batter, remains approximately constant. Let the axial deflection of the pile be Δ_L, then,

$$\Delta_L = \Delta \cos \alpha + \Delta_h \sin \alpha \qquad (44)$$

If P is the axial load on the pile,

$$P = k \Delta_L \qquad (45)$$

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[13] Supt., St. Anthony Falls Water Power Co., Minneapolis, Minn.

[14] "Erddurck, Erdwiderstand, und Tragfahigheit des Baugrundes," Berlin, 1932.

The coefficient, k, may be evaluated by such means as the author uses, or by means of loading tests on piles in the foundation under consideration.

If a computation of deflections is not important and all piles are alike, then the value of k need not be known, as the following equations can be solved directly for $k\,\Delta$. From Equations (43), (44), and (45), P may be evaluated for any pile if the three unknowns, Δ_1, Δ_2, and Δ_h, are determined (from the three conditions of equilibrium). Let V = the resultant vertical load acting on the foundation; and, H = the resultant horizontal load acting on the foundation, at the plane of the bottom of concrete; then:

$$V = \Sigma\, P \cos \alpha \ldots\ldots\ldots\ldots (46a)$$

$$H = \Sigma\, P \sin \alpha \ldots\ldots\ldots\ldots (46b)$$

and (see Fig. 11),

$$M = \Sigma\, x_2\, P \cos \alpha \ldots\ldots\ldots\ldots (46c)$$

Substituting Equation (45) in Equation (46a):

$$V = \Sigma\, k\, \Delta_L \cos \alpha \ldots\ldots\ldots\ldots (47)$$

Substituting Equation (44) in Equation (47),

$$V = \Sigma\, (k\, \Delta \cos^2 \alpha + k\, \Delta_h \sin \alpha \cos \alpha) \ldots\ldots\ldots\ldots (48)$$

Substituting Equation (43) in Equation (48):

$$V = \Sigma \left(k\, \Delta_1 \frac{x_2}{b} \cos^2 \alpha + k\, \Delta_2 \frac{x_1}{b} \cos^2 \alpha + k\, \Delta_h \sin \alpha \cos \alpha \right) \ldots\ldots (49)$$

or,

$$\frac{V\,b}{k} = \Delta_1 \cos^2 \alpha\, \Sigma\, x_2 + \Delta_2 \cos^2 \alpha\, \Sigma\, x_1 + n'\, b\, \Delta_h \sin \alpha \cos \alpha \ldots\ldots (50)$$

in which n' is the number of piles battered at the angle, α.

Similarly,

$$H = \Sigma\, k\, \Delta_L \sin \alpha = \Sigma\, (k\, \Delta \sin \alpha \cos \alpha + k\, \Delta_h \sin^2 \alpha)$$

$$= \Sigma \left(k\, \Delta_1 \frac{x_2}{b} \sin \alpha \cos \alpha + k\, \Delta_2 \frac{x_1}{b} \sin \alpha \cos \alpha + k\, \Delta_h \sin^2 \alpha \right);$$

or,

$$\frac{H_b}{k} = \Delta_1 \sin \alpha \cos \alpha\, \Sigma\, x_2 + \Delta_2 \sin \alpha \cos \alpha\, \Sigma\, x_1 + n'\, b\, \Delta_h \sin^2 \alpha \ldots (51a)$$

and,

$$\frac{M_b}{k} = \Delta_1 \cos^2 \alpha\, \Sigma\, x_2^2 + \Delta_2 \cos^2 \alpha\, \Sigma\, x_1\, x_2 + b\, \Delta_h \sin \alpha \cos \alpha\, \Sigma\, x_2 \ldots (51b)$$

From the known foundation dimensions and loads, Equations (50) and (51), may be solved for Δ_1, Δ_2, and Δ_h. Knowing these values, the stress in any pile may be determined from Equations (43), (44), and (45), or the equations may be solved for $k\,\Delta_1$, $k\,\Delta_2$, and $k\,\Delta_h$. By using these quantities in Equations (43), (44), and (45), P may be found directly. This course is only feasible when k is the same for each pile.

There will usually be at least two values of α involved in any batter-pile foundation. The most usual case involves plus and minus values of the same angle. In such a case Equation (50) would be written:

$$\frac{V\,b}{k} = \Delta_1 \cos^2 \alpha \, \Sigma \, x_2 + \Delta_1 \cos^2 \alpha' \, \Sigma \, x_2' + \Delta_2 \cos^2 \alpha \, \Sigma \, x_1 + \Delta_2 \cos^2 \alpha' \, \Sigma \, x_1' \cdots \text{etc.} \quad (52)$$

and, similarly, for Equations (51). The unprimed values of x_1 and x_2 apply only to piles battered at the angle, α, whereas the primed values apply only to the piles battered at the angle, α'.

Apparently, Krey's method yields results that are somewhat different from those of the author. Applied to Assumption (1) of the author's numerical example, the writer obtains the following comparative values:

Pile No.	Author, Table 2	Writer
1	50.9	41.7
2	2.5	11.7
3	69.5	60.3
4	21.1	30.4

Like other writers on the subject, the author assumes deflections to depend purely upon the elastic properties of the piles. This assumption is tantamount to the assumption that all piles reach a rigid surface at their tips which, of course, is rarely the case. The true pile deflections are a compound of earth settlement, yield of the pile in the soil, and elastic deformation of the soil. Since the deflections upon which the stresses are based are small, and since the relative deflections at the edge and center of a foundation in a compressible soil may be of the same magnitude as, or of a greater magnitude than, the deflections found from the author's or the writer's formulas, the stresses obtained from any formula based upon purely elastic assumptions should be weighed in the light of the probable settlements.

However, analysis such as that given in the paper is a very necessary first approach to the problem (the best now available), and is far superior, say, to the method in which stresses in the batter-piles are first computed as if the piles were vertical, after which the horizontal thrust is based upon the stress thus computed and the sine of the batter angle. Such methods may lead to dangerous conditions, through a lack of realization of the true action of the inclined piles. The author (see heading, "Two-Dimensional Systems in General"), has pointed out one such possibility. The writer encountered another case in a design having piles battered only in one direction. Under the construction dead load there was danger of marked deformations, if not failure, due to the unbalanced thrust of the batter piles arising from the vertical load, a thrust which could not be balanced until the structure had been completed and backfilled.

The dummy-pile method of accounting for restraint is an ingenious one. The writer's method has been first to make a preliminary computation of deflections, as under the author's Assumption (1), and then on the basis of the deflections thus obtained to approximate the moments and shears in the piles upon the basis of equations such as those of Cummings,[15] or Chang,[16] or Timoshenko.[17] The shear at the base of the structure thus obtained is an additional horizontal force restraining lateral movement, and the moment obtained from these formulas may be added to the moments of the external loads. The pile moments serve to redistribute the external load, often in an important manner. That this occurs in foundations containing only vertical piles, but subject to lateral loads, is not generally realized.

The author has treated only foundations loaded in a single plane. The evaluation of stresses occasioned by eccentricities about both axes of the foundations is possible for many cases. Consider a pier, as shown in Fig. 12(*a*).

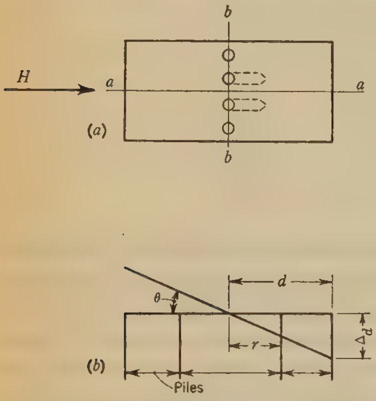

FIG. 12

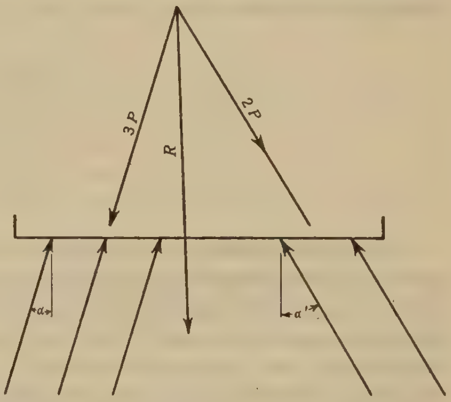

FIG. 13

The pile arrangement is assumed symmetrical about Axis *a-a*. The foundation contains vertical piles, and piles battered parallel to Axis *a-a*. The deflections due to H and V are first evaluated, neglecting eccentricity with respect to Axis *a-a*. Since the piles are symmetrical about Axis *a-a*, the moment in the plane normal to it will cause deflections symmetrical about this axis; thus:

$$M' = \Sigma\, P \cos\theta\, r = \Sigma\, k\, \Delta_L \cos\theta\, r = \Sigma\, k\, \Delta \cos^2\theta\, r = \Sigma\, k\, \Delta_d \frac{r}{d} \cos^2\theta\, r;$$ hence,

$$\Delta_d = \frac{M'}{\Sigma\, k \frac{r^2}{d}} \cos^2\theta \quad \text{.......} (53a)$$

and,

$$\Delta = \frac{r}{d} \Delta_d \quad \text{.......} (53b)$$

If these deflections cause no further deflection in the direction of H, they may be added directly to those previously determined, and from the result, the stresses may be evaluated. In the case shown, the lateral moment causes compression in the batter-piles on one side of Axis *a-a*, and tension, or rather reduction of compressive stress, in batter-piles opposite, of the same amount.

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[15] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 255.

[16] *Loc. cit.*, p. 272.

[17] "Strength of Materials," Vol. II, p. 408.

The result is that the horizontal components of the batter-piles are no longer equal on both sides of Axis *a-a*, although H does not change. A couple is formed, therefore, tending to rotate the foundation in the horizontal plane. If this couple is resisted by external restraint as, for example, by adjacent structures, no movement parallel to Axis *a-a* can occur, and the deflections may be evaluated by superposition. It is of interest that the formation of such a couple may be prevented by battering the piles in pairs, one with the load and one against the load, in each row. The horizontal components in each row due to the lateral eccentricity are thereby balanced.

In general, it will be desirable that a foundation utilizing batter-piles contain piles battered in opposition to each other. In the contrary case, the vertical load during construction or at other times when there may be no balancing horizontal load may cause lateral movement of the structure due to the horizontal component of the batter-pile stress. Where piles are battered with, and against, the lateral load, those inclined in the direction of the load may, in the absence of sufficient vertical load on the structure, be subjected to tension—that is, to extractive forces. Although in many instances piles are capable of resisting considerable extractive forces, it is usually not desirable that such forces be permitted to occur, and they will not occur if the vertical load on the foundation is sufficiently large. The effect, then, is to cause a reduction of the compressive load, rather than a tension, and the pile inclined with the load is as effective in lateral resistance as a pile inclined against it.

All the piles in a foundation containing only vertical piles may be loaded equally when subjected to vertical load alone; and rotation in the vertical plane may be prevented, by designing the group so that the centroid of the piles is on the line of action of the resultant load. Such an arrangement will be most economical in its use of piles although not necessarily most economical of superstructure, as all piles will be equally loaded.

Batter-pile foundations may be designed in a similar manner, as the author has indicated. The resultant, R, of the applied loads, excluding the part of the horizontal load taken by the earth in lateral bearing against the piles, Fig. 13, is resolved into two components inclined to the vertical at the adopted angles of batter, α and α'. Each component is divided by the permissible pile load, P, to obtain the number of piles inclined at the respective angles, α and α'. The line of action of the two components of the resultant is made to act through the centroids of the two-pile groups. All piles will thus be loaded equally and the axial deflection of each pile will be the same. The deflection of the foundation will be vertical and parallel to its unloaded position. There will be a resultant horizontal movement, however, if α and α' are unequal.

It will be noted that the batter angle, α, affects both the number of piles required and their location. These factors, in turn, affect the dimensions of the superstructure. Furthermore, there is usually no reason, other than an economic one, why all piles should be loaded equally, as such cases can be solved by the general formulas, Equations (50), and (51). The principle illustrated by Fig. 13, however, will be found of assistance in determining the most economical over-all design.

Odd Albert,[18] Assoc. M. Am. Soc. C. E. (by letter).—Although interesting and constructive, there is nothing fundamentally new in this paper (as Mr. Vetter states under the heading, "General Considerations"), except that it may be the first time the subject has been treated in the English language. As indicated in his "Acknowledgments," various pile theories were first developed in Sweden. Professor Per Gullander,[19] of the Chalmers Institute of Technology, Gottenborg, published an article on the subject in 1902. In 1911, R. Ekvall,[20] of the Royal Institute of Technology, Stockholm, added valuable material; and Professor Gullander[21] appeared again in 1912 with his first book on the subject. The previous year, Torsten Hultin, of the Chalmers Institute of Technology, read his paper on "How to Design Pile Foundations,"[2] presenting an entirely new graphical method, which has been adopted for general use.

The writer was privileged to study under Professor Gullander, and he remembers well how this famous and beloved teacher modestly stated concerning the various pile theories: "If it requires one week to design a pile foundation by the Ekvall theory, my method will require one day, and the Hultin method will only require one hour." However, he did not mention that this theory could be applied to any combination of pile groups, whereas the Hultin method required at least one axis of symmetry.

The method presented by Mr. Vetter seems to be a combination of the Ekvall and the Gullander methods, with quite a number of important improvements, which will be apparent by comparison with the following review of the original Hultin Pile Theory. The writer has taken the liberty of including a few changes, which he thinks will help to clarify this method.

The Hultin Pile Theory.—The stresses in a pile group supporting a body are caused by a very small movement of this body. This movement, no matter how complicated it may be, can always be divided into two separate movements of the entire body: (1) As a unit in two given directions; and (2) a rotation of the entire body around a selected center.

Therefore, the loads on the piles will be caused by: (*a*) The movement of the body in one direction; (*b*) the movement of the body in another direction; and (*c*) the rotation of the body around some center.

Fig. 14(*a*) shows four piles or groups of piles sloped in three directions. Assume, first, that the body moves a distance, l_h, to the left. Then each pile head will move a distance that equals the length of this movement projected in the direction of the pile. These projected distances, u, are determined graphically in Fig. 14(*b*). It will be noticed that $u_1 = 0$, because Pile (1) is vertical. Then the stresses in the piles, caused by a horizontal movement, l_h, will become:

In Pile (1) (because $u_1 = 0$)

$$P_1^h = 0 \quad \text{(54a)}$$

in Pile (2),

$$P_2^h = + k_2 u_2 \quad \text{(54b)}$$

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[18] Asst. Prof., Structural Eng., Newark Coll. of Eng., Newark, N. J.

[19] "Contributions to the Pile Theory," by P. Gullander, *Teknisk Tidskrift*, 1902.

[20] "The Design of Pile Foundations," by R. Ekvall, 1911.

[21] "The Theory of Pile Foundations," by P. Gullander, 1912.

in Pile (3) (a pull instead of a load),

$$P_3^h = -k_3 u_3 \quad \ldots\ldots (54c)$$

and, in Pile (4),

$$P_4^h = +k_4 u_4 \quad \ldots\ldots (54d)$$

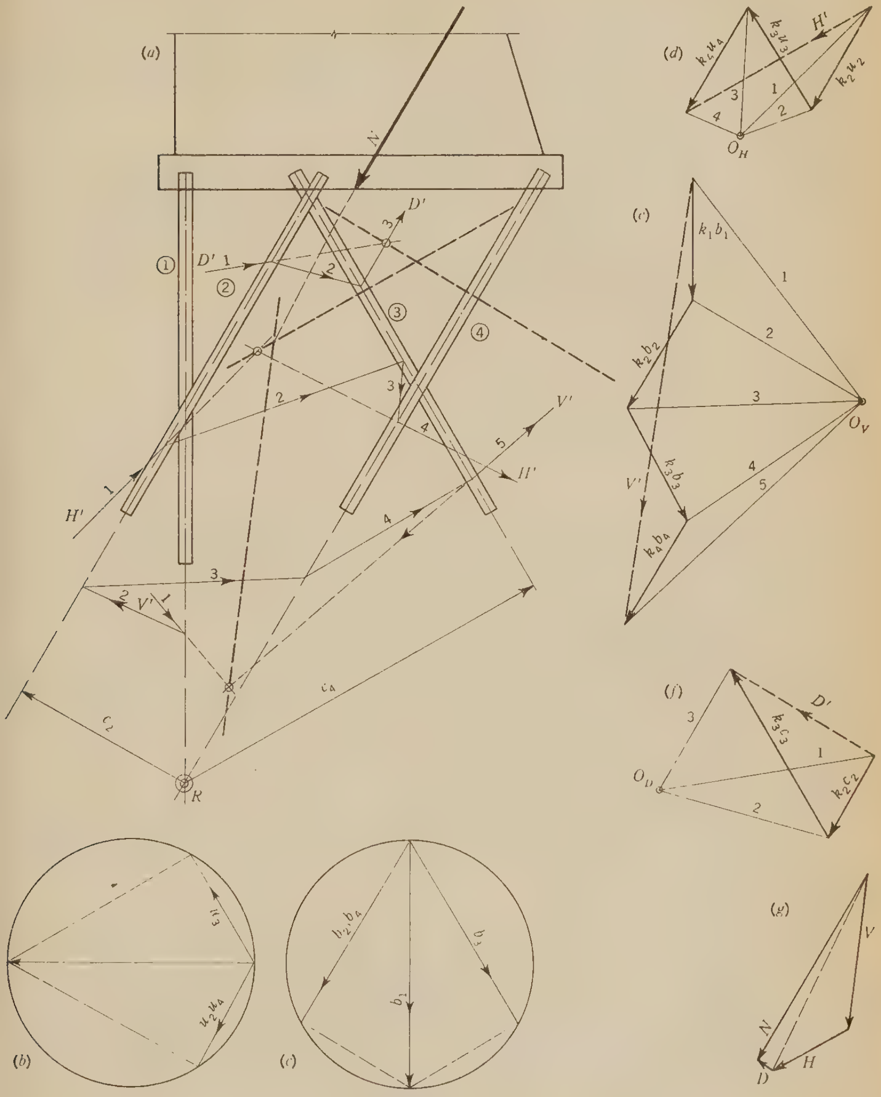
(a)
N
D'
①
②
③
④
D' 1
2
3
4
5
H'
V'
R
c2
c4
(b)
k3
u2 u4
(c)
b2, b4
b3
b1
(d)
k4u4
k3u3
H'
k2u2
OH
(e)
k1b1
k2b2
k3b3
k4b4
V'
OV
(f)
D'
k3c3
k2c2
OD
(g)
V
N
H
D

FIG. 14

in which k is a constant, depending upon the size and length of each pile. Therefore, it is only necessary to know the relation, $k_1 : k_2$, etc. Thus, if Pile (1) is twice as strong as Pile (2), $k_1 = 2\,k_2$. If all the piles are of the same strength, $k_1 = k_2 = \cdots k$.

The force polygon shown by Fig. 14(*d*) locates the string polygon in Fig. 14(*a*) (marked H'–1–2–3–4–H'). For a movement, l_h, the resultant to all the loads, H', on the piles is found to have the direction and magnitude shown in Fig. 14(*d*), and the location shown in Fig. 14(*a*) at the intersection of the first and the last strings (+ 1 and − 4) of the equilibrium polygon.

Applying the law of reciprocal displacements, as a movement of l_h causes a reaction, H', it is evident that a unit load applied along the H-direction in Fig. 14(*a*) will cause a horizontal movement of the body, thus creating stresses in the piles of $+\frac{k_2\,u_2}{H'}$, $-\frac{k_3\,u_3}{H'}$, and $+\frac{k_4\,u_4}{H'}$.

Assuming next that the body moves a distance, l_v, vertically down, each pile head will move a distance that equals the projected distance of this movement on a plane in the direction of the pile. This projected distance may be called b, and it is determined graphically in Fig. 14(*c*). (It will be noticed that $b_1 = l_v$.) Then the stresses in the piles will be:

Pile (1),

$$P_1^v = +\,k_1\,b_1 \qquad (55a)$$

Pile (2),

$$P_2^v = +\,k_2\,b_2 \qquad (55b)$$

Pile (3),

$$P_3^v = +\,k_3\,b_3 \qquad (55c)$$

and Pile (4),

$$P_4^v = +\,k_4\,b_4 \qquad (55d)$$

in which k is the same constant as in Equations (54), depending only upon the pile to which it refers. Thus, k_1 refers to Pile (1), etc. In all cases, only movements in the direction of the slope of the pile are considered, and, therefore, k may be called the unit pile resistance.

Draw the force polygon (Fig. 14(*e*)) that determines the direction and the magnitude of the resultant force, V', from all the pile loads, caused by a vertical movement, l_v, of the entire body. In the equilibrium polygon, marked V'–1–2–3–4–5–V' (Fig. 14(*a*)), the first and the last strings (+ 1 and − 5) determine the location of V'.

Therefore, a unit load applied along the V-direction in Fig. 14(*a*) will cause a vertical movement of the entire body, creating stresses in the piles of $+\frac{k_1\,b_1}{V'}$; $+\frac{k_2\,b_2}{V'}$; $+\frac{k_3\,b_3}{V'}$; and $+\frac{k_4\,b_4}{V'}$.

Finally, assume that the body rotates to the left around R. As this rotation is considered very small, the movements of the pile-heads may be considered straight, and, therefore, will be in direct proportion to the distance of each pile-head from the rotation center, R. This distance may be called c. As the position of this center is selected at the intersection of Piles (1) and (4), there will be no stresses in Piles (1) and (4) from this rotation. The pile loads will be:

Pile (1),

$$P_1^d = 0 \qquad (56a)$$

Pile (2),

$$P_2^d = +\,k_2\,c_2 \qquad (56b)$$

Pile (3) (rotation causes tension),

$$P_3^d = -k_3 c_3 \quad \text{(56c)}$$

and Pile (4),

$$P_4^d = 0 \quad \text{(56d)}$$

in which k again is the same constant as that used in the other movements.

The force polygon (Fig. 14(f)) gives the direction and magnitude of the resultant force, D', from all the pile loads caused from a rotation around the point, R. The equilibrium polygon, marked D'–1–2–3–D', gives the location of D'. Now, a unit load applied along the D-direction in Fig. 14(a) will create a rotation around R only, creating stresses in the piles, such as $+\frac{k_2 c_2}{D'}$, and $-\frac{k_3 c_3}{D'}$.

Characteristic Lines.—It will easily be seen that the three location-directions, H', V', and D' (called characteristic lines), are independent of the size of the movements, as well as the actual loading. They only depend upon the chosen directions of the two movements as well as the location of the rotation center, and the pile group itself. Therefore, this part can be computed once and for all with complete disregard to the loading. As soon as the characteristic lines are obtained, they can be used for all kinds of loading; and this is the important part in the Hultin pile theory.

Application to a Loading.—A force can always be resolved along three directions; therefore, dissolve the loading along the characteristic lines. The body will then move in two known directions, and rotate around a known center. As soon as H, V, and D are known, the stresses in various piles are easily determined. Hence, the loading resultant, N, is resolved into three components, H, V, and D, as shown in Fig. 14(g). The stress in Pile (2), therefore, will become,

$$f_2^H = +H\frac{k_2 u_2}{H'} \quad \text{(57a)}$$

In this expression, H is an actual load determined from the load, N, carried by the pile group, and given in Fig. 14(g). The ratio, $\frac{k_2 u_2}{H'}$, can easily be obtained by scaling these distances from Fig. 14(d)—first $k_2 u_2$ and then H'. Similarly, the load caused by the load, V, on Pile (2) becomes,

$$f_2^V = +V\frac{k_2 b_2}{V'} \quad \text{(57b)}$$

and that caused by Load (d) is,

$$f_2^D = +D\frac{k_2 c_2}{D'} \quad \text{(57c)}$$

Then the total load on Pile (2) will be the sum of Equations (57), with their signs. Hence,

$$f_2 = +H\frac{k_2 u_2}{H'} + V\frac{k_2 b_2}{V'} + D\frac{k_2 c_2}{D'} \quad \text{(58a)}$$

and, similarly,

$$f_1 = 0 + V \frac{k_1 b_1}{V'} + 0 \ldots\ldots\ldots\ldots\ldots\ldots \text{(58}b\text{)}$$

$$f_3 = - H \frac{k_3 u_3}{H'} + V \frac{k_3 b_3}{V'} - D \frac{k_3 c_3}{D'} \ldots\ldots\ldots\ldots \text{(58}c\text{)}$$

and,

$$f_4 = + H \frac{k_4 u_4}{H'} + V \frac{k_4 b_4}{V'} 0 \ldots\ldots\ldots\ldots\ldots\ldots \text{(58}d\text{)}$$

It will be noted that all complicated computations are eliminated from this method, and that all the stresses can be obtained by simple calculations on the slide-rule.

Mr. Vetter has selected an unusually simple case to demonstrate his method. Its full possibilities, as compared with those of the original (Hultin), cannot be properly appraised until the author has demonstrated its application to the more complicated cases.

AUGUST E. NIEDERHOFF,[22] ASSOC. M. AM. SOC. C. E. (by letter).—The author's development of his subject is based upon assumptions that may be at variance with facts. His analysis also appears needlessly complicated and leads to values that are different from those obtained by the writer using only the resolution of forces into components that act upon the piles axial and normal to the axis.

It is not feasible to neglect soil mechanics in investigating the probable stresses in a pile foundation. The soil in which the piles are driven, the bearing at the tip (if it exists), and the bulb of pressure about the pile, are as much a part of the foundation as the pile itself and should be considered in any analysis. The author attempts to bulwark his analysis by considering the elastic deformation of piles and in so doing he loses sight of the fact that the stress, and, consequently, the elastic deformation, of the pile itself is only pertinent in exceptional circumstances. These circumstances obtain only when the total stress in the pile is less than the amount the foundation soil can stand. In soft yielding soils in which piles are used it is the passive resistance of the soil that is critical rather than the stress in the pile. This is particularly true if the soil is prevented from bulging upward under lateral movement of the pile by an overlying concrete slab.

In developing a simple case the author assumes that the pile acts as a column and is connected to the pier by means of a frictionless hinge. It is stated (see heading, "General Considerations") that: "In many practical cases, such as bridge piers and gate piers in hydraulic structures, the error introduced by this assumption is insignificant." In the writer's experience on bridge and gate pier foundations, wooden piles have been extended into the concrete about 2 ft and were considered fixed. The value in fixing this end of the pile was demonstrated during tests conducted in the fine sand, silt, and clay deposits on the right bank of the Mississippi River about five miles up stream from Redwing, Minn. (These tests were authorized by the District Engineer, U. S. Army Engineers, at St. Paul, Minn., in 1934.) Round wooden piles, about 30 ft long and about 12 in. in diameter, were driven in a vertical position and tested by applying horizontal loads to the top of the pile at the ground line. One pile top was unrestrained and the deflection at the ground line for a lateral load of 19 000 lb was 1 in. A similar pile, tested in the same manner, but with the top restrained, showed a deflection of only 0.36 in. for a lateral load of 18 000 lb. These tests indicated that if the top of the pile was assumed free whereas it was actually fixed, the error introduced would be about 250% for the materials used in the experiment.

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[22] Civ. Engr., U. S. Army Engrs., U. S. Engr. Office, Portland, Ore.

The author also assumes that the frictional resistance per linear foot of pile increases rectilinearly from zero at the top of the pile to a maximum value at the tip. If the resistance to driving piles can be taken as an indication of the frictional resistance per linear foot of pile, personal observation would lead the writer to assume a parabolic variation rather than a rectilinear variation. However, in the case of vertical piles, driven in soil and supporting a concrete footing at the ground surface, the effect of lateral loads is believed to produce entirely different distribution of resistive forces. With the soil confined by the overlying slab the maximum resistance to horizontal movement is the passive pressure of the soil and is directly proportional to the pile deflection. The maximum pile deflection in this case occurs at the ground surface near the top of the pile and becomes smaller toward the tip of the pile. The distribution of resistance to horizontal loads, therefore, can be assumed as a maximum value at the top of the pile and a minimum value somewhere near the tip.

The depth in the foundation soil at which a pile can be considered fixed or hinged depends upon the material and dimensions of the pile and the character of the soil in which it is driven. Discussions[23] of a paper by Lawrence B. Feagin, M. Am. Soc. C. E., fix this point for these tests mathematically, and the paper substantiates the tests with recorded measurements. It is not necessary to assume fixation or hinge action at a point two-thirds of the pile length, as the author has done, when a series of tests will fix this point definitely.

The use of dummy piles to aid in the analysis of a complicated pile foundation with restrained pile ends is recognized as a convenient tool. However, for the more simple systems it is believed that dummy piles add to the difficulty in determining actual pile loads. The numerical example given shows that the computations and procedure are involved. Using Fig. 9, the writer computed the total vertical force acting on the pile bent at 142 kips. The net moment of all external forces above Plane Y and about the top of Pile No. 4 has been computed as 844.165 ft-kips. The eccentricity, e, of the point of application of the resultant force from the center of gravity of the system of pile tops is $\frac{844.165}{142} - 5.5 = 0.44$ ft. The vertical load taken by any one of the four piles can be obtained from the formula:

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[23] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 255.

$$P = \frac{V'}{n} \pm \frac{V' e c}{I_g} \quad \text{(59)}$$

in which c = the distance from the center of gravity of the system to the center of the pile; and I_g = the moment of inertia of the pile system about its center of gravity.

Since the piles in the example are of the same cross-section, each value of A can be considered as unity and the moment of inertia, I_g, of the pile system about Plane X equals $2(5.5)^2 + 2(1.5)^2 = 64.8$. Substituting these values in Equation (59), the vertical loads are found to be as listed in Column (2), Table 3.

TABLE 3.—PILE LOADS, IN THOUSANDS OF POUNDS OR KIPS

Pile No. (1)	Vertical load (see Equation (59)) (2)	Axial load (see Equation (60*a*)) (3)	Normal load (see Equation (60*b*)) (4)
1....	40.80	39.8	8.4
2....	36.51	35.7	7.5
3....	34.49	34.2	4.2
4....	30.20	30.0	3.7

For comparative purposes, the loads on the piles are axial (P_a) and normal (P_n) to the axis, expressed, respectively, as:

$$P_a = P \cos(\alpha \pm \theta) \quad \text{(60a)}$$

and,

$$P_n = P \sin(\alpha \pm \theta) \quad \text{(60b)}$$

The angle the resultant force makes with the vertical is $\frac{6.1}{142} = \arctan\theta$. The value of θ in the example is 2° 28′ and the value of $\cos\theta$ is so close to 1 that no correction of the vertical pile load need be made for the effect of the small horizontal force. The angle of batter of the piles is given as α and is equal to 9° 28′. Values of P_a and P_n, as computed from Equations (60), are given in Columns (3) and (4), Table 3. These values differ materially from those obtained by the author, and it is believed that the introduction of a dummy pile into the analysis obscured the author's computations.

Mr. Vetter is to be commended for a refreshing review of pertinent factors entering into the design of a pile foundation as recorded in the "Synopsis" of his paper.

A. A. EREMIN,[24] ASSOC. M. AM. SOC. C. E. (by letter).—Formulas for computing stresses in the piles of a pile foundation are developed in this paper. Mr. Vetter has simplified the computations by combining them with graphical construction, and the resulting process is simple if applied to piles in symmetrical pile footings. However, with the present application of mechanical calculating machines, a more direct method of computing stresses in more complicated cases is often desirable.

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[24] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Stresses may be computed by three equations derived from statics. Assuming the origin of the co-ordinates at a distance, y_1, above the bottom of a pile footing, the equations may be written (see Fig. 15),

$$H = \Sigma P \sin \alpha \ldots\ldots\ldots (61a)$$

$$V = \Sigma P \cos \alpha \ldots\ldots\ldots (61b)$$

and,

$$M = \Sigma P x \cos \alpha + y_1 \Sigma P \sin \alpha \,.\,. (61c)$$

in which,

$$P = \frac{E\,A}{L} r\, \Delta \phi \ldots\ldots\ldots (62a)$$

and,

$$r = (x_0 - x) \cos \alpha - (y_0 - y_1) \sin \alpha \,.\,. (62b)$$

and x_0 and y_0 are co-ordinates of the center of rotation. Solving Equations (61) simultaneously, unknown co-ordinates of the center of rotation and stresses in piles may be determined. Forces at the restrained ends of piles may be treated as explained in the paper.

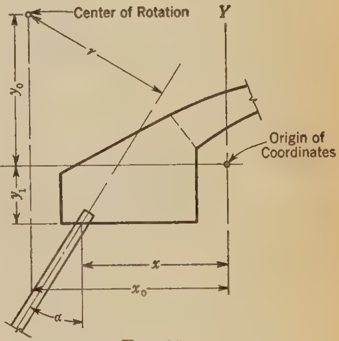

FIG. 15

Designers are often required to compute the elastic deformation in structures. Therefore, Mr. Vetter's paper is timely and creditable.

JACOB FELD,[25] M. AM. SOC. C. E. (by letter).—The purpose of piles is to transfer the base reactions of a footing to a soil that can carry such loads with settlements not exceeding the permissible values. Any resultant load can be resolved into a moment, and vertical and horizontal forces. The requirements of statics are the prime condition for any determination of what loads each pile (or group of piles) transfers to the carrying soil layers.

If all loads are vertical and the pile group is symmetrically spaced about the center of gravity of the loading, and if the footing may be considered a rigid body, the distribution of loads is uniform among all the piles. If the center of the pile group does not correspond with the center of the loading, the true distribution must take into account the eccentricity of loading, and also the compressibility of the piles and the settlement of each pile tip under the unequal loadings. The problem now falls into the class of indeterminate structures.

If all loads are vertical and if some or all of the piles are not vertical (battered) either the vertical load is transferred by skin friction or the pile is affected by a vertical couple. If the soil is sufficiently resistant to provide the skin friction in the immediate depths below the footing, no piles would be needed. In the other case, the lower end of the piles are embedded for a comparatively short distance in a soil much more resistant than that directly below the footing. A vertical couple acting on a pile can only be balanced by a horizontal couple resulting from a reverse type of deflection curve in the distance of resistant embedment.

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[25] Cons. Engr., New York, N. Y.

Unless special details are insisted upon, the usual short pile embedment into a footing is not a rigid, moment-transferring (restrained) joint, nor is it a hinged joint. Such a connection is sufficient to transfer the horizontal loads from footing to piles. A horizontal load acting on a vertical pile can only be balanced by two unequal horizontal reactions with opposite directions. In no other way can the requirements of statics be satisfied, namely, equality of forces and of moments.

A horizontal load acting on a batter pile has the same effect as on a vertical pile with the exception of the unusual case in which the resultant load is axial with the sloping pile center. As has been well proved by tests on telegraph poles, as well as full-sized piles and sheet-piling, even short embedment of piles provides large horizontal strength. The greater cost of batter piles, together with the uncertainty of the fixed direction of the resultant loads, makes it advisable to consider such horizontal strength of piles before deciding on the use of batter piles.

Driving piles in a weak soil, without being certain that there is sufficient embedment in a resistant soil layer to take care of horizontal loads, is a common error. The result is a "creep" of viaducts, tipping of abutments, twist, and tip of towers.

The author carefully enumerates a number of the fundamental ideas in pile foundations, but his paper is really a mathematical study of the design of piles as elastic and static members. The final solution of a pile-foundation problem must take into account the fact that resistance is a function of the deformation resulting from load. The assumption (see heading, "General Considerations") that the effective length of a pile is independent of the load imposed, does not agree with the known action of soils. If sufficient resistance can be developed at the upper length of a pile, the lower part will be dormant. Distinction must be made between the actual soil resistance or friction and the latent maximum values. The distribution of one is not the same as that of the other. In this respect, soils are not elastic bodies.

The author's paper is excellent if restricted to the design of trestle or tower legs, or even for that part of piles above a resistant soil layer; for sub-surface design problems, the solution is only an approximation.

JOHN M. COAN, JR.,[26] JUN. AM. SOC. C. E. (by letter).—In focusing the attention of many engineers on the idea of the "dummy pile," the author is to be commended. It is recognized as a convenient tool in that it enables the designer to predict the pile moments under certain conditions of restraint, whereas, many methods used to-day neglect or, at most, approximate these moments.

In so far as practical applications are concerned, the theory is incomplete in that certain factors pertinent to intelligent design have not been included. For example, the inability to determine, with any degree of accuracy, the location of the point of pile restraint in the foundation and the failure to take into consideration the characteristics and confinement of the surrounding soil make it advisable to design with a liberal margin of safety. Such an analysis as Mr. Vetter presents might be applied to towers, pedestals, or similarly supported bodies with laterally unsupported elements or, more specifically, to pile piers passing through alluvial or practically non-resistant materials to resistant strata.

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[26] Stress Analyst, Glenn L. Martin Co., Middle River, Md.

The author's treatment of the dummy-pile method is restricted to what amounts to two-dimensional arrangements in which the piles are parallel with, and symmetrical with respect to, a vertical plane of symmetry, and in which the resultant of the dead and applied loads lies in the plane of symmetry. If the plane of loading remains coincident with the plane of symmetry, such an analysis might be extended to include the case of a three-dimensional system in which the piles are symmetrical with respect to a vertical plane of symmetry, but not parallel to it. In this case the individual piles may be projected into the plane of symmetry, analyzed, and the results thus obtained corrected by the ratios of the true to the projected lengths. Furthermore, in the event of an arrangement having no vertical plane of symmetry, if the piles are of such section that each has equal moments of inertia about two mutually perpendicular axes, and if the resultant of the applied loads lies in a vertical plane which contains the centroid of the pile group, the piles will deflect in planes whose horizontal traces are parallel to the plane of loading. Such a condition may be analyzed as if a vertical plane of symmetry existed and coincided with the plane of loading.

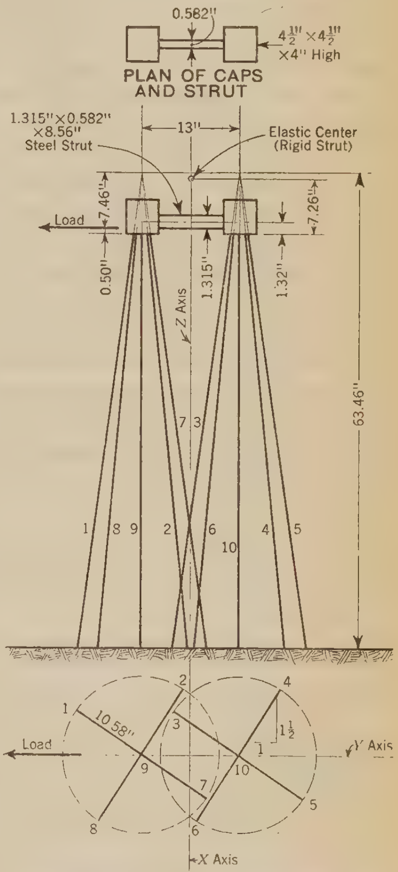

FIG. 16.—PLAN AND ELEVATION OF MODEL

The writer has made an extensive model analysis[27] of such a case and, although the pile arrangement studied was unique, it is considered worth mentioning because the axial loads and end moments obtained from a dummy-pile analysis agree reasonably well with the experimental results.

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[27] "Model and Theoretical Analyses of a Tubular Pile Bridge Pier," by John M. Coan, Jr.; an unpublished essay submitted as partial requirement for the degree of Master of Civil Engineering, at The Johns Hopkins Univ., Baltimore, Md., May, 1938.

The model (Fig. 16) was one-fortieth scale and consisted of two groups of five symmetrically arranged tubular steel piles, each having an outside diameter of 0.757 in. and a wall thickness of 0.030 in. The piles were fixed in concrete at their bases, and in mortar caps at their upper extremities. These caps were connected by a non-rigid rectangular strut which permitted the transfer of loads and caused the action of the two groups to be interdependent. The points at which the piles were fixed corresponded to the arbitrarily located points of restraint mentioned by Mr. Vetter under the heading, "General Considerations." No attempt was made to simulate the resistance to lateral displacement that the alluvial materials, through which the prototype piles are to be driven, would offer. In view of the elasticity of the strut, it was necessary to correct the results obtained from a dummy-pile analysis because they had been determined under the assumption that the supported structure was infinitely rigid. This correction was made by cutting the strut, finding the unbalanced moment on each pier cap, and then distributing this moment to the strut and to the piles in that group in proportion to their relative stiffnesses. (The batter of the piles permitted only a negligible sidesway for which it was unnecessary to correct the distributed moments.) The result of this elastic strut correction was an inappreciable change in the axial loads and an average increase in the maximum pile moments of about 20 per cent.

Three loading conditions—dead + live + impact, ice, and wind—were analyzed and, in the case herein illustrated, a 120-lb ice load, corresponding to a 200 000-lb load on the prototype pier, was applied as shown. Table 4 gives a comparison of the axial loads and maximum bending moments as determined experimentally and as computed by the dummy-pile method and corrected for the elastic strut.

TABLE 4.—EFFECTS OF A 120-POUND ICE LOAD ACTION ALONG THE *Y*-AXIS

Description	PILE NOS.									
	1	2	3	4	5	6	7	8	9	10
Axial Load, in Pounds:										
Measured........	−141.1	108.2	−168.2	93.7	135.0	−110.9	163.3	−89.2	8.4	− 7.1
Computed.......	−135.2	111.9	−159.4	87.3	135.2	−111.9	159.4	−87.3	12.3	−12.3
Maximum Moment,* in Inch-Pounds:										
Measured........	12.7	11.3	11.1	11.5	10.7	10.4	8.4	13.3	9.5	9.2
Computed.......	10.4	10.5	9.2	9.4	9.2	9.3	10.4	10.5	10.5	9.4

* The pile moments are either positive or negative.

The average difference in axial loads, except the lightly loaded verticals, is just over 3% and the average variation in the maximum moments is within 16 per cent. In this case, due to portal action, the bending stresses are considerably less than one-third the total stresses and the differences between the measured and the computed bending moments are reduced in importance. Under the condition in which equal loads were applied at the centroid of each group and parallel to the *X*-axis, the displacements of the pile groups were independent of the strut, the end moments were of a much greater magnitude, and a closer agreement between the analytical and experimental moment determinations was found, the average difference being about 1 per cent.

A. AGATZ,[28] ESQ. (by letter).—The formulas and principles of design given by the author are, in fact, applications, to special and simple cases, of the methods developed by Dr. Nökkentved.[5] It is fortunate that these applications have permitted the emphasis of certain fundamental rules and that Mr. Vetter has demonstrated the importance of the rotating moment, M.

The main disadvantage in the application of Dr. Nökkentved's general methods, or any special method derived therefrom, is that only stresses in pile foundations that have already been designed, may be determined. Except in the case of pile foundations having piles in two directions only, or having three piles only, it is not possible, by the method presented by the author, to design a foundation for which it is assured beforehand that all piles will receive approximately equal loads for a given exterior loading. Making use of the same general assumptions as those of Dr. Nökkentved and the author, it is possible, however, by using a somewhat different approach to design a foundation consisting of piles in three directions, in which the loading on each individual pile is predetermined for one, and even for two, different exterior loadings. If more than these two loadings must be resisted, it will be necessary to choose Point O at or near the points of intersection of the resultants of the various loadings so as to reduce the magnitude of the rotating moment.

Since a moment always produces a rotation about the elastic center, with resultant non-uniform pile loads, it is desirable to design the pile foundation in such a way that the most unfavorable exterior load produces translation without rotation. The elastic center of a pile system which satisfies this condition falls on the particular resultant, R. Let the resultant that produces a translation (without rotation) in a direction forming an unknown angle, η, with the vertical, be designated R_η; and, let the load on an arbitrary pile as a result of the translation be P_n and the axial deformation of an arbitrary pile, be ΔL_n, then:

$$P_n = \frac{E_n\, A_n\, \Delta L_n}{L_n} \quad\text{.......................}(63)$$

in which L_n and A_n are the length and cross-section area, respectively, of the pile, and E_n is the elastic modulus of the pile material. If all pile heads are at the same level and all pile ends likewise at the same level, at a depth, d, below the heads (see Fig. 17):

$$P_n = \frac{E_n\, A_n}{d}\, \Delta L_n \cos \alpha_n \quad\text{...................}(64)$$

in which d is the vertical projection of the pile length, and α_n is the angle of the pile axis with the vertical. Let it further be assumed that all piles are of the same cross-section area and the same material. Since, in the investigation that follows, it is the ratios between the pile loads in the various groups that are of primary interest, the factor, $\frac{E_n A_n}{d}$, then may be given the value unity, without affecting those ratios, so that:

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[28] Prof., Technische Hochschule, Berlin, Germany; tr. by C. P. Vetter, M. Am. Soc. C. E.

$$P_n = \Delta L_n \cos \alpha_n \ldots\ldots\ldots\ldots (65)$$

If the translation in the direction, η, is also unity:

$$\Delta L_n = \cos (\eta - \alpha_n) \ldots\ldots\ldots\ldots (66)$$

and, it follows that,

$$P_n = \cos (\eta - \alpha_n) \cos \alpha_n \ldots\ldots\ldots\ldots (67)$$

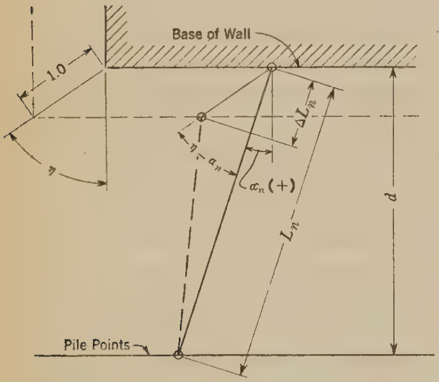

FIG. 17

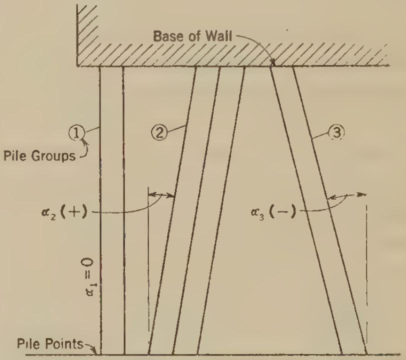

FIG. 18

If $\eta = 0$ (vertical translation), $P_n = \cos^2 \alpha_n$; and if $\eta = 90°$ (horizontal translation), $P_n = \sin \alpha_n \cos \alpha_n$. Let the loads in Pile Groups 1 and 3 (Fig. 18) be expressed as ratios, n and m, of the simultaneous pile loads in Group 2, all due to the same translation; thus, if $P_2 = \cos (\eta - \alpha_2) \cos \alpha_2$ (see Equation (67)): $\frac{P_1}{P_2} = n$; and, $\frac{P_3}{P_2} = m$.

Consequently,

$$P_1 = n\, P_2 = n \cos (\eta - \alpha_2) \cos \alpha_2 = \cos (\eta - \alpha_1) \cos \alpha_1 \ldots\ldots (68a)$$

and,

$$P_3 = m\, P_2 = m \cos (\eta - \alpha_2) \cos \alpha_2 = \cos (\eta - \alpha_3) \cos \alpha_3 \ldots\ldots (68b)$$

It will be assumed, furthermore, that the piles in Group 3 are in tension so that α_3 is negative, the numerical value in α_3 being designated $|\alpha_3|$. Equation (68b), then, is expressed as:

$$P_3 = \cos (\eta + |\alpha_3|) \cos |\alpha_3| \ldots\ldots\ldots\ldots (69)$$

Various relationships between α_1, α_2, α_3, η, m, and n may be derived from Equations (68a), (68b), and (69) as follows:

Case a.—Given, n, m, α_1, and α_2; let it be required to determine η and α_3. Then, from Equations (68*a*) and (68*b*),

$$\tan \eta = \frac{\cos^2 \alpha_1 - n \cos^2 \alpha_2}{n \cos \alpha_2 \sin \alpha_2 - \cos \alpha_1 \sin \alpha_1} \quad \text{(70a)}$$

For $\alpha_1 = 0$, Equation (70*a*) becomes:

$$\tan \eta = \frac{1 - n \cos^2 \alpha_2}{n \cos \alpha_2 - \sin \alpha_2} \quad \text{(70b)}$$

and, similarly,

$$\cos 2 \alpha_3 = \frac{q \pm \tan \eta \sqrt{1 - q^2 + \tan^2 \eta}}{1 + \tan^2 \eta} \quad \text{(70c)}$$

in which,

$$q = 2 m (\cos^2 \alpha_2 + \tan \eta \sin \alpha_2 \cos \alpha_2) - 1 \quad \text{(70d)}$$

In Equations (70) m and α_3 may be substituted for n and α_1, respectively, to determine the values of η and α_1.

Case b.—Given, $\alpha_1 = 0$, α_2, α_3, and m; let it be required to determine η and n. Similarly to Equation (70*a*):

$$\tan \eta = \frac{\cos^2 |\alpha_3| - m \cos^2 \alpha_2}{\sin |\alpha_3| \cos |\alpha_3| + m \sin \alpha_2 \cos \alpha_2} \quad \text{(71a)}$$

and,

$$n = \frac{1}{\cos \alpha_2 (\tan \eta \sin \alpha_2 + \cos \alpha_2)} \quad \text{(71b)}$$

Case c.—Given, α_1, α_2, α_3, and η; let it be required to determine m and n. Then, proceeding as in Case (*b*),

$$n = \frac{\cos (\eta - \alpha_1) \cos \alpha_1}{\cos (\eta - \alpha_2) \cos \alpha_2} \quad \text{(72)}$$

For $\alpha_1 = 0$, Equation (72) becomes:

$$n = \frac{\cos \eta}{\cos (\eta - \alpha_2) \cos \alpha_2} \quad \text{(73a)}$$

and,

$$m = \frac{\cos (\eta + |\alpha_3|) \cos |\alpha_3|}{\cos (\eta - \alpha_2) \cos \alpha_2} \quad \text{(73b)}$$

Equations (73) are of particular interest for the design of pile foundations, and numerical values of m and n for varying values of α_2, α_3, and η are given in Table 5.

It should be noted that tension in the piles of Group 3 occurs only if, $\eta > 90° - |\alpha_3|$. The allowable load on a compression pile is P_2. Then, when the svstem has no rotation:

$$P_1 = n P_2 \quad \text{(74a)}$$

and

$$P_3 = m P_2 \quad \text{(74b)}$$

If the three axes of the pile groups can be placed so that the group components of the exterior resultant are always whole multiples of P_1, P_2, or P_3, the elastic center is on the resultant. The arrangement of the piles around a group axis is immaterial. However, the number of piles in each group must equal the aforementioned whole multiple. By group axis, is understood a line parallel to the center lines of the piles in the group and going through the center of gravity of the cross-section areas of the piles. In the investigation that follows, consideration is given only to the important case in which one group of piles is vertical ($\alpha_1 = 0$).

TABLE 5.—VALUES OF m AND n FOR CORRESPONDING VALUES OF α_2, α_3, AND η ($\alpha_1 = 0°$ (SLOPE = $1 : \infty$))

α_3			α_2							
Slope	Angle	η	SLOPE = 1 : 3; AND, ANGLE = 18° 28′		SLOPE = 1 : 4; AND, ANGLE = 14° 2′		SLOPE = 1 : 5; AND, ANGLE = 11° 19′		SLOPE = 1 : 6; AND, ANGLE = 9° 28′	
			m	n	m	n	m	n	m	n
1 : 3	18° 28′	75°	−0.110	+0.495	....	....	....	....	....	....
		80°	−0.309	+0.384	−0.353	+0.440	−0.391	+0.488	−0.449	+0.528
		85°	−0.585	+0.231	−0.699	+0.276	−0.804	+0.316	−0.884	+0.354
		90°	−1.000	+0.000	....	....	....	....	....	....
1 : 4	14° 2′	80°	−0.151	+0.384	−0.173	+0.440	−0.191	+0.488	−0.208	+0.528
		85°	−0.403	+0.231	−0.482	+0.276	−0.552	+0.316	−0.616	+0.354
1 : 5	11° 19′	80°	−0.050	+0.384	−0.057	+0.440	−0.063	+0.488	−0.069	+0.528
		85°	−0.286	+0.231	−0.341	+0.276	−0.391	+0.316	−0.436	+0.354
1 : 6	9° 28′	80°	+0.020	+0.384	+0.023	+0.440	+0.026	+0.488	+0.028	+0.528
		85°	−0.204	+0.231	−0.243	+0.276	−0.279	+0.316	−0.312	+0.354

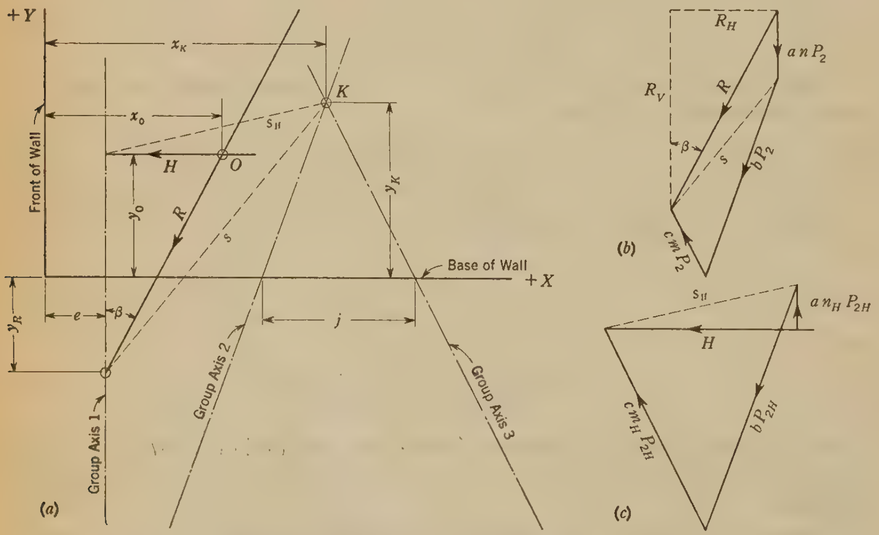

FIG. 19

From Table 5, in connection with general engineering considerations, possible values of m and n may be readily determined from assumed slopes, α_1, α_2, and α_3 of the three pile groups. The number of piles, a, b, and c, in each group may be found by means of a force polygon; thus, let a = number of vertical piles (slope, $\alpha_1 = 0$); b = number of battered compression piles (slope, α_2); and, c = number of battered tension piles (slope, $|\alpha_3|$).

In general, the magnitudes of the group reactions in the force polygon are not whole multiples of P_1, P_2, and P_3, but small remainders will occur in two of the forces. If such is the case, there will be a small rotating moment about the elastic center. As it is necessary to deal with whole numbers of piles, the nearest whole multiples, a, b, and c, may be selected and new values of m, n, and η calculated.

The known resultant, R (Fig. 19), and the pile reactions, $b\,P_2$, $a\,n\,P_2$ and $c\,m\,P_2$, must form a closed force polygon with known directions of the sides and, at the same time, must satisfy Equations (71). Values of m and n, therefore, may be obtained in terms of the pile batters, $\alpha_1 = 0$, α_2, and α_3. The number of piles, a, b, c, and the angle of R with the vertical, β, may also be determined from this condition.

Considering again the general case in which α_1 is not zero and m is negative, the force polygon will have the form indicated by $A\,B\,C\,D$ in Fig. 20. In drawing the force polygon, the scale has been chosen so that P_2 = unity; hence, $A\,B = a\,n$; $B\,C = b$; and, $C\,D = c\,|m|$. (Symbol $|m|$ indicates the numerical value of m.) Fixing a co-ordinate system as indicated, the equation for Line $D\,A$ is:

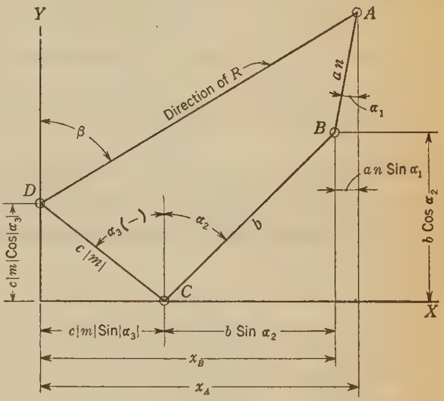

FIG. 20

$$y = x \cot\beta + c|m|\cos|\alpha_3| \dots\dots\dots\dots\dots\dots (75a)$$

and the equation for Line $A\,B$ is:

$$y - b\cos\alpha_2 = \cot\alpha_1\,[\,x - (c|m|\sin|\alpha_3| + b\sin\alpha_2)\,] \dots\dots (75b)$$

At the point of intersection, A, between the two lines, Equations (75) must both be satisfied. Designating by x_A as the abscissa to Point A, and eliminating y: $x_A \cot\beta + c|m|\cos|\alpha_3| - b\cos\alpha_2 = \cot\alpha_1\,[\,x_A - (c|m|\sin|\alpha_3| + b\sin\alpha_2)\,]$; or,

$$x_A = \frac{c|m|\cos|\alpha_3| - b\cos\alpha_2 + \cot\alpha_1\,(c|m|\sin|\alpha_3| + b\sin\alpha_2)}{\cot\alpha_1 - \cot\beta} \dots (76)$$

If the abscissa to Point B is x_B,

$$x_A - x_B = a\,n\sin\alpha_1 \dots\dots\dots\dots\dots\dots\dots (77a)$$

and,

$$n = \frac{1}{a \sin \alpha_1} (x_A - x_B) \ldots\ldots\ldots\ldots\ldots\ldots (77b)$$

and, since $x_B = c|m|\sin|\alpha_3| + b \sin \alpha_2$:

$$n = \frac{1}{a \sin \alpha_1} \left[\frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \alpha_1 \, (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cot \alpha_1 - \cot \beta} - (c|m|\sin|\alpha_3| + b \sin \alpha_2) \right]$$

$$n = \frac{1}{a \sin \alpha_1} \left[\frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \beta \, (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cot \alpha_1 - \cot \beta} \right]$$

and,

$$n = \frac{1}{a} \left[\frac{c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \beta \, (c|m|\sin|\alpha_3| + b \sin \alpha_2)}{\cos \alpha_1 - \sin \alpha_1 \cot \beta} \right] \ldots (78)$$

If $\alpha_1 = 0$, Equation (78) becomes,

$$n = \frac{1}{a} [\, c|m|\cos|\alpha_3| - b \cos \alpha_2 + \cot \beta \, (c|m|\sin|\alpha_3| + b \sin \alpha_2) \,] \ldots (79)$$

Equation (79) expresses the condition that Forces *A B*, *B C*, *C D*, and *D A*, form a closed polygon. However, the deformation formulas, Equations (71), must also be satisfied. Inserting tan η from Equation (71*a*) in Equation (71*b*), and simplifying:

$$n = m \frac{\sin 2\,\alpha_2}{2 \cos \alpha_2 \cos|\alpha_3| \sin\,(|\alpha_3| + \alpha_2)} + \frac{\sin 2|\alpha_3|}{2 \cos \alpha_2 \cos|\alpha_3| \sin\,(|\alpha_3| + \alpha_2)} \ldots (80)$$

Equation (80) shows that, with α_2 and α_3 given, and with $\alpha_1 = 0$, n is a linear function of m. If the pile foundation is to be subject to translation only and no rotation, Equations (79) and (80) must be satisfied simultaneously.

In Equation (79) the numerical value of m, designated $|m|$, has been used; and, it has been assumed that m is negative and that Pile Group 3 actually is subjected to tension. Substituting $|m|$ for m; assuming m to be negative also in Equation (80); and, equating the two expressions for n in Equations (79) and (80):

$$- |m| \frac{\sin 2\,\alpha_2}{2 \cos \alpha_2 \cos|\alpha_3| \sin\,(|\alpha_3| + \alpha_2)} + \frac{\sin 2|\alpha_3|}{2 \cos \alpha_2 \cos|\alpha_3| \sin\,(|\alpha_3| + \alpha_2)}$$
$$= |m| \frac{c}{a} \cos|\alpha_3| - \frac{b}{a} \cos \alpha_2 + |m| \frac{c}{a} \sin|\alpha_3| \cot \beta + \frac{b}{a} \sin \alpha_2 \cot \beta \ldots (81)$$

After reduction:

$$|m| = \frac{\sin 2|\alpha_3| + 2 \frac{b}{a} \cos \alpha_2 \cos|\alpha_3| \sin\,(|\alpha_3| + \alpha_2) \times [\, \cos \alpha_2 - \sin \alpha_2 \cot \beta \,]}{\sin 2\,\alpha_2 + 2 \frac{c}{a} \cos \alpha_2 \cos|\alpha_3| \sin\,(|\alpha_3| + \alpha_2) \times [\, \cos|\alpha_3| + \sin|\alpha_3| \cot \beta \,]} \ldots (82)$$

If the horizontal component of R is R_H and the vertical component, R_V:

$$\cot\beta = \frac{R_V}{R_H} \qquad (83)$$

If $\beta = 90°$, so that $\cot\beta = 0$:

$$|m| = \frac{\sin 2|\alpha_3| + 2\frac{b}{a}\cos^2\alpha_2\cos|\alpha_3|\sin(|\alpha_3| + \alpha_2)}{\sin 2\,\alpha_2 + 2\frac{c}{a}\cos^2|\alpha_3|\cos\alpha_2\sin(|\alpha_3| + \alpha_2)} \qquad (84)$$

Equations (82) and (84) hold only if m is negative, or if $\eta > 90° - |\alpha_3|$. This can be verified by computing η from Equation (71a) after substitution of $-|m|$ for m. The value of n is determined from Equation (80) after having substituted $-|m|$ for m in the same manner; thus:

$$n = \frac{1}{2}\,\frac{\sin 2|\alpha_3| - |m|\sin 2\,\alpha_2}{\cos\alpha_2\cos|\alpha_3|\sin(|\alpha_3| + \alpha_2)} \qquad (85)$$

It is evident from Fig. 20 that, for $\alpha_1 = 0$:

$$P_2\,(c|m|\sin|\alpha_3| + b\sin\alpha_2) = R_H \qquad (86a)$$

or,

$$P_2 = \frac{R_H}{c|m|\sin|\alpha_3| + b\sin\alpha_2} \qquad (86b)$$

Forces P_1 and P_3 are determined from Equations (74). The tangent, k_s, of the angle of Line s (Fig. 19) with the horizontal is determined by:

$$k_s = \frac{R_V - a\,n\,P_2}{R_H} = \frac{R_V - a\,P_1}{R_H} \qquad (87)$$

This line intersects the line of Resultant R on Pile Group Axis 1 (Fig. 19(a)). Line s is thus determined by the location of Pile Group Axis 1 and the slope is determined from Equation (87).

The point of intersection, K, between the axis of Pile Groups 2 and 3 must fall on Line s but the actual location on s may be determined from general engineering considerations.

Pile Foundation Having Its Elastic Center at a Given Point on the Resultant.—Any pile foundation that has its elastic center on the resultant of the exterior forces will be subject to translation only. However, it may be desirable that the elastic center should be located at a particular point on R. Let the point selected be O in Fig. 19(a). Apply an arbitrary force, H, to the pier, the force being given in magnitude and direction and passing through Point O. This force preferably should be horizontal. Compute $|m_H|$ from Equation (84) and n_H from Equation (85). To determine the slope of Line s_H (corresponding to Line s) rewrite Equation (87) for $R_V = 0$ and $R_H = H$; thus:

$$k_H = -\frac{a\,n\,P_2}{H} \qquad (88a)$$

Inserting P_2 from Equation (86b):

$$k_H = -\frac{n_H\,a}{c\,|m_H|\,\sin|\alpha_3| + b\sin\alpha_2} \qquad (88b)$$

Draw Line s_H with a slope, k_H, through the point of intersection of H and Pile Group Axis 1. The intersection of s_H and s is the particular point, K, at which Axes 2 and 3 must intersect to insure an elastic center at the particular point, O.

It can be shown in general that the lines, s, corresponding to the resultants, R (all of which go through the same point) also intersect in a single point. The points, O and K, are uniquely interdetermined. Instead of choosing a horizontal force through O for the determination of Point K, a force may be chosen which corresponds to some alternate possible loading on the structure. In general, it is advantageous to choose Point O at or near the points of intersection of the resultants of the various loadings so as to reduce the magnitude of the rotating moment.

Pile Foundations Having a Given Arrangement of the Battered Piles.—It is often of importance to determine the pile foundation so that the distance, j, between the points of intersection of Axes 2 and 3 with the base of the pier has a given value (Fig. 19(a)). If x_K and y_K denote the abscissa and ordinate to Point K, x_O and y_O, the corresponding quantities for Point O, are:

$$j = y_K (\tan \alpha_2 + \tan |\alpha_3|) \ldots\ldots (89)$$

or:

$$y_K = \frac{j}{\tan \alpha_2 + \tan |\alpha_3|} = j \, \frac{\cot \alpha_2 \cot |\alpha_3|}{\cot \alpha_2 + \cot |\alpha_3|} \ldots\ldots (90a)$$

$$x_K = \frac{y_K - y_R}{k_s} + e \ldots\ldots (90b)$$

and,

$$y_O = y_K - k_H (x_K - e) \ldots\ldots (90c)$$

Distance e, the abscissa to the point of intersection of Axis 1 and the base of the pier, is usually fixed by engineering considerations; and y_R denotes the ordinate to the point of intersection of R with Axis 1. For the condition indicated in Fig. 19(a), y_R is negative and should be so inserted in Equation (90b); and k_s is determined from Equation (87) and k_H from Equation (88b). Since the elastic center is located on R, Quantity y_O determines that point uniquely; and x_K and y_K determine the point of intersection of Axes 2 and 3.

By fixing the elastic center, O, at some particular point of the resultant, R, or by fixing, beforehand, the characteristic quantity, j, of the pile foundation, and from this quantity computing the position of O on R, a satisfactory pile foundation can be designed to resist any given load. The method may be used also in case it is desired to design a pile foundation so that one or two different resultants produce a definite rotating moment, M, about the elastic center. In the latter case the resultant is translated a distance, $\frac{M}{R}$. The additional pile loads due to the rotating moment are as given by Equation (37) of the paper.

Design Procedure.—The following procedure has particular reference to the design of a retaining wall for which the horizontal component of the earth pressure is independent of the width of the base. Assume that the elevation of the pile tops, the length of the unit of the wall for which the pile foundation is to be designed, the permissible pile loads, the horizontal component of the exterior forces, and the length and cross-section area of the piles, are given.

It is required to determine the width of the base slab, the resultant vertical component of the exterior forces, the batter of the piles, the number of piles, the pile loads, and the arrangement of the piles. The steps in this procedure are:

(1) Determine the horizontal component, R_H, of the exterior forces.

(2) Select the pile batters.

(3) By varying the width of the base slab, select a value of the vertical component of the exterior forces, R_V, so that the corresponding R has a direction approximately equal to the direction of the battered compression piles.

(4) Determine the required number of piles by drawing the force polygon demonstrated in Fig. 19(*b*). In determining the required number of piles in Groups 1 and 3, approximate values of $|m|$ and n should be taken from Table 5.

(5) Compute the exact value of $|m|$ from Equation (84) checking to be certain that $\eta > 90° - |\alpha_3|$.

(6) Compute the exact value of n from Equation (85).

(7) Compute Unit Pile Load P_2 of a battered compression pile from Equation (86*b*).

(8) Draw Line s by computing its slope from Equation (87).

(9) Apply a horizontal load, H, to the pier, to be placed so that it intersects R at the desired location of the elastic center. Evaluate $|m_H|$ and n_H from Equations (84) and (85), respectively.

(10) Draw Line s_H by computing its slope from Equation (88*b*).

(11) If it is desired to fix the elastic center at a given point, determine Point K as the point of intersection of s and s_H.

(12) If it is desired that the distance, j (Fig. 19(*a*)), should have a given magnitude, determine the values of y_K, x_K, and y_O from Equations (90), respectively, or by means of graphical construction.

(13) Arrange the piles around their respective axes according to general engineering considerations.

Conclusions.—The author has presented a method for direct determination of the number of piles required in a pile foundation for a pier subject to two loading conditions, provided the foundation has piles in two directions only.

In the discussion presented herein, the writer has extended the method to cover pile foundations having piles in three directions. Complete formulas and method of procedure have been given for the case in which one of the three pile groups consists of vertical piles. This special case is of considerable practical importance in connection with the design of quay walls and other retaining walls.

D. P. KRYNINE,[29] M. AM. SOC. C. E. (by letter).—The design of a pile foundation (as of any other foundation) should be accompanied by an estimation of its probable settlement. Hence the design should include two determinations: (1) Stresses (pressures) on each pile; and (2) the probable settlement of the structure from these pressures. Mr. Vetter gives a method of finding pile pressures, but does not solve the second part of the problem. Apparently, some experimental work is needed; for instance, driving and loading of experimental piles, or some other field testing.

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Methods of Finding Pile Pressures.—An historical outline of different methods of finding pressures under a pile foundation has been prepared by Professor A. Agatz [30] which, in conjunction with this paper, represents a comprehensive treatment of how the methods of design have been developed to their present stage.

The method of Culman, presented before 1870, was to break the resultant of forces acting at a pier in three components using an "auxiliary" force direction. Each component was the sum of pressures in all the piles running in that particular direction; and the stress in an individual pile was found by dividing that component by the number of corresponding piles. The well-known office method of trapezoidal pressure distribution under an eccentrically loaded retaining wall followed. Another method is to subdivide the vertical load proportionally to the distances between the piles.

These approximate methods have been replaced by methods in which the mathematical theory of elasticity is used. In his "Acknowledgments" Mr. Vetter gives several references to these methods; and his list might be amplified by the addition of Krey,[31] Jakobi,[32] and Mügge.[33] Mr. Vetter's assumptions and his general approach to the solution of the problem are similar to procedures used by previous investigators, as Mr. Vetter himself states.

The Rôle of the Earth Mass Under-Estimated.—The writer is not prepared to offer a theory of his own concerning the design of pile foundations. Therefore, he limits himself to expressing his general opinion on the methods which use elastic theories. It is assumed that the tips of the piles are in a steady position. This is because the piles are restrained or because there is a hinge either at the tip of the pile or at a certain depth as, for instance, at two-thirds the length of the pile. The former case is represented when piles reach rock or similar solid soil and the latter corresponds to friction piles. The writer doubts that, in a general case, the tips of the piles do not move. Stresses in the earth mass are not taken care of, except "passive resistance," which is sometimes mentioned. Deformations in the earth mass are not considered. In other words, the fact that an earth mass with piles driven in it in groups and subjected to lateral motion acts as a unit is neglected.

Isolated Piles Subjected to Lateral Forces.—The behavior of a group of piles under a foundation has not yet been investigated experimentally from this point of view; and some opinion thereon may be formed only by analogy with experimental data concerning the behavior of isolated piles and small groups of them. If an isolated vertical pile is subjected to the action of a horizontal force, it is bent, the "zero-point" (Z, Fig. 21) being located approximately at the third point of the restrained part, h. The accurate position of Point Z depends on several variables, among them being the elastic constants of the material of which the pile is made, and the earth material. This action would be the same whether or not the tip of the pile reaches rock, but it is not known very exactly what may happen if a heavy vertical load, P, is applied to a friction pile. It may be guessed that the lateral deflection, δ, will be smaller than without Load P.

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[30] Der Kampf des Ingenieurs gegen Eide und Wasser im Grundbau, 1936, p. 166 *et seq.*

[31] Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes, Berlin, 1932, p. 162.

[32] Ost. Wochenschr. öff. Baudienst, 1909, p. 340.

[33] Beiträge zur zeichnerischen Lösung technischer Rechnungsaufgaben. Dissert., Hanover, 1906.

If the horizontal force, H', as shown in Fig. 21, is removed, the pile will regain its original (straight) shape, but will not be vertical any more due to a non-reversible (plastic) deformation of the soil. An analogous phenomenon probably occurs in the case of a pile foundation, and it may be that these plastic deformations are much greater than the elastic deformations considered in present methods of design.

The anchor of a bulkhead is bent as shown in Fig. 22. This gives some indication on the possible behavior of battered piles. There still will be a hypothetical hinge higher than the tip of a battered pile. The writer welcomes the idea of using battered piles for resisting horizontal forces and emphasizes the necessity of making pulling tests with such piles.[34]

Reinforced Earth.—The writer believes that when there are many piles under a structure subjected to the action of lateral forces, the stressed condition becomes exceedingly complicated, and that a simplified, rather conventional method of design should be used. In a pile foundation the earth is "reinforced," something like reinforced concrete.[34]

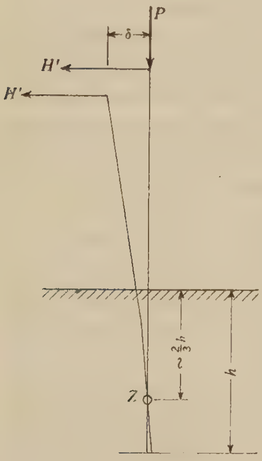

FIG. 21

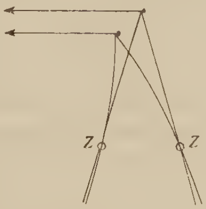

FIG. 22

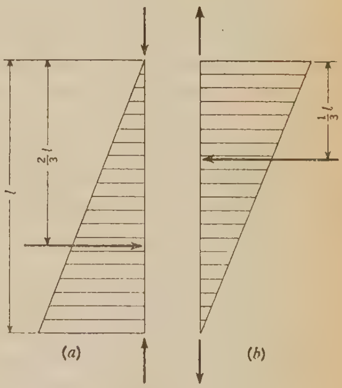

FIG. 23

Piles Designed as Columns.—In the light of several assumptions by Mr. Vetter the writer wishes to discuss his opinion as to the effective length of hinged piles. Mr. Vetter states that if the pile is driven through loose material to firm ground, the effective length of the pile will equal its length. This is not so, however, since the action of the "loose material" cannot be neglected and in reality is not neglected. To prove it, imagine piles 30 ft long standing vertically in the middle of the street, 3 ft apart, and carrying a substantial building. No engineer would dare to erect such a structure; however, in designing piles as columns exactly such a condition is considered. Cross-sections of the piles may be designed in this way. However, in computing the "effective length" some correction to the action of the adjacent soil must be introduced because in computing settlements the designer has no right to introduce arbitrary assumptions even if they are on the safe side. If he does so, his results will be in error.

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[34] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 278.

Friction Piles.—Mr. Vetter and many other writers on this subject assume the triangular friction distribution along a friction pile (Fig. 23(*a*)). This rather contradicts some existing experimental data and field observations. From several "energy-penetration curves" taken during pile-driving, it may be concluded that the friction resistance during driving is rather uniform along the pile,[35] although this statement by no means can be generalized or extended, unconditionally, to the case of static loading. However, assume that the triangular friction distribution as shown in Fig. 23(*a*) is correct. Suppose, furthermore, that instead of compression, a friction pile is subjected to tension. For instance, in Fig. 9 this would be the case with Pile 2 if the wind should blow somewhat stronger. The friction would be reversed in this case, and, to be consistent, one should imagine also the reversed friction distribution (Fig. 23(*b*)). A question arises as to whether the effective length in this case is two-thirds the length of the pile, one third, or some other value.

Wind Action.—In Fig. 9 it is necessary to consider also the reversed wind action (smaller, perhaps). This would modify the computed stresses somewhat.

Conclusion.—Mr. Vetter's paper is a valuable step in the direction toward which the design of pile foundations is being developed. This is an appreciable contribution to the rather meager English literature in this field. The writer, however, hopes for a trend toward simplifying the existing methods of design.

C. P. VETTER,[36] M. AM. SOC. C. E. (by letter).—The generous contributions which, in the form of discussions, have been made to the art and practice of the design of pile foundations, have been most gratifying. Much new light has been thrown on the various principles involved, and the writer wishes to express his appreciation.

Mr. Hill develops formulas for pile loads for the case in which all piles are parallel with each other. Apparently, it has been overlooked that under the assumption of the exterior load being resisted solely by axial pile reactions, such a foundation will be stable only if the resultant of the exterior loads is parallel with the piles. This may be seen directly from Equations (46*a*) and (46*b*) which by division yield:

$$\frac{H}{V} = \tan \alpha \qquad (91)$$

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[35] *Proceedings*, Soil. Mech. Conf., Vol. 3 (1936), paper H-16.

[36] Engr., U. S. Bureau of Reclamation, Denver, Colo.

Such a pile foundation is only rarely encountered in practice, except in the case of vertical piles supporting a vertical load ($\alpha = 0$).

Mr. Hill extends the investigations to foundations having piles in opposite directions with equal (but opposite) batters. Equation (52) is believed to cover the case correctly. However, it is incorrect to apply this formula to the writer's numerical example, in that Mr. Hill seems to have neglected the vertical piles in the example. Presumably, this accounts for the deviation in numerical values of the pile loads given in Mr. Hill's tabulation following Equation (52).

Mr. Hill next discusses pile foundations for which the resultant of the exterior forces does not fall in the plane of symmetry of the pile group and is not even parallel with that plane. The writer wishes to warn prospective designers against taking too much for granted when considering pile foundations and loadings arbitrarily distributed in space. The problem is "tricky" and highly complex. It has been solved to perfection by C. Nökkentved[5] whose work should be consulted before important designs in this line are undertaken.

Mr. Hill further states that it is desirable that a foundation utilizing batter piles contain piles battered in opposition to each other. The writer agrees fully with this assertion. In fact, in the design of the pile foundations for Imperial Dam on the Colorado River (perhaps the most extensive pile-driving undertaking in the United States containing battered piles), all foundations had piles in opposite directions. Fig. 1(*a*) shows a typical bent of a part of Imperial Dam.

Professor Albert draws attention to the valuable contributions made by Swedish engineers, about thirty or more years ago, to current knowledge of the behavior of pile foundations. The writer gave proper reference in his paper to those early pioneers in the field. Mr. Albert prefers Hultin's graphical method for the determination of pile loads for the case of hinged piles, and the writer agrees that, for one thoroughly familiar with that method, results may possibly be obtained in less time by the use of that method than by the formulas given in the paper. The difficulty with Hultin's method, as with nearly all the older methods, is that, although the applications may be speedy after the method has been mastered, the method itself is so intricate that considerable time must be spent by the designer to familiarize himself with the subject or to "brush up" on it even if he has known it once. To most engineers the designs of pile foundations come only rarely, and it would seem of considerable importance that the principles used in such designs should be formulated so simply that actual numerical work may be undertaken with the least possible delay. The expert who does little besides designing pile foundations would do well, and possibly would save time, by studying some of the other works on the subject, as referred to in the paper. He should ascertain, however, that the method he proposes to utilize rests on a sound theoretical base. The references given by the writer and by Mr. Albert are believed to be reliable in this respect.

Mr. Niederhoff appears to be mostly concerned with defending the still widely employed practice of designing pile foundations which depend for the stability of the structure on lateral resistance of the upper layers of the foundation material. The writer is definitely of the opinion, and so stated in his paper, that if the upper layers are of such consistency that they may safely be relied upon for lateral resistance, piles are not needed and should not be used. The reason is that, in driving the piles, the original firmness of the upper layers of the soil is often destroyed and so is the soil's capacity for resisting lateral loads.

Mr. Niederhoff would seem, in places, to have misunderstood some of the points brought forth in the paper, possibly because of lack of clarity on the part of the writer. The statement is made that "His [the writer's] analysis also appears needlessly complicated and leads to values that are different from those obtained by the writer [Mr. Niederhoff] using only the resolution of forces into components that act upon the piles axial and normal to the axis." The writer maintains that an analysis is "needlessly complicated" only when the same result, or a result equally correct, may be obtained by some other analysis which is easier, less complicated, or more readily understood. That results, different from the writer's, are obtained by Mr. Niederhoff's analysis is not surprising, in that he satisfies the equilibrium conditions only, without paying any attention to deformations or displacements. The relationship between a pile's axial compression and its lateral displacement does not depend only on the pile's batter and the direction of the resultant. It depends most of all on the location of the resultant with respect to the top of the piles. The movement of the resultant laterally from one side of the elastic center to the other may reverse the direction of the lateral displacement of the pile heads and, as a result, reverse the direction of the lateral resistance. It was one of the main purposes of the paper to demonstrate this, possibly surprising, but nevertheless unquestionable, fact.

Mr. Niederhoff further objects to the writer's statement that "In many practical cases, such as bridge piers and gate piers in hydraulic structures, the error introduced by this assumption [that of hinged ends] is insignificant." The sentence that follows immediately in the paper may have been overlooked: "However, in some applications, such as pile bents for trestles, the error becomes of considerable magnitude."

Under "Pile Foundations Which Are Symmetrical About a Vertical Line: Comment," the writer stated:

> "* * * if, therefore, the piles, approximately, go through the same point the rotation for a given moment would be large. The assumption that the effect of restraint may safely be neglected does not hold for this case and the stability of the pier depends almost entirely on the restraint and the stiffness of the piles."

A group of vertical test piles, as referred to by Mr. Niederhoff, may be said to represent a pile bent at its worst, in that all piles intersect at the same point (infinity). That less deflection was found for the restrained test piles than for hinged ones would seem to bear out the writer's statement rather than to refute it.

In determining the depth at which the pile is restrained (necessary for the cases where restraint is important) Mr. Niederhoff prefers to follow the analysis by A. E. Cummings, M. Am. Soc. C. E.[23] The writer can see no objection to the use of Mr. Cummings' formulas as a guide. These formulas are based on very special assumptions, probably applicable to a test pile, but only applicable to a very limited extent to piles in an actual foundation. One of the assumptions in question is that the point of contraflexure occurs at half depth. This assumption applies neither to a test pile nor to a pile in an actual foundation. However, it affects the final result very little. Of greater consequence is Mr. Cummings' assumption that the tangent to the pile's deflection curve remains vertical at the top. Any structure supported on piles will rotate under load, unless the resultant of the exterior forces goes through the elastic center. The more nearly the piles intersect at a point, the greater the rotation will be, and the greater will be the error in assuming the tangent at the tops of the piles to be vertical. From a mathematical point of view, the analysis of the same problem by Y. L. Chang[37] is preferable to that of Mr. Cummings. Mr. Chang uses a somewhat different modulus of elasticity of the soil, but that might possibly be remedied.

Mr. Eremin suggests direct algebraic solution of the three simultaneous equilibrium equations. The method is direct but somewhat tedious and time-consuming in the case of many piles. Furthermore, the sign conventions become of primary importance since there is no direct check on the work as it progresses.

Mr. Feld submits a number of general statements regarding the behavior of piles under load. In every case he seems to conclude that the behavior of a pile foundation consisting of many piles is similar to the behavior of a single pile subject to a similar load. Such conclusions are not permissible. The case can best be illustrated by an example. Assume a batter pile, *A B*, acted upon by a horizontal force, *P* (Fig. 24). Mr. Feld uses exactly this case as an illustration. In order for the pile to be in equilibrium, at least two forces, *R* and *S*, are required. They must be produced by the resistance of the soil or, in some equivalent manner, by the stiffness and embedment of the pile. If the soil offers insufficient resistance to lateral displacement, or if the pile lacks stiffness or strength, the structure will fail. If, now, a second pile, *A C*, is provided, the lateral forces *R* and *S* are not required for equilibrium and no reliance need be placed on the lateral resistance of the soil or on the stiffness and flexural strength of the piles. All that is necessary is that the piles can sustain axial loads such as *Q* and *T*. That *T* is tension is of no particular importance; as a rule there are sufficient vertical forces also acting on the structure to counterbalance the tension. It is believed that this simple example presents the advantage of battered piles in the simplest possible manner and that it answers virtually all the questions raised by Mr. Feld.

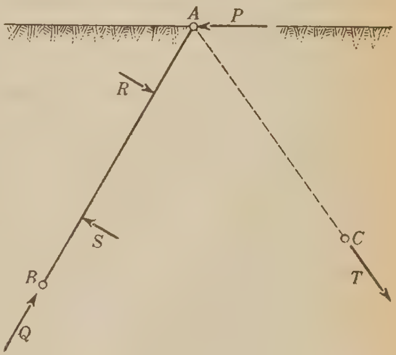

FIG. 24

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[37] *Transactions,* Am. Soc. C. E., Vol. 102 (1937), p. 272.

Mr. Coan presents an example of a three-dimensional pile group analyzed by methods similar to the writer's. The results in this particular case seem to agree fairly well with model test results. However, as mentioned previously, great caution should be exercised in applying the writer's methods to spatial pile groups and loading conditions.

Professor Agatz presents a very ingenious method for direct determination of pressures in pile foundations consisting of piles in three different directions. His method is entirely algebraical, whereas the writer's method, in the general case, is partly algebraical and partly graphical. The theoretical background for both methods is, needless to say, the same. For the special case in which one of the three pile groups is vertical, complete formulas ready for use are presented. The only criticism the writer would venture in connection with this discussion is the use of numerical values instead of algebraical values for the ratio m ($|m|$) and the angles α ($|\alpha|$). As a general rule the writer believes this to be a dangerous practice. Sign errors constitute a surprisingly great proportion of the errors made in engineering calculations and, at times, such errors are very difficult to detect. For that reason the writer is in favor of definite sign conventions and strict adherence to them. As far as Professor Agatz' discussion is concerned, this is, of course, a minor matter. His equations, although complicated in appearance, are not as complicated in actual use as might be expected. In general, the writer prefers methods which present a little easier avenue of approach but, since Professor Agatz' method is primarily applicable to retaining walls and quay walls with considerable duplication, the designer can well afford to spend considerable time in becoming familiar with this special method.

Professor Krynine calls attention to the fact that the writer's method produces pressures on each pile but not settlements on the structure due to these pressures. This deficiency was recognized in the "Synopsis" of the paper, from which is quoted: "It is the subsoil that 'carries' the load, and consolidation of soft material below the piles will inevitably contribute its share to the settlement of the structure whether or not piles are used," and further, "It may be argued * * * in particular, that considerations from the point of view of soil mechanics have been neglected in favor of those of purely structural mechanics. This argument cannot be refuted in a general manner." It would be equally futile to attempt to refute Professor Krynine's argument. The purpose of the paper was restricted and was limited to the determination of pile stresses, and, in general, to the encouragement of the use of battered piles for structures subject to horizontal loads. It did not deal, primarily, with the settlement of structures, and to attempt to predict such settlements solely from the pile stresses would obviously be erroneous. Settlements due to causes other than the elastic deformations of the piles will probably affect the pile stresses, particularly in cases where the restraint of the piles is important. Hence, contrary to the apparent opinion of some of the discussers, the methods proposed by the writer are best suited for pile foundations in which the restraint of the piles is of minor importance; that is, large bridge piers and piers for hydraulic structures. In such structures the pile stresses are likely to be least affected by secondary earth movements.

Only one other of the points raised by Professor Krynine needs to be discussed further; namely, that of the distribution along the pile's axis of the frictional forces. This question was also raised by Mr. Niederhoff. The proposed use of the triangular distribution has a very simple explanation. Assume that the coefficient of friction between the pile and the soil is constant and independent of the normal pressures and of the rate and magnitude of the relative movements. The coefficient of friction between solid substances is generally assumed to have just those characteristics. The frictional resistance to the movement of the pile would thus be directly proportional to the lateral earth pressure which, again, is usually assumed to be rectilinearly increasing with depth. As a result, the friction per unit area would increase rectilinearly with depth and, if the pile is cylindrical or prismatic, the friction per unit length of pile would also increase rectilinearly with depth. In the case of a tension pile, conditions would be exactly the same except that the normal earth pressure, according to the familiar Coulomb-Poncelet earth-pressure theory, is less in the case of a vertically upward movement of the pile than it is for a vertically downward movement. The writer does not agree with Professor Krynine that a consistent conclusion would be that, in the case of a tension pile, the frictional force would be maximum at the top and zero at the tip of the pile.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2032

STREAM POLLUTION IN THE OHIO RIVER BASIN

A SYMPOSIUM

PAGE

Foreword.
By George E. Barnes, M. Am. Soc. C. E. 814

Surveys for Stream Pollution Control.
By Harold W. Streeter, M. Am. Soc. C. E. 815

Low-Flow Regulation by Pymatuning Reservoir.
By Charles E. Ryder, Esq. 832

Sanitation Problems Incidental to Floods.
By W. L. Stevenson, M. Am. Soc. C. E. 844

Progress in Pollution Control in the Ohio River Basin.
By E. S. Tisdale, Esq. 852

Planning for Pollution Control at Pittsburgh, Pennsylvania.
By D. E. Davis, M. Am. Soc. C. E. 860

Note.—The Symposium on Stream Pollution in the Ohio River Basin was presented at the meeting of the Sanitary Engineering Division at Pittsburgh, Pa., on October 14, 1936, and published in January, 1938, *Proceedings*.

Stream Pollution Problems at Cincinnati, Ohio.
By J. E. Root, M. Am. Soc. C. E. 868

What Can Be Done About Stream Pollution?
By Abel Wolman, M. Am. Soc. C. E. 874

With Discussion by Messrs. D. E. Davis, L. S. Morgan, C. A. Holmquist, Robert Spurr Weston, Karl Imhoff, Henry D. Johnson, Jr., Charles M. Reppert, Edwin K. Morse, James H. LeVan, John C. H. Lee, W. H. Wisely, F. C. Dugan, G. M. Ridenour, Arthur W. Baum, Jr., G. R. Scott, Don E. Bloodgood, W. L. Stevenson, and D. E. Davis.

FOREWORD

By George E. Barnes,[1] M. Am. Soc. C. E.

Since 1933 the construction of sanitary engineering projects has been financed largely by Federal funds allocated as loans or grants-in-aid. Federal funding agencies act in part as co-ordinating agencies also, in reviewing studies and plans for various types of projects proposed for individual population centers and local areas. In the Ohio Basin as elsewhere, such review has made more apparent than ever the desirability of more comprehensive planning for the region as a whole, as the basis for a long-range policy on conservation and proper utilization of water resources.

The Ohio Basin, from the standpoint of size, natural wealth, and population supported, is one of the most important in the United States; and it seems appropriate at the present time to focus technical and lay interest on its peculiar sanitation problems. With this object in mind, the Cleveland Section of the Society appointed a Special Committee to work in co-operation with the Sanitary Engineering Division and to organize a Symposium on Stream Pollution in the Ohio Basin. The Committee included the writer as Chairman, and George B. Gascoigne, Rollin F. MacDowell, and George B. Sowers, Members, Am. Soc. C. E. The Committee was of the opinion that, although there may be interesting and comparatively unexplored problems in sanitary engineering, particularly in the field of industrial wastes treatment, nevertheless questions of design or treatment processes might properly be subordinated in this Symposium to an appraisal of the critical problems now existing in the water-shed, the outlook for co-ordinated planning, and the procedure for initiating and enabling sanitation improvements. Emphasis in the following papers is directed toward this end.

Mr. Streeter's paper describes methods for surveying the extent of pollution and gives the results of such a survey on the Ohio River, together with suggestions for minimum standards for river condition. Mr. Ryder's paper, discussing the operation of the Pymatuning Reservoir, shows that in certain cases it may be at least as valuable to regulate dry-season flow for dilution purposes as to invest in sewage plants complete with secondary treatment. Mr. Stevenson's paper is devoted to the extraordinary duties carried by the Pennsylvania Department of Health in time of flood, and indicates from experience many factors that must be considered in the design of sanitary engineering projects and in the organization of service agencies, if these are to function during emergencies. Mr. Tisdale's paper sets forth an excellent picture of sanitary conditions in the Ohio Basin as a whole, and is supplemented with papers by Mr. Davis on the problems at Pittsburgh, Pa., and by Mr. Root on those at Cincinnati, Ohio, the two major metropolitan centers of the Ohio Basin. The concluding paper, by Mr. Wolman, is concerned with the medium through which comprehensive planning and adequate financing may be secured. The discussions of this Symposium will be no less important than the topic papers themselves, in presenting a balanced understanding of sanitation problems in the Ohio Basin.

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SURVEYS FOR STREAM POLLUTION CONTROL

By Harold W. Streeter,[2] M. Am. Soc. C. E.

Synopsis

The effective control of stream pollution, when viewed as an engineering problem, entails not only the design of sewage and industrial wastes treatment plants, but also the formulation of comprehensive plans for restoring and maintaining entire river systems in proper condition for various water uses, avoiding the wastefulness of over-correction in some areas and the ineffectiveness of under-correction in other areas. The base data for such plans are obtainable from field and laboratory surveys of a river and its tributary drainage area.

Field surveys of this character will embrace all major sources of pollution; vital statistics of water-borne diseases in various parts of the water-shed; the extent and kind of water uses in different areas; hydrometric data for the river and its main tributaries; and information bearing on time intervals of flow in the river between various sources of pollution. Laboratory surveys will include data on the quality of stream waters at water intakes and below sources of pollution; on progressive changes in "oxygen balance" in more highly polluted river zones, and in special cases, on turbidity, alkalinity or acidity, hardness, and certain metals detrimental to the quality of the stream waters and their different uses. Biological data will embrace major groups of pollutional and non-pollutional organisms, both in the stream proper and in the bottom sediments, and likewise organisms detrimental to water supplies.

Where pollution control is designed so as to utilize the natural purification capacity of a waterway, limiting conditions of pollution should be fixed tentatively for various water uses, as the basis of interpreting the laboratory findings. Examples of such criteria are given with respect to water supply sources, nuisance prevention, and maintenance of normal aquatic life in stream waters.

Considering the Ohio River System as presenting a fairly representative large-scale problem of stream pollution control, a brief account is given of the various surveys of this river made during recent years and the main conclusions derived from them. To illustrate the application of laboratory survey data collected in this river, results are shown of an estimate of the relative responsibility of sewered population groups draining into various zones of the river for conditions of bacterial pollution at several water intakes. These results indicate that except in the winter, when the effects of natural purification are at a minimum, the major portion of sewage bacteria surviving in the river at each intake appears to be traceable to population groups draining

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into the river, either directly or indirectly, within about 200 miles up stream from that point. In winter, the effect of more distant population groups is apparent. It also is shown that acid conditions in the Ohio River and its tributaries above Wheeling, W. Va., exercise a powerful repressive influence on bacterial pollution originating in this part of the river system. With the amelioration of the acid conditions resulting from mine-sealing and other corrective measures in progress, extensive treatment of the sewage from populations now draining into this zone of the river will be necessary, in order to avoid serious overburdening of water purification plants with excessive bacterial pollution originating in sewage. Study needs to be given to the degrees and methods of sewage and industrial wastes treatment best adapted to meeting water use requirements in different parts of this river system, and also to the probable effects of large volumes of treated effluents on the stream waters and their several uses.

With the growth of public interest in stream pollution as a nation-wide problem, legislators and sanitarians have been impelled to extend their views of the subject far beyond local horizons. Until recently the sole objective of "stream cleaning" was usually the removal of local "sore spots." Little, if any, thought was given to more remote down-stream conditions, which were left to those concerned to deal with as best they could. However, it is now becoming more generally recognized that river pollution, like floods, is no respecter of local boundaries, and that if control measures are to be permanently effective they must comprehend entire drainage areas, or, at least, those portions of them in which sewage and industrial wastes impose a definite burden on streams.

This general broadening of viewpoint with respect to stream pollution and its control has brought with it a marked change in the nature and complexity of the engineering problems to be solved, not so much as regards the actual treatment of wastes, as regards the formulation of workable plans for pollution control. Such plans must be designed to restore and maintain complete river systems in proper condition for various water uses, and must avoid, as far as possible, the wastefulness of over-correction in some areas and the ineffectiveness of under-correction in other areas. It is with this latter phase of the problem and more especially with the surveys and other studies essential to a well-planned system of river-pollution control that this paper is concerned.

In a broad sense, a certain engineering analogy may be said to exist between a river system under effective pollution control and a well-planned structure, such as a bridge, which must be designed so that the maximum stress imposed on every individual member does not exceed its safe working strength. In such a river system, for example, the degree of pollution, both of the main stream and of the various tributaries, must be limited to a maximum consistent with the suitability of those streams for specified water uses, and in no case may it exceed their respective capacities for dilution and natural purification.

The limitation in pollution imposed by water use may be likened to the safe working load on a structure, and the limitation imposed by dilution and self-purification corresponds to the ultimate breaking load. A well-marked distinction is to be noted, however, between ordinary structural design and stream-pollution control in that, in the first instance, the load is predetermined and the strength or capacity of the structural members can be adjusted to support such a load, whereas, in the second instance, the limiting strength of the structure is predetermined (by the capacity for dilution and natural purification) and the load, or degree of pollution, must be adjusted accordingly.

From the foregoing analogy, it is fairly evident that as it is necessary to make a thorough study of loadings and allowable stresses in designing an ordinary engineering structure, it is equally essential in planning stream-pollution control, to make a similarly careful and detailed study of the nature and extent of the pollution of a river system and also its capacity for dilution and natural purification. The basic data for such a study are usually obtained through field and laboratory surveys of the river and its drainage area, extending over a sufficiently long time to reveal the normal seasonal fluctuations in temperature, run-off, velocities, and time intervals of flow in the streams, and other seasonal variables.

General Survey Methods

A survey of sources of pollution will include rural, urban, and sewered populations, and their trends of growth or change in different parts of the river system. It also will include waste-producing industries, listing them according to character, location, and the strength and volume of wastes produced in relation to amounts of manufactured products. All water intakes along the main stream and its major tributaries should be located. In the vicinity of the larger sewered communities, all the more important sewer outlets should also be located, and detailed information should be obtained in each case on the size of the contributing population, the volume and strength of the sewage, and the nature and extent of sewage treatment. Such a survey, which would entail visiting practically every sewered community on the water-shed, might well be supplemented by collecting available statistics on the prevalence of water-borne disease in each community (particularly typhoid fever and gastro-enteritis) and on the quality of the public and private water supplies. Drainage areas of the main stream and its various tributaries should be mapped and determined by careful planimeter measurements. These several items of data would fall under the ordinary field survey, to which the hydrometric and laboratory surveys would be supplementary.

The main objective of the hydrometric survey is to obtain detailed information as to the amount of dilution water available at different points in the river system, and its variation from day to day, or week to week, throughout the period covered by the laboratory survey. Collection of these data involves the establishment of gaging stations in the main stream and on all important tributaries, the systematic collection of gage-height records, and a sufficient number of discharge measurements at each station to define accurate rating curves. Where data are available bearing on cross-sectional and surface areas of the main stream at different stages of flow, such data are of value in estimating mean velocities and time intervals of flow at these various stages. Although usually less important, similar data for some of the major tributaries may be very useful in estimating periods of flow concentration for these tributary drainage areas. Where such data are not obtainable, velocities and time intervals of flow may be estimated from float measurements or, in very small streams, through the use of strong dyes.

The purpose of the laboratory survey is to show actual conditions of pollution, with their seasonal and other variations, in the main stream and major tributaries, having in mind maximum and minimum as well as average conditions at various times and the periods of duration of the several conditions observed. This survey should be designed to show the quality of water above and below main sources of pollution and dilution (tributaries) and also the extent of natural purification occurring in longer stretches of the river comparatively free from the disturbing influence of pollution or dilution. Particular attention should be given to the quality of water at water-supply intakes and at points in the immediate vicinity of major sources of pollution, where conditions are more likely to be unfavorable to recreational uses of the stream and where the demand for such uses is ordinarily more widespread.

For purposes of sampling, fixed points in the stream at known range line intersections are preferable to random stations. In the wider stretches of channel, three points on each cross-section may be selected and river samples collected, ordinarily at mid-depth, after checking several times the variations in river-water composition as between surface, mid-depth, and bottom. Under usual circumstances, the quality of water in a stream varies more widely at a given time in a horizontal direction on a cross-section than it does vertically at any point in such a section, although where marked sludge deposits are present, exceptions to this rule may be noted.

The number and character of laboratory determinations made in connection with a survey may vary widely with the size and complexity of the problem. However, for ordinary stream-pollution control, certain minimum requirements should be met. A detailed study should be made of the numbers of sewage bacteria (the *coli-aerogenes* group) and of their variations from day to day, or week to week, at different points in the main river and tributaries, particularly at sources of public water supplies and in the local areas affecting them. These data preferably should be supplemented by parallel observations of bacterial plate counts at the same points and in the same samples, in order to provide a check on the trends of the *coli-aerogenes* observations. The methods used in these and other laboratory determinations should follow the standard procedures described by the American Public Health Association.

In addition to the bacteriological tests, which are the most sensitive indices of significant changes in the degree of pollution of a river, systematic observations should be made of the dissolved oxygen and bio-chemical oxygen demand (B.O.D.) of the stream water at every major sampling station, in order that progressive changes in the "oxygen balance" (that is, the relation of available oxygen to oxygen demand) may be measured. The oxygen balance serves as an index of the adequacy of dilution and natural purification to dispose of more immediate pollution and also as a record of the fitness of the stream for normal aquatic life. The river water temperature should always be recorded at the time of collecting each dissolved oxygen sample, in order to provide a basis for estimating the degree of oxygen saturation, which varies with the temperature. The B.O.D. test as now made ordinarily is carried out by incubating sealed samples of the river water (undiluted, as far as possible) at a temperature of 20° C for 5 days, and measuring the loss of dissolved oxygen by the sample under these standardized conditions. Under some conditions, such as of heavy pollution or sludge deposits, B.O.D. determinations on identical samples, with incubation periods of 1, 5, 10, and 20 days, may be desirable occasionally in order to check the course of the oxidation curve through its first or carbonaceous stage. In other cases, notably where evidences of nitrification in the river are apparent, extension of the test period to 40 days may be desirable, in order to show the course of oxidation through both primary and secondary (nitrification) phases.

The addition of other laboratory tests to the routine schedule of the survey will depend largely on circumstances and on the facilities available at field laboratories. As indices of advanced oxidation in a river, the determinations of nitrite and nitrate are essential, as they show the final state of the oxygen used up in the stabilization of nitrogeneous organic matter. Determinations of turbidity and alkalinity are useful in measuring, respectively, the effects of sedimentation and of dilution in the river, the latter mainly through tributaries.

Certain other laboratory tests may be of value where special conditions or requirements are to be met. If the hardness or incrustant content of a river is an object of special interest, these determinations, together with those of calcium and magnesium, should be made regularly. Occasionally, the presence or absence of phenols in rivers from which water supplies are taken may be an important consideration. Similarly, iron and manganese are frequently matters of concern. In the streams of New England and the Southeastern States, the presence of dissolved coloring matter is often significant. In some rivers of the Western States draining lead and copper mining regions, the search for quantitative evidence of these toxic metals may be a matter of predominating importance in a laboratory survey. In every particular case, due consideration must be given to the uses made of the river water and to the presence of undesirable substances in the water. This applies particularly to the presence of substances originating in certain industrial wastes. As indices of ordinary sewage pollution, the bacterial and oxygen balance determinations are of primary importance.

The third main division of the laboratory survey is that which has to do with the biology of a river. In this connection it is not essential to stream-pollution control that an extensive or detailed biological survey of a river system be undertaken, although it is highly desirable that a sufficiently thorough examination be made of the supernatant stream and also of the bottom deposits to establish a clear record of the relative prevalence of pollutional and of non-pollutional organisms in different river zones. As these biological forms are an essential link in the chain of natural purification and as they doubtless will undergo marked changes in kind and relative numbers following the institution of sewage and industrial waste treatment, it is particularly important that these changes be recorded as a significant portion of the history of a river during and following the period of active stream sanitation measures. A fairly thorough biological survey before these measures are instituted and another after they have been completed will be well worth the time and expense required. In some instances, it will be advisable to maintain a more or less continuous check on biological conditions in a stream below major sources of pollution during the progress of initial corrective measures, in order to reveal any marked changes occurring in the balance of aquatic life and, where water supplies are involved, to furnish evidence of any tendencies toward the development of filter-clogging or taste- and odor-producing organisms which may impose an additional burden on water purification systems.

Criteria for Permissible Limits of Pollution

Although field and laboratory surveys will show the river zones in which conditions are either definitely satisfactory or otherwise for various water uses, they usually fail to reveal the limiting boundaries which lie between these two conditions; that is, in the rare instances in which these boundary conditions may prevail, the indications with respect to suitability or unsuitability for various water uses may be somewhat indefinite. If full utilization is to be made of the dilution and natural purification capacities of a given river system in undertaking a systematic program of stream pollution control, it is essential that some definite limiting criterion with respect to allowable density of pollution be adopted, tentatively at least, in order to provide a guide-post for correctional measures.

In general, the particular criterion, or set of criteria, adopted will depend on the more important uses made of a river in a particular zone, such as water supply, recreation, commercial fishing, industrial requirements, etc. In general, the suitability of a river as the source of public water supply is the predominant interest to be served. In many river zones from which water supplies are not ordinarily taken (as in the zones immediately below sewered communities), the rapidly growing pressure for enlarged recreational uses has brought to the forefront many problems of local sanitation affecting the use of streams for boating, fishing, camping, and even bathing. In relatively clean rivers now devoted principally to recreational uses, the question of preventing future impairment for such uses is a vital one in many States, affecting not only the interests of riparian dwellers, but also those of an increasing number of people from all parts of the country, whose annual migrations to these water centers in quest of rest and change are rapidly assuming the proportions of a national movement.

The results of an extensive study carried out by the United States Public Health Service in 1923–1927, led to the conclusion that where the sewage pollution of sources of public water supplies does not exceed that which is represented by a *coli-aerogenes* bacterial number of 5 000 per 100 cu cm as a rounded figure, purified water supplies conforming in quality to the primary requirements of the Treasury Department drinking water standard should be consistently attainable with the aid of efficient modern water purification systems. This figure would provide a fairly simple criterion for limiting river or lake pollution at intakes from which public water supplies are withdrawn. If taken as a yearly average, this figure would be scaled downward to some extent, depending on the character and variability of pollution at the particular water source considered, in order to ensure conformance of the purified water supply to both primary and secondary requirements of the Treasury Department *B. coli* standard. Thus, it has been noted by the writer in a previous paper,[3] that an annual average *coli-aerogenes* index of about 3 000 per 100 cu cm would represent a safer upper limit of pollution for sources of water supply located along the Ohio River, and probably also along its major tributaries.

TABLE 1.—YEARLY AVERAGE *B. coli* NUMBERS IN RAW WATERS OF OHIO RIVER FILTRATION PLANTS, 1926–1935.
(From Filtration Plant Records)

Place	AVERAGE *B. coli* INDEX PER 100 CUBIC CENTIMETERS										
	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	Mean
East Liverpool, Ohio	24 900	15 500	13 300	4 610	10 100	11 700	10 400	1 840	1 220	8 750	9 420
Steubenville, Ohio	6 230	1 640	6 690	2 820	6 430	555	1 860	338	2 060	1 370	3 000
Wheeling, W. Va.	——	2 330	6 360	3 250	4 730	1 570	1 290	388	50	854	2 080
Marietta, Ohio	1 160	1 080	1 880	916	22 500	999	2 230	3 950	1 660	1 560	3 790
Huntington, W. Va.	446	327	128	72	32	33	63	2 430	1 310	2 170	584
Ashland, Ohio	24 900	28 000	19 400	20 000	16 600	26 200	21 900	27 500	21 900	19 300	22 600
Ironton, Ohio	27 700	20 900	18 500	232	6 740	6 310	11 700	13 600	8 860	12 300	12 700
Portsmouth, Ohio	4 880	5 310	6 160	5 420	5 100	3 560	2 360	3 400	3 340	3 650	4 320
Cincinnati, Ohio	3 720	3 990	3 380	3 010	411	2 040	3 710	3 540	4 120	13 600	4 150
Louisville, Ky.	3 690	3 860	1 970	2 090	558	1 220	2 500	3 680	1 350	3 620	2 450

It is of interest to note in Table 1 the yearly average numbers of *coli-aerogenes* bacteria recorded during the 10-yr period, 1926–1935, in the raw water supplies of ten municipal filtration plants along the Ohio River.[4] If judged by the aforementioned criterion, the raw waters at East Liverpool, Marietta, Ashland, Ironton, Portsmouth, and Cincinnati, in Ohio, are shown to be definitely over-polluted, and the water at Steubenville, Ohio, has reached approximately the safe upper limit of average pollution. During the extreme drought year of 1930, conditions at all the cities named are shown to have been considerably aggravated, except at Cincinnati, where they were improved to a marked extent, owing to the effects of long storage and sedimentation in the relatively unpolluted and canalized 100-mile stretch of river above this point. Consistently, gross over-pollution is indicated at Ashland and Ironton, where the effects of heavy local pollution in zones immediately up stream are particularly manifest.

In setting the permissible limits of pollution in river zones from which public water supplies are not taken, the problem is largely one of fixing conditions satisfactory for maintaining freedom from local nuisance, supporting normal aquatic life, and, in some instances, keeping certain areas fit for bathing and camping purposes. For prevention of local nuisances, it is primarily desirable to eliminate organic sludge deposits, whether they originate in sewage or in certain kinds of industrial wastes. It also is necessary to maintain dissolved oxygen in the stream proper at all times, to prevent septic action with ensuing odors. In order to support normal aquatic life, which has a vital function in maintaining natural purification and in the general fitness of waters for recreational uses, a minimum oxygen content sufficient to support respiration in the higher aquatic animals, notably fish, is necessary. Some authorities recommend an oxygen minimum ranging from 2.5 to 5 ppm, with the additional requirement that the reserve oxygen supply present in a stream should always exceed the bio-chemical oxygen demand. A fair average for this dissolved oxygen minimum, under present general requirements, appears to be about 4 ppm. Organic sludge deposits should be eliminated not only because they interfere with the normal spawning process of fish, but also because these deposits impose a heavy oxygen demand on the overlying stream.

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[3] "Limiting Standards of Bacterial Quality for Sources of Purified Water Supplies," by Harold W. Streeter, *Journal*, Am. Water Works Assoc., Vol. 27, September, 1935, pp. 1110–1119.

[4] For convenient access to these data, the writer is indebted to H. R. Crohurst, Senior Sanitary Engineer of the U. S. Public Health Service.

As the prevention of nuisance and the maintenance of normal aquatic life in streams are closely inter-related, it would appear reasonable to assume that a single requirement sufficient to meet both conditions should be imposed in all river zones where the provision of recreational facilities for the general public and the preservation of other "amenities," such as the use of adjoining shore areas for residential purposes, may be involved. With the exception of bathing, which would require a relatively high bacterial standard of water quality, the simple requirement that the dissolved oxygen content should never be less than the oxygen demand, nor less than a minimum of about 4 ppm, should be sufficient to maintain favorable conditions, with the added proviso that all organic sludge deposits should be eliminated from the stream channel. It also may be noted in this connection that a stream conforming in all zones to a reasonable requirement for protection of sources of water supplies, such as that stated previously in this paper, would be brought more or less automatically within a favorable oxygen balance requirement such as the one just suggested. Neither of these two requirements would be adequately rigid, however, to meet even the more lenient of the various bathing water standards now being followed in different parts of the United States. It is, indeed, a debatable question as to whether under present circumstances the greatly added expense of restoring a highly polluted river to a condition safe for bathing throughout its entire course would be economically justified, although the institution of general corrective measures designed to protect sources of public water supply probably would result in the reclamation for these purposes of some areas remote from immediate sources of pollution.

From the foregoing discussion it thus appears that, in general, limiting pollution requirements may be fairly simple, consisting of the following: (1) Fixing maximum permissible numbers of *coli-aerogenes* bacteria in stream waters at public water supply intakes; (2) setting minimum oxygen and oxygen balance criteria in local areas below sources of pollution; and (3) working toward the practical elimination of all sludge deposits originating in sewage and in putrescible industrial wastes. In addition to these requirements, there would be certain special ones, applicable only in certain cases, such as the elimination of toxic, highly acid or alkaline, and taste- and odor-producing substances, not ordinarily removable by water purification processes. In the

Ohio River, acid wastes from mines and steel mills and phenolic wastes from by-product coke plants have presented major problems in this respect, although systematic efforts are being made to eliminate these sources of pollution, through mine-sealing operations and the diversion of steel and coke wastes at individual plants.

Surveys of the Ohio River

Owing to the size and importance of the Ohio River System and to the fact that it constitutes both a drainage channel and a source of public water supply for a large population, this river and its main tributaries have long been considered as presenting one of the most complex and difficult problems of stream pollution control in the entire country. Recognizing this fact, the U. S. Public Health Service, under authority of an Act of Congress passed in 1912, conducted in the years 1914–1916 a survey of pollution along the Ohio constituting probably the most detailed and comprehensive study of its kind ever undertaken, either in the United States or abroad. In connection with this investigation, a thorough hydrometric and laboratory survey was made at a series of sampling stations covering nearly the entire course of the river from Pittsburgh, Pa., to its mouth. During the summers of 1914 and 1915, a detailed sanitary survey was made of every incorporated community of more than 8 000 population in the entire water-shed.

The results of this survey, which were published in three reports,[5] revealed the main sources of pollution of the river, including both sewage and industrial wastes, the normal ranges of flow in the main stream and major tributaries, and certain definite zones of retrogression and of recovery in the density of pollution of the river. Zones of retrogression included the densely populated Pittsburgh-Wheeling Section, and stretches immediately below Cincinnati and Louisville. Zones of recovery, owing to dilution and natural purification, occur below Wheeling and in the long and relatively unpolluted stretches extending from Portsmouth to Cincinnati, from Cincinnati to Louisville, and from Louisville to the mouth of the river. The effects of acid pollution, derived from coal-mine and acid-iron wastes, were clearly apparent in the Pittsburgh-Wheeling Section, in stimulating precipitation of suspended material in the river and in exerting a well-marked bactericidal effect on sewage and other wastes discharged in that section.

A later survey of municipal water purification systems along the Ohio River, which was carried out by the Public Health Service in 1923 and 1924, indicated that the most highly polluted zone of the river from which public water supplies are taken extends from a point below Huntington to one just above Portsmouth, a distance of about 35 miles, in which the effects of sewage from Huntington are augmented by those of the combined wastes of Ashland and Ironton. Next in importance from this standpoint was the zone extending from below the mouth of the Beaver River, about 25 miles below Pittsburgh, to Steubenville, 35 miles farther down stream. In this section are the water intakes of East Liverpool and Steubenville, where increasing difficulties in producing satisfactory purified water supplies have been experienced during recent years.

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[5] *Public Health Bulletins Nos. 131, 143,* and *146*, U. S. Public Health Service, Washington, D. C.

In general, this water purification survey[6] indicated that: (1) Water filtration plants in the Huntington-Portsmouth Zone are clearly over-burdened by sewage pollution; (2) the plants at East Liverpool and Steubenville are likewise over-burdened during considerable periods and also have difficulty at all times in producing palatable effluents; and (3) those at Cincinnati and Louisville, although not seriously over-burdened, are in a marginal zone between safety and danger in this respect. Without the aid of effective and continuous chlorination, no water filtration plant located on the Ohio River, or on a large number of its major tributaries, would be able, under present conditions of pollution, to produce effluents safe for human consumption or domestic use.

In the 15-yr interval between 1915 and 1930, the completion of Government dams in the Ohio River brought that river to a state of full canalization during periods of low water. In order to ascertain the effects of this canalization, and of the natural increase in tributary population, on the state of pollution of the river, a re-survey of its mid-section (from above Cincinnati to below Louisville) was carried out in 1930–1931 by the Public Health Service.[7] This survey showed that with full canalization in effect during summer periods of low water, conditions at water intakes located in or immediately below long and unpolluted stretches of the river were improved, but that during periods of freshets, when accumulated deposits of sludge are flushed out of the channel, conditions were markedly worse than would have been expected under continuous open-channel flow. In river stretches extending below large centers of pollution, the main effect of canalization during low-water periods appeared to have been to move the critical zone of pollution-density up stream toward these centers, with a corresponding temporary improvement in conditions farther down stream.

In addition to the foregoing studies, several more localized pollution surveys confined to limited areas have been made in the Upper Ohio River, including a survey of the Weirton-Wheeling Section by the West Virginia Department of Health, in July and August, 1932; a survey of the Ohio, Allegheny, and Monongahela Rivers in and near the Pittsburgh District, by the Pennsylvania Department of Health (September, 1932, to February, 1933); and a more recent study in the Pittsburgh District, in 1935, known as the Metropolitan Main Drainage Survey.

As a basis of stream-pollution control, the data now available on the main river probably need extension in a few respects, and more detailed information is required on conditions in some of the more highly polluted tributaries, notably those draining into the upper section of the Ohio River above Cincinnati. In general, the following supplementary data would appear to be desirable:

(1) Revised estimates of total, urban, and sewered populations in various sub-divisions of the water-shed, brought up to date from the 1930 Census.

(2) Complete data on industrial wastes and on the present extent of sewage and industrial waste treatment.

(3) Revised information on present velocities and time intervals of flow in the river under low-water conditions with full canalization in effect.

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[6] *Public Health Bulletin No. 172*, U. S. Public Health Service, Washington, D. C.

[7] *Loc. cit.*, *No. 204*, U. S. Public Health Service, 1933.

(4) Extension of local main drainage surveys along lines similar to those of the recent Pittsburgh survey.

(5) A complete survey of water surface areas and rates of atmospheric re-aeration in zones of the river below major sources of pollution and, more generally, in sections of the river in which the maintenance of a favorable oxygen balance is an important objective.

(6) A survey of the location, extent, and nature of organic sludge deposits in the river, particularly during periods of canalization.

The practical significance of Items (5) and (6) has been emphasized by the results of studies[8] made by the Public Health Service at its stream pollution research station at Cincinnati. It has been found that in stream waters polluted by sewage, or by underlying deposits of sewage sludge, the surface rate of atmospheric re-aeration (which is the most important limiting factor in determining the speed of natural purification) may be retarded measurably with sewage concentrations as low as 2 to 5%, which is within the range of local pollution density in many streams, including certain sections of the Ohio River. With higher concentrations of sewage (up to 25%), this retardation may reach 60% or more of the normal re-aeration rate. It also has been ascertained from the study that this retarding effect tends to be diminished by natural purification and also by artificial oxidation processes of sewage treatment, although it apparently is unmodified by simple chlorination. It has also been observed that the rate of oxidation of sewage sludge deposits under stream-flow conditions tends to be considerably lower than the corresponding rate for sewage matters in free suspension and solution.

The practical inference to be drawn from these results is that sewage pollution imposes a double burden on streams, by increasing the oxygen demand and retarding natural oxygen recovery; and that it is necessary to take into account both these elements in estimating the benefits to be attained by sewage and industrial-waste treatment in a particular river zone. The studies also point to the importance of sludge deposits as a burden on polluted streams and to the necessity of locating and eliminating these deposits at their sources, if a given stream is to be restored to a healthy condition.

Application of Survey Data

The applications made of survey data in formulating practical measures of stream-pollution control are so numerous and varied that any comprehensive discussion of them would extend beyond the limited scope of this paper. For the present purpose, a single example will serve to illustrate how such data may be utilized in studying a specific problem.

One of the important problems connected with the pollution of a large river system in which adequate protection must be given primarily to sources of public water supply, is the relative effects which various population groups discharging sewage into the river, either directly or indirectly, may have on pollution at down-stream points, notably those at which public water supplies are taken. In a river of the size and length of the Ohio, dilution and natural purification are dominant factors in the situation, although in this particular river the effect of acidity, chiefly from mine wastes, on conditions both in the upper river and at points down stream, is an important element to be considered.

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[8] Publication pending in *Sewage Works Journal.*

From the *B. coli* data given in Table 1, it is apparent that the density of bacterial pollution of the Ohio varies widely at different water intakes and that all the factors previously noted exert their influence. The data reveal little, except by inference, as to the relative responsibility of various sources of pollution for the conditions observed.

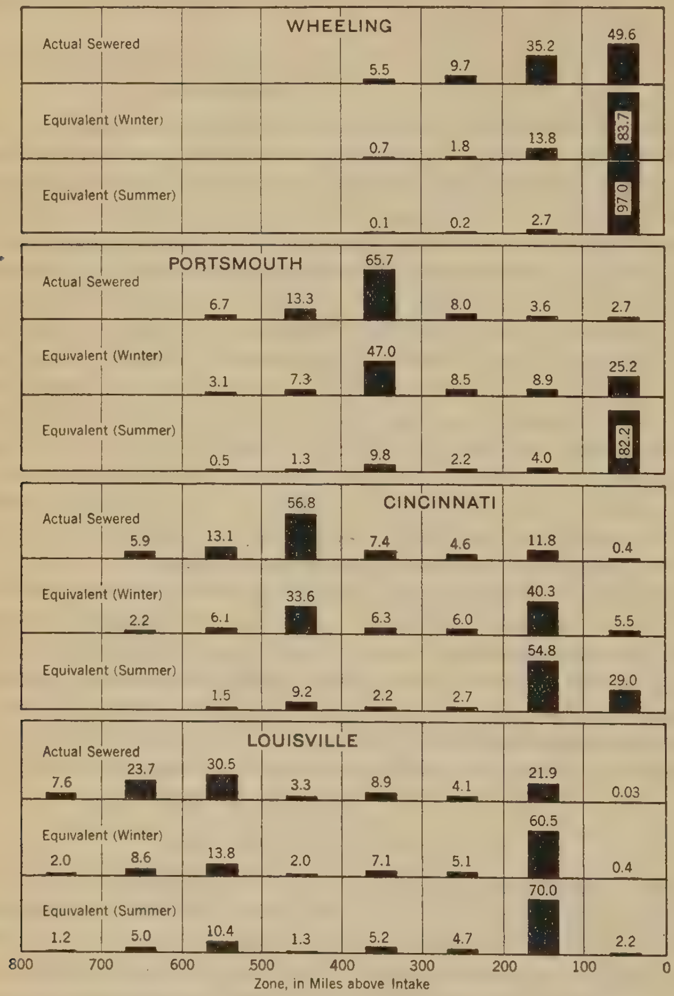

FIG. 1.—PERCENTAGE CONTRIBUTION OF UP-STREAM POPULATION GROUPS TO BACTERIAL POLLUTION AT FOUR WATER-SUPPLY INTAKES ON THE OHIO RIVER

In a paper given before the Society in 1927,[9] a method was described for determining the extent of such relative responsibility by "stepping down" the polluting effect of population groups located in successive river zones above a certain point in accordance with the observed mean time of flow from each zone to that point, and assuming that the reduction in the effect of each group would be measured along a time curve (varying with season and temperature) showing the rate of bacterial decrease in the river below sources of pollution. Curves of this kind were derived from extensive observations made by the U. S. Public Health Service in its 1914–1916 survey of the Ohio River, and were included in the paper mentioned.[9] By summing up the "stepped-down" population equivalents for all the river zones above a given point, a total equivalent population figure is obtained representing the population which, if it discharged sewage immediately above the point, would produce the same pollution effect as the actual population distributed in the various successive zones.

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[9] "Sewage-Polluted Surface Waters as a Source of Water Supply," by H. W. Streeter. (Presented at the Spring Meeting of the Society, Asheville, N. C., April 20, 1927. Not published by the Society.) Public Health Reports, June 15, 1928, pp. 1498–1522; Reprint No. 1232, U. S. Public Health Service.

In the present instance, a study has been made of the relative effects of sewered population groups draining into the river in successive 100-mile zones above the intakes at Wheeling, Portsmouth, Cincinnati, and Louisville. In Fig. 1 are shown the estimated actual sewered populations (as of 1930) in successive zones above each intake, expressed as percentages of the total sewered population above the intake. Directly beneath them are shown, for average winter and summer conditions, the relative effects of these population groups at each intake, expressed as percentages of the total "stepped-down" population-equivalent originating in the successive zones.

It will be noted that at Wheeling, a large proportion of the bacterial pollution is indicated as being due to the effect of the sewered population, aggregating about one-half the total above this point, draining into the first 100-mile zone up stream. Within this zone is included the population of the Pittsburgh District and the dense population draining into the Lower Beaver River. Passing down stream, the effect of this large population group is apparent at Portsmouth (300 to 400 miles above), at Cincinnati (400 to 500 miles), and at Louisville (500 to 700 miles), although in progressively diminishing proportion. At these three lower intakes, however, the predominant factor in the pollution of the river is shown to be the sewered population located within a river distance of about 200 miles up stream. Above Portsmouth and Cincinnati, the actual sizes of these groups are relatively small in comparison to those of the total populations draining above these points, being only 6.3% of the total at Portsmouth, and 12.2% at Cincinnati. Above Louisville, however, the effect of the Cincinnati District, located in the 100 to 200-mile zone up stream, is clearly apparent in the percentage of the total actual population in this zone (21.9%) and in its very large relative effect (60.5% of the total in winter and 70.0% in summer).

In order to obtain a rough idea as to the extent to which acid conditions in the Upper Ohio River above Wheeling might tend to modify the relative pollutional effects shown in Fig. 1, the same calculations were repeated, assuming reduction factors for all sewered population groups draining into the river above Wheeling. The factors assumed were based on the comparative results of measurements made by the Public Health Service at Pittsburgh and Wheeling in 1914, and at Cincinnati, Louisville, Peoria, Ill., and Chicago, Ill., in 1914–1916 and 1921–1922.[10] These measurements showed that because of acid conditions in the river, the measured effect of sewage from Pittsburgh and Wheeling on the *B. coli* content of the river was much less in proportion to contributing population than at the other cities noted, which were not thus affected and were considered as normal in this respect. The per capita contribution of sewage bacteria to the river in zones above Wheeling averaged 25.6% of the normal under winter conditions and 0.9% under summer conditions, these percentages being the actual ratios of the measured contribution at Pittsburgh to the mean of those observed at Cincinnati, Louisville, Peoria, and Chicago. In Fig. 2 are shown the results of this recalculation in comparison with those first illustrated in Fig. 1.

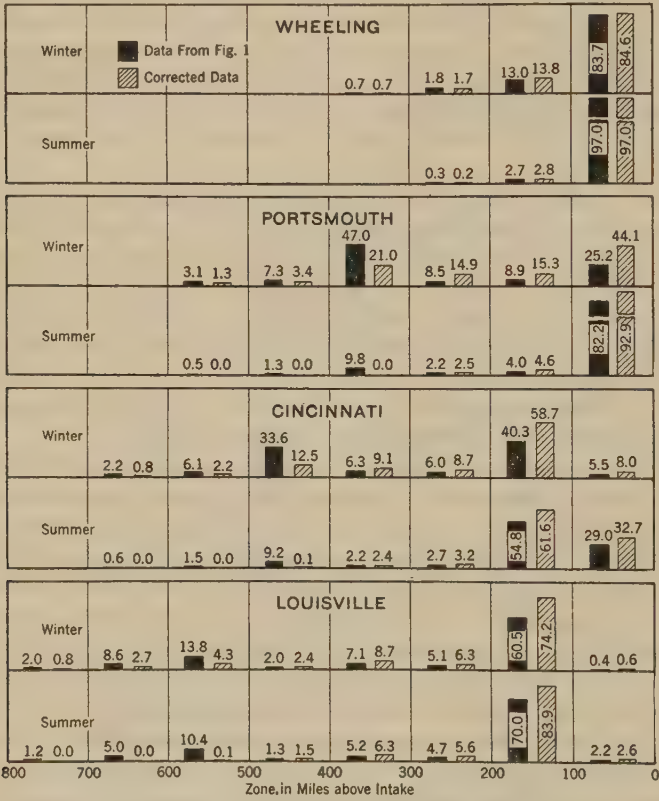

FIG. 2.—COMPARISON OF DATA IN FIG. 1 WITH CORRESPONDING DATA CORRECTED FOR EFFECT OF ACID WASTES DISCHARGED ABOVE WHEELING, W. VA.

Although the actual density of pollution in the river in terms of equivalent sewage-contributing population, was considerably lower when allowance was made for the influence of acidity above Wheeling, the relative effects of the several population groups were not modified to any large extent, except for a measurable reduction in the effect of the Pittsburgh-Wheeling Zone population group at Portsmouth, Cincinnati, and Louisville, as indicated in Fig. 2 by the smaller percentage of the total pollution originating in the 300 to 400-mile zone above Portsmouth, the 400 to 500-mile zone above Cincinnati, and the 500 to 700-mile zone above Louisville.

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[10] *Public Health Bulletin No. 143;* also, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 1366–1370.

In order to show quantitatively the estimated reductions in the total pollutional effects of the sewered populations draining into the river above the four intakes, Table 2 has been prepared, giving these totals for three assumed seasonal and flow conditions, both uncorrected and corrected for the influence of acid wastes above Wheeling.

TABLE 2.—ESTIMATED TOTAL SEWERED POPULATIONS IN 1930 ABOVE FOUR OHIO RIVER INTAKES AND THEIR CALCULATED EQUIVALENTS AT EACH INTAKE, ALLOWING FOR NATURAL PURIFICATION UP STREAM.

(Uncorrected and corrected for effects of acid wastes above Wheeling, W. Va.)

Intake	Sewered population above intake (estimated for 1930)	RESIDUAL POPULATION-EQUIVALENT AT INTAKE					
		Uncorrected for Acid Wastes			Corrected for Acid Wastes		
		Winter	Summer (high stages)	Summer (low stages)	Winter	Summer (high stages)	Summer (low stages)
Wheeling, W. Va.	2 186 500	569 150	391 280	167 080	155 800	3 680	1 465
Portsmouth, Ohio	2 846 500	150 790	34 960	18 630	86 680	31 700	16 480
Cincinnati, Ohio	3 152 400	148 850	36 010	12 040	102 320	33 430	10 700
Louisville, Ky	3 814 900	178 820	19 200	6 420	145 670	17 640	5 370

As would be expected, the most marked effect of the acid wastes is shown at Wheeling under summer low-stage conditions, although it is also apparent at all four intakes under winter conditions. The enormous decrease in the total sewered population-equivalent at Wheeling resulting from the correction for acid effect indicates the large extent to which the discharge of acid wastes above this point reduces the bacterial pollution of the river originating in sewage discharged at Pittsburgh and other points up stream. Under summer conditions the effect of acidity in the upper river would appear, from the comparative values, to exert relatively little influence on bacterial pollution of the river at Portsmouth and below this point. Under winter conditions, some effect is apparent, although of relatively small magnitude.

In Table 3 the population equivalents given in Table 2 have been converted to their corresponding numbers of *B. coli* in the river, using average volumes of flow obtained from the 1914–1916 survey by the Public Health Service and assuming per capita contributions of *B. coli* as derived by Mr. J. K. Hoskins,[11] from observations made by the Public Health Service at the cities previously enumerated. For comparison with these computed values are given the winter and summer average numbers of *B. coli* observed at each intake during the years 1926–1929, inclusive, when river conditions were more stabilized than during the drought years beginning with 1930. With a few exceptions, the calculated *B. coli* densities in the river are shown to be of about the same order of magnitude as those actually observed although somewhat higher, on the average, than the latter, possibly because the calculation has been based on average flow conditions prevailing before the river was fully canalized.

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[11] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1367.

TABLE 3.—CALCULATED NUMBERS OF *B. coli* IN OHIO RIVER WATER AT FOUR INTAKES, BASED ON POPULATION EQUIVALENTS IN TABLE 2, COMPARED WITH SUMMER AND WINTER AVERAGE NUMBERS OBSERVED DURING THE YEARS 1926–1929.

Intake	CALCULATED *B. coli* INDEX PER 100 CUBIC CENTIMETERS						OBSERVED *B. coli* INDEX AVERAGE, 1926–1929	
	Uncorrected for Acid Wastes			Corrected for Acid Wastes				
	Winter	Summer (high stages)	Summer (low stages)	Winter	Summer (high stages)	Summer (low stages)	Winter (December–March)	Summer (May–November)
Wheeling, W. Va.	49 900	238 000	400 000	13 700	2 240	3 760	2 960	4 470
Portsmouth, Ohio	6 380	10 300	19 400	3 680	9 350	17 200	5 630	5 580
Cincinnati, Ohio	5 710	9 580	10 800	4 220	8 900	9 700	3 130	4 040
Louisville, Ky.	5 260	4 440	3 480	4 290	3 200	3 710	2 180	3 150

Making due allowance for this and other possible sources of error, the quantitative relationships shown in Tables 2 and 3 and Figs. 1 and 2 appear to be fairly representative of those which can be expected under present conditions of flow and distribution of the sewered population along the Ohio River proper and its main tributaries. Among their more striking indications are the following:

(1) Under both summer and winter conditions of temperature and flow, a large proportion of the total bacterial pollution of the river at the various water intakes probably originates in wastes discharged within about 200 river-miles above these intakes.

(2) Pollution originating at more distant points up stream is a considerably larger factor in the condition of the river at these intakes during the winter than in summer, although it appears to be outweighed at all times by pollution from more immediate sources.

(3) Acid wastes in the upper river do not appear to exert much influence on the bacterial condition of the river at Portsmouth or farther down stream, but are a powerful factor within and immediately below the Pittsburgh-Wheeling Zone. In that region, in the absence of sewage treatment, the present acid condition of the river undoubtedly has prevented excessive bacterial pollution of raw-water supplies and a consequent intolerable over-burdening of water purification systems.

From these indications it might be inferred that efforts to reduce over-pollution of the river at various water intakes, or to prevent such over-pollution where it does not now exist, would logically be aimed primarily at sources of wastes discharged within about 200 river-miles above these points and, secondarily, at more important sources, such as the Pittsburgh-Wheeling District, farther up stream. The present benefit received from the presence of acid wastes in the upper river, although it appears to be confined largely to that section of the river and points immediately below it, is of such magnitude that the removal of these wastes, or their substantial reduction, doubtless will bring about a serious over-burdening of water purification plants both in this upper section and possibly at some points below Wheeling, unless provision is made for extensive treatment of sewage, now discharged into the river, both directly and indirectly, at points above Wheeling.

The space limitations of this paper do not permit a similar analysis of survey data on oxygen balance requirements, although it may be pointed out that such an analysis probably would be somewhat more localized in its character and would deal more especially with summer low-water conditions in river stretches extending within and immediately below major zones of pollution. Although a considerable amount of basic data for such an analysis is available from the two surveys made by the Public Health Service in 1914–1916 and 1930–1931, and from more recent local surveys, these data probably need considerable extension along the lines previously indicated.

In conclusion, it may be desirable to add that before any final program of corrective measures in the Ohio River Basin is undertaken, thorough study should be given to the question of adapting degrees and methods of sewage and industrial waste treatment to the various requirements to be met in relieving over-pollution of the river in different zones. These methods, in so far as sewage is concerned, may range from sedimentation and simple chlorination to complete biological oxidation, with chemical treatment possibly falling into an intermediate position. Consideration also needs to be given to the possible effects of various kinds of treated effluents on the river below their points of discharge, particularly in pooled sections under summer conditions. These and a few other similar questions are of general significance and fall more especially in the field of research, which in the long run must keep pace with that of the sanitary survey, as the two are virtually inseparable in meeting engineering problems of stream-pollution control.

LOW-FLOW REGULATION BY PYMATUNING RESERVOIR

By Charles E. Ryder,[12] Esq.

Synopsis

Pymatuning Reservoir, with a capacity of 8 365 000 000 cu ft, has been built in Northwestern Pennsylvania at a cost of approximately $3 700 000 for the primary purpose of regulating the flow of the Shenango and Beaver Rivers. A part of the project area is used as a Game Refuge, and there are opportunities for hunting, fishing, and other forms of recreation. These uses require careful evaluation of conflicting interests to determine how the project may be operated to serve best the needs of the region. Industries in the valley below require large quantities of water, especially for cooling purposes. The load on water-works filtration plants had been gradually increasing, and the State Department of Health was insisting upon the installation of sewage treatment plants to provide for the removal of settleable solids, oxidation, and chemical disinfection. Since the adoption of the reservoir plans, this high degree of treatment has not been required. Recreation interests demand that the reservoir be maintained at the highest possible elevation. Releases were made during the filling of the reservoir sufficient to satisfy the minimum industrial requirements. Since filling, tests under varying conditions of reservoir releases have been, and will continue to be, made at sampling stations established at a number of locations in the Beaver and Shenango Basins. The tests apparently show that the requirements for sanitation and public water supplies will be the criterion for releases, and a tentative operating schedule was adopted and used in 1937. Provision is made for flood regulation by the installation of flash-boards in the main spillway, and by maintaining the reservoir level below spillway elevation during the winter months. The reservoir was of material assistance in reducing flood heights during the floods of 1936 and 1937.

Pymatuning Reservoir (Fig. 3) is in the Beaver River Basin, at the headwaters of the Shenango River, in Crawford County, Pennsylvania, about 40 miles south of Lake Erie, 90 miles north of Pittsburgh, Pa., and 60 miles east of Cleveland, Ohio. When full, the reservoir covers an area of 26.5 sq miles, of which the greater part is the region formerly known as Pymatuning Swamp. It is 18 miles long, has a shore line of 70 miles, a maximum width of 2.2 miles, and an average width of about 1.5 miles. The maximum depth of water is about 35 ft and the capacity is 8 365 000 000 cu ft at the spillway elevation, 1 008 ft above mean tide at Sandy Hook, N. J.

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[12] Chf. Engr., Pennsylvania State Dept. of Forests and Waters, Harrisburg, Pa.

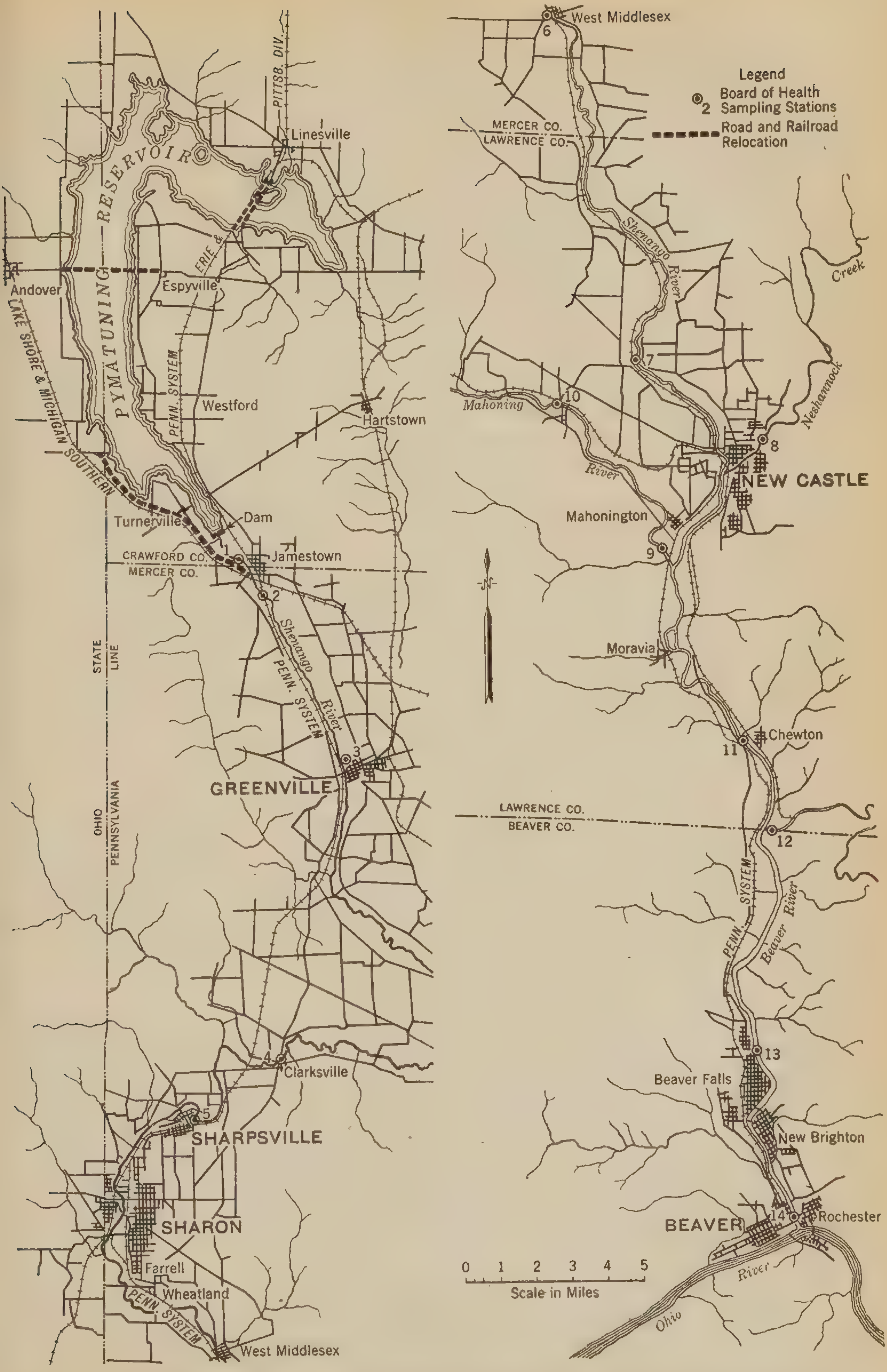
PITTSB. DIV.
Linesville
RESERVOIR
PYMATUNING
ERIE &
Espyville
Andover
LAKE SHORE & MICHIGAN SOUTHERN
PENN. SYSTEM
Westford
Hartstown
Dam
Turnerville
1
Jamestown
CRAWFORD CO.
MERCER CO.
2
Shenango
PENN. SYSTEM
River
STATE
LINE
OHIO
PENNSYLVANIA
3
GREENVILLE
4
Clarksville
5
SHARPSVILLE
SHARON
Farrell
Wheatland
PENN. SYSTEM
West Middlesex
West Middlesex
6
Legend
2
Board of Health Sampling Stations
Road and Railroad Relocation
MERCER CO.
LAWRENCE CO.
Shenango
River
Creek
Neshannock
7
10
Mahoning
River
8
NEW CASTLE
Mahonington
9
N
Moravia
Chewton
11
LAWRENCE CO.
BEAVER CO.
12
PENN. SYSTEM
Beaver
River
13
Beaver Falls
New Brighton
BEAVER
14
Rochester
River
Ohio
0 1 2 3 4 5
Scale in Miles

Fig. 3.—Pymatuning Reservoir and Parts of Shenango and Beaver River Valleys

History of the Project

The completion of the dam in 1934 culminated a long period of interest in Pymatuning Swamp. In 1868 the General Assembly of Pennsylvania ordered a detailed study of the practicability and expense of draining the area and reclaiming land for agricultural purposes. A survey was made and a favorable report submitted, but nothing further was done at that time. In 1907 the Pennsylvania State Highway Department was authorized to make a survey and study for draining the swamp and improving the highways. The resulting plans were submitted to the Water Supply Commission of Pennsylvania for approval, and hearings were held at which objections were raised on the ground that the swamp was a valuable water-storage area which helped to maintain the dry-weather flow of the Shenango and Beaver Rivers. The hearings quickened public interest in a proposal to convert the swamp into an artificial storage reservoir for increasing the dry-weather flow of these rivers, and the Commission finally refused to approve plans for draining the swamp. The Legislature in 1911 passed an Act appropriating $10 000 to the Water Supply Commission, directing it to examine the feasibility of a reservoir project. The Commission reported to the Legislature of 1913 that the construction of a storage reservoir was feasible, and that such a project would result in accrued benefits to the State at large, as well as to the communities along the stream. The estimated cost, including the dam and spillway, highway and railroad changes, purchase of land and buildings, clearing the reservoir site, and improving the shores, was $1 556 402.

Following this report, the Legislature in 1913 passed the "Pymatuning Dam Act," which directed the Water Supply Commission to erect a dam at the outlet of Pymatuning Swamp for the purpose of establishing a reservoir to conserve the water entering the swamp and to regulate the flow in the Shenango and Beaver Rivers. This Act carried an appropriation of $100 000. During subsequent sessions of the Legislature, various laws relating to the project were enacted, most of them carrying appropriations in varying amounts but frequently containing provisions which made it impossible to use the money. The first land purchases were made in 1921, but actual construction work was not started until 1931, when the Legislature appropriated $1 500 000 for the construction of the Pymatuning Dam and Spillway.

There were three major construction items, each let under separate contract; (1) The dam and spillway; (2) an embankment to carry a highway across the middle of the lake; and (3) an embankment to carry the Pennsylvania Railroad and a highway across the lake near its upper end. This latter embankment was built to form a secondary dam, thereby creating an upper lake, 2 500 acres in extent, which is maintained at an approximately constant elevation of 1 010 ft above sea level, or 2 ft higher than the spillway of the main dam. This water area and the adjoining State-owned land have been set aside as a wild life sanctuary 3 670 acres in extent.

An area of more than 20 000 acres of land was purchased in Pennsylvania for the project, and more than 5 000 acres in Ohio—the latter by private interests, principally steel companies of the Shenango Valley. About one-fourth of the lake extends into the State of Ohio.

COST OF PROJECT

The total cost of the project, as completed in 1934, was $3 717 739. In addition to the various State appropriations, $170 000 was advanced from Federal Reconstruction Finance Funds for use as a part of the clearing allotment which was completed with relief labor, and $415 000 was raised by the Pymatuning Land Company for the acquisition of Ohio lands.

Appropriations and donations were expended under the following general sub-divisions:

Surveys for feasibility of project		$ 10 000
Surveys, engineering investigations, and land appraisals		90 859
Land acquisition		1 101 756
Clearing Reservoir Site:		
State appropriations	$618 201	
Federal Reconstruction Finance Funds	170 000	788 201
Construction of dam		368 139
Construction of highway across reservoir		419 547
Relocation of railroad and highway		304 899
Relocation of State Highway Route No. 206		27 060
Reconstruction of Blair Road		1 894
Relocation of telephone lines		3 445
Reforestation of land around lake		5 745
Construction of measuring weirs on Sugar Creek and Shenango River		3 799
Improvements to buildings and grounds		6 538
Administration, supervision, and incidental expenses		170 857
Acquisition of Ohio lands by Pymatuning Land Company		415 000
Total		$3 717 739

PRESENT STATUS

On December 5, 1933, the gates were closed in the upper dam and on January 23, 1934, the four regulating gates were closed in the main dam. The upper basin, with a storage of 448 000 000 cu ft at Elevation 1 010, was filled and overflowing on March 4, 1934.

The main regulating gates in the lower dam were operated to some extent during the summer months of 1934, 1935, and 1936. During 1937 the reservoir has been operated to regulate floods and to increase low-water flows in accordance with a tentative schedule. The quantities of water released during 1934 and 1935, while the reservoir was filling, were based upon the actual quantities necessary for the operation of the industrial plants at Sharon, Pa., then operating at less than full capacity, and were much below the amounts which will be required to insure the minimum flow necessary in normal times for water supply and for sanitation purposes.

By March 24, 1936, the water reached the spillway level for the first time, and seven days later the lake attained its maximum stage at Elevation 1 008.81, due to rapid run-off from rains and melting snow. On March 28, 1936, the gates were opened to release the flood waters and to provide additional flood-storage capacity. They were closed again from May 13 to May 25, since which date they have been operated continuously until November 2 for the purpose of increasing the flow in the Shenango and Beaver Rivers. In the early part of 1937, the reservoir was operated to control floods, and from June to November to maintain a low-water flow of at least 200 cu ft per sec at Sharon, in accordance with a schedule of operations tentatively adopted.

Purpose of the Pymatuning Reservoir

The primary purpose of the reservoir, as stated in the original Pymatuning Act of July 25, 1913, and in later amendments, is to conserve the water entering the Pymatuning Swamp, and regulate the flow therefrom so as to maintain, throughout the year, as regular a flow of water as possible in the Shenango and Beaver Rivers. It was further specified that the reservoir, and the land surrounding it acquired by the Commonwealth, might be developed and used for fishing, hunting, game refuge, recreation, park, or other purposes; provided that such uses would not, in the opinion of the Water and Power Resources Board, materially interfere with the primary purpose.

The Water and Power Resources Board was vested with complete and final authority concerning the use and development of the land and water comprised within the project boundaries, and the maintenance and operation of the project. Later, it was provided that the Department of Forests and Waters, with the approval of the Water and Power Resources Board, might enter into agreements with other departments, boards, or commissions of the Commonwealth of Pennsylvania and the State of Ohio relative to the use of the reservoir and surrounding lands for fishing, hunting, game refuges, and other purposes.

It is clear that the mandate of the Legislature with respect to the operation of the reservoir was to store and conserve flood waters during the winter and spring months, and at other times when the natural flow was in excess of the needs in the Shenango and Beaver Valleys, so that such stored waters could be released to make the flow as uniform as possible during the summer and fall months, and at other times of deficient flow from the tributary drainage area. In other words, the Pymatuning Reservoir must be operated so as best to meet the needs of the valley below it for public water supply, sanitation, industrial water supply, navigation, water power, recreation, and esthetic purposes. In addition to such low-water control, it is equally certain that the Act provides for the regulation of the flow from the standpoint of the reduction of floods in the valley below the dam.

The Water and Power Resources Board must accordingly evaluate conflicting interests and determine how the project may be operated to best serve the particular needs of the valley. For example, to secure maximum flood-control benefits, sufficient storage capacity must be preserved in the reservoir to collect flood waters originating above the dam during the winter and spring seasons, and hold them back until the river down stream has receded to below flood stage. On the other hand, such dry storage capacity must necessarily reduce the total quantity of stored water which otherwise might be made available for release during dry seasons. It, therefore, reduces, to a greater or less degree, the efficiency of the reservoir as a regulator of the stream for water supply, sanitation, and other purposes. Fortunately, the water-surface area of the Pymatuning Reservoir is so great at spillway elevation that a storage depth of about 2 ft will afford complete control of the inflow under the worst known flood conditions. Hence, by providing such storage above spillway elevation the reservoir may be depended upon, with only a comparatively small loss in efficiency, to regulate the dry-weather flow.

Another very important phase of the project as a whole must be given consideration, namely, the use of the reservoir for fishing, boating, hunting, and other forms of recreation. For this purpose the lake level should be kept as nearly constant as possible. This requirement naturally is incompatible with releases during the summer season; hence, the reservoir should not be drawn down by releases more than is necessary to afford adequate stream-flow regulation. Although the Legislature was most careful to define the primary use of the Pymatuning Reservoir, nevertheless it must be borne in mind that the principal interest of the project to a large majority of the people in Western Pennsylvania and Eastern Ohio is the recreational value it will afford.

There are then three more or less conflicting purposes to consider in working out a schedule of reservoir operation. Fortunately, it appears from the studies made so far that the requirements for all three purposes can be met satisfactorily.

Need for Low-Water Control

There has long been a need for augmenting the low-water flow of the Shenango and Beaver Rivers. Prior to the completion of the reservoir the Sharon-Farrell District, 35 miles below the dam, with a population of about 45 000, had continuously been subject to shortages of water for domestic and industrial uses during dry seasons. The City of New Castle, Pa., about 20 miles below Sharon, with a population of approximately 50 000, and numerous other smaller intervening localities were likewise affected. The shortage was felt also in the Beaver Valley, where the cities and towns for a distance of 20 miles are rather closely grouped, with an aggregate population exceeding 50 000. The steel and other mills along both rivers require adequate water for economic and efficient operation. Periods of deficiency were a yearly occurrence and always curtailed operations at some of the mills. During dry seasons, water was re-used several times and as a result its temperature reached as high as 140° F. In the summer of 1933, the water was so low at Sharon that even by re-using it, there was not enough for operating purposes. The steel mills were considering closing down, when the small quantity of water that had been stored in the reservoir during construction was released and proved sufficient to tide them over the critical period.

Water-works taking their supplies from the Shenango and Beaver Rivers serve about 175 000 people, with intakes and filtration plants at Sharon, New Castle, West Pittsburgh, Beaver Falls, Pa., and New Brighton, Pa. Prior to the construction of the Pymatuning Reservoir, the Department of Health reported that the load on the water filters was gradually increasing. This condition was recognized as a menace to public water supplies, and was creating a nuisance in the Shenango and Beaver Rivers. At Sharon, flows as low as 8 cu ft per sec were experienced. Flows of less than 30 cu ft per sec were almost a yearly occurrence, and extended over periods of several months at least three times since 1930. The natural flow at New Castle was scarcely 20% greater than that at Sharon during drought seasons.

SEWAGE TREATMENT WORKS

Prior to the passage of the Pymatuning Act the Pennsylvania Department of Health properly insisted upon the installation of sewage treatment works by all municipalities discharging sewage into the Beaver and Shenango Rivers, or any of their tributaries, above the intakes of public water-works. The treatment plants were required to provide for the removal of settleable solids, oxidation, and chemical disinfection. The Department recognized, however, that with the Pymatuning Reservoir in operation, the releases during dry seasons would naturally improve the condition of the river water so that sewage-plant effluent could be safely assimilated with a lesser degree of treatment. Upon the basis of information furnished by the Water and Power Resources Board as to the amount of regulated flow which could be secured through the operation of the reservoir, it was decided that oxidation would not be required. Plans for sewage treatment works in various municipalities in the valley have accordingly been approved and works have been constructed that do not involve the high degree of treatment that otherwise would have been necessary.

LIMIT OF LOW-FLOW REGULATION

The original studies made to determine the maximum amount of regulation which could be secured through the operation of the Pymatuning Reservoir covered the period from 1908 to 1930, just prior to the actual construction of the dam. They did not include the series of dry years beginning in 1930. Within this period the most critical years were found to be 1922 and 1923, and during these years it was found that a minimum flow of 400 cu ft per sec could be maintained in the Shenango River, at Sharon, throughout the year without completely emptying the reservoir. However, to maintain this flow throughout the entire year would have meant a waste of water, as the full quantity was not necessary during the winter months. Accordingly, the calculations were revised to provide a minimum flow of 400 cu ft per sec at Sharon for the months of July, August, and September, 350 cu ft per sec through June and October, and 300 cu ft per sec for the remainder of the year.

Then came the drought years, beginning in 1930, and it was found that the reservoir, if operated on this schedule, would not have been fully replenished in 1930, 1931, or 1932, and that by November, 1933, the storage would have been exhausted.

As a result, a new analysis was made of the dependable yield which could have been secured from the reservoir had it been in operation for the critical period from 1929 to 1935. This investigation was based upon releases to maintain a minimum flow at Sharon of 200 cu ft per sec during the months of June, July, August, and September; 150 cu ft per sec during May and October; and 100 cu ft per sec during the remaining months of the year. With these releases the reservoir would have filled each year, and the maximum draw-down would have amounted to 3.9 ft. Corresponding computations covering the years 1908 to 1929 have not yet been made, but it seems probable that in normal years the fluctuation in the reservoir would hardly exceed 2 ft.

It is scarcely necessary to explain that the assumptions of minimum rates of flow to be maintained at Sharon are not to be construed as the actual flows which would have occurred throughout the period. The actual flow, particularly during the winter and spring months, would have greatly exceeded the minimum figures indicated, due to the higher rates of run-off from the tributary drainage area below the dam, and to overflow from the reservoir itself.

Tests to Determine Water Requirements

When the reservoir filled for the first time, in the spring of 1936, it became possible to release varying quantities of water and to study the resulting river conditions. Establishment of a definite program of reservoir operation has been delayed pending the completion of these tests, which are being conducted by the Engineering Bureau of the Pennsylvania State Department of Health. However, a tentative program was adopted in July, 1937.

Sampling stations were established at various locations in the Beaver and Shenango River Basins, as shown in Fig. 3. Then a series of test runs was made, with water released from the Pymatuning Dam to maintain as closely as possible a given specified flow at Sharon. The first four runs were as follow:

Dates of analytical traverses	Flow, at Sharon, in cubic feet per second	Release from dam, in cubic feet per second
June 3, 1936	111	88
June 15, 1936	215	140
July 17, 1936	240	200
August 27, 1936	490	93

The tests of the water made by the Department of Health included:

Alkalinity and pH: To determine the reaction of the stream before and after receiving industrial wastes, particularly pickling acid.
Dissolved Oxygen: To determine oxygen available to prevent nuisance by assimilating decomposing organic matter; also to determine whether stream was capable of supporting fish life.
Bio-Chemical Oxygen Demand: To determine the amount of decomposable organic matter present, and, by comparison with the results of the dissolved oxygen test, to determine whether the pollution load is greater or less than can be assimilated by the dissolved oxygen and still leave a safe oxygen balance.
Color: To determine effects of colored swamp water from Pymatuning Reservoir upon water-works.
Iron: To determine effect, other than oxygen depletion, of metallurgical industrial wastes.

The information given in Table 4, was furnished by the Department of Health with respect to sewage load and present methods of treatment in the larger towns in the Shenango and Beaver Valleys.

The studies indicated that the stream discharges tested were not large enough to provide sufficient oxygen in the diluting water to put the river in satisfactory sanitary condition below the sewage and industrial waste discharge from the Sharon-Farrell District and, at times, below New Castle and, again, at the mouth of the Beaver River; and in 1937, acting upon the advice of the Department of Health, a tentative schedule of reservoir operation was adopted to provide for a minimum flow at Sharon of at least 200 cu ft per sec, during June, July, August, and September; 150 cu ft per sec, during May and October; and 100 cu ft per sec, during the other months of the year. Further studies are to be made by the Department of Health.

TABLE 4.—PRINCIPAL SEWAGE LOADS IN SHENANGO AND BEAVER VALLEYS

Town or district	Population	Nature of effluent	Remarks
Sharon, Pa.	21 000	Treated	Removal of settleable solids
Sharon, Pa.	5 000	Raw	....
Farrell, Pa.	14 000	Raw	Primary sewage treatment works now under construction
In Ohio Near:			
Farrell-Sharon-Farrell, Pa.	3 000	Raw	....
Sharpsville, Pa.	5 000	Treated	Removal of settleable solids
Total equivalent population, raw sewage load	39 000	....	....
New Castle, Pa.	45 000	Treated	Removal of settleable solids
West Pittsburgh, Pa.	1 000	Treated	Removal of settleable solids
Elwood City, Pa.	12 000	Treated	Removal of settleable solids
Koppel, Pa.	1 000	Treated	Removal of settleable solids
Beaver Falls, Pa., and vicinity	30 000	Treated	Removal of settleable solids

At present it appears that the criterion for releases from the Pymatuning Reservoir will be the requirements of sanitation and public water supplies, and that if such needs are met, or met approximately, there will be more than sufficient water for industrial purposes. It also seems that the Sharon-Farrell District is the critical one with respect to sewage load and industrial wastes. The Department of Health is now taking steps to reduce materially both sewage and industrial pollution originating in this district.

The State has done its part in building the Pymatuning Reservoir to increase normal low flows in the Shenango River sufficiently to assimilate reasonably treated sewage and industrial wastes. The municipalities and industries should likewise be expected to co-operate in bringing about a reasonably clean stream, which will add materially to the attractiveness and economic advantage of the entire valley. It certainly should not be permissible for a small group of municipalities or industrial plants to continue to place a heavy pollution load upon the stream when, by taking reasonable measures to eliminate or reduce such loads, it would be possible to curtail greatly the releases and thus lessen the yearly fluctuation in the reservoir. There would seem to be too much interest by the public at large in the recreational use of the area for the State to countenance such gross and unwarranted pollution.

Indications point to a need, for sanitary purposes, of at least 200 cu ft per sec at Sharon and possibly more during the hot summer months. The requirements during the colder months will be much less, probably less than one-half the summer maximum. Industrial requirements for water for the Sharon District appear to be greater than for the New Castle and the Beaver Valley Districts, and these requirements during the most critical months of 1936 (June, July, and August) did not exceed 150 cu ft per sec and were closer to 100 cu ft per sec the greater part of the time. This is significant when it is recognized that those mills requiring by far the largest quantity of water were operating generally at 90% to 95% capacity during July and August.

Flood Control

Although this paper is limited primarily to the effect of the Pymatuning Reservoir in regulating stream flow for dilution purposes, it would not be complete without some reference to the flood-control aspects of the project, as the operation of the reservoir for one purpose affects its usefulness for others.

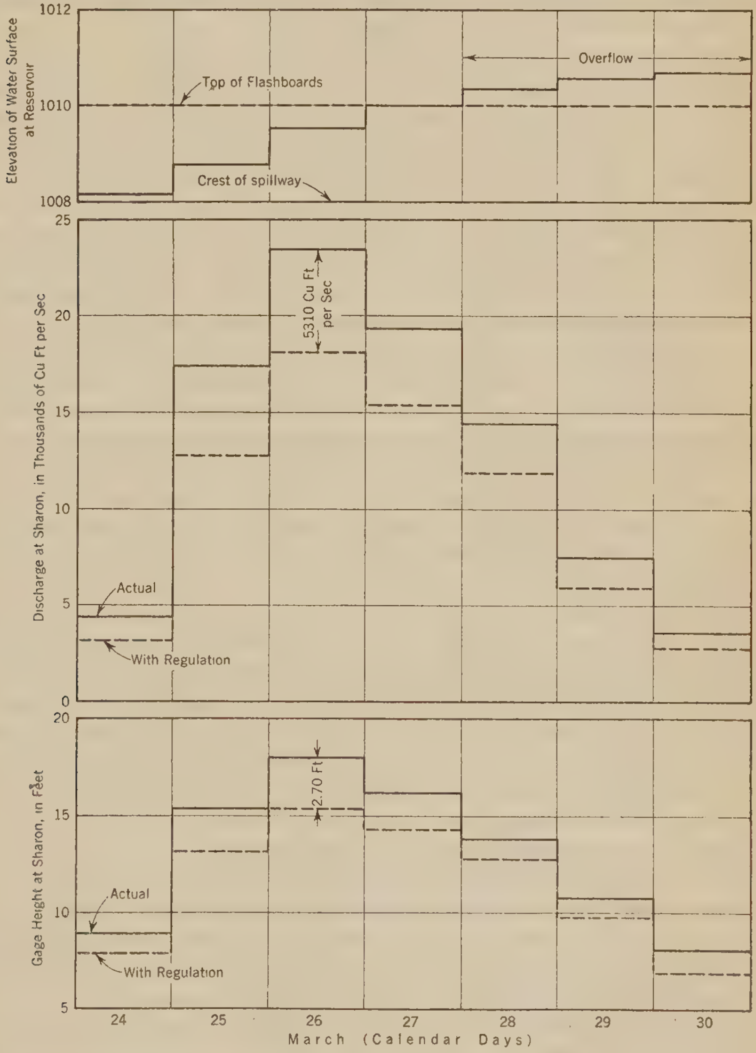

Fig. 4.—Hypothetical Effect of Pymatuning Reservoir on March, 1913, Flood at Sharon, Pa.

Thirteen disastrous floods have been experienced in the Beaver River Basin since 1806, the year of the first major flood of record. Little authentic information is available concerning them prior to 1884, although there is evidence that the flood of 1832 may have exceeded the two major floods of 1884 and 1913. This flood occurred before the valleys were densely populated and when few obstructions existed in and along the river channel.

The largest and costliest flood of recent years was that of March, 1913. Its magnitude and effect are comparable to that of the 1936 flood in other sections of Pennsylvania, and every city and town bordering the river was either completely or partly submerged. Bridges were swept away, railroad service was disrupted by numerous washouts, and dwellings, manufacturing establishments, and business places were inundated. Water-works were forced to shut down for several days, and on resuming operation pumped unfiltered water into the mains. Water entered gas mains and cut off the supply for fuel and light. The total reported loss exceeded $2 000 000.

The flood of March 17, 1936, did not extend to the Shenango and Beaver River Valleys, although other streams in the Ohio River Basin established new high-water records. Reports emanating from the valley below the dam credited the reservoir with preventing a disastrous flood at this time, but this was not true as the precipitation over most of the water-shed was in the form of snow, rather than rain. However, on March 25, a moderate flood occurred due to a warm rain and melting snow. The reservoir level was then 0.7 ft below spillway crest, and the storage capacity was sufficient to hold back the waters from the drainage area above the dam until the crest of the flood had passed Sharon. After the flood had subsided there was a small quantity of water passing over the spillway which, however, did not add to the flood height; later, the lake was drawn down gradually to provide additional storage in case another flood should follow. The reservoir reduced the discharge at Sharon approximately 25% and prevented overflow and flood damage in the business section of the city.

Studies show that 2 ft of storage above the spillway elevation of Pymatuning Dam would hold back flood waters from a storm equal in intensity to that of March, 1913 (the highest of record). Provision for such dry storage has now been made by installing a 2-ft weir in the spillway. The reservoir will be maintained at spillway level, or somewhat lower during the winter and spring months, by releases through the outlet gates. During flood flows the gates will be closed and the reservoir will then function to retard the inflow until the crest of the flood has passed out of the valley below.

The additional storage made available by the weir amounts to 1 297 000 000 cu ft, and would reduce the crest of a high flood at Sharon by 2.5 to 3 ft. Actually, the reservoir might be maintained about a foot or two below spillway crest during the flood season, and thus provide an additional factor of safety, without detriment to the project as a regulator of low-water flow.

Fig. 4 illustrates the effect the Pymatuning Reservoir would have had in reducing the March, 1913, flood, at Sharon, with a 2-ft weir in the spillway of the dam.

Benefits of Reservoir in 1936

During 1936, the first year of experimental operation, benefits derived from the reservoir at least met expectations as to the value and usefulness of the project. Tests of both its flood-control and low-flow replenishment features came quickly.

During the summer of 1936, the Beaver River and, especially the Shenango River Basin, experienced a very dry season. Although the drought was not so severe as that of 1930, nevertheless, it threatened to approach that condition and was, in effect, comparable to those of 1932 and 1934. Starting with a full reservoir, the gates were operated continuously from May 25 to November 2 to meet demands, primarily, in the Shenango Valley. During this period the average release was 100 cu ft per sec, and at no time was it less than 50 cu ft per sec. The average resultant flow maintained at Sharon was 175 cu ft per sec, and the lowest daily flow was about 100 cu ft per sec. Had there been no storage to help, the flow at Sharon could not have exceeded 30 to 40 cu ft per sec during the greater part of the time and would probably have been less over short periods.

Flood-control operations of March, 1936, have already been mentioned. Although the March run-off in that year failed to approach a record high stage, it was high enough to produce a secondary flood along many stretches of the Shenango River. Without the reduction in stage effected by the Pymatuning Reservoir, the damage in this valley would have been considerably greater than actually occurred.

At the end of 1936, consensus of opinion in the districts benefited could have been summed up in the expression, "We feel that the dam has already paid for itself."

Until the Engineering Bureau of the State Department of Health completes its investigations and submits its final report to the Water and Power Resources Board, no definite plan will be established for rates and periods of release from the Pymatuning Reservoir. Even after a plan of operation is adopted, it will probably be necessary to modify it as actual experience is gained from year to year, and as conditions change in the valley below with respect to public and industrial water supply, sanitation, etc. It is certainly to be expected that, under efficient regulation by the Sanitary Water Board, the pollution load will become no greater than it is at present, despite probable increases in population and industry.

SANITATION PROBLEMS INCIDENTAL TO FLOODS

By W. L. Stevenson,[13] M. Am. Soc. C. E.

Synopsis

In time of flood, public health authorities and municipal and water-works officials are confronted with a variety of emergency sanitation problems that demand immediate attention. It is important to be prepared in advance. This paper outlines the major factors in such planning, and makes suggestions for the conduct of activities during floods and as the waters recede. Experience in Pennsylvania in 1936 is drawn upon for many of the details.

Although flood-control reservoirs can reduce maximum flood stages, and dikes can safeguard more or less limited areas against inundation, it requires large sums of money and long periods of time to develop such projects. In fact, it is inconceivable that all rivers can ever be controlled so that flood stages will not occur at times. Therefore, preparation to meet the sanitation problems incidental to flood disasters must still be made.

For the purpose of this paper, the word, sanitation, simply means the keeping clean of the environment of Man or making it clean; and, in this sense, cleanliness means the absence of dirt resulting from Man's life and work.

Rivers draining developed areas must receive and carry away sewage, industrial wastes, and the rain-water run-off from towns and cultivated fields. Where sewage and some industrial wastes are inadequately treated, and the receiving stream at normal or low stages is too sluggish to be self-cleansing, then sludge deposits form on the bed of the river. Consequently, flood waters are always dirty, and more or less befoul inundated communities and water-works, thus creating a serious emergency sanitation problem.

The principles of sanitation are well known and in daily use. What creates the acute problem in sanitation incidental to floods and makes difficult the application of these principles, is the need for immediate action and the difficulties of transportation and communication. Seemingly everything must be done at once and everywhere at the same time, but floods prevent normal transportation and communication. Men and materials immediately needed in the stricken areas have to be taken over long detours because of inundated highways and railroads and washed-out bridges. The enormous load of business upon telephone and telegraph wires caused by flood conditions swamps their reduced capacity and makes it difficult even to ascertain the sanitary needs of the flooded areas.

The fact that floods inevitably produce these conditions makes apparent the importance of preparing and planning for preventive and corrective sanitary measures before the next floods actually arrive. This paper considers the problem in three steps: (1) Present planning and work; (2) things to be done during floods; and (3) things to be done as the waters recede. The suggested course of action is predicated upon the doctrine of home rule; that is, the primary responsibility should rest upon local authorities, with State agencies guiding and correlating activities and helping as far as possible, especially in the rural areas and smaller municipalities.

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Present Planning and Work

Water-Works.—In recent years, the trend in public water supply projects has been toward the development of impounded, relatively clean, upland sources delivered by gravity, in preference to the use of polluted river water that requires a high degree of purification, and that must be pumped as well. Those in charge of water-works subject to danger from floods should study the practicability and advisability of obtaining upland water, so as to eliminate the flood menace to low-lying water-works, and simultaneously secure a superior water supply. Even with an upland source, however, care should be exercised to safeguard all the mains at places where they may be endangered by floods; otherwise, the areas served by such mains will be no better off than if they were served by a low-level source of supply.

In Pennsylvania, during the floods of 1936, most of the communities with upland gravity supplies not only had water during the flood, but also had enough water in storage for clean-up purposes afterward. The opposite condition occurred at many of the low-level water-works, which were flooded.

Adequate storage capacity for purified water is always needed to meet drought conditions. For this purpose the storage facilities of many water-works have been enlarged since the droughts of 1930 and following years. Nevertheless, during the March, 1936, floods in Pennsylvania, many storage reservoirs were exhausted before the emergency was over, despite the fact that they were well filled before the flood, and that restrictions were enforced to prevent waste while the water-works were shut down.

Officials of such water-works should now give attention to their storage facilities, and if they find them to be inadequate, they should take the needful steps to enlarge them as a safeguard against future drought or flood.

Of the 106 water-works adversely affected by the floods of 1936 in Pennsylvania, the following parts were put out of operation: Pumps in 61 plants; chlorination apparatus in 35; clear wells in 23; filters in 13; and chemical feed apparatus in 13. Inundated motors had to be baked, which caused delay in resuming operation. Flooded switch-boards took time to clean. The flooding of filters or clear wells was most dangerous.

Water-works officials should study their plants to determine whether the vital parts can be raised above maximum flood height. If raising, in whole or in part, is found to be impracticable, then consideration may well be given to providing substantial flood guards for doors and windows of low-lying pumping stations, which could be put in place when floods occur. The building of dikes might also be considered. However, if the top of a dike is not surely above maximum flood height, then after the flood the dike will retain the flood waters and cause further delay in getting the water-works back into service. A sluice-gate set at a low elevation in such a dike would be a help in draining impounded water.

Other matters to be considered are auxiliary sources of power, duplicate and well-protected water mains under rivers (with proper valves at each end to prevent loss of water in case of pipe failure), adequately valving the distribution system to provide flexibility of operation, and emergency connection with the mains of a near-by water system.

The doctrine of home rule applies to water-works. It is the duty of water purveyors to have competent works operators, capable on short notice to take charge of emergency work, and to handle their plants successfully when disaster threatens.

Municipalities.—In some municipalities with public water supplies the residents of areas subject to flooding still obtain their drinking water from private wells. In such cases, consideration should be given to extending the public water mains into the low-lying parts of the town site. Flooded private wells are a grave menace to health, and the best way to eliminate the hazard is to replace them with a safe, piped, public water supply. Meanwhile, the local board of health should prepare a map showing the location of all private wells in areas subject to flooding, so as to be prepared, when floods come, to direct their cleansing and sterilizing without delay.

No privies or cesspools should be permitted on properties to which a public sewer is accessible, for they are an actual or potential health menace at all times, and especially when flood waters carry the night soil into water wells and into houses.

In Pennsylvania, the clean-up of flooded dwellings in 1936 was markedly expedited by help from Federal Work Relief agencies, such as the Works Progress Administration and the Civilian Conservation Corps. It may be that when severe floods next occur, these valuable aids may not be available. Therefore, municipal officials would do well to make general plans for flood clean-up work and file them away, so as to be prepared to handle the job with their own facilities and labor. Then when the emergency comes they will be ready to proceed with a minimum of State supervision or aid. In cities such plans might include the establishment of refugee centers, first-aid stations, and emergency hospital facilities. It is not to be expected that the small municipalities can handle such a problem without help.

The State.—There are some services which the State Government can and should render in flood disasters. First, it should supply forecasts of the flood stage and time of cresting, in order that local authorities may be duly forewarned. Further, the National Guard can augment local police and completely take over police duties in the rural districts to prevent looting. The State Health Department can correlate measures for the prevention of disease and take charge of public health work in the rural districts and weaker municipalities.

Considerable planning should now be done along these lines, in order to be generally prepared for future floods. This preparatory work may include urging municipal and water-works officials to make suitable and practicable plans for meeting flood disasters in their own communities.

Hydrography.—After the March, 1936, floods in Pennsylvania, U. S. Army Engineers and the Pennsylvania Department of Forests and Waters in cooperation with the U. S. Geological Survey collected as much information as possible along certain main rivers as to the maximum height reached by flood waters and the day and hour of cresting. Levels were then run to ascertain the elevation of these marks. From these data maximum flood profiles were plotted and the time of transit of the crest height from place to place was determined.

Such information is so valuable in planning for the future that where it has not been undertaken as thoroughly as in Pennsylvania it should be begun at once. (Plans should also be in readiness, in case of future high floods, for establishing high-water marks on stable structures as soon as possible after the crest has passed.) In connection with this work it would be helpful to locate the horizontal limits which the flood waters have reached in built-up communities. With such data available, municipal and water-works engineers can make plans intelligently for changing existing structures or designing new ones, as far as floods are concerned.

The maintenance of stream gaging stations at strategic points is important. At stations that are difficult of access at flood stage, auxiliary gages should be established well back from the normal channel of the river in order to assure positively continuance of the record during the period when it is most needed for flood forecasting.

The value of accurate flood forecasting to health authorities, to waterworks operators, and to the public at large, is so great as to warrant the development of a nation-wide service organized to the maximum possible degree of efficiency and dependability.

When the flood emergency comes, it is vitally essential that gage-height readings be reported promptly to the central authority. This indicates the wisdom of the special arrangements that have already been made with telephone companies in some States, to maintain telephone service as long as possible during floods between the gage readers and the central office of the hydrographers. Consideration is also being given to the use of radio for this purpose.

Health Departments.—State Health Departments should prepare for future floods by making plans for establishing, equipping, and manning first-aid stations in flooded districts, and for the distribution of needed biological products. Mobile bacteriological laboratories have proved to be valuable in the ordinary conduct of the business of State Health Departments, and they are especially valuable in flood-emergency work. Therefore, it would be well for State Health Departments not now provided with such facilities to endeavor to obtain them.

The number and location of Highway Department and National Guard mobile water tanks naturally change from year to year. It is futile, therefore, to prepare an inventory of such equipment and make definite plans, long in advance, for their use in hauling drinking water to stricken communities during a severe flood. However, where arrangements do not already exist between State Health Departments and such other branches of State Government for the emergency use of such equipment, it appears wise to make such arrangements in a general way, chiefly for the purpose of having immediate co-operation when the emergency comes.

Health Departments should also arrange with telephone companies for the maintenance, as long as possible during floods, of telephone service between the central office of the Department and the headquarters of important field agents, such as district engineers, medical officers, etc.

Things to Be Done During Floods

When it appears that floods are likely soon to occur, the State Health Department should notify all local Boards of Health as to their emergency duties, and give them advice on such matters as cleansing flooded water wells, privies, cesspools, and inundated dwellings and their surroundings. Particular emphasis should be laid on sanitation of milk stations, and warning the public to boil all water to be used for drinking purposes. Similar instructions should be issued to field medical officers of the Health Department as to their duties in rural districts where there are no local Boards of Health.

The Health Department Engineers should warn officials of water-works to store in reservoirs as much purified water as possible and take emergency measures to prevent, as far as possible, submergence of low-lying pumping stations and filter plants; also, where practicable, to make arrangements for temporary connections with the mains of near-by water systems less likely to be affected by the coming flood.

Industrial concerns are often large consumers of the public water supply, and if conditions justify, service to them should be temporarily discontinued. Householders should also be requested to save water, but hospitals and hospital laundries should not be restricted, as water is essential for care of the sick.

As soon as the State Hydrographers receive reports of rising river stages, the data should promptly be transmitted to the main office of the Bureau of Engineering of the State Health Department, whence it should be relayed to the Department's engineers in the field: First, for their information as to which rivers are rising toward flood stage; and second, in order that they may notify the low-lying water-works and communities of the coming danger. At the same time these field engineers should gather as much information as possible on river stages, from water-works or other sources, and report it back to the central office for prompt transmission to the hydrographers to augment their available data. As river stages approach flood magnitude the State Hydrographers should furnish to the Health Department Engineers their forecasts of flood-crest height and the time of cresting at various places. This procedure was used in Pennsylvania during the March, 1936, floods and proved to be of great value to the Health Department in preventing epidemics of disease.

When floods are imminent, but always before inundation occurs, every possible effort should be made by the State Health Department, by local Boards of Health, and by municipal and water-works officials to warn the public to boil all water to be used for drinking purposes. This can be done by means of newspapers, radio, printed notices posted in conspicuous places, and projected on the screen in motion-picture houses, and in all other ways that will receive the attention of the public. In foreign-language communities the warning should be given in the language understood by the citizens.

In recommending a universal warning to boil all water to be used for drinking, it is recognized that a seeming injustice is done to those public water supplies that are able to continue to furnish an entirely safe water during the flood. However, to qualify the "boil water" warnings by specifically including certain public supplies feared or known to be unsafe, and excluding other public supplies believed to be safe, would bewilder the public and lead to confusion thrice confounded. The universal warning was used in Pennsylvania in 1936 and is believed to have played a considerable part in the prevention of disease. There were no epidemics reported from the flooded area.

As soon as it can be determined which water-works are likely to be adversely affected by the coming flood, an inventory should quickly be made of available motorized water tanks for use in hauling pure drinking water to communities whose public water supply fails. These tanks can usually be obtained from State and municipal highway departments, the National Guard, and possibly from milk companies, and gasoline companies.

Where water must be hauled long distances, railroad tank cars may be used, but probably the actual distribution of such drinking water in the stricken town would have to be made by motorized or horse-drawn tank wagons. Every tank to be so used should be thoroughly cleansed and then disinfected with a chlorine solution or by steam under pressure. As a safeguard, it is wise to add a proper amount of a solution of chloride of lime to the drinking water in these tanks at the time they are filled, in order that the germicide may have adequate time to act before the water is served to the public.

Before the floods actually come, officials of municipalities and water-works should make arrangements with vendors of lime and chlorinated lime, so that when the emergency arises telegraphic orders may be placed for the immediate delivery of these disinfectants. The State Health Department should do likewise in order to insure quick delivery of such supplies to flooded rural districts and small municipalities.

Where properly trained persons are available, high-test chlorinated compounds may be used to advantage because of their more definitely known strength, lesser bulk, and better keeping qualities. However, if the chlorine compounds are to be handled by untrained persons (who are likely to use far more than is necessary), then the ordinary bleaching powder is preferable.

As the floods develop, and as soon as a reasonable forecast can be made as to the areas likely to be affected, the State Health Departments should station their mobile laboratories at strategic locations for the prompt bacteriological examination of public and private water supplies.

Things to Be Done as the Waters Recede

After a flood there are hundreds of things to be done, and all of them should be begun simultaneously. More men, materials, trucks, and tools are needed than can be furnished simultaneously throughout the flooded area. Then comes the test of previous careful planning to meet the emergency.

Officials of municipalities, water-works, and the State should, therefore, begin to plan for the clean-up just as soon as it can be forecast what places are likely to be inundated. Then rehabilitation can begin as soon as the flood crest is past. The major sanitary problems at that time are chiefly related to rehabilitating public water-works, cleansing milk stations and private wells, and cleaning up town sites.

Measures to Be Taken by Public Water-Works Officials.—The cleansing and rehabilitation of public water-works is the most important sanitary measure, for widespread disease will surely follow the drinking of a contaminated public water supply. It is not necessary to describe here the innumerable things that must be done to put flooded pumping stations back in service. In particular, however, it may be noted that filters and clear wells must not only be cleansed but sterilized—generally by means of a solution containing free chlorine. Chemical feed and chlorine apparatus which have been under water will need careful attention if they are to function properly when the remainder of the plant is ready for operation. When all these things have been done and the filter plant has been started, the water should be extra heavily chlorinated, and wasted for a while through fire plugs or blow-offs, to insure a good chlorine residual when it is again sent through the distributing system to the consumers. Meanwhile, field crews should be at work repairing broken mains, fire plugs, and house connections.

Speed in the resumption of service must not take priority over safety of the quality of water delivered to consumers. The warning to boil all drinking water should be continued after the flood until a sufficient number of samples of the water throughout the system, upon analysis, show the quality to be perfectly safe.

Cleansing Private Water Wells.—All mud, muck, and débris should be removed from the top of the well and its surroundings. It should then be pumped to waste and a strong chlorine solution applied and allowed to stand not less than 24 hours and preferably 48 hours. Where practicable, and especially where the well cover is not tight, the inside walls of the well should be scrubbed with a chlorine solution. After these things have been done, the well should again be pumped to waste until the ortho-tolidin test shows that chlorine is no longer present. A sample should then be taken for bacteriological analysis.

In this work mobile bacteriological laboratories are of great value. Following the 1936 floods in Pennsylvania, 10 000 samples of public and private water supplies were analyzed in the three mobile laboratories of the State Health Department and 2 000 in the Philadelphia Laboratory. This is about the same number as are normally analyzed during an entire year in the Department's Laboratory.

In cleansing private water wells it will be found that many of them were actually unsafe prior to the flood. After the flood is the psychological time to secure improvements in those found to be faulty in construction.

Milk Supplies.—No raw milk affected by the flood should be served raw. If a pasteurizing plant is flooded, the milk normally delivered to it should be diverted to some near-by unaffected plant the capacity of which can be increased by operating longer hours than usual. Often the employees of the flooded plant can work temporarily in the other one.

Flooded milk stations and pasteurizing plants must be thoroughly cleansed and all milk equipment, bottles, etc., sterilized. The plants should not be put back in operation until all the equipment is in good mechanical working order and all sanitary features have been checked by health authorities having jurisdiction over milk supplies.

The distribution of milk from feeding stations in the flood zone should be restricted to bottled milk.

Cleaning Up Towns.—Men, trucks, shovels, lime, chlorinated lime, and an orderly plan of procedure are needed to remove and dispose of the silt, mud, and débris left by the flood waters in inundated communities. Fortunate is the town whose water-supply reservoirs are full at this time so that an abundance of water under pressure is available for flushing.

Cellars must be pumped out, rubbish removed, and quick-lime applied. The devastation and filth in the rooms of flooded houses are appalling. Here cleansing had best be followed by the use of a solution of chlorine made from concentrated compounds or ordinary chlorinated lime.

In unsewered communities the flood waters will have destroyed privies and washed their night soil on to the surrounding ground. All exposed night soil should be sprinkled with quick-lime or chlorinated lime, according to local circumstances, and buried as soon as practicable at a location which will not menace any water supply. The prompt providing of temporary Army-type latrines is essential to prevent promiscuous defecation until permanent sanitary privies are rebuilt.

Suitable sites must be selected for dumps for the filth and débris hauled away from the flooded town. There is a strong temptation to dispose of such materials in the easiest way by dumping them into the nearest stream that may still be at high stage. If possible, this should not be done, as it only tends to befoul some place down stream, or leave the river banks dirty and littered when the stream returns to normal stages.

Conclusion

Flood emergencies try Men's souls and exhaust their bodies. Speed, sympathy, and service are essential. Above all else is needed co-operation and *esprit de corps.* The stricken people in the flooded areas must have confidence in those who are laboring to help and protect them. Of what avail is it for health authorities to warn that drinking water should be boiled and contaminated foods should not be eaten, if the public does not co-operate by heeding the warning?

With skill, zeal, and hard work on the part of those in authority, and co-operation on the part of the public, sanitation work in flood emergencies has prevented and again can prevent epidemics of disease which otherwise are sure to occur. Before the next flood strikes is the time to prepare to meet the sanitation problems it will produce.

PROGRESS IN POLLUTION CONTROL IN THE OHIO RIVER BASIN

By E. S. Tisdale,[14] Esq.

Synopsis

Progress in the control of stream pollution in the Ohio River Basin is summarized from four angles: (1) Plans for sewage treatment; (2) handling of detrimental industrial wastes; (3) development of public opinion favoring water conservation; and (4) Federal laws and compacts.

Under Angle (1), a warning is sounded that something must be done, for in critical drought periods the sewage pollution is becoming too great a burden for even modern water purification plants to cope with. Records show that $12 000 000 have been expended under Public Works Administration and Works Progress Administration programs to improve sewage disposal along the main stem of the Ohio River.

It is pointed out under Angle (2) that co-operative programs between the United States Public Health Service and the States have overcome detrimental phenol pollution and are now reducing acid mine drainage from abandoned bituminous coal mines by air-sealing. The total daily acid mine drainage is estimated to be approximately 15 000 000 lb.

Under Angles (3) and (4) the gradually awakening public opinion is touched upon. The program started as regional water planning in the Cincinnati area. Compact agreements were proposed, and the consent of Congress was obtained for States to work together to reduce stream pollution.

The development of public opinion led to the formulation of definite legislation of a national character, upon which hearings were held. This legislation did not pass in 1936, but is being actively considered by the Congress in 1937.

To obtain a tangible concept of the canalized Ohio River, one may imagine a series of steps or boxes filled with water (Fig. 5). The difference in elevation between the top step, or box, located at Pittsburgh, Pa., and the lowest one, at Cairo, Ill., is 413 ft. In length, this series of fifty-three boxes, with a Federally operated dam in the lower end of each, extends more than 900 miles. Into this canalized river the tributary streams, large and small, empty their waters. The entire drainage area is 200 000 sq miles. It embraces parts of eleven States, having a combined population of about 17 000 000. Great industrialized cities have developed along the river and its tributaries. At the head-waters lie the richest bituminous coal fields in the world and from them comes an enormous pollution load of acid mine drainage.

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The progress in pollution control in the Ohio River Basin may be gaged by the answers to four questions:

(1) What progress is being made in the reduction of pollution caused by human sewage from cities?

(2) What progress has been achieved in the control of industrial wastes?

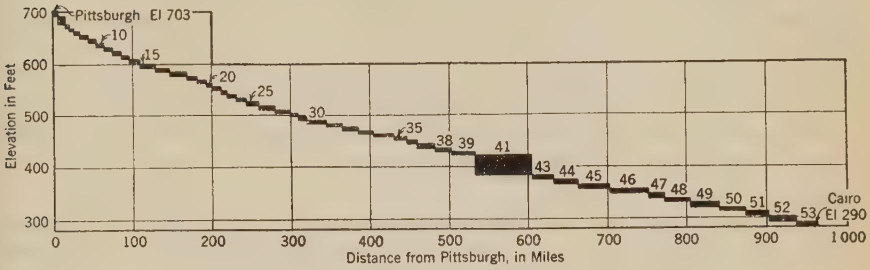

Fig. 5.—Slack-Water Improvement of the Ohio River

(3) What progress can be shown in the development of a "water-shed consciousness," involving co-operation of Federal, State, and local agencies, in conserving the waters of the basin for their varied and most useful purposes?

(4) What progress has been made which will permit co-operative effort of States and local governments in line with comprehensive Federal and interstate plans of pollution control?

Progress in Sewage Pollution Control

The first of these "What Progress" questions deals with sewage pollution. It is the most important of them all, because more than 5 000 000 people obtain their drinking water from the water resources of the Ohio River Basin and nearly 2 000 000 from the Ohio River itself. The practice of the past has been for the cities on the Ohio River and its tributaries to conduct their sewage wastes directly to the stream without purification, and it is still the prevailing method in 1936. The cities have considered it their right to utilize the dilution of the large stream as a satisfactory method of disposal.

Although pollution from this source has been increasing, the science of water purification has gone forward, and thus far sanitary engineering has triumphed. Due to the design, construction, and skilled operation of water filtration systems, the public has been protected against water-borne diseases, notwithstanding the pollution of the river. Nevertheless, that there are limits beyond which streams cannot assimilate organic wastes, particularly human sewage, was amply demonstrated during the critical drought periods of 1930–1931, when it became impossible to produce a satisfactory drinking water supply from the putrefying waters contained in some of the basins between Pittsburgh and Cairo.

In January, 1936, the National Resources Committee, interested in developing a comprehensive policy to reduce pollution of the Ohio Basin, requested the Ohio River Board of Engineers (a co-operating group of State Health Department engineers who had been working on pollution control since 1926) to prepare a statement showing the number of persons using the main stem of the Ohio River as a source of drinking water supply, the status of sewerage and sewage disposal in the municipalities along the Ohio River, and a plan for the future betterment of present pollution. The engineers' report showed that the Ohio River itself was the source of water supply for public water-works serving 1 690 000 consumers, and this did not include Pittsburgh. An encouraging note in the sewerage summary in the report was the fact that, since 1932, in five States (Pennsylvania, Ohio, West Virginia, Indiana, and Kentucky) there had been approximately $12 000 000 expended toward construction, with Federal aid, of sewage collection and sewage disposal improvements. An estimate showed that approximately $50 000 000 more would be necessary to provide primary treatment for sewage from all municipalities on the main stem of the river.

There is certainly an element of progress visible in this $12 000 000 expenditure. Since Federal financial assistance has been available for building sewers on the loan-and-grant plan, 20% of the work necessary to afford primary treatment to sewage discharged to the main Ohio River has been finished. The policy of the Health Department in at least one of the States along the river is to give no more permits for extensions to city sewerage systems until the city in question agrees to develop a comprehensive plan for sewage collection and treatment. This is a step in the right direction and if all the States on the water-shed adopt such a policy, the program of reducing sewage pollution will be speeded up.

Among the general remedial measures on sewage pollution control set out in the Board of Engineers' report of January, 1936, was this statement:

> "Every sewered municipality of reasonable size should have a comprehensive plan of sewerage approved by the state health department and should diligently proceed within the limitations of financial ability to construct the intercepting sewers. The larger communities and those situate within harmful influence of waterworks intakes should be the first to install treatment works to effect at least efficient removal of settleable matter. Thereafter, but as soon as practicable, the smaller municipalities should do likewise."

The State health authorities in States along the Ohio River feel that it is becoming increasingly difficult to produce safe, palatable water under critical conditions even with a water purification plant of modern design and skilled operation. Critical periods have occurred in 1930, 1931, 1934, and 1936. Cases of intestinal disorders were noted in the basin in the fall and winter of 1930–1931, even though an acceptable bacteriological standard for water quality was attained. This experience should be a warning that the pollution load from sewage must be reduced, and that a remedial program similar to that carried out since 1932, through the assistance of Federal funds, should be accelerated.

Progress in Industrial Wastes Control

"What progress" with respect to improved practice in industrial wastes disposal is the question which immediately follows, for it is inextricably tied in with sewage disposal. Industries located in cities commonly discharge their wastes into the city sewers. Industrial wastes as well as drainage produced by the development of mineral resources, such as bituminous coal, must of necessity be discharged into the neighboring streams.

Constructive progress in dealing with the disposal of industrial wastes in the Ohio River Basin began about 1924, when the State Health Departments of Pennsylvania, Ohio, and West Virginia met in Pittsburgh under the leadership of the late Director of Health of Ohio, Dr. John E. Monger, and agreed to work co-operatively in handling industrial waste problems where they affected public water supplies. Under the Ohio River Interstate Stream Conservation Agreement, similar policies were adopted by the State Health Departments of these three States in dealing with "phenol," a tar-acid product of the by-product coke industry which was formerly discharged into the streams, causing offensive tastes and odors in public water supplies down stream.

In 1923, H. R. Crohurst, Sanitary Engineer with the U. S. Public Health Service, made a comprehensive study of the damaging effect of phenol on public water supplies throughout the Ohio Basin. Remedial action was taken by the steel industry in one State after another as a common policy was pursued, with the result that to-day in many plants, phenol is reclaimed by the steel industry; it has become a valuable by-product rather than a trade waste. Two cardinal principles of administrative practice in industrial pollution control are illustrated by this experience: (1) The establishment of a common policy over an entire water-shed area in dealing with a detrimental trade waste; and (2) the practice of having the technical man in industry meet the technical engineer in Government service to work out the practical way of eliminating or properly treating the harmful substances discharged into streams.

The experience of State co-operation in dealing with the phenol pollution served as a guide when the question arose some years later as how best to combat the tremendous acid load which was coming into the Ohio and its tributary streams from the bituminous coal mines in Pennsylvania, Ohio, West Virginia, and Kentucky. The Board of Health Commissioners of the Ohio River Basin, a voluntary co-operating group of State Health Officials, authorized the Ohio River Board of Engineers to prepare a comprehensive plan for reducing this drainage. A report was accordingly prepared by the Board of Engineers, at Huntington, W. Va., in October, 1933, and served as a basis for the program of sealing abandoned coal mines, which was inaugurated in December, 1933, and was under the direction of the U. S. Public Health Service. The workers on this project, started under the Civil Works Administration, were unemployed miners and mining engineers. With intermittent spurts and lags, the program has gone on, until now (1937) it is well organized under the Works Progress Administration. The supervisory group of mining engineers in each State is in the employ of the U. S. Public Health Service.

The public health engineers of the States involved co-operate closely with those in the Federal service. The work in each State is under the Sanitary Engineering Division of the State Health Department, for this is primarily a program affecting public water supplies and stream pollution.

The following summary of accomplishments in West Virginia up to July 1 1936, is typical of the project as a whole:

Total number of coal mines in State	2 212
Active mines	613
Marginal mines	241
Abandoned mines	1 358
Abandoned mines sealed to July 1, 1936	405
Number of openings sealed	3 644
Average openings per mine	9

The unit costs were as follows:

Average total cost: Construction per opening	$ 89
Average total cost: Maintenance per opening	16
Total expenditure per opening to July 1, 1936	$ 105

From analyses of more than 2 000 mine-water samples, C. L. Chapman, Field Director for West Virginia, computes that by July 1, 1936, the acid drainage originally coming from the 405 air-sealed mines had been reduced by 51.1 per cent. The original daily acid production of these mines was 467 835 lb.

E. W. Lyon, Regional Director of the Mine Sealing Program, with headquarters at Pittsburgh, estimates that the total acid pollution from all mines (active, idle, and abandoned) in the Ohio Basin amounts to more than 15 000 000 lb per day. The acid load from about two-thirds of all mines—that is, the "abandoned" group—can be substantially reduced by a continuation of the present program. Again it should be pointed out that this work has been made possible by co-operative effort of the States and the Federal Government, and by Federal money apportioned on a regional basis. It is believed that the mine-sealing project is deserving of consideration in any program for water conservation in this basin.

Progress in Developing a "Water-Shed Consciousness"

The third of the "what progress" questions involves something more or less intangible—the development of a "water-shed consciousness." Progress along such lines, of course, cannot be defined quantitatively. It can be stated, however, that "water-shed pollution consciousness" in the Ohio Basin was first aroused in 1924 by the water conservation agreement between the Health Departments of Pennsylvania, Ohio, and West Virginia; and mention may be made of two more recent developments along similar lines.

The Ohio Valley Improvement Association, organized in 1893, has been interested primarily in navigation, but in 1935 it took for the theme of its Annual Meeting the control of pollution and floods on the Ohio River. This organization has been successful in securing the canalization of the Ohio River for navigation, and it is hopeful of being equally successful as it turns its energies to pollution and flood-control measures. Its members, scattered far and wide over Pennsylvania, Ohio, West Virginia, and Kentucky, form the nucleus for an appeal to the citizens and the legislatures of the several States to co-operate in a program of pollution abatement along the Ohio River.

In addition to this organization is a new one, inaugurated in Cincinnati, Ohio, on May 12, 1935, through the untiring efforts of Alfred Bettman, Chairman of the Fifth District, National Resources Committee, and Hudson Biery, Chairman of the Stream Pollution Committee of the Cincinnati Chamber of Commerce. During the winter of 1935–1936 Mr. Bettman called together representatives from the State Planning Boards of Pennsylvania, Ohio, West Virginia, Kentucky, and Indiana to effect, if possible, an Ohio Valley Planning Commission, whose purpose would be to plan for the best uses of the waters of the Ohio Basin for the future. His inspiration came from the findings and recommendations of the report of the Mississippi Valley Committee. A permanent organization was effected on March 12, 1936, with Mr. Bettman as Chairman, with headquarters in Cincinnati. As Chairman of the Stream Pollution Committee of the Cincinnati Chamber of Commerce, Mr. Hudson Biery has been successful in bringing a large number of influential civic organizations in Cincinnati to work in a united effort for an Ohio Valley Planning Commission and, for the enactment of national legislation by the Congress to permit groups of States on a water-shed to co-operate and to formulate a comprehensive plan for dealing with detrimental stream pollution.

Progress in Legislation

The fourth "what progress" inquiry deals with legislation: What progress has been made toward increased Federal and State co-operation in stream-pollution control in the Ohio Basin?

The Cincinnati group realized that a compact must be prepared, adopted by the States, and approved by the Congress before the States could work together effectively on a remedial program. Authorization of a compact by Ohio River States to work together on stream pollution control was given by the Congress in May, 1936. Thus, the ground-work has been prepared for the States to appoint commissioners to negotiate a compact to be adopted by the several State Legislatures. This is no easy task, but it is the beginning of the actual program.

The Ohio Valley is not the only water-shed or basin in the United States where interest in stream pollution is high. During 1936, from New York State and Connecticut, as well as from the Ohio Valley, measures were introduced in the Congress providing for Federal control of stream pollution. Some of the bills in Congress, particularly the Lonergan Bill, were of a drastic nature, but those prepared by the Committee of the Cincinnati Chamber of Commerce were of a more moderate type. One bill provided for a Division of Stream Pollution Control in the U. S. Public Health Service, setting up the machinery for investigating water pollution and preparing a national plan by water-sheds, also for assisting State health departments to establish proper bureaus for stream-pollution control. Another bill provided for control of stream pollution by the United States Army Engineers. Hearings were held on all these bills during March, April, and May, 1936, but none of the measures providing for national control of stream pollution was enacted by the Congress.

The Cincinnati-Louisville group associated with Mr. Bettman has furnished the motivating force to bring to a focus before Congress all the happenings in the Ohio Valley with respect to stream pollution in the past decade. The hearings on the Vinson-Barclay Bill make out a strong case for a moderate type of control, placing in the U. S. Public Health Service the functions of investigation, planning, and recommending action but retaining to the States the active control of stream pollution. This appears to be the wisest type of procedure at the present time.

OTHER FACTORS BEARING ON POLLUTION CONTROL

Mention has been made of the Board of Health Commissioners, Ohio River Basin, which is made up of the State Health Commissioners of States in the Basin. Under authority of this voluntary co-operating group of public health officials, the Ohio River Board of Engineers has thus far carried out the technical work and studies incidental to reducing sewage and industrial pollution within the Ohio water-shed. Certain points were stressed in the engineering report dealing with sanitary conservation of the water resources of the Ohio River Basin made to Mr. Bettman on January 18, 1936, which are worthy of mention. The engineers listed the priority of uses of the Ohio River in general as follows: (1) Public health; (2) drainage; (3) navigation; (4) industry; and (5) recreation.

Among general remedial measures, two important points are stressed: "Interlocking factors" and "low-water control." The following quotation is from that report:

> "*Interlocking Factors:* Improving the sanitary condition of the Ohio River in a way best serving the public interests is not confined merely to the treatment of pollution, which, in fact, is only one of the elements of a comprehensive plan. In the past such problems were approached from a single viewpoint. But the Water Resources Board have clearly shown in their reports the interrelation of other factors 'to secure the greatest total benefit from the natural resource,' and, on the other hand, to make the cost of each component benefit less than it otherwise would be.
>
> "As to sanitation, the chief factors are (1) adequate purification of public water supplies, (2) the interception and treatment of sewage and industrial wastes, (3) reduction of acidity, (4) low water control, (5) construction and operation of navigation works, and (6) the malign or beneficial influence of tributaries.
>
> "*Low Water Control:* Streams in a natural state are subject to a wide range both in stage and discharge because of the wide variation in rainfall. So far as sanitation is concerned the low flows of summer are the critical times for stream pollution of organic origin, because of lack of diluting water and high temperatures fostering decomposition. Ofttimes quite expensive treatment plants are required to prevent harmful pollution during such critical times. Therefore, means for augmenting these natural low-flows make possible economies in the treatment of pollution.
>
> "In accordance with Document 308, U. S. Army Engineers, and other authorizations of the Sixty-Ninth Congress, many studies have been made for flood control reservoirs and some of these have been designed for low water control, that is to say, for the retention in the reservoir of sufficient water to be released for augmenting the low flows of the stream below, which will be a material benefit to the sanitary condition of the streams. For example, the reservoir now being built by the Federal Government on the Tygart River in West Virginia is said to be intended to release three hundred and forty cubic feet per second of water from July to December. During extreme drought the unregulated Monongahela River below this reservoir has reached a condition of almost no flow. Where, in the public interest, it is practical so to do, flood control reservoirs in the Ohio River Basin should be designed to provide for low water control. One reason is the actual economies in the reduced cost of treating sewage and industrial wastes, and the other, and possibly higher but intangible

benefits, through the resulting protection to the public health. Low water control is an important factor in sanitary improvement of the Ohio River and its tributaries."

SUMMARY

Progress in stream-pollution control in the Ohio River Basin may be summarized as follows:

(1) Beginning with the co-operative movement between State Health Departments in the Basin in 1924, a "water-shed consciousness" was awakened. The most recent development is the organization of the Ohio Valley Planning Commission, which is working under the auspices of the National Resources Committee.

(2) New sewerage systems and sewage disposal plants to the value of $12 000 000 have been built with funds jointly provided by the Federal and City Governments in the period, 1933–1936. This represents about 20% of the total expenditure necessary to afford primary treatment to all sewage along the river.

(3) The U. S. Public Health Service and various States in the Basin are carrying on a joint program to reduce the tremendous pollution caused by acid drainage from abandoned mines. A reduction of more than 50% in acid drainage is reported from the mines already sealed. The present program is financed through the Works Progress Administration.

(4) Legislation was enacted by Congress in 1936 providing for a compact between States on the water-shed, with respect to pollution control and other water conservation measures. It is hoped that ultimately an interstate compact will be drawn up and ratified. Hearings have been held in Congress on proposed bills providing for Federal control of stream pollution, and it appears likely that a moderate bill, involving co-operation between State and Federal authorities, may be passed in the near future.

PLANNING FOR POLLUTION CONTROL AT PITTSBURGH, PENNSYLVANIA

By D. E. Davis,[15] M. Am. Soc. C. E.

Synopsis

Pittsburgh's rugged topography creates a complicated sewage disposal problem. The heavy acid load in the rivers, largely from mining operations, inhibits bacterial growth, prevents a major nuisance, and postpones the attack on the disposal problem. Primary treatment plants are indicated as meeting the requirements for a considerable period; and sites are available which will avoid the expense of costly interceptors and tunnels. River transportation can be used for conveying sludge to a central plant for final disposition.

Decided economies in first cost and in operation can be achieved if the 116 interested municipalities join forces. Public sentiment would probably support a "clean-up" program; but enabling legislation will be required.

Competent authorities have stated that the rugged topography of Greater Pittsburgh, or Allegheny County, Pennsylvania, and its multitude of governmental jurisdictions, create one of the most complicated sewage disposal problems in the United States. The Allegheny River flows from the north 30 miles, and the Monongahela River from the south 35 miles through the rugged and precipitous hills of Allegheny County to form the Ohio River, which flows westerly another 15 miles before leaving the County. Along these 160 miles of shore line are 116 separate municipalities with a total sewered population of approximately 1 200 000, representing one of the most highly industrialized sections in the country, sometimes called the "Ruhr of America."

Although most of these communities are sewered, there is comparatively little treatment of the sewage, most of which enters the stream in its raw state. To this untreated sewage are added industrial wastes with a population equivalent estimated by the U. S. Public Health Service to be nearly as great as the actual human population of the area. This pollution, at times, exercises harmful effects down stream, such that for considerable periods the bacterial counts at East Liverpool, Ohio, Steubenville, Ohio, and Wheeling, W. Va. (and in the Ironton-Portsmouth District, in Ohio, possibly affected in part by the sewage of Pittsburgh) are among the highest in the country. Commenting on this situation, H. R. Crohurst, Senior Sanitary Engineer in Charge, U. S. Public Health Service at Cincinnati, has stated:[16]

"Only by constant and efficient supervision of plant operation has it been possible to procure a water of the desired bacterial quality with the rapid changes in raw water. While it has been possible up to the present time to produce satisfactory water supplies from the upper part of the Ohio River in spite of the excessive pollution, for the reasons as outlined urban population and sewage pollution will in a comparatively short time increase to the point where it will be impossible, with all present known methods of water treatment, to produce at all times a water safe for domestic use in these downstream communities."

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[15] Engr. (The Chester Engrs.), Pittsburgh, Pa.

[16] Unpublished report.

Estimates indicate that a flow of approximately 9 000 tons of acid (as pure sulfuric acid) passes Pittsburgh daily, which causes economic loss to steel barges, pipe lines, concrete work, and other structures subject to acid attack. The Monongahela River is highly acid most of the time and the Allegheny River (up to Freeport, Pa.) much of the time during the summer.

It would appear, therefore, that there is a condition requiring early and urgent consideration; and that if steps are to be taken to clean up the Ohio River and its tributaries, Greater Pittsburgh, at the head of the river, should be one of the first communities to place its filth under control.

Early Pollution Studies

Although there is evidence that the general public is becoming increasingly interested in a general clean-up program, it has not yet evidenced its concerted intention to act. However, its representatives in the various State and National Health Departments have been alive to the problem, and as early as 1910 the Pennsylvania Health Department ordered Pittsburgh to present a report on a plan for disposing of its sewage. This report was prepared and presented in 1912, and was followed in 1916 by the classic report of the U. S. Public Health Service on "A Study of the Pollution and Natural Purification of the Ohio River." Again, in 1931–1932, this Service co-operated with the Health Departments of the various States along the river in the conduct of a similar co-operative study of pollution, the results of which are not yet published. In February, 1936, the Ohio River Board of Engineers enunciated the following policy:

> "Every sewered municipality of reasonable size should have a comprehensive plan of sewerage approved by the State Health Department and should diligently proceed, within limitations of financial ability, to construct the intercepting sewers. The larger communities, and those situated within harmful influence of water works' intakes, should be the first to install treatment works to effect at least efficient removal of settleable matter. Thereafter, but as soon as possible, the smaller municipalities should do likewise."

When, in 1933, Federal funds were being made available for various local projects, W. L. Stevenson, M. Am. Soc. C. E., Chief Engineer of the Pennsylvania State Department of Health, urged on the officials of the City of Pittsburgh the desirability of a study of its sewage disposal problem. As a result, the Metropolitan Drainage Survey was initiated under the general direction of Charles M. Reppert, M. Am. Soc. C. E., Chief Engineer of the City of Pittsburgh, and continued under his successor, Henry D. Johnson, Jr., Assoc. M. Am. Soc. C. E. Although the population embraced within the City of Pittsburgh proper represented about one-half that in Allegheny County (or Greater Pittsburgh), it was realized that it would be inexpedient for the City to attack its problem independently, without knowledge of its effect on the county-wide situation; and that in any case it should have a picture of the problem in its entirety and the City's relation thereto. Consequently, the study covered the entire County or Metropolitan Area.

The report of 1912 had not been wholly fortunate in its reception, since it had left the impression that the costs of sewage disposal were excessive. Its study was based on "considering only those methods which will be reasonably efficient and certain in stopping the passage of infectious matter." Estimates were prepared for separate sanitary sewerage systems for all river communities within Allegheny County (above Neville Island, Pa.), and for long interceptors and tunnels terminating in a pumping station delivering to a central complete treatment plant on Neville Island, about six miles below the "Golden Triangle." The total cost for Pittsburgh was estimated at $37 300 000, and for the County as a whole (above the Island) at $57 000 000, which, in terms of present-day prices, would be at least doubled. Naturally, this huge cost imposed an effective brake on affirmative action.

The Metropolitan Drainage Survey

Since 1912 material changes have altered the aspect of the problem. The use of combined sewers in handling dry-weather domestic sewage flows is better understood and more commonly practiced; the rivers are more completely canalized; Neville Island is now largely occupied by industry, and hence not available as a sewage plant site; acid waste discharge has increased; and the Health Department Engineers now consider simple primary treatment of sewage as being generally adequate, at least as the first step in the clean-up program. These changed conditions counseled a completely fresh approach to the problem.

In assembling the fundamental design data for the new study, it was decided to adopt drainage basins as the basic units without respect to their relationship to administrative areas. The population within these basins was rather accurately determined from the small Census Districts, and from the past trend in each basin the population in 1970 was predicted; a correlation between water pumpages and sewer weirings in typical sewers supplied information as to probable sewage flows; and field surveys were made of possible plant sites and intercepting sewer locations.

Influence of Acid on Natural Purification of Streams

Concurrently, studies of the bacteriological and chemical conditions in the rivers were undertaken to determine the degree of treatment which might be required at different times of the year, and under varying conditions; as well as to develop a standard by which the degree of improvement provided by future plants might be measured. A problem special to this area was posed by the relatively large acid content of the streams and its marked effect on the process of natural purification in the rivers. It was found that the acid appeared to inhibit and postpone the normal processes. Although this action was locally beneficial, minimizing nuisances that might otherwise be expected to occur, it did not eliminate the offensive organic matter, but merely transferred it down stream for the resumption of normal processes in the reaches where alkaline conditions were restored in the river.

Naturally any change in the acid content by dilution water from the new flood-control reservoir on the Tygart River, or from others to be constructed later, or by acid elimination through mine sealing, might have a marked effect on nuisance conditions at Pittsburgh. A comparison between the Pittsburgh and the Cincinnati-Louisville observations, for 1914 and 1930–1932, yielded valuable information as to the effect of acid on the inhibition of self-purification, and of the degree with various concentrations. Except for the amount of acid and the quantity of flow, the conditions in these two districts are substantially similar, since the rivers are fully canalized and the effects of sedimentation in these enormous settling basins are somewhat alike. Table 5 indicates the marked differences in the condition of the river water in the two districts for low-flow periods.

TABLE 5.—COMPARISON OF RIVER CONDITIONS IMMEDIATELY BELOW PITTSBURGH, PA. (1932) AND CINCINNATI, OHIO (1930)

(Results expressed as monthly averages)

Month	Stream Flow, in Second-Feet		Methyl Orange Alkalinity, in Parts per Million		Five-Day Bio-Chemical Oxygen Demand, in Parts per Million			Dissolved Oxygen, in Parts per Million		*B. Coli* Index per 100 Cubic Centimeters	
					Pittsburgh†						
	Pittsburgh*	Cincinnati	Pittsburgh†	Cincinnati	Calculated	Actual	Cincinnati	Pittsburgh†	Cincinnati	Pittsburgh†	Cincinnati
September	1 610	4 830	3.4	66	35	0.2	3.5	0.8	2.2	345	160 000
October	2 940	3 360	4.5	78	19	1.2	4.0	4.9	2.94	237	415 000
November	18 900	5 570	10.7	72	3	3.1	5.5	10.9	7.58	8 970	1 441 000

* Seven miles below the confluence of Monongahela and Allegheny Rivers.

† Three miles below the confluence of the Monongahela and Allegheny Rivers.

The following quotation from a report[17] by Mr. Crohurst indicates what might be expected if the acid were largely eliminated so that the rivers at Pittsburgh would become alkaline:

> "With possibly twice the equivalent sewered population at Pittsburgh and much lower flows than at Cincinnati, it is believed that the dissolved oxygen would (in Sept. to Nov. 1932) have been entirely depleted in and below Pittsburgh for a considerable distance, if normal river conditions unmasked by excessive acidity had existed."

Under such conditions it is probable that the stench would be so vile that the public would then demand sewage disposal for relief. It seems fair to conclude that if acid is materially reduced during low-flow periods, nuisance conditions will unquestionably develop.

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[17] Unpublished report.

Effect of Sludge on Down-Stream Water Supplies

The U. S. Public Health Service report also emphasized the fact that the critical period for down-stream water plants is that immediately following a rise in the river with velocity sufficient to scour out accumulated sludge banks. At such times there is a precipitous increase in bacterial counts, and the pollution load on the down-stream plants becomes excessive. The minimum effective program of sewage treatment would be that afforded by primary sedimentation plants, which would remove the settleable solids now forming the sludge banks. At the same time this would prevent local nuisances, and remove the wave or "slug" of pollution which now moves down river with freshets. The general clean-up plan was based on the construction of primary plants as being adequate for present requirements.

Physical Aspects of Proposed Treatment Plan

The development of the physical aspects of the plan was largely dictated by the unusual topography of the district. Recalling the 160 miles of shore line, it may be realized that if an effort were made to concentrate the sewage at relatively few sites, the costs of interceptors and pumping stations would be enormously expensive. Most of the communities in Allegheny County extend up the hillsides back from the rivers, and their major sewers traverse the valleys of minor streams to their outlets in the rivers. Surveys revealed that sites could be found at or near the mouths of many of these streams, and that the problem could be solved by the construction of not to exceed thirty-six plants. Further study may show that this number can be somewhat reduced.

Primary treatment plants would incorporate mechanical screens, sludge grinding, detritors, plain sedimentation with mechanism for gathering the sludge, pumps, sludge digestion tanks, and provisions for chlorination, when and if necessary. At a comparatively small increase in cost, chemical treatment could be added and peak-load conditions met, should experience indicate the need. Sludge beds would be eliminated at the individual plants in the interest of economy in construction and operation, as well as for the conservation of expensive space. The digested sludge would be delivered by gravity directly to barges (similar to those now in use for transporting gasoline) at loading docks adjacent to the plants, for delivery to storage tanks at a central sludge disposal plant equipped with vacuum drying and incineration facilities. Ample storage would be provided both at the individual plants and at the central plant, so that the barge fleet could be removed from service during two or more months in the winter when the rivers might be partly covered by ice, and to permit vacations and fleet repairs.

Economies in County-Wide Program

The necessary interceptors, diversion and control chambers, pumping stations, treatment plants, barge fleet, and central sludge disposal plant, to afford primary treatment for the sewered communities along the three main rivers and their minor tributaries in Allegheny County, can be constructed for less than $20 000 000. The possible reduction in number of plants, by re-grouping, in the interest of reduced operating expenses, might affect this estimate; but in a general way it serves to fix the scale and magnitude of the task. Decided economies are made possible by approaching the problem on a county-wide basis; for the low-cost river transportation of sludge can thereby be taken advantage of, and the single sludge disposal plant can be made available at a fraction of the cost of equivalent facilities provided by each community on its own responsibility. Moreover, the cost of a few large primary treatment plants, each serving an entire drainage basin, is less than that of a multitude of small plants, each serving a separate community. The project lends itself to unified direction and group operation, making possible expert technical supervision that, on a shared basis, will be better in quality and lower in cost than would likely be possible to individual municipalities.

The studies indicate that the proposed method of treatment would increase the oxygen content of the river at all times and would remove the danger of an oxygen deficiency at Pittsburgh. (Such a deficiency existed for a short period during the 1930 drought.) It is believed that this degree of treatment should suffice for an extended period. If secondary treatment should later be found advisable, the primary plants could continue to function as such; and a number of secondary treatment works could be provided, each of which would handle the pumped effluent from a group of primary units. There are six prospective sites, scattered over the area, which, in their present state of development, would prove acceptable locations for such secondary plants.

Such an expenditure ($20 000 000) is not an unreasonable price to pay for sewage treatment at Pittsburgh, in view of the benefits that it would confer—the removal of foul local nuisances, the reinstatement of river recreation as a relatively safe diversion, and the "good-neighbor" service of improving the quality of the raw river water for down-stream communities. By way of comparison, it may be pointed out that the estimated cost of sewage purification is about four times the cost of the George Westinghouse Bridge and its approaches, and about one-fourth the sum that would be necessary for reproducing the water-works systems in Allegheny County.

The Problem of Stimulating Public Interest

It is one thing to indicate the nature and extent of a problem and the means to its solution, but it is quite another matter to bring it to execution. The need is present, and the practical question turns on the best way to achieve the desired clean-up. It is obvious that without the whole-hearted approval of the general public, little can be accomplished. The first job is to overcome the apathy of the electorate and to carry home the realization that civic morality requires that communities dispose of their waste products without endangering health, or occasioning undue financial burdens on their neighbors below.

Time was when the drinking of unfiltered water was the general rule, and water-borne disease stalked the land. The great majority of filtration and water purification plants in the United States have been built within the easy recollection of men now in their prime. When the public became convinced that the dreaded typhoid fever was a water-borne disease, potentially affecting the health of each member of the community, it did not require much argument to develop the conviction that pure water was cheap at any price. The small increase in the water rates was much less than the ever-possible doctor and hospital bill. This was personal knowledge and elementary financial common sense.

The case for sewage purification, however, is not made out so easily. Many communities are not moved to the building of disposal plants until threatened with damage suits by lower riparian owners; then it becomes a nice calculation as to whether it is cheaper to build a plant than to pay damages. Few communities treat their wastes solely for the altruistic purpose of bettering the river for the water plants of communities below. They have projects that are much nearer their hearts and pocket-books—school houses for their own children, better streets for their personal automobiles, and bridges for streams which they themselves must cross frequently, are tangible goods whose benefits are obvious to those who pay the bills. Until these pent-up demands have been at least partly satisfied, the voter can hardly be expected to be enthusiastic about the disposition of his waste products unless they assail his nose or threaten his bank account. However, there is much evidence to the effect that these insistent needs have now been pretty generally satisfied in most communities, and if (as is predicted), the growth in population should soon slow down or even cease, the usual outlets for public moneys may dry up, and funds may be available for projects previously regarded as luxuries.

Increasing leisure for healthful outdoor pursuits should stimulate interest in cleaner streams. Fishermen are not greatly interested in cleaning up streams in some other taxpayer's county, but if they could be brought to realize that fishing could be made good in their own vicinity, they might be more willing to loosen up on their own purse strings.

The rivers here under discussion flow through a territory peculiarly devoid of lakes. However, the Ohio River and its main tributaries are fully canalized, and during low-flow periods in the summer are to all intents and purposes long inland lakes, whose waters are usually quite clear. Under Health Board control, they would lend themselves admirably to water diversions of all kinds, were the conditions more propitious. Bass could be caught at Pittsburgh less than forty years ago, and if the conviction and fear of disease were not generally prevalent, it is probable that the streams would be alive with recreational enthusiasts. Wind currents are known to carry the sewage of some communities past their own water-works intakes, thus placing an undue load on water purification processes. These are all valid arguments for affirmative action, and there is good reason to believe that the public might be appealed to on grounds of its own self-interest as to the value of clean streams. The first job then is to convince the public of its obligations, the honoring of which will yield dividends in health and recreation.

The absence of fish in the Upper Ohio River is very largely attributable to acid wastes, mostly from abandoned mines. The responsibility for its removal is obviously that of the State and National Governments. The mine-sealing program now in progress in the Ohio River Basin gives promise of a material reduction in this highly objectionable waste.

Co-operative agreements between industries and State Health Departments have been effective in setting up research programs within the affected industries, looking to the profitable reclamation of by-products which formerly polluted the streams. These programs should be expanded, but, in addition, legislation similar to the Lonergan or Barclay Bills should empower the policing of recalcitrant industries. Preferably this clean-up should be on a broad geographical basis, so as not to impose unfair competitive burdens on industries complying in good faith.

Sewage Disposal a Municipal Responsibility

The disposition of human wastes, which represent the preponderant source of pollution, is squarely up to the various communities. It is apparent that in the Pittsburgh District, the most economical and only practical way of accomplishing this result is by bringing all the communities together in a co-ordinated body, such as a sanitary district. This will require enabling legislation, since the job of coaxing voluntary co-operation among 116 separate jurisdictions is humanly out of the question. A few communities have worked out joint compacts for the construction of intercepting sewers, but the negotiations covered an extended period and the method proved unwieldy in the extreme, and would not apply to Allegheny County as a whole.

In 1927, the Pennsylvania Legislature passed a bill permitting a "Metropolitan Plan" for Greater Pittsburgh, embracing Allegheny County as a whole. Had this been adopted, it would have provided for numerous projects which would lend themselves advantageously to joint effort in their execution. However, it failed of ratification on the two occasions when it was presented to the voters. The 1935 Legislature passed a Municipal Authority Act with the intent of permitting somewhat more liberal methods of financing than those now available, so that many governmental jurisdictions could participate in improvements by accepting the financial co-operation of the Federal Government. This would have permitted the issuance of revenue bonds and the assessment of charges for service rendered (such as sewer rentals), but the provisions as to new projects terminated in 1937, without any action having been taken.

There is no reason to believe that the Legislature cannot provide means for creating the necessary machinery for advancing a unified program. The minimum provisions should incorporate joint ownership of the facilities, power of compulsory assessment (preferably of sewer rentals) for construction and operation by the central representative executive group, the right to issue revenue bonds, and preferably a provision for administration on a Civil Service basis.

Conclusion

The Pittsburgh study indicates that the rivers in its vicinity can be cleaned up at a reasonable cost for initial construction and for operation. It is believed that on a fair presentation of the problem, the public may be prepared to sanction an adequate disposal of its sewage, and that the requisite legislation may be had to effect this result.

STREAM POLLUTION PROBLEMS AT CINCINNATI, OHIO

By J. E. Root,[18] M. Am. Soc. C. E.

Synopsis

The nuisances created by sewage and industrial wastes as they have affected the condition of the Ohio River have been under observation since 1908. From time to time remedial measures have been recommended. Due to the fact that the river for the most part is the boundary line between a number of States, a correction of the pollution problem will require the joint co-operative action on the part of these States, or a number of them. Complete canalization of the Ohio River further complicates the problem, as do floods and extreme periods of low flows. The paper for the most part deals with river conditions within the Cincinnati Metropolitan Area.

Pollution of the Ohio River has received almost continuous public attention since 1908, when the subject was first investigated by a Joint Ohio River Sanitary Commission composed of representatives from Pennsylvania, West Virginia, Kentucky, Ohio, and Indiana. The general conclusion of the 1908 and 1909 investigations[19] was "* * * that the Ohio River is generally of objectionable appearance, is frequently very turbid, carries high color in the fall, and must be purified before being used for public water supplies." The river water was found to be "grossly polluted at many points along its course," and it was stated that the pollution was becoming more pronounced and more dangerous to the health of the inhabitants of Ohio River cities. A resolution, adopted at a meeting of the Joint Commission on December 1, 1909, recommended that the various municipalities along the Ohio River using the river as a source of water supply take immediate measures to discontinue the discharge of unpurified sewage into the stream.

Early Investigations

For the City of Cincinnati, J. W. Ellms, M. Am. Soc. C. E., as Superintendent of Filtration at the Water-Works, in 1910 and 1911 investigated the condition of the Ohio River water from the water-works intake up stream from the city proper to Fernbank Dam, about 20 river-miles down stream from that point. As to the effect of the sewage from the Metropolitan Cincinnati area upon the general quality of the Ohio River water and the influence of the Fernbank Dam, when in service, upon the water, Mr. Ellms summarized[20] in part:

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[18] Director of Public Works, City of Cincinnati, Cincinnati, Ohio.

[19] Rept. of the Commissioner of Health, State of Pennsylvania, 1909, Pt. II.

[20] Rept. on Investigation of Ohio River at Cincinnati, Seventy-Second Annual Rept. of City Water Works Dept., City of Cincinnati, Ohio.

"The quality of the Ohio River water entering the Cincinnati intake is subject to considerable variation, showing the greatest degree of contamination during flood periods.

"In general, the water at the intake was of better quality during the period the Fernbank Dam was in service than for the most comparable period of low water in 1911 preceding its completion.

"An exception to the above general statement is required for a brief period during which the dam was in service, because of a sudden rise in the Little Miami River, which, in discharging large volumes of water into the Ohio, retarded the flow of the latter, and even produced a back flow of water toward the intake, temporarily causing an inferior and somewhat more contaminated water to pass into the intake.

"The present contaminated condition of the Ohio River water which passes into the Cincinnati intake, whether under normal or abnormal conditions, need not be feared by consumers so long as the present purification plant is carefully and efficiently operated, since the river water can be clarified and purified and rendered a perfectly safe drinking water by the processes employed."

During July, August, September, and the first week of October, 1911, J. A. Cravan, Water Chemist of the Indiana State Board of Health, made a sanitary survey of the Ohio River between the Ohio State line and the Illinois State line, a river distance of 360 miles. In general, Mr. Cravan's investigation[21] indicated:

"(*a*) That the dissolved oxygen in the river water is not progressively depleted from Cincinnati to the Illinois State line.

"(*b*) That the bacterial content of the water between the same points is not materially higher than at the Cincinnati Water Works intake.

"(*c*) That the river water soon recovers a substantial proportion of the dissolved oxygen lost in its passage through the Cincinnati pool."

Under the direction of Henry M. Waite, M. Am. Soc. C. E., City Engineer of Cincinnati during 1912 and 1913, as part of a comprehensive plan of sewerage for the city, studies were made for sewage treatment and disposal, and a system of intercepting sewers was developed with outlets that could be adapted to a number of possible plans for sewage treatment. The problem of sewage treatment and disposal was investigated by the late Harrison P. Eddy, Past-President and Hon. M. Am. Soc. C. E. His report,[22] entitled "Report upon Necessity and Feasibility of Treating the Sewage of Cincinnati Before Its Discharge into Ohio River," was filed with the City in the latter part of 1913. Mr. Eddy based his studies of the problem upon two main considerations:

"*First.*—Consideration of the character of the water as it reaches Cincinnati, and the effect of the sewage and wastes discharged into the river above the city.

"*Second.*—The effect of the sewage discharged by the cities and towns in the immediate vicinity of Cincinnati, herein designated the Cincinnati District."

The general conclusion of the report may be summarized in the following quotation:

"In view of the fact that it does not appear that sewage treatment will be necessary at Cincinnati for a number of years, it does not seem wise to recommend any specific method of treatment or that any site be secured for a treatment plant. Methods of sewage treatment have been improved, and new methods have been devised during recent years, and much further improvement may reasonably be expected during years to come. It would, therefore, seem idle to take steps at this time which would commit the City of Cincinnati in any way to a particular method of treatment. It is necessary and wise, however, to so plan and lay out the system of intercepting sewers which may be built from time to time that they may be brought together in harmony with any method for the treatment of sewage which may be adopted; and should it later prove advisable to intercept the sewage along the Ohio River, before a sewage treatment plant is needed, the site on which such a plant is to be built should then be selected so that the intercepter may be laid out with a view to conveying sewage to that site.

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[21] Indiana State Board of Health (unpublished).

[22] Rept. on a Plan of Sewerage, City of Cincinnati, 1912–1913, p. 295.

"From these investigations, the conclusion is that Cincinnati is not at present justified in going to the expense of building and operating a sewage treatment plant, since the benefits derived therefrom would not be commensurate with the expense."

From July, 1913, until December, 1916, the Ohio River was subjected by the U. S. Public Health Service to special study as a type of large inland river presenting a complex and difficult problem in the control of sewage pollution. The purposes of this study[23] were:

"(1) To give a quantitative statement of the pollution of the river in important zones, as existing at the time of the study, with such evaluation as possible of the relative importance of individual sources or units in contributing to this pollution, and of the relation of the conditions to the public health.

"(2) To furnish the basis for estimating with reasonable precision the changes in status of pollution which may be expected in the future to result from a given change in one or more of the factors concerned; as, for example, from a given increase in the various units of population, or from a given reduction in the polluting effect of their sewage by artificial treatment; or from changes in the velocity of flow, due to the construction of additional dams, as now projected, for improvement of navigation.

"(3) To investigate the possibility of establishing definite quantitative relations between the intensity of pollution, as measured by various laboratory tests, and such obvious factors as are readily determinable by field surveys. Especially has it been the purpose, in this connection, to make a quantitative study of natural purification as related to time of flow, temperature, and other determinable factors of presumptive importance."

THE RE-SURVEY OF 1930

At the time this study was made seventeen of the proposed fifty navigation dams were completed and in operation. Dam No. 37 of the series, at Fernbank, forms the Cincinnati pool. After complete canalization of the Ohio, the U. S. Public Health Service made a re-study[24] during 1930 of that section of the stream between Dam No. 36 (above Cincinnati) and Dam No. 43 (about twenty miles below Louisville, Ky.), a river length of about 164 miles. The re-study was to determine:

"(1) The extent to which the normal increase in population and changes in industrial activities during the intervening 15 years would be reflected by changes in the sanitary condition of the stream as indicated by differences in the chemical and bacteriological findings.

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[23] "A Study of the Pollution and Natural Purification of the Ohio River," U. S. Public Health Service *Public Health Bulletin No. 143.*

[24] *U. S. Public Health Bulletin No. 204.*

"(2) The effects of canalization during periods of high water temperature and low stream discharge when the sewage and wastes flowed through a series of pools below the points of discharge.

"(3) The present sanitary condition of the river, making possible a comparison of probable conditions in 1930 as forecast by the data collected in 1915, with conditions as they actually existed in 1930.

"(4) Possible differences in the rates of chemical and bacteriological changes indicated by the two sets of observations as a result of changes in the amount and character of the pollution and in channel conditions."

The effect of canalization, as indicated by the re-study, is summarized in the following quotation from the printed report:[24]

"Canalization, at least between Cincinnati and Louisville, has had a tendency to complicate rather than simplify the problems connected with sewage disposal, possible nuisance production, the operation of water-treatment devices, and the preservation of the public health."

Pollution Control Requires Co-Operative Action

Parts of fourteen States lie within the Ohio River drainage area. The drainage area above or to the east of Cincinnati is approximately 76 300 sq miles, or slightly less than one-third the total area of the basin. When the Ohio River leaves the State of Pennsylvania it forms the boundary line between the States of Ohio, Indiana, and Illinois on the north, and the States of West Virginia and Kentucky on the south. To correct a bad condition along this river, such as pollution, therefore, becomes an inter-state problem. Actually, all the States in the water-shed share a responsibility and are contributors toward the creation of this undesirable river condition. Since 1935 considerable public agitation has developed demanding that proper corrective measures be taken. These demands were presented to the Second Session of the Seventy-Fourth Congress, with the result that in June, 1936, a Joint Resolution was passed enabling certain States—including Pennsylvania, West Virginia, Kentucky, Indiana, Illinois, Tennessee, and Ohio, or any two or more of them—to negotiate and enter into agreements or compacts for conserving and regulating the flow, lessening flood damage, removing sources of pollution of the waters thereof, or making other public improvements on any rivers or streams whose drainage basins lie within any two or more of the specified States.

Since the passage of the Federal resolution, a number of the States have arranged for and appointed Commissioners to meet jointly with Commissioners from the other Ohio River States. At the present time (1938), therefore, there is in the making the possibility of joint State action which may eventually lead to definite plans for pollution control.

Whatever the problem of stream pollution may be for Cincinnati, the measures required for its correction cannot be accomplished by the City of Cincinnati acting by itself; for the Metropolitan District consists of a number of politically independent incorporated cities and towns—about twelve north of, and ten south of, the river. The problem calls for inter-city as well as inter-state co-operation. Conversely, what the City of Cincinnati does or fails to do which may result in the creation—or, more correctly, the re-creation—of the pollution problem in the Ohio River is the co-operative negligence of a large metropolitan district with a total population of approximately 700 000, of which number approximately 470 000, or about 70%, are within the corporate limits of the city.

Each investigation of river conditions, from the first in 1908 to the most recent in 1930, established the fact that the Ohio River on its arrival at the eastern edge of the Metropolitan District was a polluted stream. In part, therefore, the conditions at Cincinnati are the result of negligence on the part of others up stream, not only on the Ohio River proper, but on practically all its tributaries; and an equitable solution for the problem at Cincinnati must pre-suppose that equal corrective measures will be taken on the part of those responsible for the polluted condition of the water as it enters the district.

The Effect of Canalization

Canalization of the Ohio River has intensified the pollution problem. The Cincinnati pool is created by Dam No. 37, located at approximately the most westerly corporation limits of the city. This pool extends twenty miles up stream to Dam No. 36, which is approximately at the most easterly corporation limits of the city. The business or central district of the city is about equi-distant from the two dams, and on the Kentucky side, the center of the developed area is at about the same point. Maximum pollution, therefore, occurs in the river near the central section of the city as well as of the Metropolitan District.

For navigation purposes the crest of Dam No. 37 is at such elevation that a minimum stream depth of 9 ft obtains at the easterly end of the pool. A rough estimate of the quantity of water in this pool at a time of zero velocity, is 16 000 000 000 gal. During periods of continued dry weather the minimum quantity of stream flow at times may be less than 3 400 cu ft per sec. For five months—July to November, 1930—during an investigation made by the U. S. Public Health Service, the average monthly stream flow was 5 000 cu ft per sec. The time required for water to pass through the Cincinnati pool, at periods of minimum discharge in 1930, was on the average 56.6 hr, or more than $2\frac{1}{3}$ days. For the extreme minimum monthly average stream discharge (October, 1930) it was estimated that the time required was in excess of 100 hr, or more than $4\frac{1}{6}$ days. At times of low flow, therefore, the pool acts as a receiving and settling tank for the sewage and the industrial and commercial liquid wastes from the Metropolitan District; and decomposition soon takes place and offensive odors are produced. There are extended periods during the summer and early fall months when the pollution from the Metropolitan District has a marked effect on the river at Cincinnati, and at such times the dissolved oxygen in the water has sometimes been completely exhausted.

Other Factors

The selection of possible sites for sewage treatment works will be no small problem. The Ohio River channel within the Metropolitan Area, is of comparatively recent geological origin. It is narrow, its banks are steep, and there are few low areas of sufficient acreage to accommodate works of the character required. When the comprehensive sewerage plan was investigated in 1912 and 1913, just five locations were considered suitable for treatment plant works. Since then the new Cincinnati Union Terminal has taken possession of one of the areas, and the Municipal Airport has acquired a second. Moreover, one of the remaining locations is near the Great Miami River, about sixteen miles from the central section of the city. To take care of the pollution on the Ohio side of the river, at Cincinnati, would require works costing approximately $15 000 000.

Another factor requiring thoughtful consideration as a part of the pollution problem is that of floods in the Ohio River. The U. S. Weather Bureau Office at Cincinnati has records of river stages from June 1, 1858, to date (January 1, 1938). In this 80-yr period, the maximum flood height was slightly less than 80 ft (79.99 ft at 4 A.M. January 26, 1937); the next highest flood reached approximately 71 ft (February 15, 1884, 71.06 ft); and the third highest was just under the 70-ft stage (69.8 ft, April 1, 1913). The records indicate twelve floods of 60 ft or more during the eighty years, or an average of one such flood every seventh year. To the city, the physical effects of flood waters begin to be felt at an elevation of about 50 ft. Such a flood may be expected to occur once every sixteen months.

Conclusion

In conclusion, it may be stated that the Metropolitan District, of which Cincinnati is the largest urban area, realizes that the Ohio River during six months of the average year is grossly polluted. To correct, in part or wholly, this unfavorable situation will require joint action by many incorporated municipalities within the Metropolitan District, as well as by the States of Ohio and Kentucky. It is realized, also, that the possible solution of the problem will be critically affected by two extreme factors—low dry-weather flows and high flood flows. The works required will cost many millions of dollars to construct, and no mean sum annually for satisfactory operation.

WHAT CAN BE DONE ABOUT STREAM POLLUTION?

BY ABEL WOLMAN,[25] M. AM. SOC. C. E.

SYNOPSIS

The elements of the stream-pollution problem are reviewed. The possible results of activity in the fields of law, science, finance, and administration are briefly evaluated. The key to present difficulties appears to be primarily in raising money for corrective measures. Opportunities in State or Federal subsidies or grants-in-aid are emphasized.

The revival of interest in the control of stream pollution is one of the significant trends in the field of water resources control. In earlier days, navigation caught the public eye and for many years dominated the scene in opening up the country. Emphasis then shifted at a later date to flood control.

One of the national results (and incidentally one of the penalties) of population and industrial growth appears in the present emphasis on the correction of stream pollution. To sanitary engineers, particularly those engaged in health department practice, this emphatic public interest is somewhat of a curiosity, since for several decades one of the primary concerns of health departments has been the prevention and alleviation of stream pollution. The diagnosis of the causes for this recent and dramatic interest of the public in this field would offer in itself an attractive field for research.

As in all matters of public concern, the answer to what can be done about stream pollution will depend upon the experience, training, temperament, and prejudices of individuals. Those who have had least to do with the problem are prone to over-simplify the solutions, while those who have struggled with the issue for a great many years are accused of delaying the wheels of progress by being too lax in the enforcement of laws and not active enough in public education. Solutions are offered in law, in science, in finance, and in administration. What are the probabilities of success in each of these fields?

Before attempting to analyze these probabilities, a few general facts should be considered.

The intensity of stream pollution is not nation-wide. It is concentrated largely in five or six of the heavy industrial States. Population distribution alone controls the localization of the problem. The degree of difficulty varies from place to place. Blanket elimination and blanket similarity of approach do not seem to fit the case. However, in each area where the situation has become acute, in regard to either domestic or industrial waste, the time has arrived for a definite position on the part of the people as to the balancing of convenience in the use of the stream.

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[25] Prof. of San. Eng., Johns Hopkins Univ., Baltimore, Md.

Many frank critics of the rush toward stream-pollution abatement are emphasizing the fact that correction, particularly in reference to industry, may militate against the advantage of the region forced to clean up. The weight to be given such considerations should be determined in each case upon the merits of the facts; that the facts should be adequately balanced should certainly be borne in mind.

LAW

Law enforcement as a device for the control of any difficulty is frequently the first recourse of certain groups of individuals. The popular phrase "there ought to be a law" exemplifies this faith in correction by legislative fiat. Experience discloses, however, that no greater progress in stream pollution abatement has occurred in those States where rigid legislation has been on the books for many years than where such laws are absent.

This situation has led some people to suggest that the difficulty lies in the inconsistency of individual State Acts. They argue that corrective measures are impossible in one State because similar corrective measures are not executed in another State. This leads to the further suggestion that the only corrective in law is a comprehensive and overlying Federal anti-pollution act. Aside from inherent fallacies in logic, many students of the problem believe that such Federal legislation is unconstitutional. More important is the belief that it would be unwise, not only because the mere passage of an act need not result in correcting difficulties, but because the long and time-consuming negotiations for stream correction are logical fields for local participation and autonomy.

State by State progress in stream-pollution control has been uneven, and uncorrelated with legislative pronouncements. If anything, it has been related primarily to the enthusiasm, ingenuity, and co-operative endeavor of individual State departments and of municipalities and industry.

It is the writer's opinion that the secret of successful stream-pollution control does not reside in the extension of legislative restriction.

SCIENTIFIC KNOWLEDGE

A review of the record discloses that solutions to many of the problems in trade waste treatment and recovery are not yet apparent. In this particular field the solution obviously lies in a more intelligent, consistent, and continued participation of industry in the search for an answer. Inertia and ignorance on the part of industry have perhaps been responsible for more delay and less accomplishment in the control of stream pollution than any other factor. The industrial sewer has been out of sight and hence out of mind. Industry's wastes have not been one of industry's major responsibilities.

However, industry cannot continue to content itself with negative resistance to extension of legislative control; it must begin to traverse more positive paths toward the development of scientific and economical methods of industrial waste treatment. On industry rests the responsibility of evaluating its contribution to the problem, of determining its method of attack, and of demonstrating its willingness to assist in the solution of one of the important problems of national economy. American industry particularly cannot afford to be accused of being unable to solve the technical problems of waste treatment, when its accomplishments in the allied fields of technological production have been as varied, as successful, and as inspiring as they have been.

As for the treatment of domestic wastes, the state of the art is sufficiently advanced to require no similar pressure. Obviously, certain technical facts are still lacking and scientific diagnoses and procedure still need to be developed, but economical and satisfactory processes of treatment are at hand to meet the issue.

MONEY

The key to the primary question of what can be done about pollution, to the writer's mind, lies in money. How can stream-pollution correction best be paid for? "Depression thinking" has given even greater emphasis to the ever-present question of how to raise money for sewage disposal and treatment. Such an enterprise is seldom popular, largely because, in most instances, the sewage nuisance is remote from the average taxpayer's eyes and nose; and in the years of depression, money for such purposes has become increasingly difficult to raise. A revival of a criterion of decency in the American public conscience is a first essential, but no less important is the development of financial techniques for raising funds.

Experience since 1933 has indicated how important an impetus may be given to sewage treatment plant construction by the artificial device of Federal grants-in-aid. More than 25% of existing sewage treatment plants in the United States have been constructed since 1933. It is difficult to believe that this is the result of chance or of a simple natural process of growth, rather than the obvious result of the artificial stimulation of activities in this field by Federal funds. The principle of grants-in-aid to stimulate special types of local endeavor is of course not new; it has been applied for more than fifty years in a variety of highway, navigation, reclamation, public health, and agricultural projects.

This experience leads the writer to suggest that perhaps the most valuable stimulus for correction of stream pollution is in the grant-in-aid principle. It need not follow, of course, that the grant-in-aid from a Federal Government is a pre-requisite, although it makes for uniformity of action throughout the country. The possibility of extending the State grant-in-aid procedure to sanitary projects is an opportunity for real progress. Such a procedure, within the State, again has precedent in the fields of education, agriculture, and highways. It has the merit of continuing and preserving the important local autonomy and responsibility which concerns so many at this time.

In other words, if money is the key to stream-pollution control, a central stimulation of activity—based, of course, upon a reasonable balancing of convenience—offers one of the soundest bases for future success. Correction is cheap, as such expenditures go, but the incentive has been missing.

SUMMARY

This brief discussion leads to the following answers, positive and negative, to the question of "What can be done about stream pollution?"

(1) Standardization of State laws on the subject is desirable. Too much faith in the efficacy of the law, however, is not warranted by past experience,

or the temperament of the American people. If the people are not ready, the law fails.

(2) Industry must show a greater interest in the problem of industrial waste. It must demonstrate that it is attempting to find economical solutions to these problems.

(3) State or Federal financial stimulation of sewage treatment plant construction offers a useful device for increasing the rate of installation of stream-pollution corrective equipment. It is perhaps the most helpful direction in which progress is to be made. Its political implications and its dangers must be carefully scrutinized. With proper control, however, it has demonstrated its possibilities of success.

(4) The permanent control of the situation lies in the co-operative effort of the public official, the private investor, and the man in the street. Intelligent appraisal of the problem and of the benefits to be derived from its correction should be the guide. Hysteria and undue charges in this field, as in any other, are not likely to produce permanent answers to the problem.

DISCUSSION

D. E. Davis,[26] M. Am. Soc. C. E. (by letter).—The paper by Mr. Streeter supplies for the first time a quantitative measure of the effect of acid in the reduction of bacteria in the Upper Ohio River. That the "step-down" method utilized in reaching these conclusions possesses validity is demonstrated by the close correspondence between the calculated and the observed results. That acid reduces the pollution load appreciably is indicated by the *B. coli* count at Wheeling, W. Va., which under winter conditions is 25% and under summer conditions is less than 1% of what might be expected under normal alkaline conditions.

The 1932 studies of the Pennsylvania State Health Department, under the immediate direction of Mr. L. S. Morgan, District Engineer, indicated that at Sewickley, twelve miles below Pittsburgh, the results are even more striking, since under the summer low-flow conditions in 1932, the *B. coli* index was 165 (compared to 3 760 for Wheeling), or only a fractional part of 1% of the anticipated results under "normal" conditions. They also indicated that for corresponding river flows, the oxygen content of the river in 1932 was better than for the 1914–15 period, in spite of a considerable increase in the population in the District. This unexpected result may have been due to the substantial cessation in the intervening period of the activities of the brewery and distillery industries, whose wastes make a heavy oxygen demand on the rivers. The increase in acid, experienced since 1914, had very decidedly reduced the bacterial load, and on these two counts the river water was better during low-flow periods than in earlier years.

Although it is known that acid reduces the "bug counts," it is not clear how this result is achieved. Whether the acid actually kills most of the *B. coli*, as chlorine does, or whether it merely makes them "play dead" like a "possum," only to come to life under alkaline conditions below, is a matter of conjecture as yet. No complete differentiation has been made as between the different strains of *coli* to lead to a certain conclusion as to whether acid may have a selective action, killing off permanently the pathogenic types and letting some other more resistant organisms continue their life cycles. An alternative possibility is that during periods of low-flow, coagulation and sedimentation effect the prompt deposition of bacteria on the river bottom, where they may lie dormant until the first freshet scours them out to increase enormously the load on down-stream filtration plants.

The rôle of plankton in the self-purification of streams under acid conditions is not yet completely understood, nor is there a definite correlation between plankton and bacteria, which might serve as a guide to the degree of purification which has taken place. It was shown in the 1914–15 studies that there was an abundance of plant life in the acid Monongahela, but that it was confined to a relatively small number of species. It was suggested that these types might have the ability of ingesting organic matter without the interposition of bacteria. It is entirely possible that some of these forms of life find optimum environments under acid conditions.

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[26] Engr. (The Chester Engrs.), Pittsburgh, Pa.

It has been noted that certain forms of grasses or "sea weeds" have begun to flourish increasingly in the Allegheny River as it has become more acid. The pools immediately below Pittsburgh (with pH usually on the acid side) are literally alive with this type of growth which flourishes on the rich river mud. The studies of the U. S. Public Health Service have shown that these plants, in generating chlorophyll by photosynthesis, give off large quantities of oxygen during daylight hours, thus measurably abetting reaeration at the surface of the water. They also exert a mechanical effect in intercepting and holding floating material, later to deposit it on the river bottom, as mud into which the plant extends its roots.

It is possible that this luxuriant growth and other plankton forms may directly break down complex organic material into simpler and more stable forms even without the aid of bacteria. In other words, under these acid conditions the absence of large numbers of the bacteria that are usually found below heavy sources of pollution may not mean that the processes of self-purification are largely postponed until more alkaline conditions obtain. The work may be taken over in large part by the river plankton.

These observations suggest the value and importance of further fundamental studies relating to the effect of acid on purification processes. They are not merely academic questions. Acid admittedly takes an economic toll in its attack on exposed steel and concrete structures, and in water treatment, but the U. S. Public Health Service has raised the question as to whether its beneficial effects on sewage may not outweigh these harmful results. A mine-sealing program is being pushed which may greatly reduce the quantity of acid entering the streams. Should this be completely successful, river conditions might become intolerable and secondary sewage treatment, at a greatly increased cost, might become necessary in some communities. Furthermore, if acid acts as a germicide, this function should not be ignored, since it contributes to the greater safety of those using the river for recreational purposes.

These questions are susceptible of fairly definite evaluation and policy determination. The time is ripe for their early consideration so that money may not be expended wastefully. To quote the author, they "fall more especially in the field of research, which in the long run must keep pace with that of the sanitary survey, as the two are virtually inseparable in meeting engineering problems of stream-pollution control."

L. S. MORGAN,[27] ESQ. (by letter).—The Pennsylvania Department of Health during the period from September, 1932, to February, 1933, conducted a sanitary survey of the Ohio River in co-operation with the Health Departments of down-stream States and the United States Public Health Service. The survey extended over the stretch of the Ohio River from its source at Pittsburgh to the Pennsylvania-Ohio State line, a distance of approximately 44 miles. Sampling stations were located at distances of three, twelve, twenty-five, and forty-four miles below the source at Pittsburgh and at distances of six and thirteen miles, and of one mile, above the mouths of the Monongahela, Allegheny, and Beaver Rivers, respectively, which are the main tributaries of the Ohio River.

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[27] Dist. Engr., Penna. Dept. of Health, Greensburg, Pa.

The main reason for conducting the survey was to determine the character and quality of the Ohio River water at and below the Metropolitan Pittsburgh District as a result of the discharge of untreated sewage in this metropolitan area.

The regimen of the Ohio River and its two principal tributaries, the Monongahela and Allegheny Rivers, is controlled by a series of navigation dams. These dams create large pools into which the sewage from a population of approximately 1 200 000 is discharged untreated in the Metropolitan Pittsburgh District from eighty-five separate municipalities, including the City of Pittsburgh proper.

The Monongahela and Allegheny Rivers drain large areas located in the bituminous coal fields of Southwestern Pennsylvania and West Virginia. The discharge of mine drainage from bituminous mines, together with the discharge of acid wastes from metallurgical plants, renders the water of the Monongahela River at its mouth acid practically at all times and the Allegheny River at its mouth acid especially during low-flow periods. These acids include free sulfuric acid, together with the sulfates of alumina and iron. The Ohio River, therefore, during low-flow periods contains varying quantities of the mineral acid and acid salts, the degree of acidity being dependent on the rates of flow and degrees of acidity of the two principal tributaries.

During periods of low flow and accompanying low velocity, the navigation pools in the Metropolitan Pittsburgh Area act as sedimentation basins in which are deposited settleable material contributed by the sewage and industrial waste discharges. Sedimentation is materially aided through the coagulation and precipitation of acid salts during periods of low flow. The passage of water at low velocities through the navigation pools and over navigation dams permits a high degree of surface aeration. Thus, during low-flow periods, these combined circumstances provide a method of sewage disposal at least roughly comparable to sedimentation.

However, because of the acid character of the river water, bacterial activity in the deposited sludge is inhibited and, as a result, the settled material does not undergo bacterial decomposition but remains for varying periods of time in what might be termed a "pickled" or "cold storage" condition until scoured out by sudden and rapid increases in stream discharge.

At such times, as velocities increase, the accumulated material is washed down stream in concentrated form and the sudden changes in the character of river water are reported as causing an undue burden on down-stream waterworks deriving their sources of supply from the Ohio River.

During the survey, determinations of the varying sanitary quality of the Ohio River water were obtained and correlated with varying rates of discharge (see Table 6). The station selected for securing such data was twelve miles below the point at Pittsburgh and below the points of discharge of practically all untreated sewage from the Metropolitan Area.

The dilution factors shown in Table 6 for the lower flows are lower than those generally recognized as necessary under ordinary circumstances to prevent nuisance conditions. In this case, the critical discharge to prevent nuisance conditions would be approximately 5 000 cu ft per sec, based on the contributing sewered population alone and disregarding entirely the equivalent population load of industrial waste pollution which, in the Pittsburgh area, would probably equal or exceed the sewered population. Nevertheless, despite the low dilution factors, general nuisance conditions do not exist in the river.

TABLE 6.—SANITARY QUALITY OF OHIO RIVER WATER BELOW PITTSBURGH, FOR VARIOUS RATES OF FLOW

(Data from Survey of 1932–1933)

Average daily discharge, in cubic feet per second	Dilution factors, in cubic feet per second per 1 000 population	Average dissolved oxygen, in parts per million	Average 5-day bio-chemical oxygen demand in parts per million	*Coli-Aerogenes* INDEX PER 100 CUBIC CENTIMETERS	
				Average	Maximum
1 500	1.2	6	0.2	160	700
2 900	2.4	7	0.5	160	670
7 200	6.0	11	2.5	290	3 400
19 400	16.2	12	2.6	4 300	36 700
44 000	36.6	13	3.5	5 300	37 000

The relatively high dissolved oxygen content at the various rates of discharge indicates that the oxygen resources of the stream are not being depleted to the point where general nuisance conditions would result. These high results also indicate that the dissolved oxygen available is not being utilized for the oxidation of organic material through bacterial activity.

The five-day B.O.D. is much lower during low-flow periods than would ordinarily be anticipated, but is more nearly normal during the higher flow. Based on the contributing sewered population the anticipated B.O.D. would probably vary from a maximum of approximately 37 ppm at the low flows to a minimum of approximately 1.5 ppm at the high flows, with the organic pollutional load uniformly distributed over the entire flow range. Assuming a reduction in organic material of 35%, equivalent to the removal obtained through sedimentation during low-flow periods, the resulting anticipated oxygen demand would still greatly exceed the observed values. It is, therefore, believed that the concentration of acid during low-flow periods renders the B.O.D. determination unsatisfactory as a yardstick for measuring the true pollutional load of organic material and that some revision in the standard procedure is necessary to obtain a true picture under such conditions.

The *coli-aerogenes* index shows clearly the inhibiting effect of the acid condition in the river at low stages. With increased discharge, the index shows a decided increase probably more nearly representing normal conditions. In the absence of acid wastes and under similar pollutional loads much higher indices would be anticipated.

These results indicate a fairly stable condition in the Ohio River below Pittsburgh during periods of low-stream flow due to the presence of acid wastes inhibiting normal biological and chemical processes of purification and preventing general nuisance conditions. With increasing stream flow, however, resulting in more normal conditions through dilution of the acid wastes, the results more nearly represent the anticipated conditions resulting from the discharge of untreated sewage from such a large population.

C. A. Holmquist,[28] Esq. (by letter).—In reviewing Mr. Stevenson's paper the New York State Department of Health finds itself in general agreement with the treatment which he has given the subject. During the July, 1935, floods, conditions in New York State were comparable with those suffered in Pennsylvania and other States during March, 1936, and the Health Department's activities and policies with respect to protection of the public health at that time closely paralleled those undertaken and followed in Pennsylvania. During the 1936 flood, conditions were less severe in New York State. However, these two flood experiences have enabled the New York State Department of Health to formulate a somewhat definite plan of action and policy to be followed in future floods.

There is one point mentioned in Mr. Stevenson's paper however, with which the writer is not in complete agreement. This is in reference to the issuance of general and unqualified "boil-water" warnings in times of such catastrophes. There are, of course, certain differences in the organization of public health work in the two States that might make the policy followed by Mr. Stevenson perfectly proper for his State and improper for New York. In New York State there are many competent local health officers and water-works officials upon whom the Health Department can always rely—who would be the first to advise the Department if anything were wrong—and upon whom it would place the responsibility for deciding whether or not a "boil-water" order should be issued. During both the 1935 and 1936 floods the Department issued promptly general "boil-water" warnings, and it is believed that such action is advisable. However, these warnings were qualified in such a way that they left to the local health officer final decision in the matter. People were advised generally to be guided by the suggestions of their local health officers and water-works officials, but, in case there was any doubt as to the safety of the supply, to boil their water or treat it with chlorine compounds of various kinds as a precautionary measure, until the exact condition of the supply could be ascertained. Had these general warnings been issued without some qualifications, the people of Albany, for example, would have had to boil their water—which would have been unfortunate in view of the known safety of that supply. Even with the qualified "boil-water" warnings there were several instances when health officers issued local warnings which were unnecessary. However, in such cases, the local health officers were uncertain as to the conditions, and there is no criticism under the conditions prevailing of "erring on the side of safety."

Mr. Stevenson has stated that the primary responsibility for protecting the public health in time of floods should rest upon local authorities. This should be emphasized in the strongest terms possible. State Health Departments and other units of State Government can furnish valuable guidance and assistance during emergencies, but local health officers, local water-works officials, and other local officials have the real opportunity of reducing to the irreducible minimum the public health hazards which invariably arise during such periods. The protection to the public health at such times is likely to be very much in proportion to the ability of the local officials to meet the problems promptly and properly.

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[28] Director, Div. of Sanitation, New York State Dept. of Health, Albany, N. Y.

Therefore, in any planning which should be done in advance to meet the public health needs of possible future catastrophes, training of local health officers and water-works operators or superintendents should be the central objective. A State Health Department should strive to provide continuously, through water-works schools, frequent inspections of supplies, etc., for the general education and training of local officials. It should seek to bring all of them to a high level of competence so that when floods occur they will know what to do, how to do it, and have the essential equipment at hand for their work.

In stressing the importance of the training of local health officers and water-works operators, the writer is not unmindful of the great need for physical improvements in supply systems that will lessen the danger of failure or contamination during floods. Water-works officials should throw overboard the philosophy of "The Arkansas Traveller," who "wouldn't repair the leak in the roof when it was raining, because then he couldn't work, or when it was not raining, because then it didn't leak." The time to make improvements to lessen the hazards to public water supplies occasioned by floods is before the floods occur. State health officials frequently come to the end of their rope in appealing and preaching to the governing boards of municipalities for favorable action on important matters. The chance of success is always enhanced considerably when there exists in the local community a health officer or local water-works man who is thoroughly trained and imbued with the proper sense of public health values. Therefore, the proper training of local officials is the important objective. Carried to the proper end effort expended in this direction will not only equip the local health officer or water-works official to respond with the maximum efficiency during any emergency but will also help to bring about many of the important improvements in public water supplies which will lessen the dangers during emergencies and which to some extent may at present be opposed by insufficiently impressed local officials.

Robert Spurr Weston,[29] M. Am. Soc. C. E. (by letter).—In Mr. Stevenson's thorough review of the sanitary problems incidental to floods, he has set down little to criticize and much to commend. His administrative experience as public health engineer for a large and varied State permits him to speak with authority. The statement of a private practitioner must necessarily be limited in comparison.

More people comprehend the practical engineering problems in the construction of flood reservoirs than the social problems involved. Imagine, for example, as might be the case on more than one river system, a series of upland valleys which are excellent sites for flood control reservoirs but which are occupied by expensive summer residences, country clubs, and the hamlets which they support. Down below are mills and cities, damageable by floods and supporting a large population; but try to apply to the situation the methods so successful on the Miami River, and then watch the summer residents rush to the State Houses and to their powerful political friends. They would say "dike or otherwise protect," or, "move low-class dwellings and shops away but leave us our homes and our æsthetic enjoyment." Again, there is the resistance of the taxpayer to benefits which accrue to riparian dwellers but not directly to him. These and other difficulties are often controlling.

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[29] Cons. Engr. (Weston & Sampson), Boston, Mass.

It is reported that when Germany decided to war against France in 1870, von Moltke took a stack of telegrams from his desk and, sending them, mobilized the Prussian Army for victory. An analogous preparation is required for the successful control of the public health during floods, as Mr. Stevenson has shown.

As he states, the safeguarding of water supplies is of greatest importance. It is far better, however, to have water-works flood-proof than to depend upon emergency measures when the floods arrive.

In Massachusetts, when the laboratory of the Department of Public Health, at Lawrence, was flooded into temporary desuetude, a number of private laboratories were called upon to assist. Viewing their results, it was rather surprising that even when the flood was at its height almost none of the tube well supplies in the flooded valleys went bad. On one day, out of eighteen samples collected, mostly from ground-waters in the flooded Charles River Valley, only one was polluted and that one only slightly, showing the value of good construction and the protection given by impervious layers in the soil.

It must be realized, however, that Massachusetts is particularly fortunate in having only one water supply from a polluted river. The problems which confront the cities whose sites dot the courses of rivers in the great valleys, where rivers are necessarily both sources of water and places of final disposal of sewage and waste, are happily absent. In Massachusetts, too, 97% of the population has access to public water supplies, and 87% is tributary to sewerage systems.

During the flood in Massachusetts, the inspiring aspect was the efficiency of the Department of Public Health; the depressing aspect was the lack of comprehension and organization shown by some of the municipal departments. Co-operation among towns under State leadership was excellent. At Lawrence, for example, hose connections were made to the hydrants of three neighboring towns and the supply was thus maintained.

Flood control is important, but safe water supplies and sewage disposal are more important. Consequently, no scheme of control should be planned without regard to the public health factors, and the judgment of the health authorities should be taken into account in any decision regarding works for flood control.

The U. S. Supreme Court, through the late Associate Justice Oliver W. Holmes, lucidly stated that "a river is not an ordinary amenity but a treasure." If so, all the treasurers, including the hydrologist and the public health engineer, should control the assets and liabilities. Furthermore, it seems best to have these treasurers appointed by the States and do the business which is necessarily inter-state by using compacts and agreements, rather than by submitting to

Federal regulation or direction. Federal data and advice, however, should be welcomed and their costs cheerfully borne.

Karl Imhoff,[30] M. Am. Soc. C. E. (by letter).—An impounding reservoir or a sewage lagoon is a natural purifying arrangement for sewage or polluted streams.[31] In Germany, many plants of this type, both large and small, are operating satisfactorily. Purification takes place according to the same rules as in the self-purification of streams. It is possible to calculate the oxygen demand so that the amount of oxygen used can be equalized with re-aeration. In making such a calculation, various factors must be taken into account:

(1) The consumption and the absorption of oxygen depend on the detention period and on the temperature. Absorption is governed, in addition, by the rate of oxygen demand.[32] Complete biological purification may be assumed to take place in 20 days detention at 20° C.

(2) Oxygen absorption is also proportional to the surface area of the pond. The greater the surface and the less the depth of the pond, the greater will be the oxygen absorption for any given volume and detention period.

(3) For all depths of more than approximately 2 ft, the depletion of oxygen and the speed of decomposition are constant. For depths of less than 2 ft, the speed of decomposition increases rapidly with decreasing depth, probably because the plant and animal life on a lighted bottom has a stronger effect. In very shallow ponds the biologic purification is completed in a few days.

(4) Sunshine generates more oxygen from green water plants than the water will absorb from the surface.

(5) A completely still water surface takes little oxygen from the air. Every wave, whether from wind or boat, increases the oxygen absorption greatly.

(6) Sludge in sewage lagoons should be eliminated as much as possible by the use of settling tanks. Some bottom sludge will accumulate anyhow, because settling occurs in quiescent lagoons, even with pre-settled sewage. The accumulation of bottom sludge is continuous throughout the year, and is due to putrefactive bacteria which, in turn, use oxygen. The greatest demand on oxygen is during the warm season, just when the flow is diluted the least. The oxygen demand of the accumulated bottom sludge is at such times generally much higher than that of the dissolved organic matter which flows through uniformly. (The three sludge lagoons in the Ruhr Valley, Germany, have the advantage of being scoured by flood waters almost every spring, and even during the summer, they are not heavily loaded with sludge. If such natural washing does not occur, it may be advantageous to dredge the bottom.)

Henry D. Johnson, Jr.,[33] Esq. (by letter).—The subject of planning for pollution control at Pittsburgh has been covered in a most able manner by

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[30] Cons. Engr., Essen, Germany.

[31] "Impounding Reservoirs as a Substitute for Biological Sewage Treatment Works in the Ruhr District," by Karl Imhoff, *Sewage Works Journal,* January, 1931.

[32] "Measures of Natural Oxidation in Polluted Streams," by H. W. Streeter, M. Am. Soc. C. E., *Sewage Works Journal,* March and May, 1935.

[33] Former Chf. Engr., Dept. of Public Works, Pittsburgh, Pa.

Mr. Davis. The writer knows of no one in that city who is more familiar with this subject than he. It seems almost obvious that Pittsburgh engineers and municipal authorities must start entertaining plans for the treatment of the sewage and industrial wastes, but whether or not the time has yet arrived for building an extensive and expensive sewage treatment system is difficult to say.

The writer is personally interested in pollution control at Pittsburgh because of his interest in the possible recreational facilities afforded by the near-by streams. Much of the enjoyment of water sports is taken away by the sewage and industrial waste pollution which exists throughout the stretches of the three rivers within Allegheny County. When one is able to cruise up stream beyond the locations of large cities or boroughs, or above large industries and above those streams which drain mines, one finds the rivers in almost their primitive state. They are beautiful and attractive beyond words. It is not to be expected that any feasible plan of sewage treatment and industrial waste control will restore the rivers to their original purity, but there is no question that the pollution could be reduced sufficiently to make safe the use of the rivers for sports.

Mr. Davis' conclusion that primary treatment by sedimentation will be the only form of treatment required for some time to come, is in general correct, although certain communities that discharge their wastes into small streams will unquestionably have to consider complete sewage treatment in the immediate future.

Recent studies on the costs of the proposed plants indicate that it would be possible to afford very definite relief at a more or less nominal figure. Earlier reports and studies have been based on the premise that the effluent from treatment plants should be bacteriologically pure. However, a comparison of the costs of establishing water purification plants for down-stream communities, with the additional cost of sewage treatment plants which would provide a pure effluent, shows that, at present, it would be more economical to provide adequate water purification plants and only primary treatment for Pittsburgh and its environs. The primary treatment of sewage would unquestionably benefit those sections of the community which border the streams, because of the consequent elimination of sludge beds with their attendant foul odors and unsanitary conditions.

As Mr. Davis points out, the topography of the Pittsburgh District is such that consideration cannot be given to sewage treatment for the municipality of Pittsburgh alone; all or most of the cities, boroughs, and townships of Allegheny County must be considered. There are 116 municipalities using the three canalized rivers for disposal, and it would be futile for any one of these municipalities to attempt a solution of the problem without the co-operation of all the others.

To avoid petty politics and party dickering the only possible way to secure action is by legislative fiat. The time is now ripe for civic bodies and various leagues of voters to support legislation leading to the desired end.

The writer is not prepared to state that the plan for sewage treatment developed by the City of Pittsburgh is the best possible one. In any event, however, it can be made the basis of a final plan, and, as such, it should be endorsed by civic bodies for the purpose of obtaining the support of the citizens. Then, when the appropriate time comes, some plan (perhaps the present one) should be endorsed by the Legislature and each municipality should be required to adhere thereto. In lieu of the foregoing it might be proper for the Legislature to establish a Metropolitan District Plan, combining the municipal bodies of Allegheny County into a Sanitary District similar to that in Chicago, and empowering the officials of the district to raise funds to carry out and supervise the construction, maintenance, and operation of treatment plants, intercepting sewers, and, eventually, of disposal plants.

Charles M. Reppert,[34] M. Am. Soc. C. E. (by letter).—The paper by Mr. Davis is a most interesting and timely discussion of one of the most important problems confronting Pittsburgh and its metropolitan area. There are some very unusual features involved, which must be fully taken into account. Inter-relationships with the acid mine drainage and proposed storage reservoirs are involved and there are many difficulties due to the nature of the topography. It is clear, as Mr. Davis states, that the situation can be met only by co-operative action of the many municipalities involved in adopting, financing, and carrying out a plan. The questions involved are: (1) The present necessity for reducing the sanitary load on the Ohio River in the interest of down-river water supplies, or for the prevention of general nuisance in the stream below Pittsburgh; (2) the need to prevent local nuisance along the shores of the rivers, particularly along the margin of the down-town business district; (3) the effect of reducing the acidity of the rivers by mine sealing; and (4) the effect of the construction of the system of proposed storage reservoirs for flood prevention and their utilization for increasing the dry-weather flow.

With regard to general nuisance at low stages in the Ohio River, below Pittsburgh, it can be stated that although a critical condition has been approached it has not been reached, probably because the increased sanitary load has been offset by the increased acid content of the rivers. It is evident that the effectiveness of mine sealing in reducing the discharge of acid mine water becomes one of extreme importance. It must be realized, however, that the acidity problem is one of control rather than elimination. Although the Monongahela River has been acid for many years, the Allegheny was alkaline until 1923; since that date it has shown a marked upward trend in acid content and in the frequency and duration of acid periods. As a matter of water conservation, the mine-sealing projects which have been carried out have been fully warranted in the public interest. On the other hand, a material reduction in the acid content during low-water periods would undoubtedly lead to a most aggravated condition of gross nuisance in the Ohio River below Pittsburgh. There is, however, a limit to the acid-reduction that can be secured by mine sealing, and it has not been demonstrated that the reduction will reach a degree that will affect the sewage disposal problem to a great extent. However, such possibility should be thoroughly investigated.

The sealing of the mines will be well worth while if the upward trend of concentration can be stopped and the extreme intensities prevented. It is evident that this particular question should receive further study, as it has great effect upon what provision should now be made for secondary treatment. If mine-sealing operations can be continued so as fully to cover the tributary coal areas, and if their effectiveness is such as to assure a very material reduction in acid content of the river water, then provision should undoubtedly be made for the early installation of secondary treatment works.

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[34] Civ. Engr., Pittsburgh, Pa.

The effect of sewage discharge from the Pittsburgh area on down-river water supplies is of the greatest importance in determining the immediate necessity for sewage treatment at Pittsburgh. At the present time it appears that this effect is of concern during rising, rather than at low, river stages, and that critical conditions obtain or are approached during rises in the Ohio River, following prolonged low-water stages, at which times the sewage sludge deposited in the beds of the rivers at Pittsburgh and held in a state of retarded decomposition is scoured out and carried down stream. It is evident that, if the pollution contributed by Pittsburgh at the present time is materially affecting the difficulty and safety of purifying down-stream water supplies, any appreciable increase in the sewage load or the occurrence of abnormally long dry-weather periods would greatly aggravate the situation. Further data, however, should be made available as to the proportion of the pollution load contributed by Pittsburgh compared with that contributed from other and local sources.

What will be the effect of the construction of the system of proposed storage reservoirs now receiving the attention of local, State, and Federal authorities? Engineers of the Pittsburgh Flood Commission have estimated that the utilization of storage during dry-weather periods will increase the dry-weather flow in the Ohio River, at Pittsburgh, about 250 per cent. If this condition is brought about, then the necessity for secondary sewage treatment will be deferred for some years, and the number of primary treatment plants needed at present might be reduced. This would enable the adoption of a construction program covering a period of years.

The existence of local nuisance along the river shores at Pittsburgh constitutes one of the strongest local arguments for undertaking primary sewage treatment without delay. Such nuisance has increased in recent years, and has become particularly offensive on the margins of the down-town business district. Public decency and civic pride demand its abatement; if it is allowed to continue and to increase, depreciation of property values will result.

The writer is fully in accord with the proposed general plan described by Mr. Davis for the installation of a number of treatment plants located with due regard to economy and topography. Primary treatment will unquestionably suffice for a number of years, but it will be necessary to locate and design the treatment works in such manner that they will fit into the future installation of secondary plants. Likewise, the areas of the sites selected should be adequate for such future needs.

The first steps have been taken in solving the problem of pollution control at Pittsburgh. The Metropolitan Main Drainage Survey Project has made available necessary sanitary data and general plans and estimates. The definition of sanitary policy for this District has been made possible. Further studies should now be made of sewage characteristics and flow, acidity, and the effect of storage reservoir construction. The selection, availability, and cost of plant sites should be investigated in detail, with due reference to the cost of collection, which is an important matter because of topography and shore conditions. Finally, a decision should be made as to the program of construction—which, for financial reasons, should preferably be spread over a term of years—and the preparation of construction plans for the works to be first undertaken should be started so as to develop fully and settle all questions as to location, economy, and cost. These steps should be taken by the local authorities who will be responsible for the construction and operation costs of the works, in order that public funds be expended only to the extent that conditions require and with proper relation to other needs.

Edwin K. Morse,[35] M. Am. Soc. C. E. (by letter).—The paper by Mr. Ryder is unusually presented and very fair, and the writer has only one comment to make on it. Most of the head-waters of the Pymatuning are swamp lands and very flat country, which, of course, make the run-off of that river much more uniform than that of any other river west of the Allegheny Mountains in the Commonwealth of Pennsylvania. To the extent that this condition affects the results of the reservoir operation, the paper might be misleading to an engineer not acquainted with the location.

In several public talks on the Pymatuning, the writer has emphasized the necessity of building a reservoir on the Cussawago River, in a water-shed six miles from the Pymatuning Reservoir, and connecting it by a tunnel and open ditch to the latter. Then, when water is required from the Pymatuning Reservoir in the low-water period the gates of the Cussawago could be opened and the water of that reservoir be permitted to flow through the Pymatuning, thereby maintaining the lake level in the latter during that part of the season when it would be a great recreation center for approximately 3 000 000 people within a radius of seventy-five miles.

The dry season in this vicinity generally takes place from about the first of July to the first of November, but the schools open early in September, and the resort would be materially vacated by the first of October.

James H. LeVan,[36] Assoc. M. Am. Soc. C. E. (by letter).—General methods and laboratory tests applicable to a pollution survey of any stream are given in this paper, and Mr. Streeter has mentioned the fact that additional tests will depend upon the circumstances of the particular survey being made. It is very necessary that consideration be given to the uses that are made of the stream and to the presence of undesirable substances in the water, particularly to the substances originating in industrial wastes. A survey being made must take into account all pollution factors of the river under consideration. Too often only dissolved oxygen (D.O.) and bio-chemical oxygen demand (B.O.D.) tests are considered. The importance of the bacteriological and biological tests is not always realized. The former give the most sensitive indices of significant changes in the pollution of a river. The latter tests will "go hand in hand" with the other tests and will aid in solving questions that arise when unusual chemical and bacteriological results are obtained.

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[35] Cons. Engr., Pittsburgh, Pa.

[36] Passed Asst. San. Engr., U. S. Public Health Service; Officer in Chg., Aedes Aegypti Control Unit, U. S. Quarantine Station, Miami Beach, Fla.

The U. S. Public Health Service is making a study of the Scioto River, from Columbus, Ohio, to its point of discharge into the Ohio River at Portsmouth, Ohio. The purpose of this survey is to observe the changes that occur in a polluted stream as the major sources of pollution are controlled by the construction of waste treatment plants. Already the City of Chillicothe, Ohio, has placed a new sewage treatment plant in operation. The City of Columbus closed its overloaded plant in 1937 and has turned all its sewage into the primary settling tanks of its large new plant, pending the placing of this activated sludge treatment plant in operation. The City of Circleville, Ohio, midway between Columbus and Chillicothe, is about to construct a plant. At this point wastes from a strawboard paper mill and several canneries enter the Scioto River. There are two paper mills at Chillicothe with wastes that discharge untreated into Paint Creek, one of the main tributaries of the Scioto River.

For this river study, it was necessary to select sampling points that would give information above and below the major sources of pollution, and above and below the river's main tributaries. Fortunately, highway and railroad bridges are situated at points where sampling stations should be established to obtain this information. As the survey progressed the results obtained showed the section where most rapid self-purification was taking place, and, consequently, where there should be the most frequent and intensive sampling. Likewise, stretches were revealed, where the river was becoming stabilized and where sampling need not be as frequent. As there is a limit to the laboratory work that can be performed by the personnel of a field laboratory, it is necessary that the officer in charge of it observe carefully to see where efforts should be concentrated and where unnecessary sampling may be discontinued. He must be on the alert for all changes that occur in the river and alter the work of the laboratory accordingly.

The following determinations are made on river samples at the Scioto River Investigation Laboratory, at Chillicothe: Temperature (at the time each sample is collected); suspended solids; methyl orange alkalinity; pH; dissolved oxygen (D.O.); 5-day bio-chemical oxygen demand (B.O.D.); bacteria growing on agar at 37°C, in 24 hr; and the most probable numbers (M.P.N.) of sewage bacteria (coliform group). In warm weather the dissolved oxygen and the bacteriological samples are transported by automobile to the laboratory in an iced container. Although the dissolved oxygen determination should be made as soon as possible after the sample is collected, the method used in this survey permits it to be done in the laboratory by a sanitary chemist instead of by a sample collector in the field. In addition, on one sample daily, B.O.D. determinations are made on identical samples with incubation periods of 3, 5, 7, 10, 12, 15, 20, and 25 days to check the course of the oxidation curve through the primary (carbonaceous) and secondary (nitrification) phases. Furthermore, one sample daily receives bacteriological examination through the completed test; the others receive the partly confirmed test using brilliant green bile (2%) broth. To save work in the field laboratory, composite samples preserved with concentrated sulfuric acid are sent monthly to the Stream Pollution Investigations' Laboratory, at Cincinnati, for nitrite and nitrate determinations. At times, these determinations are also made at the field laboratory to check up on advanced nitrification.

As part of the study of the Scioto River, a biologist makes weekly examinations of river samples. One week he examines freshly collected samples and the next week formaldehyde-preserved samples are sent to him at his Cincinnati headquarters. Mud samples are sent monthly to Cincinnati for examination. The biological forms play an essential part in the natural purification of a stream, and only by regular biological examinations can the changes in kind and relative numbers be recorded during and following a stream sanitation program. The biological examinations should never be slighted or minimized.

Unless the officer in charge has good reasons for doing so, all chemical, bacteriological, and biological examinations made on water samples should conform with the regularly accepted standard methods so that the results obtained will be comparable.

Present-day methods of automobile transportation permit samples to be transported to the laboratory with dispatch. It is more difficult to collect the samples at the correct stream points so that a representative sample will be secured each time the sampling can is lowered into the river. If bridges are not convenient, boats will be necessary unless the stream is so shallow that it can be waded. (The U. S. Public Health Service has a standard sample collecting can.[37]) Portable laboratories in automobile trailers can be used for the study of industrial wastes and untreated sewages when a large river system is surveyed.

When the study of stream pollution is undertaken nationally surveys will be made on many streams for which hydrometric data will not be available. Such situations will require the establishment of gaging stations on the main streams studied and on all their important tributaries, the regular collection of gage height records, and discharge measurements to prepare rating curves. Where such data cannot be obtained float measurements may have to be substituted, or some other method will have to be devised to obtain these data. It is necessary to know the different quantities of dilution water that enter a river system and the mean velocities and times of flow at different stages.

JOHN C. H. LEE,[38] M. AM. SOC. C. E. (by letter).—It is axiomatic that, with public opinion in support of a law, much can be accomplished; but without such support, results may be incommensurate with effort. For years, and until recently, the public's attitude toward stream-pollution prohibition has been apathetic, and even antagonistic; but, of late, the Army Engineer has been able to quote Section 13 of the 1899 River and Harbor Act, and the 1924 Oil Pollution Act, effectively.

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[37] *Public Health Bulletin No. 171;* also, Supplement No. 90, Public Health Repts., U. S. Public Health Service, Washington, D. C.

[38] Col., Corps of Engrs., U. S. Army; Div. Engr., North Pacific Div., Portland, Ore.

The Army Engineers have approached the problem along the Lower Delaware River enlisting the co-operation of industry, seeking a solution to each plant's problem that will be the least costly—if indeed an actual profit cannot be achieved from the by-products. The Army Engineers' inspectors receive welcome entrée to all plants, because the executives appreciate this attitude.

Publicity is probably as great a threat as prosecution. Industries spending thousands of dollars for public good-will are quick to realize the impending loss of that good-will, if they are posted as underminers of public health. It is not difficult to get plant executives to write their own "sentences"—what they will do to abate or remove pollution, and when—through co-operation rather than prosecution.

All that is required is patience and persistence and some persuasion. Army Engineers have contacted nearly two hundred industrial plants along the shores of Delaware River and Bay within the two years, 1936 and 1937. Without exception, all executives have responded—some more effectively, but all willingly. Impartial observers have expressed realization of improved conditions on the waters of the port, which are being sampled periodically.

In order to bring the municipalities into effective co-operation, the Army Engineers, with the written approval of the local authorities, have written into a new project report for the Delaware's channel—Philadelphia to the Sea—the requirement that annual maintenance dredging of 100 000 and 10 000 cu yd must be performed, respectively, by the Cities of Philadelphia and Camden, N. J., until they shall have made effective installations of primary sewage treatment works.

The plan for the culm-polluted Schuylkill River is based upon the principle of co-operation between the anthracite industry and the public agencies charged with the administration of the plan of recovery and improvement.

W. H. Wisely,[39] Assoc. M. Am. Soc. C. E. (by letter).—One of the authors, Mr. Streeter, does well to place emphasis on the need for consideration of stream usage in the planning and interpretation of stream studies to be used as bases for pollution-control programs. It is often also advisable to give consideration to the possibility that a stream may be developed in the future for functions not utilized at present. Rock River, in Northern Illinois, for example, is potentially a fine recreational stream, within 100 miles of Chicago. At present (1938), however, it is not extensively developed for this purpose because several other streams offering vacational advantages are closer to Chicago. In consideration of present pollution abatement requirements in the Rock River Basin, the Illinois Sanitary Water Board recognized this probable future recreational development and any stream studies will be planned with this in mind.

The appropriate engineering analogy between the river system under effective pollution control and the well-designed bridge, which is used illustratively by Mr. Streeter, can be extended somewhat further. Determination of the degree of treatment required at each source of possible pollution may be likened to the design of the structural members of the bridge, under-treatment representing a weak member affecting the usefulness of the entire structure, and over-treatment representing wasteful design.

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[39] Asst. San. Engr., State Dept. of Public Health, Div. of San. Eng., Springfield, Ill.

Where industrial pollution is of major importance in a river study, it is advisable to determine the actual operating times of the various industries during the year previous to the sampling period, in order to evaluate, properly, the residual pollution in the form of sludge deposits that may be present when sampling is begun. During sampling of the Illinois River below Peoria, Ill., and Pekin, Ill., in July and August, 1936, industrial pollution representing a combined population equivalent of 789 000 entered the river. However, several industries had operated during the winter and spring preceding the sampling period but suspended operations prior to July 1. Wastes from the latter represented an additional population equivalent of 375 000. Accordingly, it was necessary to modify computations involving the river data to allow for sludge deposits in the river during the sampling period not accountable to the pollution actually discharged during July and August.

Mr. Streeter's discussion of criteria for permissible limits of pollution of streams used for sources of water supply, commercial fishing, and industrial development is highly practical. However, there is a need for similar practical criteria to govern streams used for recreation, particularly bathing, as this usage requires much more rigid pollution control than any other.

Bacterial studies on the Upper Fox River in Illinois, in a reach used extensively for bathing and recreation, revealed that *coli-aerogenes* concentrations of 10 per 100 ml to 28 700 per 100 ml were present at some of the most popular bathing places. The program of restoration of this stream involved the requirement of complete treatment, with bathing-season effluent chlorination, at all sources of human sewage pollution so that the foregoing maximum value has been greatly reduced. Nevertheless, there may still be some question regarding the absolute protection of bathers due to combined sewer overflows and similar sources of intermittent pollution, evidencing a need for definite interpretations of bacterial analyses of the waters from this stream.

The following facts must be given consideration in the development of criteria for permissible pollution limits of streams used for bathing:

(1) A stream offering recreational advantages by virtue of its proximity to metropolitan areas and natural environment will be used for bathing almost entirely without regard for the actual existing pollution (it is very doubtful that policing of such streams as a public health measure would be practicable);

(2) It is not possible, from a practical standpoint, to maintain the water, continuously, at natural bathing places, in such condition that it will meet the usual requirements for swimming-pool waters; and,

(3) In addition to the intestinal bacteria hazard, the presence of bacteria capable of producing skin, ear, eye, and throat infections warrants consideration.

Appreciating the need for practical interpretations of bacterial concentrations in stream waters used for bathing, the Great Lakes and Upper Mississippi River Boards of Public Health Engineers have adopted by resolution, the following tabulation, indicating the "Significance of the *B. Coli* Index as a Measure of Water Pollution":[40]

Significance of Index	*B. Coli* Index, or *B. Coli* per 100 cu cm of sample
Water sample free of pollution	0
Treasury Department standard for drinking water	Not more than 1
Indicative of good water—normal for inland and Great Lakes, free of sewage pollution	10 to 100
Normal for inland streams, free of detrimental sewage pollution, might be attributed to land wash	100 to 500
Suspicious—generally indicates mild pollution in natural waters, but dangerous in proximity to fresh sewage pollution	1 000
Definite evidence of fresh sewage pollution—menace to health	10 000
Heavy sewage pollution—definitely dangerous	100 000
Normal sewage	1 000 000, or more

The foregoing classification is based on analysis of considerable data assembled by the Michigan Department of Health and the Michigan Stream Control Commission.

F. C. DUGAN,[41] M. AM. SOC. C. E. (by letter).—Stream-pollution control in the Ohio River Basin cannot be accomplished by any one State acting alone; it can result only from co-operative action by all of the fourteen States that lie partly in the Ohio River drainage area.

The *B. Coli* index of the raw water determined at the various water-works intakes along the Ohio River clearly shows that there is, at times, a very thin line of defense against outbreaks of water-borne diseases. At Ashland, Ky., it is noted that during the twelve years, 1926–1938, there were only a few months during which the average *B. Coli* index of the raw water was not greater than that which is considered as the upper limit of pollution that a water filtration plant should be required to handle. During and preceding the outbreak of gastro-enteritis, which occurred during the great drought of 1930–1931, there was not the slightest indication from the bacteriological analysis of water supplied to the inhabitants of several cities in Kentucky that the outbreaks resulted from impure water. However, it is now believed by many that the water supplies were responsible and that the causative agent was some toxin, of either chemical or bacteriological origin, which resulted from the excessive pollution by sewage and industrial wastes and which was not removed by any known method of water purification.

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[40] Rept. on Coastline Pollution Surveys of Michigan, Michigan Stream Control Comm., June, 1933.

[41] Chf. Engr., State Board of Health, Louisville, Ky.

It is possible that the acid wastes, especially in the Pittsburgh area, have prevented local nuisances from occurring, but certainly the discharge of acid wastes is causing serious economic loss because of damaging effects on steel hulls of boats, pipe lines, concrete, and other structures, and because of the increased cost of water treatment and destruction of fish life. It is known that the bulk of these acid wastes come from abandoned coal mines, and it is essential that the mine-sealing program now (1938) in progress in several States be continued until these acid wastes are reduced to a minimum.

The canalization of the Ohio River, although it is of benefit in some respects, has intensified the pollution problem and has increased the difficulty of treating the water satisfactorily for domestic and industrial uses.

Several of the States, acting under the authority granted by a Joint Resolution passed by the Seventy-fourth Congress, have appointed commissioners who are now (1938) at work drafting a compact which will eventually lead to definite plans for pollution control in the Ohio River basin.

G. M. RIDENOUR,[42] ASSOC. M. AM. SOC. C. E. (by letter).—So thoroughly has Mr. Streeter covered the mechanics of stream-pollution control surveys that little seemingly remains to be said in this respect. To the writer the paper clearly emphasizes five important facts: (1) That effective pollution-control results can be secured only by general control of water-shed areas rather than by local sections; (2) the extensive scope necessarily implies the establishment of large-scale technical organizations for each specific drainage area; (3) the eventual establishment of regulation by either intra-State compacts or by Federal bodies; (4) the important part that both applied and pure research must of necessity play in such stream-pollution surveys; and (5) the question as to when stream-pollution surveys become necessary.

Control by water-shed area is evidently the only logical method for securing effective, consistent, and economical solution of stream-pollution problems. Broad control of this kind seems to be without doubt the only fundamental method of attack. It is common experience to all those interested in stream pollution to observe cases of over-treatment and under-treatment of sewage and wastes in the same river basin within comparatively short distances, with attendant economical loss in both cases. Such situations have arisen not through intent, but through a combination of lack of adequate knowledge of river-purification capacities and long-range planning for the specific watershed under consideration. Fundamental knowledge of river-purification characteristics and capacities is the first requisite of intelligent planning and must be obtained through more comprehensive and embracing information than can be secured from over-burdened local sections. Treatment can be distributed economically only through a combination of knowledge of river-purification capacities and long-range planning in waste treatment. Mr. Streeter's plan of pollution-survey control meets both these requirements.

The maintenance of complete technical organizations for each specific drainage area or river basin is also an integral part of the program. Each basin will require the same routine control measures, regardless of the degree of pollution, and will likewise embrace the same basic branches of the science.

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[42] Asst. Prof., Dept. of Water Supplies and Sewage Disposal, Rutgers Univ.; Research Engr., Dept. of Water and Sewage Research, Agri. Experiment Station, New Brunswick, N. J.

The establishment of intra-State or Federal regulatory bodies in river pollution is only a question of time. Pollution control can only be limited, temporarily, to overburdened parts of a drainage area. The ultimate direction for most successful accomplishment of the stream pollution survey control outlined by Mr. Streeter appears to lie in a national policy-making organization and local intra-State or Federal regulatory bodies applying to each specific water-shed area. The problem cannot escape a national scope without simply producing a geographical exaggeration of present local conditions.

A phase of the program which the writer feels could be more fully stressed is the important part that research should play in such stream-pollution control surveys. This research can be directed, very profitably, to increasing certain fundamental knowledge of stream-purification agencies and principles, which can be obtained only in conjunction with large-scale programs of the kind outlined.

Of greater practical significance and of vital necessity in any stream-control survey is the part that research could play in determining, more accurately, the permissible limits for pollution control. The interest in this phase of the problem has occupied second place in relation to treatment plant devices and efficiencies, with the result that wide divergencies in stream-pollution criteria now exist under different regulatory bodies. These different requirements are sufficient to make wide differences in waste-treatment requirements, under these different regulations. This applies particularly to the maximum bacterial pollution permissible for natural bathing and recreational areas, and also to minimum dissolved oxygen requirements to prevent nuisance and maintain fish life in the stream.

The pollutional indices for drinking water supplies might also be profitably subject to further scrutiny. Under growing quantities of industrial and domestic wastes, the use of the *coli* index alone for determining the suitability of a water supply may prove inadequate for determining the quality of the water. From the health standpoint increasing concentrations of both industrial and domestic wastes in the water supplies open questions as to pathological consequences of increasing quantities of organic and inorganic toxins and their limiting tolerances. This may apply particularly in cases of the more heavily industrialized and populated sections where, by continuous re-use of water, the organic and inorganic salt contents are being continuously built up in spite of present purification methods. Indices for palatableness of water have also become an important part of water standards and promise to become increasingly so in the future. Taste, odor, and physical appearance are receiving almost equal attention to bacterial qualities under present standards of most water plants, and future improvement in this non-pathological direction will depend entirely on the development of facilities, through research, to attain these ends. The source of water supply is of equal importance to the water purification process in securing better physical and sensual qualities.

Of further practical value, research may show ways and means of improving the stream as a purification unit, depending on the fullest utilization of natural local conditions of the drainage area. The receiving stream should properly be conceded as an integral unit in the waste disposal system and as occupying as important a place in the system of waste treatment as any other treatment plant unit; and as such, it deserves the same attention toward improving operation efficiency as other plant units.

The same program research can yet play a profitable part in the design and operation of waste treatment devices to secure more economical methods of treatment. This is particularly true with respect to industrial wastes, which (economically, at the present time) appear to offer the largest unsolved problem in waste treatment. There are yet many local "sore spots" within the treatment devices themselves and it seems the doubtful part of wisdom to fail to consider inefficiencies in waste treatment processes as other than an integral problem in river pollution control. The ultimate economical solution of pollution control will not be realized until the stream, on the one hand, has been made to perform its maximum work as described by accurate pollution criteria; and, on the other hand, when the methods of waste treatment are developed to their maximum efficiency.

A final question which the term, "stream pollution control," naturally suggests is, "When does a pollution control survey, as outlined, become necessary?" The first evident reason for control measures is found naturally in those cases of grossly over-polluted streams. A second, less evident, reason is found in the border line case where the conditions are approaching pollutional limits. Probably the least evident reasons are apparent in those cases of streams that are still in satisfactory condition for normal water uses, but which by potential development possibilities may be subject to large pollutional loadings in the future. From the magnitude of the problems mentioned by Mr. Streeter in the case of the Ohio River and the tremendous simultaneous economic problem involved in the case of cleaning up of grossly over-polluted streams, it would seem that the future direction of stream control surveys should be not only toward the correction of streams already grossly over-polluted, but also toward proper maintenance of river basins that are still below such conditions.

ARTHUR W. BAUM, JR.,[43] ESQ. (by letter).—The statements by Mr. Wolman that "the intensity of stream pollution is not nation-wide. It is concentrated largely in five or six of the heavy industrial States," are debatable. The relativity of intensity varies with opinion and many people of the Pacific Northwest believe that their pollution situation is also intense. For example, Oregon certainly is not an industrial State and is surely not over-populated—with slightly more than a million people in 96 000 sq miles; but it also has its pollution problems, which are of major importance.

In the industrial States the main pollution problems center around the use of the larger polluted rivers as sources for a water supply for localized population. In Oregon, the pollution problem is studied primarily for the harmful effects it has on the fishing industry (both commercial and sport) and on the public health. With ample sources of water supply available for practically every city in Oregon, it is seldom necessary to use a known polluted stream for a water supply source.

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[43] Senior in Civ. Eng., Oregon State Coll., Corvallis, Ore.

The greatest difference in the pollution problem in one part of the United States as compared to another is represented in one's definition of "intensity." In the East, where the salmon runs have disappeared, the pollution problem is not intense to-day unless it interferes with a water supply source. The Pacific Northwest has great natural resources, especially from the viewpoint of those interested in sports and recreation, and derives a large income from this asset. More and more people are going there to enjoy the recreational facilities and the fishing offered by the streams, and as the present pollution situation is harmful to fish life and is damaging one of the region's best assets, as well as a multi-million dollar commercial industry, the pollution problem is certainly intense.

As approximately 64% of the population of Oregon resides in the Willamette River water-shed, and as the Willamette River is one of the largest tributaries of the Columbia River, it is only natural that a pollution problem is centered in this district. This area has been surveyed several times to determine the intensity of pollution present at that period when conditions are such that they should be at their worst.

The pollution situation is not a year-round problem of the same intensity, and it is only during late June, July, August, and early September of average years when there is only a precipitation of about 2 in. (and this is received in light showers), that the situation is critical. During this period the Willamette River has a minimum flow and the canneries and most other industries are running at capacity. From Salem to Portland, the oxygen content of the river gradually decreases until, in Portland Harbor, surveys show the water to have a zero oxygen content and a bacterial count at a point dangerous to the public health.[44] This "critical zone" can be considered with its two most harmful effects. It ruins a recreational area close to the major population by subjecting participants in water sports to disease and not only prevents the anadromous type of fish from passing on to the head-waters of the Willamette, but offers an insurmountable barrier to the young fish trying to reach the ocean and continue their natural cycle. Experiments have shown that a stream must have an oxygen content bordering on 3.5 ppm to sustain game fish life [45] and, if a stretch of water is present in which the dissolved oxygen content is below this point, it means suffocation to those fish trying to pass through this zone. This stretch of water in its present condition will have the similar effect on anadromous fish as cutting the bark completely off the trunk of a tree—the tree soon dies.

G. R. Scott,[46] Assoc. M. Am. Soc. C. E. (by letter).—The well-written paper by Mr. Streeter, based on detailed studies and observations of the Ohio River during the 24 yr, 1914–1938, gives a complete description of the basic data which should be collected and correlated prior to the establishment of standards of purity or requirements designed to prevent over-pollution of any river system. Instead of attempting any unnecessary restatement of the subject material in his paper, this discussion is presented solely to emphasize the need for such studies, the desirability of making them before serious conditions exist, and a general statement of the work now (1938) under way in the Tennessee Valley.

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[44] "A Sanitary Survey of the Willamette River from Sellwood Bridge to the Columbia River," *Bulletin Series, No. 6*, April, 1936, Eng. Experiment Station, Oregon State Agricultural Coll., Corvallis, Ore.

[45] "Stream Pollution in Wisconsin," Wisconsin State Board of Health, p. 249.

[46] Chf. Sanitation Div., TVA, Chattanooga, Tenn.

The natural resources of an area, whether they are the raw materials used in making finished products, or the fuel or energy necessary for the conversion, have always been the controlling factors in its development. For most industries water in some form is a necessity. Water used in the process, as steam or as the primary source of power, is usually considered a natural resource; but when used to carry away wastes, it is regarded as a fortunate convenience only—an agent that happens to be present and that is to be used for the purpose of carrying this waste material away from back doors. Many municipalities, depending as they do upon industries for financial support and population development, while at the same time providing them with their source of labor and personnel, look upon these streams with a view not much more enlightened. As a result, it is not long before such streams become little more than open sewers, having lost their valuable resources—those attributes or characteristics natural to fresh, clean streams. The water is still there and proceeds normally down its channel as before. In some cases it can be used again by others down stream after extensive and costly treatment; but in other cases it is so grossly polluted that its re-use is impracticable. This loss to the stream is accompanied by the loss of another quality, formerly considered of little value, but now being appreciated for its potentialities. This is the loss, to the area as a whole, of natural areas affording opportunity for those wholesome recreational activities which are so necessary in this energetic age for rebuilding or reconditioning the human machine so that it may be able to continue the increasing tempo of the "tread-mill." This idea, although it is persistently coming to the light, is being continually darkened by the argument that it is more important for work to be provided for the many, than that a few shall be favored by having full creels. This contention is granted; but, in general, it is quite probable that both arguments can be satisfied.

It seems logical to assume that those who can develop an intricate process of manufacture also should have the ability to devise satisfactory ways and means of disposing of waste products. Methods are already available for the disposal of most wastes and are being adopted by many industries. Others continue to discharge raw untreated wastes into running streams with the excuse that the cost of doing otherwise is prohibitive. It is recognized that the cost of known methods for the treatment of wastes from certain industries is exorbitant, and it is also realized that other industries are so handicapped by their locations that the costs of the treatment of their wastes are abnormally high. These industries should be assisted, rather than discouraged by any sanitation program.

If possible, the streams should be surveyed and studied before over-pollution occurs and the unregulated discharge of polluting wastes becomes an established practice. The data discussed by Mr. Streeter should be gathered and assembled to form a base line of existing conditions and to provide a foundation upon which to build the necessary regulatory measures. Comprehensive plans for the proper use and conservation of this natural resource, for the prevention of its thoughtless destruction, will be far les expensive and much easier of enforcement than the later remedial measures. In the preparation and establishment of these standards and regulations, due regard should be given to the economics of the problem. It should be realized that the true development of the industries and the advancement of the area are mutually dependent upon a well-rounded program in which all factors are given full consideration. It will not be just to require that all wastes be purified to meet certain standards regardless of cost; on the other hand, the uncontrolled discharge of raw untreated wastes should not be tolerated.

For the proper solution of this problem, much research work will be necessary; co-operative studies must be undertaken in which the industries should be willing to assist. By combined effort and co-operation, new solutions may be found. Although industries have made many investigations resulting in benefits to themselves by the recovery of products formerly wasted, and in benefits to the public in the way of cleaner streams, they still have a long and difficult way to go, and a way that can best be traveled at the moderate pace now possible before speed is made necessary by an aroused public through national legislation.

Early in the history of the Tennessee Valley Authority, the need for the collection of these basic data was felt by the Health Departments of the seven Valley States. At their request, the Authority has undertaken the study with the advice of the U. S. Public Health Service and with the co-operation and assistance of the States. Although the area cannot be considered intensively industrialized at present, certain areas are highly developed and the uncontrolled discharge of wastes is seriously polluting a few of the major tributaries. With the expected development of this area, peculiarly rich in a wide range of natural resources, additional streams will be ruined and conditions aggravated unless uniform procedures applying to the entire Valley are effective. This interstate nature of the problem was an added factor which prompted the States to request the Authority to direct the studies and to act as a clearing house for the assembled information.

The work was begun in 1936. All information pertinent to the study was collected from the various State Health Departments. This consisted primarily in the extent of sewers in the various towns in the Valley. No information as to the extent or character of industrial wastes, or on the present conditions of the streams, was available. It was necessary, therefore, to obtain and correlate this information before true conditions could be pictured. At present (1938), an approximate list of industries has been collected; some have been studied in detail; and plans for the future contemplate the examination of a sufficient number of the remainder to complete the necessary data. Four sections of the Tennessee River, from which weekly samples were collected from a sufficient number of stations, to define conditions of pollution and recovery, were studied for a period of one year. These sections were at

Knoxville and Chattanooga, Tenn., and at Decatur and below Wilson Dam, in Alabama. On the completion of these studies, similar studies were undertaken on the main tributaries, some by the States, and others (for example, all field and laboratory work), by the Authority.

As in other areas where such studies have been conducted in co-operation with industries, it has been necessary to develop mutual confidence before the full value of the study can be obtained. Although the program has progressed somewhat slowly, research problems in co-operation with one of the industries was undertaken in the summer of 1938.

In the execution of this work, it is the desire of all concerned to use the resulting information co-operatively and constructively in such manner as to be compatible with the achievement of the end in view, namely, the control of the discharge of wastes and sewages to insure a balanced development of this resource; that it will not be rendered unfit for re-use by municipalities and industries located down stream; that the maximum of fish life may develop; and that a sufficient number of areas safe for recreational purposes may be maintained. Although the progress of this program, as indicated by visible results, may be somewhat slower than that resulting from more drastic action, it is expected that its achievements will be more substantial and enduring. In addition, it is hoped that this project, having an early beginning before serious conditions develop and embracing as it does the entire Tennessee Valley, will demonstrate the value of such co-operative policies and action.

Don E. Bloodgood,[47] Assoc. M. Am. Soc. C. E. (by letter).—The effect of sunlight upon the dissolved oxygen in the stream is an important factor, often overlooked when samples are taken. Samples taken on White River below the City of Indianapolis, Ind., show that the dissolved oxygen may vary as much as 1.0 ppm from one side of the stream to the other, if the one side is shaded and the other is in the sun. This effect is only noted when the river is in a favorable condition for algæ growths. Samples taken at midday may have as much as 3.0 to 4.0 ppm of dissolved oxygen whereas, at night, it may drop to zero. It can be seen that samples taken during the day may not indicate the extent of pollution in a stream if the plant life in the water is abundant.

For a number of years the movement of the river water has been studied by determining the chloride concentration in the stream. The chlorides that are not removed by the purification plant pass on into the stream and, because they fluctuate from low to high with the peak flow of sewage, they are easily traceable in the stream when the ratio, of effluent to river flow, is low.

The comments by Mr. Streeter on the retardation of re-aeration by the presence of sewage in a stream are very interesting. In some work done in 1935[48] the first indications were that the saturation point of sewage with oxygen was less than that of water. This was disproved later by a further study which showed that the saturation point of sewage was the same as that of water if determined by a modified Winkler method, in which the demand for iodine was first satisfied. Whether the rate of re-aeration was slower was never determined.

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[47] Supt., Dept. of Sanitation, City of Indianapolis, Indianapolis, Ind.

[48] Rept. of Dept. of Sanitation, City of Indianapolis, Ind., March 25, 1935.

W. L. Stevenson,[49] M. Am. Soc. C. E. (by letter).—The discussions and subsequent experiences during the 1937 flood in the Ohio River, strengthens the writer's conviction that preparedness is essential for protection of the public health during floods. This means pre-planned general principles of procedure, trained personnel in Federal, State, and local health departments, systematic swift mobilization of skilled men and women with essential equipment and materials, and resourceful and trained water-works operators. Experience has also shown the need for more extensive and better facilities to those charged with the duty of forecasting flood crests so that due warning may be given and safe-guarding measures adopted where possible. Of equal importance is the need for strengthening the confidence of the public in health authorities in order that there may be the fullest co-operation.

In the final analysis, it is upon the large number of lay people whose homes are in the flooded areas that the burden falls. The most that the small group of trained engineers, physicians, and nurses can do in such emergencies is to direct, guide, advise, and help.

Mr. Holmquist, writing from personal flood experiences in New York State "emphasizes in the strongest terms" the responsibility of the local authorities; and Mr. Weston refers to the serious condition that arises when local authorities lack comprehension. Many a ship reaches its port safely in fair weather, but when storms arise and the winds rage, only a trained captain and crew can save the ship. So it is in emergency sanitation during flood or disaster.

The unqualified "boil water" notices used by the Pennsylvania Department of Health during the March, 1936, floods were criticized by some water purveyors, and Mr. Holmquist discussed this question constructively. The writer maintains his position in this matter, believing that the obvious injustice to those water purveyors who remain able to serve pure water during the emergency is far outweighed by the extra safeguard to the public health of the unqualified warning to boil all water used for drinking within the flooded areas.

It was common experience to find some private water wells within flooded areas which were dominantly furnished by a piped public water supply that remained uncontaminated during the flood. Therefore, it is better for the many to boil the pure piped water supply unnecessarily than for the few to drink, unboiled, the contaminated water from the flooded private wells.

As the flood waters receded in Pennsylvania after the March, 1936, floods, one of the heaviest tasks of the State Health Department was to direct the cleansing of polluted private wells. There was nothing spectacular about this work, such as the heroic service of many employees of public water-works menaced by the flood as the rivers crested; but it was a vital part of the entire program for prevention of water-borne disease.

D. E. Davis,[50] M. Am. Soc. C. E. (by letter).—Certain threads of thought run through many of the papers presented in the Symposium, and in the comments by the various discussers, that indicate a fairly general agreement on certain fundamental concepts. It is recognized that a broad drainage area is the proper geographical unit for approaching the solution of pollution problems, and that in many instances the co-operation of several States may need to be enlisted to make this possible. State compacts appear to offer favorable mediums for action.

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[50] Engr. (The Chester Engrs.), Pittsburgh, Pa.

A unified approach also offers advantages in the cleaning up of pollution in certain metropolitan districts, such as Pittsburgh, Pa., and Cincinnati, Ohio, where a number of separately organized communities are affected by similar conditions. Sewerage Boards, serving the group of communities and with power to raise money, assess costs, and collect rentals, offer a hopeful approach.

The rôle of research in defining the extent of pollution and the degree of purification required is highly essential. This tool can also be utilized in solving problems connected with self-purification in streams, as well as those of a special character having to do with the underlying phenomena of the action of acid on sewage.

Grants-in-aid by the Federal Government have resulted in a marked increase in the total number of treatment plants during the past few years, and the continuation of this policy in some form offers hope of further advances. All are agreed that the public in general is taking a keener interest in the vital job of cleaning up its streams, and that with a little encouragement a large measure of accomplishment may be expected.

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PRACTICAL METHODS OF RE-ZONING URBAN AREAS

By Hugh E. Young,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Robert D. Kohn, Myron D. Downs, Harland Bartholomew, Benjamin Saltzman, G. H. Herrold, and Hugh E. Young.

Synopsis

The purpose of this paper is to show why re-zoning is a present-day necessity, and to indicate some practical methods of re-zoning urban areas. The first part of the paper is devoted to an explanation of zoning experience in Chicago, Ill. The Illinois State law permitting zoning is outlined, together with the 1923 Chicago zoning ordinance and subsequent amendments in methods and policies, and then a comparison is drawn between land uses as they existed in 1923, the land-use allocations made in the 1923 zoning ordinance, and the land uses as they existed in 1936. The writer believes that this factual demonstration of conditions in Chicago, which are shown to be similar to conditions in other American cities, proves the need for re-zoning.

The latter part of the paper is devoted to the recent development of "measuring sticks" that can be used as guides in the allocation of land for various uses wherever re-zoning is undertaken. A real property inventory and land-use survey are advocated as the initial step. These are to show existing patterns of land use, areas of blight, population changes, and other factors which together make re-zoning necessary. In this part of the paper the technique of re-zoning is outlined, together with the tabulated result of a questionnaire dealing with the need for and method of re-zoning in about seventy-five American cities. The question of blight is dealt with in detail, as also is the matter of public and private housing; and the paper concludes with the writer's recommendation as to what he considers to be a practical method of re-zoning urban areas, particularly with regard to the rehabilitation of blighted districts.

Introduction

The fact that the need for re-zoning urban areas is being discussed means that considerable progress has already been made in zoning. In recent years

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Note.—Presented at the meeting of the City Planning Division, New York, N. Y., January 21, 1937, and published in March, 1938, *Proceedings*.

[1] Chf. Engr., Chicago Plan Comm., Chicago, Ill.

city planners have learned much about the technique of proportioning urban use as to amount and location. It has now become desirable to adjust some of the older ordinances to a more "even keel" by means of re-zoning, and to emphasize certain principles that should be followed in the preparation of new zoning ordinances.

Engineers should not be too critical of the work of the planners of fifteen or twenty years ago, because they were pioneers and were trying to correct conditions of urban structure that had been typical for centuries. They had to re-zone men's minds as well as the physical structure of cities.

NEED FOR ZONING

Why Zone?—Many accomplishments in art and science have been achieved and great progress has been made in recent years in architecture, engineering, and public works; and yet city congestion, blight, slums, and misery are still to be found. These by-products of human activity have ever been present as a challenge to the humanitarian instincts of Man throughout the centuries. It is only in comparatively recent years, however, that city planners have come to sense the great social need of transforming the physical structure of urban areas. Zoning is a fundamental and vital element in city planning, because it is the process that co-ordinates the social, economic, and physical structure of urban areas.

Extent of Zoning.—Since 1916, when the City of New York, N. Y., was granted enabling powers by the State Legislature, approximately 1 200 zoning ordinances have been put into effect in the United States, representing approximately 75% of the urban population of the country.

Zoning Handicaps.—As a rule, planning commissions did a creditable zoning job in the early plans, considering the handicaps involved. The art of zoning was then in its pioneer state. Often, it developed that property owners on section line streets (usually the car-line streets, and other thoroughfares) insisted upon having their residential property zoned for business use. Often, too, real estate speculators, through some device or other, obtained concessions favorable to their exploitation process. Many citizens, also, were speculatively minded and hoped, by means of zoning, to have their property transformed from ordinary to high value.

Zoning Experience.—When all the comments, pro and con, have been "put in the hopper," all must agree in the final analysis that zoning has been worth while and that it was valuable during the boom building period prior to 1929, but that it is far from perfect in its present status. The art is still crude with respect to the "tapering-off" process or the transition from a higher to a lower use and the utilization of park strips between residential sections and other uses. Although present zoning plans and ordinances seem to be satisfactory in many smaller cities, officials of most cities, both large and small, feel that re-zoning is necessary. Without doubt, zoning has exercised a beneficial influence in shaping the growth of cities, and often has served as a barrier in preventing abortive property uses.

Zoning Variations.—Variations in residential areas are detrimental because they unstabilize the zoning of the block and of the district; residential protection is destroyed and competition is set up with property already zoned for business. There are innumerable examples in which miscellaneous variations of this character are permitted, and in which penetration of unsuited uses is gradually nullifying the spirit of zoning.

In some instances entire blocks of property are zoned for business where the business structures are in decay; yet only a few blocks away commercial usage has been allowed to penetrate into residential areas. If business had been confined to the area zoned for commercial use, it would have put that property into a revenue-producing position, and would have protected the residential areas, which now are certain to disintegrate, having no stabilization either for residential or business use.

There are several variations or non-conforming uses to be found in nearly every block in Chicago, Ill., for example. It is a common occurrence that if these illegal uses are carried into Court, the cases are continued ten or a dozen times, until the matter is finally dropped.

Zoning Experience in Chicago, Illinois

The administration of a zoning ordinance in a large city is a difficult task. The mayor, aldermen, and all city officials are continually bombarded with requests for variations, which ultimately reach the Board of Appeals. The Chicago Zoning Board of Appeals has shown great fortitude and patience in this job and has also demonstrated a fine understanding of the theory and practice of zoning and the need for re-zoning; but it seems impossible, in the complexity of the situation, to maintain the integrity of the zoning ordinance 100 per cent.

The Chairman of the Board, Mr. James H. Gately, is firmly of the opinion that although it is not present practice in Chicago, a certificate of occupancy is most important for the proper functioning of the zoning ordinance; but such a certificate should not be issued by the same department that is charged with its enforcement.

Protecting Manufacturing Districts.—Mr. Gately is also of the opinion that one of the fundamental safeguards to be provided for in re-zoning is reciprocal protection for manufacturing and residential areas. He feels strongly that there should be a tapering off from manufacturing to commercial to residential use in a manner that will not impose restrictions upon manufacturing development. It has been his experience that residential areas contiguous to manufacturing property are certain to be invaded by commercial use.

The City Council should not permit variations without the prior approval of the Board of Appeals. If there is disagreement, the matter should be referred back to the Board for further consideration.

The Board of Appeals should not have the legal power to break a solid block of use, even if there is a non-conforming use in the block across the street; nor should the Council exercise such authority. Proposed zoning amendments require a two-thirds vote of the Chicago City Council to become effective in the event that a written protest against such amendment is made by 20% of the people on the frontage affected, or of adjacent frontages.

Need for Buffer Organization.—The administration of the zoning ordinance is an almost continuous problem of fighting favors. A strong buffer committee is greatly needed. No variation of any consequence would ever "get by" without political backing. In other words, any proposed variation would be taken care of expeditiously by the routine of the Board of Appeals, and would be properly disposed of, were it not for political interference.

A zoning ordinance will not be effective unless its administration is in strong hands, free from political domination. The essential need is a strong public body, such as a plan commission, to lend support to the Zoning Board of Appeals. In fact, a city plan commission is as essential as a Board of Appeals in preserving the integrity of the zoning ordinance.

Zoning Situation in Chicago.—The main purpose of the Chicago Zoning Ordinance was to protect property devoted to its highest and best use against sporadic invasion of uses which would tend to destroy the taxable value of land and buildings and impair public health, safety, comfort, morals, and welfare. In large measure, however, the ordinance has failed to provide this protection. It was not retroactive and, therefore, could not protect neighborhoods against undesirable developments already in existence.

Visions of enhanced realty values from commercial uses and greater height and volume provisions resulted in public demands for zoning classifications obviously out of line with actual and prospective land and building use. Ordinance provisions permitting additional non-conforming uses were not sufficiently strict to assure districts remaining in the class in which they were zoned. Broad discretionary powers were granted the Zoning Board of Appeals to permit variations in case of practical difficulties or particular hardship.

Chicago Zoning Provisions.—The Chicago Zoning Ordinance provides four "use" and five "volume" districts. The use districts embody residence, apartment, commercial, and manufacturing districts. The volume districts regulate and limit the height and bulk of buildings and the intensity of the use of land, the height limitations ranging from 33 ft in Volume District No. 1 to 264 ft in Volume District No. 5.

ILLINOIS ZONING LAW

The Illinois law provides for the establishment of Zoning Boards of Appeal and gives them power to: (1) Interpret the terms of the zoning ordinance or the application as made by the building commissioner; (2) vary the application of the ordinance in special cases of unnecessary hardship; and (3) recommend to the City Council changes in the zoning ordinance.

The State law does not grant Boards of Appeal the power to change a zoning ordinance or to change the classification of property under the zoning plan, but confines the power of amending the ordinance to the City Council.

1934 Amendments.—In 1934, the section of the Chicago Zoning Ordinance of 1923 dealing with the Board of Appeals was amended by the City Council to specify who should be eligible to membership; to provide for board members serving without compensation; and to incorporate certain provisions of the State Act with respect to appeals. In addition, about eighteen variations which the Board might make in specific cases of hardship were enumerated.

The power of permitting variations is in such general terms that it affords the possibility of grave abuse. No standards are defined to guide the decision of the Board, but judgment is left almost entirely to the Board's discretion. For example, the zoning ordinance specifically provides that no manufacturing use shall be permitted in a commercial district. The 1934 amendment authorizes the Board to grant permission to devote premises in a commercial district to a manufacturing use where the appropriate use of neighboring property is not injured thereby.

Variations of this character may be right and necessary to prevent hardship, even if they are in direct conflict with the language and purpose of the zoning ordinance; but there is a strong possibility of abuse by unreliable Boards when the Board constitutes the sole judge as to whether or not the non-conforming use is injurious to neighboring property. In the hands of an undependable Board the variations permitted may easily amount to such a volume of precedent as practically to nullify the purpose and intent of the entire ordinance.

Present Use Factor.—In developing the need for re-zoning, the most practicable "measuring stick" appears to be the factor of use. The research already made in this direction is of tremendous importance. Significant comparisons can be made based upon conditions in Chicago.

Fig. 1(*a*) shows the use of property in Chicago as of 1923 when the zoning ordinance was passed. At that time, the incorporated area was 201 sq miles and comprised the uses shown in Table 1(*a*). The developed area comprised 143 sq miles. The remainder of the area (58 sq miles) was occupied by water, or was vacant.

TABLE 1.—LAND USE IN CHICAGO, ILLINOIS

Uses	(*a*) As of 1923		(*b*) As Zoned in 1923		(*c*) Effect of Suburban Factors	
	Area, in square miles	Percentage of incorporated area	Area, in square miles	Percentage of incorporated area	Area, in square miles	Percentage of developed area
Single-family dwellings	23.75	11.8	5.91	2.9	62.25	26.6
Two-family dwellings	15.66	7.8	38.55	19.1	18.70	8.0
Multiple-family dwellings	9.19	4.6	26.00	13.1	15.98	6.8
Total dwelling areas	....	24.2	....	35.1	....	41.4
Commercial areas	9.87	4.9	28.24	14.0	13.61	5.8
Manufacturing areas	25.91	12.9	48.64	24.1	34.45	14.7
Special areas	4.95	2.4	....	....	11.92	5.1
Streets and alleys	46.60	23.3	46.60	23.3	69.30	29.6
Parks and playgrounds	7.06	3.5	7.06	3.5	8.06	3.4
Water and vacant property	58.01	28.8	....	....	....	....
Total	201.00	100.0	201.00	100.0	234.27	100.0

Fig. 1(*b*) shows the manner in which the area was zoned in the 1923 ordinance. The entire area of 201 sq miles was zoned for the uses indicated in Table 1(*b*). All vacant areas were defined for use of some character. Water areas were zoned for the same use as contiguous areas. A comparison of Figs. 1(*a*) and 1(*b*) shows that, although the zoning ordinance increased the total dwelling areas 1.45 times, the single-family dwelling areas were decreased to less than one-fourth the space then in actual use. Commercial areas were increased 2.86 times, and manufacturing areas were increased 1.88 times the space then in actual use.

Actual and Zoned Use Situation in 1936.—At the close of 1936, there had been little actual change in land use despite the great increase it was originally assumed would take place in manufacturing and industry. In the last thirteen years of that period 100 539 building permits were issued, which, translated into land use, is shown in Fig. 1(*c*). During that period an estimated total of 21 965 permits was issued for demolition of property, which number, translated into shrinkage of use, shows a decrement of 3.15 in percentage of developed area. The net percentage of added developed area occupied during the 13-yr period for dwelling purposes was 8.0; for commercial areas, 1.9; and for manufacturing areas, 0.46.

Number of Variations.—For this same period a number of variations occurred in the zoned use, the net interchange in the developed area (see Fig. 1(*f*)) being: An increase of 0.55% for total dwelling areas; an increase of 0.60% for commercial areas; and a decrease of 0.21% for manufacturing areas.

The graphs in Fig. 1 are of interest in that they show little agreement with the absorption of use as predicted in the 1923 ordinance. There was little change in the areas actually devoted to commercial and manufacturing use, which uses were provided for, generously, in the original ordinance.

Residential vs. Business Use.—Residential use, which was assigned the least area for expansion, showed the greatest development in the period. Demolition has not helped the situation because the largest areas thrown into disuse are commercial and manufacturing, and those are the areas that were assigned the greatest percentage of use in the 1923 ordinance. The trends are somewhat irrational, of course, because of the interrupted normal use absorption resulting from the depression years from 1929 to 1935. Considerable recession has occurred in the use of railroad and manufacturing property.

The Situation as a Whole

The percentage of developed area occupied by various land uses applies only to the area within the city limits, which does not, of course, give the "true picture" for determining the land use factors for Chicago as an entity. In order to have a proper comparison with other self-contained cities, the data should include the suburbs, because they are largely residential areas for the workers of Chicago.

Fig. 2(*b*) shows the developed area of Chicago in 1936 within the city limits, amounting to 157.77 sq miles, and Fig. 2(*a*), the developed residential areas beyond the city limits, comprising 76.50 sq miles.

Suburban Factors.—Only those suburbs have been included in this study that are integrally a part of Chicago even if located outside the corporate limits. Indicating the use for "Greater Chicago," as shown in Fig. 2, the present land uses, in percentage of developed area occupied, are as listed in Table 1(*c*).

Comparison with Independent Survey.—In Fig. 2(*d*) is shown the corresponding percentage of developed area occupied by various uses in sixteen self-

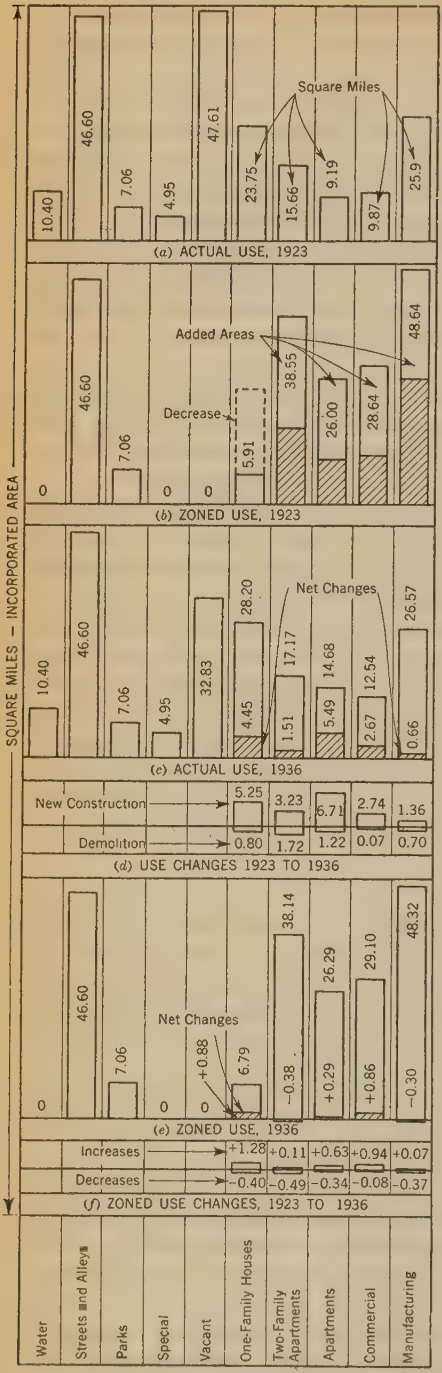

FIG. 1.—LAND USE IN CHICAGO, ILLINOIS

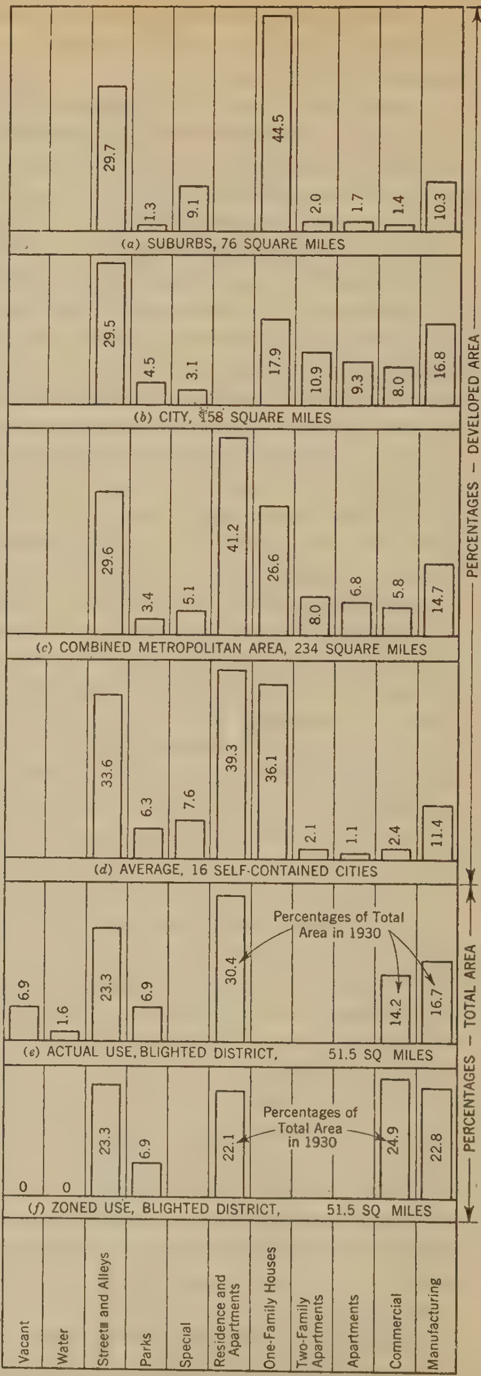

FIG. 2.—PERCENTAGES OF DEVELOPED AREA, CHICAGO, ILLINOIS

contained cities as disclosed in a survey made by Harland Bartholomew, M. Am Soc. C. E.[2] It will be noted that the percentages of occupied developed area in these sixteen cities and in Chicago are in very close agreement (see Fig. 2(*c*)). This indicates that factors determined by his survey are applicable to large urban areas, such as Chicago, with the exception of the classification of residential use. In Chicago, and probably in other similar cities, a much greater percentage must be allowed for two-family and apartment house use, and a shading of use for single-family dwellings. The ratio of all land-use areas to population, in terms of acres per 100 persons, can readily be determined from these percentages.

When the information developed by land-use surveys of other large urban centers has been compiled in terms of percentage of area developed for the various uses, city planners will have a valuable means of allocating city area, definitely, for various uses and for different classifications of city population. Therefore, any city, after it has reasonably determined its prospective future population, can zone properly for present use and can determine the proper proportion of future use absorption.

Zoning Mistakes.—When the present use of property in Chicago is compared with the zoned use, and it is found that after thirteen years practically none of the increased use anticipated in 1923 has occurred, one cannot escape the conclusion that present zoning methods are badly distorted and that Chicago must be re-zoned.

Zoning in Chicago is not only distorted with respect to an illogical allocation of usage, but also with respect to the distribution of various uses. Little consideration has been given to tapering off from highly restricted uses to unrestricted uses. There is a lack of buffer property between manufacturing use and residential use.

Chicago (and perhaps other cities as well) arbitrarily zoned major and minor streets in checker-board fashion for commercial use, and zoned areas for residential use contiguous to manufacturing use. It will be necessary to delete miles of such commercial zoning and to replace it with residential use. Furthermore, it will be necessary to re-zone areas contiguous to manufacturing property from residential to commercial use.

"Blue-Sky" Zoning.—"Blue-sky" zoning has complicated the tax machinery. If valuations are based on the present value of hundreds of miles of fictitiously zoned commercial property, where because of its blighted nature there is no income, this cannot possibly mean anything except tax delinquencies. This also applies to many areas that have been zoned for large volume residential property. The deficit thus produced must be made up in some other manner.

Surveys in Cleveland, Ohio, New York, N. Y., Boston, Mass., Los Angeles, Calif., and other cities give volumes of information on this subject. As a matter of fact, improper zoning is a blight in itself, in that it specifies a use and volume that can never be attained. While awaiting the advent of such expected use the property is often allowed to fall into disuse because it does not seem worth while to spend the money to keep it in condition for its former use.

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[2] "Urban Land Uses," by Harland Bartholomew, Harvard Univ. Press, 1932.

Re-zoning beyond the urban limits is also necessary, if for no other reason than the need of safety. The removal of the "honky-tonk," mushroom growth of "hodge-podge" commercial uses that now clutter the main highways and create a dangerous marginal friction, can be accomplished by pushing these uses, where needed for the local community, back of the highways in a manner similar to that being done on highways in some foreign countries.

Re-Zoning Needs.—One notable example in Chicago of the impropriety of zoning residential areas near manufacturing areas is the Trumbull Park Homes Project of the Public Works Administration (PWA) in the South Chicago industrial district. This is an area of 21.5 acres, the approximate boundaries of which are Trumbull Park, Bensley Avenue, and the Chicago and Western Indiana Railroad. The housing project is located in the very heart of a large manufacturing area. The Government asked for a revision in the zoning ordinance, but when it was discovered that if this area were re-zoned for residential use it would immediately render all industrial property within 2 000 ft of it useless for certain manufacturing purposes similar to those now there, the request for this zoning revision was withdrawn, although the housing project was continued.

Manufacturing property needs protection just as much as residential property, and this protection would seem to be the utilization of contiguous areas for commercial use, tapering off into residential use, the latter being removed far enough from manufacturing areas so as to avoid restricting the free use of such manufacturing areas. There is also opportunity in this buffering process for an appropriate interposition of needed park areas between residential and commercial use.

Comparison with a Hotel.—A city is a corporate entity that should be as susceptible of the intelligent application of economic principles as any business corporation. Consider a hotel, for example, and apply to it the factors of Chicago's zoning: 52% of the hotel's floor area was zoned for show rooms, restaurants, and other commercial uses, including its operating facilities, whereas only 39% of the total floor area was being used for these purposes; only 4% was zoned for single rooms, whereas 29% was being used for this purpose; 26% was set aside for double rooms and only 17% of the area was actually being so used; and 18% was zoned for suites whereas only 15% of the normal hotel area was actually being so used. Every one must appreciate how illogical such a condition is when applied to a business corporation; but few people seem to realize that it is equally illogical when applied to a municipal corporation. The proper basis for zoning is the present use factor, and anticipated future absorption of vacant property must be provided for on the basis of this factor.

BLIGHT

Description.—In the larger cities, at least, re-zoning is essential before blighted areas can be rehabilitated. This is the situation in Chicago, where the blighted district is twice the size of Manhattan Island, New York. It is the backyard of the lake-front development, and stretches virtually from Belmont Avenue on the north to 63d Street on the south, and from the "Loop" west to Kedzie Avenue. This comprises a total area of 51 sq miles, of which 44 sq miles are blighted territory. In this district, which is about one-fourth the total area of all Chicago, live two-fifths of the city's population—about 1 300 000 people. The district contains 169 000 families of the lowest income levels. Maps of this district, prepared by the Chicago Plan Commission in collaboration with the Social Science Department of the University of Chicago, show that the density of population in the 287 census tracts in the district are as given in Table 2.

TABLE 2.—Density of Population, Chicago, Illinois

Population density in persons per acre	Number of census tracts	Percentage of total
Less than 30	23	8.0
30 to 60	106	37.0
60 to 90	111	38.7
90 to 120	43	15.0
120 to 150	4	1.3
Total	287	100.0

Use Data.—Comparing the 1930 use map of this area with the zoned use, the condition is as shown in Figs. 2(*e*) and 2(*f*). The effect was to decrease the population in total dwelling areas, 27%; increase it in commercial areas, 75%; and increase it in manufacturing areas, 37 per cent. These data certainly are a strong argument for re-zoning this area. The need for rehabilitation of the area is emphasized by the fact that 70% of the district is occupied by frame structures from 30 to 60 yr old.

Home Ownership.—To what extent do the residents of this section own the homes in which they live? In the 414 census tracts of the 1930 Census the distribution is as shown in Table 3. In 72 tracts less than 10% of the families own their own homes.

TABLE 3.—Home Ownership

Extent of Home Ownership; Range of Percentages		Number of tracts	Percentage of total tracts
From:	To:		
	<1	4	1.0
1	9	68	16.5
10	19	94	22.7
20	29	123	29.7
30	39	75	18.1
40	49	31	7.5
50	59	11	2.6
60	69	8	1.9
Total	....	414	100.0

TABLE 4.—Rentals

Range of Average Monthly Rentals		Number of tracts	Percentage of total tracts
From:	To:		
*	*	4	1.0
$10	$19	37	9.0
20	29	140	34.0
30	39	97	23.5
40	49	87	21.0
50	59	22	5.2
60	69	18	4.2
70	>70	9	2.1
Total	....	414	100.0

* There are no homes in these tracts.

Rentals.—In the 414 census tracts of the 1930 Census, the median rental values were as shown in Table 4. The living quarters in one-third of the census tracts in the district yield an average monthly rental of $40, or more, and the living quarters in two-thirds of the tracts yield only from $10 to $39 per month.

The zoning volume regulations in this district are such that a building height of 66 ft at the street line and 132 ft maximum with set-backs, may prevail in more than one-half the area.

Population Changes.—One of the effects of blight and an indication of its presence, is the mass movement of residents away from a district. The boundaries of the territory of principal population decline of Chicago, 1920–1930, shown by the broad black band on Fig. 3, coincide generally with the boundaries of the blighted district. This district contained 1 060 716 residents in 1920. In the 10-yr period it lost 146 354 people, or 13.8%, leaving 914 362 people in 1930.[3] In the next four years the decline continued, the population dropping to 848 803. The total decline in this area for the 14-yr period amounted to 211 913, or nearly 20 per cent. In 1934, the boundaries of the territory of principal population decline had moved outward as shown by the broad black line on Fig. 4. The 1920–1930 territory of decline is shown bounded by a heavy broken line. The territory between these two lines contained 1 633 596 people in 1930. In 1934, this number had declined to 1 563 505, a loss of 70 091 residents, or 4.3 per cent. The 4-yr loss of the original district shows a population decline of 282 004 for both areas. Until the years 1920 and 1930, respectively, these areas had shown a steady increase in population.

Rehabilitation of Blighted Areas.—Blighted areas in Chicago must be rezoned regardless of any particular method that may be adopted for financing their reconstruction. Some of the sources of blight are the following: (1) Decentralization and moving away of factories and industries; (2) public improvements that provide outlying developments which induce the higher paid residents of a district to abandon the old and move out to the new; (3) faster transportation that enables people to live farther away from their place of employment; (4) retention of old unfit housing and lack of suitable housing facilities for lower income groups, resulting in over-crowding of run-down districts; (5) single-family dwellings occupied by a multiplicity of families; (6) removal of the established residents and the incoming of new residents with less civic interest and generally with less income; and (7) depreciated land values; weak owners; repeated change in ownership; lack of adequate maintenance because of surrounding dilapidation and fear of what adjoining property owners might do that would still further depreciate values and income.

Annual Deficits.—If city expense in Chicago were pro-rated on the basis of area and compared with revenues received from the blighted districts, the results would be a deficit of tens of millions of dollars, assuming that all taxes were paid. As a matter of fact, there are 24% delinquencies in tax payments, so that if the deficit was paid by 10% of the city area, including the high-class business and residential property, as was shown to be the case by a survey made in Boston, Mass., that fact should be some incentive for the city's leaders of industry, business, and commerce, to devote more attention to the rehabilitation of blighted areas.

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[3] "Census Data of the City of Chicago," 1920 and 1930 Rept., Social Science Dept., Univ. of Chicago.

The question is always raised: "How can water be made to run up hill?" The answer is to be found in two fundamental factors: First, vital consideration of the sources of workers' income; and, second, the ability of workers to pay rentals for living quarters of not less than "low-cost" housing standards.

Manufacturing Removals.—For years there has been a recession of manufacturing use and the contingent sources of labor, such as railroad service, etc., in the areas adjacent to the "Loop," in Chicago. The West Central Association in Chicago stated in 1936 that:

> "$30 000 000 is a conservative estimate of the amount of business which is driven away every year by the lack of good transportation arteries, the disintegration of neighborhood areas and other causes which have brought about the dry rot which has been steadily eating into the entire west side section during the last twenty years. One business concern after another is forced to move into other districts where more favorable conditions are available. In the five years between 1924 and 1929, seventy-five large industries moved from the west side to the outskirts of the city."

Manufacturing must be encouraged to remain in blighted areas. These plants must be given protection against such zoning of contiguous areas as will restrict manufacturing expansion. New manufacturing concerns must be induced to locate in blighted districts because this is an essential means of stabilizing these fast deteriorating areas. Adjustments must be made in tax valuations to bring tax rates in blighted districts down to a level as favorable as that in effect on the outskirts of the urban areas.

Income Needs.—People cannot live in any kind of housing conditions without an income. A. C. Comey, M. Am. Soc. C. E., has described the situation pointedly:

> "A slightly different approach to population planning is afforded by combining it directly with a study of activities, largely expressed in terms of numbers of people. Thus we may inquire not merely where people have been, are, and are likely to be, but also why. * * * Pursuit of livelihood is a most compelling reason "why," for most of the population."

The problem of a standard minimum wage is a national one, but it is evident that the income of a family gainfully employed cannot be less than, say, five times the rental of "low-cost" housing. Rentals, of course, will vary in different sections of the country.

There are two important factors involved, therefore, in the rehabilitation of blighted areas. In some manner, present manufacturing will need to be induced to remain in blighted areas, and should be given opportunity to expand its plant by proper zoning; additional diversified manufacturing and industry will have to be induced to locate in these districts; and it is evident that the income of a family gainfully employed must be such as to permit it to live in dwellings of not less than standard low-cost housing.

The rehabilitation of blighted areas has been an enigma for many years, and it will continue to be an insoluble problem unless these two factors are properly determined, although they seem very commonplace in the array of panaceas that have been advanced for social betterment.

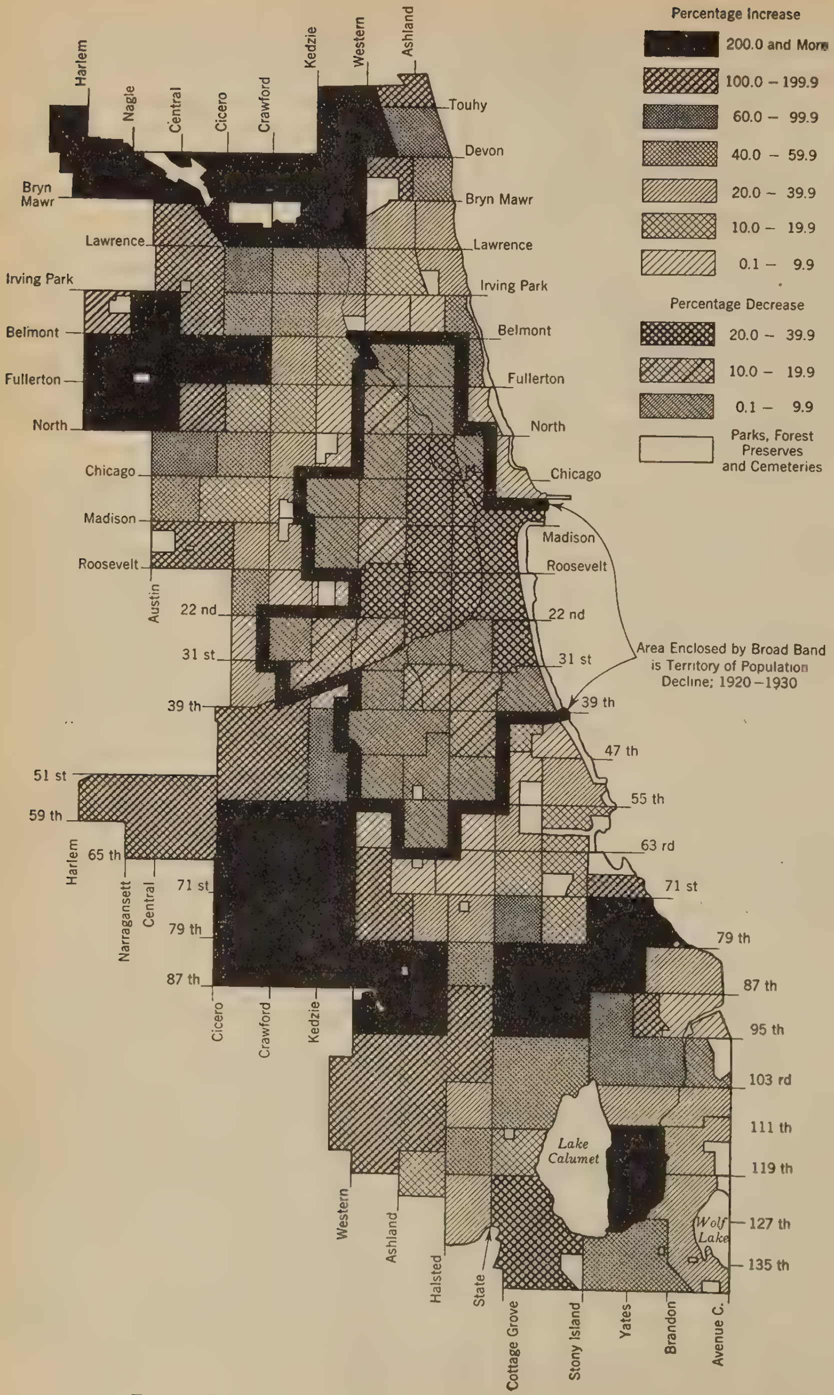
Percentage Increase
200.0 and More
100.0 – 199.9
60.0 – 99.9
40.0 – 59.9
20.0 – 39.9
10.0 – 19.9
0.1 – 9.9
Percentage Decrease
20.0 – 39.9
10.0 – 19.9
0.1 – 9.9
Parks, Forest Preserves and Cemeteries
Area Enclosed by Broad Band is Territory of Population Decline; 1920–1930
Harlem
Nagle
Central
Cicero
Crawford
Kedzie
Western
Ashland
Touhy
Devon
Bryn Mawr
Lawrence
Irving Park
Belmont
Fullerton
North
Chicago
Madison
Roosevelt
Austin
22 nd
31 st
39 th
47 th
51 st
55 th
59 th
63 rd
65 th
71 st
79 th
87 th
95 th
103 rd
111 th
119 th
127 th
135 th
Harlem
Narragansett
Central
Cicero
Crawford
Kedzie
Western
Ashland
Halsted
State
Cottage Grove
Stony Island
Yates
Brandon
Avenue C.
Lake Calumet
Wolf Lake

FIG. 3.—POPULATION TRENDS, CHICAGO, ILLINOIS, 1920 TO 1930

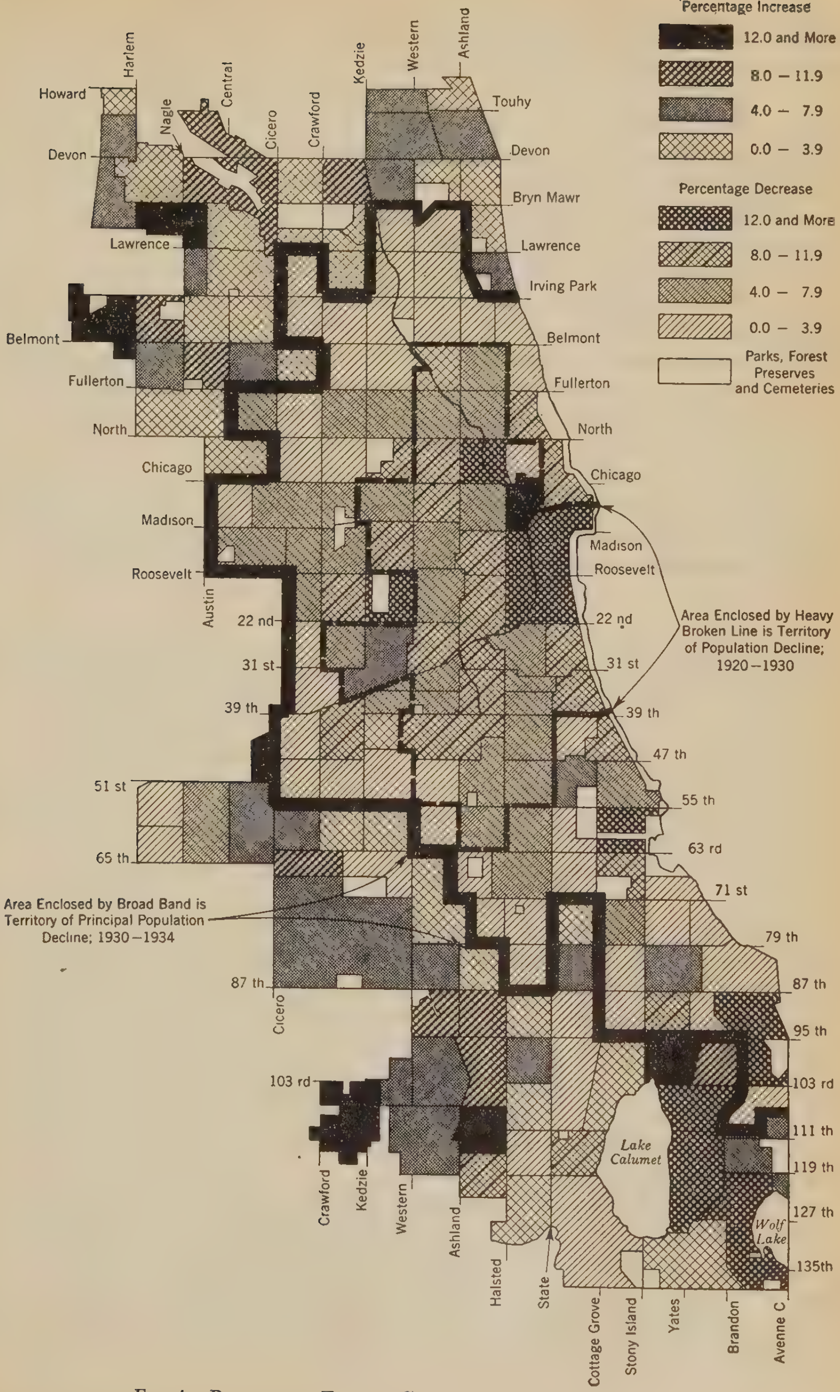
Percentage Increase
12.0 and More
8.0 – 11.9
4.0 – 7.9
0.0 – 3.9
Percentage Decrease
12.0 and More
8.0 – 11.9
4.0 – 7.9
0.0 – 3.9
Parks, Forest Preserves and Cemeteries
Harlem
Nagle
Central
Cicero
Crawford
Kedzie
Western
Ashland
Howard
Touhy
Devon
Devon
Bryn Mawr
Lawrence
Lawrence
Irving Park
Belmont
Belmont
Fullerton
Fullerton
North
North
Chicago
Chicago
Madison
Madison
Roosevelt
Roosevelt
Austin
22 nd
22 nd
31 st
31 st
39 th
39 th
47 th
51 st
55 th
63 rd
65 th
71 st
79 th
87 th
87 th
95 th
103 rd
103 rd
111 th
119 th
127 th
135th
Area Enclosed by Heavy Broken Line is Territory of Population Decline; 1920–1930
Area Enclosed by Broad Band is Territory of Principal Population Decline; 1930–1934
Cicero
Crawford
Kedzie
Western
Ashland
Halsted
State
Cottage Grove
Stony Island
Yates
Brandon
Avenne C
Lake Calumet
Wolf Lake

FIG. 4.—POPULATION TRENDS, CHICAGO, ILLINOIS, 1930 TO 1934

What Do the Studies Show Further?—Referring once more to the studies made, it is interesting to determine what they show with respect to blight and the need for re-zoning. They show that:

(*a*) In Chicago there are 44 sq miles of blighted territory piling up enormous deficits every year, which deficits must be met by the owners of property in other areas. The owners of property in this area are paying a very small share of taxes annually because they have allowed their property to depreciate and do not keep it in a condition that would give it adequate value to bear its proportionate share of city expense.

(*b*) These blighted areas may have had some prospect of development before zoning, but their prospects are now stationary because they are improperly zoned. They must be re-zoned; and there must be enforcement of demolition orders when owners refuse to make repairs, renewals, etc., necessary to make their property habitable in accordance with minimum living standards.

(*c*) The zoned use of these areas is entirely out of line with the use shown to be possible by the application of normal use absorption factors.

(*d*) There are miles of spurious .commercial frontage and the pattern of location is irrational. Fortunately, little has happened in Chicago to change the "picture" since 1923. Hence, Chicago can be re-zoned in accordance with present standards without changing anything more than the zoning plats.

A PRACTICAL METHOD OF RE-ZONING

When it comes to a practical method of re-zoning it should not be necessary to argue in behalf of the necessity of a land-use survey as a first step. Sixty-four surveys have been made in the United States and, no doubt, many more are under way. Most city planners are familiar with the startling disclosures made by these surveys.

If comprehensive land-use surveys cannot be financed, reasonably good results in fact-finding can usually be obtained from census data, use of insurance plats, assessors' cards, and other data on file in city departments.

Re-Zoning Plan.—The second step, of course, is the preparation of the official re-zoning plan based on fundamental use factors, determined from the survey and thoroughly diagnosed with respect to the average use factors of other cities. Officials have carefully estimated the future population growth of all cities so that, by applying the ratio of use per 100 population, a final total percentage of developed area can be arrived at for each use. Some cities probably have sufficient vacant property to accommodate future requirements. If not, the application of the use factors will give them a good idea of the additional area that should be incorporated. The process then becomes one of intelligent allocation of proportioned use areas.

Manufacturing Areas.—The location of manufacturing areas will be largely determined by transportation facilities. The zoned areas for this use should be checked off, and increased or decreased in size according to the use factors, but not changed in location, if the original zoning was logical. In Chicago, the location of the manufacturing areas would not be substantially changed in any re-zoning plan.

Commercial Areas.—Of course, all existing important commercial centers, areas, and frontage of major street intersections will be included as routine re-zoning. Vacant commercial frontage should be eliminated from the area zoned for commercial uses unless shown to be needed by the use absorption factor.

In allocating surplus area for future expansion of business, two important factors are involved: (1) Commercial use should be interposed between manufacturing use and residential use; and (2) proper provision should be made for proportioning the expansion of present live commercial areas. The foregoing process should proceed until the total anticipated use is absorbed.

Apartment Areas.—Multiple-family residential use should be zoned for major and minor arterial frontage. In the process of rehabilitating the blighted district these thoroughfares should receive appropriate design and cross-section development as part of the reconstruction project.

Residential Areas.—Two-family and single-family dwellings in general should occupy areas buffered by multiple-family use, parks, and commercial areas.

In examining the map of Chicago's blighted district, one may see how these factors can be applied in re-zoning the area without upsetting anything more than the 13-yr-old pages of the original zoning ordinance. Nothing much has happened since 1923 to change the "picture."

The next step, of course, is the publication of the proposed re-zoning plan, with a complete disclosure of the social, economic, and physical conditions involved and the advantages to be gained by the proposed re-zoning. That should be followed by a campaign to inform the citizens as to the benefits to be derived from re-zoning.

Summary of Answers to Questionnaire.—On December 18, 1936, the writer circulated questionnaires to 154 cities, and 102 replies were received from cities having a total population of 26 689 000, or more than 22% of the entire 120 000 000 population of the United States. The purpose of the questionnaire was to reveal the zoning situation as it exists in cities throughout the nation. This is adequate to give a true cross-section of the zoning situation throughout the entire United States. The data developed by the questionnaire are shown in Table 5.

Of the 102 cities that replied, 88 are zoned and 14 are not zoned; of the 88 zoned cities, 59 declare that re-zoning is necessary and 29 state that it is not necessary in their particular cases. Of these 88 cities, 9 have been re-zoned, according to the replies, whereas 37 others have been partly re-zoned, a total of more than one-half the entire number.

Of the 77 cities with less than 250 000 population, 39 feel the need for re-zoning, and 27 do not. Ten cities did not express an opinion one way or the other. In cities of this class the sentiment seems fairly evenly divided. In the 25 cities of 250 000 population and more, 20 report that re-zoning is needed, 2 that it is not needed, and 3 did not reply. Thus, it seems clear that the need for re-zoning is much more evident in cities of more than 250 000 population. Of these 25 larger cities, two have already been re-zoned and 10 others have been partly re-zoned. The purpose of the questionnaire was to determine the best method of gaining public approval for re-zoning. The question was propounded in the form of four alternatives.

Public Approval for Re-Zoning.—Question 4 asked if public approval of re-zoning should be based entirely upon the general advantages to be derived; that is, is re-zoning desirable?

TABLE 5.—RESULTS OF QUESTIONNAIRE TO DETERMINE THE ZONING SITUATION

CITIES GROUPED ACCORDING TO POPULATION (IN THOUSANDS)		No. of questionnaires mailed	No. of replies received	(a) NEED FOR RE-ZONING						(b) METHOD OF RE-ZONING										
From	To			No. of cities zoned	No. of cities not zoned	Need for Re-Zoning		No. of cities re-zoned	No. of cities partly re-zoned	Question 4		Question 4(a)			Question 4(b)			Question 4(c)		
						Yes	No			Yes	No	First choice	Second choice	Third choice	First choice	Second choice	Third choice	First choice	Second choice	Third choice
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
50	100	74	47	36	9	24	17	4	21	40	0	36	0	2	5	12	16	7	15	10
100	250	47	30	27	3	15	10	3	6	20	0	22	1	1	3	6	11	3	12	3
250	500	22	15	14	1	12	1	2	6	13	0	13	0	0	0	5	3	0	3	5
500	1 000	8	7	7	0	6	1	0	2	6	0	6	2	1	1	1	3	2	2	1
1 000	>1 000	3	3	2	1	2	0	0	2	3	0	3	0	0	0	0	0	0	0	0
Total	....	154	102	88	14	59	29	9	37	82	0	80	3	4	9	24	33	12	32	19

Eighty-two cities voted "yes" on this question. No city voted "no." Thus, in 100% of the cities replying, the correspondents felt that after the re-zoning plan had been completed as outlined, public and official approval should be sought on the basis of the benefits disclosed by it, with property owners being given the opportunity to voice objections, so that provision could be made for special conditions.

Re-Zoning for Highest and Best Use.—Question 4(*a*) asked if public approval of re-zoning should be based on the argument that by reducing the area overzoned for commerce, industry, and apartment use, the value of the areas properly zoned for these purposes would be increased as a result of such limitation; that is, that the smaller area for each use would increase land values.

Replies to this question were received from 87 cities. Of this number 80, or 92%, chose this method as being preferable to the remaining two alternative methods suggested in the questionnaire. Three cities picked Method 4(*a*) as their second choice; and four others picked it as their third choice.

Among cities of less than 250 000 population, 93% of those answering the questionnaire selected Method 4(*a*) as being the best; 5% of the cities selected it as their third choice; and 2% selected it as a second choice.

Upon the adoption of an official re-zoning plan there would be a logical basis for the establishment of valuations for taxation purposes, that would be consistent with the actual development of an income from property. This is a condition that is now impossible in large sections of the United States. To-day, all property is menaced because of lack of stability. In the long run, even without intensive rehousing, the corporate fund will benefit if property can be placed in its proper class of highest and best use for development, and will be stabilized through the logical forces of use requirements which operate gradually to force development, following the law of supply and demand. For this to be effective, however, variations (the "worm" that is now destroying the "fabric" of zoned use) must be exterminated by a strong city planning and housing council.

Paternalistic Method of Re-Zoning.—Question 4(*b*) asked if public approval of re-zoning should be sought because of the value of rehabilitating blighted districts through governmental financing; that is, by paternalistic methods.

Replies were received from 66 cities. Of these, 9 selected it as their first choice; 24 selected it as second choice; and 33 selected it as third choice. Thus, only 14% considered it to be of first importance, 36% of second importance, and 50% of third importance. Among cities of less than 250 000 population, 15% considered Method 4(*b*) to be the best; 34% ranked it as their second choice; and 51% felt it to be their third choice. Thus, the smaller cities clearly feel Method 4(*b*) to be the least desirable of the three alternate methods suggested. Among cities of 250 000 population and more the vote on Method 4(*b*) *versus* Method 4(*c*) for second and third place, respectively, was fairly even.

Non-Paternalistic Method of Re-Zoning.—Question 4(*c*) asked if public approval of re-zoning should be sought because of the value of rehabilitating blighted districts through non-paternalistic methods, such as some form of trustee plan, limited dividend corporation, public utility housing corporation, or other private organization, aided by governmental financing, but not under governmental control.

Sixty-three cities replied to this question. Of this number, 12 cities, or 19%, selected Method 4(*c*) as their first choice; and 32 cities, or 51%, selected it as their second choice. Thus, among all the cities voting, the majority selected Method 4(*c*) as representing their second choice among the three alternative methods.

The Housing Situation.—Summarizing the general housing situation one finds that:

(*a*) The Reconstruction Finance Corporation (RFC) was established in 1932, with power to issue building loans for limited dividend housing projects subject to State or municipal housing approval.

(*b*) The Public Works Administration (PWA) began its housing program in 1933 by making loans to limited dividend housing corporations initiated by the RFC. Seven projects were financed in this way. The PWA turned to Federal construction, under which plan fifty projects were undertaken.

(*c*) Since December, 1934, 20 States have enacted housing legislation in accordance with drafts submitted by the PWA, providing for the creation of housing authorities that have broad powers to undertake and operate housing projects, but which are without taxing power. The authorities are given the right to secure property by eminent domain.

(*d*) Now (1938) the PWA is decentralizing public housing and turning over to the State and local housing authorities the responsibility of initiating, constructing, and operating low-rent housing projects.

Housing Needs.—The following items should be kept in mind in determining the problems of the State and local authorities in attempting to initiate, construct, and operate low-rent housing projects: (1) The enactment of enabling legislation; (2) it is necessary for the municipality to furnish the housing authority with sufficient money to defray its first year's administrative and overhead expense; (3) municipalities should be empowered to grant various forms of assistance to the housing authority, in addition to tax exemptions; (4) municipalities might find it necessary or expedient to sell, lease, or donate property to the authority to furnish parks, playgrounds, and other facilities, and to re-zone or re-plan affected areas; (5) it will frequently be necessary in connection with the financing of housing projects for long-term agreements to be entered into between the housing authority and the municipality regarding the furnishing of services and payments therefor in lieu of taxes; (6) to accomplish Objectives (1) to (5), the PWA is suggesting the enactment of the housing corporation law; (7) the PWA will operate fifty housing projects; (8) it is suggested that under appropriate and effective State legislation these projects be leased or sold to local housing authorities as soon as possible; and (9) it is also suggested that, in the future, all low-rent housing projects should, in general, be undertaken by local housing authorities on a loan-and-grant basis.

The proposed plan apparently obviates some of the difficulties of the former plans for financing low-cost housing, in that the municipal housing authority is given the right to acquire property by eminent domain.

Modification of Housing Plan.—Would it not be well to consider some modification of present and proposed legislation whereby property owners and mutual interests could assemble their property into an equity or community trust and construct, own, and administer low-cost housing properties with the aid of a Government loan and grant, subject to all the requirements to be imposed by the municipal housing authority under the present arrangement? Such a plan, of course, should be based upon a limited return on the investment of the property owners. From a purely social point of view this would be much better than the present condition in which slums are subsidized by draining municipal treasuries.

A co-operative trustee plan through this kind of financing would make possible the development of much greater areas for low-cost housing than is possible under the present plan, because of the pooling of the property itself in the development. The municipality would be much better off in the long run, because it would receive revenue from a much larger area than is possible under the present plan, as rehabilitation could be made much more extensive.

Criticism of the plan because it would involve a Federal grant to a co-operative society for private profit should take into consideration that the subsidy would be for a public purpose. It has certain "strings" attached to it in that the equity trust must be operated for limited profits and must achieve low rentals consistent with those necessary to reach the levels of the standard low-cost housing. The trustee plan would employ labor and stimulate industry on a much larger scale than would be possible with the same expenditure of funds under the housing authority.

There is objection to the Government housing authority as the only means of developing housing because it weakens private initiative. This is avoided in the equity trust plan as the property rights and control would remain with the individual owners. Maintenance and operation would be more efficiently administered under the equity trust than under political control; taxation problems would be clarified, and the public would be free from the ever-growing expense of amortization due to obsolescence.

Prime requisites for this kind of plan would be re-zoning, and the selection of cities where manufacturing use exists to give employment to occupants of housing projects.

SOME OPINIONS

Many people are not in favor of paternalistic housing projects. For instance, Clifton R. Bechtel, Chairman of the City Planning and Zoning Committee of the Chicago Real Estate Board, writes:

> "The Chicago Real Estate Board has based its plan on co-operative individualism. There is nothing new or original about this. It came to Illinois with the first settlers. The individual stood on his own feet and carried on alone as far as he could go, and when the job was too big, he called in the neighbors and finished the job.
>
> "Our Committee feels the place to start is with the people who own the land. They feel paternalistic policies are bad, in that they tend to weaken the citizen. Many proposals are advocated as emergency measures, but even emergency measures should be carefully scrutinized."

Chicago Real Estate Board.—Some of the points of the rehabilitation plan advocated by the Chicago Real Estate Board are:

> "To arouse local public opinion and public spirit so the people themselves—the property owners and residents—can participate in the action."
>
> * * * *
>
> "To aid in working out the problem along sound economic lines as a commercial and not as a philanthropic proposition."
>
> * * * *
>
> "To ascertain the kind of people to live in the district; to consider those now living there; to consider those who might be induced to go there. This would mean taking into consideration the earning power of these people in making housing plans that, in cost or rentals, will fit their average income."

Lack of Private Capital.—Attempts have been made in the past to create equity trusts for the purpose of developing low-cost housing areas in which individual property is assembled into large holdings; but there has always been the complication that private capital could not be induced to take the risk because of the many unstable conditions inherent in the proposition. These conditions include instability of low-cost housing areas relative to adjacent areas and uses; the availability of employment for residents; and the ability of residents to pay the rent in such low-cost housing when constructed.

If a region is re-zoned, however, manufacturing is retained and invited to come into the blighted district, and low-cost housing projects are built in appropriate areas relative to manufacturing. There will then be large areas for residential development for housing the higher income groups along the main thoroughfares crossing the district; and it is in these areas that the assembling of property into an equity trust for ownership management and operation, or for any private undertaking, might be conducted with success, because of the stability the district would gain under the new setting.

Limited Profit Agencies.—There is also the possibility of pooling property interests in an equity trust that could obtain funds as a limited-profit housing agency, at interest rates not less than the going Federal rate. The limited-profit housing agency would be required to maintain the low-cost character of the housing, and, therefore, would not come into direct competition with private building enterprise.

The plan of pooling property interests in the formation of a limited profit housing agency would make possible low-cost housing on a much larger scale than any other co-operative limited-dividend corporation, because the Government funds would be available for costs other than that of acquiring the land.

COMMENTS ON QUESTIONNAIRE

Many interesting comments were made on the questionnaire previously referred to, among which the following were most valuable:

(1) Zoning is useful in protecting new sections from blighting influences; but alone it can accomplish little in rehabilitating run-down sections. It cannot be done intelligently unless correlated with a comprehensive city plan.

(2) Re-zoning is needed not so much to protect and increase value as to protect amenity and stability which, in turn, will be reflected in future value. Residential owners want protection of the character of their neighborhoods. Business property owners want reasonable opportunity to use their real estate profitably, consistent with the governing law of supply and demand.

(3) In re-zoning an attempt should be made to meet the following ratios: Highways and streets, 31%; apartments, 15%; homes, 37%; business, 5%; industry, 6%; and recreation, 6 per cent.

(4) Re-zoning should be along scientific lines after a careful survey of the city and of the trends in other cities of similar character; and it should be without property owners' selfish and half-baked notions. The average property owner will not devote enough time to the problem to understand it and support re-zoning in his own interest, especially in areas now over-zoned for business use.

(5) Re-zoning should be planned so that each area which shows definite trends can develop in conformity with that trend, carefully considering the relation to abutting districts of higher or lower classification, and giving special attention to the developments permitted along common boundary lines.

(6) A metropolitan committee is recommended consisting of property owners, realtors, architects, builders, traffic men, financiers, attorneys, physicians, health authorities, officials of street railway and bus companies, planning agencies, and engineers. Planners should profit by former zoning; present their findings to the City Council and have them enacted into a law with "teeth" in it; make re-zoning conditions non-political.

(7) The major objectives of zoning are born of economic necessity. Zoning is an effective stabilizing factor in real estate and is an important contribution

to enhancement of all civic values. It is a needed preventive and a civic insurance against blight and slum conditions.

(8) A practical supply-and-demand analysis (covering both the present and potential need for residential, business, and industrial uses, based upon up-to-date, accurate, detailed information, and upon comprehensive planning considerations) is the proper formula for re-zoning. This will include much more drastic limitations on the size of residential buildings, as well as on the area devoted to business.

(9) The main problem, in addition to reducing business zones, is to reduce vast areas zoned for large apartment houses, but built up with single dwellings, "duplexes," and flats, to zones protecting existing uses. The principal arguments in favor of such re-zoning are: Additional protection to home districts; stabilization of property values and community development; prevention of blight caused by evacuation of home owners to newer protected areas when apartment houses intrude; and prevention of new assessments to provide additional public utilities to replace those overtaxed by the increased population brought in by the intruding apartment houses.

(10) Re-zoning should be based on a land-use survey of the present built-up areas, and upon an estimate of the future trend of growth of undeveloped areas and their suitability for different classes of uses.

Conclusions

1.—The evidence is quite convincing that cities, particularly the larger cities, must be re-zoned if urban area uses are to be proportioned correctly and located in advance of any further city development.

2.—Zoning plans must be recast in many respects to provide an equitable and sound basis for tax valuation.

3.—Land-use surveys show conclusively that absorption of urban areas occurs in accordance with definite factors, which can be applied with reasonable accuracy in apportioning city areas for various uses.

4.—Property now static because of improper zoning must be cleared of its fictitious use as a prerequisite for its development to its highest and best use.

5.—The method of re-zoning must first be tested against a land-use survey; then an intelligent analysis of the results of the survey must be made to determine present land use and trends; third, the factors of use absorption must be determined, based on this survey, but taking into consideration surveys made in other cities of similar character; and, finally, these factors must be applied intelligently not only to present developed areas, but also in the allocation of use areas for the future absorption of vacant properties in accordance with anticipated growth.

6.—Upon completion of the fact-finding survey and the recasting of use, an official plan should then be prepared by an official planning body.

7.—A strong citizens' council should be formed to serve as an advisory group to the planning body and to throw the weight of all civic organizations back of the re-zoning plan—a body that should be free politically, and frank, firm, and thorough in its job of informing the people in regard to the need for, and benefits of, re-zoning.

8.—This council should co-ordinate its efforts and those of the planning body with the tax assessor's office in the preparation of a plan providing an equitable valuation of property for taxation purposes, developing information as to the ultimate beneficial effects that will result from having property revitalized and put in line for developing into its highest and best use.

9.—It should be the policy of the city to seek approval of re-zoning on the basis of an official zoning plan based upon a scientific analysis of land-use information. Such a plan should be used in the determination of use absorption for the present and future; and planners should develop a direct official method of putting it into operation through the instrumentality of the citizens' council, which should strive in an intelligent and aggressive manner to remove the blight of present fictitious zoning and secure the adoption of a re-zoning plan for equitable taxation.

10.—A vital factor in the re-zoning and rehabilitation of blighted areas is the correction of the conditions that are causing manufacturing and industry to decentralize. It should be the purpose of the citizens' council to re-zone and to re-value blighted district property for taxation in a manner that will retain, as well as attract, additional diversified manufacturing, a most essential requirement if employment is to be found that will make it possible for the residents in these areas to live in low-cost housing.

11.—The citizens' council should adopt a policy that will embrace the paternalistic method of rehabilitating blighted areas, because it will find application in large areas of the lowest income groups. In addition to the plan of Federal loans and grants to municipalities in the present policy of decentralizing the housing administration, consideration should be given to the alternate method of permitting property owners to assemble their property under some form of trustee plan that will permit the development of large areas of low-cost housing with the aid of Government loans and grants. The first method places the ownership and administration of a low-cost housing project perpetually with the governmental body; the second method would make possible the development of much larger areas for low-cost housing, and would place the ownership and administration in the hands of the property owners themselves.

12.—The policy adopted by the citizens' council should also include the non-paternalistic method, because this will enable property owners to pool their interest in some form of trusteeship that can operate as a limited profit housing agency. Both the paternalistic and the non-paternalistic methods of low-cost housing are necessary for the rehabilitation of blighted districts. The non-paternalistic method should include some appropriate plan that will permit the property owners themselves to become a factor in the rehousing process by pooling their interests into some form of trusteeship that will greatly amplify the scope of rehabilitation and obviate the transfer of too much real estate to the governmental authority.

Finally, with the following factors assured: (1) The stabilization of property use in blighted areas; (2) the correction of factors that are now operating to decentralize manufacturing and industry; (3) the development of low-cost housing projects by any of the methods proposed; and (4) the full co-operation of the city in the matter of utilities and the creation of badly needed park areas in locations that will serve as buffers between manufacturing property and residential property—it is a foregone conclusion that private capital will be attracted on a large scale to the construction of housing for the higher income groups. They will be especially attracted along the boulevards and major and minor streets crossing the blighted territory, particularly if proper attention is given to the design and construction of these thoroughfares, a matter that will be feasible in the rehabilitation process.

DISCUSSION

Robert D. Kohn,[4] Esq. (by letter).—In 1936, the writer came to the realization that no adequate change had been made in the zoning laws applying to the City of New York, N. Y., especially as to the density of use, although a building boom of a kind seemed to be on its way. It has not actually eventuated but much construction of a purely speculative nature was done in 1937. Such building, which planners have feared and tried to prevent, is going on with the same density that was permitted before.

The fact is that the very basis of zoning in New York City is absurd. That is, the absurdity of any scheme based on a scarcity of price placed on land when no scarcity exists. Before planners can get started on any worth while change they must make the difference between price and real value known to the public and faced by the real estate owner. For the difference is colossal. The high price is based in some measure on this very unrealistic zoning which has been accepted. The Housing Authority of New York found that districts zoned for residential purposes in New York City, if built up to the extent permitted by the zoning law, would accommodate 77 million people, and that areas provided for business and manufacturing would accommodate, if built up to the extent they were permitted by the existing zoning law, 340 million people, or, all the people in North America and South America combined.

Whenever a piece of property in a blighted area is purchased for housing purposes, the price the buyer must pay is based on the assumption that the land might be used for some purpose that the zoning law permits and to the limit permitted by the zoning law. Even if the idea with regard to trustee plans for combining properties in blighted areas were adopted they will not work as long as either the trustees have to base their plan on prices that have no relation to real values—earning values—or if the co-operative body is the State, the city, or the nation, and is forced to pay those prices. Another matter to which Mr. Young referred suggests that if the principle of earning value for land were really established, instead of scarcity price, the city would be bankrupt within one year because its tax base would disappear.

Those who are interested in this subject must begin by educating the public to the fact that there is this colossal spread between price and value; that owners can never really get such prices for their land, absurdly based on the idea that all the people in North America and South America will be working in an area such as the New York district, or the Chicago district. Planners will never accomplish anything with housing or better city planning until they realize that the entire present structure of real estate prices is built on a false basis.

Myron D. Downs,[5] Esq. (by letter).—Why is there a need for re-zoning cities zoned ten to twenty years ago? Most of the older ordinances were prepared by capable consultants, although many of them were written by city solicitors or city engineers' assistants under the direction of city councils seeking an economical method of satisfying a persistent group of citizens. Unfortunately, many consultants and the non-professional planners did not consider it essential to study other elements of the city plan prior to the recommendation of a zone plan. Too many zone plans already prepared and adopted, constitute merely maps of existing uses, no effort having been made to proportion the several types of use scientifically. It is well to bear in mind that the United States Supreme Court, in its decision in favor of the Village of Euclid, Ohio, in 1926, found zoning a constitutional exercise of the police power when based on comprehensive planning. It is not to be inferred carelessly that any map of an urban area, divided into three or four types of use districts, will withstand successfully the test of time, either in the Courts or as a medium for furthering the elements of a comprehensive and complete city plan, which should be the planner's test of a good zone plan. It would seem that the greatest need for re-zoning urban areas exists by reason of failure to plan first.

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[4] Archt., New York, N. Y.

[5] Secy., Cincinnati City Planning Comm., Cincinnati, Ohio.

Mr. Young states (see heading "Need for Zoning: Why Zone") that "city planners have come to sense the great social need of transforming the physical structure of urban areas." It is to be hoped that his observations are correct. If so, the larger cities which, twenty years ago, followed New York's leadership in zoning, will again play "follow the leader" and provide workable public agencies in the form of public housing authorities to do the work. Much of the practical difficulty involved in the operation of housing projects by the Federal Government had its beginning in the form of bad land sub-division practice and the speculative rather than investment campaigns to "own your own home."

The author properly emphasizes the need of a strong administrative branch of government if good zoning is to succeed. Liberal or inadequate enforcement is very quickly resorted to, if a poor piece of zoning has been done by the planner, or if municipal officials have not provided for adequate planning as a foundation upon which to base the zone plan. Many a community has turned to the supposed economy of charging the assistant sewerage engineer and the bridge engineer's draftsman with the task of supplying the local planning commission with a zone plan during idle winter months. Naturally, the product of their efforts is comparable with that of a planning engineer, suddenly and without special technical training, assigned to the task of designing a sewage treatment plant for a community of unknown population, or detailing the shop drawings for a bridge of unknown length, width, and loading. Use of the resulting structures, in either case, is perilous.

Advances in the quality of public planning must be gradual and ideals must be compromised. Cities with too few use districts for proper transition, can amend the use district map. In 1932 the City of Cincinnati increased its use districts from eight to ten, providing thereby an intermediate residence zone between the one-family house district and the three-story, multiple-family building district, and a new business zone restricted to local services and goods establishments, with height limitations in keeping with those of surrounding residential areas.

There is still room for improvement, in most large communities, in the public control of population density. Height, area, and bulk regulations are of relatively little assistance in regulating population to the extent that school buildings and pavements can be built with any degree of certainty that the estimated capacity will not prove inadequate long before the bonds issued for the improvement are retired. There are few large cities which do not have some area congested with a density of more than 100 persons per net acre of land, and, what is more serious, they have no legal control of density in other areas, commercial or residential. Both Chicago and Cincinnati have certain areas with densities in excess of 120 persons per acre.

High buildings always present a difficult problem to the planner, in so far as the regulation of bulk is concerned. Cincinnati has improved the bulk regulation of the central business district approximately 25% by the recent amendment (1932). However, as emphasized in 1936 by Mr. Harold S. Buttenheim,[6] the provisions are still most liberal. (The total cubical contents above the established grade must not exceed the contents of a prism having a base equal to the area of the lot and a height equal to two and one-half times the width of the street—maximum, 250 ft.)

Good zoning, and the ability to place it on the ordinance books of any city, can only result from a good quality and an ample amount of technical work by competent planners supported by community endorsement of a planning commission of proved integrity.

HARLAND BARTHOLOMEW,[7] M. AM. SOC. C. E. (by letter).—A most convincing case for the re-zoning of urban areas is presented in this paper. Surely, no one who has studied this problem in large American cities can deny its necessity. Apparently, there is now a wide realization of this need, which is indicated by the large number of cities which express themselves in favor of re-zoning in answer to Mr. Young's inquiry.

Cities will be re-zoned to best advantage only when the plan is based upon an accurate knowledge of the total land in urban use for various purposes and the annual rate of absorption. It is interesting to note how closely Mr. Young's data for the total area of land used for urban purposes in Chicago and certain suburban areas corresponds with similar values for about sixteen self-contained cities cited by the writer.[8] Had Mr. Young been able to secure data for all land in urban use in all the incorporated and unincorporated areas in the Metropolitan District of Chicago, there is no doubt but that the data would have corresponded even more closely with the average in other cities. There are well-defined laws of land use which, when thoroughly understood, will make possible far more scientific zoning practices.

In 1937, the St. Louis (Mo.) Regional Planning Commission made a land-use survey of the entire 840 sq miles within the St. Louis Metropolitan District, with the aid of the Federal Emergency Relief Administration (FERA) and the Works Progress Administration (WPA). The percentage of land used for various purposes in this Metropolitan District (which is really nothing more than a very large self-contained city) corresponds very closely with the average percentage used for various purposes in the sixteen self-contained cities (see Table 6).

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[6] "Land Overcrowding Allowed," *American City Magazine*, June, 1936, p. 81.

[7] (Harland Bartholomew & Associates), St. Louis, Mo.

[8] "Urban Land Uses," by Harland Bartholomew, Harvard Univ. Press, 1932.

TABLE 6.—LAND USED FOR VARIOUS PURPOSES; PERCENTAGES

Purpose	Average of sixteen cities*	St. Louis Metropolitan District	City of St. Louis
Single-family dwellings	36.10	29.58	20.4
Two-family dwellings	2.10		7.7
Multiple family dwellings	1.09	2.38	7.5
Commercial areas	2.38	4.27	5.3
Industrial areas	10.79	17.60	14.5
Streets	33.60	25.57	26.2
Parks and playgrounds	6.33	4.38	8.0
Public and semi-public	7.61	16.22	10.4
Total	100.00	100.00	100.00

* "Urban Land Uses," by Harland Bartholomew.

The importance of including all the area in urban land-use surveys, regardless of municipal boundaries, is revealed by Table 6. Although the percentages of land used for various purposes in the St. Louis Metropolitan District correspond fairly closely with those of the sixteen self-contained cities, it will be noted that those for St. Louis (which is only part of a self-contained city) show certain marked variations from the normal averages. Re-zoning of partial urban areas should be undertaken only after a full knowledge of total urban land-use characteristics based on comprehensive surveys.

Most city planners will agree with all Mr. Young's fourteen conclusions. It is particularly interesting to note that he concludes that both public and private enterprise will have to unite in the rebuilding of blighted districts—a conclusion that should be closely noted by the extremists who advocate one or the other of these methods of approach as the only possible means of solving these problems.

Mr. Young advocates the creation of a strong citizens' council to promote the re-zoning plan which has been prepared as the result of the land-use survey. With this the writer is in full sympathy, but the success of this council depends upon many factors. The desires of a small but influential group of property owners have caused bad distortions in original zoning ordinances. These same speculative interests have been responsible for many unwarranted changes in zoning ordinances after their adoption in many cities. What assurance is there that any sound re-zoning ordinance adopted as the result of the initiative of a strong citizens' council will not gradually be broken down by these same speculative interests? Some kind of a permanent agency is needed to insure that all zoning is soundly conceived and wisely maintained. It is for this reason that the writer advocates one step farther than Mr. Young in his approach to this problem.

A real objective view of the present American city reveals it to be divided into three different types of residential areas: (1) Slums; (2) blighted districts; (3) new sub-divisions and suburban areas.

The actual slum areas are found generally in the oldest residential districts. They are relatively small in size, constituting probably not more than 5% of the total land in urban use. There is nothing that can be done about them except tear them down completely and rebuild them upon a totally new plan. The new sub-divisions and suburban areas are also relatively small in size, constituting perhaps not more than 20% of the total land in urban use. There is no immediate problem in these new areas other than that of protecting and preserving their present character. This is fairly well provided for by deed restrictions and by zoning regulations. The greatest success with zoning has been in preserving the character of new development.

Between the slums and the new suburban developments lies a vast area of residential development in various stages of decay. These are the blighted districts. They are all potential slums. Probably the most severe indictment that could be brought against the American city of to-day is the failure to give adequate protection to residential areas, from which more taxes are collected than from all other urban land uses, commerce and industry included. Residential development, particularly of the one-family and two-family types, once developed in an American city, soon starts either a gradual or a rapid process of depreciation. These large blighted districts cannot and need not be rebuilt. They are too large and the individual buildings within them are not yet obsolete. The problem then is one of rehabilitation rather than reconstruction. Sound rehabilitation involves the development of constructive urban land policies heretofore almost completely absent in American cities.

Thus, there are three different problems within the urban residential area: (1) Reconstruction of slums; (2) rehabilitation of blighted districts; and (3) protection for new suburban development. Zoning, in itself, or re-zoning, will not accomplish either of these three objectives. It will do very much to facilitate each of these objectives and, in fact, is indispensable to each of them. Zoning, or re-zoning, however, is merely one phase of urban land policy. To accomplish the desired objectives in all three areas involves a long-time program and continuous study of relatively small unit areas of land within each of the three broad areas. The writer, therefore, favors the division of all the residential areas, slums, blighted districts, and newly developed areas, into a comprehensive plan of neighborhood units, and the formation within each unit of a citizens' organization for permanent study and planning, as well as for concerted action in all matters pertaining to the welfare of the neighborhood unit which it represents.

Stability of home ownership and satisfactory living conditions have been destroyed by uncontrolled speculation in urban real estate. One can never establish stability and desirable living conditions without control of real estate, which is consistent with normal land use. Where there is a city plan commission capable of determining broad principles of land policy, and a well organized neighborhood unit in each part of the residential area, planners can soon determine what is the proper zoning and policy of land development within each neighborhood; and when the citizens of each neighborhood realize and understand what the zoning and land policy should be, they will have a far better chance of adopting, enforcing, and maintaining appropriate zoning and other parts of a sound land policy and program.

This comment, with respect to the establishment of permanent agencies in the form of neighborhood units and neighborhood unit organizations in the interest of slum clearance, rehabilitation of blighted districts, and protection of new residential areas, as previously stated, is merely supplementary to, and in no sense believed to conflict with, Mr. Young's recommendations. The writer is in full sympathy with his recommendations, and he should be congratulated upon the preparation of so thorough and so constructive a paper.

BENJAMIN SALTZMAN,[9] ASSOC. M. AM. SOC. C. E. (by letter).—The increasingly large number of municipalities which have adopted ordinances regulating the use to which land and buildings may be put, and limiting the height and bulk of buildings, is evidence that the benefits of a sound zoning system have been recognized everywhere. The private restrictions that constituted the sole regulations prior to zoning, were haphazard and sporadic, covered only a small area, and were formulated and often distorted by the developer to secure him a maximum financial return. These private restrictions were inflexible and, under changing conditions, frequently impeded the logical growth of the neighborhood, thereby making application to the Courts necessary for their removal. As cities became larger the need for city-wide regulation became more and more essential. It was apparent that city planning and zoning are inextricably intertwined. It was difficult for a community to plan street and block layout, park and recreational facilities, water supply and sewerage systems, transit and transportation systems, port and terminal facilities, and to locate schools and hospitals when the control of the city extended only over city-owned property, and when private property, covering about two-thirds of the city's area, was permitted to develop without any control, although the manner in which private property was developed strongly influenced the direction of city planning. No plan for the development of public facilities can be complete or effective which is not based upon a comprehensive plan of control of building development on private property.

It is comparatively simple to map a proposed city along logical zoning lines. It is extremely difficult to zone land which has developed haphazardly, especially in a large city created by combining many smaller municipalities, each of which developed independently. The average zoning ordinance, as first enacted, represents a series of compromises between the city planner working along established principles of zoning and the real estate interests desirous of diverting the ordinance to those directions which might enhance the value of their property, which explains the unusually high percentage of land zoned for industry.

Almost every zoning ordinance contains "congenital" defects which prevent the fullest realization of the benefits of zoning and which contribute toward the creation of blighted areas. As time elapses the defects become glaringly apparent and real estate interests become reconciled to the idea that re-zoning is necessary. To determine the nature and the extent of re-zoning requires an examination of the ordinance, a study of what it sought to achieve, where it failed of achievement, and the factors contributing to that failure. All the symptoms must be studied, and a diagnosis made before any remedy can be prescribed.

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[9] Asst. Engr., Dept. of Buildings, Brooklyn, N. Y.

Zoning is designed primarily to reduce congestion, provide adequate light and air, keep obnoxious uses out of residential areas, lighten the burdens of traffic regulation and fire protection, ease the strain on transportation systems, reclaim blighted areas, and finally, to stabilize real estate values. Haphazard building tends to reduce taxable values, as entire districts may become depreciated, while orderly development aids in the conservation of property values.

An area that has been properly zoned will not become blighted. A residential area that has been protected against invasion by industry by a buffer zone devoted to local business use, will retain its essential character for many years. An area devoted to one-family or two-family dwellings must be protected from the congestion caused by the erection of multi-family dwellings. In New York City, the ordinance needs revision in this respect, as the only restrictions designed to keep the multi-family dwelling out of the single-family area are limitations as to height and bulk, which are, more and more frequently, legally and successfully evaded. Where such areas, even if well shielded, do decay and become blighted through obsolescence and gradual occupation by lower-income families, the remedy is not to re-zone so as to permit business or industry to enter, but rather to re-build, either with private capital or Government housing, or both.

Most blighted areas have either been improperly zoned in the first instance or have been destroyed by an invading industry which later departed, leaving the empty shells of buildings as a reminder of their existence. These are the areas to reclaim. Instead of encouraging the development of out-lying districts, which induces abandonment of the old, and removal to the new, by extension of rapid transit facilities and by the creation of other desirable features, this emphasis should be placed upon the redemption of the old. It is illogical as well as uneconomical to permit an area provided with schools, streets, fire-fighting equipment, sewers, water supply lines, and transportation facilities, in fine geographical locations, to decay and become more and more uninhabitable, with real estate values wasting away, and proceed to duplicate all this expensive development in some outlying part of the city.

Decentralization of industry may become a cause of blight, but it may also be made to contribute to progress rather than to retrogression. If industry could be confined to small areas, each surrounded by local business uses and then by residential area devoted to the homes of the workers, the strain on transportation systems would be eased and street congestion materially reduced. Unquestionably, too much area has been zoned for industrial use; segregation and the reduction of these areas to reasonable limits would fix real estate values and diminish the area subject to blight.

Re-zoning alone, however, is insufficient to correct all the negative factors that contribute to the ineffectiveness of a zoning ordinance. The defects in the ordinance may be noted and noted, and the areas to be devoted to different uses scientifically apportioned, and yet the entire system may break down in the enforcement of the law. The zoning ordinance must be administered honestly and capably to be successful. Political expediency or personal favoritism have no place either in the administration of the law, in the granting of variations, or in the framing of amendments to the ordinance or the maps.

The burden of administration and enforcement is lightened by the requirement of a certificate of occupancy for every new or altered building. The issuance of this certificate is an assurance that the building complies with all the laws applicable thereto, including the zoning ordinance, and occupancy contrary to the terms of the certificate is in violation of the law. The certificate of occupancy is a public record, and any person interested can easily determine the contents of any certificate. Illegal occupancies may be noted and corrected. The department charged with the enforcement of the ordinance should issue the certificate; any other method results in a shifting of responsibilities and bickering between the agency enforcing the law and the one issuing the certificate. In New York City, the Department of Housing and Buildings administers the zoning ordinance, with a Borough Superintendent in charge of each Borough. This Department, or its predecessors in duties, has issued certificates of occupancy since they were first required in 1914. It is only on rare occasions, and then in border-line cases of interpretation of the law, that these certificates have been challenged by any request for revocation.

In New York City the City Planning Commission has the authority to amend the zoning ordinance and the maps; the right to grant variations from the strict requirements of the law, in stipulated conditions, is vested with the Board of Standards and Appeals. Both these bodies act only after public hearing. Variations in interpretations of the zoning law are granted only after thorough investigation and upon a sound legal basis, only where the development along conforming lines would inflict unnecessary hardship or where practical difficulties intervene; and then only under such conditions as will safeguard the character of the neighborhood. Furthermore, the relief granted to an applicant must not inflict any hardship on neighboring property owners. The Courts have consistently upheld the Board in their decisions along these lines.

The full benefits of zoning or re-zoning cannot be secured unless the concomitant problem of the non-conforming use is solved. In any area previously zoned for business or industry and re-zoned for residence use, or in any area previously zoned for industry and re-zoned to permit only small local business use, there will be numerous buildings whose use does not conform to the regulations for the new district. It is the hope of the zoning engineer that eventually obsolescence and natural decay will force the demolition of these structures. Where the Board of Appeals grants indiscriminate variances permitting the modernization or enlargement of such structures, the effectiveness of the ordinance is impaired and the restrictions become valueless. The Board, therefore, at all times, must regard the interests of the community as paramount, and allow variances only where the granting of the appeal is imperative and where denial would be equivalent to confiscation of property. Unless such an attitude is maintained re-zoning is valueless. Honest and capable administration of the ordinance is the keystone of the structure of zoning; without it any zoning or re-zoning is an idle gesture.

G. H. Herrold,[10] M. Am. Soc. C. E. (by letter).—Much "water has gone over the dam" since the first zoning ordinance was drawn in 1916. Land has changed hands through the years. Lots have been purchased and many built upon in accordance with existing zoning. Skyscrapers have been constructed, filling stations established, apartments designed and built, new plats recorded, thoroughfares and parkways built, playgrounds and parks acquired; and a new conception of land tenure is under way.

To re-zone a city is quite an "upset" and should be approached cautiously and after much study. The average city in the United States that has grown naturally is divided into four classifications as to residential areas:

(1) There is the well-maintained and established high-class residential district in which properties are occupied by the owners. Each property is a specimen with an atmosphere created by the architecture and landscaping of the site and the culture of the owner;

(2) There is the neglected district lying between this stabilized residential district and the commercial or industrial district—neglected in the sense that no one is looking out for it collectively. The owners have lost faith in their investment and in the district;

(3) There is a completely blighted district or slum; and,

(4) There are the new outlying developments in which people in the stabilized districts are invited and importuned to invest.

The great good of zoning (that is, preserving the *status quo* in older districts and preventing the "mussing up" of new districts) has really been attained in the majority of cities. It is possible that the strengthening of health laws or a better enforcement of these laws in slum, blighted, or neglected districts may prove an aid to zoning. A slum district can be rehabilitated by the simple enforcement of cleanliness and structural safeguards, good municipal maintenance, and the enforcement of requirements for light and air. With this aid city planners may give the district a new lease on life under its present zoning.

All zoning ordinances provide for multiple-family dwellings. A multiple-family dwelling may be a barrack, a tenement building, a "four-plex," or a beautiful apartment building. Some definitions are necessary to segregate these different structures in zoning a city. If an apartment building was no higher than near-by residences and took the same setback; if it had an area of land, say, equivalent to 1 sq ft of open area to each square foot of floor area; if it had no common entranceways, it would be appropriate and acceptable in a residential district where houses are rented, especially if limited to locations, such as those adjoining along the side line of different zoning, facing a different zoning, or facing a park, school grounds, or institutional buildings, etc., the opportunities would be unlimited. An apartment building might be defined as a two and one-half story, multiple-family dwelling in which each apartment has its own individual entrance, etc. A tenement might be defined as a multiple-family dwelling with a central hall used in common by all tenants, etc.

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[10] Planning Engr., The City Planning Board, St. Paul, Minn.

Mr. Young shows that the great grievance is too much commercial, industrial, and apartment house zoning—a lack of balance in the proportioning of areas of zones and the placing of a value on such mis-zoned areas by the assessor that make it impossible to use it for homes.

In St. Paul, Minn., land and buildings are not assessed according to the zoning, they are assessed according to use—the zoning map being used as a guide. There is some unwarranted commercial strip zoning running through residential districts, but if residences predominate the land is assessed as residential property. This is the only rational method of assessing city property.

The only cure for poor administration, of course, is better administration. Zoning boards made up of a personnel with no planning experience are serious defects. Zoning is a part of the City Plan and can best be administered and interpreted by planning-minded people. In St. Paul, the Board of Zoning created by the Zoning Ordinance is the Zoning Committee (five members) of the City Planning Board; thus planning and zoning are tied together.

Some ordinances simply require the elimination of "strip" business zoning and the substitution of "district" business zoning to bring them into a fairly consistent situation as to proportioning the uses of property. This strip zoning, so common in existing ordinances, was brought about by members of "pressure groups," who either did not know what good zoning was or who wished to discredit zoning.

Some ordinances may be brought into line by provisions for terminating non-conforming uses after a period of years. This is an urgent need. In fact, the cure of the non-conforming evil and the reduction of footage of commercial zoning in residential developments might again carry us along over another 20-yr period.

In St. Paul, the Board of Zoning will recommend to the Council the remodeling of a non-conforming store building in a residence district, which is in distress financially, into a "triplex" or "four-plex" if it will move back to the building line of the homes and present a suitable architectural treatment.

In a city like Chicago, Ill., where changes come so rapidly, the proposed re-zoning based on a land-use survey and a proper proportioning of uses as determined by years of experience, is probably desirable, but a land-use survey must also be properly interpreted and this cannot be done by speculators.

Mr. Young has made a notable contribution to this subject of zoning and his analysis of the situation in Chicago, as well as the information gleaned through his questionnaire of December, 1936 (and which he has made available in his paper), will be of great value to all cities that are studying this matter of zoning.

The logical basis for re-zoning is the existing use of property and the anticipated normal absorption of additional property, not necessarily vacant property. This requires a sound knowledge of the trend of a city's growth. The causes of blight which were operating before zoning ordinances were devised and which were not cured by zoning must be rediscovered. Inadequate street, sewer, and water facilities, and poor maintenance were among the causes of blight. These defects must not be confused with alleged poor zoning. Suitable buffering must be considered. There must be a natural stepping up of uses between the district of no restrictions (industrial) and the district of highest restrictions (residence). The use of parkways for buffering industrial districts is probably one of the best.

Hugh E. Young,[11] M. Am. Soc. C. E. (by letter).—Data from seventy-five American cities with populations of 50 000 and more were contained in the paper. This material indicates that many people interested in the municipal problems of these cities are well aware of the need for re-zoning most of the communities. Several basic reasons were given why such a need exists. One was the failure of those who prepared the zoning ordinances to realize that zoning is fundamentally an element of city planning because it is the process which co-ordinates the social, economic, and physical structure of urban areas.

As Mr. Downs states, many zoning plans are merely maps of existing uses, and show no evidence of any attempt to co-ordinate zoning with city planning, or even to proportion the urban area between the various uses in an orderly or scientific manner. Failure to plan properly first, or to co-ordinate zoning with existing city plans, is perhaps one of the basic reasons why re-zoning is now necessary in so many large American cities, although it is not so vital in smaller communities.

Another reason was the illogical and inequitable allocation of land as between various uses, combined with improper distribution of the areas devoted to specific uses. As stated by Mr. Bartholomew, improper proportioning of land among the various uses is indicated by the fact that many cities have zoned an extensive area of land for one use or another that is unjustifiable in the light of well-known factors as to the area that is actually required in terms of acres per one hundred persons, and as to the annual rate of land absorption. Improper distribution (Mr. Bartholomew further states) is shown by the failure of many zoning ordinances to provide residential areas in appropriate relationship to places of work and business.

Mr. Kohn emphasizes a further reason outlined in the paper; namely, that much zoning is based on an incorrect assumption as to what constitutes the highest and best use of a given area, which, in turn, results in unduly high land values, or price as Mr. Kohn calls it. He points out that many zoning ordinances provide for a use in a given area that is speculative rather than factual, and for a density of use so intense as to make the assumed value of the land, if devoted to that use, far greater than its real value, if devoted to its genuine best use. Mr. Kohn believes that not only zoning, but city planning and housing progress as well, are largely dependent upon making the public and the real estate owner realize the difference between price based upon unrealistic zoning and real value based upon appropriate land use.

The paper advocated a real property inventory and land-use survey as the initial step toward re-zoning. The purpose of the survey is to show existing patterns of land use, areas of blight, population changes and other factors which together make re-zoning necessary. Mr. Bartholomew stresses the point that cities will be re-zoned to best advantage only when the re-zoning plan is based upon an accurate knowledge of the total land in urban use for various purposes and upon the annual rate of land absorption. He states that well-defined laws of land use, when thoroughly understood, will make possible far more scientific zoning practices than those of the past, and illustrates his point by giving the percentages of land used for various purposes (see Table 6): (1) As the average of sixteen self-contained cities; (2) in the St. Louis Metropolitan District; and (3) in the City of St. Louis alone. Table 6 indicates the importance of including all the area in urban land-use surveys, regardless of municipal boundaries, because the percentages for the City of St. Louis alone show certain marked variations from the normal averages.

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[11] Chf. Engr., Chicago Plan Comm., Chicago, Ill.

A considerable part of the paper was devoted to the administration of a zoning ordinance and means for maintaining the integrity of the ordinance, listing some of the bad effects of allowing improper variations therefrom. The writer showed that no zoning ordinance will be effective unless its administration is in strong hands, free from political domination; and suggested that the essential need was for a strong public body, such as a plan commission, to lend support to the Zoning Board of Appeals. Furthermore, he showed that a certificate of occupancy is most important for the proper functioning of the zoning ordinance, but that such certificate should not be issued by the same department that is charged with its enforcement.

Mr. Herrold asserts that zoning is a part of city planning and can best be interpreted and administered by planning-minded people. He adds that in St. Paul the Zoning Board created by ordinance is composed of the five members of the Zoning Committee of the City Planning Board of St. Paul.

Mr. Saltzman states that re-zoning, alone, is not sufficient to correct all the negative factors that contribute to the ineffectiveness of a zoning ordinance. He states further that, even with a good re-zoning plan, the ordinance must be administered honestly and capably if it is to be successful. Certainly, honest and capable administration of the ordinance is the keystone of the structure of zoning; without it any zoning or re-zoning activity is an idle gesture.

Mr. Saltzman states that the burden of administration and enforcement of the zoning ordinance will be lightened by the requirement of a certificate of occupancy for every new or altered building. This is because the issuance of this certificate is an assurance that the building complies with all the laws applicable thereto, including the zoning ordinance, and occupancy contrary to the terms of the certificate is in violation of the law. Certificates are a public record, so that illegal occupancies may be noted and corrected. Mr. Saltzman, however, is firmly of the opinion that the department charged with the enforcement of the ordinance should issue the certificate, because any other method would result in a shifting of responsibility, and bickering between the agency enforcing the law and the one issuing the certificate.

Mr. Downs emphasizes likewise the need of a strong administrative branch of government if good zoning is to succeed. Mr. Bartholomew advocates an even more advanced step than the writer's organization of a strong citizen's committee in order to preserve the integrity of a re-zoning ordinance. He suggests the establishment of permanent agencies in the form of neighborhood unit organizations in the interest of slum clearance, blighted district rehabilitation, and the protection of ideal residential areas. His point is that uncontrolled speculation in urban real estate has destroyed the stability and desirability of home ownership; hence it is essential to control real estate in a manner consistent with normal land use. Mr. Bartholomew, therefore, favors having the plan commission determine the broad principles of land policy, and having the citizen's committee as recommended; but in addition he recommends the establishment of well-organized neighborhood units to assist in adopting, enforcing, and maintaining appropriate zoning policies and a sound land program.

One defect in present zoning practice to which the writer called attention was the lack of adequate "buffer" areas between various uses. He stated that zoning practice is still in a crude stage with respect to the "tapering off" process—that is, the transition from a higher to a lower use, especially by means of park strips between residential sections and those used for other purposes.

Mr. Herrold agrees that suitable buffering must be considered, and that there must be a natural stepping up of uses from the less restricted districts to those which are more highly restricted. He gives it as his opinion that the use of parkways for buffering industrial districts is probably one of the best methods.

The paper demonstrated the detrimental effect of zoning variations and the problem of non-conforming use. Mr. Saltzman states that the full benefits of zoning or re-zoning cannot be secured unless the concomitant problem of non-conforming use is solved. He calls attention to the fact that any re-zoning will result in there being, in most districts, many buildings whose use does not conform to the regulations for the new district. Although obsolescence and natural decay will force the eventual demolition of some of these structures, this process will be hampered, the effectiveness of the ordinance will be impaired and the new restrictions will be valueless if the authorities allow indiscriminate variances that permit of the modernization or enlargement of such structures.

It is naturally gratifying to find so complete an agreement on the part of all those who participated in the discussion with respect to the principles set forth in the paper. One point made, however, was not discussed by any one. It is a matter of such basic importance that it should, once again, be called to the attention of the reader. That is the need, when re-zoning, to provide for reciprocal protection for all the various use districts.

Every one is aware of the need for protecting residential areas against the incursion of commerce and industry, but it is equally important, for example, to protect manufacturing areas against incompatible commercial uses, and particularly against the restrictions which often result from the presence of near-by residential use—restrictions which work a hardship upon manufacturing operations of the so-called "nuisance" type but which are, nevertheless, an important element in the city's business and financial life.

Much of the paper was devoted to the subject of blighted areas and the possibility of their redemption. Re-zoning was suggested as the first step, apportioning city areas for various uses in accordance with definite factors of land-use absorption as developed by a land-use survey, because improper zoning is a blight in itself in that it specifies a use and volume that can never be attained.

The second suggested step was the preparation of an official plan by an official planning body, and the establishment of a strong citizen's group to advise the plan commission with respect to zoning, and to throw the weight of all local civic organizations back of the re-zoning plan.

The writer took the position that both the paternalistic (public) and the non-paternalistic (private) methods of providing adequate housing facilities were necessary elements in the rehabilitation of blighted districts. The paternalistic method is that of providing low-cost housing for the lowest income groups by governmental agencies and governmental financing. The non-paternalistic method, enabling groups of property owners to redeem a neighborhood as a private undertaking, would seem to have taken hold on the public mind in recent months. State legislation is pending in Illinois, and has been suggested elsewhere throughout the nation, looking toward the establishment of "Public Service Building Corporations" with specified powers of land condemnation under the right of eminent domain. This latter feature is designed to overcome what has been heretofore the greatest obstacle to the rehabilitation of blighted areas as a private enterprise.

The public service building corporation plan, originated in Chicago as a means of interesting private capital in slum clearance housing projects, is now (1939) being considered in New York City. The Merchants Association of that city has also suggested that additional legislation be enacted which will permit the formation of local district improvement corporations. In this plan the equity would be represented principally by shares of stock given by the corporation to present owners and mortgagees in exchange for their claims on the property and its present and potential earning power.

For both the public service building corporation and the local district improvement corporation, powers of eminent domain would be granted upon proof that they were necessary to bring minority groups or their property within the scope of an approved large-scale rehabilitation plan.

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ECONOMIC ASPECTS OF ENERGY GENERATION
A SYMPOSIUM

PAGE
Progress in the Generation of Energy by Heat Engines.
By GEO. A. ORROK, M. AM. SOC. C. E. 943

Hydro-Generation of Energy.
By FRANK H. ROGERS, ESQ. 952

Improvements in the Utilization of Energy.
By JOEL D. JUSTIN, M. AM. SOC. C. E. 971

Cost of Generation of Electric Energy.
By PHILIP SPORN, M. AM. SOC. C. E. 984

Social Implications of Technological Trends in the Power Industry.
By RALPH E. FREEMAN, ESQ. 995

Electric Power in Economic Perspective.
By B. A. THRESHER, ESQ. 1001

WITH DISCUSSION BY W. S. FINLAY, JR., R. M. RIEGEL, RALPH BENNETT, J. M. MOUSSON, BEN C. SPRAGUE, V. M. MARQUIS, R. L. SACKETT, D. J. MCCORMACK, I. E. MOULTROP, M. M. SAMUELS, E. W. KRAMER, PAUL E. GISIGER, F. A. DALE, RAY S. QUICK, F. KNAPP, D. S. JACOBUS, JOEL D. JUSTIN, AND PHILIP SPORN.

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PROGRESS IN THE GENERATION OF ENERGY BY HEAT ENGINES

By Geo. A. Orrok,[1] M. Am. Soc. C. E.

Synopsis

The writer gives estimates of the total of power-producing machinery used in the United States as of 1936 and discusses hours of use and kilowatt-hours of work done annually, showing the contrast between 1880 and 1936. The size and type of units with the average economies of various types of machinery are given. The influences of high pressure and vacuum and high temperature are discussed with prediction of economies now in sight. Thermal economy and commercial economy are discussed as well as the cost of installations over the period. Tables are given for the cost of steam power.

The internal combustion engine, Otto, Diesel, and solid injection cycles, are considered and the paper ends with a review of possibilities of other cycles and other mediums of heat supply. The conclusion is that steam will be the most used medium of power generation for the near future.

In 1896 the late Robert Henry Thurston, M. Am. Soc. C. E., surveyed the field of power production and collected data on the increasing use of steam power during the Nineteenth Century.[2] It is interesting to note that his figures for 1896 show only 17 000 000 hp of power-producing machinery in the United States, divided as follows: Locomotive, 11 000 000 hp; marine, 2 000 000 hp; and stationary, 4 000 000 hp. He plotted his figures in a curve, and commented on the trends of the possible tremendous increase in power output which has since taken place.

In 1909 Henry Adams, some time Professor of History at Harvard College, wrote his essay on the Phase Rule which was published posthumously in 1919 as "The Degradation of Democratic Dogma." In this essay Professor Adams speculated as to the great increase in the use of power in the world and tried to apply the second law of thermodynamics to the power situation. His curves show the same characteristics as those of Thurston, but with the well-known Adams pessimism, he carried the curve to its logical conclusion, forgetting the law of diminishing returns with which he was without doubt much more familiar than with the second law of thermodynamics.

Among the authorities who have since contributed to power statistics both in the United States and in other countries may be mentioned, D. B. Rushmore,[3] M. Am. Soc. C. E., F. R. Low,[4] and Professor Thomas T. Reed.[5]

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[1] Cons. Engr. (Orrok, Myers & Shoudy), New York, N. Y.

[2] "The History of the Steam Engine," by Robert Henry Thurston, Appleton, 1899.

[3] "Hydro-Electric Power Stations," by D. B. Rushmore, John T. Wiley & Son, 1917.

[4] Presidential Address, A. S. M. E., Vol. 46, 1924, p. 1535.

[5] *Mechanical Engineering*, May, 1926.

In 1930, the writer brought these figures up to date[6] (for the United States only), and more recently the increases from that year to June, 1936, were shown by Dean A. A. Potter in his paper for the National Resources Committee.[7]

According to Dean Potter, the total horse-power of prime movers in the United States in June, 1936, was divided as follows:

	Horse-Power
Electric central stations	44 670 000
Industrial power plants	20 133 000
Electric railway plants	2 500 000
Isolated non-industrial plants	1 500 000
Mines and quarries	2 750 000
Agricultural prime movers	72 763 000
Automotive	965 000 000
Airplanes	3 500 000
Locomotives	88 000 000
Marine	30 000 000
Total	1 230 816 000

It is to be noted that the transportation services accounted for more than three-quarters of the power-generating machinery shown in Thurston's figures in 1896, while Potter in 1936 credits more than 91% to transportation—thus justifying Kipling's statement that "transportation is civilization." Of the 91% credited to transportation, about 8% use steam as the heat medium, and 92% use oil or gas in internal combustion engines. Of the stationary power plants (except those credited to transportation), Dean Potter schedules 56 684 000 hp in steam plants, 16 075 000 hp in water-power plants, and about 2 000 000 hp in oil and gas engine plants. Roughly, then, the stationary plant installation can be classified as 20% water power and 80% steam power.

When it comes to the actual horse-power-hours, kilowatt-hours, or work done, the figures are a little uncertain. However, it is probable that the 1935 output of the steam and water-power plants alone was not far from the equivalent of 142 000 000 000 kw-hr, of which more than 100 000 000 000 kw-hr was generated by steam. The average per capita use of power, central station plus industrial, is thus on the order of 1 115 kw-hr per yr. The equivalent kilowatt-hours of transportation power, locomotive plus automotive, is estimated conservatively at a somewhat larger amount than the stationary power, say, 150 000 000 000 kw-hr—hence, the total per capita use may be taken as the equivalent of 2 300 kw-hr per yr.

Whether the curve of prime-mover capacity is plotted from 1 800 (Thurston's curve) or from 1 500 (Adams' curve), the ascending segment is steep, and a 10-yr moving average smoothes out sufficiently the cyclic irregularities. (The curve of equivalent kilowatt-hours use follows practically the same law, the growth in use factor being very slow.) There is as yet little sign of the saturation that is evident in population curves, but this may appear in the future, particularly as the trend is toward a stationary population in the next twenty years.

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[6] Rept. on the "Status and Progress in the Art of Power Engineering," *Proceedings*, Am. Soc. C. E., March, 1930, Papers and Discussions, p. 437.

[7] Rept. of National Resources Committee, 1937.

The remainder of this paper omits from consideration the power used for transportation and that part of the stationary power that is generated by water, and is concerned solely with the stationary power-plant installation of 58 700 000 hp of heat engines. In most of this installation (56 680 000 hp), steam is the heat transfer medium.

The average annual use of this installation is about 1 730 hr, therefore, the total annual output of the steam plants is roughly 98 000 000 000 kw-hr. The central station output of 55 000 000 000 kw-hr is generated at an expenditure of about 1.45 lb of coal per kw-hr, while the industrial output of 43 000 000 000 kw-hr requires not far from 3 lb per kw-hr. The average use for both classes of plants is 2.13 lb per kw-hr, or the equivalent of 104 000 000 tons of coal per yr, roughly, 20% to 25% of the total annual coal output of the country.

In 1882, the late Thomas A. Edison considered 10 lb of coal per kw-hr a very good record for the first central station. F. R. Low, in 1891, reported to the National Electric Light Association that only one central station reported an economy as low as 4 lb of coal per kw-hr. His actual words were "250 watts per lb of coal." In 1936 the central station, at Port Washington, Wis., was consistently operated at an economy of 0.86 lb of 13 000 Btu coal per kw-hr—an over-all thermal efficiency of 30.6 per cent. In 1880, the late William Cawthorne Unwin, Hon. M. Am. Soc. C. E., the English authority on steam generation and use, considered 12 lb of Welsh coal per ihp-hr an average record in industrial establishments. This is equivalent of 16 lb per kw-hr. In 1936, industry was averaging about 3 lb per kw-hr of poorer coal and A. G. Christie[8] quotes a figure of 0.93 lb for one large modern industrial plant.

In 1880, in England, Unwin tested a Lancashire boiler and found the efficiency to be a little more than 80 per cent. Two years later, in the United States, J. C. Hoadley tested a return tubular boiler that gave 83% to 85% efficiency with a small air heater and about 79% without the heater. Power engineers have re-introduced the feed-water heater (which Edison used in the first of all central stations), the superheater, and the air heater, or economizer, making them a part of the modern boiler. They have added water-cooled furnaces, invented more than 100 yr ago, but not coming into general use until 1924. They have taken the stoker, invented about 1861, and improved it until it is nearly automatic, and they have perfected the burning of pulverized fuel. Even such fuels as lignite, or brown coal, containing 65% of water, are fired efficiently on a stepped grate. Oil wastes and acid sludges are also burned with high efficiency. Boiler tests currently show efficiencies of between 82% and 92%, and whenever it should become commercially feasible, these values can be exceeded. Year-round boiler-house efficiencies of from 78% to 88% are regularly reported by good operators. The increase in boiler-house efficiencies is necessarily small, because the law of heat transfer is fundamental. Progress has been in two directions: (*a*) Learning how to burn all the fuel at the best condition; and (*b*) adding heat traps to catch all heat not absorbed by the boiler proper.

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[8] *Mechanical Engineering*, September, 1936, p. 539.

Prior to 1900, the heat engine was the steam engine of Watt and his successors. Unwin's figure of 12 lb of coal per ihp per hr (1880) had been reduced in the following 20 yr to less than 1 lb by increase of pressures and compounding, by going back to condensing practice, and by better workmanship. A compound engine with 1 : 7 ratio cutting off at 14% stroke in the high-pressure cylinder would give about 50 expansions, and water rates varying from 11 to 9 lb per ihp-hr were being guaranteed by many builders. Poppet-valves were used if superheated steam was furnished, but the ordinary Corliss valves could take care of 30° to 50° F of superheat. The main difficulty was the demand for large units; low-pressure cylinders as large as 110 in. were made. Sometimes, two low-pressure cylinders of 96 in. each were used, as in the engines of the Interborough Rapid Transit Company, at the 59th Street Power Station, New York, N. Y., where 8 000-kw units were installed. Rolling-mill engines with two 110-in. low-pressure cylinders developed as high as 25 000 hp. All these units were of comparatively low speed, to ensure continuity of operation and low upkeep.

Following 1900, came the steam turbine—in small units at first, but rapidly increasing in size to meet the growing demand for large blocks of power. The maximum size of individual units increased from 5 000 kw in 1903 to 20 000 kw in 1912; 60 000 kw in 1925; 110 000 kw in 1928; and 165 000 kw in 1929. A three-shaft unit of 208 000 kw was installed in the same year. It is now possible to build a larger single generator unit than 165 000 kw, but the demand has not yet developed. The early turbines did not have as good a water rate as the better engines, because high vacuums were not used; the 26 in. used by engines was thought good enough. The adoption of a 28-in. vacuum raised the number of expansions to 100, and the later increase to 29 in. doubled this figure, greatly increasing the efficiency and the unit capacity. To-day, designers strive for an absolute pressure of $\frac{1}{2}$ in. to $\frac{5}{8}$ in. of mercury, and water rates of approximately 9 lb per kw-hr, or 6.75 lb per hp-hr are common, even at ordinary pressures and with temperatures below 750° F.

Meanwhile, the Power Industry has learned the value of high pressure and high superheat. Pressure has been increased to 600 lb per sq in., then to from 1 200 to 1 400 lb, then to 1 800 lb, and, in Europe, installations using steam at the critical pressure (3 200 lb) are in operation. In temperature, 825° F is now standard, 932° F (500° C) is contemplated, and at least one unit has been in operation for some time at 1 000° F. Potter lists 16 stations carrying pressures between 710 and 1 840 lb; 25 stations between 500 and 700 lb; and 174 stations between 300 and 500 lb.

Table 1 shows the increase in thermal efficiency since Watt's time (1770). The first four entries are from a paper on "The Commercial Economy of High Pressure and High Superheat."[9] In this paper and the discussion that followed, the regenerative cycle, old in engine practice, was stressed, and since that time bleeding has become general practice. The predicted economies have been obtained. As many as six stages of feed-water heating have been used, and the feed-water is now heated nearly to the saturation temperature. Steaming economizers have been used, but the practice does not seem to be increasing. The most modern plants still fall short of introducing all the heat to the medium at the maximum temperature, although in the mercury vapor installations a larger proportion of the heat is introduced at approximately that point.

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[9] *Transactions*, A. S. M. E., Vol. 44, p. 1119.

TABLE 1.—INCREASE IN THERMAL EFFICIENCY SINCE 1770

Year	Prime mover	Pressure, absolute	Temperature, in degrees Fahrenheit	Carnot cycle, actual percentage	British thermal units per kilowatt-hour	Thermal efficiency, percentage
1770	Newcomen engine	15	212	2.7	416 000	0.82
1810	Watt engine	30	300	9.17	61 000	5.60
1900	Nordberg engine	215	400	20.8	19 200	17.75
1922	Turbine	265	650	35.0	18 000	19.0
1935	Turbine*	1 245	825	58.0	11 166	30.6
1935	Turbine†	1 245	825	58.0	10 850	31.5
1935	Mercury vapor turbine	140	950	61.0	10 500	32.5
	Proposed mercury vapor turbine	240	1 025	63.5	9 500	36.0

* Port Washington, Wis., 8-month record, *Mechanical Engineering*, November, 1936, p. 697.

† Port Washington, Wis., record for single month, *Mechanical Engineering*, November, 1936, p. 697.

The cost of central stations has varied but little since Mr. Edison built the Pearl Street Station, in New York City. The writer's records, in more or less detail, cover more than 150 plants built between 1882 and 1936. The costs include land, buildings, foundations, water tunnels, coal and ash handling, boiler and turbine-room machinery and auxiliaries, the switching arrangements, and outgoing feeders to the building line. The average cost of these plants is substantially $100 per kw of capacity. Potter quotes figures collected by the Federal Power Commission, for 80 plants built since 1920, which average $108 per kw of capacity, while L. L. W. Morrow[10] lists 16 plants built since 1927 at an average cost of $114 per kw installed.

The average cost for the 250 plants in these three lists (with possibly some duplicates) is about $103 per kw of installed capacity. The installations are of many types—large and small, high- and low-pressure, light, power, and railway plants, with a small proportion of industrial plants. The lowest cost is about $60 per kw, and the highest, approximately $225 per kw. Potter lists 5 plants that cost more than $150 and 4 less than $70. Morrow's variation is from $82.50 to $145.00 per kw.

As a general rule, large plants cost a little less per kilowatt than small plants. High pressure plays little part; of the 1 400-lb plants reported by Morrow some were in the lower range of prices and some in the higher range. In favored locations, high-pressure stations have been built for as little as $82.50 per kw, and it is probable that this figure might be considerably smaller with later designs.

In the earlier years building costs were low, while machinery costs were high. Switchboards were simple and cheap. In later years, the electric costs have risen from 5% to as much as 25% of the total cost. (It should be noted that the cost of switchboard safety devices and other electrical equipment mounts rapidly with the size of the connected system.) Building costs, including cost of foundations, have increased from 10% to 40% of the total. Land has varied from 0.5% to 2.5 per cent. On the other hand, costs of machinery—boilers, turbines, piping, and auxiliaries—have decreased from 80% to 35% of the total.[11]

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[10] *Electrical World*, November 23, 1935.

There is much diversity of opinion as to the cost as well as the desirability of "out-door" stations, and this is likely to continue until more actual experience has been obtained with the design, construction, and operation of this type of plant. Meanwhile, the convenience of operation and maintenance of the enclosed plant under severe weather conditions is likely to prevent much experimentation with the new design.

Central stations using steam as the heat medium are now in operation giving yearly economies, coal burned to kilowatt-hours sent out, of as low as 11 000 Btu per kw-hr—a thermal efficiency of 31 per cent. By taking full advantage of high pressure and temperature, carrying regeneration to the limit, and using at least one reheating stage, the thermal efficiency might be raised to about 37%, or the economy to about 9 200 Btu per kw-hr. With the mercury-vapor cycle, about the same economy might be obtained ultimately. Plants with such high efficiencies would be complicated in design and difficult of operation, as well as costly. They would require much better materials than are now obtainable, and are not even contemplated at present.

A steam plant running at 10 000 Btu per kw-hr, however, may be anticipated in the near future. The new heat-resistant materials, increased knowledge of the more common metals and alloys, the recent developments in boiler and turbine design, and the demand for more efficient plant, will bring it about.

The same economy is also in sight with the mercury vapor plants. A single month's record of 10 400 Btu per kw-hr was reported from the mercury plant, at Hartford, Conn., in 1936, and designs have already been made for plants with economies even lower than 10 000 Btu per kw-hr.

But are these very efficient plants justified? What is the relation between thermal efficiency and commercial efficiency? Thermal efficiency is easily measured. Fuel bills are plain, and present-day sampling and calorimetry are trustworthy. Station watt-meters are standard instruments and quite accurate. The two ratios, coal per kilowatt-hour and British thermal units per kilowatt-hour, are well known by all operators.

Commercial efficiency is a different story, for it involves many factors subject to conflicting interpretations. The cost of the installation may be book cost, replacement cost, or depreciated cost, or even cost minus some part charged off to steam heating, general offices, or rentable property. When this cost has been delimited, the percentage of fixed charges must be determined. This usually includes interest, reserve for renewals, taxes, and insurance, lumped together as 12.5% to 15% of the prime cost; and, then, enters the use factor of the installation. The average cost of stations is, say, $100 per kw. Fixed charges at 12.5% are then $12.50 per annum per kilowatt installed. If the use of the installation is 3 000 hr per annum, the fixed charges are 0.417 cts per kw-hr; if the use is 8 000 hr, the fixed charge is 0.156 cts.

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[11] "Rôle of the Civil Engineer in Power Development," by the late Ira W. McConnell, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1928, p. 155; and the discussions by Messrs. P. Junkersfeld, Geo. A. Orrok, Joel D. Justin, and F. W. Scheidenhelm, published in *Proceedings*, Am. Soc. C. E., March, 1928, p. 923; J. A. Sargent, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., August, 1928, p. 1917; and the closing discussion in *Proceedings*, Am. Soc. C. E., for April, 1929, p. 977.

Operating charges also vary with the use factor, but not nearly to the same extent as fixed charges. With coal at the prices which ordinarily maintain on the Eastern seaboard, operating charges per kilowatt-hour may be expected to be less than fixed charges for uses of 4 000 hr, or less, per yr. Under conditions like those in the vicinity of Pittsburgh, Pa., the point of intersection may be as high as 7 000 hr. The total cost of power at the station wall is, of course, the sum of the fixed and operating charges. Hence, its magnitude always depends on the hours of use. In this connection see Table 2. It will be noticed that even with coal at $5 per ton, the fixed charges run from 36% of the total cost at 8 760 hr of use to 73% at 1 750 hr. Commercial efficiency increases with the use factor, and each plant presents a unique problem in which the prime cost of installation must be balanced against hours of use and fuel cost to secure the most favorable combination.

TABLE 2.—COST OF STEAM POWER*

Use factor, percentage	100	80	60	40	30	20	10
Hours of use	8 760	7 008	5 256	3 504	2 628	1 752	876
Fuel per kilowatt-hour, in pounds	0.773	0.777	0.786	0.804	0.821	0.857	0.964
Charges, in Cents per Kilowatt-Hour:							
Fixed	0.143	0.178	0.238	0.357	0.475	0.714	1.429
Other, except fuel	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Total, all charges except fuel	0.223	0.258	0.318	0.437	0.555	0.794	1.509
Cost of coal, in cents per kilowatt-hour:							
Coal at $1.00 per long ton	0.345	0.0347	0.035	0.0358	0.0366	0.0382	0.043
Coal at $3.00 per long ton	0.1035	0.1041	0.105	0.1074	0.1098	0.1146	0.129
Coal at $5.00 per long ton	0.1725	0.1735	0.175	0.1790	0.1830	0.1910	0.215
Total cost, in cents per kilowatt-hour:							
Coal at $1.00 per long ton	0.2575	0.2927	0.353	0.4728	0.5916	0.8322	1.552
Coal at $3.00 per long ton	0.3265	0.3621	0.423	0.5444	0.6648	0.9086	1.638
Coal at $5.00 per long ton	0.3955	0.4315	0.493	0.616	0.738	0.985	1.724

* Conditions assumed: Cost per kilowatt of installation, complete, $100; no reserve capacity; fixed charges, 12.5%; plant economy, in British thermal units per kilowatt-hour, $10\,500 + \frac{30\,000}{\text{Use factor in percentage}}$; and heat value of coal, 14 000 Btu per lb.

The Power Industry has long known that the road to commercial economy lies in the extension of the hours of use of a given installation. Edison, Crompton, Wordingham, and other early authorities have made this fact plain to every maker of power. Balanced loads, steady running, and diversified services are sought by all, for power cannot be stored except at great expense. Unfortunately, however, the tastes and habits of users of power lead to a wide variation of load in industries and power supplies. In spite of this, however, load factors have grown. Edison, at Pearl Street, was well pleased with a 12% load factor. To-day, system load factors may go as high as 40%, while certain industries maintain a load factor of 70% or more; and the trend is toward a still larger use. With better load factors, the use of higher pressures and temperatures will increase, bringing higher efficiencies and a more general distribution of the benefits attained.

Improvement in the design and efficiency of the internal combustion engine has been steady and continuous, but because of the nature of the problem the advance has not been nearly so spectacular. Ever since gasoline has been available, Otto-cycle engines have been able to develop a brake-horse-power-hour on 0.6 lb of fuel; high-speed, high-compression aviation engines now give a brake-horse-power-hour on 0.4 lb of fuel.

The best Diesel engines of the World War years used 0.36 lb of oil per bhp per hr. Two-cycle Diesel engines did not do quite so well, and 0.4 lb of fuel was a good record for them in those days. Solid-injection engines of modern design do a little better than the earlier air-injection engines. The best record that has come to the writer's attention is 0.326 lb, attained by a two-cycle solid-injection Diesel. None of these figures includes the oil used for lubrication, a portion of which is burned in the cylinder and adds to the developed heat.

Solid-injection two-cycle engines are now the standard for all sizes, although four-cycle engines are still used in sizes up to about 2 000 hp, and air-injection designs are used where low-quality fuel is to be burned. The range between 0.32 lb and 0.4 lb of fuel is small, and the heat use ranges between 6 300 and 8 000 Btu per bhp per hr, or, putting the conversion factor at 700 watts to allow for generator efficiency, 9 000 to 11 400 Btu per kw-hr for the prime mover alone. Of the 60% of the heat lost in the exhaust and jacket, about three-fourths may be recovered; the main difficulty in the way of doing so is the cost and complexity of the apparatus required.

The range of thermal economies for the oil-engine plants is much the same as that for steam plants, but the lower limit is easier of attainment. At best, the internal combustion engine would appear to have an advantage of about 10% in thermal economy. On the other hand, the installation cost is usually much higher than for steam, the average cost per kilowatt being more than one-half again as large. No very large oil-engine stations have been built, the one at Vernon, Calif. (35 000 bhp, in five units) being the largest as of 1936. About 2 000 hp has been developed in the double-acting cylinder, and twelve cylinders on one shaft have been proposed. The large peak-load engines (Copenhagen), run at high piston speeds (more than 1 200 ft per min, so high that a long life is problematical). The best figures on internal combustion engine costs and operation are to be found in the reports[12] of the Oil Engine Power Cost Committee of the American Society of Mechanical Engineers.

The steam engine, as developed by Watt, his competitors and successors, made the Nineteenth Century the century of steam. Parsons, Delaval, Rateau, and Curtiss, and their successors, in the Twentieth Century have carried on their work. Otto, Clerk, and Diesel, following other lines, have brought about a tremendous development in transportation. Meanwhile, the binary-vapor proposals of the last hundred years have been put into material form by Emmet in the mercury-vapor turbine, which has already equalled the economy of the Diesel engine. All these developments lead one to speculate on what other forms of prime movers may be developed in the near future.

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[12] *Transactions*, A. S. M. E., Vol. 57, p. 69.

On the basis of Milliken's electron theory and Rutherford's smashing of the atom a number of modern scientists and engineers have predicted that the next step will be atomic power—although Milliken's final summary on this question[13] is that power from atomic degradation can never be generated in commercial amounts.

In at least two locations (Llarderello, Italy, and Sonoma County, California), steam power is now generated by the heat of volcanic rocks, and Charles A. Parsons has proposed a well 12 miles deep to utilize the internal heat of the earth for the same purpose. The engineering difficulties of the latter proposal would be enormous, as the deepest wells to-day are less than 2 miles in depth.

In 1936 Dr. Abbott, of the Smithsonian Institute, at Washington, D. C., exhibited his solar engine, which utilizes Ericson's mirror system and is three to four times as efficient as former engines of this type. However, his apparatus is not comparable in size to Shuman's plants at Tacony, Pa.,[14] and Cairo, Egypt. The Tacony Plant, under test, developed 32 hp when the sun shone.

None of these proposals meets the criterion of a commercial power plant which must generate power when wanted, as wanted, and at a cost a little less than that at which it can be produced to-day. Engineers, scientists, and inventors are looking for a new source of power, convenient, plentiful, and cheap; but until that source appears it is certain that a major part of the world's power will be generated in heat engines using steam as the heat medium.

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[13] *Science*, September 28, 1928.

[14] *Engineer* (Lond.), July 5, 1912.

HYDRO-GENERATION OF ENERGY

By Frank H. Rogers,[15] Esq.

Synopsis

The installed capacity of hydro-electric plants has been increasing almost continuously since 1900. Concurrently, the maximum size and horse-power of individual units have grown by leaps and bounds. The limit in efficiency was closely approached by 1920, and the most important event in the history of turbine design since that time is the development of the propeller-type turbine.

This paper reviews briefly the important design features of modern high-head, medium-head, and low-head turbines; discusses the importance of laboratory testing (with special reference to low-head installations); points to six important mechanical improvements in turbine design made since about 1925; and concludes with an examination of possible future developments, such as the pumped-storage plant and the use of the propeller-type turbine for higher heads.

Various aspects of the progress in the hydro-generation of energy in the United States since the beginning of the century are indicated in Fig. 1.

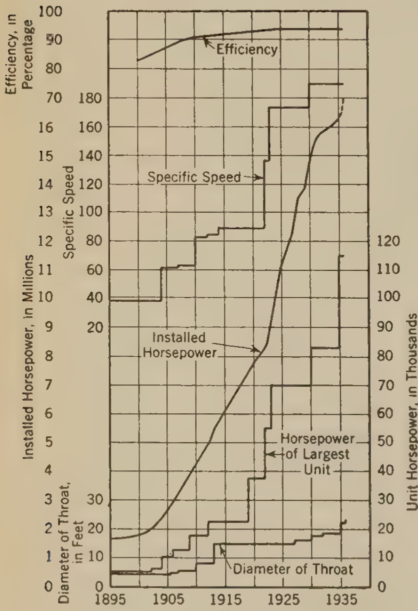

Fig. 1.—Progress Chart of the Hydraulic Turbine in the United States, 1895–1936

As for installed capacity, it will be noted that from 1900, when the first developments at Niagara Falls were undertaken, there was a rapid increase until 1922, at the rate of about 300 000 hp per yr. Then the slope of the curve rose to a still higher rate of about 800 000 hp per yr from 1922 to 1931. In the next four years the increase in installed capacity fell off considerably, during 1935 amounting to about 100 000 hp. In 1936, however, a great increase occurred, about 1 500 000 hp capacity being installed, of which probably 800 000 hp actually went into operation before the close of the year.

The demand for larger units has been met by the use of better materials and new construction methods. The highest point on the unit horse-power curve indicates the 115 000-hp Boulder

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[15] Chf. Engr., I. P. Morris Div., Baldwin-Southwark Corp., Philadelphia, Pa.

Dam turbines the most powerful units in the world. As for physical size (diameter of throat), the largest units yet constructed are those for the Bonneville Power Plant, in Oregon.

Since about 1910 a large majority of the turbines installed have been of the vertical-shaft type. This design has permitted the building of much larger units than would have been feasible with a horizontal setting. The development of the Kingsbury thrust-bearing about that time was the most important contribution to the success of the vertical unit.

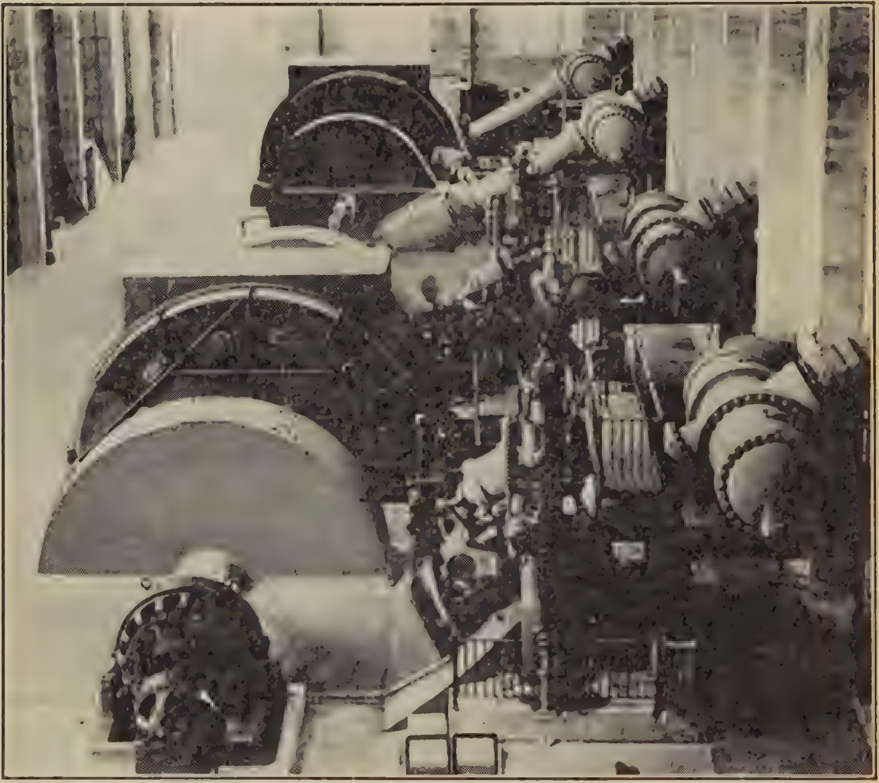

FIG. 2.—BIG CREEK NO. 2-A DEVELOPMENT, SOUTHERN CALIFORNIA EDISON COMPANY, WITH TWO 70 000-HP IMPULSE WHEELS

Little increase in efficiency has been secured since 1920, as the possible limit was closely approached at that time.

The increase in specific speed in 1922 is, however, noteworthy. In that year the large-sized propeller-type turbine, which permitted the economical development of large low-head plants, was introduced in the United States. This development is the most important in the recent history of turbine design, and this paper will devote considerable space to the problems involved in the design of this type of unit.

First, however, the progress made in the design of high and medium-head turbines will be reviewed briefly.

High-Head Turbines

The classification, "high-head turbines" is arbitrarily taken to include all units designed for heads greater than 300 ft. It comprises both turbines of the Francis type, and impulse wheels. For heads of 1 000 ft and higher the latter are used almost exclusively in spite of their lower efficiency and lower specific speed as in that head range cavitation limitations become critical, and the stresses involved in the casing and head cover would be excessive if the Francis type unit were used.

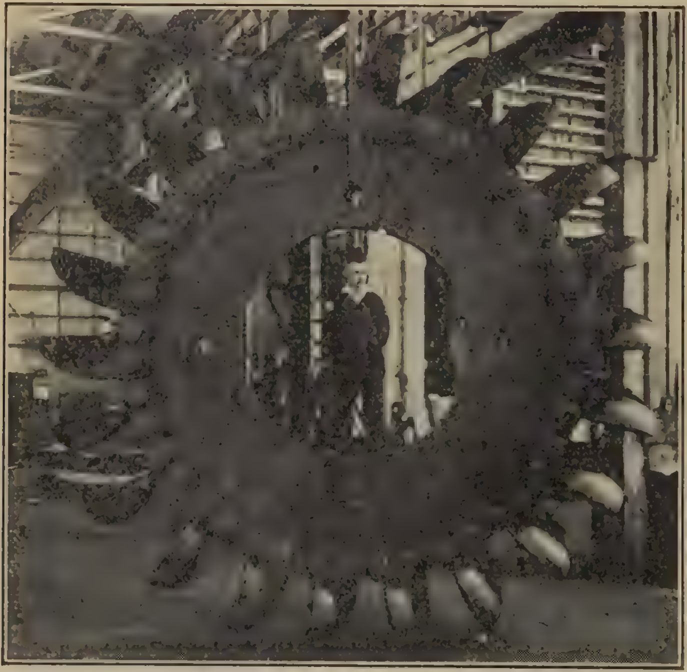

Fig. 3.—Runner, 162-Inch, for One of Big Creek No. 2-A Wheels

Impulse turbines are usually of the horizontal-shaft, single or double-runner, and single or double-nozzle type. They have been built in the United States in units as large as 70 000 hp, and for heads up to about 3 000 ft. Fig. 2 shows the two 70 000-hp impulse wheels of the Big Creek No. 2-A Development of the Southern California Edison Company. These units are of the two-runner, single-nozzle type, and operate under a head of 2 200 ft, at a speed of 250 rpm. One wheel of one of these units, with the buckets assembled, is shown in Fig. 3. The pitch diameter of the runner is 162 in. This unit is not only the largest in physical dimensions, but also the most powerful unit yet constructed for installation in this country.

Since about 1925 impulse-wheel efficiencies have been considerably improved by research in bucket and nozzle design. As an example, in Fig. 4 are shown one of the older elbow-type nozzles with relief nozzles below, and a straight-flow nozzle of improved design first developed in 1926. The water passageway in the latter, being perfectly straight, produces a jet with a minimum of turbulence and, consequently, of higher efficiency, and permits somewhat higher approach velocities to be used.

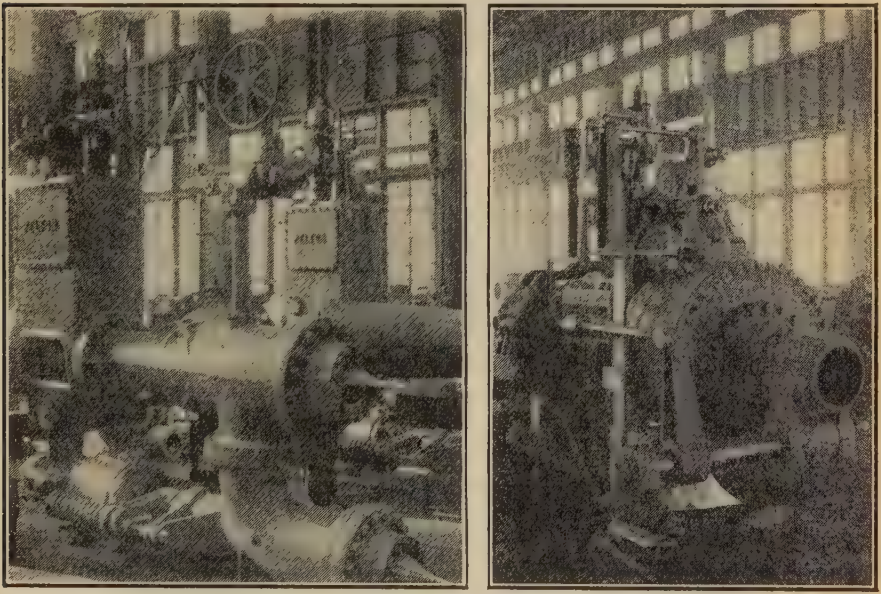

FIG. 4.—IMPULSE WHEEL NOZZLES; LEFT: ELBOW TYPE; RIGHT: IMPROVED STRAIGHT-FLOW TYPE

Reaction or Francis-type turbines have been used for heads up to about 850 ft. As examples of high-head Francis units the following may be mentioned:

30 000-hp turbine for the Big Creek Plant No. 8 of the Southern California Edison Company; head, 680 ft.
33 000-hp turbines for the Santeetlah Plant, of the Tallassee Power Company (now the Carolina Aluminum Company); head, 660 ft.
35 000-hp turbine for the Oak Grove Plant, of the Portland Electric Power Company; head, 850 ft.
49 000-hp turbine for the Waterville Development, of the Carolina Power and Light Company; head, 755 ft.
115 000-hp turbines for the Boulder Dam Plant, of the United States Bureau of Reclamation; maximum head, 590 ft.

The Boulder Dam units will operate under a wide range of head (420 to 590 ft), the average net head being 530 ft. At 420 ft each unit is guaranteed to deliver 90 000 hp, and at 480 ft and higher, 115 000 hp. Fig. 5 shows two of these turbines completely assembled in the shop. The casing has a 10-ft intake diameter, and was shop-tested to a pressure of 500 lb per sq in. One of the cast-steel runners, with an intake diameter of 14 ft 3 in., is shown in Fig. 6.

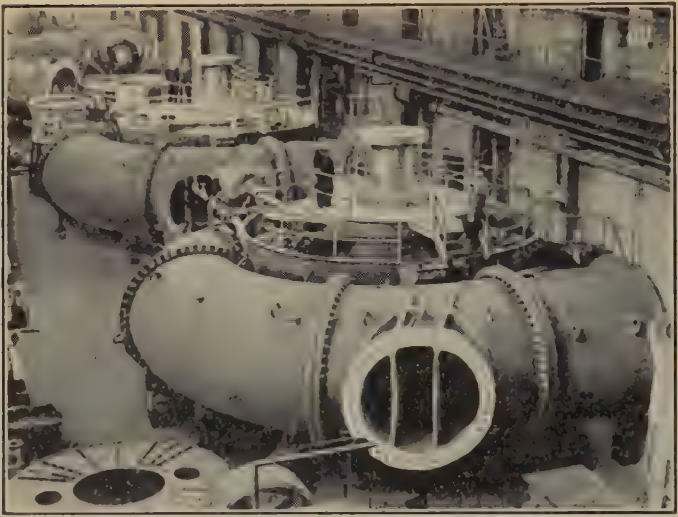

FIG. 5.—SHOP ASSEMBLY OF TWO 115 000-HP TURBINES FOR BOULDER DAM PLANT, OF THE UNITED STATES BUREAU OF RECLAMATION

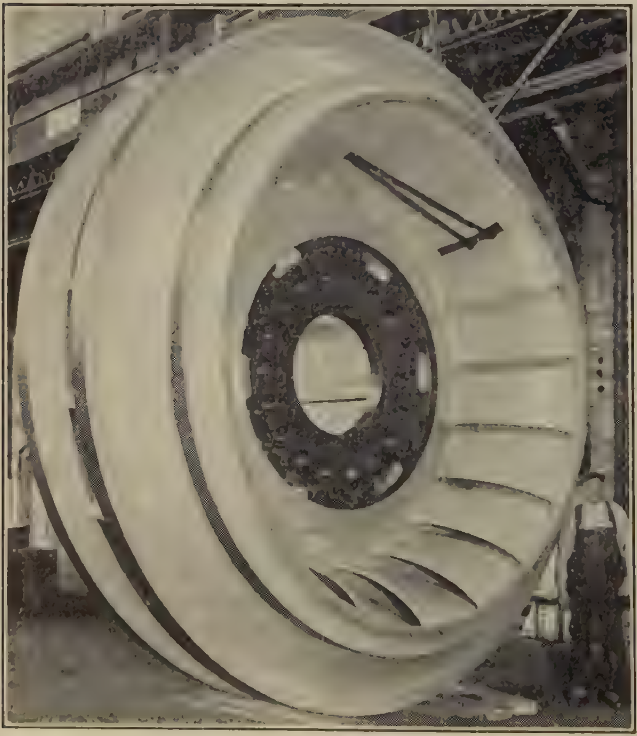

FIG. 6.—RUNNER, 171 INCHES IN DIAMETER, FOR 115 000-HP TURBINE AT BOULDER DAM

In high-head units especially, it is important to hold the leakage at the runner seals to a minimum. Any increase of the seal clearance due to wear materially affects the efficiency of the unit. A very close clearance can be used without danger of seizing due to contact, by adopting for the stationary seal an insert of soft metal such as Parson's white brass. Such a design (Fig. 7) is used in connection with two of the units of the Boulder Plant.

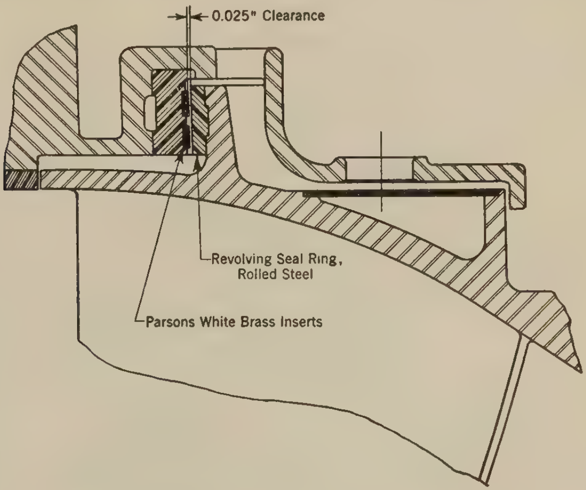
0.025" Clearance
Revolving Seal Ring, Rolled Steel
Parsons White Brass Inserts

FIG. 7.—SEAL RING DESIGN FOR TWO OF THE BOULDER DAM UNITS

MEDIUM-HEAD TURBINES

For heads ranging from about 100 ft to 300 ft Francis turbines of the vertical-shaft, single-runner type are used. For the lower heads within this range the casings are of plate steel, and for the higher heads they are of cast steel. Noteworthy installations of this type include:

Seven 54 000-hp turbines—head, 89 ft—installed in the Conowingo Development of the Susquehanna Power Company, on the Susquehanna River, Maryland.

Two 66 000-hp turbines—head, 165 ft—installed in the Norris Dam Plant, of the Tennessee Valley Authority, on the Clinch River, Tennessee.

Three 70 000-hp turbines—head, 213 ft—installed in Station No. 3, of the Niagara Falls Power Company, on the American side of the Falls.

Two 83 000-hp turbines—head, 310 ft—installed in the Diablo Plant, of the City of Seattle, on the Skagit River, Washington.

Fig. 8 is a sectional elevation of one of the Conowingo units, with its butterfly valve. The runner has a throat diameter of 16 ft 2 in., and is made in four sections secured together by a crown plate and band. The casing, of steel plate, has an intake diameter of 27 ft. The butterfly valve is probably the largest ever constructed. During periods of low flow, this installation is a peak-load plant and the units are shut down for a considerable part of the time. For this reason, leakage past the butterfly valves is an important item, and in order to reduce it to a minimum these valves are provided with a special rubber tube sealing device. When the valve is closed, water under pressure is admitted to the tube, expanding it through a slot in the valve housing so that the rubber is in contact with the outside periphery of the valve disk.

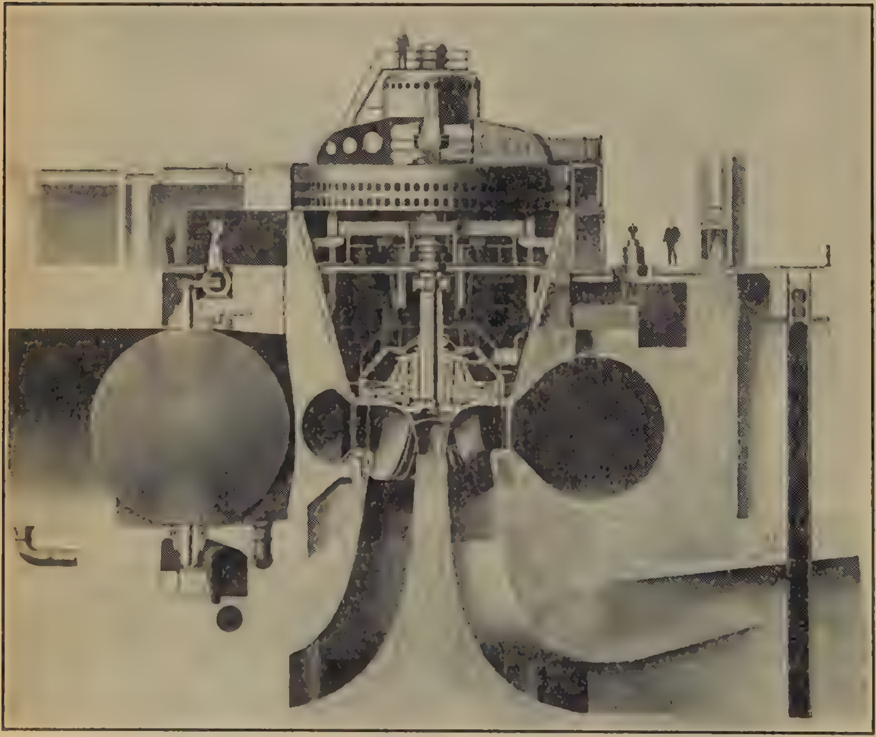

Fig. 8.—Sectional Elevation of 54 000-Hp Turbine, Conowingo Development, Susquehanna Power Company

Fig. 9 shows the shop assembly of the plate-steel casing and cast-steel stay-ring of the Norris Dam turbines. The intake diameter of the casing is 17 ft 8 in. Fig. 10 is an interior view of the stay-ring, casing, and penstock of the installed unit, and clearly indicates the field riveting at the casing joints and the connection to the stay-ring.

The two-bearing unit with umbrella-type generator is an excellent design adopted in a number of installations. Fig. 11 shows one of the two 31 000-hp units of this type in the Morony Development, of the Montana Power Company, at Great Falls, Mont. Particular attention is called to the excellent design of the water passages. The penstock is straight from the intake gates to the casing, thus reducing to a minimum the losses at the intake and through

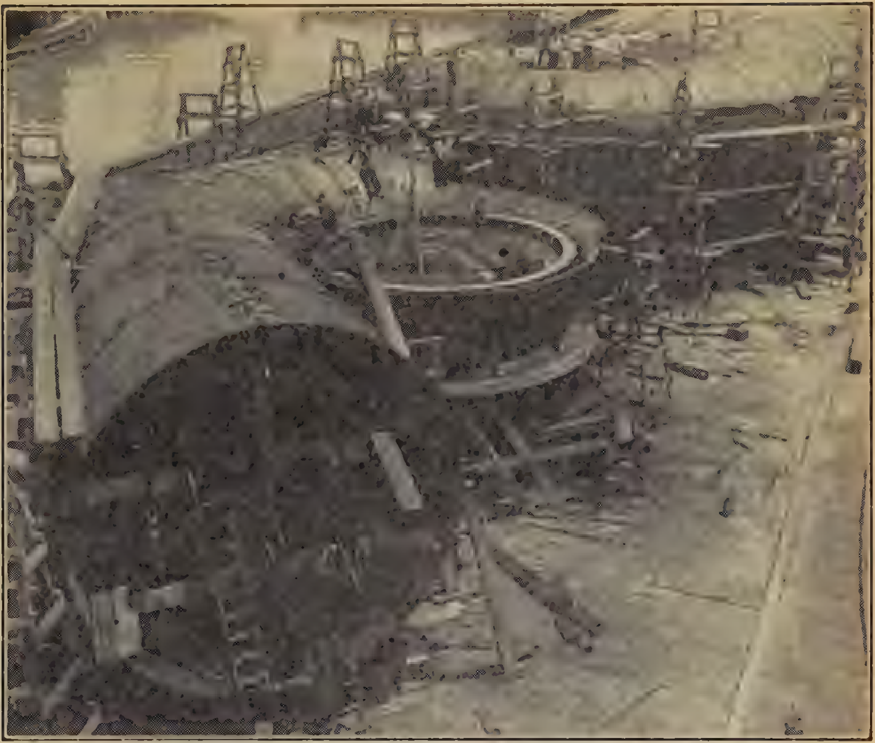

Fig. 9.—Shop Assembly of Spiral Casings and Stay-Rings of 60 000-Hp Turbines for Norris Dam Plant, Tennessee Valley Authority

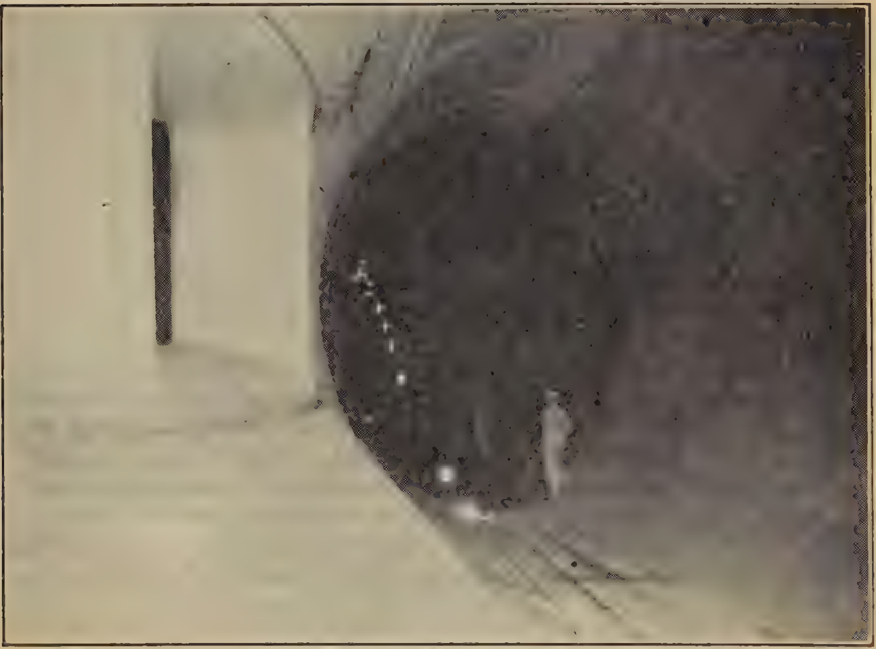

Fig. 10.—Interior View of Stay-Ring, Casing, and Penstock, Norris Dam Plant

the penstock. Turbine efficiency in the field test was 93%, and over-all efficiency from head-water to tail-water was 92.6 per cent.

LOW-HEAD TURBINES

The greatest advancement in hydro-generation of energy since 1920 has been due to the development of the high-speed propeller-type turbine. The increase in specific speed from about 90 for the Francis-type runner to 150 and higher for the propeller type has made possible the economical development of the large low-head installation, primarily by permitting the use of the lower-cost, higher-speed generator and reducing the cost of the power-house superstructure.

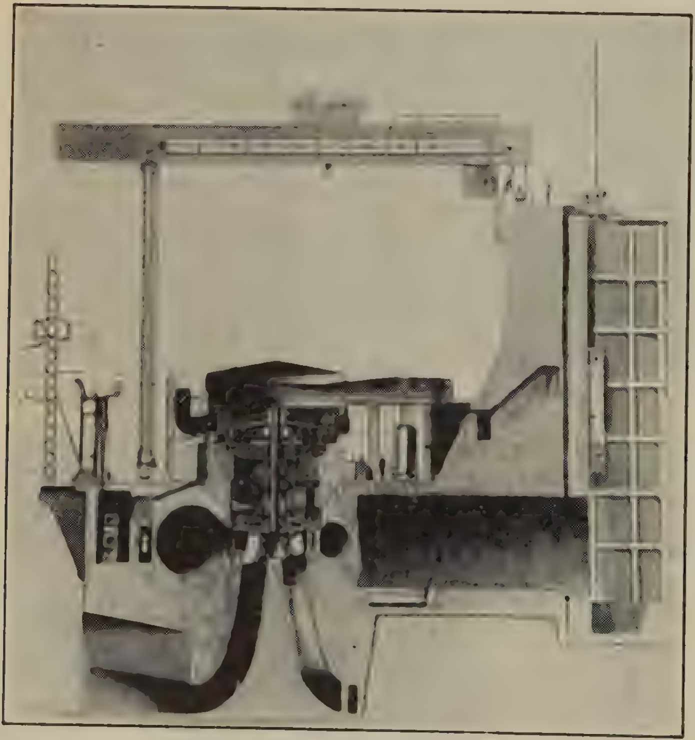

FIG. 11.—TWO-BEARING UNIT, WITH UMBRELLA-TYPE GENERATOR

The radical difference in size and shape of the propeller runner as compared to the Francis runner is shown in Fig. 12, illustrating four runners of different specific speeds. These runners are drawn to the same scale, and, under the same head, each runner would develop the same horse-power, but at a speed proportional to its specific speed. The first type would be suitable for heads up to 300 ft, the second, for heads up to 100 ft, and the last two, for heads up to 60 ft. It may be asked why the high-speed runner shown at the bottom cannot be used under the higher heads and thus obtain higher speeds for the generators. If this were attempted the velocity through the runner would be very high, and the absolute pressure would fall so low that vapor-filled cavities would form, the water would separate from the blades, and the resulting cavitation would not only reduce the power and efficiency, but cause pitting of the runner.

It is a general characteristic of turbine-runner design that, as the specific speed of the runner increases, the horse-power efficiency curve becomes steeper and the part-load efficiencies decrease. This characteristic is shown for the four runners of Fig. 12 in Fig. 13. It will be noted that for high-head installations where low specific speed runners are chosen, flat horse-power efficiency curves are obtained, whereas for low-head plants, where high specific speed runners are used, steep horse-power efficiency curves result. If it is necessary to operate the fixed-blade high-speed runner at part loads, a rapid falling off in efficiency will occur.

This inherent weakness in the characteristics of the fixed-blade propeller turbine was overcome by the development of the adjustable-blade runner known as the Kaplan type (after Victor Kaplan, of Austria, who first suggested it). In this type of unit the runner blades and guide-vanes are automatically and simultaneously adjusted in accordance with the load demand to provide

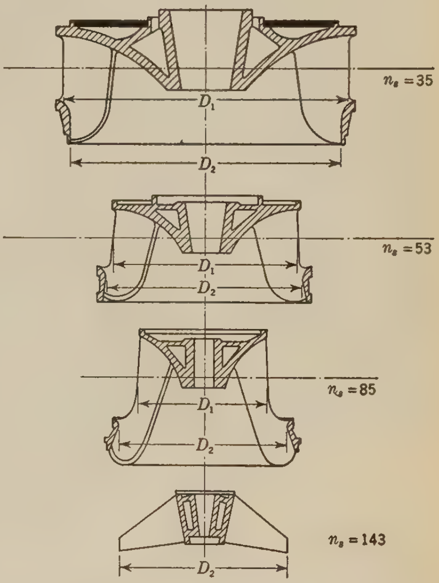

FIG. 12.—COMPARISON OF RUNNERS OF EQUAL POWER, BUT OF DIFFERENT SPECIFIC SPEEDS

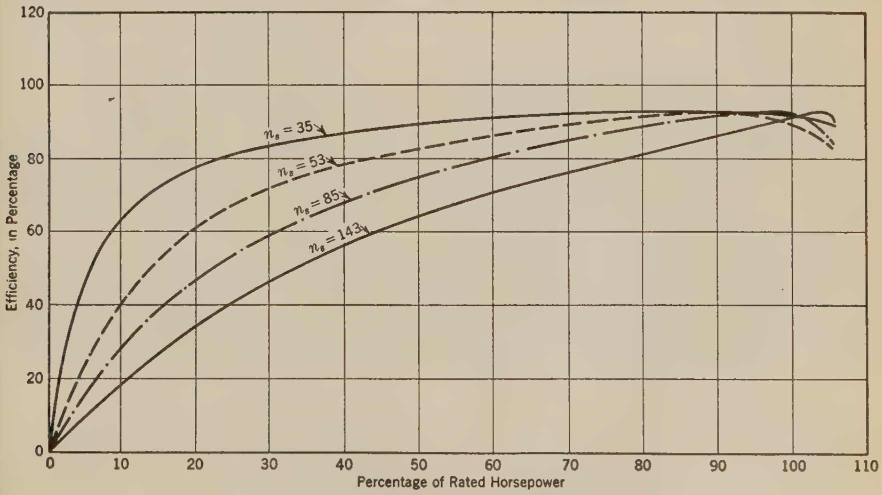

FIG. 13.—PERFORMANCE CURVES FOR RUNNERS OF FIG. 12

the best possible hydraulic relation between guide-vane opening and runner-blade position.

Fig. 14 shows a typical horse-power efficiency curve for the Kaplan-type turbine. Each of the dotted curves shows the performance of the unit if operated as a fixed-blade runner at the blade angle indicated. By automatically moving the runner blades at the same time as the guide-vanes the unit can be operated at the peak of each separate efficiency curve, so that the envelope curve (shown solid) becomes the real operating curve. The Kaplan-type turbine thus permits the adoption of high speed, yet maintains a high efficiency over a wide range in load.

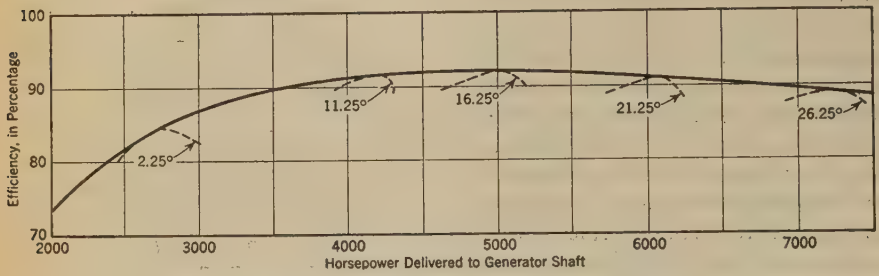

FIG. 14.—TYPICAL PERFORMANCE OF KAPLAN-TYPE TURBINE

The following installations will illustrate the fixed-blade, propeller-type turbine:

Four 13 500-hp turbines—37-ft head—installed in the Louisville (Ky.) Plant of the Louisville Hydro-Electric Company.

Four 21 000-hp turbines—32-ft head—installed in the Rock Island Plant, of the Puget Sound Power and Light Company, Columbia River, Washington.

Two 45 000-hp turbines—48-ft head—installed in the Wheeler Dam Plant of the Tennessee Valley Authority, on the Tennessee River, Alabama.

Among the Kaplan-type installations may be mentioned:

Two 7 250-hp turbines—23-ft head—installed in the London and Marmet Stations of the Kanawha Valley Power Company, on the Kanawha River, in West Virginia.

Six 42 000-hp turbines—55-ft head—installed in the Safe Harbor Plant of the Safe Harbor Water Power Corporation, on the Susquehanna River, in Pennsylvania.

Two 48 000-hp turbines—43-ft to 56-ft head—being installed in the Pickwick Development of the Tennessee Valley Authority, on the Tennessee River, in Tennessee.

Two 66 000-hp turbines—69-ft head—being installed in the Bonneville Development of the Federal Government on the Columbia River, Oregon. These turbines, with a runner diameter of 23.3 ft, are the largest as well as the most powerful units of the Kaplan type as yet constructed in the United States.

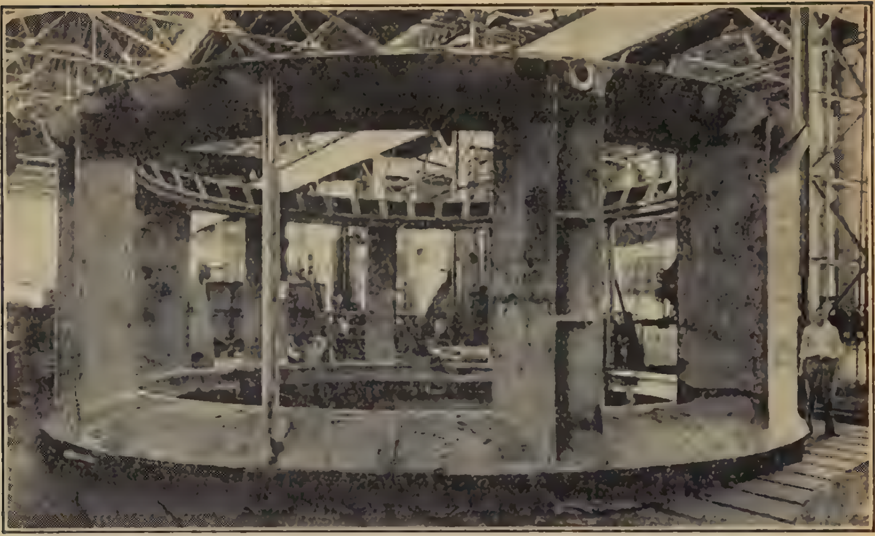

FIG. 15.—STAY-RING FOR 45 000-HP PROPELLER-TYPE TURBINE FOR WHEELER DAM PLANT, TENNESSEE VALLEY AUTHORITY

Fig. 15 shows the assembled stay-ring for one of the 45 000-hp Wheeler Dam units, and Fig. 16, the assembled runner for the same turbine.

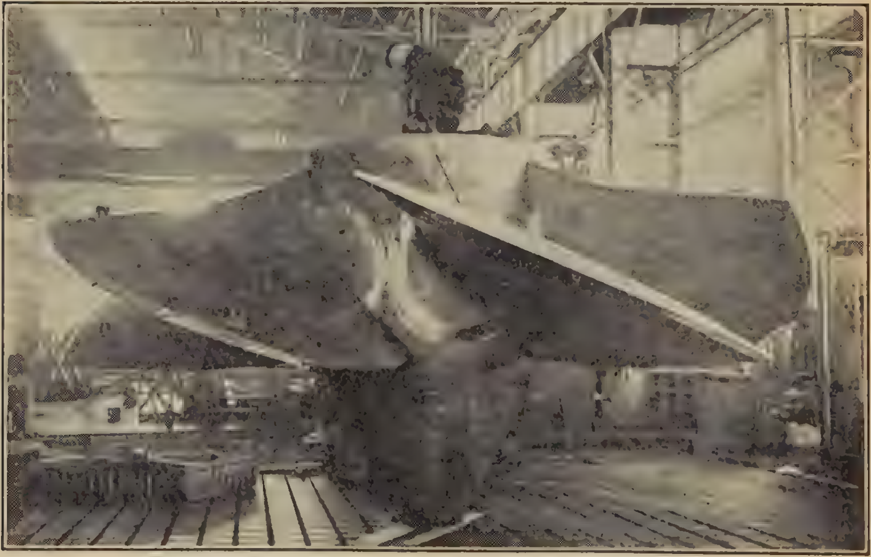

FIG. 16.—SHOP ASSEMBLY OF 22-FOOT FIXED-BLADE PROPELLER RUNNER FOR 45 000-HP, WHEELER DAM TURBINE

Fig. 17 is a sectional elevation of one of the 42 000-hp Kaplan-type turbines in the Safe Harbor Plant. At the time the first four units were installed (1931) they were the largest and most powerful Kaplan units in this country, the runner diameter being 18.33 ft.

LABORATORY TESTING

One of the important factors in the progress of turbine design and operation is the laboratory research carried on by all the turbine manufacturers. Models exactly homologous to the large units, including the intake, casing, and draft-tube, are tested over a wide range in power and speed. The model runners are usually 16 in. in diameter. Changes can readily be made in the runners, guide-vanes, casing, and draft-tube to obtain the best possible combination, before starting construction of the large units; and from the test data secured in the laboratory the performance of the large unit in the field can be accurately predicted. For the large low-head installations the intake, casing, and draft-tube are usually built by the power company in the concrete substructure, and the most efficient and economical design for the particular conditions at each

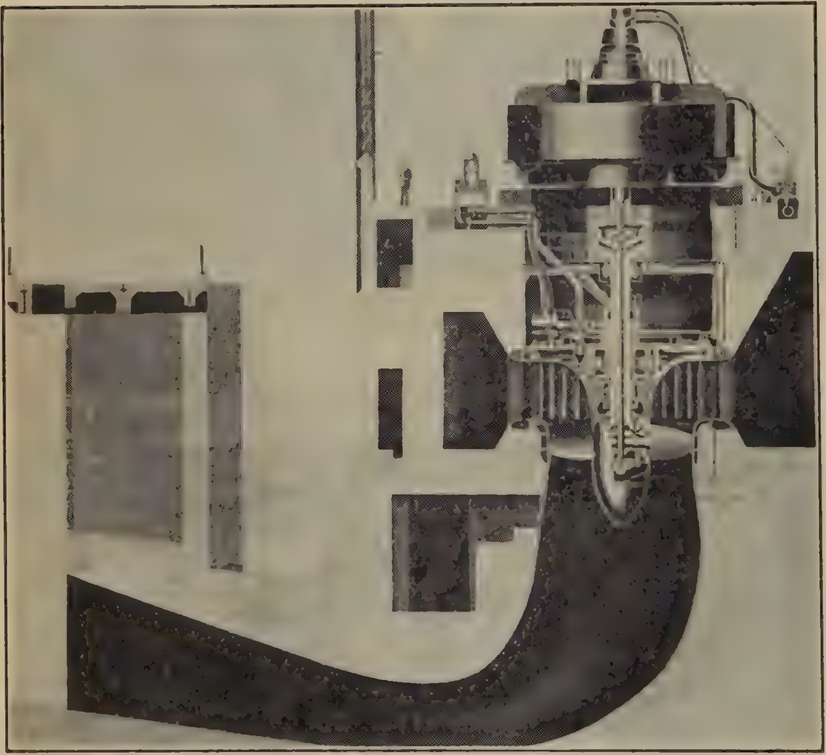

FIG. 17.—SECTIONAL ELEVATION OF 42 000-HP TURBINE, SAFE HARBOR PLANT, SAFE HARBOR WATER POWER CORPORATION

site can be determined from the laboratory test made in advance. Fig. 18 shows a model unit of the Boulder Dam turbines, and Fig. 19 shows the results of the laboratory tests made on it.

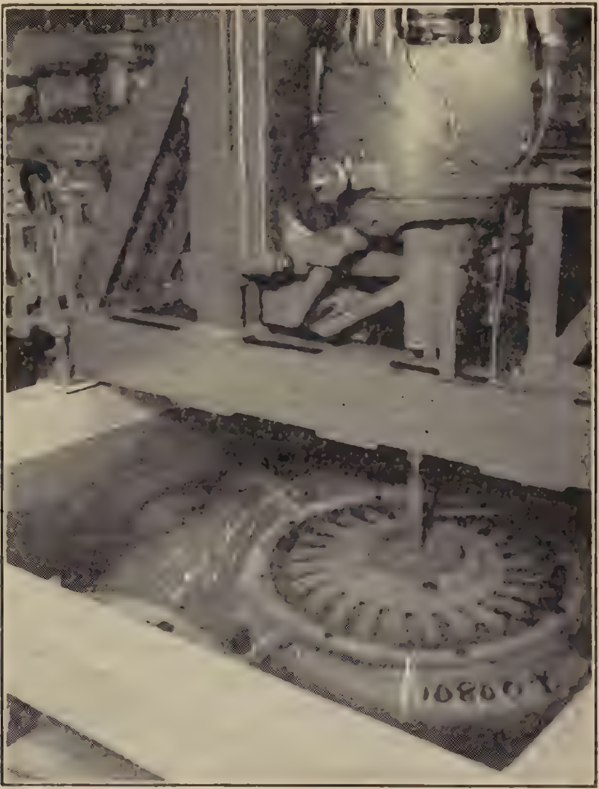

Fig. 18.—Model of Boulder Dam Turbine

Since the development of the high-speed runner the question of cavitation has assumed far greater importance than formerly, and much research has been devoted to it. Cavitation may be defined as the formation of vapor-filled cavities at points on the runner blade, or at any point within the water stream which disturbs the continuity of flow and causes the water to leave the blade surface. Cavitation reduces turbine efficiency; and due to the collapse or

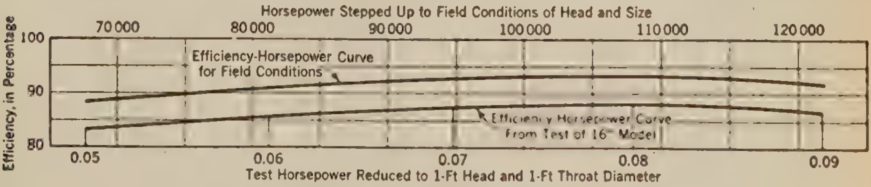

Fig. 19.—Test Results of Boulder Dam Model

"implosion" of the cavities where the flow reaches points of higher pressure, causes pitting of the runner or adjacent parts.

At a given elevation above sea level and a given temperature, the absolute pressure available is the water barometric pressure. At the runner discharge this pressure is reduced: (1) By the elevation of the runner above tail-water; (2) by the velocity head; and (3), locally, by the curvature of the flow lines. This third element is dependent on the runner design, particularly on the proportional magnitude of the blade area and blade spacing, and may differ materially on two runners of the same specific speed. Hence, for any given runner, it is necessary to determine the correct elevation with respect to tail-water, so as to assure sufficient absolute pressure at the discharge of the runner blades under the required head in order to avoid cavitation.

For this purpose special cavitation laboratories have been built by several of the turbine manufacturers, and by the Pennsylvania Water and Power Company, at Holtwood, Pa., and the Shawinigan Water and Power Company, at Shawinigan Falls, Ont., Canada. In these laboratories the elevation of the tail-water can be varied while the total head on the model turbine is maintained constant. As the tail-water is lowered, observations are made of the discharge, power output, and efficiency; the break in these curves indicates the cavitation limit.

The cavitation coefficient, σ, is defined by:

$$\sigma = \frac{H_b - H_s}{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which H_b = height of barometric water column; H_s = greatest elevation of runner above tail-water at which cavitation does not occur, with the turbine operating at a given percentage of its rated capacity. (If cavitation begins while the runner is still below tail-water, H_s is negative and represents the least depth of runner below tail-water at which cavitation does not occur.) H = total effective head on the turbine. Values of σ must be determined over the entire range of power and head through which the turbine is to be operated. In fixing the elevation of the runner in the field a factor of safety should always be allowed; that is, the value of σ used in the field should be well above the actual value determined in the laboratory.

Studies have been made of the behavior of various installations in operation, with particular reference to the avoidance of pitting, and several investigators have prepared curves to serve as a general guide to the selection of plant σ values. Such curves, for both the Francis and propeller-type turbines, are shown in Fig. 20.[16] The data shown have been collected and brought up to date by Professor L. F. Moody. The solid curves show the minimum values of plant σ recommended by the writer. The curve for propeller turbines gives the recommended limit for fixed-blade propeller units, but for Kaplan turbines σ values about 10% higher should be used. The curves are presented only as an approximate method of determining the elevation of the runner for a proposed installation; the final elevation should be fixed from the results of the cavitation tests made on a model of the runner to be used, or from field experience in homologous installations.

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[16] In Fig. 20, "Rogers and Moody, 1925," refers to paper entitled "Inter-Relation of Operation and Design of Hydraulic Turbines," by F. H. Rogers and L. F. Moody, *Engineers and Engineering*, Engrs. Club of Philadelphia, 1925; "Rogers and Sharp, 1935," refers to paper entitled "45 000 HP. Propeller Turbine for Wheeler Dam," by F. H. Rogers and R. E. B. Sharp, *Mechanical Engineering*, August, 1935; "Scoville, 1936," refers to discussion by J. D. Scoville of paper entitled "Cavitation Testing of Model Hydraulic Turbines and Its Bearing on Design and Operation," by L. M. Davis, *Transactions*, A. S. M. E., May, 1936, p. 323; and "Scoville Discussion, Cavitation Tests," refers to test points derived from preceding reference.

MECHANICAL IMPROVEMENTS

Since about 1925 there have been many mechanical improvements in turbine design. Among them at least six are deserving of special mention:

(1) The use of stainless steel for those parts subject to mechanical wear and erosive action of the water is increasing. For example, stainless steel castings or forgings are used for the shaft sleeves of water-lubricated bearings and for the stuffing-boxes. Plates of stainless steel are used in some units to protect the top and bottom surfaces of the guide-vanes, for distributor plates, and on the periphery of propeller-type runner blades. Those parts of the runner-blade surfaces and throat rings most subject to pitting are in some cases protected by pre-welding with stainless steel.

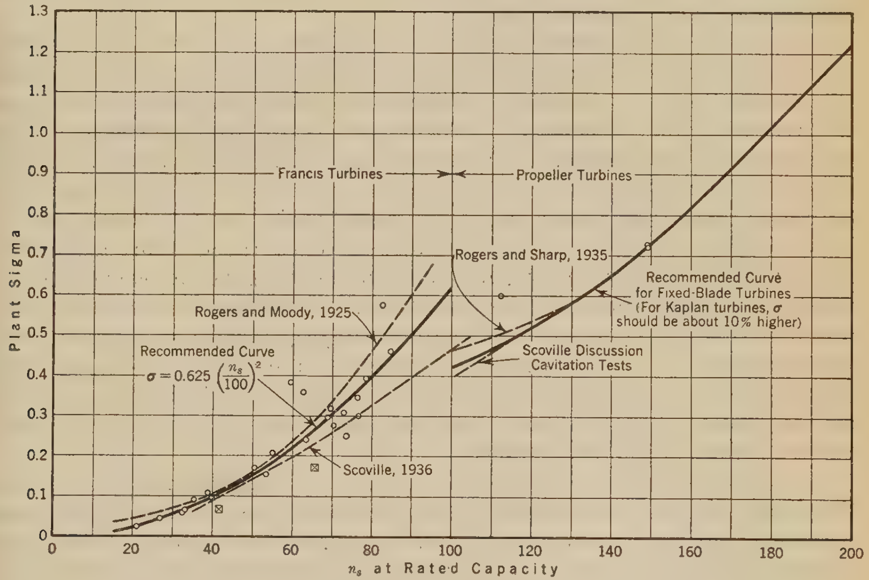

FIG. 20.—VALUES OF σ FOR USE IN DETERMINING APPROXIMATE ELEVATION OF RUNNER IN PROPOSED INSTALLATION

(2) Welded-steel construction has been used in some installations, for the guide-vanes, head covers, and stay-rings, in order to assure a more homogeneous metal than is obtainable with steel castings.

(3) Water-lubricated rubber-lined bearings have been used successfully in a large number of noteworthy low-head installations. This type of bearing was first used in one of the exciter units at the Holtwood Station of the Pennsylvania Water and Power Company, in 1924, and by 1936 about sixty of them were in use or under construction. They have proved superior to the usual

lignum-vitæ type in wearing qualities, particularly where abrasive matter is present in the lubricating water. Compared to the oil-lubricated, babbitted bearing, the rubber type is superior as regards operation, accessibility, and simplicity. It eliminates the inaccessible stuffing-box below the bearing, permitting the runner to be placed close to the bearing. The small lubricating pumps and water-drainage pumps required for the oil bearing are also unnecessary.

(4) The machining of blade surfaces of a propeller-type turbine (one of the units of the Safe Harbor Plant) has recently been tried. By this means it is possible to reproduce more accurately the curve and shape of blade desired than by the usual chipping and grinding to templates. It is not as yet possible to determine the actual benefit obtained by machining as regards either performance or avoidance of pitting.

(5) An adjustable-blade propeller turbine has recently been developed in which the runner blades are automatically adjusted by water flowing through the turbine. No inter-connection is provided between the runner blades and the governor, which actuates the guide-vanes in the customary manner. The runner blades are pivoted on roller bearings, and inter-connected through segmental gears and racks to a dash-pot piston enclosed in a cylinder that forms the upper part of the runner hub. The water flowing past the blades sets up a hydraulic moment tending at all times to open them. This moment is balanced by a reactive device, utilizing springs, weights, or balance piston, which tends at all times to close the blades. Experiments have shown, it is reported, that if the runner blades are properly proportioned and pivoted, they will assume the correct position with relation to the guide-vanes to maintain best efficiency for each load. A unit of this type, rated at 7 600 hp (23-ft head, 90 rpm), was installed in 1936 in the Marmet Plant, of the Kanawha Valley Power Company, in West Virginia.

(6) Automatic air-valves are now used in a large majority of turbines. They are operated automatically by an attachment to the turbine-gate mechanism, so that the admission of air is progressively reduced as the gates open and finally ceases at a predetermined gate position. (If pressure exists under the head cover, air may have to be introduced under pressure.) These valves improve operating conditions at small loads, and in the fixed-blade propeller-type turbines their effect is noticeable even at loads approaching normal load. The admission of air at speed-no-load condition is also of considerable benefit in synchronizing units. When the generators are operating as synchronous condensers, a great saving in power can be effected by admitting compressed air to depress the water until it clears the runner.

Possible Future Developments

At present, the Rocky River Development, in New England, is the only pumped-storage plant of any appreciable size on this Continent. However, the demand for reserve power and peak-power service, particularly near large load centers, has resulted in many studies of similar projects.

In the existing plant the motor-driven pump and the turbine-driven generator are separate units; but if and when any of the present proposals materialize, special turbine and pump equipment will be desirable to meet their requirements. Accordingly, at least one manufacturer has already designed and tested two model pump-turbine units which combine in one piece of apparatus the pumping and generating elements. This arrangement not only makes one machine do the work of two, but, in addition, does away with the elaborate hydraulic connections required when separate pumping and generating units are used. The adoption of such equipment will materially reduce the cost of pumped-storage plants.

The pump-turbine unit which has been developed is of the Francis type, with volute casing and movable guide-vanes surrounding the runner. The curves in Fig. 21 show the performance of the two model units as obtained by laboratory tests. (Both models had a diameter at the intake of about 18 in. and a throat diameter of about 12 in.) The high-head model has a specific speed, when generating, of about 26, and would be suitable for heads up to about 300 ft. The medium-head model has a specific speed of about 42, and would be suitable for heads of about 100 ft. It will be noted that when pumping, maximum efficiencies of about 85% and 87% were secured on the high-head and medium-head units, respectively; and that when generating, the maximum efficiencies were about 89% and 88%, respectively. In view of the dual performance required of the runner, these efficiencies are indeed excellent.

The two speeds required to give as nearly as possible the best efficiency for generating and pumping, respectively, are shown on the curves (Fig. 21).

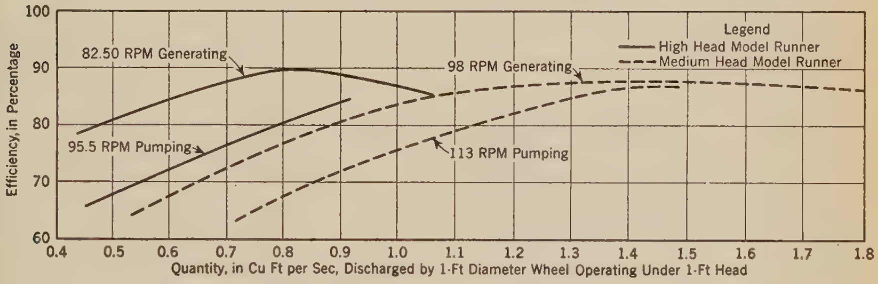

Fig. 21.—Performance of Model Pump-Turbine Units

This two-speed requirement may be met by the use of a two-speed type generator; one of the electrical manufacturers has carefully studied the requirements of such a machine and foresees no difficulty in building it. In some cases, however, it may be preferable to adopt a constant speed intermediate between the two values, and to sacrifice a small differential in efficiency.

Another promising field is the development of the propeller-type turbine for higher heads. The highest head to which this type has been applied in the United States is 69 ft (at the Bonneville Development, in Oregon). However, on the Shannon River, in Ireland, a Kaplan-type turbine has been installed to operate under a head of 106 ft. For these higher heads, runners with lower specific speed and large blade areas are required, in order to obtain low values of σ and thus avoid the necessity of locating the turbines at excessive depths below tail-water elevation. Undoubtedly, such runners will be developed to fill the gap in specific speed, between the present high limit of the Francis runner (85) and the low limit of the propeller-type runner (125). The economic advantage of higher speed as well as the better operating characteristics of the Kaplan turbine will certainly warrant the necessary research work along these lines.

With the inter-connection of large systems, automatic load control and frequency control have increased in importance. For any particular load on a station there is a most favorable division of power output between the separate units to secure minimum water demand or maximum over-all efficiency; and this optimum distribution between units can be maintained continuously and accurately only by automatic control. The same principle can be extended to load division between stations in a system. As to frequency, every station in a system must be controlled within very narrow limits. Further, if electric clocks are to operate satisfactorily, the cumulative errors caused by slight frequency changes must be compensated with a high degree of accuracy. Various devices have been developed to meet all these requirements, but further improvements and refinements in this type of equipment will undoubtedly be necessary to meet the severe demands of the large interconnected systems.

IMPROVEMENTS IN THE UTILIZATION OF ENERGY

BY JOEL D. JUSTIN,[17] M. AM. SOC. C. E.

SYNOPSIS

Recent development in the increased use and decreased cost of electrical energy is outlined herein. The paper traces the development of long-distance transmission, of regional power companies, of inter-connection between companies and of the co-ordination of steam and hydro-electric power development and shows the effect of these factors on the utilization of electrical energy.

Attention is directed to peak-load hydro-electric plants as a means of reducing the total cost of energy in a system, including both steam and hydro-electric plants.

Before the World War, power requirements of American communities were met, to a large extent, by local power companies. The major domestic use of electrical energy was for lighting. Its use in the home for refrigeration, washing machines, vacuum cleaners, etc., was relatively insignificant.

To-day, not only have all the old uses greatly increased, but additional uses for electricity in the home are constantly being found. The consumption per domestic customer has increased from an average of 268 kw-hr per yr in 1914 to 770 kw-hr for the year ending July 31, 1937.

CHANGES IN USE OF ELECTRICAL ENERGY

In 1914, manufacturing plants, to a considerable extent, owned and operated their own sources of power. American industry in that year purchased from central stations only 15% of its power requirements; and produced the remaining 85% in its own individually owned power plants. In 1936, by contrast, industry purchased about 53% of all the power it used and made in its own plants only 47 per cent.

Before the World War, a considerable percentage of all electrical energy produced was used in city and interurban electric traction; but now many of the interurban lines—and the trolley lines as well—are gone, and the electric traction load, except in the larger cities, is a relatively small percentage of the total load. There is, however, some indication that, in time, its place may be taken by the electrification of some of the steam railroads.

To-day more electrical power and less man power is being used by industry per unit of product. Electro-chemical industries, which are heavy users of energy, are being further developed. In the commercial field, there has been an increased use of electrical energy for better and more extensive lighting; and among the new uses of electricity, air-conditioning is rapidly assuming importance. A single large office building may require a motor installation of 3 000 or 4 000 hp for air-conditioning purposes alone. Stores and restaurants are finding it economically advisable to air-condition. If the present trend continues, within the next few years the office building or hotel that is not air-conditioned will be an obsolescent, low-rental structure. Air-conditioning of homes, although not yet an important factor in the increase of electrical load, will probably become so in time. The demand for it is beginning to appear.

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The increase in the variety of uses of electrical energy, the increase in the amount produced, and the decrease in the unit price, have been made possible by marked technical improvements in generation; and by a great advance in the adopted methods of utilization, whereby the maximum economic advantage of the generating facilities has been secured.

In Fig. 22 is shown the annual electrical energy output of privately owned public utilities for the period 1912 to 1936.

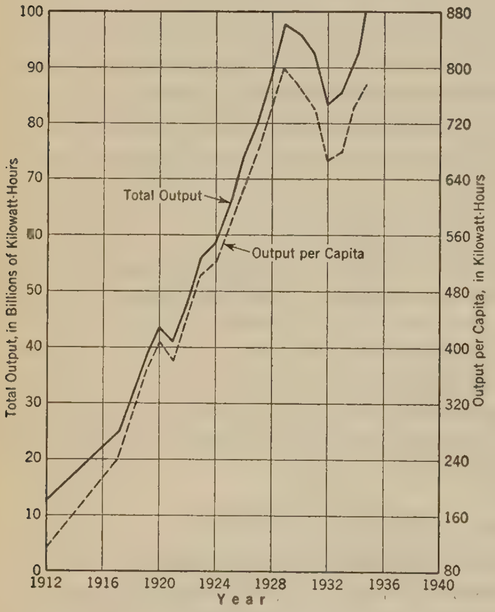

Fig. 22.—Electrical Energy Output of Central Stations in the United States, 1912–1936

Decrease in Price of Electrical Energy

The increase in output of electrical energy has been paralleled by steady reductions in its price to the consumer. In 1914, the price of electrical energy for domestic use averaged 8.2 cts per kw-hr; in the year ending June 30, 1936, the average price was 4.51 cts per kw-hr, a decline of 45 per cent. By contrast, in 1935 the cost of living was approximately 40% higher than in 1914.

Only the privately owned public utility plants reporting to the Edison Electric Institute are included in the foregoing comparison. However, such plants account for from 94% to 95% of the total output of the United States.

Cause of Decline in Price of Electrical Energy

Although Government regulation has been the cause of some individual rate reductions, the fundamental major cause of the decline in the price of electrical energy is economic.

Management, striving for greater total profits, worked for and obtained ever-increasing sales which, in turn, had a material effect in reducing costs. With the reduced costs, lower prices became possible. Lower prices stimulated greater use, and greater use permitted the introduction of more technical improvement. An endless chain whose links are technical improvement, decreased unit cost, decreased unit price, increased use, and increased total profits, was thus set in operation and has functioned successfully, with slight interruptions, to the present time. It will continue to operate to the advantage of all concerned (limited only by an apparently remote saturation point and the law of diminishing returns), unless one of the links in the chain is broken.

Community Power Companies

Prior to the World War there were relatively few regional power companies and their development was in its infancy. In general, each community or small group of communities was served by a local power company. Loads and plants were relatively small and transmission distances with a few exceptions were comparatively short. A 5 000-kw generator was considered a large unit in those days, and a 50 000-kw plant was a big one, such plants, for the most part, being confined to the larger centers of population. As there was not much inter-connection, the reserve requirements of individual systems were high, and it was not unusual for power companies to have installations equal to 150% or even 200% of their peak load for any given year.

Steam Companies and Hydro-Electric Companies

Before the World War power companies might have been divided into two classes: (1) Those utilizing steam as their main source of power, often referred to as steam companies; and (2) those utilizing water power as their main source of power, often referred to as hydro-electric companies. The useful hydro-electric installation plus the necessary machine reserve for any given water power company was limited to that for which there was always sufficient stream flow available to produce enough power to serve the peak load. Even so, the hydro-electric companies, at times, would have available energy which they could not use on their load curves. This surplus, which was produced at off-peak periods and during seasons of high stream flow, was sold to neighboring steam companies. This policy required the construction of inter-company transmission lines, and enabled the steam companies to shut down some of their steam generating units, at times, and thus save fuel.

Improvement in Long-Distance Transmission

The opportunity for utilizing low-cost surplus hydro-electric energy at important industrial centers, served largely by steam-generated electric power, put a premium on the development of the long-distance transmission of electrical energy. At the same time, it was found that some sources of hydro-electric power could be developed economically to serve primarily a distant load center, provided the cost of transmission was low enough.

These factors were largely responsible for the development of long-distance transmission. Voltages were stepped up from 66 000 to 110 000 and then to 220 000, and with the increases in capacity which these high-tension lines permitted, the cost of transformation, regulation, and control was decreased per unit of energy transmitted. Technical improvements in the generation of power by both steam plants and hydro-electric plants also led to material decreases in cost.

Regional Power Companies

A rapid growth in sales followed, and the "set-up" of the community power company proved to be an inefficient means of handling the increasing business with its extensive distribution over wide territories, and its need for rapidly increasing capital expenditures and for co-ordinated operation between the various units.

Consequently, regional power companies became more prevalent. Community power companies joined together or were absorbed to form integrated and co-ordinated systems, many of which served territories hundreds of square miles in area, with populations of several million people. The resulting economies, as indicated by the decreasing price of the product, have been very material. During the years of the great depression the tendency was retarded, but is again becoming evident with a return of more normal conditions.

Inter-Connection

Inter-connection developed coincident with the growth of regional power companies. By inter-connection, as herein used, is meant the tying together, by means of high-tension transmission lines, of two or more independently operated power systems. The economies of inter-connection ensue from savings in investment in additional generating facilities and from savings in operating expense. Against such savings must be set the increased fixed charges and operating expense for the additional transmission facilities.

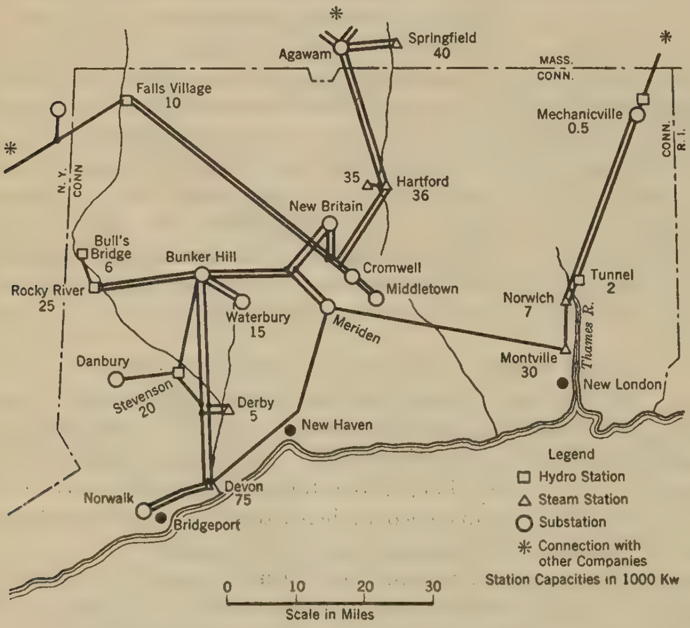
*
Springfield
40
Agawam
*
MASS.
CONN.
Falls Village
10
*
Mechanicville
0.5
CONN.
R.I.
N.Y.
CONN
35
Hartford
36
New Britain
Bull's
Bridge
6
Bunker Hill
Cromwell
Middletown
Tunnel
2
Norwich
7
Rocky River
25
Waterbury
15
Meriden
Thames R.
Danbury
Montville
30
New London
Stevenson
20
Derby
5
New Haven
Legend
☐ Hydro Station
△ Steam Station
○ Substation
* Connection with
other Companies
Norwalk
Devon
75
Bridgeport
Station Capacities in 1000 Kw
0
10
20
30
Scale in Miles

Fig. 23.—The Connecticut Valley Power Exchange

Extensive economies have been attained through many of these inter-connections. On the other hand, there have been some in which the increased annual cost of the inter-connection has exceeded the annual savings so far attained thereby. Every proposed inter-connection should be carefully analyzed to determine its potential economies.

Between 1920 and 1930, there was a rapid growth in inter-connection. In some cases very large inter-connected systems (like that shown in Fig. 23[18]) were built up, and industrial centers with a combined load of 1 000 000 kw, or more, were tied together with high-tension transmission lines. Territories covered by such systems sometimes exceed 20 000 sq miles and include populations of several million people. The power companies forming such inter-connections or power pools have entered into agreements for the purpose of operating the combined system to secure maximum economy in capital expenditures and in annual operating costs. This inter-connection of independently operated properties made little progress during the depression and even in 1937 had not been greatly revived.

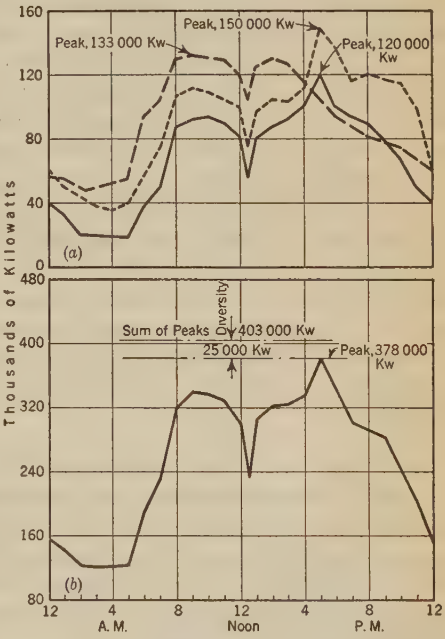

FIG. 24.—DIVERSITY OBTAINED BY INTER-CONNECTION: (a) PEAK DAY LOAD CURVES OF SEPARATE COMPANIES; (b) PEAK DAY LOAD CURVE AFTER INTER-CONNECTION

SAVINGS DUE TO LOAD DIVERSITY THROUGH INTER-CONNECTION

The load characteristics of various companies may differ materially. The load curve of a company serving a metropolitan district is very different from that of a company serving small centers and rural areas, and the load curve of a company serving an area in which heavy industries predominate will probably be very different from either of the first two. Thus, one company may have its seasonal peak in October, whereas another near-by company may have its annual peak in December. There may also be a material diversity in the daily load curves, one company having its daily peak load much earlier in the day than the other; and even where companies serve communities of a similar character, there is almost always some "incidental" load diversity—that is, a diversity for which there appears to be no special reason but which, nevertheless, occurs year after year.

If companies having load curves like those just discussed are inter-connected and operated as a single economic entity, either with or without identity of ownership, it is obvious that the combined system will not require as much installed capacity as would be the case without such inter-connection. Fig. 24 shows the load diversity obtained by a typical inter-connection of three companies.

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[18] Adapted from an illustration in "Power Supply Economics," by Joel D. Justin and William G. Mervine, John Wiley & Sons, Inc., New York, N. Y., 1934.

RESERVE CAPACITY SAVINGS DUE TO INTER-CONNECTION

Assume that two hypothetical power systems are considering inter-connection. Each has an annual peak load of 100 000 kw, and each is served by a steam plant having an installed capacity of 120 000 kw, consisting of six 20 000-kw units, one unit in each plant being in reserve. It will be assumed also that the total load diversity between the two companies is 20 000 kw. Then, if the two companies are inter-connected and operated as an economic entity, the combined load that must be served will be 100 000 + 100 000 − 20 000 = 180 000 kw. In this case, experience has determined that the required reserve must be equal to 10% of the peak load, or to the capacity of the largest unit, whichever is the greater. Of the peak load 10% is 18 000 kw, and all the units are 20 000 kw in size. Therefore, the required reserve is 20 000 kw, and the required capacity is 180 000 + 20 000 = 200 000 kw. The actual total installed capacity, however, is 240 000 kw. Thus, by inter-connecting the two companies, which separately had only enough capacity to serve their peak loads in the given year with the required reserve, a surplus capacity is made available to take care of a future growth of 40 000 kw in the demand.

As each of these companies, at the time of inter-connection, was faced with the necessity of installing an additional 20 000-kw unit, the saving due to inter-connection from this cause would be the capital sum of, say, 40 000 × $130 = $5 200 000, or a saving in fixed charges of, say, $5 200 000 × 0.14 = $728 000 per yr. In addition, there would be a saving of the fixed portion of the operating cost on the two 20 000-kw units which, because of inter-connection, it was not necessary to install. This might amount to as much as $100 000 per yr additional.

SAVINGS IN FUTURE CAPITAL EXPENDITURES WITH INTER-CONNECTION

Due to inter-connection and operation as an economic entity, material savings may be expected in future capital expenditures. Continuing the hypothetical illustration, it will be noted that, whereas 20 000 kw was probably an economic size of unit for a system with a 100 000-kw load, a unit twice as large would probably be more economical for a system with a peak load of 180 000 kw.

Therefore, when the growth in the load of the combined system requires additional capacity, the requirements may be met by installing units of 40 000-kw capacity each. The capital cost and operating cost of such units would be considerably less than those of the 20 000-kw units which the companies were in the habit of installing prior to inter-connection. The additional annual saving might be material.

SAVING IN ALLOCATION OF PLANTS TO LOAD THROUGH INTER-CONNECTION

It will be assumed that the base load of the individual companies (that is, the load which is continuous throughout the 8 760 hr of the year) is 20% of the peak load. The base load of the combined system will thus be 40 000 kw. The several units of any system practically always vary materially in efficiency, largely because of the difference in their age. It will be assumed that one of the two companies has three units which are much more efficient than any of the other units in either system (all units having a capacity of 20 000 kw).

With the combined load due to inter-connection, these three units could operate on a much higher annual capacity factor—that is, they could operate more hours per year at full capacity. The less efficient units would operate correspondingly fewer hours per year. Such operation would reduce the average unit production cost of the combined system, producing an additional annual saving. Fig. 25 shows the allocation of units to the load demand for a typical day, before inter-connection, for the company possessing the three highly efficient units. It is evident that, after inter-connection, the base load would be twice as great as that indicated in Fig. 25 and the three efficient 20 000-kw units would then be able to operate at an annual capacity factor approaching 100 per cent.

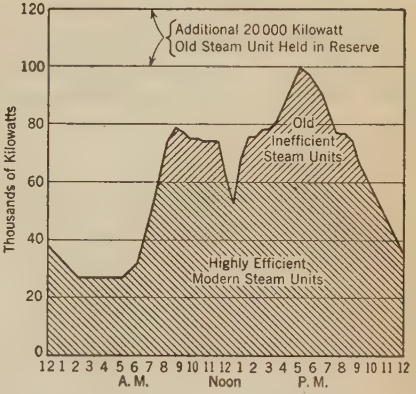

FIG. 25.—ALLOCATION OF PLANTS TO LOAD, BEFORE INTER-CONNECTION (FOR COMPANY WITH THREE HIGHLY EFFICIENT UNITS)

ANNUAL SAVINGS DUE TO BETTER USE OF HYDRO-POWER WITH INTER-CONNECTION

Many engineers think of steam and hydro-power as competitive sources of electrical energy. Occasionally, they are competitive, but over wide areas of the United States they are more usually complementary. It has been found that a system whose power supply is derived from both steam and hydro-power plants in proper proportion will have a lower over-all cost of power supply than a system otherwise identical but served solely by steam capacity.

Assume the same two hypothetical companies as before, but for one of the 20 000-kw steam units of one of the companies substitute a hydro-electric plant with an installed capacity of 30 000 kw. Further, assume that stream flow and pondage conditions are such that, at time of peak load, this plant can be relied on for a firm capacity of only 20 000 kw when operating on the load curve of the one company alone. (Firm hydro-electric capacity may be defined as that part of the total installed capacity of a hydro-power plant which, under existing conditions of load, stream flow, and pondage, is capable of performing the same function in serving the part of the load curve allocated to it, that an alternative steam plant could perform.)

As the magnitude of a load curve increases, there is a lesser amount of energy in the upper part of the load curve for any given number of kilowatts in the peak of the load. Consequently, when the two companies are inter-connected, it is found that all the installed hydro-electric capacity of 30 000 kw is firm capacity on the combined load curve.

Fig. 26 illustrates this growth of firm hydro-electric capacity. In order to plot such a diagram two preliminary steps are necessary: First, a "load-duration curve," with loads as ordinates and time as abscissas is plotted. Areas under the curve thus represent energy. By planimetering, or otherwise, energy for various numbers of kilowatts, measuring down from the peak, is determined. Second, a "peak percentage curve" is plotted using the information thus obtained. For this curve, the ordinates are the percentage of total load (measuring down from the peak) and the abscissas are the percentage of total energy for the period down to this given point. Using this curve and knowing the peak load for the given year, the energy required to serve any given part of the load may be obtained. After the peak percentage curve is plotted, the curve of Fig. 26 can be constructed readily. In some cases it may be necessary to investigate other periods than the peak-load week.

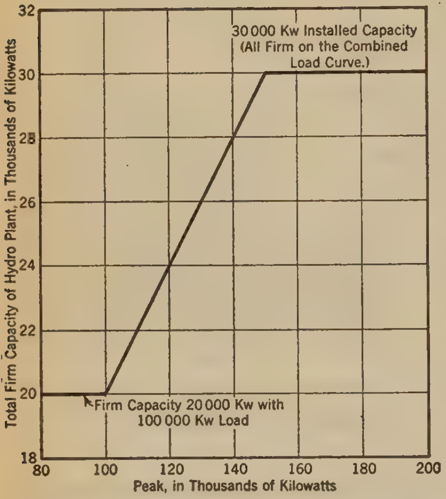

FIG. 26.—INCREASE IN FIRM CAPACITY OF 30 000-KW HYDRO-ELECTRIC PLANT WITH INCREASING SYSTEM LOAD

From the preceding discussion it is evident that by inter-connection the firm capacity of the hydro-electric plant has been increased by 10 000 kw, and that the effective capacity of the combined system has also been increased by 10 000 kw (in addition to the 40 000-kw increase previously mentioned). The corresponding capital saving of, say, $1 300 000, would accrue at such time as the companies would otherwise have had to install at least that much additional capacity.

The inter-connection of the two systems also gives an opportunity to utilize more nearly all the energy that the hydro-electric plant is capable of generating with the existing installation and the available stream flow. Without inter-connection, the base load on which the hydro-electric plant can operate continuously at periods of ample stream flow is only 20 000 kw, and, consequently, during a considerable part of the time, a large part of the available hydro-electric energy must be wasted. On the other hand, when the systems are inter-connected, the base load becomes 40 000 kw, and when ample stream flow is available the hydro-electric plant can operate continuously on the base without waste of energy.

The improvement in the utilization of available hydro-electric energy throughout the development period of the regional power companies has been very marked. Many individual power companies, with a high percentage of hydro-electric installation, had hydro-electric utilization factors of from 35 to 60 per cent. (The utilization factor is the ratio of the energy actually produced by a hydro-electric plant to the energy that could have been produced by the same plant during the same period, if there had been a market for it.) Now that these companies have been absorbed into large regional power systems, and the plants are inter-connected with load centers served largely by steam, the annual utilization factors of the same hydro-electric plants have risen in many cases to more than 90%, and, in a few cases, to practically 100 per cent.

Utilization of Peak-Load Plants

Supplying energy for the peak load of any power system is always very expensive as compared to the average cost of electrical energy. It is not unusual for the top 20% of the annual load curve to contain only from 0.5 of 1% to 1% of the total annual energy output of the system. Thus, a plant serving this part of such a load curve might have an annual capacity factor of from $1\frac{1}{4}$% to 2 per cent. As the average annual capacity factor of the system as a whole may be 35%, it is evident that the cost of providing the peak-load service may be a great many times the average total cost of energy.

In many systems, peak-load service is performed by the more antiquated steam plants in the system. Their high cost of energy production does not greatly matter because they produce very little energy. However, the maintenance and operating cost, which it is necessary to incur in order to have them ready to operate when required, and the cost of keeping them in "hot reserve" for a part of the time, are frequently high.

Peak-Load Hydro-Electric Plants

The possible improvement in the utilization of energy and the reduction in the total cost of power supply, through the use of peak-load hydro-electric plants in large regional power companies and in inter-connected systems, does not always receive the attention it deserves. The operating and maintenance cost of a hydro-electric plant is low as compared to that of a steam plant. Further, the hydro-electric plant can be put on the line at a moment's notice, thus making feasible a reduction in the amount of "hot" reserve that must be carried at the steam plants of the system.

Hydro-electric plants with ample pondage and a very high ratio of installation to available stream flow are particularly suitable for performing this peak-load service. For base load service the steam plant is generally a much cheaper source of energy. A high installation ratio usually means a low over-all cost per kilowatt of capacity, since the major part of the total investment is usually in the power plant itself ($50 to $70 per kw of installation) and the investment per kilowatt in dams and flowage is relatively small.

The economic advisability of using hydro-electric plants of the peak-load type for performing peak-load service is indicated in Fig. 27. For comparative purposes, the total unit cost of the energy output of a relatively modern steam plant, using cheap fuel, is shown by Curve *A*. Curve *C* gives the capital cost of an alternative hydro-electric plant for peak-load service, based on actual and fairly typical experience, and Curve *B* gives the total unit cost of its output. The principal assumptions are indicated in Fig. 27.

The decline in the capital cost of the hydro-electric plant per kilowatt of capacity with a decrease in the annual capacity factor actually reflects the fact that the average cost per kilowatt for a hydro-electric plant generally decreases rather rapidly with an increase in installed capacity. Although the incremental cost of steam-plant capacity is also frequently much less than the average, this holds only until the predetermined maximum capacity of the plant is reached; further, the tendency is not nearly so marked as in the case of hydro-electric installations.

It should be noted that for an annual capacity factor of 5% the total cost of energy from the hydro-electric plant would be 1.7 cts per kw-hr, while at the steam plant the total cost per kilowatt-hour would be 4.0 cts. For lower annual capacity factors the discrepancy is still greater. Although the comparison is quite typical it would vary materially for any individual case.

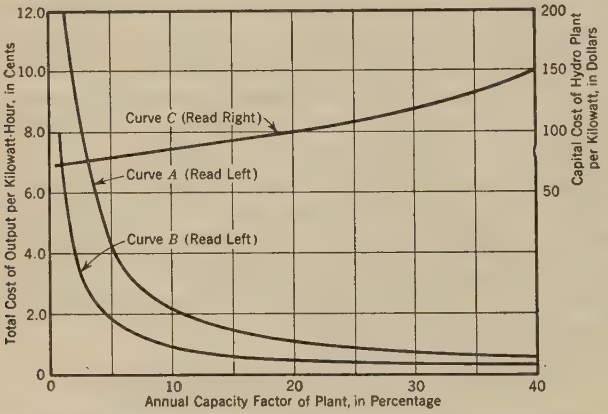

Fig. 27.—Comparison of Cost of Energy from Steam Plant and Hydro-Electric Plants on Peak-Load Service

It is not usual, however, to use new steam plants for peak-load service, as their low cost of energy production usually makes it advisable to allocate them to the base load or, at any rate, to some point lower down on the load curve. For old steam plants on peak-load service, interest charges on the investment in them should not be considered in comparisons such as the foregoing. In most cases, the fixed part of the annual operating cost for old steam plants is high enough to make up for the omission of interest charges, with the result that the comparison is frequently not any more favorable than that indicated in Fig. 27.

Run-of-river hydro-electric plants, when provided with ample pondage, are also used for peak-load purposes during periods when the stream flow is low; provided, of course, that the relation of low-water stream flow to load is such that they will be firm capacity on the load curve. It is believed that peak-load hydro-electric plants will find an increasing application, as there are many systems in which their more extensive use would prove economic. Their development, however, is limited by the availability of suitable sites near large load centers. The cost of a long transmission line, added to that of the plant itself, might more than offset the economies.

PUMPED-STORAGE HYDRO-ELECTRIC PLANTS

The pumped-storage hydro-electric plant is simply a special case of the peak-load hydro-electric plant, the main difference being that for the pumped-storage plant no source of water supply is required. In such installations the cost of spillway dams is minimized or eliminated. All that is required is a site for a pond or reservoir in the hills, another one at lower elevation, and a sharp drop between. In its simplest form (Fig. 28[18]) the pumped-storage plant consists of a head-water pond, penstocks, a power-house with turbines and pumps and a tail-water pond. "Make-up" water to take care of seepage and evaporation may be furnished by a brook or may be pumped from a distance. At peak-load periods, when the plant is in demand, water flows from the head-water pond through the turbines, generating power. During low-load periods, when the plant is not in demand, low-cost off-peak steam-generated energy or surplus hydro-electric energy flows to the plant and pumps the water back into the head-water pond, and the plant is then ready for another peak-load period. In Europe, there are thirty or forty such plants in successful operation, functioning as storage batteries for various power systems, but in

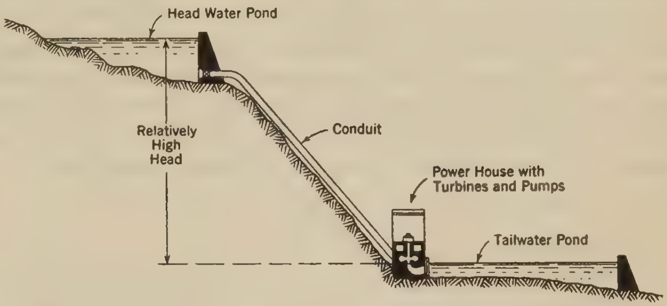

FIG. 28.—TYPICAL PUMPED-STORAGE HYDRO-ELECTRIC PLANT

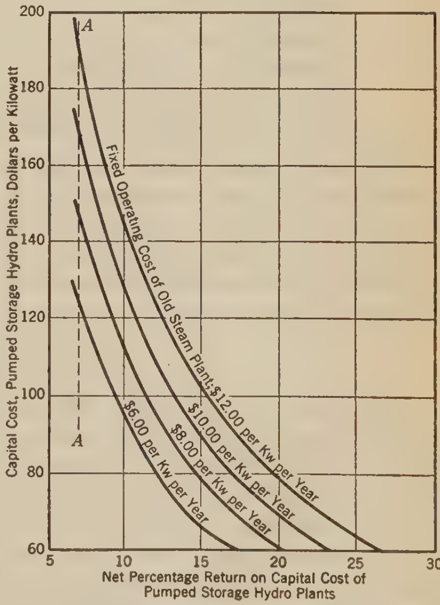

FIG. 29.—ECONOMIC LIMITATIONS OF PUMPED-STORAGE PLANTS TO REPLACE OLD STEAM PLANTS WITH 15% ANNUAL CAPACITY FACTOR

America there is only one—the Rocky River Development of the Connecticut Light and Power Company, near New Milford, Conn., with a capacity of 24 000 kw. In that plant, incidentally, the tail-water is a river instead of a pond.

Where suitable sites exist near large load centers, pumped-storage plants offer a means of reducing the cost of peak-load service and, hence, the total cost of power supply. In some such cases it will be found economical to use them to replace the antiquated steam plants already performing peak-load service. In order to make such a change economically desirable it is necessary that the total annual cost of the proposed pumped-storage plant, including interest on the investment, should be less than the annual cost of the old steam plant without considering interest on the investment. Fig. 29[18] gives curves indicating to some extent the economic limits in the application of pumped-storage hydro-electric plants for this purpose. The principal assumptions are:

Old Steam Plant:

Fixed charges	0
Renewals and replacement	$2.00 per kw per yr
Taxes and insurance	$2.50 per kw per yr
Incremental cost of energy	5 mills per kw-hr

Pumped-Storage Hydro-Electric Plant:

Fixed charges	(See Fig. 29)
Over-all efficiency	60%
Operation and maintenance	$1.00 per kw per yr
Renewals and replacements	1% on capital cost
Taxes and insurance	1% on capital cost
Off-peak energy for pumping	2.2 mills per kw-hr

In Fig. 29, the point where Line *AA* cuts each curve gives the maximum permissible cost for the pumped-storage hydro-electric plant, if the net return on the investment is to be 7 per cent.

As there are many situations where the fixed part of the operating cost of old steam plants is within the range indicated in Fig. 29, it is evident that there is room for a further application of pumped storage in the United States.

PENDING DEVELOPMENTS

In a rather general manner, this paper has discussed means of utilizing to the greatest economic advantage various technical developments in the electric-power field. With the resumption of industrial activity at an accelerated pace further technical advances may be expected.

One such advance that might produce radical improvements in the utilization of electrical energy, is long-distance direct-current transmission. A system of this type is already in experimental use between the Schaghticoke Hydro-Electric Plant of the Niagara-Hudson System and Schenectady, N. Y. Some electrical engineers are quite optimistic about its possibilities. They visualize the possibility of obtaining thereby the economic transmission of electric power to much greater distances and at considerably less cost than is possible with alternating-current transmission and transformation. The development of huge Government power plants remote from the load centers provides an additional inducement for such a development.

However, even without such startling developments as this, much will still be accomplished in the way of a better and more economical utilization of existing power-supply resources. In the existing inter-connections and power pools it has been found that, where there is an identity of ownership, the inter-connected properties come much closer to taking advantage of all the economies that are theoretically possible. Hence, with the resumption of industrial activity there will doubtless be a resumption of further inter-connection and consolidation on a territorial basis to the advantage of consumers and power companies alike.

COST OF GENERATION OF ELECTRIC ENERGY

By Philip Sporn,[19] M. Am. Soc. C. E.

Synopsis

This paper is a general discussion of the subject of the cost of electric energy, with particular emphasis on the cost of generation and the related items of transmission and distribution. It covers the economics of steam power *versus* water power and shows that, in most cases, present-day steam power can be produced more cheaply than present-day water power. It shows that there is no economy in long-distance transmission of electric energy and that sound economics require the location of the generating plant near the load. The cost of distribution is presented briefly, emphasizing the point that this item is the major cost in expenditure for electric service.

The paper concludes that the trend in cost of producing, transmitting, and distributing electric energy is downward—although no sharp dip is to be expected—and that future power requirements will be met by steam plants located economically with reference to loads.

Introduction

The cost of generating electric energy is only a small part of the total cost of electric service, which must also include transmission and distribution. Current technical and political-economic discussion on the cost of electric service is largely out of focus because this fact is not fully realized. More particularly is it neglected in much of the discussion on the cost of the generation of hydro-electric power by governmental agencies. In many cases, it is assumed that the apparently lower cost of generation of hydro-electric power can and will result in a much lower cost of electric energy to the consumer.

This, of course, is not the case. Hydro-electric power is not necessarily cheap power, and, in most cases, is not competitive with steam power. Nevertheless, it cannot be denied that the cost of generation is an item, and sometimes an important item, in the total cost of power.

The writer, in this paper, will attempt to clarify various factors in the cost of generation by different types of prime mover. It must be borne in mind that the costs are presented from a general viewpoint. The problem, of necessity, is complicated, and, within the scope of this paper, only rough studies and outlines are possible. The main object, however, has been to determine the approximate boundaries and trends of generation cost.

Present Costs of Electrical Energy

The sources of energy used commercially to-day are falling water—that is, hydro-electric power—and fuels, either in steam plants or in internal-combustion plants. Internal-combustion plants, however, have such limited application in so far as central stations are concerned that they will not be discussed herein.

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Investment Cost of Hydro-Electric Power.—Hydro-electric power has received a great deal of attention since 1933, particularly because its development has been pushed vigorously in connection with various governmental projects; and the impression has spread that, as it involves only the harnessing of "natural" power, its cost cannot be equalled by any other method. That the actual facts seldom warrant any such assumption is known to a few, but certainly is not generally realized. Most of the favorable hydro-electric sites have already

TABLE 3.—HYDRO-ELECTRIC PLANT INVESTMENT COSTS

(Data abstracted from Yearly Reports of Federal Power Commission)

Project No.	Name of company	Plant	CAPACITY, IN HORSE POWER		
			Primary*	Present	Ultimate
(1)	(2)	(3)	(4)	(5)	(6)
1 025	Safe Harbor Water Power Corp.	Safe Harbor	25 707	255 000	510 000
432	Carolina Power and Light Co.	Waterville	23 760	139 500	139 500
82	Alabama Power Co.	Mitchell	21 760	72 000	93 000
618	Alabama Power Co.	Jordan	30 400	146 800	180 000
349	Alabama Power Co.	Martin	37 750	135 000	180 000
459	Union Electric Light and Power Co.	Bagnell	15 000	201 000	268 000
346	Minnesota Power and Light Company	Blanchard	5 900	18 000	24 000
135	Portland General Electric Co.	Oak Grove	41 400	88 000	88 000
637	Chelan Electric Co.	Chelan	54 000	68 000	136 000
619	Feather River Power Co.	Bucks Creek	33 100	67 000	67 000
487	Pennsylvania Power and Light Co.	Wallenpaupack	8 148	54 000	54 000
516	Lexington Water Power Co.	Saluda	63 600	87 000	260 000
20	Utah Power and Light Co.	Soda Springs	19 820	21 000	70 000
659	Crisp County, Georgia	Crisp County	4 080	7 500	16 000
382	Southern Cal. Edison Company	Borel	3 809	14 400	14 400

Project No.	Stated cost, in dollars	COST, IN DOLLARS, PER KILOWATT OF:		FIXED CHARGES, IN DOLLARS,† PER KILOWATT OF:		INVESTMENT COST, IN CENTS, PER KILOWATT-HOUR OF OUTPUT AT:	
		Primary capacity	Installed capacity	Primary capacity	Installed capacity	90% load factor* (primary capacity)	40% load factor‡ (installed capacity)
(1)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1 025	24 995 111.74	1 350	136	145.23	14.62	1.840	0.417
432	13 764 856.51	807	139	86.75	14.94	1.003	0.426
82	10 646 056.96	673	206	72.35	22.15	0.017	0.632
618	13 047 334.50	600	124	64.50	13.33	0.818	0.380
349	17 551 299.53	650	181	69.87	19.46	0.886	0.555
459	36 024 117.21	3 350	250	360.12	26.87	4.567	0.767
346	3 451 175.25	816	266	87.72	28.59	1.112	0.816
135	9 546 593.24	322	151	34.61	16.23	0.439	0.463
637	11 067 055.53	285	227	30.64	24.40	0.388	0.696
619	9 564 549.87	404	199	43.43	21.39	0.551	0.610
487	9 070 137.00	1 555	234	167.16	25.15	2.120	0.718
516	21 661 510.44	476	348	51.17	37.41	0.649	1.067
20	3 560 872.19	251	236	26.98	25.37	0.342	0.724
659	1 267 016.11	432	235	46.44	25.26	0.589	0.721
382	3 147 501.23	1 150	305	123.62	32.79	1.568	0.936

* Power available 90% of the time.

† 10.75% of costs shown in Columns (8) and (9).

‡ Assuming that all the installed capacity is firm at 40% load factor. This assumption is necessary since Federal Power Commission data do not show actual annual kilowatt-hour output at all of these sites.

been developed, and any analysis of existing developments is likely to present a too optimistic picture of what is available at the remaining sites.

An analysis of fifteen hydro-electric plants is shown in Table 3. The data in Column (1) through Column (7) are taken from various Annual Reports of the Federal Power Commission, and are used to compute the entries in the remaining columns. The assumptions made are stated in the footnotes.

An examination of Table 3 discloses: First, the extremely high cost per kilowatt of primary capacity even among the lowest-cost projects of the fifteen analyzed; and, second, the wide variation between the highest and lowest costs. The unit costs on the basis of primary capacity are perhaps somewhat higher than they would have been if additional capacity beyond the primary had not been installed. However, the cost per kilowatt on an installed-capacity basis is still very high, running from $124 to $348. The fixed charges show variations from 0.342 ct per kw-hr to 4.56 cts per kw-hr on a 90% load-factor basis, and from 0.380 ct per kw-hr to 1.067 cts per kw-hr on a 40% load-factor basis.

All this definitely shows: (1) That hydro-electric energy, far from being "free," involves fixed charges that, in many cases, are well in excess of the fixed charges and operating costs of steam power; (2) that the costs are not predictable from general experience and knowledge of the art of generation, but depend on such factors as head, topography, stream flow, and location; and (3) that hydro-electric plants, in general, as a source of primary power, are out of the economic range. The true economic function of hydro-electric power is, without doubt, to supplement steam power. This point is covered in the paper by Mr. Justin.

Operating Cost of Hydro-Electric Plants.—The cost of operating hydro-electric plants represents only a small portion of the total cost, the preponderant portion being the fixed charges on investment, as previously discussed. Operating costs at a particular plant are more or less a fixed item and substantially independent of the kilowatt-hours produced. A recent study of operation costs of plants of various sizes is summarized in Table 4. It will be seen that the median value on the basis of the annual cost per kilowatt installed varies from $2.80 to $0.68, depending upon the size of the plant. The maximum and minimum values are $8.00 and $0.50 per kw per yr, respectively. Based upon an annual load factor of 40%, this represents a kilowatt-hour cost of from 0.8 mill to a low of 0.19 mill.

TABLE 4.—OPERATING COSTS OF HYDRO-ELECTRIC PLANTS

Installed capacity, in kilowatts	Operating Costs, in Dollars, per Kilowatt of Installed Capacity per Year			Median value of operating costs, in mills, per kilowatt-hour at 40% annual load factor
	High	Low	Median	
Up to 10 000	8.00	0.90	2.80	0.80
10 000 to 25 000	3.18	0.70	1.40	0.40
25 000 to 50 000	3.00	0.60	0.98	0.28
50 000 to 100 000	1.50	0.60	0.75	0.21
100 000 to 200 000	1.65	0.50	0.68	0.19

Steam Power Investment Costs.—The general range of costs of steam plants has been fairly well established. L. W. W. Morrow found[20] that in one group of sixteen plants built between 1924 and 1927, the range in cost, including step-up sub-station equipment, was from \$101 to \$180 per kw, the average being \$139 per kw, and in another group of sixteen plants, built since 1927, the range was from \$82.50 to \$145 per kw, with an average cost of \$114 per kw. From other data in possession of the writer on an even greater number of plants, the range of costs for steam plants built prior to 1932 was found to be from \$85 to \$140 per kw of installed capacity, with the greatest number of plants comprising the greatest amount of installed capacity falling within the range of from \$100 to \$110 per kw. There are some "sports" below and above the limits of \$85 and \$140. One plant is reported to have cost less than \$70 per kw, but this was an unusual situation; in a power system supplied in the main by hydro-electric plants, economy in steam generation was properly sacrificed to some extent in order to obtain the maximum capacity at the minimum cost. Also, in this as in a number of other very low-cost installations, the so-called "framework" features that normally are associated with a steam-power development were omitted.

The wide variation in costs thus found makes it apparent that no rational deduction can be drawn as to the "average" cost per kilowatt of steam plants. Therefore, in preparing Fig. 30, the writer has used capital costs per kilowatt of \$100, \$115, and \$130, as being representative of the costs of existing steam power plants. The annual fixed charges forming part of the total cost data in Fig. 30 were based on these capital costs. No separate investment values are plotted in Fig. 30, the two sets of data actually plotted being operating costs and total costs.

Steam Power Plant Operating Costs.—The operating costs of a steam electric power plant, unlike those of a hydro-electric power plant, form a large part of the total cost of generating. This is illustrated by Fig. 30. In this diagram the three upper curves, A_M, B_M, and C_M, show the total cost of generation at annual load factors from 10 to 90 per cent. These curves were obtained by adding the annual fixed charges per kilowatt-hour calculated at a rate of 12.75% to the annual operating costs per kilowatt-hour. An analysis of steam power plant operating costs—fuel, labor, and maintenance—discloses that for the same load factor the variations in cost per kilowatt-hour are just as wide as the investment cost.

The central station industry of the United States operates approximately 22 000 000 kw of steam generating capacity. Of this amount, more than 11 000 000 kw were installed in the years between 1923 and 1930. In that period great strides were made in the art of steam power generation, affecting the efficiency in the use of fuel and labor. This 11 000 000 kw of capacity, being the newest and presumably the most efficient, is undoubtedly generating the bulk of the power sold to-day. However, it would be just as fallacious to say that the cost in the latest plant is representative of operating costs as it would be to say that the costs in the earliest plant were representative. The answer lies somewhere in between. Curve M is an attempt to show the average operating cost and was derived after study of the costs of many representative plants.

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[20] *Electrical World*, October 27, 1928, and November 23, 1935.

The curves in Fig. 30 show clearly the effect of load factor on costs. At low load factors, say 30%, operating costs constitute only a little more than 33% of the total cost (using $115 as the capital cost), while at 80% load factor, operating costs make up 50% of the total cost.

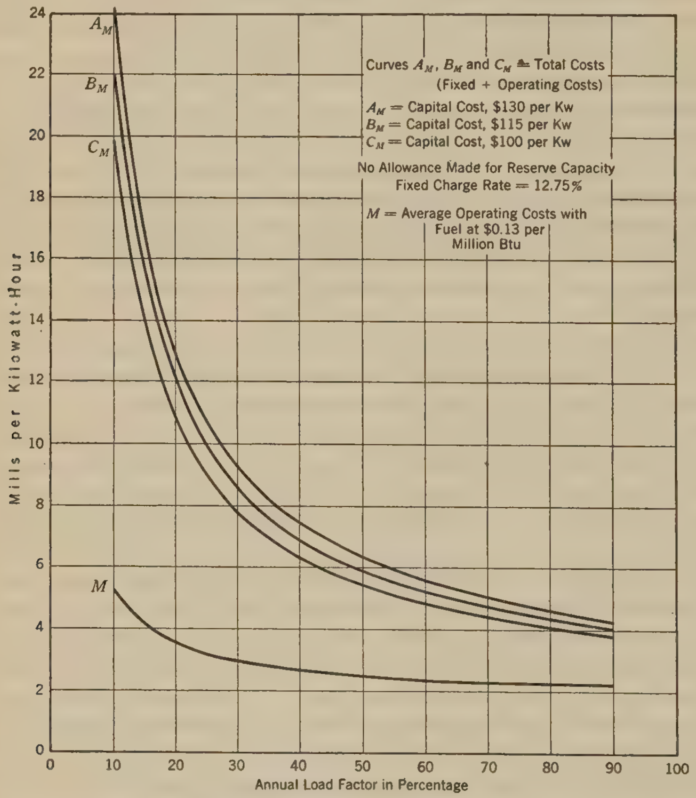

FIG. 30.—GENERATING COST OF STEAM-ELECTRIC STATIONS. THESE CURVES SHOW BASE-GENERATING COSTS AT GENERATING STATION

Cost of Transmission.—One of the most misunderstood factors in the cost of power is the cost of transmission. Those who have studied the problem closely and thoroughly understand the economics of power supply have been for the most part unable to find any sound economical basis for long-distance transmission of power. In general, power is most economical when generated nearest the load, and beyond a certain distance transmission definitely is not an economical proposition. Except in isolated cases, such as those where fuel is not easily available and sources of hydro-electric power are remote from the load, long-distance transmission of power is not practical. Hence, in economically developed supply systems transmission has been used largely as a means of integrating relatively small loads into reasonable blocks in order to take advantage of their diversity and to make possible economical generation within a limited load area.

Unfortunately, the collection of specific data on this subject is extremely difficult. An attempt is made herein, however, to present some definite data, even though they are approximate.

In Fig. 31 the cost of transmission lines, in dollars per kilowatt of capacity, is plotted against distance. Two points in connection with this curve are worthy of note: First, the tendency of the cost per kilowatt to flatten out as distance is materially decreased (this is due to the heavy cost of terminal equipment); and, second, the rapid climb of the curve, at distances of more than 200 miles, to costs approximating those of modern steam plants.

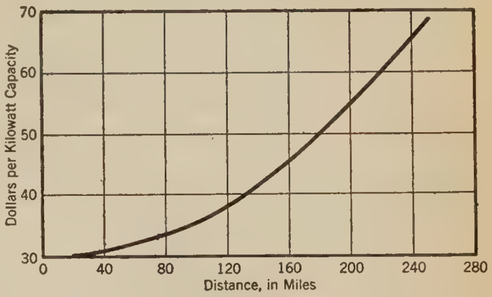

FIG. 31.—COST OF TRANSMISSION LINES

In Fig. 32 there is plotted a series of curves showing cost of transmission, in mills per kilowatt-hour, against transmission distance, in miles, for load factors of from 30 to 90 per cent. Fixed charges, taken at 12.25%, include 0.75% for operation and maintenance. The interesting point in connection with these curves is the rapid increase in the cost of transmission with increase in distance and, particularly, with decrease in load factor. Thus, at a load factor of 50% and a distance of 250 miles, the cost of transmission per kilowatt-hour is approximately that of the operating cost of generation alone in existing modern steam plant practice (see Fig. 30).

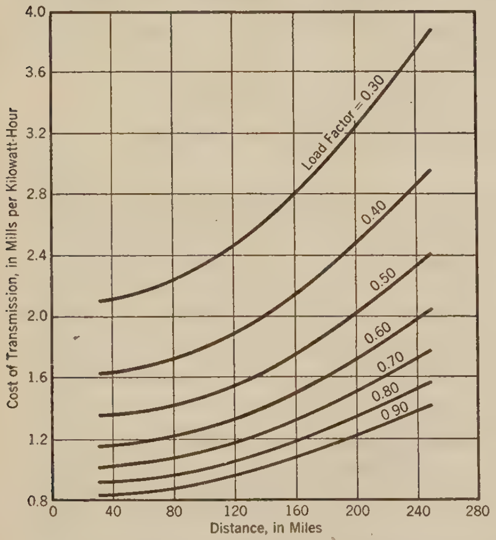

FIG. 32.—COST OF TRANSMITTING ELECTRICAL ENERGY, INCLUDING CHARGES ON TERMINAL EQUIPMENT

In Fig. 33 average freight rates per net ton of coal are plotted against airline distances. These rates are based on published freight tariffs as of October, 1936, for the shipment of coal from typical coal-producing areas to coal-consuming destinations. From these data, calculations have been made of the cost, in mills per kilowatt-hour, of transmitting energy, either as coal by freight or as electric energy. The assumption is made that coal costs $1.50 per ton and that 1 lb of this coal will produce 1 kw-hr, and that fixed charges on transmission facilities are 12.25 per cent. Losses of 10% for the transmission of electric energy have been included. The results are shown graphically in Fig. 34.

It will be noticed that except for very high load factor operation at distances of 140 miles, or more, freight transmission is more economical than electrical transmission. At load factors of 50% and less, the economy of freight transmission is particularly great. As distances are shortened a point is reached where the choice between freight and electrical transmission cannot be made solely by comparing transmission costs, since the location of the steam station (with regard to condensing water, in particular) is a major consideration: but where choice is at all free, it is obvious that it should be in almost every case in favor of freight as against electrical transmission. The actual economic distance will vary, of course, somewhat with local conditions, particularly as regards the facilities for supplying condensing water. In this connection, it is

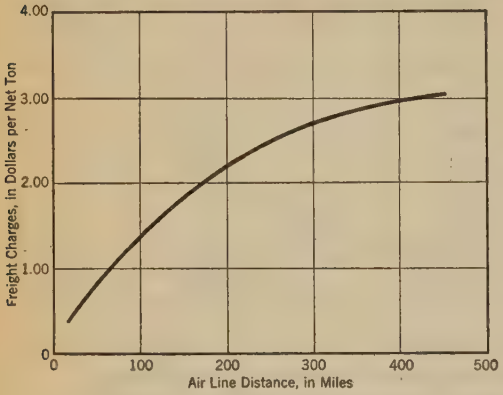

FIG. 33.—COAL FREIGHT RATES

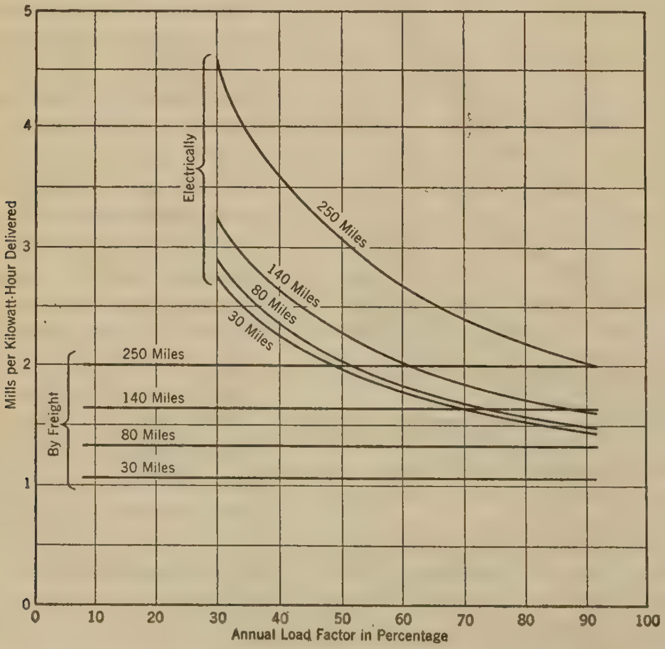

FIG. 34.—COMPARISON OF COST OF TRANSMISSION OF ENERGY BY FREIGHT (COAL) AND ELECTRICALLY

interesting to note that on the American Gas and Electric Company's inter-connected system, extending over nine States and serving about 1 200 comparatively small communities, the actual mean weighted distance of transmission under normal conditions is less than 60 miles—notwithstanding that conditions over a large part of the area (Ohio and Indiana) are extremely unfavorable from the standpoint of availability of condensing water. This definitely illustrates the fact that although inter-connection may be economically sound, long-distance transmission has no such solid economic foundation.

Cost of Distribution.—A discussion of the cost of distribution is beyond the scope of this paper. However, the point needs to be definitely impressed that, at least for residential and general factory use, the cost of distribution is the major cost of energy, despite many recent improvements in distribution facilities. In Fig. 35[21] is shown a nest of curves giving the cost of distribution, in cents per kilowatt-hour (excluding the cost of energy), for different customer densities and for average customer use of from 200 to 2 000 kw-hr per yr. It will be seen that as customer use increases these costs come down to a fairly reasonable figure, running to a low at 2 000 kw-hr of from 1 ct to 2 cts, depending upon the customer density. However, as the customer use decreases, this cost may run to as high as 10 to 20 cts per kw-hr, at very low customer densities and customer use.

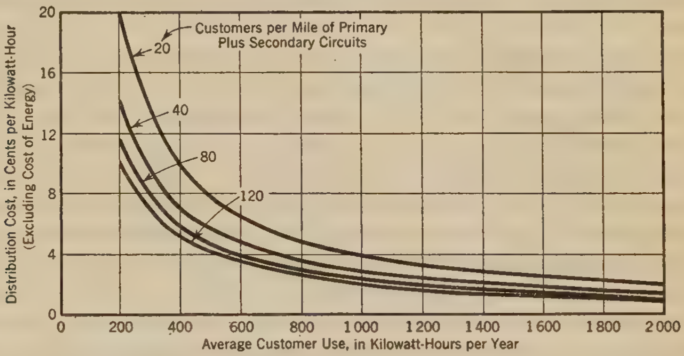

FIG. 35.—RELATIONSHIP BETWEEN CUSTOMER USE AND COST OF DISTRIBUTION FOR VARIOUS CUSTOMER DENSITIES

Total Cost of Generation.—By taking the total cost of generation as the cost of generation at the source, plus the cost of transmission to the load center, the present total cost of generation by steam plants can be obtained for any particular situation, from a combination of the data given in Figs. 30 and 32, within fairly definite limits. On the other hand, it must not be forgotten that in some respects these data have been obtained by simplifying the problem considerably beyond what is practical in an actual system.

For instance, if any electric power system is to furnish adequate and reliable service, it must provide reserve generating equipment. In Fig. 30, in developing the fixed charge item of cost, no allowance was made for reserve capacity.

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[21] "Rational Distribution of Electric Energy and Gas," by Norman R. Gibson, M. Am. Soc. C. E., World Power Conference, Washington, D. C., 1936.

In actual practice on an isolated power system with only one plant, as much as 50% of the installed capacity (100% reserve) may be required to insure continuous service. This may even double the cost of the firm capacity in the plant, that is, the capacity available for sale. Even in an inter-connected system having many generating plants, some part of the installed capacity is needed for reserve. The actual amount may vary from 15% to 30%, depending on the size of the units, the magnitude of the load served, and a number of other factors. This reduction in reserve is obtained by building inter-connecting transmission lines, and their cost must also be considered in any discussion of generating costs.

Likewise, in an actual system, transmission and distribution are frequently so closely interwoven—both lines and equipment being commonly used to perform a dual function—that an exact allocation of equipment and costs between transmission and distribution frequently becomes most difficult, if not impossible.

Nevertheless, making allowance for the simplification used, it will be noticed that for steam generation the outstanding factors are: (1) The fixed charges; (2) the production costs; and (3) the cost of transmission. At very light load factors the first item is the predominant one. As load factors rise, the production cost item comes more nearly in line with the fixed charge item, becoming approximately 60% of the total cost of power at 50% load factor and approximately 50% at 70% load factor. In large inter-connected systems serving a diversified load, annual load factors are on the order of 55 per cent. In isolated systems, serving small communities, where the load is principally that of residential customers, annual load factors are much lower, averaging about 35 per cent. The transmission cost at 50% load factor for a distance of 200 miles may substantially equal the cost of production, and for greater distances may well exceed it.

It is extremely difficult and perhaps unfair to attempt a direct comparison of costs between a single hydro-electric development and a steam plant. On the other hand, this assertion itself discloses the weakness of the typical hydro-electric development, since as a general rule no hydro-electric plant is self-sufficient. This being the case it is impossible to determine its value without considering the position it takes on the load curve, and its effect on costs of other producing sources. In general, this is a point that must be thoroughly considered before a decision as to economic feasibility of a particular hydro-electric project can actually be made.

Another way of stating this is that the hydro-electric plant does not have as much flexibility as regards load factor as the steam plant. The latter can be designed for use at high load factor, and assigned, successively, to positions on the load curve of lesser and lesser use as more economical steam plants are developed. Such shifting is impossible in the case of a hydro-electric plant, which, in general, can be developed economically only for a certain position on the load curve. Its economics are adversely affected when it is used on any other basis. In this connection the data on transmission costs shown in Fig. 32 are pertinent. When proper evaluation of the cost of transmission is made, it is obvious that the economics of hydro-electric generation are bound to be less and less sound as fuel sources are made available within reach of main load centers and as the more economical hydro-electric sites are exhausted.

FUTURE COSTS OF GENERATION OF ELECTRICAL ENERGY

Trends in Steam Generation.—From the analysis of costs previously given, it appears that one of the most important factors in lowering steam plant generation costs is the reduction of investment costs. The trend in cost of generating equipment has been definitely downward, although admittedly the trend curve has not had a very steep slope. However, there is a definite promise in the direction of, first, higher steam temperatures; and second, the use of higher speeds (which will eventually mean more efficient use of materials).

Installation of larger steam generators, in particular, offers material opportunities for reducing costs in the future. The economics of these large units, however, is contingent upon increasing the availability factors of steam generating units. Recent installations have shown availabilities as great as 94.5%, but a definite step will be made when steam generating equipment can be consistently designed and operated at availability factors of 95 per cent.[22]

Another factor tending to reduce generation cost is superposition in existing steam plants. As a general rule, however, superposition has to be handled in fairly large blocks if it is to be economical. In other words, it is of value primarily where the base loads are heavy and the load factors high. Fortunately, increased co-ordination in the generation programs of inter-connected systems can bring about an economical application of super-position in many cases where an independent program may not prove entirely feasible.

Trends in Hydro-Electric Installations.—There are no present indications of any developments that would make future hydro-electric installations more attractive economically than those already in existence. If anything, the natural trend will be toward less economical installations. The most economical sites were developed long ago, and those that have been exploited recently have, in general, been less attractive economically than those that have preceded them. Nature created only one Niagara and that has been developed.

Trends in Transmission.—The art of transmission has been improved to a remarkable degree since the World War. Continuity standards have been improved; stability limits of alternating-current transmission lines have been extended; and the lightning hazard has been brought under control. Nevertheless, a great many difficulties in the path of completely reliable transmission have not been removed, and the costs as shown herein are still high and non-competitive with other forms of transmission at long distances. Some publicity has been given to direct-current transmission, but at present the idea is still very much in the experimental stage and its economics is totally unknown. In general, although it may be stated that improvements in transmission may be expected in the years to come, no major changes are indicated at the present time.

Trends in Distribution.—The reductions in the distribution system costs made in recent years have resulted from intensive application of increased knowledge, and from the use of engineering methods and economic principles in design. Progress has been slow, but steady. There is no reason to believe that progress in the near future will be revolutionary, but there is every reason to believe that gradual progress will eventually result in lower costs than exist at present (1937).

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[22] "The Value of Proper Furnace Equipment to Power Plant Economy," by M. K. Drewry, Mid West Power Conference, Chicago, 1936.

Trends in Over-All Cost of Electrical Energy.—The future over-all costs of electrical energy, whether in generation or in generation plus transmission and distribution, can definitely be expected to be downward, although no rapid downward steps are indicated. It appears definite that future power requirements will be met most economically by steam plants, located at a reasonable distance from load centers and equipped with large units, with inter-connecting transmission to integrate and combine small loads and to reinforce and "back up" neighboring systems. As for steam-plant design, further progress along present lines—particularly in the expansion of the present limits of pressure and temperature, and the raising of the availability factor of both steam and generating units to make possible a minimum of duplication of facilities—will bring the optimum results. Here, however, as in other lines of endeavor, it will be necessary to rely upon "the inevitability of gradualness."

SOCIAL IMPLICATIONS OF TECHNOLOGICAL TRENDS IN THE POWER INDUSTRY

By Ralph E. Freeman,[23] Esq.

Synopsis

The general effects of trends in energy generation on the level of living, unemployment, overproduction, and population distribution, are treated in this paper. A plea is made for a clear distinction between the business point of view and that of the social reformer.

The purpose of this paper is to discuss some of the economic aspects of the technical matters dealt with in the preceding papers.

To begin with, two generalizations may be drawn: First, the costs of energy generation are coming down, and in spite of many obstacles are likely to be lowered still further in the future; and, second, changes are taking place in the relative efficiency of generating energy by different methods, and the trend away from earlier methods still proceeds, moving in the direction of large inter-connected steam and hydro-electric generating plants and, to a less degree, under favorable conditions, in the direction of Diesel power.

It is hardly necessary to dwell upon what is the most obvious result of cheaper power, namely, increased economic well-being. One need only compare the standard of living of to-day with that which prevailed before the discovery of the energy latent in steam to realize the economic benefits of more efficient generation of energy. When a unit of energy is produced with less expenditure of human and material resources, the amount of consumable goods at the disposal of the community is augmented. The effect upon the individual depends, of course, upon the size of the population and the distribution of wealth; but, with no changes taking place in the number of people or in the manner in which wealth is shared among them, cheaper energy means greater material well-being for the individual members of the community.

In the minds of many people in the United States, however, the growing productive capacity of an economic machine driven by cheaper power has raised some terrifying possibilities. They fear a chronic condition of glut due to the incapacity of the markets to absorb the increased output. They believe that, largely as a result of improvements in the generation of energy, the United States is entering upon a new phase of development, another industrial revolution. They claim that, whereas once the country suffered from a scarcity of goods, to-day it is overflowing with plenty. The old economic problem was how to produce enough; the new one is how to dispose of all that can be produced.

This dread of superabundance is groundless. The scientists and engineers who are engaged in raising the efficiency of the power machine system need not be disturbed. Overproduction is an economic phenomenon, not a physical one. Overproduction is unprofitable production; surplus capacity is capacity which cannot be utilized without loss. The goods now glutting the markets of many countries would immediately vanish were they offered for sale at low enough prices. At some price or other buyers could be found for all the wheat the United States is capable of growing, and for all the output of all its steel mills running at full capacity. That this drastic procedure is not feasible is chiefly because of the relatively high prices the farmers and steel-makers have to pay for their materials, supplies, equipment, etc. Congested markets are the result of disjointed prices, and these, in turn, are attributable to unbalanced relationships between one industry and another.

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To dispose thoroughly of the naive conception of a world that is spilling over with abundance would take much more space than is allotted to this paper. The delusion of plenty would be harmless were it not for the fact that it obscures the fundamental nature of the economic problem. It leads people to think of this problem in terms which clearly imply that a solution is to be found in an expansion of money incomes or in a contraction of the volume of goods produced. When people get the notion that depressions are attributable to under-consumption which can be avoided merely by distributing to the poor an increased supply of paper money, they are easily mobilized in support of ill-conceived currency, pension, and "social-dividend" schemes. When business troubles are regarded as the result of excess production, there is an agitation for reduction of output by drastic curtailment of working hours, by checking the introduction of technical improvements, and other restrictive measures.

Conditions may exist that call for additional media of exchange or less productive capacity in certain industries or even for a shorter work-day; but the fact that at times such measures may be beneficial does not imply that the basic cause of economic disturbance lies in a scarcity of money or in a superfluity of goods. At other times there may be too much money available or, in certain industries, the output of goods may be too small. Indeed, as a general rule, the ultimate source of disorder can be traced to unbalanced relationships between one industry and another that are quite independent of the total volume of output or of the total amount of currency in use. Overproduction, in other words, is a symptom of economic disease rather than its cause.

Another important economic result of generating energy at reduced cost is the further displacement of human labor in industry. Since the Eighteenth Century, when the invention of the steam engine enabled Man for the first time to make non-human energy work for him at his chosen time and place, human physical labor as the power supply of industry has been virtually eliminated in many fields. In other types of production, however, the elimination of human power awaits the invention of new machines to handle refractory materials and to deal with complex assemblies of parts, or it awaits the arrival of that relation between labor cost and energy cost which will make it profitable for the producer to substitute machine power. There is no doubt that a cheapening of non-human energy will hasten this substitution, especially if wages remain at their present levels or if, as seems likely, they continue to advance.

On the whole, such a development would be socially beneficial. Where mechanization is still in the half-and-half stage, where invention and research have absorbed only partly the old handicraft processes into the new energy system, a great deal of power has still to be provided by the human element. This makes the worker virtually a part of the machine. He is compelled to perform semi-automatic operations which are stupefyingly monotonous, often at a pace set by the machine which demands a nerve-wracking degree of strained concentration. When the difficulties in the way of complete automatism are overcome, the daily lives of many workmen will be relieved of much deadening drudgery.

Experience with the effects of reductions in the cost of energy generation in the past indicates that there is no need to fear a permanent addition to unemployment because of such developments. Cheaper non-human energy, although it displaces labor as the physical power supply, increases the demand for supervisory and other forms of what may be termed mental labor. Moreover, it enables men and women to leave the mechanized industries and enter occupations of the human service type, trade, finance, amusement, and various kinds of clerical and professional work. There is a real problem, of course, in the re-employment of the man thrown out of a job; there are obstacles to the movement of labor from one occupation to another which are serious, particularly in the case of highly specialized workers; but history clearly demonstrates that when a thousand men are displaced by an improvement in energy generation an additional thousand men are not thereby permanently placed on the relief rolls. A compensatory demand for labor arises from the enlarged social income made available by the reduction of power cost.

In addition to the economic problems arising from the trend toward cheaper generation of energy, there are many interesting economic implications in the changes that are taking place in the relative efficiency of generating energy by different means.

During the Eighteenth and early Nineteenth Centuries the direct application of steam energy brought about far-reaching transformations in the psychology of the worker and his relation to his employer. It resulted in a concentration of production because the increasingly complicated and expensive machines required greater capital and larger productive units; and since the machine compels men to limit themselves to certain branches of production, it resulted in greater specialization. It brought mass production that destroyed the individual character of the product and created uniformity in the wants of consumers. Mechanization resulting from steam power not only extended the period of production, introducing additional intermediate stages, but also increased the proportion of capital goods in the economic structure, and both these factors have enlarged the range of economic fluctuations. Mechanization has enhanced overhead costs relatively to variable costs, thereby introducing rigidities into the price structure which further accentuate industrial booms and depressions.

Are the recent changes in energy generation likely to affect any of these conditions? Consider, for example, the centralization of production in large plants in congested urban centers.

Steam power, if not converted into electrical energy, must be used where made. It is available chiefly, therefore, to the large manufacturing plant which produces power on a large scale; but electrical power can be distributed to points at a distance from the generating station, and made available at low cost to small producing units. Progress in electrification, therefore, may remove the advantage that steam power gives to large manufacturing units.

This applies also to the development of the Diesel engine, in which the economical size is not such as to favor the large generating unit. If cost conditions in a given industry and locality permit the profitable use of Diesel engines of relatively small capacity, small plants may be able to compete successfully with large ones. Whether such situations are now common, or whether there is a trend in this direction, is a matter for competent engineers to determine.

As to whether or not developments in energy generation will in fact lead to a producing system of small plants dispersed over the country, it is not yet possible to make a safe prediction. Although steam power was one of the influences which massed industry into urban centers, the concentration of production is not due to power considerations alone. Marketing facilities, the proximity of suitable labor and raw materials, and various other factors are important in determining the location of industrial plants. Where such factors as these constitute an important part of the cost of production, the electrical development is not likely to effect a great deal of decentralization.

In 1935, Mr. Daniel B. Creamer published the results of a study of population movements, entitled "Is Industry Decentralizing?"[24] This *Bulletin* discloses the major trends in the location of manufacturing industries in the United States from 1899 to 1933. It presents evidence of a moderate amount of decentralization—not so much a scattering of factories up and down the length of the land, but rather a spreading of industries from the great urban centers to their suburbs. Mr. Creamer indicates that, in many cases, particularly in the textile and boot and shoe industries, this movement has been due primarily to the possibility of hiring cheaper labor in new locations. On the other hand, it is to be observed that this migration of industry would have been impracticable in many instances had it not been possible to secure the necessary power in the new locations at reasonably low rates. In other words, although it is doubtful whether power costs in most industries are large enough to be a determining factor, they become of great importance in conjunction with other factors.

There is no doubt that a thoroughgoing diffusion or dispersion of industry would create another revolution in American social life, and in recent years the movement in that direction has given rise to an agitation that the Government should do something to encourage it. This agitation has been reflected in recent governmental policy. Various resettlement proposals and power projects have won support from those whose imaginations have been caught by the vision of a de-urbanized world. In many ways the ideal is attractive. There is, however, a danger that projects may be initiated that are out of harmony with fundamental economic and engineering principles, and that this will result in financial loss not only to the supporters of the new projects, but also to those enterprises with which those projects compete. Financial cost should not be the only guide in these matters; but before public subsidies are expended, careful engineering and economic investigation should be made to determine the cost of the expected social benefits.

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[24] "Study of Population Re-Distribution," *Bulletin No. 3*, Univ. of Pennsylvania Press, 1935.

As long as the economic regime is not centrally planned and controlled, productive activities must conform in large measure to the dictates of prices and costs. To ignore the monetary signals is to invite chaos. It must be assumed that, in the main, prices are reliable guides for the organization of the national economy. It must be assumed that changes in the amount of money offered by buyers (which influence prices) reflect changes in the wants of the community and that these wants should be satisfied. It must be assumed that technical changes (which also influence prices) reflect changes in the physical cost of production and that Society should produce its goods and services with the minimum expenditure of human effort and materials, economizing those forms that are relatively scarce.

This is the method by which, in the main, the economic activities of the United States are organized. It is the regime under which the people have advanced to their present condition of wealth. Moreover, it can be demonstrated that in many instances a departure from this means of organizing the nation's productive resources has reduced the efficiency of the economic machine. The attempts of private monopolies or agencies of Government to set prices at variance with fundamental technical or demand conditions have often been followed by serious maladjustment.

Of course, it is permissible to go behind the price structure to the ultimate conditions of physical cost and human demand and inquire whether prices reflect accurately and faithfully the relative scarcity of resources and the wants of the community as a whole. Thus, it may be argued, on the technical side, that relative prices reflect the relative scarcity of the material means of production in the present and take insufficient account of the future. In the case of energy generation, it has been argued that the use of petroleum involves the depletion of resources that cannot be replaced, whereas the harnessing of falling water is open to no such objection. Should the effects of costs and prices as determined under a private profit-seeking regime, therefore, be modified, and hydro-generation of energy be subsidized in the interests of conserving natural resources? In putting such a policy into effect much caution would be required. It is hazardous to base social plans upon future technical developments which cannot be clearly foreseen.

There is also the question of the social cost of changing techniques. When one method of energy generation displaces another suddenly and over a wide field, obsolete plant is likely to be created, and men are thrown out of work. Idle men and idle machines constitute a cost that is not taken into account in the price system, but it is a cost, nevertheless. Should this social cost be mitigated by governmental action regulating the rate of technical change or imposing a financial burden on the source of disturbance? Changes in the technique of energy generation may affect wide areas of the country and large sections of the population. Should one class or one region bear the cost of technical progress? What should be done about it?

Prices reflect the demand of those who have money, but they do not accurately express the wants or desires of the community as a whole. The price of electrical energy, for example, may be too great to enable an important group of consumers to enjoy its advantages. Should the farmers be given especially low rates to permit a more extensive use of electric power on the farms? Should the Government subsidize power projects to foster agricultural development? Should power projects be constructed by the Government in agricultural regions where private enterprise finds it unprofitable to operate? Questions such as these cannot be excluded from a discussion of the economics of energy generation.

In current controversy, however, there is much confusion and evasion of issues. Lower rates may be advocated on the ground that a greater consumption will result and that enlarged output and sale will permit lower unit costs. That is an economic argument valid within the exchange-price system; but when lower rates are advocated in order to benefit certain types of consumers or to stimulate certain other industries—that is an argument that may be valid on human or social grounds but is outside the economic plane of prices and costs. In considering the problem, these two types of approach are not always kept clearly in mind. Moreover, because accurate quantitative measurement and determination is much easier on the cost-price level than on the social level, many people (scientists and engineers in particular) refuse to admit the validity of the latter. The result is that in discussing these matters there is either a confusion of the issues involved or no meeting of minds by those who participate in the controversy.

ELECTRIC POWER IN ECONOMIC PERSPECTIVE

By B. Alden Thresher,[25] Esq.

Synopsis

The bitterness of political controversy over public utilities has obscured the real determinants of the problem, which are technical and economic. The industry might be conducted successfully under any one of many types of organization, public, private, and mixed. Regulation in the United States has placed an undue emphasis on valuations of physical property quickly outmoded by obsolescence. Both the public and private sectors of the industry are seeking the same basic solution under forms which are superficially different.

The long and bitter controversy over the regulation and control of electric power has obscured certain aspects of the problem which are of abiding and fundamental importance. The issue between public and private operation, between more regulation and less regulation, between the Securities and Exchange Commission and the holding companies—in fact, the long, running fight which has accompanied the entire period of growth in this industry—is, when seen in true perspective, a mere incident in a larger development. That larger development consists of the simultaneous growth of supply and demand. On the supply side is cheap and abundant energy flexibly distributed and readily controlled. On the demand side are marked changes in industrial organization connected with the use of cheap power in industry, and even more spectacular changes in social habits as the retail consumer adjusts himself to power in the home.

The progress of the Power Industry is not so dependent on the winds of political change as many seem to think. Behind the surface controversy over forms of organization and regulation lie the real determinants of the problem—the technical limitations of economic production on the one hand, and the social and industrial nature of the demand for electric power on the other. It is easy to take sides in the current controversy, but taking sides is a mark of the pre-scientific stage of the problem. One might, for example, deplore the abuses of monopoly, detailing the long catalogue of evils that unbounded financial peace has brought upon the industry; or one might elaborate the evils of public control, and adduce examples of the "dead hand" of Government throttling private initiative. Either of these arguments would be in the current fashion; but neither would be to the point. The real issues lie deeper. They arise out of the nature of the industry itself. They will persist under public or private control, or under a mixture of the two; and they will be settled not by the choice between these rival principles, but by the economic logic of the Power Industry itself. More important than the choice between public and private operation is the choice between a stunted power industry and a well-developed one, economically adjusted to the society in which it operates.

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Political scientists will disagree with this statement. They will argue that the issue between collectivism and individualism is fundamental and insistent. Constitutional lawyers will disagree. They will hold that the issue is between two systems of government, two opposed concepts of the tenure of rights legitimately acquired. Those in active control of the industry and immersed in its current controversies will disagree. They see their interests threatened, and few indeed are the men who can take the long view when their investments are at stake. Politicians will disagree, for the man who has to be re-elected is the last man in the world to cultivate perspective on anything.

It has often been stated that the fluctuations of the business cycle are roughly parallel in widely separated countries. Depression overtakes nations whose economic policies are widely different from each other. Similarly, when the forces of recovery are ripe, recovery appears, whether the Government policy of the moment be of the interventionist type or whether private business be left with little control.

In the development of the electric power industry throughout the world an analogous effect is evident. The industry has followed the same general cycle of technical development and consumer acceptance in all countries, although the legal, financial, and political forms under which it has developed differ widely.

In France, as in the United States, private ownership is the rule, public ownership the exception. Interestingly enough, the French power enterprises have managed to survive and attract capital without any of the so-called constitutional safeguards of which so much is made in the United States. The Parliament legislates without restraint in the matter of regulation.

In Sweden, public and private enterprise have managed to co-exist amicably. As Mr. O. C. Hormell has put it:[26]

> "While the state owns and operates an increasingly large proportion of the generating and distributing plants, the state has not taken (or been able to take) for itself any marked privileges or advantages with regard to its power development, over and above those granted to private enterprises."

In Sweden, as in Great Britain, a nation-wide distribution system yields the economies of inter-connection, and permits the public authority to control and direct power development, yet avoids the rancorous rivalry that has marred the relations between public and private agencies in the United States.

In Germany, the long-established tradition of municipal socialism in public utilities, with its roots in the Nineteenth Century, has not prevented a healthy development of private enterprises. In fact, one finds, in the post-war period, public enterprises accepting private capital and a mixed directorship.[27] In Switzerland, likewise, there is an interesting mixture of Federal, cantonal, municipal, and private ownership. The lion and the lamb have lain down together in a way that surprises Americans accustomed to the Corcyran fury of their own power controversy.

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[26] Rept., New York State Committee on Revision of the Public Service Commissions Law, 1930, p. 468.

[27] *Loc. cit.*, p. 406.

This brief summary, which might be greatly extended, suggests that there are more fundamental economic forces at work than are contemplated in the terms of the controversy as it is usually presented. Power can be distributed successfully under a wide variety of legal and economic forms. Where there is a steady and growing demand for a service, and the technical means and the labor and capital are available, the process of production somehow gets itself organized. Whether this organization takes the form of public or private enterprise is not such a fundamental fact as one might suppose. Either form, for survival, must adapt itself to basic economic realities of the industry. In all probability, both forms will persist side by side for many years, and each will gain through the presence of the other. Frequently, the claim is made that regulation has broken down. It would be more accurate to state that the concepts on which regulation in the United States is based, have never caught up with the swift development of that technical and social complex, the "Power Industry."

The economic problem involved may be clarified by asking what characteristics in any industry facilitate a smooth and continuous adjustment of supply and demand, so that at any moment an approximation to economic equilibrium can be maintained. The following ideal conditions may be advanced as making for a smooth and prompt adjustment to economic changes:

(1) The commodity sold should be reasonably uniform and standardized.

(2) It should be sold largely to ultimate consumers, rather than to producers of other goods.

(3) It should be capable of storage.

(4) Entrance into the industry should be free and should not require a heavy investment.

(5) Abandonment of production or reduction of output should be possible without heavy loss.

(6) (A corollary of Conditions (4) and (5).) Cost should be predominantly direct, rather than of an overhead nature.

(7) The industry should be in a state of steady, but not too rapid, growth.

(8) Technical progress should not be so rapid that obsolescence becomes a serious problem.

(9) The surrounding economy should be operating with an approximately stable price level.

These conditions are obviously at variance with the actualities of the electric power industry. The commodity sold is not primarily energy, but a combination of energy, usually in small amounts, with varying amounts of readiness-to-serve, and the services of a distribution plant whose importance economically far outweighs that of the energy itself. Storage is not practicable on a major scale. The very nature of the industry prohibits freedom of enterprise in the full sense, and the fixed plant is enormous in proportion to annual gross business. Downward adjustments of output are difficult, and are likely to entail heavy loss. The tendency for overhead costs to increase, as compared with direct costs, is exceptionally marked. The rate of technical development has been so rapid as to impose a heavy burden of obsolescence on the industry.

All these departures from the theoretical ideal may be classified under two heads: First, rigidities in the market structure, centering upon the problem of overhead cost allocation, and leading to a condition of natural monopoly enforced by the technical conditions of production; and, second, dynamic elements; that is, rapid qualitative change in technique and in demand.

Some examples of the rigidity caused by the difficulty of cost allocation may be mentioned briefly. First, there is the definition of the unit or commodity sold. Most problems of rate structures involve this basic question. It may be seriously questioned whether an energy unit is an appropriate one. The rich variety of multi-part rates, the attempts to separate stand-by and readiness-to-serve elements, and customer costs and distribution costs, indicate that the commodities sold often consist chiefly of elements of access and availability.

More important still is the group of maladjustments arising out of heavy overhead costs, which, in turn, are traceable to the predominance of fixed plant in the total investment. The most obvious problem under this head is that of depreciation and operating cost. In a body of fixed plant which is undergoing rapid expansion the effect of inadequate depreciation reserves can be long postponed. As long as expansion continues, retirements continue abnormally light in proportion to total investments, and true costs may long be under-estimated. In this respect a day of reckoning is surely in store for many companies otherwise conservatively managed.

It is in the matter of capital charges and rate of return that controversy has been most acute. The history of public utility valuation contains a lesson which is important not only for the power industries, but for many others as well. In this controversy both sides have taken the line of expediency and immediate self-interest, rather than that of long-range wisdom.

In 1898, when the case of Smith *v.* Ames was decided at the end of two decades of falling prices, consumers, represented by William Jennings Bryan, argued that valuation of utility property should follow reproduction cost. This, under the circumstances, led, of course, to a lower rate base. Producers (in that instance, the railroad companies) clung tenaciously to the higher original cost of construction, as representing their idea of justice.

In the long upward swing of the price level to 1920 these positions were sharply reversed. The growing utility companies went on record in great detail as favoring reproduction cost, both in practice and in theory. It came to be assumed almost as a law of Nature (so short is human memory) that a reproduction-cost rate base is always higher than the original cost.

Another complete reversal of positions might, therefore, have been expected in the price liquidation in the years following 1930. This reversal, however, was only partial and incomplete. The constitutional guaranties which seemed to hedge the reproduction-cost concept, gave to the producing interests a different kind of advantage from that arising from mere price changes.

In an industry in which costs are falling rapidly, any device that delays a normal price readjustment may be expected to offer great temporary advantage to producers. The constitutional guaranty with which the Supreme Court has hedged property devoted to public use supplied just such a device. Public clamor was partly quieted by partial rate reduction, but largely resisted by resort to the Courts. A fair return on the valuation was increased by hypothetical calculations, which assumed reproduction, at great expense, of plant which no one could logically wish to reproduce. This is not to attack the motives of the power companies. They acted in good faith in seeking legitimately an advantage that was legally and morally theirs. Their attorneys were not hypocritical in pressing for the last penny. More accurately, they were honest, but short-sighted.

As the matter has turned out, theirs was a Pyrrhic victory. The rate reductions, which were resisted so strenuously, have mostly been conceded, not in deference to any legal right of the consuming interest, but rather out of sheer expediency. The technical and economic situation in the industry has swept aside a body of legal principle. It is not that the legal principle was unjust or reactionary; in fact, it represented an honest effort by able men to do justice to all parties in a complex situation. The error lay in the failure to take account of the economic effect of technical progress in an industry in which technical progress has rushed ahead by leaps and bounds. The year 1930 found the power companies strongly entrenched in an unassailable legal position, honestly won. The year 1936 found them conceding more ground than they had gained; and they were doing so in response to economic and technological forces stronger than any body of legal rights.

This aspect of the problem must be left for a moment to consider other types of rigidity that distort the free play of market forces. The Power Industry, in common with transportation, is a proper field for class price, or, more properly, price discrimination. It should be understood that this is not a term of disparagement. Discrimination may be defined as "differences of price not attributable to differences in cost."

Discrimination is possible: (1) When there exists a monopoly, or at least a sufficient approach to monopoly to prevent competitors from breaking down a system of class prices; and (2) when the commodity sold is not susceptible of storage and resale (railroad freight rates are a classic example, and rate differences between main classes of electric power loads illustrate the same principle). What the public often fails to appreciate is that such discrimination actually benefits not merely the class of customers serving the low rates, but even the persons who apparently are discriminated against as well. In so far as a low rate brings in business not otherwise obtainable, it allows the burden of overhead to be distributed more widely, and hence prevents the high-rate business from carrying an even higher rate.

Finally, as a typical result of heavy overhead cost, one finds the phenomena of diversity and inter-connection, which have played a large part in power development. Here, again, the situation is completely at variance with that which prevails in the sale of most commodities. It is necessary to reckon not merely with the normal economies of large-scale production, but with those differences in timing that make it possible to improve the central station load factor by connecting loads of different characteristics. Both the class-price principle and the diversity principle make for a degree of economic solidarity among power customers that is not equalled in any other industry. The price Jones pays for a pair of shoes or a set of tires depends upon how many other people are willing to buy shoes or tires, but the price which Jones pays for electric energy depends not merely upon the number of other customers, but upon what time of day they use their power. Jones, furthermore, may actually benefit by having somebody else buy power cheaper than he does—although Jones would probably never be convinced of this, even if all the rate experts and economists in the world should try to demonstrate it to him.

The peculiar economies of load diversity, over and above those of mere large-scale generation of power, have led to regional inter-connection by several different approaches. It early became apparent that diversity was advantageous within a single community. To bring it about also between communities and regions has been one of the major objectives of consolidations and holding company developments since before the World War. The same logic underlies private contractual interchange agreements like the Connecticut Valley Power Exchange. The British "Grid" provides still another variant, with a public inter-connection system interposed between private power producers and retail distribution systems of which some are public and some private. The Grid thus serves as a stimulant to unified development of a national system, but leaves that system predominantly in private hands. It serves also as a regulative device by interposing the public authority as a middle-man, instead of, as in the United States, either a competitor or a policeman. Still another variant is found in the regional projects under wholly public auspices, like the Tennessee Valley Authority.

It has been well said that the human mind recognizes differences more easily than it perceives similarities. The four devices just listed are a case in point. Their relative merits are disputed so strongly that the fact that they are all expressions of a common principle is forgotten. What is really important is to get this principle recognized and utilized to the fullest extent; which device is used makes little difference.

The characteristics thus far mentioned are, so to speak, permanent differences which earmark the Power Industry as distinct from most others. Even more significant, however, is the dynamic phenomenon of rapid change, both in technique and utilization. In an industry where technical development is rapid, and utilization both by industry and the consumer is developing with equal rapidity, economic forces making for a normal equilibrium work too slowly. Costs are falling over the long period, but only with the aid of large new investments of capital. Demand is growing, but it grows faster where the desire for secure profit is tempered by willingness to postpone returns, so as to attract further business. It will be argued, however, that many other industries are in a similar condition of rapid change. Why should the Power Industry be singled out as being affected to an extraordinary degree by accelerated technical progress and by changes in the response of consumers?

The answer seems to lie in the presence of public regulation. The concepts that make up the accepted scheme of regulation in the United States are by their very nature static. The legal theory of an adequate return on the fair value of the investment is valid and sound for an industry in a stable condition. It provides, in effect, that "every tub shall stand on its own bottom"; but the moving equilibrium of a runner in rapid motion is a different phenomenon from that of a man standing still. The accepted theory laid down by the Courts finds support in the instinctive logic and sense of justice of any one who takes the trouble to reason out the problem. However, on every hand, it is seen how the technical and economic changes that keep the industry in constant ferment have already set these theories at naught.

The explanation is to be found in what has been called "the fallacy of misplaced concreteness." The Courts have put their trust in tangible equipment—that is, in property in the physical sense. It is a natural error, seeing how large, impressive, and indispensable is the physical plant necessary in this industry; but the error consists in not recognizing that the physical plant is only the passing embodiment of a technique that is undergoing rapid development. The concept of value envisaged by the Courts is of a property inherent in tangible objects; hence, the elaborate engineering appraisals that support "physical" valuations.

If there is one contribution that the economist should make above all others to the public utility problem, it is to demonstrate that value is not an inherent property dependent upon the dimensions or specifications of a tangible object, but an economic, hence partly psychological, phenomenon dependent upon conditions of use and demand. Obsolescence is not a physical but an economic process. It follows that in a situation where productive methods and productive technique are rapidly changing and improving, and where the nature of the demand is shifting, any attempt to set a value on a piece of property solely as a function of the physical nature of that property is doomed to failure. It is the misfortune of the utility industry that something as transitory and changing as its current technical methods must be embodied in fixed plant which is so permanent and expensive. This plant soon becomes a lifeless shell, when changes in method require a shift in the type of equipment.

The legal theories of regulation have never succeeded in grappling with this problem. The short-run result of such a situation has been to benefit the utility companies. Once rates are set, it has usually been possible to cut costs sufficiently that a considerable addition is made to the rate of return nominally earned on a given investment. Were this not true, all incentive would long since have been removed from the industry. The engineer can be thanked for thwarting the regulator and allowing the industry to adjust itself to the facts of social and economic change.

It seems fair to conclude that this situation is largely responsible for the contemporary emphasis on Federal power projects. It is shallow to regard the Federal program as merely punitive, or as evoked by the misdeeds and abuses of the power companies. It is true that there has been widespread exploitation and abuse, and, in this respect, the Power Industry has accurately followed the pattern set by the railroads a generation earlier. A wide margin of waste, misapplication of funds, and general inefficiency seem to be the normal price of a rapid expansion process. The phenomenal speed and effectiveness with which the railroad and power networks have overspread the continent have been offset by a heavy penalty of frenzied finance and exploitation. Similarly, the war effort of 1917 and the more recent effort to apply Federal funds to large-scale projects both led to colossal economic waste, although some net advantage appears in both cases. All this, although it seems important in the light of the political situation of 1937, will be regarded with a calm and tolerant eye by the economic historian.

It is more accurate, therefore, to regard the Federal power program, whatever its merits or defects, as a response to a long-standing maladjustment, not as a punishment meted out to malefactors. The machinery of State regulation is often said to have "broken down." More strictly speaking, it has always lagged behind the situation with which it strove to grapple. This was true even in the days of local power units, because of the rapid rate of technical change. It is doubly true now that regional inter-connections extend over areas beyond the jurisdiction of regulative authorities.

Are the people then to be content with the imposition of "yard-stick" projects that are manifestly unfair? Is the ineptness of public regulation in the past to be an excuse for unfair competition in the future? It is doubtful whether the problem can ever be solved merely by territorial division and rate adjustments between public and private enterprise. Such a solution, although desirable, is too much to hope for. In practice, some working basis will be arrived at because the demand is still expanding so rapidly as to permit, within a relatively short time, utilization of both private and public facilities without seriously overcrowding the field. As always before in the economic history of the United States, progressive expansion should provide a correction for miscalculations and errors—both those of financial exploitation by private interests, and the equally glaring ones of public policy.

A time will come, however, when economic maturity will overtake the industry. Mistakes will then be more costly because growth will be slower. When that day comes, a lesson may be learned from countries like Switzerland and Sweden. One may hope it will by then be apparent that a public temper which inclines toward moderation and co-operation is indispensable for the full development of a technical and social complex like the Power Industry. The engineer holds the key to continued progress, but unless such a temper prevails, his labors will be in vain.

DISCUSSION

W. S. Finlay, Jr.,[28] Esq. (by letter).—In any discussion of the economic aspects of energy generation, one can distinguish at least two levels—the level of pecuniary analysis and the level of social or human analysis. However, one is scarcely justified in so limiting the range of vital factors dominating the life, opportunity, and progress of the power industry.

For instance, there is a certain type of professional mind which has been identified with much that has made the industry what it is, and whose contributions are of essential importance, but which is motivated solely by the compelling thought of producing something of a highly technical order, without regard to the bearing their product may have on economic, social, or human aspects. That sort of mind can be classified as similar in its make-up to that of an artist who endeavors to reflect his ideals on the canvas upon which he paints.

Such a contribution is certainly worthy of consideration as occupying a third level. True, economics may enter into work on such a level, but history shows that it is frequently only in the last stages of development that the vision of an inventor's mind is forced into a mould that relates itself to economics. The incandescent lamp was long in existence before it was given commercial value by Edison and adapted by him to human use and service.

It may thus be possible to isolate within other levels some of the vast range of activities that compose the power industry, but is not the isolation of components working in a contrary direction to the purpose of a symposium? Is not such a purpose that of discovering, if possible, a reason for the theory that the general activities of the power industry, involved in the generation and use of electricity, cannot be isolated one from the other in levels, or in any other way? Is it not a fact that in any industry the viewpoint of one who would appraise it must be that which embraces it as a harmonious whole of men, machinery, money, materials, and more?

With the background of such an argument, the writer assumes, for purposes of discussion, that this particular form of industrial activity is indivisible into any component parts. There is a constant necessity for "give and take" among its pecuniary, social, sheerly technical, and political components. This Symposium supplies a background in indicating only a few of these interrelationships. To bind the parts together into a harmonious whole, and in a manner thoroughly satisfactory to all who have interested themselves therein, particularly in these latter days, apparently calls for a master mind beyond any yet revealed. At least, no one in the United States in whom students of the problem might have implicit confidence has supplied anything like an answer—anything that might direct the trends of effort and research, governmental or industrial, toward a single, ultimate, and ideal objective. One is impressed rather with a present tendency to tear apart in every direction—the belief on the one hand that the object of governmental authority is to destroy, and on the other hand that it supplies the only means by which to advance.

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[28] American Water Works & Electric Co., New York, N. Y.

Concentrating Objectives.—The titles of the papers in this Symposium typify in a small way the diversity of interests that motivate many minds whose theoretical objective is that of a common purpose. Long before the use of electric energy became a factor of importance in living, Man struggled to maintain life, to accumulate wealth, and thereby to promote the comfort of living. As incidentals thereto he adapted machinery to his purposes, he endeavored to transform energy from one form to another in aid of such purposes, he regimented other men in his service and, in turn, adapted his own activities in relationship with those of other individuals, groups, and nations. The whole created a civilization. Electricity entered, became a force, an element of value, a service, a means for creating wealth, and grew almost over night into an element of major economic importance, theoretically affecting human welfare not only here or there but almost universally. Certainly, so far as the United States is concerned, it has been elevated to a pedestal of national importance. Whether or not such importance is exaggerated is a matter open to discussion. Unquestionably, from the angles of politics and publicity, it is occupying, at present, rather a top position. Perhaps it is a proper part of this discussion to inject the throught that this position of importance has been exaggerated to a degree for which there exists no warrant, social or economic.

Near the beginning of his paper, Mr. Freeman states: "It is hardly necessary to dwell upon what is the most obvious result of cheaper power, namely, increased well-being." In that sentence he places his finger upon the theoretical yeast that has inflated an enormous loaf of political aspirations and popular theories that at best have an uncertain background of factual warrant. If it were not for the popular idea that an unlimited amount of electricity is a sort of Aladdin's lamp for the mere rubbing of which any individual can find relief from labor, from care, from expenditure, there would be little popular support to the political plaything that electricity has come to be. The importance and value of Mr. Freeman's key sentence depend wholly upon whether electric power can be cheap, and, thereafter, upon what contribution electric power can make to economic well-being. Without pursuing the theory to a conclusion, certain facts drawn in part from the other papers of the Symposium can be introduced to indicate the doubt of much of the social theory that supplies the background for such a key sentence. It is the writer's purpose to examine this factual background and to the extent to which the key sentence is affected by such an examination, the entire discussion is affected.

The writer may be one of a minority who holds a definite belief that the social importance of what is termed "cheap power" has been greatly exaggerated—in fact, exaggerated to such a regrettable extent as to create a demand for something that cannot be supplied. For this reason, there is bound to result deep disappointment upon the part of many, with the result that the social aspects of the general problem will take the form of resentment against those who made promises. For instance, were one honestly to examine into the net social effect of electricity as measured by the distribution of population, it might be found to be rather minor except as it is affected by and related to improvements in forms of transportation facilities.

The generation of power in bulk in certain favorably located centers reflects only a certain and peculiar set of conditions. Mr. Sporn emphasizes the economic limitations imposed upon power distribution from certain centers by his comparisons to the cost of transporting fuel for the generation of such power. Certain limited hydro-electric sources of power create special situations; but others too often tend to concentrate population in industrial areas about a source of energy, thus contributing to the difficulties resulting from such a concentration rather than supplying a solution to the problem.

To certain realistic minds, the social effects and value of electric energy supplied from a central source and distributed to the farm and rural home are distinctly uncertain and untried. To the actual costs of such service must be added the cost of electrical equipment, the fixed charges on which almost unquestionably exceed materially the actual price of electric current. The ability of the farm to carry the burden of cost will depend directly upon its producing capacity and earning ability in ways that are wholly unrelated to the availability of central station electricity to the farm.

Carrying the thought a bit further, the farmer may actually be impoverished by his attempt to carry the burden of the actual cost of such electricity. In other words, he might better continue the use of other and cheaper forms of energy plus possibly such an adjunct as an oil-engine generator set for home lighting and other minor service.

As relating simply to its social effect and value, of course, electricity in the home has a certain place of importance as a substitute for gas and oil, but there is nothing in that importance which calls for a complete re-organization of the service background of electricity as a commodity. In other words, there is no peculiar social value in service and power generated by a publicly owned central station that is not inherent in similar service and power supplied by a privately financed and operated source. In fact, the development and progress of any form of public service, be it electricity, transportation, or any thing else, have been far greater under the spur of private initiative than under governmental ownership and direction. Practically, every feature of value in power generation and service in nearly every publicly owned and operated plant in the United States had its origin under the stimulation of private investment and initiative.

In brief, the writer can find no warrant for attaching extraordinary social values to the manner in which electricity is generated and used. Yet these values have been seized upon as the basis for wholly illogical argument and masses of propaganda designed to transfer the industry in part and eventually as a whole to public ownership and operation. History has proved that under public regulation and private management, transportation plays as great a part as power—if not, indeed, a greater part—in the social life of the nation. Again, the production and distribution of food have equal, if not greater, social significance. Everything that relates to life, to ease and comfort, in general, has no more and no less social significance, and no one factor should be singled out for special treatment, control, or direction.

This general concept has been recognized for years by the man who has specialized in electricity and electrical service—that is, by the power producer, who relates every phase of his service, from the financing stage onward, to a consideration of its proper use at the point of consumption. In brief, he is the privately owned power industry of to-day.

The Technical Aspects of the Power Problem.—The various papers which have set forth individually the technical engineering features of the problem are inter-related and might almost have been combined in a unified whole. Progress in the art of power generation, as described by Mr. Orrok, is a magnificent record of achievement upon the part of a record-making and record-breaking industry; and is clearly indicative of extraordinary possibilities for the future, provided nothing is done to stifle private initiative and inventive genius. Mr. Rogers' paper illustrates the extent to which specialization in one field of power generation tends to contribute to a well-organized and forward moving whole. As an addendum, a paper might have been prepared, setting forth the extraordinarily complicated combination of the various forms of engineering comprised in a modern steam station. Thereto might have been added the complex problems of an electrical and economic type that the engineer is called upon to organize in making up a properly balanced power generating, transmitting, and distributing system.

Economics in Engineering, Generation, and Distribution of Power.—Mr. Justin has touched briefly upon some of the "high spots" of the economics of power transmission and distribution. The brevity of his presentation is possibly responsible for its apparently being rather more idealistic than practical in its concept. For instance, inter-connection of systems with the objective in mind of securing some advantage of an economic character is, and for years has been, merely a part of the ordinary day's work of the electric power industrialist. The value of these inter-connections may be based upon anything from economies in financing to insuring reliability of service to some remote customer. The history of the industry has been notable for the extent to which advantage has been taken by the various industrial units comprising the industry, of co-operative action to procure results of maximum economic interest, not only—as has been frequently over-emphasized—that accruing to the stockholder, but to the user, whether a large industry or an individual whose interest is limited to the service in his home. This fact is not as generally appreciated as it should be, and those who are prompted by a motive which might result in some form of prestige have deliberately endeavored to obscure the fact. On its part, the power industry has been backward in advertising.

The best evidence of the fact that the power industry has taken advantage to an extraordinary degree of every opportunity to cheapen and stabilize the service is the rate of progress that it has made in a very brief span of time. No industry has made, or is making, a like rate of progress.

Mr. Sporn's exceedingly well-written paper is based upon facts and figures rather than on theory. However, he cannot resist the temptation to which all engineers are subject, namely, to do a bit of forecasting. He refers in particular to probable trends of various departments of service within the industry and of economics relating particularly to production with the consequent effects on costs of the service.

There is this in favor of the reasonableness of Mr. Sporn's forecasts: The power industry of to-day is the result of years of intensive combined effort on the part of those engaged therein, which is in no way limited to engineering alone but involves everything that goes to make up a progressive industrial service—financing, sales methods and psychology, social effects, technical engineering, the perfecting of mechanical processes, etc. Unlike the average industry, it is not limited in its advancement by the advantage gained by individual units from the development of patented or special processes. Its extraordinary progress has in the main been due to the readiness—the actual anxiety—of those engaged in it to exchange ideas and information as to every phase of performance or detail of service.

This type of progress inclines one to the opinion that little or nothing has been overlooked in the way of developments that have contributed to the success of the industry, its efficiency, or performance. There are no secrets—no developments that are not fully known or made use of if needed or wanted. The industry is one which has led and directed the manufacturer in his supply of its tools of service. True, the manufacturer in his own laboratories has devised many a type of equipment of value to the industry; but the greatest laboratory has been the field of service—and the directors in that laboratory have been the power industrialists themselves, with the manufacturer as a most co-operative and able partner.

In the light of the foregoing, it is rather difficult to disassociate one's ideas as to future trends from what has been and is being accomplished. It would seem illogical to think that any sudden discovery will eliminate the great waste of heat energy that now attends the generation of electricity in steam plants; hence, one's ideas as to the obsolescence of generating machinery are developed rather in terms of something progressive than in the direction of something revolutionary that might suddenly destroy vast values in equipment, transmission lines, etc.

Nevertheless, there are those who envision the possibilities of new and wholly unknown ways of energy transformation and concentration adaptable to the service of mankind; however, if there is any precedent in history, it is that such changes can only be gradual and as the result of persistent, progressive, time-taking invention and research.

If that theory as to the future of the industry is correct, there is every reason for maintaining it as a wide open field of opportunity for private initiative and inventive genius, rather than permitting it to become a political plaything, stifled by bureaucratic control and direction. Responsibility for progress in the power industry, as in every other industry, in a democracy such as that of the United States, must rest in the application of individual effort spurred by the profit and success motive, and not in the benevolent paternalism of the political appointee or aspirant. The economics of power production still remain essentially a matter of endeavor upon the part of those who would apply themselves to intensive specialization, devoting their span of life thereto. It is not and cannot be the hit-or-miss achievement of the dilettante in politics, sociology, or reform for reform's sake.

R. M. RIEGEL,[29] M. AM. SOC. C. E. (by letter).—The advances in the art of turbine design and manufacture which have been described by Mr. Rogers, particularly those in the field of the propeller turbine, have been very important. Special consideration should be given to the extensive use of the manufacturers' laboratories, in which so many of these advances have originated. The staffs of these laboratories conduct their work with care and scientific exactness, and their product has been extremely valuable. However, the results of model tests must be verified by experience with the prototypes, and occasionally phenomena occur in the full-sized units which have been too obscure to be observed in the small models. In studying the performance of full-sized units, the power engineer functions in furtherance of the research in the manufacturer's laboratory. Other important research has been performed by interests which purchase equipment.

The use of large propeller units is comparatively new, and some facts still remain to be learned about their characteristics. The ways in which the power engineer will contribute to the art is illustrated by a case of the TVA. The two units at the Pickwick Landing Dam were designed with a horizontal splitter in the draft-tube. The two units at Wheeler Dam have no such splitter. The units at the two plants are quite comparable in size, head, and discharge, the Pickwick units being a little larger. The splitter is a difficult and expensive detail of construction in units of this size and should not be used unless it is advantageous. From the comparative performance in these two plants the TVA and the Engineering Profession will have evidence to determine its merits.

Another investigation of the TVA indicated that it was feasible to tip upward the horizontal leg of the draft-tube in units of this kind to a degree which previously had not been deemed feasible, and the specifications for the Pickwick units required that a high slope be used. In the dams of the TVA, the large propeller units require considerable rock excavation to accommodate the deep draft-tubes, and the tipping up of the horizontal leg represents a real saving in construction cost and a widening of the field of this type.

Another investigation in the laboratories of the TVA is that of a head-increasing device which may be applied to the dams on the Tennessee River. Head increasers have been developed by a number of engineers, including Tefft and Thurlow; and it may be that the proposed device can be worked out to widen the field of use of such appurtenances. It may be noted that these devices have been developed outside the manufacturers' laboratories.

The art of hydraulic generation has been advanced, furthermore, by the development in recent years of various accessories which contribute to safe and economical operation—improvements in governors, penstocks, trash rakes, gates, and similar devices. One noteworthy contribution is the "paradox gate" developed by the U. S. Bureau of Reclamation as a reserve valve for the needle-valves at Boulder Dam.[30] A modification of this type has been used for the entrance gates to the penstocks at Norris Dam. These gates, 16.5 ft by 28.5 ft in opening, are of heavy steel, and ride on caterpillar trains in a manner somewhat similar to the well-known Broome design. The seating device, however, is radically different, the gate being seated by a horizontal movement after it has been lowered into its proper position. These gates were installed in 1936, and are apparently operating satisfactorily.

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[29] Head Civ. Engr., Design Dept., TVA, Knoxville, Tenn.

[30] *Mechanical Engineering*, October, 1934.

In conclusion, it is worth while to note that co-operation between manufacturers and purchasers' engineers has led to numerous improvements in the past. Such continued co-operation will solve the problems of the future.

RALPH BENNETT,[31] M. AM. SOC. C. E. (by letter).—It is certainly true that the efficiency of power production must increase rather slowly in the future. The great advances have already been made. Only those refinements which come with detail study of specific cases and the use of more complicated equipment are left.

One of these refinements is provision for more efficiently carrying peak loads on plants especially designed for the economical delivery of power on a very low load factor. This does not mean merely improvement in direct generating efficiency to carry short peaks. It may mean an artificial time distribution of the load within the system generating plants, so that the total power sold will have the lowest cost in fuel and labor and in capital invested.

To some extent power plants having short intermittent peaks at close intervals can store heat to cover these peaks in heat accumulators. Electricity was once stored, for peak use, on central station networks in lead storage batteries; but other methods of carrying the peak have superseded the storage battery, except in the case of small isolated plants. In some countries, as Mr. Justin has stated, the use of off-peak power to pump water to high-level storages for rapid release through turbines during the station's peak, is well established. Thus, the external peak is met through a redistribution of station load within the system, fuel efficiency being sacrificed in favor of a lower total power cost.

In the typical pumped-storage plant, water is drawn from a convenient stream and returned to it after use. The reservoir is of small size. The up flow and the down flow of water are frequently through the same pipe, and the pump and the recovery turbine are commonly connected to an electrical unit which is run as either a motor or a generator. This plant is efficient under only a very limited range of conditions, but the energy thus recovered as peak power is very much more valuable than the off-peak power used in pumping the water. It is possible to recover about two-thirds of the power used.

The profitable investment in such a plant is much more than is permissible in a fuel-burning plant, since the rent of the money invested is almost the entire cost. The allowable investment may even be greater than the cost of a distant water power and a peak-load transmission therefrom, because the storage recovery plant, being closer to the point of use, will have lower transmission losses and greater reliability.

The pumped-storage peak-power plant, split into separate pumping and recovery plants connected by a high-level conduit and storage reservoir, can be used to carry a water conduit over a mountain range instead of around it or through a long tunnel beneath it. Many cities import their water through such conduits and, in many cases, the water system can be easily arranged for pumped-transit power recovery. The higher the lift the less the percentage value of the fixed losses and the greater the percentage of the pumping power recovered. If the mountain range is high and a high-level summit storage is available the system as a whole can be very efficient.

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[31] Cons. Engr., Los Angeles, Calif.

However, the conduit line used for peak-power recovery will differ materially from the classical constant-flow low-level conduits as now designed. It will need a re-arrangement of petty storages and of pumping plants such that the long conduit can flow continuously. The pumps will run on a 15 to 20-hr schedule, and the regeneration plant will make its entire recovery in only 4 to 6 hr.

If the system is operated on a day-to-day basis, the terminal and summit reservoirs need still be large enough only to impound the flow during its manipulation for peak-power delivery. However, if the summit reservoir can be large, and the terminal reservoir can be proportioned for seasonal as well as daily variations, the value of the pumped-transit system will be immeasurably enhanced. In such a system lush supplies of water and seasonal excesses of power may be combined to pump available water to the summit reservoir with secondary power and thus provide potential peak power and water for a long time ahead, independent of the fluctuating rates of the stream or of primary power production.

The use of the incoming water for power production is quite compatible with the total rate of water flow required by the city. A population unit of 5 000 will use 1 cu ft per sec of water (continuous flow), and will absorb the output of a 1 250-kw power plant.

Among the cities where this type of project would appear to be of value, four may be singled out for special mention. The situations at San Francisco, Santa Barbara, and San Diego, Calif., and at Denver, Colo., are strikingly similar. In each case, an imported water supply encounters a high mountain range close to its destination. Long low-level tunnels can be driven through these mountains at great expense, or pump-fed conduits can be carried over them and through summit reservoirs and peak-power recovery plants at relatively low cost. In each case the intervening mountains are high and the total lift correspondingly great. Such a situation is advantageous in a pumped-

TABLE 5.—POSSIBLE ENERGY RECOVERY FROM PROPOSED SAN DIEGO, CALIF., WATER SUPPLY

Item	Type of Route	
	Summit lift, no tunnels	Low-level conduit, with 25-mile tunnel
Lift, in feet	4 000	1 100
Conduit loss, in feet	100	120
Delivery head, in feet	550	550
Net regeneration head, in feet	3 350	430
Gross recovery, percentage	83.7	39
Efficiency of combined plants, percentage	76	76
Net recovery, percentage	63.61	29.64

transit system, because the fixed losses (such as conduit friction, entry head, reservoir fluctuation, and terminal reservoir elevation) are independent of the lift. This point is illustrated in Table 5, in which the power possibilities of a low-level and a high-level route for the proposed San Diego water supply are compared.

In fact, an almost perfect example of the possible advantage of such a system is supplied by the proposed importation of Colorado River water into the City of San Diego across a high mountain range, on which numerous high-level reservoirs are already in use. The generalized profile across the Coast Range (Fig. 36) illustrates the facility with which this particular diversion can be adapted to pumped-storage transit.

Steam plants in San Diego are the primary source of power. The most distant regeneration plant would be only 40 miles east of the city; and the closest, only 15 miles. Power for the large pumps would be carried by the same transmission line, and for a maximum distance of 60 miles. Other advantages include the large high-level reservoirs already in use along the proposed conduit line, and the large low-level storage and distribution reservoir at the city's edge.

Elaborate calculations covering the allowable cost and probable kilowatt and kilowatt-hour outputs of a pumped-storage plant considered as a peak load plant for a large system are contained in a discussion[32] by A. H. Markwart, M. Am. Soc. C. E. His conclusions are based on a particular, but perhaps representative, set of conditions. Under a condition of practical balance between off-peak pumping and peak-power release, from 13 to 20% of the system energy can be used for pumping. The regeneration plant would probably have a load factor of only 10 to 18%, and would generate only 8 to 12% of the total energy delivered to the system. Yet the peak which could be carried would be 30 to 50% of the system peak.

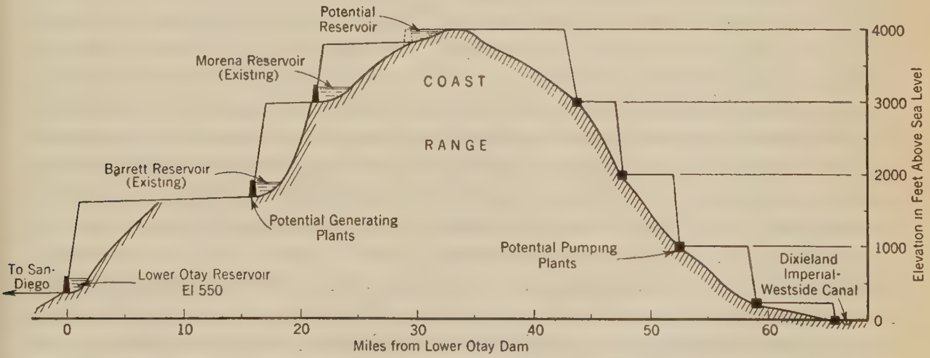
Potential Reservoir
Morena Reservoir (Existing)
COAST
RANGE
Barrett Reservoir (Existing)
Potential Generating Plants
Potential Pumping Plants
To San-Diego
Lower Otay Reservoir El 550
Dixieland Imperial-Westside Canal
4000
3000
2000
1000
0
Elevation in Feet Above Sea Level
0
10
20
30
40
50
60
Miles from Lower Otay Dam

FIG. 36.—PROFILE OF PROPOSED SAN DIEGO, CALIF., WATER SUPPLY CONDUIT, SHOWING POTENTIALITIES FOR POWER RECOVERY

Under the conditions considered by Mr. Markwart, a pumped-storage plant costing from $120 to $150 per kw, may be economically feasible. However, with pumped-transit storage, part of the money saved on conduit tunnels can be properly applied to additional plant; and although the total machinery cost may be larger, the return on the money invested in the combined water-transit and peak-power system may be much greater than the net earnings of separate plants to perform the same duties.

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[32] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 911–932.

In conclusion, it may be stated that it is possible to design a water supply system, using pumped transit and summit storage with retarded regeneration, such that intervening heights may be made to decrease the first cost of the water and power system and to increase its earnings. In general, the higher the lift, the greater will be the net recovery; and the shorter the regeneration time, the greater will be the net earning.

J. M. MOUSSON,[33] ESQ. (by letter).—The paper presented by Mr. Rogers gives an excellent review of the progress made in hydro-electric generating equipment. Since it is often widely believed that a stage of stagnation in development of hydraulic prime movers has been reached, it is gratifying to see that Mr. Rogers points out various fields in which further improvements are yet to be expected.

He mentions, for example, the ever-increasing heads under which fixed-blade propeller and Kaplan turbines are being installed, and refers particularly to the Shannon River development, in Ireland, where a Kaplan unit is operating under a head of 106 ft. It is interesting to note that, in 1937, the Electricity Supply Board of Ireland (Dublin), decided to install Kaplan turbines at the new Liffey Power Plant, where a head of more than 130 ft will be utilized.

In the field of pump-turbine units substantial progress has been made. Although Mr. Rogers refers only to laboratory studies with modified high- and medium-head Francis-type runners, an actual installation of a low-head pump-turbine unit was completed early in 1933 at the Baldeney Power Development, on the Ruhr River, in Germany.[34] The unit is operating under a maximum head of about 32.5 ft. Its design was based on laboratory studies of a modified 5-bladed Kaplan runner with fixed guide-vanes. The runner is coupled directly to a 50-cycle alternating-current asynchronous machine operating at 255 and 326 rpm when generating and running as a motor, respectively. Although capacities and size are relatively small (power output, 1 100 kw; power input, 1 300 kw; runner diameter, 59 in.), this installation seems important as a pioneer in the low-head field. The maximum efficiencies reported for turbine and pump operation, based on acceptance tests, are 90% and 78%, respectively. In view of the fact that for the water measurements propeller current meters of only one type (25 cm in pitch) were used, which even under practically ideal field conditions under-register by an appreciable amount, the foregoing prototype efficiencies should, perhaps, be corrected to 88 and 80 per cent. There is little doubt that the average maximum efficiency of the pump-turbine designs referred to by Mr. Rogers, based on a modified Francis runner, may be superior, keeping in mind that Mr. Rogers' results (see Fig. 21) were obtained on a 16-in. model and that an appreciable step-up in efficiency for the Francis type, pump-turbine may be expected. Aside from possible improvements in the Kaplan type, however, it should be noted that in spite of the lower average of the maximum efficiencies of the Kaplan turbine-pump installation referred to, better over-all efficiency may perhaps be achieved with this type of design, if the unit has to generate at part load over long periods of seasonal low-river stage, as at the Baldeney Plant.

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[33] Hydr. Engr., Safe Harbor Water Power Corp., Baltimore, Md.

[34] "Die Turbinenpumpe in Stauwerk Baldeney" ("The Turbine-Pump at the Baldeney Power Plant"), by O. Spetzler, *Zeitscrift* des Vereines Deutscher Ingenieure, Vol. 78, No. 41, October 13, 1934, pp. 1183 to 1188.

In addition to the mechanical improvements referred to by Mr. Rogers, mention may be made of the possibility of injecting air into low-head propeller-type units to reduce vibration at high-gate opening. For instance, at the Safe Harbor Plant, Safe Harbor, Pa., vibration alone was the determining factor for the gate limit under normal operating conditions. Because of the hazard involved, even short periods of over-load had to be avoided. Air injection through the head cover proved so successful in reducing vibration that more than 1 000 kw per unit, or more than 6 000 kw for the entire station, were gained in output by stepping up the gate limit without increasing the vibration above the previous intensity registered at the lower gate limit and no air injection. Air injection of 125 cu ft per min through the head cover under high loads is now a standard operating procedure at Safe Harbor.

Air injection may be used in the future not only to improve operating characteristics, but to provide a cushioning medium to render cavitation ineffective. With ever-increasing heads for propeller-type turbines, a point may be reached where even the best stainless alloy steels will no longer resist the attack from severe local cavitation. If air injection for this purpose is to be economically feasible, however, research and model tests will be essential, because with the large volume of air required it is necessary to make the injection in such manner that the major part of the air will find its way to the particular location desired.

BEN C. SPRAGUE,[35] ESQ. (by letter).—In evaluating the saving resulting from the reduction of new generating-capacity requirements accomplished by inter-connection of power supply systems, Mr. Justin assumes this saving to be equal to the total fixed charges and residual operating expenses in connection with the generating capacity additions that are avoided. Actually, the addition of new generating capacity to a power system results in very appreciable operating savings, allowance for which gives a net cost of providing additional generating capacity of appreciably less than the additional fixed expense involved. These savings are obtained by operating the new generating capacity to carry the base part of the system load, replacing a similar amount of capacity which otherwise would have been operated at or near the top of the load, and which normally would have a much higher operating cost than the new generating capacity. For example, if the new generating capacity has a coal rate of 1.0 lb per kw-hr and can be operated 7 000 hr per yr to replace capacity having an average coal rate of 1.7 lb per kw-hr, an annual saving of 4 900 lb of coal—which at $4.00 per ton would amount to $9.80—would be realized per kilowatt of new capacity added. Based upon a new capacity cost of $130 per kw and a fixed charge rate of 14%, the operating saving would amount to 54% of the annual fixed charges of $18.20.

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[35] Engr., West Penn Power Co., Pittsburgh, Pa.

As a result of the continuous improvement that has been made in power-station design throughout the past, the saving in operating cost has always been a factor to reckon with in determining the annual cost of providing additional generating capacity. This factor has had an appreciable bearing upon the economy of utilizing hydro power for carrying the peak of the system load. Because of the very low annual load factor of the load peak to which hydro power might be applied, the low operating costs obtained with new steam capacity, *per se*, have had little effect on the relative economy of steam or water-power capacity additions. Rather, it is the lower production cost of the new steam capacity, as compared with that of the capacity it could replace, which is so important a factor in steam and hydro comparisons.

Since steam station heat rates are fast approaching an irreducible minimum, it is to be expected that in the future the savings in operating cost resulting with the new steam capacity additions will be less of a factor in reducing the cost of providing additional generating capacity by steam. However, the making of superposed capacity additions has introduced a new factor into this situation. If the high-pressure boilers provided in connection with superimposed capacity additions were of the same efficiency as the low-pressure boilers they replace, the heat rate of the new capacity provided by superposing would be approximately 5 000 Btu per kw-hr. However, the new high-pressure boilers provided are usually so much higher in efficiency that the new superposed generating capacity can be loaded with an actual reduction in fuel requirements. In other words, the new generating capacity has a negative coal rate. Thus, the savings in operating cost made possible by superimposing, while not sufficient to cover the entire fixed costs involved, are sufficient to result in a low net cost of providing additional steam capacity.

Because the advent of superposing has resulted in reducing the net cost of providing additional generating capacity, it has tended to make inter-connections, peak-water power, and pumped-storage installations less economical than they were previously. However, this effect is only temporary and will be followed by a reverse effect. Superposing can only be utilized as long as low-pressure capacity remains which is economically adaptable to superposition. The low-pressure plants superposed upon will be rendered almost as efficient as new condensing capacity that could be added at present. Thus, when the time comes that further superposing is no longer economical, there will be less inefficient generating capacity in operation to be replaced by the new generating capacity. Hence, the operating cost saving will be greatly reduced. As a result, conditions may become more favorable for the development of peak hydro than they have been for some time. Considering that the need for peak hydro also increases with the load peak (because of the resulting greater amount of low load-factor peak that can be economically carried by hydro), it does not seem unreasonable to expect a resumption in development of hydro capacity after the possibilities in superposing have been exhausted.

It is thought that in many cases the difficulties in properly evaluating the reliable capacity values of run-of-river hydro-electric plants have resulted in their not being given full credit. Because during a large part of the year little output can be definitely depended upon from the run-of-river plant, its peak-carrying capacity value has been regarded as little more than that resulting with the minimum stream flow. In reasoning thus, it is forgotten that steam capacity cannot be relied upon as being available at all times. It is only by providing spare units that the probability of having sufficient capacity available to carry the load can be made great enough to meet the practical requirements. Since the availability of steam capacity and even the amount of the system peak to be carried are subject to chance variations, it is proper to evaluate water power by considering its effect upon the total probability of having power available to meet system needs.

In making such evaluation, allowances should be made for the ability to schedule steam-unit outage times so as to offset the seasonal variation in hydro-electric output, and also for the ability of the run-of-river hydro to give full output for short periods at times of emergency, by drawing upon its limited storage. Where the potentialities of remaining undeveloped water powers are properly determined, it is believed that many of them will be found economical even though they are much less favorable than Niagara.

In Fig. 26, Mr. Justin shows the firm capacity of a hydro-electric plant to increase in direct proportion to the system load. Because such a rate of increase of firm capacity would require increasing amounts of controlled energy output, it will be evident that the firm capacity of a given hydro-electric development increases at a slower rate than the system load. Usually, the firm capacity of a given hydro-electric development varies approximately as the square root of the system load, assuming, of course, that the shape of the load-duration curve remains about the same.

Mr. Sporn has made a valuable contribution in showing that it is not economical to transmit steam power over long distances. It should be stressed, however, that Mr. Sporn's comparisons are made for conditions not frequently met in actual practice. As a result, the comparisons show a much lower cost over the entire range of load factor, with transmission by freight over a distance of thirty miles, whereas it is common practice to transmit power electrically over this distance rather than resort to local generation. The failure of the comparison to correspond with what is found economical in actual practice results because it does not take account of the differing amounts of spare generating capacity required or of the necessity for inter-connection lines even when there is no normal delivery of power. Further, there is a practical necessity, in almost any case, of transmitting power for at least some distance from the generating station to the centers of distribution. Hence, if the generating station is displaced from the center of the territory supplied (in order to locate it at a coal mine, for example) the additional transmission cost involved is usually much less than the total cost of transmission over a similar distance, shown in Fig. 26.

Thus, because the actual conditions encountered are different from those assumed in Mr. Sporn's comparisons, it is frequently economical to transmit power over long distances rather than to resort to local generation involving freight delivery. Although Mr. Sporn's cost determinations necessarily had to be limited to some simple set of conditions, the variations from the conditions of actual practice should be kept in mind so as not to arrive at any wrong conclusions as to the economy of power transmission. Because Mr. Sporn's results would seem to indicate that transmission by freight is cheaper over all distances for loads of less than 90% load factor, it must not be inferred that the many lines now in operation for transmitting steam power do not represent sound investments.

V. M. MARQUIS,[36] ESQ. (by letter).—The paper by Mr. Orrok covers the progress made in the generation of energy by heat engines in such a thorough and interesting manner that it does not leave very much to be discussed. However, there are a few points on which additional comments may be of value.

It is only necessary to compare the Newcomen engine of 1770, which required an output of 416 000 Btu per kw-hr, with the present high-pressure steam turbine unit and mercury vapor units, which require an output of only 10 000 or 11 000 Btu per kw-hr, to see the great strides that have been made in the art of generation. However, the end has still not been reached. Development of materials suitable for use at higher temperatures and pressures, the use of higher speeds, and the possible further adoption of the binary cycle, should all result in a gradual increase in the efficiency of the heat engine.

The excellent results already obtainable with the steam turbine are shown in Table 2 of Mr. Orrok's paper. Mr. Orrok is dealing with the progress that has been made in the generation of energy by heat engines, and Table 2 most certainly emphasizes the results possible in a modern high-pressure plant. However, in any discussion with reference to a practical system, it must be remembered that all the plants are not of this vintage, and in discussing generation costs it is necessary to take into account not only the latest and most efficient plants in the system, but also the older plants. Some of the latter will have somewhat higher generation costs, but they are, nevertheless, perfectly good plants and cannot immediately be superseded by more efficient ones. They, of course, assume a different position on the load curve than the more modern plants; that is, they operate over the peak periods and normally have a very low annual load factor. On this basis they cannot be economically replaced by new capacity. In any practical system allowance must also be made for reserve capacity, and, as stated by Mr. Orrok, this has not been included in his calculations.

The generating costs are made up of operating costs and fixed charges. Operating costs may vary considerably with annual load factor. Again, if the total operating costs given in Table 2 are plotted against load factor, the result is practically a straight line. Although this would probably be nearly correct for a new modern plant with low maintenance and labor costs, and with a single boiler supplying all the necessary steam, nevertheless, in most plants—and especially in the older ones, with many turbine and boiler units—it is necessary to consider the fixed and variable portion of the operating costs. For example, there is a certain portion of the fuel charge which is a fixed cost and a certain portion which is variable. The same holds true to a certain extent with maintenance and labor. The net result is that the curve of operating costs is fairly flat at high annual load factors, but rises considerably at lower load factors. (This is definitely shown by Fig. 30 in the paper by Mr. Sporn.) Therefore, in a practical system that includes a variety of plants, from the most efficient "modern" type to those built 10 or 15 yr ago, operating costs at low annual load factors might be expected to be considerably higher than those given in Table 2.

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[36] With Am. Gas & Elec. Co., New York, N. Y.

R. L. Sackett,[37] M. Am. Soc. C. E. (by letter).—The fine review by Mr. Orrok emphasizes the contribution which the mechanical engineer has made in increasing the efficiency and size of generating units, while maintaining the capital cost per kilowatt practically constant. It is characteristic of modern civilization to continue development in one way or another. However, as the possible rate of increase in efficiency is now much reduced, it follows that the power industry must necessarily turn soon to those other factors of power cost—distribution and consumption—and give them more serious consideration than it has in the past.

Mr. Orrok's paper also calls to mind a modern problem in engineering education. One hears a great deal about the importance of "emphasizing fundamentals." The teacher of mechanical engineering has been compelled to emphasize them. When a student, the writer was required to design a Scotch boiler. No one asks the student to-day to design a Scotch boiler. The subjects that had to be considered as examples then were few, such as a straight-line engine, a fire-tube boiler, and a gasoline engine. To-day, the field is so complex that there is no time to go into the design of the modern boiler—or even the design of modern steam turbines, which is more important—and the teacher is compelled, in illustrating given problems, to insist more and more on the fundamental principles than on involved, time-consuming designs that after all are of specific value only in one particular field.

The writer wishes to comment also on an item which Professor Freeman mentioned but did not stress, namely, the displacement of human labor in industry as a result of reductions in the cost of power.

Americans take considerable satisfaction in the historical fact that, over a long period of years, the percentage of the population actively employed in industry has increased. However, as long as present motives continue, there is every expectation that the adoption of new labor-saving devices will also continue, and, with it, the resulting displacement of labor. The maladjustments do not concern the producer of power in large units; they are rightly the problem of the producer of goods, because it is in his plants that they occur.

That some manufacturers are already assuming the social responsibility for readjustment in such cases, is evidenced by the following incident, which came to the writer's attention some months ago: A certain rubber company was proposing to install a machine to cut out rubber soles that formerly had been cut by hand. The machine required two or three operators and would displace about forty men. The managers called in the employees and stated the facts, but said: "We are going to train you for other jobs. The installation of this machine will increase our products and the production of other departments and we will have additional jobs here and there for which you will be trained." The consequence was that there was no disturbance in that case and the employees were continued with as good or a better rate of pay.

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[37] Dean Emeritus of Eng., Pennsylvania State Coll., State College, Pa.

Certainly, the responsibility for such readjustments lies, partly at least, with engineering management. Yet engineers have done very little, from an educational standpoint, about it. Where is there an educational institution within an industry or public organization whose purpose it is to assist in these readjustments? This point is to be emphasized: These men who are displaced from jobs have little facility for re-training themselves for other kinds of work.

Consequently, there are certain social influences of considerable importance that are involved in such a readjustment, and for these reasons, it seems the responsibility is even greater on engineering management to assist in this task. Certainly, engineering can do a great deal, most of which is now being done in technical institutions, night schools, or in engineering extension courses, to reach into the industries and assist these men to better jobs—or, at least, to different ones.

D. J. McCormack,[38] Esq. (by letter).—A few supplementary statements may be added to the paper by Mr. Rogers on the hydro-generation of energy. Because of the relatively high cost of low-head hydro-electric plants (most of which are situated in the Middle West and along the Atlantic seaboard in the most densely populated section of the United States), and the widely variable flows usually encountered in such developments, it has been necessary to reduce all costs of such plants to a minimum to justify their development, or even the re-development of an existing plant. Hydro-electric engineers have been faced with the problem of cutting costs wherever possible to effect this purpose, particularly in the face of rising unit prices. In consequence, turbine builders have constantly been urged to develop higher-speed units. In this connection, the introduction of the Kaplan adjustable-blade turbine has made it economically possible to develop and re-develop many low-head water powers that otherwise could not have been considered. Records show that an adjustable-blade turbine plant has exceeded the kilowatt-hour output of a Francis-type turbine plant or a fixed-blade turbine plant, over the same year and on the same river, by 10 to 30 per cent.

Where close regulation is not required, electrical operation of the runner blades can be used. This method has the disadvantage of not being able to pick up load quickly to prevent a consequent drop in speed on sudden load demand. If the flow of the river varies quite gradually and shut-downs are permissible, hand-adjusted runner blades may be used at a slight saving in initial cost.

It has been determined at the Safe Harbor Plant, on the Susquehanna River, that the last unit installed, which has machine-finished blades, will safely carry a 5 000-hp greater load within cavitation limits than the other units, which do not have machined blades.

Impulse wheels, because of the higher efficiencies (88 to 90%) now obtainable therewith, and because of their simplicity of construction, durability, and desirability from an operating standpoint, are at present being adopted more generally than reaction turbines for heads of from 500 ft to 800 ft. It is recognized that because of the rapid wear of the runner seals and of the joints at the top and bottom of the gates under the higher heads, reaction turbines must be completely overhauled every few years in order to maintain efficiency. A well-designed impulse wheel will maintain high efficiency over a much longer period, and, if the tail-water variation is within a narrow range (as it generally is), an impulse wheel with a vertical setting can be placed very close to tail-water to increase the effective head.

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[38] Sales Mgr., S. Morgan Smith Co., York, Pa.

I. E. MOULTROP,[39] ESQ. (by letter).—The paper by Mr. Sporn is extremely timely. His fair and comprehensive treatment of the subject of generating cost should be thoughtfully read by every one interested because it gives the complete story in terms every one can understand. For the first time the three chief items in the cost of electric energy as delivered to the customer are analyzed collectively and their relative values in terms of dollars are clearly presented.

Many people, impressed by the size of some spectacular steam or hydro-electric generating station, make the erroneous deduction that the cost of such stations is the chief money outlay for the system. Reasoning from this incorrect assumption, they naturally conclude that electrical energy obtained from water, running freely down hill, must be much less costly to produce than that derived from coal, which must be bought. This would be more nearly correct if no other items were involved and the hydro-electric station was located near the load center.

The large area of the storage reservoir seldom impresses one as being an item of considerable expense. Moreover, the layman does not realize the wide range between flood and low water and the comparative duration of each, and the fact that the hydro-electric generating station must meet both extremes. Finally, in a hydro-electric development, the greater part of the money outlay is made in the initial construction; the power-house and its appurtenances are, in general, the only parts that can be built initially to just the right size to meet the demands of the existing load. Hence, considerable of the invested capital may actually be idle for a long period. This factor may be quite important, but it is seldom appreciated. The steam generating station, on the other hand, can usually be built just to meet the existing load requirements, and comparatively little capital need be tied up awaiting future demands.

The comparative costs of steam and hydro-electric generating stations, as given in the paper, are illuminating. As the cost figures are taken from Federal reports they cannot be questioned, except that one may wonder whether complete data, such as an engineer requires, are always included. Engineers familiar with both types of stations might feel that the hydro-electric station was somewhat favored.

Coming to the other two items of power cost to which the public generally gives little thought, Mr. Sporn clearly points out their greater relative importance. It is indeed unfortunate that the costs of the transmission system and of the distribution system are so little understood. If the public knew that the greater portion of the invested capital was spent in building these parts of the system there would probably be less popular agitation for hydroelectric stations.

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[39] Cons. Engr., Belmont, Mass.

It is significant that, on the average, a ton of coal can be transported about 140 miles at a lesser cost than that of transmitting the electrical energy produced by that ton of coal for a similar distance. If this were common knowledge, one would hear fewer arguments for developing a greater number of the remaining water-power sites—especially as these sites are usually quite undesirable and far from the load center.

Likewise, the cost of the distribution system is little realized. In the cities it is usually underground and, therefore, cannot be seen. Moreover, it is not generally appreciated that the cost of maintenance is high. Rural electrification, badly as it is needed, can be made possible only by developing a very cheap and none-to-reliable rural distribution system and by materially increasing the energy consumption per customer. Such a system could not be used in municipal areas.

Throughout the paper Mr. Sporn has been very conservative, basing many of his data on Government sources which must be accepted as correct. The curves of generating costs (Fig. 30) and of transmission costs (Figs. 31 and 32) are most illuminating and should be studied with care by all who are inclined to discuss superficially the subject of electrical energy costs.

The final paragraph, indicating future trends, is very sound and again conservative. The best engineering minds in the world in mechanical and electrical work are engaged in studies to simplify apparatus, reduce its cost, and improve its efficiency. One would be very rash to predict what the end will be, but utility engineers are looking forward to much more improvement. The 2 400-lb steam plant which Mr. Sporn now (1938) has plans for building is a decided step in advance. It is to be hoped that similar progress will be made in the transmission and distribution fields, and there is reason to expect that these hopes will be realized in due time.

M. M. SAMUELS,[40] ESQ. (by letter).—In presenting this discussion the writer assumes that the Symposium was not intended to prove or to disprove any political theories or opinions, but that it was intended as an engineering discussion, whose purpose can never be anything but fact finding.

Attention may well be called first to the surprising diversity of terminology among the authors of the various papers. If every one were to continue using his own favorite terms, engineers would soon be in the position of those who built the tower of Babel. True, many definitions have not yet been established as American Standards Association (ASA) standards; but the proposed ASA standards have been approved by all the respective sub-committees of the various societies and should be used by engineers. Thus, Mr. Orrok speaks of "use factor" and Mr. Justin of "capacity factor," both meaning what the proposed ASA standards term the "plant factor." Mr. Justin has his own definition for "utilization factor," whereas the proposed ASA standards define the utilization factor as the ratio of peak to capacity. Those who have served on standardization committees know that frequently a term which appears at first as the most obvious has to be dropped for one reason or another. Considerable confusion will be avoided if all engineers will speak the same language, even if some of the terms are not to their individual liking.

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[40] With Federal Power Comm., Regional Office, New York, N. Y. Any opinions expressed herein are the writer's own and have neither been approved nor disapproved by the Federal Power Commission.

Considerable confusion in the discussion is brought about by a diversity of appreciation of the significance of the various factors in the cost comparison. Credit is due to Mr. Orrok for dropping the method, previously used by him, of setting up cost comparisons on a load-factor basis and substituting for it a comparison on a plant-factor basis. Mr. Justin, too, sets up cost comparisons on the plant-factor basis. Mr. Sporn, however, presents his cost data on the load-factor basis. Although other important conditions which would give the data definite meaning are no doubt known to him, to the reader such data are meaningless. The load factor is the ratio of the average load to the peak; it is not even necessary to know the scale of the load curve to establish it. A plant of unknown size may have a peak of 90 000 kw and an average load of 45 000 kw, resulting in a load factor of 50 per cent. The same plant may have a peak of 9 000 kw and an average load of 4 500 kw, resulting also in a load factor of 50%; but the fixed charges per kilowatt-hour in the second case will be ten times as high as those in the first case. Thus, for the same power-house, and the same load factor, the fixed charges per kilowatt-hour can be made to be almost anything any one desires. Why not follow the example of Mr. Orrok and Mr. Justin and set up the comparisons on a plant-factor basis? but if load factor it must be, the data presented by Mr. Sporn would become of great value if he were to state the capacities and peaks to which the various load factors relate.

There are many other uncertainties in cost set-ups on the basis of load factor. For instance, the difference between a load factor computed from the instantaneous peak and one computed from the 60-min peak may be as much as 20 per cent. Possibly, Mr. Sporn's curves and figures are based on the assumption that peak is equivalent to capacity, in which case the load factor would be equivalent to the plant factor, and the curves and figures would assume a definite meaning, even if in reality there is scarcely a plant which, in any one year, to say nothing of the average over the life of the plant, will have a peak which is exactly equal to the capacity.

Even when comparisons are related to plant factor great caution is called for. Only in the case of an isolated plant can a comparison be made between hydro-electric and steam on the basis of the same plant factor for both. With power systems as they exist to-day, such an isolated plant situation will scarcely ever present itself. Generally, there will be a system which can absorb all the energy that a hydro-electric plant can generate, as indicated by Mr. Justin. For such cases higher plant factors must be assumed for hydro-electric than for steam. Table 6 shows that on the whole, in the United States, the plant factors of hydro-electric plants have been considerably higher than those of steam plants, which means that on the average considerably more kilowatt-hours per annum per kilowatt of installed capacity are generated by hydro-electric plants than by steam plants. This definitely indicates how little value there is in discussing such a subject in a general way. Rational plant-factor assumptions can only be made in given cases, and even then it is of great importance not to overlook the fact that the plant factor on which any cost comparison is made must be an average throughout the entire life of the plant.

TABLE 6.—APPROXIMATE OVER-ALL PLANT FACTORS* OF THE ELECTRIC LIGHT AND POWER INDUSTRY IN THE UNITED STATES

Description	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937
All hydro plants.........	45	46	49	48	41	35	39	40	39	45	45	48
All steam plants.........	30	29	29	31	29	28	21	22	25	26	32	34

* Percentages computed from basic data in *Electrical World*, January 15, 1938.

It is to be noted that the various authors in this Symposium use different percentages for fixed charges, making it difficult to compare their resulting costs. In reports published elsewhere the differences are even greater than they are in this Symposium. In many cases these variations are brought about by the fact that the several authors include different items under the term, without stating definitely what these items are. To the banker, the term "fixed charges" generally means only interest and amortization of long-term debt discount and expense. For railroads, the Interstate Commerce Commission includes also rentals. In modern utility accounting, the term is not used at all. As related to electric utilities, it is only used as a convenience in estimates, and in this case the diversity is considerable. Some include even income tax and others exclude even depreciation. Therefore, each author, when using the term, should state which items he includes, as does Mr. Orrok in his paper. The cost of money may vary considerably. The depreciation depends on the assumed life of the plant, which, in the case of new steam plants to be built, is to a great extent a matter of design policy. The assumptions vary from 15 yr to 30 yr. On the basis of straight-line depreciation this would represent a variation of one of the items in the fixed charges from $6\frac{2}{3}\%$ to $3\frac{1}{3}\%$ per annum. In 1920, the U. S. Geological Survey reports a total of 2 598 steam plants in the United States, and in 1935, only 1 491. Since the 1935 figure includes a considerable number of new plants, and since units have been removed from the remaining plants, it is evident that the great mortality of steam plants would possibly make a design policy of 15-yr life for steam plants rational. On the other hand, in 1920, the U. S. Geological Survey reports a total of 1 172 hydro-electric plants in the United States, and in 1935 as many as 1 523, an indication of a very low mortality. Thus, it is evident that, unless a definite case is at hand and all the details are presented so that they can be analyzed and criticized, the discussion will not mean a great deal.

Another feature which is scarcely ever mentioned is the comparative reliability of service. It costs more to render good service than it does to render poor service, and the cost per kilowatt-hour on a system having two outages per year cannot be fairly compared with that of one having twenty outages per year. Is a steam plant at load center more dependable than a remote water-power station with a transmission line; and what kind of a transmission line—single circuit, double circuit on the same structures, double circuit on separate structures, etc? All these features enter into cost, and unless they are known and included in the cost comparison, such comparison means very little.

As to the individual papers: Mr. Orrok's fine presentation of the progress made in fuel economy would have been much more nearly complete if figures had been presented to show the additional fixed charges involved in the higher fuel efficiency. Mr. A. E. Knowlton recently presented a tabulation[41] of large plants from which it would appear that, above a certain point, the saving in fuel brought about by higher steam pressure is offset by higher fixed charges. At the present stage of development such a point should not be overlooked.

Mr. Rogers, in his discussion of the frequency problem, fails to mention that according to Dr. W. M. White[42] this difficulty may be solved in the future by direct-current transmission.

Even if the Symposium is entitled "Economic Aspects," it would seem necessary at this time to sound a warning that, in the search for economy, reliability of service should not be overlooked. Recent widespread outages throughout the country seem to indicate that "Btu-chasing" has reduced the reliability of service of many an important inter-connected system to a level not much higher than that of an isolated little Diesel plant. Mr. Justin's paper seems to reflect the philosophy of "Btu-chasing" regardless of reliability of service.

The first sentence in Mr. Sporn's paper, calling attention to the fact that the cost of generation is only a small part of the cost of a kilowatt-hour delivered to a customer, is to the writer's mind the most important statement of his paper and possibly of the entire Symposium. Actually, the problem of deciding in an individual case whether a steam plant or a hydro-electric plant should be built, has now become a comparatively simple (although at times lengthy) problem of power engineering. It is indeed surprising that there is still so much general discussion on the subject. It is no secret that among engineers a general discussion of the question of "steam *versus* hydro," when no definite case is at hand, is considered a waste of time. This is not so with distribution. Only a few years ago engineers frowned upon distribution as being "low-brow" engineering, or no engineering at all; it was considered a job for the line foreman and storekeeper. More recently, however, engineers have been devoting considerable time to the study of distribution problems, and distribution has become respectable, even "high-brow," engineering. The results are astonishing. Definite problems have been crystallized and partly solved. Mathematical analysis has been applied to distribution studies, and the history of electrical engineering shows that when mathematical analysis is superimposed on operating experience, fine results are inevitable. It may be safely predicted that before long distribution engineering will be the most important part of electrical engineering. Professors may even consider the advisability of devoting a little less attention to machine design and more to the physical and mathematical fundamentals of distribution. The previously mentioned fault in Mr. Sporn's paper, of basing cost on load factor, is particularly evident in Figs. 33 and 34, which present a comparison of cost between shipping energy in the form of coal on the one hand and electricity on the other. It is unfortunate that these important comparisons are meaningless. No doubt, Mr. Sporn will be able to supplement curves on a plant-factor basis and thus give this important set of data a definite meaning.

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[41] *Electrical World*, October 9, 1937.

[42] "Technological Trends and National Policy," National Resources Committee, 1937, p. 281.

Mr. Sporn's expression of doubt about the economy of transmission over very long distances deserves serious attention, particularly because he is no doubt one of the outstanding and most outspoken authorities on transmission problems. Here, too, it would have been helpful if comments had been included about comparative reliability of service.

It would be interesting to know what method of weighting was applied in establishing the average transmission distance on the specific system as 60 miles, and also what the longest distance is, and what, if any, sad experiences were had with this longest distance?

Mr. Freeman's paper presents definite statements and questions without presenting a single figure to substantiate the statements or the reasons for the questions. It is impossible, therefore, to discuss this paper. One of his statements, however, is of such great importance that it deserves further elaboration: "The old economic problem was how to produce enough; the new problem is how to dispose of all that can be produced." As related to electric energy, the problem is how to fill out the load curve and thus reduce fixed charges per kilowatt-hour—in other words, how to increase the load factor for a given capacity, which, again, is another way of asking how to increase the plant factor. This problem has recently become of even greater importance A few years ago it was generally hoped that the valleys in the load curves would soon fill out and that the difference between peak and average would become less drastic. Recent indications are that on many systems the opposite has happened—the percentage increase in peak has been greater than the percentage increase in consumption. Possibly this is due to shorter labor hours in industry and the speeding up of industrial production during these short hours. Even air-conditioning, which was expected to contribute considerably to load-curve improvement, is in many cases peak load. Air-conditioning has been used thus far mostly in industrial and large commercial establishments.

This problem cannot be solved by engineering alone; it must be solved by more modern business methods and more competent salesmanship. Promotional rates at incremental costs for off-peak loads have shown good results wherever they have been tried out, and where modern sales methods were used. However, the "energy salesman" has not yet entered the home. It is a safe guess that none of the readers of this discussion was ever approached by a salesman who tried to sell him the idea of using a few more kilowatt-hours per month. It is also a safe guess that a good salesman could have sold him those few additional kilowatt-hours per month. Of course, in order to sell, a salesman must at least know his own sales prices. With the present complex rate schedules this is difficult. What is needed, therefore, is, first, a simplification of rate schedules so that the customer can check his bill; and second, a drive by experienced salesmen (possibly men who have had experience in the selling of commodities against competition) to encourage the use of off-peak energy. This, no doubt, is by far the most important social aspect of power at the present moment, and, at the same time, the most important economic aspect.

E. W. Kramer,[43] M. Am. Soc. C. E. (by letter).—There seems to be a rather common tendency among engineers in discussing the comparative merits of hydro-electric and steam power plants, to make statements that are entirely too general—even when it is assumed, as in the paper by Mr. Sporn, that the remarks apply only to the United States. The following statements are quoted from Mr. Sporn's paper:

"It covers the economics of steam power *versus* water power and shows that, in most cases, present-day steam power can be produced more cheaply than present-day water power. * * *"

* * * *

"It must be borne in mind that the costs are presented from a general viewpoint. * * *"

* * * *

"* * * that hydro-electric plants, in general, as a source of primary power, are out of the economic range. The true economic function of hydro-electric power is, without doubt, to supplement steam power. * * *"

* * * *

"It appears definite that future power requirements will be met most economically by steam plants, located at a reasonable distance from load centers. * * *"

It is believed that these quotations, although taken from their setting, represent in part what Mr. Sporn purports to prove in regard to the future of hydro-electric construction. It is the opinion of the writer that these statements are entirely too broad. They would be almost 100% wrong for the Northwestern States, such as Washington, Oregon, and Idaho, where the steam power generated is only about 25% of the total and is economical only for the reason that it makes use of the waste products from saw-mills or has value as stand-by service. The statements would be very questionable when applied to California, for the reason that there are many undeveloped water-power sites of large capacity in California that could compete successfully with steam-electric plants using natural gas or a gas-oil combination as fuel. Finally, the writer would question the general applicability of the statements in the Rocky Mountain States, in the Northeastern States, or in the Southeastern States.

In Table 3, Mr. Sporn shows very high kilowatt-hour costs for a number of hydro-electric projects under Federal Power Commission licenses. Examination indicates that he may have been led to his conclusions by applying an extremely high annual rate to the claimed costs given in order to determine the annual fixed charges. The rate used by Mr. Sporn is 10.75%, but nothing is given in the paper to show how this rate was established. Because the major part of the annual cost of producing hydro-electric power is the fixed charge, the selection of a proper rate is extremely important. If the rate selected is too high, it will, of course, tend very strongly to lead to the conclusion that hydro-electric power is more costly than steam, since the investment in steam is relatively less and the operating cost more. It is the writer's opinion that a well-established, conservatively financed and managed utility with a large and growing market should be able to hold its fixed charges on hydro-electric projects (not including fixed charges on transmission and distribution) to 8% or less. The following build-up of an annual rate, which the writer believes would be applicable to California, is presented for a hydro-electric project with an installation of 50 000 kw, or more, and for a head exceeding 300 ft, where a good part of the investment was in dams, tunnels, and penstocks:

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[43] Regional Director, Federal Power Comm., Region 5, San Francisco, Calif. Any opinions expressed are the writer's own and are not to be taken as official expressions of the Federal Power Commission.

	Percentage
Cost of money	5.5
Depreciation	0.55
Insurance and casualty reserves	0.12
Local taxes	0.7
Total	6.87

It is assumed in this case that no transmission is included, since it is understood that Mr. Sporn does not include transmission in his costs in Table 3. He deals with transmission costs later.

The depreciation figure is determined on a sinking-fund basis, interest at 5.5%, and 45-yr life. The life of the tunnels and dams, of course, will greatly exceed this, although it is probable that the life of the machinery may be less.

Justine and Mervine[44] give 6.7% and 11.5% as the usual minimum and maximum total annual rates for fixed charges, and their "illustrative cases" are based on a "typical" rate of 9 per cent. It will be noted that the writer's estimate is slightly in excess of the minimum rate given, somewhat less than the typical rate, and considerably less than the maximum. The rate used by Mr. Sporn is almost the maximum rate, and it certainly is not generally applicable to the plants given in Table 3.

The principal item, the interest rate, should not vary greatly throughout the United States for a large utility which has been conservatively financed and managed. Possibly 6% might be used. A slightly higher rate for depreciation than the 0.55% used by the writer could be used for low-head plants, where the cost of machinery and equipment represented a larger fraction of the total cost. However, assuming a much shorter life and taking obsolescence into consideration also, this item should not exceed 1% on a sinking-fund basis. This would mean an average life of slightly more than 30 yr with money at 6 per cent.

The operating and maintenance charge should not exceed $0.80 per kw-yr. This is equivalent to the rate of 0.4% on the investment for a plant costing $200 per installed kilowatt, and would bring the total annual rate (fixed charges plus operating and maintenance cost, but not including general and administrative expenses) to 7.27 per cent. The writer is somewhat familiar with the Western plants listed in Table 3 and can see no reason why a much higher rate should be applied to the capital cost of these plants if conservatively financed. It is evident that the application of this annual rate to the other hydro-electric projects shown in Table 3 will lead to very different conclusions from those of Mr. Sporn.

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[44] "Power Supply Economics," p. 153.

The foregoing discussion indicates strongly the importance of financing hydro-electric projects at the lowest possible rate, since a difference of only 1% in the interest rate will represent a difference of approximately 12% in the total annual cost and might easily be the deciding factor in determining the economic feasibility of the project.

Under the heading, "Trends in Hydro-Electric Installations," Mr. Sporn comments that the most economical sites were developed long ago, and that those which have been exploited recently have, in general, been less attractive. The inference from this statement, namely, that the sites still to be developed are even less desirable, is possibly applicable in many localities to projects requiring a medium amount of capital investment. The statement, while possibly still true to some extent, is misleading if applied too generally or to large projects (requiring investments of, say, $40 000 000, or more, to use a rough figure) such as the Boulder, Bonneville, or Grand Coulee projects in the West, or the undeveloped projects on the St. Lawrence River in the East, and on some of the larger streams in other parts of the United States. In many cases, such projects have not heretofore been undertaken on account of their size.

The writer does not wish to bring into this discussion the advantages or disadvantages of the construction of large hydro-electric projects by the Federal Government. However, the fact that the Government used a 4% interest rate in determining the charge for power from the Boulder project, and can undoubtedly borrow money at a lesser rate, should not be overlooked in considering the feasibility and probability of the construction of extremely large projects requiring an investment that, because of its magnitude and because of the many uncertainties involved, such as the growth of load to absorb the large output, could not be financed by a utility—or for that matter by a municipality, or even a State. Sound Federal financing can spread the necessary returns to recover the investment, including interest, over a long period to compensate for small initial returns. Using 4% as the cost of money, and adjusting the interest and depreciation allowances accordingly, the annual rate for fixed charges given earlier in this discussion reduces to 5.65%, which is only little more than one-half the rate used by Mr. Sporn.

It is the writer's opinion that each hydro-electric and each steam-electric project necessarily has to be considered carefully on its merits—that no general assumptions can be made as to their relative advantages or disadvantages. As a general rule, they are at their best when inter-connected. The question as to which one will supplement the other will depend on the relative magnitudes of each type connected to the system, and this relation, in turn, will vary according to the locality.

Paul E. Gisiger,[45] M. Am. Soc. C. E. (by letter).—The paper by Mr. Sporn offers a welcome clarification of the essential points pertaining to the cost of electrical energy. Although the facts and figures undoubtedly give an excellent and clear picture of the distinct elements of which the cost of power is made up, it may be questioned whether some of the conclusions drawn are not a shade too pessimistic as far as the future of hydro-electric power generation is concerned.

In the technical literature of the last fifteen or twenty years predictions of the decline of the proportion of hydro-electric power in the total power supply of the United States can often be found, but such predictions have not been confirmed by later statistics. In 1920, hydro-electric plants produced 37.1% of the total energy generated by public utility central stations. In 1924, this percentage was 33.8, and, in 1933, it was 40.7; in the 12-month period ended October 31, 1937, it was 35. These figures show variations governed largely by the annual variations in river flow, but as a general trend, it appears that water power has kept its own during a period when the efficiency of steam-generating apparatus has greatly increased and the annual output of central stations nearly doubled. One factor in this is probably the spreading recognition of the fact that the combination of hydro and steam power is very often more advantageous than the use of either one alone. In this regard, it may be noted that, in 1920, among the large cities on the Atlantic seaboard, Baltimore, Md., was the only one which received large blocks of hydro-electric power; at present (1938), Washington, D. C., Philadelphia, Pa., Providence, R. I., and Boston, Mass., are also supplied to a considerable extent with energy generated in hydro-electric plants. To cite also a foreign example of the same trend, the Metropolitan Area of Paris, France, which lies much nearer to coal fields than to water-power sites, was, until 1927, supplied to less than 1% with hydro-electric power while in 1936 the proportion of hydro-electric energy was 36 per cent.

The writer cannot agree fully with Mr. Sporn's statements that hydro-electric plants, in general, as a source of primary power, are out of the economic range, and that as a general rule no hydro-electric plant is self-sufficient. These opinions hold only within geographical and other limitations. They are undoubtedly correct for large sections of the United States, but their application to other parts of the world—for instance, to Canada—would be out of place. If a hydro-electric plant can be built at some figure near the average of the costs quoted in Mr. Sporn's paper, within a reasonable distance from the places where its output is to be used, its economic justification depends ultimately on the greater or lesser variations of stream flow. In the northeastern section of the United States these variations are, in most cases, of such magnitude that only by the most careful analysis, taking full account of all limitations and possibilities which depend upon stream flow and its improvement by storage as well as of the manner in which the energy can be fitted into the daily and annual load curve of a distribution system (or to what extent it can replace steam energy), is it possible to determine whether development of a given water-power site is economically justified, that is, preferable to steam power?

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[45] Structural Engr., Pennsylvania Water & Power Co., and Safe Harbor Water Power Corp., Baltimore, Md.

On the other hand, if a hydro-electric plant can be built at, say, $140 per kw, and the stream flow allows it to be operated at a daily and annual load factor of not less than, say, 50%, it is not difficult to show that steam power at its present, or even a higher, efficiency will cost more to generate, and a substantial margin remains to cover the cost of transmission. It is true that in the Northeastern United States there are few water-power sites that satisfy such requirements, but next door, in Canada, they are the rule rather than the exception. Consequently, steam plants produce less than 5% of all the power generated in Canadian public utility plants.

With the foregoing, it is intended to show that, for the present at least, the relative economy of steam as against hydro-electric energy depends to such an extent on place and circumstance that no general statement can be made declaring one to be cheaper, unless the specific conditions are given to which it applies.

One point which appears to be overlooked sometimes when costs of steam and hydro-electric power are discussed, is the influence of the prevailing rates of interest on the relation of these costs. It is evident that, since capital cost is normally more than 90% of the cost of hydro-electric power, whereas the cost of steam power is made up to only about 50% of capital charges, a change in interest rates will affect the cost of hydro-electric power more than the cost of steam power (see Fig. 37, in this connection). The figures on which Fig. 37 is based correspond to cost figures used in Mr. Sporn's paper. It is assumed that the kilowatt of installed hydro-electric capacity costs $160 and the kilowatt of installed steam capacity, $100. The annual cost of operation of the hydro-electric plant is taken at 75 ct per kw of capacity and all other charges exclusive of interest at 5% of cost of installation. In the case of the steam plant, the cost of operation is taken from Fig. 30 of Mr. Sporn's paper for an annual load factor of either 45.6% or 57.0%, corresponding to either 4 000 or 5 000 hr of operation per yr. All other charges exclusive of interest are assumed to be 7% of the cost of installation, making the capital charges for the steam plant 2% higher than for the hydro-electric plant, as assumed by Mr. Sporn. No transmission charges were added in case of steam energy,

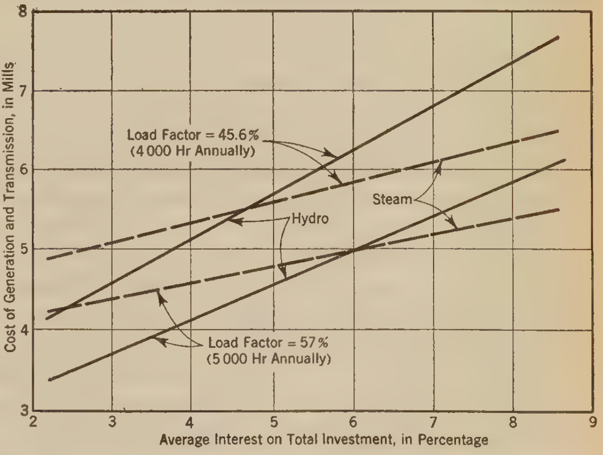

Fig. 37.—Influence of Interest Rate on Cost of Energy

while in the case of hydro-electric energy, such charges have been added for a transmission distance of 50 miles. Transmission costs for this distance and for the load factors used were taken from Fig. 32 of Mr. Sporn's paper. They were added without change to the kilowatt-hour cost resulting from a 5% interest rate, while for other interest rates they were increased or decreased in proportion to the total cost exclusive of transmission.

The resulting curves of total cost per kilowatt-hour show plainly that a reduction in interest rates is relatively more favorable to hydro-electric power than to steam power and *vice versa.*

F. A. Dale,[46] M. Am. Soc. C. E. (by letter).—The cost of generating electrical energy by means of falling water involves four major factors: (1) The cost of the development; (2) the quantity of water available; (3) the efficiency at which the plant is capable of converting the water into energy; and (4) the efficiency at which the plant *does* convert the water into energy. Mr. Sporn has emphasized the fact that the preponderant portion of total hydro-electric energy costs is the fixed charges on investment. Mr. Rogers has presented a masterly summation of what the designers have accomplished in furnishing equipment which is capable of converting a very high percentage of the water energy into electrical energy.

Additional emphasis should be placed on Factor (4) in the cost of generation, namely: That the final cost per kilowatt-hour depends to no small extent on the proportion of possible kilowatt-hours actually generated. From the point of view of cost of development, or fixed charges on investment, the investment cost for delivering water to the power plant may be 75% of the total cost of the project. The water, therefore, is certainly not free. Unlike steam plants, for which fuel can be purchased as it is used, the administration of a hydro-electric plant must purchase, in advance, the entire fuel supply for the life of the project. The water should be used as carefully and effectively as if it were being weighed out and paid for by the ton. Dividing fixed charges by kilowatt-hour output gives fixed costs per kilowatt-hour. The cost per kilowatt-hour can be decreased just as effectively by increasing the denominator of this simple fraction as by decreasing the numerator.

The designers and manufacturers have certainly done their part in furnishing plants that can convert the maximum of available energy. Unfortunately, they cannot follow through to any great extent and insure that their products will be used most effectively. Mr. Rogers mentions automatic load control devices that have been developed to insure efficient operation and which are extremely effective where applicable. The largest part of energy generation, however, is still dependent on the knowledge and ability of the operator. Much can be done to improve the results obtained by operators, especially in the smaller plants, by equipping the plants with additional devices to aid the operators. The major gains, however, must come through the initiative and demands of operating management.

Possibly, as much as a billion kilowatt-hours of energy each year slip, unused, through existing plants in the United States. The corresponding annual increase in returns would be about $5 000 000. Saving this energy is a problem which engineers will solve if given the opportunity.

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[46] Chf. Engr. and Mgr., Santee-Cooper Project, Harza Eng. Co., Charleston, S. C.

Ray S. Quick,[47] Esq. (by letter).—The interesting paper, presented by Mr. Rogers, covers the development of hydro-electric energy, including illustrations of actual machines which have made it possible to convert hydraulic power efficiently into mechanical energy for driving electrical generators.

One of the most interesting problems developed in the generation of hydro-electric energy on the Pacific slope has been an effective means of controlling water-hammer in comparatively long and high-pressure penstocks. Surge tanks are used where possible, but they do not provide an effective control near the power-house where pressures are high. The penstocks are of heavy construction and constitute a major part of the first cost of the development; therefore, their economical design is a problem of first importance. Where water economy is not required, the simplest means of controlling pressure rise is the synchronous by-pass. With this arrangement, the flow through the unit is constant and independent of load. The hand-operated needle of the impulse turbine with governor-operated deflector is a characteristic example. Most plants, however, depend upon seasonal storage, thus requiring a high degree of water economy, and it is here that the pressure regulator, or relief valve, finds its most valuable application.

In Fig. 4, Mr. Rogers has illustrated two typical forms of automatically controlled nozzles for impulse turbines, arranged for full water economy. The left-hand view shows the governor-operated power needle with auxiliary relief by-pass, whereas the right-hand view shows the governor-operated deflector with a slow-moving, power-needle follow up. With the first mentioned arrangement, the oil-pressure governor controls the opening of the power needle to suit the load. An oil-filled dash-pot is interposed in the linkage connection between the power and the relief needles in such a manner that a rapid closing movement of the power needle causes a corresponding or synchronous opening of the relief needle. The heavy closing springs on the dash-pot urge the relief needle to return to the closed position at a rate determined by the dash-pot adjustments and resulting in very little pressure rise. An opening movement of the power needle resulting from an increase in load first synchronously closes the relief needle if it is still open, otherwise simply by-passing the oil in the dash-pot and leaving the relief needle shut. As it is necessary to accommodate oncoming load with the relief needle shut, it is customary to adjust the rate of opening of the power needle to such a value that there is little pressure drop in the penstock and, consequently, no objectionable subsequent pressure rise. A sudden opening of an orifice on a high-head penstock otherwise would produce, first, a sudden pressure drop, followed by an excess pressure. These sudden surges are objectionable and must be prevented.

In the case of the needle-deflector combination, the deflector moves quickly on load rejection and bends the jet away from the water-wheel buckets. The power needle follows up at a rate permitted by the retardation of the water column in the penstock. Its rate of closure, therefore, is of the same order as that of reclosure of the relief nozzle of the aforementioned alternate system. Likewise, the rate of opening of the power needle with the deflector combination is at the same rate as the corresponding opening of the power needle of the relief nozzle system. It is thus evident that either method of controlling an impulse turbine results in complete water economy with gradual load changes and with wastage of water only for a short period, such as 1 min to 2 min following a sudden loss in load.

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[47] Chf. Engr., The Pelton Water Wheel Co., San Francisco, Calif.

When they are wide open, the relief nozzles, illustrated in the foreground of Fig. 2 of Mr. Rogers' paper, discharge a jet about 8.5 in. in diameter and have an output of approximately 35 000 hp under a 2 200-ft head. The potential destructive force of this jet is enormous. The safe conversion of this energy into turbulent flow in the tail-race has constituted a major problem of impulse turbine design. Special forms of baffles have been used in certain applications with success. The most effective form, however, is the through type of energy absorber, consisting of a circular open-bottomed bowl having a cone supported within it by radial ribs. The jet of water is concentric with the point of the cone, which acts to spread the water radially against the inside of the bowl,

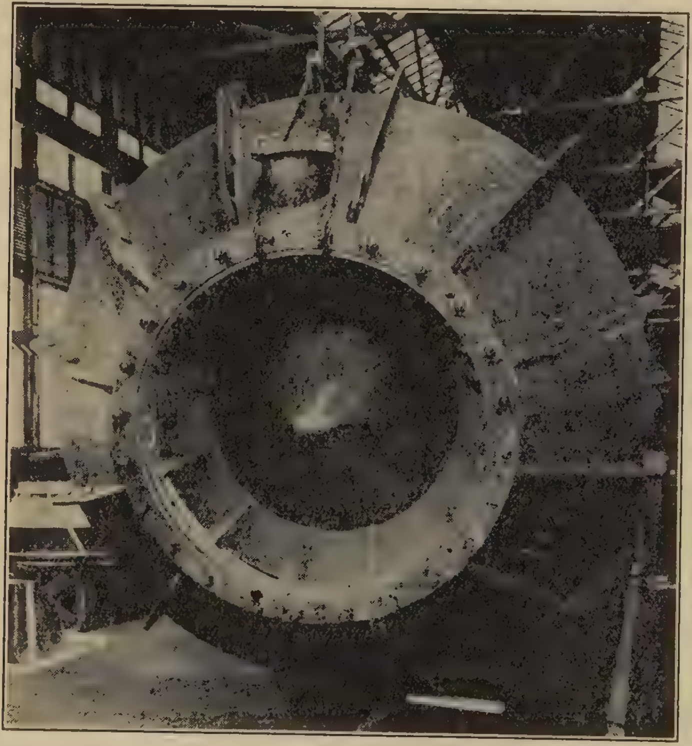

FIG. 38.—SHOP ASSEMBLY OF ENERGY ABSORBER

which, again, turns it upon itself and allows the discharge to mingle with the tail-race water without objectionable external disturbance. The energy absorber at Big Creek No. 2-A is a comparatively small structure.

In the case of the Francis turbine an outstanding installation of a pressure regulator and through type energy absorber will be found in Units N5 and N6 at Boulder Dam. Fig. 5 of Mr. Rogers' paper shows the shop assembly of these two turbines, the outlet for one pressure regulator connection being shown in the foreground. In this case, the pressure regulator is of the needle type using a pilot system inter-connected with the gate-shifting ring through a dash-pot control, and having a hydraulically operated main needle or plunger with mechanical interlock. The mechanism is designed so that should the movement of the main plunger be obstructed the governor will be able to close the turbine gates only at the rate permitted by the dash-pot timing, thus preventing objectionable pressure rise in the penstock even under such an abnormal set of conditions.

Under average conditions, each pressure regulator will have a capacity of 1 860 cu ft per sec under a 480-ft head which corresponds to more than 81 000 hp in the jet when computed at 80% efficiency. The energy of this water must be absorbed in order to prevent an objectionable disturbance in the tail-race. Fig. 38 shows the shop assembly of the energy absorber for the Boulder pressure regulator. The jet of water (which is approximately 52 in. in diameter) strikes directly upon the cone supported in the bowl, turning the water radially outward. The water is then turned back upon itself beneath the bowl from which it discharges downward as turbulent flow through a discharge elbow and concrete-lined horizontal outlet into the tail-race. The entire absorber is of heavy cast-steel construction, with ample provision for the admission of air into those regions near the jet where air may be required.

The pressure regulator and energy absorber system constitute a valuable contribution to the economical generation of hydro-electric energy, making possible a reliable means of controlling the speed and pressure rise with a penstock of minimum first cost. The pressure regulator also assists, in a most valuable way, in damping out periodic swinging surges that may be established under certain conditions of governing with a closed penstock system.

F. Knapp,[48] Esq. (by letter).—The paper by Mr. Rogers is decidedly not cosmopolitan. Only hydro-electric installations in the United States are discussed, and the only reference made to plants abroad is to the Shannon development in Ireland. The progress made in Europe (especially in the past few years) is of such magnitude as to be of special interest to American hydraulic engineers, but is entirely set aside by the author.

As far as impulse turbines are concerned, the largest unit—with 78 600 hp for a static head of 2 360 ft—is running in a Brazilian plant.[49] The 49 500-hp Pelton turbines for the Dixence plant in Switzerland, set the record of head—5 770 ft.

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[48] With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[49] "Water-Power in Brazil, with Special Reference to the São Paulo Development," by A. W. K. Billings, M. Am. Soc. C. E., *Minutes of Proceedings*, Inst. C. E., London, 1936.

There is a marked tendency to build large impulse turbines with vertical shafts and two nozzles. Typical examples are the Etzel and the Handeck plants, both in Switzerland. The straight-flow nozzle design, already incorporated in the turbines of the Dixence plant, is an old Swiss idea, introduced just prior to the World War by V. Gelpke.

Considerable difficulty was experienced for a time with cavitation of impulse-turbine buckets, and it developed that a design successful for a head of, say, 1 300 ft, was a failure for a head of 2 300 ft. Stroboscopic observation of the buckets in motion, and refined design methods, were the means of overcoming these troubles. Efficiencies of impulse turbines are now 90% or more.

The 32 600-hp Francis turbines of the Piottino plant, in Switzerland, with a speed of 750 rpm, work under a net head of 1 060 ft, and the "back-pressure" turbine of the Zapello power station in Italy operates under the unusually high head of 1 280 ft, with 33 ft of back pressure. Each of the two sets installed develops 8 800 hp at a speed of 1 500 rpm.

Designs have been prepared for a two-stage turbine, with a maximum capacity of 37 000 hp and a total head of 1 280 ft, running at 1 000 rpm. The first stage consists of a Francis runner, designed for a head of 1 100 ft; and the second stage is a propeller runner, for a head of 165 ft, with a back pressure of 15 ft. The last part of the head would be used in a single and separate turbine of the propeller type. It is clear that such an arrangement diminishes considerably the size of the units.

It is only right to mention that the development of the high-speed turbine of the propeller or Kaplan type would not have been possible without the simultaneous advance in the design of draft-tubes, inasmuch as in these turbines the water still has about 30% of its total energy, when it leaves the runner, and this energy must be regained in the draft-tube. The elbow draft-tube of the Safe Harbor turbines (Fig. 17), is typical of this development; the other types of energy regainers, such as the hydraucone and the spreading draft-tube, are now almost obsolete—mostly for economical reasons. It is interesting to note that the elbow draft-tube, as used to-day, was first developed in the United States.

As far as laboratory testing of complete hydraulic turbines is concerned, it is noteworthy that one Swiss manufacturer has installed an aerodynamic research laboratory, using air instead of water. Besides the easier testing and the smaller space requirements this research station offers the great advantage that it is now possible to measure directly the pressures on the rotating runner blades.

The writer was surprised at Mr. Roger's statement that it is not yet possible to determine the actual benefit obtained by machining the blades as regards either performance or avoidance of pitting. It seems that with the great relative velocities in high specific speed turbines there should be a certain gain in efficiency with correctly machined blades. At least, one practical case known to the writer indicates that a correct shape of the blades, such as that obtained by machining, is of decided advantage in avoiding local cavitation.

The description of the unit installed in the Marmet plant, as given by Mr. Rogers, is most interesting. A few European installations have turbines with runners of the Kaplan type, but with fixed guide-vanes of the diagonal type. The efficiencies obtained were between those of a Kaplan and a propeller turbine.

The efficiencies of the model pump-turbine unit shown in Fig. 21 are excellent and rather promising. A unit of this type, with a Kaplan runner and diagonal guide-vane, has been installed in the Baldeney plant, in Germany.

D. S. JACOBUS,[50] ESQ. (by letter),—Such a true picture of the developments that have occurred in the generation of energy are presented by Mr. Orrok that the writer will simply add some comment concerning changes in steam boiler practice. His oldest recollection in power generation is of the Corliss engine installed in the Machinery Hall of the Centennial International Exhibition held in Philadelphia, Pa., in 1876. There was an iron platform surrounding the engine at a somewhat higher level than the floor of the building and the writer can remember standing on this platform at the time of the Exhibition. What awed him most was the tremor of the platform, and the noise of the gears and, above all, the dignified motion of the engine and its great size.

The Centennial engine consisted of two beam engines of 700 normal hp each, although they could be worked up to a total of 2 500 hp if required. The two engines were connected to a common fly-wheel, which was 30 ft in diameter, with gear teeth on its outer face. The fly-wheel made 36 rpm. Steam was furnished by twenty Corliss upright boilers of 70 hp each, the steam pressure ranging from 25 lb to 80 lb per sq in., according to the amount of power required.

A modern 50 000-kw steam turbine would generate about fifty times as much power as the normal amount developed by the Centennial engine; it could be run with steam from one or two large steam boilers in place of twenty boilers; and the entire unit would occupy less space.

The late Professor Thurston, who is the first authority mentioned by Mr. Orrok, was an indefatigable worker and was noted for the exceptional accuracy of his statistics. He was an able teacher and, in addition to foreseeing the tremendous increase in power development, he impressed upon his students the necessity of dealing with the human side of engineering. In addition to specializing in steam engines he did much in contributing to the safety and improvement of steam boilers. He often referred to the tests on steam boiler explosions by Francis B. Stevens who was instrumental in having the railroad companies subscribe toward the expense of the tests. Professor Thurston served as a member of the Federal Commission that witnessed Mr. Stevens' tests. The boilers were tested in 1875 on the United States Army Reservation at Sandy Hook, N. J. A bomb-proof retreat was built for observers. Four old steamboat boilers and five new parts of boilers built for the tests were fired until the the pressure either reached a point at which they failed by leakage or they exploded. Explosions had often been attributed to some mysterious action; these tests, however, showed that they were due to simple and preventable causes.

In addition to contributing to safety in boiler construction, Professor Thurston did much in revealing the fundamental features bearing on efficiency, in papers and books that he wrote on the subject.

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Advances in the art of boiler construction have involved improvements in the furnaces and fuel-burning equipment as well as in the boiler proper. Water-cooled furnace walls have greatly reduced the time required for shutting down boilers for inspecting, cleaning, and repairs. The use of pulverized fuel has grown to a point where it is utilized in most new equipment for large coal-fired boilers in the United States. Steam pressures and temperatures of superheated steam have increased to a point at which boilers for central station and industrial plants with natural circulation are being built for 2 000-lb working pressure and more; and, forced-circulation boilers for experimental purposes are in use, that can be run at 4 000-lb pressure, which is greater than that corresponding to the critical temperature. The steam temperatures have increased correspondingly. A comparison of central power-plant boilers in 1905 and to-day is given in Table 7.

Improvements in instruments and apparatus contributed much to the attainment of present-day economies. Engineers of the "old school" will remember the uncertain results secured by the so-called barrel calorimeter which was recommended for use in the first report of the Committee on Steam Boiler Trials of the American Society of Mechanical Engineers. Professor Alfred M. Mayer, who for many years was the head of the Department of Physics at Stevens Institute of Technology, at Hoboken, N. J., and who wrote a number of important papers on his tests of inaudible sounds, etc., observed the writer operating such a calorimeter. After standing for a time without saying anything, he turned away with the remark, "Why not use a wooden horse bucket and a tin back thermometer?" After the barrel calorimeter, the so-called coil calorimeter was used. It was not until the introduction of the throttling calorimeter that results were secured of sufficient accuracy to determine the effect of different collecting nozzles in the steam main.

TABLE 7.—COMPARISON OF CENTRAL POWER PLANT BOILERS IN 1905 AND 1938

Descriptive	1905	1938
Heating surface of boiler proper, largest unit, in square feet	6 040	53 926
Heating surface of boiler, superheater, economizer, and air heater, in square feet	7 000	144 402
Working pressure, in pounds per square inch	225	650 to 2 500
Temperature of superheated steam, in degrees Fahrenheit, maximum	550	700 to 950
Maximum evaporation, in pounds per hour per boiler unit	30 000	1 000 000
Volume of boiler setting for largest boilers, in cubic feet	7 650	42 255
Height from bottom of walls of setting to center of steam and water drum for largest boilers, in feet	19	80

In the early Eighties tests of an alternating-current and a direct-current electric generator were made at the Stevens Institute for Mr. Edward Weston. There was a conspicuous lack of accurate instruments. A home-made "tangent galvanometer" was used for an ammeter for measuring the direct current, calibrating on each test by depositing copper. The "tangent galvanometer" consisted of a single circle of copper wire placed in a vertical plane, at the center of which was an ordinary magnetic compass. A voltmeter was checked by measuring the resistance of the circuit; and, a standard silver cell was built for checking voltage measurements. Both the "tangent galvanometer" and the voltmeter were affected by local magnetic influences. Finally the "tangent galvanometer" was removed to a small shed built on the campus well away from the main buildings, after which its readings were more consistent.

The troubles encountered in measuring the alternating current were still greater than in the case of the direct current and a number of methods of checking were devised. It was the extreme difficulty of obtaining reliable measurements in these tests that led Mr. Weston to develop the accurate electrical measuring instruments for which he became famous.

Much study has been made of boiler-fed water conditions. A modern steel-tube economizer requires the elimination of oxygen and carbon dioxide from the water, and necessitates the use of apparatus unknown in the older practice. The feed-water in most large central stations is as pure as the distilled water secured from a drug-store. With the higher degree of purity, troubles have developed through corrosion and corrosion fatigue of the steel, which necessitate adding chemicals to the water. Certain elements in the feed-water may lead to a characteristic cracking of the steel plates, known as caustic embrittlement, and it is necessary to provide a certain balance between the elements in the water to avoid this action. The treating of feed-water has become an art in itself, and, in order to operate an up-to-date power plant to the best advantage, the chemist must co-operate with the engineer to a greater extent than in the past.

Improvements in boiler-room auxiliary apparatus have contributed much in co-operation with boiler improvement to attain the present-day economics. The efficiency based on an accurate determination of the heat of combustion of the fuel has replaced the evaporation per pound of fuel or per pound of combustible, in reporting tests of boilers. The old-time boiler-room, where nearly everything was manually operated, has now yielded to what is more nearly an automatically, well-conducted laboratory, with a few men watching the controls of the factors bearing on combustion, steam generation rate, pressure, and temperature. Fuel feed, air supply, and fuel air ratio, boiler-feed, water level, and all pressures are controlled automatically with connections to an operating board. Its gages and meters showing the steam flow, percentage of carbon dioxide, and steam flow-air flow are symbolic of the scientific management independent of manual action, which has eliminated wastes that are bound to occur when operators must act on mere indications to guide them in making adjustments. These same elements also insure prompt response to changing load conditions and add to the safety of operation.

A marked advance has been made in the ability to predict the efficiency obtainable with the great variety of arrangements and sizes of furnaces for different fuels, boilers, superheaters, economizers, and air heaters, and the approximate rules in general use a comparatively few years ago have now (1938) been replaced by new formulations of a much more exact nature.

The Boiler Code Committee of the American Society of Mechanical Engineers has been an important factor in advancing the art and the safety of boiler construction. There are now 23 active members of the Boiler Code Committee and 40 members of the Conference Committee which is made up of representatives of those in charge of enforcing the Code in the States and municipalities that have adopted it. More than 160 men in all are continually interested and take a regular part in the work of the Boiler Code Committee, and many others having special information render invaluable help. Since it issued its first Code in 1914, the Code Committee has been successful in securing unanimous decisions in all its final actions.

Practically all steam and water drums for high-pressure boilers are now fusion welded, the old practice of riveting being correspondingly replaced. For a number of years the Boiler Code Committee was asked to formulate rules for boilers and other pressure vessels for what was at first known as autogenous welding. Certain very restricted rules were agreed on for unfired pressure vessels and for seal welding of certain joints in steam boilers. After many hearings, rules were issued in 1931 for fusion welding drums or shells of power boilers.

Radiographic examination by the X-ray or gamma ray in making non-destructive tests of the welds was the turning point in the development of fusion welding because, through its use, the safety of the welds is less dependent on the human element. The rules for fusion welding in the Boiler Code are still far from complete and are being gradually extended. This involves much painstaking work, since the use of steels and alloy steels of higher tensile strength has introduced problems requiring solution on which the best metallurgists are not yet in entire agreement.

In dealing with economy in the generation of electrical energy, one should not overlook the important saving accomplished by electrically tying in one generating plant with another and in this way insuring a continuous service with a reduced capital investment. This was one of the factors that enabled electricity to be supplied to the consumer without increased cost during the period of high prices in spite of increase in the cost of labor and fuel; and it is an invaluable aid. However, it is evident that if carried too far there may be a major shut-down due to an accident in one or more of the generating plants or on a transmission line; so there is a point beyond which it will be dangerous to go in effecting a saving by not installing spare units in a given generating station. How nearly this point has been reached, through the natural effort to avoid further investments in the present period of uncertainty as to the near future, is something of vital public interest.

JOEL D. JUSTIN,[51] M. AM. SOC. C. E. (by letter).—In view of the fact that there has always been considerable discussion as to the relative cost of transporting energy long distances, as coal by railroad, and of transporting it as electricity over transmission lines, a concrete case may be of interest.

Several years ago it was proposed to construct a large steam plant on the Susquehanna River adjacent to the anthracite mines of Pennsylvania, utilize low-value anthracite (very nearly a waste product), and transmit the electric energy to Philadelphia, a transmission distance of about 150 miles.

However, a thorough investigation of the proposal proved that it would be very much more economical to locate the proposed steam plant at, or near, Philadelphia and ship the energy as coal over the rails to the plant. This kind of transmission, by which power is transmitted long distances without take-offs, might be termed "express transmission," and it is generally uneconomical except in the case of certain low-cost hydro-electric projects. This does not necessarily mean that long-distance transmission is uneconomical, however. Often, a minor load center reached by transmission could not alone support a large, low-cost, efficient steam plant, but several of these centers inter-connected may make such a plant economically advisable. There are also other factors which sometimes justify long-distance transmission as discussed by the writer (see text beginning "Reserve Capacity Savings Due to Inter-Connection").

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[51] Cons. Engr., Philadelphia, Pa.

In considering the desirability of installing additional capacity, Mr. Sprague calls attention to the operating saving that may sometimes be effected through the installation of a new steam plant, and shows by an example that such an annual saving may sometimes cover a considerable part of the fixed charges on the new plant. The statement is correct, of course, but the weight that should be accorded to this factor is sometimes quite uncertain.

Thus, in the illustration given by Mr. Sprague, a new steam plant would operate at a capacity of 7 000 hr per yr, or on a capacity factor of 80 per cent. No steam plant operates for many years at such a high capacity factor. As shown by Mr. Samuels, the average annual capacity factor of all steam plants in the United States is about 34 per cent.

In an economic study for the purpose of determining the advisability of a capital expenditure for a new steam plant, erroneous conclusions may be reached by assuming a capacity factor of 80% as in Mr. Sprague's illustration. Although such a high capacity factor may rule for the first few years, experience indicates that the average annual capacity factor of the steam plant during its life is more likely to be something of the order of 35 per cent. Hence, such savings as those indicated by Mr. Sprague should, more properly, be considered as temporarily contributing to the carrying of the fixed charges on the plant until such time as the full capacity of the plant is required.

The foregoing is only one of the reasons why, in common with many other engineers, the writer finds it advisable, in economic investigations of this character, to make a study of the entire system, together with set-ups showing estimated results over a term of years in the future.

Based on the best available load predictions, the object is to arrange a program to meet the increasing load with increments of capacity in such a manner that the total annual operating cost (for power supply) plus annual fixed charges on new money is always at a minimum as compared to what it would be with any other plan of development. This is a "cut-and-try" method and is necessarily laborious. Conclusions thus arrived at are often at variance with short-cut methods of analysis, but are much more reliable. If load growth does not proceed as estimated, the soundness of the conclusions are not usually affected, as, generally, this merely means that the program is stepped back or accelerated, as the case may be, to meet the new load estimates.

Philip Sporn,[52] M. Am. Soc. C. E. (by letter).—It is interesting and gratifying that all the discussions are free from any great amount of "heat" and acrimony which prevail altogether too frequently in any discussion involving economics. For his own contribution to the Symposium, in particular, the writer anticipated much critical review because it dealt with that uncertainty —cost—and what is more controversial—comparative cost. He is particularly delighted, therefore, that the discussions are so generally commendatory, leaving so little to discuss in the closure.

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The writer is not a partisan of either steam power or hydro power but he is an advocate of generating power in that manner which will be most economical and, therefore, in that manner which is certain to be most productive both from a social and from a business standpoint. During the five years since 1933 it has been assumed altogether too frequently that the former (that is, social benefit) could be obtained at the expense of the latter. The writer, however, does not believe that it can. If a situation to satisfy the true economic criteria demands hydro power, it should be used for that situation, but if the situation demands the use of steam power, the choice should be steam power. The writer makes this statement because underlying some of the discussions he detects a feeling (not openly expressed) that he has not been entirely impartial in his presentation and that he has, perhaps, "tipped the scales" in favor of steam power. This, of course, is not the case. Furthermore, the writer very definitely has refrained from adopting a regional viewpoint and has been unwilling to look at power as something that should be handled from a narrow viewpoint or from the viewpoint that what is locally available is of necessity the best simply because it is "on the spot." On the contrary, he has always considered as the basic outlook for the proper engineering and social economic solution of the power problem, the principle that, regardless of where in the country the fuel or the materials must come from, as long as they are within the boundaries of the United States, in the long run the national economy would best be served by using those materials and that equipment which result in the lowest cost power. For some time to come it may be very uneconomical to develop many hydro sites and, with the grave situation in which the soft coal industry finds itself, the national interest would best be served by adhering to steam power, provided steam power gives the net over-all lowest cost of power—and by that the writer does not mean cost after deducting any subsidies either of a direct or of an indirect nature.

Again, this means that the writer is perfectly willing to look with equanimity upon the immediate present loss of white coal or hydro power, but he does not consider that a loss in the true sense of the term; nor is it of any grave consequence, provided the national economy is better served by developing some other form of power even if that involves the burning of fuel.

Mr. Sprague's point regarding certain limitations and qualifications necessary to be considered in connection with the data presented in Fig. 34, on the relative costs of freight and electrical transmission of energy, are well taken and are generally understood by every engineer who has anything to do with the power problem. That is, it is well understood that any study of the type made as the basis for Fig. 34, of necessity, fails to give consideration to the distribution angle and that from a distribution standpoint the economics of short transmission distances become more favorable to the electrical side as against the freight side. Hence, in spite of the apparent better economics obtainable by freight for transmission distances even as short as 30 miles, the mean weighted distance of transmission on the inter-connected system of the American Gas and Electric Company is actually close to 60 miles under normal conditions.

However, Fig. 34 seems very much worth studying. Taking to heart the lessons it shows would obviate the bad effects of the large mass of misinformation propagated almost daily by those who should know better as regards the feasibility or economy of long-distance transmission. Thus, quoting from a United Press Dispatch as printed in the *New York World Telegram* of July 29, 1938:

> "Mr. Ross suggested that the Columbia River on the west coast might even supply electricity to New York, 3,000 miles away, and the Niagara or St. Lawrence Rivers might send electricity to homes in Florida or for irrigating the Southwest.
>
> "Mr. Ross expressed his ideas to the Engineers' Club of Seattle. He has made a special study, at the request of President Roosevelt, he said, of the possibilities of transmitting electricity from one part of the country to the other.
>
> "The results of such a system, Mr. Ross said, would be cheaper electricity, more abundant use of it and improved service to consumers. He said the price would be reduced because a purchaser now getting electricity from a distance of 100 miles could get it as cheaply from a distance of 1,000 miles.
>
> "Direct current transmission would be used between the major plants suggested by Mr. Ross, but the present alternating current system would not be disturbed."

The writer has followed very closely every major transmission development for the past twenty years and has been intimately associated with the expansion and development of the transmission art during that period. He has particularly followed at close range the work that has been done and that is being done in the development or the attempted development of direct-current transmission, and all that he can say in connection with any opinion that economical 1 000-mile or 3 000-mile transmission of power is imminent, is that it is pure "poppycock," and that on the whole does not warrant any serious lay, let alone engineering, consideration.

Mr. Samuels seems quite unhappy about the purported inaccurate use of some of the terms such as "load factor," "use factor," and "plant factor." The writer cannot help but agree with him, but he feels that it will be necessary to wait for a younger generation to grow up and be taught before any degree of perfection is possible. The writer does not agree that 15 yr should be considered the average life of a steam plant because he does not believe that any such conclusion would be correct even if statistics may make it appear so. For 20 yr (1918–1938), the power industry has grown very rapidly. As a matter of fact, until the depression that started in 1930, regardless of the changes in the business cycle, there never was a year in which the output of electric energy by utility industries was not greater than the preceding year's output. Changes in the art of steam power generation moved forward just as rapidly. This led to a very high rate of obsolescence on steam plants, both from inadequacy and change in the art, but more so from inadequacy than change in the art. To-day, the electric utility industry has definitely passed the period of very rapid growth and is subject to the same fluctuations of the business cycle almost to the same degree as other businesses; and, although the art of steam power generation is continuing to improve, the effects of obsolescence and inadequacy probably will not be such in the next 20 yr as to limit the plants that are now being built to a life of only 20 yr. As a matter of fact, many of the developments that have taken place during the past several years have been such as to arrest, definitely, the sharp obsolescent trend of plants built in the decade, 1916–1926. Therefore, the writer believes that Mr. Samuels' selection of 15 yr for the life of a steam plant is ultra-conservative.

Mr. Kramer takes exception to some of the statements in the paper with regard to the status that hydro-electric energy holds in the present power "picture" of the United States, and specifically points out that its position in the Northwestern States is better than would be indicated by the statements that hydro plants in general as a source of power, are out of the economic range. He may be correct as regards certain sections of the Northwest; but the writer does not believe he is right as regards California, and he is quite certain that Mr. Kramer is wrong as regards the Southeastern States. Furthermore, as regards the Northwest, the writer is not at all certain that if consideration is given to the definite static conditions of hydro developments (that is, to the fact that anticipated improvements in that field are almost negligible, whereas the improvements that can be looked forward to in the steam generation art are many), the economics would not be again in favor of steam generation. However, even if Mr. Kramer were correct, it is a fact that 70% of all the energy generated and used in the United States is produced in the area east of the Mississippi River, and the writer's statement, even in its full broadness, is definitely applicable to that region.

Mr. Kramer also takes issue with the use of 10.75% as a fixed-charge rate for hydro plants, and he shows a build-up whereby he gets a value of 6.87 per cent. Even disregarding the fact that there is no general agreement on this point (and the writer, definitely, does not agree to the basis followed by Mr. Kramer in building up his fixed charge of 6.87% by the use of the sinking-fund method of depreciation), there is the further fact that what is really important is the relative value of fixed charges. Mr. Kramer does not make clear that where the writer has used 10.75% as a fixed charge for hydro-electric power he has used a fixed charge of 12.75% for steam plants and that, if a charge of 6.87% is the proper one for hydro plants, then logically 8.87% would be the proper one to use for a steam plant.

Again, Mr. Kramer takes exception to the statements that the most economical sites were developed long ago and states that this is not true if applied to such developments as Boulder, Bonneville, or Grand Coulee, or to the undeveloped project on the St. Lawrence River. His conclusion is that in many cases such projects have not been undertaken heretofore on account of their size. This disregards the fact that one of the principal reasons that they have not been developed is that they are located such a distance from a market. There is much undeveloped hydro-energy in Alaska but, for the present, at least, it will have to remain undeveloped until local markets for it are found.

It cannot possibly be brought to market economically in the United States. There are many people who believe that much of the power that is now undergoing development is uneconomical power for the reason that the cost of bringing it to the market makes the combined delivered cost so much greater than power that could be generated within reasonable distances from the market.

A number of other points have been raised which the writer would like to discuss but space will not permit. They are so important and so technical that it would involve the preparation of a discussion of a length perhaps equal to some of the papers in the Symposium. For instance, Mr. Samuels quotes Mr. Knowlton as having recently presented in the *Electrical World* a tabulation of large plants from which it appears that, above a certain point, the saving in fuel brought about by a higher steam pressure is offset by higher fixed charges. This merits considerable discussion. There is the aspect, for example, that what is special practice as regards pressures and temperatures to-day, becomes standard practice five to ten years from the time such temperatures or pressures are first brought within a practical operating range. There are many other phases of this problem that need to be discussed. The discussion of subjects such as this one would be interesting and informative but probably would not add sufficiently to the value of the Symposium to warrant the time and the space. It may very well be apropos for discussion at some other point.

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Paper No. 2035

COST OF ENERGY GENERATION

SECOND SYMPOSIUM ON POWER COSTS

Page

Elements of Cost.
By John C. Page, M. Am. Soc. C. E. 1051

Heat-Generated Energy.
By C. F. Hirshfeld, Esq., and R. M. Van Duzer, Jr., Esq. . 1060

Hydro-Generated Energy.
By H. K. Barrows, M. Am. Soc. C. E. 1088

Combined Energy Generation.
By Ezra B. Whitman, M. Am. Soc. C. E. 1115

Depreciation and Obsolescence.
By Maurice R. Scharff, M. Am. Soc. C. E. 1127

Recapitulation.
By W. F. Uhl, M. Am. Soc. C. E. 1139

With Discussion by Messrs. Joel D. Justin, Frank H. Mason, W. V. Burnell, Daniel W. Mead, A. G. Christie, William E. Rudolph, Louis Elliott, H. G. Gerdes, Leverett S. Lyon, John T. Madden, R. L. Thomas, Theodore B. Parker, Philip Sporn, L. N. McClellan, James C. Bonbright, R. Robinson Rowe, V. B. Libbey, Edwin D. Dreyfus, R. H. Parsons, F. Knapp, John C. Page, C. F. Hirshfeld and R. M. Van Duzer, Jr., H. K. Barrows, and Maurice R. Scharff.

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ELEMENTS OF COST

By John C. Page,[1] M. Am. Soc. C. E.

Synopsis

In recent years the mere mention of the word, "power," especially when uttered by a Government official or used in connection with the word, "water," has been sufficient to send large groups of Americans into hysterics. As a result it has been difficult for any one to approach this subject calmly at a time when a calm approach is needed.

Modern civilization is founded on the utilization of power in a myriad of ways. Of the total energy used in the United States, that produced by water power has remained almost constant at between 3% and 4% for fifty years. The energy used to generate electricity for public utilities in 1936 amounted to less than one-tenth of the total derived that year from mineral fuels and water power. Stated in its true perspective in this manner, the importance of power supply for public utilities appears comparatively small.

There is nothing in the situation with respect to power that cannot be rationalized. When reduced to its elements, the power question resolves itself into simple terms. This paper is concerned with elements of power cost, and the writer submits that these elements, in themselves, are not complicated. Papers will follow which will deal more specifically with individual items of cost.

It should be borne in mind—and many writers on the subject have not consistently done so—that each power development or project is a problem in itself, and that it is unsafe to generalize broadly on the power theme. "Sauce for the goose" is not necessarily "sauce for the gander" in the power field. That method of power generation which is most economical, and which, therefore, should yield the greatest social values in one area, may be the most costly in another. In any locality, changing conditions also may make the method that was most economical yesterday more costly to-morrow. These facts are self-evident to the Engineering Profession, but they have not always been so treated.

The elements of cost of electric power fall naturally into two general groups: (1) Fixed charges which include interest and amortization, depreciation and obsolescence, taxes and insurance; and (2) operating expenses, which include fuel in the case of steam and Diesel generating plants, water, oil, and operating supplies, operators' wages, superintendence, maintenance and repairs, administration and legal expense, and miscellaneous expenses.

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The elements of cost of power are essentially the same whether produced by steam, Diesel, or hydro-electric plants, except the cost of fuel, but the extent to which the various elements enter into the cost varies widely. The purpose of this paper is to outline the elementary parts involved in the cost of producing electrical energy and to indicate how the total cost of energy can be affected.

Fixed Charges

Fixed charges depend primarily upon the investment cost of the various components of the power system, and they represent a very large part of the total cost of water power. In steam plants, however, the fixed charges represent a much smaller proportion of the total cost of power.

The capital cost of hydro-electric developments varies widely and depends on the physical characteristics of the site, such as topography and location, head, stream flow, storage requirements, right of way, etc. The capital cost of steam plants also varies through a considerable but fairly well established range and depends upon the type and capacity of the plant, location, kind of fuel, and fuel economy for which the plant is designed.

The rate of interest paid on the money invested in a power project varies from year to year, depending upon the condition of the money market. At present, both public and private agencies can borrow at from 3.5% to 4% per yr. A public agency usually finances its undertakings by the sale of bonds, and provision must be made for the retirement of the bonds as well as for payment of interest. Private company financing on the other hand is somewhat more involved; funds may be obtained by sale of bonds or by sale of stock, or by a combination of the two. The principal difference between the public and the private agencies in the matter of fixed charges is that the private company endeavors to secure a so-called fair rate of return on its investment, whereas the public agency is generally concerned only with interest and amortization of the capital investment.

Depreciation depends upon the useful life expectancy of the various features of the power development. The usual practice of both public and private power agencies is to provide a depreciation reserve fund for the purpose of replacing the various components of the power system. In the case of a public project financed entirely by sale of bonds which must be retired within, say, 30 or 40 yr, it is doubtful whether depreciation should be included in the features that will have a useful life far beyond the end of the amortization period of the bond. For instance, a large concrete dam, which is a major feature of a hydro-electric development, has a relatively long life and may reasonably be expected to be useful for 50 or 100 yr, or even longer. If depreciation charges are included in the cost of such a feature during the period that its cost is being amortized, it would in effect constitute a double repayment. It is necessary, however, to include sufficient depreciation to provide for replacement of those items which will have to be replaced during the amortization period. Although a dam may have a useful life expectancy in excess of the amortization period, it may be equipped with gates and other mechanical equipment which will have a much shorter life, and will have to be replaced during the amortization period.

The useful life expectancy of different features of a power system varies considerably. For example, a steam-generating station may have a life expectancy of, say, 30 yr, whereas a hydro-electric plant may have a life expectancy of from 50 to 75 yr. The machinery and equipment in a steam plant operate at relatively high speed and at high temperature; hydro-electric plant equipment operates at slow speed and at much lower temperatures. It is customary, therefore, to assume a higher rate of depreciation for a steam plant than for a hydro-electric plant. Similarly, for other features of a power system, the depreciation in each unit should be based on the best estimate of its probable useful life.

Taxes are an important item in the total fixed charges of a private power company, and they vary considerably in different localities. A public power development on the other hand is usually exempt from taxation. In certain instances, publicly owned power systems have voluntarily included taxes in the rate base, and this seems to be desirable practice. A private power company usually must pay for franchises permitting it to operate, and it must also pay State and Federal income taxes. The public agency is not normally burdened with such charges. In general, the fixed charges are considerably higher for a private than for a public power agency.

OPERATING EXPENSES

The cost of operating hydro-electric plants is a relatively small part of the total cost of power generation, and is practically independent of the plant output. The operation costs of steam-electric plants differ markedly from those of hydro-electric plants. The cost of fuel is the major factor in the cost of operating steam plants, and the quantity of fuel consumed varies with the energy output of the plant.

Operating expenses consist principally of operators' wages, oil, operating supplies, maintenance and repairs, and fuel in the case of heat-generated energy. Fuel costs vary materially due to location, distance from source of supply, method of transportation, and competition between fuels; and the cost of a given fuel may fluctuate from year to year. Many modern steam plants are designed to burn two kinds of fuel, such as coal and oil, or natural gas and oil. This arrangement makes it possible to take advantage of the competitive situation and use whichever fuel may be cheaper at the moment. In rare instances, natural gas can be purchased at low prices during periods when surplus gas is available. Some contracts for natural gas have been made on an off-peak basis, with the supplying agency reserving the right to discontinue delivery during periods when the full capacity of its pipe line is required to supply customers other than the power plant. Under such a contract, the steam plant can take advantage of the cheap natural gas when available, but must resort to a higher priced fuel at times when natural gas is not available.

There has been a gradual improvement in the fuel economy of steam plants, due to the use of larger generating units, higher steam pressures, and higher steam temperatures; and it is now possible, with a modern steam plant designed for high economy, with units of 50 000-kw capacity, or larger, to secure a fuel economy equivalent to 11 000 Btu per net kw-hr.

Station-service requirements generally range from 4% to 10% of the installed capacity. A steam plant will require considerably more power for station service than a hydro-electric plant of comparable size, because of the necessity for operating cooling water pumps, boiler-feed water pumps, coal-handling equipment, etc.

Effect of Load Factor

The energy output of a given power plant will vary directly with the load factor or the plant capacity factor at which the plant is operated. Stated in another way, in order to produce a given quantity of energy the installed generating capacity must be increased as the load factor is reduced.

The effect of plant capacity factor on the cost of power is particularly marked in the case of hydro-electric plants because the annual cost of these plants is fixed and is practically independent of the plant output. The cost of energy generated at a particular hydro-electric plant may be 4 mills per kw-hr if the plant is operated at 80% load factor; but if it is operated at only 40% load factor, it will produce only one-half as much energy and the cost will be increased to 8 mills per kw-hr. In the case of steam-electric plants the load factor has a decided effect on the cost of power although it is not quite so pronounced. The cost of fuel represents a large part of the cost of operating a steam plant and the quantity of fuel consumed varies with the plant output. The cost of production at a particular steam-electric plant may be 4 mills per kw-hr if the plant is operated at a load factor of 80%, whereas if the same plant is operated at only 40% load factor the cost of energy may be increased only to 6 mills per kw-hr.

Effect of Power Factor

The current to be carried by the electrical equipment depends upon the power factor. If the power factor is low, additional carrying capacity must be provided in the generators, buses, transformers, circuit breakers, and transmission lines, thus increasing the cost of this equipment and increasing the cost of power. This is usually not of great importance in the case of hydro-electric developments having long-distance, high-voltage transmission lines because such lines inherently have considerable electrical capacity which tends to correct the low power factor of the load. Under these conditions the hydro-electric plant usually operates at a reasonably high power factor. Steam plants near the load center, on the other hand, do not have the advantage of the corrective effect of long-distance transmission lines, and the electrical equipment in these plants must be capable of carrying the increased current resulting from low power factor, or synchronous condensers must be provided to correct the condition.

Quality of Power Service

Certain classes of power users are much more exacting as to the quality of power service than others; for example, a metropolitan area demands service as nearly as possible free from power outages. For these areas the speed must be maintained within very close limits and the voltage must be closely regulated. On the other hand, occasional momentary outages and minor fluctuations in speed and voltage may not be of serious consequence in small communities or to isolated industrial users. The extent to which a power system is designed to insure continuity of service, close regulation of speed, and voltage, is reflected in the cost of power. As an example, the City of Los Angeles, Calif., obtains the major part of its power supply from the Boulder Power Plant, and its service requirements are most exacting. The transmission circuits are designed to be lightning proof which necessitated large expense for overhead ground wires, counterpoises, etc. Special characteristics, such as extra fly-wheel effect for low reactance and high short-circuit ratio, are incorporated in the generators to insure system stability under transmission-fault conditions. Exceptionally high-speed oil-circuit breakers, functioning in less than three cycles, are required to isolate a section of the transmission line on which a fault may occur before service to customers is affected. Special devices and equipment to control accurately the frequency and voltage are provided. All these refinements are necessary to provide the high standard of service demanded by a metropolitan area.

Another block of Boulder power is used by the Metropolitan Water District of Southern California for pumping water through its aqueduct, and in this case the service requirements are much less exacting. The District can tolerate occasional brief outages since they involve only a temporary shut-down of the pumps which can be re-started readily as soon as power service is restored. The cost of the District's transmission lines and generating equipment is considerably less than that of the City of Los Angeles because of the difference in quality of service demanded by the different classes of loads.

Spare generating equipment is an essential part of every power system. It is necessary to have sufficient spare capacity so that equipment can be taken out of service for inspection, maintenance, and repair, without interfering with delivery of power. The minimum amount of spare capacity must be adequate to permit the largest single generating unit on the system to be taken out of service. The element of dependability of the various sources of power must be considered carefully in determining the required amount of spare capacity. In general, the dependability of hydro-electric generating units is substantially greater than that of steam units, due to the fact that the former operate at comparatively slow speed and at ordinary temperatures, whereas the steam units operate at very high speed and at high temperatures.

Cost of Transmission

Generating stations must necessarily be located where conditions are suitable. A steam plant must have an adequate supply of cooling water for condensing purposes, and must be situated where transportation facilities are available for delivering fuel economically; and a hydro-electric plant must be located where water and head are available. Transmission facilities are necessary for transmitting the power from the point at which it is generated to the load centers, and the cost of transmission is an important item in the total cost of power delivered at such load centers. In the case of a hydro-electric development situated remote from the load center, the cost of transmission may be as great as, or greater than, the cost of generation.

The output of the generating station must be delivered to the transmission lines at their operating voltage and this necessitates step-up transformers and high-voltage switching and protective equipment at the generating station. The cost of the transforming and high-voltage switching equipment at the generating end of the transmission line is usually dependent largely upon the voltage of the lines and is independent of the distance of transmission.

Provision must be made for transmission-line outages, either by duplicate circuits or by stand-by generating capacity at the terminal end. If the transmission lines are designed and built in accordance with the best modern practice, such outages are infrequent.

The cost of power transmission depends principally upon the length, voltage, and capacity of the line, and upon the load factor at which the line is operated. The annual costs of a particular transmission system are practically fixed and are independent of the amount of energy transmitted; and, as in the case of hydro-electric plants, the higher the load factor the lower will be the cost of transmission. The over-all efficiency of transmission, including the voltage transformation under generating and receiving ends, should generally be better than 90 per cent.

The distance from a power development to a market is another factor that affects the cost of power. The greater the distance the greater the cost of transmission, and although a development in itself may be exceptionally low in cost per unit of capacity, it may require so large an investment in transmission lines to reach a market that the power cannot be delivered in competition with power from other sources. On the other hand, a relatively large investment per unit of capacity may be warranted in a plant situated close to a load center and requiring small investment in transmission facilities.

INTER-CONNECTIONS

Power supplies in the United States have developed from the steam engine or water-wheel supplying an individual mill or factory to the central station supplying a community; and then to the power system with a combination of steam and hydro-electric generating stations supplying power to a large territory; and, finally, to the inter-connection of power systems which makes it possible to centralize the generation of steam power at advantageous points where fuel and water for condensing purposes are readily obtainable, and to use large capacity and highly efficient generating units. These large generating plants connected by, and feeding power into, a large transmission system or network, make it possible to take advantage of diversity between the different loads on the system, minimize the stand-by capacity and spare equipment required to insure continuity of service, and secure high plant capacity factor and high fuel economy. It also makes it economically feasible to develop large-capacity hydro-electric projects and to utilize large amounts of low-cost, secondary, or seasonal hydro-electric energy which otherwise would be wasted. Inter-connection of power systems has played an important part in reducing the cost of power.

Many large power systems are supplied by a combination of hydro-electric power and steam. Steam plants are the more economical for supplying peak-load capacity due to their low investment cost per unit of capacity, whereas the hydro-electric plant may be the more economical for supplying the energy requirements of the system, notwithstanding their higher cost per unit of capacity due to their low operating cost. Normally, the hydro-electric plants are operated so as to generate the maximum energy and the steam plants are used to supply the system peak-load requirements with a minimum of fuel consumption. During periods of subnormal water supply, however, normal operating procedure may be reversed and the steam plants may be operated on base load at high-load factor to supply the deficiency in energy output of the hydro-electric plants whereas the latter are used to supply system peak-load requirements. Thus, the two sources of power supplement each other and although the total investment in generating capacity required to supply a given system peak may be greater with a combination of steam and hydro-electric than with steam alone, nevertheless, the combination results in cheaper power, especially where large quantities of low-cost secondary or seasonal hydro-electric energy can be utilized in lieu of steam-generated energy.

Market for Power

The market for power and the length of time required to absorb the power output must be carefully considered in determining the economic feasibility of a power development. This is particularly important in the case of large hydro-electric developments involving large initial capital outlay. The major part of the cost of a hydro-electric development is in the dam, storage reservoir, power-house, etc., which must be constructed before power can be produced. The annual cost of such a project may be in excess of the annual revenues for several years until a large part of the available power can be disposed of, and deficits will accumulate. The deficits accumulated during the early years may make the project economically infeasible.

Large Federal Multiple-Purpose Projects

In recent years, a number of large multiple-purpose projects, involving flood control, improvement of navigation, storage of water for irrigation and domestic purposes, silt storage, prevention of encroachment of salt water into fresh-water supplies, and power development have been undertaken by the Federal Government. These are essentially conservation projects of national scope and the power which they make available, although important from the standpoint of defraying part or all of the cost of these projects, is incidental and subordinate to the other more vital functions. These projects are too large and require too much capital outlay to be economically feasible for development by private enterprise.

The cost of power at these multiple-purpose projects depends largely upon the allocation of the cost to the various functions served. For instance, the primary purposes of the Boulder Canyon Project are to regulate the Colorado River in the interests of navigation; to provide flood protection for the Lower Colorado River Valley, including the rich Imperial Valley in Southern California, and to conserve the flood waters which otherwise would be wasted into the Gulf of California to make them available for irrigation and domestic uses. It seems appropriate to allocate part of the cost of Boulder Dam to items of major Federal interest, and Congress recognized the primary purposes of the project by providing that $25 000 000 of the cost of the project should be allocated to flood control to be repaid out of surplus revenues if any accrue.

The Bonneville Project on the Lower Columbia River serves to improve navigation as well as to produce hydro-electric power, and the Federal Power Commission is now (1938) considering the proper allocation of the cost of this project as between navigation and power. The Grand Coulee Project on the Upper Columbia River is primarily a conservation project which will provide water for the irrigation of about 1 200 000 acres of land in the Columbia Basin. The reservoir created by Grand Coulee Dam will afford some measure of flood control, will improve navigation, and will greatly increase the low-flow power output of the present and prospective power developments on the Columbia River below that point. The proper allocation of the cost of Grand Coulee Dam as between the various functions it serves, should be considered at the proper time, and the cost of power should be established on the basis of the allocation finally adopted. The Shasta Dam on the Upper Sacramento River in Northern California is another of the large Federal projects. It involves flood control, navigation improvement, salinity control, conservation of flood waters for irrigation and domestic uses, and, incidentally, the development of a large block of power. Here, again, a proper allocation of the cost of the project to the various functions it serves will be necessary in determining the cost of power.

It is useless to argue that the Federal Government should not build projects of this type because they also generate power. Social considerations demand their construction and also demand their full utilization, as well, so that the potential energy they create must be developed and made available for use.

One hears of social and of human engineers. The terms are of little moment. More important is the need for realization by the profession, of the kinship between engineering activities, as such, and the progress of an industrial, a power, civilization. Too few engineers have included in their activities a broad consideration of the social and economic objectives of the major problems upon which they have been called to work. Pre-eminent in the technical fields, all have not yet seized the opportunity to apply their abilities on the broader front.

Thought on the subject of power ranges from that of the evangelist who believes that power should be made as free as air, to that of the tycoon who views power as the silver spoon meant for the mouths of those born to the economic purple. As a professional man, the engineer has spent much of his time with the formulas of the power industry, but has devoted too little of his thought to the implications, social and otherwise, of what he has wrought. Has not the time come for the engineer to apply more of his effort to the determination of a sound basis for the settlement of the power question?

If the profession accepts, as it must, the idea that such great projects as Bonneville, Boulder, Grand Coulee, Shasta, and other dams must be built, has not the time come for the Engineering Profession to approach them calmly so as to appraise their benefits soundly both as power projects and as conservation projects?

Public agencies have demonstrated, in many localities, that they can manufacture and distribute power efficiently and successfully with public benefits. Privately owned utilities have served as admirably in many other localities. Each power project is a separate problem; because one method has rendered better service in one instance is not conclusive proof that another will not render better service elsewhere. Remembering these facts, has not the time arrived for the Engineering Profession: (1) To concede that inherently there is nothing bad in either public or private ownership of electrical utilities; (2) to accept the fact that either must earn its right to continued existence in any locality by efficient service in the public interest; (3) to lay aside prejudices; and (4) to apply itself with the precise tools it has available in technical and scientific knowledge to the job of solving the power problem?

The electrical industry is monopolistic by nature. It is self-evident that, in a democracy, a monopoly can be tolerated only as long as the people feel that it serves in the public interest. Is it not time, then, for engineers, who play so prominent a rôle in the field of power, to assist, honestly and wholeheartedly, in seeing to it that the public weal is placed foremost among the objectives of the power industry?

HEAT-GENERATED ENERGY

BY C. F. HIRSHFELD,[2] ESQ., AND R. M. VAN DUZER, JR.,[3] ESQ.

SYNOPSIS

This paper is devoted to the cost of producing electric power in fuel-burning plant. It is divided roughly into five parts: The first covers the definition of cost as used herein and certain factors affecting the cost; the second treats cost of steam-electric generation; the third refers briefly to the performance and possibilities of binary vapor cycles, with particular reference to the mercury vapor type; the fourth is devoted to cost of Diesel plant generation; and the fifth contains a very brief reference to future possibilities. Power cost, as used herein, refers to the total of operating, fixed charge, and general overhead expense at the point of generation.

INTRODUCTION

When one speaks or writes of cost it is necessary to be very specific if one's statements are to have more than general significance. Whether reference is made to commodity or service, the expression, "cost," is meaningless unless many characteristics of an actual or assumed transaction are given. Thus, if one refers to the cost of apples, one may have reference to the cost of production in a given orchard in a given year, or to the average cost of production throughout a section or over the entire country. Such a person may have reference to the wholesale or retail price at a given time and place; he may have reference to the export price at a given time and place; and so on through a great many possibilities. It is obvious that the cost under any or all of the mentioned conditions, and others, may properly be spoken of as the cost of apples, and it is equally obvious that all such costs might be different at exactly the same time.

The cost of power is an equally flexible expression. It may refer to production cost at the point of generation with or without allowance for power used in the production process. It may mean either of these limitations, enlarged by any number of other charges, such as supervision by staff members, taxes, an allowance of some kind for physical or some other form of depreciation, interest on the investment in production facilities, etc.; or it may mean cost to the producer in any of these forms at a point remote from the place of generation, as, for example, at the end of a transmission line at transmission-line voltage, or at some lower voltage. On the other hand, it may mean cost to the producer in any one of a great number of forms at some point or points in a distribution system; or it may mean cost to the consumer, including not only the total cost of the actual power to the producer at the point of delivery, but also the cost of meter reading and billing, the cost of any services other than pure power supply, a proper share of general costs of several varieties, and profit, if there is any.

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In view of such a multitude of possible meanings of the title of this paper, and in view of the resulting danger of fallacious comparisons and inferences, it is deemed necessary to define the sense in which cost of power is used herein. The expression is used to designate the total cost of power to the producer at the point of generation and ready for delivery to the transmission or distribution system which is to convey it elsewhere. By total cost is meant not only the cost of labor and fuel and other supplies, but also all supervision, all allowances for depreciation, taxes, and all other forms of overhead expanse. This may be called the "total power plant cost," or the "total cost at the power plant." The significance of the word, "total," should not be forgotten. Attention is called to the fact that cost as used herein is in terms of net output, not total generation; that is, the unit cost is determined by dividing the total cost, as defined, by that number of kilowatt-hours delivered to the line and available for transmission or distribution as the case may be.

For the benefit of those who are not familiar with the facts and economics of power supply, it is desirable to emphasize the fact that the cost of power as defined herein may represent legitimately either a very large part or a very small part of the cost to the ultimate consumer. For example, if a fuel-burning power plant is considered, which is situated in a manufacturing establishment of small extent, and is supplying all its output to that establishment, transmission costs are not involved, distribution costs are small, meter reading and billing are not necessary, and no kind of service other than power supply is given. Under such circumstances the total power-plant cost, as defined herein, represents practically the entire cost to the consumer. If, however, the power must be transmitted at high voltage to a considerable distance and there sold wholesale to a community, the power-plant cost must obviously be increased by all transmission costs, including losses, operating expenses, and fixed charges, and by the profit, in order to obtain the cost to the ultimate consumer. Neglecting the item of profit, it is quite possible for the legitimate cost of delivery at the wholesaling point to equal one and one-half times, or more, the total cost of the power plant. Furthermore, if the power is to be retailed over a wide area, even near the power plant, it is quite possible for a legitimate cost to the consumer to equal seven or eight times the power-plant cost and to vary widely each side of these values depending upon the density and character of load, the character of the territory, the kind of service rendered, etc. It is obvious that mere power-plant cost tells a very small part of the entire story and that many fallacious conclusions may be drawn with respect to power cost under different conditions if this fact is not fully appreciated.

A presentation of the data indicated by the title of this paper would be simple indeed if it were true that the total power plant cost of fuel-generated power were the same every where, or under all conditions, or even at a given fuel cost. No such simple relations exist; the cost of fuel-generated power is uniquely different for different conditioning factors. Moreover, it is practically impossible to predict how these conditioning factors will combine in any case until a specifically detailed study has been made for that case. If one is to consider such power costs in any general manner it must be done in terms of cost ranges instead of in terms of specific costs. This method has been adopted in the paper.

As stated previously, the total plant cost of power is made up of a number of cost items, a complete list, divided under three convenient and conventional headings, is as follows:

(*1*) *Operating Costs:* Fuel; operating labor; water; miscellaneous supplies and expenses; maintenance (labor and materials); and superintendence, clerical and testing.

(*2*) *Fixed Charge Expense:* Charge for depreciation; cost of money (interest, dividends, etc.); property taxes; and insurance (fire, casualty, liability, etc.).

(*3*) *General Over-Head Expense of Which Parts Should Be Allocated to Power Production:* Staff superintendence; staff accounting; general offices; stores and warehouses; transportation and communication; general and coal-testing laboratories; shops; working capital; regulatory commission expense; public injuries and damages; employee compensation insurance; social security tax; general taxes (income, surplus, sales, excise, franchise, corporation, etc.); legal and miscellaneous general expense.

Nearly every item in this long list does exist as an item of cost in any fuel-power production undertaking. Certain of them may be masked or may be "lumped in" with others so that their identities are lost, but they exist nevertheless. In small operations, laboratory expense undoubtedly would be absent. In private undertakings, such as industrial power plants as opposed to public utility activities, regulatory commission expense would not exist. With such limited exceptions, however, all the items have their equivalent in any production of power from fuel although their existence may not be appreciated and may not be shown by the method of accounting in use.

A mere survey of this list of components of total cost indicates the impracticability of treating every item in a paper of moderate length. Attention must be concentrated on the more important and larger components. Those that are slighted may conveniently be grouped into a percentage loading on others, or handled in some equally simple fashion; but their existence should not be overlooked. Furthermore, it is convenient to treat the steam and the internal combustion methods of power production separately. If this is not done the treatment becomes undesirably complex and difficult to follow.

Steam Power Costs

The methods used for producing power from steam are numerous. They all have in common the use of fuel-fired steam boilers which supply steam to prime movers of some kind; and that is about all they have in common. In general, when one refers to a steam-operated power plant, one visualizes the highly developed, condensing, steam-turbine, type of plant used in nearly all central stations and in many industrial plants. It must be remembered, however, that such plants represent only one of many classes; for example, the steam locomotives of the United States use a large part of the total fuel consumption and are just as truly steam power plants as the most elaborate central stations. Many industries use steam engines or steam turbines as devices by means of which to skim a certain amount of power from a steam supply required for process purposes. In such cases the major part of the heat originally in the fuel may legitimately be charged to process steam so that the fuel cost of the power may be reduced to comparatively low values. Similarly, under such conditions it is frequently legitimate to charge a large part of the cost of steam-producing equipment to process steam, thus diminishing the fixed charges against power.

Obviously, no paper of reasonable length could attempt to cover all such possibilities. After considerable thought the writers have decided to limit this section to the costs of fuel-generated power produced in condensing steam-turbine stations of what may be called the central station type. This type represents the nation's best achievement in the production of power as a commodity as cheaply as is compatible with reasonable continuity of supply. In adopting this course it seems wise not to neglect realities and, therefore, not to limit the treatment to the most economical central station that can be built to-day. The fact is that central station supply is now an old industry which of necessity carries with it those of the physical constructions incident to its growth which sound economics or the laws and regulations under which it operates have not yet required or permitted it to retire. An idealist might like to see everything but the latest and best retired over night. A realist takes cognizance of the fact that economic considerations do not permit this.

The fact is that the cost of steam-generated power from central stations is a composite of the cost obtained with plants of varying ages and, therefore, representing different stages in the development of the art. Naturally, each property utilizes its most economical equipment to the greatest practicable extent, but there are many limitations made necessary by physical and other factors that must not be ignored.

The writers have attempted to give a reasonably fair "picture" by considering the performances to be expected of plants operating at different steam pressures and steam temperatures, choosing the values so as to cover the entire field as it is found to-day. This has resulted in the choice of the following steam conditions:

Pressure, in pounds per square inch	Temperature, in degrees Fahrenheit
225	650
400	750
600	825
800	910
1 250	925

The values listed are to be regarded as nominal; that is, indicative of the general range in which actual values will be found. For example, 225 lb may represent any value between, say, 175 and 250 lb, whereas 650° F may represent any value between 550 and possibly 675 degrees.

It is also to be noted that no fixed and definitely predictable thermal performance can be assigned to any single combination of steam pressure and temperature. For one thing, there has been an ever-continuing development of power-plant equipment and power-plant design, so that the possibilities realizable from the use of a given steam condition have changed almost from year to year. Conditions and circumstances associated with different installations result in designing for different performances from what superficially looks like similar equipment, in order to obtain the best economic balances in different cases. Therefore, in presenting the performances from which costs may be calculated, it is necessary to use bands or ranges rather than exactly fixed or unique values.

The very marked effect of condenser vacuum upon the thermal performance of a steam-turbine plant is now generally well known. The attainable vacuum in any location is determined both by the quantity and the temperature of available circulating water and, to some extent, by its quality. In any general study of the type under discussion it is necessary, therefore, to take account in some way of the effect of average attainable vacuum. This has been done by considering a range covering the generally existing values.

It is not so commonly appreciated that the character of the load has a very marked effect upon attainable thermal and economic performances. This is a fact, however, and one fraught with complexities. Each physical unit concerned has a certain characteristic which may be defined as performance *versus* load. In general, the best thermal performance of the assembly of physical units constituting a power plant is attained near what is known as rated load, although this may be modified through rather wide limits at the time of design. Therefore, operation for long periods of time at lower rates of output must of necessity reduce the thermal performance.

This sounds, and is, simple; but it is by no means the whole story. The manner in which the load varies may be of greater significance than its average value over a given period. For example, two plants of like capacity might operate under such conditions that both of them produced in a given time, say, 24 hr, the same number of units of output. One might be operating under such conditions that it carried a perfectly uniform load throughout the 24-hr period, whereas the other produced all, or substantially all, of its output in, say, 9 hr. The characteristics of steam power plants are such that with other factors reasonably equal, the thermal performance of the steadily loaded plant might be appreciably better than that of the other. These are extreme assumptions with respect to central station operation but might be approached rather closely in industrial plants.

Another significant type of variation concerns the size of the load from minute to minute. If there is great fluctuation, the thermal performance is necessarily poorer than it would be if the same total output were produced under more nearly constant conditions. Variation from day to day is also of importance; for example, a plant may be heavily loaded during four or five days of the week and lightly loaded, or actually idle, during the remaining days. These variations and many others affect the initial design of the plant to the extent to which they can be foreseen; but, irrespective of the provisions that may be made in this direction, they affect both the thermal and the economic performance of the plant as it is operated.

Obviously, a given investment will be used to the best advantage with respect to unit fixed charges when the maximum number of units of product is produced. Moreover, a load characterized by high, short-time peaks must necessarily involve a greater investment in plant than one totaling the same number of units of output produced at more nearly a constant rate. It is evident, therefore, that the character of the load must affect the economic as well as the thermal performance.

Power engineers endeavor to express the significant characteristics of load by the use of a number of ratios known as factors. Thus, they use load factor, power factor, use factor, capacity factor, and others. Each has value and each tells something about the load; however, all combined, they still leave much of importance undisclosed. In any such general study as herein intended, it is not practicable to show completely the effects of all the possible characteristics of load. This is sufficiently difficult even in the study of an isolated case, and makes it almost impossible to compare fairly the performances obtained in as few as two isolated cases.

Under the circumstances the writers have had to content themselves with the use of one factor which may be regarded as the most general in character. By virtue of its generality it is correspondingly lacking in definitiveness. The criterion used is the annual capacity factor, or plant factor, which measures the actual output against what it would have been had the plant operated at full rated output throughout the year. This definition requires explanation. There can be little argument as to the actual annual output in any given case; but there are many different but accepted ways of computing what the output would have been had the plant operated at full capacity throughout the year. As used herein, it is assumed that steam generator capacity, prime mover capacity, and electric generator capacity, together with the capacity of all auxiliary equipment, are so well balanced that nothing prevents the operation of the main generators at full rated capacity indefinitely. The maximum possible output that is used in calculating the annual capacity factor is then based on the theoretical operation of the main generators at full rated capacity throughout the year.

In a continuing sense this maximum possible output is an unattainable performance in any real case, even if the load permitted. Equipment does wear out, maintenance must be performed, and occasional breakdowns of major character do occur. Therefore, the annual capacity factor as used herein is, in a sense, theoretical although it does have the value of being exact and relatively easily determined. In actual cases it always falls well below unity or 100%, depending upon the form in which one chooses to express the ratio.

The range used herein extends from 20 to 80 per cent. Old plants that are used only for peak and stand-by service may show an annual capacity factor well below 20%, or even below 10 per cent. When such low values are reached thermal performance is of little significance and power costs from such plants are too erratic to have much meaning in a paper of this kind, although they may have very great significance as constituting part of the power cost of a larger system. Few fuel-burning plants attain an annual capacity factor, as herein defined, in excess of 80 per cent. Even industrial plants that operate at practically full load for 24 hr per day seldom achieve such a value for any extended period. If a plant operated at full rated capacity 300 days per yr and was completely idle during the remaining days its annual capacity factor would be only about 82 per cent.

In the foregoing discussion of capacity factor, reference was made to the rated capacity of the plant. It is obvious that this must have a fairly exact value in any given case if the calculated value of capacity factor is to be properly significant. However, there are other considerations connected with capacity which may be treated in a rougher or more approximate fashion. For such purposes one may refer merely to the size of the plant, or to its capacity, meaning by that something of the order of, say, 100 000 kw instead of 25 000 kw or 300 000 kw.

Size, in this sense, has a most complicated effect upon power cost. In general, it may be stated that total power cost tends to decrease as the size of the plant is increased. This follows from many considerations such as the possible use of larger units with better thermal performance and smaller unit investment, from fewer man-hours per unit of output, and other favorable factors; the economic justification of ultra refinements of various kinds; the spreading of clerical and superintendence charges over a greater output, etc. So many apparent contradictions can be found, however, that it is not safe to assume that cost of power from a larger plant in one location will necessarily be lower than that from a smaller plant in another location. For example, a small plant may be constructed under conditions that justify the designer in planning for high economy, in making no provision for future enlargement, and in providing a minimum of spare capacity because of stand-by service from a transmission line of large capacity. If cost of power from such an installation is compared with that from a larger plant, in which costly provision has been made for relatively great expansion at some future time, and in which several relatively small units have been used to fit best to an existing load curve and supply adequate reserve capacity, it may well be that for some time the thermal, the labor, and the economic performances of the smaller plant will be better than those of the larger plant, all other things being equal. Many examples of other types might be cited as evidence of the danger of carrying a generalization too far.

For convenience, the writers have assumed plants of 150 000 to 250 000 kw in preparing the numerical data that follow. This statement means they have assumed a well balanced plant capable of delivering energy at such a rate when all equipment is available. No assumptions have been made with respect to the nominal rating of the plant because this is too complicated a consideration for general treatment. It involves choice of the size of units and many other choices that can be made only for known and reasonably specific conditions. Instead, they have lumped such variations of power costs as would occur by virtue of different causes into the margins provided by the use of ranges or upper and lower limits of costs of different types.

It is convenient to consider the different plants in the order of ascending pressures. The nominal 225-lb installation, therefore, is selected as the point of beginning. Practically all such plants of any magnitude which to-day are used for central station purposes are twenty years old or more. Therefore, they represent a comparatively early development of the art as it is now known. Just what they are capable of, with respect to thermal performance, depends upon the stage of development represented at the time of their construction and upon the changes that have been made since then, either by the improvement or replacement of existing equipment, or by the addition of new equipment.

Several curves showing thermal performances as derived from actual performance data obtained from a number of such plants, are shown in Fig. 1. The hatched area between Curves *a* and *b* indicates the probable range of thermal values for plants representing average conditions, such as typical central station load characteristics, reasonably good grades of eastern or mid-western bituminous coals, units of 20 000 kw, or larger, some modernization since initial installations, etc. Attention is called to the fact that values are all in terms of 1 in., Hg, back pressure. Practically all such plants of which the writers have knowledge are operated at comparatively low capacity factors, some as low as 10% and few as high as 40 per cent. The region beyond 40% is obtained largely by extrapolation and it is questionable in character for two reasons: (1) It is probable that many of the actual plants could not be operated continuously for the long periods of time at the outputs required to give the very high capacity factors, in spite of the fact that they frequently carry full load for shorter periods of time as required to meet the demands of the system; and, (2) the methods of extrapolation may not be very accurate.

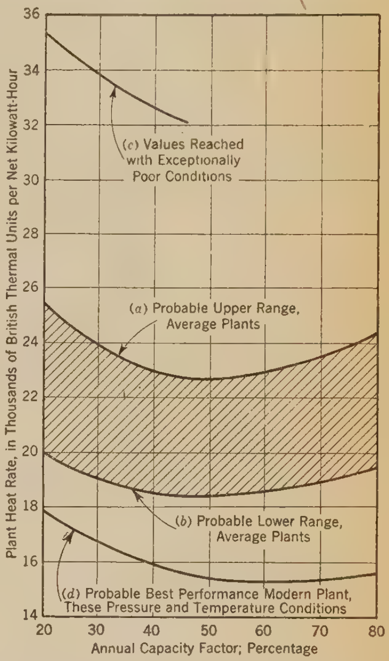


FIG. 1.—THERMAL PERFORMANCE OF A STEAM PLANT; NOMINAL PRESSURE, 225 POUNDS PER SQUARE INCH; 650 DEGREES FAHRENHEIT AND A BACK PRESSURE OF 1 INCH OF MERCURY

Curve *c* in Fig. 1 is introduced to show what may happen under abnormal conditions, such as unusual types of load, very poor or greatly varying fuel characteristics, little or no modernization, etc. The values shown on this curve have not been used in later computations and are included here only to indicate the danger of generalizing to too great an extent in such matters.

Curve *d* represents the probable performance of a plant built for these steam conditions at the present time; that is, a plant designed according to the present status of the art. It is of interest only as indicating the progress which has been made in improving thermal efficiency without reference to the effects of increased steam pressure and temperature. It is not probable that any large plant would now be built with these steam conditions, although exceptionally cheap fuel or other unique conditions might justify such a choice.

Group 1.—Fuel Costs.—The thermal performances as shown in Fig. 1 can be translated easily to fuel costs as soon as the cost of fuel is known. For the purposes of this paper, the writers have chosen to use a fuel cost at the plant of 15 cents per million Btu. This corresponds roughly to eastern bituminous coal of good grade at $4 per ton. It will become evident subsequently that correction for other fuel prices can be made easily. Curves *e* in Fig. 2 (*a*) and 2 (*b*) result from converting Curves *a* and *b* of Fig. 1, into fuel cost in mills per net kilowatt-hour. The method is obvious. It must be remembered that this limits the direct applicability of Curves *e* to 1 in., Hg, back pressure. However, methods of correcting to other pressures are given subsequently herein.

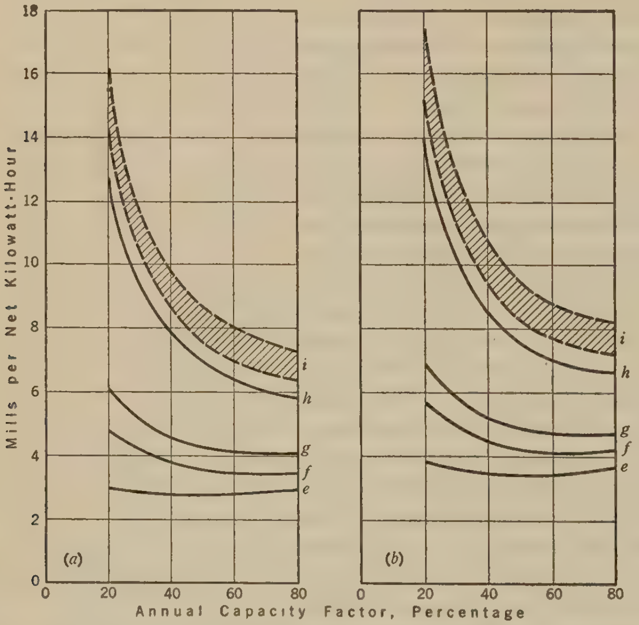


Fig. 2.—Average Lower (*a*) and Upper (*b*) Ranges of Power Cost for a Steam Plant; Nominal Pressure, 225 Pounds per Square Inch; 650 Degrees Fahrenheit; and 1 Inch, Hg, Back Pressure

Operating Labor.—The next cost item listed under operating costs is that of operating labor, in which case two major variables must be considered: (1) The output per man-hour as determined by the efficiency of labor, the type of fuel, and the design of the plant; and (2) the average wage paid. This wage varies widely with geographical location, with type of community, and with other factors.

The output per man-hour for the nominal 225-lb plant has been assumed as shown by the full line in Fig. 3. It is applicable to plants burning the better grades of solid fuel and reasonably accurate to a capacity factor of 50 per cent. Below 50%, the performance of most of these plants will fall within ± 30% of the values indicated, as shown by the two dotted lines. Above 50% much will depend upon the character of the plant, if indeed it can be operated to produce higher capacity factors. As most plants actually are, it is probable that at higher capacity factors the slope of the full line would gradually become less as shown by the dotted extension, indicating increasingly less effective use of men. The same phenomenon would probably occur to an increased extent in plants having the poorer grades of solid fuel. For plants burning liquid and gaseous fuels, the net output per man-hour at any load factor would probably be higher than shown and the slope of the line would tend to remain constant. The data available to the writers are not sufficiently extensive to permit them to be more specific in this regard.

To convert the values shown in Fig. 3 to labor cost, an average hourly wage must be selected. Under the conditions it is necessary to choose some value arbitrarily because it is practically impossible to cover all the local variations in one paper of this type. An average hourly wage of 70 cents has been used herein. Variations of this value are wide in practice.

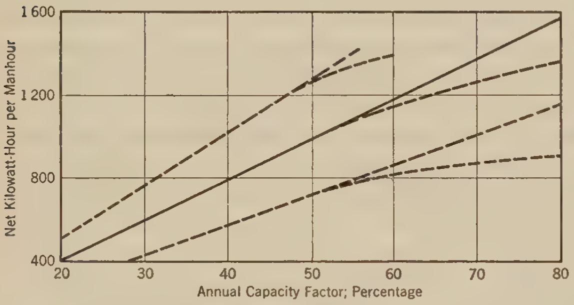


Fig. 3.—Net Kilowatt-Hour Output per Man-Hour of Operating Labor

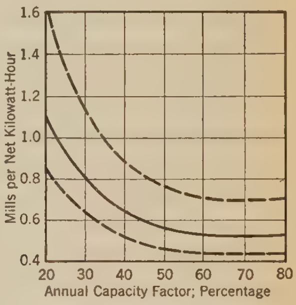


Fig. 4.—Maintenance Expense, Labor and Material

Using 70 cents as the average wage per man-hour for the plants in question, Curves *f* in Fig. 2 are obtained from the values shown by the full line in Fig. 3. These curves are plotted above Curves *e* as a base, so that the ordinates represent the combined costs of fuel and operating labor. The labor costs can be found by difference. As a matter of convenience, the cost of superintendence has been added to the operating labor cost before plotting Curves *f*. The addition is so small as not to make any visible difference in the position of Curves *f*.

Water and Miscellaneous Supplies and Expense.—The next two items of cost, namely, water and miscellaneous supplies and expense, are conveniently grouped together for this kind of generalization. For what may be considered average conditions the grouped cost may be taken at about 0.1 mill per net kw-hr, with a possible variation of ± 50 per cent. However, conditions that are abnormal may cause wider variation as, for example, when water must be purchased from a public supply or when a complicated and costly water treatment is required. The small value adopted as probable cannot be shown clearly in Fig. 2 by drawing an additional curve since this would be only 0.1 mill above Curve *f*. For this reason, the cost of water and miscellaneous supplies has been added to that of maintenance labor and material for plotting purposes.

The task of arriving at reasonable average values for maintenance, labor, and material is difficult. For one thing, standards of maintenance differ greatly from property to property and from region to region. For another, wage scales and costs of materials also vary rather widely. Furthermore, the types of design and construction effect the necessary expenditures; and, by no means least, the type of fuel and certain related factors have a very great influence. Coals with a high sulfur content, ash-fusing at a low temperature and a high water content, all tend to increase maintenance to a significant extent.

For the nominal 225-lb plants, the writers have chosen to use the values given by the full line in Fig. 4. These values are fairly representative of a high grade of maintenance in plants burning reasonably good grades of bituminous coal. The dotted lines indicate probable variations from the values assumed. With very bad fuel or very bad water, the maximum values shown might easily be exceeded. Adding the 0.1 mill for water and miscellaneous supplies and expense to the values shown by the full line in Fig. 4 gives the data for plotting Curves *g* in Fig. 2. As before, these are plotted above Curves *f* as a base.

Superintendence, Clerical and Testing.—The last item of operating costs is that designated by superintendence, clerical and testing. Although the service rendered under this item is very valuable, the outlay required to procure it is generally comparatively small. In terms of cost per kilowatt-hour it is generally about 0.1 mill. Obviously, this is too small an increment to plot in Fig. 2, and, for this reason, it was included in the operating labor charge when plotting Curves *f*.

The total operating cost is then given by the values in Curves *g* for the two limiting thermal performances assumed. It will be shown subsequently how these performances may be corrected for conditions different from some of those assumed in developing the data thus far presented.

Group 2.—Fixed Charges.—The second large group of cost items is fixed charge expense. A review of the components of this group, as given previously, will show that all are essentially functions of the investment. In fact, each is frequently expressed as a percentage of the investment. It is obvious, therefore, that they can be evaluated immediately if the investment and the various percentages are known.

In connection with investment it should be remembered that this must include, in one way or another, not only the actual plant property, such as land and improvements, buildings, and plant apparatus and equipment, but also those things necessary to convert the plant from a static to an operating entity. In the case of a coal-fired plant, for example, there must be available a sufficient stock of coal to insure power production in the face of an irregular receipt of coal. In the case of certain northern sections of the United States to which such coal is transported *via* the Great Lakes, at least a six-months' supply must be stored when navigation closes. A certain sum of money, therefore, is always tied up in stored coal and represents an investment on which carrying charges must be paid. Again, as an example, a certain amount of working capital must be available in order to pay wages and invoices as they become due. Working capital thus contributes to total investment and, in one way or another, carrying charges must be paid upon it.

The costs resulting from these types of investment are best placed in the third large group ("General Over-Head"), instead of in the second. The carrying charges resulting from the coal storage then go into the general classification of "Stores," and the costs resulting from working capital become part of a larger account of the same character.

Unit investment in power plants, that is, the investment in dollars per kilowatt of capacity, is frequently assumed to be a kind of universally applicable value easily derived. This is very far from the truth. Conditions vary greatly from year to year, from place to place, from method to method adopted or available for construction, and, to a great extent, with the conditions to be met with respect to character of fuel, character of load, provisions for reserve, etc. In years when business is slack, prices and wages tend toward low values and the efficiency of labor tends to increase. A plant built under such conditions should cost appreciably less than one of exactly the same type constructed in the same place under boom conditions. Some districts in which plants are constructed have such combinations of real estate values, building codes, construction practices, and wages that a relatively high cost of plant necessarily results. Some locations require elaborate substructure and expensive canal work, whereas others permit the simplest type of substructure and negligible cost for canals. In some cases, an ample supply of circulating water is available and, in others, expensive provisions for storing and cooling must be made. The result is that one cannot state categorically what a given type of plant should cost or should have cost. It is necessary to study a large number of facts and conditions derived at reasonable figures. How, then, is one to establish a probable investment for the class of plants concerned herein? The unvarnished truth is that one cannot. All one can do is to choose more or less arbitrarily, with full appreciation of the fact that the choice is arbitrary, or consider an entire possible range.

Most of the nominal 225-lb plants were built, or at least partly built, in times of comparatively low prices. Therefore, unless exceptional conditions prevailed in other respects, it would be expected that they should have cost less than they would, had they been built in a period of higher prices. However, in many cases, they have been enlarged during periods of higher prices and, in others, they have been rehabilitated or improved in one way or another, thus increasing the original investment both as to total and as to unit values.

Such data as are available to the writers indicate that the investment in nominal 225-lb plants, as they now exist, will probably fall somewhere between $85 and $110 per kw of capacity under what may be termed average conditions, but that there are exceptional conditions which have produced legitimately both lower and higher values. They have chosen to use $100 for the purposes of calculation, realizing that resultant values for any other condition can be obtained by direct-ratio calculation.

The percentages of this investment, which are to be assumed for each of the items of cost listed under fixed charges, have been the subjects of endless disputes. There are many different theories regarding the manner in which depreciation shall be considered. Political bodies of like character do not agree and different political bodies having jurisdiction, in one way or another, over a given power undertaking, promulgate different rules and use different theories and practices contemporaneously. Regulatory bodies are inclined to consider a high rate of depreciation proper with respect to the past when establishing a rate base, but they accept a low rate of depreciation when establishing a rate for the future on such a rate base. The United States Department of Internal Revenue treats depreciation on a standardized basis and without reference to the practices of State commissions. No agreement exists with respect to the relative values of obsolescence and physical deterioration as aspects of depreciation.

The cost of money is obviously an item that can be determined more or less accurately in any given case either historically or as of a particular time; but there is no reason why it should be the same in any two cases or under any two conditions. One company may be able to, or may have been able to, obtain the required funds at the equivalent of, say, 5.5%, whereas another, just as legitimately, may have paid 7%, or even more.

Property taxes, obviously, vary with the community and the community's practices. In regions of high property values the real estate tax is likely to be high. Personal property taxes are an item in some States and not in others.

Insurance of the types included under fixed charges generally represents a quite insignificant proportion of the total in the case of well built and well maintained properties. However, it also varies widely with the character of the property and the practices of the management.

In view of all these variations it seems best, in the present case, to lump all fixed charges into one percentage value. Such a value appears to vary between slightly less than 10% of the investment as a lower limit, to possibly as much as 14% as a high limit. The writers have chosen to use 12% for the purposes of computation, realizing again that the results can be corrected easily to any other value if it is found necessary.

With a unit investment of $100 and a fixed charge of 12% annually, the fixed charge per unit of output is obtained by dividing $12 by the kilowatt-hours produced annually per kilowatt of capacity. Values obtained in this manner are plotted as Curves *h* in Fig. 2, using Curves *g* as a base.

Group 3.—General Over-Head.—Conditions applying to the third classification of general over-head expense must vary widely from case to case. However, as has been indicated herein, this type of expense always exists even if the items to be included may vary.

It is exceedingly difficult to obtain sufficiently comprehensive data to enable one to determine for each of many properties the value of each item lumped under this heading. On the other hand, it is not nearly so difficult to obtain a close approximation to the total of all such items. Such data as are available to the writers indicate that these totals generally equal from 10% to 25% of the total of Group 1 and Group 2 when all items of general over-head character are taken into account. One might be inclined to believe that there would be a marked difference between industrial plants and public utility plants in this respect. The fact appears to be that this is not normally the case because the items in Group 3, which have no meaning in the case of industrial plants, usually represent a very small part of the total. However, the methods of accounting used may, and frequently do, tend to disguise the actual total of the costs in this group.

Practically no data are available that can be used to indicate the relation of general over-head to capacity factor. This may seem strange at first, but inspection of the items making up the general over-head will indicate that, in any given organization, most of them would tend to remain constant over fairly long periods of time even if radical changes in plant output occurred, for example, as the result of a depression or as the result of a business boom. Under such conditions the total at any time would not truly reflect what might be expected if the output remained substantially at the then value. Again, when companies are found which, respectively, operate a plant or plants at radically different capacity factors over long periods of time, the characteristics of the companies and other controlling factors vary so greatly between themselves that any variation of general over-head with capacity factor may well be assumed to be due to causes other than the respective values of the capacity factors.

Faced with such uncertainties the writers have chosen to show the effects of general over-head on total power cost as a rather wide band instead of as a line. The bands shown in Fig. 2, marked *i*, represent what the total power costs would approximate if general over-head amounted to any value between 10% and 25% of the total of Groups 1 and 2 at each capacity factor. Their purpose here is to call attention to the fact that such general over-head must be considered as an item of power cost in a going concern and that it may be expected to be of significant magnitude, rather than to state an exact value.

It will be recalled that the groups of curves in Fig. 2 have been constructed by making certain assumptions and that reference has been made to the possibility of correcting to other assumptions. The first of these corrections is concerned with variation of back pressure and its effect upon performance. Curves *e*, Fig. 2, represent fuel cost at 1-in., Hg, back pressure. For a higher back pressure the cost will be more because the thermal performance will be poorer. For a lower back pressure the cost would tend to decrease more or less, depending upon the design of the turbine and upon the average load carried.

In the case of the older types of turbine, such as would be expected in plants of the kind discussed herein, there would normally be an improvement down to 0.5-in., Hg, back pressure except at the higher loads. At these higher loads the improvement would normally cease at approximately 0.7 to 0.8 in., Hg. It follows that, for low capacity factors which would generally but not always be associated with low average turbine loads, the effect of lowering the back pressure below 1 in. might be expected to be greater than at higher capacity factors.

The differences thus involved, however, are small in comparison with many others that cannot be evaluated and it seems unreasonable to attempt to take them into account in such a general treatment as contained in this paper. Therefore, the writers have included just one curve for back-pressure correction to the performance of a nominal 225-lb plant. This is shown in Fig. 5 (*a*). The part below 1 in. is dotted to indicate the uncertain character of the correction in this region. The correction factors indicated as percentages can be applied directly to increase or decrease the height of Curves *e* in Fig. 2; or, since all curves in Fig. 2 are plotted with the next lower as base, the same percentage of the values on Curves *e* may be added to, or substracted from, the values of Curves *h*.

It seems pertinent to remark that there are few if any cases in the United States in which a normal year's performance would show an average back pressure, weighted for output or otherwise, of less than 1 in., Hg. Therefore, the negative correction is more of academic than of practical interest. This is another justification for not quibbling over the exact shape of the correction factor below 1 in.

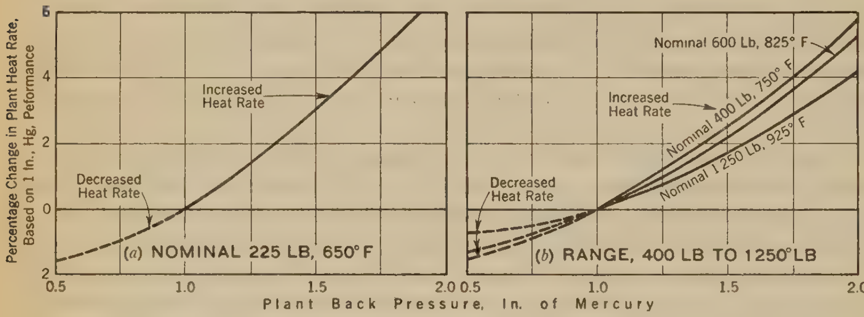


Fig. 5.—Average Change in Thermal Performance with Plant Back Pressure Based at 1 Inch, Hg

Possibly, the next necessary correction is for fuel cost. Curves *e* in Fig. 2 were calculated for a fuel price of 15 cents per million Btu at the boiler, corresponding roughly to about $4 per ton for a good grade of eastern bituminous coal. If any other value such as x cents per million Btu is considered, the values shown by Curves *e* of Fig. 2 must be multiplied by $\frac{x}{15}$. If total power cost is to be shown, it is simpler, of course, to apply the resultant correction to Curves *h* as before.

The next correction factor is for operating labor cost. The values plotted result from assumptions regarding the man-hour output and the average hourly wage. It is obvious that if the man-hour output is assumed at a lower amount, the values indicated by Curves *f* must be increased in inverse ratio, and *vice versa*. It is equally obvious that if a different wage is assumed the values indicated by Curves *f* must be increased in direct proportion and *vice versa*. In making these corrections, as indicated, the presence of 0.1 mill for superintendence and clerical costs in the values shown on Curves *f* is ignored for obvious reasons.

The second group of cost items, fixed charges, is, as stated, based on $100 per kw of capacity and 12 per cent. Alteration in either would produce a corresponding change in the values shown on Curves *h*. The correction would be performed by multiplying the ordinate value between Curves *g* and *h* by the proper ratio.

Reference has been made herein to the effect of oil and gas-firing upon the man-hour output. As a matter of fact, the use of oil or gas instead of solid fuel may have other effects as well, and some of these may result in quite different costs than those obtained for coal-fired equipment. For example, it is obvious that a gas-fired plant receiving its supply through a pipe line owned by others does not have capital tied up in stored coal; it need not have capital tied up in property on which coal is stored; it need not have coal-handling equipment; and it does not have the operating expenses incident to the handling of coal into, and out of, storage and into the plant. These and many other differences may be expected to exist, but no general treatment of the present kind can hope more than to point to that fact. One may state, in general, that, with other conditions equal, the cost of power would be expected to be lower with oil and gas-fired plant than with coal-fired plant; but the significance and importance of the expression, "with other conditions equal," must not be overlooked.

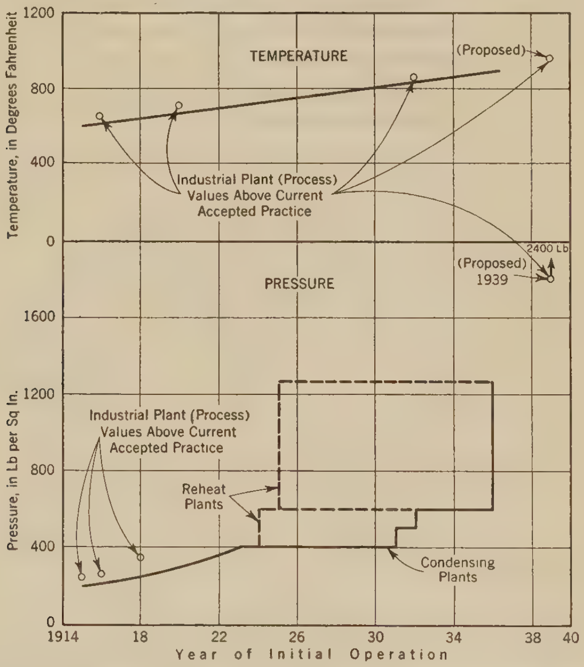


FIG. 6.—INCREASE IN STEAM PLANT TEMPERATURES AND PRESSURES

At this point the writers believe it advisable to introduce a caution. It will have been observed that although the economic performance of any plant at any one time must have a very definite value, great latitude of choice exists as soon as one attempts to generalize. The writers caution against attempts to arrive at probable power cost for a given site or for a general condition by assuming all low or all high limits among those that have been indicated. It seems to be true that in this field, as in others, one seldom, if ever, finds all controlling features most favorable or most unfavorable. Therefore, theoretical power costs, arrived at by making such assumptions, are almost certain to be ridiculous in a practical sense.

Power Costs at Higher Pressures and Temperatures.—Power costs at the higher pressures and temperatures are next considered. These steam conditions have been introduced more or less, successively, as experience, necessity, engineering and design progress, and the development of metallurgy and fabricating processes, respectively, dictated or permitted. The gradual increases of pressure and temperature in the United States are shown in Fig. 6; the year in which each step was taken is indicated. In a general manner, the thermal performance of plants improved during this upward march, is indicated in Fig. 7. Part of the improvement is due to the greater inherent value of higher pressures and temperatures; part to improved plant design, both with respect to the operating cycle chosen and to the arrangement of equipment; and part to improvements in the component equipments.

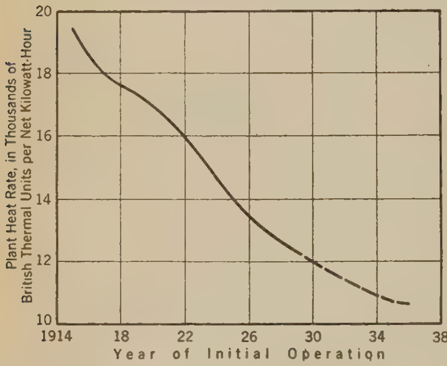


FIG. 7.—IMPROVEMENT IN THERMAL PERFORMANCES (ACTUAL RESULTS FOR AT LEAST ONE YEAR'S PERFORMANCE)

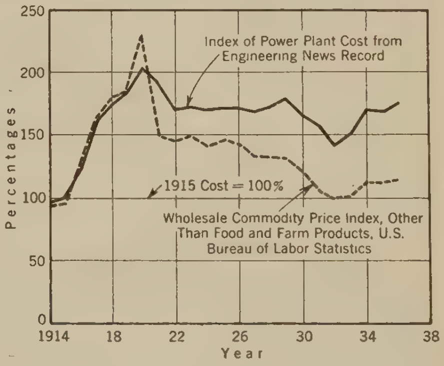


FIG. 8.—INDICES OF STEAM POWER PLANT INVESTMENT AND WHOLESALE COMMODITY COSTS

To one familiar with general price levels, study of Fig. 6 will indicate that the upward climb of pressures and temperatures occurred during a period of relatively high commodity prices, at least up to the end of the last decade. The general effect of changing commodity prices upon the cost of complete power plants is shown very roughly in Fig. 8, for the years 1914 to 1936. Naturally, a combination index such as this is not necessarily accurate with respect to any individual case; it can only reflect conditions in a most generalized or over-all manner. The circumstances in an individual case may legitimately result in wide positive and negative departures from index values.

The phenomena discussed in the immediately preceding paragraphs yield several peculiar results. First, they make it difficult to obtain a true comparison between what may be called basic investment costs of plants built to operate at different pressures and temperatures. Theoretically, one could obtain a perfect index and apply it to the actual known investment costs of different plants built at different times, thus wiping out the effect of varying commodity costs and having the data truly comparable. Practically, this cannot be done because of the impossibility of obtaining so accurate an index for application to isolated cases; because of the wide differences caused by varying local conditions and varying types of design; and because there is no known method of correcting for the extent to which development costs, engineering errors, modified production methods, etc., have been reflected in the actual finished cost in different cases. In another sense, a comparatively small number of radically different units are involved in this case, and statistical methods are not applicable.

Second, these phenomena make it practically impossible to determine the true economic values of different pressure and temperature conditions as separate from the characteristics of any given case. The thermal performances can be found both by computation and by performance statistics from actual plants; but operating expenses and fixed charges are not so readily obtainable in such form as to permit of accurate generalization.

If one tabulates historical costs of plants as they have been built during this period of shifting indices, shifting pressures and temperatures, and shifting design, there appears to have been a gradual increase of cost with increasing pressure and temperature. As nearly as the writers can estimate from the facts, if the nominal 225-lb plant investment were taken at $100 per unit of capacity as was done before, each step in pressure, as herein considered, would probably add about 5% to 7% to the required investment. On the other hand, if one were to consider what can be done in a given location under given conditions to-day, the writers estimate that, starting with the low-pressure investment as indicated previously herein, there would be a slight decrease to a minimum at about 400-lb to 600-lb pressure and then a slight increase to 1 200 lb. The differences involved appear to be small, possibly not greater than 5% to 10% at most. It is quite conceivable that, as experience is gained, as the designs are developed further, as metals are improved, and as development charges are canceled, even this estimate may be changed so as to show a negligible minimum of investment near the middle of this pressure range, or even a gradually falling investment with increase of pressure and temperature to the end.

In view of the kaleidoscopic nature of the variation of investment with pressure and temperature, the writers have thought it best to assume plant investment at $100 per unit of capacity for all cases that follow. This is to be interpreted merely as a ready means of presenting data that can be corrected easily to any set of assumed conditions, rather than as indicating that all these different plants have cost exactly $100 per kw (see, in this connection, the earlier remarks about the use of the same value in connection with nominal 225-lb plants).

The thermal performances obtained from plants operating at 400, 600, and 1 250 lb, respectively, are shown in Fig. 9. As before, the values for each pressure are given as a range to take cognizance of fact. It will be observed that each range overlaps the adjacent range, illustrating once more how difficult it is to apply theoretical values to actual conditions. The wide range of performances for 400-lb plants reflects partly the result of improvement of design and operation in the comparatively long time during which such plants have been built and, to some extent, the variations resulting from different local conditions. The upper limits of the 600 and 1 250-lb ranges are, in a sense, incorrect. They give the performances of plants that are really combinations of two pressures, the higher pressure units having been installed in existing plants operating at a lower pressure and tied in in various ways. They are included because these plants are commonly referred to in terms of the higher pressure, ignoring the existence, and the effect, of the lower pressure.

It will be observed that no performance range is shown for the 800-lb plant although the expectable thermal performance of such a plant as of to-day is indicated by a single line in Fig. 9. The values from which these performance curves for other pressures are developed, are obtained from actual operating plants. It happens that although many 800-lb plants are now (1938) being installed, operating data are not yet available.

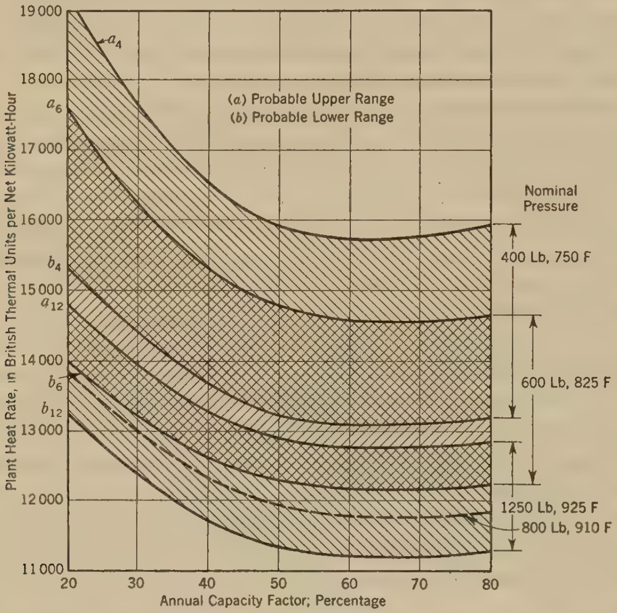


Fig. 9.—Thermal Performance; Steam Plant; Range 400-Pound to 1 250-Pound Nominal Pressure at 1-Inch Hg, Back Pressure

The power-cost data for 400-lb plants are assembled in Fig. 10, in which all curves correspond to those of Fig. 2 and are similarly labeled. (These values, however, are not directly comparable with those shown in similar curves in this paper.) Curves *e* are developed from the upper and lower values shown in Fig. 9, and using a fuel price of 15 cents per million Btu as before. Curves *f* are obtained from the line marked "Nominal 400 lb" of Fig. 11, using a wage rate of 70 cents per hr and adding 0.05 mill for superintendence. Attention is called to the fact that some actual plants are giving a measurably greater output per man-hour, as indicated in Fig. 11, and that if such values were used, the positions of Curves *f* would be lowered. Attention is also called to the fact that no allowance has been made for a probable drooping of the labor performance curve at the upper end, as indicated by dotted lines in Fig. 11.

Curves *g* are obtained from Curve *j*, Fig. 12, with the addition of 0.075 mill for water and miscellaneous supplies. It will be noted that Curve *j* represents a value between the observed limits, Curves *l* and *k*. Data available appeared to indicate its position as approximately correct for a weighted average performance.

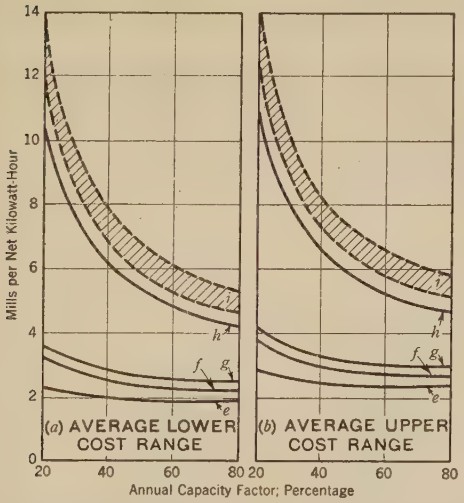


FIG. 10.—AVERAGE UPPER AND LOWER RANGES OF POWER COSTS; NOMINAL 400-LB, 750°F, STEAM PLANT AT 1 IN., HG, BACK PRESSURE

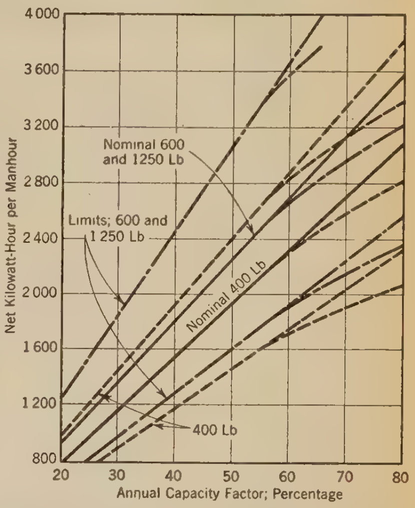


FIG. 11.—NET KILOWATT-HOUR OUTPUT PER MAN-HOUR OF OPERATING LABOR; STEAM PLANT; RANGE, 400 to 1 250-LB NOMINAL PRESSURE

Curves *h*, as already explained, are based on an arbitrary choice of $100 per kw of capacity, and 12% fixed charges. The range, *i*, in Fig. 10, is located as before in terms of percentages of the totals through Curves *h*, the lower limit representing 10%, and the upper, 25 per cent.

Corrections can be made for other conditions and other assumptions as previously noted in connection with the 225-lb plant. The correction to fuel cost for a plant vacuum other than 1-in., Hg, can be made by means of the curve for the 400-lb plant shown in Fig. 5 (*b*). All other corrections are direct ratios as previously indicated.

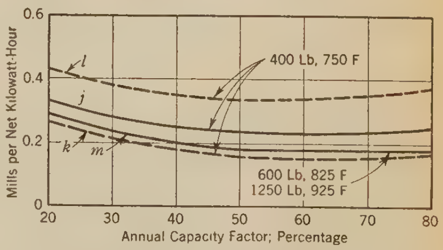


FIG. 12.—MAINTENANCE EXPENSE; LABOR AND MATERIAL; RANGE, 400-LB TO 1 250-LB NOMINAL PRESSURE

The power cost data for a 600-lb plant are given in Fig. 13. Curves *e* are developed from the values shown in Fig. 9 and by using a fuel price of 15 cents per million Btu. Curves *f* are obtained from the line marked "Nominal 600 and 1 250 lb," of Fig. 11, using a wage rate of 70 cents per hr and adding 0.05

mill for superintendence. Curves *g*, Fig. 13, are obtained from Curve *m*, in Fig. 12. This is the first case in which a range has not been shown. The facts are that plants operating at the two pressures indicated are as yet so few and are so comparatively new that the date available could best be shown by a line instead of by an area. As in the previous case, an addition has been made for water and miscellaneous supplies. For the 600-lb plant, 0.05 mill was added for this purpose.

Curves *h*, Fig. 13, are based, as before, on the arbitrary choice of an investment of \$100 per kw of capacity and 12% for fixed charges. Range *i* is located as before in terms of the totals through Curves *h* using 10% and 25% as lower and upper limits, respectively. Corrections can be made for other conditions and other assumptions as previously indicated. The back-pressure correction is indicated in Fig. 5 (*b*).

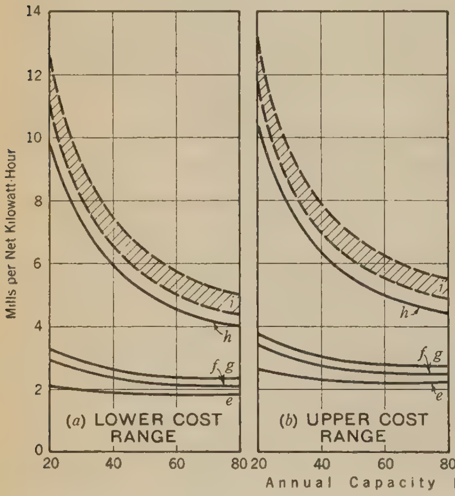


Fig. 13.—Average Ranges of Power Cost; Nominal 600-Lb, 825° Fahrenheit, Steam Plant at 1-Inch, Hg, Back Pressure

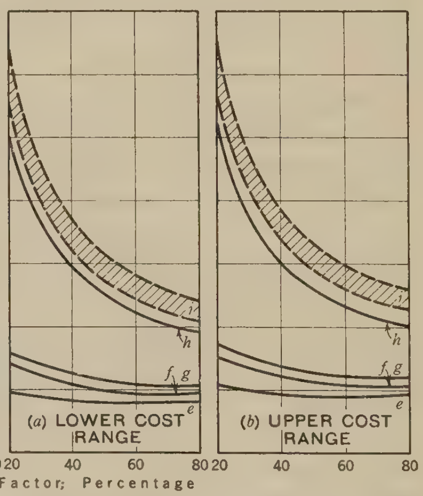


Fig. 14.—Average Ranges of Power Cost; Nominal 250-Lb, 925° Fahrenheit, Steam Plant at 1-Inch, Hg, Back Pressure

The power costs for a 1 250-lb plant are given in Fig. 14. The fuel costs, Curves *e*, are developed from the values shown in Fig. 9, using a fuel price of 15 cents per million Btu as before. Attention is called to the fact that the upper range in Fig. 9 is for what are really combination plants although commonly spoken of as 1 250-lb plants. It is probable that complete 1 250-lb plants will give a performance near the lower range under what may be called average conditions. Curves *f*, *g*, *h*, and *i*, Fig. 14, are obtained as before and by using the same values as for a 600-lb plant.

It is reasonable to assume that a 800-lb plant would give a thermal performance between those of 600-lb and 1 250-lb plants, respectively, and that all other costs would be the same when computed as has been done herein. It does not seem worth while to complicate a presentation of costs actually achieved by injecting costs based on expected performance.

Inspection of Figs. 2, 10, 13, and 14 will show that the predominating items of cost are fuel and fixed charges. The former will vary directly with the price at which thermal units are available. Strategic location in this respect is frequently possible and, under favorable conditions, from 25% to 50% of the fuel cost item can be eliminated. In fact, there are cases in which fuel is available at a price representing a still greater saving. Many who have concentrated upon the reduction of fuel cost have arrived at the suggestion of a mine-mouth plant which would eliminate all, or practically all, the freight on fuel. This subject has been discussed very extensively in recent years. There are comparatively few locations at or near coal mines where an ample condensing water supply is available and where a sizable load is within a distance permitting it to be reached with transmission lines sufficiently short so that transmission costs do not overbalance the fuel savings. Furthermore, the reliability of over-head transmission is not yet generally regarded as sufficiently high to justify depending entirely upon power transmitted in this manner in the case of loads in which a high degree of continuity of supply is required.

Little more can be written about fixed charges than has been presented already. One may hazard a guess as to unit investment in the future, but it can be little better than a guess. It seems probable that as design experience is gained and as the problems of building and installing high-pressure equipment are solved, high-pressure plants will become relatively cheaper. It also seems probable that, in terms of present-day indices, plants for all the pressures considered will be procurable at costs of less than $100 per kw of capacity under favorable conditions.

It is interesting to speculate upon the future development of steam plants. They are already down to an annual performance of approximately 11 000 Btu per net kw-hr. Power engineers know that this can be bettered by increasing the pressure and by the use of a reheat cycle, or by the increase of both pressure and temperature as metal for superheater tubes and pipe lines becomes available at a sufficiently low price to make such higher temperatures economically feasible. They are already technically possible. The designing engineer for a steam power plant can already envisage the possibility of a performance of 10 000 Btu per net kw-hr, or even a little lower than that.

MERCURY VAPOR-WATER VAPOR PLANT

Several so-called binary vapor combinations have been suggested from time to time as means for reducing the cost of fuel-generated power. Only one of the suggested combinations has thus far shown much promise of economic value. This is the plant using mercury vapor and steam, thermally in series; that is, the heat from the fuel is used primarily to produce mercury vapor and, after the vapor has passed through a turbine, heat remaining in it is used to produce steam for a second turbine.

When the construction of such plant was first undertaken on a semi-commercial scale, its computed thermal performance was so much better than the performance attained with a steam plant that the mercury vapor process held great promise. The very rapid improvement of steam plant, combined with the development of certain practical difficulties in the generation of mercury vapor, has served to retard the wide adoption of this binary process for power production. On the other hand, there has occurred an ever-increasing use of mercury vapor for process purposes.

Now it appears probable that a binary vapor power plant using mercury vapor and steam could be constructed to give a thermal performance of about 9 000 Btu per net kw-hr. Data with respect to the necessary investment are fragmentary and it is not known whether increased fixed charges would, or would not, offset the results of the improved thermal performance. Presumably, the situation will be clarified with the passage of time.

Internal Combustion Plant

Internal combustion plant is thus far restricted to the use of reciprocating engines and to comparatively small size per prime mover. Large total capacity is obtained by the multiplication of such units. In the United States a few large internal combustion plants have been built which use blast furnace gas for power production in steel plants. With the exception of these rather unique installations, gas-engine and oil-engine plants have been limited to comparatively small total capacity.

Engines using gaseous fuel have not achieved great prominence for the production of electric power, but the Diesel type using liquid fuel has been introduced very rapidly for such purposes in recent years. There are conditions under which it appears to represent the best economic solution of such problems, particularly where comparatively small capacity and high thermal efficiency are desired in combination. Only time can tell whether this is a permanent or a temporary condition. However, there are now indications that point toward the possibility of building comparatively small steam plants of high thermal efficiency, arranged so as to make them competitive with Diesel plants in similar capacities.

The prominence achieved by Diesel engines as prime movers for electricity generation during the past few years justifies consideration of the cost of power obtained from such sources. The writers have arranged cost data for Diesel plant in such ways as to parallel the treatment used in presenting data for steam plant, so that the two shall be, as nearly as possible, directly comparable. Attention, however, is called to the fact that whereas the data for steam were developed for large plants, 150 000 kw, or more, those for Diesel plants are developed for small installations, say, 10 000 kw, and less.

As in the case of steam, all graphs are plotted against annual plant capacity factor. This ratio has been defined herein as the net annual output, in kilowatt-hours, divided by the product of total rated capacity of the plant and the number of hours in the year, namely 8 760. Attention is called to the fact that this factor differs from that commonly used in presenting Diesel plant performance. Direct comparison with the data given herein for steam is permissible; direct comparison with the commonly quoted Diesel plant performance is impossible without preliminary recalculation.

A reasonable fuel cost range is given in Fig. 15. The values indicated by the two curves are based on Diesel engine fuel oil at 5 cents per gal. Corrections can be made by direct ratio for other oil prices. Such prices generally fall between 4 and 6 cents although there are cases in which oil is purchased at prices much lower than 4 cents, and others in which the higher figure is exceeded. It is to be noted that the rather wide range of thermal performance indicated is influenced more by the type of load carried, the character of operation, and the age of the engine, than by the size of the engine. In this statement, age of the engine refers both to the state of the development of the art when it was constructed and to the extent to which it was worn in use.

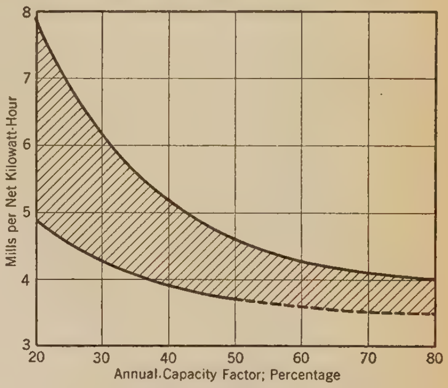


FIG. 15.—FUEL OIL EXPENSE FOR DIESEL PLANTS (OIL AT 5 CENTS PER GALLON)

The values shown in Fig. 15 have been plotted as Curves *e* in Fig. 16, corresponding to the arrangement used in presenting steam data.

Curves *f* are derived by using the middle graph of Fig. 17 for output per man-hour and an hourly wage of 70 cents, as before. They represent the cost of operating labor and superintendence plotted above Curves *e* as a base. In this case, operating labor is used partly for maintenance purposes, as is common practice in relatively small plants, so that a part of what is indicated herein as operating labor would be more properly chargeable to maintenance. Plant records do not ordinarily permit this allocation. The upper line of Fig. 17 indicates the output per man-hour that may be expected with a well-designed plant containing a few large modern units. The lower line gives values actually obtained under less favorable conditions. Corrections for different man-hour outputs and for different wage rates can be made easily by ratio computations.

Curves *g*, Fig. 16, plotted above Curves *f*, represent the summary of costs of lubricating oil, miscellaneous supplies, and maintenance to the extent that the latter is not included under operating labor as already explained. The values used for the cost of lubricating oil are taken from the middle curve of Fig. 18. The great range of costs found in practice is indicated by the upper and lower curves of that diagram. It should be noted that all these curves are in terms of oil at 55 cents per gal so that different oil prices are not directly responsible for the wide range that is shown. As a matter of fact Diesel plants are using lubricating oils costing from less than 20 cents to more than 60 cents per gal. There is some indication that the total cost of lubrication is greater with the very cheap oils than it is with the more expensive ones, but this should not be regarded as a proved fact, or as generally applicable. The problem is too complicated and is affected by too many other variables to permit of such simple generalization.

The values used for maintenance cost are taken from the middle lines of Fig. 19. It is possible that, in placing this curve as low as they have, the writers have been unduly influenced by the records of more recently installed plants which may become poorer as these plants age. It seems fairly certain that the line does not belong lower; it may belong as much as 30% higher. The upper range is indicative of what may happen with poorly built, or poorly operated and maintained, equipment.

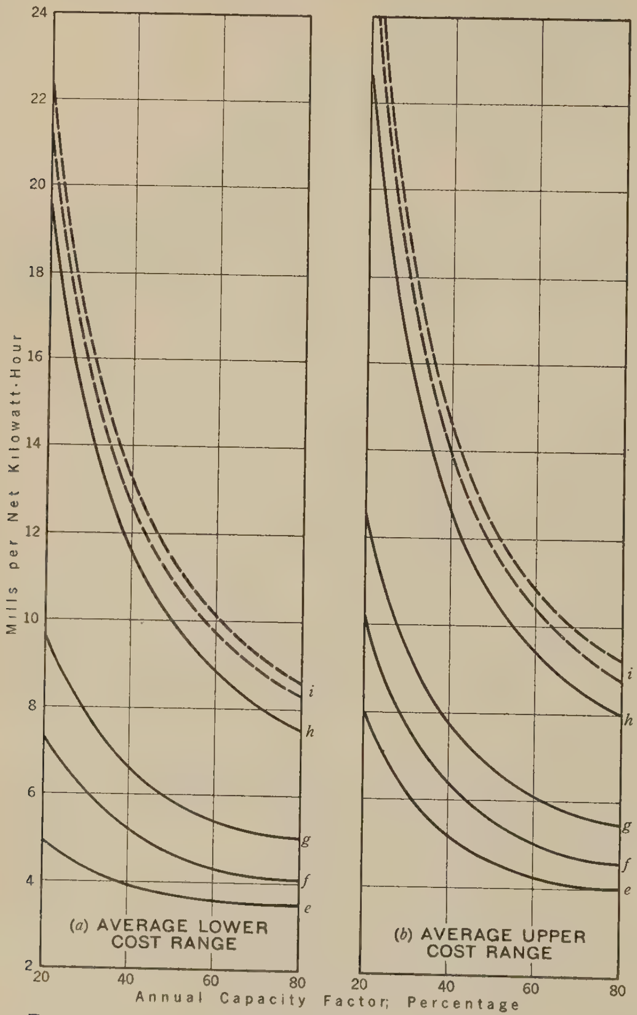


Fig. 16.—Average Ranges of Power Cost for Diesel Plants

The cost of miscellaneous supplies and expense differs greatly in magnitude, depending upon the local conditions, the grade of "house-keeping," and many other factors. The writers have chosen to use 0.15 mill per net kw-hr at all capacity factors. This appears to be a reasonable average value. Thus Curves *g*, Fig. 16, plotted above Curves *f* as a base, represent the summations of the values given by the middle lines of Figs. 18 and 19 and the constant value, 0.15 mill.

Curves *h*, Fig. 16, plotted above Curves *g* as a base, represent fixed charge expense. At any output this is obviously determined by the investment and the percentage represented by fixed charges. For investment the writers have chosen to use $135 per kw of plant capacity, which they believe represents a fair average value for a complete plant. It is obvious that, with other conditions equal, the unit cost should be greater with a large number of small units as against the same total capacity in a small number of large units. It is also a fact that many local conditions, such as character of soil, availability of water, character of the neighborhood, provision for future enlargement, etc., will all have definite effects upon investment. The choice of $135, therefore, is not to be considered as equivalent to a statement that any and all Diesel plants should be procurable at such a figure but rather that it represents a fair average of both lower and higher values.

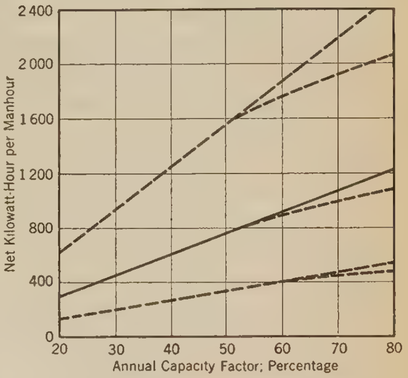


FIG. 17.—NET KILOWATT-HOUR OUTPUT PER MAN-HOUR OF OPERATING AND SUPERINTENDING LABOR FOR DIESEL PLANTS

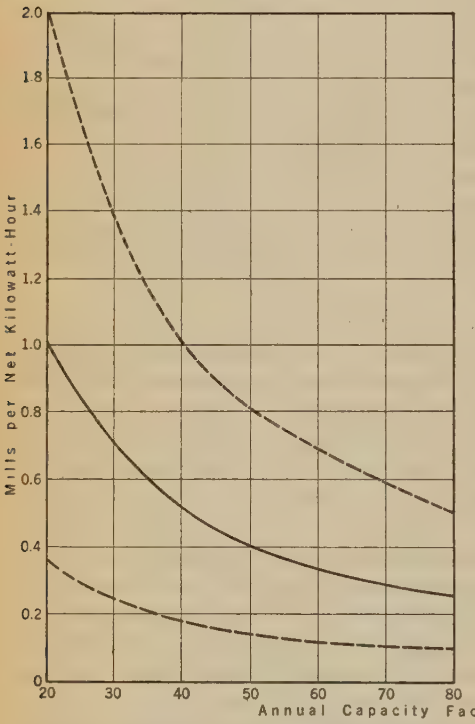
The writers have chosen to use 13% for fixed charges as against 12% for steam plant. They believe the higher rate to be justified by the shorter expectable life of Diesel equipment. It is a fact that in some cases such plants have been built recently by municipalities which obtained money at low rates of interest and that in such cases the fixed charges might legitimately be considered as less than 13 per cent. However, it is believed that there are enough operations actually costing more than this percentage to justify its use as a reasonable average value.

Curves *h*, plotted above Curves *g* as a base, then, represent fixed charge expense based on $135 per unit of capacity and 13% as the rate. Obviously, corrections can be made easily for any other value that may be chosen.

The cost of general over-head expense has been shown in Fig. 16 as Range *i*, just as it was in the case of steam plant. The range is plotted on the assumption that this form of expense will vary between 10% and 15% of the totals shown by Curves *h*. The choice of an upper limit lower than that used in the case of steam plant is a recognition of the characteristics of the smaller types of properties that would be using Diesel plant. It is not believed that a value less than 10% can be expected when all items of expense are properly accounted.

At this point the writers call attention to the caution given at the end of the section on nominal 225-lb steam plants. Here, as in the case of steam, one should not assume, without warrant, that all controlling factors will be most favorable or most unfavorable for a given case.

Comparison of expectable cost of power from Diesel plant with corresponding values for modern steam plant, as revealed herein, shows plainly that the latter is capable of producing power more cheaply both with and without the consideration of fixed charges. It should be noted, however, that this result is obtained by comparing values for very large steam plants with those for much smaller plants equipped with Diesel engines. Although not shown by data in this paper, it is true that, in the case of plants of the size represented by existing Diesel installations, the Diesel engine may still give cheaper power than the steam plant. The competitive situation in these sizes is at present in a state of flux because of many current attempts to build cheap, reliable, and thermally efficient steam equipment in the smaller sizes. Progress made thus far appears to the writers to indicate a real challenge to the supremacy of Diesel engines in this respect.



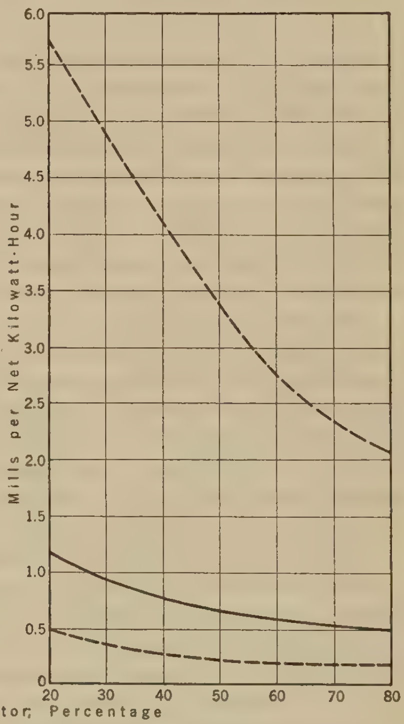


FIG. 18.—LUBRICATING OIL EXPENSE FOR DIESEL PLANTS (OIL AT 55 CENTS PER GALLON)

FIG. 19.—MAINTENANCE EXPENSE FOR DIESEL PLANTS

FUTURE POSSIBILITIES

Among other failings, history shows that Man is, in general, a very poor prophet. He has almost always demonstrated inability to imagine the possibility of change because, at the time, he could not envisage the mechanism of it. It is for this reason that the writers hesitate to assume the position of prophet and discuss the future possibilities of fuel-generated power. They recognize too well their own human limitations.

Looking back over the history of fuel-developed power, they recognize the fact that almost every major advance has been made in the face of potential economic loss; that is, the first cost of equipment was generally so high and the supplementary expense involved in making it work was so great, that it required an optimist to incur the very grave possibility of present loss in the hope of future gains. If one attempts now to look into the future with respect to presently visible means of improving thermal efficiencies, one is immediately confronted with the certainty of over-balancing fixed charges. Time after time this condition has caused studious individuals to state that the then present state of development represented the ultimate; and, time after time, they have been proved wrong by subsequent shrinkage of the predicted first cost and a resultant economic gain.

There appears to be no reason for assuming that the same kind of law will not continue to operate. If it does, it is reasonable to assume that with comparatively slight modifications of present methods along lines which can now be foreseen, thermal efficiencies of the order of about 40% may be realized commercially in comparison with the present achievement of about 30% with steam and 35% with Diesel engines. It is difficult to imagine how present types of plants can be perfected to better performance, but this does not mean that it cannot be done.

History also indicates that as Man approaches the ultimate performance with one type of equipment or method, he develops the ability to produce a new, radically different, and better one. It may be that this is indicated in the present instance by the activity in the field of fuel cells; that is, equipments somewhat similar to electric batteries in which fuel is consumed and electric power produced as a result. This development has gone on almost unnoticed by power-plant men until there is now operating in laboratory size a very promising sample of such a cell. No one can say now whether it can be enlarged sufficiently and commercialized, but careful investigation appears to indicate great probability that it may be. Many difficulties can be foreseen, but none of them appears to be of insurmountable character.

The promise is for thermal efficiencies of the order of double or more than double those presently attainable, and with a unit investment not greater than that now required, possibly even less.

Need one go further with respect to future possibilities in this line? It would seem that the chances for continued reduction of total power cost at the fuel-burning generating plant are indeed very large.

HYDRO-GENERATED ENERGY

By H. K. Barrows,[4] M. Am. Soc. C. E.

Synopsis

This paper presents an analysis of the plant characteristics and elements of cost of hydro-generated energy based upon an investigation and study of data regarding fifty-seven hydro-plants scattered throughout the United States. Attention is given particularly to hydro plants in power systems where hydro and steam power are complementary in use, and a method of evaluating such hydro-power costs is suggested, which takes into account both energy and capacity values.

Some of the larger Federal hydro-projects are also discussed and compared in cost with those of private utility companies and a brief survey is made of water-power resources not as yet developed.

Essential Features and Types of Development

The general requirement of a hydro-electric plant is a flow of water, as uniform as may be, held back usually by a dam so that it may create a head on hydraulic turbines, and by passing through them under this head or pressure, create electric energy by means of the generator, now usually direct-connected with the turbine.[5]

A hydro-development, therefore, requires a dam in most cases. Where the head utilized is only that created by the dam in what is called a "concentrated fall development" the power-house is situated just below the dam, the water reaching the turbines through a flume or penstock and again entering the river after passing through the turbines. In some cases, water from the turbines passes back to the river by a tail-race channel, which often is of considerable length.

Frequently, it is advantageous to direct the water from the dam for some distance down river by a waterway, thus obtaining additional head by what may be termed an "extended fall" development. Depending on the quantity of river flow, as well as the topography near the river and the head developed, the waterway may be an open canal, a closed pipe or penstock, or a combination of both. Wood-stave, steel, and reinforced concrete pipes or penstocks are all in use for such waterways. Occasionally, a tunnel is used for the waterway, where conditions make it economical.

The power-house is commonly rectangular in shape, with vertical turbine-generator units in a line along the length of the building and spaced to permit, in the concrete substructure, the necessary width of approach flumes to the turbines as well as draft-tubes and tail-race channels. The type of flume and turbine setting for reaction wheels depends chiefly upon the head developed, as follows:

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[4] Prof. of Hydr. Eng., Mass. Inst. Tech.; and Cons. Engr., Boston, Mass.

[5] "Water Power Engineering," by H. K. Barrows, 1934.

Head, in feet (approximate)	Type of approach flume
20 to 30, or less	Open pit
20 to 50	Concrete scroll
30 to 350	Plate-steel scroll
350, or more	Cast-steel scroll

The modern hydraulic turbine unit is commonly of the vertical, single-runner type, with a vertical generator set above the turbine and direct-connected with it. Proper control of speed is obtained by the hydraulic governor and pressure regulation (in the case of other than low heads), by a surge tank, usually now of the differential type, placed in the penstock as near to the turbines as practicable.

For higher heads, particularly 1 000 ft, or more (but depending also upon capacity), impulse wheels are used, in which the water passes through a nozzle to the buckets of the wheel. These wheels are usually set on a horizontal shaft and may be single overhung or (for larger units) double overhung, with reference to the generator supports. The general arrangement of such installations is similar to that of reaction turbines, except that no draft-tube is used, the water from the wheel entering a tail-race channel.

The power-house superstructure is usually of brick or reinforced concrete and contains the generators, switching, and other electrical equipment. It is now common practice to install the transformers, lightning arresters, and high-tension electrical equipment in an outdoor yard. In this paper, costs at the switchboard are considered and no transmission costs are included.

Following is an outline of the essential features of the hydro-electric plant as considered in this paper, and common types of construction:

1.—Dam:
  (*a*) Non-Overfall Section:
    Solid concrete;
    Hollow concrete;
    Earth (with or without core);
    Timber.
  (*b*) Spillway:
    Solid concrete;
    Hollow concrete;
    Tunnel type;
    Lateral channel type.
2.—Waterway (Including Tail-Race):
  (*a*) Canal or Open Channel:
    Lined;
    Unlined.
  (*b*) Penstock or Pipe:
    Wood stave;
    Steel;
    Concrete.

- (*c*) Combinations of Canal and Penstock.
- (*d*) Flume:
    - Wood;
    - Concrete;
    - Steel.
- (*e*) Tunnel.
- (*f*) Tail-Race:
    - Open-channel (protecting wall or embankment).
- (*g*) Surge Tank:
    - Simple;
    - Differential.

3.—Power House and Equipment:

- (*a*) Building:
    - Substructure (concrete).
    - Superstructure:
        - Brick;
        - Concrete.
- (*b*) Equipment:
    - Hydraulic:
        - Turbines and governors;
        - Relief valves.
    - Electrical:
        - Generators;
        - Switching equipment;
        - Conduits.
    - Miscellaneous:
        - Crane;
        - Station service.

4.—Reservoir Requirements; Changes in:

- (*a*) Highways;
- (*b*) Railroads;
- (*c*) Manufacturing plants;
- (*d*) Other structures;
- (*e*) Clearing.

5.—Land and Water Rights:

- (*a*) Real Estate:
    - Land;
    - Buildings.
- (*b*) Water Rights:
    - Flowage;
    - Power and riparian rights.

ISOLATED AND SYSTEM PLANTS

In this paper, attention will be given chiefly to hydro-plants in a power system in which, usually, other hydro-electric as well as steam electric plants are included. This is of importance as affecting requirements for spare units as well as auxiliary power that may be needed in the case of isolated hydro-plants in which dependence upon continuous service is essential and for which assistance from other plants in a system is not available.

A hydro-plant in a power system also makes possible a better and more economical use of hydro-electric power than can be the case with an isolated hydro-plant.

Plant Capacity with Reference to Flow

Based upon average yearly flow-duration curves constructed with the lower flows toward 100% of the time, hydro-plant capacities, as constructed, generally vary between a flow available from about 60% of the time to one available only a small part of the time. Of course, this reflects, to a large extent, the capacity or use-factor requirements.

TABLE 1.—Installed Wheel Capacities at Hydro-Electric Plants

Plant	Drainage Area, in Square Miles		Head, in Feet		Wheel Capacity			
					Discharge, in Cubic Feet per Second per Square Mile		Percentage of the Time That Discharge Is Available	
	From:	To:	From:	To:	Present†	Ultimate	Present†	Ultimate
Garvins Falls, N. H.	2 340		30		1.41		35	
Vernon, Vt.	6 300		34		2.46		21	
Fifteen-Mile Falls, N. H. and Vt.	1 600		170		7.60		2±	
New England Power Company*	250	500	66	230	3.80		21	
Harriman, Vt.	184		360		9.60		5±	
Sherman Island, N. Y.	2 782		66		2.26		29	
Safe Harbor, Pa.	26 000		55		1.95	4.00	25±	7
Bartlett's Ferry, Ga.	4 200		112		0.97	1.94	53	22
Tallulah Falls, Ga.	190		580		8.4		5±	
Great Falls, Mont.	23 000		52		0.26		75	
Holter, Mont.	20 000		100		0.30		70	
Pit No. 3, Calif.	4 320		280		0.83		17	
Oak Grove, Wash.	266		849		1.58	4.74	100	33

* Plants 2, 3, 4, and 5. † 1938.

Table 1 contains some actual wheel capacities at different plants expressed in both cubic feet per second per square mile of flow and as the percentage of time that flow is available in the average year.

As will be noted from Table 1, system hydro-plants, when not especially designed for day use, generally show a wheel capacity corresponding to the flow available about 20% to 30% of the time. The day or peak-load plants (Fifteen Mile Falls, Harriman, Safe Harbor, and Tallulah Falls) usually have wheel capacities of about 5% or less of the time flow.

Definitions

Load Factor.—The ratio of the average load over some period of time, as a day, week, month, or year, to the maximum or peak load during that time measured during a short time, as a few minutes, is termed the "load factor." Plant load factor applies to any particular hydro-plant under consideration.

System load factor applies to the power system in which the hydro-plant is an integral part.

Capacity or Use Factor.—The ratio of the average load of the hydro-electric plant over some period of time to the plant capacity, is its capacity factor, or use factor. Note that with maximum load at plant capacity, the plant load factor is the same as the plant capacity factor.

Utilization Factor.—The ratio of the average hydro-plant output over some period of time to the power available in the stream with the plant wheel efficiency and head, and as limited by plant capacity, is known as the "utilization factor." Stated more simply, this is the ratio of power output to power available.

The foregoing factors for a hydro-electric plant in a power system vary commonly about as follows (expressed as percentages):

Factors	Comparison
Yearly load	From a small percentage (often less than 10% with peak-load plants) to about 70% or more under very favorable operating conditions
Yearly capacity	From about 30 to 60%
Yearly utilization	From about 40 to 70%

Primary, or Firm Power.—Referring to hydro-power, "primary power," or "firm power" is the dependable power, or that which is continuously available at the plant under the conditions of use. It is defined by the extreme low-water flow, although sometimes it is taken as the flow available 95% of the time where low flows of record are affected by pondage, particularly at week ends.

Primary or Firm Capacity.—Referring to the use of hydro-electric power in system loads, "primary capacity" or "firm capacity" is the part of the total installed capacity that can perform the same function on that part of the load curve to which it is assigned as could be performed by an alternative steam plant. In this case the primary capacity of a hydro-plant would be based on the power available at low-water flow (or sometimes due to back-water conditions), but concentrating the use of this flow at the time of system peak by means of pondage, to replace steam capacity as far as possible.

Elements of Cost of Hydro-Electric Generated Energy

The dominating element of cost is that of the fixed charges upon cost of plant. These charges will ordinarily include:

Items	Percentage
Interest	5 to 7
Depreciation	1 to 2
Taxes and insurance	1 to 2
Total	7 to 11

A fair allowance is 10% under usual conditions. It is to be noted that a hydro-plant often includes a large proportion of essentially permanent construction, such as dams of earth or concrete, tending to lessen the necessary physical depreciation allowance. Moreover, hydro-plants are usually located in the less settled parts of the country, so that the item of taxes is less than that for steam plants in urban regions. Other minor items of cost of hydro-electric energy are attendance and ordinary maintenance items. These costs will not usually total more than 0.1 ct per kw-hr, even with relatively small plants, and for larger plants the total is usually from 0.02 ct to 0.04 ct per kw-hr. Hence, the cost of hydro-electric energy is essentially that of fixed charges.

Evidently, the cost per kilowatt-hour of hydro-electric energy will vary inversely with the plant capacity factor. Thus, a plant of 10 000 kw, costing $150 per kw, or $1 500 000, would have fixed charges of $150 000 per yr. With a capacity factor of 60% the plant would produce 6 000 kw as an annual average load, or about 52 million kw-hr per yr, costing 0.29 ct per kw-hr.

With a capacity factor of 30% there would be a yearly average load of 3 000 kw, or 26 million kw-hr, costing 0.58 ct per kw-hr, which is twice the cost for a 60% capacity factor.

Curves by means of which fixed charges for hydro-plants (in cents per kilowatt-hour) may be obtained for different plant costs per kilowatt and different plant capacity factors may be constructed.[6] From a curve based upon 10% fixed charges, the cost per kilowatt-hour for other percentages of fixed charges, can readily be obtained by proportion. As previously noted, fixed charges are essentially the cost of hydro-electric energy.

Hydro-Generated Energy in a Power System

Two aspects of hydro-electric power as used in a power system are of importance:

(1) At the wetter seasons of the year (particularly in the spring, when water is available up to and beyond plant capacity) the hydro-plant may be used to carry some, or all, of the base load part of the demand, thus saving principally in fuel cost of steam plants; and

(2) In the medium and low-water seasons the hydro-plant may be used to carry peaks, or the upper part of the load curve, thus saving in respect to steam capacity. Aspect (2) is of much importance, as in the upper part of the usual load curve the proportion of energy to that of the daily total is relatively small for a considerable range of required capacity. In other words, the peaks are of relatively short duration, with the result that a hydro-electric plant carrying, say, the upper 10% of daily energy will include approximately 20% to 40% of peak capacity, depending on the system load curve and load factor.

It should also be noted that the extent to which the system peak is already taken by hydro is of importance in the consideration of additional hydro-plants. The first hydro-plant in the system occupies the favored position at the top of the curve, hence a second plant, as it must take a lower and less advantageous position on the load curve, is usually more difficult to justify.

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[6] For an example of this style of plotting, see *Electrical World*, August 14, 1937, p. 100 (570).

Existing conditions are thus of importance in determining the economic justification of new hydro-plants.

The relations of the percentage of daily energy carried by water power and the percentage of peak load carried by the hydro-electric plant are shown in Fig. 20 for system load factors of 0.4, 0.5, 0.6, and 0.7. Each of the curves shown therein is the result of a study of groups of load curves through the given range of load factors and will serve for use in preliminary estimates and studies.

Assuming that a hydro-electric plant supplies 10% of the daily energy for a system with an average daily load of 300 000 kw, or 7.2 million kw-hr daily, the hydro-plant carries 720 000 kw-hr daily, or an average load of 30 000 kw, which latter value might represent the primary 24-hr flow at the site. Referring to Fig. 20, the hydro-plant could carry peak loads, as shown in Column (4), Table 2, for different system load factors. The assumed conditions are shown graphically in Fig. 21.

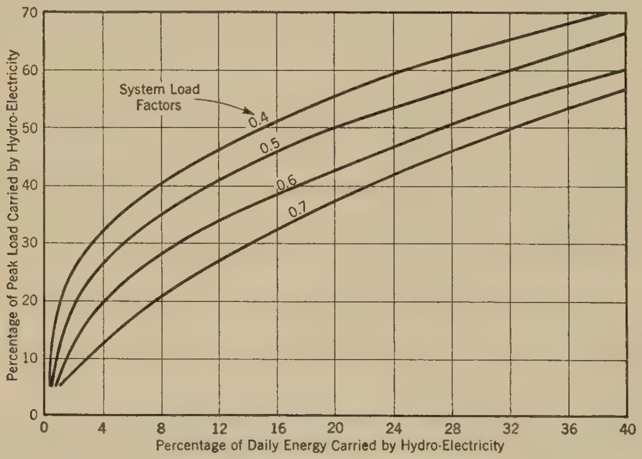

FIG. 20.—SYSTEM PEAK CHARACTERISTICS FOR DIFFERENT SYSTEM LOAD FACTORS

TABLE 2.—DATA SHOWING USE OF FIG. 20

System load factor (percentages)	Peak system load, in kilowatts	Carried by Hydro-Electric Plant		Ratio: $\frac{\text{Column (4)}}{30\,000\text{ kw.}}$	$\frac{1}{\text{Column (5)}}$ or hydro-load factors (percentages)
		Percentage of peak (from Fig. 20)	Corresponding kilowatts		
(1)	(2)	(3)	(4)	(5)	(6)
40	750 000	43.5	327 000	10.9	9.2
50	600 000	38.0	228 000	7.6	13.1
60	500 000	31.5	157 000	5.2	19.2
70	430 000	24.	103 000	3.4	29.5

In Column (5), Table 2, is given the ratio of kilowatts carried by the hydro-plant at the time of system peak load to the hydro-plant average 24-hr load. This shows that, depending on the system load factor, if used at the time of system peak load the plant can supply peak capacity of 3.4 to 10.9 times the amount available on a 24-hr steady use basis. Plant load factors will be the reciprocal of values in Column (5) × 100 and, as will be noted, will vary from approximately 9 to 30 per cent.

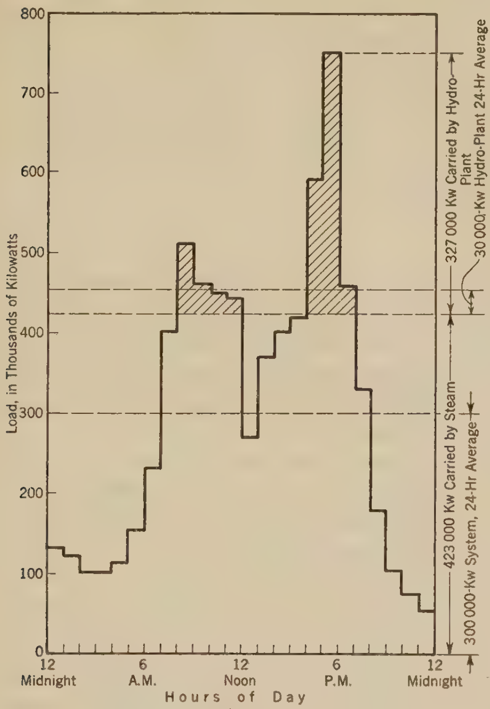

FIG. 21.—USE OF WATER POWER FOR PEAK CAPACITY; SYSTEM LOAD FACTOR, 40 PER CENT

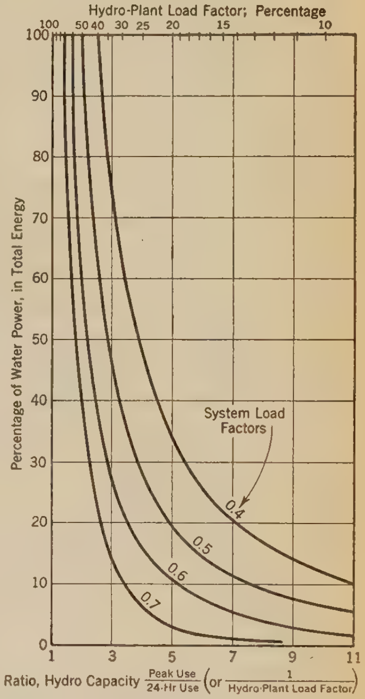

FIG. 22.—USE OF WATER POWER ON SYSTEM PEAKS

Fig. 22 is obtained by computations similar to those for Columns (5) and (6), of Table 2, for various values of percentages of water power in total daily energy (based upon Fig. 20), and shows directly, for different system load factors, the relation of peak hydro-electric capacity (when used at the time of system peak) as compared with its 24-hr use, and by auxiliary scale at the top, the corresponding hydro-load factors.

PONDAGE

Pondage is necessary at the hydro-plant, of course, in order to make possible the use of hydro-electric power at the time of the system peak. The number of hours per day during which water is being used and, by another scale, the hours of collection of water, or pondage, are given approximately by Fig. 23, for different percentages of system peak carried by the hydro-electric plant.

Referring to the previous example and assuming that the 30 000 kw of 24-hr primary hydro-electric power is developed under a head of 200 ft, requiring about 2 250 cu ft per sec with 100% utilization, the data in Table 3 are obtained.

TABLE 3.—DATA FOR THE USE OF FIG. 23

System load factor (percentages) (1)	Percentage of peak carried by hydro-plant (Column (3) in Table 2) (2)	Hours of use (Fig. 23) (3)	Hours of pondage 24 − Column (3) (4)	Pondage Required		Portion of 24-hour flow ponded $\left(\frac{\text{Column (4)}}{24}\right)$ (7)
				Cubic feet per second-hours (Column (4) ×2 250) (5)	Acre-feet $\left(\text{Column (5)} \times \frac{3\,600}{43\,560}\right)$.(6)	
40	43.5	7.0	17.0	38 000	3 140	0.70
50	38.0	7.5	16.5	37 000	3 060	0.69
60	31.5	8.0	16.0	36 000	2 970	0.67
70	24.0	8.0	16.0	36 000	2 970	0.67

Table 3 demonstrates the use of Fig. 23 and the computation of required pondage for different system load factors, where, as previously noted, the hydro-plant is carrying 10% of the daily energy, under four different system load factors. From 2 970 to 3 140 acre-ft of pondage are required, or from 0.67 to 0.70 of the 24-hr flow.

A further study similar to that of Table 3 for various percentages of the hydro-plant energy to the total daily energy output of the system shows (with only minor variation with system load factor) approximate pondage requirements as follows:

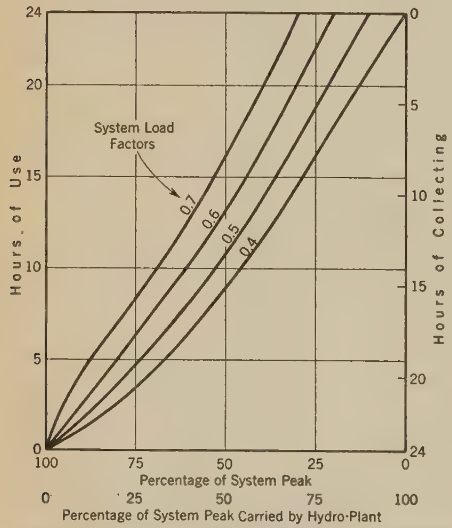

FIG. 23.—PONDAGE; TIME OF USE AND OF COLLECTION FOR DIFFERENT SYSTEM LOAD FACTORS; WATER POWER USED ON SYSTEM PEAK

Hydro-electric energy in percentage of daily energy	Pondage requirements (ratio to 24-hr flow)	Hydro-electric energy in percentage of daily energy	Pondage requirements (ratio to 24-hr flow)
2	0.85	8	0.71
4	0.78	10	0.68
6	0.74	15	0.61

Note that with less than 100% utilization of river flow, less pondage is required; but operation of the hydro-plant for peak loads will usually be with fairly complete utilization.

Although there is some approximation in the foregoing methods the general results are sufficiently accurate to indicate general pondage requirements and show that pondage, to the extent of about 60 to 85% of the 24-hr flow, is required for its peak load hydro-electric use, depending on its percentage of the total daily energy.

FIRM CAPACITY OF HYDRO-PLANT

As previously noted, the usual hydro-plant capacity in a power system corresponds to about the flow available 20% to 30% of the time. Peak or day-load plants will be at about the 5% flow or a somewhat lesser percentage (higher capacity) in some cases. The primary 24-hr flow may be taken at about the 95% flow.

The general form of the average flow-duration curve in regions where water power is in general use is shown approximately by the following two equations:

East and South:

$$\log Q = 2.45 - 0.011\,T \quad \text{(1)}$$

West:

$$\log Q = 2.52 - 0.013\,T \quad \text{(2)}$$

in which Q is the discharge expressed as a percentage of the mean yearly discharge, and T is percentage of time. Table 4 is based upon Equations (1) and (2). It will be noted that ordinary hydro-wheel capacities will be from about six to eight times the primary 24-hr power, and peak or day-load hydro-plants ten or fifteen times the primary 24-hr power.

TABLE 4.—FLOW RELATIONS

Percentage of time that flow is available	FLOW AS A PERCENTAGE OF AVERAGE YEARLY FLOW		RATIOS TO FLOW AVAILABLE 95% OF TIME	
	East and South	West	East and South	West
5	250	285	10.	15.0
20	170	182	6.8	9.6
30	132	135	5.3	7.1
95	25	19	1.0	1.0

It will be seen that, in many cases, depending upon the extent of hydro-power use and system load factor, the entire hydro-electric capacity may be firm, or largely so, when used on the system peak. Thus, in the example previously given (see Table 2), in which the primary 24-hr power was 30 000 kw, the wheel capacity would be ordinarily about 8 × 30 000 = 240 000 kw; or, if a peak or day-load plant is considered, 15 × 30 000 = 450 000 kw. The proportion of plant capacity that would be firm for different system load factors would then be as given in Table 5.

TABLE 5.—FIRM PLANT CAPACITIES

System load factor (percentage)	Carried by hydro-electric plant (Column (4), Table 2), in kilowatts	PROPORTION OF PLANT CAPACITY THAT IS FIRM	
		Plant capacity, 240 000 kilowatts	Plant capacity, 450 000 kilowatts
40	327 000	All	0.73
50	228 000	0.95	0.51
60	157 000	0.65	0.35
70	103 000	0.43	0.23

PEAK CAPACITY VALUE OF HYDRO-PLANT

The essential or important result of using water power on the peak of the system load is to save steam capacity. The cost of a steam plant is approximately \$100 per kw.[7] In comparing with water power, the ratio of fixed charges (10% for water power and 12.5% for steam power) would require a factor of 1.25, making the comparative cost of the steam plant, \$125 per kw. This may be regarded as a credit item on the cost of hydro-plant when the latter can be used for system peak loads.

Thus, to illustrate this suggested method of procedure, assuming that a hydro-plant in use for saving in system peak capacity costs \$200 per kw and that three-fourths of its capacity is firm, the credit on account of capacity saving would be, say, $\frac{3}{4} \times \$125 = \$94$ per kw, leaving a net cost of \$106 per kw for the hydro-plant. The yearly capacity factor for such a plant might be 0.5 (if used for base loads in the wetter seasons) and its fixed charges, 0.24 ct per kw-hr. With allowance for operating and maintenance costs, this would be about 0.27 ct per kw-hr as the total cost of hydro-electric energy.

This cost of hydro-electric energy would compete with fuel and attendance costs of steam power. With coal at \$3.00 per net ton and 1.5 lb per kw-hr, fuel would cost 0.225 ct per kw-hr, and attendance about 0.10 ct, or a total of 0.32 ct per kw-hr—showing some margin in favor of the hydro-plant with conditions as assumed.

This method of crediting or allowing for capacity value of a hydro-plant for peak use in a load system and regarding the hydro-electric energy output as affecting fuel saving and attendance charges enables the consideration of the total hydro-electric output and renders unnecessary the segregation of that part commonly called primary. In an actual case the steam capacity saving would be determined by taking into account the primary 24-hr power, pondage, actual wheel capacity, and the actual system load curve. The foregoing method is only approximate and is used merely for illustrative purposes. An illustrative example using actual data for the Conowingo plant is given subsequently herein.

COST OF HYDRO-ELECTRIC PLANTS

Supporting Data.—Cost data for hydro-electric plants have been obtained as a basis for study in this paper from two general sources:

(1) From the files and annual reports of the Federal Power Commission[8] at Washington, D. C., plant characteristics and data of cost for thirty-four hydro-plants in different sections of the United States and especially in the West. These factors are given in detail in Table 6 (see project numbers in Column (2)); and,

(2) By correspondence with engineers and other officials of public utility companies, as well as from personal files of the writer, giving similar data of characteristics and cost for twenty-three hydro-plants, fairly well distributed generally. These data are also given in detail in Table 6 and are

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[7] "Cost of Generation of Electrical Energy," by Philip Sporn, *Proceedings*, Am. Soc. C. E., December, 1937, pp. 1925–1938.

[8] Annual Repts., Federal Power Comm., 1923–1936, inclusive.

TABLE 6.—Characteristics and Costs of Hydro-Electric Plants

Plant No.	Name (1)	River (2)	State (3)	Date of completion (4)	Drainage area, in square miles (5)	Capacity, in thousands of kilowatts at switchboard (6)	No. of units (7)
	(a) Eastern Plants						
1	Green Island, Project (13) (a)	Hudson	N. Y.	1923*	....	6.6	8
2	Wallenpaupack, Project (487)	Wallenpaupack Creek	Pa.	1926*	240	40.0	2
3	Saluda, Project (516)	Saluda	S. C.	1931*	....	130.0	4
4	Bartlett's Ferry, Project (485)	Chattahoochee	Ga.	1926*	4 200	45.0	3
5	Jackson Bluff, Project (682)	Ocklockonee	Fla.	1930*	....	8.8	3
6	Moss Bluff, Project (177)	Ocklawaha	Fla.	1927*	....	0.5	..
7	Wyman	Kennebec	Me.	1931	2 660	72.0(b)	3
8	Weston	Kennebec	Me.	1921	....	12.0	4
9	Gulf Island	Androscoggin	Me.	1926	2 860	19.0	3
10	.... (c)	....	....	1937	690	3.0	1
11	Mollys Falls	Winooski	Vt.	1926	23	5.0	1
12	Harriman	Deerfield	Vt.	1924	184	43.0	3
13	Sherman (c)	Deerfield	Mass.	1926	234	7.0	1
14	Conowingo	Susquehanna	Md.	1928*	27 500	252.0	7
15	Safe Harbor	Susquehanna	Pa.	1932	26 000	178.0	6
	(b) Plants in the Southern and Central States						
16	Waterville, Project (432)	Big Pigeon	N. C.	1930	....	108.0	3
17	Piney, Project (309)	Clarion	Pa.	1925	....	28.8	3
18	Dam No. 7, Project (539) (a)	Kentucky	Ky.	1928*	....	2.0	3
19	Ohio Falls, Project (289) (a)	Ohio	Ky.	1928*	....	80.3	8
20	Shawano, Project (710)	Wolf	Wis.	1928*	....	0.7	1
21	Mottville, Project (401)	St. Joseph	Mich.	1923*	....	1.7	4
22	Winton, Project (469)	Kawishiwi	Minn.	1924*	....	4.0	2
23	Blanchard, Project (346)	Mississippi	Minn.	1925*	....	12.0	2
24	Twin City, Project (362) (a)	Mississippi	Minn.	1924*	....	13.4	4
25	Big Bend, Project (792)	Rapid Creek	S. D.	1930*	....	1.2	3
26	....	....	....	1923	9 827	52.5	3
27	....	....	....	1928	....	100.0	4
28	....	....	....	1926	3 000	99.0	3
29	Allegan	Kalamazoo	Mich.	1937	1 530	2.4(b)	3
30	....	....	....	1931	14 000	133.0	6
	(c) Western Plants						
31	Lower American Fork, Project (696)	American Fork Creek	Utah	1927*	....	1.0	2
32	Upper American Fork, Project (696)	American Fork Creek	Utah	1927*	....	1.2	2
33	Hyrum City, Project (946)	Blacksmith Fork	Utah	1930*	....	0.4	1
34	Logan, Project (486)	Logan	Utah	1927*	....	2.0	2
35	Santaquin, Project (665)	Santaquin	Utah	1927*	....	0.9	2
36	Stairs, Project (597)	Big Cottonwood Creek	Utah	1927*	....	1.5	2
37	Alpine, Project (671)	Alpine Creek	Utah	1927*	....	1.8	2
38	Battle Creek, Project (675)	Battle Creek	Utah	1927*	....	2.4	1
39	Kern Canyon, Project (178)	Kern	Calif.	1921*	....	8.5	1
40	Borel, Project (382)	Kern	Calif.	1925*	....	10.0	..
41	Exchequer, Project (88)	Merced	Calif.	1926*	....	25.0	2
42	Kerckhoff, Project (96)	San Joaquin	Calif.	1920*	1 488	39.0	3
43	Chelan, Project (637)	Chelan	Wash.	1927*	....	48.0	2
44	Lewiston, Project (621)	Clearwater	Idaho	1927*	....	10.0	2
45	Grangeville, Project (204)	South Fork Clearwater	Idaho	1923*	....	0.6	2
46	Wallowa Falls, Project (308)	East Fork Wallowa, Royal Purple Creek	Ore.	1924*	....	0.8	1
47	Glines Canyon, Project (588)	Elwha	Wash.	1927*	....	12.0	2
48	Ariel, Project (935)	Lewis	Wash.	1932*	....	46.0	2
49	Rock Island	Columbia	Wash.	1933	90 000	60.0	4
50	Mystic Lake	West Rosebud	Mont.	1924	47	10.5	2
51	Gorge	Skogit	Wash.	1924	1 200	55.5	3
52	....	....	....	1922	670	70.0	2
53	....	....	....	1925	4 750	81.0	3
54	....	....	....	1921	580	66.7	3
55	....	....	....	1908	1 970	65.0	6
56	....	....	....	1928	50	50.0	2
57	....	....	....	1931	260	60.0	2

* Began operation. (a) Uses Government Dam. (b) Ultimate capacity. (c) Remote control.

TABLE 6.—*Continued*

Head, in feet	Annual Output		Annual average load, in thousands of kilowatts	Pondage, or storage, in thousands of acre-feet	Ratio: Pondage to wheel capacity	Cost, in Thousands of Dollars					
	Years	Average in million kilowatt-hours				Dam	Waterway	Power-house and equipment	Highways, railways, etc.	Land and water rights	Total
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(*a*) Eastern Plants											
14	1933–1936	20.5	2.3	....	....	23	417	1 317	....	....	1 757
370	1932–1936	76	8.7	160	50	1 263	2 197	2 558	124	3 051	9 193
183	1933–1936	264	30.2	1 600	77	11 624		2 954	80	6 110	20 768
114	11 yr	218	25.0	120	10	4 228	305	1 949	166	1 199	7 847
33	1934	8.5	1.0	34	4.4	....	....	....	....	....	2 920
17	1934	1.5	0.2	....	....	....	....	....	....	....	139
139	....	280	32.0	60	3.8	....	....	....	....	....	14 000
32	1921–1936	47	5.4	....	....	....	....	....	....	....	2 200
58	....	90	10.3	29	2.6	($3 500)				1 200	4 700
41	....	15.5	1.8	6.3	2.9	250	53	223	102	95	723
350	....	5	0.6	9.0	21.0	201	215	190	....	256	862
360	....	136‡	15.5	115.0	33.0	3 720	3 150	1 620	1 210	600	10 300
78	....	25§	2.8	....	....	840	110	590	....	600	2 140
89	....	1 300	148.0	72.0	0.9	10 425	1 745	22 400	8 350	6 900	49 820
55	....	880	100.0	69.0	0.7	7 700	....	18 600	....	800	27 100
(*b*) Plants in the Southern and Central States											
860	1933–1936	325	37.0	....	....	....	....	....	....	....	13 800
80	1934–1936	55	6.3	....	....	5 871		4 380	....	389	10 640
18	1936	8.0	0.9	....	....	4	49	420	....	1	474
37	1933–1936	317.0	36.2	....	....	158		7 484	....	14	7 656
14	1933–1936	3.2	0.4	....	....	133		170	....	32	335
13	1934–1936	3.6	0.4	....	....	293	3	165	....	88	549
....	1933–1936	19.5	2.2	....	....	....	....	....	....	....	1 544
46	1933–1936	30.0	3.5	....	....	....	....	....	....	....	3 400
30	1933–1936	43.0	5.0	....	....	13	226	1 231	....	....	1 470
268	1934–1936	2.0	0.2	....	....	10	236	64	....	8	318
63	(13 yr)	266.0	30.3	....	....	($6 669)				3 802	10 471
87	(8 yr)	406.0	46.3	....	....	($9 155)				3 205	12 360
125	(10 yr)	275.0	31.4	1 380	60	($14 824)				3 458	18 282
16	....	9.0	1.0	3 500	1.4	150	....	260	24	177	611
90	....	425.0	48.5	1 200	28	6 800	180	8 000	500	3 000	18 480
(*c*) Western Plants											
....	1936	3.9	0.4	....	....	10	99	57	....	2	168
....	1936	5.1	0.6	....	....	10	113	68	....	....	191
92	....	....	....	....	....	16	24	26	....	2	68
220	1936	11.9	1.4	....	....	46	238	136	....	2	422
650	1936	2.5	0.3	....	....	3	68	67	....	2	140
370	1936	5.6	0.6	....	....	135	104	77	....	2	318
1 920	1936	3.9	0.4	....	....	2	130	65	....	1	198
1 790	1936	2.2	0.3	....	....	1	135	104	....	2	242
260	1927–1931, incl., 1936	44.0	5.0	....	....	147	1 262	534	3	41	1 987
270	....	....	....	....	....	51	1 574	364	3	112	2 104
210	1928–1931, incl.	70.0	8.0	285.0	82	3 904	215	933	8	6 400	11 460
350	1927–1931, incl., 1936	193.0	22.0	4.2	1.3	813	3 292	1 426	62	5	5 598
410	1933–1936, incl.	278.0	31.7	....	....	1 771	2 880	2 209	....	2 403	9 263
36	1933–1936, incl.	18.7	2.1	....	....	1 342	545	789	....	682	3 358
64	1933–1936, incl.	2.7	0.3	....	....	164	24	44	1	....	233
1 170	1934–1936, incl.	3.3	0.4	....	....	13	51	25	....	2	91
160	1928–1929	85.6	9.8	24.0	11	690	361	528	....	207	1 786
170	1936	225.0	25.7	225.0	28	5 045	229	2 080	397	922	8 673
32	1936	379.0	43.2	14.0	0.25	8 387	....	4 545	....	1 623	14 555
1 050	....	60.0	6.8	....	....	185	975	520	....	(120)	1 800
270	12 yr	240.0	27.4	....	....	4 424‖	....	3 557	2 385	53	10 419
454†	....	282.0	32.2	....	....	277	2 917	2 891	797	1 638	8 520
315†	....	393.0	44.8	8.4	1.1	2 073	6 268	2 143	772	355	11 611
1 149†	....	462.0	52.7	1 357.0	800	1 185	10 180	2 796	114	123	14 398
465†	....	536.0	61.2	1 466.0	360	894	4 197	2 188	101	3	7 383
2 561†	....	231.0	26.4	108.7	190	1 391	3 175	1 279	436	226	6 507
1 218†	....	291.0	33.2	155.4	108	689	5 470	1 421	168	31	7 779

† Gross. ‡ Includes lower plants. § Includes lower plants; remote control. ‖ Includes waterways.

TABLE 6.—*Continued*

Plant No.	Name	ANNUAL COSTS, IN THOUSANDS OF DOLLARS					COST, IN DOLLARS PER KILOWATT OF CAPACITY			
		During the years		Operation	Maintenance	Total	Dam		Waterway	
		From:	To:				Cost	Percentage of total	Cost	Percentage of total
	(1)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
	(*a*) EASTERN PLANTS									
1	Green Island, Project (13)	1933	1936	34.0		34.0	4	1	63	24
2	Wallenpaupack, Project (487)	1932	1936	34.0	21.0	55.0	32	14	55	24
3	Saluda, Project (516)	1933	1936	45.0	15.0	60.0	89‖	56‖	....	....
4	Bartlett's Ferry, Project (485)	1936		22.0	10.0	32.0	94	54	7	4
5	Jackson Bluff, Project (682)	1934		21.0	5.0	26.0	....	....	....	....
6	Moss Bluff, Project (177)	1934		1.2	0.4	1.6	....	....	....	....
7	Wyman	....	....	....	....	....	....	....	....	....
8	Weston	....	....	....	....	....	....	....	....	....
9	Gulf Island	....	....	....	....	....	184¶	75¶	....	....
10	....	....	....	....	....	....	83	34	18	8
11	Mollys Falls	....	....	....	....	6.0	40	23	43	25
12	Harriman	....	....	....	....	....	86	36	73	30
13	Sherman	....	....	....	....	....	120	39	16	5
14	Conowingo	....	....	105	100	205	41	21	7	3
15	Safe Harbor	....	....	116	45	161	43	28	....	....
	(*b*) PLANTS IN THE SOUTHERN AND CENTRAL STATES									
16	Waterville, Project (432)	1933	1936	25.0	23.0	48.0	....	....	....	....
17	Piney, Project (309)	1934	1936	35.0		35.0	204‖	55‖	....	....
18	Dam No. 7, Project (539)	1936		1.8	1.3	3.1	2	1	24	10
19	Ohio Falls, Project (289)	1933	1936	53.0	16.0	69.0	2‖	2‖	....	....
20	Shawano, Project (710)	1936		1.8	0.4	2.2	190‖	40‖	....	....
21	Mottville, Project (401)	1936		4.8	0.7	5.5	175	53	2	1
22	Winton, Project (469)	1934	1936	14.0	4.0	18.0	....	....	....	....
23	Blanchard, Project (346)	1934	1936	17.0	5.0	22.0	....	....	....	....
24	Twin City, Project (362)	1933	1936	114.0		114.0	1	1	17	15
25	Big Bend, Project (792)	1936		3.9	7.4	11.3	8	3	196	74
26	....	....	....	....	....	52.0	127¶	64¶	....	....
27	....	....	....	....	....	67.0	92¶	74¶	....	....
28	....	....	....	....	....	73.0	149¶	81¶	....	....
29	Allegan	....	....	....	....	....	62	24	....	....
30	....	....	....	....	....	....	51	37	1	1
	(*c*) WESTERN PLANTS									
31	Lower American Fork, Project (696)	1936		6.4	1.0	7.4	11	6	104	59
32	Upper American Fork, Project (696)	1936		3.9	0.7	4.6	8	5	94	59
33	Hyrum City, Project (946)	....	....	....	....	....	40	24	60	35
34	Logan, Project (486)	1936		9.3	2.1	11.4	23	11	119	56
35	Santaquin, Project (665)	1936		3.2	2.3	5.5	3	2	78	49
36	Stairs, Project (597)	1936		5.8	0.3	6.1	90	42	69	33
37	Alpine, Project (671)	1936		2.9	1.8	4.7	1	1	74	65
38	Battle Creek, Project (675)	1936		2.6	0.9	3.5	1	1	56	55
39	Kern Canyon, Project (178)	1931		6.2	0.5	6.7	17	7	149	64
40	Borel, Project (382)	....		....	....	....	5	3	158	75
41	Exchequer, Project (88)	1931		17.0	....	17.0	156	34	9	2
42	Kerckhoff, Project (96)	1931		27.0	11.0	38.0	21	15	84	58
43	Chelan, Project (637)	1936		24.0	6.0	30.0	37	19	60	31
44	Lewiston, Project (621)	1936		13.0	34.0	47.0	134	40	55	16
45	Grangeville, Project (204)	1936		7.4	1.6	9.0	263	70	38	10
46	Wallowa Falls, Project (308)	1934	1936	7.2	0.2	7.4	16	14	64	56
47	Glines Canyon	1928	1929	5.0	2.0	7.0	57	39	30	20
48	Ariel	1936		36.0	7.0	43.0	110	58	5	3
49	Rock Island	1936		34.0	25.0	59.0	140	58	....	....
50	Mystic Lake	....	....	....	....	....	18	11	93	54
51	Gorge	....	....	33.0	26.0	59.0	80‖	42‖	....	....
52	....	....	....	25.0	9.0	34.0	4	3	42	35
53	....	....	....	42.0	7.0	49.0	26	18	77	54
54	....	....	....	38.0	19.0	57.0	18	8	152	70
55	....	....	....	42.0	20.0	62.0	14	12	64	56
56	....	....	....	21.0	7.0	28.0	28	22	63	48
57	....	....	....	35.0	25.0	60.0	11	9	91	70

¶ Total, except land and water rights.

TABLE 6.—*Continued*

Cost, in Dollars per Kilowatt of Capacity								Annual Cost of Power, in Thousands of Dollars				Cost, in cents per kilowatt-hour of output	Annual capacity factor
Power-House and Equipment		Highways, railways, etc.		Land and water rights		Total		Fixed charges	Operation and maintenance	Water rents	Total		
Cost	Percentage of total	Cost	Percentage of total	Cost	Percentage of total	Cost	Percentage of total						
(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)
(*a*) Eastern Plants													
201	75	....	....	....	....	268	100	176	34	5	215	1.05	0.37
64	28	3	1	76	33	230	100	919	55	....	974	1.28	0.22
23	14	1	1	47	29	160	100	2 077	60	....	2 137	0.81	0.23
43	25	4	2	27	15	175	100	785	32	....	817	0.37	0.55
....	....	....	....	....	....	332	100	292	26	....	318	3.74	0.11
....	....	....	....	....	....	283	100	14	2	....	16	1.03	0.37
....	....	....	....	....	....	195	100	1 400	(60)	....	1 460	0.52	0.45
....	....	....	....	....	....	183	100	220	(20)	....	240	0.51	0.45
....	....	....	....	63	25	247	100	470	(30)	....	500	0.56	0.54
74	31	34	14	32	13	241	100	72	(3)	....	75	0.48	0.59
38	22	....	....	51	30	172	100	86	6	....	92	1.84	0.11
38	16	28	12	14	6	239	100	1 030	(40)	....	1 070	0.79	0.36
84	28	....	....	86	28	306	100	214	(5)	....	219	0.87	0.41
89	45	33	17	27	14	197	100	4 982	205	133	5 320	0.41	0.59
104	69	....	....	5	3	152	100	2 710	161	....	2 871	0.33	0.56
(*b*) Plants in the Southern and Central States													
....	....	....	....	....	....	128	100	1 380	48	....	1 428	0.44	0.34
152	41	....	....	14	4	370	100	1 064	35	....	1 099	1.99	0.22
205	89	....	....	0	0	231	100	47	3	5	55	0.69	0.44
93	98	....	....	0	0	95	100	766	69	105	940	0.30	0.45
243	51	....	....	46	9	479	100	34	2	....	36	1.13	0.52
98	30	....	....	52	16	327	100	55	5	....	60	1.68	0.24
....	....	....	....	....	....	386	100	154	18	....	172	0.88	0.55
....	....	....	....	....	....	283	100	340	22	....	362	1.20	0.29
92	84	....	....	....	....	110	100	147	114	95	356	0.82	0.37
53	20	....	....	7	3	264	100	32	11	....	43	2.15	0.19
....	....	....	....	72	36	199	100	1 047	52	....	1 099	0.41	0.58
....	....	....	....	32	26	124	100	1 236	67	....	1 303	0.32	0.46
....	....	....	....	35	19	184	100	1 828	73	....	1 901	0.69	0.32
108	43	10	4	74	29	254	100	61	(9)	....	70	0.78	0.43
60	43	4	3	23	16	139	100	1 850	(150)	....	2 000	0.47	0.36
(*c*) Western Plants													
60	34	....	....	2	1	177	100	17	7	....	24	0.62	0.47
57	36	....	....	....	....	159	100	19	5	....	24	0.47	0.48
65	38	....	....	5	3	170	100	....	....	....	....	....	....
68	32	....	....	1	1	211	100	42	11	1	54	0.45	0.68
76	48	....	....	2	1	159	100	14	6	....	20	0.80	0.32
52	25	....	....	1	0	212	100	32	6	....	38	0.68	0.43
37	33	....	....	1	1	113	100	20	5	....	25	0.64	0.25
43	43	....	....	1	1	101	100	24	4	....	28	1.27	0.10
63	27	0	0	5	2	234	100	199	7	1	207	0.47	0.59
36	17	0	0	11	5	210	100	....	....	....	....	....	....
37	8	0	0	256	56	458	100	1 146	17	2	1 165	1.66	0.32
37	26	2	1	0	0	144	100	560	38	66	664	0.34	0.56
46	24	....	....	50	26	193	100	926	30	....	956	0.34	0.66
79	24	....	....	68	20	336	100	336	47	....	383	2.05	0.21
70	19	2	1	....	....	373	100	23	9	....	32	1.20	0.50
31	27	....	....	3	3	114	100	9	7	....	16	0.48	0.48
44	30	....	....	17	11	148	100	179	7	....	186	0.22	0.81
45	24	9	5	20	10	189	100	867	43	30	940	0.42	0.56
76	31	....	....	27	11	243	100	1 455	59	....	1 514	0.40	0.72
50	29	....	....	(11)	6	172	100	180	20	....	200	0.34	0.65
64	34	43	23	1	1	188	100	1 042	59	....	1 101	0.46	0.49
41	34	11	9	23	19	121	100	852	34	....	886	0.31	0.46
26	18	10	7	4	3	143	100	1 161	49	....	1 210	0.31	0.55
42	20	2	1	2	1	216	100	1 440	57	130	1 627	0.35	0.79
34	30	2	2	0	0	114	100	738	62	72	872	0.16	0.94
26	20	9	7	4	3	130	100	651	28	105	784	0.34	0.53
24	18	3	2	1	1	130	100	778	60	260	1 098	0.38	0.55

distinguishable from Source (1) in that they do not have project numbers (see Column (2)).

Most of the items in Table 6 need no explanation. As applied to Federal projects, head, in feet (Column (8)), is the gross or static head, as the actual working heads were not available. For non-Federal projects, the head is given as the working, or net, head, except as otherwise stated. Annual output (Table 6, Column (10)) is given as an average value for such years as are available. However, where load is increasing the latest year of record is used. The values in Column (13), the ratio of pondage to wheel capacity, are obtained by dividing the pondage capacity, in second-feet-days, by the flow at wheel capacity, in second-feet.

Costs (Columns (14) to (19), Table 6) are as recorded, except as condensed into the break-down used in Table 6. Undistributed over-head costs, where they occur, have been pro-rated. Under the heading, "Cost, in Dollars per Kilowatt of Capacity" (Columns (25) to (36), Table 6) certain projects include only a small item for the cost of the dam where power is developed at a navigation dam.

Under "Annual Cost of Power" (Column (37)), note that fixed charges are taken at 10 per cent. In certain projects an item of production rental is included to allow for fees paid yearly to the Federal Power Commission. The cost per kilowatt-hour (Column (41)) is the cost of total power output, including both primary and secondary power.

Range of Conditions.—That the information in Table 6 is representative is indicated by the range of conditions that characterize the 57 plants included. Geographically, 15 are in the East, 15 are in the Southern and Central States, and the remainder (27) are in the West. In point of age, the dates of construction, or of beginning operations, are as follows: 1908 (1); 1920 (1); 1921 (3); 1922 (1); 1923 (4); 1924 (6); 1925 (4); 1926 (7); 1927 (11); 1928 (6); 1930 (4); 1931 (4); 1932 (2); 1933 (1); and 1937 (2).

Other characteristics are:

Description	Range
Capacity, in kilowatts	400 to 252 000
Head, in feet	13 to 2 561
Yearly output, in million kilowatt-hours	1.5 to 1 300
Cost of plant, in thousands of dollars	91 to 50 450
Cost, in dollars per kilowatt of capacity[9]	95 to 479
Cost, in cents per kilowatt-hour[10]	0.16 to 3.74
Yearly capacity factor	0.10 to 0.94

The fifty-seven plants have a total capacity of about 2 190 000 kw; they cost about $384 000 000, or an average of about $175 per kw. Their total yearly output is about 9.5 billion kw-hr (or about one-fourth the total public utility water power output of the United States for 1936) at an average cost of 0.44 ct per kw-hr.

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[9] Uses Government dam.

[10] Plant built in 1908.

Effect of Head and Capacity upon Cost.—The cost per kilowatt of capacity is plotted against head in Fig. 24. The size of the plant is approximated by circles of varying diameter, the numbers referring to the plant number in Table 6. In general, the unit cost decreases as the head increases and the larger plants generally show lower unit costs. By grouping plants according to head, as in Table 7, general tendencies are revealed more clearly, as shown by the dotted line drawn through the group points in Fig. 24.

TABLE 7.—ELEMENTS OF PLANT COST WITH REFERENCE TO HEAD (Fifty Plants)

Item	AVERAGE HEAD AND RANGE, IN FEET, AND NUMBER OF PLANTS							
	37 (13–64) 14 Plants		209 (78–370) 24 Plants		568 (410–860) 5 Plants		1 550 (1 050–2 561) 7 Plants	
	Cost, in dollars per kilowatt	Percentage of total cost	Cost, in dollars per kilowatt	Percentage of total cost	Cost, in dollars per kilowatt	Percentage of total cost	Cost, in dollars per kilowatt	Percentage of total cost
Dam	123	44	63	30	14	10	13	9
Waterway	13	5	53	25	59	41	85	61
	136		116		73		98	
Power-house and equipment	97	34	54	26	49	34	36	26
Highways, etc.	5	2	7	3	3	2	2	2
Land and water rights	43	15	34	16	18	13	3	2
Totals	281	100	211	100	143	100	139	100
Range of cost per kilowatt	152–479	....	124–458	....	114–193	....	101–216	....

Four plants in Table 6 are omitted from Table 7, because use is made of a Government dam, and three because the head is not known. Table 7 also shows group costs per horse-power and the percentage of the following five items of cost breakdown:

(1) The cost of the dam varies from $123 per kw for the low head groups to $13 for very high heads and is from about 40% to 10% of the total cost;

(2) The cost of the waterway varies from $13 per kw for low heads to $85 per kw for very high heads and is from about 5% to 60% of the total cost (the cost of the dam plus the waterway remains more nearly constant, varying from $136 per kw for low heads to $98 per kw for very high heads, and is from about 50% to 70% of the total cost);

(3) The cost of the power-house and equipment varies from about $100 to $35 per kw, and is about 30% of the total cost;

(4) In general, highways, railroads, etc., are relatively unimportant; and,

(5) Land and water rights vary from about $40 to $5 per kw, and are about 15% or 20% for the two lower groups, lessening to about 5% at very high heads. In the case of some of the plants in Table 7, the item of water rights is (in part, anyway) covered by yearly rental charges.

It must be kept in mind that the foregoing items are average costs based upon groups of plants as noted. Individual plants will vary greatly from the values given. The averages, however, show the trend of unit cost of different parts of the hydro-plant as the head varies.

COST OF WATER-POWER OUTPUT

Effect of Head, Capacity, and Capacity Factor upon Cost.—In Fig. 25(*a*) hydro cost per kilowatt-hour of output is plotted against head, these data being taken from Table 6. Capacity is shown approximately by the size of circles and capacity factor by radial lines in the circles as noted. The numerals refer to plant numbers in Table 6.

Although, in general, the cost per kilowatt-hour tends to lessen as head increases and to lessen with the size of the plant, the effect of capacity factor is more predominant. Note, for illustration, the group of points between 300-ft and 400-ft head, in Fig. 25(*a*). The upper points have low capacity factors and the lower points, high capacity factors. If all are reduced to, say, a 0.50 capacity factor, the points will be relatively near together. This adjustment has been made for all the points of Fig. 25(*a*) and replotted in Fig. 25(*b*) on the basis of a 0.50 capacity factor for all plants and, as will be noted, the major variations in the plot of Fig. 25(*a*) are thus largely eliminated.

The effect of capacity factor upon cost of output is also shown in Fig. 26, where cost per kilowatt-hour of output is plotted against capacity factor, irrespective of head. This also reveals more clearly the smaller cost of output from the larger plants.

Referring again to Fig. 25(*a*), the range in actual costs per kilowatt-hour is seen to vary as Table 8(*a*). From Fig. 25(*b*) the range, adjusted to a constant capacity factor of 0.50, is as shown in Table 8(*b*). The latter data reveal more clearly the range in costs due to variation in what may be called "site cost factor," or the natural conditions at sites tending toward a greater or less construction cost. The capacity, or use factor, refers to the method of plant operation and, obviously, the differences in cost in Table 8(*a*) and Table 8(*b*) are due solely to different assumed methods of operation, and consequent difference in capacity factors.

TABLE 8.—COSTS OF WATER-POWER OUTPUT, IN CENTS PER KILOWATT-HOUR

Head, in feet	(*a*) RANGE OF ACTUAL COSTS		(*b*) RANGE OF COSTS, ADJUSTED TO A CONSTANT CAPACITY FACTOR OF 0.50	
	From:	To:	From:	To:
Less than 100	0.3	3.7	0.2	1.2
100 to 500	0.16	2.2	0.3	1.1
More than 500	0.35	1.3	0.25	0.55

PRIMARY COST OF WATER POWER

Frequently, plant cost per kilowatt, as well as cost per kilowatt-hour of hydro-electric energy, is based upon the power corresponding to the 24-hr primary flow. This is logical in the case of an isolated plant, but misleading

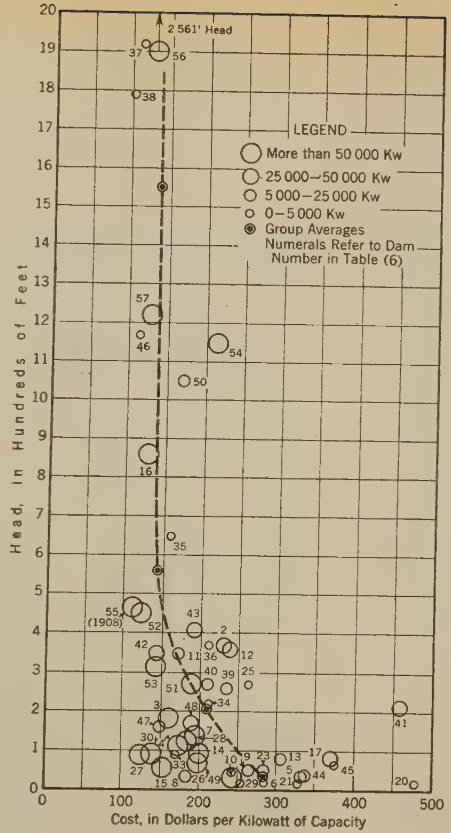

FIG. 24.—PLANT COST OF WATER POWER AS AFFECTED BY HEAD AND CAPACITY

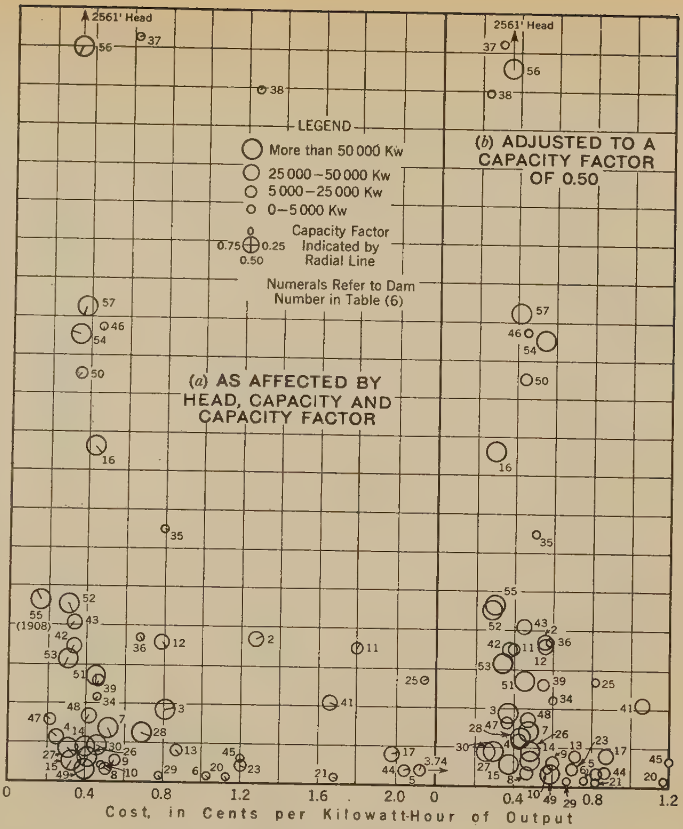

FIG. 25.—COST OF HYDRO-ELECTRIC ENERGY

where the plant is tied in with a load system. In the latter case it is better to credit the hydro-plant with its system peak capacity saving, as has been suggested, and compute the net cost of hydro-electric energy for comparison with steam power fuel saving and attendance.

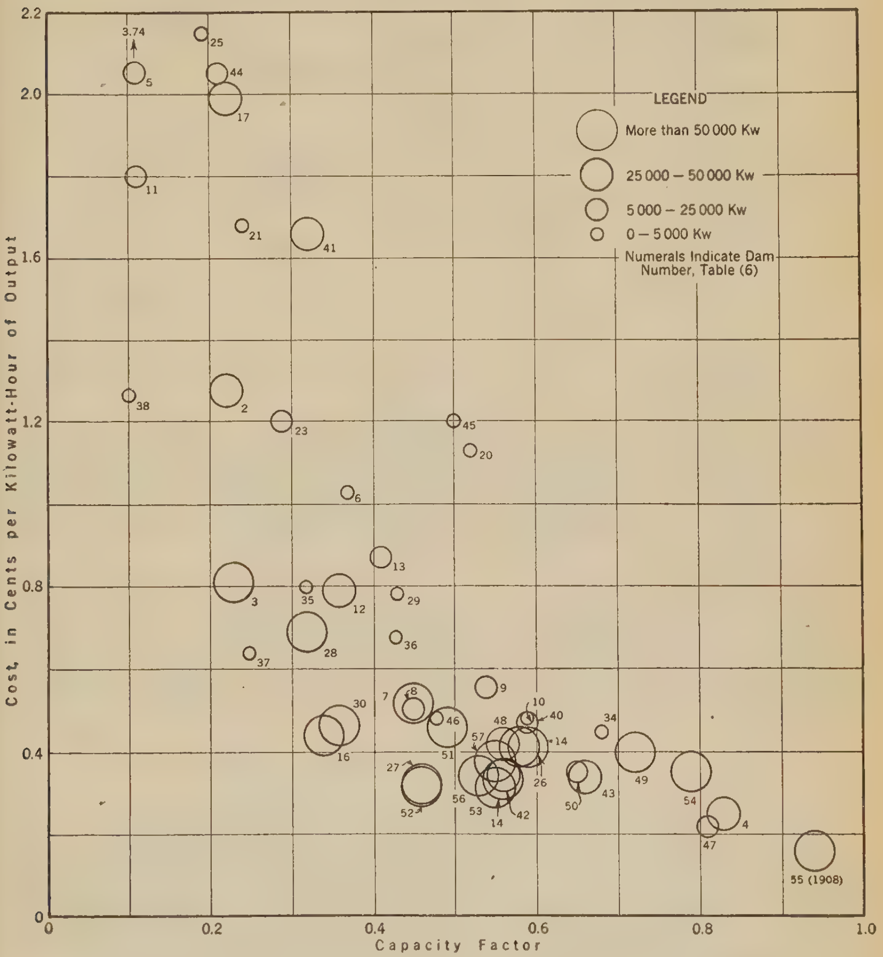

FIG. 26.—COST OF POWER AS AFFECTED BY CAPACITY AND CAPACITY FACTOR

Thus, considering the Conowingo Plant (No. 14, Table 6), in Maryland, on the Susquehanna River (The Philadelphia Electric Company System, September, 1936), to which the following additional data apply:

Average daily low-water, 24-hr power output, in kilowatts..................................	11 000
Peak capacity carried by hydro-electric plant, in kilowatts..................................	210 000
Ratio, $\frac{\text{Peak capacity}}{\text{Plant capacity}}$..........................	0.83

By the method suggested, the energy cost per kilowatt-hour of water power would be determined as follows:

Cost of plant		$49 820 000
Credit for saving in steam capacity, 210 000 kw @ $100		21 000 000
Yearly fixed charges on plant @ 10%	$4 982 000	
Credit for saving in fixed charges on steam capacity, $21 000 000 @ 12.5%	2 625 000	
Net fixed charges on plant	2 357 000	
Hydro-plant operation and maintenance	338 000	
Total yearly net cost		$2 695 000

or $\frac{\$2\,695\,000}{1\,300\,000\,000 \text{ kw-hr}} = 0.21$ ct per kw-hr, which compares with the cost of fuel plus attendance of equivalent steam power.

In the foregoing analysis no allowance has been made for the higher degree of dependability of hydro-power as compared to steam such that the hydro really replaces somewhat more than an equivalent kilowatt of dependable steam capacity. This is difficult to evaluate exactly but is perhaps of the order of 10 per cent.

Assuming an added 10% saving in steam capacity due to the greater reliability of hydro-power would, in the foregoing example, result in a steam capacity saving of 231 000 kw, and a total saving of $23 000 000, and reduce the energy cost of the water power to about 0.19 ct per kw-hr instead of 0.21 ct per kw-hr.

ADVANTAGES OF WATER-POWER IN A POWER SYSTEM

Water power used in a power system has three advantages:

(1) Hydro-plant units, large or small, can be placed in operation quickly (in a few minutes), which is important and valuable under sudden large load increases, and particularly in emergencies when some large power unit suddenly goes off the line due to accident or break-down. This quick placing in operation is not possible with steam plants.

(2) Additional, or increment, capacity for the hydro-plant with reference to its system peak use is relatively low in cost, as commonly this is limited chiefly to equipment, whereas the increment cost of a steam plant must include the entire plant. Ordinarily, increment costs for water power will be from $40 to $70 per kw of capacity, whereas for steam this will be $80 to $100 per kw.

(3) Hydro-plant units are generally more reliable than steam units, records showing, for modern hydro-plant units, that the non-scheduled outage "when in demand" is so small as to be negligible.

PUMPED STORAGE, PEAK LOAD, HYDRO-PLANTS

This special use of the hydro-plant is fairly common in Europe, but thus far in the United States only two such plants have been constructed: The Rocky River Plant, in Connecticut, and the Lake Keuka Plant, in New York.[11] There are further opportunities for the use of this method of handling peak loads and the development of a pump-turbine unit with two-speed motor-generator unit, as noted by Mr. Frank H. Rogers,[12] adds materially to their practicability and economic feasibility.

OPERATING AND MAINTENANCE COSTS, HYDRO-ELECTRIC PLANTS

The yearly cost of operation and ordinary maintenance based upon data from forty-five plants in Table 6 is shown on Fig. 27. As will be noted, the median curve shows, for a small plant (1 000 kw), costs varying between about $4.00 and $9.50 per kw yearly and averaging about $7.00 per kw. For large plants (more than 100 000 kw), the cost varies between $0.50 to $0.90 and averages about $0.75. Intermediate costs may be noted by reference to Fig. 27. Variations from the median curve for individual plants are considerable.

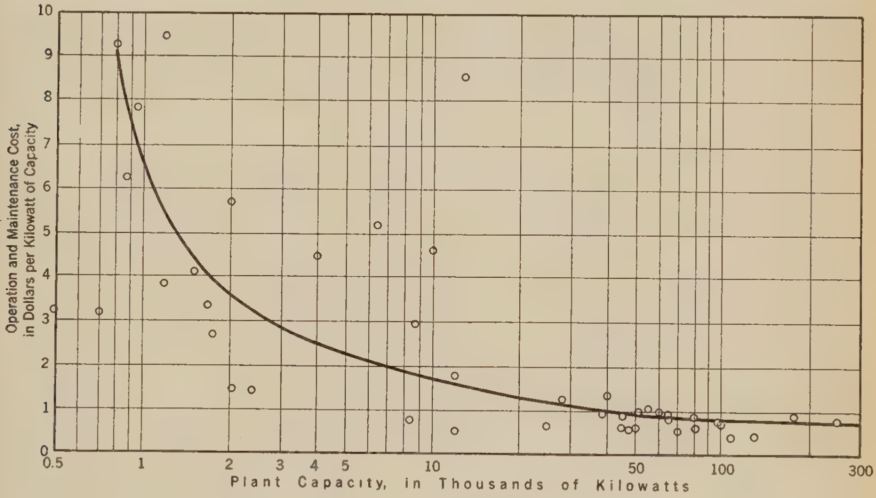

FIG. 27.—ANNUAL HYDRO-PLANT OPERATION AND MAINTENANCE COSTS, IN DOLLARS PER KILOWATT OF CAPACITY

Cost of operation and maintenance is also shown on Fig. 28, per kilowatt-hour, plotted against total yearly output. It varies between about 0.20 ct per kw-hr for small plants to about 0.02 ct per kw-hr for large plants, with considerable variation from the median curve. Harmonizing the data from Figs. 27 and 28, data of approximate average operating and maintenance costs have been determined in Table 9. These data are given both as total yearly

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[11] "Improvements in the Utilization of Energy," by Joel D. Justin, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1937, pp. 1912–1924.

[12] "Hydro-Generation of Energy," by Frank H. Rogers, *Proceedings*, Am Soc. C. E., December, 1937, pp. 1893–1911.

TABLE 9.—APPROXIMATE AVERAGE OPERATION AND MAINTENANCE COSTS OF HYDRO-ELECTRIC PLANTS

Plant capacity, in kilowatts	Cost of Operation and Maintenance		Plant capacity, in kilowatts	Cost of Operation and Maintenance	
	Total yearly cost, in dollars	Cost, in cents per kilowatt-hour (0.5 capacity factor)		Total yearly cost, in dollars	Cost, in cents per kilowatt-hour (0.5 capacity factor)
1 000	6 000	0.140	50 000	45 000	0.020
5 000	12 000	0.055	100 000	80 000	0.018
10 000	18 000	0.041	150 000	120 000	0.018
20 000	25 000	0.029	200 000	160 000	0.018
30 000	32 000	0.024	300 000	220 000	0.017
40 000	40 000	0.023	....	....	....

costs and, as costs based upon a 0.5 capacity factor, the cost per kilowatt-hour of output. Except for small plants, as has been previously noted, operation and maintenance costs are a relatively small proportion of water power costs.

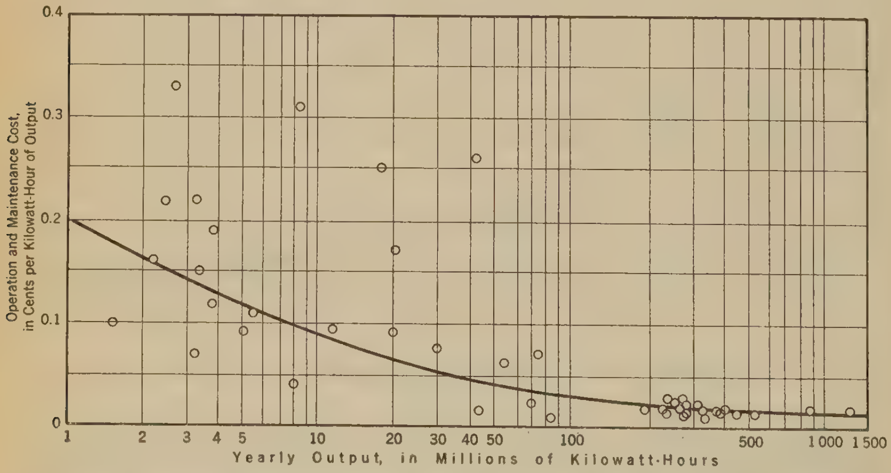

FIG. 28.—OPERATION AND MAINTENANCE COSTS, IN CENTS PER KILOWATT-HOUR OF OUTPUT

FEDERAL WATER POWER PROJECTS

Several large Federal projects for the development of water power have been under way during the past few years. A brief review of these projects in respect to cost appears pertinent in this paper. They include chiefly Boulder Dam, Bonneville, and Grand Coulee in the West, and the projects of the Tennessee Valley Authority in the South.

Boulder Dam (Colorado River).[13]—This is a combined irrigation, power, water supply, and flood-control project, costing about $79 000 000 for the dam and reservoir. The ultimate project contemplates about 1 300 000 kw of capacity under a head of about 530 ft, at a cost of about $43 000 000 for the plant. The total reservoir capacity is about 30 500 000 acre-ft, of which 9 500 000 acre-ft, or about 31%, are reserved for flood control. If the remaining 69% of the cost of dam and reservoir (or about $54 000 000), is allocated to power use, this makes the total cost for power purposes about $97 000 000, or $75 per kw of capacity. The yearly output of firm power is estimated at about 4 billion kw-hr. Fixed charges at 6.5% would be $6 300 000 per yr, operation about $700 000, or $7 000 000 total yearly cost, at about 0.17 ct per kw-hr (without transmission).

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[13] "Government Hydro versus Private Steam Power," H. R. Doc. No. 52, 75th Cong., 2d Session, 1937; see also, "Undertakings Without Precedent," *Engineering-News Record*, November 29, 1934, pp. 675–702.

Bonneville (*Lower Columbia River*).[13, 14]—This is essentially a water power project, but includes provision for irrigation. The ultimate project contemplates 432 000 kw of capacity under a head of about 54 ft, at a total cost of approximately $65 000 000, or $150 per kw of capacity. The fixed charges (at 6.5%) will be about $4 200 000, the operation and the maintenance cost about $300 000, or a total yearly cost of about $4 500 000, which is about 0.22 ct per kw-hr for an output of about 2 billion kw-hr per yr (without transmission).

Grand Coulee (*Upper Columbia River*).[13, 15]—This is a water-power project, although there is a supplemental irrigation project. The complete project is estimated to cost about $196 000 000, with about 1 900 000 kw of capacity, or slightly more than $103 per kw under a head of about 300 ft. The ultimate yearly output will be approximately 9 billion kw-hr; fixed charges (at 6.5%) $12 700 000; operation and maintenance, $1 000 000; or a total yearly sum of nearly $14 000 000, which is about 0.15 ct per kw-hr (without transmission).

Tennessee Valley Authority (*Tennessee River*).[16]—These projects include a comprehensive development of the Tennessee River by relatively high dams for the combined objective of creating power, flood control, and navigation.

Eleven major projects are planned which, in general order, going up stream are: Gilbertsville, Pickwick, Wilson, Wheeler, Guntersville, Chickamauga, Watts Bar, Coulter Shoals, Norris, Fontana, and Fowler Bend. Ultimate power installation includes about 1 800 000 kw of capacity and nearly 6 billion kw-hr yearly of primary power.

Power development alone (not including dams, reservoirs, etc.) will cost about $135 000 000, or $75 per kw of capacity. The total cost of the eleven projects, including power development cost, is estimated at about $520 000 000, or if all is charged to power, about $290 per kw of capacity.

For the ultimate development the fixed charges (at 6.5%), operation and maintenance, and total power costs would be as shown in Table 10. The cost of this water power will thus depend upon the proportion allocated to flood and navigation benefits. If dams and reservoirs are entirely allocated to these two items, the water power will cost about 0.20 ct per kw-hr; if they are entirely allocated to power, the cost will be about 0.60 ct per kw-hr.

The cost of water power from the four Federal projects is summarized in Table 11(*a*). In comparing these costs of power with those from the plants

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[14] "A Study of Wholesale Cost of Bonneville Power," Oregon State Planning Board, 1934.

[15] "Grand Coulee High Dam," *Engineering-News Record*, December 23, 1937, pp. 1021–1024.

[16] Annual Rept. T.V.A., for year ending June 30, 1936; also, First Deficiency Appropriation Bill, T.V.A., Committee Rept., 74th Cong., 2d Session, 1936, Testimony of David N. Lilienthal and Arthur E. Morgan, M. Am. Soc. C. E., pp. 113–292; Second Deficiency Appropriation Bill, T.V.A., Committee Rept., 75th Cong., 1st Session, 1937, Testimony of Arthur E. Morgan, pp. 339–451.

TABLE 10.—COST OF PROJECTS BY TENNESSEE VALLEY AUTHORITY

Item	Basis	
	Power development cost alone ($135 000 000)	Total cost ($520 000 000)
Fixed charges, in dollars	8 800 000	33 800 000
Operation and maintenance cost (including reservoir operation), in dollars	2 200 000	2 200 000
Total yearly	11 000 000	36 000 000
Cost, in cents, per kilowatt-hour (6 billion kilowatt-hours)	0.183	0.60

in Table 6, it must be kept in mind that fixed charges for the Federal water power projects were taken at 6.5% whereas the others are on a 10% basis. In Columns (6) and (7) of Table 11(*a*) a comparison of costs with these two bases is given.

TABLE 11.—COST SUMMARY

Item No.	Plant	Head, in feet	Ultimate plant capacity, in thousands of kilowatts	Unit capacity, in thousands of kilowatts	Annual output of ultimate project, in billions of kilowatt-hours	COST, IN CENTS PER KILOWATT-HOUR, FOR FIXED CHARGES OF:		Allocation; cost of dam and reservoir
						6.5%	10%	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	(*a*) FEDERAL WATER-POWER PROJECTS							
1	Boulder Dam	530	1 300	80	4.00	0.17	0.26	31% to flood control
2	Bonneville	54	432	43	2.00	0.22	0.34	To power
3	Grand Coulee	300	1 900	105	9.00	0.15	0.23	To power
4	Tennessee Valley Authority	40 to 400	1 800	23 to 45	6.00	0.18 to 0.60	0.26 to 0.90	To navigation and flood control To power
	Totals	....	5 432	....	21.00	....	....	....
	(*b*) LARGE WATER-POWER PLANTS GIVEN IN TABLE 6							
5	Conowingo	89	252	36	1.30	....	0.41	To power
6	Safe Harbor	55	178	30	0.88	....	0.33	To power
7	No. 52	454	70	35	0.28	....	0.31	To power
8	No. 53	315	81	27	0.39	....	0.31	To power
9	Waterville	860	108	36	0.33	....	0.44	To power
	Total, or average	....	....	....	3.18	....	0.36	....

The Federal plants are also quite large. It is to be noted that their total output would be about 50% of the entire public utility water-power use in the United States for 1936. Compared on the basis of 10% fixed charges, the cost per kilowatt-hour is only little less for these large Federal projects than for some of the large plants listed in Table 6, such as Conowingo, Safe Harbor, Waterville, and Nos. 52 and 53 in the West (see Table 11(*b*)). In the case of the Federal projects other than Boulder Dam the use or marketing of the power must be deferred for an indefinite time, so that an added item of accrued carrying charges should be included. This has not been allowed for in computing Table 11(*a*).

UNDEVELOPED HYDRO-ELECTRIC POWER

The total developed water power in the United States as of January 1, 1937, was about 17 million hp, or approximately 12 million kw.

The total potential water power (including developed power) available 90% of the time was about 43 million hp, or about 30 million kw. The ten leading States in this respect are:

State	Horse-power, potential water power (available 90% of time)
Washington	8 800 000
California	4 600 000
Oregon	4 400 000
New York	4 300 000
Arizona	3 700 000
Idaho	2 700 000
Utah	1 500 000
Montana	1 300 000
Tennessee	1 300 000
Alabama	900 000
Total, ten States	33 500 000

Thus, about three-fourths of the potential water power is found in these ten States (all but three of which are in the West).

It must be kept in mind, however, that the plant capacity would greatly exceed the minimum, or 90%, power, and probably 80 million hp or more of capacity would be required to utilize all the available water power in the United States. Of the total output of public utility plants in the United States in 1936, of about 114 billion kw-hr, about 41 billion kw-hr, or 36%, was hydro-electric power.[17] By general districts this was distributed as follows:

District	Billion kilowatt-hours	Percentage of total
Atlantic Coast States	17.4	43
Central States	7.8	19
Mountain States	3.3	8
Pacific States	12.5	30
Totals	41.0	100

Although many of the better sites for the development of water power have been utilized, there are still many sites that may be commercially utilized as the market for power is extended. Even in the Atlantic Coast District, where the present use of water power is greatest, there are still potential opportunities for comprehensive river developments for power, including storage, as well as for redevelopment in some cases.

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[17] "Production of Electric Power for 1936," Rept. of Federal Power Comm.

ACKNOWLEDGMENTS

This paper has been made possible through the generous contribution of cost and other supporting data for many water power plants.

Acknowledgments are made to the Federal Power Commission, through Thomas R. Tate, Chief of the Division of Power Resources and Requirements, U. S. Department of the Interior, for access to, and use of, its project files and for its kind assistance. Many engineers and executives of the public utility companies have generously responded to the request for data relating to hydroelectric projects. Mr. Arnold H. Engborg, of the writer's Office Staff, has also assisted in the preparation of this paper.

COMBINED ENERGY GENERATION

BY EZRA B. WHITMAN,[18] M. AM. SOC. C. E.

No attempt will be made in this paper to consider anything but the cost of combined steam and hydro-electric generation although the same principles must be followed in determining the cost of any other combination. The cost of combined energy generation will vary with every combination, and it is not possible to determine such cost for one combination and apply it to another, although the method of determining such cost may be the same in each case. This paper, therefore, will discuss principally the methods that must be followed in determining the cost of combined energy generation and the variable factors that affect such cost.

THE DEVELOPMENT OF COMBINED ENERGY GENERATION

The first plants for the generation of electricity by water power were constructed about 1890 and the concept of the hydro-electric plant in energy generation has undergone many changes since that time. These first plants were usually the sole source of power. This being the case, they were constructed with a capacity of generating equipment for utilizing only the minimum flow of the stream. This was the firm power that could be depended upon throughout the year. They were built in those sections of the United States where fuel costs were high and where water power was available. A number of them were built in California before the discovery of oil and gas in this section.

In order to increase the amount of firm power, storage was resorted to and large storage reservoirs were built in conjunction with the hydro-plants so that the water could be drawn upon at times of minimum flow.

At other plants, where the cost of storage was prohibitive, steam plants were built in conjunction with the hydro-plants. As the larger systems, covering vast sections of the country were developed, the hydro-plants became generating units in these systems in which both steam and water were used as sources of power.

TYPES OF PLANTS WITH COMBINED ENERGY

At present (1938) there are some plants which are mostly hydro-electric, with steam plants to be used during times of minimum flow. Such plants are the simplest ones for which to determine the cost of the combined energy generation. Many of them began to furnish power entirely from water, but as the power requirements grew, they could not supply the demand at the time of low flow, and steam plants were installed to meet these demands at such times.

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[18] Cons. Engr. (Whitman, Requardt & Smith), Baltimore, Md.

The opposite combination is that in which steam plants were first built to furnish the required power. The cost of furnishing peak power by steam is quite high, due to the fact that the boilers must be kept hot, and steam-generating units must be prepared to pick up the peak load during the day as the load increases; and when the load diminishes toward nightfall, the steam units are put out of service, but kept in condition so that they can be placed in operation again as the load increases. If there is a water-power site available that can be developed within a reasonable cost, it is much more economical to carry the peak loads by hydro-electric generation than it is with steam generation, and, therefore, a number of plants have been developed in which water is combined with steam for the purposes of carrying the peak loads.

The third type of combination is found in those plants in which large hydro-electric installations can be made and operated in connection with the steam plants so that the water-power plants, at times of high flow, carry the base load and the steam plants carry the peaks; and, during periods of low flow, the water-power plants are used to carry the peak while the steam generation carries the base load.

Firm Capacity of Water-Power Plants

In the early days of hydro-electric development, the firm capacity of a water-power plant was considered to be only the power that could be generated by the minimum flow of the stream. If the generating capacity of the plant was greater than could be operated at times of minimum flow, this capacity was looked upon as secondary power. Its sale was quite limited, and it could be used only by such industries as could afford to close down during the times of low flow.

With the combination of steam and hydro-electric power that has developed in recent years, the firm capacity of a water-power plant is looked upon in an entirely different sense. It is possible now to use practically all the power that can be developed in a water-power plant in a large system, so that this distinction between firm and secondary power no longer exists. At present, what might be termed the firm capacity of a hydro-plant can be defined as that part of the total installed capacity that is capable of doing the same work on that part of the load curve of the system that a steam plant could perform. This firm capacity varies with the shape of the load curve of the system that is being supplied by both the steam and the water. It does not necessarily come at the time of minimum flow of the stream, which might come in August or September when the load on the system is not the peak. Such a peak usually occurs during December, and in order to determine this firm capacity, the minimum stream flow for that month should be used as a basis for determining the firm capacity of the plant.

Factors to Be Considered in Determining the Cost of Combined Energy Generation

The cost of steam generation and the cost of hydro-electric generation have been covered by two other papers in this Symposium but it will be necessary to review to some extent the principles of determining such cost in order to show how the cost of combined energy generation must be determined. The first factor to be determined is the construction or capital cost of both the steam plants and the hydro-plants used in the combination.

There is a wide variation in the cost of modern steam plants. In the 1937 report of the Power Authority of the State of New York, the costs of modern steam-generating stations are shown to vary from $62 per kw to $151. On the other hand the cost of hydro-plants will vary from about $125 per kw of installed capacity to $350, and more. As this first cost of both the steam and the hydro-plants is a large element in determining the cost of the combined energy generation, it can readily be seen that it is impossible to ascertain the cost of combined energy in one system and use that cost as a measure of the cost of the combined energy in another system.

These wide variations of first costs in both steam and hydro-plants may be thoroughly justified by the conditions under which the different plants are built. Steam plants built along the Ohio River, such as those serving Cincinnati, Ohio, and adjacent areas, using the river water for condensing purposes, will be much more expensive than a plant constructed in Baltimore, Md., which uses the tidal waters for condensing purposes. At times of flood the Ohio River rises to a height of 70 ft above the normal river stage, and the foundation expense and the cost of getting the condensing water for such a plant is many times what it would be in Baltimore where the tidal range is ordinarily about 13 in., with a maximum of approximately 5 ft.

With hydro-plants, although the cost of the power-house machinery is comparatively constant, the cost of the dam construction and lands will depend upon the topography of the particular site. Then, there is the question of rebuilding roads that are flooded and, in a number of cases, relocating railroads, telegraph and telephone lines, and the flooding of villages that must be purchased.

It is necessary to determine the construction cost of both the steam plants and the hydro-plants used in the production of combined energy, in order that the fixed charges (which are a large part of the cost of energy generation in either type of plant) may be determined. These fixed charges are: (1) The cost of money; (2) taxes and insurance; and (3) depreciation and obsolescence.

The cost of money with a private company depends very largely upon the financial condition of that company. Large companies that have been soundly administered can secure their money at a much cheaper cost than is possible for the smaller companies and particularly for a new untried project. Although it is true that the large private companies can sell their bonds at a much lower rate of interest at present than was possible before the depression, on the average, the private companies have raised only about one-half their money by the issuance of bonds. An investigation of twenty-seven large utility companies shows that 51.3% of the total capitalization of these companies was the funded debt, and the other 48.7% of the capitalization was raised by the sale of either preferred or common stock. Before the depression, it was not at all unusual for companies on the average to pay between 7% and 8% for the money used for capital purposes. At present (1938), it is doubtful whether private companies could raise both senior and junior securities at a cost of much less than 7 per cent.

Taxes and insurance vary in different localities, but on the average for steam plants about 2% is usual. Hydro-plants are usually built in the country districts and have lower taxes, which will vary from 0.5% to 1.5%, with an average of about 1 per cent. In some States special taxes are placed on private utilities developing water power and, therefore, the taxes are much higher.

Depreciation and obsolescence are much heavier on steam plants than on hydro-plants. The latter have reached a point at which the machinery is already capable of operating at an efficiency of more than 90 per cent. With steam plants the development of the art has been very rapid since the beginning of the Twentieth Century. The thermal efficiency of steam engines has increased from about 18% to about 36 per cent. Taking into account both physical depreciation and obsolescence of steam plants, it is usual to allow approximately 2.5% to 4% for this item, with an average of about 3.5 per cent. With regard to the hydro-plants, there is little or no depreciation on lands and heavy structures such as dams, the greatest depreciation occurring in connection with the hydraulic and generating equipment. For hydro-plants this item is usually estimated to vary between 0.5% to 1.5%, with an average of approximately 1 per cent.

It is usual, therefore, to compute fixed charges at the present time for steam plants as approximately 12.5%, although these charges have been computed as low as 11% and as high as 15 per cent. With regard to the hydro-plants, at present the average fixed charges would be about 9% although they have been as low as 7% and as high as 11 per cent.

Operating and Maintenance Costs

In order to determine the cost of combined energy generation, not only must the construction cost be secured, but the operating and maintenance costs of both the steam and the hydro-plants must be determined. Again, these costs are widely variable. With steam plants, the largest single item is the cost of coal, which varies from approximately $1.75 at the mouth-of-mine plants to $5.00 or more in regions situated at long distances from the coal fields. The writer will not attempt to describe the many elements of the cost that enter into the operating and maintenance costs of steam plants. One of the controlling elements is the load factor and the shape of the load curve. Load factors on different plants vary from 35% to 70% which causes a wide variation in operating costs even if the two plants are identical, and both of them are able to secure coal at the same price.

Transmission Costs

Another element that enters into the cost of power delivered at the load center is the cost of transmission. As a rule, the steam plants can be built in the load centers, although this is not always the case. With hydro-plants, the rule is the opposite, most of them being built at a distance from the load center; and these transmission costs, where they occur, must be computed against either the steam or the hydro-plant in determining the cost of combined energy generation at the load center.

INCREMENT COSTS OF HYDRO-ELECTRIC PLANTS

It is not unusual in building hydro-plants to make provision for the future installation of additional units. As the load on the system grows, it becomes possible to utilize more of the flow of the river advantageously than might be possible at the time that the plant was originally constructed. The cost per kilowatt of installing such units is only a fraction of the cost of the original installation, taking into account the cost of lands, dam, and other elements over and above the actual power-house construction, with the water-wheels, water turbines, and generating and transforming equipment.

THE PROBLEM OF DETERMINING THE COST OF COMBINED ENERGY GENERATION

The cost of steam generation is the yardstick for measuring the value of hydro-electric generation. The studies that are made to determine the cost of combined energy generation are practically the same as would be made by a company that was faced with the problem of determining whether it would be better to develop a hydro-project or add additional steam capacity to its generating equipment.

The question that must be answered is whether the proposed plant will reduce the cost of power? If this can be answered in the affirmative, the plant will be built; otherwise, additional steam equipment will be added. In many modern power systems, in which both steam and water power are used as sources of power generation, the two sources are not competitive but are really complementary sources of power supply. It has frequently been found that, with the combined sources of energy generation, the power can be delivered to the customer more cheaply than would be the case had the company depended upon either steam alone or water power alone.

This may be true, where the cost of the water power is considerably more than the cost of that part of the power which is generated by steam. There have been occasions in which it was very difficult to convince public service commissions that it was in the interests of the consumers to use hydro-generated power at a considerably higher cost per kilowatt-hour than the steam power being generated in the same system. Of course, the economic use of water power, even if it costs more than the steam, is justified by the fact that it will replace that part of the power in the load curve which comes on the peaks and which it would be more costly to generate by steam. Many large steam systems have modern base-load plants that produce electricity at a low cost. The older plants, which are more expensive to operate and cost more for the production of a kilowatt-hour of power, are used only for peak and emergency purposes. The actual power produced in such old steam plants is small and may be only a small percentage (or none) of the total annual production; yet their capacity in the system is required for peak and emergency service and if the company did not have such plants, it would be necessary to build them, or new plants of equal capacity. Under these circumstances, it is much more economical to hold these old plants ready for service than to build complete new equipment for this use.

In some of the large systems, such as the Philadelphia Electric Company in 1925, there were eight steam plants and in 1923 the smallest and least economical of the plants was called upon to produce only 39 000 kw-hr out of a total of nearly 2 000 000 000 kw-hr. In a conversation with the president of one of the large Eastern power companies, for which there had been installed in about 1918 or 1919 several 20 000-kw steam-generating units, the writer was told that the company had not operated these units for several years and hoped that it never would. Nevertheless the sources of supply were such that had there been an interruption to the one of the larger sources, it would have been necessary to have started these units, which will be held in the system as a reserve for many years in the future.

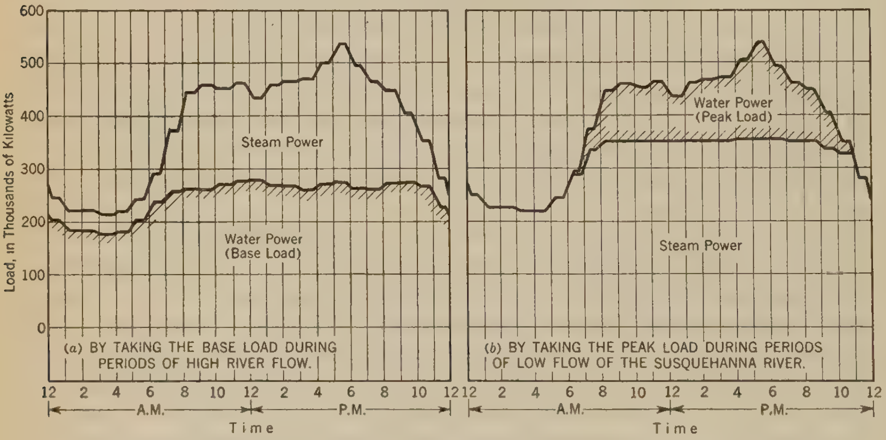

FIG. 29.—COMBINED 24-HOUR LOAD CURVE OF BALTIMORE, MD., AND WASHINGTON, D. C., SHOWING HOW SYSTEMS ARE SERVED BY WATER POWER FROM HOLTWOOD AND SAFE HARBOR PLANTS OF PENNSYLVANIA WATER AND POWER COMPANY

Fig. 29 illustrates how the modern water-power and steam-power plants are used in a complementary manner. Fig. 29(*a*) shows how the water power would be used in a period of high flow of the river to carry the base load and Fig. 29(*b*) shows how the water power is used at a period of low flow to carry the peak loads. This "hydro" is supplied from the Holtwood and Safe Harbor Plants, owned by the Pennsylvania Water and Power Company, and the Safe Harbor Water Power Corporation, respectively. The Holtwood plant was built before 1913 and the Safe Harbor plant about 1932. When the second plant was completed, Baltimore was not inter-connected with Washington, D. C. Within the past few years, however, this inter-connection has been made so that these two plants serve both Baltimore and Washington. With this increased load, the firm capacity of the combined hydro-electric plants has been considerably increased and thereby its value to the two systems is greater than it was when Baltimore alone was served.

In order to demonstrate the methods of arriving at the cost of combined energy generation, the writer presents very briefly, herewith, the method used in determining the value of the Conowingo Plant to the Philadelphia (Pa.) Electric System at the time when this project was being considered by the Maryland and the Pennsylvania Public Service Commissions.

THE ECONOMIC VALUE OF THE CONOWINGO WATER POWER IN THE PHILADELPHIA ELECTRIC SYSTEM

In 1925, application was made to the Maryland Public Service Commission and the Pennsylvania Public Service Commission for approval of the building of the Conowingo Power Plant on the Susquehanna River. The plant proposed contemplated the installation of six 50 000-hp turbines and six 40 000 000-kw generating units. This site had first been considered as a source of power more than a generation ago with Baltimore as the ultimate market for such power. Although Baltimore could have absorbed the power generated by the low flows of the river, it was not possible for Baltimore to absorb the power that could be generated economically at this site during the four months of the year when the stream flow would make it possible to operate the contemplated plant at full capacity. Philadelphia, however, with a much larger market for the power, offered the opportunity for the economic development of this site. Even with that city, it was estimated that it would not be until 1930 that the full economic value of this plant could be realized with the installation of six 40 000-kw turbines.

The study included the possibility of installing four more 40 000-kw units. The stream flow would provide for full operation of these ten units for approximately 20% of the time.

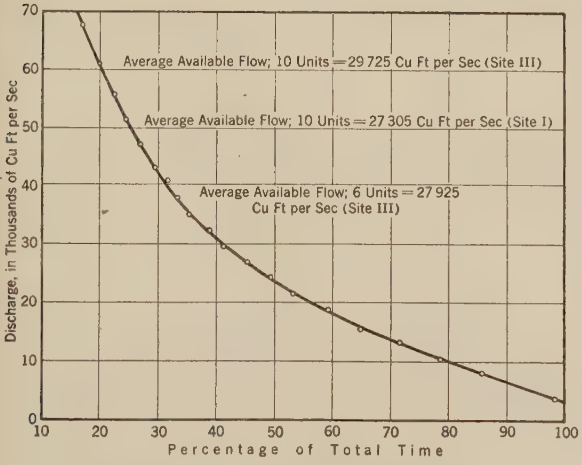

FIG. 30.—CONOWINGO DEVELOPMENT, DURATION OF FLOW CURVE, SUSQUEHANNA RIVER AT SITE III, 1891–1922

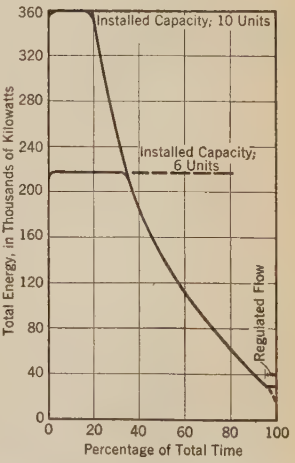

FIG. 31.—POWER DURATION CURVE, PROPOSED HYDRO-ELECTRIC DEVELOPMENT, SUSQUEHANNA RIVER, NEAR CONOWINGO, MD.

Stream-flow records were available from 1891 to 1923 and the stream duration curve shown in Fig. 30 was prepared from the flow records. A power duration curve from the data computed on an 85% over-all efficiency of the water-power plant to and including the high tension bus-bar, was also prepared (see Fig. 31).

The total rated capacity of existing steam plants in Philadelphia at the time of the hearing was 476 480 kw and the load factor on the Philadelphia Electric System in 1923 was 43.8 per cent. In 1924, it was 44.8 per cent. It was estimated that by 1930, the peak, maximum, co-incident load for the steam and water power plants would be 600 000 kw and that the steam plants could carry 405 000 kw and the water-power plants would carry 195 000 kw of this peak.

In 1930, it was also estimated that the average or typical flow-year would generate, at the hydro-electric plant, 1 278 000 000 kw-hr and that 90% of this generated power could be delivered to the system in Philadelphia, or 1 150 000 000 kw-hr. For the same year it was estimated that the steam plants would generate 1 080 000 000 kw-hr. In other words, the hydro-electric plants would produce 51.5% of the total power required and the steam plants would produce 48.5 per cent.

At that time eight steam plants were being used in Philadelphia in the generation of electricity and the maximum demand on each of these stations and the kilowatt-hours generated were shown for 1923 and 1924. Typical, daily, load curves were chosen to show the approximate variations during the year, arriving at a typical curve for each month. Stream-flow tabulations were made on a monthly basis of the average flow within the plant capacities for six and ten units, and the power available from these flows was fitted to the load curves so as to obtain the most economical adjustments between the hydro-electric and the steam plants.

Computations were then made assuming that new steam-generating equipment was built instead of the Conowingo water power. These costs were divided as follows:

Fuel	$ 9 722 000
Labor	1 394 100
Supplies	139 880
Maintenance	1 410 520
Total	$12 666 500

This cost, of course, does not include any fixed charges and amounted to 5.68 mills per kw-hr. It was estimated also that if Conowingo were not built, it would be necessary to build 250 000-kw capacity of steam-generating plant. With Conowingo constructed, it was estimated that it would only be necessary to build 100 000 kw of generating capacity. In other words, it was estimated that Conowingo would take the place of 150 000 kw of steam generation, which would cost $135 per kw to construct, or $20 250 000. Fixed charges saved on a steam plant were estimated to amount to 11.5% of this amount, which is $2 330 000. The annual charges on fuel, supplies, and equipment which were saved were estimated at $82 000.

These 150 000 kw of steam capacity, which the water-power plant could replace, were determined from Fig. 32, showing the estimated average week-day load curve for December and that part of the load which could be served by hydro-electric plants with the minimum flow of the river. This diagram also shows that with a minimum flow, the hydro-plant could be utilized to take care of 153 000 kw of the demand on the system. It was assumed, therefore, that the firm capacity of the Conowingo Plant in the Philadelphia Electric Company System would be 150 000 kw. The estimated average week-day load curve for the wet month of March, 1927, is shown in Fig. 32(*b*). As a result of these investigations, the following tabulation was prepared to show the value of the hydro-plant in the Philadelphia Electric Company's System, compared to what the cost would have been had that Company constructed new steam generation instead of the Conowingo Hydro-Plant:

Cost of producing all power requirements from steam (1930)		$12 666 500
Cost of steam power after hydro-power comes in (1930)		7 163 000
Operating saving from hydro-electric energy		$ 5 503 500
Fixed charges on $20 000 000 representing 150 000 kw saved in steam-plant investment		2 330 000
Charges on saving in coal and equipment supplies from use of hydro-electricity		82 000
Total representing what the Company could afford to pay for the hydro-plant		$ 7 915 500
Cost of Hydro-Power:		
Rentals	$4 580 000	
Operating expenses, taxes, and depreciation on hydro-plant and lines	1 430 000	
Operating expenses, taxes, and depreciation on investment in Company territory in relation to hydro-electric generation	1 126 000	$ 7 136 000
Remainder representing the saving from use of hydro-plant		$ 779 500

As the hydro-plant was estimated to furnish 1 150 000 000 kw-hr per yr, the saving on this part of the current generated at the Conowingo Hydro-Plant would amount to 0.7 mill per kw-hr or, on the total power generated of 2 230 000 000, the saving would be approximately 0.36 mill. The total cost of the water-power current delivered in Philadelphia was estimated at 6.2 mills per kw-hr. The total cost of the steam generation of all the current was estimated to cost 5.68 mills per kw-hr; but by utilizing the hydro-power for peak service during the dry flow and base-load service during the wet months, the cost of the combined energy generation was reduced nearly $800 000 per yr, although the hydro-power by itself cost 6.2 mills as compared with the 5.68 mills if all the power had been generated by steam alone. This is a typical illustration of the economic advantage of combining steam and hydro-electric generation.

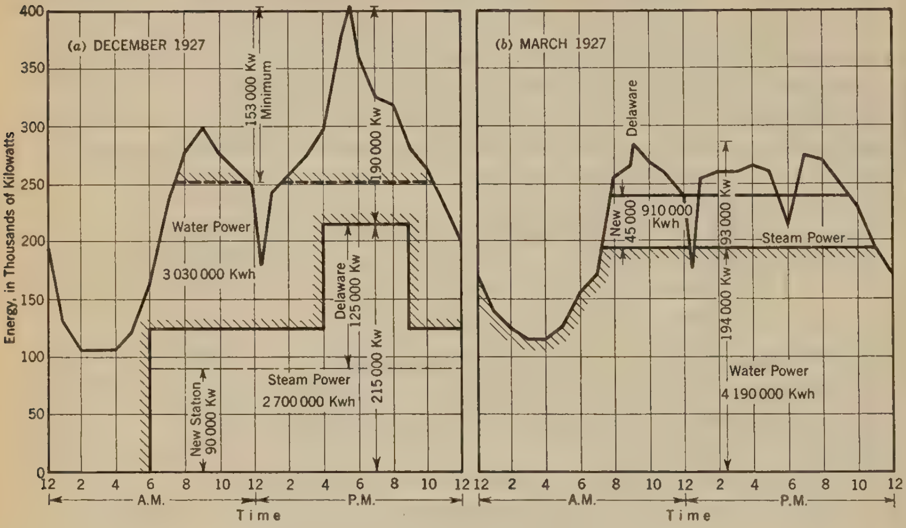

Fig. 32.—Estimated Average Week-Day Load

Federal Hydro-Projects

Federal water-power projects were started in the area covered by the Tennessee Valley Authority (TVA) with a view to providing "yardsticks" to determine whether the rates charged by private companies were reasonable. In the TVA case before the Alabama Public Service Commission, the only plant under consideration was the plant at Wilson Dam, Muscle Shoals, Ala. A steam plant costing about 12.25 million dollars was built during the World War and the hydro-plant was started; but until 1920 only approximately 3 million dollars had been spent. The project was completed in 1926 at a total cost of about 47 million dollars so that the hydro and steam plant combined cost about 60 million dollars.

It is true that the steam plant was built under war conditions, but the hydro-plant was not more than well started before 1920, and its construction was completed during the same period that a great number of private plants were built. It was testified that the Government would pay only 3% for its money, therefore, the rate of return required would be only this rate; whereas, private companies had been compelled to pay about 7% for their money. Although many of the private companies had spent considerable sums of money to provide locks and canals for navigation purposes, they had always been forced to absorb this expenditure in their capital investment for developing electricity. At Wilson Dam, one-third the cost of the entire project was charged to navigation, another one-third was charged to national defense, leaving less than 20 million dollars as the rate base on which the Government would pay 3 per cent.

With regard to the national defense, it is unquestionably true that the private power plants throughout the country engaged in furnishing power for a great number of defense projects have a value for the purposes of national defense in the same sense as the plant at Muscle Shoals; yet no credit was given to the private plants for national defense. In a number of Federal projects, there will also be allocations of the cost of the project, not only to navigation and national defense, but to flood protection, soil erosion, irrigation, and possibly for other uses.

In the aforementioned TVA case, it was also testified that the Government would pay the taxes just as a private company did, but the taxes that the Government proposed to pay were only 5% of the annual income from the plant, whereas it was shown that the private companies were paying taxes nearly three times this 5 per cent.

In practically all the operations in connection with the production and distribution of the electricity, the Government had estimated the governmental costs much below what these corresponding costs were for private companies. No private company would ever have built the great hydro-plants that have been built or are being built by the Government in the valley of the Tennessee River without first making studies as to how this power could be absorbed by the available market, in the same way that the studies were made in connection with the Conowingo Power Plant. If any such studies have ever been made, the writer has certainly never seen nor heard of them.

As the fixed charges on the hydro-plant are by far the greater part of all the charges against the cost of hydro-generation of electricity, the fact that the Government will begin with only 3% as the cost of the money, as against twice this rate for private companies, will give the Government water power tremendous advantage over any project that could be built by private capital. The Government is also in a position such that it can wait for the development of a market to absorb power which it generates and any deficits can be made up by taxation. Private companies are not in a fortunate enough position to be able to spend tremendous sums of money on projects which it may take years to develop to the point where they will pay a sufficient return on the capital invested to attract private capital.

The writer is convinced that it is useless to bicker about the comparative costs of hydro-power and steam power and of privately operated plants and Government-owned and operated plants if the greatest good of all the people of the country is to be served. After all, the cost of generating power is the smallest part of the cost of delivering the power to the customers and particularly the domestic customers. At the most, what is being argued about is a few mills per kilowatt-hour in the cost of the current as finally delivered to the customer. If the Government and the private companies would only sit down together and develop a plan for the most economical utilization of their combined power plants, it would be possible to serve the customers who may be within the territory that can be served by the Government hydro-plants to the advantage, not only of the customers, but also in such a manner that the investments by the private companies will not be greatly impaired or destroyed. Should the Government go into the TVA area and take from the private companies such cities as Knoxville, Tenn., Chattanooga, Tenn., Nashville, Tenn., Memphis, Tenn., and Birmingham, Ala., this would render useless much of the property of the private plants now serving these communities and would undoubtedly destroy the value of the investments by hundreds of thousands of individuals who own securities in these private companies.

The writer hopes sincerely, that some method may be found whereby the people will get the benefit of the Government plants and that the property of the private companies may be saved for their owners.

DEPRECIATION AND OBSOLESCENCE

By Maurice R. Scharff,[19] M. Am. Soc. C. E.

Synopsis

Depreciation should be used as a generic term for the effects of physical deterioration, obsoleteness, inadequacy and change in use or in demand, and requirement of public authority. The writer treats the engineering aspects of each of these causes of depreciation, and suggests that engineering study offers the greatest hope of progress toward their better understanding.

The determination of annual depreciation expense is then discussed. It is pointed out that such determinations heretofore have customarily been based on one or the other of two erroneous assumptions—either that all the causes of depreciation became effective in proportion to the passage of time, in accordance with a straight line or sinking-fund formula; or, that none of these causes became effective with the passage of time, or prior to the retirement of the property. The writer suggests that a rational engineering approach would recognize that some causes of depreciation do progress with the passage of time, and that others do not.

A procedure for the application of this concept to the determination of annual depreciation expense is summarized briefly, and its relation to the cost of power in competitive industry, public utilities, and governmental projects is discussed.

An opportunity is afforded the Engineering Profession to clarify the subject of depreciation by discussion of its engineering aspects. The writer proposes that civil engineers invite the co-operation of engineers in all branches of the profession in carrying out such a program.

Introduction

About a quarter of a century ago a Special Committee of the Society undertook a study of the problem of valuation of public utilities, the final report[20] on which has achieved recognition as an important contribution to the literature of this controversial subject. Not that this report ended the controversy. On the contrary, it has raged unabated to the present time, and has lost none of its vigor—as has been demonstrated by the renewal late in 1937 of the old debate over the relative merits of "prudent investment" and "fair value" as a basis for public utility rate-making. One chapter of the report was devoted to the subject of depreciation; and although twenty years have elapsed since its publication, one can still agree with the opening sentence of that chapter, in which the Committee states:[21] "Perhaps there is no single subject in connection with valuation that has caused more trouble than depreciation."

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[19] Cons. Engr., New York, N. Y.

[20] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), pp. 1311–1581.

On re-reading this report, in preparation for this paper of the cost of depreciation and obsolescence in energy generation, the writer has had the impression that if depreciation had been more clearly recognized at that time as the result of the actual incidence of specific causes, the Committee would have devoted more attention to consideration of the engineering facts regarding these causes and their effects, and would have placed less emphasis upon the relative merits of mathematical formulas for relating depreciation solely to the passage of time. Much of the discussion of the subject in the past, particularly that by accountants and economists, has proceeded from the assumption that depreciation was a simple function of cost, time, and the interest rate. More attention has often been paid to the shape of the curves and the forms of the equations for these functions than to the facts which they purported to explain.

It is the writer's proposal that depreciation be used as a generic term for the effects of several categories of causes. He proposes to discuss this suggestion in somewhat general terms, with reference to cost of energy generation only for purposes of illustration, conceiving this to be the most helpful contribution he can make to this, the Second, Symposium on Power Costs.

The categories of causes of depreciation usually listed in discussions of the subject include deterioration, wear and tear, obsoleteness, inadequacy, change in demand, change in use, and requirement of public authority. The mere enumeration of these causes, however, suggests that their effects cannot be expressed as any simple function of the passage of time, because it is common knowledge that an older unit of property or equipment may be, and often is, less deteriorated, more efficient, safer or more reliable, and less affected by change in demand, change in use, or the requirement of public authority than a similar unit of property or equipment of lesser age.

It is true that the incidence of these causes are events that occur in the universal framework of time, and that the effects of some of them are continuous, and sufficiently regular to permit formulation as simple functions of time. In most cases, however, the incidence of these causes is discontinuous, irregular, and even fortuitous with respect to time; and the engineer recognizes that their effects can be better expressed in terms of functions of physical and chemical change, abrasion, rupture, change of form or dimensions, loadings, stresses and moments, capacities, demands and uses, efficiencies, lost time accidents, service interruptions, competition of substitute services, etc., than in terms of the complicated, high-order functions that would be required to express their relation to the passage of time.

It is most fitting, therefore, that the Engineering Profession should devote attention to the development of a better understanding of the causes of depreciation and the measurement of their effects. If progress can be made in this direction, it may yet come about that accounting for costs and profits in industry and finance, and regulatory practices relating to public utilities will be brought into harmony with the facts relating to depreciation. It has too often been assumed that the meaning of depreciation is dependent upon accounting and regulatory policies and practices.

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[21] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), Chapter VI, p. 1448.

Part of the confusion of the past has been due to the fact that in accounting and finance, depreciation has been looked upon as synonymous with amortization of investment. It is a sound financial principle that the value of a capital investment is equal to the present worth of the amortization of the investment plus the return upon the unamortized portion of the investment. This principle can be complied with, however, by the consistent application of any method of amortization whatever, regardless of its relation or lack of relation to depreciation.

Confusion has also been caused by the controversies and contradictions that have grown up around the meaning of "fair value" in regulation of public utilities, and the legal measure of depreciation in determining "fair value." The Courts have performed a valuable service in emphasizing the reasonableness of the correspondence of such legal measure with the facts:[22]

> "The testimony of competent valuation engineers who examined the property and made estimates of its condition is to be preferred to mere calcuations based on averages and assumed probabilities."

Further evidence is given in the following:[23]

> "In the light of the evidence as to the expenditures for current maintenance and the proved condition of the property—in the face of the disparity between the actual extent of depreciation, as ascertained according to the comprehensive standards used by the company's witnesses, and the amount of the depreciation reserve—it cannot be said that the reserve merely represents the consumption of capital in the service rendered."

It is proposed now to discuss briefly each of the principal recognized causes of depreciation that have been enumerated as follows: (1) Physical deterioration; (2) obsoleteness; (3) inadequacy and change in use or in demand; and (4) requirement of public authority.

Physical Deterioration

Where property is used up and disappears progressively with use as, for example, the lead in a pencil or the material in a grindstone, such use is analogous to the consumption of such supplies as fuel taken from a coal pile. Such consumption, of course, can be measured in simple units of length, volume, or weight.

When the use of property can be isolated for study, it may be possible to measure the progress of deterioration as a function of such use. Wood poles, for example, have a single use—that of sustaining over-head conductors by reaction to vertical compression, on the one hand, and of supporting unbalanced span loads and lateral wind loads through resistance to bending, on the other. It is suggested that deterioration through fiber rot in such a case may reasonably be measured in terms of decline in compression and bending strength at standard fiber stresses.

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[22] McArdle *v.* Indianapolis Water Co., 272 U. S. 400 (1926).

[23] Lindheimer *v.* Illinois Bell Telephone Co., 292 U. S. 151 (1934).

The foregoing examples are cases in which simple objective measurements of the effects of deterioration appear reasonably applicable. In many types of cases, however, the relation of deterioration to objective measurements is less obvious. Thus, when a steam boiler is new it can operate under a definite pressure limitation as prescribed by the A.S.M.E. standards and boiler insurance codes. Furthermore, the tubes, shells, and headers have definite thicknesses of sound metal designed to support the anticipated stresses with reasonable factors of safety. As deterioration occurs, the thickness of sound metal is reduced. De-rating of boilers by insurance inspectors supplies an additional objective measure of the extent of deterioration.

Similarly, changes in the dielectric strength of insulation and in other physical and chemical characteristics occur in the windings of generator coils as a result of use and of changes in temperature and dielectric stress. Extensive measurements of some of these characteristics have been made by manufacturers and by engineering committees in an effort to describe them.

In these and in similar cases, however, it would be impossible, in the present state of knowledge, to secure agreement upon any standard series of objective measurements that would be generally accepted as measuring the extent of deterioration. Engineering estimates of conditions reflecting the extent of deterioration in such cases, therefore, are customarily stated as representing the judgment of the engineer as to the point where the deteriorated condition of the property in question would fall on a scale between 100%, representing condition new, and 0%, representing a condition requiring removal, if all degrees of deterioration were arranged in order of magnitude.

Such a judgment, of course, is in a measure subjective. It has been the experience of the writer, however, that such judgments of a number of competent observers of similar training and experience with respect to an identical condition tend to be reasonably consistent and to group themselves around a mean without excessive deviations.

It may be that in judging condition or extent of deterioration, engineers are in somewhat the same situation as men were when they attempted to convey to one another their relative sensations on touching similar objects at different temperature levels before it was known that temperature was proportional to molecular velocity or that a convenient scale for the measurement of temperature could be established by measuring the length of a column of mercury in a glass tube.

The time may come when the processes of deterioration will be sufficiently understood so that several objective measurements may be recognized as completely describing all the types of physical deterioration that have occurred in an item of property and may be synthesized into a determination of condition that is related as closely to objective observations as the unused length of the lead in the pencil, the remaining stone in a grindstone, the weight of fuel in a coal pile, or the remaining fiber strength of a wood pole.

The writer believes that progress in this direction depends upon engineering study, and proposes that civil engineers invite the co-operation of engineers in other branches of the profession specializing in the design, construction, and operation of power plants in undertaking to devise more complete objective standards for the measurement of the effects of deterioration of power plant structures and equipment, and of suggesting improved methods of arriving at sound engineering judgments and valid statistical inferences in the light of such objective standards as are already available.

Obsoleteness

Attention is called to the use of the word, "obsoleteness," instead of "obsolescence" as included in the title of this paper, in deference to the use of the latter term in the designation of the Sub-Committee of the Power Division, of which the writer is Chairman. The use of "obsolescence," derived from the inceptive Latin verb ending in "-sco," is misleading, since such verbs describe verbal actions commencing and progressing continuously with the passage of time. "Obsoleteness," derived from a past participle describing a completed verbal action, seems more suitable for describing a condition existing at a particular time, especially when it is intended to refer to a condition resulting from a cause which is discontinuous in time and which becomes effective upon occurrence of specific events, such as the development of new inventions or other steps in the progress of the arts.

It is suggested that the decline in the economic value of existing equipment, resulting from the development of improved equipment, which can be operated at a saving, may reasonably be used as a measure of obsoleteness. Economic value is defined as the sum of the present worths of the expected returns, where such returns are equal to the total revenues less all operating expenses (exclusive of provision for depreciation or amortization).

Progress in the arts tends to affect competitive enterprises by attracting into the field lower cost producers, with resultant lowering of prices, elimination of marginal producers, and decline of the profits and economic values of remaining enterprises using obsolete facilities.

With a relatively inelastic demand, such decline of economic value may be stated as the total of the cost of the old equipment plus the sum of the present worths of its operating expenses (exclusive of depreciation or amortization) over a reasonable period of time, minus the corresponding total for the most modern available improved equipment. The ratio of decline is this difference divided by the cost of the old equipment.

Algebraically, the percentage effect of such obsoleteness may be stated as follows:

$$D_o = 100 \frac{C - [C_e - \sum_{o}^{n} W_p \, (O - O_e)]}{C} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

in which D_o = percentage depreciation due to obsoleteness; C = cost of existing equipment; O = annual operating expenses associated with existing equipment (exclusive of depreciation); C_e = cost of the most economical modern substitute equipment; O_e = annual operating expenses associated with the most economical modern substitute equipment (exclusive of depreciation); and, $\sum_{o}^{n} W_p$ = sum of the present worths of an annual sum over a reasonable period of n years.

In some instances, improved equipment results in greater safety, greater reliability, or improved quality of product, rather than in realizable savings in operating and maintenance expense, and in such cases the measurement of obsoleteness appears less simple and has the subjective character of engineering judgments to which reference has been made in connection with estimates of some forms of physical deterioration, and the judgment is expressed as a percentage on a scale between 100% and zero. In this case, also, there would seem to be opportunity for the development of objective measurement by engineering study, and for progress toward an objective basis for judgment.

INADEQUACY AND CHANGE IN USE OR IN DEMAND

As in the case of obsoleteness, these causes of depreciation become effective as a result of specific events, such as addition of new load, loss of load, change in market, or change in method of operation. Inadequacy is rarely encountered in well managed profitable enterprises, for the reason that in such cases facilities are promptly increased to take advantage of opportunities for profitable employment of capital. Excess capacity due to decline in market or change in methods of operation, however, is not uncommon and there is opportunity for clarification of its meaning and development of improved methods of measuring its effects. The analogy of decline in price, to the extent of savings in cost of production that could be accomplished by a producer, with no more than adequate equipment, appears reasonable as a basis of measurement and on this basis the percentage depreciation due to change in demand or in use is, as before:

$$D_u = 100 \frac{C - [C_a - \sum_{o}^{n} W_p (O - O_a)]}{C} \ldots\ldots\ldots\ldots\ldots (4)$$

in which D_u = percentage depreciation due to change in use; and C_a and O_a are the cost and the annual cost of operation of equipment of no more than adequate capacity.

REQUIREMENT OF PUBLIC AUTHORITY

The effects of this cause of depreciation are rarely found in property in service, because compliance with such requirements is ordinarily compulsory, and retirement and replacement take place at the effective date of such requirement. Where property is maintained in contravention of legal requirement by competent public authority, either the property right in such property has in effect been destroyed, and depreciation due to this cause should be considered complete; or a liability is imposed upon the owner of the property, the amount of which may be taken as a measure of depreciation due to this cause.

ANNUAL DEPRECIATION EXPENSE

If the existing depreciation in the property is the sum of the effects of the several causes of depreciation that have been discussed herein, the annual depreciation expense should be the sum needed to make the reserve for depreciation representative of existing depreciation at the beginning and at the end of each year. If it were practicable to make a depreciation study as of January 1 of each year and determine the existing depreciation, it would be possible to compute the reserve requirements at such dates. Then, the annual depreciation expense would be the difference between the required reserves at the beginning and at the end of the year plus the debits to the reserve during the year. The debits to the reserve during the year are the net retirement losses, which are equal to the difference between the sum of the cost of the property retired plus the costs of dismantling and the salvage recovered.

Of course, a complete new depreciation study as of January 1 of each year cannot be made on account of the magnitude of the task and the time required to make the determination of existing depreciation. However, it is possible to approximate, within narrow limits, the probable change in the amount of existing depreciation during a year, and thus to approximate the amount of annual depreciation expense.

The difficulty of determining the annual depreciation expense arises from the discontinuous relation of some of the causes of depreciation to time. The result of this difficulty has been that depreciation due to all causes has generally been treated either as if all these causes became effective directly and solely in proportion to the passage of time, and as if depreciation were identical with amortization of investment by a straight line or a compound interest formula; or as if none of these causes was effective in proportion to the passage of time, and as if depreciation losses occurred only at the time of retirement of property.

It is suggested that a rational engineering approach to the problem would be to recognize that the effects of some causes of depreciation on some property do proceed in a simple functional relation to time and that others do not; and to attempt to formulate a plan for computing annual depreciation which will more nearly correspond with facts than either of the erroneous treatments to which reference has been made.

The writer does not minimize the difficulties of such an approach. He believes, however, that engineering study offers promise of progress in the direction indicated which is well worth the effort of attempting to overcome the difficulties that lie in the way. With this in mind he proposes to describe briefly a procedure which he has developed and used in studies made under his direction.

Of all the causes of depreciation, physical deterioration comes the nearest, and in a greater proportion of cases, to progressing approximately in proportion to the passage of time, whereas obsoleteness, inadequacy, change in use or in demand, and requirement of public authority occur irregularly, and often at the time of the retirement of a property. Admitting freely the inaccuracy of the assumption and pending the development of more complete knowledge of the relation between incidence of the causes of depreciation with respect to the particular property and the passage of time, it is suggested that a helpful guide to the determination of annual depreciation expense in many cases will be found in a computation based upon the assumption that deterioration does progress directly in proportion to the passage of time, and that the other causes of depreciation become effective at the time of retirement and can be provided for as retirement losses.

To make such a computation it must be recognized that the physical condition determined by an engineer, in the manner previously described, is the result of two offsetting influences: (1) Deterioration, which acts to lower the condition of the property; and (2) the replacement of parts through maintenance, which tends to restore the condition of the property.

The physical condition of the property as found is the net result of the effect of these opposing influences during the history of the property. Every dollar spent on maintenance, however, does not improve the condition of the property. Some maintenance expenditures tend only to prevent deterioration and others affect only the location or arrangement of property. The cost of replacing parts on maintenance, also, because of its piecemeal character, may be substantially greater than the cost of the parts replaced as portions of the cost of property used in operations.

If it is assumed: (1) That a constant proportion of maintenance expenditures has been effective in restoring deterioration; (2) that maintenance expenditures have been spread evenly over the property whose condition has been determined; and (3) that parts replaced on maintenance tend to deteriorate at the same average rate as all the property whose condition has been determined, then it is suggested that the relation of these several factors may be expressed by the equation:

$$K_n = 100 \left[1.00 - X\,Y_n + \frac{b_n}{B_n} (I)\,(1.00 - X\,y_n) \right] \ldots\ldots\ldots (5)$$

in which K_n = percentage condition resulting from physical deterioration; X = average annual rate of deterioration in the past; Y_n = weighted average age in years of dollars representing the property; $\frac{b_n}{B_n}$ = ratio of cumulated maintenance expenditures (b_n) to fixed capital balance, representing the property (B_n); I = ratio of maintenance expenditures effective in restoration of condition; y_n = weighted average age in years of cumulated maintenance expenditures; and n = subscript indicating year, January 1 of which is the basis of the study.

Equation (5) may be solved for X, which is the average rate of deterioration independent of the offsetting effects of maintenance. The reciprocal of X may be considered as an average indicated useful physical life, which the property would live if the constant rate of deterioration continued from 100% to 0% condition, and if it were retired solely because of physical deterioration.

With the same assumptions as before, and with amounts budgeted for additions, retirements, and maintenance expenditures for an ensuing year following the date for which existing depreciation has been determined, the annual depreciation expense, D_a, may be stated as follows:

$$D_a = [B_{n+1}\,(1.00 - K_{n+1}\,M_n) - B_n\,(1.00 - K_n\,M_n)] + R_n \ldots\ldots (6)$$

in which M_n = the net percentage effect of all causes of depreciation other than deterioration; R_n = the estimated net retirement loss in the ensuing year; and K_{n+1} is expressed by Equation (5) for $n = n + 1$ (the subscript,

$n + 1$, indicates merely that the figures to which they are applicable are those of a year later than n).

The writer has found this formulation useful (although it is inchoate), in estimating the costs required to be budgeted for depreciation expense during a year following a determination of existing depreciation. Where depreciation reserves have been established in accounts that are the same proportion of the book cost of depreciable property as the percentage depreciation at the beginning of the year determined by engineering study in the manner described, this procedure will tend to preserve such correspondence between reserves and actual depreciation at the end of the ensuing year.

The procedure, of course, is capable of indefinite refinement, as information and a better understanding of the facts of the particular case become available. If there is ground for anticipating that deterioration of some part of the property will not proceed in proportion to the progress of time, correction may be made on this account. If the causes of obsoleteness, inadequacy, or change in use or in demand are known to be in operation in a manner that will result in additional depreciation during the ensuing year, due to these causes, without justifying retirement, a proportionate allowance may be made for these factors. It may be desirable in any case to increase the estimated requirement for depreciation expense by a percentage for contingencies to allow for the obvious limitations on the accuracy of the procedure. If such a procedure should be followed for a number of years it would probably be desirable, possibly at intervals of five to ten years, to check the correspondence between the resultant reserves and the actual depreciation of the property by a re-examination of its condition and by new engineering studies of obsoleteness and of the effects of change in demand or in use.

It does not seem worth while, at this time, to labor the point, or to attempt to develop, in any more detail, the solution of the practical problems that will be encountered in the application of this procedure. The purpose is to emphasize the opportunity for a contribution by the Engineering Profession to a rational solution of this problem which it is believed will have application to competitive industry, to public utilities, and to the determination of the cost of power developed by public projects, to which it is now proposed to make brief reference.

COMPETITIVE INDUSTRY

In competitive industry the prices of products tend to be determined by equating utility to the marginal consumer to cost to the marginal producer. Profit is the difference between price, as so determined in the market, and cost of production. Neither price nor profit has any necessary relation to the investment in fixed facilities of the particular enterprise under consideration; and the value of the enterprise is the present worth of its prospective earnings including profit and recoupment of investment at such time and in such manner as the profitable operation of the enterprise in the competitive market may make possible.

The policy of management, however, as to amortization of investment may be based on considerations other than the occurrence of actual depreciation and retirement losses. Thus, if a uniform product is manufactured and sold, it may be desirable that the entire cost of amortization of investment—or possibly that all costs—shall be spread over the total number of units of the product, regardless of the time when, and the rates at which, such costs or losses are actually incurred. On the other hand, a more limited equalization of the effect of depreciation on cost of production in accordance with some other plan may be desirable because of some other special consideration. It has seemed to the writer that it is for this reason that depreciation and amortization of investment have been so generally considered as identical in accounting practice.

It is suggested, however, that knowledge of the actual time and rate of occurrence of losses due to depreciation, based upon competent engineering study, will be helpful to management even when amortization policies are not controlled by depreciation. The tendency to extend governmental supervision over financial and accounting practices in connection with the issue of securities, moreover (as illustrated by developments under the Security and Exchange Acts), suggests the possibility that even in competitive industry a closer correspondence between such practices and the actual facts of depreciation may be necessary or desirable at some time in the future.

Public Utilities

In the public utility industry the situation seems to be quite different. The principle has been established that public utility rates are to be based upon allowance of gross revenues from sales of service equal to operating expense, plus annual depreciation, plus a reasonable return on the fair value of the property used and useful in the public service. Fair value, as prescribed by the Courts, must be based upon consideration of reproduction cost less depreciation, prudent investment less depreciation, and other pertinent measures of value. The tendency of the Courts, in prescribing these general requirements, has been to insist that depreciation deducted in determining fair value shall be "actual depreciation," and that there must be a reasonable correspondence between such "actual depreciation" and the reserves resulting from the annual depreciation included in operating expense.

The freedom of competitive industry to amortize investment in any manner suggested by the special conditions of the industry is limited by the definite requirement that depreciation charges and reserves shall correspond as closely as possible with the facts as to depreciation losses and the condition of the property from time to time, taking into account all the effective causes of depreciation.

The tendency toward clarification of this requirement has received a marked impetus in the two years, 1936–1937, through adoption by various Federal and State regulatory commissions of new classifications of accounts requiring (to quote the language of the classification of the Federal Power Commission, for example) that "each utility shall record as at the end of each month the estimated amount of depreciation accrued during that month on depreciable electric plant."

The method of approach herein outlined complies with these special requirements which are applicable to the public utility industry. Even the suggested methods of measuring the effects of obsoleteness and excess capacity due to change in demand or in use, derived from the analogy of the economic effect of such changes on price and economic value in the competitive field, seem to be directly applicable. The regulation of semi-monopolistic public utility enterprises constitutes, in effect, the substitution of an artificial simulation of competitive conditions to make up for the lack of the influence of free competition on prices.

The references previously made to opportunities for refinement of methods of calculation, with increasing knowledge of facts and their relations, are especially applicable in the present case. Some equalization of requirements for annual depreciation expense by averaging over reasonable periods of years may also be desirable. In special cases, as where the property of a public utility consists of a single long-lived unit, like a hydro-electric dam, it may even be desirable, as a matter of public policy, to equalize such costs by a compound interest formula over a long period of years.

However, even such departures of amortization provision from depreciation, if desirable, cannot be made intelligently, and cannot avoid that unreasonable disparity between existing depreciation and reserves which has been condemned by the Courts, unless the facts are fully understood on the basis of proper engineering investigation and analysis. Such engineering investigation and analysis may well provide a more generally accepted basis of determining amortization based upon average depreciation experience for the purpose of making economic comparisons of different types of design and of the relative costs of steam and of hydro-electric power. The importance of such development is illustrated by the widespread discussion that followed the publication in November, 1937, of the report of the Power Authority of the State of New York on "Government Hydro *v.* Private Steam Power." In this report it was shown that the average fixed charges of four modern base-load steam-generating stations, furnishing power to distributing companies under contract, included depreciation varying from 1.90% to 3.25 per cent. The Authority adopted 2.0% as a reasonable comparative allowance for depreciation and renewals on both public and private water power plants, and 3.0%, based on sinking-fund accrual at 7% interest, as a reasonable comparative allowance for private steam generating stations.[24]

Cost of Power Produced by Governmental Projects

As in the case of competitive industry, the amortization of investments in public projects of any character may be controlled by considerations of policy unrelated to the occurrence of depreciation in the property representing such investments. It is not intended to suggest that it is a function of this Society, or of any other engineering organization, as such, to discuss such governmental policies. It is suggested, however, that, as in the case of competitive industry, knowledge of the engineering facts relating to the occurrence of depreciation in such property will be helpful to all those concerned in such policies, including governmental officers, tax-payers, and voters; and it is obvious that if helpful comparisons are to be made of costs of power produced by governmental projects, and of those produced by private agencies, it will be helpful to include costs of depreciation in both on a strictly comparable basis.

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[24] H. R. Committee on Rivers and Harbors, Doc. No. 52, 75th Cong., 2d Session, pp. 26, 37, 38.

CONCLUSION

The purpose of the writer in presenting this discussion of depreciation has been primarily to emphasize the engineering aspects of the problem, to which he believes sufficient consideration has not been given in the past, and to suggest that an opportunity be afforded the Engineering Profession to clarify this highly controversial subject by discussion of its engineering aspects. He ventures to express the hope that civil engineers will take the lead in this engineering activity, and will invite the co-operation of engineers in all branches of the profession in carrying forward a program which, he is convinced, can be made of great value, both to the profession and to the public.

RECAPITULATION

By W. F. Uhl,[25] M. Am. Soc. C. E.

Synopsis

The final paper of a Symposium presumably is useful if it summarizes the salient facts revealed in each paper and points out divergencies of opinion where they may be found. It can also properly present conclusions and some of the economic aspects of the subject-matter.

It was thought best to confine the Second Symposium on Power Costs to generation only, in order that the issue might not be confused. Later, it is hoped that the more complex problem of cost of transmission and distribution may form the subject of a Third Symposium.

A study of the papers comprising this Symposium indicates that the cost of generating power is indeed a variable quantity. This is a well-known fact which it should be unnecessary to discuss at length, but many recent statements emanating from sources that should be well informed indicate that the belief is abroad that power should be produced at a fixed price or that a "yardstick" of some kind can be set up to measure the cost of power.

It is clear that, considering all the variables, it must be a mere accident if the cost of power generated in any two or more stations is the same. No attempt has been made in this paper to summarize facts and conclusions separately.

Elements of Power Cost

Recent papers relating to the cost of power indicate that some of the fundamental principles involved are sometimes overlooked. These principles are well established. An honest difference of opinion can exist regarding their application but they may not be disregarded except as a convenient means of promoting unsound projects. Engineers cannot properly become promoters unless they acknowledge such activities.

The total cost of generating power consists of fixed charges and operating costs. These terms are well understood. Fixed charges may vary between wide limits, depending on cost of money (interest), depreciation and obsolescence experience, taxes, and possibly some other minor items. Operating costs can be more readily determined, but where costs are to be determined for the future, there can be honest disagreement on such items as maintenance, fuel, labor, etc.

With steam power plants the major uncertainty is the variable cost of fuel and the increasing cost of attendance. During recent years the rapid advance in the art of steam power generation makes it difficult to estimate depreciation and obsolescence. With hydro-plants the major uncertainty is the available stream flow. This is highly speculative in most cases and, at best, can only be based on records covering past experience where such records are available. Extreme drought and flood conditions are seldom known.

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[25] Hydr. Engr., Chas. T. Main, Inc., Boston, Mass.

Quality of power as reflected by outages, voltage, and speed control, should receive consideration. Comparative reliability of operation must also be considered and the importance of reliability under certain conditions stressed.

Engineers with experience in the power field are thoroughly familiar with all items that must be considered in arriving at power costs, but very often there is a marked difference of opinion regarding the total fixed charges to be included in such costs. The principal items usually included in fixed charges are: (1) Cost of money; (2) depreciation and obsolescence; (3) taxes; and (4) insurance.

(1).—Cost of Money, or as More Commonly Referred to, Interest.—This item can vary within wide limits, depending principally upon financial conditions, the credit of the borrower, the popularity of the business, the hazards involved, and other matters. Some municipalities and boards, set up by legislative enactment for special purposes, have borrowed money for power plants at interest rates as low as 3% to 3.5%, whereas the average rate of interest for private concerns has generally varied between 6% and 8%, although a part of their money might be borrowed at rates as low as 4% to 6 per cent.

(2).—Depreciation and Obsolescence.—These items depend largely on the economic life of a plant which, in turn, depends principally on the design, materials, workmanship, and quality, in the construction and equipment, the extent to which the plant is used, its operation and maintenance, and the state of the art. It can be readily seen that there can be a wide difference of honest opinion regarding such matters.

Present experience indicates that the economic life of a steam plant is probably from 20 to 35 yr, whereas a well-designed and constructed hydro-plant may have an economic life of from 40 to 50 yr, or longer. In other words, the average rate of depreciation may be as high as 5% for a steam plant and as low as 2% for a hydro-plant.

Where the investment in plant is amortized by a sinking fund, it is necessary to establish the term of the useful life of the plant and the probable rate of return on the fund set aside, both of which items are speculative.

The prevailing practice of superposing high-pressure steam turbine units on old low-pressure plants which may have already been operated 10 to 15 yr, or more, complicates the depreciation problem in such cases. Overhauling the older units may be justified economically where they act as the low-pressure part of the new plant, since they now have an additional useful life of a more or less indeterminate character.

The future advance in the art of steam-power generation alone can determine whether the new superposed unit will have a useful life fixed by the remaining life of the older units or whether the life of the older units will be fixed by the useful life of the new high-pressure unit. The problem is not unlike that which presents itself when old hydro-plants are redeveloped and many structures are found suitable for continued use.

(*3*).—*Taxes.*—The principal taxes to be considered are property taxes to City, Township, County, and State. A portion of the owner's Corporation Franchise, State, and Federal income taxes may also be properly chargeable to power generation.

Property taxes on hydro-plants are often very much less in the country than the property taxes on steam plants which are generally situated near load centers. Social security and unemployment assessments must also be included. Taxes may vary from 1.5% to 4%, or more.

(*4*).—*Insurance.*—Insurance is a relatively minor item but is becoming more important as liability insurance has been gradually increasing. Fire, electrical, boiler, and fly-wheel insurance, as well as insurance against damage by the elements, are often carried. The amount of this insurance depends largely upon individual judgment. Generally, the insurance carried on hydro-plants and their equipment is materially less than that carried in connection with steam plants. Insurance usually varies from 0.25% to 0.75 per cent.

A summary of the foregoing items, constituting the annual fixed charges on power plants, indicates that the total may be as low as about 7%, or as high as about 18% of the capital cost. Fixed charges on hydro-plants usually vary from 8% to 11%, and on steam plants from 12% to 15 per cent.

The principal items usually included with operating cost are fuel, water, labor, maintenance, oil and supplies, repairs, superintendence, accidents and damages, general and miscellaneous expense, management, legal expense, etc.

Operating costs for hydro-plants are relatively small and do not vary much with the output of the plant. For steam plants, operating costs are relatively high and vary greatly, depending largely on the capacity factor of the plant, and for the most part on the cost of fuel. On the other hand, the total annual fixed costs for hydro-plants generally are higher than for steam plants, due to greater capital cost per unit of dependable hydro-plant capacity.

Where more than one plant carries the load of an inter-connected system, the average cost of generation must be considered, as well as the generating cost of some particular plant which may be operated at a high load and capacity factor. Money tied up in coal storage and working capital may also be considered a charge against the cost of generating power.

HEAT-GENERATED ENERGY

Published cost records and individual experience show that the cost of steam power plants varies widely. Conditions under which they are built, their location, reserve capacity, load conditions, nature of service to be rendered, and other matters, have a marked effect on the capital cost. In studying the capital cost of steam plants, it is important that item for item be compared so that conclusions will not be misleading.

Data published on the cost of plants do not always state what is included. Besides the usual itemized account, including the plant rating, the number, size, and type of generating and boiler units, steam pressure and temperature, type of service, fuel used, number of outgoing lines, voltage of lines, condensing water conditions, etc., it is important to know something about land, buildings, foundations, water tunnels, coal and ash-handling equipment, or equipment for servicing other fuel where used, boiler and turbine-room auxiliaries, switching arrangements, outgoing feeder lines, step-up sub-station equipment, etc. The extent of reserve capacity should also be stated.

Published costs show the following: One list of 150 plants built between 1882 and 1936 is stated to show an average cost of \$100 per kw.[26] Another list of 80 plants built since 1920 shows an average cost of \$108 per kw. Five of these plants cost more than \$150, and four less than \$70 per kw.[27]

Another list of sixteen plants built between 1924 and 1937 shows a range in cost, including step-up sub-station equipment, from \$101 to \$180 per kw, the average being \$139 per kw.[28] Still another list of sixteen plants built since 1927 shows a range of cost from \$82.50 to \$145.00 per kw, with an average cost of \$114 per kw.[29] These costs do not include any reserve capacity. Adequate reserve capacity may add from 20% to 40% to the cost of dependable generating capacity.

The records of cost of steam plants indicate that the size of the plant has little if any effect on the unit cost and the same is true of steam pressure and temperature. However, in general, it may be stated that the total power cost tends to decrease as the size of plant is increased.

Station service requirements for steam power plants are from 5% to 8% of the installed capacity. This service includes pumping, fuel handling and preparation, fans and other auxiliaries, and lighting. The power requirements for station service may be supplied in different ways, depending on the heat cycle. The necessary power may be supplied by the main unit and delivered through house-service transformers; it may be produced by house-service generators; or, the auxiliary equipment may be steam driven.

When steam power plants form a part of an inter-connected power system, a portion or all of their output may be transmitted over high-tension lines, thus requiring step-up sub-stations. Many of the large new steam power plants are some distance from the larger load centers and in such cases the entire output may require transmission at higher than generator voltages. In making comparisons between cost of plants, these questions must receive consideration.

The cost of the unit output of the steam power plant, assuming that the fixed charges are agreed upon, depends principally on the capacity and load factors and the cost of fuel. It will be found that the unit cost of generating steam power will vary just as widely as the capital cost.

Particular attention should be called to the difference in cost of steam power from plants designed with full auxiliaries and reserve capacity and those designed for less reliable service. Fixed charges per kilowatt-hour of output from modern steam power plants costing about \$100 per kw and using 12% fixed charges, may vary from 0.17 to 0.7 cent, or more, depending on the capacity factor, which may vary from 80% to 20%, or less. Fuel costs per kilowatt-hour of net output will vary from about 0.17 cent at 80% capacity factor to 0.3 cent at 20% capacity factor, with fuel costing 15 cents per million Btu and with a 1-in. condenser back-pressure.

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[26] "Progress in Generation of Energy by Heat Engines" (First Symposium on Power Costs), by Geo. A. Orrok, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1937, p. 1888.

[27] From records of the Federal Power Comm.

[28] *Electrical World*, October 27, 1928.

[29] *Loc. cit.*, November 23, 1935.

Total costs per kilowatt-hour composed of fixed charges, operation, maintenance, and fuel, will vary from about 0.4 cent at 80% capacity factor to 1.1 cents at 20% capacity factor for plants costing $100 per kw, with fixed charges at 12%, and fuel costing 15 cents per million Btu.

These costs are for a net output on low-tension terminals of large modern plants of 150 000 to 250 000-kw capacity with steam pressures from 400 lb to 1 250 lb, and temperatures from 650° F to 925° F, and without provisions for reserve capacity. General over-head expense will add from 10% to 25% to the foregoing unit costs.

Diesel and gas engine installations for the generation of electric energy are mostly made with relatively small units and for comparatively small plants. The total installation of such plants is also small compared with the water power and steam power installations.

Based on an investment cost of $135 per kw, 13% fixed charges, and 5 cents per gal for fuel oil, the total generating costs per kilowatt-hour net output from Diesel engine plants (includingg eneral over-head expense) will vary from 2 to 2.5 cents for operation at 20% capacity factor, to about 0.8 cent for operation at 80% capacity factor. A suitable reserve capacity will add somewhat to these costs.

HYDRO-GENERATED ENERGY

Comparisons between capital costs of hydro-plants are even more difficult to make than comparisons between costs of steam plants, since the natural conditions to which hydro-plants must be suited are more variable and generally more difficult than the steam-plant locations which are subject to some choice. The installed capacity of hydro-plants with relation to the available stream flow also varies widely, depending largely on the load to be served and the available pondage and storage.

The cost of economic hydro-electric developments varies from $100 to $300 per kw of installed capacity. The cost of the greatest number for which data are available varies from $150 to $200 per kw. Higher costs than $300 per kw may sometimes be justified under special load conditions. In general, unit cost decreases as head and size of plant increase.

One of the principal difficulties in establishing the true capital cost of hydro-plants from which the full output cannot be used immediately, is the added capital cost due to fixed cost on the unused part of the plant. Generating units can be added from time to time, but the major cost is in the water rights and structures other than the power-house.

A comparison of hydro-plant costs based on the installed capacity is almost meaningless since plants are found which can operate to capacity only 3% to 5% of the time, due to stream flow limitations, whereas some plants can operate to capacity from 75% to 100% of the time on the available stream flow.

The earlier hydro-plants were developed for private industrial use at a time when the art of steam power generation was relatively uneconomical. As the power load of the industries increased and continuity of power supply became more important with the advance of the industrial age, steam power was usually installed to supplement the water power during the seasons of low stream flow. Many of these industrial hydro-plants have been redeveloped in recent years, and generation and distribution of output have been changed from mechanical to electrical. Some of the old structures built for the original hydro-plants, such as dams, canals, conduits, and tail-races (and, in some cases, even the power-house), are utilized to a greater or less extent in the redevelopment.

The cost of such redevelopment generally varies from $75 to $100 per kw of installed capacity. The cost of operation and maintenance of these industrial plants is very low. Many are operated without continuous attendance. Pumped-storage hydro-plants can sometimes be built to carry the peak load of a system and thus improve the load factor on the remainder of the plants in the system. They may also be relied on for emergency reserve capacity. For some reason this type of plant has not been adopted extensively in the United States, but quite a number of them have been built in Europe. Natural conditions which permit construction of such plants at reasonable cost and where their output does not require excessive transmission costs, are factors that have an important bearing on the economic advantage which such plants might have over other peak capacity.

The cost of hydro-generated energy, assuming agreement on fixed charges, may be readily determined if the usable output is known, since costs other than fixed charges are nominal. Operation and maintenance costs per kilowatt-hour of output depend largely on the size of the plant and somewhat on its design and construction. For plants of from 5 000 kw to 10 000 kw capacity, these costs may vary from 0.1 to 0.05 cent per kw-hr and for large plants they may be as low as 0.02 to 0.01 cent per kw-hr. Total generating costs consisting of fixed charges computed at 10% and operation and maintenance vary from 0.25 cent to 3 cents and more per kw-hr. Although, in general, the cost per kilowatt-hour tends to lessen as the head and the size of the plant increase, the effect of capacity factor is more predominant.

STEAM AND WATER POWER COMBINED

The earlier hydro-plants were usually developed as a main source of power. This is true with respect to both industrial and public utility hydro-plants. As the loads increased it was generally found more economical to install a supplementary steam plant to be operated during low-water seasons rather than a second water-power plant. Such supplementary steam plants were built at low cost without much regard for economy of operation since they were not in use much of the time.

The utilization factor of most hydro-plants operated by themselves is generally very low unless a portion of their output is sold as dump power. In some cases in the West and in many places in Canada, hydro-plants always did, and still do, carry the entire power load of very large systems. It is true that occasionally during extreme droughts the output must be curtailed, but usually arrangements are made to drop some of the low-price load which is taken on with the understanding that this can be done.

Water power and steam power are not often economically competitive, but ordinarily are complementary if used to best advantage. Under modern conditions, water power finds its greatest economic use where it supplements steam power, since it is found that in many cases where sufficient flow and pondage are available, incremental hydro-capacity costs less than incremental steam plant capacity. It also has certain advantages as reserve capacity since hydro-plant units can be started and their output placed on the outgoing lines in a few minutes whereas steam units must be kept hot and revolving for some time before they can be put under load. Hydro-capacity can also be used for power-factor correction at small cost.

Hydro-electric energy output should cost no more than the fuel cost of an equal amount of steam-generated energy if credit is allowed for that portion of the hydro-plant capacity which is capable of carrying peak loads or which acts as reserve capacity. Usually, hydro-plants are not developed unless they show some margin of economy over steam.

Small water-power plants near load centers or near the ends of transmission or distribution systems often have a value which they would not have if they were situated at a distance requiring the transmission of the energy generated to a distant market. If they were near a load which would otherwise be supplied over a transmission line from a distant power plant, there is a saving in transmission losses and there are certain advantages due to possible voltage control and power factor correction. If pondage is available, such plants also have a capacity value. There is also the additional advantage that during transmission difficulties at least a part of the local and near-by load can be supplied from such a plant.

It is often claimed that Niagara Falls furnishes a water power that is sufficient in itself and does not require a supplementary source of power, since the stream flow is such that a practically constant output of water power is available. However, even Niagara power cannot be fully utilized, except as a part of a large system having a substantial amount of steam power, since a load requiring a constant output has not yet been discovered.

Niagara, "tied in" with steam power, can carry a large base load, and thus can be utilized quite fully; whereas, if operated by itself, there would be many periods when much of the available water would be wasted due to lack of demand. Ultimately, every water power can be more fully utilized if operated in conjunction with heat energy plants.

Economic Importance of Cost of Power

Available statistics indicate that in 1936 the total stationary power plant installations in the United States amounted to about 75 000 000 hp, divided as follows:

Steam plants	56 684 000
Hydro-plants	16 075 000
Oil and gas engine plants	2 000 000
Total	74 759 000

An average cost of $100 per hp (about $135 per kw) would make the total investment in stationary power plants about $7 500 000 000 as of 1936. The actual investment is probably much larger, and the annual investment in such plants has become one of the major items of national business.

If the cost of power is compared with the cost of other commodities for which there is a similar universal demand, it would appear that the unit cost of its generation is unimportant, particularly since such cost is only a small part of the cost of delivering it to the ultimate consumer, as in the case of public utility service.

The total power generated by the public utilities alone, during 1937, was about 125 billion kw-hr. A difference of 1 mill per kw-hr represents a sum of $125 000 000. This indicates that power use in the United States has reached such proportions that small savings in generation are extremely important. However, it is much more important that an abundant and reliable supply of power be available than a source of cheap power which may be unreliable.

By far the greatest power capacity in this country is in the automotive field, probably about 1 billion hp. The power produced by this automotive equipment can certainly not be classed as cheap power, but automobile and airplane engines to-day furnish a supply of reliable power, a condition one would scarcely care to change if reducing the cost of this power resulted in less reliability.

Cost comparisons have often been made between power generated in different parts of the country and between steam and water power. Assuming that all other factors affecting cost of power are considered, such comparisons may be very misleading unless a careful analysis is made of the quality of the power supply. Quality embraces such matters as continuity of supply throughout the seasons, outages of short duration, speed and voltage control, etc. In making power-cost comparisons, the quality of the power supply must be given due consideration.

Power cost is a relatively unimportant item to many industries but the continuity and quality of power supply may be extremely important when profitable business is being processed. Reduced production or spoiled materials are more important than cost of power to most industries.

The ratio of power cost to the total cost of production is small in most industries. It is less than 10% of the total cost of manufacturing in all except perhaps ten or a dozen industries. In many industries the cost of power is less than 1% of the cost of manufacturing.

The ratio of the power cost to the total value of products for 100 different industries averages about 2.64 per cent.[30] The following information on the cost of power was reported by one newspaper:[31]

> "A shade over 10 per cent of the total cost of manufacturing our paper goes to the Ontario Hydro-electric Commission at Niagara Falls for purchased power, and a shade less than 9 per cent covers the cost of making the paper and its delivery in Chicago. * * * If our mill was fully equipped with modern machinery, the cost of power could be brought down to about 6.6 per cent. * * * The ink factory power cost is 2.1 per cent of the cost of manufacturing. * * *

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[30] *Magazine of Wall Street*, 1926, Vol. 38, p. 568.

[31] *Public Service Magazine*, Vol. 53, No. 4, October, 1932, pp. 101–102.

In the Tribune Tower Building all electricity for lights, elevators, etc., etc., amounts to 1.5 per cent of the total revenue received from the building. * * * The cost of electricity for operating the Tribune WGN Radio Station and Studio is 2.5 per cent of the total cost of maintaining the Studio. * * * Electricity for the warehouse and dock amounts to 0.7 per cent of the total cost of operating the property."

Power is one of the few commodities the cost of which has steadily decreased while the cost of other commodities has been doubled and trebled. Even during the 1929 depression when the demand decreased materially, and in spite of ever-increasing taxation, the cost of power to the consumer was steadily reduced. The Consolidated Edison Company of New York, serving both gas and electricity to the great metropolis, reports that its tax bill is now (1938) about $1 000 000 per week, or $52 000 000 per yr. It is the largest single item of expense except labor.

The tax increase in recent years is astonishing—as much as 70% in seven years. In 1930, taxes took 12 cents out of every dollar of income. In 1937, taxes took 20 cents of every dollar paid by the consumer for service.

The rates charged by the Company have been lowered sharply in these years, so much so that the bill to consumers was $44 000 000 less in 1937 than it would have been on 1929 rates; but if the Company paid no taxes and the entire tax saving were passed on to the consumers, their bills could be reduced 17% more.

Power production as an industry was practically unknown until the efforts of Edison, Thompson, and their contemporaries, established electricity on a commercial basis about 1883, and it was not until about twenty years later that records were obtainable for comparisons as to the cost of power, its use, and its influence upon the welfare of the country.

TABLE 12.—COST OF POWER, SHOWING GROWTH OF THE POWER INDUSTRY IN THE UNITED STATES

Description	1912	1922	1936
Generating capacity, in kilowatts	5 165 400	14 313 350	34 466 000
Invested capital, in thousands of dollars	2 062 655	4 817 000	13 300 000
Thousands of kilowatt-hours of energy generated	11 569 100	44 422 000	108 854 000
Operating cost, in thousands of dollars*	151 270	533 068	934 200
Taxes, in thousands of dollars	13 147	73 772	272 258
Number of domestic customers	3 101 000	10 211 230	21 648 300
Kilowatt-hours of energy sold per customer	264	359	720
Average rate, in cents per kilowatt-hour	8.9	7.4	4.74
Number of people living in electrically lighted houses	14 000 000	44 000 000	89 700 000
Percentage of population	16	39	70
Ratio of investment to receipts	6.85	4.50	6.00
Investment, in dollars per dollar of annual taxes	157.0	65.50	49.0

* Does not include fixed charges.

The economic aspects of the cost of power may be reflected by a few comparisons (see Table 12) which show the growth of the power industry in the United States during the last half century, the increasing additions of the invested capital to the national wealth, and the increasing contributions through taxes to the support of the municipal, State, and Federal Governments.

CONCLUSIONS

Some of the more important conclusions may be summarized as follows:

(1) The total cost of producing power consists of fixed charges and operating costs. Fixed charges are the principal cost in producing hydro-electric power and are also a major item of cost in producing steam power.

(2) The capital cost of power plants, both steam and water power, is variable, depending upon many factors, but principally on the location and nature of service to be rendered.

(3) The cost of power generation is also variable, depending principally on the cost of fuel and the output of the plant in the case of steam plants, and upon the capital cost and the available stream flow in the case of hydro-plants.

(4) Reliability of power supply is an important factor which may affect both the capital and operating costs of power plants.

(5) Steam and water-power plants are seldom economically competitive but are ordinarily complementary if used to best advantage.

(6) When investments are made in large new power plants, the output can seldom be absorbed immediately and, in the case of some of the large hydro-plants, not for many years. In such cases the yearly fixed charges and maintenance costs on the unused part of the plants, less credit for increased economy of operation, if any, must be added to the capital cost of the plant until it is fully utilized.

(7) Particular attention should be called to the difference in cost of steam power from plants designed with full auxiliaries and reserve capacity, and those designed for less reliable service.

(8) Abundance of reliable power is of much greater importance than cheap power from unreliable sources.

(9) The power industry is of increasing importance to the welfare of the nation and has made tremendous contributions to its wealth and income.

DISCUSSION

Joel D. Justin,[32] M. Am. Soc. C. E. (by letter).—It is hoped that Mr. Whitman's paper, as well as others included in this Symposium, will help to dispel the romance which so often obscures discussions of economics in relation to power. If the public is to secure power at the minimum practicable cost, certain simple rules of economics should govern the undertaking of new projects regardless of whether private capital or Government capital is involved. Thus, if an ill-advised capital expenditure is made, the cost of power is increased, and the public pays for it either as consumer, taxpayer, or investor. Some of these simple rules of economics in relation to power supply are discussed herein.

The capacity available in any given power system should at all times exceed the existing load by a margin (reserve capacity) sufficient to insure continuity of service and, in addition to the foregoing, there should be, as nearly as may be, just enough "surplus capacity" to take care of the growth of load until the next contemplated increment of capacity can be installed and placed in operation. To maintain a proper balance between available capacity and load it is necessary to plan. "Planning" to supply future needs is not an innovation of the last few years as some seem to think. The power companies and, in fact, most industrial corporations have done it for decades. Many of the public utility corporations maintain groups of engineers whose sole function is to determine as nearly as possible the future needs of the system and "plan" to meet them in the most economical manner.

When additional capacity is required its installation should be predicated on thorough economic studies by competent engineers in order to determine the capacity required, the most desirable and economic location for it, and also the kind of capacity that should be installed—that is, whether it should be steam, Diesel, or hydro-electric power. Such a study will involve not only the general design and cost estimates of proposed plants, but also the problems of transmission, distribution, and sub-stations. Sometimes, it will be found that although a huge new steam turbine in a central plant would produce the required additional power at a lower cost at the switch-board, nevertheless, when transmission and distribution are considered, it may be found that the additional annual cost will be a minimum if several smaller, less efficient, units are added at a higher cost per kilowatt in remote plants at local load centers.

Although it may be surprising to some engineers, it is believed that economically the Diesel engine may perform an important function in some large power supply systems. Thus, in the case of a long distribution line, the end of which is many miles from one of the large central stations of the system, if the load on this distribution line increases greatly, the increased demand may be met by enlarging the capacity in the central station and the distribution line. Sometimes, however, it is found that the alternative of installing a

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[32] Cons. Engr., Philadelphia, Pa.

Diesel plant out near the end of the line is more economical because it is then not necessary to increase the investment in the line.

When considering whether or not a hydro plant, or a steam plant, should be installed, many factors must be given consideration if the most economic solution is to be reached. Mr. Whitman has stated some of these factors. The capital cost of a proposed hydro plant may be so great that the average cost of energy produced by it is greater than that of any steam plant in the system but, nevertheless, by reason of its peak-carrying capacity, such a hydro plant may be the most economical increment of capacity that could be installed in the given situation. One engineer referred to this problem as the hydro-paradox. In other words, it costs more, but it is cheaper, sometimes.

Some engineers, charged with the responsibility of planning development to meet future power requirements, have a leaning toward the biggest and best and may take pride in feeling that their system has the most efficient and largest units. A large unit with a minimum production cost per kilowatt-hour is not always the most economical increment of capacity for a given system. For one thing, fixed charges on unneeded capacity may more than eat up the saving in production cost. The use of extremely large units also increases the reserve capacity which it is necessary to have. A reserve requirement, which has been found suitable in many systems, is that the available reserve at any time should be either equal to the largest unit, or to 10% of the load, whichever is larger. Thus, for a 165 000-kw unit, there should be at least 165 000 kw of reserve capacity ready to go on the line in case of a break-down. For a system having a load of more than 1 000 000 kw, this may be no hardship, but for a system having a load of, say, 200 000 kw, it means that units of such great size are entirely uneconomical in spite of low production cost per kilowatt-hour.

When an additional plant is required, that increment of capacity (usually either steam or water power) should be selected which will result in the total annual cost of power supply for the entire system being lower over a term of years than it would be if any alternative increment of capacity were added.

Throughout most of the United States the development of power systems on an economic basis has resulted in the inter-connected combined power systems which Mr. Whitman has discussed. The power supply of these systems is derived from both steam and hydro-electric plants.

Even some engineers are prone to discuss the thesis: "Is steam power cheaper than water power?" Such questions should be relegated to the high school debating societies because no general and truthful answer is possible. In sections where topographical, water supply, and load conditions are favorable, it is usually economic to supply a part of the requirements of the increasing load by the construction of hydro plants. The peak-load carrying advantages of water power are so great that it appears improbable that any near-by development will crowd the hydro plant out of the economic picture. Thus, in 1922, 31.0% of the total capacity of privately owned public utility plants was hydro capacity. In 1937, the corresponding value was 27.5 per cent. The latter may have been adversely affected by the extensive construction of water power plants by the Federal Government.

Power supply investigations for various systems have convinced the writer that, for most territories, there is a fairly definite economic hydro ratio which may be defined as that ratio of hydro capacity to annual peak load that will give a minimum total annual cost of power supply for the given system.[33] This economic hydro ratio may range from 100% (as in some sections of Canada where fuel is very expensive and suitable hydro sites with abundant water supply are common) to a very small percentage in territories where fuel is very cheap and where favorable water-power sites are almost non-existent. The interesting fact is, that for considerable territories covered by inter-connected systems, the economic hydro ratio appears to be quite constant for that section.

Mr. Whitman makes the statement in connection with a Government hydro project (see heading, "Federal Hydro-Projects") that "it was testified that the Government would pay only 3% for its money, therefore, the rate of return required would be only this rate; whereas, private companies had been compelled to pay about 7% for their money." When all factors are considered, 3% Government money is in reality somewhat more expensive than the 7% private money required by privately owned utilities for capital expenditures.

In order to determine the true cost of money for a proposed Government project some such set-up as the following should be made. (It is herein assumed that Government construction and operation are just as efficient as if the project was privately owned and that in each case the proposed project would be loaded to the same extent):

Cost of Money to the Government for a Power Project Amounting to $1 000 000 (The Term, "Government," as Used Herein Means All Government—Federal, State, and Local):

(1) Interest on Government investment in power project:

3% on $1 000 000 .... $30 000

(2) Taxes on privately owned public utility which Government will have to forego if it builds and operates the project itself; 15% of revenue (in many cases this is more than 16%) which, on a 1 to 6 ratio would be:

15% of $166 700 .... 25 000

(3) Individual Federal income taxes of investors on dividends and bond interest of alternative privately owned project which Government would have to forego if it builds and operates the project itself. Since such investors are ordinarily in the higher than average income groups, this will probably average 15% of the 7% return received by them, or,

1.05% of $1 000 000 .... 10 500

(*Carried forward to next page*).

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[33] "Power Supply Economics," by Joel B. Justin and William G. Mervine, John Wiley & Sons, Inc., New York, 1934, p. 142.

(4) State and local taxes on the income of investors which Government will have to forego if it builds and operates the project itself. In Pennsylvania, this takes the form of a personal property tax, 0.4% State plus 0.4% county on values, or, 0.8% on $1 000 000	8 000
Total annual cost of money to Government for $1 000 000 power project	$73 500

Hence, on this basis, the true cost of money for the Government project is 7.35% as compared to 7% for the privately owned project.

Item (3) is the writer's estimate as he does not know of any statistics on the subject. In the case of any individual investor, this item would be the highest increment of the income tax which he pays. There is also the question of multiple taxation in some cases which would be very difficult to evaluate. Item (4) is precise for the Commonwealth of Pennsylvania, except that it does not apply to the securities of corporations incorporated in Pennsylvania, but may vary greatly elsewhere.

Although it is believed that the foregoing tabulation approximates the truth in many cases, it is presented merely to illustrate one of the set-ups which the Government authorities should make as the result of extensive study to determine whether or not it is advisable to construct a Government project. This statement assumes, of course, that the only reason for the Government to build a power project would be to obtain power for the people at the lowest practicable total annual cost.

Frank H. Mason,[34] M. Am. Soc. C. E. (by letter).—Each author of this Symposium has performed a service in helping to give a broader view of the subject. Professor Barrows covers the field of hydro-generated energy with the exception of pumped storage plants, which are not common in the United States. This paper is valuable in revealing such variations in the cost and the output of different types of plants as the following: The higher head plants normally cost less; the capacity and capacity factor have a great influence on the cost of the plant and the cost of power; and, the increase in size of the plant generally lowers the cost of operation and maintenance in terms of the capacity or the output.

Professor Barrows also gives the reason for, and value of, over-capacity of a plant to some systems, and the advantage of using water power for peak capacity in a combined hydro-electric and steam system. He gives a summary of what other engineers have done and think, as a comparison and aid. Even a casual reading of his paper shows that the variations in cost of a hydro plant and the output are dependent on the locality, the type of plant, and the object of its construction.

Too many people in the past have sought, or quoted, an average price for construction and for generation without knowledge of the requirements of the service, or the location of the plant. The Government proposed to have a yardstick for the measurement of cost. The examples, either because of their large size, the subsidized cost, or the lack of information on the cost, have failed to furnish material for a fair comparison.

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[34] Chf. Engr., New England Public Service Co., Augusta, Me.

The fifteen years since 1923 have brought about a remarkable advance in the type and knowledge of equipment for hydro-generation and power development construction. Who has brought this about and paid for the cost of present-day advancement? Certainly no one but the private company, the manufacturer, and the private laboratory. If the private company could rebuild its plant with present, up-to-date equipment at subsidized values and with low interest rate, and then operate without much, if any, taxation, a low cost would be obtained; but, after all, the real costs depend on the sale of output and not on a plant's capacity to generate power. Some of these Federal plants for years to come might just as well be back in the woods of Canada because the plants will not have a sale for power. Consequently, the actual output costs will rise, or there will be years of fixed charges to be met.

It has taken years of time and much expense to acquire loads, and to perfect equipment. Even now, the increase in demand for a better and surer type of service, the unknown question of obsolescence, and especially the increased taxation, leave the future unknown. It is certainly a proper time for a complete review of the subject of power costs.

W. V. BURNELL,[35] ESQ. (by letter).—In his paper, Mr. Scharff has made a distinct contribution to the troublesome subject of depreciation. He has lifted the veil of mystery that has long enshrouded the subject by demonstrating that many facts, which hitherto have been determined largely by judgment, are, after all, susceptible of reasonably accurate and reliable determination. If the business of the electric industry is to be governed hereafter in accordance with the facts concerning existing depreciation, much is to be gained from a careful consideration of Mr. Scharff's proposals.

In any forward view of the matter, however, it is incumbent upon the management of electric utilities to inquire also into the regulatory aspects of the problem. No matter how sound and well supported a professional determination of the facts may be, it must first be accepted by the regulatory body and the Courts, before it can safely be adopted by a regulated utility as a business policy.

As to the attitude of these regulatory bodies, if one may judge from the revisions that have been made recently and which are currently being made in the accounting systems which they prescribe, it is leaning definitely in the direction of a radical departure from previous practice. There is ample reason for the conclusion that the commissions, generally, are now in favor of a depreciation policy which is essentially actuarial in nature and based on some application of the principle of life expectancy. Along with this trend of thought, there goes the parallel conviction that the depreciation policy which is to be adopted for the purpose of utility accounting should correspond with the basis adopted for the determination of permissible income.

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[35] Appraisal Mgr., Stone & Webster Eng. Corp., New York, N. Y.

Depreciation is defined as loss in value. The purpose of depreciation accounting is to provide a means by which the losses may be recouped by the investor. In principle, it matters little how the recoupment is provided for, just as long as the investor receives the income to which he is entitled during the life of the investment and the principal when the income terminates.

Until recently the electric industry, in accordance with the privilege extended to it by most of the regulatory commissions, has seen fit in most cases to follow the retirement reserve policy. To a considerable extent this policy has relieved the present customer of depreciation charges to compensate for losses in value which, although anticipated as a future probability, have not yet occurred. Under this policy, some of the losses brought about by improvements in the art have been assessed against the prospective beneficiaries of those improvements. The tendency now seems to be to require the present customers to accept a much greater share of the burden.

In this apparent transition from one depreciation policy to another, the industry is confronted at once with two important considerations. In the first place, there is a disposition to make a retroactive application of the new rule of the game and, by this process, to assess against the utility a loss in value for which the present investor would have been compensated if the new rule had been in effect hitherto. This is not "cricket" by any means, but the industry nevertheless is confronted with the danger. In the second place, the newly adopted rule implies the necessity of increased annual appropriations to the depreciation reserve and a relatively higher schedule of rates to provide for them.

As to the question of retroactive application of a new rule which deprives the industry unfairly of some part of its property value, there seems to be no available alternative except for the utilities to insist upon their rights. As to the policy to be followed in the future, however, the industry has been "placed on notice" that the old rule may no longer be honored. It may be that a contention can be made successfully against the new rule, but all will agree that at least there is considerable doubt about it.

It may be stated also that the industry is confronted with a number of new competitive aspects of the situation which are purely economic in character. Aside from any such economic aspects, however, and in the light of the regulatory trend alone, would it not be the part of wisdom for the electric utility companies to "drop an anchor to windward" by a careful analysis of their operating charges for depreciation, and also as to the sufficiency of their charges to the customers for service to provide for the higher cost of power which may result from the new regulations?

DANIEL W. MEAD,[36] PAST-PRESIDENT AND HON. M. AM. SOC. C. E. (by etter).—In 1924 the writer was drafted by the Power Division of the Society to prepare a paper on "The Economics of Hydro-Electric Development"[37] for the Spring Meeting on April 9, at Atlanta, Ga. For simplicity the analysis was confined to single independent hydro-electric developments and should perhaps be somewhat modified when considering developments in connection with extensions of properties already in operation and with combined projects having two or more objectives. It is obvious that, when considering new developments made in connection with extensions of power properties already in operation, conditions are somewhat changed. For example, the long and expensive process of developing a market may be entirely eliminated because the power of the new plant, upon completion, may be turned at once into an operating distribution system. In combined properties having two or more objectives, the distribution of first costs and operating costs may be very complex and indeterminate. Appendix A of the writer's paper[38] includes a simplified analysis of "Factors of Economic Expediency"; Appendix B, "Capitol Costs"; and Appendix C, "Annual Expenses."

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[36] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

[37] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 158.

The writer has re-examined these analyses and finds no additions which he would make at this time and nothing which he would eliminate. Similar outlines for plants which develop power from heat sources can readily be made. The writer believes that very little difference in opinion is possible among power engineers as to what should, in the general case, be included in such an analysis.

In view of the discussion now before the public as to the actual and relative costs of hydro-electric and steam-electric power, and in view of the pretense of the Federal Administration to establish so-called "yardsticks" to measure the legitimate cost of power as generated by public utilities, it seems fortunate that the subject of power cost is now open to discussion in the form of the present Symposium.

To those familiar with power-cost analysis the subject cannot be regarded as greatly complicated; but to those unable to make such an analysis, the subject is complex on account of the numerous factors which enter into its consideration, all of which must be properly evaluated in order to determine the proper answer to any particular problem.

No simple rule, measure, or criterion can be applied under the great variations in circumstances that occur when the entire United States is considered, and often wide variations of conditions occur within quite narrow geographical limits. This is particularly true when comparing public and private power plants as to first cost and cost of operation.

Mr. Page indicates some of the differences in cost between power plants constructed by public and private agencies such as "taxes," "franchises," and "State and Federal income taxes." The matters of "interest on first cost" of the project and of "interest during construction" are also of great importance and are seldom considered in Federal projects. Even if they are considered, the considerable difference in interest rates (often urged as a condition favorable to public ownership) would make plant and power costs not fairly comparative.

It is unfortunate that the public, including both State and Federal legislators, seems to regard this matter of power costs as comparatively simple and easily determined because, in fact, it is a problem for the trained power analyst. The idea of the average legislator on this subject is illustrated by the following example: About 1910 the State Legislature of Wisconsin passed a water-power law by which an attempt was made to confiscate the privately built water powers of Wisconsin on the ground that they were natural resources which legally belonged to the State. Prior to the passage of this law, extended hearings were held by a joint committee of the House and Senate of the State and the writer spent considerable time in trying to explain to this committee the basis of hydro-electric and steam-plant power costs. Among the questions asked the writer, was: "How much does it cost to develop a horse-power from water?" The writer explained that such cost would vary largely with local conditions, size of plant, nature of equipment, and the point at which the resulting power is to be measured. For example, the cost of installation might vary from $35 per hp, measured at the turbine shaft of a high-head impulse wheel in the Far West to $350 per hp, for a low-head plant in the East with power measured at the switch-board of the customer, say, 100 miles from the power-house; and that similar differences in steam-power cost depended on the size and character of equipment and the great variation in cost of fuel and labor at different points in the country. The writer thinks that it was the opinion of at least the so-called "progressive" members of the Committee that he was endeavoring to obscure the issue rather than to clarify it; when, in fact, he was doing his utmost to present this somewhat complex subject fairly and in the simplest language possible. He explained that the question is similar to such questions as, "What does it cost to build a house?" and can be answered only with specific conditions in mind.

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[38] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 186.

Assuming that the writer's estimate of hydro-electric costs is fairly accurate, at least for the purpose of illustration, it is quite evident that the $35 per hp cost of power development at the turbine shaft in the Far West, high head plant is not an honest nor a far "criterion" or "yardstick" for the cost of power development from a low-head plant in the Central or Eastern States where the actual cost of development was $350 per hp, delivered at the switch-board of the customer 100 miles distant. Neither is the reverse true.

In discussing "Public Ownership of Power" and the so-called "yardstick," Arthur E. Morgan, M. Am. Soc. C. E., states[39] that "it is imperative that records and accounts be honest, and fair and open, and that there be no hidden elements of subsidy. The very fundamental element of such comparison is honesty, fairness, and openness in measurement." He might have added that the circumstance of development of all plants compared to this "yardstick" should be essentially similar or identical.

It is obvious, of course, that all plants competing for business in any given community must furnish power at a competitive price regardless of their cost of construction or of the cost of delivering power. If they cannot compete they must go out of business. There cannot be any real competition between Federal and privately owned plants, however, as the latter are regulated and controlled by the Government and can be eliminated at any time by Government fiat in which the factors that of necessity control private investments may receive little consideration. The United States attorneys on the case between the TVA and the private utilities which was heard at Chattanooga, Tenn., in 1937, stated that, if Congress so directs, the Federal Government can furnish power for nothing. This is doubtless true, but in so doing the Administration can scarcely expect engineers or even the public to accept such prices as a fair and honest "yardstick" for the measurement of public utility charges.

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[39] *Atlantic Monthly,* September, 1937, p. 342.

The so-called "yardstick" is purely fantastic and actually meaningless except as a political catch word or to the man who assumes that the problems of the cost of power plants and of power generation are simple, and of such uniform and constant character under all circumstances that a criterion can easily be established that is widely applicable. This idea is entirely erroneous. All power engineers know that, in general, the only way to determine with any degree of accuracy the fair cost of a power plant or of power generation is to determine all the circumstances under which the power is generated, delivered, and used.

The determination of the cost of a power plant and the fair price for the power which it furnishes, by building a competing plant under entirely different circumstances and operating it together with many other objectives whereby both the first cost and the cost of operation will be hopelessly obscured, is an idea which would never occur to an engineer. Such a method of determining power costs is perhaps fairly comparative to the way in which most political questions are now solved.

While the determination of power costs is complicated to the layman, the principles involved are so simple that in the main the writer would expect little difference among qualified engineers as to those matters which must enter into any fair consideration of the costs from any single plant or combination of plants. Differences in opinion concerning power cost arise, however, not in the theory but in the application of principles of concrete cases.

When a hydro-electric power development is only one of a number of objectives undertaken in a common development and it becomes necessary to separate the costs of power development from other objectives, real or imaginary, that are included in the combined project, there probably always will be difference of opinion, even among engineers, as to the proper proportion of costs to be distributed to the various objectives. Fair and just proportion of charge, which should be assigned to any one objective, cannot always be fully demonstrated and will always result in a wide difference in opinion.

For example, in the Grand Coulee project on the Columbia River, in Washington, the objectives are said to be navigation, flood protection, irrigation, and power. In the Tennessee Valley the objectives are said to be navigation, flood protection, power, reforestation, erosion prevention, the manufacture of fertilizer, and national defense. In each case power is assumed to be incidental to more important objectives that are legitimate undertakings of the Federal Government, which cannot undertake power development as a main or principal objective. It seems quite evident that in all cases where more than one objective is declared in the enabling act, the distribution of fair and just costs to each objective is difficult, if not impossible, and must to a large extent, be, and remain, uncertain. For this reason the attempt to establish, as a criterion or "yardstick" for measuring either the cost of development or the cost of power, any power plant having an uncertain cost of construction and an equally uncertain cost of operation is fallacious. Such a criterion is of no value in measuring the true or fair cost of development of the cost of power for plants built under quite different circumstances.

It is evident that there may be various points of view in particular cases. If, for example, the present Tennessee Valley improvements, in their present form, were needed and warranted for flood-protection and navigation purposes, and after construction and completion, power plants were added, the fair cost of development might include only power-house equipment, transmission, etc. Such cost certainly should include interest, engineering, supervision, freight and mail subsidies, etc. Even then it could hardly be considered a fair "yardstick" to measure the cost of power to an independent development where power only is the main objective and many other factors are involved.

In the Tennessee River case the high dams are probably an actual detriment to navigation when compared with low dams built purely for navigation purposes, and have only a limited value for flood protection. As a matter of fact, the height of dams was undoubtedly determined from the power standpoint. Suitable navigation and all flood protection warranted by the condition on the Tennessee River could have been furnished at a fraction of the cost which has apparently been charged to these objectives; and the objective of water power apparently has been charged with less than one-third the costs which the writer believes should be fairly attributed to it.

Neither the Federal Administration, Congress, the State legislatures, nor any other authority, corporation, or individual, can in any way establish a fair and just measurement of what power will cost, or the fair and equitable price at which it should be reasonably sold, except by considering the individual circumstances of construction and operation.

In other words, the so-called "yardstick" for the cost of power is a purely fantastic and impossible ideal, and when attempted is only fairly applicable to the plant or combination of plants created for such an unnecessary purpose. Such a term sounds well to the layman and, therefore, is useful to the politician to mislead the people and to secure popular approval; but it is practically meaningless to the engineer except perhaps in the case of plants when the conditions of duplication may be so uniform that almost identical results can be expected to obtain.

False estimates of the cost of power for either Federal, State, or private power plants will only deceive the engineer where costs are unknown or where "hidden elements of subsidy" cannot be determined. A clear and honest statement of facts and conditions or an investigation of these conditions, if permitted, will demonstrate to any qualified engineer or board of engineers, the fair cost of a power plant or power plants and the fair cost of power which such plants can or do deliver to their customers.

On the other hand, by unfair and dishonest statements and inaccurate or erroneous records of conditions and costs, the public may be deceived—"all of the public part of the time and part of the public all of the time."

The writer agrees with Mr. Page that it is time for engineers "to assist, honestly and whole-heartedly, in seeing to it that the public weal is placed foremost among the objectives of the power industry." He also believes that there is no less of a duty due from the administration and its officials to deal frankly, honestly, and intelligently with the public.

Mr. Page's paper is a frank and fair statement of the factors that enter into the problem of power costs, and the writer finds little to which he desires to take exception. He is unable to agree, however, with the apparent assumption that many of the "large Federal multiple-purpose projects" are warranted at this time, even on the basis of doing something possibly to aid in recovery from the depression. To add to the burden of the taxpayers of the nation the enormous sums made necessary by many of these entirely unwarranted projects can only prolong the depression and make recovery more difficult. The lack of information concerning these Federal Government multiple-objective projects and district power plants leads to the conclusion that they have not been duly considered but have been hastily adopted as projects which would furnish opportunities for labor and might possibly aid in recovery from the depression. It is evident that in most, if not all, cases there has been no proper and detailed consideration of their plans, cost, desirability, necessity, or soundness prior to their adoption and authorization. Under such conditions it must be expected that many ill-conceived and unwarranted projects have been included and that great waste will be entailed.

Undoubtedly, much misinformation has been issued on the subject of power costs—some of it propaganda, to mislead the public for or against either private or public ownership and operation of power plants. Some of the information on this subject is furnished without sufficient explanation to limit its use in a proper manner.

A committee of the U. S. Chamber of Commerce in 1936 published a table[39] entitled "Federal Power Projects" which gives: (1) The name of the project; (2) the approximate expenditures to June 30, 1935; (3) the ultimate cost (estimated); and (4) the ultimate horse-power (estimated). Any one familiar with power matters should know that all these projects are multi-objective in character and a reading of the report so indicates; yet in a paper read at the Sixth Midwest Power Conference held in Chicago, Ill., on April 22, 1936, this table was taken, unmodified, as a basis for estimating the costs of Federal hydro-electric power plants and power costs. This unwarranted use of the data destroys almost the entire value of an otherwise excellent paper and leads to suspicion of intended propaganda adverse to Federal hydro-electric power projects. The paper is so denounced in a "letter of submittal" and in a report sent to the President of the United States by the Chairman of the New York Power Authority[40] in which the public development of hydro-electric power projects is presented in an equally fair and logical way.

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[39] Report of Committee on Water Resource Policies, March, 1936, p. 22.

[40] H. R. Doc. No. 52, 75th Cong., 2d Session.

Every power engineer knows that the profession is entirely ignorant of the probable cost of those Federal Power Projects, much less as to what costs will ultimately be charged to them by the Federal Power Authorities. It is doubtful if the public or even the Engineering Profession will ever know what fair cost should be attributed to these power installations or power costs in these Federal multi-objective installations where many of the objectives are largely imaginary and often introduced only to assure the legality of Government development.

The writer believes that the question of hydro *versus* steam power plants can never be settled by such general one-sided and unfair presentation based on unknown or imaginative data. All power engineers know that each problem is a question of local conditions and must be studied and solved as such. After all, the power market or the probable demand for power should be the true reason for any installation and this important matter is ignored in many of these discussions as though power were of value, whether used or not.

Even if the first cost of the development of these projects could be established, it is certain that quite a different result would be found if the delays and consequent cost of market development were considered. Many a private development has failed of success due to long delays in the establishment of a profitable market. In Federal projects such expenses seem never to be considered, as they can be met at the expense of the taxpayer, who will probably never appreciate how much he is called upon to pay for the development of unwarranted and ill-advised Government projects, not only for power but also for navigation, irrigation, flood protection, and recovery.

It is certainly true that honesty, intelligence, frankness, and fair dealing should be introduced into all public and private relations; for to the present time neither Government authorities nor business interests have a monopoly on either honesty and frankness or misrepresentation and propaganda.

A. G. CHRISTIE,[41] ESQ. (by letter).—The elements entering into the cost of heat-generated energy have been clearly described in the paper by Messrs. Hirshfeld and Van Duzer. Their cost estimates represent a painstaking analysis of available data and a careful selection of average values for inclusion in the curves. Messrs. Hirshfeld and Van Duzer leave little for discussion other than to amplify some of the factors they have considered. They emphasize the fact that any costs presented in the paper are not representative of the industry as a whole but are only the logical deductions that one derives from the data assumed in each case. They stress the fact that conditions of location, load, and fuel vary so widely that costs under the several conditions cannot be equal; nor are costs truly comparable without consideration being given to correction factors for these variables. The methods of applying corrections are also stated.

Consider fuel costs, for example. Messrs. Hirshfeld and Van Duzer assume 15 cents per million Btu. In many localities fuel may be obtained for about 10 cents per million Btu. In such a case, the fuel cost would be much less than that assumed by Messrs. Hirshfeld and Van Duzer for the same station efficiency; but there will be some question as to whether the expected capacity factor of the plant with this low priced fuel would warrant as efficient and as expensive a plant as Messrs. Hirshfeld and Van Duzer assume. If a less efficient and less expensive plant is used, both fuel costs and fixed charges will be affected. At the other extremity, the writer has designed a plant for a foreign location where the fuel cost was about 50 cents per million Btu. Considerable money was spent to secure maximum efficiency from the equipment installed at this plant. A diagram[42] published in January, 1938, shows the remarkable reduction of coal per kilowatt-hour over the last few years. The curves indicate large increases in total kilowatt-hour output with almost stationary total coal consumption.

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[41] Prof. of Mech. Eng., The Johns Hopkins Univ., Baltimore, Md.

TABLE 13.—UNIT COSTS OF THREE TYPICAL STEAM-ELECTRIC POWER PLANTS

Item No.	Description (1)	Cost,* in Dollars per Kilowatt of Capacity: Plant A (2)	Plant B (3)	Plant C (4)
1	Plant capacity, in thousands of kilowatts	70	70	185
2	Number of units	2	2	6
	Cost per Kilowatt of Capacity, in Dollars:			
3	Land, fill, etc.	4.55	2.94	12.60
4	Building substructure	15.03	5.33	4.97
5	Building superstructure	3.17	8.95	21.40
6	Building trim	1.15	0.88	1.78
7	Building equipment	0.45	0.35	2.72
8	Mechanical equipment	24.45	22.38	30.03
9	Electrical equipment	24.90	20.56	28.68
10	Service equipment	7.85	2.80	16.45
11	Substation	4.16	1.40	10.67
12	Engineering and construction	27.12	17.46	20.42
13	Total cost per kilowatt	112.83	83.05	149.72

* Except Items Nos. 1 and 2.

Referring to investment costs, Table 13[43] gives data on the cost of three different stations. Plant A, situated on the Ohio River, has the lowest coal cost but the highest cost of substructure (probably due to the incidence of floods). Plant C, in a locality with high land values, has also the highest coal costs. Plant B has fuel costs intermediate between Plants A and C. Note that: (1) Building substructure varies from $4.97 to $15.03 per kw; (2) building superstructure from $8.17 to $21.40; (3) mechanical and electrical equipment from $42.94 to $58.71 per kw; and (4) total cost from $83.05 to $149.72 per kw. Attention is drawn to such actual values to demonstrate the influence of such different factors as location, fuel costs, and foundations upon individual items of plant cost and consequent influence upon total plant cost.

Plant locations are often chosen for reasons that outweigh sub-soil conditions. Thus, Cahokia Station, at St. Louis, Mo., is supported upon very long piles, whereas the Avon Beach Plant near Cleveland, Ohio, is built upon solid rock which had to be blasted away to make room for proper cooling water intakes.

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[42] *Power Plant Engineering,* January, 1938, Fig. 1.

[43] "Power Supply Economics," by Joel D. Justin, M. Am. Soc. C. E., and William G. Mervine, John Wiley & Sons, Inc., New York, N. Y., 1934.

It has been suggested that the total cost could be decreased and the fixed charges lowered by the elimination of the building superstructure. This part of the plant cannot be eliminated entirely for obvious reasons. In certain localities some savings may be effected by this change. In an Australian plant designed by the writer only a roof is provided over the plant to keep off the sun and shed water during a few weeks per year of rain. Outdoor plants are not justifiable in cold climates.

Sometimes plant location and resultant construction cost are fixed by factors other than simply the supply of power. Thus, all plants supplying both power and district heating are located and planned with reference to the heating load. A unique case of the influence of external factors occurs at the power plant in Edmonton, Alberta, Canada, which is adjacent to the city's water-works and filter beds, on the Saskatchewan River. Water is pumped through the surface condensers of the power plant before delivery to the settling basins and filters of the water-works. In winter, vacuum is regulated to provide water at 50° F to 60° F to the settling basins as desired. This water does not cool below freezing in passing through the filter beds and in the pipes under the streets to the various consumers. Although low temperatures are experienced at times, this waste heat from the exhausts of the steam turbines in the electric plant prevents losses from frozen water pipes throughout the city, and even aids in better functioning of the sewers and sewage disposal plant. This system has also been embodied in the new plant now building at Edmonton.

Another factor in the initial cost of plant is the degree of expediency in its construction. Overtime added $8 per kw to the cost of one plant with which the writer was associated, owing to the urgent need of this plant capacity, and union rates for overtime work.

These facts support the contentions of Messrs. Hirshfeld and Van Duzer that every station is an individual problem and that the designer of a power plant must analyze and balance the effect of cost variables before arriving at a design that will produce power for the least total cost.

Even when such a plant is completed the costs secured may be modified by changes in assumed conditions, such as modifications of load curves, changes in fuel costs, inter-connection with other systems, or supersedure, by more economical plant on other parts of the system. For instance, Gould Street Station, in Baltimore, Md., was designed to take a large part of the electric system's base load. However, the subsequent development of the Safe Harbor Hydro-Electric Plant has completely upset the economic assumptions upon which the Gould Street Station was planned.

An analysis of the distribution of costs in the paper for the 400-lb, 600-lb, and 1 250-lb plants, with 40% annual capacity factor and average lower costs, indicates that fuel constitutes approximately 26.2% of costs; labor, 7.5%; maintenance and supplies, 3.6%; fixed charges, 53.7%; and general expense, 9 per cent. The predominance of fixed charges as an element of production cost is evident from such data. Fixed charges may be reduced as already suggested by minimum building structure, simplicity, and standardization of design, by inter-connection, and by avoidance of undue haste.

Messrs. Hirshfeld and Van Duzer indicate that whereas the cost per kilowatt of steam and electric generating equipment has tended to decline, the cost of electrical switching, protective, and transforming equipment has increased, leading to a constant total cost over a period of years. This has been a general experience of plant designers.

It has been suggested that lower plant costs might be secured by the adoption of certain standardized designs for steam plant equipment. Some progress has been made in the matter of standardized turbine sizes and operating conditions in Great Britain, where 650 lb, 850° F, are used widely for plant conditions, with 30 000-kw turbines at 3 000 rpm and 50 000 to 60 000-kw units at 1 500 rpm. Little progress has been made in standardization in the United States owing to the prevalence of superposition, the development of condensing turbines of greater capacity at 3 600 rpm, and the increased pressures and temperatures used in the newer stations. When, and if, these developments are complete, consideration should be given to standardized equipment; but standardization tends toward stagnation. Decreases in the cost of production have followed from pioneering which has always been an expensive undertaking to those who depart from accepted conditions.

Messrs. Hirshfeld and Van Duzer do not consider reserve equipment, which is particularly needed in the case of an isolated plant. When a system has several plants, reserve equipment must be provided in one of these and the cost of maintaining this reserve must be spread over the system output. System inter-connections permit the use of less reserve on an individual system and thus tend to reduce production costs. The authors have confined their paper to production costs only. The public thinks only of the bills they pay. The cost of distribution service and of transmission are usually equal to, or several times greater than, production cost. For instance, the cost of meter rental, meter repair and calibration, meter reading, billing, and collecting is frequently stated as $6 per customer per yr. If this is a domestic customer whose total consumption is 600 kw-hr per yr, the cost of this service alone is 1 cent per kw-hr. In other words, customer service equals or exceeds production cost in the case of many domestic consumers. The consumer's attention should be directed to means whereby these larger items of service and transmission costs may be reduced, rather than to the low charges for production alone. Greater individual consumption by all customers will greatly lower their costs of service.

At the moment, Diesel plants have an advantage over steam plants when small capacities are needed. However, maintenance increases rapidly with age and the useful life is not great.

In the discussion of future possibilities Messrs. Hirshfeld and Van Duzer refer to a new "fuel cell" which is under development. Engineers will be interested in more complete information on this interesting research.

Messrs. Hirshfeld and Van Duzer have presented a clear unprejudiced analysis of the cost of heat-generated energy. This discussion cannot be closed better than by quoting their last sentence: "It would seem that the chances for continued reduction of total power cost at the fuel burning generating plant are indeed very large."

William E. Rudolph,[44] M. Am. Soc. C. E. (by letter).—The Second Symposium on Power Costs is a compilation of valuable data and conclusions as applied to electrical energy generation in the United States. The writer feels that one might go a step further with the discussion by carrying it into Latin-American countries, where important power-plant construction continues in progress, in many instances under the direction of North American engineers and with equipment supplied by North American manufacturers. For, whereas power problems of the more densely populated regions of Latin-America have been solved in much the same manner as in the United States, there would appear to be considerable diversity of practice in interior regions where the demand for power comes principally from the mining industry. In one country—Bolivia—for example, two leading mining companies adopted opposite policies during 1937; one arranged to operate its mines and mills almost entirely by water power, relegating existing Diesel plants to stand-by, "because of the high cost of transporting fuel oil and repair parts"; the other installed Diesel units for operating its principal mill, "to secure maximum safeguard against interruptions of service." Under such circumstances, executives of Latin-American mining enterprises, where expenditures for power generally range between 25% and 33% of the total costs, should find good "food for thought" in this Symposium.

Two important features of the background affecting both capital investments and operating expenses in Latin-American countries are quite different from those in the United States, as follows:

(1) Labor costs are considerably lower. Depreciation of certain Latin-American currencies in terms of the dollar has accentuated this condition in some countries (Venezuela being a noteworthy exception).

(2) Materials costs are likely to be higher, particularly on manufactured products and fuel, because of transportation expenses and import duties.

In mountainous regions away from the coast, these factors contribute to make the comparison between hydro energy on the one hand, and steam or Diesel energy on the other, more favorable to the former; and yet, even here, the two forms of generation are complementary rather than competitive.

A more recent example of hydro-electric plant construction costs is the Kilpani Station on the Rio Yura, in the southwestern part of Bolivia. This installation, with a firm capacity of 4 000 kw, was placed in operation during May, 1937, to supply current to mining establishments at Potosi and Pulacayo, and is typical of plants in mining districts where relatively small tonnages of ores of higher priced metals (in this case, tin and silver) are treated. It utilizes a head of 985 ft, having slightly less than 2 miles of canal, and 4 000 ft of penstock. Its unit cost of $74 per installed kw, exclusive of transmission lines, was divided as follows:

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[44] New Canaan, Conn.

Description	Cost	Percentage of Total
Dam	$ 2	3
Waterway	36	49
Power-house and equipment	27	36
Roads, etc.	8	11
Land and water rights	1	1

All these unit costs are lower than the costs for the composite of seven higher head plants tabulated by Professor Barrows, except that of road work which is four times as large. This outstanding item reflects ruggedness of terrain and inaccessibility of the region in which the plant was built, factors which hold not only in most parts of Bolivia, but also throughout the Andes.

Obviously, lower costs of dam and waterway, as compared to those in the United States, were due to lower wages, which at official peso exchange in Bolivia at that time averaged about 35 cents U. S. currency per day for peons, and about 75 cents for skilled labor. The dam was earth with a clay core, and the canal was masonry and concrete with rock and sand convenient to the site; and these items benefited most from such labor rates. That power-house and equipment costs were lower than in the United States was due to a temporary condition whereby machinery was purchased from German manufacturers at extremely low prices, through the use of export marks. However, inability of these manufacturers to guarantee early deliveries at the present time, and unfortunate experiences with some of the equipment which they have supplied, would indicate that North American manufacturers have in no way lost their South American market. Costs of land and water rights were low because the site was in a region of 14 000 ft altitude inhabited only by Indian communities.

Attendance and maintenance expenses of operating water-power installations in Latin-America are generally lower than in the United States, again because of the lower scale of wages. However, the item of depreciation in the case of plants furnishing power for mining enterprises, must include consideration of the depletion of ore reserves, causing ultimate loss of load. On the other hand, operators of steam and Diesel installations away from the coast must cope not only with first cost of oil or coal, but also with high transport expenses from the nearest seaport. For the greater part mines are located at elevations of 9 000 to 16 000 ft in the Andes, and the crustal movements which here caused rock structures to become impregnated with veins of metals also provided the steep gradients which permit power to be generated from falling streams for extracting such metals. Rise and fall matter little to a high-tension line, but the toll which such rise and fall exact upon the delivery of oil or coal to remote spots in the Andes represents no small item of the cost of producing Diesel or steam power.

The hydro-plant installation in mining regions of the Andes often has the disadvantage of being remote from the center of operations, entailing difficulty of access for replacements or repairs. Hence, unscheduled interruptions, while less frequent than at Diesel and steam plants, are likely to be considerably more costly because of time losses. Destructive landslides and floods sometimes result from heavy showers in these regions of seasonal rainfall, and abnormal droughts also occur at long intervals. Against such emergencies and transmission-line interruptions, enterprises having only a single source of water energy find it advantageous to install additional Diesel or steam units, or, as is more often the case, to hold in reserve the Diesel or steam units which probably preceded the hydro-plant. Most feasible, economically, is the combined installation in which the Diesel or steam units carry only occasional peak loads during normal years when there is plenty of water, but where they carry the base load during dry years when it is necessary to conserve storage for the minimum stream-flow period just prior to the next rainy season. Inter-connected systems are rare in mining regions, due to obstacles of distance; a recent interconnection involved the installation of frequency changing equipment.

Investments in power plants so large that the output cannot be absorbed immediately are rare in Andean mining regions, because of the high rate of return upon capital required for projects in foreign countries. Much of the expansion in South American mining enterprise during more recent years has been paid for out of current earnings, and with the application of this practice to power plants, the large project is likely to be deferred, while a group of smaller projects, although less advantageous in the aggregate, might proceed. These larger projects may not count upon Government aid, as part of public works programs to reduce unemployment, most Latin-American countries lacking both the money and the need for such expenditures.

LOUIS ELLIOTT,[45] ESQ. (by letter).—The analysis presented by Messrs. Hirshfeld and Van Duzer gives as good a view of thermal-plant energy cost as is possible within the limits of a short paper. With the assumptions made, the stated range of energy production costs, as summarized in curve form in Figs. 2, 10, 13, 14, and 16, gives a reasonable idea as to what is to be expected from steam and Diesel stations in the United States. General considerations, and detailed factors bearing on energy costs, are well covered by the authors. It is particularly desirable to call attention, as is done in the paper, to the magnitude of general overhead—assessed at from 10% to 25% of specific production costs and fixed charges. These overheads, discussed under Group 3, are summarized in the "Introduction."

Engineers studying and utilizing the data presented will doubtless keep in mind that the costs cited cover a wide range with respect to the assumptions made, but apply, as to investment basis, to only one unit cost of plant—$100 per kw of capacity. Therefore, in considering energy costs derived, allowance must be made for spare capacity, which is necessary on every power system, as well as for any variation of unit investment, under actual conditions, from the $100 assumed as a round figure.

Papers of the character presented by Messrs. Hirshfeld and Van Duzer—and this remark applies to sections of the Symposium covering hydro-electric as well as thermal-electric stations—are of general educational and historical value, but should be used with caution as a guide in studying any specific problem. It can scarcely be too strongly emphasized that in analyzing a specific case and determining the economic type and size of a power development, only general guidance can be obtained from a broad study of this kind, and that the most careful analysis of special conditions obtaining, detailed estimates of costs and economies, and broadest judgment, must be brought to bear before a proper decision can be reached. Such a study, for any given case, calls for a command of the best information with regard to modern power equipment and systems, and familiarity with the technical tools of study, such as the various forms of load and duration curves.

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[45] Cons. Mech. Engr., Ebasco Services Inc., New York, N. Y.

The "art" of designing and operating a power system, whether it be supplied by hydraulic or thermal plants, or by a combination of the two, has become a quite highly developed specialty, and analyses and decisions with respect to power development should not be entrusted to men who have not, through years of training and experience, become thoroughly conversant with the body of data and practice that has been developed. Investigations and reports with respect to power systems or developments, founded upon elementary conceptions and incomplete data, or made on an emotional basis, may well arrive at erroneous conclusions.

H. G. GERDES,[46] M. Am. Soc. C. E. (by letter).—All opinions advanced and estimates made are entirely those of the writer, and all data cited have been taken from published sources. Statements made herein have neither been approved nor disapproved by the Federal Power Commission.

Natural Basis of Complementary Use.—The principal thesis expressed by Mr. Whitman—namely, that the cost of power from combined steam and water power sources is frequently lower than from either source alone—is a paradox, the answer to which, unfortunately, does not appear to be quite clear to the public at large.

In view of recent apparent controversy between advocates of steam and water power, the fundamental principle cannot be emphasized too often that steam and hydro-power sources are more often naturally complementary rather than competitive. This affinity proceeds out of the extremely varying public demand for electricity, whereby engineers are enabled to couple unlike sources of power together in varying degrees to meet this demand with greater economy than from either source alone.

An exception to the rule of complementary economy of unlike power sources occurs where a constant power source such as Niagara Falls attracts a constant power consumption load of like nature. Exceptions, not to the rule, but to the practice of incorporating a steam plant as one of the unlike power sources, occur in the Pacific Northwest where base-load, low-head hydro-plants on the larger rivers may be combined with peak-load, high-head storage hydro-electric plants on near-by mountain streams.

Phases of Combined Power Use.—Growth in the development of combined power use has been marked by many changes, generally characterized, however, by exhibition of the complementary function described. In California, steam plants have successively served major hydro-systems as load center stand-by capacity, peak-load capacity, and latterly as base-load carriers. In the Northwest, steam plants have been constructed on hydro-systems as peak-load carriers, as stand-by reserve, and as quickly added increments of capacity. In the Southeast, steam plants have largely functioned as stand-by reserve and intermediate incremental construction on systems that are predominantly hydro-electric.

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[46] Cons. Engr., Harza Eng. Co., Charleston, S. C.

In the Middle Atlantic section of the United States, where factories operated by steam power were common before the opening of the electrical era, the growth in central station steam capacity by incremental steps has been a natural evolution, and the complementary advantages of hydro-power have been realized only in recent years.

Factors Favoring Combined Operation.—Peak-load hydro-plants have been more often economically feasible in large sizes of 50 000 kw and upward. Low unit cost demands a large installation, which may be so greatly in excess of the reliable stream flow as to be available only a few hours per week. Interconnection of utilities into large systems increases the usefulness and economic feasibility of such peak-load hydro-plants both by allowing large incremental steps in new capacity and by permitting hydro-peak operation on a higher and sharper weekly load peak in which the same number of stream-flow kilowatt-hours available may be utilized in a shorter time, making firm a larger capacity.

Quite properly, Mr. Whitman remarks that the cost of furnishing peak power by steam is expensive. A renewal of search for peak-load, hydro-plant sites may be expected as soon as load growth and super-position complete the conversion of certain existing obsolete steam plants into base-load carriers.

Combined Hydro-Steam Energy Cost.—Two fairly typical illustrations, drawn from the operations of hydro-steam systems, are presented herein. In these systems the total load is large and the annual peaks both high and sharp. The load will absorb without waste, large blocks of river-run fuel saving power; hydro installations with capacities as great as ten times the low stream flow have been made, and the determining factor is usually the kilowatt-hour storage available to firm the plant capacity on the system peak.

It will be understood that although the data given are believed typical and fair, many simplifying assumptions have been made to bring the examples within the bounds of a simple comparison for the purpose of this illustration. The cost data are comparative and do not include items not affecting the comparison, such as general overhead, etc.

Example 1.—On a large Eastern combined power system, a peak-load hydro-plant of one-third the total system capacity operates at about 20% load factor (1 750 kw-hr per kw-yr) to cover the system weekly peak load. Additional stream flow, in excess of required peak-load storage, averages 3 600 kw-hr per kw of installed capacity annually. The annual load factor of the system is about 50% and the resulting annual load factor of the steam plant is 44.5 per cent. Typical fixed and operating costs have been assumed, as shown in Table 14 for the purpose of illustrating the cost of combined energy generation and delivery to the load center as compared with the cost of serving an independent portion of the same load by either means alone. The steam station is assumed to be at the load center.

From Table 15(*a*) the comparative cost of combined steam and hydro-energy delivered at the distribution network is seen to be 6.24 mills per kw-hr (Item No. 10) as compared with 6.85 mills (Item No. 7) for the entire steam generation, resulting in a saving of 9% (Item No. 11). On a 3 000 million kw-hr annual output system, the indicated saving of 0.61 mill amounts to the substantial sum of $1 830 000 per yr.

TABLE 14.—TYPICAL EASTERN HYDRO-STEAM; ASSUMED CAPITAL AND ANNUAL CHARGES ON DEPENDABLE CAPACITY

Item No.	Description	Capital cost, in dollars per kilowatt	Annual Cost								Total
			Fixed					Operating			
			Percentage return on investment	Taxes and insurance, in percentage of total cost	Percentage depreciation	Total		Operation and Maintenance		Fuel,* in mills per kilowatt-hour	
						In percentage of total cost	In dollars per kilowatt	Constant, in dollars per kilowatt	Variable, in mills per kilowatt-hour		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1. . . .	Hydro-electric power plant	200	6	1.5	1	8.5	17.00	1.60	. . . .	. . . .	18.60
2. . . .	Transmission and reception	40	6	1.5	1	8.5	3.40	0.40	. . . .	. . . .	3.80
3. . . .	Steam power plant.	120	6	2.5	3	11.5	13.80	5.00	0.30	2.25	18.80+2.55

* $4.50 coal at 1 lb per kw-hr.

Based on the following plant and load data, the typical load factor diagram in Fig. 33(*a*) shows the position on the annual load diagram of the three classes of power available:

Hydro-plant peak-use factor $= \dfrac{1\,750}{8\,760} = 20.0\%$

Hydro-plant fuel-saving use factor $= \dfrac{3\,600}{8\,760} = 41.0\%$

Total hydro-use factor $= 61.0\%$

Steam plant use factor $= \dfrac{7\,790}{8\,760 \times 2} = 44.5\%$

System load factor $= \dfrac{1\,750 + 3\,600 + 7\,790}{3 \times 8\,760} = 50.0\%$

Hydro-plant energy $= \dfrac{5\,350}{8\,760 \times 3/2} = 40.8\%$

Steam-plant energy $= \dfrac{7\,790}{8\,760 \times 3/2} = 59.2\%$

Total energy 100.0%

The three classes are as follows:

(1) Firm hydro-energy rendering the installed capacity firm on the weekly load curve of the system (assumed at 1 750 kw-hr per kw, or 20% plant factor);

TABLE 15.—COMPARATIVE COST OF COMBINED ENERGY GENERATION

Item No.	Description (1)	Capital cost, in dollars (2)	Percentage of capital cost (3)	Cost of Power			
				In dollars per kilowatt-year (4)	Mills per Kilowatt-Hour		
					50% load factor (4 380 hours) (5)	61% load factor (5 350 hours) (6)	44.5% load factor (3 900 hours) (7)
	(a) TYPICAL EASTERN HYDRO-STEAM SYSTEM						
	Hydro:						
1	At switch-board	200	9.3	18.60			
2	Transmission and reception	40	9.5	3.80			
3	Power loss in transmission (10%)	...	...	2.25			
4	Total, ready to distribute*	240	10.3	24.65	§	4.60†	
	Steam:						
5	Fixed components	120	15.7	18.80	4.30		4.82
6	Variable components	...	...	...	2.55		2.55
7	Total, ready to distribute*	120	...	...	6.85		7.37
	Combined Power:						
8	Hydro: 40.7% × 4.60	...	...	...	1.87		
9	Steam: 59.3% × 7.37	...	...	...	4.37		
10	Total, ready to distribute*	...	...	...	6.24		
11	Percentage of saving through combined use	...	...	...	9.0		
	(b) HIGH-HEAD AND LOW-HEAD HYDRO-SYSTEM						
	Low-Head Hydro:						
12	At switch-board	104	5.8	6.00			
13	Transmission and reception	30	7.0	2.11			
14	Power loss in transmission (10%)	...	...	0.81			
15	Total, ready to distribute*	134	6.7	8.92	2.04	1.37†	7.6‡
	Peak-Load Hydro:						
16	At switch-board	60	6.2	3.71			
17	Transmission and reception	30	7.0	2.11			
18	Power loss in transmission (10%)	...	...	0.58			
19	Total, ready to distribute*	90	7.1	6.40			5.46‡
	Combined Use:						
20	Low-head hydro, 1.37 × 89.3%	...	...	...	1.22		
21	Peak-load hydro, 5.46 × 10.7%	...	...	...	0.59		
22	Total, ready to distribute*	...	...	...	1.81		
23	Percentage of saving through combined use	...	...	...	11.0		

* Includes only the comparative items stated. † 74.4% load factor (6 520 hr). ‡ 13.4% load factor (1 170 hr). § Only 20% load factor hydro firm.

(2) Excess hydro-energy available during months of high stream flow, carrying the base load during the time available (assumed as 3 600 kw-hr per kw per average yr); and,

(3) Steam energy carrying base load and semi-peak during months of high stream flow (assumed as the 7 790 kw-hr per kw-yr shown on the annual load curve of the system).

The foregoing substantial saving is made although the hydro-investment is $240 per kw as compared with $120 for steam capacity.

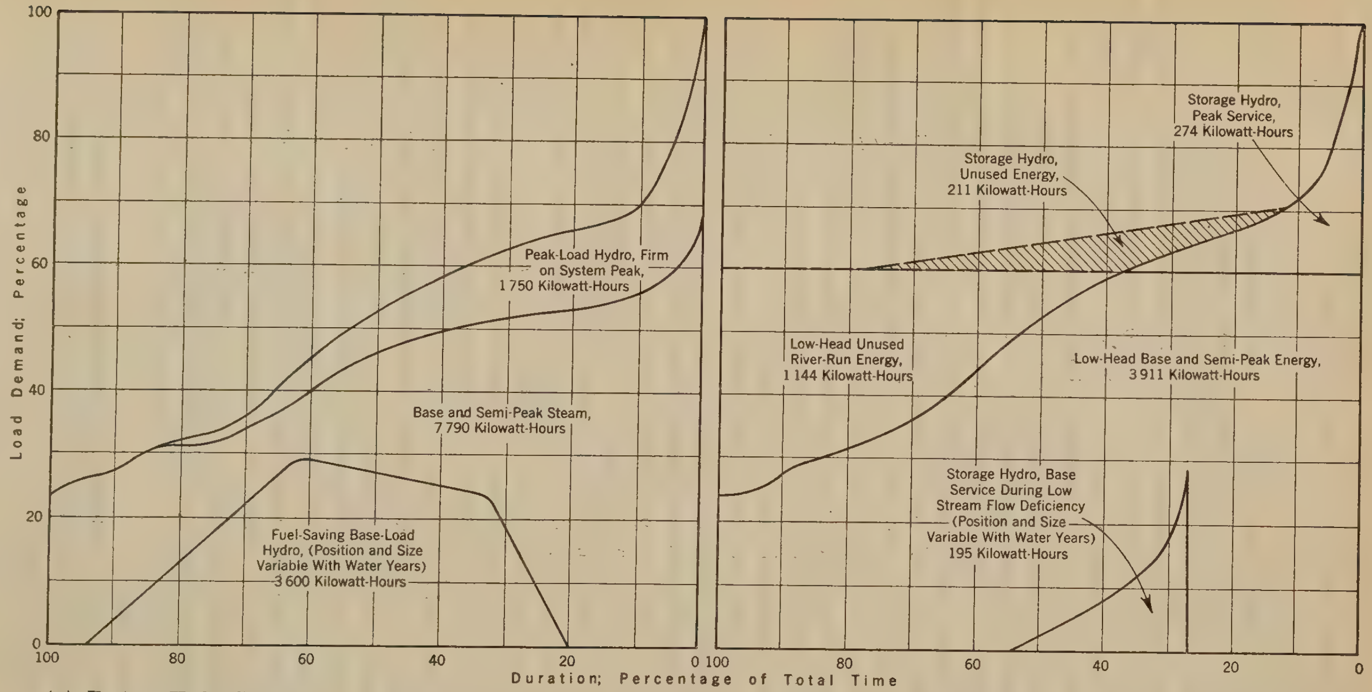
Example 2.—On a somewhat smaller Eastern system in which conditions are similar except that hydro-capacity cost is only $145 instead of $200 per kw, with an operating cost of $0.80 per kw, the comparative cost of combined power at the distribution network is 5.79 mills per kw-hr, a saving of 16% through combined energy generation. The incremental saving of 1.06 mills amounts to $2 120 000 annually on a 2 000 million kw-hr system.

Complementary Hydro-Systems.—The advantages of combining power sources of inherently different characteristics in one system are by no means limited to the large fuel-fed networks of the East. Although only a few have been built thus far, a number of opportunities exist for the complementary operation of base-load run-of-river and peak-load storage hydro-plants in the same system. Some of these complementary operations, however, are subject to the economic and operating disadvantages of connecting transmission lines more than 100 miles long to the mutual load center.

Power from low-head plants on Western rivers may be combined with peak-load hydro-power from high-head storage plants, and it appears of interest to study the cost of possible combined operation. Large sustained river flow would permit a high plant utilization factor at these low-head installations. Deficiencies in power output (which are ordinarily short but fairly severe) ranging from 10% to 50%, would usually occur during the summer flood period of reduced head and winter freezing periods of low water. During the winter low water, rated capacity would not be impaired, but operation at a plant factor less than 100% would become necessary.

Due to the low average head, the cost of power plant and machinery is relatively higher, and the cost of providing peaking capacity would be, therefore, greater than at higher head plants. The high utilization factor, and the large proportion of total cost required for power-plant building and machinery, appear to "earmark" these low-head river-run plants as natural base-load power producers. Operation on a 50% load factor, for instance, would require wastage of nearly one-half the power of the river otherwise within the plant capacity. Conversely, combined operation together with a peak-load storage plant would enable the fullest use of available power and greater earnings upon the larger unit investment necessary in the low-head plant. The function of the high-head plant would be to carry all peak load in excess of the low-head plant capacity and all seasonal capacity (winter) or energy (summer) deficiencies of the latter.

Example 3.—As an illustration of the advantages of combined use a study of possible combined high and low-head operation is presented herewith on the basis of the following data:



(*a*) Eastern Hydro-Steam System; Allocation of Hydro and Steam on Annual Load Curve (Diagram Represents 3 Kilowatts at 50% Load Factor).

(*b*) High-Head and Low-Head Hydro; Allocation of Peak and Base Hydro on Annual Load Curve (Diagram Represents 1 Kilowatt at 50% Load Factor).

FIG. 33.—TYPICAL COMBINED LOAD-FACTOR DIAGRAM

Peak hydro use factor $= \dfrac{274}{0.4 \times 8\,760} = 7.8\%$

Peak hydro deficiency use factor $= \dfrac{195}{0.4 \times 8\,760} = 5.6\%$

Total peak hydro use factor $= 13.4\%$

Low-head hydro use factor $= \dfrac{3\,911}{0.6 \times 8\,760} = 74.4\%$

High-head plant: Peak + deficiency energy $= \dfrac{274 + 195}{8\,760 \times \frac{1}{2}} = 10.7\%$

Low-head plant: Base energy $= \dfrac{3\,911}{8\,760 \times \frac{1}{2}} = 89.3\%$

Total energy 100.0%

The following assumptions as to cost and operations are made:

(1) Low-head power-plant cost, $104 per installed kw with an installed cost of transmission and reception facilities at $30 per kw;

(2) High-head storage development with 1 700 kw-hr per kw-yr of available energy, at an installed cost of $60 per kw plus transmission and reception facilities at $30 per kw;

(3) Interest 3.5% and amortization through a 40-yr, 3.5% sinking fund; and

(4) Combined operation on a 50% load factor similar to Fig. 33(*b*).

Fig. 33(*b*) illustrates the complementary function of the peak-load storage hydro, both in carrying 40% of the combined annual peak load and in operating on base-load from storage during the winter period of low stream flow. The comparative cost of power at the distribution net is calculated at 1.81 mills (Item No. 22, Table 15(*b*)) for combined use as compared with 2.04 mills (Item No. 15) from the aforementioned low-head plant, a saving of 11% (Item No. 23). With an average annual output of 3 000 million kw-hr, the 0.23 mill saving would amount to $690 000 per yr.

Unused hydro-electric energy shown as the shaded area at the left in Fig. 33(*b*) might be utilized by raising the plant factor, either by developing a higher load factor or by combination with additional semi-peak generating capacity.

General Comments.—The combination of power sources of unlike characteristics, as described herein, so as to fit a system load shape best, represents (in common with load-factor increase through diversity in use) a mutual reduction in reserve capacity. This is one of the advantages of grouping power systems through inter-connections over natural market areas.

Leverett S. Lyon,[47] Esq. (by letter).—In their public aspects power and the costs of power are subjects of great consequence to economists. Mr. Uhl has given an excellent discussion of these costs. He has enumerated the important items, both fixed and direct, and has emphasized the significance of each. He has indicated that the engineer must not overlook them in analyzing projects—that if he does he changes from an engineer to a promoter.

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[47] Executive Vice-Pres., The Brookings Inst., Washington, D. C.

Although it is impossible to overstate the importance of costs from the point of view of constructing projects, it is from another point of view that they become of great concern in public policy, and to economists. The economist's interest in cost, in engineering, and in any other matter, is essentially in terms of public policy in relation thereto. It is the failure to realize that the economist is concerned with public policy which transcends the limits or boundaries of any given industry, that causes many people to say that economists never agree. Such a position is untenable. It comes from confusing so-called economists with those who are what some like to refer to as professional economists. There is a difference between the farm economist, the labor economist, the business economist, the practical economist, and those who more truly represent the profession. The difference is that the latter type are interested in the nation's welfare rather than the welfare of any particular industry or group.

The professional economist is concerned with the question of how to use the nation's resources to the best advantage in the public interest. This means securing the largest possible output for the smallest input—a good engineering concept. In more detail, this means that if input can be had at a lower cost in one form than in another the lower cost form is the one to use.

Questions of cost thus viewed become questions of public policy and, therefore, of interest to the economist as costs find reflection in prices. In ordinary business, such as manufacturing or merchandising, or in any other competitive field, the costs of an individual company may have no direct relation to the prices at which it can sell its product. If there are five manufacturers of a given commodity for example, competing in a given market, and each has a cost that is different from the others, the one with the lowest cost will get as much of the business as he desires or can supply—unless the others sell below so-called "costs." It does the high-cost competitors no good to complain that their costs will not permit them to compete profitably. Prices simply are not made that way in competitive markets. In such a situation a high-cost manufacturer has no choice but to forget part of his costs, to close his eyes to the nicely worked-out "cost system" of his accountants, and to remember that competition came before accountants and will live long after them unless the nation becomes essentially socialistic.

In such a situation the manufacturer "closes his eyes" to some of his fixed charges. This is not difficult if his fixed charges are not bonds or other borrowed money on which there is a definite interest charge. If there is no such charge he will come to the conclusion that his investment is not worth as much as he thought it was—that it is indeed only worth what he can earn on it.

If he has a fixed charge in the form of bonds which leads to a foreclosure or a re-organization, his fixed charge after the re-organization will no longer be what it was before. By the ritual of a Court procedure or of a re-organization committee his costs will have been reduced. Accordingly, if he must think of his prices in terms of cost, all is now made easy. Although the original investment in the company has not been changed, the bookkeeping can now be changed so that "book costs" may now be in line with selling prices.

Of course, there is a limit below which prices will not be reduced even in a competitive market. In general, a manufacturer will hesitate to go below his direct charges although he may be quite willing to conclude that anything he can make above them is better than nothing. Many manufacturers find it profitable to go even below their direct charges at times in order "to keep in the game during a period of sharp recession." It may pay well to take a loss even on direct charges for a time with a view to being "in the picture when the situation is better."

The idea that, in all industry, prices should cover costs is persistent. The business man finds it difficult to escape from the feeling that "my investment is in this business, I am entitled to a return." This seems an entirely natural point of view. Indeed, it is a natural reaction to the situation, but there is an important idea of great social significance which opposes it. This idea is the one already stated, namely, that the resources of the nation should be used so as to give the greatest possible output at any given time for the smallest possible input. This really means that a buyer, whether he is a consumer buyer, or a business buyer, should be in a position to purchase, at any time, at as low a price as any one will offer at that time. A consumer who buys a hat should be able to purchase it at the lowest price for which any one is willing to make that kind of a hat. A power company should be able to buy its coal, its machinery, construction materials, and to borrow its money at as low a rate as any one at any given time is willing to furnish those things. No one has ever yet presented another plan so well calculated to make the resources of a nation at all times give the largest total yield to all the people as this plan of economic organization.

Alternative plans that have been presented have certain merits. Their fundamental weakness is that without such a measure as a competitive market gives, of what the price should really be, there is no satisfactory way of knowing what it should be. For example, to claim that prices should cover costs leads into the greatest difficulties. Whose costs, for example? On such a basis (that a manufacturer is entitled to his costs) there would be no incentive to keep costs down. Indeed, if he believes he should have a profit as a percentage of his cost, higher cost would be an advantage.

In the field of power, however, the competitive method of price determination will not apply. The reason is fundamentally the same as the reason in favor of competition in other fields. The people cannot afford to get along without competition in most fields; they cannot afford to have competition in this one. In other fields, without competition, goods cost too much. In the power field, with competition, goods cost too much. It has been discovered long ago that competition in the production of power for communities (and of many other commodities which are produced by so-called public utilities) will either result in building so much equipment that a charge based on costs will be higher than is necessary, or that the competitors will form a monopoly and charge an extortionate rate or that, in fear that competition will cut prices so low that the return on investment will be less than it would be in other fields, the desired facilities will not be built. In other words, in the field of power there is what economists call a "natural monopoly."

Since a monopoly cannot be trusted to create prices in the public interest, some social regulation of rates has been established in every State. A social regulation is directed primarily at the question of what price should be charged. Since the commissions which have been given this responsibility have no competitive tests they have groped, none too happily, in an effort to find a satisfactory method of relating costs to prices. This is not the time nor the place to describe the trials and tribulations that have beset the public utility commissions. Every one knows something of their struggles to determine what costs should mean—not in terms of expenses for building a project, but as a basis for rate-making. Shall they take all costs into account? Will all costs mean production costs? If so, production when? Will production mean original production costs or will it mean reproduction now? Or will it mean that beautiful but vague concept, cost on the basis of prudent investment?

All Mr. Uhl's observations on variables are relevant for consideration in trying to relate cost to prices. One good example is his comment on obsolescence. If one wishes to compute the cost of producing power as a basis for determining the prices at which it should be sold one would need to include a charge for obsolescence; but this, in turn, as Mr. Uhl has shown, depends upon the calculations which one makes as to how rapidly new equipment (even new inventions) may come into the field to supplant the old. Even if everything which the past tells is included, something new may happen. There may be an Edison, and coal may be made relatively obsolete in many communities. Gas may replace electricity. Some physicist may "pull" a power ray out of a laboratory which will make hydro-electric plants old-fashioned almost over-night.

What happened to the canals when the locomotive was invented is a matter of history. The struggles of the railroads and the Interstate Commerce Commission in an effort to find a rate for railroad traffic that will take care of the costs of the railroad companies is a problem of the present day. They are now at that nightmarish stage at which no one seems to know—even if they wish to cover all costs—whether it will be better to push the rates up or to push them down. "Verily, the way of the price fixer is hard" even when he does the fixing with the aid of the law.

Quite aside from these deeper issues, however, there are others which are of public importance when prices must be related to costs. It is always proper to ask whether the costs are fair or unfair. Are they really the necessary costs or are they "padded"? Is the enterprise capitalized at the minimum necessary to furnish the service or is it over-capitalized? Are officials over-paid or otherwise? To what extent are holding companies and similar financial and managerial mechanisms an effective means of organization and operating, and to what extent, if any, are they means of adding costs, positions, and salaries, on which rates are to be computed? Such questions should properly be asked and answered, not in an atmosphere of acrimony and controversy, but of careful consideration of the accounting, engineering, and economic issues involved.

It is the raising of such issues of costs as these which has introduced the now much discussed "yardstick" as a proposed method of determining what prices should be. Of course, the so-called "yardstick" is not a new idea. Every method that can be applied to determining what prices shall be—even competition—is a yardstick; but the great extension of Government operations as a means of comparing costs does offer, in size at least, something new. It also introduces new difficulties by the half-dozen if not by the score To such problems as how to measure costs, reproduction costs, and the like, it introduces that most confusing of all cost concepts, joint costs. To illustrate: If a dam is built as a means of creating power and, at the same time, as a means of providing irrigation, and, at the same time, for purposes of flood control, and, at the same time, to lift a backward community into an industrial community, how much of the costs of the project—of each item of fixed costs—such as Mr. Uhl mentioned, should be charged against the cost of power?

Finally, Mr. Uhl's excellent paper refers to the contributions of the power industry to the American nation. As indicative of these contributions he cited the investment and the contribution through taxes which had come from this industry. These criteria of contribution are worthy of mention, but they do not do justice to the power industry. Although it is politically helpful to point out the tax payments of a public utility, its real contribution to the nation must be measured in quite different terms.

Man's effort, from the earliest recorded time, has been to increase the output which he can secure from his input in labor and toil. It is as the power industry has added to this output or lessened this input that its contribution has been made. Mankind has always had resources at his disposal. He has progressed on power—as he has been able to utilize natural forces to supplement or replace his own puny efforts. It must be in calculations of the additions—or, perhaps, better—the multiplications of Man's unaided power which have come through the power industry that one can best estimate what it has given to the world. It has added a series of problems, and some of them are difficult, and it has not been done without cost, but the credit balance for the power industry is an outstanding one.

JOHN T. MADDEN,[48] ESQ. (by letter).—One unfortunate circumstance in the economic life of this nation is the frequent necessity of attempting to settle important questions by steering a middle course between two evils. In order to avoid Scylla, or the evil of monopoly, on one hand, and Charybdis, or the evil of unrestrained competition, on the other hand, legislators pursue the middle course, and public utilities become a regulated monopoly. This middle course does not always mean "clear sailing." In avoiding twin evils, other vexing difficulties are often encountered. To shift metaphors, a veritable witch's broth has been brewed in this public utility stew, combining engineering, economics, finance, law, and politics, and there are some who say that too little of that important condiment—ethics—has been used.

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[48] Dean, School of Commerce, Accounts, and Finance, New York Univ., and Director, Inst. of International Finance, New York, N. Y.

There is no doubt that an incalculable amount of economic waste could have been avoided if politics had been kept out. Better cooks would have left out the complexities, uncertainties, and paradoxes of the law. Sound principles of economics, combined with ethics, would have avoided such dishonest and shoddy financing as there was. The engineering contribution seems to have been sound; at least, there is little condemnation on this score. However, although perfection in this area of human activity seems as unattainable as in others, on the whole, the results are fairly satisfactory; accomplishment is not as poor as some have alleged.

The story of this industry is the same old tale of industrial development in the United States, and it follows the usual pattern. A new industry produces a service which later comes to be regarded as essential. Progress is slow at the outset, proceeding in lame and halting fashion. For a time the venture is highly speculative, but, nevertheless, it manages to attract to itself the capital, skill, and energy of those venturesome souls always present in an acquisitive society. The industry reaches its "adolescent" period, making the customary errors of youth. Ultimately, it attracts the attention of politics and the law steps in. Controversies arise, and instead of resolving them with the aid of ethics and economics in simple and direct fashion, differences increase. The penalty for economic success is to have one's "head chopped off" in appropriate "Alice in Wonderland" style.

Despite the controversy and criticism of regulation and rate-making, the summary data in Table 12 are most impressive. The wonder really is that such progress could have been made in view of all the difficulties placed in the path of the electric light and power industry. Mr. Uhl concludes his paper with the statement that the "power industry is of increasing importance to the welfare of the nation and has made tremendous contributions to its wealth and income." This is a temperate statement, appropriate and becoming in the engineer. The record of progress from 1912 to 1936 is merely indicative of the potential capacity of the industry to contribute to the future wealth, income, and social betterment of the nation, if legitimate expansion could proceed. In fact, the record cited serves to "highlight" the serious damage to the national economy and welfare wrought by those who have used the industry as a political football. Mr. Uhl did not stress the fact that the power industry is a most important arm of the national defense, and those whose aim is to wreck it, or whose actions impede its capacity to serve the nation in the event of war, will have much to answer for at the bar of public opinion.

This Symposium has demolished the pretensions of those who have supported the alleged value of "yardstick" rates established by public plants scattered about the nation. The variable factors are so many and so incapable of being reduced to a common denominator that any comparisons would be worthless.

In his discussion of rate theories,[49] Professor James C. Bonbright has stated that "whatever the proposed remedy, the disease to be cured is generally diagnosed as one caused by the focal infection of the fair value doctrine," which he characterizes as Court-made law. The writer does not subscribe to his view. Despite the criticism of the fair value doctrine, relatively few rate controversies have reached the Courts. The legislature imposed regulation; and, to continue the medical metaphor, the regulatory "treatment" was to be administered to "patients" or "cases," no two of which, as shown by this Engineering Symposium, are alike biologically, chemically, or physically. While the legislature developed its "science of medicine," it was left to the commissions and the Courts to practice "the art of healing." There is a vast difference between the science of medicine and the art of healing. In this Symposium, the engineers who have studied the cost of power generation have shown that, although there is a certain general uniformity in the "anatomy" of physical plants, they vary considerably. The economist knows that the investment costs—"the flesh and glands"—vary far more than the "anatomy." In establishing the fair value doctrine, the Court left leeway for varying and individualistic treatment of the "patients." The physician who prescribes only one remedy for all his patients does not command much respect.

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[49] "The Valuation of Property," by James C. Bonbright, McGraw-Hill, 1937, Vol. II, Chapter XX, p. 1079.

Proficiency in the art of regulation has improved greatly, just as the art of healing has shown great advance. Despite the "focal infection" which Professor Bonbright finds, the pragmatic test of the Court-made law reveals that the "patient" got along fairly well, although alternative treatment was not employed. The "doctor's" expenses, such as appraisals, legal and engineering services, and Court costs, were not unduly expensive considering the financial circumstances of the "patient." Furthermore, before 1932, this growing industry had made significant advances in its engineering technique; it extended the benefits of its service to countless hamlets and many thousands of persons who have not even yet had the benefits of railway service; it raised at reasonable cost all the capital required for its development; the cost of that service was greatly reduced; and, a sound basis has been laid for still further progress if the politicians will cease harassing the industry. Abuses have existed, of course; "write-ups" and "write-downs" have occurred on the books, but despite the allegations of watered capitalization, so commonly made, the fact is that no one knows either the prudent investment cost or the fair value of the existing plant, and no one will know until an inventory and an appraisal are made.

Economists would find it difficult to point to any other business institution that has made such forward advance in so short a period, with equally beneficial social consequences. In fact, if social engineering were half as proficient as civil, electrical, and mechanical engineering have been, there would be no power issue of any consequence to-day.

Whatever may be the outcome of the prudent investment or fair value doctrines, or whatever may be the outcome of "yardsticks" and "big sticks," more attention is to be given to the preparation and maintenance of continuing fixed property records. Engineers will devote more attention to, and perfect, their techniques in measuring the effect upon fixed property of the several causes of depreciation. Assume that the engineers accomplish all that is to be desired in these matters; assume that a technique for measuring the extent of wear and tear has been developed with reasonable accuracy; and, assume that engineering experts develop a foresightedness as to obsolescence—there is still one more question to be answered. Is the object of the business struggle to preserve intact the mere dollar capital invested, or shall it be to protect the "real" invested capital in terms of purchasing power? The distinction is important and this problem is no less baffling than other problems connected with this difficult question. The gyrations of price levels in the United States since the World War vitally affect the accounting process, which attempts to record financial facts in terms of a variable "yardstick." It will be observed that the computation of depreciation under either the fair value principle or the prudent investment principle does not touch upon this point, particularly if by the latter principle is understood that, in so far as depreciation is concerned, the sum to be charged to operating expenses is the proportion of mere dollar investment that is to be amortized.

Recent experience bears out the suggestion of Mr. Scharff that in competitive industry it will be necessary or desirable to have a more accurate determination of the monetary sums to be set out in the accounts for the various causes of depreciation. In too many instances the depression beginning in 1929 revealed an impairment of dollar capital through neglect in this respect.

There is another cause of depreciation beside those commonly mentioned in discussions on the subject; namely, shifting in population, or shifting of industries, which may completely alter the character and extent of a company's business, and increase its obsolescence factor. The perfection of air-conditioning may have a serious effect upon the popularity of seaside resorts in the summer. There still remains much work to be done in a more accurate study of the market, and the results of such a study might indicate an increase or decrease in the provision for obsolescence. Apparently, Mr. Scharff recognizes the difficulty of shifting the loss due to obsolescence; it may be placed upon the present generation of rate payers, upon a later generation; or it may fall upon the owners of the enterprise. In some cases it may be shifted to employees who will accept reduced wages to compensate for the loss to an employer who finds himself in an unfavorable competitive position with other manufacturers with more modern equipment.

In 1906, Woodrow Wilson stated that the automobile would be responsible for an increase in socialism since the vulgar display of the wealth of those who owned automobiles would create dissatisfaction. Although this fear proved groundless, the automobile did injure the revenues of railroads and electric railways. This Symposium displays no present fear on the part of engineers that radical changes in the art are in prospect. One can easily dream of a revolution that would jeopardize the entire investment in electrical properties. Stranger things have happened. If more extensive public ownership is to come eventually, the loss would then fall upon taxpayers. Perhaps taxpayers should think more than once before they rush heedlessly into Government ownership.

Briefly, then, it appears that engineering science has so definitely established comparative costs and uses of hydro and steam projects that Government and finance should be governed by these conclusions. Further progress in engineering studies on depreciation and obsolescence will determine the final disposition of this vexing question, but the writer concludes that, in the future, greater allowances should be made for this item of fixed cost. It now appears that too much of the burden is likely to be shifted to the investor, whereas, in all good conscience, a substantial portion should fall upon the consumer.

The interest problem comes down to this: Either Government development and ownership with its lower rates, or private development with interest rates fixed under the usual market conditions. As for taxes, the burden is shifted either to suppliers of the things the companies buy, to their customers, or to the investors; but when the average tax burden of the industry is 15% of the revenue, and higher in some instances, it becomes an extremely important matter. Unquestionably, it must go into costs, but the important question is whether it is to be recovered from the consumer or from the investor. If an attempt is made to recover so much from the investor that the rate of his return is lowered, he will transfer his attention to other investments and the capital needed for development will not be forthcoming.

The problem of joint costs is insoluble, and it is the same as that faced by the Australian sheep raiser who sells mutton, pelts for ladies' coats, and wool.

Incidentally, the comparisons made with foreign countries are of little validity in the present case. The social and economic milieu abroad, particularly, and the legal philosophy are so different that little is gained from their experience in solving problems in the United States.

If, as some believe, the nation is on the threshold of an era of decentralization, any prophecy of the social and economic consequences is futile; but it does suggest this: It would seem that proposals to establish, under Government ownership, various large units scattered throughout the nation as "yardsticks" is untimely, because it appears to commit the Government to a wasteful economic venture at the "tail end" or closing phase of engineering development. Certainly, if decentralization is to be the significant operating practice in the future, the Government should hesitate in the establishment of "seven little sisters" to the TVA.

Regulation in itself is an adequate instrument for social or economic problems. In fact, it is the best instrument possible, if used properly. With equal facility, one can either shave himself or cut his throat with a razor; it all depends upon the way one uses the instrument. Even in those cases where qualified economists, or capable engineers, sat on commisions, they found themselves confronted with political problems rather than ethical or economic ones. Many serious controversies have grown out of the abuse, and not the use, of regulation.

R. L. THOMAS,[50] ESQ. (by letter).—It is significant to the writer that Mr. Scharff, in referring to a 25-yr old report[21] on depreciation, selects as one statement with which there is still general agreement the comment that "perhaps there is no single subject in connection with valuation that has caused more trouble than depreciation." There seems to be something inherently controversial in the subject—something that causes blood pressures as well as voices to be raised. There is no subject on which it is more difficult to keep discussion within reasonable bounds—no subject so surrounded and obscured by "fog and smoke." Using another metaphor, the Hon. Clyde L. Seavey, Acting Chairman of the Federal Power Commission, in an address before the American Bar Association in August, 1936,[51] aptly referred to the "inferno of depreciation."

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[50] Project Engr., Safe Harbor Water Power Corporation, Baltimore, Md.

Engineers, more than any other class, should be striving to cut through the "haze"—to find a way out of the "inferno." They should bend their efforts toward simplifying rate-making procedure in the public utility field, provided that such simplification does no injustice to either the rate-payer or the utility.

Mr. Scharff has done a real service in emphasizing the need for consistency between determination of depreciation for valuation and rate-base purposes and annual depreciation expense. His proposal that the actual depreciation be determined annually by mathematical formulas, with periodic checks by field inspection and engineering investigation, is theoretically desirable and is a logical method if it is necessary to use a depreciated rate base. Its practical application, however, is another matter. Even in the case of the simplest of the causes of depreciation—physical deterioration—the writer regretfully doubts that it will be possible to obtain general agreement on a series of "objective measurements" which will be recognized as measuring the extent of physical deterioration. With respect to the other causes, it seems even more likely that determination of the extent of depreciation will remain a matter of judgment. As a matter of fact, there are many cases in which it is impracticable to distinguish between several of the causes.

Now, there are undoubtedly many cases where there is no other "way out," and it is necessary to make the best possible determination of depreciation by a combination of "objective measurements," investigation, and judgment. A better way out of the "fog," when it is feasible, is the use of an undepreciated rate base. The complete sentence in Mr. Seavey's address[51] is: "And, in order to escape as far as possible from the inferno of depreciation, the California Commission used an undepreciated rate base with a correlative adjustment in expense."

Undoubtedly, the "correlative adjustment in expense" refers to the sinking-fund method of depreciation accounting which has been used in California for a long time. Under this method the total annual accretion to the depreciation reserve consists of two components—one the annuity which is chargeable to the rate-payers as an operating expense, and the other the interest on the reserve, which is not chargeable to the rate-payers, but is a deduction from income. It is not necessary to know the "actual" depreciation. The annuity, as Mr. Scharff states in his reference to special cases, equalizes the annual depreciation expense over a long period of years.

Five years of intimate knowledge of the use of the method at a large hydro-electric project has convinced the writer that the statement often made, that the method is complicated, is entirely unfounded. As compared with an annual or even a periodic determination of actual depreciation it is extremely simple.

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[51] *Public Utilities Fortnightly*, October 8, 1936, p. 546.

Another method of using an undepreciated rate base is to determine the depreciation expense charged to the rate-payers by some simple and more or less arbitrary, but fair, method and credit the rate-payers with interest on the accumulated depreciation reserve at an arbitrary interest rate. This is the method used in the District of Columbia under the Consent Decree of the Supreme Court of the District of Columbia.

An objection sometimes made to the undepreciated rate base is that it is contrary to "accepted law." This statement was made recently by a well-known engineer before a Public Service Commission; yet it is common practice for electric utilities, including the company with which the aforementioned engineer is connected, to establish, among themselves and with large customers, annual payments for the use of facilities (as, for example, in interchange) by applying a fixed percentage to a constant sum of money. The rate base and the payment go on year after year without deduction for depreciation. Without attempting to argue the legal questions, the writer believes that if the method is equitable it will be found to be legal "in the long run."

THEODORE B. PARKER,[52] M. AM. SOC. C. E. (by letter).—In his paper on "Cost of Hydro-Generated Energy," Professor Barrows has developed a very effective method of analyzing such costs, and has tabulated cost data relative to a great variety of representative plants. Hydro-electric developments differ so widely in their general design, and must conform so frequently to special requirements, that it has been considered a difficult task to secure a common basis for comparison of costs. It is interesting, therefore, to find that Professor Barrows has managed to reduce a considerable number of variables to what appears to be a common denominator.

His method appears admirably adapted for making general comparisons of different plants. For a detailed study of a given power system it is naturally more accurate to use the actual characteristics of the system and its parts; that is, to determine the comparative costs of two alternative schemes for supplying equivalent generating capacity, the most reliable method is to compute the total system cost under each plan. "Short-cuts" or approximations are frequently misleading.

One of Professor Barrows' conclusions is that, "although, in general, the cost per kilowatt-hour tends to lessen as head increases and to lessen with the size of the plant, the effect of capacity factor is more predominant" (see heading, "Cost of Water-Power Output: Effect of Head, Capacity, and Capacity Factor upon Cost"). This statement can scarcely be disputed. On the other hand, the economy of high-head plants is sometimes more apparent than real. Plants with heads greater than 400 ft (with the exception of Boulder Dam and similar recent developments) generally require the use of long conduits. These conduits are frequently wood pipes or flumes which depreciate rapidly and require considerable maintenance. Where more permanent materials are used, there is still a greater proportional annual expense than for plants with the power-house directly at the dam. This tends to render incorrect a comparison of such plants based on the same fixed charges.

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[52] Chf. Engr., TVA, Knoxville, Tenn.

At a high-head plant the waterway frequently represents a large proportion of the investment. Its capacity must equal the capacity of the plant, with only minor allowance for load factor. Increment cost at such a plant, therefore, is high, and may affect the lower cost of generating equipment. For certain purposes it may be found more economical to secure additional peak capacity at the low-head plants of a system, for which the energy cost may be high but the capacity increment cost low.

In Table 6 Professor Barrows includes certain high-head plants numbered 31 to 39, inclusive. These are comparatively small developments in Utah which were constructed prior to 1912 by various local interests, and, later, taken over by the Utah Power and Light Company. They were built at a time when the general cost level was lower than at present. The waterways are mostly of wood, and have required extensive maintenance and replacement which are apparently not reflected in the annual costs given in Table 6. Cost data from this group, therefore, are not a very reliable basis for appraising present-day, large-scale developments.

A study of Table 6 and the supporting text is most interesting and serves again to emphasize the fact that where exceptional economy is secured the cause is frequently some special condition rather than any general relation of head and flow. For example, the Chelan Project (Plant No. 43) owes its value largely to an opportunity to raise the level of a large lake by means of a relatively short dam at its outlet.

The effect of capacity variation is shown clearly in the case of the Rock Island Development (Plant No. 49), on the Columbia River. For the four present units the total cost is given as $14 555 000 and the unit energy cost as 0.40 cent per kw-hr. Since six additional units can be added at a cost not much exceeding $1 000 000 each, it is apparent that this will greatly reduce the cost per kilowatt-hour. Owing to the unusually well sustained flow there would be a large increase in total energy, but naturally not in proportion to installed capacity.

Table 6 shows a considerable number of reasonably large modern projects which develop energy at what might be regarded as high unit cost. This, it is assumed, must be justified by favorable load factors, high cost of power from other sources, or other local conditions. In general, it has been the writer's experience that outside of special conditions, such as existing lakes and unusually favorable topography, the most effective factor in securing economy is a high ratio between low flow and length of dam. Such a condition allows the most effective use of the structures built, and the greatest return on the investment.

In recent years, low-cost hydro-energy production has been obtained at developments where there are large, well-sustained stream flows, and where the power houses are directly at the dams. This is a most important factor in the very low costs obtainable at the large Federal developments on the Columbia River. It is a smaller, but still an important, factor in the relatively low costs at the plants on the Susquehanna River where the flow is large but not so well sustained. Many of the successful Canadian developments are situated on rivers having a large volume, with flow sustained by natural lakes on their head-waters.

Where long conduits are used there is a definite limit to the economy possible, except where a large drop can be obtained in a relatively short distance. This sometimes occurs where there is an opportunity to tunnel through a divide or to effect some similar short cut. To secure low cost per kilowatt-hour with long conduits, it is necessary to have a high capacity factor—that is, a high rate of use of the facilities provided.

In a given power system it is frequently desirable to determine where the cheapest generating capacity can be obtained; that is, to determine which plant can best be over-developed for use on the peak of the load. Other conditions being favorable, the lowest incremental cost is frequently found at a plant with moderately high head, but with the power house directly at the dam.

The most vital consideration affecting hydro-electric costs is the rate at which fixed charges are computed. Professor Barrows has taken fixed charges on the Federal hydro-electric projects at 6.5%, whereas the others are on a 10% basis. Of the fixed charges the largest factor is the interest rate, which varies with economic conditions and with the agencies involved. During recent years considerable Government financing has been effected for less than 3 per cent.

Depreciation (that is, purely physical depreciation) varies greatly with the type of development. Where most of the investment is in structures and equipment of limited life, the total average depreciation may exceed the 2% maximum rate given by Professor Barrows. On the other hand, there are certain types of development in which a large part of the investment is in items having practically no physical depreciation. An example of such items is the investment in reservoir facilities, consisting of cost of land, clearing operations, reconstruction of railroads and highways, and back-water protection.

In addition to the tabulation of cost data for fifty-seven hydro-electric developments, a summary is given of estimated costs for several of the larger Federal projects recently completed or under construction. Included among these is a summary of estimated costs for eleven existing or proposed major projects of the Tennessee Valley Authority.

Unit energy costs are given for two conditions: (1) Those based on power development cost alone, given as $135 000 000; and (2) those based on total cost, given as $520 000 000. Fixed charges are taken in each case at 6.5 per cent. The resulting unit costs are given as 0.183 cent and 0.60 cent per kw-hr, respectively. It is thus inferred that the cost of energy to the TVA lies between these limits.

The actual cost of energy to the TVA depends upon a number of factors, not all of which can be discussed adequately herein. It seems appropriate, however, to emphasize several factors. The actual interest rates paid by the Government during the TVA construction period have been very low. These rates are a matter of official record and can be verified easily. The physical depreciation of the TVA projects as a whole will be extremely low. More than one-third of the cost is in the reservoirs, with negligible depreciation, and much of the remainder is invested in massive structures of concrete and earth. The estimated annual depreciation of the dams, reservoirs, and generating stations (excluding transmission lines) will be less than 1% of the total cost.

Among the eleven projects listed as costing \$520 000 000 is the Gilbertsville Project, which, if built, would be utilized mainly for flood control. Its estimated cost is more than \$110 000 000, which provides for only a moderate generating capacity. This can scarcely be classed primarily as a power development, and no theory of allocation would justify charging any considerable part of the total cost of this development to power. With Gilbertsville excluded, it is apparent that the upper limit of 0.60 cent per kw-hr would be reduced considerably even on the basis of 6.5% fixed charges. The effect of reducing the rate of fixed charges is obvious.

When final costs are available and definite allocations have been made, it should prove very interesting to analyze the TVA and other recent Government developments, and to compare them with the projects listed in Table 6. Meanwhile, the writer wishes to congratulate Professor Barrows on having developed a method which makes such comparisons practicable.

PHILIP SPORN,[53] M. AM. SOC. C. E. (by letter).—The question of true production economics is one to which engineers must pay more and more attention if more goods are to be available for distribution. Although the production of electric power and electric service has been developed to an unusual degree on sound technical and economic lines, the subject will bear further study and analysis. There have been periods in the past when the power business was growing at such a rate that almost the entire effort had to be centered upon the task of meeting demands. In such periods some mistakes—particularly economic mistakes—may have been made, but these did not show up and their cost was quickly absorbed in the growth of the business; but, as business approaches technical and economic maturity, the cost of such even rare mistakes is more apparent and it becomes necessary to scrutinize more closely the economic merits of what is done in the way of building new capacity. Although Mr. Whitman's paper traverses familiar ground, it may well be that the very repetition of these principles will "bring home," not only to engineers but to others who are or who may become associated with, or who are in contact with, the power business, the need for careful economic analysis in any power situation.

This subject of the cost of combined energy generation is one that requires, above all, careful analysis. So much has been said and written on the subject of steam *versus* hydro energy (with prejudice substituted for facts and figures), that fundamentals are being overlooked in the desire to "make out a good case" for whichever side its author has chosen.

In reviewing the elements entering into the cost of energy as enumerated by Mr. Whitman, the writer is impressed by the fact that not enough emphasis has been placed upon cost of transmission and its effect on the cost of the delivered energy. After all, that, and not the bus-bar cost at the generating station, is the item of greatest interest. Those who have studied the problem closely and thoroughly understand the economics of power supply have been, for the most part, unable to find any sound economical basis for long-distance transmission of power.[54] In general, energy is most economical when generated nearest the load. Beyond a certain distance, transmission definitely is not an economical proposition; and, only too often, hydro developments require some long transmission to reach the load. Long-distance transmission of electric energy had its early development in areas where fuel was scarce and costly and there was available hydro power which could be developed cheaply. The early days of the electric power development in California are a case in point. However, even there, this situation has changed with the discovery of oil and gas supplies near the load. The writer feels that this question of transmission cost is very definitely a limiting factor in the development of large hydro-electric projects. Therefore, when a question arises as to whether a hydro-plant or a steam plant should be built to meet the growth of load, one should be very careful not to overlook the cost of transmission.

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[53] Vice-Pres. in Chg. of Eng., Am. Gas & Elec. Service Corp., New York, N. Y.

In illustrating the economic principles of selecting the source of power, Mr. Whitman chooses the well-known situation at the Conowingo Plant, on the Susquehanna River. He could not have picked a better one. Everything was in favor of fitting that plant into the load curve economically: (1) Short transmission distance (only about 60 miles) to a large load center; (2) access to a large metropolitan load, rapidly growing, in which fuel costs were relatively high; and (3) an existing steam system in which there was sufficient steam-plant capacity to firm the larger part of the Conowingo capacity. One wonders how the Conowingo Plant, if proposed to-day, would appear in the light of the latest developments in steam-plant design. Mr. Whitman states that a new steam plant was estimated at that time to cost $135 per kw. The most pessimistic engineer, to-day, would expect to reduce this cost by at least $25 per kw, and if he were in an optimistic mood, by as much as $45 per kw. Mr. Whitman does not state what efficiency was estimated for the plant, but certainly the efficiencies (that is, the fuel consumption) estimated in that day for the new plant were not as good as those that would be estimated to-day with all the new knowledge of high-pressure and high-temperature operations. In 1925, engineers were thinking of plants with a fuel consumption of 13 500 Btu per kw-hr. To-day, 10 000 Btu per kw-hr is "within sight."

This leads to another thought. The common impression is that hydro-plants never wear out and, once built, are useful for all time. Granted that hydro-plants and equipment show the effects of wear and tear slowly, one still must not overlook the influence of obsolescence on these plants, an effect that has been accelerated in the ten years since 1928 by the very great improvements in generation of energy by steam. All those conversant with the facts admit freely that the great improvements in the art of generation of energy by steam have made the development of new hydro-plants less attractive; but it is often forgotten that the same advance in steam generation has made present hydro-plants less of a bargain than they appeared to be when built.

Of course, the plants are built, the money has been spent, and they will continue to be operated until their cost becomes prohibitive in relation to some new future developments. The writer wishes to emphasize this point on obsolescence to show that it may be a much more important factor in the determination of the economic value of hydro-plants than is generally assumed. However, he does not wish to be misunderstood in regard to hydro-electric power development. He has no prejudice against it; nothing but a desire to examine the facts clearly. The American Gas and Electric Company has in operation about 125 000 kw of hydro-electric capacity fitted into what is basically a steam system, with about 850 000 kw of installed steam capacity.

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[54] *Proceedings*, Am. Soc. C. E., December, 1937, p. 1929.

The writer would like, also, to mention the subject of Federal hydro-projects which Mr. Whitman covers in the closing paragraphs of his paper. In concentrating on engineering calculations, one should not lose sight of some of the social implications in the development of large hydro-plants by the Federal Government. As indicated in the principles and data presented by Mr. Whitman, hydro developments are generally feasible, economically, only when they are made complementary to steam developments. To make them directly competitive, some form of subsidy is necessary and, if this subsidy is made by the National Government, the national economy cannot help but suffer. Moreover, a subsidy that brings this about definitely tends to reduce the national pay-roll.

Although it is well known to most engineers connected with the development of power, it is not generally appreciated that in a hydro-electric project, the principal cost of the energy is the fixed charges. Very little will go into operating expenses in the form of labor and materials.[55] In a steam-generating plant, on the contrary, fully 50% of the total cost of power goes into continuously recurring expenses for material, fuel, and labor. It has been estimated that for every billion kilowatt-hours generated each year by hydro, there is less paid out for labor and material by the approximate sum of $2 300 000. This amount is obviously reflected in further reduction in purchasing power and pay-rolls through the well-known downward spiral, so that it is entirely probable that this process results in as much as $10 000 000 to $15 000 000 of annual decrease in the national pay-roll for every billion kilowatt-hours of hydro-electric energy generated. Taking, as a fair assumption, the maximum possible hydro-energy that might be developed by subsidy along these lines at 20 000 000 000 kw-hr per yr, it can be seen that the production of such an amount of energy would mean a reduction in the annual pay-roll of between $200 000 000 and $300 000 000. This most certainly is not a negligible item in the national economy; particularly, since it will result in more expensive power to every one and the only beneficiaries of the development along these lines will be those who will be provided with additional means and outlets for investments of their capital in tax-exempt Government securities.

L. N. McClellan,[56] Esq. (by letter).—The cost of power developed in connection with the large multi-purpose projects that have been undertaken in recent years by the Federal Government depends upon two important factors: (1) The extent to which the cost of these projects is allocated to purposes other than power, such as flood control, improvement of navigation, conservation of water for irrigation and domestic uses, etc.; and (2) the period of time required to absorb the full power output. These facts have been stated by Professor Barrows and Mr. Page.

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[55] *Proceedings*, Am. Soc. C. E., December, 1937, p. 1927.

[56] Chf. Elec. Engr., U. S. Bureau of Reclamation, Denver, Colo.

Congress recognized the flood control benefits of the Boulder Canyon Project and allocated \$25 000 000 of the cost of the project to flood control to be repaid out of 62.5% of revenues in excess of those required to amortize the remainder of the cost of the project. The other 37.5% of the excess revenues is to be paid to the States of Arizona and Nevada in lieu of taxes. The Boulder Canyon Project Act provides that all funds advanced for the construction of the project are to be repaid in 50 yr with interest at 4% per annum. All Boulder Canyon power has been contracted for and the power contracts provide a definite date for each power contractor to start taking power and each contractor has a 3-yr period in which to absorb the full amount for which he has contracted. Therefore, this project is on a definite basis as to the allocation of cost and period required to market the power. The estimated cost of the Boulder Canyon Project, with full installation of generating equipment, is \$143 000 000, including interest during construction. The estimated annual cost of the Boulder Canyon Project is given in Table 16 (Item No. 1), assuming that amortization of the \$25 000 000 allocated to flood control is included in the rate base.

TABLE 16.—ESTIMATED ANNUAL COST OF BOULDER CANYON PROJECT

Item No.	Assuming that the amortization of the \$25 000 000 allocated to flood control is:	Interest rate, percentage (1)	COST, IN THOUSANDS OF DOLLARS: Interest and amortization (2)	Depreciation (3)	Operation and maintenance (4)	Payments to Arizona and Nevada* (5)	Total annual cost (6)
1	Included in the rate base.........	4.655	6 657	802	841	700	9 000
2	Not included in the rate base.....	4.263	5 030	880	841	700	7 451

* In lieu of taxes.

Revenue from stored waters will average about \$250 000 per yr, and that from secondary energy will average \$350 000 per yr, assuming that the present rate of 0.5 mill per kw-hr for secondary energy remains unchanged. Deducting these sums from the total annual cost leaves \$8 400 000 as the annual cost chargeable to firm energy. The average annual output of firm energy during the 50-yr amortization period will be 4 120 million kw-hr, and the average cost of firm energy will be 2.04 mills per kw-hr delivered at the high-voltage bus at Boulder Power Plant. The present rate for use of falling water for generation of firm energy is 1.63 mills per kw-hr, in addition to which the power contractors are required to repay the cost of the generating equipment and to bear the cost of the operation and maintenance of the power plant. At 55% load factor these additional costs amount to about 0.57 mill per kw-hr making the total cost of power under the present contracts about 2.2 mills per kw-hr.

There has been considerable agitation on the part of the contractors for Boulder power to have the interest rate reduced from 4% to 3.5%, which they contend would be more in line with the present rate of interest paid by the Federal Treasury on borrowed money, and to defer repayment of the $25 000 000 allocated to flood control until the remainder of the cost of the project has been repaid. These changes would require additional legislation and if they were made the annual costs would be reduced as indicated in Item No. 2, Table 16.

Deducting the revenues from stored water and secondary energy amounting to $600 000 per yr from the total annual cost of $7 451 000, would leave $6 851 000 as the cost of firm energy. On this basis the cost of firm energy would be 1.66 mills per kw-hr delivered at the high-voltage bus at Boulder Power Plant. This corresponds with 1.7 mills per kw-hr reported by Professor Barrows for the cost of Boulder Canyon energy. Reducing the rate of interest from 4% to 3.5% per annum and eliminating the $25 000 000 allocated to flood control from the power rate base would reduce the cost of firm energy delivered at the high-voltage bus at Boulder Power Plant about 18.6 per cent.

The Grand Coulee Project on the Columbia River in Eastern Washington is the largest of the multi-purpose Federal projects. It will make available almost twice as much firm energy as the Boulder Canyon Project and no part of this power has been contracted for to date (July, 1938). The project is a combination irrigation and power development, and power revenues are expected to repay a large part of the cost of the irrigation development as well as the entire cost of the power development.

The Grand Coulee Dam and Reservoir will serve both irrigation and power and, because of this dual purpose, it has been tentatively proposed to allocate one-half the cost of the dam and reservoir to irrigation and one-half to power. The total estimated cost of the project is $425 000 000. Of this sum, $282 000 000 is the cost of the irrigation development which is to be repaid in 40 yr without interest (conforming with the established plan of repayment of Federal Reclamation projects), and $143 000 000 is the cost of the power development which is to be repaid in 40 yr with interest at 3.5% per annum. It is estimated that the water users will be able to repay about $23 000 000 of the construction cost of the irrigation development in addition to the cost of operation and maintenance, leaving about $243 000 000 of the cost of the irrigation development to be repaid out of power revenues.

With the 5 000 000 acre-ft of active storage in the Columbia River Reservoir, the firm energy output of the Grand Coulee Power Plant will amount to 8 138 million kw-hr per yr. A considerable period of time will be required for the market within economic transmission distance to absorb this large amount of energy and for the purpose of determining the cost of power it is assumed that about 20 yr will be required to market all the firm energy. It is believed that this is conservative, and it is expected that the full output will be absorbed in a considerably shorter period. If so, the deficits in the early years and the cost of power will be reduced somewhat. In addition to the firm energy there will be large amounts of seasonal or secondary energy which it is proposed to use largely for pumping water for irrigating the lands of the Columbia Basin Project. It is assumed that the water users will pay 0.5 mill per kw-hr for secondary energy used for pumping purposes.

An analysis of the financial operation of the Grand Coulee Project is shown graphically in Fig. 34, which is based on the assumptions that the firm energy will be absorbed at the rate of 720 million kw-hr during the first year of operation, increasing at the rate of 360 million kw-hr per yr until the twenty-second year when the annual output will be 8 138 million kw-hr. The irrigation development will be 50 000 acres in the first year and will increase at the rate of 50 000 acres per yr until the total of 1 200 000 acres has been developed.

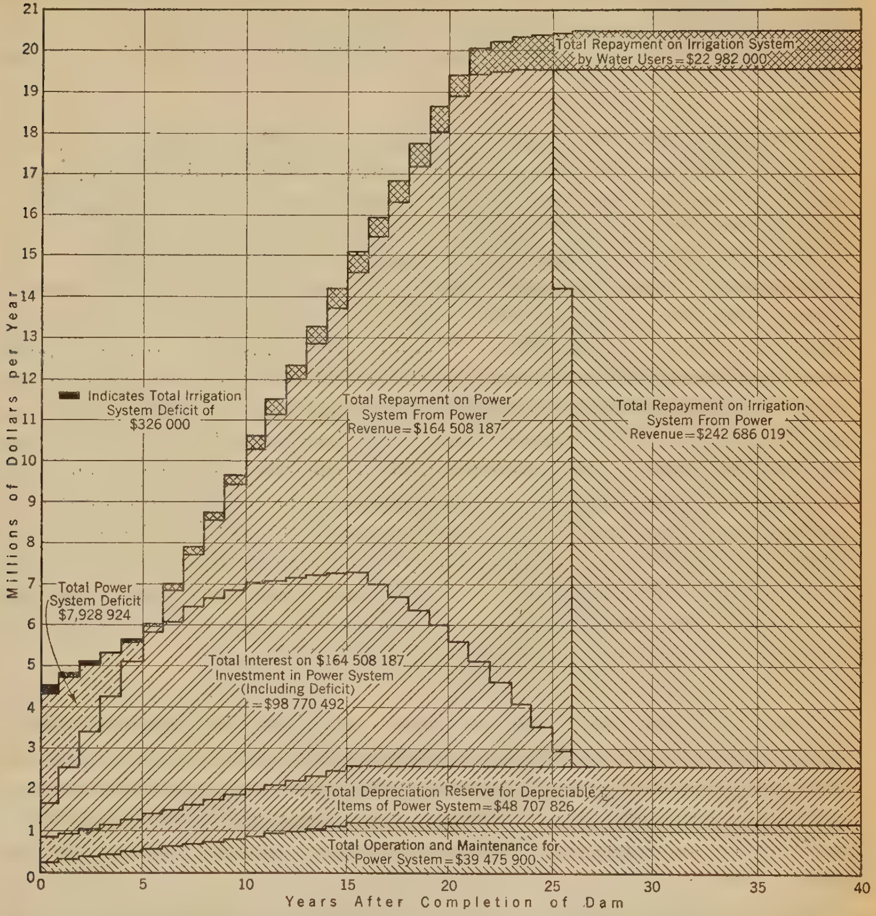

FIG. 34.—POWER DEVELOPMENT, COLUMBIA BASIN PROJECT

Interest is computed at 3.5% per annum on the power system investment and no interest is charged against the irrigation system. Fig. 34 indicates that during the first five years of operation the revenues will be insufficient to meet the annual interest charges on the investment in the power development, and deficits amounting to about $8 000 000 will accumulate which will have to be paid off with interest in subsequent years, adding to the cost of power.

Interest payments will reach a maximum in the sixteenth year, after which they will reduce rapidly as the investment in the power development is amortized. The cost of the power development will be amortized in 26 yr and the cost allocated to irrigation will be repaid in about 41 yr. This study indicates that the cost of firm energy delivered at the high-voltage bus at Grand Coulee Power Plant will be 2.25 mills per kw-hr if repayment of the cost of the irrigation development, to the extent of about $243 000 000, is included in the power rate base.

Professor Barrows considered the cost of Grand Coulee power on the assumptions that the investment in the power development only is included in the power rate base and that the total energy available is disposed of; under these conditions he found that the cost of energy would be 1.5 mills per kw-hr based on the total output, including both firm and secondary energy.

The propriety of subsidizing irrigation development with revenues from power might be subject to question, but it must be borne in mind that the water users cannot, and should not, be expected to shoulder the entire cost of an irrigation development, such as the Columbia Basin Project. If the proposed cost of firm energy is competitive with the cost of energy from other sources in the Northwest and the power users are able to secure power from the Grand Coulee Dam as cheap or cheaper than it can be obtained from other sources they will not suffer by the proposed subsidy of the irrigation development. In fact, they will benefit from the general increase in the wealth and commerce of the Northwest as a result of the development of the 1 200 000 acres of the Columbia Basin Project. Considering the Columbia Basin Project as a whole it would seem entirely proper to apply power revenues to the repayment of the cost of part of the irrigation development in order to place the project on a sound financial basis and make it economically feasible.

JAMES C. BONBRIGHT,[57] ESQ. (by letter).—Although Mr. Scharff is thinking primarily in terms of proper "financial" accounting for depreciation, the writer wishes to add some comments in terms of the related but distinct problem of "cost" accounting. What a cost accountant properly treats as a cost is often something quite different from what a company auditor properly treats as one.

First, the writer wishes to discuss the question why any allowance for depreciation need be made in estimating, say, the average kilowatt-hour cost of generation by a given steam plant at 60% load factor. Presumably, the answer is that the economic life expectancy of the plant assets must be prophesied in order to determine the total number of units of energy against which the initial cost of constructing the plant must be charged. If a new turbine were required for each kilowatt-hour of energy to be produced, no allowance for depreciation would be needed. The entire cost of the turbine would then be charged directly to each kilowatt-hour, with the result that electricity would be prohibitive except for experimental purposes. On the other hand, if the turbine and its parts were indestructible, no depreciation allowance would be required, and the entire fixed charges on that asset would be limited to interest and taxes. In fact, however, neither of these extremes will be realized. The turbine will have, say, twenty years of useful life, and then it will be retired. Therefore, in cost calculations, allowance must be made for this 20-yr life expectancy as the only means of estimating what total volume of energy should assume the burden of the cost of buying and installing the turbine.

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[57] Prof. of Finance, Columbia Univ., New York, N. Y.

The writer is well aware of the hazardous nature of forecasts as to the future retirement dates of turbines and other assets used in the production and supply of electricity. The difficulties are so great that a plausible, although not convincing, case can be made for a refusal to make the forecasts for purposes of standard, "financial" accounting. In "cost" accounting, however (which is the only type of accounting that concerns the subject of this Symposium), there is no escape from the necessity of forecasting the future; and an assertion that life expectancy cannot be estimated is tantamount to an assertion that the cost of energy generation is indeterminate.

To restate the problem: In cost analysis, the problem of allowing for depreciation is the problem of cost pro-ration. It is the problem of tying up a given capital outlay—say \$1 000 000 for a steam turbine built in 1937—with that estimated volume of energy which can economically be produced by the use of the asset for which the outlay was made. Strictly, depreciation is not a cost, because the entire cost of the turbine was in fact incurred in 1937, when it was paid for by the purchasing company, and no accounting mechanism by which this cost is gradually amortized over a period of years can alter the historical fact that the full cost was actually incurred in that year. Depreciation is a factor, however, that should determine what units of output should be held responsible for the total outlay and hence is a necessary factor in the calculation of average unit costs.

The foregoing statements are very general and, perhaps, very obvious; but the following is not so obvious and seems worthy of more attention than it has received. It concerns the allowance for depreciation due to prospective obsolescence in estimates of energy cost designed to show which of any alternative methods of producing energy is the most economical. Current practice in this respect is sometimes in error. Suppose that a manufacturer making large-scale use of electrical energy is in doubt as to whether to build a private steam plant or a private hydro-plant, since the alternative cost estimates come quite close together. The hydro-plant, say, would be scrapped in fifty years, because of silting and other elements of physical depreciation, but would not become obsolete before that date. The steam plant, with adequate replacement of wearing parts, would also have a 50-yr physical life but would become obsolete in twenty years.

Now ordinary practice would require that the cost of the steam plant be written off in twenty years, whereas the cost of the hydro-plant would be written off only in fifty years; and this mere difference in life expectancy might seem to turn the scales in favor of the hydro-plant because of its gentler liability to depreciation. Water power might appear to cost 5 mills; steam power might be shown to cost 5.5 mills. Therefore, the data would seem to say: "Let the manufacturer build the hydro-plant rather than the steam plant."

Is there not something wrong with this picture? Why should the choice lie with the hydro-plant simply because water power has already come close to realizing its maximum efficiency whereas a much better steam plant may be available in the near future? To be sure, if the manufacturer can afford to postpone building any power plant, his expectation that a better steam plant will be available within a few years may possibly justify delay in building; but if he must build some plant now, why should he decide against the steam plant merely because the hydro-plant, once built, will take a longer time to become obsolete than the steam plant?

This is a problem that the writer has not yet solved to his own satisfaction; but he suggests that, in calculating alternative costs, the engineer should not burden his putative steam-plant costs with any allowance for obsoleteness antedating the retirement date of the hydro-plant, unless he also makes a specific allowance for the lower operating expenses that the manufacturer will be able to enjoy during the remaining life of the hydro-plant if he now builds a steam plant, retires it in twenty years, and then builds a more efficient new steam plant. In practice, however, an engineer can scarcely forecast the operating expenses of a steam plant to be designed twenty years in advance of the present time. His ignorance, therefore, may force him to assume that the annual expenses of steam generation, during the longer life of a hydro-plant, will remain what they are to-day; but if he makes this assumption, he should not burden his steam-plant cost with an allowance for a premature obsoleteness which, should it materialize, would put the owner in a position to take advantage of the very improvement that will have rendered his present steam plant obsolete.

To state the point in another way: In choosing between the construction of a high-cost hydro-plant with a probable useful life of fifty years, and the construction of a lower-cost steam plant with a probable useful life of twenty years, the choice cannot be based intelligently on a mere comparison between the average cost of the hydro energy for the longer period and the average cost of the steam energy for the shorter period. Instead, the comparison should be between the cost of the hydro energy over the 50-yr estimated life of the hydro-plant, and some average cost of energy for the same period produced, first by a present-day type of steam plant and then by whatever type of new plant will become economical when this steam plant is retired. However, if it is impossible, for practical reasons, to estimate what it will cost to produce energy by a new plant designed twenty years from the present date, and if, therefore, the cost analyst must assume the continuance of the present annual steam operating expenses for the entire 50-yr period, he should not contradict this assumption by calculating an annual allowance for steam-plant depreciation that assumes a state of obsoleteness within twenty years. If the steam plant should indeed become obsolete in twenty years, that very fact would mean that the company which built this plant could then reduce its energy costs by installing a new plant. No similar opportunity to take advantage of rapid improvements in technique will apply if the company decides to build the hydro-plant, because the costs of hydro generation are largely fixed costs that are unavoidably "sunk" as soon as the plant is constructed.

The writer is not at all sure that he is right in this tentative solution of the foregoing problem; but the problem is a real one and has not yet received adequate study by specialists in cost accounting. In general, the problem arises because obsolescence, unlike physical deterioration, is only the adverse side of a good situation—of an opportunity to reduce costs in the future. The question arises as to whether cost accounting should not take direct cognizance of this good side instead of giving its sole attention to the bad side. To the important questions which Mr. Scharff has suggested for study, the writer would add this further question as offering an almost virgin field for engineers interested in private and public cost accounting.

It is pertinent to raise a question as to the definition of that multi-significant word, "depreciation." For practical reasons, the writer is inclined to favor the recognition of a number of alternative definitions, each of which might be characterized by some distinguishing adjective or adjunct. Although the restriction of the word to just one meaning would have obvious advantages, the difficulty of finding words for other accepted meanings is formidable. Engineers would probably find it difficult, if not impossible, to induce accountants, appraisers, and Courts to limit the use of the term to one single sense.

Mr. Scharff's proposal to use the term as an expression for the combined effect of a number of specified "causes" or phenomena (wear and tear, obsolescence, etc.), raises the question: Effect on what? In ordinary appraisal and accounting usage, depreciation is treated as something measurable in terms of dollars. Consequently, it is usually taken to denote a decline in value due to certain causes. Must one not say, therefore, "combined effect on value"? And must one not, then recognize that, since "value" itself is a word of many meanings, "depreciation" is indeterminate without reference to the type of "value" to which it is applied?

Finally, there is a distinction between depreciation as an economic value concept and deterioration as a physical concept. Referring, for example, to wooden poles, Mr. Scharff suggests (see heading "Physical Deterioration") that "deterioration through fiber rot in such a case may reasonably be measured in terms of decline in compression and bending strength at standard fiber stresses." Yet, the decline in the value of the pole resulting from the process of rotting (depreciation) is by no means closely correlated with the decline in compression or bending strength. Similarly, an ornamental candle, half used up (or even burned half-way down to the top of its candlestick) is worth less than one-half its value new, because of the serious impairment in its ornamental qualities. The writer suspects that appraisal engineers, in their testimony in valuation cases, are sometimes guilty of making unwarranted transfers from engineering concepts such as that of "per cent condition" to economic concepts such as that of "per cent of value new." Precisely what is meant by "per cent condition"? The writer has never found an appraisal expert who could answer that question with even a decent approach to scientific precision.

R. Robinson Rowe,[58] M. Am. Soc. C. E. (by letter).—Under the heading, "Undeveloped Hydro-Electric Power," Professor Barrows has tabulated the "horse-power, potential water power (available 90% of time)" for the ten leading States. The writer has been greatly impressed by the quantity of cheap water power available in Southeastern Alaska (sometimes called the "Panhandle"). Although not proposing Statehood for all or any part of Alaska at this time, it is suggested that the southeastern part has an area comparable to that of many States, a population density not unlike that of Nevada, a climate less rigorous than that of many States, and social, political, and economic qualities equivalent to those of many States at the time of admission.

Comparison with power resources of leading States seems in order and the comparison is more striking if expressed in terms of power density. This density has been computed on the basis of political boundaries, but, if data were available, and density were computed to water-shed lines, the comparison would be still more impressive. Less than 5 000 hp of that surveyed is available from water that has crossed the political boundary of the area. The data in Table 17 have been compiled from Professor Barrows' table, a survey[59] of Southeastern Alaska made for the U. S. Forest Service by the writer, and data from publications of the U. S. Geological Survey.

TABLE 17.—Water Power Density

Item No. (1)	State, or region (2)	Horse-power, potential water power* (3)	Area, in square miles (4)	Horse-power per square mile (5)
1	Washington	8 800 000	69 127	127
2	New York	4 300 000	49 204	87
3	Connecticut	330 000	4 845	66
4	Oregon	4 400 000	96 699	45
5	Arizona	3 700 000	113 956	33
6	Idaho	2 700 000	83 888	32
7	Tennessee	1 300 000	42 022	31
8	California	4 600 000	158 997	29
9	Southeastern Alaska	790 000	35 527	22
10	New Hampshire	200 000	9 031	21
11	Utah	1 500 000	84 990	18
12	Alabama	900 000	51 998	17

* Available 90% of time.

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[58] Assoc. Bridge Engr., State Dept. of Public Works, Sacramento, Calif.

[59] Report to be published shortly by the U. S. Forest Service and the Federal Power Commission.

Summarized, the survey of Southeastern Alaska power yielded the following totals:

Hydro-Power Potential:		
Average yield, in horse-power		1 007 590
Primary yield, in horse-power		790 470
Hydro-Power Developed:		
Average yield, in horse-power	36 064	
Abandoned yield, in horse-power	−8 190	
Net, active yield, in horse-power		27 874
Installed:		
Turbines, horse-power		40 676
Generators, kilovolt-amperes		25 811

Many of these powers can be developed for less than $50 per hp, as the physiography is favorable to low cost and most power-house sites are at protected tidewater. Storage reservoirs can be developed from "hanging lakes," in elevated valleys left by receding glaciers close to tidewater. Heads average 400 to 500 ft, with a maximum of 2 400 ft. Conduit lengths generally approximate five times the head and mountain spurs will usually permit about 70% of the conduit to be a low-pressure or a moderate-pressure line. Precipitation is high, sea-level annual means ranging from 27 in. at Skagway to 180 in. at Little Port Walter, and annual means of precipitation and run-off show less departure from norms than for the Pacific Coast States.

V. B. Libbey,[60] Assoc. M. Am. Soc. C. E. (by letter).—The writer doubts whether a technician entering the forum constituted by the *Proceedings* of the Society to discuss "depreciation" as applied to units of public utility property is free to write his own definition of the word, and he believes it imperative that this point should be brought out in the discussion of Mr. Scharff's paper.

The larger phase, of course is whether or not the concept presented is necessary or usable in the philosophy or technique of rate determination or other problems in engineering economics. This discussion will be limited to a consideration of the first proposition, with the larger question entering only incidentally.

"Depreciation" (the word must remain in quotation marks until a definition is adduced) of units of a public utility property is a most important factor in the determination of rates (especially as the rates are determined by administrative boards) in that it appears as an element of "value" and again as an element of expense. As stated by the Special Committee on Valuation and quoted by Mr. Scharff:[21] "Perhaps there is no single subject * * * that has caused more trouble than depreciation." Continuing (in the following sentence not quoted by Mr. Scharff) the Committee stated: "This has been due to various causes perhaps not the least of which has been confusion in the use of the term."

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[60] Supt., Hawaiian Contracting Co., Ltd., Honolulu, Hawaii.

The deliberate introduction of confusion into rate proceedings in the forum constituted by public utility commissions acting as judicial bodies and subject to review by the Courts seems to be an accepted—if not highly approved—technique followed, in the legal approach to rate-making by counsel employed as advocates of low rates or high rates. In such instances, the proceeding partakes of a trial, and the facts are to be determined on the weight of the polar evidence presented by advocates approaching the matter from the two extremes, and employed and paid in accordance with their ability to influence the decision.

On the contrary, in the engineer's forum the object must be to eliminate confusion. The engineer seeks to determine the operations to which the concept is subject and, if possible, fix the concept by fixing the operations. In this respect the engineer must maintain an attitude toward depreciation much the same as that which he maintains toward length, velocity, and other phenomena with which he deals.

Even in proceedings before commissions confusion as to the use of the term has been greatly reduced, and among economists, engineers, accountants, etc., the concept of the word has been crystallized to mean loss of value arising from causes that bring about ultimate retirement.

In a remarkable manner the Special Committee on Valuation, beginning its work in 1911 and reporting in 1916, viewed "depreciation" in exactly this manner. Only in 1909 had the United States Supreme Court, with a background of legal philosophy as to property and value and its body of laboriously built precedent, recognized "depreciation" as a loss of value. Even so the Court had left it as a metaphysical concept not clearly related to its causes. The Committee of engineers trained in an objective approach, went directly to the causes. Accustomed to the use of mathematics as a tool for the transposition of an unusable set of "knowns" to a usable "expression," the Committee set forth a method of arriving at "depreciation" which satisfied the requirements of fairness and convenience that were involved in the concept and were believed to conform to law. If "remarkable" carries a suggestion of surprise, then "noteworthy" is a better word, because the engineers acted only in accordance with the customary engineering method of approach.

The following is a definition of "depreciation" that may be obtained from the text of the report of this Special Committee by a grouping and re-arrangement of words and phrases, with only slight modification:

> "Depreciation," as applied to depreciable electric plant, means the loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of electric plant in the course of service from causes which are known to be in current operation and against which the utility is not protected by insurance. Among the causes to be given consideration are wear and tear, decay, action of the elements, inadequacy, obsolescence, changes in the art, changes in demand and requirements of public authorities.

The foregoing does not embrace the entire concept of the Committee but, in so far as it goes, it is in strict accord with it. It is the definition of depreciation formulated by the Federal Power Commission[61] in its new classification of accounts mentioned under the heading, "Public Utilities," in Mr. Scharff's paper.

This is also (except that "utility" is substituted for "electric" as the adjective qualifying "plant") the definition appearing in the Uniform System of Accounts recommended for adoption by the National Association of Railroad and Utilities Commissioners[62] and it is already quite generally adopted by State regulatory commissions. It is almost identically the definition of depreciation adopted by the Interstate Commerce Commission after extensive hearings on the matter of Depreciation Charges of Telephone Companies and Steam Railroad Companies.[63] It is almost, word for word, the definition appearing in Uniform System of Accounts for Telephone Companies prescribed by the Federal Communications Commission.[64]

To the layman, at least, it is a very accurate definition of depreciation as viewed by the Bureau of Internal Revenue in enforcing the provisions of the Income Tax Law.[65]

It is thus seen to be the definition of "the various administrative bureaus and commissions, necessarily called * * * into existence by the increasing complexities of our modern business and political affairs."[66]

As was not the case in 1916, when the report of the Special Committee on Valuation was rendered, this definition is now clearly accepted by the Supreme Court. In the Lindheimer *v.* Illinois Bell Telephone Company Case, Chief Justice Hughes, speaking for the Court says:[67]

> "Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence."

The engineers, the administrative bodies, and the Courts have thus tied depreciation to the essential observable operation—ultimate retirement. This is the connecting link between the abstraction, loss of value, and its causes without which there would be a "missing link." It is the integrating medium.

The definition does not fix the operation by which depreciation is to be measured, but it leaves the way open. At this stage, with past practices as to amortization so diversified it is probably expedient, reasonable, just, and equitable that this should be so. The engineer and other technicians who have a working acquaintance with mathematics can set up a rate of return and anticipated lives in years that can be agreed upon or determined by the weight of the evidence, and can develop formulas for present worth of future returns (value) that will permit arriving at loss of value. This, in the mind of the writer, will be "true depreciation." The annual charge to expense for depreciation so computed will be less than that under the "straight line" method of computation for expanding enterprises, such as power industries, and will permit lower rates that, in turn, will promote additional consumption.

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[61] "Uniform System of Accounts Prescribed for Public Utilities and Licensees Subject to the Provisions of the Federal Power Act," Federal Power Commission (Approved June 16, 1936), Definition 13, p. 5.

[62] "Uniform System of Accounts for Electric Utilities," National Assoc. of Railroad and Utilities Commrs. (Copyright, 1937), Definition 13, p. 3.

[63] "Depreciation Charges of Telephone Companies, No. 14700"; and "Depreciation Charges of Steam Railroad Companies, No. 15100"; Interstate Commerce Commission; decided July 28, 1931; 177 I. C. C., p. 422.

[64] "Uniform System of Accounts for Telephone Companies," Prescribed by the Federal Communications Commission in accordance with the provisions of the Communications Act of 1934; issue of June 19, 1935; Definition "J," p. 3.

[65] Revenue Act of 1934, Sections 11, 21, 22, and 23, Regulation 86, Article 23, Treasury Dept., U. S. Bureau of Internal Revenue.

[66] Mr. Justice Sutherland, Jones *v.* S. E. C. 298 U. S. 1 (1936).

[67] Public Utility Repts. (New Ser.), 292 U. S. 151 (1934).

Meanwhile, there are utilities which have followed a "retirement method" of accruing depreciation upon which it would work a great injustice to allow a return upon a "value" so computed. There are other utilities which have followed the "straight line method" of accruing depreciation. To compute rates on the higher value that would obtain and to include in expense the higher depreciation allowances that accure in the later life of units would impose an unfair and unreasonable burden on consumers.

As Mr. Scharff observes (see "Introduction"):

> "It is a sound financial principle that the value of a capital investment is equal to the present worth of the amortization of the investment plus the return upon the unamortized portion of the investment. This principle can be complied with, however, by the consistent application of any method of amortization whatever, * * *."

Equitably and reasonably the method chosen for amortization may be conceived of as determining loss of value and thus depreciation. This is the case if "anticipated returns" is the measure of value, and a "lack of relation to depreciation" becomes impossible.

Under the definition of depreciation presented herein and holding to the "present worth" concept of value, the rate-base of a utility may be fixed, whatever the method of amortization used in the past, and substantial justice will be done to owners and consumers. Acceptance of the definition and the "present worth" concept of value results in a workable plan for the inclusion of depreciation in rate determination or other problems in engineering economics that involve depreciation.

EDWIN D. DREYFUS,[68] ESQ. (by letter).—An important contribution to the problem dealing with depreciation is presented by Mr. Scharff who offers a novel plan for regularly checking up on the extent to which depreciation has actually progressed.

From a theoretical aspect, it is a simple matter to assume certain life probabilities and then make mathematical calculations for renewal or retirement accruals, more commonly called "depreciation." On such premises, a perfect balance can be obtained on a paper basis, as between the annual accrual rate and the accumulated reserves. The result of such calculations has the appearance of precision, and possibly the application of well informed judgment also may be implied accordingly. Extensive treatises have been published attempting to prove that accruals may be determined rather definitely. However, close investigation will reveal clearly that the depreciation problem is not quite so simple. As things happen to work out in Nature, there are always particular conditions or characteristics that Man must cope with and satisfy according to the practical necessities of the situation. The following will demonstrate this contention. A new business venture does not develop its full earning power at its inception; nor is it able to maintain earnings at required levels up to the time of final abandonment. Furthermore, it is not to be presumed that any business may be continued in perpetuity no matter how essential or vital it may be at the moment. Through the ingenuity of Man, the possibility of a substitute being developed is always in the offing.

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[68] Engr., Pittsburgh, Pa.

The fact must always be remembered that through governmental regulation, the return is limited on the one hand, and that there is, on the other hand, no corresponding guaranty of the investment or protection against losses. By dealing too closely with various features of the regulatory problem, regulators, actuarians, accountants, engineers, and others might readily produce an undesired effect, causing real injury and thereby discouraging progress.

The estimating of allowances for annual accruals for depreciation is generally approached on theee much discussed bases: Straight-Line Method; Sinking Fund Method; and Retirement Method. There are some variations, but it is not important for the practical objective to include them in this discussion. The first two are based upon assumed life tables and the third, the Retirement Method, involves a careful gauging of the requirements from time to time—such as business experience in the past, progress in the arts, changes in governmental activity, and other influences would dictate.

In order that a better idea of the true depreciation problem may be had, the life of a business enterprise (and particularly one of a highly technical character such as that under consideration) is divided (see Fig. 35) into four stages: (1) Pioneering Period; (2) Accelerating Expansion; (3) Possible Arrested Growth; and, (4) Possible Serious Decline.

It is generally known that it requires time to place a new type of operation on a paying basis. A certain pioneering or developmental period is required to establish the business. In the initial stage of operation, since the plant is new, renewals and replacements will not be necessary for some time. To set up accruals immediately when physical conditions did not require it, or when earnings did not permit, would be unwise and, possibly, financially embarrassing, it being presumed that the project was conceived on a sound and economical basis taking into account the forecast of the growth in business and improvements in economy.

The second stage represents the time when the demand for services has rapidly increased until the territory or market has been fairly well saturated. As advance takes place in this period, the business is able to pay a fair return and accumulate a fund for retirements. Conservative management is guided by these practical conditions and gauges its reserve situation accordingly. It may speed up or slow down the rate of accumulation to the extent that it is able to foresee future changes. The percentage of reserves in relation to the investment to be protected may be maintained at a certain level for a period of time; or it may be gradually or rapidly increased as the circumstances may warrant.

When the third stage is reached and the growth of the business is definitely arrested, extra caution must be exercised. Many technological businesses follow what is termed the biological or *S*-curve as typified by the investment curve, *A*, in Fig. 35. In this day of science and invention, a business is "placed on warning" when its volume does not keep pace with the growth of the population and the markets for goods and services. If a company delays too long in accumulating reserves, in proper degree and during the productive period, for the protection of the investment, it may readily happen that it will be overtaken by disaster. When the fourth stage is reached, any attempt to overcome shortcomings of the past would be futile.

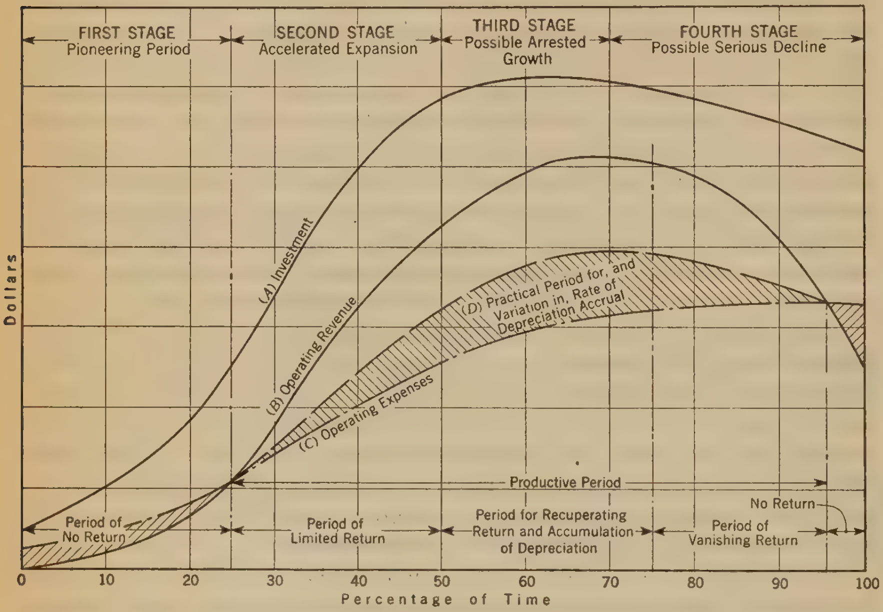

FIG. 35.—POSSIBLE DEVELOPMENTS REQUIRING CONSIDERATION IN GAGING THE ACCOUNTS FOR RESERVES FOR ULTIMATE RETIREMENT

The point that this illustration emphasizes is that as a result of various business situations the proper rate of depreciation accrual is often vastly different from that given by theoretical formulas based upon the assumed life of the items of property. The rate of accrual must vary necessarily with the productiveness of the business. It is for this reason that those responsible for management, frequently prefer to use a certain percentage of gross reserve to provide for maintenance and the annual accruals for renewals and retirements. If large sums are spent in any one year for maintenance and repairs, they have the effect of restoring the values in the property and, therefore, result in a lesser need for accruals.

Fig. 36 shows, relatively, the degree of protection provided for a given investment throughout the economic life of the enterprise under three methods of accruing for depreciation: Curve (1), Straight Line; Curve (2), Sinking Fund; and Curve (3), A Practical Method of Retirement. The first two methods do not deal with realities, and the inevitable consequences would be that they would fall far short of protecting the investment.

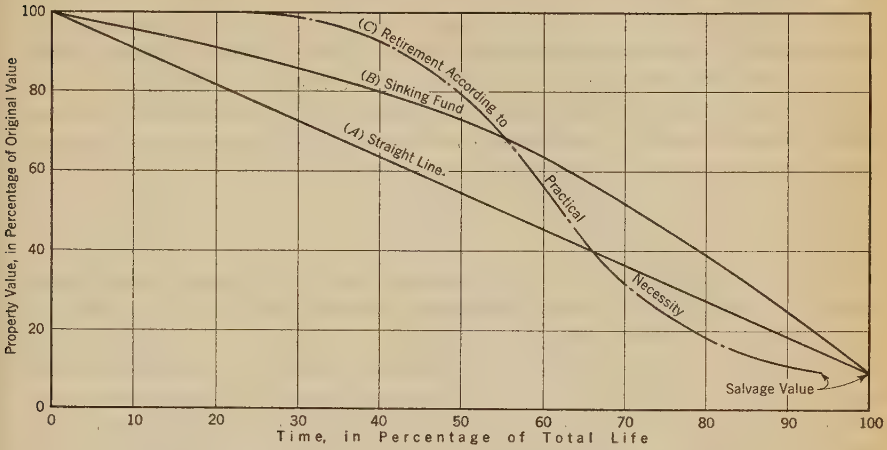

FIG. 36.—PROGRESS IN PROTECTING THE INVESTMENT IN PROPERTY BY DIFFERENT METHODS OF DEPRECIATION

The adoption of a theoretical treatment of depreciation for regulatory purposes without allowances for various practical features, may unwittingly and unlawfully deprive the owners of a substantial part of their property.

R. H. PARSONS,[69] ESQ. (by letter).—Since most of the operating charges of a given hydro-electric system, such as labor, management, maintenance, etc., are almost independent of the load, and as the capital charges are also constant, it is reasonable to assume, as Mr. Page does (see heading, "Effect of Load Factor"), that the cost of producing hydro power will be inversely proportional to the annual load factor. It must be remembered, however, that the consumer of electricity pays only for current at his own terminals, so that the capital charges and operating expenses of long transmission lines usually will have to be added to those of the hydro station before the minimum selling price of the power can be determined.

As far as fixed charges are concerned, of course, the same conditions apply to a steam power station, but the latter has the advantage that it can be built and extended as required by the demand at the time, thus reducing idle capital expenditure to the minimum; and the greater freedom in the choice of its location avoids the cost of long transmission lines. The fixed charges on a steam plant, therefore, are always substantially less than those of a hydro plant, so that as regards these charges only, the cost of steam-generated power to the consumer will be much less affected by a poor load factor on the system. In 1928, Professor A. G. Christie[70] suggested a formula of the type: British thermal unit per kilowatt-hour $= a + \frac{100\,000}{L}$, as showing the effect of the load factor on the heat consumption of a power station. This formula, however, is open to the serious objection that it takes no account of the magnitude of the output. Indeed, it would imply that any station running steadily for a year at half load would have the same heat consumption per kilowatt-hour as if it had been running steadily at full load, for the load factor would be the same (100%) in both cases. The effect of the load factor on efficiency can be determined more properly by the rational formula[71]:

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[69] Engr., C. A. Parsons & Co., Ltd., Newcastle-on-Tyne, England.

$$H = a + b\,K + \frac{c\,K}{L} \qquad \text{(7)}$$

in which H denotes the annual heat consumption of a power station in British thermal units; K is the annual output in kilowatt-hours; and L is the load factor. By substituting recorded values of H, K, and L, in Equation (7), three simultaneous equations are obtained from which the values of the constants, a, b, and c, can be obtained. In general, the value of c is quite small, from which it follows that the efficiency of a power station producing a given annual output is not, in practice, appreciably affected by the load factor.

Referring to the remarks of Mr. Whitman the point that is often overlooked when comparing the capital costs of hydro and steam plants for a given duty, or when considering the part to be played by each in a combined system, is the advantage the steam turbine possesses in respect to overload capacity. By judicious design it is possible to construct a large steam turbine for a maximum output equal to practically double the most economical output, and to attain this result with only a very slight sacrifice of efficiency at the higher loads. In power stations, the use of turbines with large overload capacities was fully dealt with in a paper[72] read by Sir C. A. Parsons and R. Dowson at the World Power Conference, at Berlin, Germany, in 1930. It can be shown that turbo-generators having a continuous rating of 75% instead of the more usual 25% in excess of the most economical rating would provide a very large emergency capacity for about one-third the cost of installing extra plant for such emergency, thus giving a lower total cost per unit generated, which after all is more important to the consumer than spectacular thermal efficiency results.

In his recapitulation of the various features of the papers, Mr. Uhl, referring to the depreciation and obsolescence of steam and hydro plants, states that the average rate of depreciation may be as high as 5% for a steam plant and as low as 2% for a hydro plant (see "Elements of Power Cost: (2) Depreciation and Obsolescence"). Taking his own estimates of life, however, it is equally true that the depreciation of a hydro plant may be as high as 2.5%, and that of a steam plant as low as 3 per cent. There are turbo-generators running to-day in British steam stations which were installed 34 yr ago (in 1904), and it is found economical to keep them in service in spite of their obsolete steam conditions and design. Mr. Uhl (see heading "Steam and Water Power Combined") claims advantages for hydro plants over steam plants in the matter of starting-up time. It is certainly true that to start up a steam station with everything cold, would normally take several hours, but such a requirement is so rare that it need scarcely be considered. A turbine, equipped with slow-speed turning gear, can be put on the line from the cold condition in a matter of minutes should this be necessary, and boilers are now being developed which will provide steam from a cold start almost as quickly.

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[70] "Peak Load Problems in Steam Power Stations," by A. G. Christie, read before the meeting of A.S.M.E. at its meeting in New York, N. Y., December, 1928 (F.S.P. 51–24).

[71] *The Engineer* (London, England), August 13, 1937.

[72] "The Use in Power Stations of Steam Turbines Having with Their Auxiliaries Large Overload Capacities," by the Hon. Sir Charles A. Parsons and R. Dowson, read at the Second World Power Conference, Berlin, Germany, 1930.

F. Knapp,[73] Esq. (by letter).—Power is stated by Mr. Uhl to be one of the few commodities the cost of which has steadily decreased whereas the cost of other commodities has been doubled or trebled. This statement requires some explanation. Public utilities combine two characteristics. They serve to supply the population with essential needs, such as those provided for by power stations, gas-works and water-works, railways, tramways, mail, and similar undertakings. The other and essential characteristic is the impossibility, both of a technical and economic nature, of allowing free competition because of the tremendous importance of the necessary capital outlay and the requirements of the right of way. The monopolistic nature of the public utilities is practically unavoidable and explains the acrimonious discussions among those favoring either the private monopoly controlled by the Government, or the public monopoly. At any rate, any form of monopoly can be tolerated only as long as it serves the public interest. Experience in the United States, where the system of the privately owned utilities predominates, has shown the weak spots and the difficulties of an effective control, leading to rather unhealthy conditions. In the case of the public utilities owned by the Government, the management comes, daily and hourly, in close and sensitive contact with the public, thus submitting the undertaking to an extremely efficient control. The prestige and authority of this type of public utility often represent a political weapon of outstanding importance.

The danger of a monopoly arises from the removal of the natural law of supply and demand, thus offsetting completely the regulation of the cost of the goods as established by competition. Making the assumption that the monopolist wishes to obtain a profit as great as possible (an assumption which is not far from the truth), the question arises as to just what level of cost will be necessary for him to reach his purpose. Choosing a high cost, the profit per unit sold rises, but the market probably becomes smaller. On the other hand, a large market permits him to reduce the cost per unit. Forced with this alternative, the monopolist certainly chooses the level that will make the product of cost per unit times the quantity sold, a maximum. This product being different for different cases, the monopolist will try to find this cost level by cut-and-try methods. Of deciding influence is the "elasticity" of the demand. It is now a well-established fact that the demand for power is extremely elastic; that is, a reduction of the cost to the ultimate consumer is accompanied by a considerable increase of the demand to such an extent as to make up for the loss caused by the original reduction. The significance of the elasticity of the demand for the fixing of the cost level makes it evident that the managements of every monopoly, and especially those of the electrical utilities, base their politics of cost upon a close (in most cases, statistical) knowledge of the coefficient of the elasticity of demand. Thus, in recent years, the cost of power is being reduced by economic reasoning, and has absolutely nothing to do with a philanthropic point of view.

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[73] With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

As stated by Mr. Page, there is nothing bad in either public or private ownership of electrical utilities, provided the public interest is being served. There are too many examples to prove that this has not always been the case. Nothing is gained by arguing about a few mills per kilowatt-hour in the cost of the current as finally supplied to the customer. What is necessary is a broad consideration of the social and economic objectives of the electrical utilities and the need of placing the public interest above any other consideration.

JOHN C. PAGE,[74] M. AM. SOC. C. E. (by letter).—The Second Symposium on Power Costs should prove most helpful in clearing away much of the haze that has surrounded some of the questions involved. The problem of computing the "obsolescence" factor, for example, has received particular attention, which has been clarifying. Only a few points raised in discussion require comment.

There seems to be general agreement, among those who participated, on certain fundamentals. These are: (1) That it is impossible to determine, on a general basis, whether steam or water power is cheaper, since each development must be considered separately, taking into consideration physical, economic, and technical facts which vary from place to place and from time to time, the sum total of which may favor either steam or water power in a given instance; and (2) that the electrical industry is monopolistic by nature, and whether privately or publicly operated, must serve in the public interest in self-justification.

Mr. Mason asks who has brought about the advancement in the type and knowledge of equipment for hydrogeneration in the past fifteen years, and answers that it certainly included no one but the private company, the manufacturer, and the private laboratory, thereby excluding all public agencies from any credit for this valuable public service. The record will show that many valuable contributions of such equipment for hydro plants have been made by designers in public employ. Certainly the designing section, Bureau of Reclamation, through its laboratory work and otherwise, has improved high-pressure reservoir outlets (to name only one example); and others could be mentioned. Other public agencies also have made important contributions. In the advance of the industry by scientific means, there seems to be sufficient credit for all. In the general advance, there has certainly been no lack of co-operation between engineers and designers (no matter where or by whom employed), and the manufacturers.

Professor Mead states that interest during construction is seldom considered in computing the cost of Federal projects. This statement is true, unless too much emphasis is given to the word "seldom." Interest during construction is not computed on every Federal project. It was computed in the rates for Boulder Dam, to name one, and projects financed by loans from the Public Works Administration carried interest during construction, although the grant which usually accompanied the loan served to offset, in part, at least, the interest charges. The rates at which power is sold by all projects of the Bureau of Reclamation that remain under Federal control are computed to include interest on the capital cost chargeable to power.

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[74] Commr., U. S. Bureau of Reclamation, Washington, D. C.

In this discussion, Professor Mead intimates that many of the Federal multiple-purpose projects, which include water power, are unwarranted. He indicates that information concerning their cost and their probable returns is not obtainable, and that the conclusion must be reached that they were not adequately planned. The writer calls attention to the contribution by Mr. McClellan, in discussing this Symposium, in which information as to the cost and repayment of the Boulder Dam and Grand Coulee Dam projects is presented. Data similar to these are available on other projects constructed by the Bureau of Reclamation. Although Mr. McClellan's discussion dealt primarily with cost and with returns, the planning and studies affecting these factors were not made more carefully than the investigations of other phases and features of the projects.

There may be instances, as Professor Mead hints, in which the desire to create work on useful projects outweighed other considerations as to desirability, necessity, or soundness, but the writer knows definitely of no such project. Those constructed by the Bureau of Reclamation (and the writer is qualified to speak authoritatively only for the Bureau) may have been begun quickly at the outset of the work programs; but that is no indication that thorough investigation had not preceded them. They were all carefully planned in advance, and some of them had been awaiting construction for long periods. Whether the conclusions upon which the plans were adopted were erroneous, only some historian in the future can say. The preparatory studies, however, were the best that could be made by able and qualified engineers at the time. The writer does not believe that any of these irrigation-power projects will be proved later to be unwarranted expenditures.

C. F. Hirshfeld,[75] Esq., and R. M. Van Duzer, Jr.,[76] Esq. (by letter).—Both Messrs. Christie and Elliott have reiterated a fact which the writers have emphasized—that judgment must be used in arriving at power costs from the data presented in this paper and others comprising the Symposium. The concrete examples contained in Mr. Christie's discussion, showing how the initial plant cost can vary widely with different local conditions, serves further to illustrate that generalizations as to power costs cannot be made.

The writers made no direct mention of reserve equipment and its effect on power costs because of the complications that would have been introduced. The subject could very well serve as the topic for a complete paper. The fact that reserve equipment is necessary for proper maintenance of service is one of the important reasons why few fuel-burning plants show annual capacity factors much in excess of 60 to 70 per cent.

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[75] Chf. of Research, The Detroit Edison Co., Detroit, Mich. Mr. Hirshfeld died April 19, 1939.

[76] Engr., The Detroit Edison Co., Detroit, Mich.

It is unfortunate that the various ratios or factors used to express the significant characteristics of load have never been clearly defined by a recognized engineering body. The writers have observed in several of the papers and discussions that the same terms are used by the several writers with totally different meanings implied. For instance, the terms load factor, capacity factor, and use factor are ratios expressing entirely different characteristics of load. This fact should be recognized when comparing power-cost data.

H. K. Barrows,[77] M. Am. Soc. C. E. (by letter).—Mr. Mason's reference to over-capacity of plants recalls an unusual example of this condition in the Cobble Mountain plant of the City of Springfield, Mass., which is one of the features of their Little River water-supply system. This plant develops power from water for municipal supply as well as surplus water now available. Power use is during the eight hours of the day, at such times as needed for peak use in the power system of the Turners Falls Power Company, the lessees of this plant. To adjust conditions of flow to the 24-hr municipal supply needs, there is an intervening balancing reservoir between the power plant and the municipal intake.

The drainage area available is 45.5 sq miles and a net head of 330 to 420 ft, with complete regulation by storage. The wheel capacity is 33 000 kw under the maximum head of 420 ft, which corresponds to 1 160 cu ft per sec and about 25 cu ft per sec per sq mile of wheel capacity. As far as the writer is informed this is the highest flow per square mile of wheel capacity of any plant in the world, corresponding as it does to what would be a flood flow of this river. The yearly output is about 20 000 000 kw-hr, which gives the low capacity factor of about 0.09.

This plant is an example of what might be called a "super-peak load" plant —used only when most needed to help carry seasonal peaks in the load system. It cost about $4 200 000 including dam and reservoir, or $140 per kw, on the basis of 30 000-kw capacity. The power-plant alone, including penstock, surge tank, power-house, and equipment, cost $1 100 000 or $37 per kw.

Mr. Parker calls attention to the effect of long conduits as tending to increase the cost of high-head plants. As he notes, this tends to make the waterway for such plants a large proportion of the investment. On the other hand, this excess cost is often compensated by a smaller cost of dam, as well as of power-house and equipment, as will be noted by reference to Table 7.

The capacity of waterway is fundamentally based upon loss of head under average operating conditions; that is, wheel capacity times capacity factor, keeping in mind that true average losses should be used and not the loss corresponding to average flow or use of water. A low capacity factor, with a long penstock, however, may mean a somewhat greater carrying capacity in order not to lessen, too greatly, the maximum plant output.

For wooden pipes or flumes a greater allowance for depreciation should undoubtedly be made. Of course, it was impracticable for the author to vary this factor in individual cases for lack of detailed information as to structures.

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[77] Prof. of Hydr. Eng., Mass. Inst. Tech., and Cons. Engr., Boston, Mass.

Mr. McClellan has contributed an interesting diagram (Fig. 34) of costs and tentative financing for the Grand Coulee Project on the Columbia River.

Mr. Rowe has called attention to the water-power resources of Southeastern Alaska, where, evidently, conditions are favorable to development, as compared to the leading States in this respect.

The author appreciates the interest shown in his paper and hopes that it may be of some service in the field of water power.

MAURICE R. SCHARFF,[78] M. AM. SOC. C. E. (by letter).—Professor Bonbright has raised some most interesting questions. The writer agrees with him that it is a common error, in making economic comparisons of comparable installations, to disregard the relative probability of the development of obsolescence in different types of equipment and its resultant effect of probable frequency of replacement. The specific problem which he submits, of the correct comparison of a hydro plant to be scrapped in 50 yr and a steam plant to become obsolete in 20 yr, is an intriguing one. The writer suggests as a theoretical solution that the comparison should be made on the basis of the sum of the present worths of the total costs of the alternate projects, including capital costs and operating costs, for a period of time equal to the least common multiple of the assumed periods of replacement, or 100 yr. The writer agrees fully with Professor Bonbright as to the impossibility of anticipating the developments in steam power generation at the end of periods of 20, 40, 60, and 80 yr, respectively. Such an impossibility, however, does not prevent the submission of a suggestion as to the theoretically correct basis of comparison.

Mr. Burnell and Mr. Dreyfus have called attention helpfully to some of the practical management, and financial, considerations that must inevitably affect the practical determination of depreciation policies. The writer believes, however, that it will be most helpful for the engineering profession to continue the discussion and clarification of the engineering and economic aspects of the problem, leaving these practical financial and management considerations for application to the final engineering conclusions.

Mr. Thomas has emphasized the practical advantages of the application of the sinking fund, or compound interest method of treating depreciation to a large hydro-electric project. The writer has not intended to express any substantial disagreement with this point of view.

Mr. Libbey apparently agrees fully with the writer in his suggestions as to the relationship between annual and accrued depreciation and the equality between the value of the capital investment and the present worth of its amortization, plus the return upon the unamortized part of the investment. He seems to the writer, however, to state the conclusion from this premise in reverse when he suggests that the method chosen for amortization may be conceived of as determining loss of value and thus depreciation. It would seem more correct to the writer to say that the loss of value or depreciation should determine the method chosen for amortization.

In conclusion, the writer is gratified by the interest that has been shown in the subject and expresses the hope that engineers may continue to study the engineering and economic aspects of this difficult problem.

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ECONOMICS OF SEWAGE TREATMENT

By George J. Schroepfer,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Charles Gilman Hyde, and Frederic Bass.

Synopsis

A general analysis of the economics of various basic processes of sewage treatment is presented in this paper which draws attention to a number of influencing and controlling factors that affect the comparison of the various processes as applied to a specific situation. Data are presented on construction and operation costs and on the reductions accomplished by the various processes indicative of the usual conditions and of the range to be expected.

Introductory Statement of the Problem

Sewage treatment processes or combinations of processes are available for accomplishing varying degrees of treatment, including relatively complete removal of pollution, to meet the requirements of practically any condition. The number of processes available presents the problem of determining for a particular situation the method and degree of treatment which most economically and practically satisfies the needs of that definite problem. From a purely economics viewpoint, and for a fixed set of conditions, there is only one solution to the problem. However, a number of factors other than cost, affect the decision in a number of instances. A variety of considerations well known to sanitary engineers, serves to determine the degree of treatment necessary in a particular problem. Furthermore, the method capable of accomplishing the degree of treatment determined upon, is dependent upon a still larger number of influencing practical considerations. These practical factors may be of controlling importance and may make necessary the elimination of certain methods from considerations; or they may be of lesser import and serve merely to affect the judgment of the engineer in the determination of the most desirable and economical form of treatment.

Without question the most important item affecting the economic analysis of the sewage treatment problem is the determination of the degree of treat-

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ment necessary. In view of the fact that this question is definitely one which must be answered for every particular problem, no detailed discussion will be made herein. The fact that the degree of treatment required may vary from practically nothing to almost complete removal of pollution, necessarily affects the selection of the process of treatment in view of the limitations of the various processes now in use. The further fact that the cost varies with the degree of treatment provided, makes this latter consideration fundamental. The analysis of the type of process which will most economically provide the necessary degree of treatment is generally, from the standpoint of cost to the community, of much less importance than the determination of whether any intermediate, or relatively complete, treatment is necessary.

The aim of this paper is to show, in a general manner, the economic relationship between the various methods of treatment as well as between the different processes involved within the particular method.

Economics Problems Involved

In the selection of the degree and method of treatment best adapted to a particular situation, a variety of economics problems present themselves. Before proceeding with detailed discussion of the problems, it is well to consider them in general terms.

Disposal by Dilution.—The most common and generally the most economical manner of disposing of sewage and waste matter is to discharge it into the available receiving bodies of water—ocean, lake, river, or creek—either without treatment or after varying degrees of pollution removal.

As a natural drainage channel, a watercourse receives the normal surface pollution of the area tributary to it, and by the processes of natural purification converts the organic matter into stable, non-putrefactive matter. The processes underlying this change are physical, chemical, bacteriological, and biological in character, and depend for their effectiveness upon the existence of a proper and sufficient quantity of the elements and organisms which accomplish the changes. When, however, because of the growth of communities along such watercourse and their consequent discharge of sewage and trade wastes into the river or stream, the organic load becomes too great, the oxygen supply in the river is reduced, with a consequent impairment in the activity of the desirable organisms. When the balance between the oxygen resources and the oxygen demand of the organic matter discharged into the stream becomes unfavorable, some form of treatment becomes necessary to supplement the natural processes. The time when treatment becomes necessary and the extent of treatment required are dependent on the unfavorableness of the balance and the use to which the river will be put.

When adequate dilution is available, proportionate to the sewage and wastes discharged into it, the disposal of the sewage by emptying into a near-by lake or watercourse offers an economical solution to the sewage disposal problem. The proportionate degree of pollution that the river can receive is dependent on the characteristics of the receiving body of water, and the use to which it is put. In any event, there is an upper limit to the capacity of the stream or lake to receive polluting material and when that condition is reached, treatment to some degree, to relieve the unsatisfactory condition, becomes necessary.

Preliminary Treatment.—Many problems present themselves in connection with the economics of preliminary treatment. An important consideration is the most economical period of sedimentation when considered in relation to the reduction in suspended solids and bio-chemical oxygen demand (B.O.D.). The comparison between fine screening and sedimentation is worthy of consideration.

Secondary Treatment.—The advantage and economy of one type of secondary treatment as compared with others capable of performing the required degree of treatment present a number of problems. Within the various types themselves a number of considerations are of importance, such as the effect of various rates of application on the cost of trickling filters, the economical period of aeration in the case of the activated sludge process, and the effect of the addition of varying quantities of chemical in the case of chemical treatment. In all cases consideration must be given to the varying reductions accomplished, and to the controlling effect, in special cases, of the limitations on maximum reductions capable of accomplishment by the various processes. The extent to which preliminary treatment is carried before the secondary process is begun, as well as the type of such treatment, is an important economics problem.

Sludge Disposal.—One of the most important factors in arriving at a decision as to the economics of various types of treatment is the method of sludge disposal. Of the various methods available in any project, a study of the economics and applicability of such methods must be made, having in mind the local influencing or controlling factors. What might be a satisfactory and economic method under the conditions of one situation, might be objectionable or extravagant in another, stressing the fact that in sludge disposal, perhaps more than in sewage treatment, every situation must be considered by itself. Generalizations are difficult, and likely to be misleading when applied to sludge disposal. It is for this reason that the writer has refrained from evaluating the various sludge disposal methods in a general manner; for example, it is apparent that the controversial subject of the economics of sludge digestion or no digestion, in connection with disposal by vacuum filtration and incineration, cannot be answered in general terms when it is realized that variations in a large number of influencing factors definitely affect the results of the investigation. As a further example, vacuum filtration and incineration would not be justified in a situation where climatic and soil conditions and favorable plant environmental factors make disposal of digested sludge on sand drying beds particularly advantageous and economical. The variety of methods of sludge disposal available to the engineer, with their consequent questions of fertilizer production, power production with gas engines, and utilization of heat in incineration, considered in relation to characteristics of sewage and sludge, very definitely make the solution of the sludge disposal problem a specific one, with known conditions, rather than one capable of generalized discussion. The estimates shown herein for the various types of treatment are based upon digestion and air drying on open sludge beds.

However, the substantiating costs from various plants include a number of different methods of sludge disposal, such as digestion and drying, vacuum filtration, incineration, fertilizer production, lagooning, etc. Although the writer has similarly analyzed the costs for various methods of sludge disposal, and realizes the importance of this factor in the determination of the economics of sewage treatment, it was felt to be impracticable to include the analysis in this paper because of length limitations.

Miscellaneous.—For reasons of brevity a number of accepted forms of treatment, such as pre-aeration in connection with trickling filters, chlorination, intermittent sand filters, and Imhoff tank treatment, are not discussed, or if mention is made of them, the statements are of a general nature.

METHOD OF COMPARISON

To compare the merits of the different processes, accomplishing as they do, varying reductions in the strength of the influent sewage, the total annual charges of treatment by the various processes have been reduced to a cost per 1% reduction in suspended solids and B.O.D. Although, in special cases, bacteriological and nitrogen analyses may be important, the suspended solids and B.O.D. determinations are believed to furnish the most reliable general index of the efficiency of varying treatment processes.

It is the fact that the economic relationship between the various processes is tied in with, and depends very closely on, the analytical results obtained, that explains the large quantity of material included herein which is not strictly economics. Costs, analytical results obtained, adaptability, and other influencing or controlling factors must be carried "hand in hand" in the determination of the most favorable and economical treatment for a particular situation.

From a cost standpoint the various processes have been compared on the basis of the total annual charges per million gallons of sewage treated per 1% reduction in either suspended solids or B.O.D. Aside from the varying reductions accomplished by the different processes already discussed, this unit or yardstick does not take into consideration the fact that the higher the degree of treatment the greater the cost, as will be shown subsequently herein. In other words, the cost of removing the last 10% of suspended solids or B.O.D. in a sewage is proportionately much greater than the first 10%, as is generally recognized. This fact must be realized by the reader in comparing the costs on the unit used herein or, otherwise, conclusions will be drawn from the data which are contrary to the facts.

A further variation which the writer's unit does not definitely show is the effect of size of plant on the costs per unit or reduction. For various reasons the costs per million gallons in the larger plants are generally lower than in the smaller ones, and this variation is greater in some types of treatment than in others. This variation must also be considered in studying the data herein.

With these exceptions the method of evaluating the various processes is believed to be proper and correct and when cognizance is taken of the effect of these factors, a true picture of the cost and place for the various processes can be attained.

General Statement on Cost Data

The construction cost estimates presented herein are based on a level which, in general, prevailed from 1923 to 1930. They are believed to be reasonably close to the *Engineering News-Record* cost index of 200. Although, consciously or subconsciously, the writer had the general conditions existent in Minneapolis and St. Paul, Minn., in mind in preparing the estimates, the cost data are believed to be of general applicability in so far as cost level is concerned, except for unusual conditions. Substantiating cost data from actual plants bear out this statement. The previous statement applies, as noted, to the general cost level, but it is not intended to cover the multitude of factors which by their favorable or unfavorable effects might make one form of treatment more economical in a particular situation than another.

The term, total annual charges, as used in the paper, includes both operating and maintenance costs and fixed charges on the investment. Fixed charges include interest and amortization on bonds issued which is the usual method of financing sewage works improvement. A value for fixed charges of 6% of the construction cost annually has been used, realizing, however, that the magnitude depends for a particular situation on the form of the bond issue, the bond period, and the interest rate, which is influenced by a number of economic factors. This percentage represents an approximately average rate and is approximately the rate commonly secured. The effect of higher and lower rates will be shown in a later comparison.

Bond amortization should more properly be considered as a depreciation item which takes into account the comparative lives of the various plants, rather than one of bond retirement which may cover long or short periods, depending on local requirements and public feeling at the time of issuance. Allowance has been made for this fact by considering a relatively long bond period, and providing in the estimates for operation and maintenance cost an item of "Maintenance, Supplies, Repairs, and Renewals" to cover all such costs as might be necessary to maintain the plant in a first-class operating condition.

Characteristics of Sewage Considered

Although an attempt is made to make the comparisons and conclusions of general applicability, the condition and situation in Minneapolis and St. Paul have been uppermost in the writer's mind. This fact is obviously both necessary and beneficial, since the effect of certain features, such as characteristics of the sewage, capacity of plant, unit costs, etc., would make an abstract generalization worthless, if not even delusive. The characteristics of the sewage considered have an important effect on the results of any economic study of sewage treatment.

Except where specifically stated to the contrary herein, sewage having the following characteristics will be assumed:

Suspended solids, in parts per million	185
Five-day B.O.D., in parts per million	200

The foregoing values are the approximate strengths of the sewage from Minneapolis and St. Paul as determined in the latter part of 1927 and the early part of

1928, and because of their use in a number of economic studies to which reference will be made herein, have been used in this study. Except for the fact that the suspended solids content is rather low, the foregoing characteristics represent a sewage of normal strength, as evidenced by the fact that the average of the annual results at forty-two representative plants in the United States was approximately 239 ppm on a suspended solids basis and 198 ppm on a 5-day, B.O.D. basis.

ESTIMATED UNIT CONSTRUCTION COSTS

The estimates of construction costs of the various treatment plants used in this analysis are intended to be reasonably close to, but in general slightly higher than, the actual costs, since in such a general investigation as this, where fully developed plans and specifications cannot be prepared, it is obviously difficult to make a detailed and complete list of quantities to which unit prices can be applied. Being mindful of this fact, contingency items were added to provide for inevitable omissions, so that the estimates will show costs which will probably be found to be slightly higher, rather than lower, than actual bid prices; 15% was added to the estimated construction costs for engineering, contingencies, and omissions.

Because of the impracticability of presenting detailed estimates of costs of all the structures and equipment which make up a complete treatment plant, the estimated costs used herein are presented as unit cost estimates (that is, per cubic foot of tank, per acre-foot of filter, etc). This manner of presentation lends itself to a comparison with substantiating costs at actual existing plants. The following unit costs have been used in the estimates, except in the case of smaller plants, where somewhat higher costs have been used:

1.—Settling Tanks.—Estimates are based on cost of 54 cents to 62 cents per cu ft of effective capacity, which includes cost of tanks, mechanisms, influent and effluent channels, and various appurtenances, the value depending upon whether it is used in connection with trickling filter or activated sludge plants, and whether preliminary or final settling tanks are under consideration. In substantiation of the foregoing costs, an average of eight plants, most of which were constructed from 1930 to 1934 was 51 cents per cu ft, the lowest unit cost being 35 cents and the most expensive, 66 cents.

2.—Trickling Filters.—Estimates are based on a cost of $14 000 per acre per ft of depth, exclusive of cost of covers. An average of five trickling filter plants constructed from 1923 to 1929 was $13 170 per acre per ft of depth, the lowest being $10 060 and the highest, $15 600.

3.—Aeration Tanks in Connection with an Activated Sludge Plant.—An estimated cost of 52 cents per cu ft of effective aeration tank capacity was selected, this cost including operating galleries, but excluding air compressors, return and excess sludge pumps, power plant, air filters, and appurtenances. In substantiation of this value the average of four plants constructed from 1927 to 1931 was 51 cents, the lowest being 45 cents and the highest, 57 cents.

4.—Power Plant, Air Compressors, Sludge Pumps, Air Filters and Appurtenances in Connection with Activated Sludge Plant.—The estimated cost of these items is $7 500 per million gallons capacity. The average of six plants constructed from 1927 to 1934 was $8 800, the lowest being $5 100 and the highest, $12 200.

5.—Sludge Digestion Tanks.—Estimates are based on 35 cents per cu ft of effective capacity, including tanks, sludge pumps, galleries, heating plant, mechanisms, covers, etc. An average of nine plants constructed from 1928 to 1933 was 32 cents, the lowest being 21 cents and the highest, 44 cents.

6.—Sludge Storage Tanks.—Estimates are based on a cost of 20 cents per cu ft of effective capacity, including covers. An average of six plants constructed from 1928 to 1933, some of which were uncovered, was 18 cents, the lowest being 12.5 cents and the highest, 27 cents.

7.—Sludge Drying Beds.—Cost estimates are based on 60 cents per sq ft of open beds, the hand-cleaned type, including transportation facilities. In substantiation, the average of twenty-two plants, constructed from 1925 to 1933, was 55 cents per sq ft, the lowest being 26 cents, and the highest, $1.03. Considering only the nine larger plants the costs averaged 48 cents per sq ft. Substantiating costs do not include hauling equipment.

SCREENS AND GRIT CHAMBERS

General.—Although not productive of as high removals, proportionately, as other units of the treatment plant, the removal of the larger material by screens and the heavier material by grit chambers is none the less important. The efficient and economical operation of the remainder of the plant makes the inclusion of these units a necessary adjunct to a well designed plant, especially if the sewage is from a combined system.

The early installations of screens and grit chambers were simple, involving usually manual removal of screenings and grit; but recently mechanical and automatic equipment has become available and its use in connection with later plants has resulted in more efficient operation of these units.

Bar Screens

Removal Accomplished by Bar Screens.—A study has been made of the operating results of 117 bar-screen installations, in which the average removals varied with the size of the opening, somewhat as follows:

Clear opening, in inches	Average quantity of screenings, in cubic feet per million gallons
2.00	0.9
1.50	1.2
1.00	3.1
0.75	5.3
0.50	8.6

Usually the maximum peaks are 200% of the average quantity, but in some cases they range as high as 500 per cent.

With raw sewage containing approximately 200 ppm of suspended solids, approximately 1.5% of the suspended solids may be removed with screens with 1-in. clear openings and about 3.5% with screens of 0.5-in. openings.

Method of Disposing of Screenings.—In forty-nine plants concerning which data were available, sixteen plants disposed of screenings by incineration, eleven by grinding, twenty-one by burial and one by digestion. The costs for incineration range from $1.07 to $9.30 per ton, and by burial from $0.50 to $5.00.

Grit Chambers

Removal Accomplished by Grit Chambers.—In view of the varying designs of grit chambers, as well as of the characteristics of the sewage entering them, it is to be expected that the percentage of suspended solids removed will vary accordingly. The average quantity of screenings removed in thirteen plants investigated was 3.7 cu ft, or 0.14 cu yd per million gallons, the minimum removal being 1.16 cu ft, and the maximum, 7.0 cu ft per million gallons. The foregoing data are based on yearly, or at least on monthly, averages. Daily maximums have been observed as high as 80 cu ft per million gallons.

Based on the average removed of 3.7 cu ft per million gallons, and a raw sewage containing 200 ppm of suspended solids, grit chambers may be expected to remove about 5.5% of the solids contained in the sewage reaching them.

Total Quantity of Solids Removed by Screens and Grit Chambers.—With screens with 1-in. clear openings and grit chambers of usual length and cross-section proportionate to flow, it is reasonable to expect a total removal of about 7 cu ft per million gallons, corresponding to a total reduction in suspended solids of about 7% for a sewage of normal strength. The removal of B.O.D. is proportionately much less.

Construction Cost of Screens and Grit Chambers.—A wide range in the cost of building screens and grit chambers is indicated, dependent on the simplicity or elaborateness of the installation, the costs varying in twelve installations investigated from approximately $150 to $1 200 per million gallon capacity. The data indicate a total average cost, including mechanisms, approximating in the smaller installations, a value of $1 000 per million gallon capacity, and ranging in the larger installations to $600 per million gallon. A still further variation exists in the data on operation costs of screen and grit chambers.

In explanation of the foregoing data it should be emphasized that the flow capacities are those of the grit chambers or of the intercepting sewers entering the plant, and are usually several times the capacity of the remainder of the plant. The costs of screen and grit chambers are included in those of the basic processes mentioned subsequently.

Fine Screens

Removals Accomplished by Fine Screening.—Data have been collected on the removals of suspended matter accomplished by fifteen fine screen installations, with clear openings from 0.20 in. and smaller. The data indicate an average removal of suspended solids of 5% with screens having clear openings of 0.2 in., 6.5% for screens with $\frac{1}{8}$-in. openings, increasing to 10% for screens with $\frac{1}{16}$-in. openings, and to 20% from screens with 0.02-in. openings. Most of the fine screen installations have screens with openings from $\frac{1}{8}$ in. to $\frac{1}{16}$ in. with the latter size predominating.

Construction and Operation Cost of Fine-Screenings Plants.—A study of seven installations indicates a construction cost range of from $5 000 per million gallon capacity in the smaller plants to $4 000 per million gallon capacity in the larger plants, and an operating and maintenance cost ranging from $2 per million gallon treated for smaller plants to $1 per million gallon in the larger plants. For installations with $\frac{1}{16}$-in. clear openings, the total annual cost per million gallons per 1% reduction in suspended solids is as given in Table 1.

TABLE 1.—TOTAL ANNUAL CHARGES OF FINE SCREENING PLANTS

Sewage treated, in million gallons daily	Total annual charges	Total annual cost per million gallons per 1% reduction in suspended solids
10	$10 000	$0.26
25	21 300	0.23
50	38 000	0.21
75	53 000	0.195
100	66 200	0.18

SEDIMENTATION

Reductions Accomplished by Sedimentation.—It is well known that the reduction accomplished by sedimentation is influenced by a large number of factors. A more complete discussion of the effects of the various factors was published[2] in 1933. These factors may be summarized as follows: (1) The characteristics of the liquid, under which can be included specific gravity and viscosity; (2) the characteristics of the solids, which include size, shape, specific gravity, concentration, coagulation, etc.; (3) the characteristics of the design, comprising detention period provided, velocity of flow, depth, and ratio of length to depth, inlets and outlets, baffling, etc.; and, (4) miscellaneous effects, such as currents caused by winds, eddies, and difference of temperature, biological activities, etc.

TABLE 2.—PERCENTAGE REMOVAL OF SUSPENDED SOLIDS AND FIVE-DAY BIO-CHEMICAL OXYGEN DEMAND SETTLING FOR VARIOUS PERIODS

Detention period, in hours	RAW SEWAGE STRENGTH, IN PARTS PER MILLION*								REMOVALS FOR SEWAGE OF AVERAGE STRENGTHS USED IN THIS ANALYSIS	
	50 to 100		100 to 200		200 to 300		300 to 400			
	S.S.	B.O.D.	S.S.	B.O.D.	S.S.	B.O.D.	S.S.	B.O.D.	S.S.	B.O.D.
0.5	26	12	34	16	40	18	42	20	35.5	12.5
1.0	36	19	45	24	51	27	53	30	45.5	21.0
1.5	43	24	50	30	56	33	60	36	51.0	26.5
2.0	47	27	54	34	60	37	63	39	55.0	31.0
2.5	50	30	57	36	62	40	65	41	57.5	34.0
3.0	52	32	59	38	64	42	66	43	59.5	37.0

* S.S. = suspended solids; and B.O.D. = five-day bio-chemical oxygen demand.

For a particular sewage one of the most important factors affecting the reduction accomplished by sedimentation is the detention period. In Table 2 the relationship between detention period and reduction in suspended solids and 5-day B.O.D. for sewages of various strengths is shown. The reduction increases sharply for a period of time, depending on characteristics of the sewage, and then gradually diminishes until a point is reached at which an increase in detention period does not result in an appreciable increase in solids reduction. From an economic standpoint, a balance exists between the cost of treatment and the reduction accomplished beyond which it is uneconomical to provide settling capacity. Furthermore, there is the possibility that under adverse conditions too long a period of settling may result in a degradation of the effluent because of septic action and the resulting gas buoyance of particles.

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[2] *Sewage Works Journal*, Vol. V, No. 2, March, 1933.

The substantiating data used as a basis in preparing Table 2 consist of information available to the writer from more than thirty sedimentation plants scattered throughout the United States. A total of approximately 140 values was considered, each representing a monthly, up to a yearly, average. The settling periods shown are based upon theoretical, computed detention periods.

TABLE 3.—CONSTRUCTION, AND OPERATION AND MAINTENANCE, COSTS OF SEDIMENTATION PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	CONSTRUCTION COST			OPERATION AND MAINTENANCE COST		
	Usual lower cost	Estimated in this analysis	Usual higher cost	Usual lower cost	Estimated in this analysis	Usual higher cost
10.....	$240 000	$325 000	$400 000	$17 000	$23 000	$29 000
25.....	550 000	725 000	900 000	38 000	50 000	62 000
50.....	980 000	1 300 000	1 600 000	68 000	90 000	110 000
75.....	1 350 000	1 800 000	2 250 000	94 000	125 000	160 000
100.....	1 720 000	2 300 000	2 900 000	120 000	155 000	200 000

Construction and Operating Costs of Sedimentation Plants.—In Table 3 estimates of construction costs and operation and maintenance costs are shown for plants of various capacities. These costs denoted as "estimated in this analysis" are based on a sedimentation period of 2.5 hr. Included also are usual lower and higher cost values which are approximately 25% below or above the estimated cost shown. Of twelve plants on which substantiating construction costs are available, the costs at one plant were less than the usual lower limit. Substantiating operation and maintenance cost data are available from ten plants of which four were less than the usual lower limit.

Economical Period of Sedimentation.—Because both the removals accomplished and the cost of sedimentation plants vary with the detention periods provided (but not in the same proportion) it is apparent that for a particular problem an economical period of sedimentation exists. A summary of data relative to this determination is shown in Table 4, in which the fourth and fifth columns contain the annual cost per million gallons per 1% reduction in B.O.D. and suspended solids, respectively. On a B.O.D. basis the economical period of settling is approximately 3 to 3.5 hr whereas, on a suspended solids basis, the economical period is approximately 1.25 to 1.50 hr. The foregoing statements apply to sedimentation in plants in which this is the sole process of treatment, and not to cases in which it is used in conjunction with secondary processes. Furthermore, it should be stated that for sewages with different settling characteristics as affected by detention period, or where costs are materially different, the aforementioned indications must be modified; and there may be situations in which higher reductions than are indicated to be economical are necessary or desirable, or other cases where the minimum of treatment is indicated, in which cases the economic study, of course, has no application.

TABLE 4.—Construction Costs and Total Annual Charges; Sedimentation Plant of 100 Million Gallons Daily; Average Flow with Various Detention Periods

Detention period, in hours	Estimated construction cost	Estimated total annual charges*	Annual Charge per Million Gallons per 1% Reduction In: 5-day B.O.D.†	Suspended solids
0.5	$990 000	$173 500	$0.380	$0.134
1.0	1 325 000	203 500	0.265	0.123
1.5	1 595 000	228 000	0.236	0.122
2.0	1 845 000	250 000	0.220	0.124
2.5	2 070 000	269 000	0.216	0.128
3.0	2 290 000	288 000	0.213	0.132
3.5	2 500 000	304 500	0.213	0.137
4.0	2 700 000	321 000	0.217	0.143

* Includes fixed charges @ 6 per cent.
† Bio-chemical oxygen demand.

As a general statement, however, it can be said safely that where the pressing need is simply removal of solids, a short period (1 to 1.5 hr, or less) is indicated as being economical, and in cases where higher B.O.D. removal is desired detention periods of 2.5 to 3.5 hr may be economical. It should be mentioned in the latter connection, however, that such higher reductions in B.O.D. can usually be secured more economically through a variety of secondary processes.

Trickling Filters

Reductions Accomplished by Trickling Filters.—As in other processes the reduction accomplished by trickling filters varies in the different plants, depending on the characteristics of the sewage and design features. Data have been collected on the over-all reduction accomplished by eighteen trickling filter plants, based generally on annual averages. The weighted reduction, discarding data on plants in which the sewage was of unusual characteristics or concerning which a duplication of data existed on the same plant, is 85% on a B.O.D. basis and 84% on a suspended solids basis. On a B.O.D. basis, the maximum reduction accomplished in any plant was 94.5% and the minimum, 80%; whereas on a suspended solids basis the maximum reduction accomplished was 92.3% and the minimum, 70.5 per cent.

It is interesting to consider also the efficiency of the filter itself. In 1931[3] data indicative of the reductions from the point of application on the filter (effluent of preliminary sedimentation) to the point of leaving the final tanks (if included), were presented in some detail by the late J. A. Childs, M. Am.

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[3] For a more complete discussion of this matter the reader is referred to *Civil Engineering*, Vol. 1, No. 15, December, 1931, p. 1391 (see especially Figs. 1 to 4).

Soc. C. E., and the writer. This information was collected in an attempt to determine economical filter loadings and rates of application. Reference to the source will reveal the following general observations: (1) Within limits, the percentage reduction in 5-day B.O.D. accomplished by trickling filters is independent of the strength of the sewage applied to the filters; (2) also within limits, the efficiency is independent of the rate of application; and (3) the efficiency is not, within a reasonable range, influenced by increases in combined volume and strength (loading).

These statements must be tempered in their conclusiveness in that they apply only within the limits for which data were collected, which, however, covered a rather wide range in both volume and strength. It is obvious that there must be an upper limit to the values for which they apply, but on a B.O.D. basis, that limit had apparently not been reached at any of the plants investigated, as no reduction in efficiency was in evidence.

These deductions make economic studies very difficult, since they do not indicate the most economical loading. If it were possible to load a filter continuously at some of the higher values indicated, low cost would result. Other factors, however, such as depth of bed, climatic conditions, characteristics of the sewage, size and character of the filtering media, method of application, dosing and rest periods, as well as many others, affect the magnitude of the economical loading in varying degrees. The data collected from the fifteen plants[4] considered in this study indicate an average reduction of the 5-day B.O.D., by secondary treatment with trickling filters of 75%, the lowest efficiency being 49% and the highest, 88 per cent.

It may be of interest to mention that since the collection of the foregoing data in 1929, trickling filter plants have been operated on a large-scale experimental basis with rates up to 20 and even 25 million gallons per acre per day. If it is demonstrated definitely that such high sustained rates of application are practicable in actual operation, trickling filters will be in a most advantageous position as far as the economics of treatment are concerned.

Construction, Operation, and Maintenance Costs of Trickling Filter Plants.—The estimated construction cost of trickling filter plants is shown in Table 5.

TABLE 5.—CONSTRUCTION, OPERATION, AND MAINTENANCE COSTS OF TRICKLING FILTER PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	CONSTRUCTION COST			OPERATION AND MAINTENANCE COST		
	Usual lower limit	Estimated in this analysis	Usual higher limit	Usual lower cost	Estimated in this analysis	Usual higher cost
10.....	$900 000	$1 200 000	$1 500 000	$22 000	$30 000	$38 000
25.....	2 130 000	2 850 000	3 550 000	47 000	62 000	78 000
50.....	4 130 000	5 500 000	6 850 000	83 000	110 000	138 000
75.....	6 000 000	8 000 000	10 000 000	115 000	155 000	195 000
100.....	7 900 000	10 500 000	13 200 000	145 000	195 000	245 000

The tabulation of estimated costs is based upon a filter rate of 2.0 million gallons per acre per day on a filter 8 ft deep.

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[4] *Civil Engineering*, Vol. 1, No. 15, December, 1931, p. 1391 (see especially Fig. 3).

Of twenty-six plants on which substantiating construction costs are available, the cost of one plant was higher than the usual higher cost and the costs at seven plants were lower than the usual lower limit.

As an indication of the effect of higher rates of application on the construction cost of trickling filters, it is estimated that with a rate of application of 3.0 million gallons per acre per day the construction cost of a complete trickling filter plant will be approximately 85% of the cost with a rate of 2 million gallons, whereas with a rate of 5.0 million gallons, the construction cost will be approximately 70% of the cost with a 2.0 million gallon rate. Some reduction in operation and maintenance cost would also be effected.

The estimated operation and maintenance cost of trickling filter plants substantiated by actual costs at other plants is shown in Table 5. Of twenty-two plants on which operation and maintenance costs are available, the costs at two plants were higher than the usual higher limit and those at two other plants were lower than the usual lower limit. As an indication of the distribution of the annual operating and maintenance costs the average of six trickling filter plants, varying from a capacity of 5 to 55 mgd, was as given in Table 6.

TABLE 6.—DISTRIBUTION OF ANNUAL OPERATING AND MAINTENANCE COSTS; AVERAGE OF SIX TRICKLING FILTER PLANTS

Item	Cost, in dollars per million gallons	Percentage of total cost
Labor and supervision	4.15	72
Maintenance, supplies, repairs, and renewals (includes power)	1.65	28
Total	5.80	100

Balance Between Various Processes.—Cost estimates have been prepared to determine the theoretical economical balance between the sedimentation and the secondary processes. It is evident that, unless the costs by both preliminary and secondary processes are identical, a variation in the detention period provided in the preliminary tanks will affect the over-all construction and operation and maintenance costs and the over-all reductions. Data relevant to this point, indicate that, at usual design rates, to increase the period of preliminary sedimentation from 1 hr to 2 hr results only in increasing the over-all reduction by about 2%, accomplished by raising the total annual charges by approximately 4% to 5 per cent. From a theoretical standpoint computations indicate that on a B.O.D. basis, the most economical period of sedimentation is the shortest one. However, there is a practical limit to the possibility of shortening the period of preliminary sedimentation as controlled by operation difficulties with filter nozzles and clogging of beds. It appears that a detention period in preliminary tanks of approximately 1.5 hr is a desirable and practical one for ordinary sewage. This has been demonstrated and is in accord with usual practice in design in this regard.

ACTIVATED SLUDGE PROCESS

Reductions Accomplished.—The activated sludge process is capable of accomplishing somewhat greater reductions, at least on a B.O.D. basis, than other processes. Data representing, in most cases, yearly or longer averages from nineteen activated sludge plants have been collected indicating an average reduction of 91.7% of 5-day B.O.D. and in suspended solids of 89.5 per cent. A weighted average (excluding certain plants in which the sewage was of unusual characteristics or, for other reasons, the data were not considered representative) indicates an average removal of B.O.D. of 92.5% and of suspended solids of 91.0 per cent.

The foregoing data refer to reductions accomplished by complete activated sludge treatment, including preliminary treatment. The reductions accomplished by the secondary process alone, including aeration and final sedimentation, for various periods of aeration of mixed liquor (sewage plus return sludge) for sewage of normal strength and characteristics are as follows:

Aeration period, in hours	Percentage reduction of 5-day bio-chemical oxygen demand
3	74.0
4	84.5
5	91.0
6	93.0
7	95.0

The foregoing reductions are substantiated by the average reductions in seventeen activated sludge plants. A total of thirty-five values covering operating periods usually of 1 yr, as well as of test plants, have been considered in the determination of the average values given. The list indicates that up to a period of approximately 5 hr, the reductions in B.O.D. increase markedly, after which an increase in the period of aeration does not result in appreciable increase in removals.

TABLE 7.—CONSTRUCTION, OPERATION, AND MAINTENANCE COSTS OF ACTIVATED SLUDGE PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	Construction Cost				Operation and Maintenance Cost			
	Usual lower limit	Average for Aeration		Usual higher limit	Usual lower limit	Average for Air		Usual higher limits
		5 hours	6 hours			0.6 cubic foot per gallon	1.0 cubic feet per gallon	
10....	$ 560 000	$ 700 000	$ 800 000	$ 940 000	$75 000	$ 90 000	$110 000	$125 000
25....	1 280 000	1 600 000	1 800 000	2 120 000	150 000	185 000	220 000	255 000
50....	2 500 000	3 200 000	3 500 000	4 200 000	250 000	310 000	370 000	430 000
75....	3 700 000	4 750 000	5 200 000	6 200 000	340 000	420 000	495 000	570 000
100....	5 000 000	6 300 000	6 900 000	8 200 000	430 000	520 000	615 000	710 000

Construction, and Operation and Maintenance, Costs of Activated Sludge Plants.—The estimated construction costs of activated sludge plants along with the estimated construction cost of activated sludge plants with detention periods of 5 and 6 hr are shown in Table 7, together with the usual lower and higher costs of this type of plant. An idea of the variation in construction costs is shown by the fact that of twenty-one plants on which construction costs are available, the costs at two plants were less than the usual lower limit and at one plant more than the usual higher limit shown.

Estimated operation and maintenance costs for activated sludge plants based on the use of 0.6 and 1.0 cu ft of air per gal are also shown. Of twenty-four plants on which operation and maintenance costs are available, the costs at five plants were less than the usual lower limit and at one plant, more than the usual higher limit shown. The operation and maintenance costs are based on a power rate of 1.0 cent for the larger plants, increasing for the smaller plants in accordance with usual schedules. In substantiation of the operation and maintenance costs shown in Table 7, and to furnish an indication of the composition of costs, the following costs at a number of representative activated sludge plants are shown: (1) Power costs: The average of fifteen plants in which power rates varied from 0.7 cent to 1.51 cents per kw-hr was $6.50 per million gallon treated; (2) labor and supervision: The average of fourteen plants was $6.35 per million gallon; and, (3) maintenance, supplies, repairs, and renewals: The average of fourteen plants was 1.74% of construction cost annually.

Economical Period of Aeration.—The fact that the reduction accomplished by activated sludge treatment is affected by the period of aeration, somewhat as shown previously, suggests the possibility of the existence of an economical period of aeration, at least from a theoretical standpoint. In Table 8 data of this nature are shown. The most economical period of aeration with normal sewage is calculated to be approximately 4.5 hr. The importance of determining the economical period of aeration from a cost standpoint, either in the design or in the operation stage, can be shown by the following: Considering secondary treatment alone (without preliminary treatment) an increase in the aeration period from 4.5 to 6 hr, increases the percentage reduction, 5%, the construction cost, 15%, and the total annual charges per unit of reduction, 5 per cent. At 4.5 hr, the total annual charges for secondary treatment are estimated to be $0.210 per million gallon per 1% reduction in 5-day B.O.D.

TABLE 8.—CONSTRUCTION COSTS AND TOTAL ANNUAL CHARGES—SECONDARY TREATMENT* BY ACTIVATED SLUDGE PROCESS—PLANT CAPACITY 100 MILLION GALLONS DAILY WITH VARIOUS DETENTION PERIODS

Period of aeration, in hours	Percentage reduction in 5-day bio-chemical oxygen demand of influent	Estimated construction cost	Fixed charges @ 6%	Annual maintenance and operation cost	Total annual charges	Total annual charges per million gallons per 1% reduction in 5-day bio-chemical oxygen demand
3.0	74.0	$3 445 000	$207 000	$402 000	$609 000	$0.226
4.0	84.5	3 845 000	231 000	425 000	656 000	0.213
4.5	88.0	4 005 000	240 000	436 000	676 000	0.210
5.0	91.0	4 235 000	254 000	447 000	701 000	0.212
6.0	93.0	4 640 000	278 000	471 000	749 000	0.221
7.0	95.0	5 040 000	303 000	494 000	797 000	0.230

* Preliminary treatment (that is, screen and grit chambers and preliminary settling) not included; disposal of sludge from secondary process included.

A shorter aeration period, approaching that mentioned previously, is substantiated by recent designs and operating data. It was only a few years ago that plants were designed and operated with detention periods in aeration tanks of from 6 hr and to 8 hr. More recently, periods of 5 hr and even shorter are more usual in design. It is important to realize that, in some cases, longer periods are necessary or desirable either because of unusual characteristics of the sewage, or because of the controlling features of the installation.

Balance Between Preliminary and Secondary Treatment.—Data relative to a determination of the balance between various processes are shown in Table 9, based on various periods of preliminary sedimentation, and an aeration period of 4.5 hr. The data indicate that from a theoretical standpoint the most economical reduction of B.O.D. is accomplished with the shortest period of sedimentation. From a practical standpoint, there is a limit to the extent to which preliminary sedimentation can be reduced as affected or controlled by difficulties in aeration tanks and sludge handling. Substantiation for the conclusion that secondary treatment is more economical than the preliminary processes, where a high degree of treatment is necessary, can be found in practice by the fact that relatively short periods of sedimentation or even fine screening has been provided in the larger plants. It appears that a practical and desirable range in period of preliminary treatment is from 0.5 hr to 1 hr. The method of disposing of sludge affects the economics to a certain extent.

TABLE 9.—TOTAL ANNUAL CHARGES OF ENTIRE ACTIVATED SLUDGE PLANTS WHEN PRECEDED BY VARIOUS PERIODS OF SEDIMENTATION PLANT CAPACITY, 100 MILLION GALLONS DAILY

(Based upon detention period of 4.5 hours with 20% return sludge in aeration tanks, and 1 cubic foot of air per gallon of sewage)

Period of sedimentation preceding aeration, in hours	Total over-all reduction in 5-day biochemical oxygen demand	Total annual charge per million gallons per 1% reduction in 5-day biochemical demand	Period of sedimentation preceding aeration, in hours	Total over-all reduction in 5-day biochemical oxygen demand	Total annual charge per million gallons per 1% reduction in 5-day biochemical demand	Period of sedimentation preceding aeration, in hours	Total over-all reduction in 5-day biochemical oxygen demand	Total annual charge per million gallons per 1% reduction in 5-day biochemical demand
0.25	89.5	$0.264	1.50	91.9	$0.281	3.00	93.2	$0.294
0.50	90.3	0.269	2.00	92.5	0.285	3.50	93.4	0.298
0.75	90.7	0.273	2.50	92.9	0.290	4.00	93.6	0.302
1.00	91.1	0.276	....	....	......	....	....	......

Air Requirements of Activated Sludge Process.—Data have been collected from fourteen plants on the quantity of air required in the activated sludge process. These data indicate that the quantity used for normal sewage varies from 0.4 cu ft to 2.5 cu ft per gal of sewage. The data show that of the total amount of air added, only 1% to 10% is actually used for oxidation of organic material in the sewage and even a portion of this may be derived from surface aeration. The remainder (from 90 to 99%) is used for agitation. The relation between influent strength in parts per million of 5-day B.O.D. and the cubic feet of air per pound of 5-day B.O.D. removed for the same data, is somewhat as follows:

Influent strength, in parts per million of five-day bio-chemical oxygen demand	Air required, in cubic feet of air per pound of five-day bio-chemical oxygen demand removed
100	1 700
200	900
300	650
400	500

In general, the quantity of air required per pound of 5-day B.O.D. removed, is greater for the weaker sewages than for the stronger ones. The possible economy of mechanical means of aeration, or a combination of air and mechanical methods is suggested from a study of the foregoing data for exceptionably weak sewages.

Period of Final Sedimentation.—The usual method of expressing the loading of final settling tanks following aeration is in terms of gallons of sewage per square foot of tank area per day. As an indication of the usual design practice, eleven activated sludge plants investigated, provide an average rate of approximately 900 gal of average sewage flow (not including return sludge) per sq ft per day, with the lowest design rate approximately 600 and the highest approximately 1 200. The average design rate at five of the larger plants was 830 gal per sq ft per day. Review of operating records indicates that rates from 800 to 1 000 gal per sq ft per day are practical and desirable rates for normal sewage. The rate selected should be based on a consideration of variations in flow (maximum and minimum with respect to average) and depth of tank. For example, with a 900-gal rate a 10-ft depth of tank provides a detention period of 2 hr; with a 12.5-ft depth, a period of 2.5 hr; and with a 15-ft depth of tank, 3 hr. In general, exceptionably shallow tanks should be designed with lower rates of application.

Chemical Treatment

Reductions Accomplished by Chemical Treatment.—One of the features of chemical treatment is its susceptibility to variation in reduction accomplished by varying the chemical addition. Data showing the reduction in suspended solids and B.O.D. from the results of experimental plants by various investigators and at six actual plants with different chemical additions have been collected. In these studies the chemicals added have been expressed in dollars per million gallons rather than parts per million because of the variety of chemicals used and their varying cost and effectiveness. Although a rather large range in reduction is indicated the data are believed to be sufficiently well substantiated and possible of attainment especially for well designed and well operated plants. Unusual sewages, of course, present an exception. The difference in effectiveness of chemical treatment at various plants should at least suggest the desirability of careful investigation before decision on the practicability of this type of treatment for a particular problem, as well as of the type of chemicals and various design features. The data collected (see Table 10) indicate the reductions in B.O.D. and suspended solids with various additions of chemicals for normal sewage and with reasonable detention periods in

TABLE 10.—REDUCTIONS ACCOMPLISHED BY CHEMICAL TREATMENT

Percentage removal of:	PERCENTAGE REDUCTION FOR THE FOLLOWING CHEMICALS ADDED, IN TERMS OF DOLLARS PER MILLION GALLONS			
	3.00	6.00	9.00	12.00
Bio-chemical oxygen demand..........	52	63	72	77
Suspended solids....................	69	77	84	88

the flocculating and settling tanks. Chemical costs used in comparing various plants are actual costs, except where data were not available, in which case actual chemical additions were used, applying the following chemical costs per ton:

Lime..............................	$10
Chlorine..........................	50
Copperas..........................	17
Ferric chloride...................	50

Based on the additions of chemicals at the various plants, and on the foregoing costs, the average chemical cost per million gallons in the various plants investigated is $8.35; and the average reduction in B.O.D. was 67.3%; and in suspended solids, 83.8 per cent.

Construction, Operation, and Maintenance Costs of Chemical Treatment.—The estimated construction costs of chemical treatment plants are shown in Table 11. Of thirteen plants on which costs are available the costs at two plants are less than the usual lower limit and at two plants more than the usual higher limit. The construction costs of chemical treatment plants is likely to vary widely depending on the provisions of the design.

TABLE 11.—CONSTRUCTION, AND OPERATION AND MAINTENANCE, COSTS OF CHEMICAL TREATMENT PLANTS OF VARIOUS CAPACITIES

Plant capacity or sewage treated, in million gallons daily	CONSTRUCTION COST			OPERATION AND MAINTENANCE COST*		
	Usual lower cost	General average*	Usual higher cost	Usual lower cost	General average	Usual higher cost
10.....	$ 380 000	$ 500 000	$ 620 000	$26 000	$35 000	$44 000
25.....	750 000	1 000 000	1 250 000	52 000	69 000	86 000
50.....	1 300 000	1 750 000	2 200 000	84 000	112 000	140 000
75.....	1 800 000	2 375 000	3 000 000	115 000	153 000	190 000
100.....	2 200 000	2 950 000	3 700 000	143 000	190 000	240 000

* Exclusive of chemical added.

Data on operation and maintenance costs are shown in Table 11. Of six plants on which operation and maintenance costs are available the cost at one plant was more than the usual higher limit. In view of the varying cost of chemicals depending on the reduction desirable or necessary, the costs with various additions of chemicals have been added to the costs shown in Table 11 to prepare Table 12. Including chemicals the total estimated costs to accomplish various reductions are as shown in Table 12.

The costs shown in Table 12 are subject to the same variations with respect to usual lower and upper limits as shown in Table 11.

TABLE 12.—ESTIMATED COST OF REDUCTION BY CHEMICALS

REDUCTIONS ACCOMPLISHED		ANNUAL OPERATION AND MAINTENANCE COST				
Percentage of bio-chemical oxygen demand	Percentage of suspended solids	10 mgd	25 mgd	50 mgd	75 mgd	100 mgd
56	73	$48 000	$105 000	$185 000	$260 000	$330 000
69	83	65 000	140 000	255 000	360 000	475 000
79	89	80 000	180 000	330 000	465 000	625 000
82	91	85 000	190 000	60 000	515 000	680 000

EFFLUENT FILTERS

Removals Accomplished by Effluent Filters.—In the past several years, a number of installations of effluent filters consisting of beds of magnetite sand, usually of 3-in. thickness, cleaned by the action of a solenoid passing over the bed, have been made. Experimental data are also available on modified rapid sand filter beds, as used in water purification practice, and also on beds 12 in. deep, cleaned by jets of water; but to date (1938) no large-scale installations of these latter types have been made for which continuous long-time operating records are available.

Although filters of the previously described types may have a function as the principle method of treatment, their important application at present appears to be in conjunction with other basic processes, such as sedimentation, trickling filters, activated sludge, and chemical treatment, and only such use will be discussed herein. For this reason they are not considered in the comparison of basic treatment processes described subsequently. Data have been collected on the reductions accomplished by effluent filters (magnetite sand type) in certain plants. It will be observed from Table 13, which summarizes the data collected, that the reductions accomplished are a function of the strength of the filter influent, much the same as for trickling filters, except

TABLE 13.—REDUCTION ACCOMPLISHED BY EFFLUENT FILTERS FOLLOWING SEDIMENTATION AND CHEMICAL TREATMENT

Filter influent strength —bio-chemical oxygen demand and suspended solids, in parts per million	REDUCTION ACCOMPLISHED, IN PARTS PER MILLION			
	Following Sedimentation		Following Chemical Treatment	
	Bio-chemical oxygen demand	Suspended solids	Bio-chemical oxygen demand	Suspended solids
25	....	1	....	6
50	2	10	2	30
75	8	27	8	55
100	15	45	15	80
125	22	63	22	105

that the reduction is not a fixed percentage of the strength of the filter influent. The table is based on sewage of normal characteristics. As might be expected, the reductions in B.O.D. are somewhat lower than the reductions in suspended solids.

As in the case of trickling filters, the reduction accomplished does not appear for the same grain size to be affected by the rate of application, at least up to a limit of 4, or possible 5, gal per sq ft per min. A considerable variation occurs in practice in the filter rate varying in nine installations investigated from 1.8 to 3.0 gal per sq ft per min, the more usual rates being between 2.0 and 2.5. The rate to be chosen for design purposes should include a consideration of the reductions desirable or necessary, characteristics of filter influent, the sand size, variation in flow, and certain other factors.

Construction, Operation, and Maintenance Costs of Effluent Filters.—Data have been collected on the estimated construction costs of effluent filters, substantiated by costs of five installations and estimates from manufacturers. The cost of the equipment installed is estimated to be approximately 70% to 75% of the total installation cost. With upward flow filters constructed integrally with settling tanks, the extra cost of structures may be approximately 10% to 20% of the installed equipment cost. Based on rates of application of 2.5 to 3 gal per sq ft, the estimated construction cost per million gallons with separate structures vary approximately from $4 000 per million gallon of capacity for the larger installations to $5 000 per million gallon for the smaller installations.

Although data were available on some items of operation and maintenance costs, no actual substantiating data are available as to the total costs. They are believed, however, to be reasonably high for usual conditions. For various plant capacities the estimated operation and maintenance costs vary from approximately $1.00 per million gallon treated in the smaller installations to $0.70 per million gallon, in the larger installations.

Relative Costs as Compared with Other Processes.—This feature of cost comparison can best, and more accurately, be illustrated by application to a specific problem. A summary of data on an investigation of this subject prepared by the writer in connection with the Minneapolis-St. Paul project, is presented for this purpose. The design capacity of the plant is 134 mgd and the characteristics of the sewage are as assumed herein. The costs include only those chargeable to sewage treatment, and do not include the cost of sludge disposal, which was considered to be reasonably equal per ton of solids for sludge produced by other processes. On a suspended solids basis, the cost of filtration is approximately equal to chemical treatment if between 15 and 20 ppm are removed by filters, and to plain sedimentation if approximately 23 to 24 ppm are removed. On a B.O.D. basis, filtration is more economical than chemical treatment if between 12 and 15 ppm are removed, and as economical as sedimentation if approximately 16 to 17 ppm are removed. Based on the reductions by filters as shown in Table 13 and on the reductions expected by sedimentation and chemical treatment, it was estimated that the reductions accomplished by effluent filters for the Minneapolis-St. Paul project would be 18 ppm during chemical treatment and 31 ppm during plain sedimentation on a suspended solids basis; and 8 ppm during chemical treatment and 21 ppm during plain sedimentation on a B.O.D. basis. As will be explained subsequently, chemical treatment is required in Minneapolis and St. Paul only about 10% of the time, and, therefore, the removals during plain sedimentation are of more interest. The addition of filters in the Minneapolis-St. Paul project is expected to reduce the quantity of suspended solids in the plant effluent by approximately 6 000 tons annually.

COMPARISON OF VARIOUS PROCESSES

General.—From what has previously been mentioned it is apparent that a wide variation in construction, operation, and maintenance costs, and in reductions accomplished, obtains for the basic processes described herein. It is evident also that it is possible to obtain removals to meet the requirements of a particular situation, from nothing to relatively complete treatment by some of the basic processes or combinations with other processes. Having arrived at what seem to be reasonable basic cost and reduction data for normal conditions, this section of the paper is devoted to a comparison of the various processes, and a discussion of certain factors that affect their relative economy and thus serve to alter the general indications presented in the comparisons.

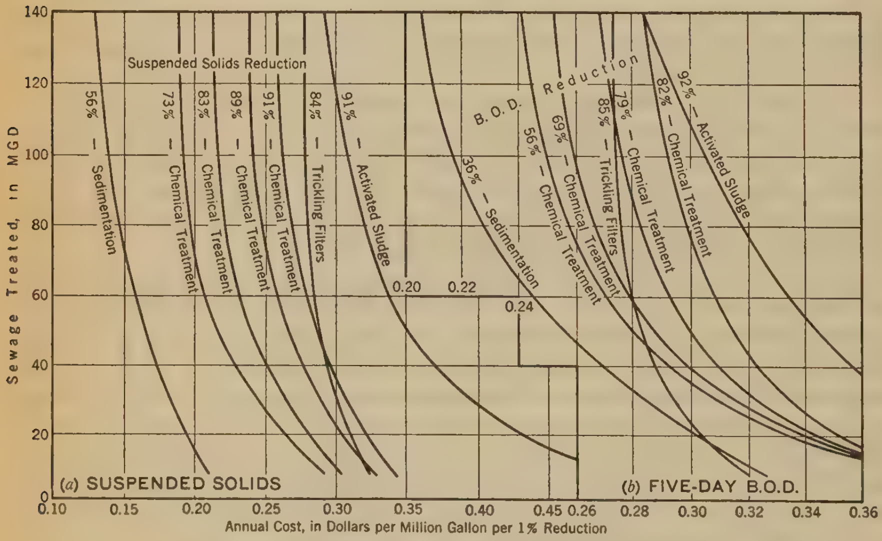

FIG. 1.—ANNUAL CHARGES OF VARIOUS METHODS OF TREATMENT

Comparison of Processes on Basis of Suspended Solids Reduction.—Using as a basis, the cost estimates and removals accomplished by the various processes, as already described, the estimated total annual costs per million gallons per 1% reduction in suspended solids, are shown for normal conditions, in Fig. 1(*a*). Selecting from this diagram three plant sizes for comparison, the costs may be summarized as shown in Table 14(*a*). It will be observed that on a suspended solids basis, the various processes, as applied to normal conditions, may be arranged in order of ascendancy of costs per unit of reduction, as follows: (1) Sedimentation; (2) chemical treatment to various degrees; (3) trickling filters; and (4) activated sludge. This arrangement does not hold for the smaller plants in which trickling filters may be more economical than some degrees of chemical treatment.

TABLE 14.—COMPARISON OF VARIOUS PROCESSES

No.	Treatment process	(a) SUSPENDED SOLIDS REDUCTION				(b) BIO-CHEMICAL OXYGEN DEMAND REDUCTIONS				(c) ANNUAL COST, IN DOLLARS, PER MILLION GALLONS PER 1% REDUCTION OF 5-DAY BIO-CHEMICAL OXYGEN DEMAND WITH FIXED CHARGES OF:		
		Percentage of suspended solids	Annual Cost, in Dollars, per Million Gallons per 1% Reduction for Plant Capacity (in Million Gallons Daily) of:			Percentage of bio-chemical oxygen demand	Annual Cost, in Dollars, per Million Gallons per 1% Reduction for Plant Capacity (in Million Gallons Daily) of:			5%	6%	7%
			20	50	100		20	50	100			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Sedimentation........	56	0.19	0.17	0.14	36	0.30	0.25	0.22	0.20	0.22	0.24
2	Trickling filters........	84	0.31	0.28	0.27	85	0.30	0.28	0.27	0.24	0.27	0.30
3	Activated sludge.......	91	0.43	0.35	0.31	92	0.39	0.34	0.30	0.28	0.30	0.32
4	Chemical treatment...	73	0.27	0.22	0.19	56	0.33	0.28	0.25	0.27*	0.28*	0.29*
		83	0.28	0.23	0.22	69	0.34	0.29	0.26			
		89	0.30	0.26	0.24	79	0.34	0.30	0.28			
		91	0.32	0.28	0.26	82	0.35	0.31	0.29			

* For a removal of 79% B.O.D.

Comparison of Processes on Basis of Bio-Chemical Oxygen Demand Reductions.—Data relevant to this comparison are shown in Fig. 1(*b*) and for various selected plant sizes in Table 14(*b*). It will be observed that, on a B.O.D. basis, the relative economy of the various processes is less marked. For a plant of 100 mgd capacity, the order of economy for normal conditions is as follows: (1) Sedimentation; (2) chemical treatment to a B.O.D. removal of 77%; (3) trickling filters; (4) chemical treatment with B.O.D. removal greater than 77%; and (5) activated sludge treatment. However, for a plant of 20 mgd capacity the indicated order is: (1) Sedimentation; (2) trickling filters; (3) chemical treatment to various degrees; and (4) activated sludge.

The foregoing statements are irrespective of reductions accomplished. For processes accomplishing approximately the same degree of treatment the order of economy is altered. It should be emphasized also that the processes capable of the highest reduction, in general, cost more per unit of reduction.

Effect of Various Influencing Factors on the Comparison.—In view of the fact that the cost of the various processes accomplishing substantially the same removal are remarkably close, the necessity of investigating a number of influencing factors which affect the comparison in a particular situation, becomes apparent. It is important also to call attention to certain of these influencing factors which affect the comparison because, to this point, the analysis has of necessity been general, and the writer well realizes the impossibility of stating proper generalizations of many features of sewage treatment.

Effect of Degree of Treatment Capable of Accomplishment by the Various Processes.—The statement has already been made that the cost per unit of removal of the first 10% of pollution in a sewage or waste is less than the last 10%, indicating that processes capable of high removals, when compared on that basis will, in general, cost more per unit of removal than processes capable only of low removals. This fact can be shown very clearly by considering 35 (Fig. 1(*b*)) for a plant of 100 mgd capacity. For this condition, the costs can be summarized as in Column (9), Table 14(*b*). Thus, to effect the removals capable of accomplishment by activated sludge (that is, 92%) results in an increased cost per unit of removal of 35% over the cost by sedimentation. This fact must be borne in mind when considering the unit costs as presented herein.

It is because of the conditions stated herein that the degree of treatment necessary for a particular situation has such an important effect on the type of treatment selected and, therefore, upon the costs. The requirements of a situation may be such as to necessitate the adoption of a process or combination of processes capable of accomplishing high removals, in which case the cost of processes which result only in low removals is not pertinent.

Effect of Fixed Charges on Cost.—As already mentioned, this analysis has been based upon a uniform rate for fixed charges of 6 per cent. If, because of variation in interest rates or other considerations previously mentioned, these rates vary markedly, an important effect on the comparison of the various processes is observed. Table 15 illustrates the effect of this factor for a plant of 100 mgd capacity.

TABLE 15.—EFFECT OF FIXED CHARGES

Treatment	Total Annual Charges with Fixed Charges of:		
	5%	6%†	7%
Sedimentation	$260 000	$285 000	$310 000
Trickling filters	750 000	857 000	964 000
Activated sludge	955 000	1 024 000	1 093 000
Chemical treatment*	775 000	804 000	833 000

* For removals of 79% B.O.D. and 89% suspended solids.
† From Table 16 and used generally in this analysis.

A low rate of fixed charges reflects favorably on the cost of processes more expensive in first cost and lower in operation and maintenance costs. Thus, for the conditions assumed, a reduction in fixed charges from 6% to 5% reduced the total annual charges of the trickling filter process by 12.5%, whereas the reduction in the activated sludge processes is 6.7%, and in the chemical treatment process only 3.6 per cent. The total annual charges per million gallon treated per 1% reduction in B.O.D. shown in Fig. 1(*b*), with various rates of fixed charges, are as given in Table 14(*c*), for a 100 mgd plant. It will be observed that whereas the cost per unit reduction of B.O.D. might be lower for the trickling filter process with fixed charges of 6%, with fixed charges of 7%, or more, the chemical treatment process becomes more economical. The effect of this factor in a particular installation must be considered in connection with studies of the economical form of treatment.

Effect of Sewage Characteristics.—The fact that the characteristics of the sewage may be such as to be more favorable to one type of treatment than to other types, must be considered in comparing processes for a specific situation. In fact, in some cases, the presence of inhibitive wastes makes some processes of the biological type impractical. Even in more normal sewages, strength and other characteristics affect both construction and operation costs of sewage treatment and sludge disposal. This influencing factor can be viewed more clearly by considering the individual processes—sedimentation, trickling filters, etc.

Sedimentation.—A number of considerations of sewage characteristics, such as its settleability, temperature, concentration, etc., are likely to affect the reductions capable of accomplishment and, therefore, for a given reduction, the period of detention necessary. It is well known, for example, that one plant with a detention period, of, say, 1.5 hr, may accomplish as great or a greater reduction than another plant providing a period of 2.5 hr. At least, part of this difference in behavior may be attributed to the sewage characteristics.

The processes following sedimentation, such as effluent filters, trickling filters, activated sludge, affect the detention period which should be provided, as already mentioned.

Considering equal reductions as being accomplished, a decrease in the detention period from 2.5 hr to 1.75 hr, reduces the construction costs of the entire plant by approximately 15 to 20 per cent. Assuming a decrease in construction cost of 15% and no reduction in operation and maintenance costs, such a decrease in detention period reduces the total annual charges of a plant of 100 mgd capacity by 8%, or the cost per unit of reduction in B.O.D. from 22 cents to 20 cents per million gallon.

Trickling Filters.—It is likely that, within limits, the rate of application of sewage on a trickling filter can be increased without affecting the reduction accomplished. Thus, it is conceivable that under both favorable design conditions and sewage characteristics it is possible to increase the average rate of application from 2 million gallon per acre per day as considered herein to as high as 5.0 million gallon in which case the construction costs will be reduced by approximately 30 per cent. Without considering possible reductions in operation and maintenance costs, the total annual charges will be reduced by approximately 22 per cent. In other words, the total annual charges per million gallons per 1% reduction in B.O.D. shown in Fig. 1(*b*), might be reduced from 27 cents to 21 cents. The effect of such a decrease in costs in the comparison of various processes is apparent.

Activated Sludge Process.—It may be possible under favorable conditions to reduce the detention period from 6 hr to 5 hr, and the quantity of air from 1.0 cu ft of air per gal to 0.6 cu ft. Under these conditions the construction costs might be reduced by as much as 9% and the total annual charges by as much as 12%, in which case the total annual charges per million gallons per 1% reduction in B.O.D., would be reduced from 30 cents to 26 cents.

Chemical Treatment.—In view of the varying effect of the addition of chemicals as affected by the characteristics of the sewage observed at various plants, it is apparent that, with a sewage particularly amenable to chemical treatment, the quantity of chemicals necessary to produce a given reduction may be materially less than for another less favorable sewage. If a reduction in chemical costs of $4 per million gallons can be effected for a sewage to which chemicals can be applied advantageously, a saving in annual operation and maintenance charges of approximately $150 000 annually for a plant of 100 mgd capacity will result, equivalent, in a condition where the chemical costs are reduced from $12 to $8 per million gallon for a 79% reduction in B.O.D., to a decrease in total annual charges approximating 19%, or to a decrease in the cost per million gallons per 1% reduction in B.O.D. from 28 cents to 23 cents.

Effect of Seasonal Variations in Degree of Treatment.—The preceding discussions are based on year-round treatment by the various processes. In certain situations, either because of variation in pollution, dilution capacity requirements, and possibly for other reasons, the necessity of a high degree of treatment is seasonal in character. Preliminary treatment may suffice for a part of the time, but for certain critical periods a higher degree of treatment is necessary. For such a condition, depending upon the percentage of time that such secondary treatment is necessary, it may be stated as a general principle that treatment processes with low initial costs and higher, proportionately, operation costs are the most economical. It is clearly evident, for example, that in a case in which a secondary treatment is indicated for 10% of the time

TABLE 16.—CONSTRUCTION COSTS AND TOTAL ANNUAL CHARGES OF VARIOUS TREATMENT PROCESSES: SECONDARY TREATMENT FOR VARIOUS PERIODS OF TIME; PLANT CAPACITY 100 MILLION GALLONS DAILY

No.	Type	(a) GENERAL				(b) TOTAL OPERATION AND MAINTENANCE, WITH SECONDARY TREATMENT FOR:			(c) TOTAL ANNUAL CHARGES, WITH SECONDARY TREATMENT FOR:		
				Percentage Reduction							
	(1)	First cost, in thousands of dollars (2)	Fixed charges @ 6%, in thousands of dollars (3)	Suspended solids (4)	B.O.D (5)	12 months (6)	6 months (7)	2 months (8)	12 months (9)	6 months (10)	2 months (11)
1	Plain sedimentation	2 500	150	56	36	135	135	135	285	285	285
2	Trickling filters	10 700	642	84	85	215	175	148	857	817	790
3	Activated sludge	6 900	414	91	92	610	375	215	1 024	789	629
4	Chemical Treatment; Cost of Chemicals per Million Gallons:	2 900	174	..	..	....	....	....	....	....	....
4(*a*)	$4.00	....	....	73	56	335	235	170	509	409	324
4(*b*)	8.00	....	....	83	69	485	310	195	659	484	369
4(*c*)	12.00	....	....	89	79	630	385	220	804	559	394
4(*d*)	14.00	....	....	91	82	700	420	230	874	594	404

(as in Minneapolis and St. Paul), a process with high fixed charges on construction is definitely uneconomical as compared with a process with low first costs and proportionately higher operation costs for a short period of time. Data indicative of this variation for a plant of 100 mgd are shown in Table 16. With secondary treatment for twelve months, chemical treatment (at least with lower chemical additions) is the most economical, with trickling filters next and activated sludge most expensive. On a basis of two months' secondary treatment, however, chemical treatment is still more economical, activated sludge is next in cost, and trickling filters the most expensive. Here, again, the effect of fixed charges on construction on the economics is clearly demonstrated. Sedimentation during the entire year is included in the foregoing estimates, and no allowance is made for the maintenance of key men for the secondary treatment processes on the payroll, or for the periodic operation and checking of equipment as may be necessary in a number of cases. This has been done in a later estimate applicable to the Minneapolis-St. Paul situation.

The degree of treatment necessary during critical dilution conditions here, again, plays an important part in the determination of the type of treatment.

Effect of Commodities Cost.—As has already been mentioned construction costs vary under different conditions. In addition, operation and maintenance costs may vary depending upon cost of power, chemicals, labor, etc. In some cases, this variation may assume controlling importance in the selection. The effect of power production from sludge gas may be an important factor, especially in the selection of the type of sludge disposal. For reasons clear to the reader, especially the large variation possible in these factors and their relative effect on the various processes, the effect of commodity cost variation on the economics of treatment will not be discussed in detail, but has been mentioned in view of its important effect, in some cases, in the selection of the economical treatment process.

Effect of Hydraulic Head Available.—In certain instances, circumstances may be such as to make certain processes uneconomical when consideration is given to the hydraulic head available for operation of the treatment plant. As an example, the head necessary for operation of trickling filters may not be available in a specific problem in which case pumping of the sewage may be necessary. Even if sufficient head is available, its value in reducing the cost of intercepting sewers which would be possible in connection with other processes must be considered in the analysis. For example, in connection with the Minneapolis-St. Paul project it was determined that every foot of head saved in the plant was worth approximately $50 000 in reducing sewer sizes, when applied to the intercepting sewer system.

Effect of Design Features.—A rather wide range in the construction cost of a treatment plant can be expected as affected by a number of design features, such as provisions made for: Odor control, flexibility and dependability of operation, mechanical and automatic operation of equipment, etc. The personal opinions of the designer with respect to the conservativeness and the care with which the structural design is made, may affect the comparison between the processes. Furthermore, as knowledge has advanced (as it very markedly has during the 10 years, 1928–1938), economies have been affected in both design and operation. In the activated sludge process for example, the commonly accepted width of aeration tanks until 1930 was about 15 to 18 ft, whereas in recent years widths as great as 30 ft and more have been adopted, resulting in reduced structure costs and in air equipment and requirements. Similar economies have been, and will continue to be, made in other processes of sewage treatment.

Miscellaneous Factors Affecting the Comparison.—The writer has attempted to mention some factors of general application that affect the comparison of the economics of various treatment processes, but has by no means covered all such factors. The practical consideration that funds for operation are ordinarily difficult to procure may be considered sufficient justification in certain cases to affect a theoretical selection, and lead to the choosing of a process costing comparatively more to construct with attendant lower operation and maintenance costs. The method of sludge disposal selected or determined to be economical should be considered, together with the type of treatment, and, in certain cases, may exert an important influence on the selection. The size of the plant constructed as related to present requirements is a factor to be considered in the comparison. In special cases climatic and foundation conditions may have an influencing or even controlling effect on the selection. For various reasons all such factors cannot be covered in a paper of this nature and, therefore, the writer wishes to point out summarily that the generalizations made herein must be considered in the light at least of the major factors that affect economics of sewage treatment.

An idea of the effect of the various factors may be gained from a consideration of the substantiating data on construction and operation costs. Considering first construction cost: Of the 71 plants for which substantiating data are available 53, or 75%, are within a range of 20% above and below the estimated curves of costs shown in the curves. Of the 62 plants in which operation and maintenance costs are shown, 38, or 61%, are within the same range. Although this would indicate a fair substantiation of the estimates prepared for average conditions, it also serves to reveal the effect of a large number of factors that affect the costs in individual cases.

Economics of Various Processes as Applied to Minneapolis-St. Paul Project.—As a result of the relatively long period available for preliminary investigations in the "Twin City" project, both by the Metropolitan Drainage Commission over a period of five years, and the Minneapolis-St. Paul Sanitary District for two years, an unusual opportunity was presented for a careful determination of the relative economy of various processes and degrees of treatment. In various stages of the investigations, sedimentation, trickling filters, activated sludge, and chemical treatment were considered progressively. It was determined early that a high degree of secondary treatment was not necessary, and that preliminary treatment would suffice for all but a relatively small proportion of the time. The almost continuous record of river conditions over a 9-yr period was an invaluable aid in these determinations.

As a result of the investigations it was determined that preliminary treatment represented by sedimentation should be provided during the entire year, and that provision should be made to accomplish a reduction of as much as 70% of B.O.D. during certain critical periods estimated to be approximately 10% of the time. For periods of ten to fifteen years no secondary treatment whatever may be necessary, based on present river standards. The estimated costs as determined for the "Twin City" project are shown in Table 17. The estimates indicate the relative lower first cost of the chemical treatment process as compared with the activated sludge process. Based on year-round treatment the total annual charges of the activated sludge process (Treatment No. 3, Table 17) is $35 000 less expensive than the chemical treatment process (Treatment No. 1, Table 17). However, on a two-months basis (which is more than is estimated as necessary) chemical treatment will cost about $155 000 less annually than activated sludge treatment. Under different conditions the effect of the various influences previously mentioned may be such as to alter the determination.

TABLE 17.—ESTIMATED CONSTRUCTION COSTS AND TOTAL ANNUAL CHARGES OF VARIOUS TYPES OF TREATMENT PLANTS FOR MINNEAPOLIS-ST. PAUL, MINNESOTA

Treatment No.	DESCRIPTION OF TREATMENT IN TERMS OF MILLION GALLONS DAILY TREATED			Percentage reduction in five-day biochemical oxygen demand	Construction cost	TOTAL ANNUAL CHARGES (FIXED CHARGES AT 6%), WITH SECONDARY TREATMENT FOR:		
	Chemical (1)	Sedimentation (2)	Activated sludge (3)	(4)	(5)	Two months (6)	Six months (7)	Twelve months (8)
1...	134	....	..	70	$3 376 000	$521 100	$659 100	$937 600
2...	94	40		70	5 201 800	625 600	748 600	976 900
3...	....	134	84	70	6 149 700	677 900	747 900	902 000
4...	....	134	..	33⅓	2 796 300	413 000	413 000	413 000
5...	....	134		92	7 342 000	767 800	845 400	1 032 000

SUMMARY AND CONCLUSION

In an attempt to arrive at a solution of some of the economic problems involved in sewage treatment, the writer has presented data on the construction, operation, and maintenance costs, and reductions accomplished by some of the basic processes of sewage treatment, and has attempted to draw attention to certain factors which, in a particular situation, may alter such generalizations which of necessity have been made herein. Certain processes and combinations of processes have not been discussed because of space limitation and the writer wishes expressly to emphasize that this has not been done because such processes are not, in his opinion, of economic importance.

In conclusion, it should be stated: (1) That, although the writer has attacked the problem in a general manner, the effect of influencing factors in affecting conclusions relative to a definite project must be considered; (2) that in the writer's opinion the effect of such factors dictates that almost every project be analyzed separately; (3) that the fundamental and controlling economic factor is the degree of treatment necessary in a particular project; (4) that sufficient and well proved processes are available to satisfy, economically, the requirements of almost any problem that may arise; and (5) that the advances in the last five years, and the improvements that are certain to occur in the future, are of such magnitude as to require careful study in each project in order to reach an economical solution.

Acknowledgments

Because it has been found impractical to acknowledge information and data furnished by others at its particular place of use, the writer wishes, collectively and summarily, to acknowledge his indebtedness for all such data used in this analysis. Data have been extracted from the engineering publications on the subject; they have been secured by direct visits to a number of treatment works; and by correspondence with the officials in charge, for all of which the writer expresses his gratefulness.

Special acknowledgment is due to the Metropolitan Drainage Commission and to Mr. Childs, its former Chief Engineer, whose assistance in connection with the preliminary studies, is noteworthy; and to the Board of Trustees of the Minneapolis-St. Paul District (Mr. C. C. Wilbur, Chief Engineer), for information relative to the "Twin City" project used in this paper.

DISCUSSION

Charles Gilman Hyde,[5] M. Am. Soc. C. E. (by letter).—Referring to the economics problem, in general, practically every engineering project demands consideration from three economic standpoints: Justification, selection, and proportioning.

In the usual case, private and quasi-public undertakings, such as manufacturing enterprises, office buildings, hotels, railway, water, gas, and electric systems, etc., must eventually justify themselves, in so far as their owners are concerned, on the basis of financial returns only. Enterprises undertaken by the public, such as water supply and highway systems, should also justify themselves from the economic standpoint, in considerable part, at least, if not wholly. However, many private, quasi-public, and public enterprises of the type in question yield intangible returns of distinct social and esthetic value that cannot be expressed in monetary terms. In those instances, where such dividends are relatively large, expenditures on the part of the public are justifiable either in the form of subsidies or of direct investment, as the situation may suggest. Rarely, however, does it appear that public monies are thus contributed to private undertakings even if the returns therefrom are of great social benefit.

The Economic Justification of Sewerage and Sewage Treatment.—It so happens that investments in sewerage and sewage treatment works can seldom be economically justified because their benefits are not to be reckoned in dollars. It is unquestionable that public health and comfort are immeasurably and directly improved by sewerage and to a less degree by sewage treatment. However, there seems to be no financial "yardstick" (for instance, in terms of the value of the human lives which might be saved by any particular system of works) by which the advantages of this indispensable public service can be measured. Occasionally, the treatment of sewage may lessen the cost of treatment of water taken from a source which would otherwise be more grossly polluted or possibly too polluted to permit of its use for water supply purposes. Occasionally, also, fish and shell-fish industries of commercial value may become possible where, except for such treatment, they would not. In such cases some form of estimate can be made of the economic value of sewage treatment.

It is believed, however, that in most cases and for the most part sewage treatment must be justified upon social and esthetic, rather than upon economic, grounds. Thereby odors are eliminated, unsightliness avoided, recreation in the forms of boating, bathing, fishing, and picnicking made possible, water sources rendered more available and amenable to successful treatment for industrial, domestic, and other public purposes, and, in general, a better and more satisfactory environment vouchsafed.

Problems which, perhaps, may be looked upon as coming within the category of economic justification sometimes present themselves in connection with the design of sewage treatment works. For instance, the question may arise: Can the dehydration of sludge and its preparation as a fertilizer be justified on the basis of the expected sales? Since the sludge must be disposed of in some fashion, however, this problem may also be regarded, and perhaps more properly, as one of economic selection—selection as between different methods of securing a final or end result.

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Economic Selection in Sewage Treatment.—The paper deals only with problems of economic selection in sewage treatment planning and design. It is a noteworthy contribution of extreme value because of the truly enormous mass of worthy fundamental data that have been brought together and critically examined, because it is virtually a pioneer discussion of the major phases of this important matter, and because in no other single place, known to the writer, can the material or any substantial part of it be found. It appears that much of this information has never been published, at any rate in available form. Mr. Schroepfer deserves the gratitude of the profession for the faithful, untiring effort that he has put forth over a long period of time to bring these data together. He is to be congratulated upon their careful analysis and excellent presentation. His discussion of the many factors to be considered in this particular group of problems in economic selection is so comprehensive that little need be said further on that score.

Economic selection implies a choice, on the basis of involved costs, between different methods or machines for accomplishing a given, pre-determined result. In sewage treatment the desired character and condition of the effluent must determine the processes and mechanisms which are available and which can be utilized. The selection as between two or more possibilities will be made, wholly or in part, as the case may be, upon an economic basis. The most significant criteria of performance will ordinarily be the removal of suspended solids and bio-chemical oxygen demand, as suggested and used by the author. There will usually be other factors, not readily expressible in monetary units, which should be given due consideration, and these may be controlling in some instances. The author has wisely stated that such factors demand attention and that each local problem should be analyzed independently with respect to all influencing circumstances and conditions.

The total annual cost in every case should constitute the criterion of economic selection. This cost must comprehend all fixed and operating charges. The fixed charges must include interest, insurance, taxes (if any), depreciation, and maintenance. The operating charges must comprise all labor and overhead, and supplies of every kind not overlooking power, heating, and lighting costs.

Unfortunately, published data on sewage treatment costs too frequently present operating costs only, and omit the fixed charges. Both groups of charges should be presented, properly segregated. The total investment in the works covered by the operation cost data should be stated, together with the rate of interest thereon and the depreciation allowances. Maintenance expenses should be shown as a separate item. The author is to be complimented on the fact that he has taken both main categories of expense into due account.

It frequently happens that the investment costs of projects, selected to yield comparable effluents, are widely different. It is not always an easy matter to convince administrative authorities and the public that works requiring the larger initial investment may actually be the cheaper in the long run, all factors considered. It happens, also, that funds for the economically more advantageous scheme are often unavailable, or are assumed to be.

Different mechanical equipments may involve different types and shapes of structures which, in turn, may represent different investment and perhaps different annual costs. In such cases, if the performances are equivalent, the selection should be made upon the economic basis, unless local conditions dictate to the contrary.

It must be obvious that problems of economic selection present themselves not only with respect to entire projects, but also with respect to numerous features of these projects. For example, such problems arise in connection with the selection of screening, pumping, and aeration equipment, of types and shapes of settling basins and their mechanisms, and of sludge-handling and disposal procedures and equipments.

Economic Proportioning or Dimensioning in Sewage Treatment.—So much has been spoken or written upon the general subject of economic proportioning that designing engineers should be well aware of its importance and thoroughly acquainted with the calculation procedures. As in the domain of economic selection, however, there are many factors, aside from those of cost, which must be considered in determining the proper shapes, proportions, and dimensions of sewage treatment plant structures and equipments. The characteristics of sewage and sludge, of course, must always be borne in mind. Dependability is frequently of more importance than mechanical efficiency. Features and works that might appear to be most advantageous from an economic standpoint might actually perform so badly and demand so much operating attention that they might prove in reality (that is, in service) to be the more costly. The primary effort of the designer must be directed toward securing, with ease and assurance, the treatment effects that are sought. Economic proportioning must be a secondary, although still an important, consideration. There are many features of sewage treatment plants, nevertheless, which may be subjected to analysis to determine the most economic proportions. These features may be both structural and hydraulic.

Among other functions economic proportioning has to do with the determination of the most advantageous plant elevations. In this single item of planning there is frequently to be found a signal opportunity for cost saving. There may be one controlling elevation which will represent the greatest overall economy for the project. That elevation should be sought. Any other might involve more pumping, more excavation, more difficult and expensive foundations, less flexibility in operation, or some other costly or undesirable feature.

Effective sedimentation imposes severe depth limitation. Sludge-removal equipment may control the width of settling basins. Lengths must be sufficient to prevent excessive short circuiting. Certain shapes of basins require much more area than others. The economic dimensions of two basins in a single structural unit will usually differ from those of a single unit; and the economically most advantageous dimensions of several basins in a single structural unit will vary from those of a smaller number of basins. Unfortunately, however, the dimensions that will produce the best effluents may not vary in such manner.

One of the most common, and perhaps the best understood, problems of economic proportioning in sewage works design, as well as elsewhere, is that of the determination of the most economic diameter of force mains. In this case, where the volume of sewage or sludge will increase with the growth of the contributing district, the annual costs of pumping should be calculated for various pipe sizes and for various dates throughout the anticipated life of the pipe. The mean minimum cost will define the most economically advantageous diameter. Here again, however, the characteristics of the sewage or sludge must be considered. Pumps and pipe lines most economically proportioned for water may be so small as to become clogged with sewage or sludge. Moreover, the friction factor, due to grease accumulation, may increase with age. The entire subject is assuredly one of great interest and importance.

Frederic Bass,[6] M. Am. Soc. C. E. (by letter).—In the voluminous literature on sewage treatment relatively little is found on the economic phase, and for that reason alone this admirable paper is welcome. The author has industriously and painstakingly accumulated a great quantity of data on costs and performances of the larger plants of the United States, and has arranged and correlated them in a manner adaptable to various situations under average conditions.

As the obvious purpose of this paper is to present data that will be of assistance to the engineer in selecting the degree and type of treatment for a particular case, the writer believes that emphasis should be placed on: (1) The need of more extended study of the requirements of each situation; and (2) the effect of local factors that may alter the relative economy of types of treatment, as indicated by the average costs given. The author states (see "Comparison of Various Processes: General"): "Having arrived at what seem to be reasonable basic cost and reduction data for normal conditions, this section of the paper is devoted to a comparison of the various processes, and a discussion of certain factors that affect their relative economy and thus serve to alter the general indications presented in the comparisons." Furthermore (see heading, "Comparison of Various Processes: Miscellaneous Factors Affecting the Comparison"): " * * * 75% [of the plants] are within a range of 20% above or below the estimated curves of costs shown in the curves"; and 61% are within the same range for costs of operation and maintenance. The specific causes of variation from the average are mentioned: Fixed charges, sewage characteristics, seasonal variations in degree of treatment, commodity costs, head available, and design features. However necessary it may be for the designer to be in possession of average over-all costs of construction and operation, it is of prime importance for him to keep in mind the possible variations from the average due to local conditions.

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The author recognizes that the fundamental economic consideration is the degree of treatment necessary. This most important factor has quite often been given scant attention, particularly in smaller plants. In many cases it is apparent to the engineer that primary treatment only is required for all conditions, or that, on the other hand, continuous secondary treatment is necessary, but there are many cases in which only extended laboratory-controlled observation can determine the degree of treatment required to maintain a given standard in the receiving body of water and the varying degrees of treatment required in a stream of widely fluctuating discharge. The standard may be that of a minimum dissolved oxygen content in the stream alone, or in combination with suspended solids—or, in some cases, in addition, a bacteriological standard. At present there appears to be difference of opinion with respect to dissolved oxygen content standards in the receiving body of water, and until this question is fully answered in each doubtful case, the economic degree and type of treatment cannot be determined. Relatively few designing engineers possess the equipment for making the investigations necessary; the expansion of the services of the U. S. Public Health Service and of State Health Departments in making continuous surveys of sewage-polluted streams and from them establishing definite conditions to be met by treatment plants would, in the end, contribute very greatly to the economic design and operation of all treatment plants. At present (1938) such investigations are limited to the larger cities. Investigations by Federal, State, or by inter-State authorities have the further advantage of treating natural drainage districts as a whole, allotting to each community its proper share of the burden of cost.

As such investigations multiply and are made generally available the primary economic factor of degree of treatment required will tend to become limited within narrower ranges. In the case of the Minneapolis-St Paul plant (to which the author calls particular attention) long and detailed studies of the Mississippi River were made under the direction of the late J. A. Childs, M. Am. Soc. C. E., but the uncertainties in predicting the effect of some of the factors, particularly the rate of oxygen absorption by sludge deposits, required a considerable factor of safety to assure the required dissolved oxygen content in the river after the plant was in operation. If more extended investigation resulting in more firmly established and generally accepted technique had been available at that time, more precise interpretation of the situation would have been possible. From the economic standpoint, the decision as to the minimum dissolved oxygen content to be maintained in the receiving stream, whether it be 2 ppm or 4 ppm is vitally important. Accumulation of further data that will throw light on this factor will be of great value. Having such data, all the information in this paper may be used to its fullest extent.

One of the noticeable characteristics of the data presented is that practically all of them are of the last decade (1927–38) and many of the last two or three years. Even during the period when the Minneapolis-St. Paul plant was being designed, new and important information concerning developments in all types of treatment was forthcoming. In 1928, a trickling filter plant would have been designed and, in 1933, an activated sludge plant. The development of the chemical treatment process and effluent filter in this period contributed to the principle of flexibility in degree of treatment as required by the fluctuations of stream flow.

The feature of this paper showing the costs of operating secondary processes for fractions of the year suggests emphasis upon the economic values of this principle and that other combinations than those mentioned may be suitable in certain cases. In Table 16 the cost of effluent filters (if regarded as a secondary process) might be added, as such filters appear to be available in connection with plain sedimentation when that is not at all times sufficient, and when higher degrees of treatment attainable by other secondary processes are not required. From the data given of such filters, it appears that with plain sedimentation, 31 ppm suspended solids and 21 ppm B.O.D. would be removed by filtration. These values must be added to the 56% of suspended solids and 36% B.O.D. removed by plain sedimentation to obtain the over-all removal. From the costs of filtration shown in Table 18, on a plant of 100 mgd capacity, the fixed charges at 6% would be $24 000 and the operation and maintenance cost, $25 550, a total of $49 550.

TABLE 18.—ADJUSTED COSTS OF FILTRATION

Sewage treated, in million gallons per day	Construction cost, in dollars per million gallons per day	Operation and maintenance costs, in dollars per million gallons
20	4 500	0.95
50	4 200	0.80
100	4 000	0.70

Another combination that might be considered is that of short-period aeration (perhaps 2.5 or 3 hr) followed by effluent filters, if and when data are available to show the reductions by filtration of the suspended solids and bio-chemical oxygen demand from the effluent of the secondary settling basins of the activated sludge process.

In the section on trickling filters, the author has given as a basis the costs of construction and operation-maintenance at a rate of 2.0 mgd per acre. He has mentioned the fact that large-scale experiments (at Minneapolis, Minn., and Chicago, Ill.) have used rates as high as 20 and 25 mgd per acre. These experiments have resulted in securing bio-chemical oxygen demand removals from 65% to 75% and, in some cases, more. If, in practice, an average of 70% bio-chemical oxygen demand removal is attained (which seems likely) and a total construction cost of from 50% to 60% of a plant having a capacity of 2 mgd per acre with an operating cost perhaps 5% greater than that of the slower rate plant, values in Table 19, based on costs given in the paper for trickling filters at the lower rate, would apply. A curve might be added to Fig. 1(*b*) based on Table 19 which would appear quite closely parallel to the trickling filter curves, but to the left of them.

Although these last two illustrations of possible combinations cannot be substantiated by costs from actual large-sized installations (in view of the rapid rate at which new processes and combinations of processes are developing under adequate laboratory control) it is in that direction that sanitary engineers may look for advances in the near future.

In treatment plants with capacities of approximately 5 mgd and more, it is reasonably certain that careful and continuous laboratory control will be secured and that the performances cited in this paper will be achieved at the costs indicated. At present, information is necessarily obtained from them. Unfortunately, in considering the smaller plants, which obviously are greater in number, such control and information are lacking in many cases. In these plants the actual performances of the various types are likely to be so variable and to differ so much from their potential performances that not only is economic comparison more difficult, but certain types, because of the need of their more careful operation, due to their sensitiveness to varying conditions, should be eliminated altogether from consideration. In order to secure true economy in sewage treatment in the smaller cities, generally, the engineer must recognize the need for educating the public and its representatives in the value of acquiring necessary data for the correct solution of engineering projects.

TABLE 19.—COST OF HIGH RATE TRICKLING FILTERS

Plant capacity, in million gallons per day	Estimated construction cost	Estimated operation and maintenance	Fixed charges	Total annual cost	Annual cost, in million gallons	Cost; 1% per million gallons per 5-day B.O.D. reduction
10	$ 660 000	$ 31 500	$ 39 600	$ 71 100	$7 110	$0.278
20	1 265 000	52 500	75 900	128 400	6 420	0.251
50	3 025 000	115 500	181 500	297 000	5 940	0.232
100	5 885 000	226 300	353 100	579 400	5 794	0.227

This paper suggests that a great mass of unpublished data on the economic aspect of sewage treatment is existent and that this rapidly advancing science would benefit by its publication.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2037

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

By Y. L. Chang,[1] Esq.

With Discussion by Messrs. Hans Kramer, E. W. Lane, Joe W. Johnson, J. E. Christiansen, W. H. Huang, Lorenz G. Straub, Hunter Rouse, and Y. L. Chang.

Synopsis

This paper describes a laboratory investigation of the problems of flume traction and transportation. Authoritative literature on the subject is also reviewed. Although the experiments are not—and cannot be expected to be—of the high accuracy obtainable in some classes of laboratory work, it is hoped that the results, when suitably corrected and refined, will provide a close guide to a scientific conclusion.

The subject is presented under three main headings: Part I is concerned principally with the tractive force required to cause initial movement of débris; the laboratory investigation by the writer is described in detail, an equation for critical tractive force is presented, and the results obtained by other investigators are tabulated and shown to conform generally to the same equation. Part II deals with the laws of transportation by traction, and Part III, with the laws of transportation by suspension. In each case the writer's experiments are reviewed briefly, while the major part of the presentation consists of theoretical analyses and discussions of the work of other experimenters. Formulas are suggested for making rough estimates of the traction load and the suspended load in either natural streams or regularly shaped channels. The nomenclature is summarized in the Appendix.

This paper is an abstract of a thesis[2] entitled "Hydraulics of Flume Traction and Transportation".

PART I.—FLUME TRACTION

The first part of the investigation was directed to determining the critical tractive force for a series of uniform sands of various sizes, and to studying the relation between the variables involved.

Note.—Published in November, 1937, *Proceedings*.

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[2] The thesis has been submitted as a partial fulfillment for the degree of Doctor of Philosophy to the Victoria Univ., Manchester, England. Original copies are also filed with the Board of Trustees, Nanking, China, and in the Library of the National Tsing Hua Univ., Peiping, China.

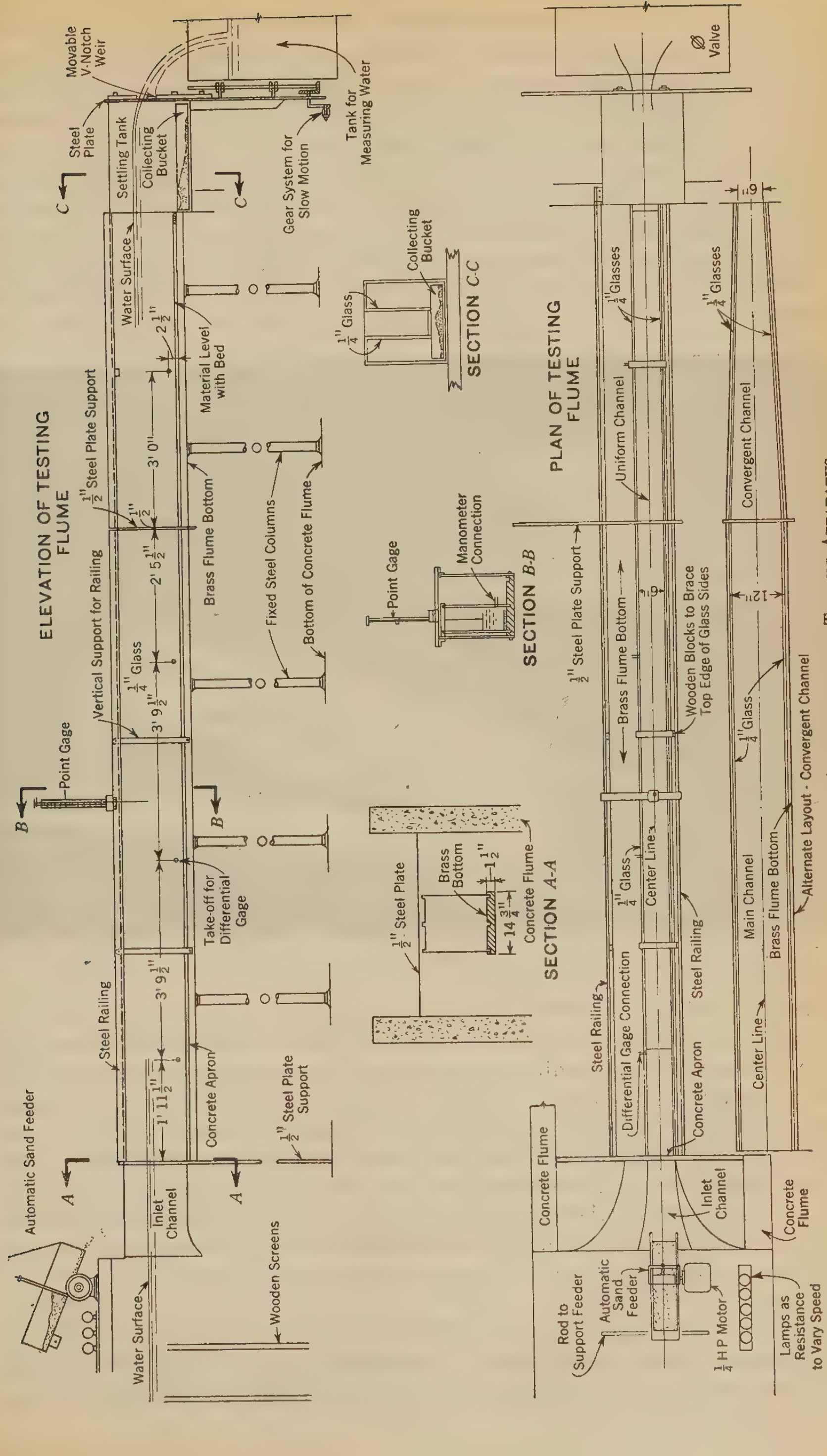
ELEVATION OF TESTING FLUME
Movable V-Notch Weir
Steel Plate
Settling Tank
Collecting Bucket
C
C
Gear System for Slow Motion
Tank for Measuring Water
Water Surface
$2\frac{1}{2}''$
Material Level with Bed
3' 0''
$\frac{1}{2}''$ Steel Plate Support
$\frac{1}{2}''$
$2' 5\frac{1}{2}''$
Brass Flume Bottom
Fixed Steel Columns
Bottom of Concrete Flume
Vertical Support for Railing
$\frac{1}{4}''$ Glass
$3' 9\frac{1}{2}''$
Point Gage
B
B
Take-off for Differential Gage
$3' 9\frac{1}{2}''$
Steel Railing
$1' 11\frac{1}{2}''$
Concrete Apron
$\frac{1}{2}''$ Steel Plate Support
Automatic Sand Feeder
A
A
Inlet Channel
Water Surface
Wooden Screens
Collecting Bucket
$\frac{1}{4}''$ Glass
SECTION C-C
Point Gage
Manometer Connection
SECTION B-B
$\frac{1}{2}''$ Steel Plate
Brass Bottom
$\frac{1}{2}''$
$14\frac{3}{4}''$
Concrete Flume
SECTION A-A
PLAN OF TESTING FLUME
Valve
$\frac{1}{4}''$ Glasses
$\frac{1}{4}''$ Glasses
6''
Uniform Channel
Convergent Channel
$\frac{1}{2}''$ Steel Plate Support
Brass Flume Bottom
6''
12''
Wooden Blocks to Brace Top Edge of Glass Sides
$\frac{1}{4}''$ Glass
$\frac{1}{4}''$ Glass
Center Line
Steel Railing
Main Channel
Brass Flume Bottom
Alternate Layout - Convergent Channel
Steel Railing
Differential Gage Connection
Concrete Apron
Center Line
Concrete Flume
Inlet Channel
Concrete Flume
Rod to Support Feeder
Automatic Sand Feeder
$\frac{1}{4}$ H P Motor
Lamps as Resistance to Vary Speed

FIG. 1.—GENERAL ARRANGEMENT OF TESTING APPARATUS

DESCRIPTION OF APPARATUS

The laboratory flume and appurtenances are shown in Fig. 1. Three different arrangements were possible: (1) A uniform channel, 18 ft long and 12 in. wide; (2) a uniform channel, 18 ft long and 6 in. wide; and (3) a uniform channel, 12 ft long and 12 in. wide, followed by a 6-ft length of channel converging to a width of 6 in. at its down-stream end. The convergent section was provided in order to locate more exactly the point (and, hence, the velocity) at which movement commenced. The point-gage, Pitot tube, and templet for smoothing the bed were arranged to slide on rails on top of the flume.

The point-gage could be read to 0.005 in. The drop in water-surface elevation from point to point along the flume was measured accurately by means of a toluene differential gage, inclined at an angle of α from the horizontal. This arrangement of the apparatus produced a magnification of 46 to 1 when $\alpha = 10°$, and readings were taken with a traveling vernier microscope (Fig. 2) reading to 0.01 mm. Proper corrections were made for temperature, which was observed on a mercury thermometer in contact with the gage.

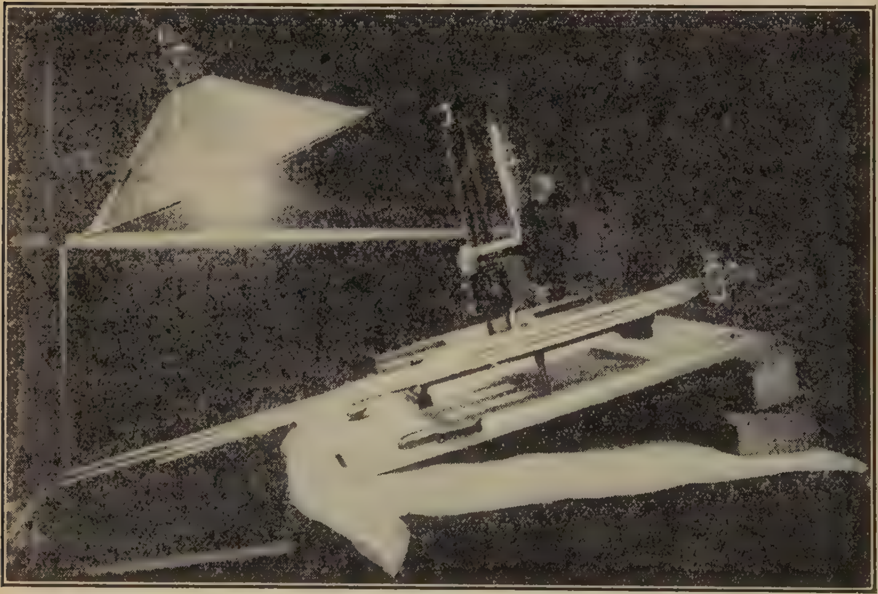

FIG. 2.—TOLUENE DIFFERENTIAL GAGE (INCLINED) WITH TRAVELING VERNIER MICROSCOPE

The water discharge could be measured either by means of a diaphragm in the feed-pipe or, volumetrically, in the tank at the down-stream end of the flume. Short match sticks, weighted to float vertically, were used for observations of surface velocity, and a Pitot tube connected to a toluene gage provided accurate readings of sub-surface velocities at any desired point.

PROCEDURE

A special feature of the present experiment is that the bed of the flume was horizontal, instead of inclined to the same slope as the water surface.

The latter arrangement is theoretically sound, by the reasoning of uniform flow and parallel tractive force, but the necessity for it is doubtful. In model experiments the slope should never be sufficient to produce shooting flow; the permissible limit, for depths commonly used, is about 0.002. The actual velocity required to initiate movement of the samples used in the present experiments was much smaller than that corresponding to the limiting slope, the average value being about one-fourth of it. To mold the bed exactly to such a slope is very difficult, even with accurate apparatus, and, for smaller slopes, it is entirely impracticable.

With the bed horizontal, it would be possible to correct to the condition of uniform flow by subtracting the loss of potential head due to acceleration of flow, from the observed drop of surface level. However, in the present experiments, with a smooth unriffled bed, this loss never exceeded 0.0002 ft, and, being smaller than the deviation of the static readings from the differential gage, was neglected. Over a riffled bed, any such correction is meaningless, for the surface waves are of the same order of amplitude as the riffles.

In beginning a test run, the bed material was placed in the flume, wetted thoroughly, and made perfectly level and smooth by means of the brass templet and a trowel. Water was then introduced very gradually so as not to disturb the smooth bed (even tiny surface waves may produce riffles especially in shallow depths). After the flume had filled sufficiently, the test run was begun, the depth of flow being controlled by the movable weir plate at the end of the settling tank, and by the valve in the feed-pipe.

When sand movement began, the water discharge was kept constant for a while to make sure that the movement was due to the drag of the flow rather than to local irregularities. Then the flow was increased a little to generate stronger movement of the bed. This procedure was continued until data for "general movement" were obtained. (The term "general movement" applies to that condition in which grains of mean diameter are in motion so frequently that the numbers are not countable. Such motion should be general, not local. It is not strong enough to affect bed configuration and does not result in transportation of an appreciable quantity of material.)

In the uniform channel the slope of the water surface can be measured directly by the differential gage; in the convergent section the slope varies at different points, and can be determined more accurately by calculation, taking into account the different roughnesses on the bed and the sides.

Mechanical Properties of Test Samples

In all, twelve commercially sieved samples were tested in the flume, each sample consisting of grains of more or less uniform size. The average grain diameter varied from 8.090 mm for the coarsest sample to 0.134 mm for the finest. The principal mechanical properties are listed in Table 1.

The method of determining the grain diameters requires some explanation. For each sample, 150 particles were selected arbitrarily and examined under a microscope. The two diameters, D_1 and D_2, are those lying in the plane parallel to the horizontal glass, and were measured during this examination. For the coarsest materials, D_1 and D_2 were measured by projecting the images of the grains on to a screen. The diameter, D_3, is the vertical dimension of the grain. In general, the shortest of the three, D_3, cannot be measured under the microscope, and must be computed from the grain volume, as measured by the displacement of a definite number of grains in water. For this computation, the general shape of the grains must be assumed. Each sample appeared to be composed of grains of two general shapes, which may be designated as "spherical" and "cubical," respectively. An examination of about 50 grains by eye permitted an approximate determination of the relative number of grains of the two shapes. The demarcation is not definite, and the estimate in Column (9), Table 1, is rough, but it does permit the computation with reasonable accuracy of D_3, as well as of the mean diameter, D, and the ratio, o, of the longest to the shortest diameter. For fine grains the volume of which was not measurable, D was assumed equal to the mean of D_1 and D_2, and o was taken as $\frac{D_1}{D_2}$.

TABLE 1.—MECHANICAL PROPERTIES OF TEST SAMPLES

Sample designation (1)	Specific gravity* (2)	Average volume of 1 grain, in cubic centimeters (3)	GRAIN DIAMETERS, IN MILLIMETERS				o (ratio of longest to shortest diameter) (8)	Approximate relative number of spherical and cubical grains (9)
			D_1 (4)	D_2 (5)	D_3 (6)	D (7)		
6 205	2.547	0.2375	12.100	8.400	3.760	8.090	3.21	2 : 1
6 204	2.558	0.1195	9.880	6.960	2.790	6.540	3.53	2 : 1
6 203	2.567	0.0619	8.150	5.490	2.240	5.290	3.65	2 : 1
6 202	2.590	0.02195	5.790	3.990	1.523	3.770	3.74	2 : 1
6 201	2.633	0.00800	3.580	2.707	1.320	2.540	2.71	2 : 1
6 200	2.634	0.00264	2.465	1.789	1.016	1.760	2.43	3 : 1
6 199	2.640	0.00033	1.171	0.884	0.544	0.886	2.16	3 : 1
6 198	2.643	....	0.612	0.396	....	0.396	1.55	....
6 197	2.645	....	0.312	0.226	....	0.226	1.40	....
Pumice†	2.053	....	0.350	0.230	....	0.289	1.52	....
Pumice W‡	2.016	....	0.401	0.234	....	0.317	1.72	....
Emery	3.890	....	0.240	0.180	....	0.210	1.33	....
Sand α	2.520	....	0.155	0.113	....	0.134	1.37	....

* Determined by displacement of water in specific gravity bottle. Each value listed is a mean of three determinations.

† Composed of black and white particles.

‡ Separate analysis of the white particles of the pumice sample. About 95% of the pumice particles transported, and collected in the settling tank, and practically all the pumice particles in suspension, were white.

The uniformity modulus, M, has been proposed by Hans Kramer,[3] M. Am. Soc. C. E., as a convenient expression for the distribution of sizes of the particles in a sand mixture. The uniformity modulus of the samples used in this experiment was not determined, but a rough plotting of the microscopic readings of the grain sizes showed that all samples had nearly the same distribution. They are assumed, therefore, to have the same uniformity modulus.

EQUATION OF VARIED FLOW

As many observations were taken in the convergent channel, application of the principles of non-uniform flow[4] is necessary to evaluate slope and also to study the tractive force. The general expression for non-uniform flow is:

---

[3] "Sand Mixtures and Sand Movement in Fluvial Models," by Hans Kramer, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 798.

[4] "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E., N. Y., 1934, p. 24.

$$S = S_0 - \frac{dy}{dx} = \frac{V^2}{C^2 R} - \frac{Q^2}{g B^2 y^3} \frac{dy}{dx}$$

the nomenclature being explained in the Appendix.

For a rectangular section, since $Q = V B y$,

$$\frac{dy}{dx} = \frac{S_0 - \frac{V^2}{C^2 R}}{1 - \frac{V^2}{g y}} \qquad (1)$$

For a horizontal bed, $S_0 = 0$; and reducing to Manning's form,

$$\frac{dy}{dx} = \frac{1}{C^2 R \left[\frac{1}{g y} - \frac{1}{V^2} \right]} = \frac{n^2 V^2}{2.21 R^{4/3} \left[\frac{V^2}{g y} - 1 \right]} \qquad (2)$$

in which C has been replaced by $\frac{1.486 R^{1/6}}{n}$. Further,

$$S = - \frac{dy}{dx} = \frac{n^2 V^2}{2.21 R^{4/3} \left[1 - \frac{V^2}{g y} \right]} \qquad (3)$$

Hence, for any value of V less than $\sqrt{g y}$, the slope of the water surface in the convergent channel can be calculated by the ordinary Manning formula for uniform flow, and divided by the term, $\left[1 - \frac{V^2}{g y} \right]$, to correct for the non-uniform flow condition.

MANNING'S *n* FOR THE SAMPLES

The value of n for the flume as a whole is a function of n_g (the roughness coefficient of the glass sides) and n_s (the coefficient for the sample composing the bed). It was desired to determine separately the value of n_s.

The Manning formula, with R replaced by $\frac{A}{p}$, may be written,

$$V = \frac{1.486 A^{2/3} S^{1/2}}{(n^{3/2} p)^{2/3}}$$

In the present case, the denominator of the fraction may be equated as follows:

$$n^{3/2} p = n_s^{3/2} B + 2 n_g^{3/2} d \qquad (4)$$

in which d is the depth of flow. Then, if n_g and n are known, n_s can be computed.

There are two methods of determining n_g: (1) Experimentally: Assuming the brass bottom to have the same roughness as the glass, observations on flow through the flume with the sand bed removed will yield the desired value; and (2) by calculation from a derived formula.

J. Allen[5] has derived a formula for the roughness coefficient in terms of the Reynolds number, which reduces to,

$$n = \frac{R^{1/6}}{35.3\left(\frac{R\,V}{\nu}\right)^{3/40}} \qquad \text{(5)}$$

The relation holds provided the bed and sides of the channel are smooth.

Results of Methods (1) and (2) proved to be in reasonable agreement, and gave a mean value for n_g of 0.0103.

In the present experiments, there are two cases of uniform flow and one of non-uniform flow. The corresponding equations for n_s are:

1.—For uniform channel, 1 ft wide:

$$n_s = [(1 + 2\,d)\,n^{3/2} - 2\,dn_g{}^{3/2}]^{2/3} = [(1 + 2\,d)\,n^{3/2} - 0.00208\,d]^{2/3}$$

2.—For uniform channel, 0.5 ft wide:

$$n_s = [(1 + 4\,d)\,n^{3/2} - 4\,dn_g{}^{3/2}]^{2/3} = [(1 + 4\,d)\,n^{3/2} - 0.00405\,d]^{2/3}$$

3.—For the convergent channel (any width):

$$n_s = \left[\frac{(B + 2\,d)\,n^{3/2} - 0.00208\,d}{B}\right]^{2/3}$$

This expression is identical to one derived independently by R. E. Horton,[6] M. Am. Soc. C. E. He extended the method to a trapezoidal canal section, for which he gives the equation:

$$n = \left[\frac{n_b{}^{3/2} + 2\,z\,n_l{}^{3/2}}{1 + 2\,z}\right]^{2/3} \qquad \text{(6)}$$

in which n_b and n_l are the separate values of n for the bottom and the sloping sides, respectively, and z is the ratio of the length of one of the sloping sides to the width of the bottom. This formula is particularly applicable to natural canals with different roughness of bed and sides.

Values of n_s for the twelve test samples are given, with other pertinent data, in Table 2.

RELATION BETWEEN MANNING'S *n* AND SIZE OF PARTICLES

Manning's equation is dimensionally sound only if the friction does not depend upon viscosity (that is, only if the turbulence is fully developed) and n has the dimension of $(\text{length})^{1/6}$. It will be shown later that the Chezy coefficient is equal to $\left(\frac{g}{\kappa}\right)^{1/2}$, in which g is the acceleration of gravity and κ, the coefficient of skin friction, a dimensionless factor. Hence, C is of the dimension, $\frac{(\text{length})^{1/2}}{(\text{time})}$. Expressing Manning's n in terms of C, $n = \frac{1.486}{C} R^{1/6}$. Since n is independent of time, the constant, 1.486, must be of the same dimensions as C; hence, n is of the dimension, $(\text{length})^{1/6}$. The experimental Equation (5) gives the same result.

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[5] "Streamline and Turbulent Flow in Open Channels," by J. Allen, *Philosophical Magazine*, Vol. 17, 1934.

[6] "Separate Roughness Coefficients for Channel Bottom and Sides," by R. E. Horton, *Engineering News-Record*, Vol. 111, 1933, pp. 652–653.

TABLE 2.—MANNING'S n FOR SAMPLES

(Experiments done in a uniform channel, 0.5 ft wide)

Sample designation	S	Depth of flow, d	R	V	n	n_s	n_s (mean)
6 205	0.00060	1.070	0.341	1.110	0.0160	0.0276	0.0258
	0.00125	1.046	0.332	1.585	0.0161	0.0258	
6 204	0.00092	1.000	0.333	1.470	0.0148	0.0222	0.0225
	0.00140	0.950	0.328	1.740	0.0152	0.0227	
6 203	0.00094	1.041	0.338	1.505	0.0148	0.0225	0.0222
	0.00109	1.125	0.346	1.696	0.0144	0.0218	
6 202	0.00121	0.948	0.327	1.712	0.0144	0.0210	0.0210
	0.00119	0.666	0.285	1.443	0.0154	0.0211	
6 201	0.00128	0.871	0.193	1.460	0.0122	0.0187	0.0190
	0.00253	0.294	0.134	1.320	0.0148	0.0194	
6 200	0.00047	1.080	0.342	1.190	0.0132	0.0186	0.0186
	0.00059	0.658	0.284	1.093	0.0142	0.0186	
6 199	0.00019	0.875	0.318	0.746	0.0128	0.0166	0.0166
	0.00066	0.492	0.248	1.130	0.0136	0.0166	
6 198*	0.000133	0.960	0.329	0.700	0.0116	0.0141	0.0141*
	0.000166	0.736	0.298	0.715	0.0119	0.0141	
6 197	0.00025	0.817	0.189	0.711	0.0109	0.0127	0.0128
	0.00026	0.837	0.191	0.730	0.0109	0.0129	
Pumice	0.000206	0.282	0.132	0.478	0.0116	0.0129	0.0134
	0.000081	0.518	0.167	0.352	0.0115	0.0138	
Emery	0.000417	0.453	0.161	0.842	0.0107	0.0120	0.0120
	0.000588	0.196	0.109	0.743	0.0112	0.0119	
Sand, α	0.000089	0.438	0.158	0.387	0.0106	0.0113	0.0114
	0.000154	0.357	0.146	0.481	0.0107	0.0113	

* Experiment done in a uniform channel, 1 ft wide.

Strickler has proposed the equation,

$$n = 0.0150\ D^{1/6} \qquad (7)$$

in which D is the median diameter of the grains of the surface layer, in millimeters. Logarithmic plotting of the values of n for the samples shows exceedingly good coincidence to Equation (7). The equation of the straight line best fitting the plotted points (Fig. 3) is

$$n = 0.0166\ D^{1/6} \qquad (8)$$

D in this case being the mean diameter of the bed material, in millimeters. The difference in the value of the constant is probably due to the different geometrical shapes of the grains tested.

AVERAGE TRACTIVE FORCE

For conditions of uniform flow at depth, d, the tractive force per unit area opposed to the bed resistance is

$$T = w\ S\ d \qquad (9)$$

The expression[3] presupposes uniform flow and infinite width of channel. Referring to Fig. 4, the total tractive force in an irregular channel is

$$\Sigma T = T_1 + T_2 + T_3 + \cdots = w\,S\,(d_1\,p_1 + d_2\,p_2 + d_3\,p_3 + \cdots) = w\,S\,A$$

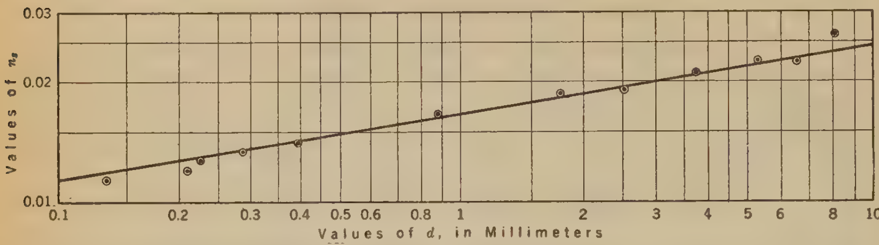

FIG. 3.—RELATION BETWEEN MANNING'S n AND MEDIAN DIAMETER OF SAND PARTICLES COMPOSING BED OF EXPERIMENTAL FLUME

and the average tractive force per unit area becomes,

$$\overline{T} = \frac{w\,S\,A}{p} = w\,S\,R \qquad (10)$$

The average unit tractive force in a rectangular flume should be

$$\overline{T} = w\,S\,d\,\frac{B}{B + 2\,d} \qquad (11)$$

According to Schoklitsch[7] an additional coefficient is required for channels in which the ratio of width to depth is less than 30. In such a case the average tractive force becomes, $\overline{T} = \frac{30}{32}\,w\,S\,d$, which is about 6% smaller than $w\,S\,d$. Schoklitsch[7] has suggested a similar, but more complete formula,

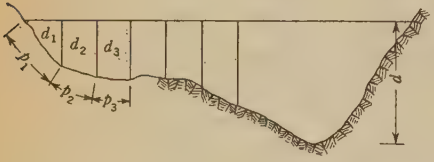
d_1
d_2
d_3
p_1
p_2
p_3
d

FIG. 4

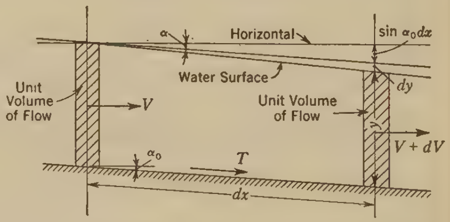
α
Horizontal
$\sin\alpha_0 dx$
Water Surface
dy
Unit Volume of Flow
V
Unit Volume of Flow
$V + dV$
α_0
T
dx

FIG. 5.—TRACTIVE FORCE IN ACCELERATING FLOW

$$T = \alpha\,w\,S\,d \qquad (12)$$

in which $\alpha = \dfrac{1 + \dfrac{d}{B}\,i}{1 + \dfrac{2\,d\,n_l}{B\,n_b}\sqrt{1 + i^2}}$, in which n_b and n_l are, respectively, the Manning n for the bottom and the sides, and i is the horizontal over the vertical of the sides. When $n_b = n_l$ and $i = 0$, $\alpha = \frac{B}{B + 2\,d}$, which is identical to Equation (11).

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[7] "Wasserbauliche Strömungslehre," by P. Nemenyi, Leipzig, 1934; or, "Über Schleppkraft und Geschiebebewegung," by A. Schoklitsch.

EFFECT OF NON-UNIFORM FLOW ON TRACTIVE FORCE

Assume the flow to be non-uniform but accelerating (the flow in the flume with horizontal bed is of this kind) in a channel of infinite width (Fig. 5). The change in energy between two sections may be expressed as follows: Increase of kinetic energy + work done in overcoming bed resistance = decrease in potential energy.

For a column of water of unit volume:

$$w\,\frac{y}{g}\,\tfrac{1}{2}[(V + dV)^2 - V^2] + T\,dx = w\,y\,\frac{\sin\alpha + \sin\alpha_0}{2}\,dx \ldots\ldots (13)$$

in which T is the tractive force on the bed, assuming that all frictional loss is due to bed resistance. Neglecting higher orders of the derivative than the first and reducing, Equation (13) becomes,

$$T = \left[w\,y\,\frac{\sin\alpha + \sin\alpha_0}{2} - \frac{w\,y\,V}{g}\,\frac{dV}{dy}\,\frac{dy}{dx} \right] \ldots\ldots\ldots\ldots (14)$$

Taking $V = \frac{q}{y}$, in which q is the discharge per unit width of the channel,

$\frac{dV}{dy} = -\frac{q}{y^2} = -\frac{V}{y}$. Also, $-\frac{dy}{dx} = \sin\alpha_0 - \sin\alpha$; and

$$T = \left[w\,y\,\frac{\sin\alpha + \sin\alpha_0}{2} + w\,\frac{V^2}{g}\,(\sin\alpha - \sin\alpha_0) \right] \ldots\ldots\ldots (15)$$

For a sloping bed with uniform flow, $\sin\alpha = \sin\alpha_0 = S$:

$$T = w\,S\,y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

For accelerating flow, $\alpha > \alpha_0$; for decelerating flow, $\alpha_0 > \alpha$. For accelerating flow on a horizontal bed, $\sin\alpha_0 = 0$:

$$T = w\;S\left(\frac{y}{2} + \frac{V^2}{g}\right) = \frac{w\,S}{2}\left(y + 4\,\frac{V^2}{2\,g}\right) \ldots\ldots\ldots\ldots (17)$$

therefore, for high depth with comparatively small velocity, the tractive force on a horizontal bed is one-half that on a sloping bed with the same inclination of the water surface. In case $\sin\alpha_0 = -\sin\alpha$, that is, when the bed has an adverse surface slope, there is only a tractive force by the velocity head, namely, $w\,S\left(4\,\frac{V^2}{2\,g}\right)$.

SUGGESTED EQUATION FOR CRITICAL TRACTIVE FORCE

The critical tractive force, T_0, has been defined as that tractive force which brings about general movement of the average grains of the bed material. This force, which is constant for a certain material, is not determined by either d alone or S alone, but by a constant product, $w\,S\,d$. Considering the tractive force as a function of the mechanical properties of the bed material, it can be written, $T_0 = F$ (grain size, grain distribution, grain form, apparent weight per unit volume in water) $= F\,(D^x\,M^{-1}\,n^\alpha\,\rho_e{}^y)$ in which Manning's n is assumed to be a function of the size and shape of the grains. It has been shown by Kramer[3] that tractive force is inversely proportional to M, which is a pure numerical ratio. Victor L. Streeter,[8] Jun. Am. Soc. C. E., has suggested that the coefficient of roughness varies directly with the depth and slightly with the interval of the grooves; for larger and more irregularly shaped grains, the grooves on the surface of the material would be expected to be deeper and larger. Therefore, it can be assumed that n, having a dimension of $(\text{length})^{1/6}$, takes the place of $(D\,o)^{1/6}$. By the method of dimensional analysis the following exponentials may be equated: $1 = y$; and $-1 = x - 2\,y + \frac{\alpha}{6}$. From Manning's formula, however, the tractive force, T, is proportional to n^2; therefore, $T_0 = c\,D^{2/3}\,\rho_e\,n^2\,M^{-1}$, in which c is a constant, and if n is assumed proportional to $(D\,o)^{1/6}$,

$$T_0 = \frac{c\,D\,\rho_e\,o^{1/3}}{M} \qquad (18)$$

For uniform, equal sized grains of material, $M = 1$, and for spherical grains, $o = 1$, and the formula reduces to

$$T_0 = c\,\rho_e\,D \qquad (19)$$

which is dimensionally correct.

CRITICAL SIZE OF SAND FOR CRITICAL TRACTIVE FORCE RELATIONSHIP

Table 3 shows in detail the observations made on a typical sample to determine the critical tractive force, and Table 4 summarizes the results of tests on all samples. In Fig. 6, the mean values of T_0 are plotted logarithmically against the values of $\rho_e\,D\,o^{1/3}$, M being assumed to be unity for uniform material or a constant for uniform distribution. It will be noted that for sand of a larger size than No. 6198, a line inclined at 45° to the X-axis joins up the points smoothly; for sand less than that size the points are better fitted by a line inclined at approximately 26.5 degrees. This fact indicates that there is a critical value of the function, $\rho_e\,D\,o^{1/3}$, above which it is directly proportional to the critical tractive force, and below which it is proportional to the square of the latter quantity. It happens that these two lines intersect at the value of T_0 for $\rho_e\,D\,o^{1/3} = 1.0$. Expressed symbolically,

$$T_0 = 0.0175\left[\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}\right]^{\beta} \qquad (20)$$

in which $\beta = 1$ for $\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}$ greater than 1, and $\frac{1}{2}$ for $\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}$ less than 1.

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[8] "Frictional Resistance in Artificially Roughened Pipes," by Victor L. Streeter, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 681.

TABLE 3.—DETAILED TABULATION OF OBSERVATIONS ON A TYPICAL SAMPLE *

Run No. (1)	B, in feet (2)	d, in feet (3)	A, in square feet (4)	R, in feet (5)	Q, in cubic feet per second (6)	V, in feet per second (7)	$\frac{V^2}{2g}$, in feet (8)
1	0.496	0.728	0.3615	0.1852	0.224	0.620	0.006
2	0.495	0.609	0.3265	0.1800	0.214	0.654	0.007
3	0.495	0.591	0.2950	0.1760	0.212	0.753	0.009
4	0.496	0.454	0.2250	0.1603	0.134	0.596	0.006
5	0.496	0.282	0.1400	0.1320	0.081	0.575	0.005
6	0.496	0.242	0.1200	0.1224	0.071	0.591	0.005
7	0.496	0.143	0.0710	0.0905	0.048	0.680	0.007
8	0.496	0.125	0.0618	0.0830	0.041	0.663	0.007
9	0.495	0.461	0.2285	0.1613	0.131	0.572	0.005
10	0.495	0.301	0.1490	0.1357	0.081	0.546	0.005
11	0.495	0.250	0.1240	0.1245	0.084	0.677	0.007
12	0.496	0.175	0.0870	0.1027	0.049	0.566	0.005

TABLE 3.—(*Continued*)

Run No. (1)	R (9)	n (10)	S‡ (11)	$w\,S\,d$, in pounds per square foot¶ (12)	T,‖ in pounds per square foot¶ (13)	Movement (14)
1	8 410	0.01107	0.000204	(0.00933)	(0.00478)	General motion
2	8 650	0.01098	0.000230	(0.00956)	(0.00457)	General motion
3	9 775	0.01046	0.000286	(0.01070)	(0.00560)	General motion
4	7 100	0.01105	0.000227	(0.00642)	(0.00338)	General motion
5	5 690	0.01110	0.000276	0.00495	0.00256	Occasional motion
6	5 350	0.01094	0.000311	0.00480	0.00254	Occasional motion
7	4 675	0.01077	0.000600§	(0.00562)	(0.00320)	General motion
8	4 250	0.01136†	0.000677§	(0.00558)	(0.00323)	General motion
9	6 820	0.01056	0.000234§	(0.00681)	(0.00352)	General motion
10	5 640	0.01063	0.000220§	0.00420	0.00220	Occasional motion
11	6 440	0.01071	0.000384§	(0.00617)	(0.00333)	General motion
12	4 480	0.01136†	0.000371§	0.00417	0.00226	Occasional motion
....	....	....	....	0.0075	0.00395 $= T_0$	....

* Sand α; $\sigma = 2.520$; $D = 0.134$ mm.

† Assume $n = n_s$ for small depth.

‡ $S = \left[\frac{n\,V}{1.486\,R^{2/3}}\right]^2 \div \left[1 - \frac{V^2}{g\,d}\right]$.

§ Calculated from $n^{3/2} = \frac{0.00116\,B + 0.00208\,d}{B + 2\,d}$.

‖ $T = \frac{w\,S}{2}\left[d + 4\,\frac{V^2}{2\,g}\right]$ for channel of horizontal bed.

¶ The values in parentheses are taken to compute the mean value of the critical tractive force, T_0.

TABLE 4.—MEAN VALUES OF CRITICAL TRACTIVE FORCE FOR TEST SAMPLES

(*M* is assumed equal to unity)

Sample No.	$\frac{\sigma - \rho}{\rho}$	D, in milli-meters	$o = \frac{D_e}{D_s}$	$o^{1/3}$	$\frac{\sigma - \rho}{\rho} D\, o^{1/3}$	$w\,S\,d$, in pounds per square foot	T_0, in pounds per square foot
6 205	1.547	8.090	3.21	1.475	18.450	0.3620	0.18500
6 204	1.558	6.540	3.53	1.523	15.520	0.2780	0.13900
6 203	1.567	5.290	3.65	1.540	12.770	0.2220	0.11100
6 202	1.590	3.770	3.74	1.552	9.300	0.1830	0.09100
6 201	1.633	2.540	2.71	1.395	5.790	0.0970	0.04850
6 200	1.634	1.760	2.43	1.345	3.870	0.0699	0.03490
6 199	1.640	0.886	2.16	1.293	1.880	0.0327	0.01660
6 198	1.643	0.396	1.55	1.157	0.753	0.0143	0.00757
6 197	1.645	0.226	1.40	1.119	0.415	0.0114	0.00635
Pumice	1.053	0.289	1.52	1.150	0.350	0.0098	0.00510
Emery	2.890	0.210	1.33	1.100	0.668	0.0142	0.00767
Sand α	1.520	0.134	1.37	1.111	0.226	0.0075	0.00395

For ordinary quartz sand, $\sigma = 2.65$; $\rho = 1$; and $o = 1.75$. Hence, the size of sand corresponding to the critical value of the function is approximately 0.5 mm. For spherical grains, $o = 1$; therefore, $D = 0.6$ mm (approximately). In other words, for ordinary quartz sand grains larger than 0.5 to 0.6 mm in mean diameter, the critical tractive force is proportional to $\rho_e\, D\, o^{1/3}$; for smaller sands it is proportional to the square root of that function.

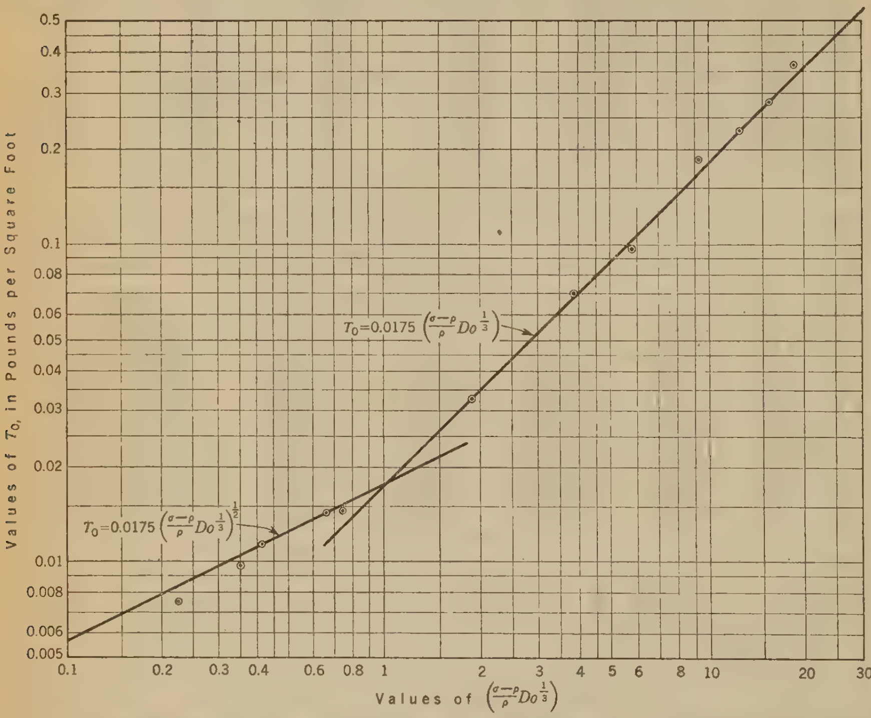

FIG. 6.—CRITICAL TRACTIVE FORCE (CHANG EXPERIMENTS; HORIZONTAL BED; *M* ASSUMED UNITY)

Pertinent experimental data from all available sources are summarized in Table 5, and plotted in Fig. 7, with the uniformity modulus taken into account. (The values of o are not available and are considered unity.) To reduce to the condition of uniform flow, the value of T_0 of the writer's experiment should be reduced to one-half; but here, instead, the numerical values of $(\rho_e\, D\, o^{1/3})$ have been doubled to correspond to a uniform modulus of 0.5. Here, again, the results suggest the existence of a critical size of sand with respect to the critical tractive force relationship, although the demarcation of the two lines is, in this case, by no means distinct. A reason for this lack of definition will be advanced subsequently.

The two equations for T_0, represented by the lines on Fig. 7, are:

For $\left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right] > 2.0$:

$$T_0 = 0.00450 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right] \qquad \text{(21a)}$$

TABLE 5.—DATA ON CRITICAL TRACTIVE FORCE DETERMINATIONS BY VARIOUS INVESTIGATORS

Item No. (1)	Investigator* (2)	Sample designation (3)	T_0, in pounds per square foot (4)	σ (5)	D, in millimeters (6)	M (7)	$\frac{\sigma-\rho}{\rho}\frac{D}{M}\,o^{1/3}$† (8)	$\frac{\sigma-\rho}{\rho}\frac{D}{M}$ for $M = \frac{1}{2}$ (9)
1	H. Kramer	I	0.01067	2.70	0.706	0.358	3.352	2.400
2		II	0.00800	2.70	0.558	0.461	2.055	1.898
3		III	0.01005	2.70	0.800	0.414	3.285	2.725
4	F. Schaffernak	*A*	0.01538	2.65	1.536	0.564	4.495	5.060
5	A. Schoklitsch	*B*	0.11160	2.60	6.520	1.000	10.430	20.860
6		*C*	0.05400	2.60	4.000	1.000	6.490	12.980
7		*D*	0.02318	2.60	2.256	1.000	3.618	7.236
8		*E*	0.00903	2.60	1.240	1.000	1.985	3.970
9		*F*	0.00574	2.60	0.916	1.000	1.467	2.934
10	H. Krey	*G*	0.00492	2.68	0.376	0.825	0.766	1.265
11		*H*	0.00574	2.61	0.526	0.869	0.976	1.695
12		*J*	0.00656	2.57	0.800	0.778	1.613	2.510
13	Prussian Experiment Institute	*K*	0.01046	2.65	1.154	0.265	7.200	3.810
14		*L*	0.00861	2.65	0.846	0.278	5.020	2.785
15		*M*	0.01190	2.65	0.836	0.366	3.780	2.765
16		*N*	0.01148	2.65	0.744	0.399	3.075	2.453
17		*O*	0.00492	2.65	0.244	0.435	0.926	0.805
18		*P*	0.01148	2.65	0.806	0.357	3.720	2.660
19		*Q*	0.01025	2.65	0.686	0.424	2.670	2.265
20	H. Engels	*R*	0.02050	2.65	1.484	0.229	10.650	4.880
21	G. K. Gilbert	*S*	0.00800	2.69	0.576	0.896	1.085	1.945
22		*T*	0.01251	2.69	1.906	0.714	4.507	6.340
23		*U*	0.06050	2.69	3.710	0.867	7.230	12.530
24		*V*	0.10050	2.69	5.296	0.820	10.900	17.860
25	E. Indri	*a*	0.00369	2.63	0.324	0.690	0.765	1.055
26		*b*	0.00636	2.70	0.474	0.481	1.673	1.610
27		*c*	0.04920	2.70	2.500	0.818	5.295	8.660
28		*d*	0.08370	2.70	4.000	0.777	8.750	13.600
29	Waterways Experiment Station	*e*	0.00970	2.65	0.586	0.280	3.460	1.936
30		*f*	0.00880	2.65	0.541	0.439	2.038	1.790
31		*g*	0.00880	2.65	0.525	0.539	1.607	1.733
32		*h*	0.00840	2.65	0.506	0.406	2.055	1.670
33		*i*	0.00800	2.65	0.483	0.438	1.820	1.595
34		*j*	0.00600	2.65	0.347	0.643	0.891	1.145
35		*k*	0.00660	2.65	0.310	0.525	0.975	1.023
36		*l*	0.00470	2.65	0.205	0.560	0.604	0.677
37		*m*	0.05800	2.65	4.077	0.566	11.870	13.470
38	Y. L. Chang	6205	0.18500	2.547	8.090	1.000	18.450	36.900
39		6204	0.13900	2.558	6.540	1.000	15.520	31.040
40		6203	0.11100	2.567	5.290	1.000	12.770	25.540
41		6202	0.09100	2.590	3.770	1.000	9.300	18.600
42		6201	0.04850	2.633	2.540	1.000	5.790	11.580
43		6200	0.03490	2.634	1.760	1.000	3.870	7.740
44		6199	0.01660	2.640	0.886	1.000	1.880	3.760
45		6198	0.00757	2.643	0.396	1.000	0.753	1.506
46		6197	0.00635	2.645	0.226	1.000	0.415	0.930
47		Pumice	0.00510	2.053	0.289	1.000	0.350	0.700
48		Emery	0.00767	3.890	0.210	1.000	0.668	1.336
49		Sand *α*	0.00395	2.520	0.134	1.000	0.226	0.452

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* Data of Table 5 were taken from "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," U. S. Waterways Experiment Station, *Paper 17*.

† *o* assumed unity for others than the writer's results.

and for $\left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right] < 2.0$:

$$T_0 = 0.00635 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right]^{1/2} \quad \text{(21}b\text{)}$$

At the intersection, $\frac{\sigma - \rho}{\rho} \frac{D}{M} = 2.0$; hence, for $M = 0.5$, $D \sim 0.5$ mm. If the value of $M = \frac{1}{2}$ be substituted into the equations they become identical:

$$T_0 = 0.00900 \left[\frac{\sigma - \rho}{\rho} D\right]^{\beta} \quad \text{(21}c\text{)}$$

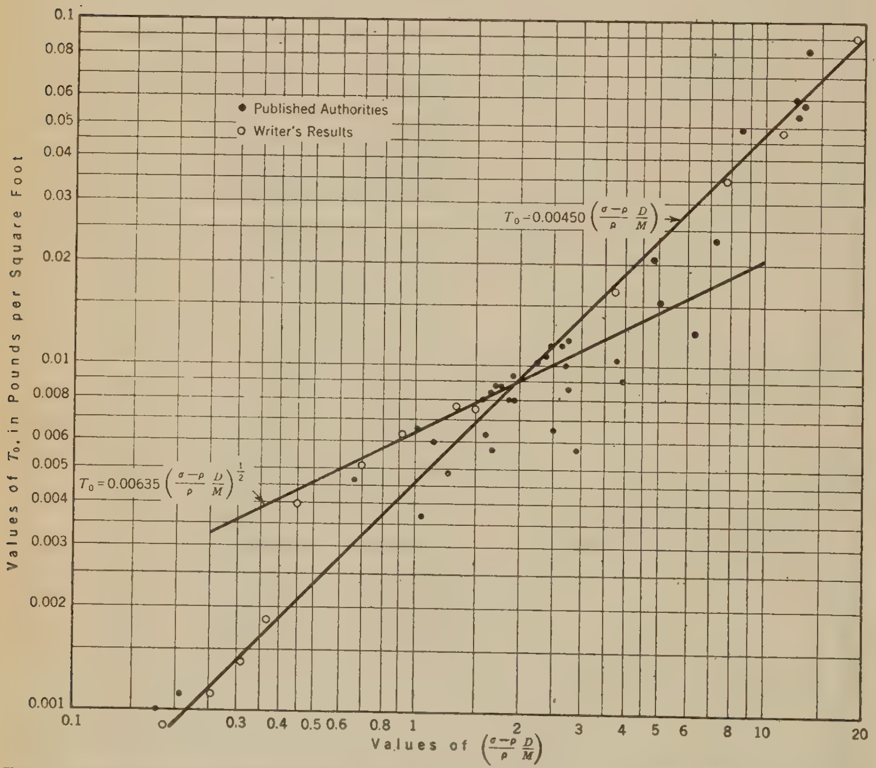

FIG. 7.—CRITICAL TRACTIVE FORCE (DATA FROM VARIOUS SOURCES; *o* ASSUMED UNITY; PLOTTING ADJUSTED TO $M = \frac{1}{2}$)

The numerical coefficient in Equation (21*c*) is just about one-half that in Equation (20), showing that the tractive force needed to move débris on a sloping bed parallel to the water surface is only one-half that needed on a horizontal bed. It also indicates that the uniform modulus, M, has practically nothing to do with the critical tractive force, and could be omitted.

The equation suggested by Kramer for critical tractive force is,

$$T_0 = 0.00339 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right] \quad \text{(22)}$$

Equation (22) has been modified by the United States Waterways Experiment Station[9] to

$$T_0 = 0.00595 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M} \right]^{0.5} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

It is clear that both formulas are derived from the same data; then, if both are right, there must be a criterion separating the available data into two sets satisfied, respectively, by Equations (22) and (23). In this connection it is interesting to note the following quotation from the Station's report:[10]

> "There must be some limiting size of particle at which the larger size no longer moves first, and above which the finer materials are the first to be placed in motion. While no data were recorded in these experiments which would tend to locate this limiting size, it is believed that its value is in the vicinity of 0.6 mm."

In a paper by Morrough P. O'Brien,[11] Assoc. M. Am. Soc. C. E., the existence of such a criterion has also been outlined.

TURBULENCE AS AN INTERPRETATION

Dimensionally, the idea that the critical tractive force is proportional to $\rho_e\, D\, o^{1/3}$, taking o as a dimensionless ratio, is correct. In case it is proportional to $[\rho_e\, D\, o^{1/3}]^{1/2}$, the coefficient by which the function is multiplied will no longer be dimensionless. The turbulence of the flow is suggested as an explanation of this variation. The action of the tractive force on the bed material is equivalent to the motion of a fluid resisted by a tangential stress at the wall in a channel or in a pipe. It is reasonable to presume that this unit stress, T_0, will be a function of the depth, d (or hydraulic radius, R), density, ρ, viscosity, μ, and mean velocity, V; that is, $\tau_0 = C_1\, d^x\, \rho^y\, \mu^z\, V^k$. By dimensional analysis, the form of this function[12] is found to be

$$\tau_0 = C_1\, d^{k-2}\, \rho^{k-1}\, \mu^{2-k}\, V^k \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

For laminar flow, $k = 1$, and is constant because the flow is independent of the roughness of the wall. For turbulent flow, the value of k varies from 1.75 for extremely smooth channels to a maximum of 2 for channels which are rough. For $k = 1$, $\tau_0 = C_1 \frac{\mu}{d} V = K_1\, V$, and for $k = 2$, $\tau_0 = C_1\, \rho\, V^2 = K_2\, V^2$. These relations are sketched logarithmically in Fig. 8, and can be expressed in the form, $\tau_0 = K\, V^\alpha$, in which $\alpha = 1$ or 2.

Tractive force in turbulent flow varies with the square of the mean velocity, whereas tractive force in laminar flow varies directly with the mean velocity; consequently, if it is reasonable to presume that the tractive force in turbulent flow is a certain function of the sand, then the tractive force in laminar flow will be the square root of the same function, because in such flow the function of sand is equivalent to the reaction of the shearing stress due to velocity. Therefore, in the general equation,

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[9] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," U. S. Waterways Experiment Station, *Paper 17*, 1935, pp. 29–30.

[10] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," U. S. Waterways Experiment Station, *Paper 17*, 1935.

[11] "Notes on the Transportation of Silt by Streams," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, 1936.

[12] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, Assoc. M. Am Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1936, p. 21.

$$T_0 = C\left[\frac{\sigma - \rho}{\rho} D\,o^{1/3}\right]^{\beta} \qquad (25)$$

in which C is a constant, the experimentally determined values of $\frac{1}{2}$ and 1 for β appear to have a rational justification. Equation (25) is also plotted in Fig. 8. The discontinuity of the curve is here assumed to occur at the same ordinate as that of the curve showing the tangential stress relationship. With regard to the latter curve, there is a zone of uncertainty in the transition region between laminar and turbulent flow. If the relationships of tractive force with velocity and sand properties are justified, there must be also a zone of uncertainty in the curve of tractive force. The nature and the extent of this zone are not definite.

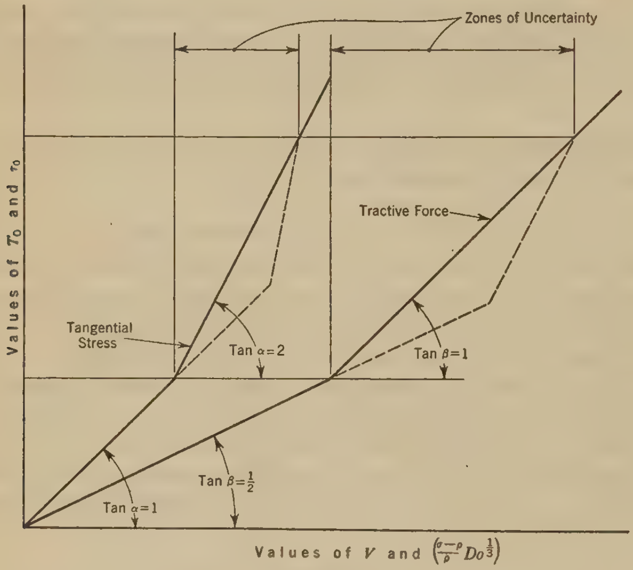
Zones of Uncertainty
Tractive Force
Tangential Stress
Tan α=2
Tan β=1
Tan β=½
Tan α=1
Values of T_0 and τ_0
Values of V and $\left(\frac{\sigma-\rho}{\rho} D o^{\frac{1}{3}}\right)$

FIG. 8

At the lower critical point, the shearing stresses in laminar and turbulent flow are identical,

$$\frac{\tau_s}{\tau_t} = \frac{\frac{2\mu}{R} V}{\frac{f}{4}\rho\frac{V^2}{2}} = \frac{16}{f}\,\frac{\mu}{V R \rho} = \frac{16}{f}\,\frac{1}{\mathsf{R}} = 1$$

Assuming 0.01 to be the average value of f, the roughness factor, in the experiment, R = 1 600; while Allen[6] gives 1 400 as the lower criterion. The upper criterion is uncertain; according to the writer's experiment it may be as much as ten times the value of the lower criterion. (As shown in Fig. 8, the zone of uncertainty on the curve for sand properties is considerably larger than that on the curve for velocity.) The same problem may be explained from another point of view by the theories of frictional drag given in hydro-mechanics.[13]

SUMMARY: PART I

(1) In the convergent channel the water surface slope can be calculated by Equation (3).

(2) When the glass-sided flume used in the writer's experiments has bed material leveled on the bottom, Manning's n_s for the sand can be computed by,

$$n_s = \left[\frac{(B + 2\,d)\; n^{3/2} - 0.00208\,d}{B} \right]^{2/3}$$

assuming that Manning's n_g for the sides of the flume is 0.0103.

(3) The relationship between Manning's n and the mean diameter, D, in millimeters, of the grains is well expressed by Equation (8).

(4) For channels in which the ratio of width to depth is less than 30, Equation (12) may be used to find the tractive force on the bottom of either rectangular or trapezoidal sections.

(5) Water flowing at relatively low velocity in a deep channel with horizontal bed exerts a tractive force on the bottom equal to approximately one-half that exerted in uniform flow. In other words, the resistance to motion of particles on a horizontal bed is nearly double their resistance on a sloping bed having the same inclination as the water surface.

(6) A complete formula for the critical tractive force in terms of the mechanical properties of the sand is Equation (25).

(7) There is a critical size of sand at which the power index theoretically changes suddenly. For quartz sand the critical mean diameter of grain is about 0.5 mm. Turbulence of flow best explains this criterion.

PART II.—TRANSPORTATION BY TRACTION

PROCEDURE

The experiments on transportation by traction were performed in the flume ½ ft wide. Damp material was placed on the bed and leveled. Water was then admitted gradually, to the desired depth; and the flow was adjusted to a condition just above that causing general motion. The period for measuring sand discharge—that is, the time between inserting and withdrawing the collecting bucket—was chosen to be nearly that during which a constant rate of sand movement prevails. For small sand discharges a long period is desirable for the sake of accuracy. For greater discharges the period should be as short as possible, since the surface of the bed changes its form from instant to instant. The dry weight of the collected sand, divided by the measuring period, gives the rate of transportation, G.

The sand was fed automatically, but not accurately. A box, 18 in. by 6 in. by 6 in., filled with the sample material was supported on a rotating shaft connected directly to a ¼-hp motor the speed of which could be roughly controlled by a resistance composed of lamps. In the middle of the box there was a baffle-plate that could be moved vertically by screws. The rate of discharging the sand, therefore, could be approximately adjusted by the opening of the baffle-plate, through which the sand was sliding. The criterion to adjust the feeding, however, was based only upon the visual observation of the movement of the sand in the channel.

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[13] "Applied Hydro- and Aeromechanics," by L. Prandtl and O. G. Tietjens, N. Y., 1934, p. 86.

The depth measured was in all cases the depth to the original smooth bed surface. Slope measurements were made as in Part I. With the higher flows, however, more trouble was experienced in attaining a stable reading. As soon as riffles developed, the differences of readings on the differential gage increased suddenly, and thereafter continued to increase throughout the run. This means that the roughness of the bed was increasing constantly from the time the riffles were formed, other factors being considered almost unaltered. However, the exact rate at which the slope varied is unknown. The mean value of several slope observations, taken desirably at equal time intervals, was assumed to represent the average condition during the run. The length of time of collection varied from 15 min to 1 hr, depending mainly on the intensity of movement. During each run, at least two measurements of water discharge were taken, most of them directly from measuring tank. Types of flow could be easily distinguished with light on the opposite side of the glass flume.

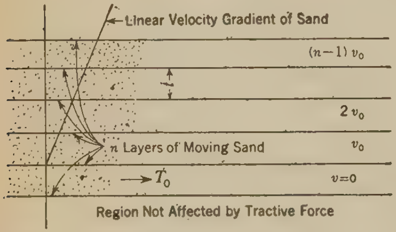
Observations on sand movement and riffle development were carefully noted at different stages. Riffles usually appeared first on the bed at the two ends of the channel, indicating disturbances in those regions, and at isolated points where there were irregularities of flow. Their formation could not be controlled.

DU BOYS' FORMULA

The formula originally derived by du Boys[14] in 1879 is of the form:

$$G = \psi\, T\, (T - T_0) \quad \text{(26)}$$

in which T is the unit tractive force on the bed; T_0 is the critical tractive force required to initiate general movement; and ψ is a function of the size, etc., of the sand grains. Equation (26) can be written in many other forms by means of Manning's formula.[15] The derivation of Equation (26) was based upon the assumption that sand is moved in layers subject to uniform tractive force, and that the velocity gradient in the moving sand layers is linear. The coefficient of friction of sand in water is also considered constant. According to Fig. 9, the sand does not move at the layer subjected to the critical tractive force. The total tractive force consumed per unit area of sand is, $w\,S\,d = f\,(\gamma - w)\,n\,t$, in which f is the coefficient of friction of the sand in water; n, the number of layers of sand, which depends on the tractive



Linear Velocity Gradient of Sand
$(n-1)\,v_0$
t
$2\,v_0$
n Layers of Moving Sand
v_0
T_0
$v=0$
Region Not Affected by Tractive Force

FIG. 9.—CONDITIONS ASSUMED IN DU BOYS' EQUATION

force and the sand properties; and t, the thickness of each layer. The rate of sand transportation is, therefore,

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[14] "Le Rhone et les rivieres a lit affouillable," par P. du Boys, *Annales des Ponts et Chaussées*, Vol. 2, 1879.

[15] "Hydraulic and Sedimentary Characteristics of Rivers," by L. G. Straub, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1932.

$$G = t\,w\,[0 + v_0 + 2\,v_0 + 3\,v_0 + 4\,v_0 + \cdots (n-1)\,v_0] = n\,(n-1)\,\frac{w\,t\,v_0}{2}$$

No layer will be moved on which the force is less than the critical tractive force required to overcome the frictional resistance of the bed; therefore, $T_0 = f\,(\gamma - w)\,t$. Hence, $n\,T_0 = w\,S\,d$, and $n = \frac{w\,S\,d}{T_0} = \frac{T}{T_0}$, so that,

$$G = \frac{w\,t\,v_0}{2}\frac{T}{T_0}\left(\frac{T}{T_0} - 1\right) = \frac{w\,t\,v_0}{2\,T_0^2}\,T\,(T - T_0) = \psi\,T\,(T - T_0) \quad \ldots\ldots (27)$$

Using Chezy's equation, $V = C\sqrt{S\,d}$,

$$G = \psi\,w^2\,\frac{V^2}{C^2}\left(\frac{V^2}{C^2} - \frac{T_0}{w}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Equation (28) indicates that transportation of sand will not occur until $V > C\sqrt{\frac{T_0}{w}}$; thereafter, the rate of transportation will be approximately proportional to the fourth power of the velocity.

The foregoing derivation applies only to the case in which materials are being dragged on a smooth bed; load due to saltation, which is closely associated with the formation of riffles, is not included. Although the derivation is not very sound, the resulting expression checks well with most experimental formulas, and is satisfactory for estimating sand movement in natural streams.

Sand Movement in Terms of Advance of Riffles

The foregoing discussion applies only to a smooth bed. For a riffled bed, a better interpretation is not yet practicable.

In this paper, the velocity of sand movement is considered to be the same as that of the progression of the riffles. Of course, this assumption is only applicable in the velocity range in which the riffles are progressing in a downstream direction. The plotted results (Fig. 10) of the experiment, although the points are considerably scattered, could still indicate the following relation between the advancement of riffles and the velocity of the flow:

$$v_r = [0.000125 \text{ to } 0.001085]\,V^5 \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

both velocities being in feet per second. (The inclination of the lines is not absolute, because of the scattering of the plotted points, but they are approximately consistent to one another.) It is interesting to compare these results with those of Owens and Deacon.[16] The former gives $G \propto V^6$, whereas the latter indicates $G \propto V^4$. According to Gilbert's[17] observations on natural streams the power of V is much less than 6. His formula, based on flume tests, is, however, of the form $G \propto V^4$. Such deviation could probably be attributed to the different size of flume used.

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[16] "The Settlement and Transport of Sand in Water," by J. S. Owens, *Engineering*, 1912.

[17] "The Transportation of Débris by Running Water," by G. K. Gilbert, *Professional Paper 86*, U. S. Geological Survey, 1914.

It is observed that saltation is strongest when the riffles are the largest. Very roughly it can be set forth, that,

$$G \propto [(\text{height of riffle})^{\alpha} \times (\text{velocity of riffle})^{\beta}]$$

If it may be assumed that the size of a riffle, ξ, is equivalent to that of a grain,

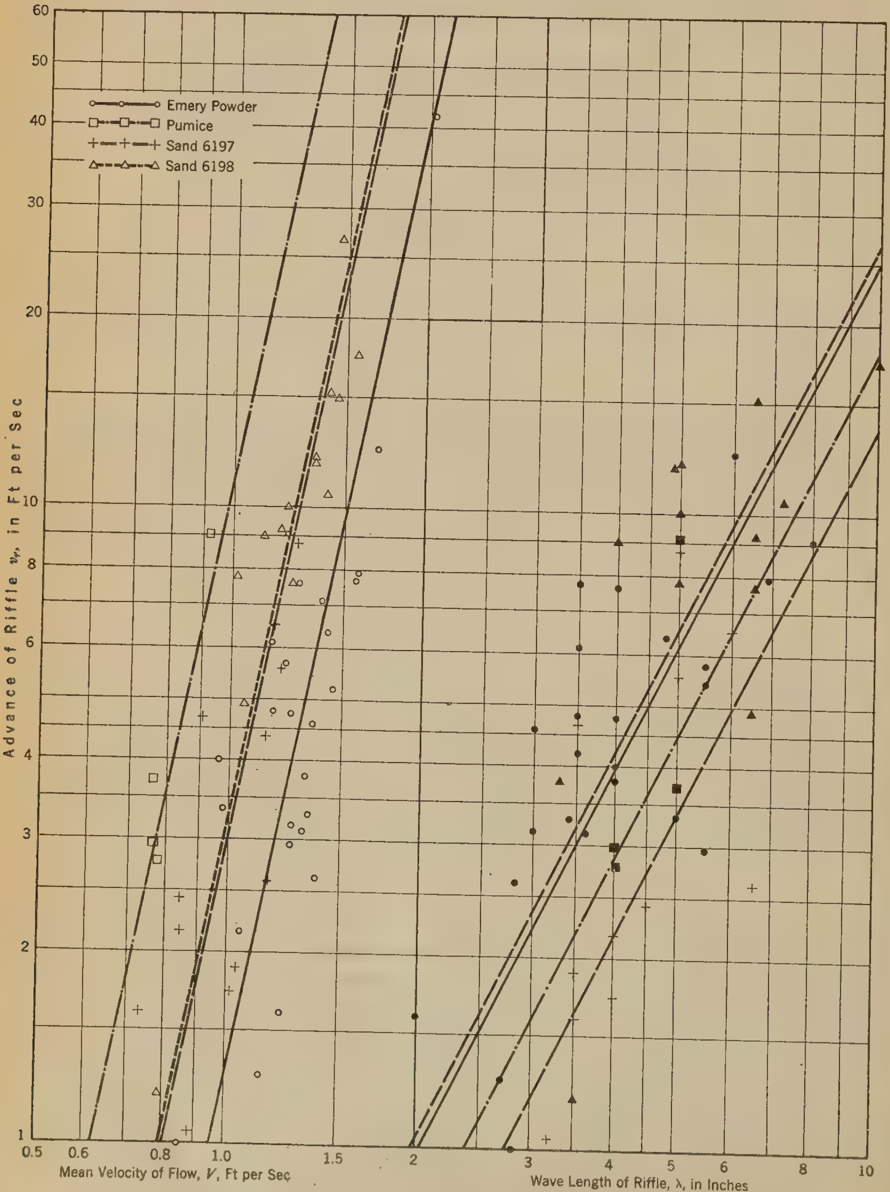

FIG. 10.—ADVANCE OF RIFFLES

and, consequently, that Manning's n is a direct measure of the size of the riffle, then the quantity of sand moved must be also a function of the size of the riffles. The United States Waterways Experiment Station found[10] also that the size of the riffles increased with the percentage of fine material in the mixture; but, obviously, the rate of transportation of a finer mixture is greater unless cohesion comes appreciably into effect. Observations[3] on the Rhine indicate that there is a general increase in the value of Manning's n with an increase in bed-load movement. It has been shown that Manning's n is dimensionally a function of the size of the particles to the one-sixth power. Actual plottings (as in Fig. 11) of the relation of G and n show that G is proportional to n^6, indicating that the exponent, α, should be unity. The deviation of points at higher values of n is due to the restraining effect of the narrow flume; that is, the narrowness limits of the development of larger riffles. When β is also unity, as assumed, the equation becomes:

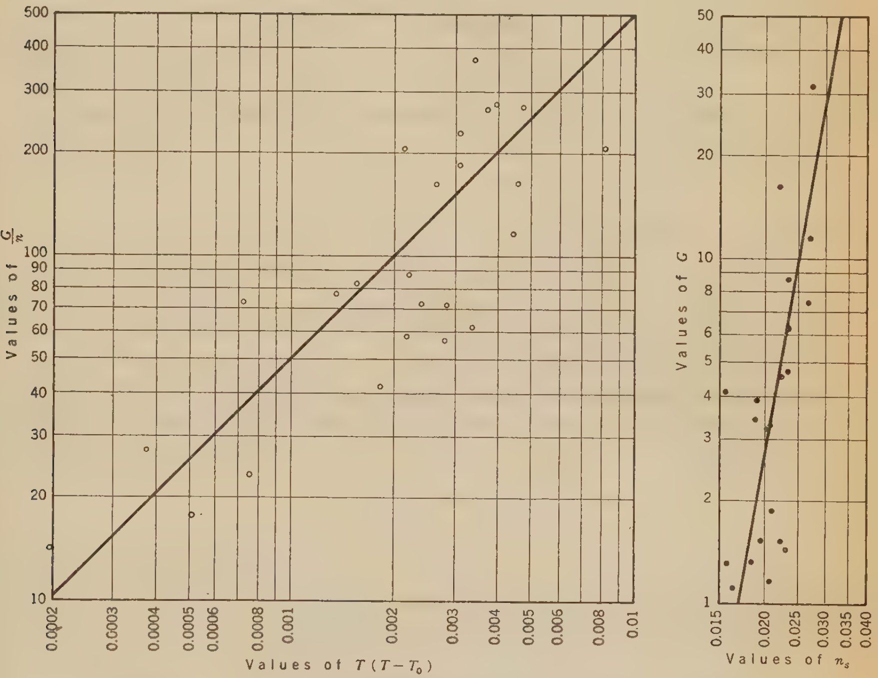

FIG. 11.—DATA ON TRANSPORTATION BY TRACTION: EMERY POWDER

$$G \propto \xi V^5 \qquad (30)$$

Using Chezy's formula for an infinite width of channel,

$$G \propto n^6 \frac{(S\, d)^{5/2}}{n^5} \propto n\, T^{2.5} \qquad (31)$$

The appearance of n in Equation (31) is justified dimensionally by noting that $T \propto n^2$.

EXPERIMENTAL FORMULA

A formula of the du Boys' type appears to fit best the data obtained in the present experiment, although the plotted points are so scattered that no equation will join them closely. When $\frac{G}{n}$ is plotted against $T(T - T_0)$ logarithmically, as in Fig. 11, lines with an inclination of 45° evidently satisfy the general requirements. The equations of these lines for three of the bed materials tested, read:

For emery,

$$G = 50\,000\, n\, T\, (T - T_0) \quad \text{(32a)}$$

For 6197,

$$G = 100\,000\, n\, T\, (T - T_0) \quad \text{(32b)}$$

For pumice,

$$G = 210\,000\, n\, T\, (T - T_0) \quad \text{(32c)}$$

Since the numerical coefficient in Equations (32) must be the function, ψ, it can be written that

$$G = \frac{K\, n}{T_0^2}\, T\, (T - T_0) \quad \text{(33)}$$

in which T_0 is defined by Equation (20) and K is a constant.

PUBLISHED AUTHORITIES

The following representative formulas given by other authorities are briefly reviewed herein as comparisons:

I.—A. Schoklitsch.—Using additional experimental data of his own, but mainly the classic flume measurements by Gilbert, Schoklitsch developed the following formula[18] for uniform quartz grains of approximately the same density:

$$G = \frac{436.5}{\sqrt{D}}\, S^{1.5}\, B\, (q - q_0) \quad \text{(34)}$$

in which G is the bed-load movement in pounds per second, D is the grain diameter, in millimeters, and q_0 is the critical discharge to move sand per unit width of channel, in cubic feet per second. The value of the latter is

$$q_0 = 0.00021\, \frac{D}{S^{4/3}}$$

II.—M. Fabre.—The results[19] obtained by the laboratory at Zurich, Switzerland, on gravels ranging between charcoal, the density of which approximates that of water, and baryta (protoxide of barium), of specific gravity greater than 4.0, lead to the following general relationship:

$$G = 2.16\, \frac{S^{1.5}\, Q}{\rho_e^{1/3}} - 20.70\, D\, \rho_e^{7/9} \quad \text{(35)}$$

The equivalent critical discharge is $Q_0 = k\, \frac{\rho_e^{7/9}\, D}{S^{3/2}}$

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[18] "The Schoklitsch Bed Load Formula," by S. Shulits, Assoc. N. Am. Soc. C. E., *Engineering*, June, 1936.

[19] "Mesure des debits solides des cours d'eau," par M. Fabre, *Annales des Ponts et Chaussées*, Vol. 8, 1934.

III.—Meyer-Peter.—In studying the results obtained with gravels of the same density but of different diameters, and with different slopes, the Swiss engineers[20] have arrived at a relationship of the form:

$$G = (a\, S\, Q^{2/3} - b\, D)^{3/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

and the equivalent critical discharge is

$$Q_0 = k \left(\frac{D}{S} \right)^{3/2}$$

in which a, b, and k are constants.

IV.—G. K. Gilbert.—The final expression by Gilbert[17] for bed-load movement, in which the slope, discharge, and fineness (reciprocal of D) are considered as of primary importance, is

$$G = k\, F\, (S - S_0)^{0.93 \text{ to } 2.37}\, (Q - Q_0)^{0.81 \text{ to } 1.24} \left(\frac{1}{D} - \frac{1}{D_0} \right)^{0.50 \text{ to } 0.62} \ldots (37)$$

or, approximately,

$$G = \frac{a}{D^{0.58}} S^{1.59}\, Q^{1.02} - b \ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which F is a shape factor, which is constant for a given ratio of depth to width of channel, and k, a, and b are constants. Those notations with the subscript, 0, are values for competent conditions to cause transportation.

V.—United States Waterways Experiment Station.—Data on the rate of sand movement obtained from experiments at the U. S. Waterways Experiment Station[10] conformed to the following relationship (after the formation of riffles):

$$G = \frac{k}{n} (T - T_0)^{1.5 \text{ to } 1.8} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

The contradiction of Equation (39) with that of the writer seems that of the reciprocal arrangement of Manning's n. In *Paper No. 17*, of the Experiment Station,[10] it has been concluded that "in general, the finer the sand mixture, the larger was the riffle size, and, consequently, the higher the value of n". Now, from all formulas given herein, G is always inversely proportional to a certain function of D; therefore, the sand movement is greater for finer material. If the finer material is more favorable for higher riffle development, then the sand movement must vary directly instead of inversely with Manning's n if a first power is used.

Exner's[21] expression for dune (riffle) velocity is

$$v_r = \frac{k}{(d - \xi)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which ξ is the height of the surface of the dune, in feet. This, again, indicates that the larger the dune, the faster it will travel.

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[20] "Neure Versuchsresultate über den Geschiebetrieb," von E. Meyer-Peter, H. Favre, und A. Einstein, *Schweizeriiche Bauzeitung*, No. 103, 1934, *Professional Paper 86*.

[21] "Uber die Wechselwirkung zwischen Wasser und Geschiebe in Flussen," von F. M. Exner, Wien, Aka. Sci. Ber., Vol. 134, 1935.

VI.—C. H. MacDougall.—The form of equation that has been found to fit the data accumulated at the Massachusetts Institute of Technology[22] is

$$G = a\,S^b\,(Q - Q_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

The constants, a and b, are interrelated, and also dependent upon D and M. They range from 100 to 1 000 and from 1.25 to 2.00, respectively.

VII.—H. Nakayama.—With certain restricted conditions, possible variations of bottom slopes, discharges, modes of flow, etc., were experimented with, and the results[23] obtained yielded the relation:

$$G = (\alpha + \beta\,S)\,Q\,(V^2 - V_0^2)$$

in which α is found to be small compared with $\beta\,S$. Further, assuming that V_0^2 is small compared with V^2, and may be neglected,

$$G = \beta\,S\,B\,d\left[\frac{1.486}{n}\right]^3 (S\,d)^{3/2} = \frac{K}{n^3}\,T^{2.5}$$

Manning's n may be taken as proportional to $D^{1/6}$ or $\xi^{1/6}$, so that, $G = \frac{K}{\sqrt{D}}\,T\,(T - T_0)$, or,

$$G = \frac{K}{\sqrt{\xi}}\,T\,(T - T_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

VIII.—S. D. Chyn.—From experiments made at the Massachusetts Institute of Technology on sands bigger than No. 48 mesh ($d > 0.5$ mm), S. D. Chyn[24] has obtained the following formula:

$$G = \psi\,V^p\,(T - T_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

in which ψ is a function of the sand. The exponent, p, being a function of Manning's n, has three distinct values for any one material, corresponding, respectively, to a smooth bed, to a bed being dragged, and to a riffled bed. That is, before riffles have developed, p is a function of the diameter of the sand grains; after their formation, it is a function of the size of the riffles.

IX.—F. Schaffernak.—Materials used in Schaffernak's[25] tests were obtained from the Danube River; experiments were performed both on samples of uniform size and on definite mixtures. The tests indicated that after the critical bottom velocity is exceeded, the capacity of transporting sand is directly proportional to the square of the bottom velocity. In connection with the possible application to conditions in natural streams, it is concluded that "for granular materials smaller than 5 mm, in a natural stream with surface slopes remaining approximately constant, du Boys' formula is applicable for determining the capacity of the stream."

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[22] "Bed Sediment Transportation in Open Channels," by C. H. MacDougall, *Transactions*, Geophysical Union, 1934.

[23] "Some Model Investigations of the Motion of the Sand Along a Self-Formed Channel," by H. Nakayama, *Journal*, Faculty of Engineering, Tokyo, Vol. 13–6, 1923.

[24] In unpublished communication with the writer.

[25] "Neue Grundlagen für die Berechnung der Geschiebeführung in Flusslaufen," von F. Schaffernak, 1922.

ESTIMATE OF BED-LOAD MOVEMENT IN A RIVER SECTION

The general formula (Equation (20)) derived previously can be applied to a natural stream for a rough estimate[26] of the rate of its transportation. Further, it has been shown that the critical tractive force is constant for a certain material, and that it may be expressed as

$$T_0 = 0.0175 \left[\frac{\sigma - \rho}{\rho} D\, o^{1/3} \right]^{\beta} = w\, S\, d_0$$

If the slope of the stream and the value of T_0 for its bed material are known, d_0 can be computed. In Fig. 12 (a), it is evident that motion and hence transportation of bed material will take place only at and below the depth, d_0—that is, on the surface, $C\ E\ D$—and that there is no motion on the sides, $O\ C$ and $D\ F$. Actual observations by Kurzmann[27] support this theoretical consideration. The area, $A\ C\ E\ D\ B$, is thus called the effective area. Using the general formula and taking O as origin,

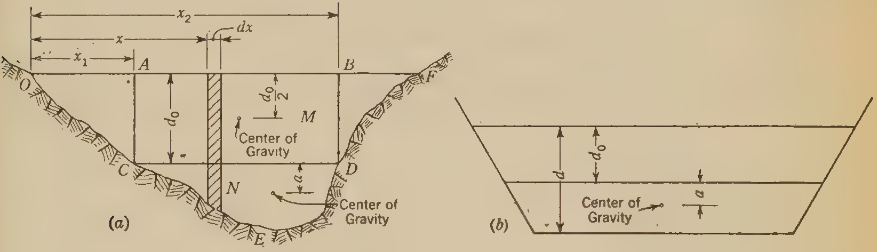

FIG. 12

$$G = \psi \int_{x_1}^{x_2} n\, T\, (T - T_0)\, dx = \psi \int_{x_1}^{x_2} n\, w^2\, S^2\, d(d - d_0)\, dx$$
$$= k \left[\int_{x_1}^{x_2} d^2\, dx - d_0 \int_{x_1}^{x_2} d\, dx \right]$$

The first integral is an expression of the moment of area about the surface. Dividing the total effective area into a rectangle, $A\ B\ C\ D$, called M, and an area, $C\ E\ D$, called N, the center of gravity of which is calculated and located at a distance $(d_0 + a)$ below the water surface,

$$2 \int_{x_1}^{x_2} (d\, dx)\, \frac{d}{2} = 2 \left[\frac{M\, d_0}{2} + N\, (d_0 + a) \right] = M\, d_0 + 2\, N\, (d_0 + a)$$

The second integral is simply an area $(M + N)$, and,

$$d_0 \int_{x_1}^{x_2} d\, dx = d_0\, (M + N)$$

Combining,

$$G = k\, [M\, d_0 + 2\, N\, (d_0 + a) - (M + N)\, d_0] = k\, N\, (d_0 + 2\, a) \ldots (44)$$

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[26] "Sand-Carrying Capacity of Streams," by Y. T. Sah, *Proceedings*, Eng. Soc., Tsing Hua Univ., Peiping, China, 1932.

[27] See "Wasserbauliche Strömungslehre," by P. Némenyi, 1933, p. 139.

from which it is seen that the quantity of sand moved is proportional to the area below the critical depth, d_0, times approximately the mean depth of the total effective area.

The application to a trapezoidal section (Fig. 12 (*b*)) is still simpler. Let A be the total effective area of the section; A_0, the upper trapezoid; and N, the lower trapezoid below the depth, d_0. Since a is approximately equal to $\frac{(d - d_0)}{2}$, then

$$G = \psi\, n\, w^2 S^2 N\, (d_0 + 2\, a) = \psi\, n\, w^2 S^2 d(A - A_0) = k\, d^{0.5} S^{1.5} (Q - Q_0) \quad \text{. . (45)}$$

which approximates the general form of Equations (32).

Sorting Phenomenon

In the present experiment on transportation, at the stage of initial dragging or rolling, it was observed that the coarser material was being dragged or rolled more rapidly than the finer. L. G. Straub,[28] Assoc. M. Am. Soc. C. E., noticed the same phenomenon in his experiments on flume traction. He observed that a surprisingly uniform gradation took place from one end of the flume to the other, despite the fact that the original material was of quite uniform size particles.

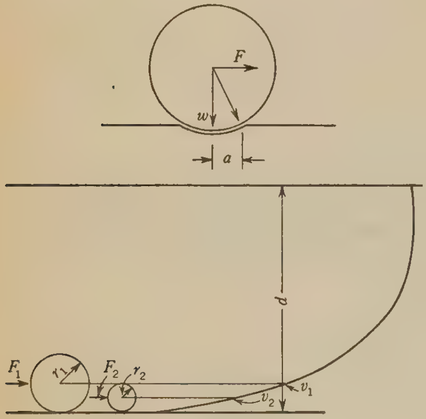

FIG. 13

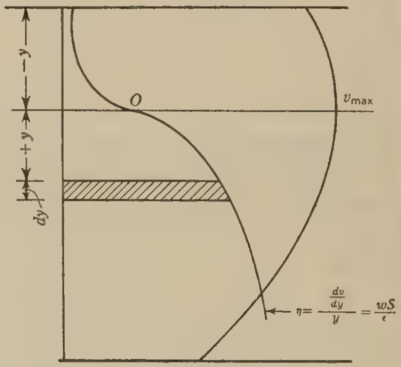

FIG. 14

The cause of this sorting is illustrated in Fig. 13, which shows two particles of different sizes on the bottom of the flume. The velocity distribution curve is horizontally exaggerated. Consider, first, the particles sliding on the surface. In turbulent flow the pressure on the particle is proportional to the velocity squared. By equating the pressure of the bottom velocity head on a spherical particle to the frictional resistance of the bed at impending motion,

$$\pi\, r^2\, w \frac{V_b^2}{2\, g} = f \frac{4}{3} \pi\, r^3\, (\gamma - w)$$

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[28] "Some Observations of Sorting of River Sediments," by L. G. Straub, *Transactions*, Am. Geophysical Union, 1933.

and

$$V_b = \left[\frac{4}{3} f \, 2 \, g \frac{\sigma - \rho}{\rho} r\right]^{1/2} \propto \sqrt{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

If the velocity distribution curve is of parabolic form, $v = k \, r \, (2 \, d - r)$, and if, in addition, r is small compared with $2 \, d$, $v \propto r$. If $r_1 = 2 \, r_2$, the larger sphere will be moved by a velocity only 1.414 times as great as that required to move the smaller one. Actually, the velocity, v_1, at a distance, r_1, from the bed, is likely to be more nearly $2 \, v_2$, for the velocity increases almost linearly near the bottom. Hence, the coarser particle is more easily and quickly moved than the finer one.

Next, consider particles rolling along. Equating the moment of the frictional resistance of the bed to that of the water pressure on a particle

$$\pi \, r^3 \, w \frac{V_b^2}{2g} = \frac{4}{3} \pi \, r^3 \, (\gamma - w) \, a$$

in which a, as shown in Fig. 13, is called the coefficient of rolling resistance and may be assumed, in the case of a soft bed, to be proportional to the radius of the particle, that is, $a = F \, r$. In this case, V_b is again expressed by Equation (46). Therefore, the sorting phenomenon is independent of the mode of movement; it exists either in sliding or in rolling.

In experiments on transportation of a graded sand, it is usually found that a large percentage of the coarser constituents is sorted out and transported. That this should occur can be demonstrated from the general equation,

$$G = \frac{k}{(D)^{2\beta}} T \, (T - T_0)$$

in which β varies from $\frac{1}{2}$ to 1 according to Equation (20). If the change in T_0 due to the change of D is negligible in comparison with T, then, approximately,

$$G = K \left[\frac{a}{(D_a)^{2\beta}} + \frac{b}{(D_b)^{2\beta}} + \frac{c}{(D_c)^{2\beta}} + \cdots\right] = K \, \Sigma \frac{p}{(D)^{2\beta}} \quad \ldots\ldots (47)$$

in which a, b, c, $\cdots$, and p are the percentages by weight of the material transported. To have G a maximum, differentiate with respect to D, taking both G and p as variables, and equate to zero:

$$DG = K \left[-\frac{2 \, \beta \, p}{D^{1+2\beta}} dD + \frac{1}{D^{2\beta}} dp\right] = 0$$

$$\frac{dp}{p} = 2 \, \beta \frac{dD}{D}$$

or

$$\frac{p_1}{p_2} = 2 \, \beta \left(\frac{D_1}{D_2} - 1\right) + 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Equation (48) is the general formula for sorting in sand transportation. Note, for example, that if the coarser particles are twice the diameter of the finer ones (that is, if $D_1 = 2 \, D_2$ and $\beta = \frac{1}{2}$ for fine sands), then $\frac{p_1}{p_2} = 2$. In other words, two-thirds of the material transported will consist of coarse particles and one-third of fine particles.

Summary: Part II

(1) An equation of the du Boys type is found to fit best the present experimental data. The specific form suggested is $G = \frac{K\,n}{T_0^2} T\,(T - T_0)$.

(2) The advance of riffles is proportional to the fifth power of the mean velocity of the flow.

(3) Sorting always takes place in graded sands, irrespective of the mode of motion of the particles, the coarser particles moving in greater quantities than the finer.

PART III.—TRANSPORTATION OF MATERIAL IN SUSPENSION

Method of Experiment

Suspension was visible in the flume as soon as saltation began. When saltation was strong and accompanied by dune formation of the bed material, bands of particles were lifted vigorously and dispersed into suspension. The present observations were usually made under the latter conditions, and with the bed continuously changing, and are, therefore, only approximate. However, the results were consistent enough to illustrate the principles reviewed and developed in this part of the paper.

In addition to the data required in the study of transportation by traction, it was necessary in the present case to measure the velocity distribution of the section and to obtain samples of the suspended load. Pitot tube readings were taken at vertical intervals of about 1 in. in the axial plane of the flume. At the same time samples were taken at prescribed depths by means of a glass tube, $\frac{1}{4}$ in. in diameter, equipped with a suction bulb. These samples were transferred to a flask, in which the suspended particles soon settled to the bottom, and the concentration, in particles per cubic centimeter, was obtained by direct count of the particles and measurement of the volume of water. No attempt was made to investigate the size of suspended particles microscopically.

During these tests, feeding at the up-stream end was stopped to ensure that all the suspended load was supplied from the bed itself.

Lifting Force Near Bottom

The lifting force on a particle resting on the bed may be considered as due mainly to the pressure of a velocity head. The particle will tend to lift when this vertical force just balances its effective weight in water. If V_l is the velocity of flow near the bottom required to produce this lift, the balance can be expressed mathematically by,

$$w \frac{V_l^2}{2\,g} \frac{\pi\,D^2}{4} = \frac{\pi\,D^3}{6} (\gamma - w)$$

whence

$$V_l^2 = \frac{4}{3} g \frac{\sigma - \rho}{\rho} D \ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

Comparing Equation (49) with Equation (46),

$$\frac{V_l}{V_b} = \frac{1}{\sqrt{f}} \qquad (50)$$

in which f is the coefficient of friction between particles in water. The present experiments yield an average ratio of 1.4; this will correspond to $f = 0.51$. The value of f usually varies with ρ_e and D.

Gilbert observed in his studies of flume transportation that vortex rings with transverse axes occur near the bottom of the flume. Thus, the phenomenon of lifting can also be interpreted in a mathematical way[29] by "circulation." Since the radius of the vortex ring is so large compared with the size of the grain, the vortex ring may be considered as an imaginary large hollow cylinder. By the interrelated action of the horizontal velocity and the circulation, two equal forces are created to support the particle; one is due to the change of momentum and the other to the difference of velocity head above and below the particle. The final form of total lift is $\rho\ V\ \Gamma$, in which ρ and V are the density of the liquid and the velocity of flow, and Γ is the circulation, which is shown in aeromechanics, to be proportional to V. This agrees with the previous result that the lifting force is proportional to the square of the velocity.

DISTRIBUTION OF SUSPENDED SEDIMENT

The general equation[30] for distribution of suspended sediment is,

$$c = c_0\, e^{\omega \rho \int_0^y \frac{dy}{\epsilon}} \qquad (51)$$

in which ϵ, defined by $-\dfrac{w\,S\,y}{\dfrac{dv}{dy}}$, is the Austausch coefficient; ω is the settling velocity of the solid particles; c is the sediment concentration at the elevation, y, and c_0 is the sediment concentration at the elevation of maximum velocity. If ϵ is a constant, Equation (51) can be expressed as

$$c = c_0\, e^{\omega \rho y / \epsilon} = c_0\, e^{\alpha y} \qquad (52)$$

y being measured downward from the surface. The equation developed by Hurst[31] from experiments with sand stirred up into suspension by propelling the water in a cylinder, is of a similar form. He considers that in any layer, dy, the weight of the particles will balance the pressures due to the impact of particles from above and below. The result is:

$$c = c_0\, e^{\beta y} \qquad (53)$$

in which $\beta y = \dfrac{3\,g}{\bar{u}^2} \dfrac{\sigma - \rho}{\rho} \displaystyle\int_0^y dy$, and $\bar{u}$ is the absolute mean velocity of the particles. Equations (52) and (53) show agreement in that both α and β

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[29] "Prandtl-Lanchester Theory," *Engineering*, Vol. 67, 1924.

[30] "Distribution of Silt in Open Channels," by J. E. Christiansen, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1935.

[31] "The Suspension of Sand in Water," by H. E. Hurst, *Proceedings*, Royal Soc., London, Vol. 124 1929.

depend on the size and specific gravity of the grains and the turbulence of the flow. In the closing discussion on his paper entitled "Modern Conceptions of the Mechanics of Fluid Turbulence", Hunter Rouse,[32] Assoc. M. Am. Soc. C. E., has constructed a set of theoretical sediment distribution curves based on the assumption that the universal logarithmic velocity curve actually extends to the free surface of a wide stream. The general equation governing the sediment distribution is,

$$\frac{c}{c_a} = \left[\frac{y\,a}{(d - y)\,(d - a)}\right]^z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

in which $z = \dfrac{\omega}{k\,V\sqrt{\dfrac{f}{8}}}$. The equation is applicable down to some arbitrary depth $(d - a)$, beyond which the suspension becomes mixed with the tractive load. In the parameter, z, k is an universal constant having an empirical value 0.4 and f is the roughness coefficient of the channel.

Thus, a general conclusion can be drawn that whether the velocity distribution is parabolic or logarithmic, the vertical distribution of suspended sediment in turbulent streams is exponential.

EXPERIMENTAL VERIFICATION

Tests have been made on sediment distribution as described; the results plotted do not agree with the theoretical Equation (51), because this equation is presumably derived for a considerably wide channel suffering no side restriction. Assuming ϵ to be a constant (as it is for a parabolic velocity distribution), and S, the energy gradient, to be practically constant at any depth, y, then Equation (51) can be written in the form,

$$c = c_0\, e^{kw\,Sy} = c_0\, e^{\phi\, T}$$

which means that the sediment distribution is an exponential function of the tractive force at the depth. This is particularly in agreement with Bottomley's idea[33] that the silt transportation force depends mainly on the frictional resistance of the bed, or, in other words, the tractive force on the bed. The flume used in the present experiments is very narrow and it is evident that the depth, d, cannot take the place of the hydraulic radius, R, and, therefore, the exponent, $\omega\,\rho \displaystyle\int_0^y \frac{dy}{\epsilon}$, must be adjusted accordingly. Using Equation (12), assuming an average value for d of $\frac{1}{2}$ ft, and taking $n_b = 2\,n_l$, the average tractive force becomes

$$\overline{T} = w\,S\,d\,\frac{1}{1 + \dfrac{2\,d}{B}\,\dfrac{n_l}{n_b}} = \frac{1}{2}\,T$$

Even this equation, however, will not apply exactly in the present case, because both the depth and the channel roughness vary continuously; hence, the exact correction is unknown. Several trials of plotting have shown, however, that the exponent, $\frac{\omega\,\rho}{2}\int\frac{dy}{\epsilon}$, fits the experimental data better than any other.

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[32] *Proceedings*, Am. Soc. C. E., January, 1937, p. 116.

[33] "A New Theory of Silt and Scour," by W. T. Bottomley, *Engineering*.

Before proceeding further, it is necessary to compute the settling velocity of sand in water. Rubey's formula,[34] reduced to a temperature of 15° C, is

$$\omega = \frac{7}{D}\left[\sqrt{134.5\,(\sigma - 1)\,D^3 + 1} - 1\right]$$

D being expressed in millimeters and ω in millimeters per second. The values of ω for pumice, Sand No. 6197, and emery are, respectively, 0.0886, 0.0853, and 0.1137 ft per sec.

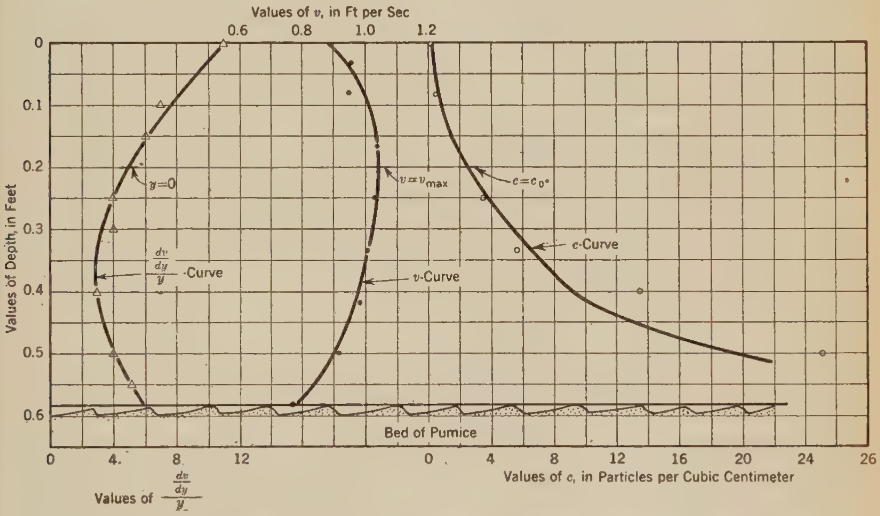

FIG. 15.—DISTRIBUTION OF PUMICE IN SUSPENSION (ENERGY GRADIENT, S = 0.000088

The velocity curve is first plotted as shown in Fig. 14. The point of maximum velocity is taken as the origin, with positive values of y being measured downward therefrom. Values of $\frac{dv}{dy}$ are scaled directly from the curve, divided by the corresponding values of y, and plotted as the η-curve. The latter is indeterminate at the point of maximum velocity, but the upper and lower branches can be joined by a smooth continuous curve. The area of the η-curve to any co-ordinate, y, is equal to, $w\,s\int_0^y\frac{dy}{\epsilon}$, and can be measured by planimeter, the values always being taken as positive. From these graphical data, silt distribution curves can be computed once the specific concentration, c_0, and the value of ω are known. Curves for the various bed materials tested (see Fig. 15 for a typical example) were prepared on that basis. The velocity distribution curve was first plotted and the η-curve computed. Then, the observed concentrations were plotted and joined with a tentative curve, and the abscissa for $y = 0$ was taken as c_0. Theoretical values of c for other depths were then computed and plotted. If this curve did not agree with the original trial curve, the latter was modified accordingly—that is, varied to give a new value for c_0—and the computations were repeated.

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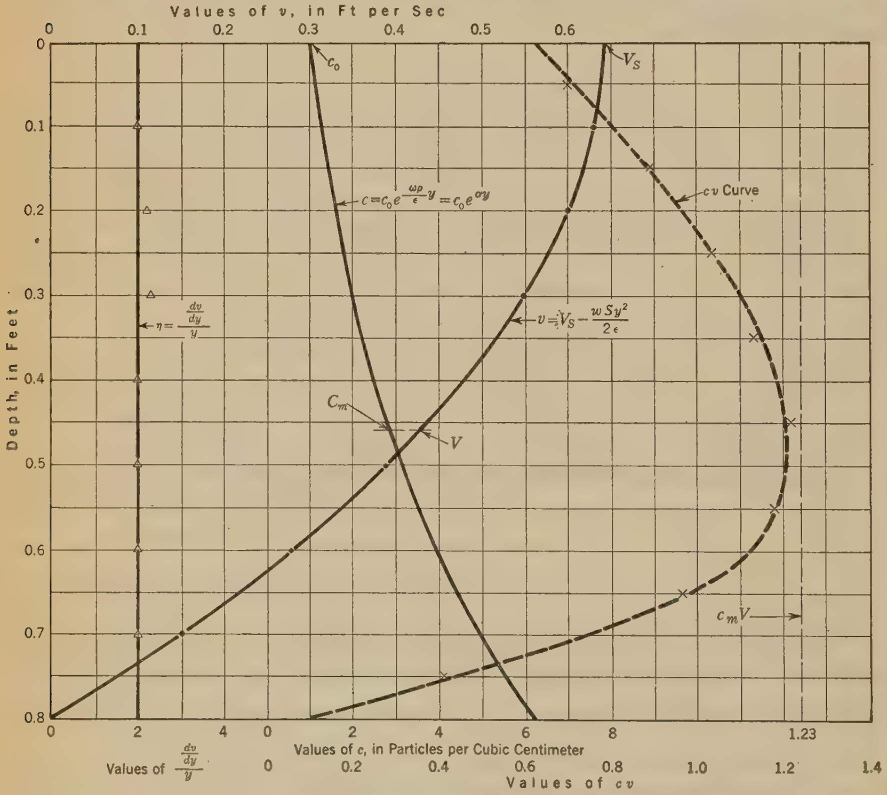
[34] "Settling Velocities of Gravel, Sand, and Silt Particles," by W. W. Rubey, *American Journal o Science*, Vol. 25, 1933.

TOTAL LOAD IN SUSPENSION

The following theoretical study assumes a parabolic velocity distribution, with maximum velocity at the surface (Fig. 16), and the silt concentration at the surface is taken as c_0. Then the mean value of the silt concentration throughout the depth is



. 16.—Silt Distribution and Total Suspended Load in Stream with Parabolic Velocity Distribution

$$c_m = \frac{1}{d}\int_0^d c\,dy = \frac{c_0}{d}\int_0^d e^{\alpha y}\,dy = \frac{c_0}{d}\left[\frac{e^{\alpha y}}{\alpha}\right]_0^d = \frac{c_0}{\alpha\,d}(e^{\alpha d} - 1) \quad \ldots\ldots(55)$$

in which $\alpha = \frac{\omega\,\rho}{\epsilon}$. For the case plotted in Fig. 16, the following values have been assumed: $V_S = 0.64$, $d = 0.80$, $S = 0.00155$, $\omega = 0.0575$, and $c_0 = 1$; these are more or less the average conditions of the experiment

$$\omega \rho \int_0^y \frac{dy}{\epsilon} = \omega \rho \int_0^y \frac{\frac{dv}{dy}}{w S y} dy = \frac{\omega}{g S} \int_0^y \frac{\frac{dv}{dy}}{y} dy = \frac{\omega}{g S} \eta y = \alpha y$$

For a parabolic velocity distribution curve, η is a constant, its value in the present case being 2. Therefore, $\alpha = \dfrac{0.0575 \times 2}{32.2 \times 0.00155} = 2.303$; $\alpha d = 1.843$; and, substituting in Equation (55), $c_m = 2.88$ particles per cubic centimeter. Equating c_m to c to locate the depth of the point of mean concentration, $e^{\alpha y} = 2.88$; whence, $y = 0.46 = 0.575\, d$ from the surface.

Assuming that the vertical co-ordinates of the mean velocity and the mean concentration are nearly the same, the total suspended load, P, may be expressed approximately by,

$$P = c_m q = c_m V d = \frac{2}{3} \frac{c_0}{\alpha} (e^{\alpha D} - 1) V_S \dots\dots\dots\dots (56)$$

in which c_0 is in pounds per cubic foot. Substituting the numerical values for an illustration, $P = 0.985$ (pound per second per foot of width).

If the velocity distribution is parabolic but the bottom velocity is not equal to zero, the following approximation may be used:

$$V = \frac{1}{d} \int_0^d v\, dy = \frac{1}{d} \int_0^d \left[V_S - \frac{w S y^2}{2 \epsilon} \right] dy = \left[V_S - \frac{\eta d^2}{6} \right]$$

and

$$P = \frac{c_0}{\alpha} \left[e^{\alpha d} - 1 \right] \left[V_S - \frac{\eta d^2}{6} \right] \dots\dots\dots\dots (57)$$

Equations (56) and (57) do not give the exact total suspended load even when the co-ordinates of the mean velocity and the mean concentration are at the same elevation; the resulting value is always somewhat too high. However, the following exact solution may be applied to any cases as long as the velocity distribution is parabolic and the silt distribution exponential:

$$P = \int_0^d c_0 v\, dy = c_0 \int_0^d e^{\alpha y} \left[V_S - \frac{\eta y^2}{2} \right] dy$$

$$= \frac{c_0 V_S}{\alpha} \left[e^{\alpha d} - 1 \right] - \frac{c_0 \eta}{2 \alpha^3} \left[(\alpha d)^2 e^{\alpha d} - 2 (\alpha d - 1) e^{\alpha d} - 2 \right] \dots (58)$$

This is the general equation for total load in suspension. c_0 is the concentration at $y = 0$; that is, at the surface where the velocity is a maximum. If $v = 0$ at the depth, d, $\dfrac{\eta}{2} = \dfrac{V_S}{d^2}$; therefore,

$$P = \frac{c_0 V_S}{\alpha^3 d^2} \left[2 e^{\alpha d} (\alpha d - 1) - (\alpha d)^2 + 2 \right] \dots\dots\dots\dots (59)$$

Substituting the numerical values, $P = 0.755$ (pound per second per foot of width). Curves of v, η, and c are plotted in Fig. 16. The ordinates of V

and c_m happen in this case to be at the same elevation. Their product gives the vertical line, $c_m V$; the area bounded by this line, the surface, and the axes, gives the total load of suspension by Equation (56). More accurately the product of c and v at various elevations should be plotted to obtain the curve, $c\,v$, the area to the left of which is the total load of suspension by Equation (59).

Straub[15] suggested the following (Equation (60)) derived from typical velocity-depth curves and typical suspended sediment-concentration curves, to determine the total quantity of silt transported in suspension per second:

$$P = \left[\tfrac{5}{8}\, c_{0.2d} + \tfrac{3}{8}\, c_{0.8d}\right] q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

In the present theoretical case, $c_{0.2d} = 1.5$, and $c_{0.8d} = 4.3$, so that $P = 0.855$ (pound per second per foot of width) which is 13.2% greater than the value by the exact solution, and 13.2% less than the value by the approximate solution. For practical purposes Straub's empirical formula is probably of the right order. For a rough estimate of the quantity of silt transported by a river it is highly recommended, because it is fairly simple to make two soundings at the proper depths, to collect the silt concentration samples. However, for a theoretical parabolic distribution of velocity, Equation (58) is convenient and more accurate. Of course for a refined determination, an arithmetic integration of the product of V and c at numerous elevations is needed.

EFFECT OF SILT TRANSPORTATION ON VELOCITY AND STREAM ENERGY

Transportation by traction affects the mean velocity of the stream by changing the roughness of the bed and, consequently, by varying the rate of dissipation of stream energy. Transportation by suspension, if not of appreciable quantity, will not affect the mean velocity very much. In rivers such as the Yellow River, however, with a maximum silt content of approximately 10% by weight during flood flows, and the Wei and Fen Rivers, tributaries of the Yellow River, with maximum silt contents as high as 46% and 20%, the suspended load will affect the mean velocity much more seriously than the bed load.

Evidently, the silt load in suspension will affect the mean velocity mainly in two respects: First, by changing the energy content of the stream; and, second, by changing the density and viscosity of the fluid. Changes in kinetic energy along the course of a stream due to the addition of silt are relatively small; further, little energy is consumed in supporting the débris. Therefore, it may be assumed that the change of kinetic energy due to the addition of silt is nil. Let $k\,w\,V\,Q$ be the kinetic energy of a clear stream. If V_1 is the mean velocity of the same stream with a quantity, $\alpha\, Q_1$, of silt in suspension, then the kinetic energy is represented by $k\,[w\,Q_1 + \alpha\, Q_1\,(\gamma - w)]\,V_1$. Equating the two expressions by assuming that the cross-sectional area remains the same, and solving for V_1:

$$V_1 = \sqrt{\frac{w}{w + \alpha\,(\gamma - w)}}\,V = \frac{V}{\sqrt{1 + \alpha\left(\dfrac{\sigma}{\rho} - 1\right)}} \quad \ldots\ldots\ldots (61)$$

Taking $\sigma = 2.65$, $\rho = 1$, and $\alpha = 10\%$ (for the Yellow and Colorado Rivers), $V_1 = \frac{V}{1.079}\cdot$ The velocity of flow is thus retarded by 7.4 per cent. For the Wei River with $\alpha = 46\%$, $V_1 = \frac{V}{1.326}$; that is, the velocity is decreased by 24.5 per cent.

This diminution of velocity of flow by silt content may serve to explain the "saturation" stage of stream capacity in supporting débris, for it is clear that silt will be transported in suspension only as long as the stream velocity exceeds the competent velocity to cause lifting up of particles.

Gilbert's observations[17] showed the same result. He concluded that under all conditions, streams are retarded by their suspended loads. The average of the thirteen velocities found for the loaded streams was 10% less than the corresponding average for unloaded streams.

In a paper published by Rubey[35] further light is thrown on this subject. By assuming that the change of kinetic energy is negligible, and equating the loss of potential energy of the flow (water + load) to the energy consumed in friction plus that consumed in transporting the load, he derived the following expression:

$$(W + L)\, g\, S\, V = \kappa\, \rho_f\, p\, V^4 + L \frac{\sigma - \rho}{\sigma} g\, \omega \ldots\ldots\ldots\ldots (62)$$

in which ρ_f is the density of flow (water + load); W and L are the weights of water and load, passing the section each second, respectively; κ, the skin friction coefficient of the channel; and p the wetted perimeter. When $L = 0$, $W\, g\, S\, V = \rho_f\, Q\, g\, S\, V = \rho_f\, A\, g\, S\, V^2 = \kappa\, \rho_f\, p\, V^4$; and $V = \left[\frac{g}{\kappa} R\, S\right]^{1/2}$, which is the familiar Chezy equation with its coefficient, C, expressed as $\left(\frac{g}{\kappa}\right)^{1/2}\cdot$ Substituting $C^2\, R\, S$ for V^2 in Equation (62),

$$\frac{\frac{L}{W}\frac{\sigma - \rho}{\sigma}\omega}{\left(1 + \frac{L}{W}\right) S\, V} = 1 - \frac{\kappa}{g} C^2 = \delta \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

When $L = 0$, $C = \sqrt{\frac{g}{\kappa}}\cdot$ When there is suspended load, C will be less than $\sqrt{\frac{g}{\kappa}}\cdot$ In the twenty-two experiments of Gilbert on débris-laden streams recalculated by Rubey, the values of $\left(1 - \frac{\kappa\, C^2}{g}\right)$ range from 0.007 to 0.070, with a median value of 0.025; that is, under the conditions of these experiments, an average of 97.5% of the total stream energy appears to have been lost in friction, and only 2.5% in transporting débris. In this equation $\frac{L}{W}$ is the fractional weight of the load in the stream. In the case of the Yellow River flood in 1931, with

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[35] "Equilibrium Conditions in Débris-Laden Streams," by W. W. Rubey, *Transactions*, Am. Geophysical Union, 1933.

39% loess suspension, δ was 0.0613 (taking $\sigma = 2.63$; $\omega = 0.004$ ft per sec, $S = 0.000135$, and $V = 84$ ft per sec).

Another substitution of the Chezy formula into Equation (63) gives,

$$\frac{\dfrac{\sigma - \rho}{\sigma}}{C\left(1 - \dfrac{\kappa}{g} C^2\right)} \times \frac{L\,\omega}{\rho_f\,Q} = S^{3/2}\,R^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

Equation (64) states the general relationships that the graded slope varies directly as the quantity of débris that must be carried, directly as the average size of the material, and inversely as the discharge of the stream.

Let $X = \dfrac{\text{mean depth}}{\text{mean width}}$. Then $R = \dfrac{A}{p} = \dfrac{Q}{C\sqrt{R\,S}\,p} = \dfrac{Q\,X}{C\,R^{3/2}\,S^{1/2}}$, and $R^{1/2} = \left[\dfrac{Q\,X}{C\,S^{1/2}}\right]^{1/5}$. Substituting this in Equation (64),

$$S^{7/5}\,X^{1/5} = \frac{\dfrac{\sigma - \rho}{\sigma}}{C^{4/5}\left(1 - \dfrac{\kappa}{g} C^2\right)\rho_f} \times \frac{L\,\omega}{Q^{6/5}} \ldots\ldots\ldots\ldots\ldots (65)$$

In this expression the settling velocity,[34] ω, is proportional to $D^{1/2}$ for sand particles coarser than 1.55 mm in diameter, and to D^2 for sand particles finer than 0.145 mm in diameter. Consequently, the equation can be written in the form, $L \propto \dfrac{S^{\alpha}\,Q^{\beta}\,X^{\gamma}}{D^{\phi}}$.

Table 6 lists values of these exponents as obtained from several sources:

TABLE 6

Source	α	β	γ	ϕ
Theoretical from Equation (65)	1.40	1.20	0.20	0.50–2.00
Mean in Gilbert's empirical equation*	1.59	1.02	0.47	0.58
Rubey's result on Illinois River†	1.50	1.25	0.25	....
Gibson's experiment on Severn Model‡	....	1.00	....	0.77
Present result of the writer§	1.40	1.20	....	0.50–2.00

* "The Transportation of Debris by Running Water," by G. K. Gilbert, U. S. Geological Survey, *Professional Paper 86*, 1914.

† "Equilibrium Conditions in Debris-Laden Streams," by W. W. Rubey, *Transactions*, Am. Geophysical Union, 1933.

‡ "Construction and Operation of a Tidal Model of the Severn Estuary," by A. H. Gibson, H. M. Stationery Office, London, 1933.

§ By assuming that all tractive transportation is lifted in suspension, that is, $L = G$.

SUMMARY: PART III

(1) Theoretically, the force required to lift a particle from the bottom of a stream is about 40% greater than that required to move it along the bed.

(2) The general equation for vertical distribution of silt in two-dimensional flow is of the form of Equation (51).

(3) The mean silt concentration throughout a depth, d, may be computed by Equation (55), in which $\alpha = \dfrac{\omega\,\rho}{\epsilon}$.

(4) The total suspended load in a stream may be well estimated by the empirical formula, Equation (60). Equation (58) is an exact equation, assuming a parabolic distribution of velocities.

(5) Silt transportation tends to decrease the mean velocity of flow. However, the energy consumed in supporting the load is only a small fraction of the total energy of the stream, which is mainly dissipated in overcoming the frictional resistance of the channel.

ACKNOWLEDGMENTS

The writer is indebted to Professor A. H. Gibson for his advice and interest in the problem, and for offering every possible facility in the use of the Whitworth Engineering Laboratory of Victoria University, at Manchester, England. Mr. J. Allen helped to perform the experiment and corrected the original manuscript. Mr. S. D. Chyn, of Tsing Hua University, Peiping, China, read the paper and offered helpful criticism. The writer wishes also to express his sense of indebtedness to the Board of Trustees, Nanking, China, for the scholarship which enabled him to carry out the research.

APPENDIX

NOMENCLATURE

The following nomenclature is used throughout the paper:

A = area of cross-section, in square feet;

a = vertical distance from d_0 to center of gravity of N; also, an exponent;

B = surface width, in feet;

C = Chezy coefficient; also, a constant;

c = constant in tractive force equations; elsewhere, concentration of suspended sediment, in particles per cubic centimeter; or in pounds per cubic foot;

D = mean diameter of a sand particle, in millimeters;

d = depth in feet; d_0 = depth at which critical tractive force obtains;

e = base of natural logarithms;

f = coefficient of friction in water;

G = rate of sand movement, in pounds per second;

g = gravity acceleration, in feet per second per second;

i = slope of side of trapezoidal channel, horizontal over vertical;

k = a coefficient;

L = total weight of suspended load passing a given section in 1 sec;

M = Kramer's uniformity modulus (dimensionless); also, area of channel above d_0;

N = area of channel below d_0;

n = Manning's n; also number of layers of sand (in explanation of du Boys' formula);

o = ratio between longest and shortest diameter of a sand grain;

P = total suspended load, in particles per second per centimeter of width; or in pounds per second per foot of width;

p = wetted perimeter; also, percentage of material of a given size in a mixture of sand that has been transported by traction;
Q = discharge, in cubic feet per second;
q = discharge per unit width of channel;
R = hydraulic radius, in feet;
$\mathsf{R} = \frac{V\,d}{\nu}$ = Reynolds number (dimensionless);
r = radius of sand particle;
S = slope of water surface; S_0 = slope of bed;
T = unit tractive force, in pounds per square foot; $\overline{T}$ = average unit tractive force over entire wetted perimeter; T_0 = critical unit tractive force;
$\bar{u}$ = absolute mean velocity of suspended particles;
V, V_1 = mean velocity, in feet per second; V_s = surface velocity; V_b = bottom velocity; and V_l = lifting velocity;
v = velocity at any point, in feet per second; v_r = advance of riffles, in feet per second;
W = total weight of water passing a given section in 1 sec;
w = weight of water, in pounds per cubic foot;
X = ratio of mean depth of channel to mean width;
x = length measured along the bed;
y = vertical distance, variable;
z = ratio between length of sloping side and bottom width of trapezoidal channel;
α = angle between water surface and horizontal; also, coefficient in Schoklitsch equation; also, an exponent; in Part III, $\alpha = \frac{\omega\,\rho}{\epsilon}$;
α_0 = angle between bed and horizontal;
β = an exponent to the expression $[\rho_e\, D\, o^{1/3}]$;
γ = unit weight of sand; also, an exponent;
$\delta = \left[1 - \frac{\kappa}{g} C^2\right]$, in Equation (63);
ϵ = Austausch coefficient $= -\frac{w\,S\,y}{\frac{dv}{dy}}$;
$\eta = \frac{\frac{dv}{dy}}{y}$;
κ = coefficient of skin friction (dimensionless);
μ = viscosity of fluid;
ν = kinematic viscosity of fluid;
ξ = height of riffle, in feet;
ρ = specific gravity of fluid; ρ_e, effective specific gravity of submerged sand particles; ρ_f, the specific gravity of flow (water + load);
σ = specific gravity of sand particles;
ω = settling velocity of sand particles in water, in feet per second.

DISCUSSION

Hans Kramer,[36] M. Am. Soc. C. E. (by letter).—This paper, based on flume experimentation and correlation of data from the observations of previous investigators, adds another valuable chapter to the field of knowledge regarding the complex phenomena of sand movement. Because of its intimate relation with, and frequent reference to, the writer's previous work[37] in the same field, this discussion will be confined to Part I of Mr. Chang's paper, which treats the subject of "Flume Traction."

The author made use of a horizontal flume bed instead of one inclined to the same slope as the water surface because the necessity for the latter arrangement was considered doubtful. However, from a mathematical analysis of his observed data, Mr. Chang concludes "that the tractive force needed to move débris on a sloping bed parallel to the water surface is only one-half that needed on a horizontal bed." The hypothesis and conclusion appear irreconcilable. Professor Hubert Engels, the pioneer and dean among investigators in this special field, emphasized[38] the importance of uniform flow and the interrelation between bed slope and water-surface slope in experimentation of this nature. It is unfortunate, therefore, that Mr. Chang's observations have been beclouded unnecessarily by the introduction—or lack of elimination—of this important variable.

A similar comment is applicable to the uniformity modulus, M. The author states that the uniformity modulus of samples used in his experiments was not determined but, nevertheless, concludes—again by mathematical manipulation—"that the uniformity modulus, M, has practically nothing to do with the critical tractive force, and could be omitted." Inasmuch as Mr. Chang's experimentation purposely utilized only sands of uniform grain size ($M = 1$), whereas natural sand mixtures vary greatly in their gradation—and in M—it would appear that his extrapolated conclusion as to the effect of the uniformity modulus is beyond the limit of his own observations.

The painstaking determination of the mechanical properties of test samples, including microscopic measurements, deserves commendation. It is regrettable, however, that the determination of grain size by the standard method of sieve analysis was not included. It is believed that the grain size of sand used by other investigators, as listed in Column (6), Table 5, is based on the conventional sieve analysis curve. Inasmuch as standard sieving procedure results in larger apparent sizing than absolute physical measurement, Mr. Chang's size determinations are not directly comparable with those tabulated for other investigators.

It is also noted that Mr. Chang has adopted as a definition for "general movement" a set of conditions which are more comparable to the writer's definition for "medium movement" than to his definition of "general movement." A cursory review of the writer's observed data indicates that the magnitude of the critical tractive force based on his "general movement" is approximately 20% greater than that based on "medium movement." This variance in a basic definition, which forms a criterion for comparison of results, would appear to require the application of a corrective factor in the correlation of observations.

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[36] Capt., Corps of Engr., U. S. Army; Dist. Engr., U. S. Engr. Office, Conchas Dam, N. Mex.

[37] "Sand Mixtures and Sand Movement in Fluvial Models," by Hans Kramer, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 798.

[38] "Versuche über den Reibungswiderstand zwischen strömenden Wasser und Bettsohle," by H. Engels, *Zeitschrift für Bauwesen*, 1912.

Mr. Chang's experimentation and treatise, while constituting another important step in the diagnosis of the complex phenomena of sand movement, may also be regarded as further proof of the difficulty of reconciling and correlating data observed and conclusions derived by different experimenters under widely divergent conditions. His attempt to simplify the problem by choosing to experiment with uni-granular sand, thereby eliminating the vexing variable of grain-size distribution, is a step in the right direction. However, it is believed that the ultimate, basic solution to this problem—both mathematical and physical—will not be clearly and convincingly obtained until the scientific viewpoint and procedure of the experimental physicist are adopted. The writer is convinced that further research to determine the independent variables which govern the very complex phenomena of sand movement by traction and the fundamental relation between these variables must seek simplification and idealization of the phenomena. To this end, it is suggested that future experimentation with artificial spherical or spheroidal grains—probably of glass—with colors, markings, etc., for statistical identification may develop the simple fundamentals of flume traction without the complicating influences of accidental and uncontrollable variations inherent in the use of natural sand specimens.

E. W. LANE,[39] M. AM. SOC. C. E. (by letter).—A very valuable contribution to the science of the transportation of solids by flowing water is presented in this paper. It seems to the writer that the reason for some of the lack of agreement of different experimenters on the values of critical tractive force for the movement of the various sizes of material is probably that initial movement is not a function of tractive force alone but of a combination of tractive force and turbulence. The traction is an average force over a considerable area, but the forces that move the first particles are probably intense local forces, acting only over a small area of the bottom, having a magnitude greater than the average force acting. The initial movement, therefore, would depend not only on the magnitude of the average force but also on the range above the average of the variable forces, due to the turbulence in the water. Therefore, initial movement would depend on both tractive force and turbulence. These two factors would also influence other degrees of movement, but it does not seem likely that turbulence would be of relatively as great importance in them as in the initial movement.

In his summary of Part 1, Item (5), the author concludes that the "resistance to motion of particles on a horizontal bed is nearly double their resistance on a sloping bed having the same inclination as the water surface." This would indicate that the motion of bed load is very sensitive to changes in the bed slope. The bottom of a stream where bed load is in motion is usually rough, with slopes both upward and downward in the direction of flow. Even in a laboratory flume there are sand waves such that the sand moves up slopes considerably steeper upward in the direction of flow than the water surface slopes downward in this direction. Sand waves were observed in the Mississippi by the late J. B. Johnson, M. Am. Soc. C. E.,[40] with an average height of 5 ft, and an average length of about 330 ft, indicating an upward slope of 1 in 66. The surface slope was probably only about one two-hundredths of this inclination. The depths of water ranged from 13 to 30 ft, with movement apparently throughout this range. It would be interesting to extend the analysis that led to the foregoing conclusion to see if it would indicate any motion at all up the slopes of the Mississippi sand waves. The effect of bottom slope on the bed load carried in a stream may be a factor that will make it very difficult to correlate field and laboratory results.

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[39] Associate Director in Chg., Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

The author shows that a change in the law of bed-load transportation occurs at a grain size for quartz of about 0.5 mm, and states that this is best explained by turbulence. Since the law governing the settling rate of quartz grains changes at about the same size, may the change in the bed-load law not be related to the change of settling rate? There is good evidence that the law of transportation of material in suspension shows a similar sharp break at about this size.

The author concludes (see "Summary": Part III, Item (5)) that "silt transportation tends to decrease the mean velocity of flow." This conclusion is the opposite of that reached by A. B. Buckley, Jr.,[41] from observations in the Nile Valley. Surprisingly low roughness values have been obtained on some of the silt-laden rivers of North China by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., and others, which support Mr. Buckley's conclusions. This is apparently a field which justifies much further research.

Joe W. Johnson,[42] Assoc. M. Am. Soc. C. E. (by letter).—In this paper the author has presented an excellent review of the problem of transportation by flume traction. He has also presented original data and conclusions that are welcome contributions to the knowledge of this phenomenon. Of particular interest to the writer is the section of the paper treating the formulas for rate of transportation. Therein is given a review of the published formulas and a new formula which is of the du Boys[14] type.

The writer reviewed a large number of formulas representing the data obtained from various experiments. Perhaps the most common type of formula is that of du Boys (Equation (26)), in which rate of movement is plotted against a function of tractive force. Also very common is the Schoklitsch-MacDougall type (Equation (41)), in which rate of movement is assumed to be proportional to excess power.[22] Authorities undertaking flume studies have usually presented their data in one of the common types of formulas, but, in many instances, with slight modifications. The result of such studies has been that there are approximately as many formulas describing the rate of sand movement as there are authorities on the subject. It was with this situation in mind that an investigation was made in 1937 to determine, for a certain set of published flume observations, whether a particular method of presenting data offered distinct advantages over other methods. In order to obtain an unbiased opinion in this determination, recourse was made to the methods of statistical analysis.

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[40] Chf. of Engrs'. Rept., 1879, Pt. III, p. 1967.

[41] "The Influence of Silt on the Velocity of Water Flowing in Open Channels," by A. B. Buckley, Jr., *Minutes of Proceedings*, Inst. C. E., Vol. 216, p. 1813.

[42] Asst. Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

Flume-study observations published in United States Waterways Experiment Station *Paper No. 17* were used in the computations. These data are for U. S. Waterways Experiment Station Sand No. 1 which was tested in a concrete-lined flume, 2.416 ft wide and with slopes of 0.0010, 0.0015, and 0.0020. Only those data in which "general movement" occurred were used in this study. From the results of these measurements, computations were made to permit the data to be plotted according to the criteria of du Boys,[14] Chang, Meyer-Peter,[20] U. S. Waterways Experiment Station,[10] MacDougall,[22] and O'Brien.[43] It is noted that in Fig. 11 the author plots $\frac{G}{n_s}$ versus $T\ (T - T_0)$, whereas in the general statement of his formula (Equation (33)), $\frac{G}{n}$ is adopted as the dependent variable. In the discussion to follow, the parameters in Fig. 11 have been used. Figs. 17, 18, and 19 show the results of plotting the computed data according to various authorities. The curves were fitted by the method of least squares and their "goodness of fit" was tested by the χ^2-test. The equations for the various groups are, as follows:

The U. S. Waterways Experiment Station (Fig. 17):

$$G_1 = \frac{29\,400}{n}\,(d\,S - d_0\,S_0)^{1.41} \qquad (66a)$$

Du Boys (Fig. 18(*a*)):

$$G_1 = 20\,860\,[T\ (T - T_0)]^{0.87} \qquad (66b)$$

Chang (Fig. 18(*b*)):

$$G_1 = 480\,600\,n_s\,[T\ (T - T_0)]^{0.74} \qquad (66c)$$

O'Brien, for $S = 0.0010$ (Fig. 19):

$$G_1 = 2.4 \times 10^{-5}\left(\frac{V}{R^{\frac{1}{3}}}\right)^{14.4} \qquad (66d)$$

O'Brien, for $S = 0.0015$ (Fig. 19):

$$G_1 = 1.8 \times 10^{-6}\left(\frac{V}{R^{\frac{1}{3}}}\right)^{16.0} \qquad (66e)$$

and O'Brien, for $S = 0.0020$ (Fig. 19):

$$G_1 = 3.5 \times 10^{-6}\left(\frac{V}{R^{\frac{1}{3}}}\right)^{14.3} \qquad (66f)$$

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[43] "Notes on the Transportation of Silt by Streams," by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, Pt. II, 1936, p. 431.

The MacDougall data resulted in curves having the following equations (illustrations not included):

For $S = 0.0010$ (G_1 plotted against discharge per foot width of flume):

$$G_1 = 40.47\,q - 4.78 \quad \text{(67a)}$$

For $S = 0.0015$:

$$G_1 = 73.20\,q - 6.18 \quad \text{(67b)}$$

and, for $S = 0.0020$:

$$G_1 = 98.62\,q - 6.26 \quad \text{(67c)}$$

in which G_1 = rate of sand movement, in pounds per hour per foot of width.

The general formulas of du Boys, Chang, U. S. Waterways Experiment Station, and MacDougall have been reviewed by the author and further discussion is not necessary. Perhaps it is desirable, however, to discuss briefly the Meyer-Peter formula (Equation (36)) inasmuch as it was developed from measurements on material of uniform granular size. The observations on material ranging from 3.17 mm to 28.8 mm were obtained by the laboratory at Zurich, Switzerland, and from the data of Gilbert.[17] The result of these studies, stated generally by Equation (36), is:

$$\frac{G_2^{\frac{2}{3}}}{D'} = 1.323\,\frac{q^{\frac{2}{3}}\,S}{D'} - 0.335 \quad \text{(68)}$$

in which G_2 is the cubic feet of bed material transported per foot width of channel per second, and D' is the diameter of bed material, in feet. A plot of the data observed at the U. S. Waterways Experiment Station results in the formula $\left(\frac{q^{\frac{2}{3}}\,S}{D'}\text{ plotted against }\frac{G^{\frac{2}{3}}}{D'};\text{ graph not included}\right)$:

$$G_2 = (1.66\,q^{\frac{2}{3}}\,S - 0.147\,D')^{\frac{3}{2}} \quad \text{(69)}$$

Equation (68) was intended primarily for use in movable bed-model studies. The advantage of this relation is obvious when it is noted that dividing both sides by $g^{\frac{1}{3}}$ gives a resulting equation that is non-dimensional and in agreement with the Froude law of similarity.

In flume tests on Columbia River material, Professor O'Brien[43] found that experimental data showed considerable variation when rate of movement was plotted against the following parameters: Velocity at a constant distance from the bottom, mean velocity, the U. S. Waterways Experiment Station method, or the method of Schoklitsch. In all cases the disagreement was greater than could be accounted for by inaccuracy of measurement. Good results, however, were obtained by plotting rate of movement versus $\frac{V}{R^{\frac{1}{3}}}$, as shown in Fig. 19. Professor O'Brien[44] also reported a second method, namely, plotting $\frac{G}{f}$ against $\frac{V}{R^{\frac{1}{3}}}$, that showed slightly less variation than the first.

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[44] "Laboratory Study of Transportation of Columbia River Bed Materials," *Technical Memorandum No. 11,* U. S. Tidal Model Laboratory, Berkeley, Calif., January 28, 1936.

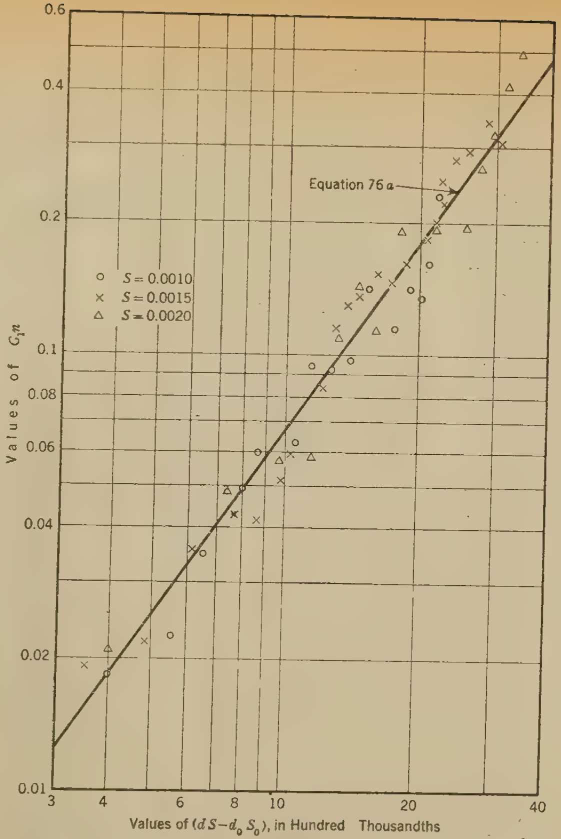

FIG. 17.—DATA ON TRANSPORTATION BY TRACTION, ACCORDING TO U. S. WATERWAY EXPERIMENT STATION; EQUATION (39) OR EQUATION (66*a*)

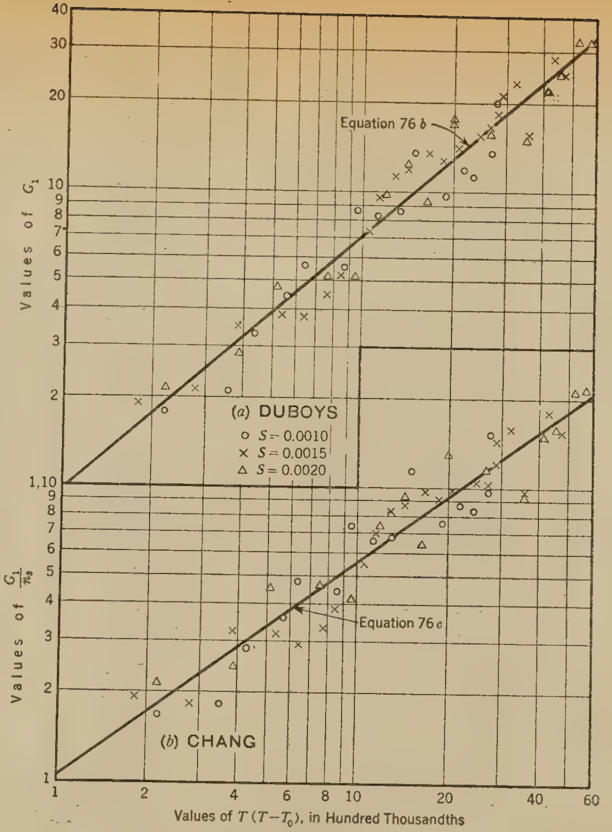

FIG. 18.—DATA ON TRANSPORTATION BY TRACTION; EQUATIONS (26) AND (33)

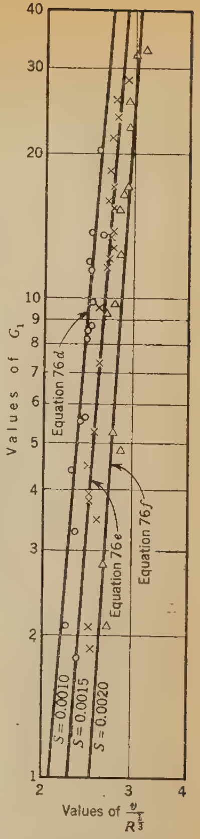

FIG. 19.—DATA ON TRANSPORTATION BY TRACTION, ACCORDING TO O'BRIEN; EQUA-

From a glancing examination of the curves shown in Figs. 17, 18, and 19, it is difficult to determine which type of plotting gives the best fit; however, by the application of the χ^2-test, personal opinion is eliminated and an unbiased measure of the goodness of fit is obtained. Table 7 shows values of χ^2 computed for the various criteria. An interpretation of the significance of the computed values of χ^2 requires the use of tables from which the probability of fit can be obtained. Reference to probability tables[45] shows that in 99 out of 100 trials, in random sampling, one should obtain a fit as bad as or worse than that observed, if the real distribution is normal. For these various plots which show such excellent fit, it is to be noted that the probabilities of occurrence are so near the same magnitude that it cannot be truthfully stated that a particular method of plotting is superior to any one of the other methods. The reason for favoring a particular form of plotting, therefore, appears to be only in the ease and accuracy with which certain variables can be measured.

TABLE 7.—COMPARATIVE VALUES OF χ^2

Reference No.	Authority	Number of observations	Values of χ^2	Reference No.	Authority	Number of observations	Values of χ^2
1	U. S. Waterways Experiment Station	52	16	4	Chang	52	23
2	du Boys	52	18	5	Meyer-Peter	52	25
3	MacDougall	52	20	6	O'Brien	52	46

In flume studies there are three sources of error which, in many instances, have seriously limited the value of the results. These sources of error are: (*a*) The use of the water-surface slope instead of the slope of the energy gradient; (*b*) the neglect of the retarding effect of the channel side-walls; and (*c*) the use of relatively short collection periods during which the bed material is trapped. An accurate determination of the slope of the energy gradient requires the measurement of the velocity distribution at each end of the experimental reach. Owing to the relatively short time of a run, this observation is often eliminated and the error involved in slope determination is fairly large for the usual experimental conditions. It is of interest to note that Gilbert[17] was undecided as to the proper value of slope to use and stated:

> "I do not find it easy to decide which slope should be regarded as the true correlative of capacity for traction, but as all our laboratory data include the débris slope, while the determinations of water slope were relatively infrequent, the discussion of the results has adhered almost exclusively to the former. If the water slope is the true correlative, then the use of the débris slope involves a systematic error."

Professor O'Brien and Lieut. B. D. Rindlaub[46] support Gilbert's selection in the statement,

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[45] "Statistical Methods for Research Workers," by R. A. Fisher, Oliver & Boyd, Lond., 1936.

[46] "The Transportation of Bed Load by Streams," by M. P. O'Brien and B. D. Rindlaub, *Transactions*, Am. Geophysical Union, June 1934, Pt. II, p. 593.

"It is to be noted that the slope of the bottom is more nearly equal to the slope of the energy-gradient than is the slope of the water-surface and partly for this reason, the data of G. K. Gilbert show less scattering than the data of more recent experimenters who have criticized Gilbert for not measuring the slope of the water-surface in all of his experiments."

Consideration of the retarding effect[47] of the channel side walls is necessary in order to correlate the data from flumes of various widths or to correlate accurately the data from a particular flume whose width to depth ratio is relatively small. A striking illustration of the importance of side-wall resistance is afforded by Fig. 20, which shows rates of transportation of Gilbert's Grade *B* sand in flumes of various widths. It is obvious that consideration of side-wall resistance is essential if a general expression for describing rate of transportation by traction is expected. MacDougall[22] attempted to eliminate the effect of the side walls mechanically by placing diversion walls on the experimental flume. Meyer-Peter[20] in his formulation of Equation (36) eliminated side-wall disturbances on the basis of the method given by Einstein.[48] The latter method gives that part of the total discharge which is supposed to be acting upon the bed. The results from Einstein's method are similar to the Schoklitsch[7] method defined by the author in Equation (12). In the data herein discussed, a reduction in scattering of plotted points was not effected by the introduction of α into any of the various parameters used in Figs. 17, 18, and 19.

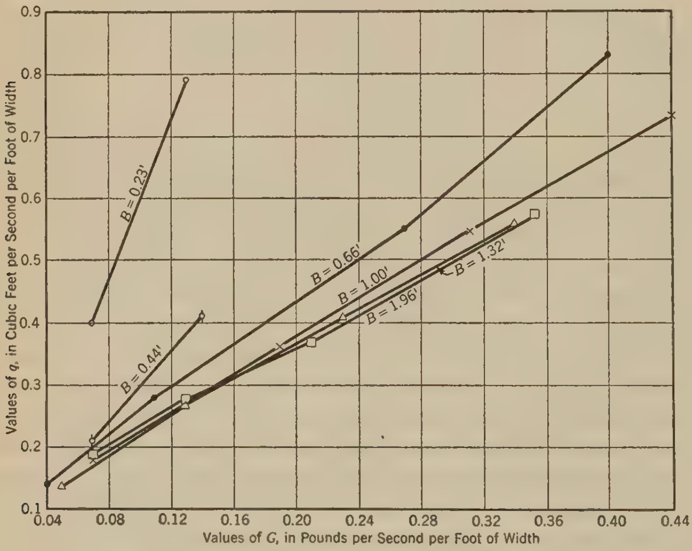

FIG. 20.—DATA ON TRANSPORTATION OF GILBERT'S GRADE *B* SAND IN FLUMES OF VARIOUS WIDTHS; SLOPE = 1 PER CENT

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[47] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 519.

[48] "Der Hydraulische oder Profil-Radius," by A. H. Einstein, *Schweizerische Bauzeitung*, No. 8, Vol. 103, February 24, 1934.

The use of relatively short collection periods when observing rates of bed-load movement probably accounts for most of the scattering of experimental data. For example, during calibration tests on bed-load traps for use on the Rhine, Einstein[49] observed the rates of bed-load movement shown in Fig. 21.

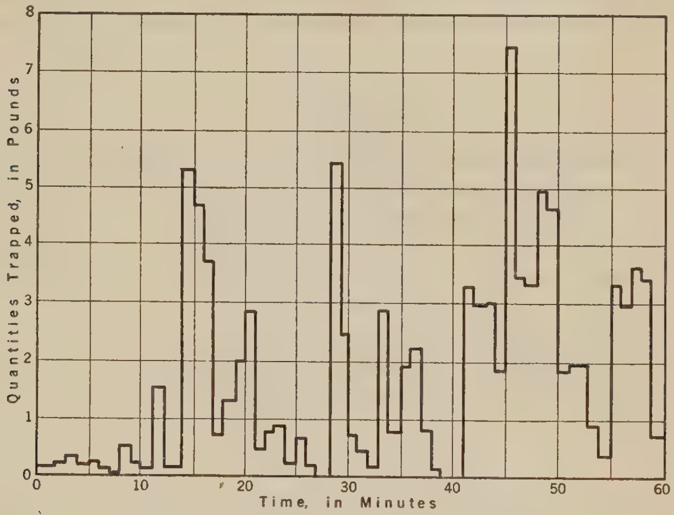

FIG. 21.—TREND OF TRAP RESULTS IN THE COURSE OF ONE HOUR, WITH CONSTANT HYDRAULIC CONDITIONS

It is obvious from an examination of this diagram that relatively long collection periods and recourse to statistical analysis are perhaps necessary to obtain reliable and consistent data on the movement of granular material by flowing water.

J. E. CHRISTIANSEN,[50] ASSOC. M. AM. SOC. C. E. (by letter).—The results of an interesting laboratory study of flume traction and transportation of sediment by suspension, combined with a review of literature on the subject, is presented in this paper. Since the writer has been interested primarily with transportation of silt by suspension, this discussion will deal chiefly with Part III, "Transportation of Material in Suspension."

Equation (49) is derived to express the bottom velocity required to lift a particle from the bed. This assumes that the full impact of the velocity, V_l, is applied in a vertical direction to the projected area of the particle. By definition, the author implies that V_l is the bottom velocity in the direction of flow (horizontal), and not the vertical velocity, or vertical components of the velocity. The derivation of this equation is similar to the usual derivation of an equation for settling velocity of spheres in a fluid,[51] except that the author makes no mention of the coefficient of resistance of the particle, which is unity for particles of only one size. Direct reasoning would indicate that to lift a particle from the bed would require the vertical components of the bottom velocity due to turbulence to exceed the settling velocity of the particle.

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[49] "Calibrating the Bed-Load Trap as Used in the Rhine," by A. H. Einstein, *Schweizerische Bauzeitung*, Vol. 110, No. 14.

[50] Asst. Irrig. Engr., Coll. of Agriculture, Univ. of California, Davis, Calif.

[51] "Distribution of Silt in Open Channels," by J. E. Christiansen, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1935, pp. 478–485.

In the general formula for the distribution of suspended sediment (Equation (51)), the origin of y must be taken at the point of maximum velocity to be consistent. The Austausch coefficient is correctly defined by the expression,

$$\epsilon = \frac{\tau}{dv \div dy} \dots\dots\dots\dots (70)$$

in which τ is the shearing force per unit area. The definition of ϵ as given by the author applies only to a wide shallow stream for which the surfaces of equal velocity are essentially horizontal planes. The Austausch coefficient has the same physical significance for turbulent flow as the coefficient of viscosity has for laminar flow. Assuming that τ is a function of y, any selection for the origin of y, other than point of maximum velocity, indicates a shearing force at the point of maximum velocity and an infinite value of ϵ.

The writer believes that the method of computing the values of ϵ and τ used by Leighly[52, 53] could be used advantageously in the laboratory when the velocity distribution can be determined. This method involves the expression,

$$\epsilon = m\,g\,S \div \frac{dv}{dy} \dots\dots\dots\dots (71)$$

in which m is the mass of water between a unit surface of equal velocity and the region of maximum velocity, bounded on the sides by surfaces normal to the surfaces of equal velocity.

The author states (see "Experimental Verification") that "tests have been made on sediment distribution as described; the results plotted do not agree with the theoretical Equation (51) because this equation is presumably derived for a considerably wide channel suffering no side restrictions." This statement appears misleading because no such assumptions are made in its derivation.[54] The only assumptions concern the evaluation of ϵ. For a deep narrow flume, ϵ is not correctly expressed by the equation given by the author. From the subsequent remarks, it appears that the exponential curve did fit the observed data when a coefficient was applied to the exponent. Another factor that may have affected the value of the exponent, $\omega\,\rho \int^{y} \frac{dy}{\epsilon}$, is a possible error in the value of ω. Although Rubey's equation appears to fit Richards' experimental data[55] very well, it should be noted that Richards determined the size of the particles by means of sieves. Knapp[56] and Alexander

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[52] "Toward a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams," by John B. Leighly, Univ. of Calif., *Publications on Geography*, Vol. 6, No. 1, 1932, pp. 1–22.

[53] "Turbulence and the Transportation of Rock-Débris by Streams," *Geographical Review*, Vol. 24, 1934, pp. 453–464.

[54] "Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation," by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1933.

[55] "Velocity of Galena and Quartz Falling in Water," by R. H. Richards, *Transactions*, Am. Inst. Min. Engrs., Vol. 38, 1908, pp. 230–234.

[56] "New Apparatus for Determining Size Distribution of Particles in Fine Powders," by R. T. Knapp, *Industrial Engineering Chemistry*, Vol. 6, 1934, pp. 66–71.

and Jacob[57] have reported that settling velocities of particles computed from microscopic measurements of diameters are too high, and that they agree better with data on spheres when diameters are determined with sieves.

Concerning the size of particles, the author states (see "Part I," heading, "Mechanical Properties of Test Samples"), "for fine grains the volume of which was not measurable, D was assumed equal to the mean of D_1 and D_2." For Samples 6198 and 6197, however, the value of D as given in Table 1 is the same as for D_2. The writer has been unable to check the values of ω given from the formula and values of D as given in Table 1, or from the mean value of D_1 and D_2. It appears possible that the difficulty in making the equation fit the experimental data might be due, at least partly, to incorrect determinations of both ϵ and ω. In the writer's study[58] it appeared that Equation (51) was in very good agreement, without modification, with all the available published data that were analyzed.

The author's discussion of total load in suspension is interesting, but it must be kept in mind that these equations are valid only for silt particles with a uniform settling velocity. Silt found in suspension in natural streams includes particles of various sizes and settling velocities. For each size fraction, the concentration at any depth can be expressed by Equation (51) in terms of the known concentration at one depth, the turbulence of the stream as expressed by ϵ, and the settling velocity of the silt, ω. The total concentration of all size fractions cannot be expressed by a similar exponential equation, however, and, consequently, the total load in suspension is not correctly expressed by Equations (56) to (59).

Assume a general case of a natural stream for which the vertical velocity distribution is determined with a current meter, and a sample of silt is collected at one or more depths. The silt samples can be analyzed for distribution of size of particles, and a curve can be plotted showing this relationship graphically. The average settling velocity for each size fraction between rather narrow limits can be estimated. From the velocity distribution, $\frac{dv}{dy}$, and slope, S, values of ϵ can be determined by graphical integration. Curves can then be constructed showing the size distribution of silt in a vertical section for each size fraction. The total concentration at any depth is, $c_T = c_1 + c_2 + c_3 + \cdots$, in which c_1, c_2, etc., are the concentrations for each size fraction. A curve for c_T plotted against y can then be plotted, but the relationship cannot be expressed by a simple exponential equation. The total load, P, is correctly expressed by the relation,

$$P = \int_0^d c_T \, v \, dy \qquad (72)$$

and the integration can be performed graphically.

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[57] "Mechanical Analysis of Finely Divided Natural Phosphates," by L. T. Alexander and K. D. Jacob, *Technical Bulletin 212*, U. S. Dept. of Agriculture, 1930, pp. 1–24.

[58] "A Study of the Distribution of Silt in Open Channels," by J. E. Christiansen, Assoc. M. Am. Soc. C. E. Thesis submitted in partial fulfillment for the degree of Civil Engineer, Univ. of California, 1935. (Not published; on file in the Library, Univ. of California.)

The writer questions the validity of Equation (61) and the reasoning that precedes it. He can see no more reason for assuming that the kinetic energy of a stream is independent of the silt carried, than for first assuming that the velocity is not affected. It would seem that the principal effect of silt on the velocity and kinetic energy would be due to a possible change in the smoothness of the bed, and the possible effect on the Austausch coefficient, ϵ. Few data are available on this point. Buckley[59] observed that the velocity increased during periods of high silt concentration, other factors remaining constant. In deriving the general expression for silt distribution, the basic assumption is made that ϵ is not a function of silt concentration; and this assumption is probably justified for ordinary concentrations found in many rivers, but it may not be justified for the exceptionally high concentrations sometimes found.

Referring to the "Summary," Part III, the writer cannot agree with Conclusions (1), (3), (4), and (5). The reasoning leading to Conclusion (1) does not appear sound for the reasons previously mentioned. In connection with Conclusion (2), it must be kept in mind that Equation (51) expresses the relationship only for a single-size fraction, and that the distribution curve for a graded silt is not a simple exponential curve. Conclusion (3) is misleading. The first part of the expression, Equation (55), $c_m = \frac{1}{d}\int_0^d c\,dy$, is valid, but the latter part of the expression applies only to the specific case of parabolic velocity distribution, and maximum velocity at the surface, where it is seldom found. The statement concerning the location of the point of mean silt concentration is based on a calculation for specific conditions, and does not hold, generally. For the assumptions used by the author, a general expression for the position of the mean silt concentration in a vertical section can be obtained by equating,

$$e^{\alpha y} = \frac{1}{d}(e^{\alpha d} - 1) \ldots\ldots\ldots\ldots\ldots\ldots (73a)$$

from which,

$$y = \frac{1}{\alpha}\ln\left(\frac{e^{\alpha d} - 1}{\alpha d}\right) \ldots\ldots\ldots\ldots\ldots\ldots (73b)$$

The position of mean silt concentration, therefore, is a function of α as well as d. Furthermore, the use of the term, "mean silt concentration," to refer to the average silt concentration in a vertical section is misleading. Mean silt concentration is more properly defined by the expression,

$$c_m = \frac{P}{q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

The total amount carried in a vertical section of uniform width is expressed correctly by the equation,

$$P = \int_0^d c\,v\,dy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

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[59] "The Influence of Silt on the Velocity of Water Flowing in Open Channels," by A. B. Buckley, *Minutes of Proceedings*, Inst. C. E., Vol. 216 (II), 1922–23, p. 183.

in which both c and v are functions of y. The first statement in Conclusion (4) is not verified by the data presented. The second statement is valid only for a single size fraction, and cannot be applied to the total silt concentration of a stream. Conclusion (5) is the result of an assumption that may not be valid, and is in direct conflict with certain previous observations.

W. H. HUANG,[60] ESQ. (by letter).—The lucid explanations which the author has given to some of the phenomena observed in his experiments are impressive. For example, he interprets Exponent β in Equation (20) and Equation (21c) as turbulence, and shows that sorting will take place in graded sands irrespective of the mode of motion of the particles, and that the coarser particles will move in greater quantities than the finer ones.

Attention is called to Equation (13), in which it appears that the term corresponding to the decrease in potential energy is incorrect. Mr. Chang takes the average of the drop of the water surface and that of the bed in the reach, dx, as the decrease in potential energy, and thus he obtains $w\,y\,\frac{\sin\alpha + \sin\alpha_0}{2}\,dx$. It is known, however, that throughout the depth at any section the potential energy is equal to that at the water surface. The decrease in potential energy, therefore, should be: $w\,y\cos\alpha_0\tan\alpha\,dx$; and Equation (13) should read:

$$\frac{w\,y}{2\,g}\left[(V + dV)^2 - V^2\right] + T\,dx = w\,y\cos\alpha_0\tan\alpha\,dx \ldots\ldots (76)$$

In deriving Equation (15) Mr. Chang refers to Fig. 5, which is also incorrect. Taking x as positive toward the right and y as positive upward, the depth of water at the left section should be taken as y, and the increment of depth at the right section should be taken as $(-)\,dy$. Using these correct notations, $\frac{dV}{dy} = \frac{-V}{y}$, $\frac{(-)\,dy}{dx} = \cos\alpha_0\tan\alpha - \sin\alpha_0$, is obtained and,

$$T = \left[w\,y\cos\alpha_0\tan\alpha + w\frac{V^2}{g}(\sin\alpha_0 - \cos\alpha_0\tan\alpha)\right] \ldots\ldots (77)$$

For a sloping bed with uniform flow,

$$T = w\,y\sin\alpha_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78a)$$

and, for a horizontal bed,

$$T = w\tan\alpha\left(y - \frac{V^2}{g}\right) \ldots\ldots\ldots\ldots\ldots\ldots (78b)$$

Therefore, for great depth with comparatively small velocity, the tractive force on a horizontal bed is approximately equal to that on a sloping bed with the same water-surface slope (not 0.5 as the author has concluded). Therefore, the values of T_0 in Table 4 should be recalculated, and Equation (20) should be modified accordingly.

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[60] Prof. of Hydr. Eng., National Central Univ., Chungking, China.

Equation (76) may also be considered as Bernoulli's equation with each term multiplied by the factor, $w\,y$; thus:

$$T\,dx = w\,y\,H_f \quad \text{(79)}$$

in which H_f is the loss of head in the reach, dx.

Since,

$$H_f = S_e\,dx \quad \text{(80)}$$

in which S_e is the energy gradient; therefore:

$$T = w\,y\,S_e \quad \text{(81)}$$

By Manning's formula,

$$S_e = \frac{n^2\,Q^2}{2.21\,y^{3.33}} \quad \text{(82)}$$

which, substituted in Equation (81), yields,

$$T = \frac{w\,n^2\,Q^2}{2.21\,y^{2.33}} \quad \text{(83)}$$

or,

$$T = w\,y\,\sin\alpha_0\left(\frac{y_0}{y}\right)^{3.33} \quad \text{(84)}$$

in which y_0 is the depth of normal flow.

For channels other than the rectangular cross-section of infinite width as assumed, the average tractive force may be represented by the following formula: $\bar{T} = w\,R\,S_e$; or,

$$\bar{T} = \frac{w\,n^2\,Q^2}{2.21\,R^{\frac{1}{3}}\,A^2} \quad \text{(85)}$$

Equation (85) may be used to calculate the tractive force in experimental flumes where the walls are roughened to give approximately the same roughness as that of the bed. When the roughness of the walls differs widely from that of the bed, as in the author's flume, more experimentation is needed to justify the use of Equation (4) and Equation (6).

LORENZ G. STRAUB,[61] ASSOC. M. AM. SOC. C. E. (by letter).—Many differences of opinion have been expressed from time to time relative to the transportation of sedimentary materials by rivers. Laboratory studies have aided in arriving at conclusions, but even in that case there is lack of agreement between various investigators. The author has presented an interesting review of several noteworthy studies which have appeared in engineering literature during the past several years; he also compares results of his own observations with results of earlier investigators. He is to be commended on his approach to the problem and his effort to give due consideration to the dimensional analysis of his theories. However, the writer cannot agree with the logic of some points in the theoretical development and the author's conclusions. He is, therefore, taking the liberty of discussing a few of these questions in some detail, without, however, giving special consideration to other features of the paper.

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[61] Prof. of Hydraulics and Administrative Asst., Coll. of Eng. and Architecture, Univ. of Minnesota, Minneapolis, Minn.

The data pertaining to the relation between Manning's n and size of particles are of considerable interest. They conform very closely to results obtained in connection with studies by the writer for roughness factors in laboratory experiments on sediment transportation. Of course, in case of a lack of uniformity of motion of the bed material (as, for example, when pronounced riffle formation takes place) there is considerable change in the value of the roughness coefficient. Presumably, the author's data are for relatively low velocities.

Exception is taken to the analysis of the "effect of non-uniform flow on tractive force." Equation (17) and the conclusions drawn therefrom appear to be illogical and seem to be mathematically unsound. Although the traction force can be written (as is usually the procedure) as a function of the water-surface slope, depth, and unit weight of the water, this conception is useful primarily as a simple means of visualizing tractive force in the case of uniform flow. Tractive force can be written in terms independent of the slope of the water surface or of the stream bottom, if due account is taken of the roughness of the bed and the velocity of the stream.[62] Subsequently, the writer will indicate one possible relation of this character, showing that for a given roughness and velocity of flow the traction force is independent of the slope of the stream bottom.

Apparently, the author assumes that Equation (15) is correct because for the special case of a sloping bed with uniform flow the usual expression for traction force is obtained. However, although the equation for the special case happens to reduce to the correct form, the general form (Equation (15)) is mathematically incorrect.

Detailed consideration will now be given to Equation (17). Assume that super-critical velocity takes place. In the case of a horizontal bed the value of S must be negative. Equation (17) would thus indicate a negative tractive force or consequent sediment transportation in a direction opposite to that of the flow of the water. Other similar illogical deductions would result from a close analysis of the physical significance of the relation. A careful analysis of the type suggested by the author for a horizontal bed will lead to the equation:

$$T = w\,S\left(y - \frac{V^2}{g}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

instead of the expression represented by Equation (17). Here, in the case of super-critical flow (shooting velocity), S likewise becomes negative, but the expression in the parentheses becomes negative simultaneously, so that the value of T remains positive. It is impossible for flow to go through the energy critical on a horizontal bed. For this hypothetical condition in Equation (86), mathematically the value in parentheses $\left(y - \frac{V^2}{g}\right)$, would become zero and the slope, S, would become infinite; this conforms to the physical phenomenon just mentioned. In the case of non-uniform flow on a horizontal bed of infinite width the slope of the water surface in the Manning formula might be expressed as:

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[62] "Hydraulic and Sedimentary Characteristics of Rivers," by Lorenz G. Straub, *Transactions*, Am. Geophysical Union, Thirteenth Annual Meeting, National Research Council, 1932, pp. 375–382.

$$S = \frac{V^2}{C^2\, y^{\frac{1}{3}} \left(y - \frac{V^2}{g} \right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

in which the value of C corresponds to the expression, $C = \frac{1.486}{n}$, in Manning's formula. Substituting this value for the slope of the water surface into Equation (87) it will be found that:

$$T = w \frac{V^2}{C^2} \frac{1}{y^{\frac{1}{3}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

Thus, the traction force is shown to be a function of the velocity, depth, and roughness. The expression for the traction force in Equation (88) is the same as would be obtained by substituting from Manning's formula for uniform flow the value of $S = \frac{V^2}{C^2\, y^{\frac{4}{3}}}$ into the equation for tractive force for uniform flow, $T = w\, S\, y$.

The use of narrow channels in the experimental study of the effect of tractive force upon sediment transportation invariably involves certain limitations and necessitates corrections due to side-wall effects. The writer has experienced considerable difficulty in this regard even in the case of channels as wide as 12 in. Unless corrections are made for side effects the results are likely to be appreciably in error.

Reference is here made to the analytical treatment of the observations as indicated in Table 3. The side influence has been taken into account in computing the water-surface slope in Column (11). However, the computation for the tractive force (Column (13)) is incorrect, first in that Equation (17) was used and, second, in that the traction on the side walls has been neglected. Computation by Equation (17) gives tractive forces on the stream bed which are too low, while neglecting the resistance due to the sides of the channel gives an indicated tractive force which is too high. These two errors of opposite sign in this particular case have a more or less counter-balancing influence upon the result, so that the tractive force indicated in the tabulation seems to be in the proper order of magnitude. Nevertheless, if the results are scrutinized it will be observed that the computed critical tractive force for sediment transportation is indicated in the tabulation to be greater for the deeper streams of water. This is due to the fact that the traction resistance on the sides of the channel has greater influence as the depth of flow increases.

In connection with extensive experimental work on sediment transportation the writer has observed the phenomena noted by the author relative to an apparent change in the law of transportation for finer sediments. The change is similar to that indicated by the author; however, thus far an abrupt critical from one law to another has not been noted. A change somewhat of the type indicated in Fig. 6 seems likely although the critical point does not seem to be accurately defined by experimental results. In the writer's opinion the visual determination of traction force is likely to be unsatisfactory for use in the application of the du Boys formula for determining the quantity of material transported. It will be found that the introduction of a value for the critical tractive force or its equivalent in terms of the velocity, roughness, and hydraulic radius of the stream into some form of the du Boys formula (which is considerably different from that usually recorded from direct observation) will more nearly

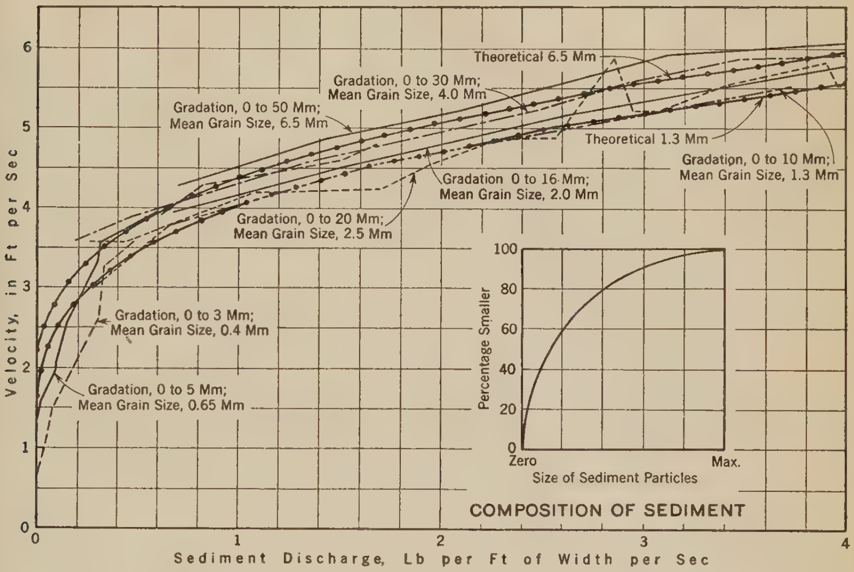

FIG. 22.—COMPARISON OF BED SEDIMENT TRANSPORTATION THEORY WITH SCHAFFERNAK EXPERIMENTS

correspond to the actual movement of the sedimentary load. This has been previously cited by the writer in connection with studies of the sedimentary characteristics of the Missouri River.[63] It is generally found that the observed critical tractive force indicating "general motion" recorded by most observers is too low to give best results in the application of the du Boys formula. A typical example is indicated in Fig. 22, which represents the comparison of actual sediment transportation observed by Schaffernak,[64] with a theoretical analysis by the writer. The theoretical curves in Fig. 22 are plotted only for two of the series of samples tested by Schaffernak. It is interesting to note that although the experiments by Schaffernak were made with water flowing at super-critical velocities, the same theoretical analysis was applicable as for sub-critical velocities, provided due account is taken of the proper roughness factor for the particular material concerned.

The theoretical curves in Fig. 22 were computed using the following formula derived by the writer for bed-load transportation:

$$G = \psi\, C^{-4}\, d^{-\frac{2}{3}}\, V^2\, (V^2 - V_0^2) \ldots\ldots\ldots\ldots\ldots\ldots (89)$$

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[63] "Missouri River Report," Appendix XV, U. S. Serial No. 9829, H. R. Doc. No. 238, 73d Cong. 2d Session.

[64] "Neue Grundlagen für die Berechnung der Geschiebeführung in Flussläufen," by Dr. F. Schaffernak, 48 pp., Franz Deuticke, Leipzig and Vienna, 1922.

in which C corresponds to the coefficient in Manning's formula—that is, $C = \frac{1.486}{n}$; d is the depth of flow; and V is the velocity of flow. The value of V_0 is determined from experimental data and corresponds approximately to the velocity at which transportation begins when computed by the relation,

$$T = w \frac{V^2}{C^2} \frac{1}{d^{\frac{1}{3}}} \qquad (90)$$

Thus, in Equation (89), V_0 is a function of critical tractive force as well as of the roughness of the stream bed and the depth of flow. The value of ψ is an experimental coefficient referred to as the "sediment characteristic" and is dependent upon the mechanical composition of the bed-load material; and G represents the sedimentary load. Equation (89) can also be written in other forms for use in accordance with the hydraulic factors available. The formula seems to give very good results for a wide variety of flow conditions. All the empirical constants are dependent entirely upon the mechanical composition of the sediment, provided riffle formation is not pronounced. The parameters have been determined by the writer for various types of sediment using experimental data compiled by various investigators and data collected in connection with sediment transportation studies at the University of Minnesota. In the case of the two theoretical curves superimposed upon the experimental data in Fig. 22, the characteristic values for the particular sediments were based upon the mean size of particle. Thus, for the gradation from 0 to 50 mm, the theoretical curve was computed for the diameter, 6.5 mm, and for the sedimentary material ranging in size from 0 to 10 mm, the mean size of 1.3 mm was used. For the coarser material, $\psi = 28\,000$, where G is measured in pounds per second per foot breadth of channel; $T_0 = 0.115$ lb per sq ft; and $n = 0.021$. In the case of the curve for the finer material the values are $\psi = 92\,000$, $T_0 = 0.0275$, and $n = 0.0165$. It should be noted that the critical velocity, V_0, at which sediment transportation begins is a function of the depth and roughness as well as of the critical tractive force; the latter being determined from data on rates of sediment transportation rather than simply being a visual determination of the beginning of sediment movement.

There are certain difficulties involved in any theoretical analysis when the riffle formation becomes a major importance. The difficulty appears to be primarily in the formulation of an accurate value for the roughness coefficient. This is particularly true in the case of shallow depths of flow. There are different modes of transportation when riffles form. Three types of bed-load movement can be clearly distinguished. In one instance, riffles progress forward after the fashion of sand dunes, practically all particles coming to resent periodically when reaching the leeward side of the dune. For this case the writer has found that the bed-load can be computed readily by simply taking into account the shape, height, and velocity of the riffles. Another extreme is the condition under which no riffles form—that is, when the bed is quite smooth. For this case the roughness coefficient has been reasonably well fixed and the bed-load can be computed with a reasonable degree of accuracy. There is an intermediate condition between these two extremes in which undulations or riffles of a rounded form appear on the surface of the bed.

Here there is a progressive movement of the riffles, but there is also general movement of the stream bed. Computation based solely on the height, shape, and speed of the riffle in such instances is found to give a sedimentary load less than the actual load. These different formations are related at least in part to the Froude number for the condition of flow. Thus, for a Froude number of unity or flow near the energy (velocity) critical, very pronounced dune-type riffles will frequently be observed with certain types of sand. The mechanical composition of sand also influences the character of the sediment transportation.

HUNTER ROUSE,[65] ASSOC. M. AM. SOC. C. E. (by letter).—A comprehensive treatment of sediment transportation is not an easy task, for at the present time the subject is in a pronounced state of flux. Moreover, for want of more conclusive experimental evidence, every man's opinion seems to differ from that of the next. The writer, therefore, confines his remarks to one portion of the author's mathematical derivation and to several phases of the general problem which warrant amplification in the light of research results not mentioned in the paper.

Perhaps the safest way to arrive at a general equation for tractive force is to follow the development of the energy equation for gradually varied flow. The basic expression,

$$E = \frac{V^2}{2\,g} + y + h_0 \quad \text{.......................(91)}$$

is usually differentiated with respect to distance in the direction of flow:

$$\frac{dE}{dx} = \frac{d\left(\frac{V^2}{2\,g}\right)}{dx} + \frac{dy}{dx} + \frac{dh_0}{dx} \quad \text{...................(92)}$$

The term at the left then represents the local rate of change of total head. Designating a downward slope as positive, $-\frac{dE}{dx}$ will equal S_e, the slope of the energy line. For want of accurate knowledge as to the rate of energy loss in gradually varied flow, this slope is necessarily assumed to be the same as that for uniform flow at the same mean velocity and depth;[66] thus,

$$S_e = \frac{V^2}{C^2\,y} \quad \text{.........................(93)}$$

The error involved in this assumption is probably not serious, in particular if the flow is in the tranquil state. Such an assumption as to the rate of energy loss is tantamount to making the boundary shear equal to that in uniform flow for the same values of y and V, because, regardless of how the rate of energy dissipation may be distributed over the flow section, the resulting shearing stresses are transmitted undiminished to the fixed boundary. It then follows that,

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[66] "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph Series, McGraw-Hill Book Co., 1932, p. 25.

$$\frac{dE}{dx} = -S_e = -\frac{V^2}{C^2\,y} = -\frac{T}{w\,y} \dots\dots\dots\dots\dots\dots (94)$$

The second term of Equation (92), showing the rate of change of velocity head, is written in terms of depth and mean velocity as follows:

$$\frac{d\left(\frac{V^2}{2\,g}\right)}{dx} = \frac{d\left(\frac{q^2}{2\,g\,y^2}\right)}{dx} = -\frac{q^2}{g\,y^3}\frac{dy}{dx} = -\frac{V^2}{g\,y}\frac{dy}{dx} \dots\dots\dots (95)$$

The third and fourth terms may be combined to yield the slope of the water surface,

$$\frac{d(y + h_0)}{dx} = -S_w \dots\dots\dots\dots\dots\dots\dots\dots (96)$$

while the fourth term alone represents the slope of the channel bottom:

$$\frac{dh_0}{dx} = -S_0 \dots\dots\dots\dots\dots\dots\dots\dots\dots (97)$$

Upon introducing these several equivalent values, the differential equation may be rewritten with the intensity of boundary shear as the dependent variable:

$$T = w\,y\left(\frac{V^2}{g\,y}\frac{dy}{dx} - \frac{dy}{dx} + S_0\right) \dots\dots\dots\dots\dots\dots (98)$$

Equation (98) is the general expression for tractive force in gradually varied flow, and is recommended in place of the author's Equation (15).

In the case of non-uniform flow in a horizontal channel—under which condition the author's experiments were conducted, $-S_0$ is zero and $\frac{dy}{dx}$ becomes equal to $-S_w$; whereupon,

$$T = w\,S_w\left(y - \frac{V^2}{g}\right) \dots\dots\dots\dots\dots\dots\dots\dots (99)$$

Equation (99) should replace the author's Equation (17).

Since the author so evidently sought to present an inclusive résumé of past research in this field, it is unfortunate that he failed to include a very valuable paper by A. Shields[67] which successfully answers more than one troublesome question touched upon in the paper. In this investigation it was shown that the relative magnitudes of tractive force and the resistance to motion of uniform sediment grains could be expressed dimensionlessly in the form, $\frac{T}{(\sigma - \rho)\,w\,D}$. (The writer presumes, despite evidence to the contrary, that the author's symbols for specific gravity of sediment and fluid, σ and ρ, have their usual dimensionless significance, as distinguished from w, the specific weight of water.) This ratio, therefore, represents a fundamental parameter in bed-load movement, subject to the possible inclusion of additional factors for shape and grading of the material. Through comparison with Nikuradse's analysis of flow in artificially roughened pipes,[68] it was then proved that the beginning of movement from a leveled bed of uniform grains should depend upon a single additional parameter, $\sqrt{\frac{g\,T}{w}}\,\frac{D}{\nu}$. This parameter, furthermore, is proportional to the ratio of grain diameter to boundary-layer thickness, and as such has already been used to co-ordinate the measurements of pipe resistance for various conditions of relative roughness.[69] In other words,

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[67] "Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau, Heft 26, Berlin, 1936. Translations are on file at the Engineering Societies Library, 33 West 39th Street, New York, N. Y.; National Hydraulic Laboratory, Washington, D. C.; and the U. S. Waterways Experiment Station, Vicksburg, Miss.

$$\frac{T_0}{(\sigma - \rho)\,w\,D} = \varphi\left(\frac{D}{\delta}\right) \qquad (100)$$

Shields made careful measurements of initial movement on beds of pulverized amber, lignite, granite, and barite, each of uniform size, and plotted his results dimensionlessly—together with those of other investigators using sand—as shown in Fig. 23. Despite considerable variation in specific gravity, diameter, and conditions of flow, the plotted values are seen to follow a single functional trend. At the left the laminar boundary layer still envelopes the particles on the bed, and the initial motion is not dependent upon turbulence; conditions of flow are, therefore, similar to those along a smooth boundary. When the particles begin to protrude through the laminar layer, initial effects of boundary roughness are noticeable, and the curve gradually approaches a minimum value. An upward trend begins when particles become sufficiently large to disrupt the laminar flow around them. An asymptotic limit is finally reached when the boundary layer no longer exists, owing to the turbulent wake produced by bed roughness of relatively great magnitude. The form of the curve closely approximates that resulting from Nikuradse's measurements on rough pipes.[68, 69]

Although the problem of initial movement has also been approached in terms of boundary-layer thickness by W. W. Rubey[70] and others, Shields' analysis appears to provide the solution sought by Professor Chang (compare with Figs. 6 and 7). Shields further noted that the relative dimensions of D and δ also govern the form of the initial bed irregularities. Particles moving within the laminar layer show a pronounced tendency to form ripples. Sediments slightly disrupting the boundary layer form "scales"—that is, longer, lower ripples. The ratio of length to height of the bed undulations increases steadily with the parameter, $\frac{D}{\delta}$, until finally the entire surface of the bed moves fairly uniformly as the curve of initial motion approaches its horizontal asymptote.

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[68] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, VDI Forschungsheft 361, 1933. See "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 485.

[69] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, Eng. Societies Monograph Series, McGraw-Hill Book Co., 1938, p. 251.

[70] "The Force Required to Move Particles on a Stream Bed," by William W. Rubey, U. S. Geological Survey, *Professional Paper 189–E*, 1938.

In Fig. 23 values of the bed-load parameter greater than the minimum evidently represent stages of movement beyond the initial phase. Shields, therefore, used this parameter in the form, $\frac{(T - T_0)}{(\sigma - \rho)\, w\, D}$, to develop an equation for rate of transport. The remaining dimensionless parameter is composed of the

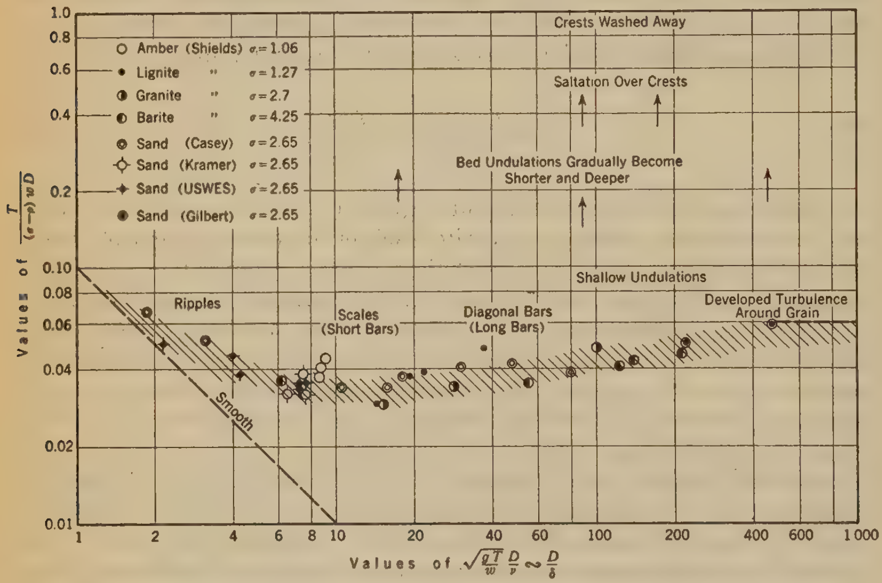

Fig. 23.—Dimensionless Plot of Initial Bed-Load Movement for Sediment of Uniform Size, According to A

ratio of weight discharges of sediment and water, the specific gravities of sediment and water, and the slope. In order that the viscosity may have no appreciable influence upon the movement, the analysis is restricted to conditions in which the ratio of height to length of the bed undulations is low. Measured rates of movement for the five different types of material, plotted in Fig. 24, are seen to follow the equation,

$$\frac{G}{Q\,S}\,\frac{\sigma - \rho}{\rho} = 10\,\frac{T - T_0}{(\sigma - \rho)\,w\,D} \quad \text{.................... (101)}$$

over nearly a thousand-fold range on either scale. The scattering of points, so characteristic of experimental results of this nature, is attributed to long-period cycles of bed movement; in addition, points for shooting flow lie somewhat higher than the others. This relationship is not to be considered an ultimate statement of bed-load transport, for the influence of shape and grading of the sediment grains still remains to be determined.

With regard to the author's treatment of suspended load, the writer would call attention to the fact that the validity of Equation (51) has been proved experimentally.[71] On the other hand, it remains to be seen whether the magnitude of ϵ determined from the velocity gradient in turbulent flow will correspond numerically to that for the sediment concentration at the same elevation. Otherwise, abundant evidence of a logarithmic velocity distribution in wide channels would seem to justify the form of the suspended load function published by the writer.[32]

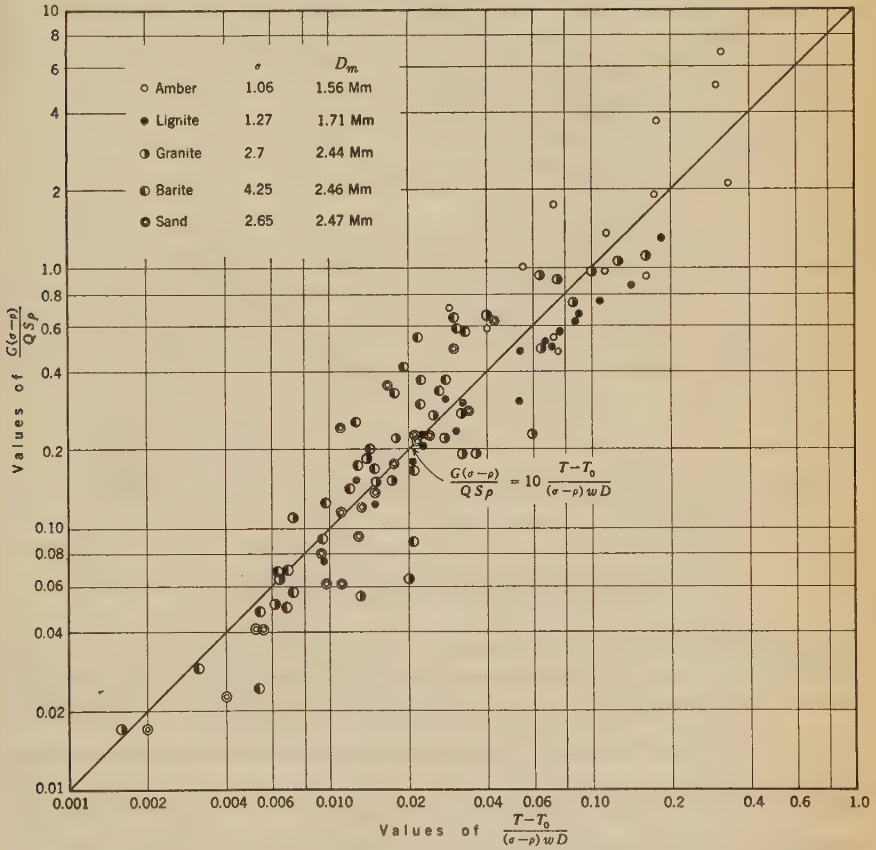

FIG. 24.—DIMENSIONLESS PLOT OF BED-LOAD TRANSPORTATION FOR SEDIMENT OF UNIFORM SIZE, ACCORDING TO A

The author attempts to integrate a simplified distribution function in order to express the total suspended load carried by a given flow—the total load, that is, of a given particle size. Obviously, in the case of a graded sediment, the process must be repeated for every different size. It is possible to simplify this procedure for specific conditions in a manner which may also prove to have general application. If the fall velocities of the various sizes of sediment at the level, a, follow a normal distribution curve (that is, linear when plotted on logarithmic probability paper), the ratio of total concentration, C, at any higher level to that at the level, a, will follow closely the distribution curve for the

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[71] "Experiments on the Mechanics of Sediment Suspension," by Hunter Rouse, *Proceedings*, 5th International Congress for Applied Mechanics, 1939.

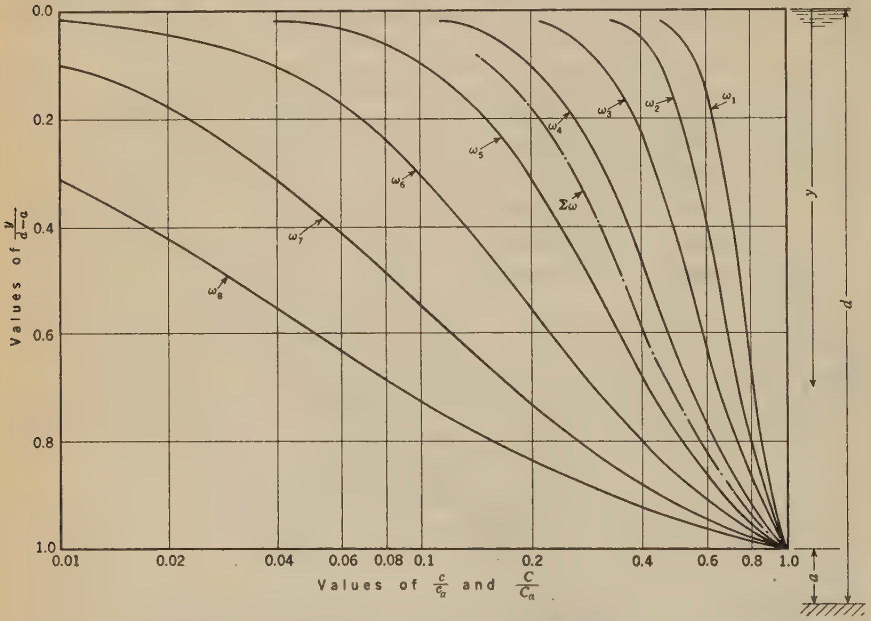

Fig. 25.—Relative Distribution of Total Suspended Load in the Case of a Normally Graded Sediment at Level a

particle size having the geometric mean fall velocity. This relationship is shown schematically in Fig. 25 for the distribution function of Equation (54), although it should hold equally well for any other analytic relationship between ϵ and y. Evidently, the distribution of $\frac{C}{C_a}$ (shown as a broken line in Fig. 25) is practically the same as that of $\frac{c}{c_a}$ for the geometric mean value of ω. Graphical integration according to the assumed (or measured) velocity distribution curve would then be a relatively simple matter.

Y. L. Chang,[72] Esq. (by letter).—The valuable criticisms and ingenious contributions to the discussion have been of great benefit. Captain Kramer does not agree on the hypothesis of the tractive force on the stream bed by a non-uniform flow. The writer believes, however, that the investigation of the influence of non-uniform flow (although it complicates the problem) is undoubtedly necessary in order to study the actual condition of debris transportation in natural water courses. In the derivation of the expression for the decrease of potential energy, the same correction has been made independently by Professors Huang, Straub, and Rouse, and a revised conclusion has been drawn regarding the tractive force due to an accelerating flow in a horizontal channel bottom. The writer derived these equations on the basis of certain assumptions and certain reasoning which were not given in detail in the paper.

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[72] Associate Prof. of Civ. Eng., Tsing Hua Univ., Yunnanfu, Yunnan, China; and Cons. Engr. to the Provincial Government of Kweichow, Kweiyang, Kweichow, China.

First, the writer considers the acceleration due to a convergence of stream lines in a non-uniform flow. The hypothesis is shown diagrammatically in Fig. 26. Professor Bakhmeteff has stated[73] that "when the stream lines are substantially inclined toward the cross-sectional plane, the acceleration $o\,a$ may have a noticeable component $o\,a'$ in the cross-sectional plane, the effect of which again will be to modify the distribution of pressures as caused by gravity alone." In the writer's derivation of the expression of potential energy between the stretch dx a mean excessive pressure of $\frac{w\,dy}{2}$ exerted on the down-stream cross-sectional plane is included. Moreover, Professor Bakhmeteff definitely shows that "elementary hydromechanics teaches that the distribution of pressure in a cross-section of moving fluid will obey the hydrostatic law and will be affected solely by gravity, when, and only when, flow takes place in such a manner that the fluid filaments have no acceleration components in the plane of the cross-section." Therefore, in the case of accelerated flows, it is obviously unsound to conclude that throughout the depth at any section the potential energy is equal to that at the water surface.

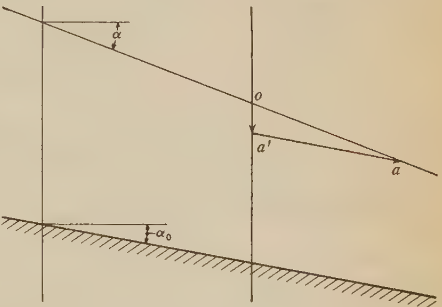

FIG. 26

Secondly, the experimental results of tractive force obtained in this particular flume are more likely consistent with the foregoing theory. However, this has been clarified by Professor Straub in his discussion that computation by Equation (17) gives values of tractive force that are too low, whereas neglecting the resistance due to the side-wall of the channel gives values that are too high. These two deviations have a more or less counterbalancing influence upon the result, so that the tractive force indicated in Table 3 seems to be in the proper order of magnitude.

In illustrating the effect of converging flow the following derivation is worth studying. Let $A\,B$ in Fig. 27 be a short stretch, within and near which the surface curve can be considered as a straight line with a slope α, and converges with the bottom at an angle $(\alpha - \alpha_0)$. The section that approximates the equipotential section at Point A most reasonably is an arc $A\,c$ struck with Line $A\,C$ as a radius. Call this arc length s; then,

$$s = r\,(\alpha - \alpha_0) \qquad \text{(102)}$$

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[73] "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E., Engineering Societies Monographs, 1932, p. 29.

in which r is the radius. The relationship between the vertical depth y and s could be formulated thus:

$$\overline{A\,c} = y \frac{\cos\alpha}{\cos\left(\dfrac{\alpha - \alpha_0}{2}\right)} \quad \text{(103)}$$

$$r = \frac{\overline{A\,c}}{2} \frac{1}{\sin\left(\dfrac{\alpha - \alpha_0}{2}\right)} \quad \text{(104)}$$

and,

$$s = \frac{y}{2} \frac{(\alpha - \alpha_0)\cos\alpha}{\sin\left(\dfrac{\alpha - \alpha_0}{2}\right)\cos\left(\dfrac{\alpha - \alpha_0}{2}\right)} = \frac{y\,(\alpha - \alpha_0)\cos\alpha}{\sin\,(\alpha - \alpha_0)} \quad \text{(105)}$$

Equating the energy expressions:

$$\frac{w}{2\,g}\left[(V + dV)^2 - V^2\right] + \frac{T}{s}\,dx = w\,dx\sin\alpha \quad \text{(106)}$$

As the volume considered is a unit with curved height s, the area subjected to tractive force should be $\frac{1}{s}$. From Equation (106) the expression of tractive force becomes

$$T = \left(-\frac{w\,V}{g}\frac{dV}{dx} + w\sin\alpha\right)s \quad \text{(107}a\text{)}$$

or,

$$T = w\,(\alpha - \alpha_0)\left[y\frac{\cos\alpha\sin\alpha}{\sin\,(\alpha - \alpha_0)} - \frac{V^2}{g}\right] \quad \text{(107}b\text{)}$$

For uniform flow, $\alpha = \alpha_0$, and

$$\operatorname*{Limit}_{\alpha=\alpha_0} T = w\,y\cos\alpha\sin\alpha \quad \text{(108}a\text{)}$$

When α is small,

$$T = w\,y\,\alpha \quad \text{(108}b\text{)}$$

For non-uniform flow on a horizontal bottom, $\alpha_0 = 0$, and

$$T = w\,\alpha\left(y\cos\alpha - \frac{V^2}{g}\right) \quad \text{(109}a\text{)}$$

When α is small,

$$T = w\,\alpha\left(y - \frac{V^2}{g}\right) \quad \text{(109}b\text{)}$$

The writer believes that this derivation is more rational than the others previously given. As an arc of equal potential is considered there is no accelerating component in the directions normal to the stream lines. In Fig. 27 one observes that the potential drop of the water surface within the stretch dx, $dx\sin\alpha$, is evidently less than $dx\cos\alpha_0\tan\alpha$, when two vertical depths $A\,a$ and $B\,b$, instead of two circular arcs $A\,c$ and $B\,d$, are taken for reference. This supports the writer's original assumption that the difference of potential energy between the reach dx is less than that represented by the vertical drop from a to *b.* One can easily note, however, that the derivation is based on certain assumptions, such as:

(1) There is no excessive pressure due to convergence of flow on the arc $B\ d$;
(2) The distribution of static pressure is linear along the arcs.

One can also note that there is a question whether the tractive force should vanish when super-critical velocity occurs in a flume with a horizontal channel bottom. Unless assumptions of this nature could be proved and established definitely, one can scarcely venture to conclude that the foregoing expressions are the correct ones. For the time being, perhaps, it can be presumed that the derivation in the original paper, with due consideration for the convergence of flow, is possibly of the right order for high accelerating flows, whereas the revised expressions are more suitable for cases not much different from parallel flow.

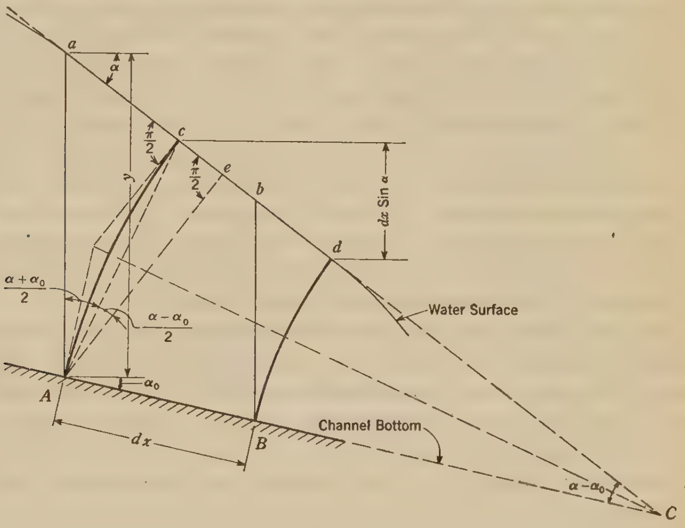

FIG. 27

Professor Lane takes a practical view of the situation pertaining to the bed-load movement in the Mississippi River. Accepting Equation (107*b*) as the correct expression and using Professor Lane's data, it can be shown that there will be motion upward on the sand slopes when the mean velocity of flow does not exceed $\sqrt{g\ y}\sqrt{\dfrac{\cos\alpha\sin\alpha}{\sin(\alpha-\alpha_0)}}$; or, numerically 1.44 ft per sec, above which sediments should move against the direction of flow. This only reveals one of the ambiguities encountered in these latter revisions. Actual records can be found in a report of the Mississippi River Commission.[74] The observation at Bullerton gives: $\alpha_0 = -\dfrac{4.4}{300} = -0.01466$; $\alpha = \dfrac{\alpha_0}{200} = 0.0000733$; $y = 10$ to

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[74] "The Improvement of the Lower Mississippi River for Flood Control and Navigation"; prepared under the direction of T. H. Jackson, President, Mississippi River Commission, by D. O. Elliott, U. S. Corps of Engineers, Vol. 1, Table XXIII, St. Louis, Mo., 1932.

30 ft; and $V = 3.5$ ft per sec. The report also states[75] that "the quantity of material comprising sand waves was very small as compared with the quantity of material transported in suspension by the river. In the observations on the Fulton Section, Plum Point Reach, it was estimated that the quantity of material composing the sand wave was equal to not more than 2.5% of that carried in suspension."

In discussing sand dune progression the wave action of the water in the vicinity of the dunes should also be considered. A detailed investigation has been reported elsewhere.[76]

Regarding the question as to whether the silt load tends to decrease or increase the mean velocity of flow, the writer believes that the problem will be: Which is the more prominent factor—the viscosity of flow or the change of kinetic energy due to the suspension? The variation of the state of turbulence with the presence of silt in flow also might be one of the factors involved.

Mr. Johnson expresses regret that, with so many experiments and observations available, a general formula is still lacking that will express the quantity of debris movement. Painstakingly, he traces through a method of statical analysis to determine whether a particular formula offers distinct advantages over other formulas. He also reveals, correctly, the sources of error that are likely to affect the result in flume studies; but thus far, the application of these remedies in laboratory testing is not at all easy.

The part of the paper concerning the transportation of silt in suspension has been improved in several respects by Mr. Christiansen's discussion. He defines the validity of the suggested equations clearly. Other parts of the discussion, pertaining to the total load in suspension, and to the effect of silt on kinetic energy or on the Austausch coefficient of a stream, are also helpful. With regard to Samples 6 198 and 6 197, it is obvious that D is assumed equal to D_2, because of the flatness of the grains observed under the microscope.

Referring to Equation (33), the writer wishes to add the following equations so as to facilitate comparison with the other representative expressions reviewed under the heading "Published Authorities." Knowing that $T = 1.609\ w^2\, n^{6/5}\, s^{7/5}\, Q^{6/5}$, it is not difficult to convert Equation (33) into the following approximate form:

$$G = \frac{C\, n^{11/5}}{T_0^2}\, s^{7/5}\, Q^{3/5}\, (Q^{3/5} - Q_0^{3/5}) \ldots\ldots\ldots\ldots (110)$$

in which $Q_0^{1.2}$ varies as $k_1 \frac{\rho_e\, D_0^{1/3}\, 2\,\beta}{n^{1.2}\, s^{1.4}\, M^{2\beta}}$. When ρ_e, n, and M are constant, and $\beta = \frac{1}{2}$,

$$Q_0^{1.2} = k_2 \frac{\rho_e\, D}{s^{1.4}} \ldots\ldots\ldots\ldots (111)$$

in which C, k_1, and k_2 are constants.

In addition to the authorities cited in the paper, the writer wishes to include two more recent publications on debris transportation:

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[75] "The Improvement of the Lower Mississippi River for Flood Control and Navigation"; prepared under the direction of T. H. Jackson, President, Mississippi River Commission, by D. O. Elliott, U. S. Corps of Engineers, Vol. 1, Table XXIII, St. Louis, Mo., 1932, p. 121.

[76] "Stauraumverlandung und Kolkabwehr," by A. Schoklitsch, Julius Springer, Wien, 1935.

X.—H. J. Casey.—In the summary of his paper,[77] Casey gives two formulas for the rate of transportation, based on experiments with quartz sand (specific gravity, 2.65) with surface slopes ranging from $\frac{1}{400}$ to $\frac{1}{1\,200}$:

$$G = 0.367\, s^{9/8}\, (q - q_0) \quad \text{(112a)}$$

and

$$G = 0.333\, s\, (q - q_0) \quad \text{(112b)}$$

in which $q_0 = k \frac{D^{0.75}}{s^{1.25}}$, G and q are units of volume, and k is a constant. Equation (112*a*) is for uniform sand 0.5 mm $< D <$ 3.0 mm, whereas Equation (112*b*) is for mixtures.

XI.—A. Shields.—In his paper, Shields[78] attempts to connect the bed-load movement with turbulence and dynamic similarity. He obtains a dimensionless equation of the form:

$$\frac{G}{Q}\left(\frac{\sigma - \rho}{s\,\rho}\right) = 10\, \frac{T - T_0}{(\sigma - \rho)\, w\, D} \quad \text{(113)}$$

Professor Rouse has discussed this work in some detail.

It may be interesting to demonstrate that there is a critical depth at which the mean load will be a maximum with all other hydraulic factors remaining unchanged. Using Equation (55) and equating to zero, the first differentiation of c_m with respect to d, one finds:

$$\alpha\, d\, e^{\alpha\, d} - e^{\alpha\, d} + 1 = 0 \quad \text{(114)}$$

This condition could be satisfied only when $d = 0$. Therefore it may be assumed that the shallower the depth the heavier the concentration of silt in suspension will be, provided that other factors beside d remain unchanged.

Observations in the Yungtingho,[79] near Peiping, China, have made it possible to derive two empirical formulas for the total quantity of silt transported in suspension; one in terms of the flow discharge Q:

$$P = a\, Q^{1.54} \quad \text{(115a)}$$

in which $a = 0.0095$ as an average (its maximum and minimum are, respectively, 0.0363 and 0.0026); and the other in terms of the flow velocity V:

$$P = b\, V^{3.89} \quad \text{(115b)}$$

in which $b = 2.158$ as an average (its maximum and minimum are, respectively, 45.60 and 0.293). Both Q and V are in metric units and P is expressed as a percentage by weight. By combining Equations (115*a*) and (115*b*) one finds that V varies as $d^{0.656}$, which is not far from that given by Manning's formula.

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[77] "Über Geschiebebewegung," by H. J. Casey, Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Heft 19, 1935.

[78] "Anwendung der Ähnlichkeitsmechanik und Turbulenzforschung auf die Geschiebebewegung," by A. Shields, Mitteilung der Preussischen Versuchsanstalt fur Wasserbau und Schiffbau, Berlin, Heft 26, 1936.

[79] "A Complete Scheme for Controlling Yungtingho," Tientsin, China, Vol. 1, 1933, pp. 119–143.

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THE THREE-POINT PROBLEM IN A CO-ORDINATED FIELD

By R. Robinson Rowe,[1] M. Am. Soc. C. E.

With Discussion by Oscar S. Adams, George D. Whitmore, John R. Jahn, and R. Robinson Rowe.

Synopsis

An improved method of solving the classical three-point problem, particularly adapted to orientation or location in a previously co-ordinated field, and to calculation by machine instead of by logarithms, is offered in this paper. The probable normal and position errors due to errors of observation are evaluated to measure the precision of the location and, in case more than three points are observed, to aid in the selection of the best three-point set or to weight the adjustment of all sets. A graphical adjustment of the "four-or-more-point" problem is outlined and a practical problem is solved to illustrate the use of the formulas.

The Problem

In Fig. 1, Points O, A, B, C, and D are triangulation stations with known co-ordinates in a plane system. If an observer at Point P measures the angle, $O\,P\,A = \alpha$, he has limited his position locus to the circle, $O\,P\,A$. If he then measures Angle $O\,P\,B = \beta$, he determines his position as the intersection of the circles, $O\,P\,A$ and $O\,P\,B$, unless these two circles coincide. Of course, the circles intersect at two points, but one will be Point O; and it can be shown that Angle $A\,P\,B = \beta - \alpha = \theta$ determines a third circle which passes through the intersection at Point P. Hence, the observer should measure all three angles and adjust the results before further computation; or he should measure the two angles defining the circles with the best intersection.

If one measures Angles θ and $C\,P\,D$, he defines two position-locus circles ordinarily intersecting at two points. His position is fixed if he can decide between them—by a compass bearing, for instance. Such data may give strong locations, but they will be weak if the circles are nearly tangent, or if they are coincident.

Note.—Published in June, 1938, *Proceedings*.

[1] Associate Bridge Engr., State Dept. of Public Works, Sacramento, Calif.

With four points, A, B, C, and D, visible, it is best to measure the three angles, θ, $B\,P\,C$, and $C\,P\,D$, and also Angle $A\,P\,D$ for check and adjustment. Then, Angles $A\,P\,C$ and $B\,P\,D$ may be calculated, so that six position-locus circles are defined. Unless all data are exact, there will be four independent points of intersection, due to errors of observation at Point P, errors in the previous co-ordination of the signals, and to the plane projection distortion of the field. If the four intersections are tolerably approximate, considering the probable error of angular measurements, a mean position can be determined by least squares or by the graphical method outlined herein. Otherwise, it may be assumed that at least one of the signals was not accurately located; or, that it has since shifted in position. Such a shift should be anticipated when depending upon old co-ordinates of spires, due to uneven subsidence, translation by earthquake, or altered reconstruction after fires.

If but one signal is in error, a fifth signal can be used to determine which should be rejected. Then, a complete set would determine ten circles and ten independent intersections. A graph of these intersections would show four points tolerably approximate, the other six being erratic and dependent upon the signal which must be rejected.

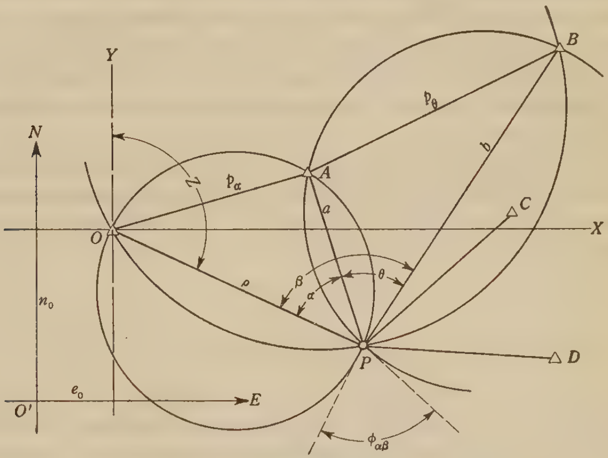

FIG. 1

Use of six or more signals increases the labor of solution, but there is the possibility that more than one station must be rejected or that some locus circles will intersect at very acute angles, too small for the desired precision. The field data to all visible signals should be obtained. In general, for observations on n signals, the number of locus circles, N_c, is:

$$N_c = 0.5\,n\,(n-1) \quad \ldots\ldots (1)$$

and the number of independent intersections, N_i, is:

$$N_i \doteq \frac{n!}{6\,(n-3)!} \quad \ldots\ldots (2)$$

Along the coasts, many areas have been co-ordinated by the United States Coast and Geodetic Survey and the United States Engineer Office. Re-establishment of the instrument points is not always simple, but co-ordinated spires are usually abundant. The three-point method affords a practical device for: (1) Connecting to the U. S. Coast and Geodetic horizontal datum (plane and geographical co-ordinates); (2) determining azimuth without astronomical observations, particularly when the sun or the stars are obscured, or when they are not in favorable position; or (3) controlling a stub traverse by three-point ties near each end, which, incidentally, serves to check base lines or rate the tape used in the traverse. The solutions of connections required in a resurvey of a part of the mean high-tide line of Coronado, South Island, California, in 1929, led to the development of the methods described in this paper.

SOLUTIONS

The graphical solution on the plane-table is known to all topographers and is explained in most surveying texts. The methods in this paper are not applicable to such work. The usual trigonometric solution requires: (*a*) Computing closing courses and distances between the observed signals; (*b*) solving a series of three triangles which depend upon the locus circles (and two more triangles for a check); and (*c*) computing latitudes and departures from one signal to the instrument position. Reference is made to any surveying textbook for details of the operations.

A simpler method is derived as follows: In Fig. 1, the given co-ordinates of triangulation stations at Points O, A, and B, referred to rectangular Cartesian axes, $O'N$ and $O'E$, are (e_o, n_o), (e_a, n_a), etc. By the linear transformation, the formulas,

$$x = e - e_o \qquad (3a)$$

and,

$$y = n - n_o \qquad (3b)$$

refer all co-ordinates to the auxiliary origin, O, such that $O'N$ is parallel to OY and perpendicular to OX, and the co-ordinates are reduced by subtraction to $O = (0, 0)$, $A = (x_a, y_a)$, etc. To determine the position of Point P, Angles α and β have been measured. Let the azimuth of OP be Z = Angle YOP.

Determining the equations of Lines OP, AP, and BP, Fig. 1, from slope forms, and remembering that $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$,

Line OP:

$$\frac{y}{x} = \frac{y_p}{x_p} = \cot Z \qquad (4a)$$

Line AP:

$$\frac{y - y_a}{x - x_a} = \frac{y_p - x_p \tan\alpha}{x_p + y_p \tan\alpha} \qquad (4b)$$

and, Line BP:

$$\frac{y - y_b}{x - x_b} = \frac{y_p - x_p \tan\beta}{x_p + y_p \tan\beta} \qquad (4c)$$

and, these lines being concurrent at Point P, simultaneous solution of their equations will determine the position of P. Substituting Equation (4a) in

Equations (4*b*) and (4*c*) to eliminate x_p and y_p, then:

$$(x^2 + y^2) \tan \alpha = x\,(x_a \tan \alpha + y_a) + y\,(y_a \tan \alpha - x_a) \tag{5a}$$

and,

$$(x^2 + y^2) \tan \beta = x\,(x_b \tan \beta + y_b) + y\,(y_b \tan \beta - x_b) \tag{5b}$$

Dividing Equation (5*a*) by Equation (5*b*) and simplifying:

$$\frac{y}{x} = \frac{(x_b - x_a) \tan \alpha \tan \beta + y_b \tan \alpha - y_a \tan \beta}{(y_a - y_b) \tan \alpha \tan \beta + x_b \tan \alpha - x_a \tan \beta} \tag{6}$$

Dividing the third and fourth parts by $\tan \alpha \tan \beta$ and substituting Equation (4*a*):

$$\cot Z = \frac{x_a - x_b + y_a \cot \alpha - y_b \cot \beta}{-\,y_a + y_b + x_a \cot \alpha - x_b \cot \beta} \tag{7}$$

At Point P in Equation (5*a*), $x = x_p$; $y = y_p$; $x^2 + y^2 = x^2 \csc^2 Z$; and, $y = x \cot Z$. Substituting these values in Equation (7):

$$x_p{}^2 \csc^2 Z \tan \alpha = x_p\,(x_a \tan \alpha + y_a) + x_p \cot Z\,(y_a \tan \alpha - x_a) \tag{8a}$$

$$x_p = \sin^2 Z\,[x_a + y_a \cot \alpha + \cot Z\,(y_a - x_a \cot \alpha)] \tag{8b}$$

$$y_p = \cos^2 Z\,[\tan Z\,(x_a + y_a \cot \alpha) + y_a - x_a \cot \alpha] \tag{8c}$$

and,

$$l = O\,P = \sin Z\,(x_a + y_a \cot \alpha) + \cos Z\,(y_a - x_a \cot \alpha) \tag{8d}$$

For the most direct solution, solve Equation (7) for Z; then Equation (8*b*) for x_p; and then:

$$y_p = x_p \cot Z \tag{9}$$

If Z is not required, solve Equation (7) for $\cot Z$; and, for Equation (8*b*), use

$$x_p = \frac{1}{1 + \cot^2 Z}\,[x_a + y_a \cot \alpha + \cot Z\,(y_a - x_a \cot \alpha)] \tag{10}$$

In Equation (7) there will be two solutions for Z differing by 180°; but if the wrong Z is selected, then l from Equation (8*d*) will be negative and the same point will be determined by either choice of Z. The other equations are free from ambiguity except in singular cases. In the insoluble case with O, A, B, and P lying on a circle, Equation (7) gives $\cot Z = \frac{0}{0}$. If $\alpha = 0°$ or $180°$, $\cot Z = \frac{\infty}{\infty}$, the apparent indeterminacy having been introduced by the division by $\tan \alpha \tan \beta\ (= 0)$. This case is best solved by a simple resection triangle; or, determining Equation (7) by limits:

$$\cot Z = \frac{y_a}{x_a} \tag{11}$$

and, substituting Equation (11) in Equation (5*b*),

$$x_p = \frac{x_a}{x_a{}^2 + y_a{}^2}\,[x_a\,x_b + y_a\,y_b + \cot \beta\,(x_a\,y_b - x_b\,y_a)] \tag{12}$$

To complete the solution, co-ordinates to the origin, O', can be computed by reversing the transformation of Equations (3) by:

$$e_p = x_p + e_o \qquad \text{(13a)}$$

and,

$$n_p = y_p + n_o \qquad \text{(13b)}$$

If true azimuth is desired, Z must be corrected for the difference in longitude represented by the distance, e_p.

RELATIVE WEIGHTS AND FIGURE STRENGTHS

In Fig. 1, with Angle θ observed between given Signals A and B, the equation of Circle $P\ A\ B$ is:

$$\begin{aligned} x^2 + y^2 - x\,[x_b + x_a + (y_b - y_a)\cot\theta] - y\,[y_b + y_a - (x_b - x_a)\cot\theta] \\ + x_b\,x_a + y_b\,y_a + (x_a\,y_b - x_b\,y_a)\cot\theta = 0 \qquad \text{(14)} \end{aligned}$$

The tangent to this circle at Point P is:

$$\begin{aligned} & x\,[2\,x_p - x_a - x_b + (y_a - y_b)\cot\theta] \\ & \quad + y\,[2\,y_p - y_a - y_b - (x_a - x_b)\cot\theta] + 2\,x_a\,x_b \\ & \quad + 2\,y_a\,y_b - x_p\,(x_a + x_b) - y_p\,(y_a + y_b) \\ & \quad + \cot\theta\,[2\,x_a\,y_b - 2\,x_b\,y_a + x_p\,(y_a - y_b) - y_p\,(x_a - x_b)] = 0 \qquad \text{(15)} \end{aligned}$$

The slope of the circle and tangent at Point P is:

$$\lambda_{p(ABP)} = -\frac{2\,x_p - x_a - x_b + (y_a - y_b)\cot\theta}{2\,y_p - y_a - y_b - (x_a - x_b)\cot\theta} \qquad \text{(16a)}$$

and the slope for Circle $O\ A\ P$, substituting α for θ and $x_b = y_b = 0$:

$$\lambda_{p(OAP)} = -\frac{2\,x_p - x_a - y_a\cot\alpha}{2\,y_p - y_a + x_a\cot\alpha} \qquad \text{(16b)}$$

In the general triangle, $A\ B\ P$, let the three angles be A, B, and θ, and let the opposite sides be b, a, and p_θ, respectively. If there was a small error, $\Delta\theta$, in the measurement of θ there will be little change in the slope from Equation (16a). Practically, the tangent is in error by a normal distance, ΔN, such that:

$$\begin{aligned} \Delta N &= \tfrac{1}{2}\,p_\theta\,[\Delta(\csc\theta) + \Delta(\cot\theta)\cos(A - B)] \\ &= -\frac{p_\theta\,\Delta\theta}{2\sin^2\theta}\,[\cos\theta + \cos(A - B)] \\ &= -\,p_\theta\,\Delta\theta\,\frac{\sin A\sin B}{\sin^2\theta} = -\,\Delta\theta\,\frac{ab}{p_\theta} \qquad \text{(17)} \end{aligned}$$

If the error in seconds of arc is expressed by δ'', and N is the absolute normal error:

$$N_\theta = \frac{(\delta'')\,a\,b}{206\,265\,p_\theta} \qquad \text{(18)}$$

In the use of Equation (18) in the solution of four-or-more-point systems, it is necessary to solve for N only to two or three significant figures. Having approximated P from one set of three points and λ for any pair of those points, a, b, and p can be scaled; δ'' can be determined as the probable error of that measurement; and, N can be computed. Now, the angle, $\theta \pm \delta''$, has limited the locus for P to a band of width, $2\ N$, slope, λ, centered on Point P, since a short part of the locus circle is essentially a straight line. Intersection of this band with bands pertinent to other combinations of points should further limit this locus area. If probable errors for all sets are then gradually reduced in the same ratio until only one point is common to all bands, there will result an acceptable solution which will generally be very close to the least squares solution. The reduction can be done rapidly by cut-and-try on a large-scale graph.

If the bands do not have a common area, at least one error will exceed the computed probable error. To obtain a solution, gradually increase the widths of all bands in the same ratio until there is just one point common to all. In either case, the common point will be the intersection of the edges of three bands. If two of these critical bands are nearly parallel, the solution can usually be improved by letting these two nearly parallel band edges determine their median as a line locus; then the width of all other bands can be decreased gradually in proportion to their normal errors until the common area of these remaining bands has only one point on the determined line locus. This adjustment will have increased the departure from the nearly parallel bands by a small amount, but will have decreased the departure from most of the other bands by a relatively large amount.

Equation (18) can be used reciprocally for weighting a series of angles observed in a four-or-more-point system. The relative strength of different such figures is found by holding δ'' constant; and, since constants are unnecessary in strength comparison, the relative figure strength may be expressed as:

$$s_\theta = \frac{p_\theta}{a\ b} \qquad \text{(19)}$$

To obtain the absolute position error from a three-point observation with only two angles measured (such as Angles α and θ) compute N_α and N_θ from Equation (18), measure or compute $\phi_{\alpha\theta}$, the acute angle between slope tangents, $\lambda_{p(OPA)}$ and $\lambda_{p(APB)}$, from Equation (16a) or Equation (16b). Then, the absolute position error, $\Delta_{\alpha\theta}$, is the long semi-diagonal of a parallelogram with one angle, $\phi_{\alpha\theta}$, and altitudes, N_α and N_θ; or,

$$\Delta_{\alpha\theta} = \csc \phi_{\alpha\theta} \sqrt{N_\alpha^2 + N_\theta^2 + 2\ N_\alpha\ N_\theta \cos \phi_{\alpha\theta}} \qquad \text{(20a)}$$

and, reciprocally, the relative strength of the three-point figure compared to others is:

$$S_{\alpha\theta} = \frac{p_\alpha\ p_\theta \sin \phi_{\alpha\theta}}{a \sqrt{l^2\ p_\theta^2 + 2\ b\ l\ p_\alpha\ p_\theta \cos \phi_{\alpha\theta} + b^2\ p_\alpha^2}} \qquad \text{(20b)}$$

If all three angles, α, β, and θ, were measured, the probable errors of each observation are interdependent and there would be three solutions for Equations (20). If the angles and probable error are balanced, the three normal-error bands will intersect with a hexaparallelogram for a common area, and the position error is the greatest of the three semi-diameters,

$$\Delta_{\alpha\beta\theta} \underset{>}{\equiv} N_\theta \csc \theta_{\beta\theta} \qquad (21a)$$

or, $N_\alpha \csc \phi_{\alpha\beta}$, etc. Reciprocally, the relative strength of the three-point figure with all three angles measured, is,

$$S_{\alpha\beta\theta} \underset{<}{\equiv} S_\alpha \sin \phi_{\alpha\theta} \text{ (etc.)} \qquad (21b)$$

Example

Fig. 2 shows positions of seven spires observed from Point P, which was taken arbitrarily on the roof of the Guymon Building, in San Diego, Calif.

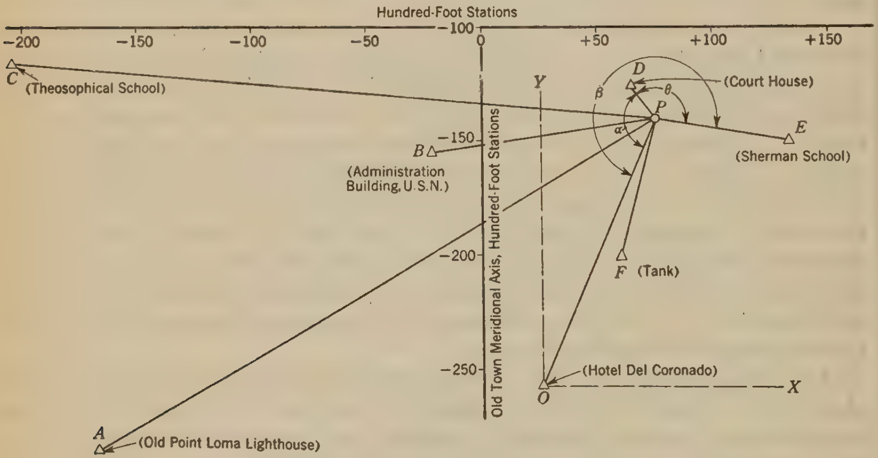
Hundred-Foot Stations
−200
−150
−100
−50
0
+50
+100
+150
−100
C
(Theosophical School)
D
(Court House)
β
θ
P
α
E
(Sherman School)
Y
−150
B
(Administration Building, U.S.N.)
Old Town Meridional Axis, Hundred-Foot Stations
−200
F (Tank)
−250
O
(Hotel Del Coronado)
X
A
(Old Point Loma Lighthouse)

Fig. 2.—Location of Problem Stations, San Diego County, California

Plane co-ordinates were taken from the "Map of San Diego Harbor, Calif., Harbor Lines," of the U. S. Engineer Office, dated September 24, 1927, which data had been computed from the U. S. Coast and Geodetic Survey's triangulation of many years before. Angles were turned with an engineer's transit equipped with a 30″ vernier. To illustrate the method of weighting, repetitions were purposely varied in the range from 4 to 20. The basic data and the transformation of Equation (3a) are given in Table 1.

TABLE 1.—Basic Data and Solution of Equation (3a)

No.	Signal Mark		Co-Ordinates				Observed angle, clockwise from 0
	Spire name	Point (see Fig. 2)	Old Town Origin		Transformed		
			East	North	X	Y	
63	Hotel del Coronado....	O	+ 2 469.5	−25 641.5	0.0	0.0	0° 0′ 0″
3	Point Loma Light House	A	−16 850.1	−28 687.8	−19 319.6	− 3 046.3	35° 31′ 52″
574	Administration Building	B	− 2 196.7	−14 417.6	− 4 666.2	+11 223.9	64° 02′ 40″
534	Theosophical School....	C	−19 543.7	−11 587.1	−22 013.2	+14 054.4	71° 43′ 05″
33	Court House..........	D	+ 6 466.4	−12 618.6	+ 3 996.9	+13 022.9	119° 18′ 17″
39	Sherman School........	E	+13 228.2	−14 985.0	+10 758.7	+10 656.5	256° 43′ 00″
65	Tank................	F	+ 5 941.4	−20 033.7	+ 3 471.9	+ 5 607.8	351° 05′ 30″

A first approximation, p_1, is computed from the three-point Triangle $O\ D\ E$, and Equations (7), (8*b*), and (9):

Given: x_d, y_d, x_e, y_e, α, and β, from Table 1
$\alpha = 119°\ 18'\ 17''$; $\cot \alpha = -0.5612822$
$\beta = 256°\ 43'\ 00''$; $\cot \beta = +0.2360829$

$$\begin{array}{lrr}
x_d = & +\ 3\,996.9 & \\
y_d \cot \alpha = & -\ 7\,309.52 & -\ 3\,312.62 \\
-x_e = & & -10\,758.7 \\
-y_e \cot \beta & & -\ 2\,515.82 \\
\text{numerator} & & -16\,587.14 \\
-y_d & -13\,022.9 & \\
x_d \cot \alpha & -\ 2\,243.39 & -15\,266.29 \\
y_e & & +10\,656.5 \\
-x_e \cot \beta & & -\ 2\,539.95 \\
\text{denominator} & & -\ 7\,149.74
\end{array}$$

$\cot Z = \dfrac{-16\,587.14}{-\ 7\,149.74} = +2.3199641$
$Z = 203°\ 19'\ 04.9''$
$\sin Z = -0.3958345$
$x_p = (-0.3958345)^2\,(-3\,312.62 + 2.3199641 \times 15\,266.29) = +5\,030.31$
$y_p = 2.3199641 \times 5\,030.31 = +11\,670.14$

The foregoing outline would complete the solution if no other signals had been observed, and Equation (13*a*) would give P = East 7 499.81, South 13 971.36. From Equation (16*a*), $\lambda_{p(ODP)} = -\dfrac{2 \times 5\,030.31 + 3\,312.62}{2 \times 11\,670.14 + (-\ 15\,266.29)} = -\ 1.6563$; and, similarly, $\lambda_{p(OEP)} = +\ 0.211$; and, $\lambda_{p(DEP)} = -\ 0.945$. From field data not given, the probable angular errors were computed to be: $\delta_\alpha'' = 4.7$; $\delta_\beta'' = 5.9$; and $\delta_\theta'' = 7.6$. By scaling Fig. 2, $l = 12\,700$; $P\ D = 1\,700$; $P\ E = 5\,800$; and $O\ D = 13\,600$. Then, from Equations (18) and (19): $N_\alpha = \dfrac{4.7 \times 12\,700 \times 1\,700}{206\,265 \times 13\,600} = 0.036$ ft. Similarly, $N_\beta = 0.142$; $N_\theta = 0.051$; $S_\alpha = \dfrac{13\,600}{12\,700 \times 1\,700} = 0.00063$; $S_\beta = 0.00020$; and $S_\theta = 0.00071$. Hence, in this three-point set, Triangle $P\ D\ E$ is the strongest figure, but the greater accuracy used in measuring the angle in Triangle $P\ O\ D$ gives the latter a greater weight.

The angles between slope tangents were measured by protractor as $\phi_{\alpha\beta} = 70.8°$; $\phi_{\alpha\theta} = 15.5°$, and $\phi_{\beta\theta} = 55.3°$. From Equation (20*a*),

$$\Delta_{\alpha\beta} = \csc 70.8° \sqrt{0.036^2 + 0.142^2 + 2 \times 0.036 \times 0.142 \cos 70.8°} = 0.177$$

and, similarly, $\Delta_{\alpha\theta} = 0.322$; and, $\Delta_{\beta\theta} = 0.220$. Hence, if only two of the three angles are to be measured, α and β would give the greatest accuracy. With all three angles measured, the position error is reduced to $\Delta_{\alpha\beta\theta} = 0.142 \csc 70.8° = 0.151$. For comparison with other three-point sets, the relative strength of the figure is $S_{\alpha\beta\theta} = 0.00063 \sin 15.5° = 0.000166$.

In routine calculations, the experienced computer will usually be able to select the strongest figures by eye. It may be of interest, however, to compare calculated strengths for the great variety of figures shown in Fig. 2. With calculations similar to the foregoing, using a slide-rule or a graph where less than four significant figures are carried, the data in Table 2 were obtained for the 21 position-locus circles. Note that two distant points (such as Points *A* and *C*) give low precision, but that either one combined with a near point gives great precision.

TABLE 2.—RELATIVE VALUES OF COMPUTED STRENGTH

Signals	Probable error, in seconds of arc	Slope	Normal error for 1″	Probable normal error	Strength, $10^5 S$	Signals	Probable error, in seconds of arc	Slope	Normal error for 1″	Probable normal error	Strength, $10^5 S$
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
O A	2.6	+50.45	0.090	0.23	5.4	*B C*	6.2	+ 0.120	0.072	0.45	6.7
O B	3.4	− 0.941	0.050	0.17	9.8	*B D*	5.7	− 1.86	0.009	0.05	54
O C	5.3	−13.64	0.065	0.34	7.5	*B E*	6.6	− 0.093	0.018	0.12	27
O D	4.7	− 1.656	0.008	0.04	63	*B F*	5.4	− 2.37	0.030	0.16	16
O E	5.9	+ 0.211	0.024	0.14	20	*C D*	7.1	− 1.45	0.009	0.06	56
O F	4.2	+ 9.69	0.058	0.24	8.3	*C E*	7.9	− 0.161	0.023	0.18	21
A B	4.2	− 0.185	0.066	0.28	7.4	*C F*	6.7	+51.5	0.030	0.20	16
A C	5.6	− 3.28	0.22	1.23	2.2	*D E*	7.6	− 0.945	0.007	0.05	71
A D	5.2	− 1.48	0.008	0.04	58	*D F*	6.3	− 1.93	0.007	0.04	70
A E	6.0	− 0.059	0.024	0.14	20	*E F*	7.1	+ 0.597	0.020	0.14	24
A F	4.8	+14.07	0.035	0.17	14	....	....	....	....	....	....

Data for the thirty-five independent intersections of these twenty-one circles are given in Table 3. Certain three-point sets may be rejected from further consideration: (*a*) Because of large departure of co-ordinates in Columns (2) and (3), Table 3, from the approximate centroid at E. 7 499.5, S. 13 971.0 (note that *O A F* is 30 ft away); (*b*) because all three intersection angles are small

TABLE 3.—COMPUTATIONS OF INDEPENDENT INTERSECTIONS

Three-point set	Co-ordinates of P_1 from East 7 490, and South 13 900		Intersection angles			Position	Strength	Three-point set	Co-ordinates of P_1 from East 7 490, and South 13 900		Intersection angles			Position	Strength
	East	South	$\phi_{\alpha\beta}$	$\phi_{\alpha\theta}$	$\phi_{\beta\theta}$	Probable error, $\Delta_{\alpha\beta\theta}$	$10^5 S_{\alpha\beta\theta}$		East	South	$\phi_{\alpha\beta}$	$\phi_{\alpha\theta}$	$\phi_{\beta\theta}$	Probable error, $\Delta_{\alpha\beta\theta}$	$10^5 S_{\alpha\beta\theta}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
O A B	10.85	71.69	47.9	80.7	32.8	0.38	5.3	*A B F*	9.35	71.41	83.6	56.6	27.0	0.37	6.3
O A C	10.75	76.95	5.4	18.2	12.8	5.57	0.5	*A C D*	8.50	69.60	17.0	17.6	0.6	4.07	0.6
O A D	10.82	73.04	32.3	35.2	2.9	0.79	3.2	*A C E*	8.94	71.03	69.6	63.9	5.7	1.40	2.0
O A E	10.86	71.14	76.9	87.8	15.3	0.53	5.3	*A C F*	9.29	72.18	21.1	18.1	3.0	3.95	0.7
O A F	11.46	41.45	4.7	2.9	1.8	7.65	0.3	*A D E*	9.49	71.06	52.6	12.6	40.0	0.22	13.0
O B C	10.33	71.20	42.5	50.1	87.3	0.59	5.1	*A D F*	9.38	70.89	38.1	6.6	31.5	0.35	7.0
O B D	8.91	69.87	15.6	18.4	2.8	0.82	3.0	*A E F*	9.37	71.05	89.3	34.2	55.1	0.25	11.0
O B E	10.37	71.24	55.2	38.0	17.2	0.47	6.0	*B C D*	9.67	71.28	68.5	62.2	6.3	0.51	5.9
O B F	8.55	69.53	52.6	23.8	28.8	0.50	4.0	*B C E*	10.17	71.22	12.1	15.9	3.8	1.81	1.8
O C D	10.42	72.37	26.9	30.4	3.5	0.67	3.8	*B C F*	9.31	71.32	73.9	82.1	24.0	0.46	6.6
O C E	10.33	71.25	82.3	76.7	21.0	0.39	7.2	*B D E*	9.61	71.17	56.4	18.3	38.1	0.19	16.8
O C F	9.53	60.15	10.1	5.3	5.8	3.69	0.8	*B D F*	7.97	68.14	5.4	0.9	4.5	3.18	1.3
O D E	9.81	71.36	70.8	15.5	55.3	0.17	16.6	*B E F*	9.23	71.13	61.8	36.1	82.1	0.20	15.8
O D F	8.57	69.31	37.0	3.7	33.3	0.62	4.3	*C D E*	9.56	71.13	46.3	12.0	34.3	0.32	11.8
O E F	8.33	71.67	72.2	18.9	53.3	0.43	6.6	*C D F*	9.32	70.78	35.7	7.2	28.5	0.48	7.3
A B C	9.04	71.36	62.5	17.3	79.8	1.25	2.2	*C E F*	9.32	71.08	82.0	39.9	58.1	0.28	13.5
A B D	9.79	71.49	45.5	51.2	5.7	0.40	5.8	*D E F*	9.44	71.01	19.2	74.2	86.6	0.15	23.0
A B E	6.50	70.88	7.1	5.2	1.9	4.22	0.7	....	....	....	....	....	....	....	....

(note *O A F*, *O C F*, *A B E*, and *B D F*); (*c*) because of the large position error in Column (7), Table 3, or the low figure strength in Column (8); or, (*d*) because of any combination of Causes (*a*), (*b*), and (*c*). If all angles are measured to nearly the same precision, figure strength is the easiest determinant; but if field precisions were quite varied, position error is a better comparison. First, however, one must determine the position more closely from the more precise sets (say, *O D E*, *B D E*, and *D E F*, giving a mean of E. 7 499.62, S. 13 971.18) and list any intersections that fall much farther from this position than their calculated position errors. In this example, all intersections seem tolerable except for twelve of the fifteen that are dependent upon Signal *O*, which is sufficient evidence for the rejection of all data depending upon *O*. A study of the large-scale plot of these intersections makes the conclusion more obvious.

Eliminating these intersections and four others that have probable position errors greater than 3 ft, the other sixteen intersections are plotted to a large scale in Fig. 3. Bands the width of the probable normal error on each side of the lines do not intersect with an area common to all. Increasing each band width by 45%, a common point, P_1, is found on the edges of bands of *B F*, *B D*, and *A B*, and within all other bands. As *B D* and *B F* are nearly parallel, draw the median to the edges of those bands. By diminishing the width of all other bands until the width of each side is 2% greater than the probable error, the diminished common area just touches the median at P_2. This point is accepted as the solution; its departure from position-locus circles is about 1.50 times the probable error for two circles; 1.02 times for two others; and, less than the probable error for the other twelve. The least squares solution, which is too long for inclusion herein, gives Point *Q*, distant 0.03 ft from Point P_2 and has a probable error of 0.13 ft.

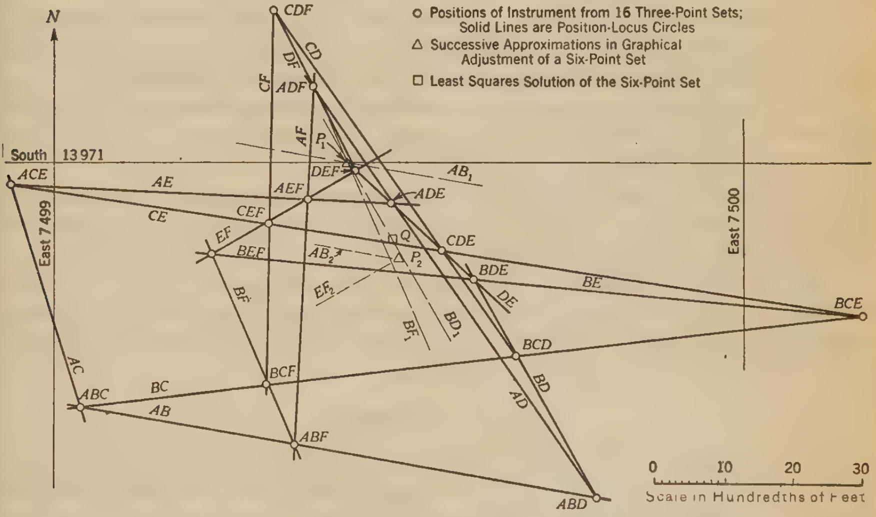
○ Positions of Instrument from 16 Three-Point Sets; Solid Lines are Position-Locus Circles
△ Successive Approximations in Graphical Adjustment of a Six-Point Set
□ Least Squares Solution of the Six-Point Set
N
CDF
CD
DF
CF
ADF
AF
P_1
South 13 971
ACE
AE
AEF
DEF
AB_1
ADE
CE
CEF
East 7 499
EF
BEF
AB_2
Q
P_2
CDE
East 7 500
BDE
BE
DE
BCE
EF_2
BF
BF_1
BD_1
BCD
AC
BCF
BD
AD
ABC
BC
AB
ABF
ABD
0
10
20
30
Scale in Hundredths of Feet

FIG. 3.—GRAPHICAL SOLUTION OF A SIX-POINT SET

Before concluding, the rejected signal, O, must be disposed of. Computing closing azimuths from P_2 to each of the other signals, and then Azimuth $P_2\,O$ from these and the measured angles, the resulting six values may be expressed as 203° 19′ 13″.5 ± 4″.2, with a weighted mean of 203° 19′ 13″.7. The closing azimuth from P_2 to the given position of O is 203° 18′ 58″.9. The difference, 14″.8, is the angle subtended at P_2 by the position error of O. As the distance is about 12 708 ft, the normal projection of this position error is 0.92 ft. Measurement of the position-error projection from another instrument point would determine the absolute error, and, thereafter, the station could be used.

It should not be concluded that the other six signals have been proved correct. Any of them might have had a shift in such a direction that its normal projection from P is small; or a distant signal might have an error that would not be appreciable until the survey approached it.

SUMMARY AND CONCLUSIONS

The salient features of the paper may be summarized as follows:

(1) The three-point problem may be solved by formulas, without ambiguity;

(2) The use of spires, co-ordinated by responsible surveys, affords a simple method of connecting to other systems or controlling spur traverses;

(3) If such spires have not been observed recently, shifts due to several causes should be anticipated (use of a fourth spire affords a check; and, a fifth spire will serve to eliminate one which is in error);

(4) Methods of analysis of four-or-more-point systems have been outlined fully (the experienced computer may substitute his judgment for formulas expressing relative strengths of figures); and,

(5) The graphical method of compromising intersection errors by representing each line by a band of width proportional to its probable error is also applicable to ordinary triangulation, particularly in the convergence of second-order surveys to locate third-order stations.

ACKNOWLEDGMENTS

The formulas were developed while the writer was associated with Tom J. Allen, Assoc. M. Am. Soc. C. E., and appreciation is expressed for his co-operation. The field observations for the illustrative example were made by Eric V. A. Quartly, Assoc. M. Am. Soc. C. E.

DISCUSSION

Oscar S. Adams,[2] Esq. (by letter).—Many papers dealing with the three-point problem can be found in periodicals devoted to engineering and surveying. Any treatment of the subject by means of co-ordinates must necessarily depend upon the same principles. The paper by Mr. Rowe is developed in a very complete manner and it should be useful to any surveyor who may wish to take advantage of this method of connection to control.

The U. S. Coast and Geodetic Survey is following the practice of computing the plane co-ordinates of all the triangulation stations on the State plane-co-ordinate systems. In the end every station will have its plane co-ordinates given as well as its geodetic position. For some States this work is already practically completed and, in course of time, this will be true of all the States. In any region that lies near the various arcs of triangulation, tanks, steeples, and other such objects, are located by intersection observations. This work should form a good basis for the application of the three-point fix.

Besides the work of the Coast and Geodetic Survey, the U. S. Geological Survey, the Corps of Engineers of the U. S. Army, and other surveying and engineering organizations have located many prominent objects in the vicinity of their operations. All these stations can be used indiscriminately if they are all computed on the same basis and are mutually dependent for their datum. Of course if they are not so co-ordinated, each would have to be used independently of the others. The Coast and Geodetic Survey developed the State plane-co-ordinate systems with the hope that all other organizations would make use of them in their work and that all work would be tied to the control net of the country. If this hope should be realized, all work of the various organizations would be on a common basis, and all results would have a permanent value for any succeeding work. If this could be accomplished there would be a great mass of data available for use in many regions where now there is great lack of such control.

The three-point fix has a practical and valuable use in hydrographic surveying, in determining locations offshore, and in plane-table work in topographic mapping. A three-point location, however, even when dependent on accurately located stations, is not of sufficient accuracy to justify its use in the extension of control, except for surveys of the lower order. The use of intersection stations is not advocated as the basis of control for second-order surveys and little dependence can be placed, as far as control is concerned, on a three-point location which is based upon tanks, steeples, water towers, or intersection stations of any kind.

In army operations in time of war, much use is made of this method of determining positions. The Allied Armies in France had a method of computation for the three-point fix dependent upon the grid system in use in the World War Zone. This method, of course, depended upon plane co-ordinates.

The author is to be congratulated upon the excellence of his development of the computation formulas. The treatment is logical and the resulting formulas would be easy of computation with an ordinary calculating machine. The discussion of the accuracy of the results is very well done. As a final word, it is desired to repeat the warning against the use of this method where any accuracy of location is essential. As already stated, it has its use—and an important use—but as control for further work, it is not to be employed even under the most favorable circumstances.

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George D. Whitmore,[3] M. Am. Soc. C. E. (by letter).—As shown by the author, three-point triangulation has a definite place in modern surveying and mapping practice. For its successful use, however, its peculiar characteristics and limitations must be recognized. For example, it may be said that, in general, a horizontal position established by three-point triangulation (resection) is never as strong as the position established from the same basic control stations by "cuts" (intersection). Furthermore, it is true that there are many instances in which the position from a three-point determination might be uncertain or even unsolvable, whereas the same position might be strong if the intersection method of "cuts" were used. In many cases, the strength or weakness of a three-point location is not immediately apparent to the field man, this being a rather serious disadvantage if the basic control stations are scarce and if advance planning of the field work is not practicable.

Three-point triangulation can often be used to advantage, however, under the following conditions, if too high an order of accuracy is not required:

(1) When it is not practicable to occupy the basic control stations (for example, if the basic control includes a large percentage of spires, cupolas, stacks, towers, etc., impossible to occupy with an instrument for "cut-in" work without an expensive building for instrument support);

(2) When the basic control is already signaled (in this case, under the three-point method, no new construction would be required except for instrument supports at the three-point stations);

(3) When the ratio of new supplemental stations to existing basic stations is high (obviously, it would be cheaper to construct a few signals at the existing basic stations for three-point observations than to construct many signals at the new supplemental stations for "cut-in" observations); and,

(4) When flexibility in the observing schedule is required.

During the two years, 1937–1938, in connection with a topographic mapping project in the Tennessee Valley, more than 700 three-point stations have been established as detailed horizontal control. The terrain being mapped under this program includes several thousand square miles in the Appalachian Mountains of Eastern Tennessee and Western North Carolina, and is generally heavily wooded and quite inaccessible. There are many peaks with elevations that exceed 5 000 ft, the floors of the valleys averaging about 1 200 ft. The basic control already in existence consists of the first-order and second-order triangulation of the U. S. Coast and Geodetic Survey, with about one station to each 50 sq miles; and third-order triangulation of the U. S. Geological Survey, with an average of one station for each 12 sq miles. It is necessary to supplement this basic control with additional stations averaging about one for each 4 sq miles. Three-point triangulation is the principal method used to establish this supplemental control.

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The mapping of this area is by the stereo-topographic process, using aerial photographs. Because of the characteristics of this method, the supplemental control stations must be established in certain restricted locations, namely, in the common overlap areas of several photographs, and close to some definite point which can be identified on all the overlapping photographs. Thus, there is very little latitude in selecting the locations of these supplemental control stations. Furthermore, the observing schedule is such that it is not practical to "cut in" all supplemental stations with one visit to each basic control station. If the "cut in" method were used, many return visits to these basic control points would be required. Under the three-point method, only one trip is made to each basic control station, at which time specially designed pyramid signals are erected, and precise vertical angles measured for the determination of elevations on these basic stations.

In order to meet the accuracy required by the maps, the horizontal position of each supplemental control station must be correct to within 10 ft in relation to the basic control, the horizontal plotting scale of the map being either 1 : 10 000 or 1 : 15 840. To obtain this fourth-order accuracy in the three-point locations, repeating theodolites are used, equipped with verniers reading to 0° 0′ 20″. Six repetitions direct and reverse are taken on each angle, including the angle closing the horizon. The lengths of the sights occasionally run as high as 10 miles, and average 4 to 5 miles. Usually four or more basic control stations are visible from each three-point station, all near-by visible basic stations being observed, of course.

Early in this program the advantages of making all computations and adjustments of three-point stations in a co-ordinated field were recognized. The State-wide plane co-ordinate systems originated by the Coast and Geodetic Survey were adopted as the computing medium. It is interesting to note that the formulas developed by the computing staff for the solution of the three-point locations are substantially the same as those developed by the author. A simple and "fool-proof" computation form, based on Equation (10), has been in use for some time. The angular value of Z is obtained by an inverse solution as a part of the routine check on the work. The research in developing these computing and adjusting methods was done principally by W. O. Byrd, Assistant Mathematician, working under the close direction of C. C. Miner, Assoc. M. Am. Soc. C. E., head of the Surveys Computing Staff of the Tennessee Valley Authority.

The formulas developed by the author to determine the "relative weight and figure strengths" are very useful and have an important application. For the successful utilization of three-point triangulation, it is absolutely necessary to avoid weak figures. The Surveys Computing Staff has made extensive use of this principle, not so much in the refined adjustment of the three-point figures, as in selecting the strongest figure from which to establish each three-point position.

The experiences of this same organization indicate that a refined or delicate adjustment of a three-point position is not justified unless the basic control is of a high order of accuracy, and located so as to provide several reasonably strong figures for each three-point location. When these conditions do not exist, the method used has been to select the strongest figure and base the determinations entirely upon that figure. The remaining pointings are then used as checks on the work. These checks are made rapidly and consist of comparing the observed directions with the computed directions.

Occasionally, it has been found desirable to determine the most probable value of a three-point position by taking advantage of all observations to all basic stations. The least squares method of adjustment,[4] is very satisfactory and can possibly be made in less time than the "cut-and-try" method advanced by the author. An experienced computer can complete such an adjustment with pointings on five stations in less than 2 hr.

The method of establishing elevations at three-point stations in connection with the topographic mapping project may also be of interest. Elevations are required to be correct to within 4 ft, with respect to the nearest basic vertical control, the contour interval being 40 ft, with half-interval, 20-ft contours being shown in all flat and critical areas. Elevations are first established on the basic control stations, either by spirit leveling, or by vertical angles of a high degree of precision. To determine the elevation of a three-point station vertical angles to each of the visible basic control stations are read, the measurement consisting of two readings with telescope direct and two readings with telescope reversed, using a vertical circle reading to $0° 0' 20''$. Then, utilizing the horizontal lengths computed from the three-point problem, the observed vertical angles, and a field-determined coefficient of refraction, the difference of elevation between the three-point station and each basic control station observed upon is computed. Each basic control station observed thus furnishes an elevation for the three-point station, the final elevation of the new station usually being the mean of the elevations from the various basic stations.

JOHN R. JAHN,[5] ESQ. (by letter).—In bringing this valuable graphical device into a more exact form for use in balanced surveys, the author is to be congratulated. His method is ingenious and should be of considerable value in the work of the larger surveying organizations that employ men who are sufficiently skilled in the mathematics of geodesy so that errors of manipulating the formulas, and errors of technique in the analysis, do not defeat the straightforward solution of the problems. In the illustrative example, the basic data, with the exception of that applying to Station *O*, are of uniformly high accuracy. Had this not been the case, the analysis would probably have failed, due to ambiguity in the many sets of possible locations for Station *P*. The writer has spent much time attempting to find single points that would satisfy the directional data for more than three-point groups. Part of the trouble was caused by slightly inaccurate angular data and part by slightly inaccurate station positioning. The author's method of working toward the most reasonable position by the use of unit bands was attempted without success, although the finer analysis that Mr. Rowe uses was not then known to the writer.

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[4] *Special Bulletin No. 138*, U. S. Coast and Geodetic Survey, p. 114.

[5] Civ. Engr., Riverside, Calif.

In the example, the observed angles were adjusted to seconds. This indicates the use of an instrument more accurate than the ordinary engineer's transit, reading to minutes. Multiple pointings can be taken and the average value used. However, to be of use to the average trained surveyor, the writer believes that a simpler method of analysis is necessary to enable the surveyor to add this "three-point problem" to his mental equipment. For several years, the writer has been attempting to formulate a procedure, using the mathematics of analytic geometry in co-ordinated field data, in order that field survey points might be referenced to established landmarks, such as flag-poles, steeples, windmills, etc., whose "positions" could be determined closely. This is particularly valuable in open country, where near-by reference points are not always at hand. With a transit line tied into co-ordinated landmarks by three-point or multiple-point directional data, the recovery of old stations would be considerably aided.

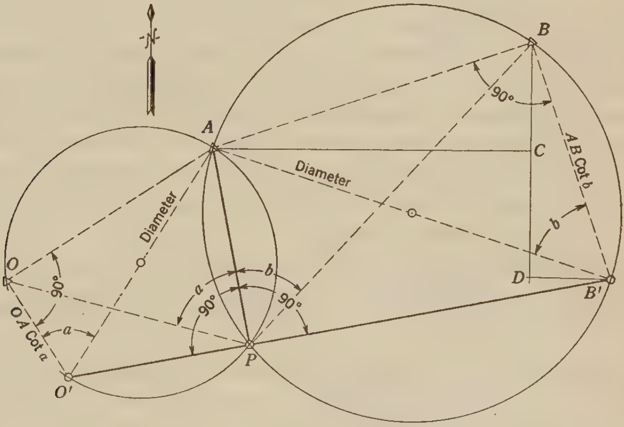

FIG. 4

In Fig. 4 which is similar to Fig. 1, Point P lies at the intersection of the two determining circles passing through Points O and A and through Points A and B. By constructing Point O' such that $\overline{OO'} = \overline{OA} \cot a$, Circle $O\ A\ P$ is divided by Diameter $A\ O'$, since $O'\ O\ A$ is a right angle and Chord $O\ A$ subtends Angle a; hence, Angle $A\ P\ O'$ is a right angle. By like construction $\overline{BB'} = \overline{AB} \cot b$, and likewise Angle $A\ P\ B'$, is a right angle. Hence, $O'\ PB'$ is a straight line and Point P is located at the intersection of two lines, one through Points O' and B', the "closing line" of a traverse having a slope,

$$m = \frac{dy}{dx} = \frac{\text{Difference of northings}}{\text{Difference of eastings}} \quad \ldots\ldots\ldots\ldots\ldots (22)$$

and the other through Point A and having a slope at right angles to the former.

If Line $O'\,B'$ is expressed by the equation,

$$y = m\,x + c_B \qquad \text{(23a)}$$

and the line through Point A is expressed by the equation,

$$x = -\,m\,y + c_A \qquad \text{(23b)}$$

the two equations may be solved for x_P and y_P from the combination; that is, $x_P = -\,m^2\,x_P - m\,c_B + c_A$, or,

$$x_P = \frac{-\,m\,c_B + c_A}{1 + m^2} \qquad \text{(24)}$$

This would be laborious if the lengths of Lines $O\,A$ and $A\,B$ were to be computed as the square root of the sum of the squares of co-ordinate differences. However, Triangles $A\,C\,B$ and $B\,D\,B'$, representing the longitude and latitude differences of Lines $A\,B$ and $B\,B'$, are similar to each other since corresponding angles are equal. Then, $\dfrac{\overline{AC}}{\overline{BD}} = \dfrac{\overline{BC}}{\overline{DB'}} = \dfrac{\overline{AB}}{\overline{BB'}}$; but $\overline{BB'} = \overline{AB}\cot b$, so that "difference of northings, $\overline{BB'}$" (or BD) = "difference of eastings, $\overline{AB}$" $\cot b$ (or $\overline{AC}\cot b$); and, "difference of eastings, $\overline{BB'}$" (or $\overline{DB'}$) = "difference of northings, $\overline{AB}$" $\cot b$ (or $\overline{BC}\cot b$). It follows that Line $O'\,B'$ is the closing line of a traverse, $O\,O'$, $O\,A$, $A\,B$, and $B\,B'$, and the signs of the several courses can be visualized and set down without likelihood of error. It is to be noted that this analysis is the same as that leading to Equation (7). Fig. 4 shows Angles a and b as acute, or less than 90 degrees. If either or both had been obtuse, or greater than 90°, Lines $O\,O'$ or $B\,B'$ would have extended on the opposite sides of Line $O\,A$ or Line $A\,B$ from Point P.

A work sheet using the author's data for Signals D, E, and F is illustrated by Table 4. Let $m = \dfrac{-\,11\,991.9}{-\,15\,699.1} = +\,0.763859 = \cot 52°\,37\tfrac{1}{2}'$; from which the

TABLE 4.—COMPUTATION FORM FOR ALTERNATE SOLUTION

Signal (See Fig. 2)	Azimuth	Included angle	Differences of:		Co-ordinates		Remarks
			Northings	Eastings	North	East	
F'.....	....				....	....	Not required
		....	−413.3*	+5 837.8			
F.....	194° 24'.5				−20 033.7	+5 941.4	Given
		128° 12'.75	+7 415.1	+525.0			
D.....	322° 37'.25				−12 618.6	+6 466.4	Given
		137° 24'.75	−2 366.4	+6 761.8			
E.....	100° 02'				−14 985.0	+13 228.2	Given
		....	+7 356.5	+2 574.5			
E'.....					−7 628.5	+15 802.7	From Signal *E*
....	....	....	−11 991.9	−15 699.1	....	....	To balance

* 413.3 = 525 × cot 128° 12¾'.

true bearing of Line $D\,P$ is $270° + 52°\,37\tfrac{1}{2}' = 322°\,37\tfrac{1}{2}'$ (as read, $322°\,37\tfrac{1}{4}'$; and, therefore, the correction to the angles is $+\,\tfrac{1}{4}'$).

At Point E', Equation (23a), for Line $E'\,F'$, yields: $y = +\,0.763859\,x - 19\,699.5$ ($c_{E'} = -\,15\,802.7 \times 0.763859 - 7\,628.5$).

At Point D, Equation (23*b*) for Line $D\ P$ yields $x = -0.763859\ y - 3\ 172.4$; and, solving for x, $x = -0.58348\ x + (-0.763859 \times -19\ 699.5) - 3\ 172.4 = \frac{+11\ 875.2}{+1.58348} = +7\ 499.5$; and, $y = (+0.763859)(+7\ 499.5) - 19\ 699.5 = -13\ 971.0$ (see Case I, Fig. 5). A form sheet for the foregoing computation (see Table 4) makes the solution straightforward, and with the aid of natural functions and a computing machine, the computation can be made in 5 min.

As the author states, the analysis of data on multiple pointings involving more than three stations brings in many possible solutions. It will be noticed that in the foregoing analysis any station (D, for example) may be taken for the central, or reference line. If perpendiculars are taken, and auxiliary points, such as E', etc., are computed for each pair of points (such as ED), a series of such auxiliary points will range along the true line passing through Point P and normal to Line DP. Using the author's data Table 5 gives such results. A first approximation is the "range-line," $C'\ E'$:

TABLE 5.—STATIONS REFERRED TO STATION *D*, CO-ORDINATES; *N* − 12 618.6; AND, *E* + 6 466.4

To Station	Included angle	dN	dE	$d'N$	$d'E$	Co-Ordinates	
						North	East
E.......	137° 24′ 43″	−2 366.4	+6 761.8			−14 985.0	+13 228.2
E′......				+7 356.5	+2 574.5	− 7 628.5	+15 802.7
F.......	231° 47′ 13″	−7 415.1	−525.0			−20 033.7	+5 941.4
F′......				+413.3	−5 837.9	−19 620.4	+103.5
O.......	240° 41′ 43″	−13 022.9	−3 996.9			−25 641.5	+2 469.5
O′......				+2 243.4	−7 309.5	−23 398.1	−4 840.0
A.......	276° 13′ 35″	−16 069.2	−23 316.5			−28 687.8	−16 850.1
A′......				−2 543.8	+1 753.2	−31 231.6	−15 096.9
B.......	304° 44′ 23″	−1 799.0	−8 663.1			−14 417.6	−2 196.7
B′......				−6 007.5	+1 247.5	−20 425.1	−949.2
C.......	312° 24′ 48″	+1 031.5	−26 010.1			−11 587.1	−19 543.7
C′......				−23 761.6	−942.3	−35 348.7	−20 486.0

Co-Ordinates *C′*..........	− 35 348.7	− 20 486.0
Co-Ordinates *E′*..........	− 7 628.5	+ 15 802.7
Change.................	+ 27 720.2	+ 36 288.7

and, the slope, m, $= \frac{+27\ 720.2}{+36\ 288.7} = +0.763880$.

Using the co-ordinates of Point E', the equation of the "range-line" is,

$$y = +0.763880\ x - 19\ 699.9 \qquad (25)$$

When the values of x for each of the auxiliary points is substituted in Equation (25) and the y-value computed, the departure of the value in Table 5 can be ascertained. Since the line passes through Points E' and C' these departures are zero. The other departures are F', + 0.4; O', − 1.0; A', + 0.5; and, B', − 0.1. The departure of the value for O' is considered as excessive and is rejected. A new line may be chosen that will more nearly equalize the departures of the tabulated data. Such a line would pass 0.3′ north of Point C' and

0.1′ south of Point *E′*. This can be seen on a graph of exaggerated scale (Fig. 6) in which "*x* (East)" is plotted: 1″ = 5 000 ft; and, "difference of *y* (North)" = 1″ = 1 ft (the scale in Fig. 6 has been reduced for publication). The new equation for the "range-line" would then be,

$$y = +0.763867\,x - 19\,699.8 \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Since Station *D* may have been in error, it will be necessary to choose another "range-line," a quadrant removed from the pointing to *D*. Consider, for example, a set of data referred to Station *B* and determine a range line that will distribute the computed errors approximately equally on either side. An analysis gives a line normal to Line *P B* having the equation,

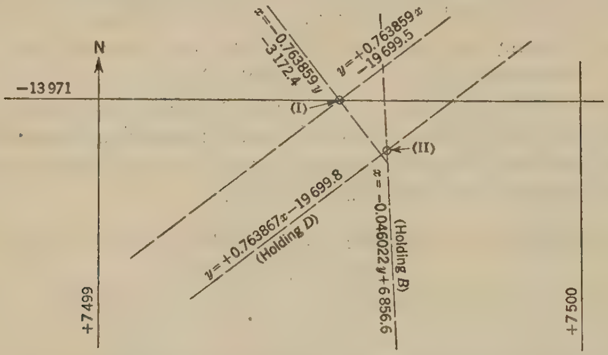
N
−13 971
x = −0.763859 y −3 172.4
y = +0.763859 x −19 699.5
(I)
(II)
y = +0.763867x −19 699.8 (Holding D)
x = −0.046022 y + 6 856.6 (Holding B)
+7 499
+7 500

FIG. 5.—SOLUTION BY ANALYTIC GEOMETRY: CASE I = SIMPLE 3-POINT FIX; AND CASE II = BALANCED 6-POINT FIX

$$x = -0.046022\,y + 6\,856.6 \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

and the solution of the two equations for *x* and *y* (Case II, Fig. 5) gives the probable position of the point as: North − 13 971.1; and, East + 7 499.6.

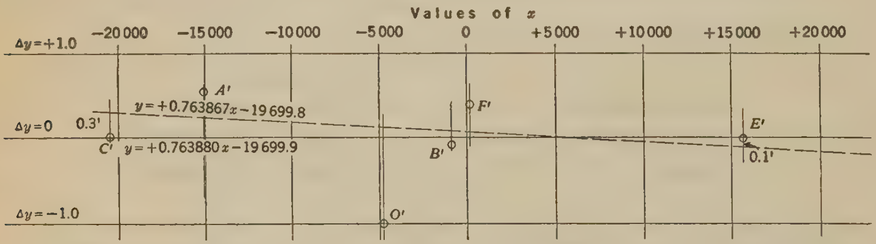

FIG. 6.—POSITION OF AUXILIARY POINTS WITH RESPECT TO "RANGE-LINE"

The writer feels that the careful use of the "three-point-problem" can have a distinct value in giving an independent check on survey work, particularly in certain types of reconnaissance work where it may not be economically possible to loop the survey into closed traverses. For this purpose it would be necessary, of course, to refer the co-ordinates of the basic control stations to the meridian of the initial station.

R. ROBINSON ROWE,[6] M. AM. SOC. C. E. (by letter).—The limited discussion of this paper may be summarized as: (1) Acceptance of the principles and formulas as correct; (2) statements of particular cases for which the methods are, or are not, adaptable; (3) doubt that the proposed method for the reduction of a four-point problem (or more) is economical; and (4) alternate forms for tabulating the solutions. The writer desires to express his appreciation of the thoughtful and helpful contributions of Messrs. Adams, Whitmore, and Jahn; with them, he is in substantial agreement.

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[6] Associate Bridge Engr., State Dept. of Public Works, Sacramento, Calif.

Mr. Adams has stressed the increasing availability of co-ordinated spires in certain localities. His further comment on precision of co-ordinates would have been in point; but, regardless of original precision, spire co-ordinates cannot be considered valid until some check has been effected, due to deflection of structures. When so checked, however, accuracy sufficient for the next lower order of survey can be assured. If Mr. Adams implied the contrary in his final sentence, the writer cannot agree.

For example, the probable error of the location P_2 was not greater than 0.16 ft (0.13 + 0.03 ft), which is less than one-ten-thousandth part of the distance from the nearest spire and a much smaller part of the distance from the nearest instrument point of the prior triangulation. If another point, say 1 mile from P, were to be fixed in similar fashion, and if these two points were connected by traverse with satisfactory closure, all points of the survey should merit a fourth-order rating.

Spire co-ordination by the U. S. Coast and Geodetic Survey should be appreciated by surveyors because of their instant availability. Not only are instrument stations of the Survey often obliterated or covered by buildings, but residential construction on eminent hills frequently interferes with important sight lines.

The writer agrees with Mr. Whitmore that a five-point fix can be adjusted rapidly by the least squares method, if computers are familiar with the method and the five points are proved; but if one of the five points is rejectable, the method may introduce fallacy. Furthermore, if Mr. Average Surveyor should have occasion to adjust such a fix once in a year, or twice in a lifetime, he would probably select three points by "eeny-meeny-miny-mo," with one chance in ten of selecting the strongest figure. The semi-graphical method should appeal to him; admittedly, the writer's example would have been more clear if only five points had been studied.

Mr. Jahn has suggested an additional use for the three-point fix—the recovery of old instrument stations which had been referenced to several spires. Probably he refers to buried monuments, and for such objects the point is well taken. It was interesting to learn that he had also conceived of an instrument sight line as a band rather than the conventional geometric line; the width of the band should not be a "unit," but should be proportional to the probable error. His alternate computation forms and method of adjusting three-point and six-point fixes are worthy of consideration. Although the writer still prefers his own, he realizes that each computer will find certain arrays of figures more natural to his psychophysical processes. Although Mr. Jahn's six-point solution departs from the least squares solution by 0.11 ft (compared to 0.03 by the method of the paper), the lesser precision may be adequate and obtainable more economically.

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TRANSACTIONS

Paper No. 2039

TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE

By G. W. Howard,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Fred R. Brown, Josef E. Montgomery, Elliott J. Dent, David L. Neuman, Morrough P. O'Brien and R. G. Folsom, R. L. Vaughn, M. P. Durepaire, Pierre F. Danel, H. S. Gladfelter, and G. W. Howard.

Synopsis

Tests on 4-in. pipe, transporting water mixed with sand and small gravel, are described in this paper. Although a description of the flow characteristics of sand and water mixtures and gravel and water mixtures in a 4-in. pipe constitutes the main part of the paper, an attempt is made to generalize the findings in regard to other sizes of pipe in so far as it is believed to be applicable. A discussion is given of existing general formulas for the transportation of materials in pipe lines, and it is attempted to prove that the type of material to be transported and the experience of the designer should have more bearing on the solution of a problem than any solid-transportation formula now extant.

No new formulas are presented. An insufficient number of investigations have been made to justify the use even of the formulas now in existence. The attempt is made, however, to consolidate all known facts concerning the transportation of sand and water mixtures in order to show that, at present, only general characteristics can be determined for this phenomenon.

Introduction

One of the earliest discussions on the transportation of sand mixtures in pipes is given in a paper entitled "Works for the Purification of the Water Supply of Washington, D. C.,"[2] by the late Allen Hazen and E. D. Hardy, Members, Am. Soc. C. E. Only a brief treatment is presented in the paper, however, because the topic of sand transportation in filter washing is incidental to the primary subject. The pipes for which friction curves are given had diameters of 3 and 4 in. Types of transportation and other flow characteristics

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Note.—Published in September, 1938, *Proceedings*.

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[2] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 307.

are not considered, as friction losses were the only features that were of interest. A discussion of the paper of Messrs. Hazen and Hardy by Nora Stanton Blatch, Jun. Am. Soc. C. E., provides more definite data than any other available reference on the subject.[3] Using the results from tests on 1-in. pipe as a basis for her discussion, Miss Blatch compared the data of Hazen and Hardy with additional data from the Mississippi River Commission on the dredges *Alpha*, *Beta*, and *Gamma*. Due to the fact that the test line was equipped with a transparent section, Miss Blatch covered, in detail, the types of movement that occur at different velocities. An analysis of all data led to a general formula for head loss in pipes ranging in diameter from 1 in. to 32 in., and for all variations of solid concentration. Her findings are generally used as references in textbooks on hydraulics.[4]

The data available from pipe-line dredges are so few that it might be considered surprising until the difficulties in making accurate field measurements are considered. Unstable pumping conditions with subsequent variations in pipe-line velocities and solid concentrations make data of this nature extremely difficult to correlate. Due to the fact that the load in the pipe line of a dredge varies so frequently, it is almost impossible to obtain consistent pressure-gage readings; therefore, an average value of velocity and solid concentration is all that can be hoped for in tests on large pipe lines. Discussions of tests on pipe-line dredges have been presented more frequently in recent years,[5] but no sets of curves, such as were presented by Miss Blatch, have been included in any of these papers.

Although friction losses for pipe lines of hydraulic dredges have been determined,[6] few instances have occurred in which these lines were loaded with solids. Since actual design work for this plant must consider the material that is being moved, the losses found for water alone have definite limitations.

The purpose of this paper is to present the findings from tests on 4-in. pipe carrying varying concentrations of sand and gravel. These findings are believed to be of general interest to those engaged in work dealing with the transportation of solids through closed conduits, and the results presented herein are given with the view of advancing the limited knowledge in this field.

PROGRAM OF EXPERIMENTS

Two separate series of tests were performed, each on a different material. The investigation of the sand and water mixtures was more complete than that of the gravel mixtures. This was due to the fact that the specific purpose of the gravel experiments, to which the data presented herein are only incidental, had been limited in scope, because of findings from the sand tests. During each series the following conditions were varied within certain limits: (*a*) Pipe-line velocities; and (*b*) solid concentrations.

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[3] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 403.

[4] "Hydraulics," by Ernest W. Schoder and Francis M. Dawson, Members, Am. Soc. C. E., McGraw-Hill Book Co., 1934, p. 259; also "Hydraulics and Its Applications," by A. H. Gibson, D. Van Nostrand Pub. Co., 1930, p. 210.

[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 625; "Hydraulic Dredging and Methods of Placing the Fill," *Engineering News-Record*, August, 1935; and "Hydraulic Fill at Quabbin Dike," *Engineering News-Record*, June 18, 1936.

[6] "Friction in Dredge Pipes," by James H. Polhemus, M. Am. Soc. C. E., and John R. DuPriest, *Transactions*, A. S. M. E., Vol. 49–50, 1927–28, HYD–50–7; also, "Velocity Tests in Hydraulic Dredge Pipes," *Engineering News-Record*, January 6, 1921.

DESCRIPTION OF THE APPARATUS

General.—The apparatus used to maintain a constant solid concentration consisted of a sump, a pump, and a variable head tank that discharged through the 4-in. test line back into the sump. This caused a uniform circulation of any mixture and provided a means for obtaining constant pressure and an even distribution of solids over the cross-sectional area of the pipe. The arrangement of this apparatus is shown diagrammatically in Fig. 1.

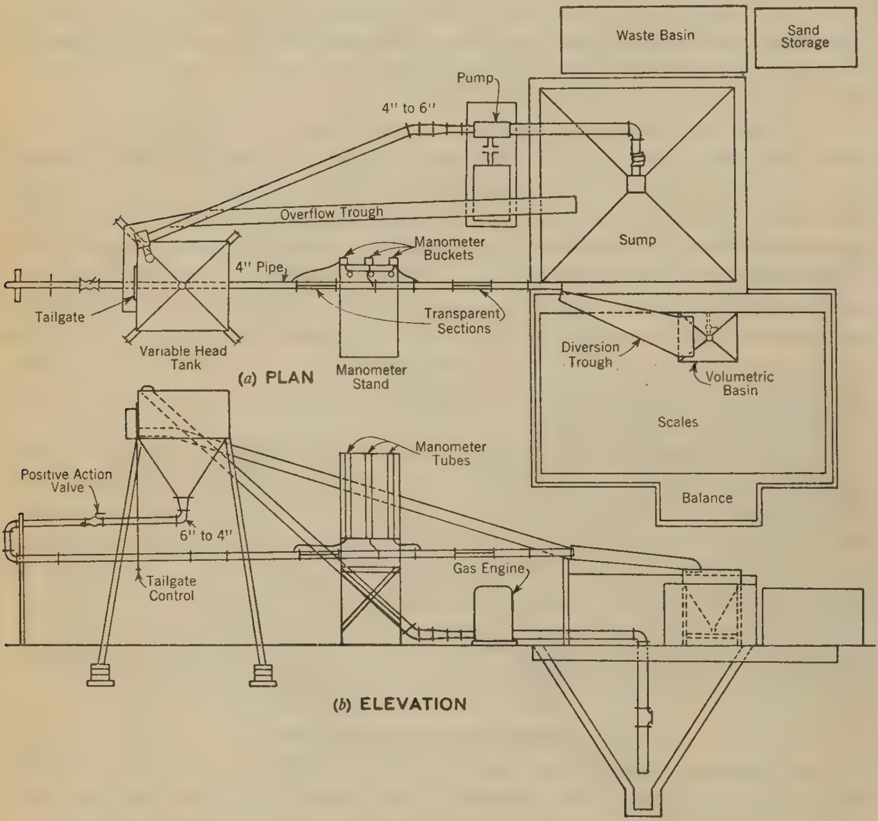
Waste Basin
Sand Storage
Pump
4" to 6"
Overflow Trough
Manometer Buckets
4" Pipe
Sump
Tailgate
Transparent Sections
Variable Head Tank
Diversion Trough
(*a*) PLAN
Manometer Stand
Volumetric Basin
Scales
Manometer Tubes
Positive Action Valve
Balance
6" to 4"
Gas Engine
Tailgate Control
(*b*) ELEVATION

FIG. 1.—APPARATUS USED ON TESTS FOR SAND AND GRAVEL

Test Line.—Starting from the variable head tank the 4-in. line extended 8 ft horizontally and then curved downward and reversed its direction through two 90° ells. The upper 8-ft section was equipped with a positive-action cock to give control over the lower velocities. After reversing direction, the 4-in. line extended a distance of $86\frac{1}{4}$ diameters and discharged into the sump. The lower $43\frac{1}{3}$ diameters of the line were used as the test section. Two 2-ft sections of transparent pyralin pipe were installed in the test line and three open-end manometers were connected to the test section. Due to the fact that considerable material was usually carried on the bottom of the pipe, it was not considered practical to use a manometer ring or four manometer taps. Two openings, at 45° with the horizontal plane, were used at each connection. The up-stream connection was $43\frac{1}{3}$ diameters from the end of the line, and the lengths of the other connections were varied for different tests; the readings of these two gages were used as a check on the up-stream manometer.

Solid Concentration Determination.—The device used to determine the percentage of solids of the mixture consisted of a volumetric basin supported on scales. A diversion trough, which was moved under the end of the pipe when samples of the mixture were taken, was attached to the volumetric basin.

Concentrations of material over the cross-sectional area of the pipe were determined by inserting a tube, $\frac{7}{16}$ in. in diameter and bent to an angle of 90°, into the end of the pipe. To insure comparable locations for all samples, the pipe was held in place by a plate that could be raised or lowered between two hinges in order to give vertical control. Horizontal control was secured by a wing nut which held the tube firmly in place. Three ranges across the pipe were used for observations: One at the center of the pipe, and the other two at one-half the distance to the edge of the pipe on either side of the center range. This apparatus is shown in Fig. 2.

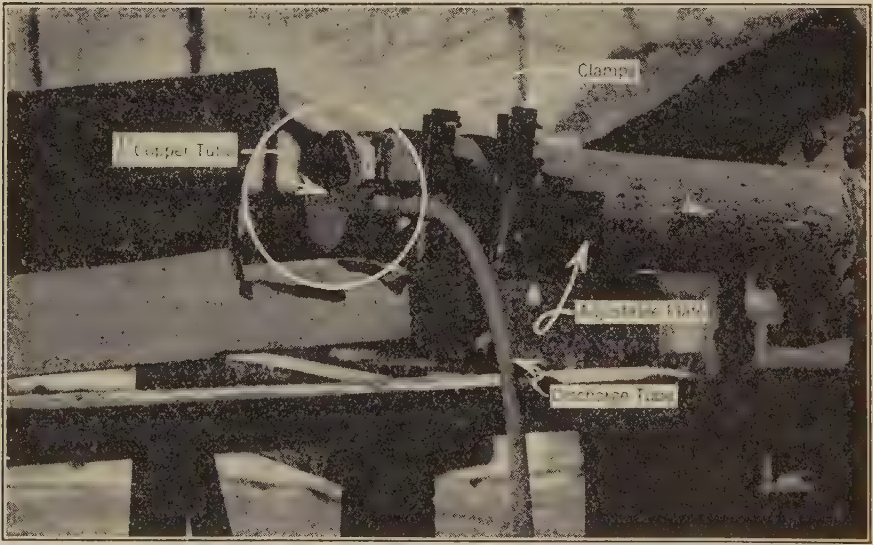

FIG. 2.—APPARATUS FOR DETERMINING THE CONCENTRATION OF SOLID MATTER

PROCEDURE OF EXPERIMENTATION

The material tested is known at the United States Waterways Experiment Station as Pearl River sand, an analysis of which is shown on Fig. 3. The gravel in Fig. 3, known locally as "pea gravel," had the general specification that no particle should be larger than $\frac{1}{4}$ in.

Sand Tests.—The velocity was first set at about 5 ft per sec and a solid concentration introducd into the circulating system that caused about 10 lb per sec to be discharged at this velocity. (The percentage of solids used for these investigations was apparent; that is, included water in the voids. This resulted in a specific gravity for wet sand of 2.08 and wet gravel of 1.94, instead of the 2.65 customarily used.) When the condition had been obtained and checked, a complete set of manometer readings was taken and the velocity increased to about 6 ft per sec. Another set of manometer readings was taken at this increased velocity. In this manner the data for the complete velocity range obtainable with the apparatus were obtained for a sand discharge of 10 lb per sec. The equilibrium set up in the system was such that, when the velocity was increased, although the percentage of solids decreased, the same weight of sand was discharged. Clear water was used in the manometers and this pressure was later converted to feet of mixture of the solid concentration.

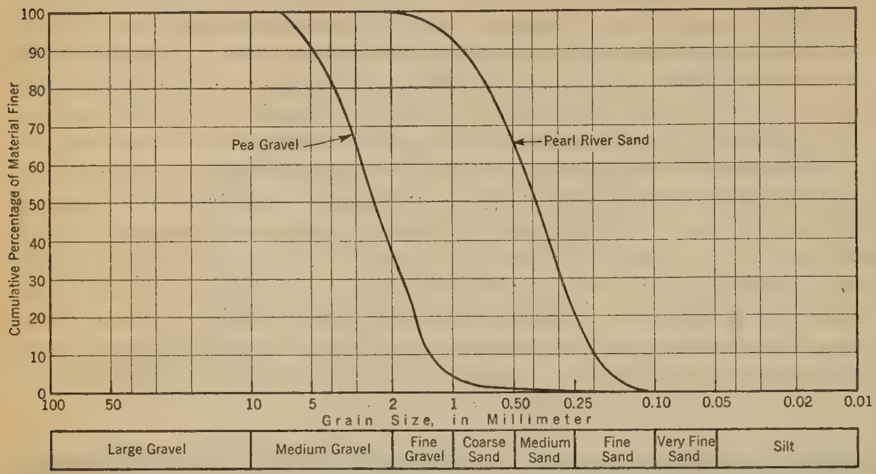

FIG. 3.—MECHANICAL ANALYSIS OF PEARL RIVER SAND AND PEA GRAVEL, TESTED IN A FOUR-INCH PIPE

After obtaining the data for the 10-lb per sec discharge, the velocity was reduced and material added until the discharge was 15 lb per sec. The foregoing testing procedure was followed, for the new sand discharge, throughout the entire range of the carrying capacity of the pipe, with the sand discharge increased in 5-lb increments.

The distribution of solids over the cross-sectional area of the pipe was determined by inserting the bent tube sampler and taking observations at different places in the section. Besides obtaining data for the percentage of solids passing a certain point, information was also obtained relative to the velocity in the pipe at that location.

Gravel Tests.—The primary difference between these tests and the sand tests lay in the fact that the testing range was considerably decreased. Velocities of from about 5.5 to 8.5 ft per sec constituted the entire range covered in the investigation. Since an equilibrium similar to that obtained during the sand tests occurred during this investigation, it was possible to observe the entire velocity range for any given weight of material discharged without changing the quantity of gravel in the circulating system. Gravel discharges of from 10 to 30 lb per sec were used and the same test routine was followed as that used for the investigation of sand. Considerably more difficulty was experienced in obtaining check readings for these tests on gravel than for those with sand. The distribution of solids over the cross-sectional area could not be obtained on account of the fact that the size of the gravel was such that it would plug the sampling device. Although the particles would enter the tube, the curved outlet caused blocking.

Method of Transporting the Material

Sand.—Three distinct types of transportation were found for sand and water mixtures in pipes. The first of these was at low velocities, where practically all material was on the bottom of the pipe. A layer of sand was built up and the material rolled over this bed in much the same manner as in an open flume. The second type of movement was one in which the sand layer that was built up at low velocities would move forward slightly and then halt, while other material would still move freely over this bed. This spasmodic movement can be termed most correctly, "jerking," because the material along the bottom moved freely one instant and was quiet the next. The third type was one in which the material was moving over the entire cross-sectional area, although the greater percentage was concentrated in the lower part of the pipe. These three types of movement cause head-loss determinations for sand and water mixtures naturally to be quite complex, because of the indeterminate line of differentiation. For a 4-in. pipe transporting the sand that was used in the studies at the U. S. Waterways Experiment Station, the type of movement in which the material is on the bottom of the pipe will generally occur for velocities of less than 6.0 ft per sec; the type for material that jerks along the bottom, for velocities of from 6.0 to 7.5 ft per sec; and the type for material that is moving at velocities greater than 7.5 ft per sec. Although the

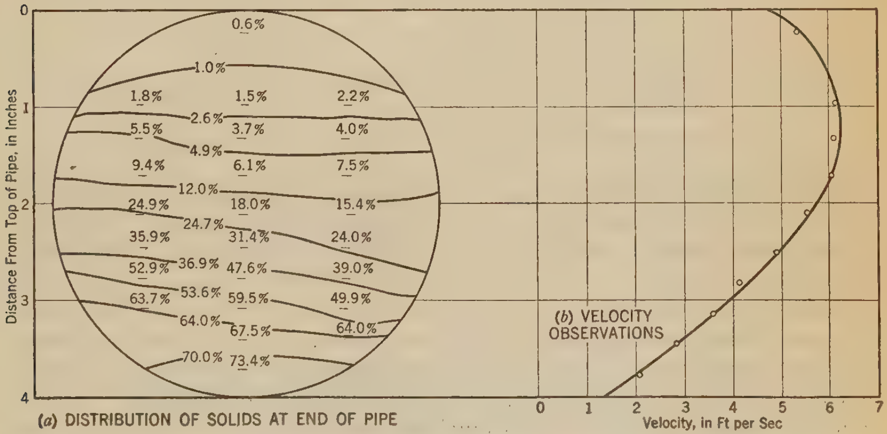

Fig. 4.—Distribution of Solids and Velocity Observations in the Cross-Section of a Plain Four-Inch Pipe

foregoing values will vary somewhat with solid concentrations, it is possible to use them for general purposes.

The manner in which the material is concentrated over the cross-sectional area is shown in Fig. 4, together with the variation of vertical velocities. These velocity observations are considered as weights rather than true values because it was not possible to obtain a satisfactory calibration of the sampling device, due to the variation of the solid concentrations, coefficient of friction, and mean grain size of material transported at various locations in the pipe. General data relating to these observations are:

Solid matter discharged, in pounds per second	14.6
Percentage of solids	14.4
Velocity of flow in pipe, in feet per second	8.88

Although it can be seen that the greater concentration of material is on the bottom of the pipe, this does not mean that the greater quantity of material is carried in this area. Fig. 5 presents a curve showing the distribution of solids in a vertical section and a curve showing the volume of material discharged from this vertical section. The volume curve was obtained by measuring the areas between contours shown on Fig. 4 (by planimeter) and multiplying these areas by a value obtained from the velocity curve.

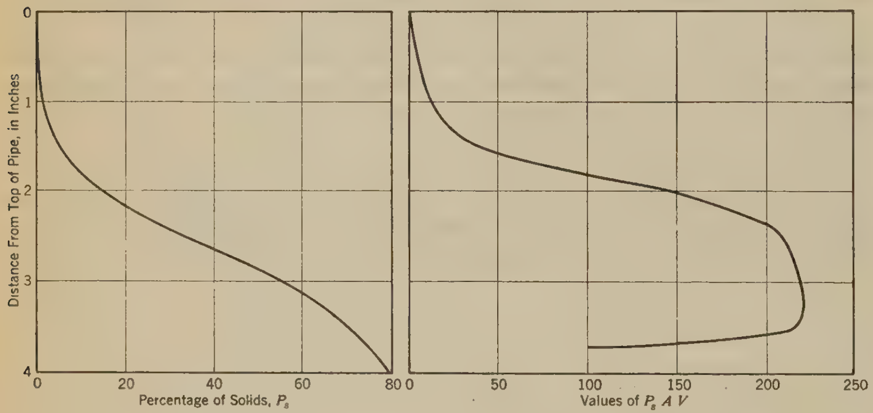

Fig. 5.—Curves of Solid Concentration and Quantity of Material Discharged

Gravel.—Types of movement similar to those observed for sand were found for gravel, with the exception that the first type (that in which the material rolled over a motionless layer) covered such a small range that it should scarcely be considered in a class by itself. The stage at which the material "jerked" along the bottom extended over a much greater velocity range than had been observed for sand, but, due to the very gradual change from this range to the one in which all particles were in motion, it was not possible to determine a definite upper transition velocity. The lower transition velocity was determined to be about 5.0 ft per sec. At velocities less than this value, for the solid concentrations that were investigated, the pipe became blocked frequently.

This was caused by the lower gravel particles becoming motionless on the bottom and the other particles piling rapidly on top of each other instead of rolling along over a motionless layer as did the sand particles.

Head Loss Variations

Curves showing changes in head loss for sand and gravel mixtures in the 4-in. pipe are presented in Fig. 6. These curves show tendencies that are to be expected from the aforementioned types of movement. At velocities where material is settled or greatly concentrated on the bottom, there is a greater head loss per foot of pipe than for slightly higher velocities, because of the pressures created by the constriction of the cross-sectional area. The minimum head loss was obtained during the velocity range in which the jerking movement occurs, since the constriction was being removed. Curves of the losses for sand (Fig. 6(*a*)) show this condition more clearly than do those for gravel (Fig. 6(*b*)), because of the small velocity range investigated for gravel and the fact that gravel was found to travel along the bottom in a high concentration at all velocities. The curves for sand show points of minimum loss and the gradual increase in head loss at high velocities. The gravel curves, however, show only the great losses found at low velocities and the gradual decrease in head loss for a corresponding increase in velocity.

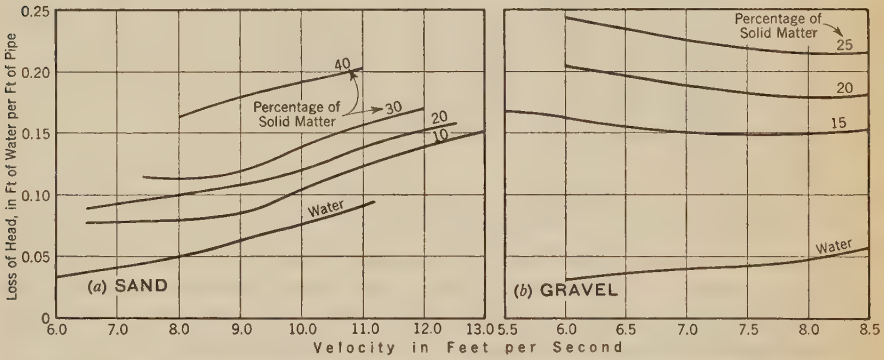

Fig. 6.—Head Loss Curves for Various Sand and Gravel Mixtures

When applied to water, exponential pipe formulas are more accurate than the Darcy formula,

$$h_f = \frac{f\,L\,V^2}{2\,g\,D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

which fact more than offsets the extra computation required. Changes in head loss for pipes carrying sand or gravel ,however, can best be shown by the formula,

$$f = \frac{2\,g\,D\,h_f}{L\,V^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and the results from an investigation of f-values will be presented before consideration is given to any exponential formulas. (In Equations (1) and (2): h_f = loss of head due to friction; f = a friction factor; L = length of pipe, in feet; V = average velocity, in feet per second; g = acceleration due to gravity; and D = mean inside diameter of pipe, in feet.)

The Common Formula for Head Loss.—Values of f for various concentrations of material are presented in Table 1. (It should be noted that these values are determined from a head that is expressed in feet of mixture rather than in feet of water.) Inspection of Table 1(*a*) shows decrease in f-values for increases in velocity as well as increases in f-values for practically all increases in solid concentration. Observations through the transparent section of the pipe indicated that these variations in f could be expected, because of changes in transportation characteristics that were observed.

TABLE 1.—VARIATIONS IN THE FRICTION FACTOR, f,* FOR CHANGES IN SOLIDS CONCENTRATION IN A FOUR-INCH PIPE

Velocity, V, in feet per second	(a) Values of f for the Following Percentages of Sand in the Mixture							(b) Values of f for the Following Percentages of Gravel		
	10	15	20	25	30	35	40	15	20	25
5.5	....	....	....	....	....	....	....	0.089	....	....
6.0	....	....	....	....	....	....	....	0.084	0.102	0.118
6.5	0.035	....	....	....	....	....	....	0.069	0.084	0.096
7.0	0.030	0.031	0.033	....	....	....	....	0.058	0.070	0.082
7.5	0.027	0.028	0.030	....	0.034	....	....	0.050	0.060	0.068
8.0	0.024	0.026	0.027	0.029	0.030	....	0.038	0.044	0.051	0.058
8.5	0.022	0.024	0.026	0.026	0.027	0.029	0.036	0.040	0.045	0.051
9.0	0.019	0.023	0.024	0.025	0.025	0.029	0.033	....	....	....
9.5	0.020	0.022	0.023	0.023	0.024	0.028	0.031	....	....	....
10.0	0.020	0.021	0.021	0.022	0.023	0.026	0.029	....	....	....
10.5	0.020	0.020	0.020	0.021	0.022	0.024	0.027	....	....	....
11.0	0.020	0.020	0.020	0.021	0.021	0.023	0.025	....	....	....
11.5	0.019	0.019	0.020	0.020	0.020	0.022	....	....	....	....
12.0	0.019	0.019	0.019	0.019	0.019	....	....	....	....	....
12.5	0.018	0.018	0.017	....	....	....	....	....	....	....
13.0	0.017	....	....	....	....	....	....	....	....	....

* All values of f are computed from a head that is in feet of mixture rather than in feet of water.

TABLE 2.—VARIATIONS IN FRICTION FACTOR, f,* FOR CHANGES IN SOLIDS CONCENTRATION IN PIPES OF VARIOUS SIZES

Velocity, V, in feet per second	One-Inch Pipe			Three-Inch Pipe			Four-Inch Pipe		
	Values of f for the Following Percentages of Solid Matter:								
	10	20	30	10	20	30	10	20	30
4.0	0.060	0.084	0.098	0.070	0.081	0.098	0.076	0.100	0.123
5.0	0.046	0.054	0.066	0.048	0.059	0.069	0.055	0.071	0.085
6.0	0.038	0.041	0.050	0.039	0.047	0.053	0.044	0.055	0.064
7.0	0.036	0.037	0.041	0.034	0.039	0.043	0.037	0.045	....
8.0	0.034	0.034	0.038	0.030	0.034	....	0.033	0.038	....
9.0	....	....	....	0.028	0.030	....	0.029	0.034	....

* All values of f are computed from a head that is in feet of mixture rather than in feet of water.

Investigation of the variation of f-values when gravel was transported shows characteristics similar to those obtained for sand. These values are shown in Table 1(*b*). The changes in values of f can be seen to follow the same trends that were found for sand.

Investigation of the 1-in. pipe by Miss Blatch and the 3-in. and 4-in. pipe of Hazen and Hardy shows tendencies similar to those obtained from the 4-in. pipe at the U. S. Waterways Experiment Station. Computations of f-values, made for the 1-in., 3-in., and 4-in. pipes are presented in Table 2. The weight of wet sand in these tests was assumed equal to 129.5 lb per cu ft.

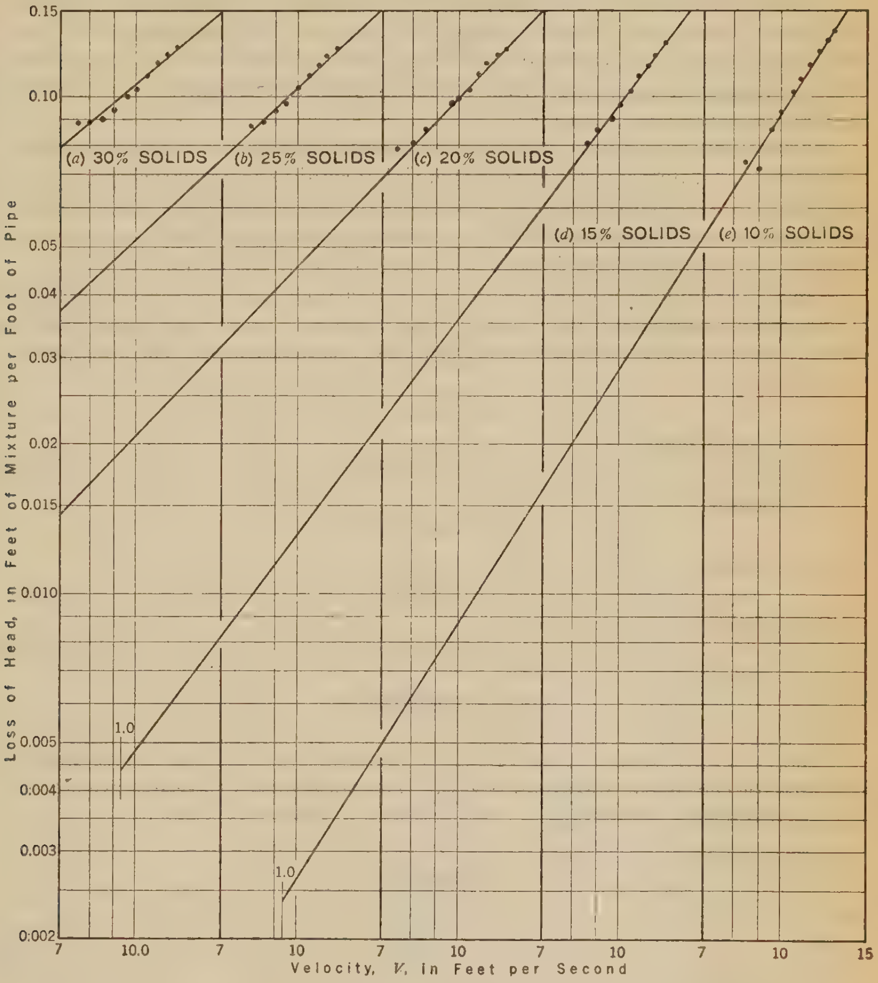

FIG. 7.—LOGARITHMIC PLOT OF HEAD LOSSES FOR VARIOUS SAND CONCENTRATIONS

The only data available for f-values in clear water referred to the 1-in. pipe, and these values were found to range from 0.037 at 3.0 ft per sec to 0.035 at 11.0 ft per sec. Values of f for the 4-in. pipe at the U. S. Waterways Experiment Station varied from 0.0206 at 5.4 ft per sec to 0.0164 at 11.2 ft per sec.

Results from the 4-in. pipe for the Hazen and Hardy investigations cannot be compared with data obtained at the U. S. Waterways Experiment Station, because of the differences in pipe and sand. The test pipe described herein has two 2-ft pyralin sections and one 2-ft sheet metal section in the test line. No sand analysis curves were presented in the Hazen and Hardy paper.

These results show the differences that can be expected in head losses due to the type of material that is being transported. It can be seen from a review of these data that a very fine sand should be transported with much less head loss than a large sand or small gravel. Changes in the gradation of the materials are the primary causes of variations, and it is due to these changes that it is not possible to obtain a formula for head loss that is applicable to any but specific gradations of material.

Exponential Formulas.—Attempts were made, without success, to determine a satisfactory approach to an exponential formula for pipes carrying sand. Due to the fact that head losses were believed to vary with the three types of sand movement, the attempts at developing exponential formulas were confined to the velocity range in which all particles were in motion.

Logarithmic plots of varying solid concentrations were made and the results of these plots are shown in Fig. 7. From these curves it was found that the slope of the lines varied with changes in solid concentration, the formulas of the type,

$$h_f = m\,V^x \qquad (3)$$

being, for 30% solids:

$$h_f = 0.0160\,V^{0.830} \qquad (4a)$$

for 25% solids:

$$h_f = 0.0118\,V^{0.950} \qquad (4b)$$

for 20% solids:

$$h_f = 0.0089\,V^{1.050} \qquad (4c)$$

for 15% solids:

$$h_f = 0.0043\,V^{1.360} \qquad (4d)$$

and, for 10% solids:

$$h_f = 0.0024\,V^{1.590} \qquad (4e)$$

The next step was to plot values of the intercept of these curves against the slope of the line as shown on Fig. 8. The equations for these curves (of the type of Equation (3)) being:

For Curve (1),

$$m = 0.0044\,P_s^{1.740} \qquad (5a)$$

and for Curve (2),

$$x = \frac{6.4}{P_s^{0.595}} \qquad (5b)$$

(In Equations (5*a*) and (5*b*): m = the intercept; P_s = the solid concentration; and x = the exponent for the general equation.) Although this action provided a satisfactory set of curves which were applicable to all solid concentrations, it was realized that this formula would yield data that were satisfactory for only one size of pipe transporting only one particular class of sand. A family of curves might yield data applicable to general cases, but the foregoing data considered only one of the types of movement. Since data were lacking for all

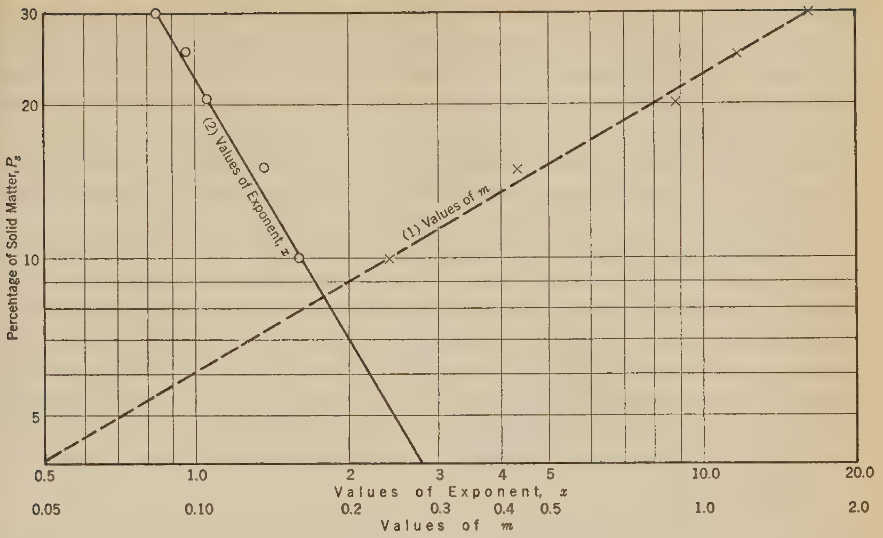

FIG. 8.—LOGARITHMIC PLOT OF EXPONENT AND *m*-VALUES FOR VARIOUS SAND CONCENTRATIONS

except one set of curves, and it was believed that this method would result in more complications relative to transition velocities for the types of movement, efforts to obtain an exponential formula were abandoned.

THE ECONOMICAL VELOCITY FOR SAND TRANSPORTATION

The velocity at which any given volume of sand per hour can be transported through a given length of pipe with the least expenditure of power per unit volume of sand transported, has been called by Miss Blatch[7] the "economical velocity." Curves taken from diagrams in her discussion can be found in several references on sand transportation, and it is the information contained in these curves rather than the definition itself that will be treated herein.

In order to determine an economical velocity for transporting material in any given size of pipe, the following factors should be considered: (1) The length of line to be used; (2) the energy required to produce the desired velocity; (3) the type of material to be transported; and (4) the solid concentration that will cause a minimum loss of head at the velocity that is desired. The resultant economical velocity, therefore, is a delicate balance among these factors.

Of the aforementioned considerations, it is believed that the type of material to be transported, Factor (3) is most pertinent. A comparison of Figs. 6(*a*) and 6(*b*) shows clearly the great effect on head loss that will result from variations in gradation of the material. These differences in grain sizes have a decided effect on all transportation characteristics and the results obtained from one class of material cannot be expected to hold for another. The fact that the data from the U. S. Waterways Experiment Station fail to check those of Hazen and Hardy is attributed directly to this cause. Therefore, it is suggested that exceeding care be taken in the use of curves giving "economical velocities" or "velocities at which all material is in motion," as these curves fail to consider the changes in the physical composition of the material to be carried. Too great emphasis cannot be placed on this point, because a blind acceptance of values from curves of unknown origin will lead to unusual and unexpected results.

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[7] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 406.

Although no method for determining the "economical velocity" or "the velocity at which all material is in motion" is presented in this paper, it is desired to emphasize the fact that for each type of material there will be variations in characteristics that will definitely affect the economical velocity. It is recommended that for all exact design purposes a test line be used to determine the characteristics of the material, rather than a curve or formula, because any data that can be obtained relative to the characteristics of a particular material in a pipe will be of far more value than all theoretical approaches to the solution.

Transferability of Experimental Results

Results from the investigations on 4-in. pipe cannot be applied, by means of any model law, to larger pipe used in dredging. General variations of type of movement, head losses, and method of transportation, discussed previously, occur in a similar manner in larger pipe. Because of the constant variations in the material pumped by dredges, the results from 4-in. pipe can be considered only qualitatively, when applied to the larger pipe lines. Velocity determinations in regard to the ranges of the different classes of movement are of primary importance in attempting to apply results from smaller pipe to the larger sizes. Observations through the transparent section of the test line showed that a velocity range of from about 5.0 ft per sec to 8.5 ft per sec in the 4-in. pipe would cover all types of movement that occur in the discharge lines of dredges. It can also be expected that, depending on the length of the discharge line of the dredge, the material in the line will move in accordance with one of the three types of movement previously described. For long discharge lines, where the velocity drops to about 14 or 15 ft per sec in a 32-in. line, or to 9 or 10 ft per sec in a 20-in. line, the material will settle on the bottom of the pipe, provided sand is being pumped. As the length of the line is decreased and the velocity is correspondingly increased, the types of movement change until the material is being literally blown through the line, passing through exactly the same stages of transportation as those found in the 4-in. pipe.

The determination of an economical velocity for the discharge lines of dredges cannot be made directly from characteristics of the transported material, however, because it is usually true that power developed by the pumping machinery and the fuel consumption have more bearing on this velocity than head losses in the line. This fact makes the economical velocity for each dredge an individual problem which cannot be solved until a sufficient period of operation has elapsed to furnish data relative to output and fuel consumption for various pump speeds. Although it can be found that the minimum head loss for the discharge line would occur at a certain velocity, it must also be considered that a greater quantity of material might be moved, at a higher velocity and with a slightly greater fuel consumption, but with no real increase in operating overhead. It can be seen, therefore, that, besides the theoretical conditions for the economical velocity in the pipe line, the consideration of operating costs and depreciation must necessarily play a greater part in determining velocities for use in large pipe lines.

CONCLUSIONS

In attempting to generalize the findings of this investigation, the conclusions are confined to a range in which as many statements as possible can be confirmed by data from previous investigations. This is done, whenever possible, to prevent the forming of conclusions from only one set of data.

The investigations described herein, with information available from other studies conducted on somewhat similar lines, justify the following conclusions:

(1) Sand is transported in pipes by rolling along the bottom at low velocities, "jerking" along the bottom at medium velocities, and by having all particles in motion at velocities greater than the range in which "jerking" occurs;

(2) The largest quantity of material is transported in the lower third of a pipe rather than along the bottom;

(3) For pipes carrying sand, values of f in Equation (1) will decrease with an increase in velocity;

(4) Values of f will increase with an increase in solid concentration, for any velocity;

(5) A general formula for use in determining head loss in pipes carrying sand is definitely dangerous for use by any person who has not had considerable experience with the use of the formula;

(6) The economical velocity for transporting solids depends upon the character of the material to be carried, and each class of material will probably have a different economical velocity for the same size of pipe;

(7) A pipe line, transporting material of a large grain size at low velocities, will become blocked much more frequently than pipes carrying material of a small grain size, because the larger particles tend to become locked, obstructing other particles, and do not form a smooth bed over which the material can move; and,

(8) The transfer of results from a small pipe line to a line of greater diameter must be qualitative and not governed by any law of corresponding velocities.

ACKNOWLEDGMENTS

The investigations described herein were conducted at the U. S. Waterways Experiment Station, at Vicksburg, Miss., in connection with an investigation dealing with methods of increasing the capacity of pipe lines transporting

solids. They were made under the direction of F. H. Falkner, Lieutenant, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E. The writer wishes to thank the present Director of the Station, Paul W. Thompson, Lieutenant, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E., for his co-operation in making this paper possible and for suggestions in connection with its preparation. He also wishes to thank Joseph B. Tiffany, Jr., Jun. Am. Soc. C. E., and V. G. Kaufman, for reviewing the contents prior to final assembly.

DISCUSSION

Fred R. Brown,[8] Jun. Am. Soc. C. E. (by letter).—Despite the findings of recent worth while investigations, the information on the transportation of material in pipe lines is still rather meager. Hence, there are not at hand data sufficient for the development of a formula with comprehensive application to the movement of solids through pipe lines. If such a formula can be determined its development can follow only upon the collection of further data, and the correlation of these data with those previously obtained.

TABLE 3.—Variations in Friction Factor, *f*, for Changes in Solids Concentration in a Thirty-inch Pipe

Velocity, V, in feet per second	Values of f for the Following Percentages of Solid Matter:			Velocity, V, in feet per second	Values of f for the Following Percentages of Solid Matter:			Velocity, V, in feet per second	Values of f for the Following Percentages fo Solid Matter:		
	Clear water	10	20		Clear water	10	20		Clear water	10	20
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
4.......	0.030	....	0.265	8.....	0.025	0.052	0.074	12.....	0.022	0.029	0.043
5.......	0.029	....	0.175	9.....	0.025	0.043	0.063	13.....	0.022	0.028	0.039
6.......	0.027	....	0.125	10.....	0.024	0.038	0.053	14.....	0.021	....	0.037
7.......	0.026	....	0.094	11.....	0.023	0.032	0.047	..	....	....	....

It is with this point in mind that the computations in Table 3 are offered. The data for these computations were taken from the curves of the dredge *Alpha*,[9] presented in the discussion by Miss Blatch,[7] and bear out Mr. Howard's findings on 1-in., 3-in., and 4-in. pipes in that the friction factor in pipes transporting material increases with the percentage of material and decreases with the velocity. These values of *f* may be incorporated in Table 2, compiled by Mr. Howard, thus presenting *f*-values in 1-in., 3-in., 4-in., and 30-in. pipes. All values in Table 3 are computed from a head that is in feet of mixture rather than in feet of water.

The magnitude of velocities and the percentage of solids used in deriving the foregoing *f*-values were determined by the volumetric method. Velocities obtained by other methods (salt solution, gage stick, etc.), for pipes transporting material not in suspension, are not true average velocities. Hence, such velocities will introduce appreciable errors in any computations involving velocity. Likewise, there are numerous ways of determining the percentage of solids pumped, but percentages determined by volumetric measurement are the most accurate. Errors in velocity or the percentage of solids will increase as the percentage of solids increases, if other than the volumetric method of measurement is used. For this reason, any data that might be incorporated with those of Mr. Howard in an attempt to arrive at a formula for pipes transporting material should have the velocities and the percentages of solids determined volumetrically. Data obtained otherwise would not be comparable with that presented so ably by Mr. Howard.

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[8] Jun. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[9] "Dredges and Dredging on the Mississippi River," by the late J. A. Ockerson, Past-President, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XL (December, 1898), p. 215.

JOSEF E. MONTGOMERY,[10] ESQ. (by letter).—One of the most interesting points developed by Mr. Howard is that which shows the increase in values of Darcy's f with decrease in velocity for flow in a 4-in. pipe carrying water and entrained sand or gravel. Below certain values, the relation of f to velocity is much different for solids-laden water than for clear water. The cause of this difference is shown by comparison with f-values for flow of clear water in similar pipes. Data suitable for such comparison are found in the results of previous work[11] by Fred C. Scobey, M. Am. Soc. C. E., published in 1930. Using the data obtained by Mr. Scobey, curves have been plotted in Fig. 9 to show the relation of values of f in Equation (2) to mean velocity for the flow of clear water in a 4-in. pipe. Representative data obtained by Mr. Howard have been likewise plotted. At higher velocities all values approach the same relation to velocity, although the scope of the investigation was not sufficient to provide for full comparison.

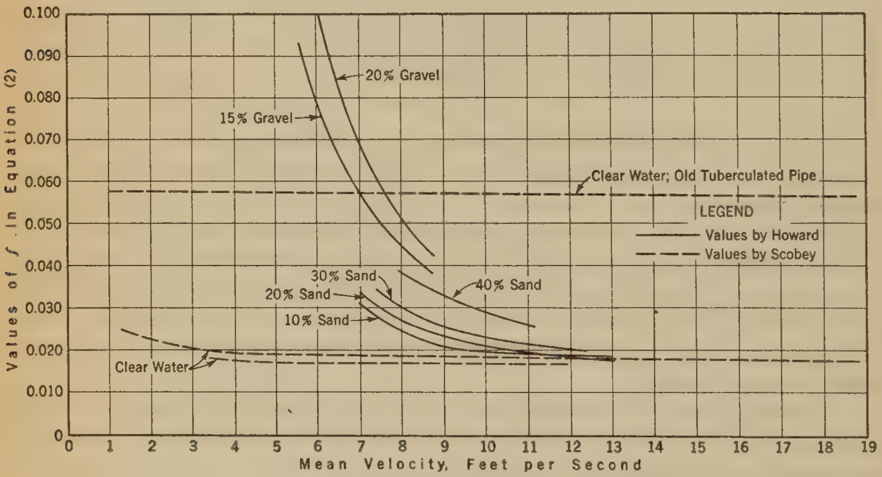

FIG. 9.—COMPARATIVE VALUES OF DARCY'S f.

The curves for the runs with gravel are least similar to the curves for clear water, but Mr. Howard states that some gravel remained motionless on the bottom during the full range of velocities observed. Curves for the sand approach those for clear water, and two of the curves coincide at higher velocities. It is apparent that the highest values of f occur for the lowest velocities and highest solids concentration, in which some solid material was observed to be at rest on the bottom of the pipe. The relation of f-values to velocity for sand-laden water approaches that for clear water in those runs in which all material was known to travel in suspension.

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[10] Engr. Aide, U. S. Waterways Experiment Station, Vicksburg, Miss.

[11] "The Flow of Water in Riveted Steel and Analogous Pipes," by Fred C. Scobey, *Technical Bulletin, No. 150*, U. S. Dept. of Agri., Washington, D. C., pp. 35 and 38.

The data show that entrainment of solids (that is, sand or gravel) is not alone sufficient to produce appreciable effect on the variation of f with velocity, but that it is deposition of such material at the bottom of the pipe that affects the relation. This change is not due to the pressures created by constriction of cross-sectional area, as stated by Mr. Howard under "Head Loss Variations," since the friction factor, f, is generally unaffected by pressure changes. The explanation is found in the fact that values of f are very sensitive to any deviation from the nominal area value used in the computation of this coefficient.

Comparisons of this type can be put to practical use in dredging and similar operations because they indicate the possible existence of motionless solids on the bottom of a pipe and the consequent waste of energy. Curves of the type shown for gravel in Fig. 9 show the excessive energy loss due to insufficient velocity for that particular concentration. If extended by additional data, these curves would indicate further the optimum velocity for transportation of such material.

ELLIOTT J. DENT,[12] M. AM. SOC. C. E. (by letter).—This paper is of particular interest to the writer as it supplies a more complete picture of the manner in which sand and water mixtures flow through pipe lines than any of the reports that had previously come to his attention. With the information presented, combined with that previously available, and combined further with data furnished by Mr. Howard by letter, it is possible to picture the behavior of quite a wide range of mixtures flowing in a 4-in. pipe. Although such a portrait may not be applicable to full-sized dredge pipes in a quantitative sense, the data do afford a qualitative picture for the benefit of dredge designers and operators. These laboratory results will also constitute an excellent guide for the preparation of a program for further investigations.

The author emphatically calls attention to the effect of the size of the material handled, and gives an analysis of the sand and gravel used in the experiments reported upon. The importance of this feature cannot be overestimated and it is to be hoped that, in all future investigations, this practice will be continued. In this paper, as in most others on the subject, the friction losses due to the flow of water only have been inadequately described.

In a relatively unexplored field of this nature it is desirable that the factual data as well as the adjusted results be published. Students of the subject should be free to use their own methods for adjusting, compiling, and presenting the facts and the conclusions derived therefrom. For this subject there is no standardized system of presentation and, pending the time when a satisfactory system may be evolved, a wide variety of viewpoints is most desirable.

In this and other papers on the subject references are made to the manner in which the pipe lines are completely blocked off when the velocity falls below a certain value. The author has very kindly furnished the following data showing the blocking-off points in sand tests of a 4-in. pipe, for some of the mixtures handled:

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[12] Col., U. S. Army (*Retired*), Washington, D. C.

Velocity	Percentage of sand concentration
5.7	26
7.0	27
7.5	34
8.2	52

In adjusting these data the writer has assumed that another point would correspond to a velocity equal to 00 and a percentage of solids equal to 00. The points were plotted and a smooth curve was drawn conforming as closely as possible to the data. The values for 15%, 20%, and 25%, were then taken from the curve. It is hoped that, in future investigations, the blocking-off points will be carefully established.

In presenting his adjusted data the author has reduced the losses of head to terms involving the specific gravity of the mixtures handled and has computed and tabulated the values of f in the Darcy formula. This method does not afford a simple or readily visualized picture of the case.

A formula for the loss of head, which is better known than Equation (1) of the paper, is the Chezy formula:

$$V = C\sqrt{R\,S} \qquad (6)$$

in which R = the mean hydraulic radius $= \frac{D}{4}$, in feet; S = the sine of the angle of slope $= \frac{h_f}{L}$; and, C = a factor often erroneously referred to as a constant.

Combining Equations (1) and (6):

$$V^2 = \frac{2\,g\,D\,h_f}{f\,L} = C^2\,R\,S \qquad (7)$$

Substituting S for its equivalent, $\frac{h_f}{L}$, and R for its equivalent, $\frac{D}{4}$:

$$V^2 = \frac{8\,g}{f}\,R\,S = C^2\,R\,S \qquad (8)$$

Therefore, $\frac{8\,g}{f} = C^2$; and, $f = \frac{8\,g}{C^2}$. It is obvious that phenomena that cause a variation in the values of C in the Chezy formula will cause corresponding variations in the values of f in the Darcy formula.

It is well known that, in the Chezy formula, the values of C vary with the slope, hydraulic radius, and roughness of the inner surface of the pipe. About two generations ago a formula for computing the value of C was devised and is known as the Kutter formula. It is laborious to solve and many tables and diagrams have been published to assist in this process. Similar tables and diagrams can be prepared for finding the values of f in the Darcy formula if it is desired to do so.

In dealing with the flow of water through pipe lines, the value of f is a complicated function of roughness, diameter, and slope. In dealing with the flow of mixtures of sand and water, the author has added to the complications by making f a function of roughness, slope, diameter of pipe, coarseness of material, percentage of material in the mixture, and specific gravity of the mixture.

When dealing with water alone the values of C in the Chezy formula were so involved that a simplified expression was urgently needed. About a generation ago the late Allen Hazen and the late Gardiner S. Williams, Members, Am. Soc. C. E., devised the following exponential formula for computing the friction due to the flow of water in pipe lines. The nomenclature used is the same as that in Equation (6):

$$V = C\,R^{0.63}\,S^{0.54}\,0.001^{-0.04} \qquad (9)$$

A slide-rule was devised for the convenient and rapid solution of Equation (9), and tables and diagrams have been prepared and published in handbooks and elsewhere. The writer uses this method for computing the losses of head in pipe lines. Mr. Howard states that, for water only, the exponential formulas are to be preferred; but for mixtures of sand and water—a more complicated condition—he uses the Darcy formula.

The normal problem confronting the dredge designer or operator is to determine the loss of head due to a given velocity when the other items of the problem have been determined or assumed. The author calls attention to the difficulties encountered in attempting to devise an exponential equation or equations that would give reliable results.

In Fig. 6 Mr. Howard has plotted certain data, and it is believed that an extension of this graphic method may result in a very useful diagram. If existing information is plotted now, the diagrams will be of immediate use to designers and operators. The diagrams will also indicate the nature of the information that should be secured by additional investigations in order to increase the sum total of knowledge on the subject.

In Fig. 10 the writer has plotted certain fairly reliable information with respect to the friction losses due to the flow of various concentrations of various kinds of materials, at various velocities, in a 4-in. pipe. Some adjustments of the basic data have been necessary. The curve for water only has been based on the value, $C = 140$, in the Williams-Hazen formula. This conforms fairly well with the author's observation that $f = 0.0206$ when the velocity was 5.4, and $f = 0.0164$ when the velocity was 11.2.

Curve (1), 15% loess, was plotted from data supplied by the author. The effective size (10% of the material is finer than this size) was taken from a diagram also furnished by Mr. Howard. Curve (2), 15% sand, was obtained from data published by John A Vogelson,[13] M. Am. Soc. C. E. The suspended material was described as bar sand, with an effective size of 0.26 mm. The average of the runs used gave the percentage of sand as 15.97, and it was assumed that for 15% of sand the loss of head due to the presence of the sand would be proportional to the percentage of solids present, or $\frac{15}{15.97}$ times the observed loss.

Curves (3), 15%, 20%, and 25% sand, were computed from the data presented in Table 1 of the paper, the curves being then extended to indicate the rise in loss of head as the blocking-off points were approached. The blocking-off points were obtained from a smooth curve plotted as already explained, Curve (4), 15% sand, was plotted from data published by Mr. Vogelson[14] adjusted in the manner described for Curve (2), 15% sand. The analysis of the sand was as given by Mr. Vogelson.[15] The data for Curves (5), 15%, 20%, and 25% gravel, were taken from Table 1 of the paper and reduced by the foregoing method.

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[13] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 391.

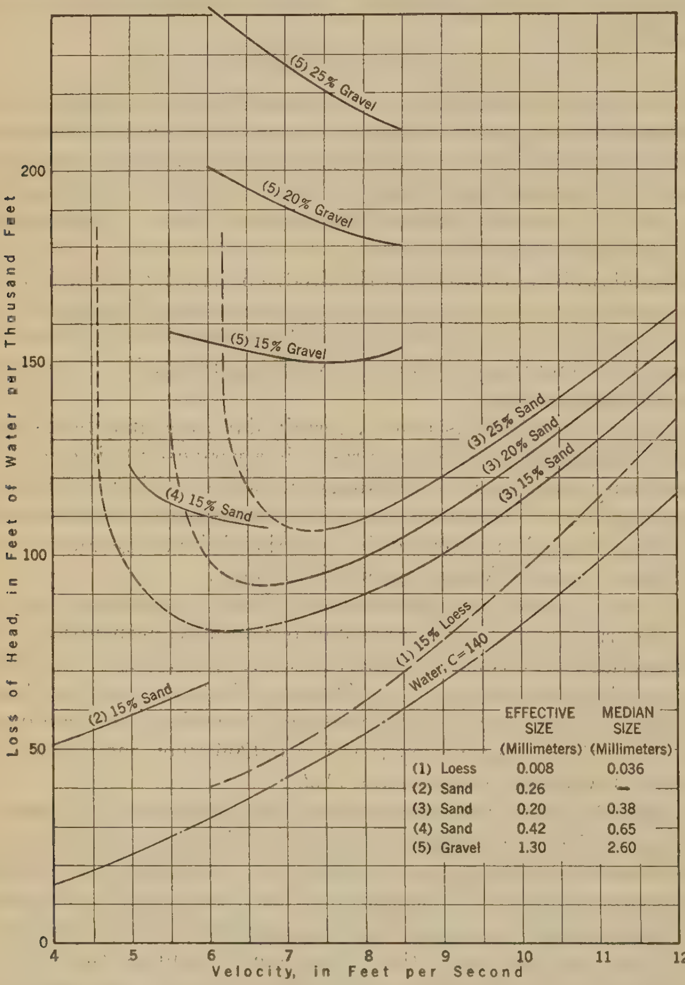

FIG. 10.—FRICTION IN A FOUR-INCH PIPE; MIXTURES OF SILT, SAND, GRAVEL, AND WATER

These are the only tests on 4-in. pipe that have come to the writer's attention, in which sufficient data have been published to make it possible to include them in the diagram, which is now sufficiently complete to warrant certain conclusions. Other data with reference to other pipe diameters might be cited to confirm the conclusions qualitatively although they have not been in sufficient detail to warrant the construction of special diagrams for those sizes.

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[14] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 396.

[15] *Loc. cit.*, p. 395.

The following conclusions may be drawn as a result of the study of Fig. 10:

(*a*) For each variety and each percentage of sand, there is a blocking-off velocity below which the pipe clogs and the flow is shut off. The heavier the percentage of sand, or the coarser the material, the higher will be the blocking-off velocity.

(*b*) With a fixed percentage of a given material the loss of head decreases as the velocity increases from the blocking-off velocity to some point where the total friction is a minimum. The velocity corresponding to the minimum value of the friction loss increases with increase in the percentage of material and with the coarseness of the material.

(*c*) For velocities higher than those corresponding to the lowest friction losses, the loss-of-head curves approach the curve for water only for a certain distance, and then become substantially parallel to that curve. (Curve (1), 15% loess, and other data not published herein, indicate that with very high velocities the loss-of-head curves diverge from the water curve).

(*d*) A study of the curves and the effective sizes of the materials indicates that the effective size may not be a suitable measure of the coarseness of the material transported. This appears reasonable as the effective size is determined by the finest 10% of the material. The curves show that the effect of 15% gravel is much greater than that of 25% sand; or, in other words, that a small proportion of coarse material may have a greater effect that a large proportion of fine material. It may develop that a better index will be the size of opening in a sieve on which 25% of the coarsest material would be retained.

In practical dredging operations the muds and silts are well distributed throughout the entire cross-section of the pipe line whereas the sands and gravels concentrate near the bottom. Attention is particularly invited to the author's lucid description of the manner in which the material was moved through the transparent sections of the pipe during the experiments reported. These observations made on the 4-in. model are believed to apply qualitatively to the full-sized dredge pipes. For best results in practical dredging operations, the type of flow should be as indicated in Curves (3), 15%, 20%, and 25% sand, with velocities of 8.0 ft per sec, or more. For loess the velocities may be 6.0 ft, or less; and for gravel they should exceed 8.5 ft by an undetermined margin.

In an effort to find a practical working formula to be used in dredge design, the writer, in 1915,[16] suggested the following:

$$h = h_w + h_s = h_w + A\,N \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which (in the notation of the paper): h = the total loss of head due to the flow of a mixture of sand and water per 1 000 ft of pipe line; h_w = the loss due to the flow of water only at the same velocity; h_s = the excess loss due to the presence of the sand; A = a factor depending upon the character of the sand and the diameter of the pipe line; and, N = the percentage of sand present in the mixture.

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[16] *Professional Memoirs*, Corps of Engrs., U. S. Army, March–April, 1915.

This formula puts into algebraic form a rule suggested by Miss Blatch[17] for determining the loss of head at the "economical velocity" and the rule given by Mr. Hazen[18] for transporting sand in 3-in. to 6-in. pipes. For conditions of flow corresponding to the best dredging range (velocities of 8 ft per sec to 12 ft per sec, in Fig. 10), the values in Table 4(*a*), for the terms in Equation (10) may be taken from Fig. 10, or computed. The mean value of *A* (Column (5) Table 4(*a*)) is 2.14. If this value is assumed as correct and if the losses of head are computed by Equation (10), the comparison between the computed and observed values will be as given in Table 4(*b*).

TABLE 4.—VALUES OF TERMS IN EQUATION (10)

Mean velocity, V, in feet per second	Percentage of sand, N	(*a*) FOR CONDITIONS OF FLOW CORRESPONDING TO BEST DREDGING RANGE				(*b*) COMPARISON OF COMPUTED AND OBSERVED VALUES				
		Total loss, h	Loss, h_w, due to water flow	Loss, h_s, due to sand	Factor A	Loss, h_w, due to water flow	Loss, h_s (=2.14 N), due to sand	Total Loss, h		Percentage error
								Computed	Observed	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	25	109	54	55	2.20	54.0	53.5	107.5	109.0	−1
	15	90	54	36	2.40	54.0	32.1	86.1	90.0	−4
10	25	133	82	51	2.04	82.0	53.5	135.5	133.0	+2
	15	114	82	32	2.13	82.0	32.1	114.1	114.0	0
12	25	163	115	48	1.92	115.0	53.5	168.5	163.0	+3
	15	147	115	32	2.13	115.0	32.1	147.1	147.0	0

The relatively small errors that would result, in this model test, from the assumption that, within the range simulating practical dredging velocities, the value of A is dependent solely upon the character of the material and the diameter of the pipe line, are most encouraging. It is hoped that additional data for testing this method will become available, and that, if the method is found to have merit, the working values for *A* will be established.

Practical dredging velocities have increased greatly in recent years due to the development of better machinery and other equipment. Fig. 10 shows how higher velocities make it possible to transport greater percentages of sand. In a 24-in. pipe line a velocity of 24 ft should be within the range of modern equipment and should be at the call of the crew whenever the need for it arises. Lower velocities would be adequate for pipe lines of smaller diameters. In a 1-in. brass pipe a velocity of 5 ft per sec was shown by Miss Blatch to be ample for sand having a median diameter of about 0.63 mm. In a 3-in. pipe with sand having a median diameter of about 0.65 mm, Mr. Vogelson found that a velocity of 8.5 was sufficient; and, with the same sand in a 4-in. pipe a velocity of 7.0 ft per sec was inadequate. Precise observational data for full-sized dredge pipes are not available and, as stated by the author, they are probably not obtainable.

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[17] *Transactions,* Am. Soc. C. E., Vol. LVII (1906), p. 400.

[18] American Civil Engineers' Pocket Book, 1912 Edition, p. 935.

With respect to the diameter of the pipe lines it is known that, for the small sizes, the values of A vary rapidly with the diameter. It is doubtful, however, whether the variation on this account for dredge pipes of 12 to 34 in. in diameter are large. A comparison of Miss Blatch's experiment with a 1-in. brass pipe and Mr. Vogelson's experiment with a 3-in. pipe gives the following:

Description	One-inch brass pipe	Three-inch iron pipe
Suitable velocity, V, in feet per second	5.0	8.5
Head Losses in Feet:		
h	280	141
h_w	100	87
h_s	180	54
Percentage of sand	17.78	17.78
Factor A	10.10	3.04

The best available information is to the effect that, with similar sand, the values of A for pipe lines 12 in. to 34 in. in diameter would be between 1.0 and 1.5.

With respect to the effect of coarseness of the sand upon the values of A, the writer knows of no better information than that shown in Fig. 10, for 4-in. pipes. For full-sized dredge pipes the values are relatively small and the range of values is correspondingly small, provided always that the velocities are greater than those for the minimum friction losses on the head-velocity curves.

As long as a dredge can pick up a full load for its pipe line the most economical velocity will be the maximum obtainable with the equipment. An increase in velocity results in the transportation of a larger volume of mixture and the percentage of solids in the mixture may also be increased. The increase in power required to produce the extra velocity will cost much less than the value of the increase in pay work.

This paper is a valuable and timely contribution to the subject. If it encourages the publication of additional data and promotes a better understanding of the problem, future experimental work can be aimed at the solution of the remaining essential problems and progress in the direction of the final solution will be greatly facilitated.

David L. Neuman,[19] M. Am. Soc. C. E. (by letter).—The definition of "economical velocity" quoted, but not used, by Mr. Howard is of little value in dredge operation. The definition given is that (see "The Economical Velocity for Sand Transportation") "velocity at which any given volume of sand per hour can be transported through a given length of pipe with the least expenditure of power per unit volume of sand transported * * * ." Such "economical velocity" will not result in minimum unit total costs. Fuel costs are a minor part of the total costs of dredging operations. For forty-five pipe-line, cutter-type, hydraulic dredges, varying in size from 12 in. to 30 in., the average fuel costs for 1937 was 13.3%; the average for 12-in. dredges was 8.3%; 20-in. dredges, 11.2%; and, 28-in. dredges, 17 per cent. Generally, minimum unit costs are obtained with maximum output per pumping hour.

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[19] Maj., Corps of Engrs., U. S. Army, Grosse Pointe Village, Mich.

For many materials there is a rather narrow range of velocities in which maximum output is obtained. In a few instances it has been observed that a change of average velocity of 1.5 ft per sec, above or below the velocity producing the maximum output, would reduce the output as much as 40 per cent. This change in output is due to the change of velocity in the suction pipe. In soft or light material, especially, a small change in suction velocity may strongly effect the output. Theoretical considerations indicate that such results could be expected. The "economical velocity," as previously defined, would fall below the range producing maximum output, and the fuel savings through operation at the "economical velocity" would not offset the reduction in output caused by it. However, within the range of velocities producing maximum outputs fuel economy may be important. Many dredge-masters operate under the theory that maximum output is obtained with maximum obtainable discharge velocities. Frequently, outputs with existing dredges can be increased materially by decreasing the discharge velocities.

The discharge velocities must be sufficiently high to carry off, without danger of blocking the line, all the material that can be picked up on the suction side of the pump. Such velocities should be of Mr. Howard's third type of flow in which material is moving over the entire cross-sectional area of the discharge line. Average discharge-line loadings vary from about 12% to 20%; for short periods of seconds duration, loadings as great as 50% solids occur. The values are apparent percentages, which include the water in the voids for the material in place on the bottom. Indications are that for average loading in sand the third type of flow is obtained for values of velocity greater than 15 ft per sec in a 12-in. line, and greater than 19 ft per sec for a 24-in. line. In a 20-in. discharge line, it has been noted that even at a velocity of 22 ft per sec the line could be loaded so heavily with sand that some of the material could be heard moving along the bottom of the pipe. Table 1(*a*), for sand in a 4-in. line, indicates that at 12 ft per sec the loss of head, in feet of mixture, between 10% and 30% solids, was not increased by an increase of percentage of solids. Pipe-line blocking, even in coarse material, including boulders with mean diameters up to about 70% of the pipe diameter, can be avoided for the most part by the use of sufficiently higher discharge velocities. Velocities of 21 ft per sec for a 12-in. line to 30 ft per sec for a 30-in. line are suggested. In dredge design for increased higher discharge velocities, the suction diameter must be correspondingly increased to avoid the reduction of the picking up power of the suction line through too high suction velocity. Practical experience first indicated the need of small suction velocities with relatively high discharge velocities, and dredge design in recent years has followed the trend of increasingly larger suction diameters compared to discharge diameter with increased pump power for the higher discharge velocities. Apparently, minimum suction velocities have been taken at from 12 ft per sec to 14 ft per sec. It is believed that suction velocities might be safely reduced to 8 ft per sec and the output increased by such reduction.

The results of Mr. Howard's investigation could be used effectively if comparative results were obtainable for large size pipe such as that used for dredge discharge lines. Dependable and accurate gages permitting a continuous record of discharge velocities and percentage of solids passing through the discharge line have been developed and used since 1937 in connection with dredging operations. With the use of such gages in dredging operations results similar to those published by Mr. Howard are obtainable. With such gages installed on a dredge the range of discharge velocities producing maximum output and minimum unit costs can be determined on the site of the work, during actual dredging operation, without interference with such operation.

A most worth while investigation pertaining to obtaining increased output and decreased unit costs in hydraulic dredging can be secured through investigation of the action around the suction-mouth opening and in the suction pipe itself. Such investigation might well include the determination of favorable suction velocities for picking up various materials, head losses in the suction line, effective design of the suction-mouth opening, and the relation of suction-line vacuum and velocities to picking up and transporting ability. Such studies can best be performed in the laboratory where visual observation of suction-line action is possible.

MORROUGH P. O'BRIEN,[20] M. AM. SOC. C. E., AND R. G. FOLSOM,[21] ESQ. (by letter).—The writers have published[22] a paper on the transportation of sand in pipe lines, which presented experimental results on a number of sands. They have correlated these results with previously published data on materials ranging in specific gravity from 2.6 to 3.5 and median diameters from 0.0005 to 0.067 in. As this paper[22] draws conclusions which are in conflict with those of the author, a summary, supplemented by certain information not previously published, may be of interest.

The experimental equipment used by the writers was similar to that shown in Fig. 1 and about the same accuracy could be obtained with it. Nominal sizes for wrought-steel pipe were 2 in. and 3 in. The specific gravity of the mixture was determined by weighing measured volumes. The head loss was measured by differential manometers and expressed in feet of the flowing mixture. Runs were made with clear water before and after the series of runs with each sand. The test pipes were horizontal. During some of the runs, a 3-ft length of glass tubing of proper diameter was inserted in the pipe lines.

Conclusions pertinent to a discussion of Mr. Howard's paper were:

(1) At a given sand concentration, there was a certain velocity below which sand tended to accumulate on the bottom of the pipe, ultimately causing clogging if the flow conditions were maintained constant;

(2) Above this critical velocity, the head loss of the sand-water mixture, expressed in feet of the mixture, was found to be the same as for fresh water within the limits of accuracy of the experimental data; and,

(3) The ratio of the pressure drop (horizontal line) for clear water and sand-water mixtures was equal to the ratio of specific gravities.

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[20] Prof., Mech. Eng., and Chairman, Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[21] Asst. Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

[22] "The Transportation of Sand in Pipelines," *Univ. of California Publications in Engineering,* Vol. 3, No. 7, November 12, 1937, pp. 343–384.

If these conclusions are valid, they simplify the computation of head losses with sand-water mixtures because it is necessary only to know the friction loss in the same pipe with clear water. In order to test this relationship, the writers recomputed the published data of Miss Blatch,[23] W. B. Gregory,[24] N. Mikumo,[25] E. J. Dent,[26] M. Am. Soc. C. E., and unpublished data on a few dredges operated by the United States Engineer Department. It was found that all the laboratory experiments agreed with the conclusion that, above a certain critical velocity, the head loss for a sand-water mixture, expressed in feet of the mixture, was the same as the head loss for water in the same pipe. Below this critical velocity, the head loss remained very nearly constant as the apparent velocity, $\frac{Q}{A}$, was decreased. This phenomenon, observed first in the Blatch tests, was ascribed by the writers to accumulation of sand in the pipe to an extent which approximately held the true velocity constant. This conclusion was confirmed by the trend of the curves obtained by Professor Gregory on slurry so fine that it did not settle fast enough to reach equilibrium between runs.

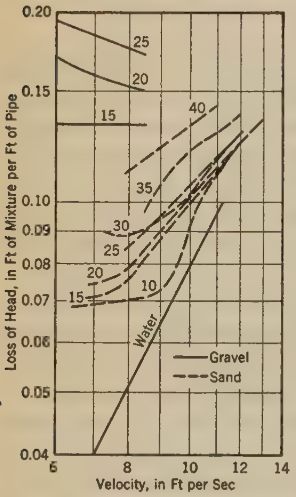

FIG. 11.—COMBINATION OF FIGS. 6(*a*) AND 6(*b*), EXPRESSED IN TERMS OF HEAD LOSS, IN FEET OF MIXTURE PER FOOT OF PIPE.

Fig. 11 shows all the author's data on sand and gravel mixtures and the friction curve for clear water. The trend of these curves is toward the "water" line; unfortunately, the experiments were not carried to sufficiently high velocities to determine whether the curves for mixtures are asymptotic. The data are in general agreement with the writers' conclusions previously mentioned.

There are a number of points in the paper under discussion which require clarification. In Fig. 4, the maximum velocity shown is approximately 6.3 ft per sec, whereas on the following page the velocity of flow is given as 8.88 ft per sec. The exact diameter of the test pipe before and after the experiments is not stated. Although the head loss with clear water is shown in Figs. 6(*a*) and 6(*b*), the friction loss or friction coefficient is not specified. The "water" curves in Figs. 6(*a*) and 6(*b*) appear to differ slightly, raising a question as to whether this difference represents a real change in the pipe characteristic.

In introducing Equation (1), the author states that "when applied to water, exponential pipe formulas are more accurate than the Darcy formula

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[23] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), pp. 400–408.

[24] "Pumping Clay Slurry Through a Four-Inch Pipe," *Mechanical Engineering*, Vol. 49, pp. 609–616.

[25] "On the Hydraulic Conveyance of Slime Pulps (Copper Ore)," *Journal*, Soc. of Mech. Engrs. of Japan, Vol. XXXVI, pp. 825–831.

[26] "Pipe Line Dredges," *Professional Memoirs*, Corps of Engrs., U. S. Army, Vol. VII, No. 31.

* * *." There may be a question of convenience involved, but certainly not of accuracy if Equation (1) is properly used with f as an experimentally determined function of the Reynolds number.

Fig. 7, showing the lines of head loss as a function of velocity, is misleading. The points show a tendency (pronounced in the case of 30% of solids) to break away from the line, and extrapolation as shown is not justifiable.

The distribution of sand concentration shown in Fig. 4 may be explained at least quantitatively in terms of mechanism of turbulent flow. Field measurements by Colonel Dent[26] gave the same type of sediment distribution curve as Fig. 5. A conclusion drawn from this theory is that the settling velocity of the material being transported is its most important characteristic; the sieve analysis, specific gravity, percentage of voids, and other measurable characteristics are important only in so far as they provide a means of estimating the settling velocity. To complete the data, it is desirable that the author measure the mean settling velocity of the pea gravel and Pearl River sand used in his experiments. Using approximate methods based on the sieve analysis, the writers have estimated the mean settling velocities of the author's materials as: Sand, 0.19 ft per sec; and pea gravel, 0.76 ft per sec.

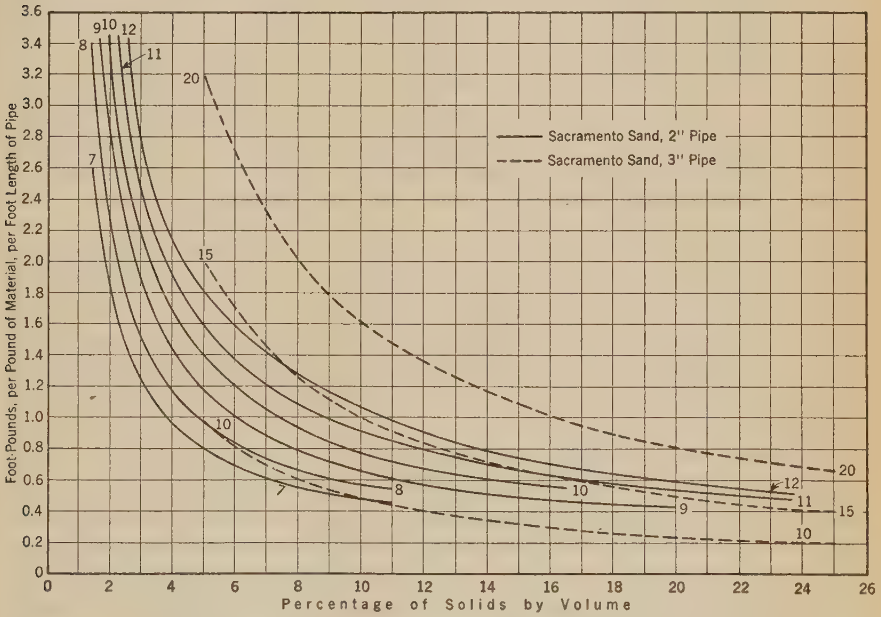

FIG. 12

One of the obstacles to an understanding of the mechanism of sand transportation in pipe lines has been the failure to separate driving unit, pump, and pipe line in field tests. As a consequence, scores of over-all tests of dredges have been made without yielding data of general significance. A similar situation appears to exist in determining the economical operating conditions. The pipe line can, and should, be regarded as a separate unit requiring a certain power input in order to operate at a specified velocity and concentration. To this power may be added the loss in the pump, driving unit, transformers, and all other elements of the energy system, and the total power utilization may then be converted to terms of cost. To this cost, computed for a range of practicable operating conditions, must be added all over-head, interest, and other charges to obtain the total cost of the project. To be sure, some of these items are difficult to estimate, but they do not tend to make the pipe-line problem any less definite, as the author implies. As a matter of fact, the true situation seems to be that no one is really concerned about the economical velocity. Avoidance of clogging in non-uniform materials, specified dates for completion of the work, the fixed notions of operators, and other practical considerations control pipe-line velocities; but these do not make it impossible to come to some conclusion as to what the most economical velocity would be if one is interested in knowing it.

The experiments conducted by the writers were initiated for the purpose of comparing the difficulty of dredging in three locations. Representative samples were secured for the tests and the laboratory results had to be compared on a basis which would represent the relative dredging costs for the pipe line alone. For this purpose, the writers computed the work required to deliver sand, per foot of pipe per pound of dry material. The resulting diagram for one sand appears in Fig. 12, from which it is evident that, considering the pipe line alone, the most economical velocity is the lowest obtainable without clogging. Increases above this velocity are to be justified by other factors, and it should be possible in handling large quantities of uniform material to come to some reasonable conclusion as to the economical velocity.

R. L. Vaughn,[27] M. Am. Soc. C. E. (by letter).—It is encouraging to note that an important agency has begun to devote some of its attention and facilities to the sadly neglected subject of hydraulic dredging. In view of the magnitude of the sums expended annually for dredging, the nearly complete lack of scientific knowledge concerning the operation is astonishing but quite understandable by any one who has attempted to conduct investigations covering any phase of it.

The author states that the experiments were a part of an investigation into means whereby the capacity of pipe lines might be increased. A legend persists to the effect that, in the early days of hydraulic dredging, efforts were made to accomplish this end by inserting spiral baffles in occasional sections of pipe. These baffles were reported to be angle-irons of comparatively small size, but of considerable length (probably about 16 ft). Also, at one time, it was contended that spiral riveted pipe had an advantage in this respect. The idea, of course, was to impart a whirling motion to the mixture and thus prevent undesirable concentrations in the bottom of the pipe. Spiral pipe failed to substantiate its claims to superiority and baffles did not survive the initial trials.

The "apparent percentage of solids" is a convenient factor in dredging work since it furnishes a ready means by which the output of a dredger in operation may be calculated. It is a very treacherous factor for use when estimating the performance of a dredger in advance and is likely to give a misleading idea of the true properties of any mixture to which it is applied. Two banks of sand, of identical mechanical grading, may have substantially different percentages of voids, depending upon the circumstances under which they were deposited. Obviously, if 1 cu yd of sand from each bank, in turn, is mixed with 9 cu yd of water, the result in both cases will be a mixture containing "10% of solids"; and yet the two mixtures may be substantially different. Similarly, the entire series of clays, from fluid muds to hard clays, may be regarded, for dredging purposes, as simply fine-grained matter mixed with more or less water. Any one of them, by the admixture of the requisite quantity of water, and with sufficient agitation, can be reduced to a fluid mud. If a soft mud bank is excavated by a dredger and pumped at the rate of 4 cu yd of water to each yard of mud, the only trouble with the mixture containing "20% solids" will be to prevent its escape from the fill into some navigable waterway where it is certain to irritate the United States District Engineer. On the other hand, if a stiff clay bank is excavated and pumped at the rate of 4 yd of water to one of clay, the resulting mixture of "20% solids" will irritate a number of people. Use of this and other redundant or misleading phrases, such as "bulk density" and "bulk specific gravity," should be avoided in engineering work. In the present case it is not clear whether the percentage was computed on the basis of a volume of sand with the voids filled with water, or on the basis of an equal volume with the voids filled by air.

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[27] Cons. Engr., San Francisco, Calif.

Efforts to correlate observed phenomena on a false basis account for at least some of the troubles encountered by investigators attempting research on a subject which is at best most difficult and confusing. Two properties of any pumpable mixture are of primary importance; one is the weight, and the other is the grading of the particles. Another important factor, not solely a property of the mixture, is the ratio between the diameter of the larger particles and the diameter of the pipe through which the mixture is forced.

It has generally been assumed that a mixture of water and solid particles will act exactly as a liquid of equal weight, and that all the fundamental laws of hydraulics may be applied to it. This can be true only as long as the mixture is a uniform one; when material begins to settle in a pipe there is no longer one mixture but an infinite series of them grading from top to bottom; and when material begins to roll or drag along the pipe the area of the flowing stream is reduced, and most of the hydraulic laws no longer are valid. There is thought to be a critical velocity which will keep the mixture uniform, but no one has discovered any formula for it. Both the weight of the mixture and the grading of the particles seem to be involved. Where the larger particles are balls of clay or porous objects having a unit weight materially different from that of the solid matter in the mixture, the density of the particles will also be an important factor.

The effect of the grading is well known. It is notoriously easier to pump a mud than a sand, and there is a marked difference in pumping sands depending upon whether they are fine or coarse. On the other hand, any dredging man will comment feelingly upon the difficulty of pumping clay balls. However, no quantitative data on this subject have been presented to the profession.

The importance of the size of the pipe has received more recognition in recent years. Formerly, it was thought that so long as a velocity above the critical one was maintained the output would be directly proportional to the weight of mixture, velocity, and area of pipe. With the advent of high-powered dredgers of large size it has developed that, all things considered, clay can be pumped through large pipe lines much more effectively than through small ones. The fact is known but not the fundamental laws governing it. Something other than mere friction loss of head seems to be involved.

Eventually, probably, some property of mixtures analogous to the viscosity of liquids will be evolved; and, probably also, something resembling the "effective size" and "uniformity coefficient" which have been evolved in connection with studies of the flow of water through soil. Whoever discovers these properties will have the privilege of naming them. The soil mechanics investigators have invented an imposing glossary to describe newly discovered and hitherto unsuspected properties of earths. Dredging specialists certainly have the right also to a jargon of their own whenever they can discover properties to which to apply the words. It is suggested that appropriate names for some of the new units would be "Gilmores," "von Schmidts," and "McMullins," in honor of some of the early pioneers. Surviving old timers would probably be unanimously in favor of naming a particularly objectionable characteristic after the late Alonzo P. Bowers.

The author mentions difficulties arising from the variable nature of the mixture in a dredger pipe line. In part, these difficulties are due to practical considerations involved in the operation of the dredger. Clear water must be pumped while "fleeting." When digging, the material will cave down on top of the suction at an irregular rate so that it is impossible even to approximate a constant feed. Some factor other than those due to operating conditions also seems to be involved. Occasions have arisen in which it was possible to introduce reasonably uniform material into a pipe line by controlled means at a constant rate. In some instances efforts were made to determine the loss of head. It was found impossible to maintain a constant mixture and, at the same time, attain any value approaching the maximum capacity of the set-up. Results of observations were about as satisfactory as have been all other observations concerning the hydraulic performance of an hydraulic dredger.[28]

It is unfortunate that a motor was not used in the present experiments rather than a gas engine, since with an electric drive it would have been possible to determine the power used at various times. The author states that the velocity was first set at about 5 ft per sec and a solid concentration introduced. What happened to the velocity, and what steps if any were taken to bring it back to 5 ft per sec? How was the velocity determined in the first place?

After a state of equilibrium had been attained and some observations made, the velocity was increased but the rate of discharge of solids was kept constant. Again, how was the velocity measured, and how was the mixture controlled? From an inspection of Fig. 1 it is not clear how the procedure described could have been followed readily. On the other hand, with this arrangement it would have been easy to make a mixture of any predetermined consistency, and keep it circulating at any desired speed. After observations had been taken on one constant mixture at all velocities desired another mixture could have been made and observations again taken at predetermined velocities.

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[28] *Engineering News-Record*, June 16, 1921, p. 1035.

It was only necessary to make tests on gravel at velocities ranging from 5.5 to 8.5 ft per sec, which emphasizes the importance of the relation between size of particles and size of pipe. On a dredging scale it would be impossible to obtain satisfactory results when pumping this gravel at a velocity as low as 8.5 ft per sec.

It is not clear where the data pertaining to solid matter discharged, percentage of solids, and velocity of flow in pipes, came from (see heading, "Method of Transporting the Material: Sand"). The information is supplied in connection with Fig. 4 which shows a maximum velocity of only slightly more than 6 ft per sec, while the tabulated velocity is 8.88 ft per sec.

Mention is made of difficulties in determining the velocity distribution shown in Fig. 4. Evidently, the sampling device was used as a Pitot tube. Were the several pressure readings in each case reduced to feet of head of the particular mixture which existed at each of the several points where observations were taken? A small reaction plate could be calibrated and substituted for the Pitot tube, thus avoiding clogging troubles. Such a disk must have a considerable size relative to the size of the particles in the mixture, and in this case could not have been used for gravel as it would necessarily have been about as big as the pipe.

The author used feet of mixture rather than feet of water in the preparation of Tables 1 and 2. It is to be regretted that he did not follow this same correct procedure when drawing Fig. 6. He states that where material settled to the bottom of the pipe there was a greater loss of head than for slightly higher velocities because of increased pressures caused by the constrictions. The explanation is incorrect. Where constrictions occurred the local velocity, perforce, rose because the same volume was passing all points, and it was this increase in velocity which caused the increased loss of head. The pressure had nothing to do with it; and, in fact, there must have been a local drop in pressure at the constricted points, or the case for Bernoulli's theorem and all Venturi meters is weak indeed.

Changes observed through the windows are advanced as explaining the variations in values of f in Table 1. Aside from the rolling and jerking, what changes, if any, were observed? The rolling and jerking were reported to occur at rather definite velocities, whereas the variations in Table 1 appear to be quite regular throughout the entire range of velocities.

The opinion is expressed in the paper that it is impossible to obtain a formula for loss of head that is applicable to any but a specific gradation of material. In other words a separate formula is needed for each batch of dirt. The writer does not agree. He believes that when research has disclosed the inter-relationship between the essential variables, weight of mixture, gradation, specific gravity of particles, diameter of pipe, and velocity, a satisfactory formula can be written containing factors that reflect the influence of each variable.

A general criticism of the paper and of nearly all other investigations of the kind is that the true nature of the problem is not recognized. The essential study is that of the flow of a mixture. When solid particles settle to the bottom of the pipe and begin to jump, slide, and roll, they no longer follow the law of flow of mixtures; nor does the water entrapped within them follow that law. The problem with respect to such accumulations is: What can a certain flowing mixture do in the matter of shoving material along the bottom of the pipe? The mixture itself is whatever remains actually flowing. The essential factors are the weight, gradation, velocity, etc., of the flowing stream. It is no more correct to include the rolling and dragging material than it would be to conduct dynamometer tests on a ship's model in a tank while the model was dragging an anchor behind it, and the results are about equally valuable.

From the accounts given, it is suspected that in these tests, at least at the lower velocities, the velocity was considered to be the result of dividing the total discharge by the area of the pipe, and that the percentage of solids was determined from the volume of sand and water in the sampling tank. Reflection will show that whatever sand was scraped into the tank had nothing to do with the quality of the mixture, and that the velocity should be that of whatever was flowing, not some quantity derived by including materials that did not flow and areas through which flow did not occur. It cannot be expected that the fundamental laws governing flow will be disclosed when these non-relevant items are reflected in the observations.

Figs. 7 and 8 appear much more concordant and orderly than most plottings of data of the kind, and it is possible that the attempt to evolve a formula was abandoned without due consideration.

Recently tests of transportation of sands in 2-in. and 3-in. pipes were conducted by Morrough P. O'Brien, M. Am. Soc. C. E., and Mr. R. G. Folsom, at the University of California.[29] Those tests form a valuable supplement to those described by the author.

The California experimenters point out that Stoke's law does not apply to particles of sizes frequently encountered in dredging practice, but, nevertheless, they find that, for sands, so long as the critical velocity is exceeded, the flow is a turbulent one, and therefore, as was to be expected from the laws of turbulent flow, the viscosity or density of the mixture is not a factor in the problem. They also found that, above the critical velocity, the loss of head expressed in feet of mixture was the same as it would be for clear water. All the available evidence, including that contained in the paper, tends to sustain these findings and the conclusion is important even if it is at variance with ideas previously held in dredging circles.

Messrs. O'Brien and Folsom also observed the performance of the pump and found that the pressure developed was not directly proportional to the weight of the mixture pumped. This is entirely in accord with the fact that Stoke's law does not apply to such mixtures. The general theory of centrifugal pumps indicates that the head developed with any liquid is constant, that is, that the pressure for any given discharge will be directly proportional to the weight of the liquid. When dealing with dredger pumps it has generally been assumed that the mixture would behave as a true liquid. In a previous discussion the writer questioned this assumption.[30]

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[29] Univ. of California Publications in Engineering, Vol. 3, No. 7, pp. 343–384.

Under actual operating conditions on a dredger it is difficult if not impossible to maintain a constant mixture for a time sufficient to permit a determination of the weight of the mixture, the head developed, the power input, and the discharge. The observations at the University of California are the only ones known by the writer to bear directly on this problem. If the author should make any further tests it is to be hoped that he may be able to include the performance of the pump in his observations.

Messrs. O'Brien and Folsom concluded that the most important single property of a mixture was the settling velocity of the particles composing it. A turbulent flow is one consisting of a combination of linear flow and cross-flow. It is suggested that further investigation might reveal a relation between average velocity, cross-velocity, and diameter of pipe line, and further that it might be found that whenever the transverse velocity exceeded the settling velocity the flow became fully turbulent. In this manner it might become possible to predict, for any given pipe, the minimum velocity which would maintain turbulent flow for a mixture containing clay balls, cobbles, shells, and similar particles, the settling velocity of which could be determined.

As far as the writer is aware no observations have been taken of settling velocities of particles in any liquid but water. It might be found that the velocity of a clay ball, for instance, falling through a liquid mud was much lower than that falling through water, and, consequently, that the critical velocity for a mixture of mud and clay was less than that for clay alone.

Possibly the problem of transporting material in pipe lines is not one involving turbulent flow only. Experience suggests that the greatest rate of output may be attained when some of the material is carried in suspension, while, at the same time, some is also moved by being dragged or rolled along the bottom. This would involve two different types of flow in the same pipe at the same time and obstacles in the way of arriving at any laws governing the situation are obvious. If a mixed flow is the one which will transport the most material, reported troubles in determining the maximum or economic weight of the mixture would be explained since flow under this condition must be somewhat unstable.

All the author's formal conclusions and his remarks under the headings, "The Economical Velocity for Sand Transportation" and "Transferability of Experimental Results," are exceptionally sound and pertinent. They are endorsed unreservedly as far as they go.

The economical velocity for a dredger involves factors not mentioned by the author. Fig. 13 shows tests conducted with Columbia River sand on the dredgers, *Wahkiakum* and *Multnomah*. These are sister machines, with both suction pipe and discharge pipe 24 in. in diameter. It will be noted that as the pipe-line velocity increased, not only did the percentage of solids fall, but so did the output of the dredger, until the highest attainable velocity was approached. The answer to this seeming paradox lies in the dredger—not in the pipe line nor in the flowing stream. By increasing the power used, the pressure and velocity can be raised to almost any reasonable extent; velocities of 35 ft per sec and working pressures of 125 lb are not unknown. On the suction side of the pump, however, the pressure is absolutely limited to 1 atmosphere. Nothing can be done to increase it, for which reason, in theory, the proper place for the pump is out on the ladder immediately behind the cutter.

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[30] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 651.

This pressure of 1 atmosphere must do several things. It must overcome entrance losses, and friction losses, and must create velocity head. If the center of the pump is above the water-line it must lift all water pumped this additional distance. Most important, the excavated material must be lifted and fed into the pump. If a disproportionate fraction of the total available pressure is used to accomplish the former objects, little or nothing remains available to bring in pay dirt.

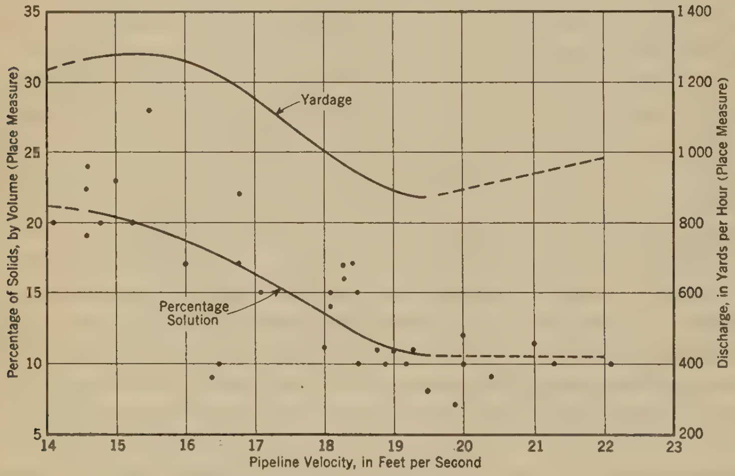

FIG. 13

The illustration is no isolated instance. The fact has been confirmed many times. On some electric dredgers the speed of the pump, and consequently the pipe-line velocity, may be varied at will, within rather wide limits. Power consumption is immediately visual through indicating watt-meters. By trial it has been found frequently that lowering the velocity would actually increase the output materially, not only when pumping sand but when pumping clay.

As far as the dredge operator is concerned, the economic velocity is the one that will get out the most material. The unit cost for "wear and tear" is about constant; unit costs for labor, over-head, interest, depreciation, etc., are inversely proportional to the output; and, although the unit cost for power may be slightly higher for the higher outputs the increase, if any, is seldom enough to offset the other gains. Of course, this is not the true problem of the economical velocity in the pipe line and the dredge operator spends his spare time wondering what would happen if the diameter of the suction pipe or the diameter of the discharge pipe were increased. Eventually he builds a new machine in which the size of both is increased. After a comparatively brief operating interval it is decided that more power is needed, which is duly installed, and the operator is right back where he started except that he has a larger dredge.

Finally, the announced object of the experiments was to devise means whereby the flow of mixtures in pipes might be improved. Results of early-day experiments indicated that any device inserted in the pipe would constitute an obstruction causing more harm than good. Experience indicates that boosters are more effective than dredge pumps. For illustration, a combination of a 1 500-hp dredger pump and a 500-hp booster out in the line is likely to do better work than a 2 000-hp pump on the dredger without a booster. The reason the booster is effective is thought to be that it stirs the mixture.

Boosters are heavy machines, expensive to install; they require considerable labor for attendance, and are anything but mobile pieces of equipment. Their use is avoided wherever possible. In special instances, especially where electric power is readily available, it might prove feasible to fit a few sections of shore pipe with small motor-driven centrifugal pumps arranged to draw practically clear water from the top of the pipe line and inject it into the bottom again under considerable velocity. Devices of this kind, located about a half mile apart along a long pipe line, might be found to be surprisingly effective and to require much less power than one would be inclined to suppose. The writer has not heard of anything of the kind, but it would not cost much to try the idea.

M. P. DUREPAIRE,[31] ESQ. (by letter).—In certain respects, sand transportation in dredge pipes is still in a crude, empirical state. A better understanding of the basic laws governing this technique is needed because great sums of money are spent each year in pumping sand mixtures through pipes in conditions which are often far from the best possible.

The paper by Mr. Howard is a welcome contribution to current knowledge on the subject in the wealth of experimental data and engineering remarks contained in it. Experiments with the same problem have been in process for several years in France, and a brief summary of the unpublished results thus far obtained should constitute a constructive discussion of Mr. Howard's paper. Tests on the flow of sand mixtures through water pipes were started in 1935, under the direction of engineers in charge of Nantes harbor. These tests were conducted in a steel pipe of 52 mm, inside diameter (2.05 in.). Loire River sand was used, screened so that the maximum grain size was 0.3 mm (0.012 in.). The volume of the voids was 43% and the specific gravity was 1.945 kg per cu decimeter (121 lb per cu ft).

The test layout was a closed circulating system operated with a centrifugal pump of adequate design. Two glass sections were provided for observing the conditions prevailing during each run, and the following conclusions were reached:

(1) When all the load is in suspension and within the range of concentration encountered in the experiments (to 40%), the loss of head (measured in feet

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[31] Ingénieur des Ponts et Chaussées, Nantes, France.

of the mixture under test) was the same as with clean water, regardless of the sand concentration, except at the stage when deposition is impending.

(2) For each concentration, a critical velocity was found corresponding essentially to that in which deposition occurs in a stage of decreasing velocities, this critical velocity being approximately that at which the minimum loss of head occurs. With the grain size used in the Nantes experiments, the loss of head was never found to be less than that with clear water.

(3) It was concluded that, theoretically, jerking motion such as that described by Mr. Howard, and the well-known phenomenon of sudden blocking of the entire section, was a problem involving stability of flow conditions encountered with mixtures. This stability condition depends on the characteristics of the kinetic head. These phenomena can be avoided entirely by using a specially designed centrifugal pump. No jerking motion was observed in the glass sections of the Nantes experiments, whereas it seems impossible to avoid that type of motion using a gravity head such as that in the Howard experiments.

(4) The loss of head in the partial deposition phase appears to be a very complex phenomenon. It was found in the partial deposition phase that, for a constant concentration of sand, the loss of head is greater when the total discharge decreases. At the same time the height of deposited material increases. The deposit in a given run was of rather uniform depth because no jerking motion was taking place. The same depth could be obtained in a fairly wide range of discharge with different concentrations of the flowing mixture. It was found that, for a given constant depth of deposit, the loss of head varied as the square of the discharge, whatever the concentration.

(5) Within the duration of each run, the deposition phenomenon did not seem to be a reversible phenomenon; that is, it requires higher velocity to pick up material than the velocities which prevailed when the material was deposited.

This means that in the partial deposition phase, the loss of head for a given discharge and a given concentration may have a very different value compared with the condition prevailing in accelerating or decelerating the flow. In the phase under discussion, the phenomenon cannot then be represented by a single curve, but covers a zone. In undertaking to study this feature more closely, it is essential to use a well designed pump.

The foregoing conclusions were deduced from the laboratory experiments. As far as actual dredge lines in the field are concerned, the writer agrees with the general conclusions reached by Mr. Howard. However, although due regard must be given to all costs (such as running costs, amortization, etc.), the most important guides in designing new dredges and dredge lines are the results obtained on pipes in the laboratory, transporting material similar to that to be used in the field.

The economical velocity is practically the aforementioned critical velocity, and this must be obtained with as high a concentration as feasible. The minimum energy per unit of sand volume, obtained for each concentration when the mixture is flowing with a critical velocity, diminishes when concentration increases. This energy would reach a true minimum for high concentrations which would not be generally feasible with the technical equipment now available.

Common experience with dredge lines has shown that the most serious difficulty in running dredging equipment arises from the danger of sudden plugging of the line when pumping in the partial deposition phase. Such plugging is likely to occur not only in long lines pumping coarse material, but also in any length of pipe or size of material. This danger of plugging causes most dredge lines to be run at velocities that are far from economical; and until now the so-called economical velocity has been only of academic interest when the cost of re-starting flow in a plugged pipe was taken into account.

The fact that, with proper design, the pump can be given special characteristics that avoid the possibility of jerking and plugging is, then, of prime interest. Consider one of the older dredges in Nantes harbor. It has a main engine of 1 000 hp with a centrifugal pump of a type that could pump Loire River sand only for a distance of approximately 900 m to 1 000 m (2 950 to 3 300 ft), using a 580 mm (23 in.) pipe. At this distance, the sand volume pumped was limited to about 575 cu m (750 cu yd) per hr, under conditions in which sudden plugging was a real menace. The entire design of this dredge was made according to what is considered good normal practice.

In the new pumps, with the same engine, the pumping distance is now 1 800 m (5 900 ft) with the same pipe diameter. At this distance, the sand volume pumped is normally 1 100 cu m (1 440 cu yd) per hr, with the same grade of sand as previously used. The danger of sudden plugging is now practically eliminated.

Since the concentration of the mixture thus pumped was more than 25% (bulk volume), the efficiency of the pump was about 80 per cent. There was much less wear in these pumps than in the older ones.

In the same general research, the suction heads have been perfected so that higher concentrations can be pumped. This is of major interest when dealing with fine sand, which can then be pumped away easily. Such fine sand, especially when mixed with muddy silt, is not so easily picked up and carried in suspension because natural deposits are often quite compact. A special suction head was designed using the entire flow of water to disintegrate the soil, the energy being supplied by the pump itself. A great uniformity is obtained without the risk of plugging. Concentrations are greater than 20% when working in compacted sand of very high impermeability. The best dimension for the suction pipe has been derived for each condition.

The foregoing shows conclusively that the study of sand transportation in dredge pipes is most effective when theoretical knowledge of hydrodynamics and hydraulics is combined with experiments on small size test pipes. Although accurate measurements of the performance of big dredge lines in the field are difficult to obtain, such observations of field behavior and the interpretation of phenomena observed are essential.

The transportation of sand in pipes is a complex problem in the partial deposition phase. The flow condition depends largely for its stability on the characteristic of the kinetic head; and, although the flow conditions might be as described by Mr. Howard in certain pumps, the writer feels that, with a proper design, the flow condition could be much improved, and thus a genuine economical velocity could be used. As both the pump and the suction head must be able to produce the high concentration that the pump now is able to drive through the line, it seems difficult to study the behavior of the line alone.

Although the laws governing all these phenomena appear to be extremely complex, the results noted thus far are gratifying. The difficulty of the task is no reason for abandoning research. Even if the final goal is still far removed, it must be reached, step by step, and investigators are still working in a stage where great strides can be made.

A broad exchange of ideas between workers in this field will foster progress and Mr. Howard deserves credit for making his results available to the Engineering Profession.

Acknowledgment.—The researches conducted at Nantes, mentioned in this discussion, were formerly conducted by Mr. R. Siegfried, under Chief Engineer M. Notté, and later by the writer. Technical advice came from Messrs. Louis Bergeron and Paul Bergeron, who designed the pumps, and also supplied valuable technical advice.

PIERRE F. DANEL,[32] ASSOC. M. AM. SOC. C. E. (by letter).—Knowledge of turbulent flow has been much advanced in recent years, both through theory and experiment. Researches on sediment transportation in open channels that have been reported show encouraging results. Laws governing sediment transportation in pipes have much in common with those of open channels, and, as long as the basic differences are understood, much can be derived from one field to benefit the other. Among the most important differences is the effect that the free surface in open channels has in damping turbulence. This effect will create a very different distribution of time, average velocities, turbulence intensities, and sediment concentrations.

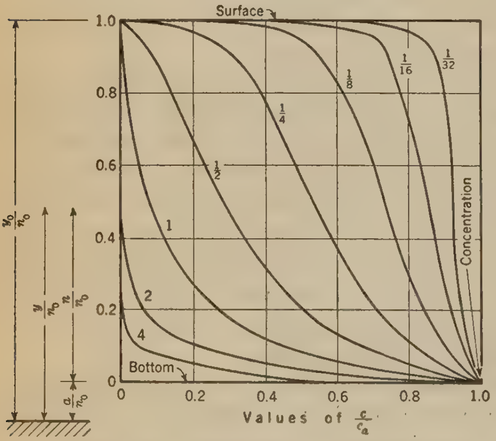

FIG. 14

Fig. 14 shows some distribution curves of different typical sediment concentrations, arranged according to their relative state of turbulence.[33] The effect of the free surface damping out turbulence is apparent as the concentrations diminish rapidly near the free surface.

Fig. 15 shows the approximate concentration distributions for a closed pipe. The bottom part of the curves is similar to that of open channel but the top is quite different. When all the sediment is in suspension in the pipe, and if the concentration distribution is fairly uniform (see Curve *A* or Curve *B*, Fig. 15), the loss of head in the depth of the mixture is the same as that of water (as reported by Miss Blatch,[2] Mr. Gregory,[34] and at Nantes, France). In this case, the conditions are nearly those of an homogeneous fluid.

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[32] Research Engr., Neyret-Beylier Piccard Pictet; Director of Hydr. Laboratory, Ecole des Ingenieurs Hydrauliciens, Grenoble Univ., Grenoble, France.

[33] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, Assoc. M. Am. Soc. C. E., McGraw-Hill Book Co., New York, N. Y., 1938, p. 342.

When the mean velocity is decreased the concentration distribution will no longer be uniform and, at the same time, when deposition occurs, the concentration distribution curve is of a type similar to Curve *C* or Curve *D*, Fig. 15. When the maximum size of the particles is small enough, a curious phenomenon may occur, as it did in experiments reported by Mr. Gregory. The loss of head (in depth of mixture) in a limited range of velocities may be less than that of clear water. This may be explained as being due to the damping effect of density gradation on turbulence, just as the density variations with height cause that well-known calmness of the atmosphere at sunset. The atmosphere is fairly homogeneous, however, whereas the mixture is not. If the particles are coarse their continued falling through the fluid creates a turbulence of its own, and there is little likelihood that the density variation can reduce turbulence to the point where the loss of head is less than that of water. When, on the contrary, the maximum size is such that the free fall of the grains would occur in laminar conditions (that is, particles of less than 0.1 mm), it is quite possible to get a loss of head, less than that of clear water, in a very limited range. In the experiments conducted by Mr. Howard, the grain sizes are too large to permit such a phenomenon to occur, but that must not be taken as a general proof that its occurrence is impossible.

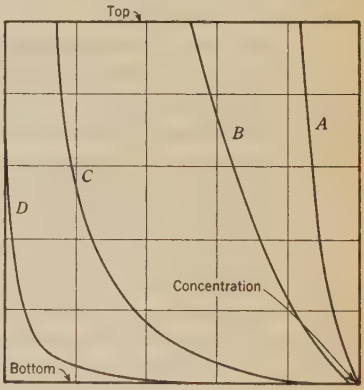

FIG. 15

In an open channel, the influence of a sudden deposition somewhere cannot be transmitted faster than surface waves, and the pressure changes involved are always small, whereas in closed pipes the pressure changes are transmitted at the velocity of sound in the pipe which is many times that of the speed of surface waves, and the pressure changes involved may be fairly great.

The jerky motion described by Mr. Howard, and the sudden plugging of the dredge pipe, do not have an exact parallel in open flumes, which shows that the comparison must not be extended too far. However, in mountain streams, mud flows of very high concentrations are stopped suddenly, sometimes in a manner that reminds one of the plugging of dredge pipes. In open channels only gravity flow is considered, whereas in closed pipe the kinetic head is generally produced by a pump, and flow conditions may vary considerably according to the pump characteristics, as accidental disturbances may or may not be smoothed out.

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[34] *Mechanical Engineering*, June, 1927, p. 609.

The design of pumps giving a stable performance outside of the cavitation range is a great step toward economy in the transportation of sand in pipes because, not only can a greater quantity be pumped, but the wear is much less.

In connection with the construction of dams the writer has been engaged for sometime in laboratory tests by throwing stones in flowing water. The theory of S. Isbach[35] has been checked and its scope has been broadened considerably. The almost perfect check between theory and experiment is very encouraging and, for material that is sufficiently coarse, the possibility of solving the deposition problem of gravel transportation in dredge pipe is not remote.

H. S. Gladfelter,[36] M. Am. Soc. C. E. (by letter).—Dredge engineers and others interested in the hydraulic transportation of solids will find this paper valuable because it furnishes information on a subject about which there is very little published.

The data and conclusions arrived at by the author have been obtained from laboratory tests under conditions that were controlled during the entire test period. The pipe being tested was straight, of uniform cross-section, smooth on the inside; and it was possible to control the velocity, grain size, and percentage of solids. Compare these almost ideal test conditions with those encountered in testing large dredges engaged in actual dredging operations. In the latter case, long pipe lines, seldom straight, are made up in sections 50 ft to 60 ft long, connected by rubber sleeves, ball joints or tapered joints, which introduce either contracted or expanded sections in the pipe lines. Furthermore, the percentage of solids, pipe-line velocity, and character of the material may vary greatly within short periods. The presence of trash in the system intake also may cause test periods to be interrupted. These conditions render any test to determine pipe-line characteristics very difficult and may account for the meagerness of published information on the subject.

During the three years, 1936, 1937, and 1938, the writer made a number of tests on 32-in. pipe-line dredges of the dust-pan type operated by the Memphis Engineer District on the Mississippi River, in the maintenance of its navigable channel. They included the determination of the carrying capacity of several types of pipe lines. The observations made during these tests were suggested by the author, and were generally similar to observations made on the 4-in. test pipe at the U. S. Waterways Experiment Station. A comparison by the author of the test results obtained in the laboratory with those obtained in the field on the 32-in. dredge should furnish valuable information.

In Fig. 4 the author shows the distribution of sand in the 4-in. test pipe. The distribution of sand in the 32-in. pipe tested is shown in Figs. 16(*a*) and 16(*b*). This condition was found to exist with pipe-line velocities of 14.78 and 21.85 ft per sec, respectively, and the observed percentage of solids under these conditions was 13.3 and 18.2, respectively. Examination of the distribution of sand in the two sections of pipe shows that the sand is transported in the manner described by the author as the first method. The writer has found that low pipe-line velocities will result in plugged pipe lines. The earlier dredges built by the Mississippi River Commission were operated at pipe-line velocities ranging from 13 to 18 ft per sec with resulting percentages of solids ranging from 10 to 15. When these dredges were operated at a high rate of advance, with increased percentage of solids pumped, frequent plugging of the pipe line resulted. In dredges now (1939) operated by the Memphis District, velocities of 25 to 26 ft per sec pumping water, and 23 to 25 ft per sec pumping solids, are obtained.

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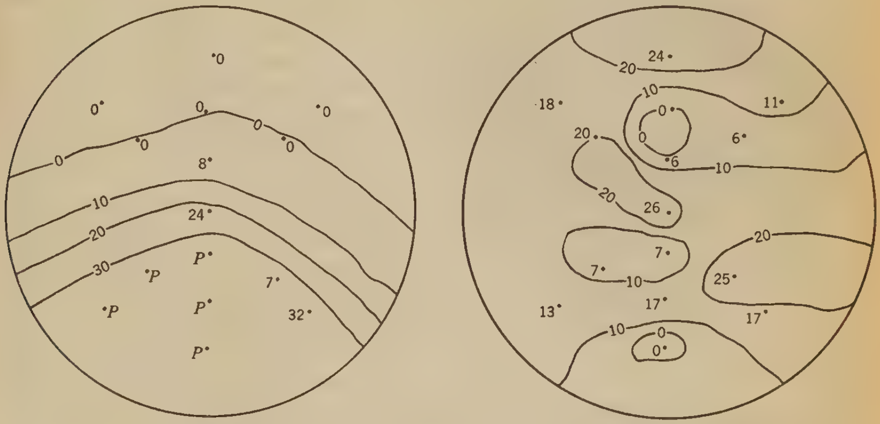
[35] "The Construction of Rockfill Dams by Dumping Stones in Running Water," S. Isbach, Leningrad, 1932. Translated by A. Dovjikov, U. S. Engr. Office, Eastport, Me., September, 1935.

[36] Senior Engr., U. S. Engr. Office, Memphis Dist., Memphis, Tenn.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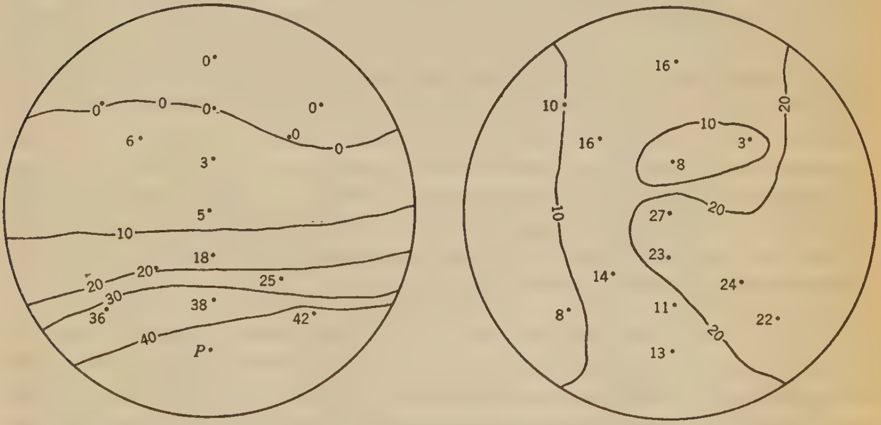
(*P* Denotes Plugged Sampler)
(*a*) PLAIN PIPE; 150 RPM

(*b*) RIFLED PIPE; 150 RPM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(*c*) PLAIN PIPE; 175 RPM

(*d*) RIFLED PIPE; 175 RPM

FIG. 16.—DISTRIBUTION OF SOLIDS IN 32-INCH PIPE

To improve the carrying capacity of pipe lines used for transporting heavy material the Memphis District installed rifles or vanes on the inside of the pipe line of one of the dredges of the district, and included tests of this type of pipe with tests of plain pipe. The effect of the rifles was to reduce the concentration of solids in the bottom of the pipe and prevent plugging that may occur at low pipe-line velocities. These rifles are shown in Fig. 17. The test of this type of pipe showed concentration of solids as in Figs. 16(*c*) and 16(*d*). These tests were made at the same pumping speeds obtained in tests that produced the concentrations of solids shown in Figs. 16(*a*) and 16(*b*). In this test the pipe-line velocities were 15.79 and 20.30 ft per sec, respectively, and the average percentage of solids was 18.1 and 24.7 respectively. This distribution of solids in the rifle-fitted pipe is more uniform than in the plain pipe, and is a result that may be expected in the third type of transportation of sand referred to by the author.

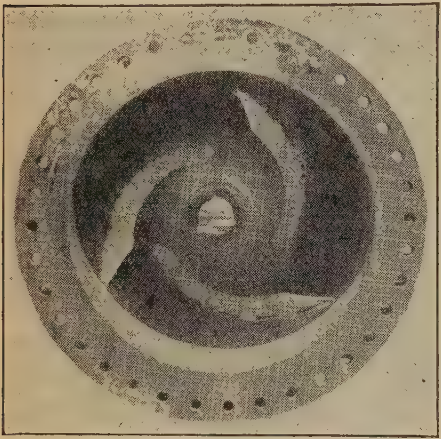

FIG. 17.—TYPICAL RIFLED PIPE

Mr. Howard has referred to the "economical velocity" for sand transportation, and gives four factors that must be balanced carefully for its determination. This procedure may be permissible when a new dredge is being designed. After its construction, however, the pipe-line velocity can be changed within limits by changing the speed of the prime mover or by changing the pump impeller. One condition which is not referred to by the author in determining the economical velocity and which would appeal to dredge operators is the cost of pipe-line and pumping equipment maintenance. When pumping a high percentage of solids containing abrasive material, the wear on the pipe line, pumps, etc., may be excessive at high velocities and it may be found more economical to operate at reduced velocities.

G. W. HOWARD,[37] JUN. AM. SOC. C. E. (by letter).—It is desired to emphasize the fact that the data presented were obtained in connection with an investigation which was concerned primarily with methods for increasing the capacity of pipe lines transporting solids. The tests in question did not stress flow through plain pipe and, therefore, are not as complete as in the case of a study of plain pipe alone. The primary purpose was the development of the design of rifling referred to by Mr. Gladfelter.

Certain features of the apparatus have caused some concern. The most important of these are the method of introducing material into the tank and the sampling apparatus. Fig. 18, a general view of the apparatus, shows: (*a*) The line into the upper tank through which the material from the sump was pumped, (*b*) the upper tank, (*c*) the test line, (*d*) the trough for diverting the discharge, (*e*) the volumetric basin, and (*f*) the scales. Fig. 19 presents details of the sampling tube. It should be noted that this equipment was not used as a Pitot tube, but was simply an arrangement for taking representative samples from definite locations in the cross-section. Velocities at these various places were obtained volumetrically with this apparatus, the mixture being discharged into a standard graduate. As was originally stated, velocities determined in this manner were considered merely as weights "because it was not possible to obtain a satisfactory calibration of the sampling device." Average velocities in the pipe were greater than velocities obtained through the sampling device. The principal use of this sampling device was to de-

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[37] Junior Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

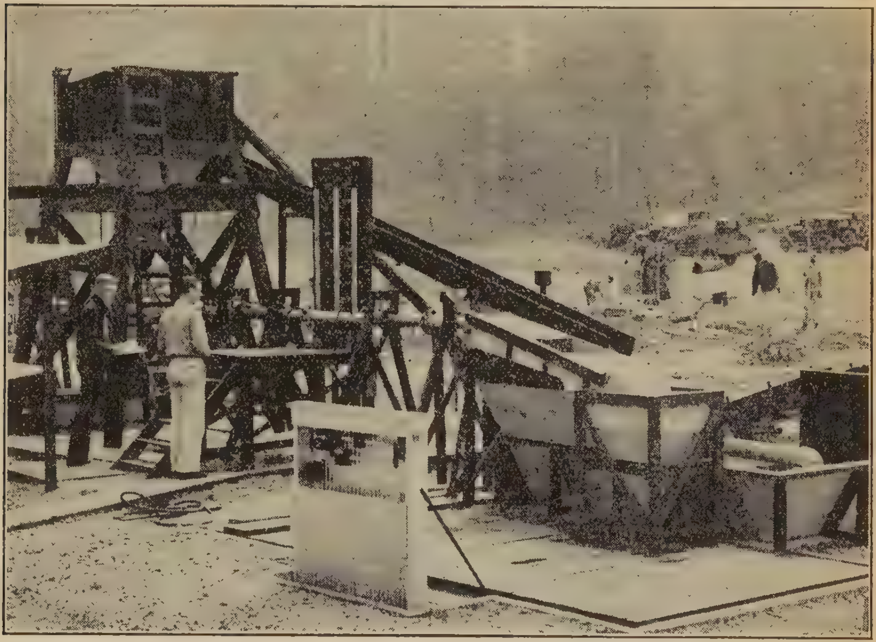


FIG. 18.—GENERAL VIEW OF TESTING APPARATUS

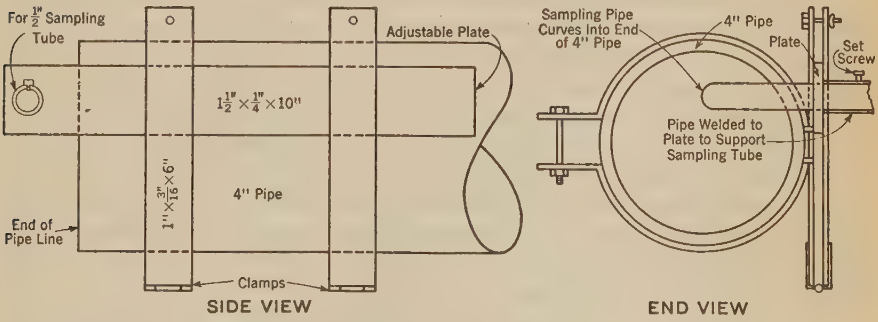

For ½" Sampling Tube
Adjustable Plate
1½"×¼"×10"
1"×3/16"×6"
4" Pipe
End of Pipe Line
Clamps
SIDE VIEW
Sampling Pipe Curves Into End of 4" Pipe
4" Pipe
Plate
Set Screw
Pipe Welded to Plate to Support Sampling Tube
END VIEW

FIG. 19.—SAMPLING DEVICE

termine solid concentrations at any given point in the cross-section. Average velocities in the test line were obtained by diverting the discharge from the test line into a volumetric basin and weighing the trapped sample. The velocity in the test line was controlled by means of a positive-action valve which was located just below the upper tank.

Of primary interest in the preparation of the paper was the gathering of data available on the subject of sand transportation in pipe lines. The additions made by Messrs. Brown, Durepaire, and Gladfelter tend to further this purpose. Messrs. Montgomery and Vaughn have shown that values of f are sensitive to changes in area; thereby, they clarified a misleading explanation of a phenomenon to which insufficient space had been given. Thanks are due to these gentlemen for clearing this point.

Colonel Dent prefers the Chézy formula to that of Darcy. This seems to be a matter of personal preference, because the value of C would of necessity become also a function of "coarseness of material, percentage of material in the mixture, and specific gravity of the mixture," which is objectionable to Colonel Dent but unavoidable under the existing circumstances.

Equation (10), suggested by Colonel Dent, is of particular interest. It is gratifying to find that the data which were presented were of value in providing an additional check on his formula. A compilation of Factor A would be of considerable value to all those engaged in the study of transportation of material in pipe lines.

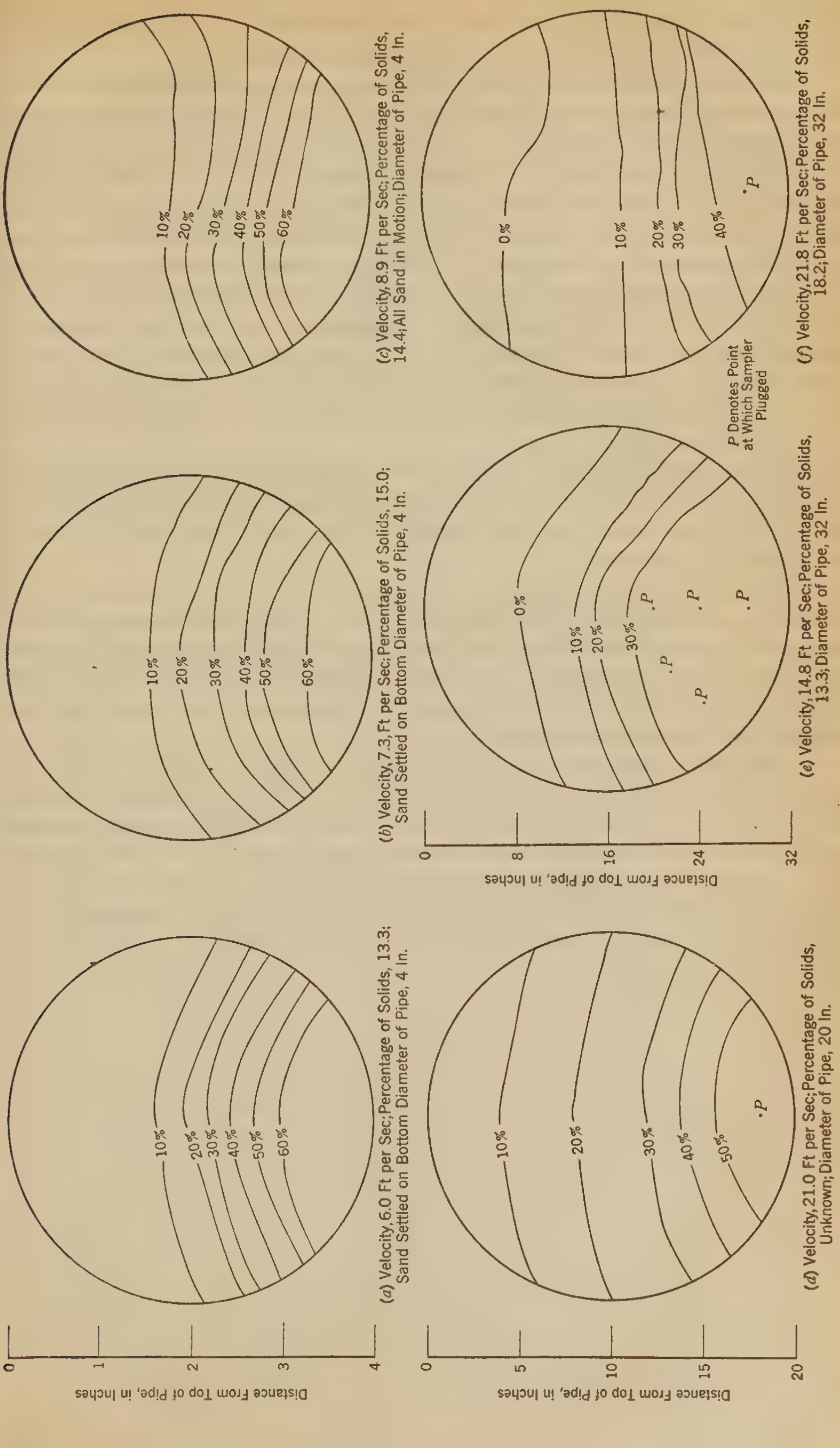
Conclusions presented by Professors O'Brien and Folsom should not be considered at variance with those advanced in the original paper, because of the different range of velocities used for the two series of tests (velocities for the California tests were greater than for the writer's tests). Certain differences which seem apparent, possibly, may be reconciled as follows:

(1) The California tests indicated that, for a given sand concentration, there was a velocity below which sand tended to accumulate on the bottom of the pipe, ultimately causing the pipe to block off, if flow conditions were constant. The writer's tests on 4-in. pipe indicated that material was carried along the bottom of the pipe in the lower velocity range; but, if a certain concentration was exceeded for a given velocity, the pipe would block off.

(2) The California tests indicated that the head loss of the sand-water mixture was the same as for fresh water. The writer's tests on 4-in. pipe indicated that the value of the friction factor decreased with increases in velocity. Indications, from Table 1, are that the values of the friction factor tended to approach those for clear water.

Mr. Vaughn objects to "the apparent percentage of solids." This factor is admitted to have its limitations, but is one that is used with satisfaction in dredging practice on the Mississippi River. Furthermore, it is satisfactory for use in comparing flow characteristics of identical mixtures through two different types of pipe, as was the case for the study in question.

The implication that a general formula is impossible was not intended, as was interpreted by Mr. Vaughn. It was intended, however, to state that,



(*a*) Velocity, 6.0 Ft per Sec; Percentage of Solids, 13.3; Sand Settled on Bottom Diameter of Pipe, 4 In.

(*b*) Velocity, 7.3 Ft per Sec; Percentage of Solids, 15.0; Sand Settled on Bottom Diameter of Pipe, 4 In.

(*c*) Velocity, 8.9 Ft per Sec; Percentage of Solids, 14.4; All Sand in Motion; Diameter of Pipe, 4 In.

(*d*) Velocity, 21.0 Ft per Sec; Percentage of Solids, Unknown; Diameter of Pipe, 20 In.

(*e*) Velocity, 14.8 Ft per Sec; Percentage of Solids, 13.3; Diameter of Pipe, 32 In.

(*f*) Velocity, 21.8 Ft per Sec; Percentage of Solids, 18.2; Diameter of Pipe, 32 In.

FIG. 20.—DISTRIBUTION OF SOLIDS IN PLAIN PIPE

from existing data, such a formula is not practicable. Colonel Dent shows that this formula may become an actuality in the future.

Mr. Vaughn indicates also that the correct nature of the problem has not been understood by investigators of the subject, and that the essential study is that of the flow of a mixture. It is granted that the study is that of the flow of a mixture, and this was realized all during the study. Mr. Vaughn, however, failed to offer any tangible means of remedying the testing procedure. For the purpose of the specific study the procedure was satisfactory and resulted in satisfactory conclusions. Criticism can be offered for all investigations after completion, but the value of the investigation lies in the application of the results. This particular investigation was for the purpose of proving that the "legend" referred to by Mr. Vaughn (that is, use of rifling to increase capacities of dredge pipes) could be applied practically. The remarks of Mr. Gladfelter enlarge on the applicability of rifled pipe for dredges in the Mississippi River.

Although Mr. Durepaire feels that an "economical velocity" can be obtained with pump design, the use of this velocity as stated by Major Neuman "will not result in minimum unit total costs." This fact is particularly true in large dredging operations.

Mr. Danel has provided further interesting information in connection with the variations in head loss. The remarks by Mr. Gladfelter are quite pertinent and explain some of the reasons why the data originally reported upon were obtained in such a manner. The primary objective of the tests was the comparison of all types of transportation of materials in two pipes: One that was plain, and one that was rifled.

Distribution of material in a pipe is always of interest, and the data presented by Mr. Gladfelter add considerably to those already presented. Fig. 20 shows the distribution of material in 4-in., 20-in., and 32-in. pipe, prepared from material furnished by Mr. Gladfelter, and additional information obtained by the writer. The velocities in the various sizes of pipe were observed by different methods: Figs. 20(*a*), 20(*b*), and 20(*c*) were measured volumetrically; Fig. 20(*d*), by a velocity stick; and, Figs. 20(*e*) and 20(*f*) were measured by the salt-solution method.

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TRANSACTIONS

Paper No. 2040

PRINCIPLES APPLYING TO HIGHWAY ROAD-BEDS

By Ira B. Mullis,[1] Esq.

With Discussion by Messrs. A. F. Greaves-Walker, H. Z. Schofield, W. H. Campen, Bernard E. Gray, W. J. Turnbull, Bert Myers, E. Neil W. Lane, and Ira B. Mullis.

Synopsis

The purpose of this paper is to present, for criticism and discussion, a process of reasoning and development in road-bed design and construction rather than to set forth definite rules and data; to apply the laws of mechanics of materials to the solution of road-bed problems; and to stimulate research and thought in the rational design of road-beds. The mechanical properties of typical earthy formations are discussed with reference to their suitability for road-bed use. Construction methods and experience are presented and discussed.

Essential Structural Requirements

With respect to the design, construction, and use of any structure, certain physical properties are required. These properties and their inter-relationships constitute that branch of science known as Mechanics of Materials. Other branches of science are frequently required to aid in the explanation or interpretation of certain physical facts; but with respect to structures of all kinds, the mechanics of materials furnishes a direct means of approach.

The first essential to the design of all structures, including road-beds, is the determination of the specific structural requirements, such as type and magnitude of load to be supported by the several members of the structure. A satisfactory road-bed must possess the following requirements:

(*a*) Compressive strength in excess of that of the maximum wheel load to be used;

(*b*) Effective pore space so limited in volume that it cannot receive sufficient water to produce plasticity;

Note.—Published in September, 1938, *Proceedings*.

[1] Assoc. Highway Engr., U. S. Bureau of Public Roads, Omaha, Nebr.

(*c*) Water content so restricted in quantity as to cause the road-bed to remain an elastic solid; and

(*d*) Surface protection against the destructive forces of traffic, weather, and erosion.

To satisfy the foregoing requirements, both designer and builder must possess adequate knowledge of the structural requirements of road-beds and must be familiar with the mechanical properties of the available materials to be utilized.

GASES

Every one knows that matter exists in three states: Gases, liquids, and solids. Gases belong to that form of matter which is elastic and tends to expand indefinitely. The gas that affects road-beds most is air. Its chief characteristic, as it relates to road-beds, is that of expansibility. If a pint of liquid is poured into a quart bottle, it does not expand and fill the bottle. In contrast, an inflated pneumatic tire is punctured, and the escaping air expands and occupies considerably more room than it had before.

During the period of road-bed construction the engineer is concerned with the elimination of air from earthy materials. This removal is essential before earth particles can unite. After a road-bed has been completed the presence of a limited quantity of air in its pores restricts the quantity of water that these pores may receive. In case of freezing, the air is either compressed or removed, and room is thereby furnished for the expansion of the freezing water within the pores without disrupting the solid mass. Thus, it will be seen that a limited air content in the fine pores of a road-bed is an element of safety. For this reason the better grades of sand-clay gravel roads do not fail under frost action.

LIQUIDS

A liquid is a body of matter, such as water, which offers resistance to any change in size, but none to change of form. The only liquid to be considered herein is water which, when pure, has a specific gravity or density of 1.00 gram per cu cm at 4° C. Within the scope of this paper this value is satisfactory for all atmospheric temperatures above the freezing point. Pure ice, at the freezing point, has a density of 0.9175. With the density of water considered as unity, therefore, it will be seen that 1 unit-volume of pure water, upon freezing, becomes 1.08+ units, an expansion of 8 per cent. However, under field conditions, this expansion (including entrapped air) is generally considered to be about 10 per cent. When freezing water is rigidly confined, its force is enormously destructive. The only known method of avoiding this destruction to road-beds is that of restricting the quantity of water that may enter them.

The forces exerted by water are both gravitational and molecular. The gravitational force is too well known to require more than mention. The molecular forces, because of their finite scope and the conditions under which they act, frequently evade interpretation. They include adhesion, cohesion, and chemical affinity. Discussion of how these forces act is beyond the scope of this paper, but knowledge of the conditions under which they apply is essential.

As a constituent of road-beds, water is essential at all times. Its importance as a means of removing air during the period of construction has already been stressed; but the removal of air is not enough. Particles must be cemented together, if all their potential strength is to be utilized. On the other hand, the excessive use of water is to be avoided; otherwise, a weak and excessively porous or fissured structure is obtained. "Spare the water and increase the weight of the roller" is generally a safe maxim when compacting earth, but the presence of excessive volumes of air in the compacted mass should not be tolerated.

SOLIDS

A solid body is one which offers resistance to any force that tends to change either its form or its size. The ultimate particles of a solid are considered as being bound together by some kind of intermolecular force which causes them to resist forces tending to separate them.

The theory of solids includes the concept of continuity of matter, but, in a strict sense, this is never true. Even atoms contain much open space. Although steel is an excellent example of an elastic solid, it possesses a crystalline structure. When crystal size in metals or non-metals is reduced and the crystals are more closely arranged, strength is increased. Moreover, it has been shown that certain metal surfaces can be ground and polished to planes so perfectly fitted as to bring them within range of molecular forces sufficient to offer considerable resistance to their separation.

Bodies of earth within the definition of that of a solid are to be found occurring, both naturally and artificially, everywhere. It is this state of aggregation of earth particles that is most desirable for road-beds.

To solidify masses of earth, the constituent particles and molecules must be united. This process involves the removal of air, the development of cementing properties, and close molecular arrangement of all constituent particles. In this process, water is essential both as an aid in the removal of air and also in the preparation of the clay molecules for the uniting process. Another essential is adequate pressure. With the aid of water and pressure, the molecules are brought into that state of aggregation in which intermolecular force-functioning becomes possible. The mass may now be considered as having been "welded" into a solid body.[2] Upon completion of the welding process it may be considered as a malleable solid possessing considerable compressive strength.

Of course, small masses of clay particles may also be united into strong bodies by the simple process of thorough wetting and mixing, followed by controlled drying. This method may be termed "aqueous fusion" and is somewhat analogous to the fusion of metal scraps into a solid body. Under certain conditions, such as that of constructing a sand-clay mat, the aqueous fusion process may be used advantageously in highway work; but its use should be restricted to granular or thin masses where adequate drying is possible during the process of construction. This method should not be attempted in heavy clays where the development of shrinkage fissures will certainly cause trouble, unless they are solidly filled during the period of construction.

With respect to the manner in which solids resist forces, hardness, brittleness, elasticity, and plasticity may be mentioned as being important mechanical properties applying to road-beds.

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[2] "A Treatise on Metamorphism," by C. R. Van Hise, *Monograph 47*, U. S. Geological Survey, 1904.

Hardness, in the broadest sense, is that property of any material that causes it to resist deformation or abrasion by external forces. This characteristic depends, first, upon the nature of the chemical substances it contains; and, second, upon the state of molecular aggregation. The quantity of water present in rocks and minerals is an important factor of hardness. Hydrous minerals are invariably softer than those of a similar character except that they contain less water; for example, quartz is harder than opal. The only chemical difference between quartz and opal is the presence of water in the opal in varying percentages ranging from about 2 to 13.[3] The softness of wet brick, wet stone, and other non-ferrous materials in contrast to the same materials in the dry state are too well known to require discussion. It may be mentioned, however, that clays are hardest when in the densest and dryest condition; and, of course, they soften and also weaken in proportion to increases of porosity and water content.

Brittleness is characteristic of materials having little or no plasticity. Such materials have a very low elastic limit and this is poorly defined. Failure of this class of materials under stress, therefore, is sudden and generally without warning. Dry clays are characteristically brittle and such materials are not well adapted to road use. Moreover, dry particles are easily sheared, form dust, and, therefore, produce traffic hazards.

Elasticity is that property of solids which causes them to recover or tend to recover change of form or size upon removal of the deforming force. Within the elastic limit, stress and strain are proportional; but when the pressure exceeds that of the elastic limit of a given material, plastic flow has begun.

One of the essentials of any traveled way, whether it be path or road, is that of elasticity. The road must be able to resist wheel loads adequately and also to recover after their passage. The elastic limit of the road-bed must be adequate for all loads under expected conditions of weather.

Inasmuch as bone-dry clays are characteristically brittle and wet ones plastic, obviously there must be a state of moisture at which a given clay may be caused to attain its maximum elasticity. The methods for developing elasticity will be discussed later.

Plasticity may be described as follows: A body is said to be plastic when it has the ability to change its shape, even to a marked extent, without rupture. A perfectly plastic material is one on which an applied load, however small, produces a permanent deformation. Plasticity in the wide sense of the word, according to the New Century Dictionary, means the possession of a structure weak enough to yield to an influence, but strong enough not to yield all at once. The word is defined by Alfred B. Searle[4] as "that property of a material by means of which it may be deformed or changed in shape and yet retain that shape when the deforming force is removed." Heinrich Ries[5] defines plasticity as "the property which many bodies possess of changing form under pressure, without rupturing, which form they retain when the pressure ceases, it being understood that the amount of pressure required and the degree of deformation possible will vary with the material." The late Mansfield Merriman,[6] M. Am. Soc. C. E., defined a material as plastic when it has no elasticity, so that the smallest forces cause permanent deformation.

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[3] "Textbook on Mineralogy," by E. S. Dana, Fourth Edition, 1932.

[4] "The Chemistry and Physics of Clay and Other Ceramic Materials," by Alfred B. Searle, Second Edition, p. 274.

[5] "Clays, Their Occurrence, Properties and Uses," by Heinrich Ries, Second Edition, p. 119.

Work done by Bennett H. Levenson and verified by Charles Terzaghi,[7] M. Am. Soc. C. E., showed that pressures as small as 5.6 lb per sq in. were sufficient to produce plastic deformation in all wet bodies of earth tested (including both fragile and tough clays). His experiments also showed that plastic deformation occurred at equal moisture contents in duplicate bodies of earth regardless of the magnitude of the loads applied. The loads applied were 5.6, 12.0, 16.0, and 32.0 lb per sq in., respectively. In all cases, the bodies were free to deform in one or more directions.

Yield Point Due to Water Content.—The moisture content at which earth and other wet ceramic bodies change from the elastic to the plastic state is known as the yield point due to water content. By this definition it will be seen that earth containing water, equivalent to that of its yield point, will behave as a plastic body. All unconfined plastic bodies not only deform when loaded, but continue to do so without any increase in the applied load. This yield point of plastic clay bodies under load may be said to be analogous to the yield point of a loaded mass of red-hot steel which is also plastic. Both metallurgists[8] and structural engineers[9] recognize the effect of applied heat in the reduction of the strength of metals. Gustav Tammann states, "With increasing temperature the tensile strength of metals decreases at first slowly, then rapidly so that at a red heat most metals have only a very small tensile strength." The law of "plastic yield" governs in both cases. Moreover, it is evident to the physicist that in the processes of drying wet clay and of cooling hot steel, both first lose their plastic properties and then pass into states of elasticity. It must also be remembered that beyond certain limits of dryness of the clay or of the coldness of the steel the elastic properties disappear and the bodies are then said to be brittle. The effect of water content on the several states of coherent earth is illustrated in Fig. 1. By substituting the words, "Temperature increasing," instead of the words, "Water content increasing," on the X-axis of the diagram, the effect of temperature on the strength of metals may be illustrated.

By definition of Ries,[5] the yield point of plastic clay is equivalent to Albert Atterberg's "lower plastic limit," [10] which is restricted by the point of rupture or rolling-out limit. However, Atterberg's method of determination is made in the reverse order, that is, by a process of moisture decrease instead of that of moisture increase. A comparison of the essential rupture feature in the Atterberg test with other definitions of plasticity reveals the harmony of the definitions of all investigators of plasticity. Furthermore, the yield point due to water content of fine sands and silts was found by Levenson[7] to approximate

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[6] "Mechanics of Materials," by Mansfield Merriman, Tenth Edition, p. 44.

[7] "Simplified Soil Tests," by Charles Terzaghi, *Public Roads*, Vol. 7, No. 8, pp. 153–162.

[8] "A Textbook of Metallography," by Gustav Tammann, Third Edition, 1925, p. 129.

[9] "Textbook of the Materials of Engineering," by H. F. Moore, Fifth Edition, pp. 36–45.

[10] International Rept. on Pedalogy (1911); also, "A Study of the Atterberg Plasticity Method," by Charles S. Kinnison, National Bureau of Standards, *Technical Paper No. 46*.

75% of Atterberg's "lower liquid limit" value. Moreover, in many types of plastic earth the relationship between these limits is such that 75% of the lower liquid limit value frequently approximates the lower plastic limit value, but important exceptions to this relationship are to be found.

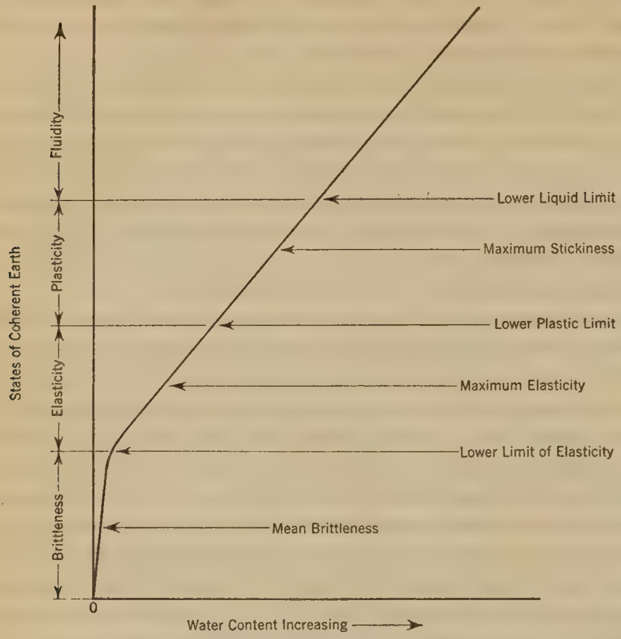
States of Coherent Earth
Fluidity
Plasticity
Elasticity
Brittleness
Lower Liquid Limit
Maximum Stickiness
Lower Plastic Limit
Maximum Elasticity
Lower Limit of Elasticity
Mean Brittleness
0
Water Content Increasing

FIG. 1.—ILLUSTRATION OF EFFECT OF WATER CONTENT ON THE SEVERAL MECHANICAL PROPERTIES OF EARTHS

GRANULAR MASSES

Material in a granular state is exemplified by sand and gravel deposits, which are characterized by their limited compressibility and the ease with which they receive and lose water. Owing to the surface tension of water, moist sand under traffic is more stable than dry sand.

The ease with which pores or voids in masses of coarse sand and gravel receive the discharge water is too well known to require discussion. In contrast to this, the pores in masses of very fine sand and silt are noted for retaining much of the water so readily received. Assuming a usual bulk density of sand and gravel to be 1.70 and that of silt to be 1.36 and both having an absolute density of 2.65, the former will have a pore space of 35.8% and the latter, 48.7 per cent. To fill the voids in the mass of sand and gravel with water would require a quantity equivalent to 21.0% of their dry weight; however, probably less than 5% of it would be retained. To fill the voids in the mass of silt would require a quantity of water equivalent to 35.8% of its dry weight and most of this water would be retained against gravitational drainage. The water retained in the mass of sand or gravel would be an asset in aiding traffic over it, but the water retained in the mass of silt would convert it into a quagmire.

LOOSE, AMORPHOUS, PLATE-LIKE, FIBROUS, AND CELLULAR MATTER

Loose material, regardless of other attendant structural properties, should have no place in a road-bed. Pore space is always large in loose materials and when the individual openings are so small that the force of surface tension is greater than that of gravity, water is removed only through evaporation.

Dry Clay.—The brittle properties of consolidated masses of dry clay have already been discussed under the heading, "Solids." Loose masses of dry clay are composed of particles or chunks containing much air. In this state, the mass can be neither compacted nor welded into a solid. Dry masses of earth utilized in the formation of road-beds should be expected to contain many air-filled cavities which are likely to contribute to future saturation and settlement. Dry earth should be utilized in road-bed construction only for blending or conditioning wet earth so as to obtain little variation of moisture throughout the structure.

Although loose deposits of moist clay or well-graded mixtures of sand and clay have possibilities for being converted into road-beds of considerable strength and durability, even these should have no place in a road-bed until they have been well compacted. Masses of earth particles which do not conform to the definition of a solid as given by the physicist cannot function as a part of a satisfactory road-bed.

Wet Clay.—Whether it is in a plastic or fluid condition, wet clay is generally in a state of expansion; it has very little supporting capacity and, therefore, should not be utilized. When it must be placed in road-beds, it should be used sparingly and distributed so that excess water will become absorbed by contiguous dryer layers of earth during the period of road-bed construction.

Road-beds constructed of wet earth shrink upon drying and, therefore, lead to the development of shrinkage fissures. These fissures become water-filled during rainy seasons and thereby contribute in large measure to the never ending cycle of excessive moisture changes and their attendant instability.

Plate-Like Particles.—Plate-like fragments or particles belong to one of three general classes: Brittle, elastic, or flexible matter. Of these, brittle, plate-like matter, such as spalls of rock, may best be utilized only when sandwiched between layers of moist, compact earth so that bond is provided for these incoherent fragments.

Cellular Matter.—Another type of brittle matter is that of diatomaceous or radiolarian earth. Both these types consist of shells of siliceous matter mixed with greater or less quantities of earth. These shells or frustules have various shapes, but all are more or less cylindrical, cup-like, cellular, or rod-like in form and have many openings which give the particles a kind of lace-like appearance. The pore space within, and also between, these minute shells forming the mass is enormous, and all pores are so small as to resist gravitational drainage when filled with water. Due to the fragility of the particles and also to the enormity of the pore space, neither diatomaceous nor radiolarian earth is satisfactory for use in road-beds.

Volcanic ash deposit contains enormous pore space also. Its utilization in the formation of road-beds, therefore, should be restricted to the minimum. In no case should it be placed in the upper part of the road-bed nor on road-bed slopes where it may be subjected to erosion.

Micaceous Earth.—The only minerals that possess marked elasticity so that they spring back when deformed, are the micas. These mica flakes when associated with common earth tend to produce a fluffy mass. Even the presence of small quantities of mica flakes increases mass porosity. Although the individual flakes possess considerable elasticity, the mass cannot be considered an elastic solid. Due to lack of stiffness and other structural weaknesses, bodies of micaceous earth may be utilized only when better material is not available. However, if this material is utilized, the upper 2 ft or more of the road-bed must be formed of select earth if maintenance troubles are to be averted.

Peat.—Spongy cellulose matter such as peat, humus, leaves, and straw has an absolute density of about 1.5. The absolute density values obtained for peat deposits from five localities along the Atlantic Coast and one from the State of Washington as determined by Messrs. I. C. Feustel and H. G. Byers[11] ranged from 1.105 to 2.161, the mean value being 1.533. Abnormally high density values are obviously indicative of mineral contamination to a considerable extent. In further substantiation of the absolute density of cellulose matter being about 1.5, the research personnel of the United States Forest Products Laboratory states that the absolute density of the actual wood substance of all structural timbers is about 1.54.[12]

The bulk density values of dry peat as given by Messrs. E. K. Soper and C. C. Osbon[13] are as follows: Turfy peat, 0.11 to 0.26; fibrous peat, 0.24 to 0.67; earth peat, 0.41 to 0.90; and, pitchy peat, 0.62 to 1.03. The bulk density values of dry peat from localities already mentioned, as determined by Messrs. Feustel and Byers,[11] ranged from 0.06 to 1.21, the mean value being 0.53. The effect of the presence of considerable quantities of mineral matter is again apparent in its influence upon density increase. A tabulation of thirty-three commercial woods tested in the U. S. Forest Products Laboratory shows bulk density values, when corrected to a 0% moisture content, ranging from 0.29 to 0.65, with a mean value of 0.44. It becomes apparent, therefore, that the only essential difference between the structural properties of timber and those of peat consists of difference in porosity and bond between fibers in the respective materials.

Due to its enormous pore space and also to its marked affinity for holding water,[11] [12] deposits of peat are to be avoided as road locations when possible.

MEASUREMENTS OF THE PROPERTIES OF ROAD-BED MATERIALS

The design of road-beds, like that of other structures, must be based necessarily on the mechanical properties of the materials to be utilized. The usual location survey must be supplemented by a materials and profile survey which includes soundings, collection of cores and samples of all strata, and, finally, the measurements of the mechanical properties of the specimens and samples collected. For obvious reasons no designing should be started until all design data, including materials tests, are complete.

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[11] "Physical and Chemical Characteristics of Certain American Peat Profiles," by I. C. Feustel and H. G. Byers, *Technical Bulletin No. 214*, U. S. Dept. of Agriculture.

[12] "Wood Handbook," Forest Products Laboratory, U. S. Dept. of Agriculture.

[13] "The Occurrence and Uses of Peat in the United States," by E. K. Soper and C. C. Osbon, *Bulletin No. 728*, U. S. Geological Survey.

DENSITY

The term, density, is used in two senses: (1) Absolute density, for expressing the weight per unit volume of constituent particles of a solid or mass, exclusive of voids; and (2) bulk density for expressing the weight per unit volume of mass, including voids. In both cases, the numerical value of density is expressed in grams per cubic centimeter.

Bulk density is frequently expressed in the English system as weight in pounds per cubic foot, but the metric system is to be preferred for these measurements since absolute density is always expressed in that system. Furthermore, when bulk density is expressed in the English system, the two terms, absolute density and bulk density, must be expressed in the same system before pore space can be computed. Moreover, the metric system is better adapted to scientific work since it does not involve the awkward volumes and weights of the English system.

Absolute Density.—The term, "absolute density," is frequently called absolute or true specific gravity and in geology, "mineral specific gravity." In all cases it signifies the ratio of weight of a given volume of dry matter, unaffected by porosity, to the weight of an equal volume of distilled water. Absolute density may be determined by any one of several well-known methods, all of which require precise weight and absolute volume measurement of the matter being investigated.

Bulk Density.—The term, "bulk density," signifies the unit weight, in grams per cubic centimeter, of dry matter being investigated, including its contained pore or void space. It is synonymous to the term, "bulk specific gravity," and also to the term, "rock specific gravity," frequently used by geologists. Bulk density measurements may be performed by any one of several well-known methods, but the use of kerosene in the overflow volumeter (shown in Fig. 2) is recommended for volumetric measurements in the laboratory. This volumeter was designed by Mr. L. L. Marsh of the Kansas State Highway Commission. It is of the siphon type, dripless in performance, and capable of giving precise results on specimens of considerable size. The balance

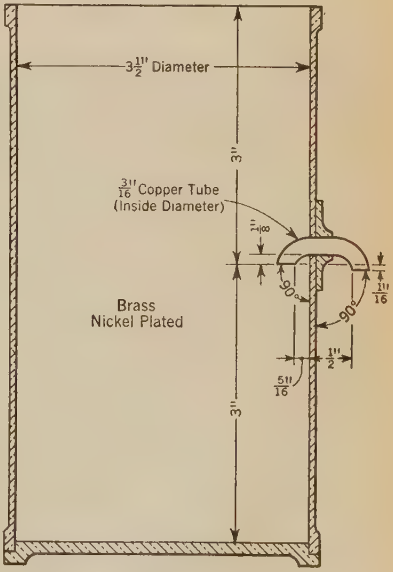


FIG. 2.—OVERFLOW VOLUMETER

should be accurate to 0.1 gram for laboratory work and to 0.5 gram for field work.

Volumetric measurements of earth in cut and also in fill are readily made by the use of a 4-in. post-hole auger for obtaining the sample, and its volume may be obtained by filling the hole with common S.A.E. 30 oil or dry, screened sand, of known density, from the spout of a pot containing a known weight of the material utilized. A table for converting the weight of these materials into metric volumes is a recommended convenience, based on the formula,

$$\text{Bulk density} = \frac{\text{Dry weight, in grams}}{\text{Bulk volume of mass, in cubic centimeters}} \quad \ldots\ldots (1)$$

Subdivision of Bulk Density.—Inasmuch as bulk density measurements are utilized in three distinct states or conditions of compactness, it becomes necessary to restrict its meaning to suit its precise application to a given problem. The term, bulk density, therefore, has been subdivided into three brief expressions: "Cut density," "fill density," and "standard density."

The term, "cut density," will be applied to all bulk density measurements made on specimens of undisturbed rock or earth taken from "cut," "borrow," or other sources from which "fill" material is to be obtained.

Fill density will be the term applied to all bulk density measurements made on specimens taken from "fill," after the fill material has been placed in its final position or in a given state of compactness. Further restrictions of this term can be made as desired by adding a limiting adjective, such as "rolled," "tamped," etc.

Standard density of earth utilized in road work may be defined as that state of compactness in which all pores have been reduced to the minimum without crushing any of the constituent particles, and all water has then been removed. Samples for this determination obviously must be representative of the material sampled. Deposits containing rock fragments too large for this determination are given special consideration through other measurements. For obvious reasons, the size of the sample taken must be proportional to the size of the largest rock particles in it.

The sample is prepared for test by vigorous boiling. This serves three purposes: It breaks up the clod structure without crushing the rock fragments; it reduces the labor involved to the minimum; and, it removes all air from the pores and thereby permits the wet mass to dry and shrink to a reproducible numerical value of density. Boiling should be continued until the water content is reduced to the minimum boiling limit. When the boiling process has been discontinued, more water may be removed by pouring the wet mass into a hot, porous, plaster-of-Paris mold similar to those used in the pottery industry. Finally, the sample is dried in an oven. The dried and cooled specimen is then weighed, soaked in kerosene, and its volume measured in the kerosene-filled overflow volumeter. This method of testing has been developed by the Iowa State Highway Commission to the extent that close check results are obtained and many tests can be completed by two operators within 24 hr of their receipt in the laboratory. Moreover, the usual laborious work of drying and pulverizing the material prior to testing is entirely eliminated by this improved technique.

Application of Density Measurements.—Density measurements are utilized as follows:

(1) The ratio of shrinkage or swell which earth and rock masses undergo when transferred from "cut" to "fill" may be determined in advance by means of density values obtained from representative specimens of materials in place and also from the density value designated for the proposed road-bed;

(2) A means is furnished for computing pore space in bodies of earth in any state of compactness or wetness; and,

(3) Density measurements furnish a simple means of expressing the quality of earth compaction in numerical terms.

Standard density is utilized as follows: (*a*) As a distinctive and important property essential in the classification of earths proposed for structural use; (*b*) as a reproducible density, essential for specimens subjected to the hardness test; and (*c*) as a means for comparing the mechanical properties of thoroughly "welded" clays utilized in road-beds, with those utilized in the manufacture of clay products the mechanical properties of which have been measured. Experience has shown that the elastic limit and the yield point due to water content are to be preferred as standards of compaction instead of any standard based on density alone.

POROSITY

All matter is more or less porous. The large pores in deposits of gravel cannot escape detection. The pores in deposits of silt are not so obvious, but water readily finds its way into them. Even iron is known to be porous and, therefore, allows water under enormous pressure to be forced through it.

The invariable weakness of highly porous masses made up of minute cells which collect and retain large quantities of water has already been discussed. Therefore, attention will be directed to the discovery and measurements of pore space for the purpose of controlling it.

The measurement of pore space is sometimes attempted by the water-absorption method. When the pores are large and open, the water-absorption method may be saisfactory; but where pores are more or less closed and tortuous, the entrance of water is restricted. Hence, in the one case, the volume of water absorbed may be equivalent to that of the total pore space; but, in the other, only a small percentage of the total pore space may be filled.

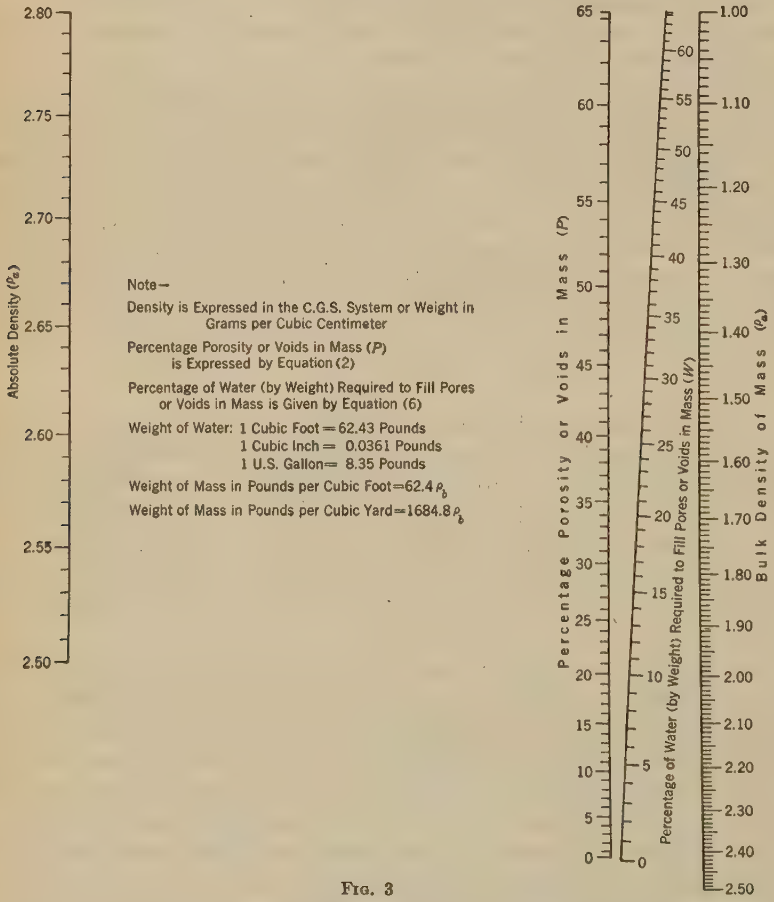
The most precise method of determining pore space is by computation. The essential measurements required for this method are: (*a*) Numerical value of absolute density of the constituent particles, ρ_a; and (*b*) numerical value of bulk density of the mass, ρ_b. Then, from the well-known relationship between the two types of density, the volumetric percentage of space occupied by pores, P, is expressed by the following equation:

$$P = \left(1 - \frac{\rho_b}{\rho_a}\right) 100 \qquad (2)$$

Obviously, Equation (2) may be applied either to cut, fill, or standard porosities by the substitution of actual numerical values for ρ_b and ρ_a. The volumetric percentage of solid matter, V_s, in a given road-bed is expressed by the following equation:

$$V_s = \frac{\rho_b}{\rho_a} 100 \ldots\ldots\ldots\ldots (3)$$

The percentage porosity of a given deposit having been determined by formula, then that part of the space actually occupied by water may be computed from the determination of percentage of moisture by weight made on a representative sample of that deposit. Since this value is based on the dry weight of earth, the volumetric percentage is,



Note—

Density is Expressed in the C.G.S. System or Weight in Grams per Cubic Centimeter

Percentage Porosity or Voids in Mass (P) is Expressed by Equation (2)

Percentage of Water (by Weight) Required to Fill Pores or Voids in Mass is Given by Equation (6)

Weight of Water: 1 Cubic Foot = 62.43 Pounds
1 Cubic Inch = 0.0361 Pounds
1 U.S. Gallon = 8.35 Pounds

Weight of Mass in Pounds per Cubic Foot = $62.4 \rho_b$

Weight of Mass in Pounds per Cubic Yard = $1684.8 \rho_b$

Fig. 3

$$V_w = P_a \rho_b \ldots\ldots\ldots\ldots (4)$$

and the percentage of space remaining must be occupied by air and is,

$$V_a = 100 - (V_s + V_w) \ldots\ldots\ldots\ldots (5)$$

The percentage of water by weight (P_w) required to fill the pores and voids completely in a mass of a given state of bulk density is equal to the percentage of voids in that mass divided by its bulk density; or,

$$P_w = \frac{P}{\rho_b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

To facilitate the computation of pore space in earth masses and also to determine the quantity of water required to fill the pore space in these masses, the nomograph shown in Fig. 3 has been devised. This nomograph, which is based on Equations (2) and (6), can be used to demonstrate the inter-relationship of density, pore space, and water capacity of earth masses. This is a major concept of this paper.

Theoretical water capacity is the quantity of water necessary to fill the actual pore space in a mass of matter of known porosity. It may be expressed on a basis of either total dry weight or total dry volume. In this paper it is expressed as a percentage of dry weight.

TABLE 1.—CHARACTERISTICS OF CERTAIN FORMATIONS ENCOUNTERED IN ROAD BUILDING

Item No.	Material (1)	Density: Absolute (2)	Density: Bulk (3)	Porosity (4)	Percentage of water, by weight, required to fill the pores (5)	Characteristic bearing capacity (6)
	(*a*) MATERIAL IN ITS NATURAL STATE					
1	Average peat bed	1.50	0.20	86.7	433.5	Low
2			0.60	60.0	100.0	
3	Mica flake bed	2.88	0.40	86.1	215.2	
4	Diatomite bed	2.15	0.45	79.1	175.7	
5			0.96	55.4	57.7	
6	Volcanic ash bed	2.60	0.92	64.6	70.2	
	(*b*) MATERIAL IN "STANDARD" CONDITION OF COMPACTNESS					
7	Rock flour, or silt bed	2.65	1.20	54.7	45.6	Medium to low
8			1.50	43.4	28.9	
9	China clay, or kaolin bed	2.62	1.30	50.4	38.8	
10	Common sand bed	2.65	1.60	39.7	24.8	High*
11			1.80	32.1	17.7	
12	Common heavy clay bed	2.65	1.70	35.9	21.1	High
13			2.10	20.8	9.9	
14	Sand-clay road surface	2.65	2.16	18.5	8.5	
15			2.16	18.5	8.5	
16			2.22	16.3	7.3	
17			2.26	14.8	6.5	

* When moist.

Actual water capacity is the total quantity of water contained in a mass of earth that has ceased to absorb available water. Trial determinations for verifying the possibility of inducing further absorption may be made by means of shallow water-filled wells cut in the mass to be tested. Moisture determinations around wells, at short increments of distance from their centers, should be started after the rate of subsidence of water in the wells has been materially reduced or after a suitable interval of time of percolation from the wells. From these data and from the pore space previously determined, the percentage of pore saturation may be determined both for given distances from free water in the wells and also after given intervals of time of percolation.

Table 1 is shown for the purpose of familiarizing the reader with important inter-relationships of density, porosity, and water capacity of certain formations encountered in road building. (In the case of Items Nos. 14 to 17, Table 1, the determinations were made during the first "spring thaw." The actual mean percentage of water held was 2.4, the maximum was 2.7, and the minimum, 2.2.)

BRITTLENESS, ELASTICITY, AND PLASTICITY

The effects of stress are deformation and rupture. The mechanical properties of earths, like metals, may be classified, in a general way, in three groups: (*a*) Resistance to deformation and rupture; (*b*) capacity for deformation without rupture; and (*c*) work of deformation or rupture.

In Group (*a*), which may be designated by the general term, "hardness," are included the elastic limit, modulus of elasticity, compressive strength, transverse strength, shearing strength, endurance limit under repeated stresses, Brinell hardness, etc.

Group (*b*) may be covered by the term, "plasticity," and includes the relative compression without rupture in the compression test, the deflection at failure in the transverse test as well as the general properties denoted by the term, "malleability."

The work of deformation and rupture is the product of the resistance to deformation by the amount of deformation, and can be computed from the complete data of any test in which both load and deformation are observed continuously or at close intervals. It is represented by the area under the stress-strain curve.

The method used in the measurement of brittleness, elasticity, and plasticity is that usually followed to determine stress-strain relationships. Stress-strain tests are performed either on specimens in the laboratory or on the road-bed during construction. However, a sufficient number of both laboratory and field tests should be made for correlation after which field tests made by means of the roller should be adequate.

Tests on molded and dried specimens of known density can best be made in the laboratory. It is known that dried specimens possess an ultimate strength, show little elasticity, and fail suddenly under load. It is also known that very little moisture is required to develop elastic properties in earth in a well-compacted state, but exact information is lacking. Crusts of sand-clay roads carrying heavy traffic have been examined by the writer in which the moisture content was less than 4 per cent. The elastic properties of the crusts were not measured, but their condition under the traffic carried would lead one to infer considerable stiffness. Research on the quantity of water needed to change a brittle earth into an elastic one is needed.

Although the exact quantity of water necessary to develop the maximum elastic properties of compact bodies of earth is not known, it is known that an excess quantity not only weakens the elastic properties, but continues to do so in proportion to the increase in water content, until the yield point due to water content is reached. This is the point at which the material changes from the elastic to the plastic state as previously discussed.

Several investigators, including C. C. Williams,[14] M. Am. Soc. C. E., the late John H. Griffith,[15] M. Am. Soc. C. E., and others, have applied the principles of elasticity to the solution of problems in the mechanics of earth solids.

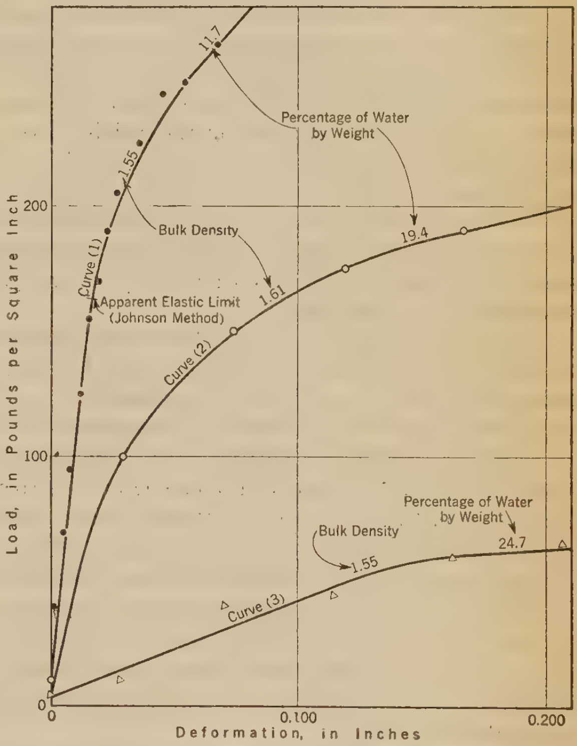


FIG. 4.—STRESS-STRAIN CURVES PLOTTED FROM TESTS ON A MASS OF EARTH COMPACTED TO INDICATED DENSITIES, AND WITH GIVEN MOISTURE CONTENTS

The application of the stress-strain method to the study of the elastic and plastic properties of bodies of a given earth is shown in Fig. 4. Earth utilized in these tests had been uniformly tamped in a cylindrical container of adequate diameter and depth for loading a 1-sq in. circular bearing-block. To prevent undue restraint against plastic flow, the distance between the bearing-block and the confining walls of the cylindrical container was spaced several inches. The loading was applied at a constant rate on all bodies tested.

Curve (1) in Fig. 4 was compacted to a density of 1.55, with a water content of 11.7 per cent. Its apparent elastic limit, as computed by the Johnson method,[16] is 164 lb per sq in. It will be noted from the several curves shown in the diagram that the stiffness of the bodies tested decreased with increasing water content. This is to be expected. Although Curve (2) is the densest body of earth tested, its water content causes it to show less stiffness than any, except the one having 24.7% of water. It will be noted that the stiffest and also the most plastic bodies tested had equal densities of 1.55; but the stiffest one contained only 11.7% and the most plastic one contained 24.7 per cent. With a water content of the latter amount, no elasticity whatever is indicated. This is to be expected since the water content in this case is 24.7% and that of the yield point due to water content is 20.1. A comparison of the water content represented by Curve (2) with that at the yield point due to water content, shows a difference of only 0.7 per cent. It will also be seen that this curve indicates little, if any, elastic properties. It does, however, indicate plastic flow under a limited load.

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[14] "Foundations," by C. C. Williams, in "Civil Engineering Handbook," by L. C. Urquhart, M. Am. Soc. C. E. and others, pp. 632–659.

[15] "Physical Properties of Earths," by John H. Griffith, *Bulletin 101* (Iowa State College Eng. Experiment Station).

[16] "Johnson's Materials of Construction," by M. O. Withey and James Aston, Fifth Edition, 1935, pp. 9–10; also "Standard Specifications for Highway Materials and Methods of Sampling and Testing," adopted by the Am. Assoc. of State Highway Officials, 1935, pp. 291–303.

Rapid stress-strain tests may be made by means of a sheepsfoot roller resting on the spot to be tested. Deformation readings may be made on the feet at both ends of the roller at frequent intervals of time by means of a wye-level and rod. From the data thus obtained stress-strain data may be computed. Elastic limits in excess of the weight of the roller may be determined by constant increases in the weight of the roller, applied by means of bags of earth of known weight.

The term, "ultimate strength," is applied to the crushing strength of molded and dried specimens only. The maximum utilizable strength of a moist compact body of earth will be considered as being equivalent to its compressive strength at the elastic limit.

The term, "elastic limit," may be applied appropriately either to laboratory specimens or to full-sized field tests. In either case it is the unit stress below which the material would fully recover its form upon removal of the load. It may also be defined as the limit of proportionality between stress and strain.

For use in this paper, the yield point due to pressure will be regarded as being coincident with that of the elastic limit. In its 1921 report the Special Committee of the Society on the Bearing Value of Soils recognized the analogy of the yield point due to pressure in soils to that of the yield point of steel.[14]

The "yield point due to water content" is defined as the moisture content at which clay or clay-like bodies change from the elastic to the plastic state, so that the smallest forces cause permanent deformation. The moisture content at this point, as previously stated, is equivalent to Atterberg's lower plastic limit in clays and equivalent to 75% of his lower liquid limit in silts and other fine friable materials.

In regard to the inter-relationship of mechanical properties, both science and experience reveal that the magnitude of energy contained in a body is the measure of its capability to resist forces. Although one may not know the exact magnitude of a given form of energy possessed by a certain body, well-known laws aid in estimating the capacity of these bodies to resist certain types and magnitudes of forces. For example, the mechanical energy stored in hard or dense materials is known to be greater than that of soft or less dense ones. As a further example it may be stated that the hardness of steel, as determined by the Brinell method, has a direct relation to the tensile strength, and is equal to the product of a coefficient, C, into the hardness number.[17]

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[17] Mechanical Engineers' Handbook, by William Kent, Ninth Edition, Rev., 1916, p. 365.

Moreover, if a given material shows considerable tensile strength, one may reasonably expect it to be able to offer considerable resistance to shear, compression, etc.

Other conditions being equal, dense materials, including clays, clay products, concrete, stone, wood, and metals are stronger and harder than their more porous respective equivalents. With respect to the strength of earths, Williams states:[14] "The supporting capacity of sands, gravels, clays and other earths, within the range of each soil type, varies approximately with the weight per cubic foot." He states furthermore that a committee of the Society "from a study of the influence of colloidal content of earths upon their behavior under pressure, found the strength of loam and clay specimens increased with the percentage of colloids contained."

Gumbo clays, noted for their high liquid limit (water-fusing point), possess the highest tensile strength of any known clay. In contrast, kaolins have low liquid limits and also are among the weakest of clays. Moreover, the strongest metals generally have a high fusing point, a good example of which is tungsten wire, the strongest metal known, which has a fusing point of 6 152° F. In contrast to the strength of tungsten wire, lead is one of the weakest of metals, and it also has one of the lowest fusing points of metals.

The relationship between density and elasticity of earth masses is quite marked. Loose masses (low density) under stress manifest no elastic properties, but solidified masses (high density) show elastic properties as do other solids under stress. Increasing the density of a given body is equivalent to increasing its elasticity.

Considerable experimental data on the inter-relationship of the several manifested forms of energy in given bodies have been published by Griffith.[18] He shows certain relationships between the strength of bricks and rocks and their absorption and hardness. In each of the *Bulletins* mentioned he discusses the structural organization of matter and shows the similarity of laws governing different forms of energy, including elasticity, plasticity, hardness, density, etc. In support of his experiments and hypotheses, he cites the findings of others among whom are included A. E. H. Love[19] and A. Nadai.[20]

Detailed studies based upon experimental evidence of the relationship between stress and strain in masses of earth of certain density and moisture content should reveal a wealth of information to the highway engineer. In like manner, the effect of admixtures of sand and clay on the development of structural properties of road-beds could also be determined.

DESIGN OF ROAD-BEDS

Obviously, the design of road-beds should be based on the essential structural requirements outlined and discussed in detail in this paper. Information obtained from large transportation companies, tire manufacturers, and producers of heavy equipment show that the pressures in balloon tires now rarely exceed 75 lb per sq in., with a tendency toward much lower pressures. Inflation pressures of tires do not now exceed 100 lb per sq in. Therefore, it seems safe to assume a maximum wheel pressure on the road to be 100 lb per sq in. Due to the limited use of solid rubber tires, these tires are not to be considered.

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[18] "Dynamics of Earth and Other Macroscopic Matter," by John H. Griffith, *Bulletin No. 117*, Iowa Eng. Experiment Station, 1934; also, "Physical Properties of Typical American Rocks," *Bulletin No. 131*, Iowa Eng. Experiment Station, 1937.

[19] "Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, Second Edition, p. 1.

[20] "The Phenomena of Slip in Plastic Materials," by A. Nadai, *Proceedings*, A.S.T.M. (1931), Pt. 2, pp. 11–26; also "Plasticity," by A. Nadai, McGraw-Hill Book Co., New York, 1931.

Factor of Safety Against Pressure.—To avoid the possibility of over-stressing the road-bed, the application of a factor of safety seems essential. Following the usual custom, it would seem that the minimum factor of safety recommended should be not less than 2. Based upon the maximum tire pressure of 100 lb per sq in. and a factor of safety of 2, the road-bed should be designed for pressures not less than 200 lb per sq in.

Elastic Limit Required.—To meet the load requirement, obviously the material of which the road-bed is to be made must be capable of being formed into a solid having an elastic limit not less than that required for the given load and safety factor against pressure, or not less than 200 lb per sq in.

Stiffness Required.—Obviously, the stiffness of a road-bed should be developed adequately to resist the load for which it is designed; but before any numerical value of stiffness can be set up as a standard, considerable field and laboratory data must be obtained. Moreover, until a satisfactory standard of stiffness can be established the designing engineer should make every effort possible to develop much needed data on this feature of road-bed design.

Development of Hardness, Elasticity, and Stiffness.—The properties of hardness, elasticity, and stiffness may be developed in wet coherent material in either of two ways. A reduction of water content will develop these properties to some extent, provided the pore space is not too voluminous; but since the capacity for holding water is predicated on the available pore space, these properties of strength may best be developed by increasing the density of the road-bed under controlled moisture conditions.

If the road-bed is too wet to be compacted, its density may be increased by the well-known method of adding dry sand, gravel, or other hard rock fragments and pounding the added material into the wet spongy mass. By this process, densities of 2.20 or more may be developed in a layer of desired thickness. Obviously, the greater the thickness and density of the layer thus built, the greater will be its resistance to mechanical forces. Sand, fine gravel, or fine crushed stone is to be preferred since fine pores offer greater resistance to the passage of wet clay than coarse ones. Aggregates graded in sizes from that of silt to that of gravel are essential in the rapid choking of the pores. Moreover, the aggregate must be cemented into a solid mass by means of clay or other adhesives.

Where a thin layer is sufficient, the work may be accomplished by the use of a tamping roller applied to thin layers of aggregate. Where rigid and dense layers of considerable thickness are to be placed across swamps, the use of a heavy rammer of the pile-driver type is essential for constructing the lower part of the densely constructed stratum.

Factor of Safety Against Water Absorption.—This factor of safety must be determined by actual test prior to the work of designing. From actual compaction tests, determine a density which gives an actual water capacity, less than the quantity of water required to produce the yield point due to contained water. Then, from these data,

$$S_W = \frac{s_{yw}}{P_a} \qquad (7)$$

in which S_W = the factor of safety against water absorption; s_{yw} = the yield point stress due to water content; and P_a = the actual water capacity of pores, expressed in terms of percentage by weight. The true criterion for determining what density is essential for a road-bed is the relationship between actual water capacity and the yield point due to contained water. This relationship seems to be expressed best in the form of the "factor of safety against water absorption." It is this relationship which controls the water softening of the road-bed. That compacted bodies of earth offer a greater resistance to water softening than bodies of loose earth was probably first discovered by prehistoric Man as he walked along a path loosened here and there by some burrowing animal just before a rain; but since the writer is not aware of an existing method for determining the critical point of softening and then guarding against its occurrence, he has been forced to apply the method now termed, "factor of safety against water absorption."

Many bituminous mat-road surfaces which had failed due to the apparent weakness of the road-bed have been examined. Of these failures, no case is recalled in which the bulk density of the road-bed was in excess of 1.50. Based on this value of bulk density and on an absolute density of 2.65, the pore space is found to be 43.4% and the theoretical water capacity, therefore, is 28.9 per cent. A pore space of this magnitude will probably have an actual water capacity almost as great as its theoretical capacity. Since the yield point due to water absorption in many clays is less than 28.9%, the reason for the occurrence of many failures in road-beds having a density less than 1.50 becomes evident.

As a basis for further study and research, the writer offers the following limiting values for use in designing road-beds:

(*a*) Do not allow the pore space in the road-bed at depths greater than 2 ft below subgrade, to exceed 40% of the volume;

(*b*) Reduce the pore space in the upper 2 ft of the road-bed so that its theoretical water capacity shall be not greater than 70% of the lower plastic limit of the earth used in forming the road-bed.

Materials Balance Factor.—As previously stated, this factor is based upon the ratio of density of mass in a fill to that in a cut. The results to be expected from the use of this factor depend entirely upon the precision of the original weighted density values obtained and also upon the precision with which the compacting specifications are executed.

Road-Bed Protection.—Experience teaches that thoroughly compacted deposits of earth do not change in volume unless acted upon by outside forces.

Bench-marks set below the frost line in wet earth or even on tree roots have been found to remain stable unless they come within range of landslides or destructive earthquakes. Even deposits of clay subjected to enormous pressures through long ages do not bulge when the pressures are removed by the excavation of the overlying deposits. Moreover, there is no such thing as a "natural density" for any given material, regardless of its environment. For example, a given type of clay should be expected to have densities proportional to its compacting environment—that is, they should be low in arable fields, higher in old pastures, and still higher in well-trodden paths, or in barnyards.

The agency most destructive to compacted road-beds is weather. It matters not how well a road-bed may be compacted, if it is left exposed to weather, the exposed part should be expected to become less dense. Therefore, the greater the number and magnitude of weather changes, the greater the change to be expected in the original density.

To prevent the weather from producing marked changes in road-beds, they must be shielded against its influence. Ground-water must be intercepted and surface water must be diverted from the entire road-bed, including the shoulders. A pavement or bituminous surface alone is not sufficient protection against the weather. Pavement cracks generally develop and joints are seldom water-tight. For this reason provision should be made to prevent excessive moisture changes near the subgrade and in the shoulders. Some form of effective stabilization seems to be the only method of preventing road-bed softening and plastic flow in the region of the subgrade which leads to pavement sag, rupture, and faulting.

CONSTRUCTION METHODS AND EQUIPMENT

Organized Personnel.—Before any grading work is to be started, the resident engineer should organize the inspection service and make certain that everything is ready for construction operations. All data that have been collected must be compiled on the plans and shown as part of the road-bed design. The moisture conditions of the material to be excavated must be ascertained immediately before construction work is to be started.

Systematic Inspection.—The system of inspection must be planned so that an inspector is constantly on duty while road-bed construction is in progress. The minimum number of tests allowable should be specified. Results of tests should be recorded by station and elevation.

Testing Equipment.—Field-testing equipment should be portable in type and located conveniently near the center of construction activity.

Control of Materials and Methods of Construction.—The essential qualifications for the proper execution of this work are: (*a*) Thorough knowledge of the inherent structural properties of the road-bed materials to be found; (*b*) methods by which these materials may best be utilized in the construction of the road-bed; and (*c*) essential diplomacy and firmness of purpose in dealing with the contractor.

The inspectors should be competent to advise with the contractor in all details of the work. They should aid in planning the work so that construction operations may be made to fit the weather with the least delay, but under no circumstance should they allow weather conditions to lower the quality of the work. Basin-like areas must not be permitted—either in excavation or in embankment—at any time. These areas may collect water and hinder the work. All embankments should be completely compacted and finished with a smooth surface before closing the work at any time.

Conclusions

Five pertinent conclusions are supported by the foregoing text:

(1) The structural properties of earth masses are enhanced by increasing their density;

(2) The resistance offered by earth masses to the penetration of water increases with increasing density;

(3) The porosity of earth masses can generally be decreased to a certain value by means of a roller; but the porosity of granular masses such as gravel, sand, and silt, can be reduced best by the use of suitable admixtures followed by heavy rolling;

(4) A satisfactory standard of compaction is that based on a suitable factor of safety against water absorption; and

(5) Surface protection against the ravages of weather on clay-like matter is essential, regardless of the degree of compaction.

Acknowledgment

The principles set forth in this paper are from various sources. Most of them are well known, especially in other fields of work, and all are believed to be in accord with fact. Working hypotheses were first written and then tested by a series of observations and researches, most of which were done on the field, covering hundreds of miles of roads of the several types of materials already described. Conclusions were not formed until these hypotheses had been either abandoned or verified by competent evidence.

The work was done as a part of the writer's official duty as a Supervising Engineer in the United States Bureau of Public Roads on Federal Aid work. Clifford Shoemaker, M. Am. Soc. C. E., District Engineer, had long recognized the need for more stable road-beds and, in 1931, directed the writer to give special attention to this feature of the work. For this opportunity and for many helpful criticisms the writer acknowledges his grateful appreciation. Acknowledgment is also made to the Engineers of the several State Highway Departments in the District for their splendid co-operation in the collection of data used in the development of this paper.

DISCUSSION

A. F. Greaves-Walker,[21] Esq. (by letter).—An excellent argument for the necessity of a better knowledge of ceramic materials and ceramic fundamentals on the part of highway engineers is presented in the paper by Mr. Mullis, which describes the properties of the ceramic materials used in highway construction very well. A study of these properties should be helpful to all highway engineers, and particularly to those who are confronted with the problem of constructing long-lived secondary highways.

There is no question about the structural properties of earth masses being enhanced and made more highly resistant to water penetration by increasing their density. Considerable research work has been done by ceramic engineers in an effort to produce maximum density in both plastic and non-plastic bodies. In plastic bodies (clay), it has been found that the addition of acids or bases, depending on the pH-value of the clay, will increase both the plasticity of the mass and the dry density. Common or waste acids, or such bases as lime, calcium chloride, salt cake, soda ash, etc., may be used, but care must be taken to add the correct quantities. This depends on the pH-value of the material. Under any circumstances only small quantities are required to give desired results.

Much research has also been done on particle packing, and from the results it is possible to calculate beforehand the percentages of the coarse, medium, and fine fractions necessary to obtain the maximum density with any particular material or materials. Clays vary widely in their mineralogical composition and especially in their content of "clay substance," the colloidal hydrous aluminum silicate which gives to clay its peculiar properties. The content of "clay substances" controls the bonding power. Many clays which to the "feel" are extremely plastic have low bonding power, due to a low content of "clay substance." When used as binders such clays produce "open" or porous bodies when dry, and thus readily absorb water.

Pressure is very important in producing dense bodies when either clay-bonded or non-plastic masses are considered. With increased pressure the water of plasticity can be reduced increasingly which, in turn, reduces shrinkage where clay bonds are used.

The effect of pressure on density varies considerably with the materials. Non-plastics like crushed stone which have hackly fractures, offer great resistance to movement under pressure, whereas smooth materials like gravel offer much less. Consequently, under equal pressure, the gravel will form the denser mass provided both are composed of fractions of the same size.

Mr. Mullis' investigations are unquestionably along lines that will result in greatly improved road-beds and fills. In the manufacture of a good ceramic product and the making of a good road-bed there is much common ground.

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[21] Pres., The Inst. of Ceramic Engrs.; Prof., Ceramic Eng., Univ. of North Carolina, Raleigh, N. C.

H. Z. SCHOFIELD,[22] ESQ. (by letter).—It is of much interest to note that the conclusions (notably Conclusions (1) and (2)) of this paper are verified by results of researches on the effect of de-airing the earth materials used in the stiff-mud manufacture of structural ceramic products.

By stiff-mud extrusion from a laboratory brick machine, Messrs. J. O. Everhart, C. R. Austin, and W. C. Rueckel[23] prepared specimens of three shales, a glacial surface clay, an alluvial surface clay, three coal-formation clays, a stoneware body, and a table-ware body. Duplicate specimens were then prepared in which the materials, immediately before extrusion from the brick machine, were de-aired by direct evacuation of air from the chamber of the brick machine. The effect of de-airing (and the consequent reduction in porosity and increase in density) on the physical properties was then determined. De-airing increased plastic strength both in tension and compression; it increased the dry transverse strength and modulus of elasticity; and, in all but one case, it caused a marked increase in the water-slaking time of the dry specimens.

W. H. CAMPEN,[24] ASSOC. M. AM. SOC. C. E. (by letter).—Much care has gone into the preparation of this paper. Mr. Mullis should be especially commended: (*a*) For his analysis of the steps involved in densifying water-soil mixtures; (*b*) for the classification for road construction of a number of typical soil deposits; (*c*) for clearly showing the effect of water content on the structural properties of soils; and (*d*) for the application of the information in the design of road-beds.

He assumes that a maximum pressure of 100 lb per sq in. is applied to surfaces on highways and then suggests that the entire upper 2 ft of the road-bed be constructed to support this load. There is no doubt in the writer's mind that, if all the soil or soil-mixture courses within this thickness are compacted so as to restrict their water capacity to 70% of their plastic limit, the resistance required will be developed. However the pore space called for is lower, and the thickness is higher, than is necessary for satisfactory road-beds. Some tests made in connection with the construction of the runways of the Omaha (Nebr.) Airport and other roadways will support the writer's contentions.

Assume that a road is required to carry, safely, and at all times of the year, a vehicle having 10 000-lb wheel loads and 100 lb per sq in. of air pressure in its tires. Assume, furthermore, that a soil is available which will pass a No. 40 sieve (with a plastic limit of 20); and a soil-gravel mixture containing material 27% smaller than a No. 40 sieve (plastic limit, 20). The relation between the load-bearing value and the water content, in dry weight per cubic foot, or pore space, is shown for these materials in Table 2. Column (3) is obtained by dividing the percentage of water used by the water content of the sample at plastic limit, and multiplying by 100. The plastic limit is determined on the part of the mixture that passes a No. 40 sieve.

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[22] Research Director, National Paving Brick Assoc., Washington, D. C.

[23] *Bulletin No. 74*, Ohio State Univ. Eng. Experiment Station, November, 1932.

[24] Pres. and Mgr., Omaha Testing Laboratories, Omaha, Nebr.

The mixture is compacted in a mould 8 in. in diameter and 4 in. deep. Without removing the base of the mould a round bearing plate, 2 in. in diameter, is placed in the center and loaded until a $\frac{1}{8}$-in. penetration is produced. The load, supported for 5 min at this penetration, is converted to pounds per square inch (see Column (7), Table 2). Readings can be taken at other penetrations but $\frac{1}{8}$-in. is taken as representative of surface failure.

TABLE 2.—RELATION BETWEEN LOAD-BEARING VALUE AND WATER CONTENT

WATER CONTENT EXPRESSED AS A:			AGGREGATE		Percentage of air voids (6)	Load-bearing value, in pounds per square inch (7)
Percentage of the Dry Weight		Percentage of the plastic limit (3)	Weight per cubic foot, dry (4)	Percentage by volume (5)		
By weight (1)	By volume (2)					
(a) SOIL (SPECIFIC GRAVITY, 2.62)						
25.5	39.3	127	96.2	59.0	1.7	13
24.5	38.8	122	98.6	60.5	0.7	18
22.7	37.2	113	102.5	62.9	0.0	30
20.3	35.7	101	105.6	64.8	0.0	41
19.6	33.8	98	107.4	65.9	0.3	55
18.3	32.3	91	110.0	67.5	0.2	100
17.0	30.1	85	111.0	68.1	1.8	200
16.5	28.9	82	109.1	67.0	4.1	....
(b) SAND-GRAVEL BINDER MIXTURE (SPECIFIC GRAVITY, 2.61)						
10.4	21.3	180	127.0	77.8	0.9	13
8.1	17.2	137	132.3	81.2	1.6	42
7.4	16.1	124	134.7	82.7	1.2	200
6.4	13.7	106	133.5	81.9	4.5	320

As shown by Mr. Mullis, the load-bearing value (he calls it the resistance to pressure) increases as the density increases, or as the pore space decreases, it being understood that the pore space is nearly filled with water. It is to be noted also that at maximum density the load-bearing value is 200 lb per sq in. for both materials, and that the water content is 85% of the plastic limit in the case of the soil (Column (3), Table 2), and 124% in the case of the soil-gravel mixture (allowance is made for 1% absorption in the coarse aggregate). Since a resistance of 200 lb per sq in. satisfies Mr. Mullis' requirements, it would be wasteful to compact with less water and to a higher density. It might be added that the results obtained with the two materials given are representative of mixtures in general within the writer's experience. These tests have been made on soils having plastic limits of from 15 to 33, and on soil-coarse aggregate mixtures having plastic limits of 10 to 30 and maximum sizes as great as 2 in.

As to the thicknesses required: Since it is good common practice to use a coarse aggregate mixture, covered with a bituminous mat, on the upper part of the road-bed, assume that the upper part of the road will be composed of 2 in. of mat and 6 in. of a soil-gravel mixture. The mat will reduce the 100-lb load by distribution so that only 57 lb will be delivered to the base. The base (Table 2(b)), packed to maximum laboratory density, can safely handle this load, and, in turn, will deliver a load of about 17.5 lb per sq in. to the sub-grade. For adequate protection the sub-grade should be constructed to support twice this load, or 35 lb. The soil shown in Table 2(*a*) will do this job well if compacted with a water content equivalent to 101% of its plastic limit, or to 95% of laboratory density.

The next question is: How thick shall the compacted sub-grade be made? The full 16 in. demanded by Mr. Mullis would deliver a load of about 7 lb to the 24-in. level (it is assumed that the compacted soils have a load distribution ability equal to 50% of the coarse aggregate mixture). There are many locations where this type of construction is needed because of the water content of the natural sub-grade. There are other locations (and they are much greater in number) where the soils have pore spaces of about 50%, but where water can be prevented from reaching them. At these locations the water content is such that high load support prevails. In these cases, then, advantage should be taken of the high sub-grade support and the construction should be limited to compacting the upper 6 in. to 90% of laboratory density. As a matter of fact, many heavily traveled highways are built on this principle and would fail if enough water could reach the sub-grade to fill the pore space.

Mr. Mullis' requirement of not more than 40% of pore space in materials below the 2-ft level is reasonable except when applied to certain types of clay; but, as has been stated herein, this should only be considered where water might be accessible. In connection with this condition, it must be borne in mind that the load-bearing value at any given water content varies with the plasticity index. For instance, a clay that has a plasticity index of 60 has a load-bearing value of 100 lb per sq in. at a 26% water content, whereas a soil having a plasticity index of 30 has only a 25 lb per sq in. load-bearing value at the same water content.

The construction and successful performance of runways at the Omaha Airport will serve as an example of the writer's contentions. A typical cross-section shows 1-in. sheet asphalt, 2-in. asphaltic concrete binder, 6-in. soil-gravel mixture, and 6-in. compacted soil. The base was compacted to a maximum density and the sub-base (the upper 6 in. of the sub-grade) was compacted to at least 95% of maximum density. The undisturbed sub-grade is saturated with water and has a maximum pore space of 50 per cent. A modern airplane having a wheel load of 13 500 lb and 51 lb of pressure in its tires has operated for two years on parts of the runways without any signs of failure.

Bernard E. Gray,[25] Esq. (by letter).—It is indeed encouraging to find attention being given in larger measure to the problems of earth foundations, because through application of even the broader principles governing soil densities, great saving in construction costs, particularly of highways, may be obtained. Mr. Mullis has outlined these principles in a concise and logical manner.

It is unfortunate indeed that so many engineers have only a slight knowledge concerning soil behavior and are little concerned about it. This attitude arises from a fundamental misconception in regard to the place of soil in the construction "picture." To many, "dirt is dirt," and their designs are usually made massive to take care of the worst anticipated conditions, with no thought or provision for possible modification of the soil itself. To check the truth of this statement, it is only necessary to watch some heavy grading job, and to note the blithe manner in which the weathered and more stable soil is placed at the bottom of a fill, and the raw, unstable soil on the top. The writer assumes no virtue in this regard, having built many, many miles of highway over past years in just the same wrong manner, but, in the light of present knowledge, such practice should be modified, and the valuable stable soils should be saved and utilized in the sub-grade where they belong. When engineers begin to think of relative values in soils as they do for the various manufactured structural materials, then they will be truly on the way to making some genuine savings in foundation designs.

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[25] Chf. Engr., The Asphalt Inst., New York, N. Y.

Shear necessity is the most compelling motive to action, and about 1918 the writer, in making a study of old survey notebooks covering highway locations in Virginia and West Virginia, was impressed with the comments therein respecting the character of the ground traversed. The party chief had carefully noted his proposed location with especial reference to soils and their behavior under temperature and moisture changes. Some of these soils had remarkable stability, as evidenced by their solid condition under wagon traffic, so much so that when the writer later built modern type roads, largely on these old locations, he experimented by placing variable thicknesses of macadam over the most stable soils. In one particularly good section, only 3 in. of limestone macadam was built using the compacted soil as a base, except for a thin layer of screenings and dust placed over the soil as a leveling course. This section is still (1938) under traffic and its behavior has been better than some of the 8-in. and 10-in. adjacent sections having sub-grades of less bearing power.

On some of the secondary roads, bituminous surface treatments were placed directly on these soils and served traffic adequately for a number of years before additional thickness was required to distribute the increasing wheel loads better. Taking advantage of these favorable conditions it was possible to improve the system as a whole many years sooner than would have been the case had conventional designs only been used.

There is little to criticize in Mr. Mullis' paper, but perhaps some emphasis on certain paragraphs may be helpful, as follows: Under the heading, "Loose, Amorphous, Plate-Like, Fibrous, and Cellular Matter," he states: "Loose material, regardless of other attendant structural properties, should have no place in a road-bed." Nothing could be more true, and yet it is continually disregarded. Many bases have failed, not because of poor quality, but because they were not consolidated, and the surface placed thereon became distorted later as further compaction occurred under traffic. It is common practice to write specifications as to thickness of such courses, and quite uncommon to have any standard as to density of the completed work. Without such density control, thickness is a meaningless measurement. Time and again, thicknesses have been increased, when all that was required was to obtain uniform density of the thinner layer. This conclusion is borne out in the statement by Mr. Mullis in the text following Equation (7):

> "Many bituminous mat-road surfaces which had failed due to the apparent weakness of the road-bed have been examined. Of these failures no case is recalled in which the bulk density of the road-bed was in excess of 1.50. * * * the theoretical water capacity, therefore, is of 28.9 per cent."

Regardless of pavement type, it should be recognized that stabilized road-beds are required for long life and low cost of repairs.

Under the heading, "Brittleness, Elasticity, and Plasticity," it is stated that: "Loose masses (low density) under stress manifest no elastic properties, but solidified masses (high density) show elastic properties as do other solids under stress."

The rational design of the so-called "flexible type" pavements is predicated upon this axiom. It applies both to the road-bed and the pavement. When road-beds (sub-grades) are designed for a given minimum density under the most adverse conditions, it is then a simple matter to calculate the depth of pavement required to distribute wheel loads so that its bearing capacity is not exceeded. In respect to wheel loads, the comment under "Design of Road-Beds" should be noted. Balloon tires have modified pavement design. Present wheel loads are so distributed over a large contact area that there is little, if any, more destructive action under the heaviest vehicle than was formerly experienced under a two-horse wagon. As Mr. Mullis states, present tire pressures are rarely in excess of 75 lb with the trend toward less pressures. Thus, a design load of 200 lb per sq in. gives an ample margin of safety.

Under the heading, "Design of Road-Beds: Road-Bed Protection," it is stated that:

> "The agency most destructive to compacted road-beds is weather. * * * A pavement or bituminous surface alone is not sufficient protection against the weather. Pavement cracks generally develop and joints are seldom watertight. For this reason provision should be made to prevent excessive moisture changes near the sub-grade and in the shoulders."

This is a most pertinent comment, and should have much influence on design. The increased use of bituminized shoulders and sub-grades is recognition of the need for preventing water from softening sub-grade or shoulder support. A striking example of the merit of such protection is to be found in Connecticut, where during the flood of September, 1938, practically no damage occurred to pavements where such shoulder protection was used.

Although the inherent values of designed sub-grades are not yet widely appreciated, a few engineers have begun to build roads by such means. Some question has arisen in respect to the rational design of flexible pavements making use of variable sub-grade support. Valuable contributions on this subject have been made by W. S. Housel,[26] Assoc. M. Am. Soc. C. E., W. S. Downs,[27] A. T. Goldbeck,[28] and G. E. Hawthorn,[29] Members, Am. Soc. C. E., and many others. Although there may be some difference of opinion as to the manner of stress distribution in soils, the conventional distribution for an elastic solid is certainly conservative for designed bituminous mixtures of known stability. Accordingly, the writer suggests a formula to control design thickness that gives results which, in the light of road behavior, are more than adequate for modern traffic.

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[26] "Design of Flexible Surfaces," by W. S. Housel, 23d Annual Highway Conference, Univ. of Michigan, February, 1937.

[27] "The Cost of Providing Highways Suitable for Various Classes of Vehicles," by W. S. Downs, *Special Bulletin*, Univ. of West Virginia, September, 1933.

[28] "Researches on the Structural Design of Highways by the United States Bureau of Public Roads," by A. T. Goldbeck, *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 264.

[29] "A Method of Designing Non-Rigid Highway Surfaces," by G. E. Hawthorn, *Bulletin 83*, Univ. of Washington, August, 1935.

In the usual equation, the pavement thickness is calculated by assuming that the load is applied at a point, whereas actually it is applied over a considerable area depending upon the character of the vehicle and its tire equipment. Although this area varies with different loadings and different pressures (in pneumatic tires), Table 3 shows the average conditions in round numbers.

TABLE 3.—AREA OF TIRES IN CONTACT WITH ROAD

Description	UNIT WHEEL LOAD, IN THOUSANDS OF POUNDS:				
	2	4	6	8	10
Area of tire or tires in contact with road, in square inches	25	57	75	95	112
Radius of equivalent circle, in inches	2.8	4.2	4.9	5.5	6.0

L. W. Teller and J. E. Buchanan,[30] Assoc. Members, Am. Soc. C. E., indicate that maximum load intensities occur in two transversely symmetrical zones rather than at the geometric centers of the areas, and the differences in load per square inch are not as great as was formerly supposed.

It is apparent, therefore, that in place of a cone of distribution, there is actually the frustum of cone, and that the true radius of the area of load distribution over the base or sub-grade is equal to the thickness of the pavement plus the radius of tire-area contact.

In Fig. 5, let W = wheel load, in pounds; s = safe bearing value of the subgrade, in pounds per square inch; t = required thickness of pavement, in inches; r = radius of cone; and e = the dimension shown.

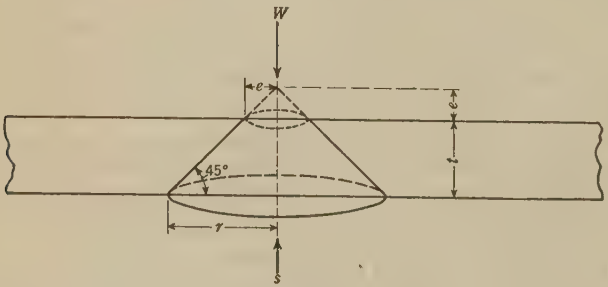

FIG. 5

Then,

$$W = \pi r^2 s \quad (8)$$

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[30] "Experimental Determinations of the Variations in Pressure Intensity Over the Contact Area of Tires," Highway Research Board, December, 1937.

but, $r = t + e$; and $W = \pi (t + e)^2 s$; that is,

$$t = 0.564 \sqrt{\frac{W}{s}} - e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The bearing capacity is composed of two separate factors of strength, one which is dependent upon the perimeter of the area, called perimeter shear, the other dependent upon the area itself, termed the strength of the pressure bulb. This latter factor often has been the only one given consideration in determining the thickness of flexible pavements, and yet in bearing areas of small size, the factor of perimeter shear, in giving support value to the sub-grade, may be the larger of the two. Professor Housel,[26] who has made a number of studies in this regard, gives the following equation for these relationships:

$$W = A\,S = m\,P_A + n\,A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which W = total load carried on a given area, in pounds; A = area, in square feet; P_A = perimeter of area, in feet; m = perimeter shear, in pounds per linear foot; n = developed pressure, in pounds per square foot; and, S = bearing capacity, in pounds per square foot $= m \frac{P_A}{A} + n$.

In 1932, Roland Vokac,[31] Assoc. M. Am. Soc. C. E., then with the Michigan State Highway Laboratory, reported some very informative studies in connection with the design of low-cost bituminous mixtures. Making use of Equation (10) (expressed in inches instead of feet to conform to the size of suitable laboratory samples) he found that the values of m and n could be determined for any mixture through the use of a simple laboratory machine. Numerous mixtures of varying behavior under traffic were tested and the laboratory results consistently confirmed the road behavior, not only relatively, but in accurate fundamental terms of bearing capacity. Also, contrary to the general assumption that the highest concentration of pressure always occurs under the center of the loaded area, it was found that up to certain loads there is actually a negative value for n because the perimeter shear value, m, is so high that all the load is carried at the edge of the bearing area. As the load further increases this shear value builds up first, followed later by the increase of pressure resistance until both become constant, at which point the ultimate strength of the material is reached.

Professor Housel also shows that,[26] "The importance of the time element in interpreting the results of test on all varieties of soils cannot be over-emphasized." By this is meant the difference in the effect of load distribution between a load suddenly applied and then immediately removed, and one applied gradually over a longer period of time. An interesting example of this difference is familiar to all in the incident of school days in sliding across thin ice without difficulty where it would have been impossible to have walked across without breaking through. Still another factor contributing to the supporting power of a sub-grade is the effect of surcharge due to the weight of a superimposed layer of well bonded material. This is particularly marked in sub-grades which depend principally upon internal friction for stability and in which there is little cohesion, as in the case of loose sand.

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[31] "Studies in the Proportioning of Low Cost Bituminous Mixtures of Dense Graded Aggregate Type," Tenth Annual Asphalt Paving Conference, 1932.

Although this discussion could be prolonged indefinitely without exhausting its potential values, it is hoped that some additional attention may have been focused upon the paper of Mr. Mullis. It is believed that the following conclusions can be derived from reading it:

(1) Although various types of road surfaces have been used over a long period of time, only in the past few years has there been any effort to appraise conditions under which they should be selected and utilized in relation to traffic needs.

(2) The most important part of the road structure (the natural soil foundation) has been almost completely ignored in making an engineering analysis. Even now only a few States follow any definite procedure in this regard.

(3) There are two methods of designing a road surface: (*a*) To make it so thick that it stands in spite of a bad foundation; and (*b*) to place a wearing course exactly thick enough to supplement the sub-grade support. The first method often entails unnecessary expense.

(4) There is always a way to utilize local materials for road construction; it only requires intelligent engineering study to develop the requisite technique. This is particularly true in respect to sub-grade stabilization.

(5) The load supporting capacity of a sub-grade depends upon two characteristics: Internal friction and cohesion between the particles making up the soil. These vary according to the moisture content (especially the latter), and beyond a critical point result in reduced capacity to carry load.

(6) Control of moisture content, then, is of the greatest importance and yet engineering attention to this matter is often more a matter of talk than of action.

(7) Having controlled moisture through appropriate methods according to job conditions, it is then required to determine the thickness of surfacing for traffic. This involves two studies, one as to the minimum support to be derived from the sub-grade, either natural or as modified, and the other as to the capability of the selected surface, to transmit load without distortion. Soil classification has proceeded to the point where a relatively few general groups cover all locations, so that all that is required is to apply methods for measurement already established in other fields. There are several laboratory machines for determining the stability of wearing course mixtures. There only needs to be a greater use made of these methods and facilities.

(8) To utilize inherent soil values to best advantage, work should be done in a uniform manner. By proper attention to grading operations the finished sub-grade can be made of uniform load-supporting character, with resulting uniformity in wearing course. It is less expensive to prepare a high support sub-grade with a relatively thin surface course to meet a given traffic load, than to build a thick surface on a "hit or miss" sub-grade. What seem to be low prices for unclassified excavation in road-grading operations are often actually very costly because good soils are frequently wasted.

(9) According to well-known laws it is possible to use a formula for the determination of the necessary thickness of wearing course to meet any condition. That this formula is not the same as the one used in designing rigid pavements is simply a recognition of the fact that there are basic differences between elastic and plastic substances.

W. J. TURNBULL,[32] ASSOC. M. AM. SOC. C. E. (by letter).—In this paper, the author has presented a clear picture of the functions of a soil in a road-bed. He has reviewed the various states of matter and the way in which each one influences the action of a soil in the road-bed, when placed at varying degrees of compaction. The definitions of the various characteristics of a soil, as well as the various soil types, are believed to be especially good.

The gist of the paper, as interpreted by the writer, is to the effect that increasing the soil density: (1) Enhances the structural properties of the soil; (2) increases its resistance to water penetration, thus acting to decrease weathering ravages; and, generally, (3) improves the desirability of the soil in all respects as the principal component in a road-bed. In general, this is true. However, some soils are known to have more detrimental swelling properties at the higher densities, and, consequently, this feature should not be neglected in the study of a soil to be used in a road-bed.

Mr. Mullis introduces the term "yield point due to water content" as marking the end of the elastic state and the beginning of the plastic state. He also states that it is approximately equal to Atterberg's lower plastic limit in clays, and to 75% of Atterberg's liquid limit in silts and fine, friable materials. Further, the inference is made that the plastic state is undesirable. The writer feels that the early stages of the plastic state are not undesirable provided the strain produced in the road-bed on the application of the maximum wheel load is not beyond an amount which is harmful to the surface-protection mat. This is analogous to the fact that a structure may withstand successfully 2 in. of settlement, whereas 4 in. would be very harmful. The early stages of the plastic state may even be more desirable than the early stages of the elastic state, due to the fact that the latter borders near brittleness; whereas the strain in the former state produced by the maximum wheel load is not detrimental to the wearing surface. As stated by the author, a brittle (dry) road-bed is undesirable unless the soil possesses high tensile strength, which very seldom if ever occurs. Furthermore, the elasticity of a soil increases with increasing density, other factors remaining equal. The preceding statements have been made on the assumption that a soil placed at a given density passes through the brittle state into the elastic state and into the plastic state with increasing moisture content.

In the design of the road-bed, the author indicates the "factor of safety against water absorption" as the ratio of the "yield point due to water content" (lower plastic limit) to the actual water capacity of the soil. He further indicates that the top 2 ft of a road-bed should have a factor of safety of 1.43 (70% of yield point), and that the pore space of all sub-grade soil below the

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[32] Soils Engr. and Chf. of Laboratory, The Central Nebraska Public Power and Irrig. Dist., Ogallala, Nebr.

2-ft depth should not be greater than 40 per cent. Assuming a true specific gravity of 2.65, the dry weight of the soil below the 2-ft depth would be about 99 lb per cu ft. Assuming a true specific gravity of 2.65 and a lower plastic limit of 18% moisture, the dry weight of the soil with a factor of safety of 1.43 above the 2-ft depth would be about 124 lb per cu ft. It would be impractical, if not impossible, to obtain the latter weight by rolling, if the road-bed was to be constructed of a wind-blown silt loess material of uniform grain size. The only practical way to obtain this density would be by using suitable admixtures, which immediately brings the question of economy into the picture. The dry weight of 99 lb per cu ft is quite possible to obtain even in most poorly graded silts. It would seem to the writer that, in general, the factor of safety against water absorption as set by the author for the upper 2 ft may be somewhat high; however, for certain soils, it may be entirely practicable and satisfactory. It is impossible to lay down "hard-and-fast" rules that are applicable to all soil types. Certain general conditions undoubtedly apply to all soils. When used in a road-bed, however, each should be studied in detail and the various requirements modified to fit the individual soil.

The author is to be commended for presenting a definite method to follow in the design of road-beds. Such a method should tend to reduce the practice of "hit and miss" which has existed, all too often, in the past.

BERT MYERS,[33] Esq. (by letter).—The highway engineer concerned with the economics of highway design will be interested in the limits suggested by Mr. Mullis for the physical properties of a satisfactory road-bed, and the cost of building a structure that will meet these requirements. He will also be interested in learning whether soil to which these properties have been imparted will continue to possess these properties after being exposed to varying conditions of temperature, stress, and opportunity for the absorption of moisture (see Table 4).

It is suggested that the road-bed should have an elastic limit of not less than 200 lb per sq in. The method for determining this elastic limit is not described. If it is to be determined by subjecting a small sample of soil, such as might be cut from the road-bed, to compression without horizontal restraint, this value would seem to be high, and difficult and expensive to attain.

It is suggested that the pore space in parts of the road-bed more than 2 ft below the sub-grade should not exceed 40% of the volume. This limit seems entirely practicable and reasonable. Another suggestion is that the pore space in the upper 2 ft of the road-bed be reduced to such an extent that its theoretical water capacity is not greater than 70% of the lower plastic limit of the earth used.

The Bureau of Soils Surveys, Iowa State Highway Commission, has conducted some studies to determine the probable behavior of soils in such a condition under field conditions (see Table 5). Samples have been taken from stabilized gravel base courses and their density and moisture content determined. The results of these studies indicate that this material, which had a distribution of particle sizes and degree of compaction such that the theoretical water capacity was below the limit suggested by Mr. Mullis, did not become less dense or absorb moisture when exposed to the conditions existing on the road. Samples taken from bases that contained 8 to 10% of moisture at the time they were constructed, were found to contain from 3 to 7% moisture two years later. This was true even where the moisture content of the subgrade soil, immediately beneath the base material, was as high as 24 per cent.

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[33] Engr., Materials and Tests, Iowa State Highway Comm., Ames, Iowa.

A laboratory study along this same line was conducted as follows: Samples of three different soils common in Iowa, and mixtures of these soils with pit-run gravel, were formed into cylindrical specimens 4.5 in. in diameter by 9 in. high. The specimens were compacted to the highest density that seemed practicable with present road-building equipment. The specimens were moulded in rubber tubes which covered the specimens throughout the test. The lower end of the specimen was held in the tube by a perforated metal disk and blotting paper. The specimens were frozen in a freezing chamber and then thawed in the laboratory air with their lower ends immersed in water. The freezing and thawing treatments were alternated ten times. The volume and weight of each specimen were determined after each alternation. The results of this series of tests are given in Table 4.

TABLE 4.—RESULTS OF FREEZING AND THAWING ON THE DENSITY AND MOISTURE CONTENT OF SOIL SPECIMENS

Specimen	COMPOSITION (PERCENTAGES)		Percentage passing a No. 40 sieve	ORIGINAL CONDITION		CONDITION AFTER FREEZING AND THAWING		Plastic limit	Water capacity*
	Clay	Gravel		Density	Percentage of water	Density	Percentage of water		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A	Loess		100	1.84	15.1	1.73	20.4	19.4	89
B	Loess		100	1.77	17.5	1.68	21.0	20.9	93
C	Glacial clay		77	2.01	10.7	2.00	12.2	12.1	100
D	30	70	40	2.16	8.1	2.18	8.0	12.8	67
E	10	90	26	2.20	7.4	2.20	7.5	13.0	60
F	30	70	41	2.10	9.8	2.10	10.0	15.2	67
G	13	87	30	2.17	8.3	2.18	8.3	14.1	60

* Percentage of plastic limit.

It will be noted that specimens in which the theoretical water capacity was greater than 70% of the lower plastic limit of the material (Column (10), Table 4) decreased in density and increased in moisture content under the test procedure. In these specimens the final moisture content was slightly greater than the lower plastic limit.

Specimens with theoretical water capacities of less than 70% of the lower plastic limit of the material did not decrease in density or gain materially in moisture content during the test. Specimens that did not decrease in density or increase in moisture content were composed of mixtures of soil and pit-run gravel.

The values in Column (4), Table 4 would indicate that, in order to make these characteristic Iowa soils comply with the suggested requirement, they

TABLE 5.—CHARACTERISTICS OF SOIL TYPES

Item No.	Soil type	Position	ANALYSIS (PERCENTAGES)				DENSITY				ATTERBERG			Road-bed moisture (percentage)	PERCENTAGE OF WATER, BY WEIGHT, REQUIRED TO FILL PORES			Percentage of road-bed pore space filled with water	Ratio road-bed moisture (lower plastic limit)
			Gravel	Sand	Silt	Clay	Absolute	Proctor	Standard	Road-bed	Lower liquid limit	Lower plastic limit	70% of lower plastic limit		Proctor	Standard	Road-bed		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1	Clay	New cut	2	35	33	30	2.70	1.86	1.95	1.66	33.7	14.9	10.4	19.6	16.7	14.3	23.2	84.5	131.5
2	Clay	New cut	3	31	30	36	2.74	1.72	2.00	1.69	35.7	17.5	12.3	20.4	21.5	13.5	22.7	90.1	116.6
3	Clay (gumbotil)	New cut	0	5	35	60	2.73	...	2.13	1.44	64.0	22.0	15.4	23.9	...	10.3	32.8	72.9	108.6
4	Clay	New cut	1	18	50	31	2.66	...	1.93	1.32	49.5	21.9	15.3	25.9	...	14.2	38.2	67.9	118.3
5	Clay (shale)	Old road-bed cut	0	2	25	63	2.75	...	1.96	1.62	51.0	17.0	11.9	25.2	...	14.7	25.4	99.2	148.2
6	Clay loam	New cut	1	35	39	25	2.57	...	1.66	1.16	52.0	27.6	19.3	31.3	...	21.3	47.3	66.1	113.4
7	Clay loam	New cut	2	36	36	26	2.61	...	1.67	1.47	38.0	21.0	14.7	22.0	...	20.9	29.7	74.1	104.8
8	Clay loam	Old road-bed cut	0	45	31	24	2.82	...	1.82	1.73	21.5	15.7	11.0	16.3	...	19.5	22.4	72.5	103.8
9	Clay loam	Old road-bed cut	0	49	22	29	2.69	...	2.00	1.83	37.0	15.0	10.5	14.2	...	12.8	22.5	63.1	94.7
10	Clay loam	New cut	2	41	28	29	2.67	1.81	1.96	1.68	37.0	16.7	11.7	20.5	13.6	17.8	22.1	92.7	122.8
11	Loam	Borrow-pit	1	43	41	15	2.62	...	1.69	1.33	38.4	21.6	15.1	23.8	...	21.0	37.0	64.4	110.2
12	Loam	Borrow-pit	0	47	36	17	2.64	1.82	1.68	1.37	27.6	16.2	11.3	19.3	17.1	21.6	35.1	54.9	119.1
13	Loam	Old road-bed cut	1	44	38	17	2.68	...	1.69	1.44	25.9	16.7	11.7	19.3	...	21.9	32.1	60.1	115.6
14	Loam	Road-bed fill	0	35	49	16	2.69	...	1.77	1.78	23.4	17.2	12.1	16.0	...	19.3	19.0	84.2	93.0
15	Loam	New cut	1	40	44	15	2.69	...	1.75	1.67	23.6	15.1	10.6	12.5	...	20.0	22.7	55.1	82.8
16	Silty clay	New cut	0	1	69	30	2.69	1.74	1.82	1.37	41.0	19.4	13.6	25.5	20.3	17.8	35.8	71.2	131.4
17	Silty clay (muck)	Rejected borrow	0	7	52	41	2.35	...	0.99	0.69	98.0	74.0	51.8	101.6	...	58.4	102.3	99.3	137.4
18	Silty clay (peat)	Rejected sub-grade	1	16	...	...	1.92	...	0.77	0.34	164.0	146.0	102.2	241.7	...	77.8	242.1	99.8	165.5
19	Silty clay	Old road-bed cut	0	1	57	42	2.70*	...	1.85	1.51	51.0	20.0	14.0	26.0	...	17.0	29.2	89.0	130.0
20	Silty clay	Old road-bed cut	0	4	57	39	2.71	...	1.78	1.45	49.0	20.0	14.0	26.6	...	19.3	32.1	82.9	133.0
21	Silty clay loam	New cut (top-soil)	0	4	68	28	2.63	1.52	1.60	1.26	48.7	26.6	18.6	28.2	27.8	25.5	41.3	68.1	106.0
22	Silty clay loam	Old road-bed cut	0	1	71	28	2.68	...	1.77	1.63	36.0	19.0	13.3	19.9	...	19.2	24.0	82.7	104.7
23	Silty clay loam	Old road-bed cut	1	24	50	25	2.64	...	1.62	1.50	36.5	19.6	13.7	20.1	...	23.8	28.8	69.9	102.5
24	Silty clay loam	Old road-bed cut	1	20	52	27	2.70	1.80	1.73	1.60	33.0	20.0	14.0	16.5	18.5	20.8	25.4	64.9	82.5
25	Silty clay loam	Road-bed fill	0	4	74	22	2.69	...	1.62	1.63	35.0	22.0	15.4	22.9	...	24.6	24.2	94.6	104.1

TABLE 5.—*Continued*

Item No.	Soil type	Position	Analysis (Percentages)				Density				Atterberg			Road-bed moisture (percentage)	Percentage of Water, by Weight, Required to Fill Pores†			Percentage of road-bed pore space filled with water	Ratio road-bed moisture (lower plastic limit)
			Gravel	Sand	Silt	Clay	Absolute	Proctor	Standard	Road-bed	Lower liquid limit	Lower plastic limit	70% of lower plastic limit		Proctor	Standard	Road-bed		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
26	Silty loam	Rejected borrow	0	18	62	20	2.60	...	1.63	1.24	48.8	29.3	20.5	27.5	...	22.9	41.2	65.2	93.9
27	Silty loam	New cut	0	3	78	19	2.71	...	1.71	1.62	26.4	18.9	13.2	20.1	...	21.6	24.8	81.1	106.3
28	Silty loam	Old road-bed cut	4	29	53	14	2.70*	...	1.79	1.67	22.7	16.8	11.8	18.1	...	18.8	22.7	79.7	107.7
29	Silty loam	Old road-bed cut	0	2	78	20	2.70	1.80	1.72	1.36	29.6	19.6	13.7	34.9	18.5	21.1	36.5	95.8	178.1
30	Silty loam	Road-bed fill	1	30	52	17	2.71	...	1.73	1.50	27.2	17.7	12.4	19.9	...	20.9	29.7	67.0	112.4
31	Silt	New cut	0	1	81	18	2.70*	1.74	1.74	1.59	26.6	19.2	13.3	25.8	20.4	20.4	25.9	99.8	134.4
32	Silt	New cut	0	1	80	19	2.72	...	1.70	1.65	26.7	19.4	13.6	21.5	...	22.1	23.8	90.3	110.8
33	Silt	New cut	0	1	81	18	2.71	1.84	1.63	1.65	28.1	20.5	16.0	22.9	17.4	24.4	23.7	96.7	111.7
35	Sandy clay loam	Old road-bed cut	9	48	24	19	2.68	1.96	...	1.69	24.3	13.2	9.3	14.3	13.7	...	21.8	65.6	108.3
36	Sandy clay loam	Old road-bed cut	10	46	24	20	2.68	...	1.80	1.57	31.0	15.0	10.5	12.7	...	18.2	26.3	48.3	84.7
37	Sandy clay loam	Road-bed fill	3	51	23	23	2.69	...	1.97	1.69	30.9	12.4	8.7	14.2	...	13.6	22.0	64.5	114.5
38	Sandy clay loam	Old road-bed cut	7	47	26	20	2.69	...	1.85	1.82	30.0	16.0	11.2	15.3	...	16.9	17.7	86.1	95.6
39	Sandy clay loam	New road-bed back-fill	2	51	25	22	2.68	...	1.92	1.83	32.0	13.0	9.1	11.6	...	14.8	17.3	67.1	89.2
40	Sandy loam	New cut	9	53	20	18	2.69	...	2.02	1.76	21.9	11.8	8.3	12.2	...	12.3	19.7	62.1	103.4
41	Sandy loam	Borrow-pit	4	62	26	8	2.67	...	1.82	1.58	...	...	...	4.2	...	17.5	25.8	16.2	...
42	Sandy loam	Borrow-pit	0	53	34	13	2.66	1.85	1.72	1.53	22.5	13.9	9.7	6.7	16.5	20.5	27.8	24.2	48.2
43	Sandy loam	New cut	1	51	31	17	2.66	...	1.68	1.41	25.0	15.5	10.9	16.7	...	21.9	33.3	47.9	107.7
44	Sandy loam	Old road-bed cut	6	50	25	19	2.69	...	1.91	1.81	26.5	12.9	9.0	14.2	...	15.2	18.1	78.6	110.1
45	Sand	Borrow-pit	2	88	5	5	2.68	1.88	1.74	...	...	...	...	...	15.9	20.2	...	...	...
46	Gravelly sand	New cut	16	68	10	6	2.74	2.02	...	...	15.6	13.0	9.1	...	13.0	...	...	...	...
47	Sand	Old road-bed cut	8	79	8	5	2.75	...	1.75	1.61	15.8	14.7	10.3	15.7	...	20.8	22.0	71.4	106.8
48	Sand	Old road-bed cut	2	82	10	6	2.67	...	1.85	2.02	...	...	...	7.4	...	16.6	12.0	61.7	...
49	Sand	Old road-bed cut	0	87	9	4	2.66	1.80	...	1.85	...	...	...	12.9	18.0	...	16.5	78.2	...
50	Gravelly clay loam	Sub-grade back-fill	21	36	27	16	2.71*	2.00	1.97	1.77	36.0	17.0	11.9	12.6	13.5	13.7	19.6	64.3	74.1
51	Gravelly clay loam	Back-fill borrow	23	42	21	14	2.71*	2.07	1.99	...	31.0	15.0	10.5	...	11.4	13.4	...	...	...
52	Gravelly sandy loam	Back-fill borrow	35	48	9	8	2.68*	2.16	2.06	...	23.6	14.1	9.9	...	9.0	11.2	...	...	...
53	Gravelly clay loam	Sub-grade back-fill	32	22	26	20	2.70*	2.01	1.85	1.90	51.0	22.0	15.4	7.2	12.7	17.0	15.6	46.2	32.7
54	Gravelly sandy loam	Rolled stone stabilized base	52	25	14	9	2.66*	2.12	...	2.20	22.7	13.3	9.3	6.6	9.6	...	8.0	82.5	49.6
55	Gravelly sandy loam	Soil aggregate stabilized base	31	54	10	5	2.66*	2.17	...	2.26	18.8	12.3	8.6	4.8	8.9	...	6.6	72.7	39.0

* Estimated.

† Densities corresponding to Columns (15), (16) and (17) are given in Columns (8), (9) and (10) respectively.

would need to be mixed with a sufficient quantity of coarser material to reduce the percentage passing the No. 40 sieve to about 40. In some localities, the cost of importing such a proportion of coarse material and mixing it with the soil locally available would be very great. This suggests that it might be cheaper to use the local materials to produce the best road-bed possible without the addition of coarser materials, and make up for the deficiency in the road-bed by constructing a stronger surface course.

Mr. Mullis has made a valuable contribution to the literature on the subject of the design of road-beds by presenting a paper in terms with which every engineer is already familiar.

E. NEIL W. LANE,[34] JUN. AM. SOC. C. E. (by letter).—Settlement of a stabilized road-bed will occur as erosion proceeds and as the road-bed approaches the natural condition of top soil. The rate and amount of settlement will naturally vary as the intensity and frequency of the erosional processes.

Since all roadway economics is based on the economical life of the wearing surface, it is at once apparent that the most desirable road-bed is one that will serve its useful life over a period equal to the economical life of the wearing surface. This implies, of course, that it is quite as necessary to give the same careful attention to the design of the road-bed as it is to the layout of the surface course. Little information is available, it seems, as to the useful life of different stabilized road-beds. The effect on the useful life of the various weather factors, the type of surface protection (as under paving or forming its own wearing surface), the location of the road-bed (as to slopes and depths), and the amount and types of loads have been scarcely questioned as far as accurate detailed study is concerned. Soil stabilization is a young science, and there is naturally much confusion in classifying and interpreting the results of such existing examples and studies as have been made and in setting proper questions for the basis of future studies. The latter are of utmost importance at this time, especially since the most valuable tests are the results of experience under scientific conditions.

In this paper, Mr. Mullis has raised several open questions and has collected and interpreted a great mass of fundamental data. He is to be commended for his painstaking efforts and for the general and basic perspective from which he has viewed the subject. The use of available materials and economical design seem to be underlying key-notes of the paper.

Many state specifications require the control of moisture and pore space but make little or no provision for placing the soil according to its ability to perform a service. Quite often they require that all the embankment be compacted to within 90% of the optimum compaction as shown by the Proctor density test. In doing this, it is generally found desirable to keep the moisture content slightly below (rather than slightly above) the optimum moisture as determined by the Proctor method, this being due to the fact that the usual construction equipment works more efficiently under these conditions. It would be better, the writer believes, to specify that the main body of fills be compacted to a stated lower density limit or such higher density as may be obtained by compacting the fill with a standard procedure and using a moisture content equivalent to 80% of the lower plastic limit, as shown by Atterburg's tests. Atterburg's tests are simple and thoroughly reliable, as have been proved. Moreover, it is believed that they are more adaptable to rapid field use in unskilled hands and with a minimum of equipment. Atterburg's tests do not allow as much variation of method and results as does the Proctor method.

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[34] Junior Engr., U. S. Dept. of Interior, National Park Service Eng. Div., Omaha, Neb.

The use of a sheepsfoot roller is obviously better than the smooth-wheeled roller for use on the major portion of a fill, although the latter is necessary at and near the surface. The specifying of the two types of rollers for each project is a nicety to be developed by the future.

As a result of present-day experiences with field conditions and specifications, it would seem desirable to propose certain changes in the control of the construction of stabilized road-beds for the sake of a cheaper construction cost for an equivalent road-bed as far as useful life is concerned. These propositions are not as economically important for shallow cuts and fills as they are for heavy earthwork and side-hill construction.

It has been noticed that many road-bed troubles occur at transition points between a cut and the adjacent fill. This is undoubtedly caused by the rapid change in thickness and density of the road-bed at such points. In the case of rigid pavements an extreme condition can be shown for the purpose of illustration, although the results are noticeable in the non-rigid surfaces with a more deteriorating effect.

Assuming that the span between suitable bearing in cut and fill is the practical equivalent of two slab lengths of pavement and that the joint between these slabs is approximately over the thinnest portion of the stabilized road-bed, it is easily seen how undesirable variations in the density of this thin portion of the road-bed can cause serious surface misalignment in the case of rigid paving and a gradual disruption in the case of the non-rigid surfaces.

To overcome this difficulty it would seem desirable to excavate the ends of a cut first to such a depth as is necessary to obtain a natural soil with suitable density, or to excavate to some depth as would seem judicious, and replace this soil with stabilized material. The material excavated at the ends of the cut should be hauled to the deepest portion of the fill, and replaced by soil from the middle of the cut. This eliminates duplicate hauling but necessitates an additional handling charge which the writer believes would be more than offset by a decrease in maintenance and replacement expenses.

The variation of density with the depth of fill is a subject which Mr. Mullis has treated rather lightly in view of its importance, although there is a dearth of information on this subject. It is reasonable that the density at any depth of a stabilized road-bed should vary as the load above it and inversely as the width of the base. Since the bearing width increases at a greater rate than the depth, appreciable savings in road-bed construction could be obtained by using a less dense stabilization in the wider portions of the fill which make the larger portion of the embankment. Due to the desire to distribute concentrated wheel-loads and cut down the action of capillary water adjacent to wearing surfaces, it is naturally desirable to require the most rigid control over stabilization in the first few feet below the surface course. This region, being the most exposed and least confined portion of the road-bed, should be the most dense. However, the depth of this layer of maximum density need not be the same for all conditions. With a protective covering of concrete paving one could quite reasonably expect a satisfactory performance with a high density layer of, say, a 2-ft minimum thickness whereas under a bituminous mat it would obviously be much better to have a minimum depth of 4 ft or 5 ft.

The use of a thin layer of granular stabilization (2 in. to 6 in.) between concrete paving and the high density layer is being studied with a view of decreasing the action of moisture on the bottom surface of the slab. It is expected that the moisture coming through the joints, cracks, and at the sides of slab will spread over a wide area on reaching the granular layer, thus producing a light and uniform percolation downward through the road-bed. In this way the layer will act as an insulator. A secondary effect will be to decrease the bond between the slab surface and road-bed, thus lowering internal stresses in the paving due to temperature changes and creep from hillside location.

IRA B. MULLIS,[35] ESQ. (by letter).—In this paper a new method is offered for designing road-beds. It follows well-known mechanical laws and the writer applies them in a manner similar to that followed by structural engineers.

The discussers have enhanced the value of many phases of the paper, and especially: (*a*) By contributions of basic data; (*b*) by pointing out certain structural relationships between road-beds and ceramic products; and (*c*) by the presentation and discussion of methods of design. The discussions will be commented upon topically and in the order of their appearance in *Proceedings*.

Method of Approach.—Professor Greaves-Walker states that the investigations are unquestionably along lines that will result in greatly improved road-beds and fills. He also states that in the manufacture of a good ceramic product, and in the making of a good road-bed, there is much common ground. In this connection he also stresses the value of ceramic principles which he believes should be applied to the structural design of road-beds.

Mr. Campen expresses approval of: (*a*) The analysis of the steps involved in consolidating water-soil mixtures; (*b*) the classification for road construction of a number of typical soil deposits; (*c*) the method used for showing the effect of water content on the structural properties of soils; and (*d*) the application of the information in the design of road-beds. Mr. Gray expresses his belief that the paper outlines the broader principles of road-beds in a concise and logical manner and recites his own experience in partial support thereof.

Mr. Turnbull says that the paper presents a clear picture of the functions of a soil in a road-bed, and that it reviews the various states of matter and the manner in which each one influences the action of a soil in the road-bed when placed at varying degrees of compaction. He also states that the definitions of the various characteristics of a soil, as well as the various types, are believed to be especially good. Mr. Myers says the paper is a valuable contribution to the literature on the subject of the design of road-beds and especially so because it is written in terms with which every engineer is already familiar. Mr. Lane states that the writer has raised several open questions and interpreted a great mass of fundamental data, but he does not express any view as to the validity of the interpretations.

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[35] Assoc. Highway Engr., U. S. Bureau of Public Roads, Omaha, Nebr.

Density, Porosity, and Moisture Content.—The density feature of the paper has been discussed at greater or less length by six contributors, and all agree that the strength of a road-bed increases with increasing density. This agreement is reached by a group of engineers representing a varied field of experience and research.

Professor Greaves-Walker says that there is no question about the structural properties of earth masses being enhanced and made more highly resistant to water penetration by increasing their density. Mr. Schofield says that Conclusions (1) and (2), relative to the effect of increasing density on the structural properties of earth masses and the resistance offered to the penetration of water, reached by the writer have been verified by Messrs. Everhart, Austin, and Rueckel.[23] Researches by these gentlemen also show that the effect of de-airing clay bodies (and the consequent reduction of porosity and increase of density) increased plastic strength, the dry transverse strength and modulus of elasticity; and, in all but one case, it caused a marked increase in the water-slaking time of the dry specimens.

Mr. Campen states that resistance to pressure increases with increasing density of earth masses, and shows this relationship in Table 2. Mr. Myers contributes Table 5 which gives a detailed concept of the relationships of density, porosity, and water capacity of certain types of earth. In this table certain relationships appear to be common, if not universal. Of these, the following will be mentioned: (*a*) That the percentage of road-bed pore space filled with water during wet weather is generally high; (*b*) that the largest percentages of moisture are found in road-beds of low density (high porosity); (*c*) that the smallest percentages of moisture are found in road-beds of high density (low porosity); and (*d*) that the percentage of road-bed pore space filled with water appears to be reduced by the presence of considerable quantities of coarse matter, such as sand and gravel.

Mr. Turnbull is in accord with the belief that increasing soil density enhances its structural properties, increases its resistance to the penetration of water, and generally improves its desirability in all respects for road-bed use. However, Mr. Turnbull states that certain soils are known to have more detrimental swelling properties at the higher densities. The writer believes that such detrimental swelling occurs only in rare cases, none of which has come under his observation; and, he also believes that no type of earth can be compacted to an extent which impairs its resistance either to pressure or to weathering. The basis for this belief will now be exemplified by means of the inherent properties of a certain clay when compacted to given densities. The clay selected is a gumbotil listed as Item No. 3, Table 5, by Mr. Myers. The discussion is as follows:

It is assumed that this gumbotil is to be placed in a road-bed in each of two states of compactness. It is also assumed that the magnitude of the compacting energy is ample for producing a density of 2.13 (Column (9)), and a water capacity of 10.3% (Column (16)). It is further assumed that the density required for Section *A* is 2.00, with a moisture content not in excess of 12%, and for Section *B* a density of 1.69, with a moisture content not in excess of 22 per cent.

Under these assumptions, the pore space in Section *A* was reduced so that its water capacity was only 1.7% in excess of that of its approximate shrinkage limit. Moreover, since the hygroscopic coefficient[36] of most clays is about 13%, this would aid in preventing further loss of moisture under field conditions. It will be seen, therefore, that the moisture range of earth in Section *A* is limited on the one hand by its hygroscopic coefficient and on the other hand by the restricted capacity of its pores. The capacity of the mass to shrink or swell, therefore, is reduced to the minimum, and its bearing capacity remains high as long as the mass is protected against weathering.

Section *B* was constructed with a water content of not more than 22% and hence, if it contained that quantity of water, was plastic at that time. Furthermore, regardless of the moisture content during construction, the pore space was such that the mass would become plastic whenever a sufficient quantity of water was available. Therefore, the pores, due to their large volume, would offer little resistance to the penetration of water. Assuming that a given drying would reduce the moisture content of the mass in Section *B* to 12%, which value is equivalent to that of Section *A*, the moisture loss since the time of construction would be 10%—a quantity ample for the development of shrinkage fissures of such size or number as would aid materially in the rapid distribution of water when available.

It may be seen, therefore, that the clay, consolidated as in Section *A*, is in a most favorable condition for resisting the loss of water either by evaporation or percolation. Moreover, since the restricted pore space is nearly filled at all times, the amplitude of moisture change is indeed small. By a similar process of reasoning, it may also be seen that the less compact mass of earth in Section *B* undergoes greater changes in volume and moisture content, and a more rapid loss of density when unprotected from the weather, than does Section *A*.

General Requirements for Density.—Within the decade prior to 1939, the necessity for developing considerable density in road-beds and earth dams had become increasingly apparent, and several standards of density have been set up. In 1932 the writer proposed the use of "standard" density as the ultimate density or ideal to be attained. In 1934 one State with which the writer was associated in Federal-Aid road work specified that road-beds must be compacted to 90% of "standard" density. Other standards of compaction have also become more or less popular, but the writer will discuss only his own unsatisfactory attempt to utilize "standard" density as a standard of compaction. Although this standard is an advance in improving the work, it was found to be unsatisfactory, inasmuch as the pore space at standard density bears no constant relationship to the quantity of water required for the production of critical softening or yield point due to water content. This lack of constancy of relationship may be exemplified by test values from Table 5 and shown in Table 6.

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[36] "Principles of Soil Technology," by Paul Emerson.

TABLE 6.—INCONSTANCY OF RELATIONSHIP BETWEEN THE LOWER PLASTIC LIMIT AND THE PERCENTAGES OF WATER REQUIRED TO FILL PORES IN MASSES AT STANDARD DENSITY
(Item and Column Numbers Refer to Table 5)

Item No.	Column (12)	Column (16)	Ratio	Item No.	Column (12)	Column (16)	Ratio
1	14.9	14.3	0.96	17	74.0	58.4	0.79
3	22.0	10.3	0.47	19	20.0	17.0	0.85
5	17.0	14.7	0.86	31	19.2	20.4	1.06
8	15.7	19.5	1.24	42	13.9	20.5	1.47

Density Below the 2-Ft Level.—Of those who discussed this phase of the paper, the following comments were made: Mr. Campen states that the writer's suggestion that pore space in a road-bed at depths greater than 2 ft below subgrade should not exceed 40% of the volume is reasonable, except when applied to certain types of clay. He also states that locations are to be found where road-beds having a 50% pore space are entirely satisfactory for heavy loads because he believes water can be isolated from them. Mr. Myers states that an upper limit of 40% for pore space at depths of 2 ft below the subgrade seems to be entirely practicable and reasonable.

The writer's reason for suggesting an upper limit of 40% for pore space in a road-bed deeper than 2 ft below subgrade (when a lower percentage is preferable) is a two-fold one: (*a*) It is desirable to keep the cost of compaction as low as possible, but without sacrificing too much strength and durability; and (*b*) on a basis of an absolute density of about 2.65 for the vast majority of road-bed materials and the 40% pore space suggested in the paper, the bulk density would be 1.59 and the water required to fill the pore space would be 25.1% of the dry weight. Furthermore, since many earthy materials do not become plastic with less than 20% to 25% of water, the writer believes, on the basis of test data, that pore space in this part of the road-bed would commonly fail to become filled with water to an extent which would produce a plastic state. However, where conditions or materials are such as would produce instability, the upper limit of the 40% suggested should, of course, be lowered.

Density Above the 2-Ft Level.—Mr. Campen states that there is no doubt in his mind that, if all the soil or soil mixture courses within the upper 2 ft are compacted so as to restrict the water capacity to 70% of their plastic limit, the resistance required will be developed; but he also says that the pore space called for is lower and the thickness is higher than necessary and states his reasons for disagreement. Mr. Turnbull points out the difficulty, or even the impossibility, of compacting certain poorly graded sediments to densities suggested by the writer for use in the upper 2 ft of the road-bed. He also says that it would seem to him that the factor of safety set for the upper 2 ft may be somewhat high; but he admits that, for certain soils, it may be entirely practicable and satisfactory.

Mr. Myers states that, in studies made under his supervision, on stabilized gravel base courses, which had a distribution of particle size and a degree of compaction such that the theoretical water capacity was below the limit suggested by the writer, these base courses did not become less dense or absorb additional moisture when exposed to the conditions existing on the road. He also states that samples taken from bases that contained 8% to 10% of moisture at the time they were constructed were found to contain only 3% to 7% of moisture two years later. Mr. Myers also conducted laboratory studies on the porosity-water content relationship through ten cycles of freezing and thawing, after which it was found that the specimens in which the theoretical water capacity was greater than 70% of the lower plastic limit of the material decreased in density and increased in moisture under the test procedure. Specimens with a theoretical water capacity of less than 70% of the lower plastic limit of the material did not decrease in density or gain materially in moisture content during the test.

Since no objection has been expressed relative to the 200-lb per sq in. load requirement proposed for road-beds, comments on this proposal will be omitted. It should be suggested, however, that the use of a sheeps-foot roller with a unit weight of not less than 200 lb per sq in. must be used for developing this strength. Furthermore, adequate compaction of earth demands that the unit weight of the roller must be increased with increasing plasticity. Moreover, the continued application of this unit weight to road-bed layers, until penetration ceases, would constitute an adequate test for compressive strength and uniformity of compaction.

The writer's reasons for suggesting a 2-ft thickness of special compaction for the uppermost part of the road-bed are as follows: (*a*) Road-beds in cuts are frequently composed of stratified, non-uniform or loose material which cannot be compacted adequately nor rendered water-tight without removal; (*b*) underground channels for water movement, if present, must be destroyed; (*c*) the uniformity of cut and fill sections of a road-bed should approach a constant density; (*d*) the cost of developing a given strength in a 2-ft thickness of road-bed by this method is commonly less than the cost of a 3-in. or 4-in. layer of granular material; and (*e*) road-bed compaction is a means of reducing the essential thickness of granular bases. A reduction in the thickness of this layer should be permitted only when this is warranted by the strength and water-tightness of underlying layers.

Mr. Campen and Mr. Turnbull have expressed doubt as to the necessity for restricting the pore space in the uppermost 2 ft of the road-bed to 70% of the lower plastic limit for the purpose of preventing critical softening. In reply to these objections, the writer would call attention to the authoritative definitions of the term "plasticity" recited by the writer under the heading "Solids." Moreover, the factor of safety suggested is only 70%, or as stated by Mr. Turnbull, 1.43. The writer had considered the advisability of suggesting the restriction of pore space to that of the lower plastic limit and depending upon entrapped air content to act as the sole factor of safety; but subsequent experience demands a more adequate factor of safety. Tests have frequently shown that pores sometimes become 100% saturated, although the values are commonly lower (Table 5, Column (18)). Since engineers usually recognize the need of various means for compensating for small variations in the uniformity of structural materials, it would seem that a value of 1.43 is not too great, especially since a factor of safety of 2 is in common use for structural steel, which is quite uniform in quality as compared to most clays.

The inability to compact poorly graded materials to certain densities, as pointed out by Mr. Turnbull, was implied although not adequately presented in the original paper. Obviously, size gradation is an essential to the production of strength and resistance to the penetration of water. A comprehensive method for the predetermination of ideal grading for maximum density of earth mixtures has been described by Charles H. Lee, M. Am. Soc. C. E., in 1938.[37] Although Mr. Lee's work is based upon the well-known experiments on aggregate for concrete by Sanford E. Thompson and the late William B. Fuller, Members, Am. Soc. C. E., in 1903,[38] supplemented by those of A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and F. E. Richart, M. Am. Soc. C. E., in 1919 to 1923,[39] he shows the application of this law to earth mixtures. Mr. Lee has also developed a splendid form of graphical diagram based on Professor Talbot's equation and includes five cycles, ranging in size from 0.001 to 152.4 mm.[40]

TABLE 7.—EFFECT OF PLASTICITY INDEX ON RESISTANCE TO EARTH FAILURE

Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)
36	200+	40	14	200+	30	8	200+	64	4	82	2	2	197	7
34	200+	5	14	200+	31	7	200+	120+	4	117	3	2	117	29
28	200+	120+	13	200+	49	7	200+	120+	4	129	7	2	136	60
31	200+	120+	13	200+	11	7	200+	120+	4	186	18	1	6	1
27	200+	60	13	200+	9	6	200+	120+	4	158	32	1	18	1
25	200+	5	12	200+	7	6	128	120+	3	158	120	1	22	2
22	200+	8	12	200+	120+	6	200+	33	3	162	2	1	71	2
20	200+	2	12	200+	120+	6	200+	12	2	43	1	0	6	1
16	200+	120	11	200+	53	5	200+	4	2	169	2	0	13	1
16	200+	120+	11	200+	53	5	146	83	2	65	3	0	12	1
16	200+	6	10	200+	120+	4	59	1	2	68	3	0	12	3
15	200+	20	9	200+	48	4	165	2	2	148	5	0	8	6

Plasticity Index-Strength Relationship.—Mr. Campen makes the statement that the load-bearing value at any given water content varies with the plasticity index. In partial support of this statement, it may be safely stated that highly plastic clays, such as gumbos, commonly show high strength, but china and other clays of low plasticity are generally weak. The reader, however, should not infer that, because a certain clay shows a plasticity index of greater magnitude than that of a certain other clay, its cohesive strength is to be considered as being universally superior in resisting all types of forces. For the purpose of exemplifying any relationship of plasticity index to resistance of earth failure, Table 7 is shown. Cementation and absorption cylinders were 1 in. by 1 in. in size, molded under the usual pressures and dried under standard conditions.

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[37] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 1–61.

[38] *Loc. cit.*, Vol. 59 (1907), p. 67.

[39] *Bulletin No. 137*, Eng. Experiment Station, Univ. of Illinois, October 15, 1923.

[40] *Proceedings*, Am. Soc. C. E., September, 1938, pp. 1438–1441.

The results of these tests indicate that, as a rule, the cylinders having low plasticity indices offered less resistance to blows and also to absorption of water than did those having high plasticity indices. However, it is also evident that the resistance to blows or slaking offered by the cylinders is not often indicated by the plasticity index.

Road-Bed Density and Pavement-Thickness Relationship.—Mr. Gray's contribution of design for "flexible type" pavements is sound in principle and his discussion is so practical and full of common sense that it should receive considerable attention. Moreover, his conclusions concerning the relationship between road-bed stability and pavement durability are quite apropos, regardless of the type of pavement laid.

In closing the discussion, the writer would stress the following essentials:

(*a*) Develop the road-bed material into the strongest and most water-resistant structure possible;

(*b*) Utilize selected local materials for enhancing the quality of the uppermost part of the road-bed; and,

(*c*) Complete the roadway with a pavement or wearing surface adequate for all requirements.

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WATER-SOFTENING PLANT DESIGN

By W. H. Knox,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Philip Burgess, A. Elliott Kimberly, L. R. Howson, Charles P. Hoover, M. H. Klegerman, Rollin F. MacDowell, D. E. Davis, and W. H. Knox.

Synopsis

In Ohio, the water supplies available in sufficient quantities to justify their development for municipalities are hard, almost without exception. The ground-waters used for municipal supplies range from about 60 ppm at a few supplies, derived from the sandstone region in the southeastern part of the State, to about 2 000 ppm in the northwestern part. The hardness of the surface water supplies does not vary through such a wide range. The average hardness of the Ohio River water throughout the year is about 100 ppm and the average hardness of the Lake Erie water is about 115 ppm. Most of the surface waters from streams within the State have an average hardness between 100 and 400 ppm; the maximum and minimum hardness of these surface waters fluctuates considerably above and below these averages.

Thus, it is seen that Ohio has been, and still is, a natural field for water softening. The first water-softening plant in the United States was installed in 1903 at Oberlin, and the first large plant in the United States was placed in operation at Columbus, in 1908. Very few plants were installed during the next fifteen years, there being a total of only six municipal water-softening plants installed in Ohio prior to 1924. Since 1924 the number of municipal water-softening plants has increased rapidly and, when plants now (April 1, 1938) under construction are completed, there will be 93 plants in the State. A total of approximately 4 800 000 population is supplied by the 450 municipal water-works systems in Ohio and 1 450 000 of these persons are supplied with softened water.

It is required by State statutes that the plans for water-works improvements (including water-treatment plants) shall be submitted to, and shall have received the approval of, the State Department of Health before construction of such works is undertaken. In reviewing the plans for such works since 1924,

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Note.—This paper was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in May, 1938, *Proceedings*.

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the writer has had an excellent opportunity to observe the trend of the design of water-treatment plants in the State. This experience has been supplemented by visits to plants to observe the results being obtained and by the summaries of daily reports of operation that are submitted to the Department monthly. The size of the existing plants varies from the smallest plants, which have capacities of only 8 000 to 10 000 gal daily, to the Cincinnati Plant which has a rated capacity of 160 mgd. The plants in Ohio represent almost every type that can be constructed.

In this paper, an attempt has been made to avoid discussion of plant operation, the chemistry of the processes, and comments on patented equipment, except in so far as these items do affect the design of a plant.

Although the first water-softening plants were installed almost as early as the first rapid sand filtration plants, many more of the latter were built during the early years of the Twentieth Century. As a result, the design of the earlier water-softening plants was patterned somewhat after the design of water-filtration plants. The defects of such design and the need of special features for water softening were discovered by the operation of the plants and, hence, there has been a gradual change in design so that at present a softening plant is distinctly different from a filtration plant in which softening is not a part of the process. In this paper, only the special features of water-softening plants, which are not also features of filtration plants, will be emphasized.

SOURCE OF SUPPLY

If an existing well-water supply has a hardness of more than 700 to 800 ppm, another source of supply should be sought if a water-softening plant is to be installed. The cost of chemicals for water softening is, roughly, 1 cent per 1 000 gal for every 100 ppm the hardness is reduced. Hence, the cost of softening becomes burdensome when a very hard water is to be treated.

AERATION

For surface waters, aeration is sometimes an assistance in the removal of tastes and odors. For ground-waters it is advisable if considerable carbon dioxide or sulfur occurs in the water.

In recent years, patented aerators have been introduced as substitutes for the spray nozzles, coke trays, and step aerators. Limited tests have shown that such patented aerators require just as much head as, and are not any more efficient for aeration or removal of carbon dioxide than, non-patented devices. However, the patented devices are much more sightly, occupy less space, and cause less splash than non-patented devices. Therefore, consideration should be given to patented devices if their cost is reasonable.

Provisions should be made so that the aeration devices can be by-passed, the aerator devices cleaned, and the cleaning water drained to waste.

LIME-SODA WATER SOFTENING

The devices for handling chemicals—for adding chemicals to the water and for mixing them—are of major importance at a lime-soda softening plant.

These features should be considered first and the remainder of the plant built around these units.

Chemical Feed Devices.—Dry-feed machines have almost entirely superseded solution-feed devices for such plants except in small municipalities that have industrial or railroad types of softening plants, in which solution-feed devices are generally used.

In small plants it is desirable to provide an extra machine that can be used for more than one chemical; at larger plants, duplicate machines should be provided for each chemical. The machines should be placed as close as possible to the point of application so that the chemical solution can flow by gravity from the machines through short hose lines to the points of application. If this is not feasible, the chemical solutions should be conveyed in an open trough, or in hose lines supported continuously. If pipes are used to carry chemical solutions, such pipes should be accessible and should be provided with unions or other devices so that they can be removed and replaced quickly. Crosses at changes in direction to permit hose connections for flushing will assist in keeping the lines used for some chemicals clean; for lime, such flushing is useless.

Pneumatic equipment should be used for handling chemicals at large softening plants if the chemicals must be elevated. This is the most economical method of handling large quantities and permits the receipt of chemicals in bulk, which reduces the cost. For the plant of medium size a separate second-story room should be provided for the storage of chemicals in bags. Provisions should be made for elevating them to the storage room either by a hoist or by a bag conveyor. They can be dumped through openings in the floor of the storage room to the hoppers of the dry-feed machines. It is essential that connections to the hoppers be dust-proof.

For very small plants it is satisfactory to store chemicals in a room adjacent to the dry-feed machines and to carry the supply to the hoppers as needed.

Quick or Flash-Mixing Devices.—At extremely large plants it is desirable to obtain an intimate and thorough mixing of the chemicals with the water to insure that the water passing to all the subsequent treatment devices shall be of uniform quality. The hydraulic pump is used in a patented process as a flash mix at the Baldwin Water Purification Plant, at Cleveland, Ohio, and at the water-softening plant at Cincinnati. Various mechanical patented devices will also produce a flash mix. They have also been installed at some moderately large plants in Ohio. The writer questions the need of flash-mixing devices at medium or small plants, since a thorough mixing is obtained so quickly in the mechanical mixing devices at a water-softening plant.

Mixing Devices.—Baffled mixing chambers were used at the earlier plants. Mechanical mixing devices are preferable and have been installed at most of the plants constructed since 1924. Prolonged mixing is necessary to secure as complete reaction as possible and to avoid waste of chemicals. A detention period of 30 to 40 min is desirable and in no case should the detention period be less than 20 min.

A wide variety of stirring devices have given fairly satisfactory results. In many of the earlier plants, they consisted of horizontal or vertical paddles rotated through a vertical shaft by a motor on top of the mixing chamber. Recent trends are toward patented mixers. The devices should provide velocities between 1 and 1.5 ft per sec.

When the plant is large enough to justify the installation of two mixing chambers, it is desirable to have the two basins arranged so that they can be operated either in series or in parallel. It is necessary, of course, to have provisions for draining the chambers completely, preferably by gravity. Large-sized hose connections should be provided for flushing sludge from mixing chambers, clarifiers, and settling basins.

Clarifiers.—The quantities of sludge produced in water-softening plants are much greater than at plain purification plants. Unless provisions are made for frequent removal, the capacity of the settling basins is soon decreased materially by accumulated sludge, in which case storage capacity must be provided in the design. For this reason, and also due to the fact that, in many instances, small quantities of sludge can be disposed of more easily than accumulated quantities, the settling basins at most water-softening plants are equipped with devices for continuous or frequent removal of sludge.

The settling basins at a water-softening plant (and also at a filtration plant) should be true settling basins rather than coagulation basins; that is, the chemical reactions and coagulation should be practically completed before the water enters the settling basins. This item, together with the fact that the precipitate in lime softening is heavier and settles rapidly, permits much shorter detention periods for settling basins equipped with devices for sludge removal than the long detention periods formerly provided for coagulation basins at water-purification plants.

Either a rectangular basin with a longitudinal flow, or a circular basin with radial flow, gives satisfactory results. Square basins with transverse flow tend to short circuit and, as a result, no basins of this type have been installed recently.

Most engineers are guided, to some extent at least, by the typical layouts furnished by the manufacturers of the sludge-removal equipment which they propose to use. Competition among manufacturers can be obtained on bids for sludge-removal equipment for either rectangular or circular basins. There is one item that must not be overlooked in the design of a plant where clarifiers are used: In most clarifiers the effluent discharges over a weir, and this causes a practically constant water level in the clarifiers; in most coagulation basins the effluent is drawn off through submerged orifices, or over submerged weirs, which permits a considerable fluctuation of the water level in the basins. Such fluctuation is not provided when the clarifier effluent discharges over a weir. Hence, the only storage of water available for filtration is that in the carbonation chamber and in the filters themselves, above the level of the wash-water troughs. This storage is quite limited and may not be sufficient to provide the necessary "cushion" between the low-service and the high-service pumps. Lack of such storage may cause difficulties in operation unless automatic devices are installed to secure co-ordination of the various devices. This "cushion" or storage may be secured in clarifiers by causing the effluent to discharge through submerged ports which will permit the water elevations to fluctuate.

If clarifiers alone are used as settling basins, a detention period of 2 hr is generally sufficient to secure a satisfactory clarified effluent. Some of the chemical reactions are not entirely completed within this period; nor would they be in several times this detention period. This deficiency is partly taken care of by recarbonation of the water.

It is essential that provisions be made for completely draining the clarifiers, preferably by gravity, in order to secure access to the sludge-removal mechanism. In colder latitudes a cover or superstructure over the clarifier may be necessary to avoid ice troubles, especially if a surface water is to be treated.

Plain Settling Basins.—At small plants it may still be desirable for various reasons, to install settling basins without sludge-removal equipment. Such basins should be designed and operated in a manner similar to the coagulation basins of a purification plant. A detention period of 4 to 6 hr should be provided for such basins.

Efforts to withdraw sludge through a manifold system or series of small openings in the floor of such basins while the basins are filled with water have been unsuccessful at two Ohio plants; such withdrawal of sludge can be, and is being, accomplished at industrial types of water-softening plants when the depth of water is at least as great as the diameter of the tank. Since sludge from plain settling basins is generally disposed of by discharging it into streams during high flows, it is essential that the basin be set sufficiently above flood levels so that it can be completely drained into the stream during flood periods.

Sludge Disposal.—This is one of the chief difficulties of lime-soda water softening. The disposal of lime sludge into a stream does not constitute "pollution" such as that caused by the direct discharge of sewage, but it does cause unsightly deposits on the banks and bed of the stream. However, this is still the most common method of sludge disposal; it is satisfactory only when ample flows and velocities in the receiving stream are available. One Ohio plant is fortunate in having a near-by abandoned quarry into which sludge can be discharged. Another large plant discharges the sludge into up-ground lagoons surrounded by earth embankments. Other plants discharge the sludge alternately into two or more lagoons, allowing one lagoon to dry out while the other one is in use; at these plants some of the accumulated sludge is sold or is given away to farmers for use as "agricultural lime." The latter method is fairly satisfactory when the sludge produced can be absorbed by local demands. Limited experiments have been made on sludge drying by drum filters. Experiments are also being made of incinerating the sludge to convert it to calcium oxide so that it can be used again in the water-softening process.

In sewage treatment, due to the offensive nature of the sludge produced, the disposal problem had to be solved, and the engineers and equipment manufacturers worked on this problem until, at present (1938), satisfactory sludge disposal can be provided for any sewage plant. A satisfactory means of disposing of lime sludge must be found within the near future, or a lime-soda type of water-softening plant will often not even be considered for this reason alone.

Recarbonation of Water.—Lime softening produces a supersaturation of normal carbonates of calcium and magnesium in the water. In order to avoid delivering an incrusting water, it is necessary to convert the normal carbonates into bicarbonates, and this is accomplished by applying carbon dioxide to the lime-softened water.

Before passing to the filters, the effluent from the clarifiers or settling basins is passed through a chamber in which carbon dioxide is applied to the water. On account of the high solubility of carbon dioxide gas, especially in caustic water, only a brief detention period is required. For structural reasons such chambers are generally compartments adjacent to, and of the same depth as, the settling basins. Although the reactions are practically instantaneous, a detention period of 5 to 10 min, or even a longer period, is generally provided.

Carbon dioxide is applied through grids of perforated pipes usually laid on, or slightly above, the floor of the chamber. If the chamber is of excessive depth, the grids are supported about 10 ft below the water surface in order to avoid excessive pressures. One item, sometimes overlooked, is that the carbon dioxide pipe from the compressor should enter the chamber at an elevation above the water surface in order to avoid siphon action. If the inlet and outlet are at about the same elevation, a transverse baffle should be installed to prevent short circuiting of the water.

Various methods of generating carbon dioxide gas are used. At most of the larger plants operated almost continuously, a coke furnace is used to produce carbon dioxide. The heat generated is frequently used for heating water. At smaller plants, especially those not operated continuously, it is difficult to keep a small coke fire burning and some other fuels, such as oil or natural gas, are desirable. Flue gases from coal fires are likely to contain phenols unless the combustion is quite perfect; the designer should be cautious in deciding to use flue gas if the water is to be chlorinated. If coke is used as a fuel the flue gas must be passed through a scrubber and a dryer: if gas is used as a flue, the flue gas need only be passed through a cooling tank. It is very essential that the gas passing to the compressor is dry.

The devices should be sufficient to provide at least 300 lb of carbon dioxide per million gallons of water treated. The capacity of the compressor required varies with the percentage of carbon dioxide in the flue gas, the character of the water treated, and the degree to which recarbonation is to be carried. Careful investigation should be made of the type of compressor to be used.

The recarbonation equipment is one item in a plant for which duplicate units are not absolutely necessary. Since many lime-softening plants have operated for years without recarbonation devices, there is ordinarily no reason why they cannot be discontinued for a few hours or a few days without any unsatisfactory results.

Gravity Filters.—The design of filters for a lime-soda type of water-softening plant is practically the same as the filters of a purification plant. There are several features, however, that require special attention. Filter influent pipes, even where recarbonation is practiced, tend to incrust rapidly with normal carbonates. Such pipes should be designed over-size to permit longer periods between cleaning. They should also be designed with as few changes in direction as possible; flanged crosses instead of bends should be placed at changes in direction to permit rapid access for cleaning. Where small pipes must be used, they should be provided with flanges or unions so that the entire pipe can be removed for cleaning. Since the influent water is not corrosive, the filter influent pipes may be made of light-weight steel, which is much less expensive for larger sizes than cast-iron pipe. For a similar reason, steel wash-water troughs do not corrode at a lime-softening plant. Since the filter sand will begin to incrust as soon as the plant is in operation, it is obviously unnecessary to specify that the sand shall not be acid soluble as generally called for in the specifications for ordinary filtration plants.

The removal, regrading, and replacement of sand in filters, is an item that is not peculiar to a softening plant but one that has been neglected in the design of both filtration plants and softening plants. A few features included in the design would greatly facilitate handling of filtering material: Ordinarily, it can be conveyed by a water ejector; a high-pressure water line at least 2 in. in diameter should be available at each filter; a permanent pipe line in the pipe gallery should extend to a storage room for filtering materials; there should be provision for temporary rigid or flexible connections to this pipe line at each filter; a room should be provided in which a bin, sufficient in size to hold the filtering material for one filter, can be placed; and, the filtering material from a filter can then be ejected to a sand washer and gravel grader from which the sand and gravel would pass to the storage bins. In returning the filtering material to the filters, the operation is reversed. Provision of such equipment would result in improved operation of plants.

Pressure Filters.—Pressure filters should not be installed unless they cannot be avoided. The sand in such units becomes incrusted, is not frequently inspected, and it is practically impossible to spade it. As a result the water frequently passes through cracks in the filters instead of through the filter media. Since pressure filters are only intended as clarifying devices at lime-softening plants, there is no sanitary objection to their installation where the water to be softened is a well water of satisfactory sanitary quality; pressure filters must not be used where bacterial purification of the water is necessary. The type of plant feasible with pressure filters is frequently of considerably less cost than a plant of similar capacity with gravity filters, and this fact may occasionally justify the use of pressure units.

ZEOLITE WATER SOFTENING

Although zeolite softening units have been used for household and industrial installations for a considerable length of time, the adaptation of this process for municipal water softening is of comparatively recent date. The first municipal plant in Ohio was installed at Lowellville in 1927. Such a plant quite often consists almost entirely of assembled equipment furnished by the manufacturer. For this reason there is frequently very little actual designing to be done by the engineer. Sometimes, his chief duty may be only to determine whether zeolite treatment is the best and most economical process for the water to be treated, and to write specifications so that the plant will be adequate and so that competitive bids will be received.

A number of the earlier municipal zeolite plants were "sold" directly to municipal officials by representatives of equipment companies. In these cases no investigation was made by a disinterested engineer to determine whether the type of plant offered was best suited to local conditions. As a result, some zeolite plants have been installed in places where they are, to say the least, not the most suitable and, in some cases, the plants have been absolutely unsatisfactory. These installations have been, and still are, a detriment to the zeolite-softening field. Reputable manufacturers of zeolite and zeolite equipment now prefer and generally insist that the municipality installing the water-softening plant be represented by a competent engineer experienced in this work.

A zeolite plant has several distinct advantages over a plant of the lime-soda type. The chief merits of this type of plant are: (1) The absence of a sludge disposal problem; (2) lower first cost; (3) simplicity of operation; (4) expert supervision of plant is unnecessary; and (5) single pumping, directly from the well to the distributing system, can be practiced in many instances.

There are also disadvantages to zeolite softening which may be listed as follows: (1) Zeolite softening alone cannot be used where bacterial purification is required; (2) waters containing objectionable proportions of iron and manganese are not suitable for zeolite treatment unless these minerals are removed prior to zeolite softening; (3) some types of zeolite treatment require a large proportion of wash and rinse water; and (4) zeolite treatment alone provides no pH-correction.

Plant Capacity.—It is impossible to rate the daily softening capacity of a zeolite unit by its surface area or by the volume of zeolite in the unit. Its daily capacity depends, among other things, on the number of times the unit is regenerated during 24 hr. Therefore, the engineer must be careful to give proper weight to this factor when considering units of various manufacturers.

Iron Removal Prior to Zeolite Softening.—If the untreated well water has an iron content very much in excess of 0.5 ppm, it is essential to remove it before softening; and it can ordinarily be accomplished by aeration and filtration. Iron and manganese in solution can be removed by base exchange, by regenerating the zeolite with salt. This principle is used at several plants where at least three zeolite units are installed. In these cases, two (or more) units are operated as iron-removal and softening units, while one unit is over-run after its capacity for softening is exhausted. This single unit is thus only effecting iron removal; the mixed water from the three units is delivered to the distributing system in order to avoid serving a water of zero hardness. In this case the operation of the various units is staggered so that each unit in turn is over-run. The writer has observed one plant successfully operated in this manner. At this plant it requires expert operation to deliver a satisfactory water of uniform quality. At some small villages where this type of plant has been installed and where only ordinary operation is provided, the iron content and the hardness of the plant effluent often vary through wide ranges.

pH-*Control.*—Zeolite treatment has no effect whatever on the pH-value. Waters containing iron ordinarily contain large quantities of carbon dioxide. At some plants, where iron removal and water softening are effected by base exchange, the effluent is practically free from iron. However, there has been no change in the pH-value of the water, and the calcium or incrusting material has been removed; as a result, the water re-dissolves iron either from deposits in the mains or directly from the piping system with the result that the water delivered to the consumer may have a greater iron content than the water delivered directly from the wells.

Adjustment of the pH-value of the water will eliminate these troubles. Aeration of the water will release the carbon dioxide and raise the pH-value. The pH-value of the water can also be adjusted by introducing alkali to neutralize the carbon dioxide. In that case, there must be a constant check on the pH-value of the water delivered. Since the alkali feeders are generally pressure devices similar in principle to the "alum pots" of pressure filters, it is rather difficult to secure a uniform rate of feed. At small plants it is almost impossible to secure the degree of attention necessary to obtain control of the pH-value by alkali feed.

The foregoing operation experiences are cited to emphasize the precautions necessary in the selection of the type of plant to be installed. Zeolite softening has a wide and increasing field. It is important that it shall not be handicapped and discredited because the water is unsuitable or because it has not been properly prepared for zeolite treatment.

Selection of Type of Plant.—After having decided that a zeolite type of water-softening plant is to be installed, the designing engineer must then weigh the merits of the various types. Among the items to be decided are: (*a*) Gravity *versus* pressure units; (*b*) kind of zeolite to be used; and (*c*) up-flow *versus* down-flow units. Some of these questions are inter-related, and a decision on one question may establish the type of unit to be installed. Especially for small plants, it is advisable for the designer to give serious consideration to the installation of the standard pressure units of a reputable manufacturer rather than to install gravity units of his own design.

If pressure units are to be installed, the direction of flow is established by the manufacturer whose equipment is selected. At all except one of the gravity zeolite plants in Ohio the units are the up-flow type. At the one gravity plant, where green sand or natural zeolite is used (synthetic zeolite is used in the others), the units are of the down-flow type.

Gravity zeolite units have (to a lesser degree) the same advantages over pressure units that gravity filters have over pressure filters; namely, that the operation of, and conditions in, the units can be observed at all times. The design of gravity zeolite units has been patterned after the design of gravity, rapid-sand filters. In a down-flow unit the design may be practically identical with the design of a gravity filter. In an up-flow unit the direction of flow is reversed and, therefore, changes in design are necessary. Zeolite softening units are generally deeper than gravity filters. The gravel layers supporting the zeolite are similar to the gravel supporting the sand in a filter. In down-flow units (where green sand is used) the depth of zeolite does not generally exceed 3 ft. In up-flow units the loss of head through them is considerably less than in the down-flow units and the zeolite may be 4 ft to 6 ft deep, and occasionally of even greater depth; similar depths are used for down-flow synthetic zeolite units. Greater depths of zeolite permit economy in the quantity of water required for washing and rinsing. Frequently, considerations defining the vertical limits of the plant structures, such as the loss of head available and the lowest elevation at which natural drainage can be secured, determine the depth of the zeolite units.

The rate at which water is to be passed through a unit determines the area needed. In down-flow units the rate is generally from 2.5 to 4 gal per sq ft per min. In up-flow units the rate may be 4 to 6 gal per sq ft per min, or even somewhat higher.

In the design of the concrete up-flow softening units there are several significant items. The collecting or effluent troughs (corresponding to the wash-water troughs of a gravity filter) need to be designed only for the maximum rate of flow through the unit. During regeneration of a unit the brine and rinsing water pass downward. The design of the water-distributing system or the rinse water collecting system in the bottom of the unit requires special attention to insure that no brine remains after it is rinsed. A system of perforated laterals such as those used in gravity filters is not satisfactory because the brine is heavy; it would collect below such laterals during regeneration and would gradually pass out after the unit is again placed in service, thus causing a salty water to be delivered from the unit. In order to avoid this condition the strainer system should be designed as a system of laterals with perforations on top of the laterals in which strainers are placed. After the laterals and strainers are installed the entire strainer system should be covered with a layer of grout to an elevation just below the openings. Since the so-called re-wash valve is used every time a unit is regenerated, it should be as accessible as the others.

Proportioning Devices.—It is not satisfactory to deliver the water from the softening units directly to the distributing system since it is of practically zero hardness. Hence, some means must be provided to mix hard water with the effluent from the softening units in order that water of the desired degree of hardness may be delivered to the distributing system.

At a small plant this may be accomplished by installing a hard-water by-pass line on which a valve and a meter are placed. The hardness of the mixed water can be adjusted sufficiently by throttling the valve on the by-pass line. Zeolite equipment companies now furnish automatic proportioning devices to control the quantity of water by-passed; the installation of such devices is almost a necessity for larger plants and is desirable even for small plants.

If the raw water contains objectionable quantities of iron, and if zeolite treatment is provided to remove iron as well as for softening, the by-passed water may also have to be treated for iron removal, in order to prevent the mixed water from having an objectionable iron content. Since the hardness of a well water varies only slightly throughout the year, the proportioning devices need only to be changed infrequently after they are once set.

Salt and Brine Devices.—A considerable economy can be effected if salt is purchased in bulk instead of in bags. For this reason almost all plant administrators purchase salt in bulk. A desirable method of storing salt is to provide a basin into which it can be dumped from a truck or a railroad car. Such a basin should be provided with a tight cover, preferably of concrete. It should have a capacity sufficient to hold at least a month's supply, and should be kept filled with water. The salt rests on layers of sand and gravel over the floor which should drain to a gutter in which there are half tile or other brine-collecting devices.

At most plants, where there are gravity zeolite units, the concentrated brine passes from the salt-storage tank to a brine-measuring tank where the brine is diluted with water to the desired strength for regenerating the zeolite. The measuring tank is usually of a capacity sufficient to provide for the regeneration of one unit. If the brine cannot pass by gravity from the salt-storage tank to the brine-measuring tank, it is desirable to place the brine pump at such an elevation that there will always be a positive head on the suction of the pump. The concentrated brine may be ejected to a softening unit; in this case, the water from the ejector provides the dilution to reduce the brine to the desired strength for regeneration.

Back-Wash and Rinsing.—Down-flow units must be back-washed in the same manner as gravity filters, although generally at a lesser rate. If the hard water is clear and free from iron, it may be desirable to back-wash and to rinse the units with hard water instead of softened water. The designing engineer should secure accurate information regarding the percentage of water required for back-washing and rinsing. This feature is a factor that is often neglected in deciding the relative merits of lime-soda *versus* zeolite softening.

Conclusions

The public in Ohio is just becoming aware of the fact that softened water can be made available in almost any locality at a cost only slightly greater than the rates being charged for hard water. The field for water softening is practically unlimited. This offers water-works engineers an opportunity to serve the public; and it also offers an opportunity to design and install treatment plants for municipalities where previously it has been thought that water-supply and water-treatment problems had been definitely solved.

Since it is desirable to profit as much as possible by past experiences, the writer offers the comments in his paper in the hope that they may be of assistance to engineers engaged in this type of work. In accordance with Society policy, he has purposely refrained from comments on specific patented methods, processes, or equipment. It is suggested that discussers observe the same limitations.

DISCUSSION

Philip Burgess,[2] M. Am. Soc. C. E. (by letter).—To a striking degree, the valuable data contained in this paper should be very helpful to engineers who are concerned with the design of water-softening plants. It is to be regretted that more data could not have been presented which would substantiate some of the author's conclusions in a helpful manner.

Mr. Knox states that he is emphasizing only the special features of water-softening plants that are not features of filtration plants. However, his discussion contains valuable data with respect to many features that are included in both types of plants. It is not an uncommon experience for an engineer to be called upon to convert a rapid sand filtration plant into a water-softening plant. Such a problem includes generally the addition of more storage and of devices for the application of much larger quantities of chemicals, and the disposition of the resulting large quantities of sludge entailed by the softening process.

Furthermore, the author states that the pneumatic equipment is preferable for handling chemicals at large softening plants if the chemicals must be elevated. In general, pneumatic equipment is desirable because it eliminates the dust problem and permits ground storage of chemicals in bulk; at the same time it provides means for elevating the chemicals from ground storage to elevated tanks, or hoppers, through which the chemicals are fed by gravity as required. The writer would suggest also the use of automatic weighing machines as an adjunct to the dry-feeding equipment.

That feature of a water softening, as well as of a modern rapid sand filtration, plant which has been least appreciated, is the necessity for proper mixing of chemicals after their application with the raw water. Old plants frequently contained baffled mixing devices which, as the author states, are unsatisfactory because they do not permit the required flexibility of operation. Furthermore, it is difficult to secure proper distribution of the water as it leaves such tanks and enters the settling tanks. Modern devices of the flocculator type permit flexibility of operation and also proper distribution of the water to the settling tanks.

Reaction tanks should permit flexibility of operation at proper velocities, which the author states should be between 1 and 1.5 ft per sec. Experience indicates that slow velocities are not effective and higher velocities are harmful because they break up the coagulated floc.

The writer is interested in the design of two plants, each of 10 mgd capacity, wherein it is not now proposed to soften the water but which are to be designed so that softening may be added at minimum cost. This means that suitable devices are to be provided for storing and handling the larger quantities of chemicals and that the basins are built to allow sludge removal devices to be added when desired. No other changes are essential.

Rapid sand filters designed for filtering softened water should be equipped with devices which will permit washing at unusually high rates. This tends to decrease the deposition of after-deposits on the sand and is helpful in eliminating sand troubles and cost of maintenance. Furthermore, the filter sand may be comparatively coarse and inexpensive and need not be more than 24 in. deep. The filters are not required to improve the pathological quality of the water, but act rather in the nature of finishing devices wherein the coagulating reactions are completed, and some suspended matter is removed from the treated water. They seldom accomplish any substantial bacterial removal.

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[2] (Burgess & Niple), Columbus, Ohio.

On the contrary, most of the solids in a softening plant are removed in the settling basins and it is of major importance that the engineer design these basins so as to permit proper displacement of the water with continuous flow through them. Inlet and outlet devices should be efficient and the basins should be properly designed with respect to depth, surface area, and velocity of flow. The author has not discussed these features of design, and it is hoped that he may add some data pertaining to them in his closing discussion.

The matter of providing covers for the basins and for the reaction tanks is of importance. It is noted that frequently engineers will cover the mixing tanks and will not cover the settling basins. In cold climates, the reverse is necessary because the settling basins must be covered to prevent trouble from ice which would not be experienced in the reaction tanks where the water is flowing at comparatively high velocity.

In designing re-carbonation devices, the engineer should appreciate that the gas supplied is of a toxic or poisonous nature and the features for handling it should be designed accordingly.

The author might have stated in his comments on the design of zeolite softeners that the wash water required in such plants may be 15% to 20% of the total quantity of water softened as compared with 3% to 4% of wash water generally required for the usual water purification plant. On the other hand, the disposal of sludge is the unsolved problem in connection with the operation of lime softeners and there is no such problem in connection with the zeolite plant.

The writer is not in accord with the author to the effect that zeolite plants should be designed by the manufacturer who is now in a position similar to that of the manufacturer of filtration-plant equipment some years ago. The engineer should inform himself so that he is capable of designing such plants and should not be dependent upon the manufacturer.

In conclusion, the writer believes that the author has rendered a distinct service to the profession in presenting much of the basic data required for the proper design of water-softening plants.

A. Elliott Kimberly,[3] M. Am. Soc. C. E. (by letter).—Engineers interested in water-softening plant design will appreciate Mr. Knox's timely and informative review of current design practice. The author has had the unusual opportunity of examining plans for water-softening plants submitted during the twelve years, 1925–1937, to the Department of Health of a State in which ground and surface waters greatly exceed acceptable economic limits of hardness. This paper emphasizes the important, essential differences between water-softening and water-purification plant design, especially as regards the application of chemicals, the substitution of mechanical agitation for baffling, and the maintenance of continuously effective settling tank capacities through effective mechanical means for continuous sludge removal.

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[3] Cons. San. Engr., Columbus, Ohio.

Experience has shown definitely that effective softening and the economic use of softening agents at lime-soda softening plants are dependent upon proper mixing at non-depositing velocities, generally obtained by the use of mechanical devices, assisted by a certain percentage of sludge return. Such a plan insures a maximum softening effect without waste of chemicals.

An important unsolved problem in lime-soda softening at municipal plants is the matter of sludge disposal. Doubtless, many engineers recall the difficulties which attend efforts to flush from a settling basin a compacted sludge containing from 13% to 15% of solids, the accumulation of many months, awaiting sufficient stream flow, and the serious problems which arise when no stream is available for dilution, or when no quarry-pits or other places of disposal are available. Due to the lime-sludge disposal problem, lime-soda softening is frequently rejected in favor of zeolites, even in cases where the former may be definitely more adaptable to raw-water conditions. The solution of this problem lies in lime recovery.

In 1915, Charles P. Hoover, Chemist-in-Charge of the Columbus Water Works, conducted extensive experiments at Columbus, Ohio, and actually operated the Columbus plant for a period of 4 hr with 5 000 lb of lime produced at a practical lime recovery plant. The softening results are stated as normal and satisfactory. The recovery plan was as follows:[4] The settling basin precipitate is pumped into a thickener from which the material is fed directly into a rotary kiln. The kiln used in these tests is 20 ft long and 34 in. in diameter and was oil fired. The sludge contained 18% of solids and from it was produced quick lime containing from 63% to 73% of water soluble calcium oxide.

A lime recovery plant, in successful operation in England, comprises duplicate agitated mixing tanks into which the sludge from the settling tanks is pumped.[5] The mixed sludge containing about 25% of solids is pumped to a vacuum filter, which reduces the water content to about 50 per cent. The filter is 8 ft in diameter by 8 ft wide and operates under a 16-in. to 20-in. vacuum. The filter cake discharges by gravity into a rotary kiln 86 ft long and 6 ft in diameter, operating at the rate of 1.33 rpm. From 12 to 14 hp are required to drive the kiln, which is oil-fired.

The capacity of the kiln is about 17 tons per day. The oil consumption is stated as 27% of the lime produced. The maximum temperature reached is about 1 500° C. Provisions are made for preventing dust by passing the evolved gases through a suitable dust collector. The results obtained at this plant are said to be highly satisfactory.

In the fall of 1935, studies were made by the Nichols-Herrshoff Company to determine the possibilities of incineration of lime-sludge reduced in volume by thickeners and vacuum filters, following the treatment successfully applied in the incineration of sewage sludge.

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[4] Forty-fifth Annual Rept., Div. of Water, City of Columbus, Columbus, Ohio, 1915.

[5] *The Industrial Chemist,* July, 1933.

The need for a satisfactory practical method for the disposal of lime-sludge is evident. The successful application of filtration and incineration may solve the problem, obviously, to effect marked economies in water-softening plant operation. Many of the Ohio water supplies softened with lime and soda ash contain substantial quantities of magnesium (average 9% of the total hardness of Ohio ground-waters) and, consequently, sludge calcination results in the production of an admixture of calcium and magnesium oxides.

In a private communication with the writer, Mr. Hoover points out that, by a change in design which makes possible a two-way application of lime (additional mixing basins and clarifier units), it is possible to precipitate lime sludge containing practically no magnesium. This result is accomplished merely by holding the potential of hydrogen (pH) of the water at a point below that required to precipitate the magnesium. Since, ordinarily, considerably more reclaimed lime is produced at a plant than is required to soften the water, the two-way lime dosage plan suggested by Mr. Hoover should have definite application in practice, in cases where lime recovery is planned at softening plants treating water relatively high in magnesium salts.

Lime-soda softening was used in 1936 to effect a reduction of fluorides.[6] Briefly stated, it has been determined that, in the presence of magnesium salts, the application of lime to a water containing fluorides effects a fluoride reduction which, between the limits of 1.5 and 3.5 ppm, initial fluorides, varies as the square root of the magnesium removal. Clinical data indicate that fluorides on the order of about 1.0 ppm have no deleterious effect on teeth enamel. Mottled enamel is endemic in about twenty-eight States and, therefore, the advantages of lime treatment, with or without complete softening, appear definitely important.

Quite recently, a new type of zeolite has been developed which produces either carbonic, sulfuric, or hydrochloric acid, depending upon the composition of the raw water. Where carbonates predominate, a definite quantity of carbonic acid is produced. Using this zeolite, a certain percentage of the raw water is by-passed through a filter and, later, is admixed with the lime-softened water to adjust the potential of hydrogen. The process has the advantage that the quantity of carbon dioxide produced is constant and that no phenols or other objectionable materials are introduced into the water.

Prior to 1937, installations were limited to commercial practice, but in the latter part of May of that year a small unit went into operation at a softening and fluoride reduction plant at Mt. Victory, Ohio. The efficiency of this so-called "hydrogen zeolite" unit has been investigated by the Division of Engineering of the Ohio Department of Health. The results are said to justify the statement that the equipment is satisfactory.

Recarbonating with "hydrogen zeolite" is of much interest to the waterworks designer since, with the use of zeolite, the quantity of carbonic acid is fixed so long as the composition of the well water remains unchanged, whereas using flue gases or burning gas or coke, the percentage of carbon dioxide produced is variable, thus preventing a close control of recarbonation.

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[6] *Journal*, Am. Water Works Assoc., January, 1937, Vol. 29, pp. 9–25, inclusive.

L. R. Howson,[7] M. Am. Soc. C. E. (by letter).—The fact that approximately one-half the municipal water-softening plants of the United States are found in Ohio offers more than passing tribute to the contributions of the Ohio State Department of Health in acquainting the citizens of Ohio, not only with the necessity of securing a bacterially satisfactory water, but with the advantages to be secured from a reduction in the hardness as well. Without question, water softening in Ohio has paid large economic dividends.

The engineer who undertakes to design water filtration works is frequently aware of the ultimate, if not the present, desirability of softening, as well as filtering, the supply. When that is the case, the design should be made so as, economically and practicably, to install sludge removal equipment and other facilities necessary to the adaptation of the filtration plant to water softening. In the Milwaukee, Wis., plant (capacity 200 mgd), mixing design and periods follow softening practice. The settling basins are designed so that sludge-conveying equipment can be installed economically at a later date. It is believed that within the next decade, more or less, many plants treating Lake Michigan water with a hardness of 125 ppm will include softening.

With respect to aeration, it is the writer's belief that when used with surface waters for the removal of tastes and odors, some of the more positive and complete methods of control, such as activated carbon, are to be preferred. An experience has come to the writer's attention in which one of the inspirator type of aerators was installed for carbon dioxide removal before the lime treatment of a well supply. Red water troubles resulted, due to the presence of oxygen absorbed by the water. By-passing the inspirator type of aerator and using a simple splash type overcomes the red-water trouble in this plant.

In the moderate-sized softening plants chemicals, in general, are stored overhead. In the larger plants, elevated storage becomes costly and it is usually desirable to provide ground storage with only a limited overhead storage, transfers from ground to elevated storage being made at frequent intervals through dust-proof conveying equipment. Vacuum fan exhausters are frequently installed to remove dust from dry-feed machines during the charging operation.

Mr. Knox properly calls attention to the fact that mechanical mixing devices must be provided for water softening, and that the mixing period must be relatively long. Experience with stirring devices of both the horizontal-paddle, vertical-shaft type and the horizontal-shaft, flocculator type, leads to the opinion that the latter is much to be preferred. It has been found difficult to prevent the accumulation of sludge deposits where the vertical-shaft, horizontal-paddle type of equipment is used.

Where flocculators are used in softening, the peripheral velocity is ordinarily from 2 ft to 3 ft per sec, compared to from 1 ft to 1.5 ft per sec where alum is used for plain filtration.

Where softening is practiced in conjunction with the filtration of a sometimes turbid surface supply, provision for series operation is desirable. At times of high turbidity the first stage is used for the removal of turbidity with the second operated as single-stage softening treatment. Two-stage recarbonation is also being frequently adopted.

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[7] Cons. Engr. (Alvord, Burdick & Howson), Chicago, Ill.

As Mr. Knox states, sludge disposal is one of the big unsolved problems in water softening and on that account is probably one of the greatest deterrents to the adoption of lime-soda softening in some cities. It has been found practicable at some localities, by continuous removal of the sludge in small quantities, to discharge it into the sewer system. Although the volume of sludge is considerable the effect of the lime in sweetening the sewage and the sewage sludge is beneficial providing the quantity of sludge is not out of proportion to the sewage into which it is discharged.

In the smaller and moderate-sized softening plants the building heating system quite frequently consists of oil-fired or gas-fired boilers. With oil, two boilers are commonly provided—one of sufficient capacity to provide all the carbon dioxide requirements, and the other of sufficient capacity to furnish all the heat required. There have been a few installations in which two burners have been installed under only one boiler, but the difficulty of securing good combustion and of controlling the carbon dioxide content of the gases under widely varying demands is difficult to overcome.

Filtration rates as high as 3 gal per min per sq ft are commonly used in softening plants treating well water. With proper pre-treatment the sand should not require replacement for many years.

Mr. Knox is to be congratulated on having condensed his broad observations on water softening into such a concise, yet comprehensive, discussion.

Charles P. Hoover,[8] Esq. (by letter).—The lime-soda ash water-softening plants that have been built in the past were not designed to produce softened water of any predetermined alkalinity or pH-value. This is probably due to the fact that, to this time, no definite values have been agreed upon for the calcium carbonate alkalinity and the pH-value of the softened water.

What should they be? Results of experimental work, conducted during 1935 and 1936 at the Columbus, Ohio, Water Softening and Purification Works, lead to the conclusion that if the softened water is to be in contact with galvanized plumbing, the potential of hydrogen should range from about 7.8 to 8.2, and the calcium carbonate alkalinity should be sufficient to produce saturation at these pH-values. In other words, the water should be in chemical balance to calcium carbonate. A pH-value of 7.8 to 8.2 means a water free from normal carbonates.

Low-alkalinity water containing all, or nearly all, normal carbonates (especially in hot water) is more corrosive to galvanized metal than water containing no normal carbonates. Results of recent experiments indicate that normal carbonates (calcium and magnesium) deposit an uneven scale, in hot water tanks from $\frac{1}{32}$ in. to $\frac{1}{16}$ in. thick, in a year's time which is only somewhat adherent; whereas, the bicarbonates of calcium and magnesium, although they produce a thinner scale, produce one that adheres much better to the metal. The bicarbonates, therefore, produce a much better protective coating on galvanized surfaces than the normal carbonates.

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[8] Chemist in Chg., Water Softening and Purification Works, Columbus, Ohio.

When recarbonation of lime-softened water was first adopted and accepted as an essential part of water softening, about 1925, it was thought that its use would eliminate all filter-sand troubles, and all incrustation and corrosion problems in the distribution system.

Water-softening treatment, it is believed, can be made to eliminate distribution troubles, but thus far no practical means has been found to eliminate incrustation of filter sand entirely and, at the same time, produce the desired alkalinity. Tentative results from incomplete experiments indicate that the following procedure seems promising as a means for removing or crystallizing colloidal precipitates, and thus prevents them from crystallizing on, and coating, the sand grains:

(*a*) The addition of precipitated calcium carbonate, water-softening sludge, or other finely divided inert substances to the softened carbonated water, then stirring and settling before filtration;

(*b*) Blanket filtration of the softened carbonated water through precipitated calcium carbonate, water-softening sludge, or other finely divided inert material; and,

(*c*) If water is carbonated and then allowed to settle for a long period of time before being filtered, sand troubles will be eliminated, but the settling time must be measured in hours and days rather than in minutes.

The difficulty is that precipitates resulting from water-softening reactions are least soluble at a high pH-value (about 9.4) and if enough carbon dioxide is added to bring the potential hydrogen below this value more of the precipitate stays in solution, and colloidal precipitates are dissolved. The alkalinity or the carbonate hardness is thus increased beyond that desired. Therefore, the addition of carbon dioxide to softened settled water just ahead of the filters in sufficient quantity to stabilize it (so as to prevent incrustation and cementing together of the sand grains) cannot be practiced if it is desired to produce water of low alkalinity. This means that until a satisfactory procedure for removing colloidal precipitates is developed, sand filters will have to be resanded every ten or twelve years. Proper adjustment of the pH-value by the addition of carbon dioxide to lime-softened water should be made following filtration.

A careful study of Mr. Knox's paper by engineers who design water-softening plants should prevent them from repeating mistakes that have been made in the past. There are many items that might profitably be discussed; at least four of them should not be ignored:

(1) If a plant is designed to soften surface water that becomes extremely muddy during flood times, provisions should be made to coagulate and remove the bulk of the mud before the softening chemicals are added. The flow line would be to: (*a*) Add coagulant; (*b*) mix; (*c*) settle; (*d*) add softening re-agents; (*e*) mix; (*f*) settle; (*g*) carbonate; (*h*) filter; and, (*i*) adjust pH.

(2) Too much emphasis cannot be registered against the use of pressure filters for filtering lime-softened water. At a water-softening plant, it is

necessary to spade and break up the sand in the filter beds frequently, and it is almost impossible to do this where pressure filters are used.

(3) The importance of accurate chemical feeding devices decreases in almost direct proportion to the size of the plant; that is, the smaller the plant, the more necessity there is for accurate control. Automatic controls are especially desirable in small plants. Devices for weighing the softening reagents are preferable to measuring devices. The necessity for making provisions that will prevent arching of chemicals in hoppers is also very important.

(4) During the summer, there will be sweating from pipes and exposed concrete in plants treating cold well water. The difficulties from this source can be minimized by providing proper insulation and drainage; and it is especially important that the elevation of the filters be low enough that the level of the water in them may be kept below the elevation of the operating floor.

M. H. Klegerman,[9] Assoc. M. Am. Soc. C. E. (by letter).—Preliminary testing for the purpose of providing basic design factors is probably more readily possible in the field of water supply conditioning than in many other fields of engineering design. Such preliminary testing, or pilot-plant installations, may provide facilities varying from a simple laboratory jar-mixing set, to a model plant handling a substantial volume of water and equipped with diverse mechanical aids entering into the different steps in the treatment. The desirability of such procedure should be emphasized, if only but to confirm, in any particular instance, the suitability of accepted design factors for treatment of the water supply in question.

In a recent investigation for the treatment of the water supply of a Southern municipality, involving "excess lime" followed by recarbonation, a small pilot plant was installed which provided principally for: (*a*) Automatic chemical feed; (*b*) mechanical mixing with means for varying mixing speed and time of mixing; (*c*) "stage mixing" (that is, varying the speed of mixing from "fast" to "slow" as the flow proceeded from inlet to outlet of the mixing compartments); (*d*) settling, with provisions for varying the detention period; and (*e*) sand filters.

Without entering into a detailed discussion of the various factors derived from this investigation (which were principally of value to the particular water supply under consideration) it is of interest to illustrate at least one of the items noted which has application to the paper. That item pertains to the question of speed and time of mixing and time required for settling. The author has stated that where mechanical devices are used (see heading, "Lime-Soda Water Softening: Mixing Devices"), a 30-min to 40-min mixing time is desirable and that velocities provided by mixing devices should be between 1 and 1.5 ft per sec. As for the settling period, he states that a detention of 2 hr is generally sufficient to secure a satisfactory clarified effluent (see heading, "Lime-Soda Water Softening: Clarifiers").

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[9] Associate, Alexander Potter, New York, N. Y.

Although these factors are generally confirmed by the tests conducted by the writer, these tests, nevertheless, indicated that mixing speed, mixing time, and settling time, were not independent elements, but bore a relationship; that is, various combinations of these three elements were capable of producing equivalent results.

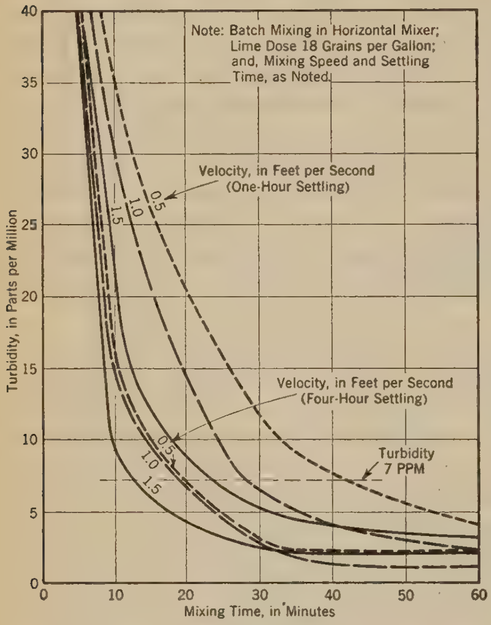

Fig. 1.—Relation Between Mixing Speed, Mixing Time, and Settling Time

In illustrating this relationship, the writer will follow the policy adopted by the author of avoiding discussion of the chemistry of the process, and will use "turbidity" which is an index of the clarity of the settling-tank effluent as a basis of comparison. (The water supply in these tests emanated from limestone springs; it was clear and colorless so that any turbidity present in the settled water resulted only from the reacting chemicals.)

Fig. 1 indicates the average results of a series of "batch" tests, made in a rectangular mixing tank provided with power-driven mixing paddles mounted on a horizontal shaft, mixing being at right angles to the direction of flow. Means were provided for mixing at three different speeds. (The speed of mixing refers to the paddle-tip speed.)

The test procedure involved filling the mixing tank, introducing the previously determined lime dose (the same for each test), then operating the mixing device at a speed of 0.5 ft per sec for one series of tests, 1.0 ft per sec for another, and 1.5 ft per sec for a third. Samples were dipped from the mixing tank at intervals of 10 min for each test series, thus providing samples representing 10-min mixing at a given speed, 20 min, 30-min, etc. Two samples were taken at each interval, one being permitted to settle in a jar for 1 hr and the duplicate sample for 4 hr.

As will be noted from the curves, the same "end-point" as determined by turbidity measurements was obtainable with a number of variations. For example, assuming that the desired turbidity of the clarified effluent is, say, 7 ppm, the combinations in Table 1, in so far as this water supply is concerned, were found possible. These data indicate that, in general, for the same speed of mixing, cutting the mixing time in half requires quadrupling the detention time in the settling basins for equivalent clarity of effluent (compare Series *a* and *d* and Series *c* and *f*). It will also be noted that substantial reduction in time of mixing is possible at increased mixing speeds.

For example, the same result is obtained with 24-min mixing, at a speed of 1.5 ft per sec and 1 hr settling, as is obtained with equivalent mixing time (that is, 20 min) with only a speed of 0.5 ft per sec but with 4-hr settling (compare Series *c* and *d*). The cost of structures would be far greater in the latter case (Series *d*) than in the former (Series *c*). Similarly, sludge removal equipment would be more costly in the tanks providing for longer detention. Mixing equipment costs would be about the same, except that the drive units for the faster speed of mixing (Series *c*) would have to be powered substantially higher than that of the lower speed (Series *d*). Likewise, power consumption in the faster mix would obviously be much greater than that in the slower, the power requirements varying as the cube of the mixing speed. Thus, selection of final factors involves economic consideration.

TABLE 1.—COMBINATIONS OF MIXING SPEED, MIXING TIME, AND SETTLING TIME TO PRODUCE A TANK EFFLUENT OF IDENTICAL TURBIDITY

(For This Illustration, Seven Parts per Million Was Selected as the Turbidity)

Speed of mixing, in feet per second	(*a*) SETTLING TIME, 1 HOUR		(*b*) SETTLING TIME, 4 HOURS	
	Series	Time of mixing, in minutes	Series	Time of mixing, in minutes
0.5	*a*	42	*d*	20
1.0	*b*	28	*e*	19
1.5	*c*	24	*f*	13

It is the writer's opinion that most conditions involving mixing of chemicals with water for coagulation warrant the provision of variable speed devices, particularly in view of the changing characteristics of the water in many instances, requiring, in turn, varying dosages of lime, soda ash, etc., for softening. The ease with which most mechanical mixers may so be equipped merits their adoption over baffling and hydraulic methods not readily controllable.

The author has given the profession a valuable "bird's-eye view" of standard practice in the field of water-softening plant design. It would be of additional value if data could be presented relative to filter-washing practice in which lime or lime and soda ash are the treating chemicals. Does incrustation of filter sand with lime warrant differences in filter-washing provisions not required in ordinary rapid sand-filter practice?

ROLLIN F. MACDOWELL,[10] M. AM. SOC. C. E. (by letter).—Very concisely, Mr. Knox has covered the main considerations in the selection of the type and the design of municipal water-softening plants. The writer offers a few additional suggestions from his experience in this field.

Municipal water softening is a relatively new field in public service, but has already proved to be practical and economically sound. It is no longer considered sufficient to provide only a clean water, free from pathogenic bacteria, and thus the number of water-softening plants constructed is accelerating rapidly. As of January 1, 1938, there were 362 municipal plants installed, or under construction, in the United States, of which Ohio had 90, or approximately 25 per cent.

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[10] Cons. San. Engr., Cleveland, Ohio.

Mr. Knox states (see "Sources of Supply") that, if the hardness is more than 700 to 800 ppm, another source of water supply should be sought. Although this is true from a standpoint of cost, it does not mean that it is impractical to soften water containing 800 ppm, or more, of hardness. It is sometimes impossible to find another and softer source of supply. At Elmore, Ohio, the hardness of the raw water is about 1 060 ppm. This supply is softened to 70 ppm without difficulty and a satisfactory, palatable water is produced. The cost has not proved to be a burden. In fact, it was not necessary to raise the water rates in order to finance the cost of operation.

Aerators.—The writer agrees that it is not necessary to provide patented aerators in order to secure satisfactory results. Furthermore, non-patented aerators can be designed so as to be sightly. The patented aerators are usually entirely or partly of metal construction, which corrodes rapidly under the operating conditions that are usually most favorable to oxidation, thus causing the metal to corrode. Coke aerators can be made of concrete and cypress wood construction which is more permanent and requires less maintenance. Incidentally, if aerators are provided following softening, aluminum should not be used in the construction as the soft water pits this metal badly.

It is certainly true that sludge disposal "is one of the chief difficulties" of lime-soda water softening. This is especially the case in those municipalities, usually small, which are not on or near a sizeable stream into which the sludge can be discharged without sight nuisance. In such cases, and because of the sludge problem alone, the zeolite method of softening is usually advisable.

Type of Plant.—For every proposed plant careful consideration should be given by the designing engineer to all the factors involved in the determination of the type of plant to be provided. There are some factors, such as the sludge problem with lime-soda plants, which may make zeolite advisable aside from other considerations. On the other hand, if the raw water is from a surface source, or otherwise subject to pollution, or if the raw water has an objectionable fluoride content, the lime-soda method must be utilized; but, usually, the problem is largely an economic one, including first cost as well as cost of operation.

With the small plants having capacities as great as, say, 0.5 mgd, the saving in labor and the cost of technical control in the operation of zeolite plants quite often make this method advisable. This is especially true if automatic regeneration can be provided, thus eliminating the necessity for regular and frequent attention to the plant.

Troughs and Piping.—In gravity zeolite plants, because of the highly corrosive character of the salt brine used for regeneration, special attention should be given to the materials used for troughs and piping. For this reason, steel troughs are unsatisfactory, but concrete and asbestos cement have been used with success. For the collector and brine distribution systems, cast-iron pipe or material containing chromium and nickel is satisfactory.

Control Equipment.—With both gravity and pressure zeolite plants certain control equipment is found desirable. Automatic proportioning of the by-passed hard water is worth while, especially if the hardness of the raw water varies or two sources of raw water, having different degrees of hardness, are used.

With pressure zeolite plants in which the water is pumped from the source of supply or from a collecting basin directly through the pressure softeners, or through pressure filters and softeners, to the distribution system and elevated storage, provisions should be made to eliminate the surge in the pipe lines due to rapid starting of the pumps. For this purpose a slow opening discharge gate-valve, of the hydraulic type, operated electrically through the pump starter, is found to be effective.

Salt Storage.—The most economical method of purchasing and handling salt for regeneration is in bulk carload lots. A minimum carload is 40 tons. The wet salt storage basin, therefore, should have a capacity of at least 40 tons of dry salt, with ample free-board for float-valve control of the feed-water. Preferably, however, the basin should be divided into two compartments, each of which holds 40 tons of salt, in order that one compartment may be filled at any convenient time while the other is in service.

Brine pumps, which are usually of the centrifugal type, should be entirely brass-lined and mounted. If they cannot be located conveniently as to elevation to provide a positive head on the suction side, satisfactory self-priming units can be secured.

D. E. Davis,[11] M. Am. Soc. C. E. (by letter).—This paper will repay the careful study of those contemplating the construction of a softening plant. It brings into sharp focus a long first-hand experience of the essential design features that should be incorporated if success is to be attained. Communities have been slow to recognize that softening plants are a good investment; but at chemical costs of 1 cent per 1 000 gal per 100 ppm of hardness removed, as quoted by the author, combined with reasonable fixed charges, it is usually found that soap savings alone more than justify the expenditure for a plant.

With good reason, the author stresses the desirability of short, direct lines for conveying chemicals to the mixers, with adequate provisions for cleaning, since lime in particular tends to build up a dense coating. No plant feature is more important than the mixing chamber, preferably of the mechanical type in which the raw water and the chemicals are brought into intimate contact at a speed of rotation less than that which will break the floc, but fast enough to permit the building up of a large aggregate which will quickly settle in the clarifiers. The flow of water may well be in an upward direction, thus maintaining the floc longer in suspension while it is being built.

Perforated baffles at the entrance and exit contribute to even flow through settling basins, and provide the "fly-wheel" or "cushion" of storage for meeting fluctuations in softener rates, the advisability of which is stressed by the author. The combination of a clarifier followed by a settling basin should give better and more flexible operation than a clarifier alone.

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[11] Engr. (The Chester Engrs.), Pittsburgh, Pa.

The disposal of the sludge, as the author shows, is one of the distinct limitations to the use of the lime-soda softening process under certain conditions, and more positive methods of meeting this situation may well be one of the design features in future plants.

Coke used for generating carbon dioxide for recarbonation should be low in sulfur content in order to prevent corrosion in compressors and lines. The author's suggestions relative to the design of piping which conveys water with a normal carbonate content is well taken, since it will usually be necessary to clean the deposit from these lines at intervals of several years.

The most careful study of the hydraulics of a zeolite plant is essential if head losses and sand rise are to be properly handled. The relationships between varying hardness, sand volumes, storage capacities, and wash and rinse waters require most careful study if the design is to prove successful.

In listing the disadvantages of zeolite, the author states that "objectionable proportions of iron and manganese are not suitable for zeolite treatment unless these minerals are removed prior to softening." This scarcely holds true for manganese in quantities as great as 7.0 ppm, and averaging 5.0 ppm for several years, because zeolite has been shown capable of removing these quantities continuously by base exchange and without damaging the green sand. However, manganese is not removed beyond the end of the period of zero hard water, but appears immediately with the first sign of exhaustion of the zeolite.

Professor W. F. Langlier has shown that changes in the calcium content of a water may have a material effect on its corrosive qualities.[12] Since in zeolite softening, calcium is removed or reduced, the resulting soft water is likely to be corrosive. This is sometimes offset by an increase in pH which accompanies the action of some zeolites.

When dealing with a pure, clean, but hard water, a zeolite plant offers many advantages, including low first cost and "fool-proofness" in operation. The choice of type of softening plant for a given situation can only be determined after a careful examination and weighing of the various determining factors.

W. H. Knox,[13] M. Am. Soc. C. E. (by letter).—The seven discussions do not disagree with, or contradict, the writer's statements. Mainly, they amplify various items mentioned briefly in the paper, and thus give valuable additional information.

In an article of restricted length on such a broad subject only the salient features could be included. A textbook would have to be written to discuss details of all features of a water softening plant. Hence discussions could be continued indefinitely, since the design of such plants is changing continuously. Considerable time has elapsed since the paper was prepared and some minor items would be changed due to progress in the art.

The number of water softening plants is rapidly increasing, especially in the Middle West. If the present paper has helped in the understanding of the problems involved it will have served a useful purpose.

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[12] "The Analytical Control of Anti-Corrosion Treatment," by W. F. Langlier, *Journal*, Am. Water Works Assoc., Vol. 28, No. 10, p. 1500.

[13] Asst. Engr., State Dept. of Health, Columbus, Ohio.

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Paper No. 2042

AERODYNAMICS OF THE PERISPHERE AND TRYLON AT WORLD'S FAIR

BY ALEXANDER KLEMIN,[1] ESQ., EVERETT B. SCHAEFER,[2] ESQ., AND J. G. BEERER, JR.,[3] ESQ.

WITH DISCUSSION BY MESSRS. ELLIOTT G. REID, KARL ARNSTEIN, AND W. B. KLEMPERER.

SYNOPSIS

There can be no better way of beginning this paper than to quote the following recommendation from the pamphlet, entitled "Standards of Design for Structural Steel," (1)[4] published by the United States Navy Department:

> "For structures of unusual character or magnitude where the wind is an important factor, the actual distribution and intensity of positive and negative wind pressure should be investigated, preferably by wind tunnel tests of a scale model, and the stresses calculated on that basis should be used if they exceed those determined from the formula given herein."

Certainly the Perisphere and the Trylon, the "theme" buildings at the New York World's Fair, come under the designation of structures of unusual character and magnitude—since the Perisphere is a hollow spherical building with a diameter of 200 ft, and the Trylon is a tapered triangular prism-like object, 675 ft high. The group, with a variety of supports for the sphere collars and columns, is shown in Fig. 1. The difficulty in estimating the wind loads, and especially their distribution, on these two buildings was all the greater since they are in close proximity to one another. Because of mutual interference they must be considered as a single aerodynamic unit, since no prior experimentation was available as to the aerodynamic effects on a sphere close to the ground, and since, in general, the contemplated project is without parallel in structural history.

Accordingly, the senior writer immediately suggested the advisability of a systematic wind-tunnel investigation. The paper is an account of these experiments, accompanied by theoretical considerations.

NOTE.—Published in May, 1938, *Proceedings*.

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[4] Numerals in parentheses, thus (1), refer to items in the Appendix, List of References.

The experimental results constitute an addition to the meager information available on the aerodynamics of buildings, and may find application in the design of other structures, such as circular gas tanks, triangular towers, etc. In general, the technique of the investigation should be of interest to structural designers who have to deal with wind loads.

From a theoretical point of view the main interest of the paper lies in the mathematical study of the air flow around a sphere when in proximity with the ground, and the comparison of the calculated results with those obtained experimentally.

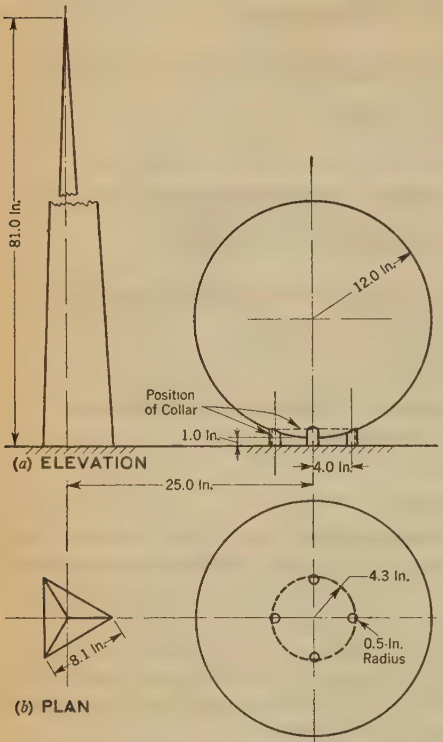


FIG. 1.—MODELS OF PERISPHERE AND TRYLON (MODEL SCALE, 1 : 100)

THEORETICAL CONSIDERATIONS

In examining the results of the tests of pressure distribution over the Perisphere, a useful check is provided by theoretical calculations of the potential flow past a sphere in free air, and of the potential flow past a sphere close to the ground. There is also scientific interest in comparing the actual flow with the potential flow.

Flow Past a Sphere in Free Air.—For a sphere moving in free air with Velocity V, the velocity potential is given by,

$$\phi = -\frac{V a^3 \cos \theta}{2 r^2} \qquad (1)$$

in which, a = radius of the sphere; r = distance from the origin to any point; and θ = angle between Vector r (Fig. 2) and the direction of motion of the sphere. If the sphere is at rest, and the air is flowing past it with Velocity V, the velocity potential becomes,

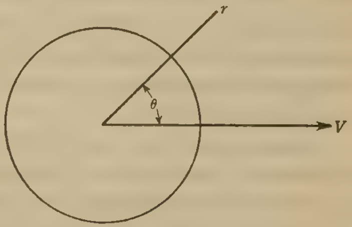


FIG. 2

$$\phi = -\frac{V a^3 \cos \theta}{2 r^2} - V r \cos \theta \qquad (2)$$

The velocity at right angles to Vector r is, therefore,

$$V_n = \frac{\partial \phi}{r\,\partial \theta} = \left[\frac{V\,a^3}{2\,r^3} + V\,r\right]\sin\theta \quad \ldots\ldots\ldots\ldots\ldots (3)$$

and over the surface of the sphere, the tangential velocity over any great circle whose diameter is along the direction of the general flow is,

$$V_t = \frac{3}{2}\,V\sin\theta \quad \ldots\ldots\ldots\ldots\ldots (4)$$

with a maximum velocity of $\frac{3}{2}\,V$; in which, $\theta = 90$ degrees.

If p_∞ is the static pressure at infinity, where the flow is undisturbed by the presence of the sphere, and p_n is the normal pressure at any point on the sphere, then by Bernoulli's equation,

$$p_\infty + \rho\,\frac{V^2}{2} = p_n + \frac{V_n^2}{2} \ldots (5)$$

and,

$$p_n - p_\infty = \rho\,\frac{V^2}{2} - \rho\,\frac{V_n^2}{2}$$

$$= \rho\,\frac{V^2}{2}\left[1 - \left(\frac{3}{2}\sin\theta\right)^2\right] \ldots (6)$$

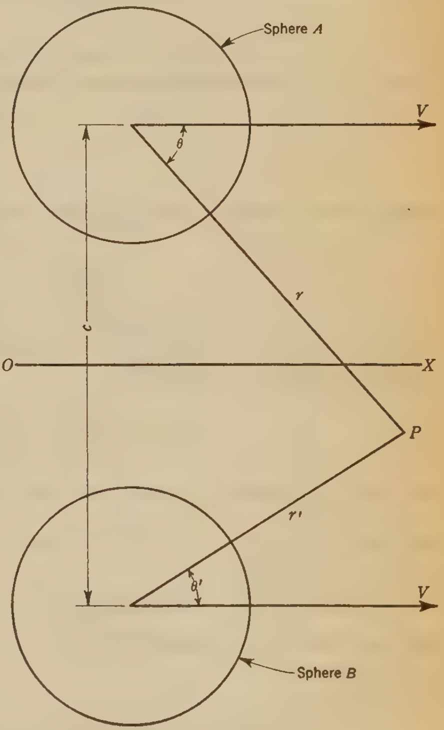


FIG. 3

Two Spheres of Equal Diameter Moving with Equal Velocity Along Parallel Lines.—In this case (see Fig. 3) it is clear that there will be no flow across the horizontal plane whose trace is the axis, $O\,X$. Therefore, the flow will be the same as if Sphere A or Sphere B were moving with the same velocity, V, in the presence of the boundary or ground, $O\,X$. To find the velocity potential in this case is the best introduction, therefore, to the problem of the flow past a sphere in the presence of the ground. In finding this velocity potential, an approximate treatment must be adopted because of the mathematical difficulties involved.

Thus, if the velocity potential, ϕ_1, for Sphere A, moving with Velocity V, is expressed by Equation (1), then near Sphere B (see Fig. 2):

$$\phi_1 = -\frac{V}{2}\,\frac{a^3\,r\cos\theta}{r^3} = -\frac{V}{2}\,a^3\,\frac{r'\cos\theta'}{r^3} = \frac{V}{2}\,\frac{a^3\,r'\cos\theta'}{c^3} \quad \ldots\ldots\ldots (7)$$

since $r \cos \theta = r' \cos \theta'$. The normal velocity over Sphere B is,

$$V_n = \frac{\partial \phi'}{\partial r'} = -\frac{V}{2}\frac{a^3 \cos \theta'}{c^3} \dots\dots\dots\dots\dots\dots\dots (8)$$

that is, into Sphere B. Since there can be no velocity normal to a solid body, a space doublet is applied at the center of Sphere B having a velocity potential,

$$-\left[\frac{V}{2}\frac{a^3}{c^3}\right]\frac{a^3}{2(r')^2} \cos \theta' = \phi_2 \dots\dots\dots\dots\dots\dots\dots (9)$$

which induces a velocity,

$$\frac{-\partial\left(\frac{V}{2}\frac{a^3}{c^3}\frac{a^3}{2\,(r')^2}\cos\theta'\right)}{\partial r} = \frac{V}{2}\frac{a^3 \cos \theta'}{c^3} \dots\dots\dots\dots (10)$$

in which $r = a$; thus canceling the normal velocity across the surface of Sphere B. The value of ϕ_2 near Sphere A is (approximately):

$$\phi_2 = -\frac{V}{4}\frac{a^6}{c^3\,(r')^2}\cos\theta' = -\frac{V}{4}\frac{a^6\,r' \cos\theta'}{c^3(r')^3} = -\frac{V}{4}\frac{a^6\,r \cos\theta}{c^6} \dots (11)$$

which gives a normal velocity over the surface of Sphere A equal to,

$$V_n = -\frac{V}{4}\frac{a^6 \cos\theta}{c^6} \dots\dots\dots\dots\dots\dots\dots\dots (12)$$

It can be readily seen that by applying a space doublet at the center of Sphere A having a potential,

$$\phi_3 = -\left(\frac{V}{4}\frac{a^6}{c^6}\right)\frac{a^3}{r^2}\cos\theta \dots\dots\dots\dots\dots\dots\dots (13)$$

the normal velocity expressed by Equation (12) will be canceled. Although this again introduces a velocity normal to Sphere B $\left(\text{since } c > a; \text{ and } \frac{a^6}{c^6}\right.$ $\left.\text{is very small}\right)$, the approximation need be carried no further, and one can say that, due to the motion of Sphere A, the velocity potential near it is represented by,

$$\phi_4 = -\frac{V}{2\,r^2}a^3\cos\theta - \frac{V}{4}\frac{a^6\,r\cos\theta}{c^6} - \frac{V}{8}\frac{a^6}{c^6}\frac{a^3}{r^2}\cos\theta \dots\dots\dots (14)$$

and also that, due to the motion of Sphere A, the velocity potential near Sphere B may be represented with sufficient approximation by,

$$\phi_5 = -\frac{V}{2}\frac{a^3\,r'\cos\theta'}{c^3} - \frac{V}{4}\frac{a^6}{c^3\,2\,(r')^2}\cos\theta' \dots\dots\dots\dots (15)$$

Hence, the velocity potential near Sphere A due to the combined motion of both spheres must be, by considerations of symmetry,

$$\phi_6 = -\frac{V}{2\,r^2}a^3\cos\theta - \frac{V}{4}\frac{a^6\,r\cos\theta}{c^6} - \frac{V}{8}\frac{a^6\,a^3}{c^6\,r^2}\cos\theta - \frac{V}{2}\frac{a^3\,r\cos\theta}{c^3} - \frac{V}{4}\frac{a^6}{c^3}\frac{\cos\theta}{r^2} \qquad (16)$$

Air Flow Past a Sphere in Presence of the Ground.—To bring the air into motion and the spheres to rest, it is now only necessary to add the term $-V\,r\cos\theta$ to Equation (16) and the complete expression for ϕ_7 is the result. When $r = a$,

$$\phi_7 = -\frac{V}{2}a\cos\theta - \frac{V}{4}\frac{a^6}{c^6}a\cos\theta - \frac{V}{8}\frac{a^7}{c^6}\cos\theta - \frac{V}{2}\frac{a^4\cos\theta}{c^3} - \frac{V}{4}\frac{a^4}{c^3}\cos\theta - V\,a\cos\theta \qquad (17)$$

and the tangential velocity is:

$$\frac{\partial\phi}{a\,\partial\theta} = V\sin\theta\left[\frac{1}{2} + \frac{1}{4}\frac{a^6}{c^6} + \frac{1}{8}\frac{a^6}{c^6} + \frac{1}{2}\frac{a^3}{c^3} + \frac{1}{4}\frac{a^3}{c^3} + 1\right]$$
$$= \frac{3}{2}V\sin\theta\left[1 + \frac{1}{2}\frac{a^3}{c^3} + \frac{1}{4}\frac{a^6}{c^6}\right] \qquad (18)$$

For maximum velocity $\theta = 90°$ in Equation (18), and $\sin\theta = 1$. It is seen that the tangential velocity is greater for the sphere in the presence of the ground, than for the sphere in free air. Another interesting phenomenon is that the velocity, theoretically, is dependent only on the value of θ, and is the same at symmetrical points above and below the sphere, when in the presence of the ground, and that the stagnation points are not changed by the ground board. Furthermore, the velocity will be greater as the distance, c, between the centers becomes smaller.

The pressure at any point can be determined from the formula,

$$p_n - p_\infty = \rho\frac{V^2}{2}\left\{1 - \left[\frac{3}{2}\sin\theta\left(1 + \frac{1}{2}\frac{a^3}{c^3} + \frac{1}{4}\frac{a^6}{c^6}\right)\right]^2\right\} \qquad (19)$$

Points at Which the Tangential Velocity Is Equal to That in the Undisturbed Air Stream.—In the examination of test results, it is of some interest to determine where the tangential velocity is equal to the velocity of the undisturbed air stream. This occurs when $p_n - p_\infty$, as expressed by Equation (6), equals zero; that is, when $\theta = 41.7$ degrees. For the sphere in proximity with the ground, substituting $[1 + K]$ for $\left(1 + \frac{1}{2}\frac{a^3}{c^3} + \frac{1}{4}\frac{a^6}{c^6}\right)$ in Equation (19), the value of θ will be given by,

$$1 - \left[\frac{3}{2}\sin\theta\,(1 + K)\right]^2 = 0 \qquad (20)$$

so that θ will have a smaller value than for the sphere alone.

EXPERIMENTAL METHODS

Model Dimensions and Reynolds Number of the Tests.—The wind-tunnel models of the Perisphere and Trylon were built to a scale of 1 : 100, with only external appearance represented. The main model dimensions were: Sphere diameter, 2.00 ft; height above ground board, 0.08 ft; Trylon height, 6.75 ft; and, Trylon width at the base, 0.68 ft.

In aerodynamic experimentation, scale effects (that is, the difference between air flow over the model and the air flow over the larger full-scale object) must always be considered. The effective scale of the model is represented by the Reynolds number, **R**, which in the case of the sphere assumes the form,

$$\mathbf{R} = \frac{V\,D}{\nu} \qquad \text{(21)}$$

in which V = velocity of the air stream, in feet per second; D = diameter of the sphere; and, ν = coefficient of kinematic viscosity.

The resistance coefficient of a sphere is conveniently represented by the equation,

$$C_D = \frac{R}{\left(\frac{\rho\,V^2}{2}\right)\frac{\pi\,D^2}{4}} \qquad \text{(22)}$$

in which R = resistance of the sphere, in pounds; ρ = density of the air $= \frac{\text{specific weight per cubic foot}}{g}$; and, D = diameter of the sphere, in feet.

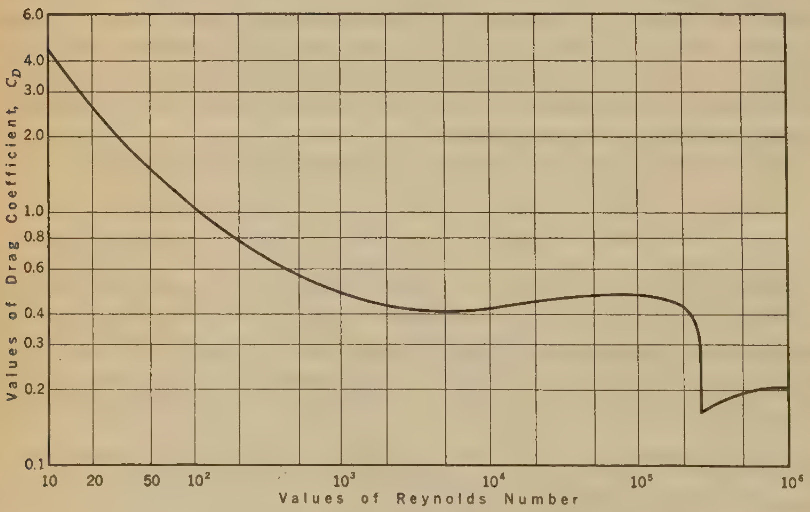


FIG. 4.—VARIATION IN SPHERE DRAG COEFFICIENT WITH REYNOLDS NUMBER

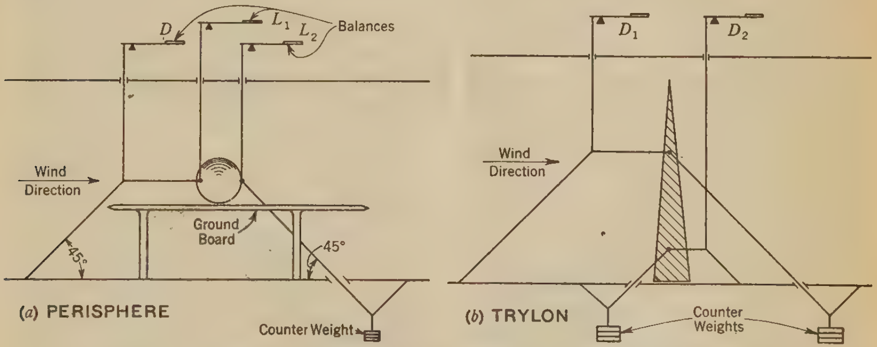
In Fig. 4 the values of the drag coefficients are plotted against the Reynolds number, the curve being obtained from prior tests. It will be seen that, although there is considerable variation in C_D at the lower Reynolds number, the tests at the higher values of **R** are definitely in an asymptotic region. This indicates that the critical region in which the character of the flow changes has been passed, and that the pressure distribution tests, made at an effective Reynolds number of 1 833 880, were definitely in the region where the flow has assumed the character of complete turbulence in the boundary layer. This indicates that the results of the pressure distribution tests may be applied with confidence to full-scale conditions.

It was only possible to make the force tests on the sphere up to a maximum of 30 miles per hr, but in the pressure distribution tests, where it was possible to brace more securely, the indicated air speed of the tunnel was 70 miles per hr, giving a Reynolds number, $\frac{V\,D}{\nu}$, equal to $6\,378\,(70 \times 1.467) \times 2 = 1\,390\,914$.

The "Effective Reynolds Number" is the best basis of comparison between free air and wind-tunnel results since it includes the effect of the tunnel turbulence, and is defined as the actual Reynolds number multiplied by the turbulence factor which, in this case, equals $1\,390\,914 \times 1.4 = 1\,833\,880$ since the turbulence factor of the 9-ft wind tunnel was 1.4.

Because of the large forces encountered, the drag measurements of the needle were made at lower air speeds than the pressure distribution tests on the sphere, namely 40, 50, and 59.4 miles per hr. Since the needle is not a stream-lined body, its resistance does not change appreciably with the Reynolds number and, therefore, the results of the tests are also applicable to full scale without correction.

Methods of Measuring Air Forces.—The methods of measuring air forces were entirely conventional and are illustrated by Fig. 5. Fig. 5(*a*) shows the sphere suspended in the presence of the "ground board" which was used to simulate the ground effect experienced by the Perisphere. Force tests for the sphere were also made on the same set-up with the ground board removed, to simulate "free air" conditions. With the arrangement shown in Fig. 5(*a*), the sum of the readings of Balances L_1 and L_2 gives the vertical or lift force on the sphere. A sufficiently heavy counterweight is used as shown, so that the wires leading to the lift balances are always in tension. The front wire takes



(*a*) PERISPHERE (*b*) TRYLON

FIG. 5.—SCHEMATIC DIAGRAM OF SUSPENSION SYSTEM FOR FORCE TESTS

up all the drag, and since it is inclined at 45° to the horizontal, the vertical wire running to the front balance, *D*, transmits a force equal to the drag or air resistance. It is clear also that with this arrangement moments can be computed readily about any point. "Tare" measurements and corrections are made without the wind stream, in obvious fashion.

Drag tests of the Trylon, or needle, were run with the needle in the presence, but just free, of the ground, with the tunnel floor representing the ground The special mounting of the Trylon during the drag tests is illustrated diagrammatically in Fig. 5(*b*).

The vertical center of pressure (point of application of resultant drag force) is determined by obvious calculation when the forces in the two drag wires and their points of application are known.

Measurement of Pressure Distribution on Sphere, (2), (3).—The pressure distribution was measured on one hemisphere only as it may be assumed that the air flow is symmetrical about a plane through the center of the sphere, normal to the ground line and parallel to the direction of the air stream. This system was particularly convenient inasmuch as it permitted one-half the sphere to be free of pressure orifices, thus leaving a clear space for the exit of the pressure tubes, and avoiding flow disturbing influences.

The pressures on the surface of the hemisphere were measured at thirty-seven points disposed as follows (see Fig. 6): All pressure holes lay on the surface of the sphere along the traces of great circles cut by planes passed through the center of the sphere. The "meridian," or VI-XII, circle was the trace of a vertical plane passed through the center and two poles. When the

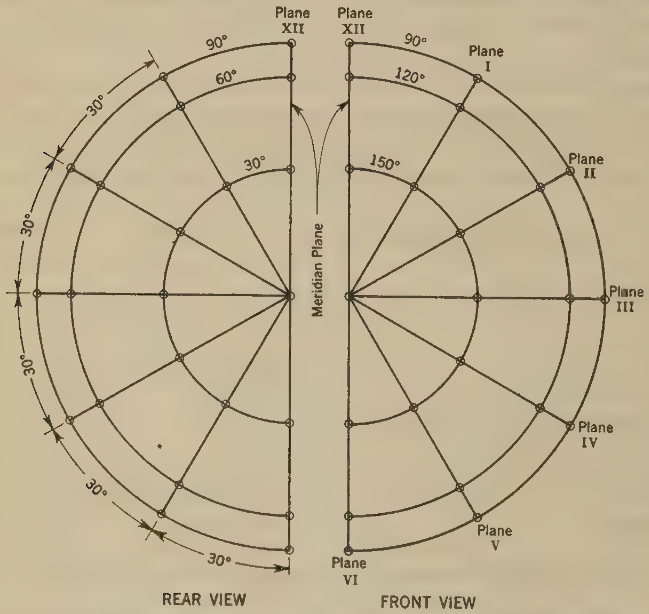


Fig. 6.—Distribution of Pressure Holes Over Surface of Model Sphere

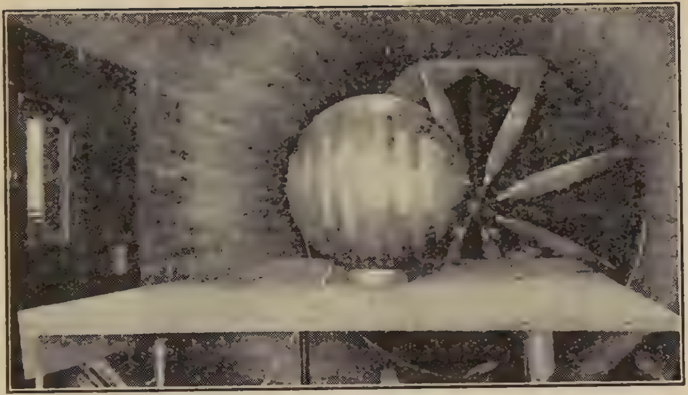


FIG. 7.—GENERAL ARRANGEMENT OF MODEL IN TUNNEL

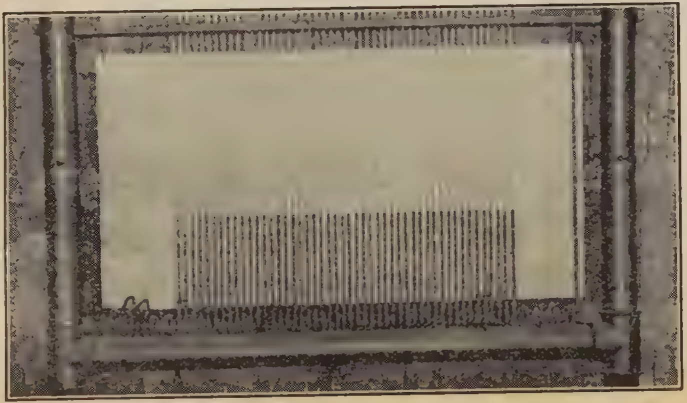


FIG. 8.—MULTIPLE MANOMETER FOR PRESSURE MEASUREMENT

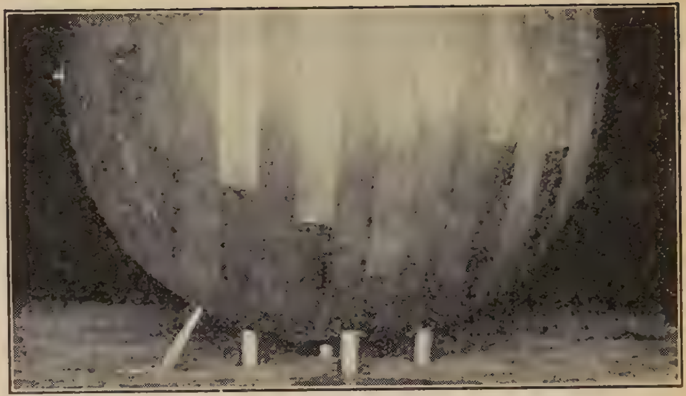


FIG. 9.—SPHERE ON COLUMNS

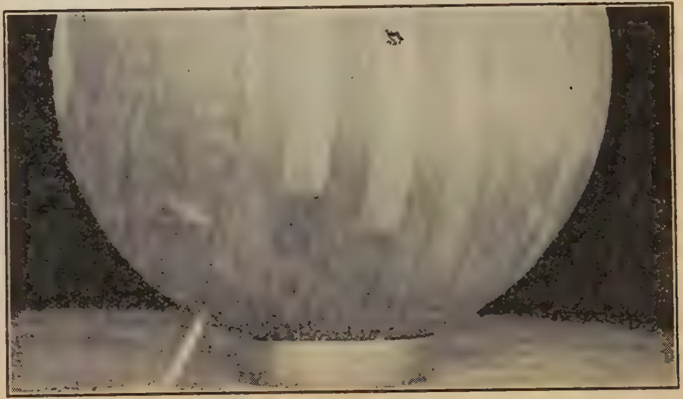


FIG. 10.—SPHERE ON COLLAR

sphere was mounted in the tunnel this "meridian" plane was coincident with the vertical plane of symmetry of the tunnel. The planes designated as Planes I, II, III, IV, and V were all at 30° intervals from the "meridian" plane. The pressure holes on each great circle were placed at 30° intervals, with two points common to the "meridian," or VI-XII, plane and to all other planes. This system of defining planes and points was selected because theoretical calculations are made conveniently for points thus arranged on the surface of a sphere.

The pressure orifices were small brass tubes, set flush with the sphere's surface, gathered together in the hollow interior, and led out through a single hole on the far side of the surface that did not contain pressure holes.

The leads from the pressure orifices were connected to the multiple-tube manometer with a common fluid reservoir. There were several additional pressure leads connected into the manometer which were used to measure the static pressure of the undisturbed air stream in the test chamber in order to have a basic reference for the calculation of the absolute pressures on the sphere surface. The pressures of the undisturbed air stream were measured on the sides of the channel well ahead of the position of the sphere. A permanent record of the fluid-pressure heads indicated by the colored liquid in the glass manometer tubes was obtained by means of positive printing paper placed behind the tubes and exposed to a carbon arc light. The use of this procedure made it quite a simple matter to run a series of pressure tests, take the permanent records of each, and, later, make the conversions to the desired pressure units.

Fig. 7 shows the general arrangement of the model in the wind tunnel; Fig. 8, the multiple manometer set-up outside the tunnel; Fig. 9, the model on the four-column supports; and Fig. 10, the model on the collar support. Pressure distribution tests were made with the sphere suspended without the ground board (to simulate what may be called "free air conditions"). Subsequently, the sphere was tested in the presence of the ground board; in other runs, it was tested for pressure distribution in the presence of the ground board and the Trylon combined.

FORCE TESTS ON TRYLON

Drag Tests on the Trylon.—The tests on the Trylon were made in two attitudes relative to the wind direction (see Fig. 11): (1) With one of the pointed edges into the wind; and (2) with one of the flat sides into the wind. It was realized, of course, that the greatest air resistance would occur with a flat side into the wind, but the condition of a pointed edge into the wind was deemed worthy of investigation. The results of the drag tests on the Trylon may best be expressed in the form of coefficients of drag, as exemplified by the American absolute drag coefficient, C_D, of aeronautical practice, which is defined similarly to Equation (22) for the sphere, namely,

$$C_D = \frac{D}{\left(\rho \frac{V^2}{2}\right) S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which S = the area of a flat side projected on a vertical plane, in square feet. Although the drag tests of the Trylon were run at several tunnel air speeds, the coefficients for all speeds were sufficiently close to permit an average value to be chosen for each of the two attitudes tested. This coincidence of values had been expected on theoretical grounds. The values are:

Attitude of Trylon	Drag Coefficient, C_D
Pointed edge into wind	0.879
Flat side into wind	1.433

They substantiated the anticipation that the Trylon with the flat side into the wind would show the greater drag, since the body with an edge into the wind is really of crudely semi-streamlined form, whereas with the flat side into the wind the flat-plate effect is predominant. The accepted coefficient for the flat-plate drag is C_D = 1.28, as used in aeronautical engineering practice. The value obtained for the Trylon with the flat side into the wind is C_D = 1.433, about 12% greater than for the flat plate alone. The difference between the two may be justified in view of the fact that the Trylon, being a solid of three dimensions, presents some surface area to the wind stream, so that, in addition to the flat-plate drag effect, there is a drag due to the surface areas of the body, with the probability of turbulent flow on the two down-stream sides.

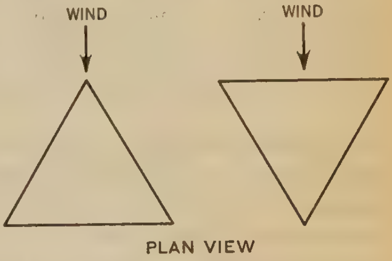


Fig. 11.—Two Positions of Trylon Relative to Wind

The center of pressure position of the drag forces acting on the Trylon, with a flat side into the wind, was shown to be at an average value of 28.50 in. from the base and along the vertical axis. This point too seems logical since it is within 1.5 in. of the centroid of area of the vertically projected triangle of a flat side of the Trylon.

The full-scale air resistance or drag and upsetting moment about the base of the Trylon may be computed easily for any given air speed by substitution in Equation (23) for the solution of force, and the moment by the usual formula, in which the center of pressure position shown by the tests would be used as the moment arm. If it is assumed that the air speed is 100 miles per hr, the value, C_D = 825 000 lb, would be obtained for the full-scale body with flat side into the wind, and an area projected on the vertical of 22 500 sq ft.

The wind load at 100 miles per hr is thus seen to be 36.6 lb per sq ft of projected area. For buildings higher than 100 ft the Building Code ordinances of New York, N. Y., call for an assumed wind pressure of 20 lb per sq ft of exposed surface from the top of the building down to the 100-ft level. If the flat-plate coefficient of 1.28 is used, this is equivalent to 78 miles per hr. The results of the tests on the Trylon, computed for a wind of 78 miles per hr, would show a loading of about 22.3 lb per sq ft for the entire length of the projected area.

FORCE TESTS ON THE PERISPHERE

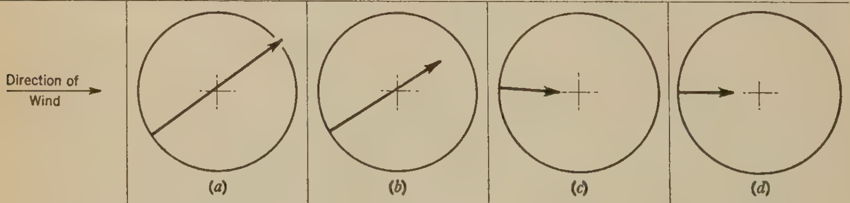
Drag Tests in Free Air.—The coefficients of drag, C_D, for the free air test (that is, without the presence of the ground) are given in Table 1(*a*). These values agree with results obtained in other laboratories, and they indicate the attainment of an asymptotic condition of flow which is not greatly altered by transposition to full scale. No transverse or lift force is found, of course, under this condition of test.

TABLE 1.—FORCE TESTS ON PERISPHERE

Air speed, in miles per hour	(*a*) IN FREE AIR		(*b*) WITH COLLAR AND GROUND BOARD			(*c*) WITH COLUMNS AND GROUND BOARD		
	Drag, *D*, in pounds (1)	Coefficient, C_D (2)	Drag, *D*, in pounds (3)	Lift, *L*, in pounds (4)	Coefficient, C_D (5)	Drag, *D*, in pounds (6)	Lift, *L*, in pounds (7)	Coefficient, C_D (8)
15	0.305	0.169	1.040	0.73	0.573	0.885	0.45	0.489
20	0.620	0.193	1.875	1.34	0.583	1.560	0.92	0.485
25	0.930	0.185	2.880	2.07	0.573	2.455	1.47	0.488
30	1.445	0.200	4.140	2.98	0.573	3.500	2.14	0.485

Drag and Lift or Transverse Force Tests in the Presence of Ground Board and Collar, and of Ground Board and Columns.—In these tests the sphere was supported by wires from the balances in the manner described under the heading, "Experimental Methods," with just sufficient clearance between the sphere and the collar, and the sphere and columns, to permit force measurements to be made. The results of these measurements are given in Table 1(*b*) and Table 1(*c*) and in the vector diagrams of Table 2 which indicate the direction

TABLE 2.—FORCE VECTORS ON PERISPHERE, AT 30 MILES PER HOUR



Item No.	Description	IN THE PRESENCE OF:			Free air test
		Ground board and collar (*a*)	Ground board and columns (*b*)	Ground board only (*c*)	(*d*)
1	Coefficient of drag, C_D	0.573	0.485	0.300	0.200
2	Resultant force vector, C_R	0.705	0.569	0.303	0.200
3	Angle θ between C_R and the horizontal plane	35°.8	31°.4	−5°.77	0°
4	Perpendicular distance, *e*, from the center of the sphere to C_R, in inches	0.30	0.27	0.24	0

of the resultant force. This resultant force is expressed in coefficient form as C_R, the coefficient being defined by an equation similar to that used for C_D (see Equation (23)), but with the resultant force, R, used instead of the drag, D. The resultant force has been calculated in the usual manner from the drag, D, and the transverse or lift force, L.

It is clear that the introduction of the ground board and the collar, or of the ground board and columns, serves to produce an appreciable transverse force in an upward direction—that is, a "lift" in aeronautical terminology. The production of such transverse forces is readily explainable. The introduction of the collar below the sphere naturally retards the flow on the lower surface, and also removes suction effects over a considerable part of the lower area. The presence of the column produces a similar effect but not to so large an extent, and hence the resultant vector is less inclined away from the horizontal in the case of the columns, than in the case of the collar, which blocks the air flow so effectively. It will also be noticed that for the sphere in the presence of the ground only (that is, without collar or columns (Table 2(*c*)), the transverse force is small, but in a downward direction as shown by the resultant force vector which points downward. It is apparent that for this case the air flow is speeded up between the ground board and the lower half of the sphere giving rise to high suction effects in this region and, hence, a downward or negative lift force.

The foregoing conclusions seem to be substantiated by the pressure distribution tests for all the cases, the results of which are given subsequently under the heading, "Pressure Distribution Tests on Sphere." The transverse forces are probably of little importance to the structural engineer. It is most interesting, however, to compare the drag coefficients under the various conditions of the test (see Item No. 1, Table 2).

The great increase in the drag is explainable partly by the increase in speed over the surface of the sphere when in the presence of the ground (as indicated under "Theoretical Considerations" and substantiated by the pressure distribution tests), partly by the general disturbance of the flow due to the obstructions, and partly by the additional or induced drag which follows on the production of lift.

From a structural engineer's point of view it is important to note that wind forces on the sphere are greater in the case of the collar than in the case of the columns, although even in the very worst condition the direct drag force on the sphere is less than one-half the force that would be experienced on a flat plate of the same area as the projected area of the sphere.

Pressure Distribution Tests on Sphere

The following pressure distribution tests were run:

Test No.	Description
1	Free air test of sphere alone.
2	Sphere alone in the presence of the ground board.
3	Sphere on four columns, 1 in. in diameter, with ground board.

Test No.	Description
4	Sphere on collar, 8 in. in diameter, with ground board.
5	Sphere on collar, with ground board and with the Trylon 25 in. on one side.
6	Sphere on collar, with ground board and with the Trylon 25 in. in front.

The distance of 25 in. was measured from the center of the base of the Trylon to the center line of the sphere as shown in Fig. 1. The results of these pressure distribution tests are best shown by the diagrams in Figs. 12, 13, and 14. The pressure distribution diagrams for all the tests are plotted around 360° for the meridian plane (VI-XII) and around 360° for the equator plane (III-IX). In addition, corresponding diagrams have been plotted for Planes I-VII and II-VIII for the test of the sphere alone in the presence of the ground board, so that the pressure distribution over the entire sphere is represented for that case as shown in Figs. 13(*a*), 13(*b*), 13(*c*), and 13(*d*).

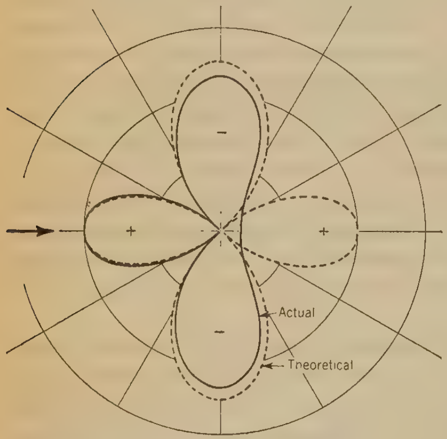
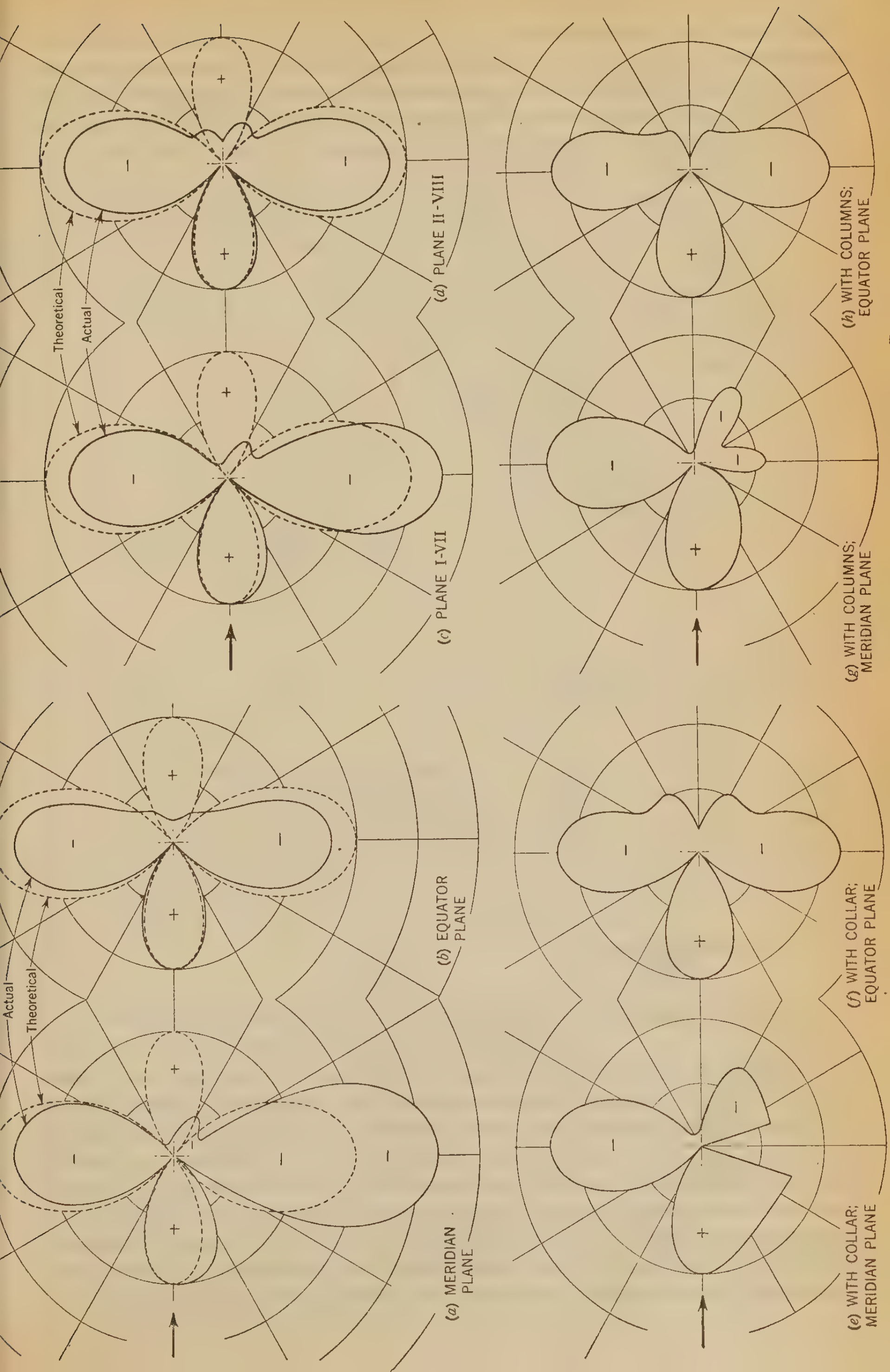


FIG. 12.—PRESSURE DISTRIBUTION OVER ANY GREAT CIRCLE PLANE OF SPHERE IN FREE AIR

The method of plotting the results of the pressure distribution tests utilizes a form of polar diagram in which the wind direction is indicated by the heavy arrow, the radial lines, the 30° divisions, on which the pressures were measured, and the concentric circles, represent radii of $\left(\frac{p_n - p_\infty}{\rho \frac{V^2}{2}}\right)$ equal to 0.5, 1.0, and 1.5, respectively. The expression, $\frac{p_n - p_\infty}{\rho \frac{V^2}{2}}$, is the same as that used in Equation (19). The test data were reduced to this pressure-unit ratio because it is non-dimensional and, therefore, will be the same for all velocities beyond the critical range. It will be noted that on the pressure plots certain parts of the distribution curves are marked plus (+) and others minus (−). The plus (+) sign means that the pressure-unit ratio was positive, thereby indicating a region of positive pressure acting normal to the sphere surface; and, the minus (−) sign shows the negative pressure or suction area normal to the sphere surface. Pressures, of course, are referred to the static pressure of the undisturbed air stream as a base.

Under the heading, "Theoretical Considerations," the flow for a perfect fluid was investigated for the cases of the free-air sphere, and for the sphere in


Theoretical
Actual
(d) PLANE II-VIII
(c) PLANE I-VII
Actual
Theoretical
(b) EQUATOR PLANE
(a) MERIDIAN PLANE
(h) WITH COLUMNS; EQUATOR PLANE
(g) WITH COLUMNS; MERIDIAN PLANE
(f) WITH COLLAR; EQUATOR PLANE
(e) WITH COLLAR; MERIDIAN PLANE
+
−

the presence of the ground board. The pressure ratios have been computed by Equations (6) and (18), and it is interesting at this point to compare the theoretically calculated and the measured pressures over the sphere. Therefore, in Figs. 12, 13(a), 13(b), 13(c), and 13(d) the measured pressures are shown by the full line and the theoretical pressures by the dotted line.

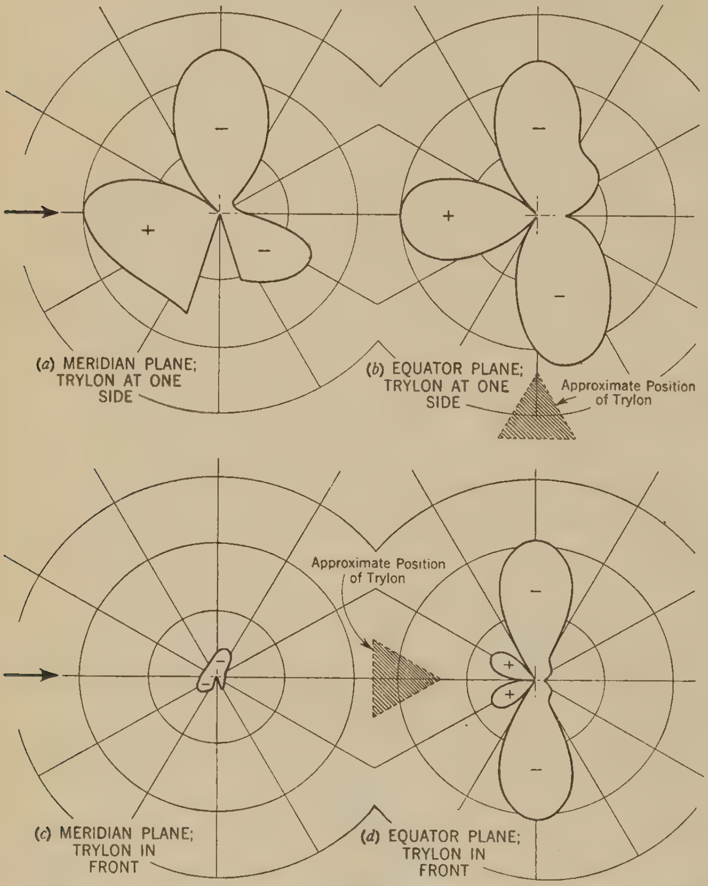


Fig. 14.—Pressure Distribution Over the Sphere; in the Presence of Trylon, Ground Board, and Collar

Sphere in Free Air.—An examination of the pressure diagram for the free-air sphere (Fig. 12) reveals that the computed and measured pressures are in excellent agreement for the area included between about ±60° either side of the horizontal plane of symmetry on the windward side. It will also be observed that on the windward side of the sphere, the points where the tangential velocity is equal to the undisturbed air stream velocity (or where the pressure ratio is equal to zero) check for both the calculated and measured pressure patterns. At the rear of the sphere surface the theoretical pressures for a perfect fluid have the same positive values as for the corresponding area on the front. The measured pressures on the rearward side of the sphere show that, instead of a region of positive pressures which would be present in a perfect fluid, there is a region of negative pressures. This discrepancy may be explained as follows: In a perfect fluid such as was assumed for the theory, the flow around any great circle of the sphere would be symmetrical. If this were so, the symmetrical pressure distribution diagram indicated by the dotted lines of Fig. 12 would be obtained and the sphere would have zero drag or resistance. In the imperfect fluid that air is, this ideal condition cannot be realized as the air moves over the sphere surface with viscous drag, with the building up of a boundary layer; ultimately, the tearing away of the boundary layer on the rear half; the creation of an eddying region; and, consequently, a drop in pressure behind the sphere. This actual flow is represented diagrammatically in Fig. 15. It is interesting to note that the maximum suction developed experimentally is somewhat greater than the theoretically calculated value. This does not necessarily indicate that the tangential velocity is higher, since loss of pressure is experienced due to viscosity.

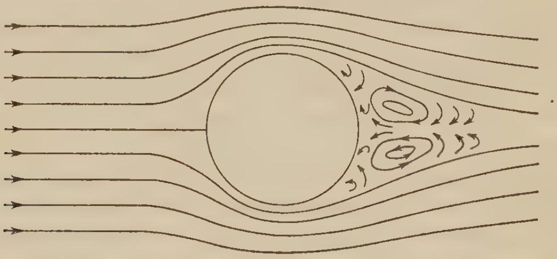


FIG. 15.—DIAGRAM OF ACTUAL AIR FLOW AROUND A SPHERE

Sphere in Presence of the Ground.—The comparison of the theoretically calculated and measured pressures for the sphere in the presence of the ground board is shown in Figs. 13(*a*), 13(*b*), 13(*c*), and 13(*d*) in four plots so that all the great circles used are represented. The plots show that the theoretical and measured pressures agree very well for the part of all the great circles that is on the upper forward half of the sphere. In this region too, the points at which $\left(\frac{p_n - p_\infty}{\rho \frac{V^2}{2}}\right) = 0$ are in good agreement for both the theory and practice. On the rear half of the sphere the same general discrepancies are noted as were shown for the free air sphere. The most interesting points in the comparison occur on the lower side of the sphere that is in close proximity with the ground board. Here, it will be seen that the greatest difference between theory and measurement is shown for the meridian plane and the least for the equator plane. Upon following the general effects on all planes from the lower half of the meridian up to the equator, it can be seen that the discrepancies between theory and measurement tend to lessen progressively. The general conclusions that might be drawn from the measured pressure distribution diagrams are:

(1) Hydrodynamic theory indicates that, in a perfect fluid, the presence of the ground board should cause a greater tangential velocity over the entire

surface of the sphere, and not only on the lower part of the sphere near the ground board. A study of the suctions in Figs. 12, 13(*a*), and 13(*b*), indicates that experiment is in accord with the theory.

(2) On the lower half of the sphere, in the presence of the ground board, there is greater discrepancy between theory and experiment. A boundary layer is built up over the ground board within which the air is retarded. This is equivalent to a construction of the air passage between sphere and ground, and explains the much higher velocity on the lower part of the meridian plane. Figs. 13(*a*) and 13(*b*) indicate that the maximum suction attains relatively very high values. For points that are farther away from the direct influence of the ground this effect is lessened.

(3) The presence of the ground board intensified the eddying region at the back of the sphere, on its lower half.

(4) The generally greater tangential velocities (and, hence, greater skin friction), the far greater tangential velocities over the parts of the sphere nearest the ground, and the intensification of the eddying regions, confirm the results of the drag tests, which showed that the drag of the sphere in the presence of the ground board is far greater than in free air.

(5) From the pressure distribution diagrams it is clear that the transverse, or lift, force would be in a downward direction. This was shown to be the case by actual measurement, the results of which have been given in Table 2.

Sphere in Presence of the Ground and Collar, and in Presence of Ground and Columns.—The pressure distribution diagrams for a sphere in the presence of a ground board and a collar, and in the presence of a ground board and columns, are plotted in Figs. 13(*e*), 13(*f*), 13(*g*), and 13(*h*) for the meridian and equator planes only. These two planes or great circles were chosen as being the most representative. Study of the diagrams of Figs. 13(*e*), 13(*f*), 13(*g*), and 13(*h*) leads to the following conclusions:

(6) The introduction of either the collar or the column changes the character of the flow appreciably on the meridian plane. The positive pressures on the meridian plane at the windward side of the sphere now extend far below the equator. The introduction of either columns or collars impedes the flow, as would be expected, and hence creates the pressure region. The production of pressure below the equator explains the fact that the transverse force or lift on the sphere is now upward, as shown by the force tests.

(7) The introduction of both the collar and the columns disturbs the flow considerably and produces intensification of the eddying region at the back of the sphere, below the equator. This explains the production of a new region of suction which was not present in the case of the sphere in free air, or in the presence of the ground.

(8) At and above the equator, the pressure distributions in the presence of the ground and in the presence of the ground and collar, or column, are very similar. It is on the lower half of the sphere that the introduction of the collar or a column produces the most marked effects.

(9) The intensification of eddying and the production of a new region of suction when a collar or column is introduced explain, adequately, the greater drag of the sphere as compared with its drag in the presence of the ground alone.

(10) The introduction of the collar affects the drag and pressure distribution far more than the introduction of the columns, which still provide a passage for air flow close to the ground. From the point of view of structural loads, mounting on columns is preferable to mounting on a collar.

(11) The introduction of the collar or column produces great asymmetry of pressure on the meridian which the structural engineer would no doubt take into account, taking care to examine local effects.

Sphere on Collar with Trylon at Side.—The pressure diagrams of Figs. 14(*a*) and 14(*b*), illustrating the aforementioned condition, constitute a composite diagram for the equator plane, since only 180° of the equator plane was provided with orifices. In plotting the pressure at the equator plane the 180° on the Trylon side was obtained from actual observed data. On the far side of the equator plane away from the Trylon, data were taken from the test with collar but no Trylon, the assumption being that the presence of the Trylon would not appreciably affect the pressure distribution on the far side.

The introduction of the Trylon on the side does not cause marked variations in the pressure distributions. A twisting moment about the vertical axis of the sphere would be introduced, but this should not be serious.

Sphere with Collar and Trylon Ahead.—The pressure distribution for this case is illustrated in Figs. 14(*c*) and 14(*d*). The Trylon acts very powerfully as an aerodynamic shield. On the meridian plane pressure effects practically disappear, with all the orifices under a very slight degree of suction. The shielding effect on the equator plane is very marked. The pressure on the windward side of the sphere almost disappears, and the suction diminishes to a value of $\left(\dfrac{p_n - p_\infty}{\rho \dfrac{V^2}{2}}\right) = -1.0$ (approximately).

General Conclusions

From the point of view of the structural engineer some general conclusions appear to be of interest:

(12) For even a semi-stream-lined object, such as a sphere, it is impossible to design for air loads on the basis of some empirical rule for wind pressure. The same point of the structure may, at one time, attain a maximum pressure, and, at another time, a high suction value.

(13) Intense variation in pressure differences from point to point must be expected and allowed for in design.

(14) Instead of endeavoring to apply general rules, a systematic survey of pressure distribution is necessary, with subsequent integration of loads along planes in three directions, and integration of local load distribution. It is understood that Shortridge Hardesty, M. Am. Soc. C. E., has made use of the data herein presented for the most painstaking load integrations.

(15) The heaviest loads will be encountered when the Trylon is at the side of the sphere when the wind is considered as striking the nose of the sphere.

(16) The only points that will always be in high suction are those toward the upper surface of the sphere.

(17) The shielding effect of the Trylon should be taken into consideration with saving in weight of certain members.

(18) In general, structural members would have to withstand tension and compression loads under different wind conditions.

(19) The maximum positive pressure will be experienced at the equator and will never exceed $\left(\frac{p_n - p_\infty}{\rho \frac{V^2}{2}}\right) = 1.0.$

(20) Since the static pressure inside the sphere should be about equal to the outside static pressure it is inconceivable that pressures in excess of those shown in the pressure diagrams should ever exist. This being the case, and if 100 miles per hour is assumed as the maximum possible outside air velocity, then the maximum positive pressure on the sphere would be 25.6 lb per sq ft, and the negative pressure about −30.8 lb per sq ft on the sides of the equator plane.

ACKNOWLEDGMENTS

The investigation reported herein was undertaken at the request of the Construction Department of the New York World's Fair Corporation. For their active co-operation, thanks are due especially to John P. Hogan and L. B. Roberts, Members, Am. Soc. C. E., Chief Engineer and Assistant Chief Engineer, respectively, of that organization. Thanks are also due to Mr. Hardesty for his critical guidance in the conduct of the experimentation. All the major and auxiliary apparatus in the laboratories of the Daniel Guggenheim School of Aeronautics, College of Engineering, New York University, was made available for this purpose.

APPENDIX

LIST OF REFERENCES

Some of the following references are cited in the paper. The remainder have a general significance in relation to the entire paper.

(1) "Standards of Design for Structural Steel," *Bulletin 12 Yb*, U. S. Navy Dept., September, 1934.

(2) "Speed and Pressure Recording in Three-Dimensional Flow," by F. Krisam, *Technical Memorandum, No. 688*, National Advisory Committee for Aeronautics.

(3) "Druckverteilung an der luftumströmten Kugel," by O. Krell, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, February 28, 1931.

(4) "Flow and Drag Formula for Simple Quadrics," by A. F. Zahm, *Technical Report, No. 253*, National Advisory Committee for Aeronautics.

(5) "Wind Stresses in Buildings," by Robins Fleming.

(6) The Code of Ordinances of the City of New York Building Code, Section 54.

(7) "Wind Pressure on a Model of the Empire State Building," by H. L. Dryden and G. C. Hill, *Journal of Research*, National Bureau of Standards, April, 1933.

(8) "Wind Pressure on a Model of a Mill Building," by H. L. Dryden and G. C. Hill, *Journal of Research*, National Bureau of Standards, April, 1931.

DISCUSSION

Elliott G. Reid,[5] Esq. (by letter).—It is the writer's opinion that several phases of the work reported in this paper should not be allowed to pass unchallenged.

The theory presented is not only inapplicable but highly misleading. To be sure, the distribution of pressure over a large part of the surface of a sphere which moves through an unlimited mass of air is satisfactorily predicted by the methods of classical hydrodynamics. Therefore, determination of the velocity potential for the motion of a sphere parallel to a plane boundary might be expected to enable qualitative determination of the boundary interference effects. However, the mathematical complexity of this problem is so formidable that Lamb[6] gives only "the first steps in the approximation" and, of course, not even Neumann's[7] rigorous solution covers the practical problem which is further complicated by interference effects arising from the foundation structure of the Perisphere. These facts would seem to justify, if not to compel, the omission of all but qualitative theoretical treatment.

Nevertheless, the authors actually present a substantial reproduction of Lamb's approximate solution—which is entirely inapplicable when the sphere is close to the boundary surface. It appears that the significance of the substitution of c^3 for r^3 has escaped them because they conclude (following Equation (18)) that "the velocity, theoretically * * * is the same at symmetrical points above and below the sphere, when in the presence of the ground," and that "the stagnation points are not changed by the ground board." Inspection of the potential flow pattern for this case[8] demonstrates not only that both conclusions are unfounded but that the experimental results (Fig. 13(*a*)) are in qualitative agreement with hydrodynamic theory.

The significance of the results of force and pressure distribution tests on the Perisphere is highly questionable because the authors' interpretation of Fig. 4 is not justifiable. They err first by assuming the numerical values to be correct. The important portion of the curve (that is, that for which $R > 3 \times 10^5$) is substantially that determined by Wieselsberger[9] who supported his spheres approximately as illustrated by Fig. 5. The importance of the interference effects which characterize such installations was first demonstrated by the writer[10] in 1923; subsequent investigations by Wieselsberger[11] and Flachsbart[12] confirm his findings and thus invalidate that part of the curve upon which the authors predicate their extrapolation of model test results to full scale. According to the most reliable information available,[13] the value of C_D at $R_e = 750\,000$ is about one-half that shown by Fig. 4. Even more important, however, is the authors' erroneous assumption (also based on Fig. 4) that an unchanging flow pattern exists at all values of R_e greater than 1 400 000. The fallacy is exposed by Jacobs' tests[13] which show that C_D increases continuously while R_e varies from 1 400 000 to 2 600 000. (It is noteworthy that in the case of the full-scale Perisphere the value of R_e would be 140 000 000 at a wind speed of 70 miles per hr.) Since neither theory nor experiment justifies such procedure, use of the unaltered model pressure data for "the most painstaking load integrations" seems likely to prove love's labor lost.

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[5] Prof. of Aerodynamics, Dept. of Aeronautics, Stanford Univ., Stanford University, Calif.

[6] "Hydrodynamics," by Horace Lamb, Cambridge, 1906, Third Edition, pp. 124–125.

[7] "Hydrodynamische Untersuchungen," by C. Neumann, Leipzig, 1883.

[8] "Applied Hydro- and Aeromechanics," by L. Prandtl and O. G. Tietjens, New York, 1934, Fig. 58, p. 109.

[9] "Further Information on the Laws of Fluid Resistance," by C. Wieselsberger, National Advisory Committee for Aeronautics, *Technical Note No. 121*, 1922.

[10] "The Resistance of Spheres in Wind Tunnels and in Air," by D. L. Bacon and E. G. Reid, National Advisory Committee for Aeronautics, *Technical Report No. 185*, 1923.

[11] "Ueber die Verbesserung der Stromung in Windkanalen," by C. Wieselsberger, read at the 108th Session of the Japanese Soc. of Mech. Engrs., March 19, 1925.

[12] "Recent Researches on the Air Resistance of Spheres," by O. Flachsbart, National Advisory Committee for Aeronautics, *Technical Memorandum No. 475*, 1928.

Discussion of the pressure distribution tests would be incomplete without reference to the explanation which appears in the last two sentences under "Pressure Distribution Tests on Sphere: Sphere in Free Air." The authors appear to have explained a non-existent phenomenon, that is, in Fig. 12, the maximum suction developed does not exceed the calculated value.

Turning now to the Trylon, if that were to be an isolated structure and if the variation of wind speed with height were unimportant, the model test results might provide a rational basis for estimating the total wind loads on the full-scale structure. However, since the interference of the adjacent Perisphere would undoubtedly modify—and, with some wind directions, would probably augment—these forces, the omission of interference tests makes the validity of the recorded results somewhat dubious. No occasion for surprise is evident in the result, $C_D = 1.433$; narrow rectangular plates have long been known to experience relatively large resistances. Finally, since information concerning the distribution of wind loads was sought, it seems regrettable that tests were not made to determine the severity of the pressure gradients near the edges of the Trylon.

KARL ARNSTEIN,[14] ESQ. (by letter).—The wind tunnel tests and theoretical consideration of models of the Perisphere and Trylon that the authors are making add more light to the problem of designing structures of unusual nature to withstand aerodynamic loads. These tests are interesting to the writer because they are somewhat parallel to problems encountered in designing the large airship hangar at Akron, Ohio, for which the senior author conducted wind tunnel tests on a hangar model which were later confirmed to a very reasonable degree by full-scale tests made at a later date on the hangar itself. These full-scale tests and related design considerations have been published.[15]

There is one conclusion made by the authors which should be scrutinized more closely. In Conclusion (20) it is suggested that the static pressure inside the sphere should be about equal to the outside static pressure. In large buildings such as the Perisphere or an airship hangar, ample ventilation must be provided to accommodate changes in temperature and barometric pressure and for other considerations. For any static loading condition the internal static pressure of the building must equal the local pressure at the ventilator or other openings, such as doors, which, in turn, must correspond to the local external pressure at that point. Since the external pressure pattern may change greatly for any fixed point on the surface of the building, depending on the direction of the wind, it is difficult to imagine the internal pressure remaining at any constant value.

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[13] "Sphere Drag Tests in the Variable Density Wind Tunnel," by E. N. Jacobs, National Advisory Committee for Aeronautics, *Technical Note No. 312*, 1929.

[14] Chf. Engr., Goodyear-Zeppelin Corp., Akron, Ohio.

[15] "Wind Pressures on the Akron Airship Dock," by K. Arnstein and W. Klemperer, *Journal of the Aeronautical Sciences*, January, 1936.

In the case of the airship hangar it was assumed that the internal pressure could vary from the greatest ventilating suction to the greatest over-pressure to be expected from any local opening.

W. B. KLEMPERER,[16] ESQ. (by letter).—The subject of the paper is of interest to the aeronautical engineer especially because of its close analogy with the problems arising from the groundhandling of large free balloons. Every opportunity to supplement theoretical investigations with experimental research, therefore, is keenly appreciated.

The approximate theoretical treatment of the Perisphere in representing the effect of a mirror image sphere by a mere increase of the axial doublet strength, as adopted in the paper, is a close approximation only where the ground distance is large compared to the radius. However, the closer the ground distance the poorer the approximation becomes. It neglects the flow constraint near the ground and leaves no room for any vertical force acting on the sphere without pedestals.

With the ground clearance only a small fraction of the radius, the first-order mirror doublet almost doubles the velocity increment at the bottom of the sphere, whereas it contributes only about 4% at the top. The body generated by a space doublet does not stay spherical when the flow of a parallel mirror doublet in its proximity is added to a general parallel flow.

Therefore, the observed vertical nonsymmetry of the pressure distribution need not be interpreted as a discrepancy between theory and experiment. Thus the high suction observed in the bottom region may not all be due to the ground board's boundary layer restricting the passage, as was suggested in Conclusion (2), but also to a genuine Venturi effect which can be embraced by a classical theory taking the nonsymmetry of the mirror-flow system with respect to the sphere center level into account.

Such a theory requires the introduction of successively mirrored doublets, or sources and sinks, eccentrically located in the spheres. Lamb postulates an attraction between the sphere and the ground, for the ideal fluid.[17]

The fact, however, remains that as soon as circulation is created about the sphere by ground surface friction, etc., a lift force is generated which may outweigh the Venturi attraction. The same effect may be caused by a large-scale wind boundary layer extending high above the sphere, much larger than would correspond to the relatively thin boundary layer adjoining the ground board in the wind tunnel. The real sphere, therefore, may have a greater lift than the model. This additional lift could be expected to be essentially proportional to the average vertical wind gradient throughout a layer comparable in thickness to the height of the structure.

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[16] West Los Angeles, Calif.

[17] "Hydrodynamics," by H. Lamb, Fifth Edition, pp. 124, 174.

The question of what pressure will actually prevail inside the spherical building would appear to be worth while to answer by full-size experiment. The tendency to keep out draft favors leeward ventilation and most likely works toward lowering the interior pressure below atmospheric static. An exposition building of this kind might lend itself to the installation of recording gages.

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Paper No. 2043

RELATION OF RAINFALL AND RUN-OFF TO COST OF SEWERS

By John A. Rousculp,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The preliminary design of storm sewer systems often requires a number of laborious studies involving various plans and perhaps various combinations of the two design factors—rainfall frequency rates and run-off percentage. It is the purpose of this paper to describe the development of a method, using time-saving charts, based on a study of relative costs of a system, which can be a valuable aid in preliminary design work.

The relative costs of storm sewers, as studied herein for various combinations of design factors, are based on the application of such combinations to an assumed district. The selection of this assumed district was based upon a study of the concentration of area of a number of actual districts.

The rational method of estimating run-off was used in computing the various cases. Explanation is made of the method of determining the assumed district, and a sample computation is given to illustrate the method of determining the run-off, Q, for one of the cases. The results of the run-off Q-computations for all cases are plotted on charts showing the run-off, Q, for corresponding area. These charts are used as the basis for computing relative sewer-capacity requirements in connection with the study of cost relations. A discussion of results is given, together with examples suggesting some practical uses of a study of this nature.

The writer does not intend that the charts, as developed, should cover the full range of design stipulations that may be encountered, or be of widespread applicability. Rather, it is hoped that the development, as a method, may be helpful to other engineers in making similar studies to satisfy various special requirements.

Introduction

In the design of storm-sewer systems, the run-off at various points and the cost are dependent on the values adopted for the rainfall frequency rates and the run-off percentage. The use of various values for either or both of these

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Note.—Published in June, 1938, *Proceedings*.

[1] Senior Designing Engr., Div. of Eng., City of Columbus, Columbus, Ohio.

design factors will result in corresponding values of computed run-off and cost. In order to study the effect of such design factors it is necessary to re-compute the entire system for each set of values. For example, suppose a design has been made on the basis of 10-yr rainfall and 40% run-off, and the cost estimated. Then, if for some reason, the estimated cost on a design basis of a 5-yr rainfall and 30% run-off is required, the system must be re-computed entirely. The relations of run-off and cost resulting from the use of various combinations of design factors as determined by this study can be used to determine such relative cost quickly, and to lessen the computing work greatly. Hence, the results of this or similar studies should be especially useful to engineers when confronted with preliminary design problems of this kind. Clearly, with such a quick means of estimating the relative costs, more thorough and comprehensive preliminary studies would be possible. Since it was necessary to vary slope and area to determine the effect of the rainfall and run-off factors, it was convenient to analyze also the relative effect of the slope and area factors on run-off and cost.

Selection of Typical District

As a guide in the selection of the typical district, several representative sewer districts were used (see Fig. 1). The area concentrating and time of concentration, in minutes, were computed for a number of collection points along the main trunk sewer in each district. The areas concentrating for each district were then plotted in Fig. 2, and a curve for each district was drawn.

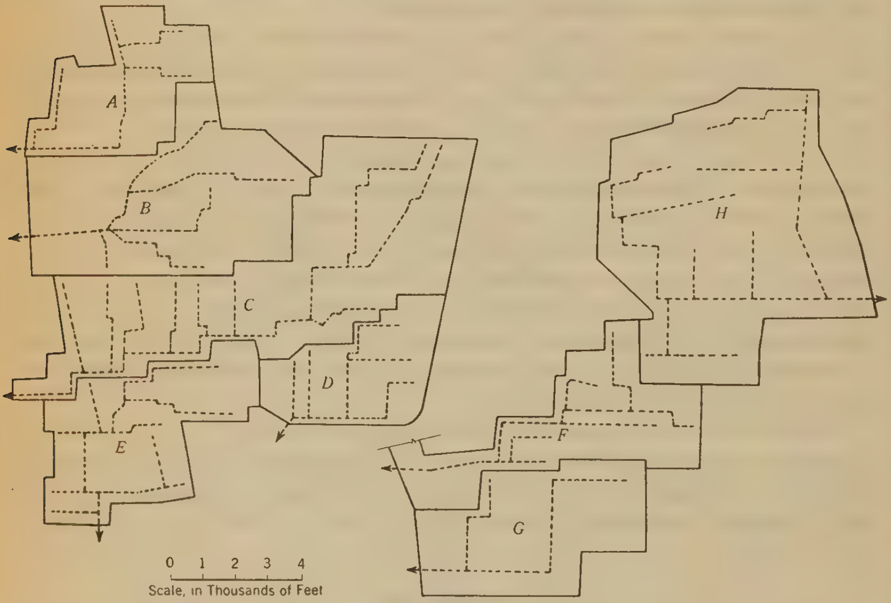


Fig. 1.—Outline of Various Districts Used in Study of Concentration of Area, Showing Location of Principal Sewers

A smooth curve, which was assumed to represent approximately an average of the curves of the various districts, was then drawn. This average curve represents the typical district, and areas concentrating with respect to time, as given by this curve, are as follows:

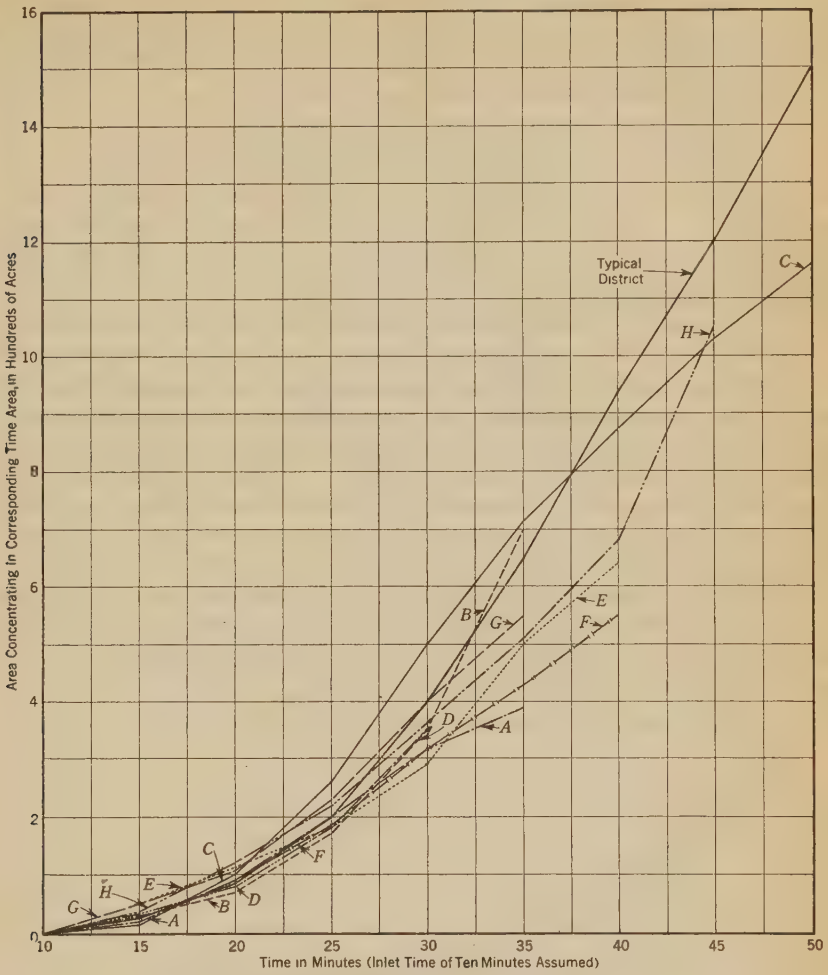


FIG. 2.—CONCENTRATION-OF-AREA CURVES OF VARIOUS DISTRICTS USED AS GUIDES IN SELECTING TYPICAL DISTRICT

Time, in minutes (based on $S = 0.003$)	Area, in acres, used for typical district	Time, in minutes (based on $S = 0.003$)	Area, in acres, used for typical district
15	25	35	650
20	80	40	940
25	200	45	1 200
30	400	50	1 500

The next step was to convert time in the typical district to distance, since the element of distance between points of concentration is necessary in a design problem. In computing the time element for the representative districts, velocity was based on sewer sizes required when using a design basis of 10-yr rainfall intensities, $C = 30\%$, and $S = 0.003$. Hence, these same conditions were used as a basis in computing distance from the foregoing list of time and area concentrating for the typical district. The results of the calculations to determine distance are given in Table 1. Note that the acres accumulating appear in a different order than in the foregoing list. This change makes the typical district more comparable to an actual design problem, and the distances shorter between the up-stream points of concentration where the velocities are smaller and changing rapidly. It was found that much larger distances between points of concentration could be used at the lower end of the system (where flow is large and where velocities do not change to any extent even for considerable increase in tributary area) than at the upper end of the system, for the same relative accuracy in computing time of concentration. It was desired to use as many points of inlet as were required for reasonable accuracy, and no more, for the reason that each additional point used would add materially to the computations.

TABLE 1.—AREAS CONCENTRATING AND DISTANCES BETWEEN POINTS OF CONCENTRATION

Area, in acres	5	5	10	20	30	40	60	80	100	150	200	200	300	300
Accumulated area, in acres	5	10	20	40	70	110	170	250	350	500	700	900	1 200	1 500
Distances between successive points of concentration, in feet	440	530	780	830	850	1 100	1 200	1 450	1 950	2 350	2 200	3 800	3 600	....

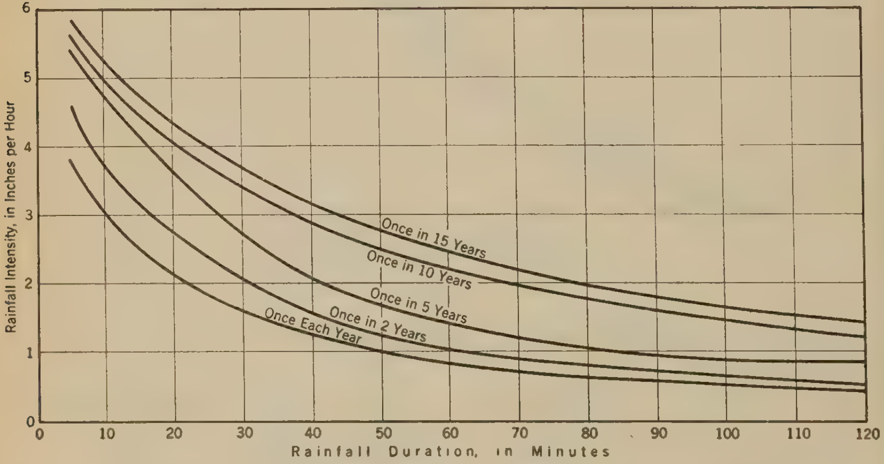


FIG. 3.—INTENSITY, DURATION, AND FREQUENCY RAINFALL CURVES FOR COLUMBUS, OHIO

SCOPE OF STUDY

In the usual storm-sewer design a basis is selected as, for example, 10-yr rainfall intensities and 30% run-off. The sewer slopes usually vary somewhat in each separate problem, and, hence, slope is not a part of the design basis. In this study, however, four different slopes were used, principally to determine whether the relative results due to the rainfall and run-off factors were affected by the slope factor. The range of factors covered in this study are presented in Table 2.

TABLE 2.—RANGE OF FACTORS COVERED BY STUDY

Rainfall intensities, yearly bases	10	5	2
Run-off percentages	30	40	50
Slopes	0.001	0.003	0.01 and 0.02

The total number of possible combinations of these factors is 36, and, hereinafter, they are referred to as cases. Hence, 36 sets of computations were made, using the typical district areas and distances, as given in Table 1. The rainfall intensities used are given in Fig. 3. Table 3 is an example of the computation to determine the run-off, *Q*, for one of the 36 cases. The values of *Q* thus determined for all 36 cases were summarized and plotted on Fig. 4.

TABLE 3.—SAMPLE COMPUTATION; 10-YEAR RAINFALL; 30% RUN-OFF; AND 0.003 SLOPE

Accumulated area,* in acres (1)	Distances* between successive points of concentration, in feet (2)	Time of concentration of area based on Column (2) and Column (6), in minutes (3)	Rate of run-off ($I \times C$), in cubic feet per second per acre (4)	Run-off, *Q*, Column (1) × Column (4), in cubic feet per second (5)	Velocity of flow in sewer required for *Q* in Column (5), in feet per second‡ (6)
5	440	10†	1.47	7.3	3.5
10	530	+2.1 = 12.1	1.42	14.2	4.3
20	780	+2.2 = 14.3	1.35	27.0	5.0
40	830	+2.6 = 16.9	1.28	51.0	5.8
70	850	+2.4 = 19.3	1.23	86.0	6.5
110	1 100	+2.2 = 21.5	1.18	130	7.5
170	1 200	+2.4 = 23.9	1.13	192	8.3
250	1 450	+2.3 = 26.2	1.09	272	9.0
350	1 950	+2.7 = 28.9	1.05	368	9.7
500	2 350	+3.3 = 32.2	0.99	495	10.3
700	2 200	+3.8 = 36.0	0.93	650	11.0
900	3 800	+3.3 = 39.3	0.88	792	11.5
1 200	3 600	+5.6 = 44.9	0.81	970	12.0
1 500	....	+5.1 = 50.0	0.75	1 125	....

* See Table 1. † 10-min inlet time assumed; velocities based on Kutter's $n = 0.013$. ‡ Sewer size not given since velocity was taken directly from flow curves.

TABLE 4.—SEWER COST PER LINEAR FOOT

Sewer diameter, in inches (1)	Depth to sewer invert, in feet (2)	Sewer cost, in dollars per linear foot (3)
12	10	2.00
15	10	2.60
18	10	3.30
21	11	4.10
24	11	4.80
30	11	6.40
36	12	8.10
48	13	11.70
60	13	15.40
72	14	19.60
84	15	23.90
96	16	28.20
108	17	32.70
120	18	37.00

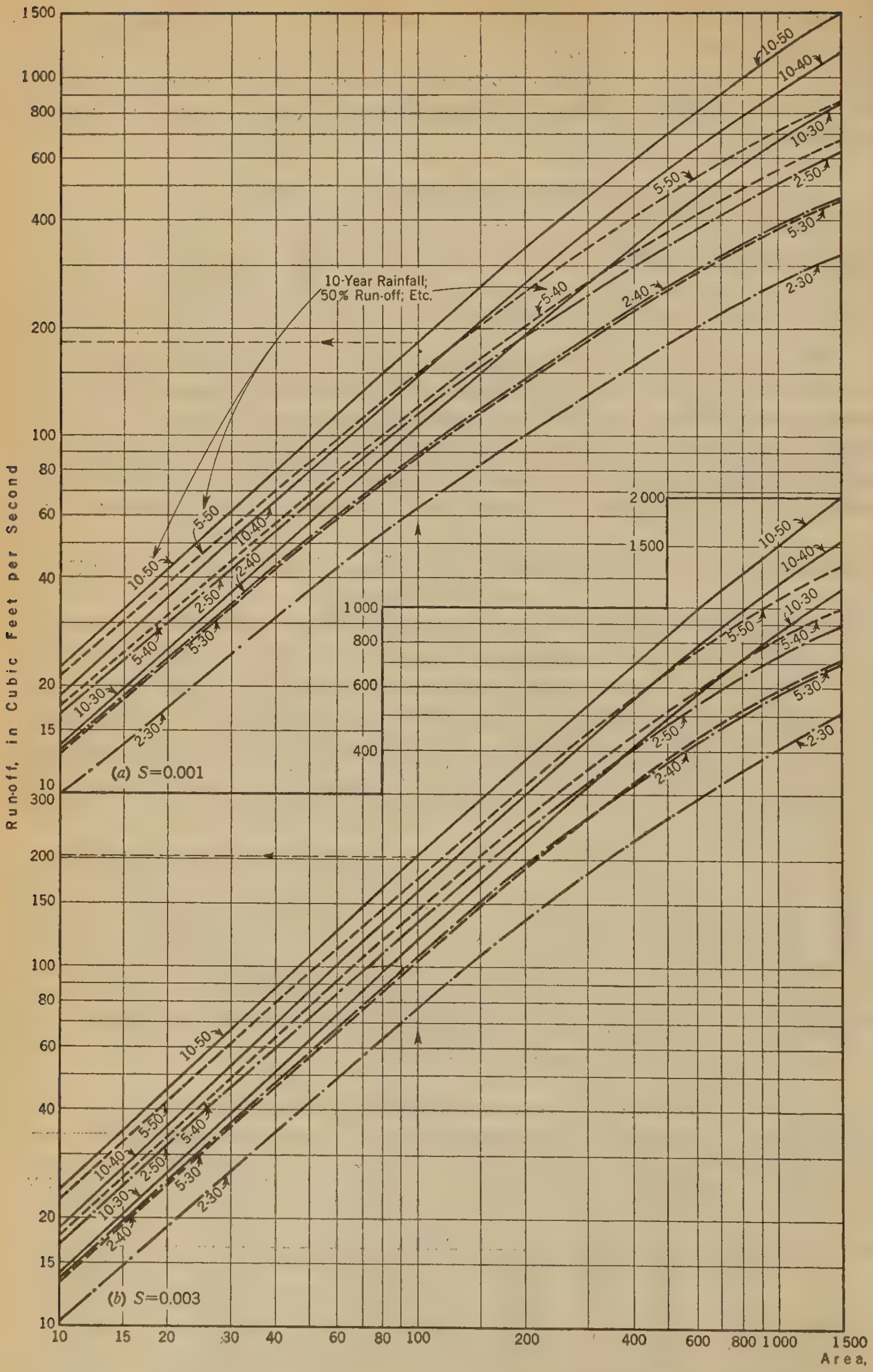
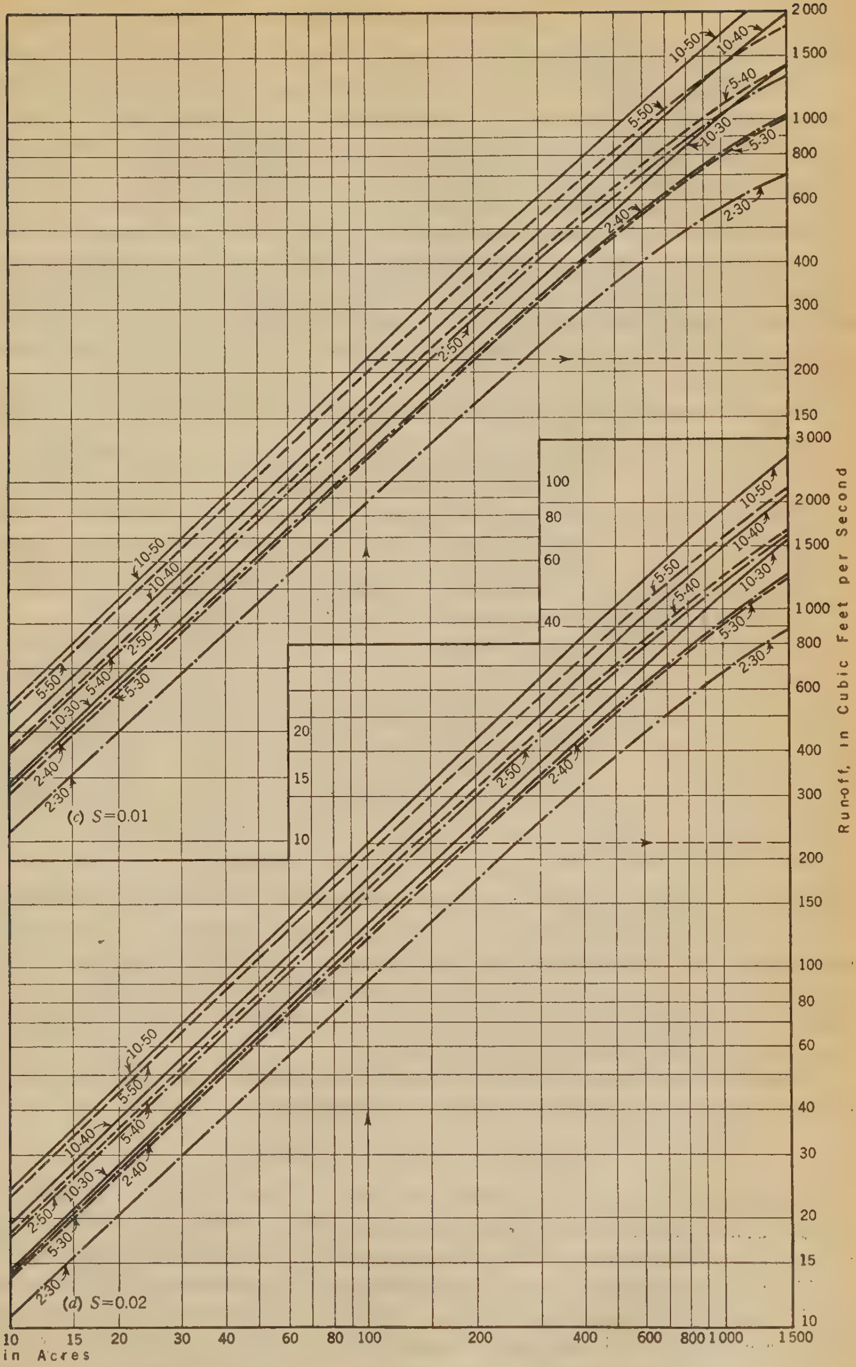


FIG. 4.—RUN-OFF FOR VARIOUS RAINFALL



Curves and Run-Off Coefficients

Cost Computations

The costs per linear foot of sewer given in Table 4 were used as the basis of the cost studies. The depths in Column (2) were considered as average and the costs per linear foot (Column (3)), as representative of the average 1936 prices. Although prices varying considerably from those selected could be justified as being representative of various localities, it is believed that any such variation would have but slight effect on the relative costs. The next step was a tabulation setting acreages opposite the capacity, in cubic feet per second, of the various sewer sizes for each of the 36 cases. The areas were read from the curves representing the various cases in Fig. 4. In reading these values, the chart being used was entered on the run-off side with the value of the sewer capacity in cubic feet per second, the corresponding area being read from the proper curve.

Table 5 is a sample tabulation on the basis of a 10-yr rainfall and $S = 0.001$, giving the values for 3 of the 36 cases. Values of area in the tabulations (of which Table 5 is an example), where the acreage values were less than 10 acres, could not be read from Fig. 4, but were computed by using the 10-min rainfall intensity with the proper run-off coefficient. Such computations give results approximately correct, since the concentration time of all areas smaller than 10 acres may be assumed to be 10 min.

TABLE 5.—Areas Corresponding to Sewer Capacities—10-Year Rainfall; and $S = 0.001$

Sewer diameter, in inches (1)	Sewer capacity* ($S = 0.001$), in cubic feet per second (2)	Area, in Acres, for the Following Run-Off Percentages			Sewer diameter, in inches (1)	Sewer capacity* ($S = 0.001$), in cubic feet per second (2)	Area, in Acres, for the Following Run-Off Percentages		
		30% (3)	40% (4)	50% (5)			30% (3)	40% (4)	50% (5)
12	1.1	0.75	0.55	0.43	48	45.6	37.0	27.0	21.0
15	1.9	1.3	0.95	0.75	60	83.0	75.0	53.0	42.0
18	3.2	2.2	1.6	1.3	72	135	130	90.0	72.0
21	5.0	3.4	2.5	2.1	84	202	210	143	112
24	7.0	4.7	3.6	2.9	96	287	325	215	165
30	12.9	9.1	6.6	5.3	108	392	480	310	240
36	21.1	16.0	11.5	9.0	120	518	710	455	340

* Based on Kutter's $n = 0.013$.

From the tabulations, of which Table 5 is an example, the curves of Fig. 5 were developed, these being the basis of the cost studies.

Computing Costs per Acre

Preliminary to computing costs per acre, it was necessary to select a typical sewer layout, from which to secure sewer lengths per acre serving various size areas, against which to apply costs. The layout adopted was one of 100-acre units made of blocks of approximately 4 acres each, which were 350 ft wide and 500 ft long (see Fig. 6). These blocks were arranged in rows, 4 blocks wide and 6 blocks high, to make up the 100-acre units. This would actually be about 96 acres, but was taken as 100 acres to simplify computa-

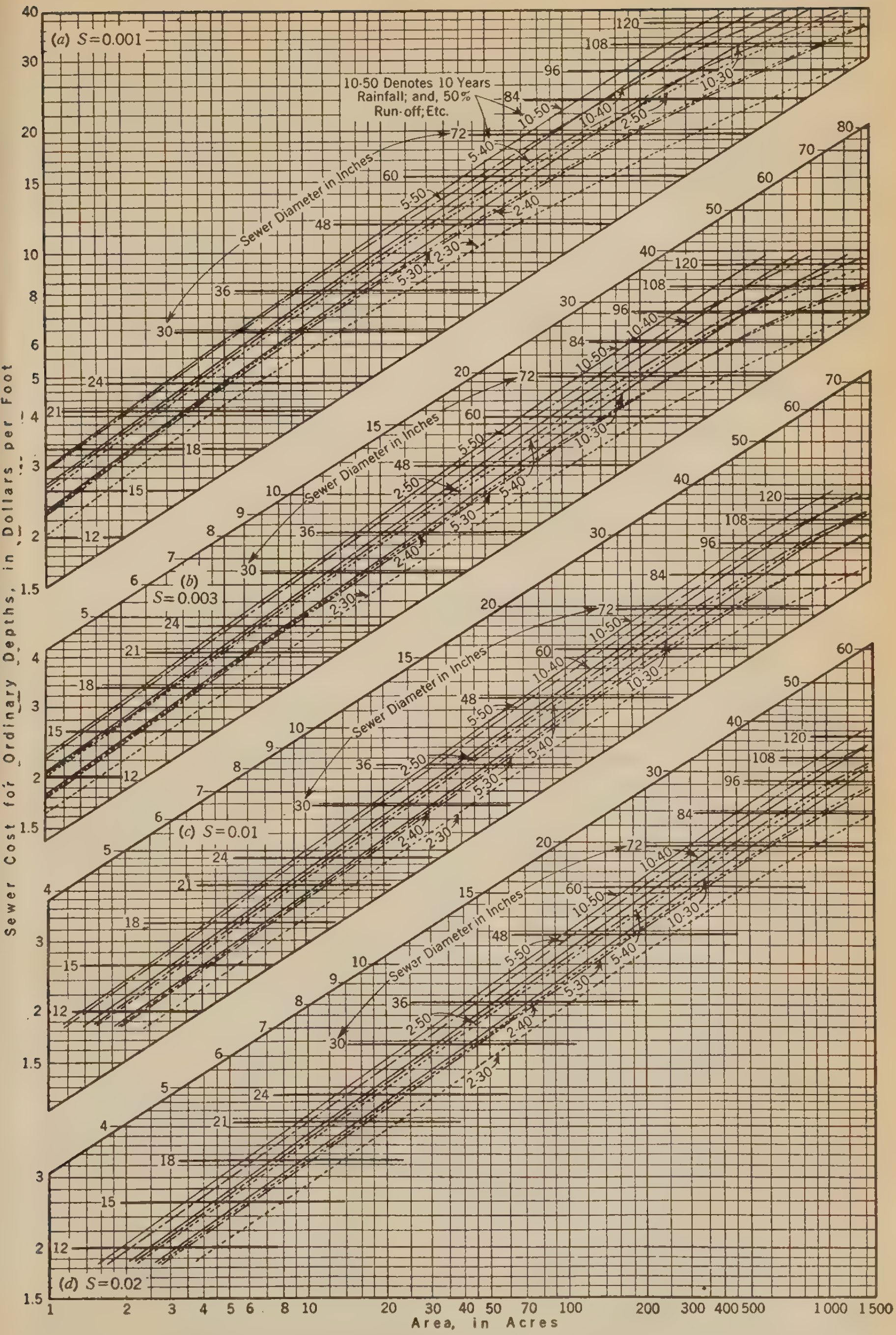


FIG. 5.—COST AND SIZE CURVES FOR TWO-YEAR, FIVE-YEAR, AND TEN-YEAR RAINFALL FREQUENCIES AND RUN-OFF COEFFICIENTS OF 0.30, 0.40, AND 0.50

tions. In this arrangement, the 4-block width of the unit was 4 × 530 ft = 1 400 ft, and the 5-block length was 6 × 500 = 3 000 ft. The collecting lines were arranged with one line to each row, extending the width of the blocks; these lines connected with a line which ran along the 2 500-ft side of the 100-acre unit. This line and the corresponding ones for each 100-acre unit connected with the main line serving all the units, by extending along the 1 400-ft width of the units. This layout, with the main trunk running along the lower edge of the 100-acre units, was estimated to represent, closely enough, the typical district used in computing the run-off, Q, as shown in Table 3. The arrangement in blocks and units made it possible to compute costs per acre for the various cases with a minimum of labor. Table 6(a) gives the computed lengths of sewer per acre for various areas served, and accumulated sewer lengths per acre. Using the sewer lengths per acre given in Table 6(a) and costs per foot taken from Fig. 5, the costs per acre were computed. Table 6(b) is an example of the computations for one of the 36 cases.

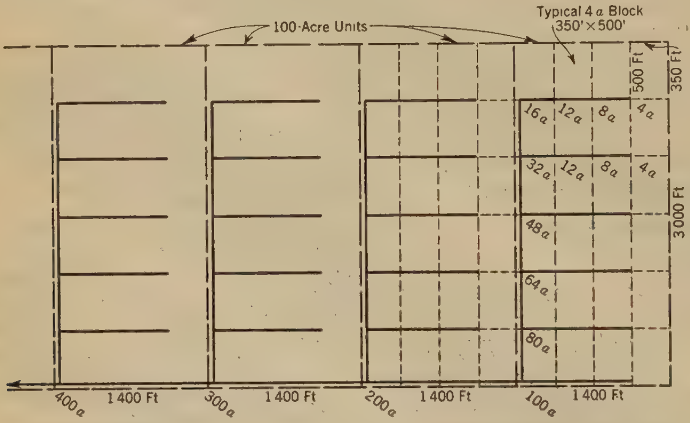


FIG. 6.—ASSUMED LAYOUT USED AS A BASIS OF SEWER LENGTHS PER ACRE

Using the results of the cost computations, curves representing costs per acre for 32 of the 36 cases were plotted on Fig. 7. The studies of cost per acre do not give values for areas smaller than 100 acres, as the purposes of the study are amply served without dealing with smaller areas.

The principal purpose of computing costs per acre, as given by Fig. 7, was to relate or compare costs of the various cases. Although the lengths per acre used may vary considerably from those for an ordinary problem, they are so proportioned for the various areas that the relative costs are fairly representative of the ordinary problem. To compare results in any ordinary problem with the results of this study, merely determine the accumulated sewer lengths per acre as in Table 6(a), and proportion the results to a comparative basis.

Note that in connection with computing costs per acre, the use of the interpolated costs per foot as taken from Fig. 5 was necessary to eliminate the 5% to 10% variation or error that would result from using the cost per foot of the sewer size which came nearest to serving the area in question.

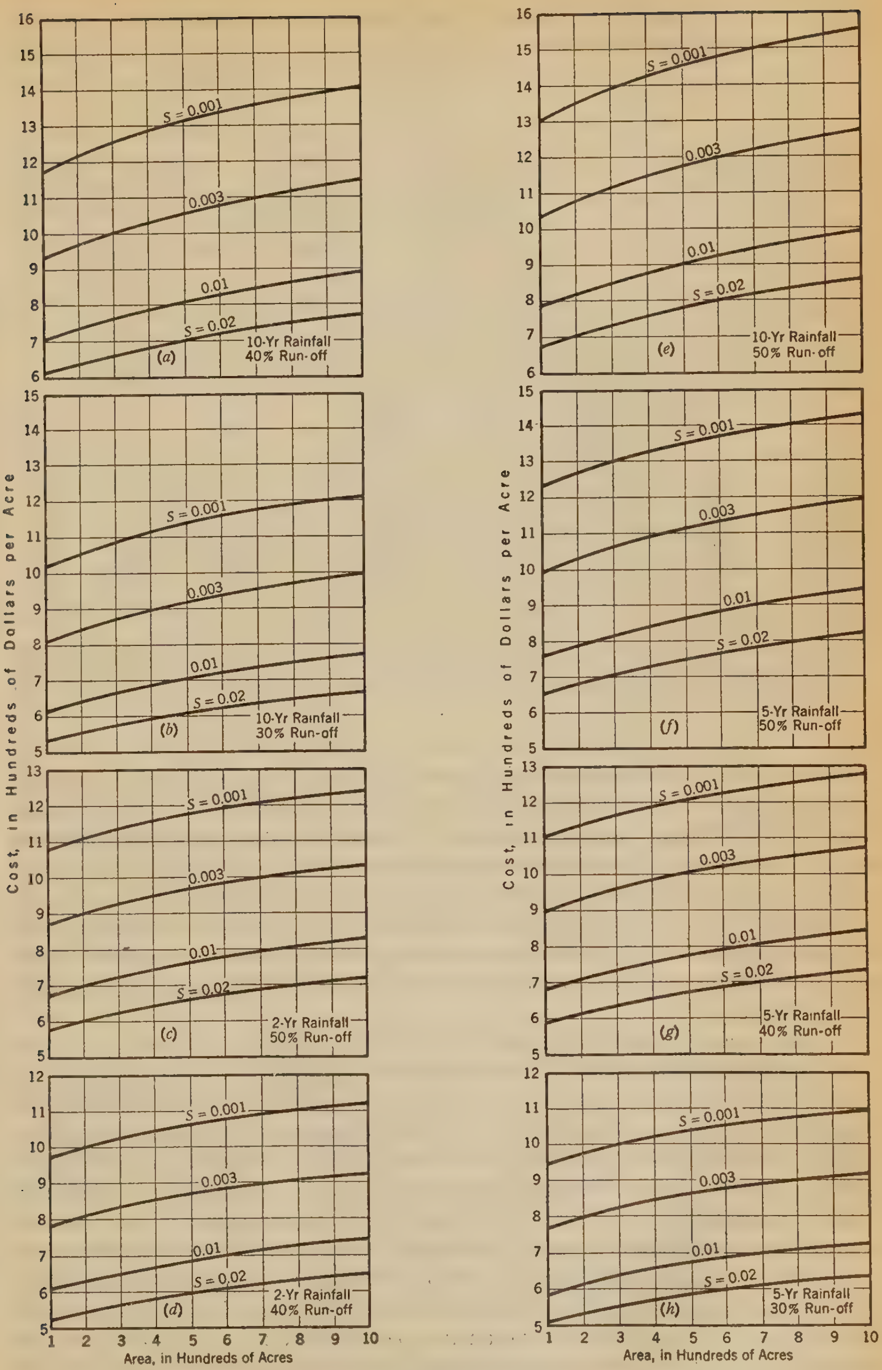


Fig. 7.—Costs per Acre

TABLE 6.—Costs and Lengths of Sewers

	(*a*) Lengths		(*b*) Costs‡				
Area served, in acres	In feet per acre	Accumulated lengths, in feet per acre	In dollars per foot	Total, in hundreds of dollars (Column (2) × Column (4))	Accumulated Costs		
					In hundreds of dollars	In Dollars per Acre	
						Column (6) ÷ Column (1)	$610 + Column (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4	350 ft per 16 *a* = 21.9	21.9	3.40	....	....	74*	....
8	350 ft per 16 *a* = 21.9	43.8	4.70	....	....	103*	....
12	350 ft per 16 *a* = 21.9	65.7	5.60	....	....	123*	....
16	500 ft per 80 *a* = 6.25	71.9	6.40	....	....	40*	....
32	500 ft per 80 *a* = 6.25	78.2	8.70	....	....	54*	....
48	500 ft per 80 *a* = 6.25	84.4	10.30	....	....	64*	....
64	500 ft per 80 *a* = 6.25	90.7	11.60	....	....	72*	....
80	500 ft per 80 *a* = 6.25	96.9	12.80	....	....	80*	610
100	1 400 ft per 100 *a* = 14.00	110.9†	14.10	197	197	197	807
100 to 200	2 800 ft per 200 *a* = 14.00	110.9	19.10	267	464	232	842
100 to 300	4 200 ft per 300 *a* = 14.00	110.9	22.80	319	793	261	871
100 to 400	5 600 ft per 400 *a* = 14.00	110.9	25.80	361	1 144	286	896
100 to 500	7 000 ft per 500 *a* = 14.00	110.9	28.20	394	1 538	307	917
100 to 600	8 400 ft per 600 *a* = 14.00	110.9	30.00	420	1 958	326	936
100 to 700	9 800 ft per 700 *a* = 14.00	110.9	31.80	445	2 403	343	953
100 to 800	11 200 ft per 800 *a* = 14.00	110.9	33.20	465	2 868	358	968
100 to 900	12 600 ft per 900 *a* = 14.00	110.9	34.80	487	3 355	372	982
100 to 1 000	14 000 ft per 1 000 *a* = 14.00	110.9	36.10	505	3 860	386	996

* Actual cost, Column (2) × Column (4). † No increase in accumulated sewer lengths per acre for areas greater than 100 acres, for a layout on which sewer lengths are based (see Fig. 6). ‡ For 10-yr rainfall, 30% run-off, and 0.003 slope.

Effect of Various Factors on Cost

Using the costs per acre from curves in Fig. 7, an analysis of relative costs was made and plotted in the form of curves in Fig. 8. The relative costs for each factor were determined by using various combinations of the other three factors and then running through the range of values of the factor under consideration with each combination. In the cases of slope and run-off coefficient, it was found that relative costs for the factor under consideration did not vary materially with the various combinations of the three other factors. Hence, the cost relations for each of these factors are represented by one curve.

In the cases of the rainfall and area factors, it was found that there was some variation of cost relation due to different combinations of the other three factors, and limiting curves were necessary to represent the cost relations for these factors. In the case of the rainfall frequency relations, the upper limiting curve is shown for slopes of 0.01 and 0.02 (steep slopes) and an area of 100 acres (small area), whereas the lower curve is for slopes of 0.001 and 0.003 (flat slopes) and an area of 1 000 acres (large area). The analysis revealed that only slight difference was due to variation of run-off coefficients. The difference due to variation of area from 100 acres to 1 000 acres was found to be about the same as that due to variation of slopes from 0.02 to 0.001. Hence, these factors should be taken into consideration in using the chart to estimate decrease or increase in cost due to the use of a lower or higher rainfall frequency. For example, for a 2-yr frequency, a slope of 0.002, and a 500-

acre area, the value would be found slightly above the lower curve; or, the cost for a 2-yr frequency would be about 80% of the cost for a 10-yr frequency.

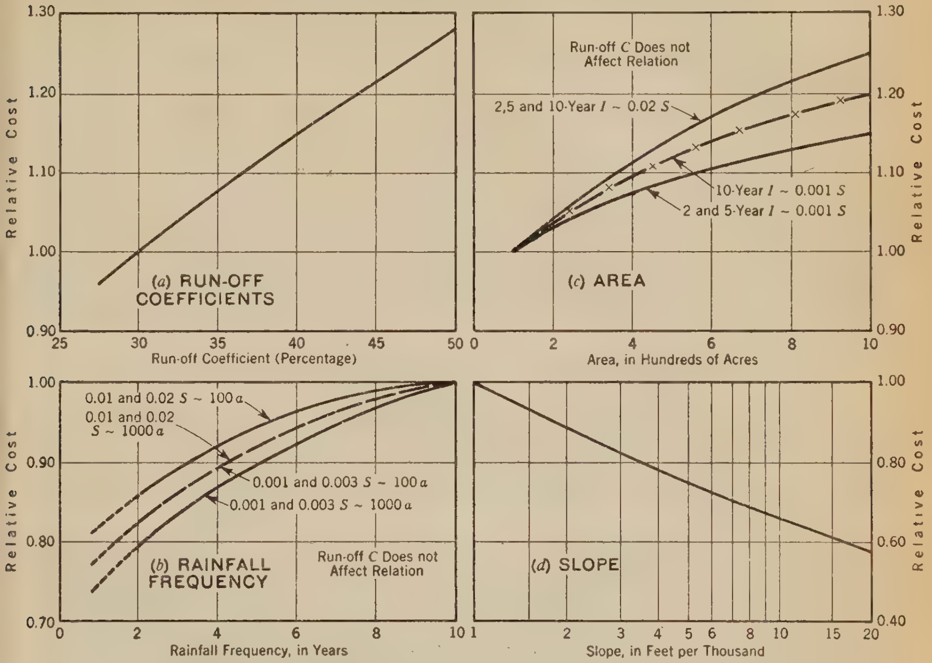


FIG. 8.—RELATIVE EFFECT OF VARIOUS FACTORS ON COST

In the case of area and slope factors, the cost relation curves are of no practical use other than as a matter of general interest because there would be a variation of these factors only in separate problems; for example, in the case of slope, if in a certain problem the slopes were 0.01 instead of 0.001, the cost relation would be 64%, or 36% less. In the case of area on the basis of a 10-yr rainfall and a 0.001 slope, the cost relation for 1 000 acres would be 120% of that for 100 acres, or 20% more.

EXAMPLES

To illustrate the practical use of the results of this or similar studies, a number of examples are given.

Example 1.—Assume a 10-yr rainfall, a 30% run-off, and a slope of 0.003. The design of a main trunk in accordance with these data is indicated in the following tabulation:

Areas to be served, in acres	Read run-off, in cubic feet per second, from Fig. 4(*b*)	Read diameter, in inches, from Fig. 5(*b*)
20	27	33
50	62	42
100	118	54
200	220	72
400	405	90

Example 2.—Assume the case of an area, for which it is desired to determine the capacity required, or the size of a main trunk outlet, without computing the time of concentration for the system. The area is 200 acres; the slope, −0.01; the frequency, a 5-yr rainfall; and a 30% run-off. Fig. 4(*c*) gives 215 cu ft per sec as the capacity required; or, Fig. 5(*c*) gives 54 in. as the required diameter.

Example 3.—Suppose the cost of a system of 1 000 acres has been computed on the basis of a 10-yr rainfall, a 30% run-off, and a slope equal to 0.001; and, the cost is found to be $1 000 per acre. If the relative cost on the basis of a 5-yr rainfall should be desired, Fig. 8 gives the 5-yr cost relation as 90% of that for 10 yr; hence, the cost would be 90% × $1 000 = $900 per acre.

Example 4.—Suppose the cost per acre of a system computed on the basis of 40% run-off has been found to be $800 per acre. To find the cost on the basis of 30% run-off, Fig. 8 gives the cost relation as 115 for 40% and 100 for 30 per cent. Hence, the cost would be $\$800 \times \frac{100}{115} = \$700$ per acre.

Example 5.—By the use of Fig. 7, the cost relations for any problem with any combination of factors can be determined. Assume a problem with 500 acres, and $S = 0.01$. Then, suppose it is desired to know the comparative cost computed on the basis of a 10-yr rainfall and a 40% run-off as against a 2-yr rainfall and a 30% run-off. From the 0.01-curve in Fig. 7, for 500 acres (10-yr rainfall, and 40% run-off), the cost per acre is $810. From the 0.01-curve in Fig. 7 for 500 acres (2-yr rainfall, and 30% run-off), the cost per acre

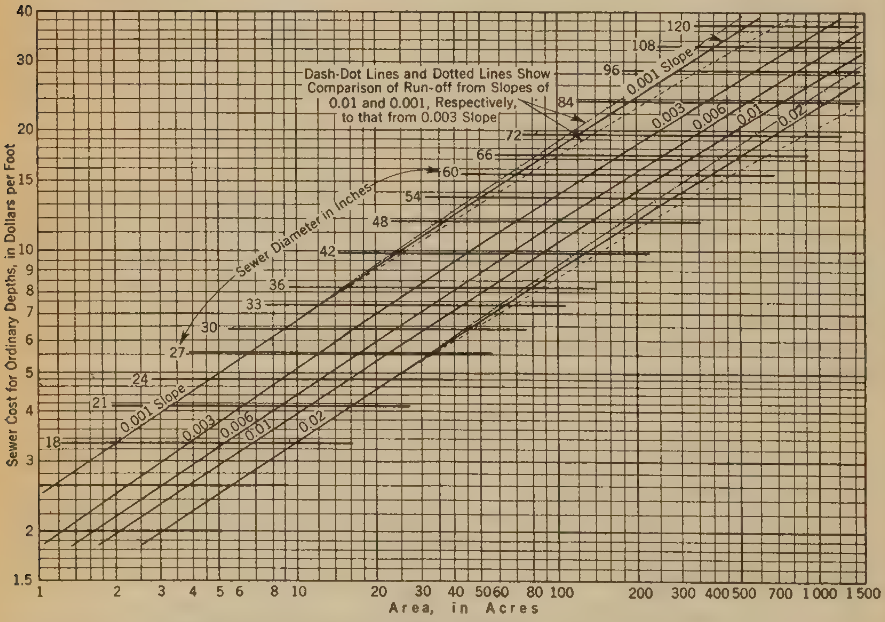


FIG. 9.—DESIGN CHART FOR STORM SEWERS; BASED ON TEN-YEAR RAINFALL; 30 PER CENT RUN-OFF; $S = 0.003$; AND KUTTER'S $n = 0.013$

is \$600. Hence, if a problem had been solved on the basis of a 10-yr rainfall, and a 40% run-off and if the cost had been found to be \$800 per acre, the cost on the basis of a 2-yr rainfall and a 30% run-off would be $\$800 \times \frac{600}{810} = 800 \times 74\% = \$595$ per acre. This example should make clear that the use of the costs per acre in Fig. 7 is as an index of cost for comparative purposes and not for direct answers, in cost per acre, for a particular problem.

DESIGN CHART

As a result of this study, the writer has developed a chart (Fig. 9) for use in storm-sewer design which has proved to be a valuable aid in preliminary design and in checking designs. The sample-design problem in Table 7 is solved by the customary method and the diameters required are found to be identical with those determined by Fig. 9.

TABLE 7.—DESIGN BY THE USUAL METHOD

(Design basis: 10-year rainfall; 30% run-off; $S = 0.003$)

Accumulated area, in acres (1)	Distances between inlet points, in feet (2)	Time, in minutes (3)	Rate of run-off* $I \times 30\%$, in cubic feet per second per acre (4)	Run-off, in cubic feet per second, Column (1) × Column (4) (5)	Diameter† of sewer required, in inches (6)
30	....	17	1.29	39	36
60	700	17 + 2.5 = 19.5	1.21	73	48
140	900	19.5 + 2.5 = 22	1.18	165	63
180	700	22 + 2 = 24	1.13	203	69
200	600	24 + 2 = 26	1.10	220	72
250	600	26 + 1 = 27	1.08	270	78
350	800	27 + 2 = 29	1.04	364	84
600	2 500	29 + 5 = 34	0.96	576	102

* See Fig. 3 for rainfall intensities. † Sewer sizes based on 0.013 value of Kutter's *n*.

ACKNOWLEDGMENT

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TRANSACTIONS

Paper No. 2044

OBSERVED EFFECTS OF GEOMETRIC DISTORTION IN HYDRAULIC MODELS

By Kenneth D. Nichols,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Herbert W. Ehrgott, E. H. Taylor, Herbert D. Vogel, and Kenneth D. Nichols.

Synopsis

Geometric distortion in hydraulic models is a systematic change from geometric similitude involving the use of exaggerated depths or slopes. In many cases the advantages of distorted models more than compensate for their disadvantages; hence distorted models are commonly used in many countries. Questions of costs and benefits, size of prototype, laboratory facilities, time, and requirements of turbulent flow and adequate tractive force in the model may make geometric distortion necessary or desirable in hydraulic laboratory models. For both technical and economic reasons the degree of geometric distortion is usually greater for large than for small prototypes.

In this paper, studies of specific models are summarized and analyzed for effects of geometric distortion in the behavior of the models. Comparative data are presented for an undistorted and a distorted model of the same prototype showing effects of slope distortion and depth distortion upon velocity conversion factors, from model to prototype. Data are presented for other models showing effects on bed formations of variations in bed materials (including light-weight materials), and slope and depth distortions. Selected studies of flood control and river navigation in the Mississippi River System are summarized and analyzed. Brief reference is also made to observations, pertinent to effects of geometric distortion, in many other hydraulic studies.

Analysis of the data showed that a lesser degree of distortion may be required in movable bed models if light-weight materials are used to simulate the stream bed. The analysis also outlined representative methods of compensating for distortion and effecting hydraulic similarity in specific details when over-all similarity cannot be expected.

Note.—Published in June, 1938, *Proceedings*.

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On the basis of this study recommendations are made concerning the use, design, and operation of distorted models.

INTRODUCTION

Definition of Geometric Distortion.—Geometric distortion in hydraulic models is a systematic change from geometric similitude involving the use of exaggerated depths or slopes.

The general term, "geometric distortion," should be sub-divided into "depth distortion" and "slope distortion." In depth-distorted models the depth scale, $d_r = \frac{d_m}{d_p}$, is greater than the length or horizontal scale, $L_r = \frac{L_m}{L_p}$. The degree of depth distortion, $b = \frac{d_r}{L_r}$, is the ratio of the depth scale to the length scale. The degree of slope distortion is the slope scale, $S_r = \frac{S_m}{S_p}$. In slope-distorted models the slope scale is other than unity, and S_r may be greater than or less than b. Depth distortion and slope distortion may be used independently, or in combination. In practice, depth distortion is usually accompanied by slope distortion of equal or greater degree. Other types of geometric distortion have been proposed, but their use is not common enough to warrant further discussion.

Advantages and Disadvantages of Distorted Models.—The advantages of distorted models may be listed as follows:

(1) In order to satisfy the minimum requirements of tractive force, turbulent flow, or measurable depths for a model of any prototype, a larger horizontal scale is necessary if the model is undistorted than if distorted. (Comparisons between a distorted model and an undistorted model apply also to models with a high degree of distortion and with a low degree, respectively.) The distorted model will have the following advantages over the undistorted model: (*a*) It will occupy a smaller space; (*b*) less water will be needed; (*c*) the laboratory facilities and personnel required will be less; (*d*) a shorter time may be required for the model study; and (*e*) the cost of the model study will undoubtedly be less.

(2) For very large prototypes the cost of the smallest practical undistorted model may be prohibitive, whereas a smaller distorted model of lesser cost may be economically justifiable and may serve the desired purpose.

The disadvantages and limitations of distorted models are:

(1) A smaller distorted model usually will depart from true similarity in more details of model performance than the smallest practical undistorted model of the same prototype;

(2) Owing to the lack of all-around similarity, a distorted model must be designed to obtain hydraulic similarity in specified details; hence, it cannot be used for all-around studies;

(3) A distorted model usually needs more extensive field data for verification purposes than an undistorted model: and.

(4) Because of the uncertainty of many of the effects of distortion the dangers of a model study being a failure are greater for a distorted model than for an undistorted model.

Degree of Geometric Distortion Used in Practice.—For any prototype the questions of costs and benefits, size of prototype, laboratory facilities, time, and requirements of turbulent flow and adequate tractive force in the model may make geometric distortion necessary or desirable. These factors combined with certain natural characteristics of rivers usually require a greater degree of distortion for large prototypes than for small prototypes. This is borne out in experience as follows: Considering chiefly costs and benefits, in practice the length scale used in hydraulic models decreases as the size of the prototype increases. For example, in hydraulic laboratories in Prague and Zurich one finds a length scale of 1 : 50 used and suitable for the small rivers in Czecho-Slovakia and Switzerland; at Karlsruhe Laboratory, in Germany, a scale of 1 : 170 or 1 : 200 for a Middle Rhine River model; at Berlin, Germany, 1 : 200 for a Middle Elbe River model and 1 : 500 for a Lower Elbe River model; at Budapest, Hungary, 1 : 500 for the Danube River; and at Vicksburg, Miss., 1 : 450 to 1 : 2 000 for the Mississippi River.

Due mainly to technical reasons and to a change in the characteristics of rivers with size, the depth scale for these models does not fluctuate over such a wide range. In the model a certain minimum depth and slope are usually necessary to insure turbulent flow, measurable depths, or sufficient tractive force. Moreover, for most rivers, as the size of the river increases, the slope and the grain size of the bed material decrease, and the ratio of width to depth increases. Consequently, for larger prototypes the depth scale must be proportionately greater than the length scale.

In the foregoing models, the range in depth scales was only 1 : 50 to 1 : 200, compared to a range in length scales of 1 : 50 to 1 : 2 000. The range in depth distortion was from 1 to 20, and the range in slope distortion was from 1 to 26. Models of the larger prototypes were built with the greater degree of geometric distortion.

DESIGN OF DISTORTED MODELS

Design of River Models.—The design of a geometrically distorted model, like most engineering design, involves the careful balancing of many conflicting factors, and requires much sound judgment if the finished product is to be useful. The procedure in hydraulic model studies is as follows:

I. Preliminary Work.—

(*A*) Study all aspects of the field problem and determine those phases for which model tests may be of value. Consider the limitations of distorted models. Decide on the general type, scope, and purpose of the model study.

(*B*) Secure and assemble all necessary field data. Special data may be necessary for the verification of a distorted model.

II. Design of Models.—

(*A*) Tentatively, select the extent of model area, and the horizontal and vertical scale ratios. If a movable bed model is considered, also tentatively select the bed material.

(*B*) Modify this tentative design on the basis of: Laboratory facilities available (particularly the water supply and space available); turbulent flow requirements; the limits of accuracy of measurements; roughness requirements to obtain the desired velocity scale (for movable bed models substitute tractive force requirements to move bed material); the probable effects of geometric distortion; and a comparison of the costs of the model with the value of benefits to be derived from the model.

(*C*) Prepare adequate plans for the verification and operation of the model. In the preparation of these plans particular attention must be given to the probable effects of distortion.

(*D*) Complete the design of the model.

III. Construction of Model.—

(*A*) Use construction methods that will permit subsequent necessary alterations to be made. In movable bed distorted models it may be desirable to modify the degree of distortion after the model is built.

IV. Operation of Model.—

(*A*) Conduct adjustment and verification tests. Additional field data may be necessary to complete the verification tests. Special observations should be made to determine the effects of geometric distortion.

(*B*) Conduct formal model tests pertaining to the field problem. For these tests the model should be operated in accordance with special operating conditions determined during the verification tests. During all the tests make observations to determine the effects of geometric distortion, particularly the effects on the comparative powers of the model.

V. Report.—

(*A*) Report and interpret the model test data and observations. The interpretation of the effects of distortion on model results is of major importance.

Other Fundamental Principles.—In every step of the foregoing outline the importance of the effects of geometric distortion is emphasized. Evidently, the design of a distorted model cannot be outlined independent of the necessary preliminary work and subsequent construction, operation, and report. The designer of a distorted model must consider all phases of the study during the design phase; particularly, he must anticipate the effects of distortion in the subsequent verification and operation of the model and in the interpretation of the test data.

In addition to the foregoing procedure the following four fundamental principles of distorted model design and operation are presented:

(1) Field data must be available for a model verification, and definite plans must be made for this verification. The model operator must assure himself by appropriate comparisons of model and prototype behavior that the model faithfully simulates the prototype in the particular details being studied.

(2) A distorted model should be designed for a particular problem. A given model can only be used to solve problems closely related to the one for which it was designed. If the study involves several different problems, two or more models or a larger model may be necessary.

(3) The effects of scale, and of depth and slope distortion, should be analyzed carefully in conjunction with the fundamental laws of hydraulics in order to avoid costly mistakes in the construction and operation of the model, and the interpretation of model results. In considering these effects it should be remembered that the primary aim of most distorted models is to effect hydraulic similarity in specific details when over-all similarity cannot be expected.

(4) Light-weight bed materials may be used in movable bed models in order to reduce the degree of geometric distortion necessary.

It is evident that a knowledge of the effects of geometric distortion is of major importance in the design, operation, and interpretation, of distorted models. Because most knowledge concerning the effects of distortion is obtained from specific model studies, the experience of the laboratory personnel must be considered in model design in some such way as are the laboratory facilities. The experience of the designer undoubtedly influences his design.

ABSTRACTS OF MODEL STUDIES

Fourteen hydraulic studies pertaining to the observed effects of geometric distortion are summarized in Table 1 and further described in the following text. Items Nos. 4 and 13 refer to studies in Berlin, Germany (Preussische Versuchsanstalt für Wasserbau und Schiffbau); Item No. 7 was from the University of Manchester, Manchester, England; and the remainder were tests made at the United States Waterway Experiment Station, at Vicksburg, Miss., Item No. 14 being in part from Cornell University, at Ithaca, N. Y. Items Nos. 4, 7, 10, 11, and 13 had sand beds; Item No. 12 had a light-weight material for the bed; Item No. 14 had both fixed and sand beds; and the remainder were all fixed beds.

Item No. 1, Table 1.—Tests of the St. Clair River sills were conducted in a tilting flume, supplementary flume tests being conducted in conjunction with the model study to determine the back-water effect of submerged sills (21) (33).[2] The conclusions reached were: (1) The back-water produced by any given sill varies as the square of the velocity of approach; (2) tests conducted with equal Froude's number gave approximately similar results regardless of scale; (3) the shape of the sill, particularly that of the up-stream face, influences the back-water produced by the sill, the back-water being greater for a vertical face than for a sloping face; and, (4) for the large spacings proposed, the effect of one sill is not reduced by the addition of more sills.

Items Nos. 2 and 3, Table 1.—Models were constructed of a 3-mile stretch of the St. Clair River (just below Lake St. Clair) to determine the back-water effects of a series of submerged sills (21) (33). The relation between depth scale and length scale for the distorted model is expressed by:

---

[2] Numbers in parentheses refer to the Bibliography in the Appendix.

$$d_r = L_r^{0.75} \qquad \text{(1)}$$

In the distorted model the sills were built to an undistorted scale of 1 : 30. It was impossible to roughen the bed surface sufficiently to obtain the required Froudian Velocity Scale, the relation between the velocity scale and the depth scale being:

$$v_r = 1.15\, d_r^{0.5} \qquad \text{(2a)}$$

instead of,

$$v_r = d_r^{0.5} \qquad \text{(2b)}$$

TABLE 1.—Data Pertaining to Model Studies

Study No.	Type of study	Remarks*	Model Characteristics			
			Length ratio, L_r	Depth ratio, d_r	Degree of depth distortion, b	Degree of slope distortion, S_r
1.....	Flume tests; St. Clair River	Average sill was 12 ft high in 42 ft of water; (21) and (33)	1 : 16 1 : 60 1 : 100 1 : 120	1 : 16 1 : 60 1 : 100 1 : 120	1.0 1.0 1.0 1.0	¶ ¶ ¶ ¶
2.....	Model tests; St. Clair River	Approximately 3 miles of river reproduced in both models; (21 and (33)	1 : 100	1 : 30	3.3	3.3 : 1
3.....	Model tests; St. Clair River		1 : 100	1 : 100	1.0	1.0 : 1
4.....	Flume tests; Elbe River	Active bed movement but stable banks; (27) and (43)	1 : 200 1 : 200	1 : 40 1 : 200	5.0 1.0	8 : 1†† 13 : 1‡‡
5.....	Model study; Ohio River	Coney Island model study, Dam No. 36; (21) and (44)	1 : 250	1 : 60	4.2	4.2 : 1
6.....	Model study; Mississippi River	Helena-Donaldsonville model; (21) and (44)	1 : 2 000	1 : 100	20.0	20 : 1
7.....	Model study; Severn Estuary	Tidal model of the Severn Estuary; (12)	1 : 8 500 1 : 8 500	1 : 100 1 : 200	85.0 42.5	85.0 : 1 42.5 : 1
8.....	Model study; Mississippi River at Chicot Landing, Miss.	Effect of distortion on the distribution of velocity in a model stream; (21) and (34)	1 : 1 000 1 : 1 000 1 : 1 000	1 : 100 1 : 167 1 : 250	10.0 6.0 4.0	10 : 1 6 : 1 4 : 1
9.....	Model study, Mississippi River, at Vicksburg, Miss.	Effect of distortion on the distribution of velocity in a model stream; (21) and (34)	1 : 1 000 1 : 1 000 1 : 1 000 1 : 1 000	1 : 100 1 : 125 1 : 167 1 : 250	10.0 8.0 6.0 4.0	10 : 1 8 : 1 6 : 1 4 : 1
10.....	Model study; Mississippi River	Model of Fitler Bend; (21) and (44)	1 : 500 1 : 500 1 : 500	1 : 150 1 : 150 1 : 150	3.3 3.3 3.3	26.2 : 1 19.0 : 1 15.7 : 1
11.....	Model study; Mississippi River	Robinson Crusoe Island model; (21) and (44)	1 : 1 000	1 : 125	8.0	15 : 1
12.....	Model study; Mississippi River	Memphis (Tenn.) Depot model; (21) and (44)	1 : 450 1 : 450	1 : 150 1 : 150	3.0 3.0	20.0 : 1 12.5 : 1
13.....	Flume test; Berlin, Germany	Effect of geometric distortion on bed load movement in a bend; (27)	† †	‡ §	X¶ 3X¶	§§ ‖‖
14.....	Flume test	Bifurcated flume tests; (4), (21), (36), and (44)	†^{a}	‖	**	¶¶

* Numbers in parentheses refer to corresponding items in Appendix. † Width of flume = 6.6 ft. †^{a} Width of flume = 2.0 ft. ‡ d_m = 0.16 ft. § d_m = 0.49 ft. ‖ d_m = 0.25 ft to 0.58 ft. ¶ Variable. ** Varies. †† S_m = 1 : 800. ‡‡ S_m = 1 : 150. §§ S_m = 1 : 400. ‖‖ S_m = 1 : 1 200. ¶¶ Velocity varied from 0.47 ft per sec to 0.99 ft per sec.

This shows that the so-called Law of Compatibility, based on equal roughness coefficient in model and prototype (that is, when d_r is expressed by Equation (1) the velocity scale is expressed by Equation (2*b*)), is of little practical value.

The undistorted model, operated in accordance with Equation (2*b*) indicated less back-water than the distorted model. When operated in accordance with Equation (2*a*), two conclusions were indicated:

(*a*) Sills with vertical faces, or with slopes of 1 on 1, indicated approximately the same back-water as the distorted model, thus supporting the contention that when velocity and roughness requirements can be satisfied a model with a low degree of distortion may be used for this type of problem; and,

(*b*) Sills with faces sloped at 1 on 3 indicated about twice as much back-water as the distorted model. For these latter tests, in the distorted model, the toes of the sills were almost touching and the sills no longer retained their individual action; hence, the back-water was reduced.

Conclusion (*a*) confirms the belief that, in a depth-distorted model, an overfall structure should be built undistorted to the depth scale; but Conclusion (*b*) illustrates that there are limits to the application of this rule.

Item No. 4, Table 1.—Supplementary flume tests were made in the Prussian Experiment Institute for Hydraulic Engineering and Ship Building, in Berlin, Germany, for a model study of the Elbe River (27) (43). These tests were made to determine the relative effects of depth and slope distortion, and the effect of change in cross-section of an impermeable training dike on the scour resulting from the dike. In the first tests, the flume was considered to be a depth-distorted model; but the cross-sections of the dikes were built to various scales $\left(L_r = \frac{1}{40};\ d_r = \frac{1}{40};\ L_r = \frac{1}{60};\ d_r = \frac{1}{40};\ L_r = \frac{1}{80};\ d_r = \frac{1}{40};\ \text{etc.} \right)$, representing depth distortions, b, of 1, 1.5, 2, 3, 4, 5 and ∞.

Two findings pertaining to geometric distortion were:

(1) The change in cross-section of a dike such as would be caused by depth distortion will increase the resulting scour around the dike. The change in slope of the dike from the shore end to the outer end such as caused by depth distortion will also increase the relative scour. The tests did not indicate that the increase of scour would be in the correct ratio to the increase in depth scale; and,

(2) An increase in slope distortion will increase the scour resulting from a dike.

Item No. 5, Table 1.—The purpose of the so-called "Coney Island Model Study" was to find a method of eliminating the back-lash eddy along the lock approach below Dam No. 36 in the Ohio River (21) (44). Similarly, to Items Nos. 2 and 3, the relation between the depth scale and the length scale is expressed by Equation (1). In the model, the dam was built to an undistorted scale of 1 : 60. Two findings are pertinent:

(1) As in Items Nos. 2 and 3 it was determined that sufficient roughness could not be applied in order to reduce the velocity scale to that of the Froudian requirement. By applying roughness, n_m was increased from 0.013 to 0.021. The resulting velocity scale was approximately 20% to 25% too great.

(2) In spite of excess velocities a good reproduction of the back-lash eddy was obtained, and the model was suited for the study.

Item No. 6, Table 1.—A model of a 600-mile stretch of the Mississippi River (see Fig. 1) was constructed in order to determine the comparative value of various flood-control measures (21) (44). The model was particularly effective in giving a vivid picture of the flood problem for the entire Lower Mississippi Valley, by the reproduction of past major floods. Considering this aspect, the model might be termed to have an educational value, which is often under-estimated when evaluating the benefits to be derived from a model study. The conclusions were:

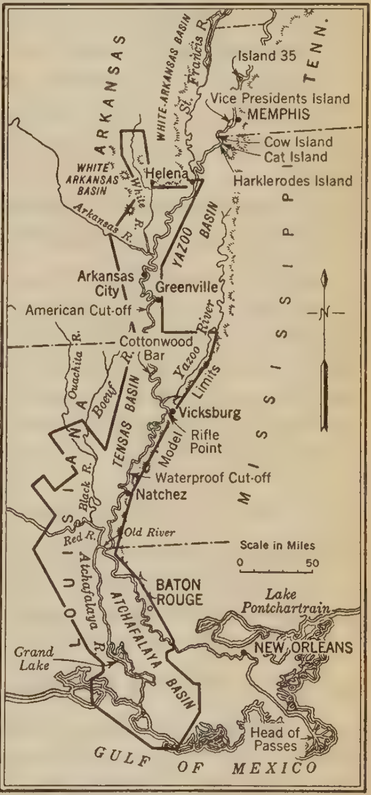

ARKANSAS
WHITE-ARKANSAS BASIN
St. Francis R.
Island 35
TENN.
Vice Presidents Island
MEMPHIS
Cow Island
Cat Island
WHITE ARKANSAS BASIN
White R.
Helena
Harklerodes Island
Arkansas R.
YAZOO BASIN
MISSISSIPPI
Arkansas City
Greenville
American Cut-off
Cottonwood Bar
Ouachita R.
Yazoo River
Limits
Boeuf R.
TENSAS BASIN
LOUISIANA
Vicksburg
Rifle Point
Model
Black R.
Waterproof Cut-off
Natchez
N
Red R.
Old River
Scale in Miles
0
50
Atchafalaya R.
ATCHAFALAYA BASIN
BATON ROUGE
Lake Pontchartrain
Grand Lake
NEW ORLEANS
Head of Passes
GULF OF MEXICO

Fig. 1.—Model of the Mississippi River from Helena, Ark., to Donaldsville, Miss. (Model Scales: Horizontal, 1 : 2 000; and Vertical, 1 : 100).

(1) Before the application of roughness it was determined that the model discharge scale varied with stage from a value of 1.484×10^{-6} to 0.823×10^{-6}. This variation could not be tolerated because the length of the model was so great that the corresponding varying time scale introduced serious errors into the form of a flood wave.

(2) Computations for selected sections of the model revealed that the scale for hydraulic radius varied from 1 : 118 to 1 : 527.

(3) In order to obtain similarity of hydraulic capacity for all parts of the model, variable intensities of roughness were applied and a fixed discharge scale of 1 : 1 500 000 resulted. This discharge scale is in excess of the Froudian scale of 1 : 2 000 000.

(4) In spite of extreme distortion and the resulting difficulties of adjustment, it was possible to verify the model by reproducing recent major floods accurately. The wealth of field data which are available for the Lower Mississippi River System permitted this verification.

Item No. 7, Table 1.—The purpose of this study was to test the effects of a barrage in the Severn River (12). Various sizes of sand were used in an effort to improve the verification. Three findings are to be noted:

(1) The type of sand used as a bed material influences the bed configuration. The height of banks formed in the model was found to be a function of the diameter of the sand grains.

(2) The model having a vertical scale of 1 : 200 was in closer agreement with the prototype than the model having a vertical scale of 1 : 100.

(3) In spite of extreme distortion a good verification was obtained.

Items Nos. 8 and 9, Table 1.—These studies were part of a general research program to determine the effects of geometric distortion (21) (34). They were conducted in existing models, changes in distortion being made by remolding the channel. No effort was made to adjust the roughness to obtain any particular velocity scale. The results showed that:

(1) The degree of distortion affects the distribution of velocity and, therefore, would affect the relative horizontal distribution of energy and the effective tractive force; and,

(2) There appeared to be greater discrepancy between the prototype and the model for a high degree of geometric distortion than for a low degree.

Item No. 10, Table 1.—This was a navigation study to determine methods of improving one or the other of the two channels at Fitler Bend, at Point Lookout, La. (Fig. 2 is a river survey made from September 2 to October 6, 1933). A low degree of depth distortion was considered necessary in order to test various dredging proposals. Light-weight bed materials were not being used at U. S. Waterways Experiment Station at the time of this study.

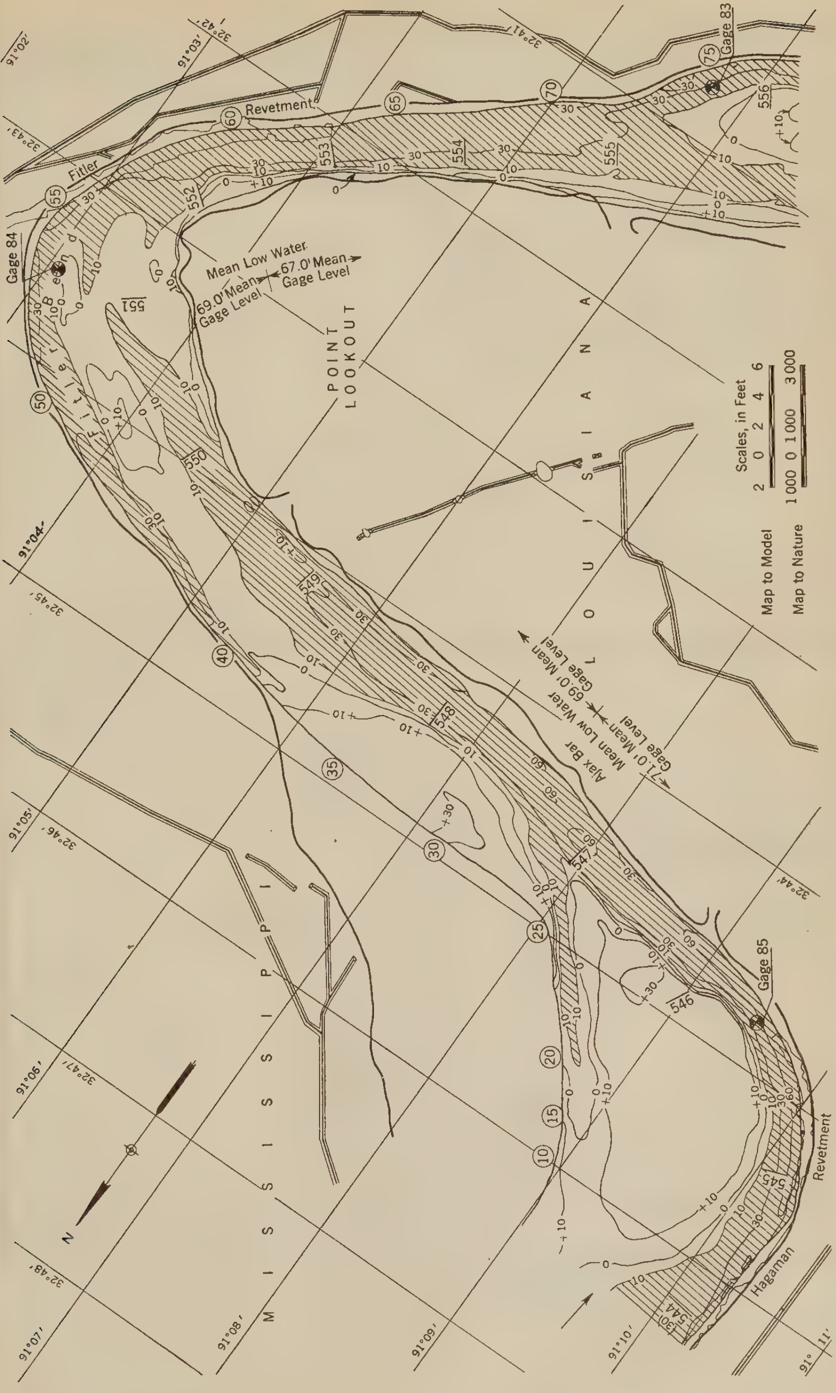
Three general conclusions may be drawn from this study (21) (44):

(1) For a slope distortion of $\frac{26.2}{1}$ the adjustment tests indicated that the resulting velocities were too high. The resulting thalweg was very definitely on the outside of the bend, whereas a survey of the prototype showed greater development of the inside channel.

(2) For a decrease in slope distortion the model verification was improved. Verification tests were run with a slope distortion of $\frac{19.0}{1}$ and $\frac{15.7}{1}$. The latter slope furnished the better verification of natural conditions. In both cases the movement of sand was decreased, but there still remained sufficient tractive force to provide adequate movement of the bed materials.

(3) These tests show that a change in slope distortion may result in changed bed configuration particularly in a change of thalweg alignment. They also indicate the importance of adequate field data for supporting verification tests.

Items Nos. 11 and 12, Table 1.—The Robinson Crusoe Island model (Item No. 11) was designed and operated for a study at, and up stream from, the City of Memphis, Tenn. (21) (44). Because the model included the Memphis Depot area an attempt was made to use this existing model for the Memphis Depot Study (Item No. 12); but it was soon determined that the existing model was not suited for the study, and a new model was designed for the latter case. Items Nos. 11 and 12 were both navigation studies for channel improvements at Memphis, Item No. 11 being up stream from Memphis, and Item No. 12 in the approach channel to the dock of the United States Engineer Office (known as Memphis Depot). Fig. 3(*a*) shows the river survey made of the Memphis Depot area in July and August, 1935. Five conclusions are offered:


POINT LOOKOUT
Fitler Bend
Fitler
Revetment
Gage 84
Gage 83
Gage 85
Mean Low Water
69.0' Mean Gage Level
67.0' Mean Gage Level
Ajax Bar
Mean Low Water
69.0' Mean Gage Level
71.0' Mean Gage Level
LOUISIANA
MISSISSIPPI
Hagaman
Revetment
Scales, in Feet
Map to Model
Map to Nature
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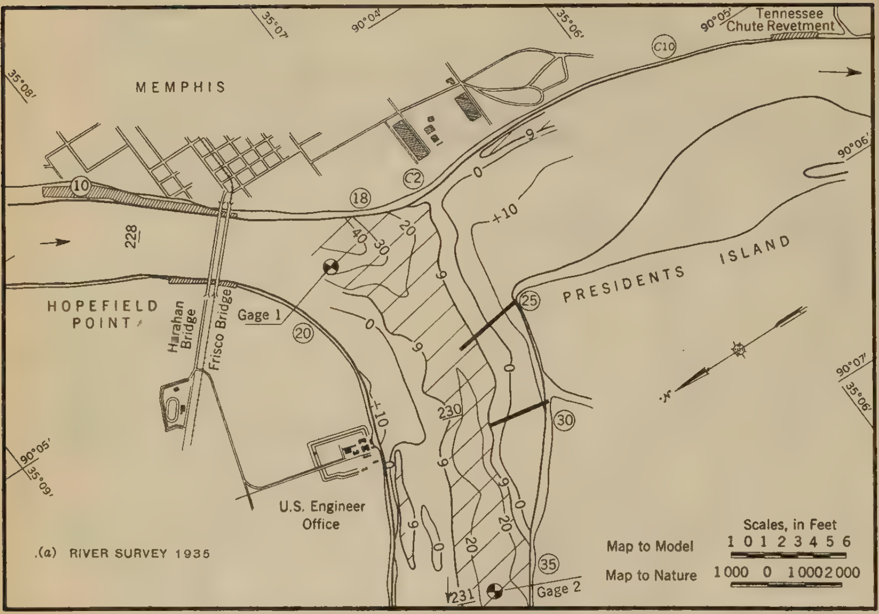
75

FIG. 2.—1933 SURVEY OF FITLER BEND (MISSISSIPPI RIVER), FOR CHANNEL IMPROVEMENT (MILES 543.7 TO 557.2) BELOW CAIRO, ILL.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Tennessee Chute Revetment
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35°08'
MEMPHIS
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C2
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90°06'
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228
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ISLAND
PRESIDENTS
25
HOPEFIELD POINT
Harahan Bridge
Frisco Bridge
Gage 1
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90°07'
35°06'
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230
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90°05'
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U.S. Engineer Office
20
0
Map to Model
Scales, in Feet
1 0 1 2 3 4 5 6
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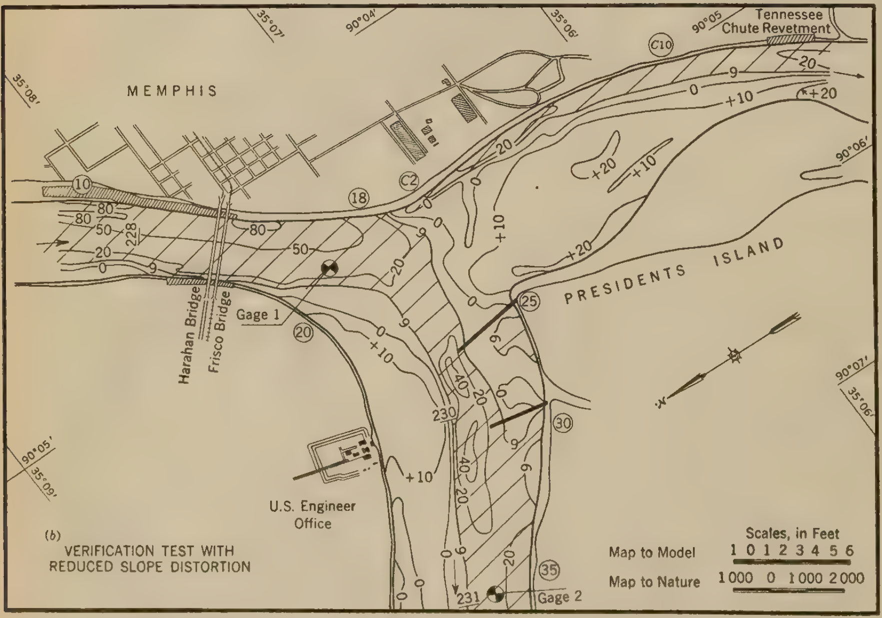
Map to Nature
1000 0 1000 2000
231
Gage 2

(a) RIVER SURVEY 1935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ISLAND
PRESIDENTS
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Harahan Bridge
Frisco Bridge
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Gage 2
Scales, in Feet
Map to Model
1 0 1 2 3 4 5 6
Map to Nature
1000 0 1000 2000

(b) VERIFICATION TEST WITH REDUCED SLOPE DISTORTION

FIG. 3.—MEMPHIS DEPOT MODEL STUDIES FOR CHANNEL IMPROVEMENTS

(1) The Robinson Crusoe Island model was not suited for studying the area in the vicinity of the head of President's Island. Excessive bar building occurred on the left bank, forcing the thalweg to the center of the left-hand channel; and an excessive attack on the head of President's Island resulted.

(2) The Memphis Depot model, as first constructed, was also unsatisfactory due to excessive slope distortion $\left(S_r = \frac{20.0}{1}\right)$. (See Fig. 4.)

(3) Fig. 3(*b*) shows the improvement resulting in the verification tests due to a reduction in slope distortion. The scale was reduced, successively, from $\frac{20.0}{1}$ to $\frac{12.5}{1}$. Fig. 3(*b*) is for a slope distortion of $\frac{12.5}{1}$. A comparison of Figs. 3(*b*) and 4 with Fig. 3(*a*) shows the improvement resulting in verification tests due to a reduction in slope distortions.

Other data are:

Duration of test (Test No. 13, Run 5), in hours	27.5
Slope in prototype	0.00008
Total slope in model	0.0010
Horizontal scale of model	1 : 450
Vertical scale of model	1 : 150
Actual discharge scale	1 : 317 000
Actual velocity scale	1 : 4.7
Theoretical velocity scale, v_r	1 : 12.2
Ratio: $\frac{\text{Theoretical velocity scale}}{\text{Actual velocity scale}}$	1 : 2.6
Mean grain size of bed material, in millimeters	1.040
Specific gravity of bed material	1.85

(4) For this study excellent year-by-year field surveys were available for the area involved, and a more accurate verification thereby resulted.

(5) These tests show the advantages of using a light-weight bed material in combination with moderate depth and slope distortion instead of sand in combination with greater depth and slope distortion.

Item No. 13, Table 1.—In Berlin (27), a study of two models was conducted to determine the relative merits of depth and slope distortion in a model in which the radius of curvature of the channel was 6 m (approximately 20 ft). The following were the most significant findings:

(1) Although the tractive force available in the two models was equal (tractive force expressed as the product of depth times slope), greater bed movement occurred in the model having the greater depth.

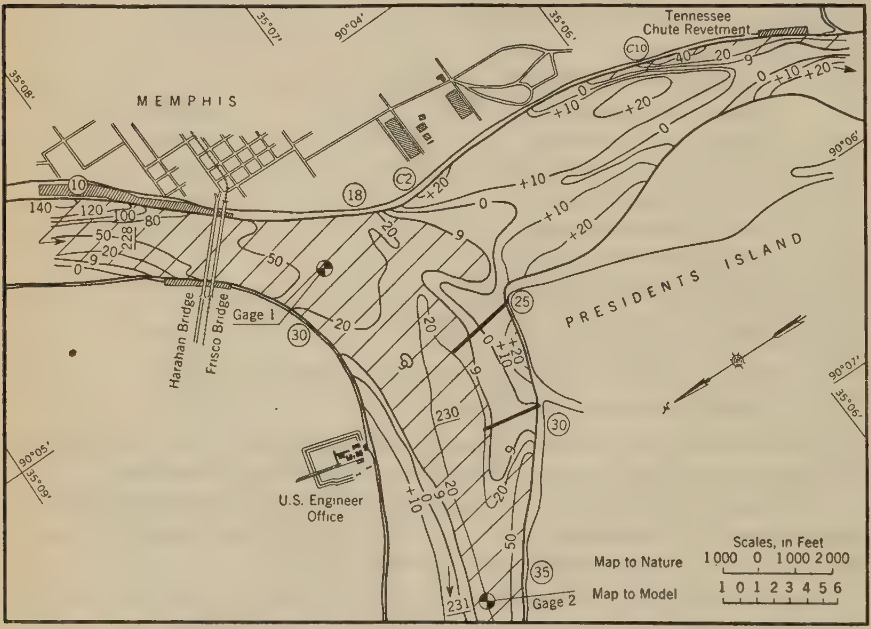
(2) Shape and alignment of sand riffles indicated that greater depth created a greater tendency for diagonal currents or helicoidal flow.

(3) The size of the sand riffles increased with the increased depth.

(4) In the two models the location of the thalweg remained approximately the same; the depths of scour varied approximately with the depth scale. The secondary channel on the inside of the bend was not so apparent in the depth distorted model. This latter is probably due to the increased movement of sand to the inside of the bend, caused by the increase of diagonal currents.

Item No. 14, Table 1.—In the bifurcated flume tests of Item No. 14, tests were conducted with a fixed bed and with a sand bed, various types of sand being tested (4) (21) (36) (44). The purpose was to study the bed-load action at the fork of a stream. One branch of the flume model was a continuation of the approach channel and the other, or side channel, made an angle 30° with the main channel. The approach and two branch channels were of equal rectangular cross-section. (See references given for other types of channels and variation in the angle between the channels.) Pertinent results of these investigations are:

(1) The tests indicated clearly that for equal distribution of flow the water entering the side channel is mainly that flowing along the bottom and one side (nearer the side channel) of the approach channel. The zone of top currents turning into the side channel usually extends over less than 50% of the top width; the zone of bottom currents entering the side channel extends over the greater percentage of the bottom width.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Tennessee Chute Revetment
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MEMPHIS
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C2
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PRESIDENTS ISLAND
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Gage 1
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Harahan Bridge
Frisco Bridge
230
U.S. Engineer Office
90°07'
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90°05'
35°09'
Scales, in Feet
Map to Nature
1 000 0 1 000 2 000
Map to Model
1 0 1 2 3 4 5 6
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Gage 2
231

Fig. 4.—Effect of Excessive Slope Distortion, Memphis Depot Model Study

(2) The data show clearly that for a given depth of flow, as the velocity (or slope distortion) increases, the zone of bottom currents turning into the side channel increases in width. For example, for one velocity the zone of bottom currents entering the side channel may cover 70% of the bottom width of the approach channel, whereas for a higher velocity the zone may cover 80% or 90% of the width.

(3) For equal distribution of water flowing down the two channels, the percentage of bed load entering the side channel was greater than that entering

the straight channel. The percentage of bed load diverted increased as the velocity increased.

(4) For otherwise like conditions, it was shown that the bed material mixture influences the percentage distribution of bed load entering the two channels. As the bed material mixture becomes finer, and a greater percentage of the material moved is in suspension, the percentage entering the side channel decreases.

Analysis of Data and Observations

The foregoing observations, and the data in Table 1, together with some other observations not included, may be classified and analyzed under two major headings: (1) Effects of geometric distortion; and (2) behavior of bed materials. Under the first classification are discussed changes in form, changes in hydraulic capacity, changes in flow characteristics, secondary currents, and miscellaneous effects. Under the second classification are discussed the general behavior of bed materials, and the use of light-weight bed materials.

Changes in Form.—The most obvious effect of depth distortion is in the change in the form of the cross-section of the channel and of the structures. Change in form influences the back-water effect of structures and also, in movable bed models, varies the scour pattern caused by such structures. The ratio of width to depth in model and prototype is not the same, and the relative area of cross-section affected by side-wall influence on hydraulic flow is thereby altered. In movable-bed, depth-distorted models the side slope of the bed material is steepened, which may increase the rate of scour of the material; or, in cases where the angle of repose of the submerged material is exceeded it may result in an undesirable sloughing. Depth distortion changes the shape of the channel in the model, thereby causing the scale for the hydraulic radius to be a variable. The hydraulic capacity of the channel is thus affected. All the foregoing effects of depth distortion must be considered in model design, operation, and interpretation.

Hydraulic Capacity of Channel.—Both depth distortion and slope distortion influence the hydraulic capacity of an open channel. The hydraulic capacity of a channel is its capacity to carry water, as measured by the rate of flow through a given reach for a given stage. Based on Manning's formula for velocity the capacity of a channel in model and prototype is similar only when,

$$Q_r = \left[\frac{L\,d\,R^{0.67}\,S^{0.50}}{n}\right]_r \qquad (3)$$

in which Q_r = discharge scale $= \frac{Q_m}{Q_p}$; R_r = hydraulic radius scale $= \frac{R_m}{R_p}$; and n_r = coefficient of roughness scale $= \frac{n_m}{n_p}$.

In depth-distorted models it has been shown that R_r varies from cross-section to cross-section and also in the same cross-section for different stages; hence, if Q_r is to remain constant, n_r must vary within each cross-section and from cross-section to cross-section. It is common practice to vary n_r by roughening a model by trial and error.

In the Helena-Donaldsonville model (Item No. 6, Table 1) the roughness varied from rough stuccoed concrete to extremely smooth concrete. In fact, in a few isolated sections it was impossible to make the concrete smooth enough and the section was enlarged arbitrarily.

If the slope of the model is also distorted, the roughness must be further increased to compensate for the greater slope, or the discharge scale will increase. In many studies the actual value of the discharge scale is relatively unimportant as long as a similarity of hydraulic capacity is obtained; but in other studies it is desirable to comply with Froude's requirement for the velocity scale. (Supplementary flume tests showed that compliance with Froude's law was desirable for the St. Clair River model tests.) For a distorted model this requirement is met only when Equation (2*b*) holds. Combining Manning's formula and Froude's law, it can easily be shown that this formula applies only when,

$$\frac{R_r^{0.67}\, S_r^{0.50}}{d_r^{0.5}\, n_r} = 1 \qquad \text{(4)}$$

(The criterion (35), $v_r = d_r^{0.5}$ when $d_r = L_r^{0.75}$, is based on the assumption that $n_r = 1$, which is seldom true.)

When the degrees of depth and slope distortion are equal, it follows that greater roughness must be used as the degree of distortion increases. For such a distorted model of a rectangular channel the roughness scale should increase as the width-depth ratio of the prototype, the degree of distortion, or the horizontal scale of the model, increases (21). Consequently, the velocity scale must often exceed the value indicated by Equation (2*b*). Where this requirement must be satisfied, one must roughen the model to extremes, reduce the depth and slope distortion, or reduce the slope distortion.

Changes in Flow Characteristics.—Slope distortion and depth distortion will cause changes in the characteristics of flow. Variation of the depth or slope distortion will affect the magnitude of velocity and also the velocity distribution, such as to change the form of the vertical and horizontal velocity distribution curves. Another effect of distortion is to alter the direction of currents. Particularly, the relative divergence of surface and sub-surface currents may be changed. Distortion may cause eddies to occur in a model where none exists in the prototype; or it may affect the size, shape, location, and intensity of existing eddies or rollers. The intensity of turbulence or the pulsating character of the flow may be altered. Another common effect of distortion is an undesired increase in the transverse slope of the water surface. Many experimental data are available to show specific instances of such changes, but insufficient data are at hand to provide a basis for the formulation of definite laws concerning such changes.

Of particular interest is the effect of geometric distortion on the shape of the vertical velocity curve. Experimental data and field observations on natural streams show definitely that depth, slope, form of channel, and roughness affect the form or shape of the vertical velocity curve. For example, the following statements have been fairly well substantiated by field and laboratory observations:

(*a*) The vertical velocity curve approximates a parabola, the axis of which is horizontal and passes through the point of maximum velocity.

(*b*) As the roughness of the bottom increases, the depth of the filament of maximum velocity and that of mean velocity in any vertical, is less. The ratio of surface to bottom velocities also increases.

(*c*) For the range of model slopes and model depths commonly used in hydraulic laboratories, a change in slope or depth distortion will result in a change in the vertical velocity distribution. The exact nature of the law of variation has not yet been determined conclusively.

(*d*) Side-wall effect on the vertical velocity curve is apparent to a point about 2.5 times the depth from a side wall.

(*e*) A rapidly rising channel bottom—that is, a bottom of adverse slope—tends to make the velocity distribution more uniform, whereas a rapidly falling bottom tends to make the ratio of upper to bottom velocities increase.

Hence, a change in the combination of depth and slope distortion will change the vertical velocity curve in some unpredictable manner. The change in distribution of velocity becomes of particular importance when a bend section or divided flow around an island or obstruction is considered.

Secondary Currents.—When water flows in an open channel secondary currents usually exist. From the standpoint of hydraulic models the most important of these are the cross-currents near the bottom of a stream, which are set up in a channel whenever there is a change in direction of flow such as that caused by a curved channel, a divided channel, a large obstruction, deflecting dikes, etc.

One explanation for the cross-currents is as follows: Usually in an open channel the surface currents are moving faster than the bottom currents. Hence, when a change in direction occurs, it takes less force to change the direction of the bottom, than the surface, currents. At a bend in the channel a transverse slope is set up due to the change in direction. This slope acts as an unbalanced force on each filament of current and deflects it so that the radius of curvature of each filament is approximately proportional to the square of its velocity. In a bend section the radius of curvature is generally less for the bottom than for the top currents and this phenomenon is a part of helicoidal flow.

In general, such cross-currents exist for any change of direction. For example, in Item No. 14, Table 1, the bulk of the water entering the side channel was that water which had been flowing near the bottom of the approach channel. In these tests the horizontal angle between the direction of the top and bottom currents varied from zero to 180 degrees.

In movable bed studies the engineer is particularly interested in the direction of the bottom currents because they are the principal determining factor in the direction and amount of movement of the bed.

Inasmuch as geometric distortion affects the magnitude of velocity and also the form of the vertical velocity curve, it is logical to assume that the nature of the cross-currents will also be influenced. Items Nos. 10, 12, 13, and 14, in Table 1, and other studies, illustrate, directly or indirectly, the effect of a change in distortion on the secondary currents.

Analysis of these studies indicates that usually greater depth or slope distortion will increase the divergence of surface and bottom currents, and that the divergence is more marked for an increase in depth distortion than for an equal increase in slope distortion. Other factors, such as horizontal velocity distribution, relation between wave velocity and mean velocity, roughness, effect of one bend on a lower bend, etc., greatly affect the nature of flow. Additional data are needed before conclusive relationships can be established.

Miscellaneous Effects.—In addition to the specific examples referred to in Table 1, countless other examples of discrepancies introduced by geometric distortion could be presented. Many of these discrepancies are unimportant or may be properly weighed in interpreting model results. Among the more important, especially in movable bed models, are the following effects which have been observed at the U. S. Waterways Experiment Station (in all cases the name of one representative model or reach of a Mississippi River model is given in parentheses, see Fig. 1 for the geographic location of many of these studies):

I. Excessive Scour:
  (*A*) At a bend (Hotchkiss Bend);
  (*B*) At the end of deflecting dikes (Island 35);
  (*C*) At the head of sand-bars, tow-heads, and islands (Cat Island);
  (*D*) At relatively narrow sections (American Cut-Off); and,
  (*E*) At a point of junction of two streams (Vice-President's Island).
II. Improper Location of Scour:
  (*A*) At a bend (Head of Passes); and,
  (*B*) Displacement down stream below a sill or submerged dike (Head of Passes).
III. Unstable Side Slopes:
  (*A*) Steep bank occurring in prototype (Memphis);
  (*B*) Dredging holes or cuts (Cat Island); and,
  (*C*) Sand dikes (Rifle Point).
IV. Other Effects:
  (*A*) Excessive bar building at a relatively wide section (Waterproof Cut-Off);
  (*B*) Displacement down stream and relative straightening of crossing (Cow Island);
  (*C*) Introduction of eddy and a resultant bar at a point of sudden widening of the channel (Cottonwood Bar);
  (*D*) Improper thalweg alignment and sand deposits in a bend (Cat Island);
  (*E*) Improper division of flow and excessive development of one channel and excessive deterioration of the other in cases of divided flow around an island (Harklerodes Island).

Behavior of Bed Materials.—In most model studies the movement of the bed material is by rolling or sliding along the bottom; hence, the movement of material and the resulting bed configuration is largely influenced by the bottom currents. Inasmuch as the direction of bottom currents is influenced by distortion, the bed configuration will also be affected.

A change in the bed material may change the bed configuration. This is illustrated in the Severn Estuary model (Item No. 7, Table 1). An explanation is as follows: A change in the grain size or the specific weight of the bed material will affect the rate of movement and may affect the type of movement (that is, movement by riffles to movement by banks, or true bed-load movement to movement in suspension, etc.). Any change in the nature of movement will increase or decrease the roughness, which, in turn, will affect the magnitude of velocity and the cross-currents. Hence, the bed configuration may be changed by merely substituting one bed material for another. One may take advantage of this factor during the verification of a model. If one bed material will not provide a successful verification, a change of bed material may materially improve the verification.

Light-Weight Bed Materials.—For a given hydraulic model study, the degree of geometric distortion necessary may be reduced by the use of light-weight bed materials instead of natural sand (see Fig. 5 and Table 2). Many laboratories have resorted to the use of such materials. In Europe, broken briquettes of coal, pumice, and amber are being used successfully. The U. S. Waterways Experiment Station has operated models with haydite, coal, and gilsonite and the use of (I) limed rosin has been considered.

TABLE 2.—CHARACTERISTICS OF MODEL BED MATERIAL, CURVES IN FIG. 5

Curve No. (see Fig. 5)	Material	Specific gravity, ρ	Mean grain size, D_g, in millimeters	Slope of test, S
1	Gilsonite	1.028	3.112	0.0003
2	Gilsonite	1.028	1.165	0.0003
3	Gilsonite	1.052	3.526	0.0003
4	Gilsonite	1.070	3.553	0.0003
5	Gilsonite	1.052	1.249	0.0003
6	Gilsonite	1.070	0.899	0.0003
7	Wood rosin (5.5% lime)	1.111	1.317	0.0003
8	Wood rosin (5.5% lime)	1.111	2.780	0.0003
9	"Slack" bituminous coal	1.36	1.081	0.0005
10	"Slack" bituminous coal	1.36	1.081	0.0010
11	"Semi-anthracite" coal (briquettes)	1.30	1.168	0.0010
12	"Carona" coal (bituminous)	1.31	1.097	0.0010
13	"Carona" coal (bituminous)	1.31	1.097	0.0005
14	"Semi-anthracite" coal (briquettes)	1.30	1.168	0.0005
15	"Semi-anthracite" coal (briquettes)	1.30	2.610	0.0005
16	"Semi-anthracite" coal (briquettes)	1.30	2.610	0.0010
17	"Slack" bituminous coal	1.36	2.481	0.0010
18	"Slack" bituminous coal	1.36	2.481	0.0005
19	"Carona" coal (bituminous)	1.31	2.008	0.0010
20	"Haydite" (⅛ in.)	1.85	0.908	0.0005, 0.0010, and 0.0015
21	"Haydite" (No. 6)	1.85	1.040	0.0005, 0.0010, and 0.0015
22	Fitler sand	2.65	0.485	0.0010

Light-weight materials have the advantage over heavier materials in that, for a given grain-size, movement occurs at a lower tractive force and that coarser light-weight materials may be used instead of heavier sand to reduce riffling and the degree of distortion. However, the apparent specific gravity

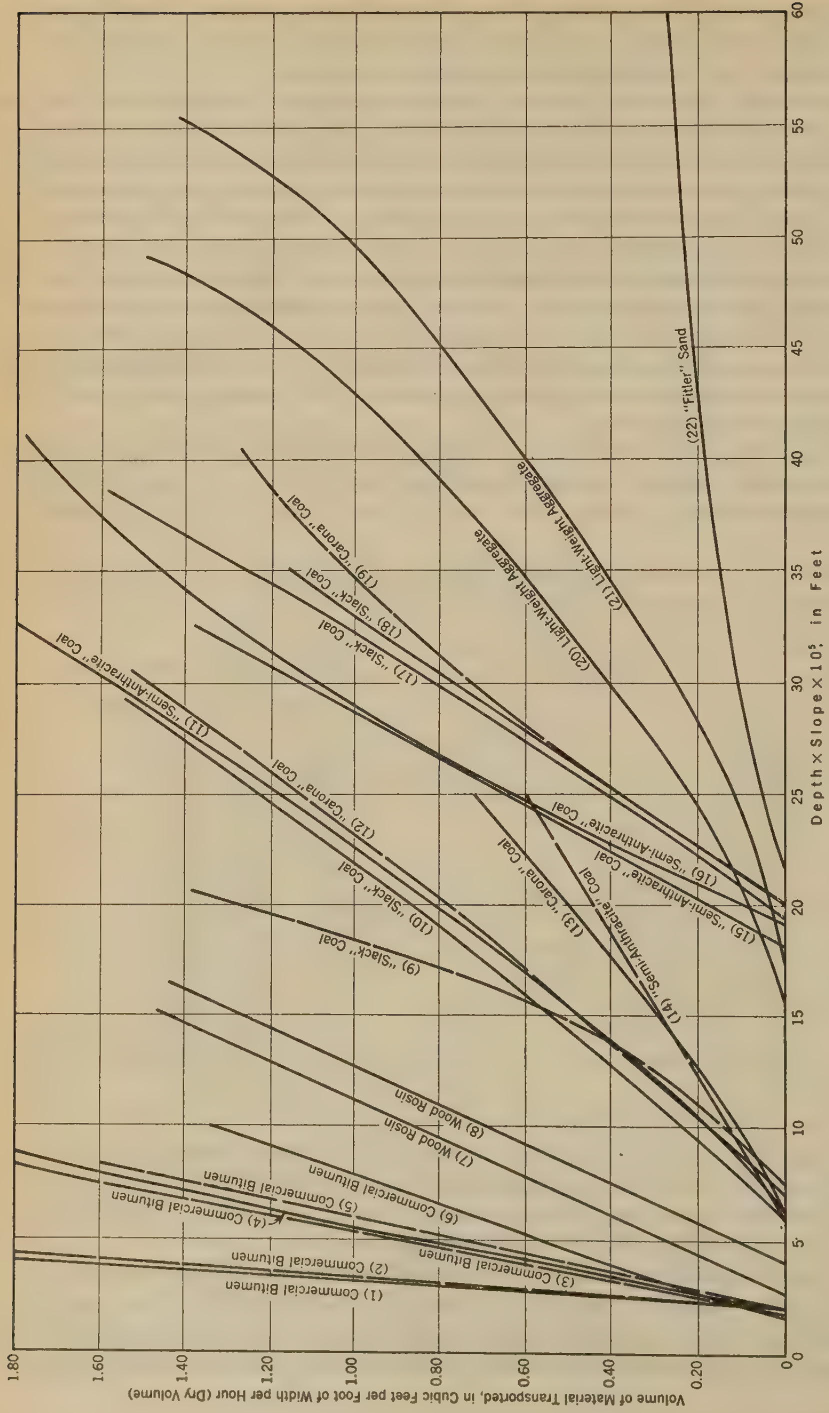

FIG. 5.—RESULTS OF FLUME TESTS ON LIGHT-WEIGHT BED MATERIALS

and size of coal, pumice, and haydite grains change after continued use. Amber and gilsonite are very costly and are extremely difficult to use. Because of their light weight these materials are not suitable for studying wave action on beaches or certain types of river problems where bars are exposed. Unless gilsonite is soaked in water, shaped rapidly to conform to the model bed, and flooded almost immediately, tiny air bubbles attach themselves to the grains and cause the material to float. However, the use of light-weight materials is feasible, and it is recommended as a substitute for extreme geometric distortion. Experience indicates that a combination of moderate depth distortion, moderate slope distortion, and light-weight bed material is better than the use of one extreme to the exclusion of the others.

Conclusions

Summary.—This study shows the following: (*a*) There is a definite need for constructing geometrically distorted models, particularly because such models are often suitable when a geometrically similar model is out of the question on account of cost or other considerations; (*b*) geometrically distorted models should be used with extreme caution; (*c*) arbitrary measures may sometimes be taken to compensate for some of the effects of distortion; (*d*) light-weight bed materials may be used to reduce the degree of distortion necessary; (*e*) in general, a change in the degree of depth or slope distortion, or in the bed material, may result in a change in model results; and finally (*f*) owing to the complex nature of many of the effects of distortion the importance of model verification cannot be over-emphasized.

Recommendations.—On the basis of this study the following recommendations are made: (1) Geometrically distorted river models should be avoided whenever suitable undistorted models can be constructed at reasonable cost; (2) distorted models should be designed for specific purposes and they should not be used for general studies; (3) the designer should weigh, intelligently, the probable effects of distortion in order to avoid failures or reconstruction costs; he should consider carefully the advisability of using light-weight bed materials instead of extreme depth or slope distortion; and (4) the operator should assure himself by appropriate comparisons of model and prototype behavior that the model faithfully simulates the prototype in the particular details being studied.

Acknowledgments

The writer wishes to express his thanks to the Institute of International Education for a fellowship to study this subject in Germany; to Professor H. Seifert, Director, Preussische Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Germany, for offering valuable suggestions and for making available data from that laboratory; to the Directors, past and present, and the Staff, of the U. S. Waterways Experiment Station, Vicksburg, Miss., for advice and assistance; and to F. T. Mavis, M. Am. Soc. C. E., Head of the Department of Mechanics and Hydraulics, State University of Iowa, Iowa City, Iowa, under whose direction this study was completed.[3]

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[3] The paper is based on data contained in "The Observed Effects of Geometric Distortion in Hydraulic Models," a thesis presented by the writer to the State University of Iowa in 1937, in partial fulfillment of the requirement for the degree of Doctor of Philosophy.

APPENDIX

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DISCUSSION

HERBERT W. EHRGOTT,[4] ESQ. (by letter).—The general attitude toward models expressed in this paper may be described as an optimistic skepticism, a wise attitude for all hydraulic experimenters to adopt. When working for a "crank," it is politic to make a close study of his foibles, and to discover the limits beyond which it is not safe to go. No one will question the statement that hydraulic models—particularly open-channel models—are "cranky." The author has undertaken to evaluate the queer quirks of distorted models, while persuading them to yield valuable information despite their shortcomings.

In his discussion of Studies Nos. 2, 3, and 5, Table 1, Lieutenant Nichols shows that the so-called Law of Compatibility is of little value in cases of submerged dikes in open-channel models. This conclusion is correct, and is understandable, when it is considered that the law of compatibility (Equation (1)) was derived for the case in which: (*a*) Model and prototype roughnesses are equal ($n_r = 1$); (*b*) ratio of hydraulic radii is equal to depth ratio ($R_r = d_r$); and, (*c*) there is no supplementary slope $\left(S_r = \frac{d_r}{L_r}\right)$. Of these three assumptions only the third is true in the three studies cited. The hydraulic radius ratio in distorted models is never equal to the depth ratio, and varies from section to section and from stage to stage within each cross-section. The hydraulic roughness ratio in open-channel models can seldom be made equal to unity, as is apparent from Table 3 in which are given all roughness coefficient data for small channels, available to the writer.

Referring to Item No. 15, Table 3, the gravelly earth had the following composition:

Type	Percentage
Arable earth	50
Sand	30
Fine gravel (3 to 10 mm)	10
Gravel (10 to 30 mm)	10
Total	100

In Item No. 20, the value of n decreases with increasing grain size, but not in direct proportion; for example:

Mean grain diameter, D_g, in millimeters	Roughness coefficient, n
0.205	0.026
0.35	0.021
0.56	0.013

The writer is inclined to be skeptical of the value of any quantitative results to be obtained in model studies of open-channel models involving submerged works, in which the channel is distorted, while the submerged sills, dikes, or

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[4] Capt., Corps of Engrs., U. S. Army, Care of Dist. Engr., Binghamton, N. Y.

TABLE 3.—OBSERVED VALUES OF MANNING'S ROUGHNESS COEFFICIENT n FOR SMALL MODELS

Item No.	CROSS-SECTION		Alignment	BED MATERIAL			Number of observations	Roughness coefficient, n
	Shape	Variation between sections		Type	Mean grain diameter, D_g, in millimeters	Movement		
1	Trapezoidal[a]	Uniform	Straight	Clay; smoothed and tamped[g]	....	None	7	0.0105
2	Rectangular[b]	Uniform	Straight	Smooth cement	....	....	95	0.0105
3	Rectangular[b]	Uniform	Straight	Fine sands; smooth bed	<0.6	Weak	52	0.0107
4	Natural	Slight	Curved[e]	Smooth cement	....	....	1	0.0130
5	Semi-circular	Uniform	Straight	Smooth cement	....	....	1	0.0132
6	Trapezoidal[a]	Uniform	Straight	Tamped clay	....	None	9	0.0135
7	Trapezoidal[a]	Uniform	Curved[e]	Tamped clay	....	....	15	0.0145
8	Natural[c]	Uniform	Straight	Sand	....	....	3	0.0146
9	Natural	Average	Curved[e]	No fines[n] ($M = 0.75$)	1.06	Waves	6	0.0148
10	Trapezoidal[a]	Uniform	Straight	Tamped clayey earth[h]	....	....	3	0.0155
11	Rectangular[b]	Uniform	Straight	Sand 9;[n] ($M = 0.57$)[i]	4.0	General	50	0.0161
12	Trapezoidal[a]	Uniform	Straight	Tamped clay[j] and rounded river stones, 10 to 12 cm[k]	....	None	4	0.0182
13	Natural	Moderate	Curved[e]	Haydite; towhead section	1.04	General; waves	6	0.0201
14	Natural[d]	Extreme	Curved[f]	Smooth concrete	....	....	4	0.0203
15	Trapezoidal[a]	Uniform	Straight	Gravelly earth	....	None	9	0.0205
16	Natural	Average	Curved[e]	Slag coal ($\rho_g = 1.36$)	1.08	Waves; general	17	0.0210
17	Rectangular	Uniform	Straight	Heavily stuccoed concrete	....	....	5	0.0214
18	Natural	Average	Curved[e]	Heavily stuccoed concrete[l]	....	....	6	0.0230
19	Natural	Average	Curved[e]	Fine sands	<0.6	Waves and riffles	55	0.0230 to 0.0260
20	Rectangular[b]	Uniform	Straight	Fine sands[m]	<0.6	Waves and riffles	....	0.013 to 0.026

[a] Compound trapezoidal section for proposed Canal d'Alsace. [b] Tilting flume at U. S. Waterways Experiment Station. [c] Stream allowed to scour to equilibrium within fixed banks. [d] Helena-Donaldsonville Model before roughening. [e] Moderately. [f] Extremely tortuous. [g] With extreme care. [h] Including 30% ovoidal calcareous particles which protruded as the clay was washed away. [i] Smooth and scouring bed; no riffles. [j] In the deep section. [k] In the shallow section. [l] Or very much roughened. [m] *Paper 17*, U. S. Waterways Experiment Station. [n] M = Kramer's uniformity modulus.

dams remain undistorted. Such problems involve both Froude's and Reynolds' Laws of Similarity, acting together in undetermined ratios (Eisner's "Zusammenwirkungsgrenz"). Only a geometrically similar model of this type may be counted upon to give results that are transferable to Nature. This is not to say that model studies such as Study No. 3, Table 1, are without value. Quite the contrary. The action of different types of submerged works, as compared with each other, may be revealed clearly by such a study; and a general knowledge of the effects of a moderate degree of vertical distortion indicates that such relative action will probably be closely similar to the corresponding action in the prototype. It is to be hoped that if and when the sills are built in the St. Clair River, a re-check will be made on models to answer the question of transferability of results in problems of this ambiguous type.

In connection with Study No. 6, Table 1, the writer has made a study of the variation of the hydraulic radius ratio from cross-section to cross-section and from stage to stage. Forty typical river cross-sections were plotted to various distortions, and the ratios of hydraulic radii for each section and for each distortion were calculated. It was found that:

(1) The hydraulic radius distortion varies approximately parabolically with the depth distortion;

(2) The hydraulic radius distortion varies with the ratio, $\frac{\text{Width}}{\text{Maximum depth}}$; and,

(3) The width-maximum depth ratio varies with stage, in a quite uniform manner, regardless of exact shape of cross-section.

There was some indication that shape of cross-section had some importance, but the indications were so slight and erratic that it was felt that inclusion of this factor would constitute an unjustifiable refinement.

The author gives a corrected "law of compatibility," into which none of the three assumptions made in the case of Equation (1) enters. This formula (Equation (4)) may be clarified, and its application simplified, if it is rewritten:

$$\left(\frac{d_r}{L_r}\right)\frac{1}{S_r\, L_r^{0.33}} = \left(\frac{R_r}{L_r}\right)^{1.33}\frac{1}{n^2}\,\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

In this form the three adopted scales built, or to be built, into the model appear on the left-hand side, whereas the resulting discrepancy from dynamic similarity, due to differing hydraulic radius distortions for each stage and cross-section, are given on the right-hand side, together with the degree of roughness distortion to correct for the discrepancy. Equation (4) written in this form (see Equation (5)) makes it perfectly clear why adjustment of a fixed bed model, such as the Helena-Donaldsonville Model (Study No. 6, Table 1), involves "cut-and-try" variation of the roughness throughout the model, and at various stages. Theoretically, the exact roughness to be applied for each section and each stage could be computed from Equation (5) (assuming that Manning's equation is strictly applicable to the model, which it is not); actually, it is simpler to apply approximately the correct average roughness and then smooth or roughen the model locally to "iron out" the numerous humps and sags. Equation (4) would seem to indicate that, within the limits imposed by: (*a*) The streaming-shooting limit; (*b*) the laminar-turbulent limit; (*c*) the hydraulic roughness limit (n_m cannot be made smaller than 0.009, nor greater than 0.026, without actually constricting the cross-section); (*d*) the limit imposed by the angle of repose of bed material; and, (*e*) bed movement limit (tractive force limit); any combination of vertical, slope, and hydraulic roughness distortions which makes the left-hand member equal to unity, may be used.

Since the depth distortion, once chosen, is usually invariable, and since the hydraulic radius distortion for any particular section and stage is fully determined by the shape of the cross-section and the vertical distortion, it follows that if Froude's law is to be approximated, a model with constant roughness (at different stages) must be operated with varying slope distortion. Conversely, if a model is to be operated with constant slope-ratio, the roughness ratio must be made to vary with stage so as to keep Equation (4) in balance.

In practice, of course, these considerations impose difficulties, often insuperable, so that the model cannot be brought to operate in compliance with Froude's law. The usual practice is to operate with constant roughness and slope ratio, and a discharge scale which varies with stage. (One model study completed at the United States Waterways Experiment Station, in 1938, was made with a variable slope scale, with excellent results.)

In Conclusion (3) of his description of the Helena-Donaldsonville model, Lieutenant Nichols gives the discharge scale, and states that it is greater than that required by Froude's law. If the reference is to Equation (3) it should be explained that this is not Froude's law, but an equation of compatibility between Froude's law and Manning's formula; Froude's law proper does not apply to open-channel models. If a different formula for C were used, which is applicable equally to both model and prototype, it is possible that the discharge scale would be found to check closely with that required by theory. By plotting the value of Chezy's C for a large number of observations taken from models of all sizes and descriptions, as well as from small, medium, and large canals, flumes, and rivers, against the corresponding values of the product, $V\,R$, the writer has developed the formula,

$$C = \frac{1.81}{n^{0.9375}} (V\,R)^{0.1175} \quad \text{(6a)}$$

or,

$$V = \frac{1.96\, R^{0.700}\, S^{0.567}}{n^{1.062}} \quad \text{(6b)}$$

which appears to fit models as well as prototypes. It would be interesting if the author would compute the value of Q_r for the Helena-Donaldsonville model, using Equation (6b) for comparison with the empirically determined value of $\frac{1}{1\,500\,000}$.

Under Study No. 13, Table 1, the author cites a case in which the same model was tested with two different depths, to determine the effect of distortion in a sharp bend. His first conclusion is that although the tractive force available in the two models was equal, greater bed movement occurred at the greater depth (smaller slope). This is extremely interesting as indicating that a true criterion of tractive force should include the factor of depth to a greater degree than in DuBoys' law—a conclusion which the writer has reached independently by a study of the tractive force investigations of the U. S. Waterways Experiment Station. The tractive force, according to DuBoys' law, is expressed by the product, $\rho\, d_x\, S$; in which d_x is the depth of flow at the particular point in question, and S is the general water-surface slope of the stream, the resultant of all the dynamic forces acting throughout the stream in that vicinity. DuBoys' expression for tractive force does not consider velocities, except as they are tied to d_x (through R), and S. Bed velocity is generally acknowledged to be of first importance in the transportation of bed materials, but practical means of measuring bed velocities have not been developed. Since they are determined by the velocity distribution, and the hydraulic resistance to flow near the boundaries, it would seem that a true criterion of tractive force should be intimately linked with Chezy's formula for mean velocity. A preliminary analysis of the flume traction studies at the Waterways Experiment Station shows that an equation of the general form:

$$G = \frac{K\,C^3}{\rho_g}\,(d\,S - d_0\,S_0)^{1.5} \qquad \text{(7)}$$

adequately represents the relationship between the solid flow and fluid flow in a small, straight, rectangular channel with smooth sides. In Equation (7): G is the solid flow, in cubic feet per foot width of flume per hour; K is an empirical constant representing the effect of shape and size of bed material, and coefficient of friction between the material and the bed (includes the factors, g and ρ_1); C is the Chezy roughness coefficient; ρ_g is the (wetted) specific gravity of the bed material; $d\,S$ is the depth-slope product for the particular hydraulic conditions under consideration; and, $d_0\,S_0$ is the corresponding product for conditions obtaining at inception of "general" movement.

It will be noted that according to Equation (7) the quantity of material moved will increase as the cube of the friction coefficient. If Manning's formula for C is used, $G \approx \frac{R^{0.5}}{n^3}$. The result is the same as that observed in the results of the author's Study No. 13, Table 1. Of course, the results of flume traction studies cannot be applied directly to natural open-channel models, or to full-sized streams; but it is to be expected that as knowledge of the mechanics of bed-load movement expands, some formula of the general form of Equation (7) will be found to apply. The author's observations on the effects of distortion on velocity distribution are very pertinent. It is here that the crux of the entire question of evaluation of distortion effects is to be found. The vertical velocity curves and the corresponding isovel plots are the resultant of all the complex dynamic effects of sinuosity of channel, variation in size and shape of cross-section, absolute roughness, type and quantity of bed-load movement, etc. If a relationship could be established between the isovel plot, the bottom velocities, and Chezy's friction coefficient, C, one may anticipate a rational design of open-channel models. Herein lies a fruitful field of investigation.

The writer would like to plead with all engineers who may have occasion to make stream measurements, to make every effort to report at the same time all pertinent information as to: (*a*) Water temperatures; (*b*) rate of change of surface slopes; (*c*) configuration of the reach, both longitudinal and lateral; (*d*) variation in shape and size of cross-section along the stream axis; (*e*) mean grain size of bed material, or careful description of channel roughness; (*f*) type of bed-material movement (smooth, riffling, sand waves, super-wave smooth, anti-dune), together with average height, wave length, and speed of sand waves; (*g*) quantity of bed load; and, (*h*) kineticity of flow (this requires taking two velocity cross-sections and the slope between them). The accumulation of a widely distributed body of such data should lead to a much more useful table of n-values, and eventually to a definitive solution of the entire problem of open-channel flow.

Study No. 7, Table 1, is one of the most interesting model studies reported to date. Professors A. H. Gibson and J. Allen (see author's bibliography reference (12))[5] made an exhaustive study of the suitability of various bed materials for this particular model, which involved open-channel flow (unidirectional), tidal flow in a funnel-shaped estuary, and tidal bore. Any conclusions which they draw should be applied to other models only with the greatest caution, since the two vertical distortions used were far greater than has been generally accepted as a safe limit for vertical distortion. Despite such excessive distortions, extremely good verification was obtained, particularly as to height of sand banks and depth of pot-holes. Lieutenant Nichols stresses the importance of a careful verification; in the opinion of the writer this cannot be over-emphasized. The reliance to be placed in any results obtained with new dispositions built into the model depends entirely upon the accuracy with which the model was able to reproduce known conditions or known changes in Nature. In the study of the Severn Estuary about one-half the bed materials tested (which included sands, emeries, and pumices) failed to reproduce the correct position of the thalweg just below a sharp bend. Some of the materials that produced good results in the $\frac{1}{200}$-model failed in this respect when used in the $\frac{1}{100}$-model; on the other hand, one sand, which gave the incorrect thalweg in the $\frac{1}{200}$-model gave the correct position in the model with smaller vertical distortion. It is evident that some undetermined relationship between the size and specific gravity of the bed material, and the tractive forces developed by the flowing water (dependent upon the vertical and slope distortions) is effective in such a model, and that a fine balance must exist between the solid and fluid flows if similarity of bed-load movement is to be attained. It is significant that for the three materials which produce the best results, the values of the product, $d_g (\rho_g - \rho_1)$, were nearly equal.

The author is to be congratulated upon his paper, which avoids all matters of conjecture and deals only with actually observed effects of geometric distortion. Hydraulic models are here to stay, and within the limits imposed by the particular laboratory, and funds available, distortions are unavoidable; it is a question of fundamental importance, therefore, to determine what allowances are to be made for each such distortion in the interpretation of the measured model results. The paper is a valuable contribution to current knowledge in this field, so far as vertical and slope distortions are concerned. Much remains to be written, particularly in the field of bed material distortion.

E. H. TAYLOR,[6] JUN. AM. SOC. C. E. (by letter).—The fact that geometric distortion of river models is theoretically invalid has long been recognized. Solution of the model equations for a number of effects—for example, backwater, tractive force, or transverse slope at a bend—leads to the conclusion that $L_r = d_r$, or that the distortion ratio should be unity. Tests conducted at the Hydraulic Laboratory of the University of California at Berkeley, Calif., during 1935 and 1936 support Recommendation (1) that "geometrically distorted river models should be avoided whenever suitable undistorted models can be constructed at reasonable cost."

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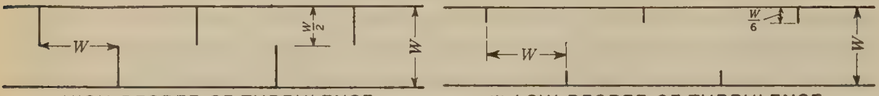
[5] *Proceedings*, Am. Soc. C. E., June, 1938, p. 1101.

[6] Instructor, Mech. Eng., Univ. of California, Berkeley, Calif.



(a) HIGH DEGREE OF TURBULENCE (b) LOW DEGREE OF TURBULENCE

FIG. 6.—ARRANGEMENT DESIGNED TO PRODUCE TURBULENCE

In an attempt to determine the effect of geometric distortion upon the bed configuration of a movable bed model, an arbitrary prototype was constructed in a channel 1 ft wide, and its model was built in another channel to a horizontal reduction scale of two. The model was constructed so that it could be operated as either a distorted or as an undistorted model. Arbitrary rates of flow were set up in the prototype so as to produce definite bed configuration about certain simple obstructions which could be easily modeled in the smaller channel. The obstructions, or baffles, about which the flow was made to pass, were built to cause either a high degree of turbulence, or to allow the flow to proceed smoothly. Shown in Fig. 6 are plan views of two of the arrangements used, in which w = channel width (either 12 in., or 6 in.).

No attempt at "verification" of the prototype bed configuration was made, but discharges and lengths of time for the model were calculated on the basis of the Froudian law of similarity. At the conclusion of each test, the elevations of the prototype and model beds were compared at a large number of corresponding points and an error curve was prepared. Fig. 7(*a*) shows the relative correspondence of the distorted and undistorted model to their prototype obtained from a typical set of runs. The agreement between models and prototypes was found to be superior for the condition represented by Fig. 6(*a*). Fig. 7(*b*) shows the relative correspondence of undistorted models of Arrangements *A* and *B* to their respective prototypes. It was concluded that:

(1) Undistorted models more truly represent their prototypes than distorted models; and,

(2) If Froude's law is used as the model criterion, models of highly turbulent systems are more faithful than models of relatively smooth flows.

The writer's understanding of "model verification" of movable bed models is as follows: A known hydrograph covering a certain period of time in Nature is simulated in the model. Several trials may have to be made, adjusting the model discharge, slope, or time, until a certain combination of hydraulic conditions is obtained whereby the model reproduces the natural bed condition. The model is then said to be adjusted, and the assumption is made that these experimentally determined hydraulic conditions fix the proper ratios for discharge, time, etc., for the particular model. The model is then assumed to be capable of indicating future changes in bed conditions which would be due, perhaps, to proposed training works.

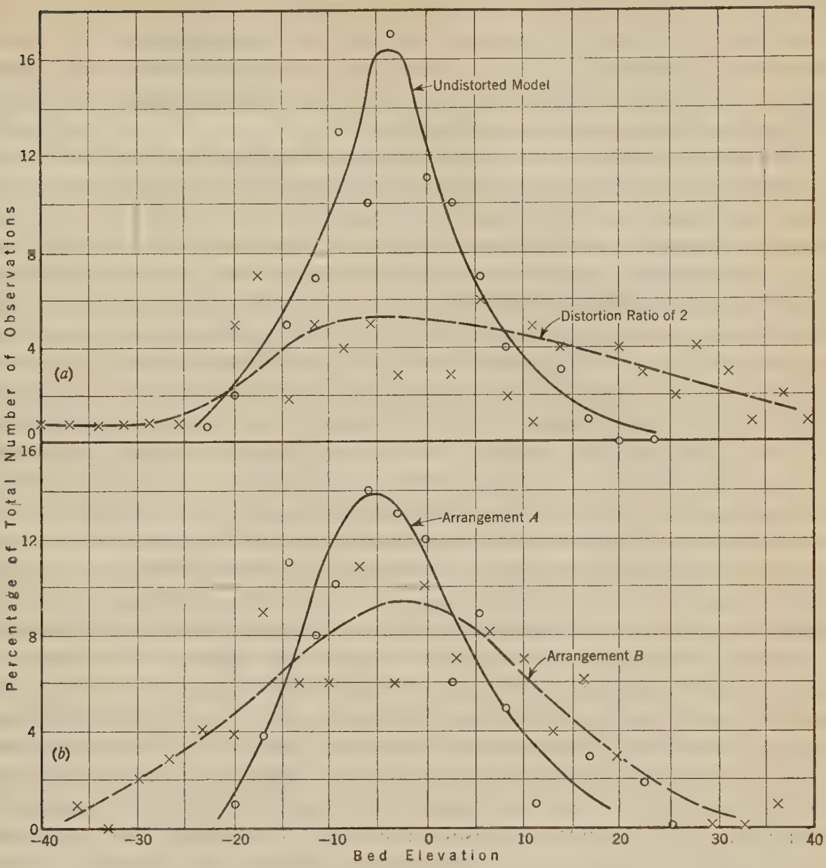


FIG. 7.—VARIATION OF BED ELEVATION AT CORRESPONDING POINTS OF MODEL AND PROTOTYPE IN PERCENTAGE OF MAXIMUM DIFFERENCE IN BED ELEVATION

It may be reasoned, however, in view of the foregoing Conclusion (2) that the degree of turbulence, or nature of the flow system, is a factor that influences the discharge or time scale necessary to produce similarity of bed configurations between a model and its prototype. Thus, the introduction or removal of groynes and dikes in the model stream would effectively alter the turbulence which would seem to invalidate the experimentally determined model ratios. The matter of movable bed model "verification" impresses the writer as being somewhat uncertain.

HERBERT D. VOGEL,[7] M. AM. SOC. C. E. (by letter).—A rising vote of thanks is due Lieut. Nichols from the harassed brotherhood of hydraulic model testers, for he has laid low their ghost of despair and given them new courage with which to meet their difficulties. Fear engendered by ignorance is dispelled by the light of knowledge; hence, there should result from his paper a bolder and more intelligent approach to related problems. The analyses contained in the paper are worthy of serious study by all investigators of hydraulic phenomena. Although it is not desired to take material issue with Lieut. Nichols on any of the points raised by him, the following random thoughts, based on experience, appear to be in order.

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[7] Capt., Corps of Engrs., U. S. Army; Instr. The Army Engr. School, Fort Belvoir, Va.

Whether one likes it or not, it must be admitted that the ideal model is as much a figment of the imagination as the perfect fluid. Being forced at the start to use plain, every-day water in a model, and to keep it on this planet where the effects of gravity cannot be changed, one must relinquish any fond hope one may have harbored for the achievement of true dynamic similitude. It is not meant to imply by this statement that an investigator should don the eternally dark cloak of a defeatist; on the contrary, he must assume the attitude of an opportunist, ready to meet each newly developed situation squarely, anxious to try the untried, and determined to leave no stone unturned in his search for the ultimate truth. Above all, he must face facts as they are instead of as he would like them to be. There is no place in research for wishful thinking. The model is a handy, if relatively expensive, tool for determining, in advance, the probable effects of proposed changes in a prototype river or harbor. It should be expected to reveal such effects with accuracy and clarity. By a process of verification it can be tested as to trustworthiness, and, in this, all surety and hope repose. Although it is true that individual forces and accelerations of the prototype may fail of exact reproduction in the model, it is possible, nevertheless, to design and operate the latter so as to make it duplicate known effects. Thus, by "cutting and trying," by varying the variable conditions, by adjusting as the need directs, and by resorting to necessary distortions, the investigator can overcome dissimilarities engendered by the use of water and a garden-type of gravity. Perhaps he cannot determine the individual effects of each and every force, but—what is more important—he can come to an evaluation of resultant effects. If the model has been tested faithfully as to its capability to reproduce past occurrences, there is no reason to suppose that it will fail to predict the future with an equal degree of accuracy.

In general, allowing for differences in materials comprising beds and banks, every alluvial river may be considered as a rough model of every other. Traction in each must be sufficient to move the bed materials down stream; otherwise, there would be no river. If depths are less in the smaller rivers, slopes must be correspondingly greater; and, similarly, unless slopes are greatly exaggerated there must be definite increases in relative depths. Of course, there are definite limits to the slopes that may be obtained. Consider, briefly, the Elbe River, in Germany, as compared to the Lower Mississippi River, slopes of the former being greater on an average of from 25 to 70 per cent. If one may look upon the former as an approximate model of the latter, one should expect to find an exaggeration of the vertical scale. The horizontal scale, based on bankful measurements, is about 1 : 7; the vertical scale, similarly considered, is about 1 : 4. This indicates that a vertical exaggeration of about 1.75 has been applied by Nature herself.

Referring to Lieut. Nichols' statement that models of the Elbe River have been built to horizontal scales of 1 : 200 and 1 : 500, these values would correspond to scales of 1 : 1 400 and 1 : 3 500 for the Mississippi River. The vertical scale of the larger Elbe model (1 : 40) produced an exaggeration, in depths, of 5. Considering that Nature had already applied an exaggeration of 1.75, the total exaggeration was 5 × 1.75, or 8.75. For a model of the Mississippi River, with a horizontal scale of 1 : 1 400, this would permit the selection of a vertical scale as large as 1 : 160, approximately. In the Elbe model, slopes were exaggerated by the factor, 8, in order to produce movement of the bed; but because of the warping that results therefrom in over-bank areas, this is no longer judged the best practice. Without supplementary slope to produce bed movement, it is probable that a vertical exaggeration of about 7 would have been necessary. This fact, translated to Mississippi River terms, would justify an exaggeration of more than 12, all of which tends to show that greater distortion may be tolerated in models of large, rather than in those of small, rivers. Extending the theory to small mountain streams, one can see that little or no distortion would be in order. It is fortunate, of course, that for such streams it is not needed, whereas the reverse is true for large rivers.

It is doubtful if any one should, or could, establish an ultimate permissible limit of distortion. Lieut. Nichols has wisely refrained from so doing, although he has indicated discrepancies that have been observed in models greatly exaggerated as to depths and slopes. The entire problem resolves itself eventually into a question of money. If information is desired relative to the effects of proposed improvements in a certain reach of river and the facts are not obtainable by analytical methods, a model test is the obvious means available. The question of how much to spend is then pertinent. Certainly no test should be undertaken that will cost more than it can be hoped to save. Having determined the maximum expenditure permissible, it then remains to select suitable scale values. The horizontal scale is determined largely on a basis of costs; the vertical scale on a basis of depths that will be needed in the model. Distortion (or vertical exaggeration) is the result, and if an arbitrary limit is imposed, it may be that a model study will be precluded. If no worthy results are possible it would be better, of course, to abandon plans for a model then and there; but in almost every case some beneficial data may be expected. It is then necessary to weigh the estimated value of anticipated results against the cost thereof, in dollars and cents, and to arrive at a final decision on that basis.

A word of warning should be added in closing as an addition to the recommendations of Lieut. Nichols, all of which are concurred in. Models of spillways and similar structures should be made geometrically similar, in every respect, to prototypes. Distortion of any kind applied thereto results in a change of basic shape, and so alters frictional coefficients that direct comparisons are impossible. In an open-channel model, frictional differences are adjusted by the process of verification previously referred to, but in the case of spillway models it frequently happens that the prototype exists only in drawings or in imagination; hence, no check is possible. Since the heights of such structures are usually great with respect to length sand widths, there is little temptation, if any, to exaggerate vertical dimensions. In case of an excessive length of crest, a sectional model will generally suffice.

KENNETH D. NICHOLS,[8] JUN. AM. SOC. C. E. (by letter).—The frank criticism and constructive comments in the discussions are appreciated. The writer is in practically complete agreement with the views expressed by Captain Ehrgott, and his discussion aids materially in clarifying the original paper. He presents some interesting and valuable data on the observed values of n in small models. The observation and gradual collection of more of such data will greatly facilitate the work of the model designer in the future.

Captain Ehrgott implies that Equation (4) might be applicable in the design of movable bed models. In the writer's opinion Equations (3) and (4) have little application in the average movable-bed model study. They are intended primarily for consideration in design and operation of fixed-bed models used for hydraulic studies. In the average movable-bed model of a river the bed-movement considerations are so much more important than certain hydraulic requirements that the latter must be subordinated. Only in certain types of river tidal models do hydraulic and bed-movement requirements necessitate equal consideration. In general the purpose of the model should be kept firmly in mind and the study confined to rather narrow limits. This limitation has brought about the necessity for having different techniques for fixed-bed models and for movable-bed models.

The writer is in complete agreement that Equation (3) is not Froude's law, but an equation of compatibility between Froude's law and Manning's formula. Conclusion (3) of Item No. 6 is a comparison of the empirically determined discharge scale with the "so-called" Froudian discharge scale as determined by the application of Equation (2b)—not Equation (3).

Captain Ehrgott's remarks on Item No. 7 are particularly appreciated. The additional evidence of the variable results obtained by changing bed materials cannot be repeated too often nor over-emphasized. Complete and careful verification of each and every model is an absolute requirement of sound model technique.

Mr. Taylor's discussion gives the impression that he has accepted a defeatist attitude concerning the use of distorted models. The writer is in agreement that within certain ranges "undistorted models more truly represent their prototypes then distorted models," but as stated in the original paper, for a large prototype very frequently the smallest practical undistorted model is economically unjustifiable from the standpoint of cost whereas a smaller, but yet practical, distorted model may be of definite value and also a good investment economically. A distorted model is not a perfect tool, but it may be better than no tool.

Mr. Taylor's skepticism of verification as being somewhat uncertain is justifiable to a certain extent. Verification may be uncertain and when carelessly and improperly done, it may lead to placing faith in an absolutely untrustworthy model. However, the large number of successful model studies has shown that, in the hands of experienced personnel, a distorted model, properly verified, is of definite value for aiding in the solution of difficult hydraulic problems.

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[8] Lieut., Corps of Engrs., U. S. Army; Instr., Dept. of Civ. and Military Eng., U. S. Military Academy, West Point, N. Y.

Captain Vogel has presented an excellent discussion of some of the common-sense principles that govern the conception and use of hydraulic models. It is well occasionally to rise above the petty arguments, pro and con, concerning the exact accuracy of hydraulic models and to view the problem as a whole to decide whether or not the results have been of benefit. Moreover, the importance that Captain Vogel attributes to economic considerations cannot be over-emphasized. Practical considerations and experience should outweigh minor theoretical arguments that the tool is imperfect.

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TRANSACTIONS

Paper No. 2045

BEAM CONSTANTS FOR CONTINUOUS TRUSSES AND BEAMS

By George L. Epps,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. C. W. Deans, George W. Lamb, A. A. Eremin, Leo Legens, and George L. Epps.

Synopsis

The material presented in this paper deals with the determination of the elastic constants used in various methods of analyzing continuous beams and trusses. By rotating the ends of the beam in turn and computing the elastic curve, the constants are determined for a span. This method simplifies greatly the process of determining stresses in continuous trusses and girders.

Introduction

The method of determining beam constants by elastic weights, described herein, was first developed for use in the analysis of continuous beams of variable moment of inertia and, later, was extended to include continuous trusses. Since the application of this method to continuous trusses is more involved, it will be explained in detail. The simpler application to continuous beams will be indicated afterward.

The usual method of analyzing continuous trusses is to assume various reactions as redundants and compute the elastic curves with loads in their places. In Fig. 1(*a*), Curve *A E F D* is the deflection curve for a unit load at Point *B*; Curve *A G F D* is the deflection curve for a load of sufficient magnitude at Point *C* to produce a deflection at that point equal to *C F*; and the

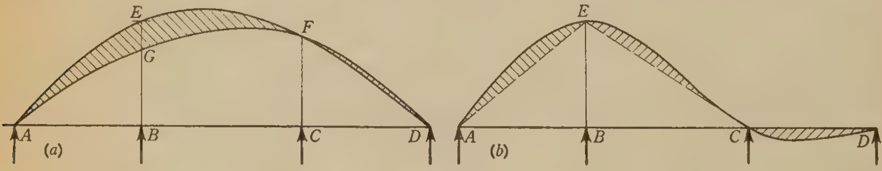

Fig. 1

Note.—Published in October, 1938, *Proceedings*.

[1] Senior Engr., Design Dept., State Highway Comm., Topeka, Kans.

shaded area between these two curves represents the ordinates of the required influence line to some scale. These deflections are several times larger than the influence ordinates that are obtained from them and, therefore, must be determined quite exactly if the ordinates are to be accurate.

A very clear illustration of the necessity for great exactness is given in an example of a 3-span continuous girder analyzed by Mr. V. L. Pierce[2] in 1935. In Fig. 2 the deflections at the pier, R_2, for the loads shown are 4.2666 and -3.2666, and the influence ordinate is the difference, or 1.0000. The larger of these deflections is more than four times the required ordinate and must be accurate to 0.025% if the ordinate is to check within 0.1 per cent. In a 5-span continuous girder, analyzed in the Design Department of the Kansas Highway Commission, the larger deflections were thirty-three times the ordinate, thus requiring very precise calculations to obtain the correct value of the influence ordinate. Thus, it will be seen that the deflection (and hence, the necessary precision of calculations) is increased in proportion to the fourth power of the number of spans.

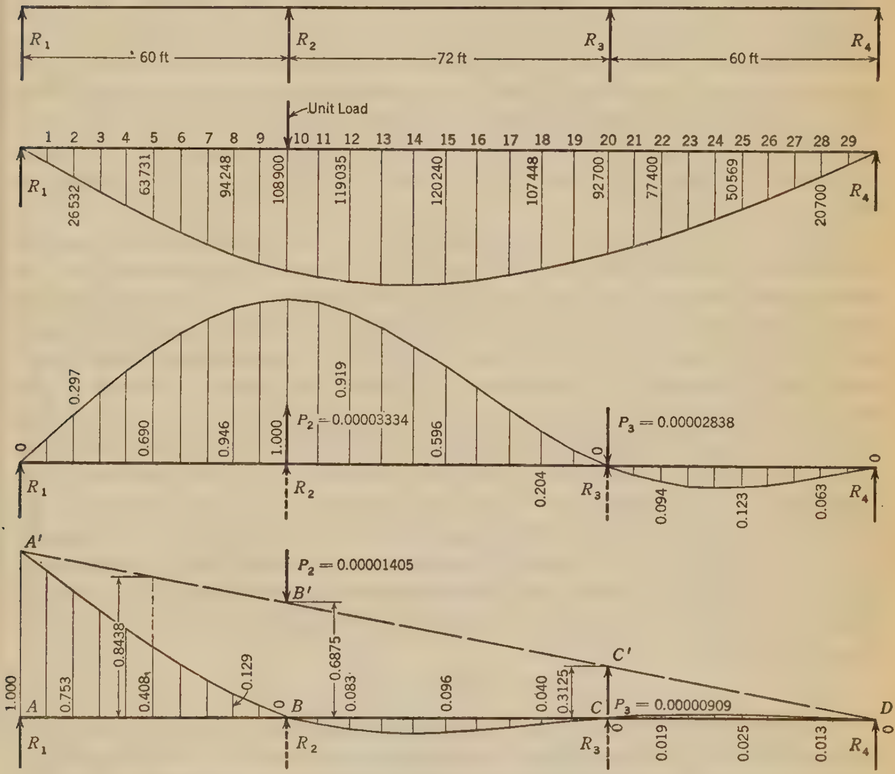
R1
R2
R3
R4
60 ft
72 ft
60 ft
Unit Load
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
26532
63731
94248
108900
119035
120240
107448
92700
77400
50569
20700
0
0.297
0.690
0.946
1.000
0.919
P2 = 0.00003334
0.596
0.204
P3 = 0.00002838
0.094
0.123
0.063
A'
P2 = 0.00001405
B'
0.8438
0.408
0.129
0.6875
0.083
0.096
0.040
0.3125
C'
1.000
A
0.753
B
C
P3 = 0.00000909
D
0.019
0.025
0.013

FIG. 2.—DETERMINATION OF INFLUENCE LINES FOR THE REACTIONS OF A CONTINUOUS BEAM

The method explained herein makes possible the application of moment distribution, slope deflection, conjugate points, and similar methods, to the analysis of continuous trusses. Using these methods the moments are determined at each pier. From these pier moments the changes in reactions due to continuity are found. These changes in reactions are indicated by the shaded areas of Fig. 1(*b*). The dashed line of this diagram is the influence line for R_B if there is no continuity at the piers, and the solid line or curve becomes the influence line for R_B of a 3-span continuous beam. The part of the influence ordinate between the dashed and solid lines is not more than one-third the total ordinate, and, consequently, an error of 0.3% in this part will reduce to 0.1%, or less, of the total. Inasmuch as slide-rule computations can easily be made to an accuracy of 0.3%, this method readily lends itself to the use of the slide-rule. The first time the method was applied by the writer to a continuous truss (a 5-span bridge) the influence lines for moments, shears, and reactions at each pier were obtained in one week less than the customary office time. The reaction lines thus found, checked within 0.1% with those computed by other designers who were using the method of redundant reactions. The method of redundant reactions required two weeks to find the influence lines for reactions only.

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[2] "Simplified Solution of Influence Lines for the Reactions of Continuous Beams," by V. L. Pierce, *Civil Engineering*, December, 1935, p. 798.

In the analysis of a continuous truss or girder by slope-deflection or moment distribution methods, the stiffness, carry-over factor, and influence line for fixed-end moments are required for each end of each span. All these quantities may be obtained by the use of elastic weights. According to Müller-Breslau's principle, if a unit rotation is applied to the end of a beam, the opposite end being freely supported, the deflections produced will be the ordinates of the influence line for fixed-end moment. The moment required to produce this rotation is defined by Hardy Cross,[3] M. Am. Soc. C. E., as the stiffness of the beam. The rotation at the far end of the beam is the carry-over factor for moments from the far end to the near end of the beam. The foregoing constants may be used directly in the slope-deflection equations and, with proper adjustment, for the case in which both ends are fixed, in the moment distribution method. All calculations are made using elastic weights as described by D. B. Steinman,[4] M. Am. Soc. C. E., or by Hale Sutherland and Harry L. Bowman, Members, Am. Soc. C. E.[5]

THEORY

Consider the second span of a 5-span continuous truss bridge with members that are symmetrical as to length, but not as to cross-section, so that the elastic constants and fixed-end moments are different for the two ends. Two sets of computations are required. However, those for only one end will be shown herein.

Fig. 3 shows the truss span diagrammatically, the lengths of the members, their moment centers, and the external forces used in the calculations. These forces are assumed and may have any values consistent with the theory of statics. As will be seen, the left end is subjected to a moment and the right

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[3] "Continuous Frames of Reinforced Concrete," by Hardy Cross and Newlin D. Morgan, John Wiley & Sons, Inc., 1932, pp. 83 and 99.

[4] "Truss Deflections Accurately Determined by Elastic Weights," by D. B. Steinman, *Engineering Record*, May 13 and 20, 1916.

[5] "Structural Theory and Design," by Hale Sutherland and Harry L. Bowman, John Wiley & Sons, Inc., 1930, p. 152.

TABLE 1.—STRESSES AND CHARACTERISTICS OF THE TRUSS SHOWN IN FIG. 3.

Bar	Stress, s, in pounds	Area, A, in square inches	Length, L, in inches	Values of $\frac{sL}{A}$	Radii, r, in inches	ELASTIC WEIGHTS, $\frac{sL}{Ar}$, AT THE FOLLOWING PANEL POINTS:												
						8	9	10	11	12	13	14	15	16	17	18	19	20
$U_8 U_9$	−600	21.16	150	−4 260	180	−23.62	....	....	....	....	....	....	....	....	....	....	....	....
$L_8 U_9$	−94	19.11	234	−1 150	115 96	+10.00	−12.00	....	....	....	....	....	....	....	....	....	....	....
$L_8 L_9$	+673	19.11	153	+5 390	147	....	−36.62	....	....	....	....	....	....	....	....	....	....	....
$U_9 L_9$	+66	11.77	150	+840	1 500	....	−0.56	....	....	....	....	....	....	....	....	....	....	....
$L_9 L_{10}$	+663	19.11	151	+5 230	149	....	−34.98	....	....	....	....	....	....	....	....	....	....	....
$U_9 L_{10}$	+9	15.59	202	+120	111 100	....	+1.08	−1.20	....	....	....	....	....	....	....	....	....	....
$U_9 U_{11}$	−666	19.11	300	−10 440	135	....	....	−77.50	....	....	....	....	....	....	....	....	....	....
$L_{10} U_{11}$	−11	21.16	202	−108	100 89	....	....	+1.08	−1.21	....	....	....	....	....	....	....	....	....
$L_{10} L_{11}$	+679	19.11	151	+5 350	119	....	....	....	−44.80	....	....	....	....	....	....	....	....	....
$U_{11} L_{11}$	+67	11.77	120	+684	1 200	....	....	....	−0.57	....	....	....	....	....	....	....	....	....
$L_{11} L_{12}$	+675	19.11	150	+5 290	120	....	....	....	−44.10	....	....	....	....	....	....	....	....	....
$U_{11} L_{12}$	−96	11.77	192	−1 570	94	....	....	....	−16.75	+16.75	....	....	....	....	....	....	....	....
$U_{11} U_{13}$	−600	24.73	300	−7 290	120	....	....	....	....	−60.70	....	....	....	....	....	....	....	....
$L_{12} U_{13}$	+96	13.24	192	+1 390	94	....	....	....	....	−14.86	+14.86	....	....	....	....	....	....	....
$L_{12} L_{14}$	+525	25.36	300	+6 210	120	....	....	....	....	....	−51.80	....	....	....	....	....	....	....
$U_{13} L_{14}$	−96	11.77	192	−1 570	94	....	....	....	....	....	−16.75	+16.75	....	....	....	....	....	....
$U_{13} U_{15}$	−450	29.11	300	−4 630	120	....	....	....	....	....	....	−38.60	....	....	....	....	....	....
$L_{14} U_{15}$	+96	11.77	192	+1 570	94	....	....	....	....	....	....	−16.75	+16.75	....	....	....	....	....
$L_{14} L_{16}$	+375	24.73	300	+4 560	120	....	....	....	....	....	....	....	−38.00	....	....	....	....	....
$U_{15} L_{16}$	−96	15.59	192	−1 190	94	....	....	....	....	....	....	....	−12.63	+12.63	....	....	....	....
$U_{15} U_{17}$	−300	19.11	300	−4 710	120	....	....	....	....	....	....	....	....	−39.20	....	....	....	....
$L_{16} U_{17}$	+96	13.24	192	+1 390	94	....	....	....	....	....	....	....	....	−14.86	+14.86	....	....	....
$L_{16} L_{17}$	+225	19.11	150	+1 760	120	....	....	....	....	....	....	....	....	....	−14.72	....	....	....
$U_{17} L_{17}$	+22	11.77	120	+228	1 200	....	....	....	....	....	....	....	....	....	−0.19	....	....	....
$U_{17} L_{18}$	+226	19.11	151	+1 790	119	....	....	....	....	....	....	....	....	....	−14.92	....	....	....
$U_{17} L_{18}$	−123	24.73	202	−1 010	89 100	....	....	....	....	....	....	....	....	....	−11.31	+10.03	....	....
$U_{17} U_{19}$	−133	19.11	300	−2 090	135	....	....	....	....	....	....	....	....	....	....	−15.48	....	....
$L_{18} U_{19}$	+99	15.59	202	+1 280	100 111	....	....	....	....	....	....	....	....	....	....	−12.80	+11.51	....
$L_{18} L_{19}$	+60	27.86	151	+324	149	....	....	....	....	....	....	....	....	....	....	....	−2.17	....
$U_{19} L_{19}$	+6	11.77	150	+72	1 500	....	....	....	....	....	....	....	....	....	....	....	−0.05	....
$L_{19} L_{20}$	+61	27.86	153	+336	147	....	....	....	....	....	....	....	....	....	....	....	−2.28	....
$U_{19} L_{20}$	−94	19.11	234	−1 150	96 115	....	....	....	....	....	....	....	....	....	....	....	−12.00	+10.00
Totals	....	....	....	......	....	−13.62	−83.08	−77.62	−107.43	−58.81	−53.69	−38.60	−33.88	−41.43	−26.28	−18.25	−4.99	+10.00

end is free to rotate. The stresses are computed by statics and are presented, with the areas, lengths, and radii for the various members, in Table 1. From these values the elastic weights were computed as indicated. The sign convention used assumes that strains which produce a lowering of a joint have negative elastic weights, and *vice versa.*

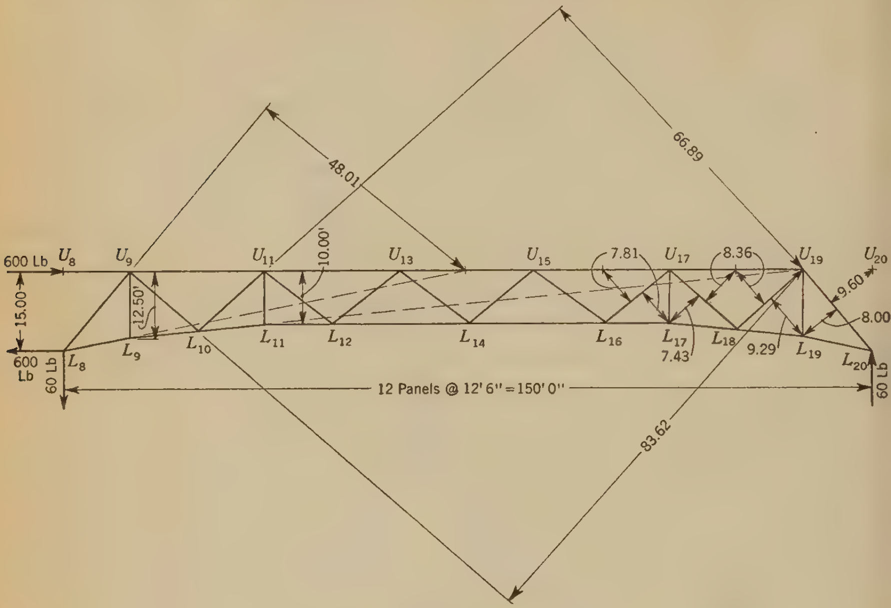
600 Lb
U_8
U_9
U_{11}
U_{13}
U_{15}
U_{17}
U_{19}
U_{20}
48.01
66.89
10.00'
7.81
8.36
9.60
8.00
15.00'
12.50'
600 Lb
60 Lb
L_8
L_9
L_{10}
L_{11}
L_{12}
L_{14}
L_{16}
L_{17}
7.43
L_{18}
9.29
L_{19}
L_{20}
60 Lb
12 Panels @ 12' 6" = 150' 0"
83.62

FIG. 3

These elastic weights are applied to a simple or conjugate beam as shown in Fig. 4. In Table 2, the first operation is the determination of the left reaction of the elastic weights by taking moments at the right end of the span. Beginning with the left reaction the shears are computed between load points. These shears are the slopes at successive points of the elastic curve of the truss under this loading.

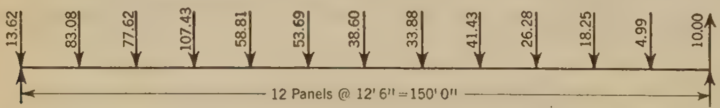
13.62
83.08
77.62
107.43
58.81
53.69
38.60
33.88
41.43
26.28
18.25
4.99
10.00
12 Panels @ 12' 6" = 150' 0"

FIG. 4

As indicated previously the end rotation or slope must be unity to obtain the beam constants. Consequently, the first column of elastic shears (Column (5), Table 2) is divided by the end slope of 362.84 to obtain the second column of elastic shears (Column (6), Table 2). From these, the moments (Column (7), Table 2) in the conjugate beam are found by summation. These moments must be multiplied by the panel length to obtain the correct value of the fixed-end moments (Column (8), Table 2). The carry-over is the last shear in Table

2 (Column (6)), or the slope of the far end of the beam, 0.509. The stiffness is $\frac{600 \times 15}{362.84} = 24.79$.

TABLE 2.—DETERMINATION OF FIXED-END MOMENTS AT POINT 8.

Panel Point: (1)	Elastic weight (2)	Arm No. (3)	Moment (4)	ELASTIC SHEAR (5)	ELASTIC SHEAR (6)	$\frac{H_8}{12.5}$ (7)	H_8 (8)
				362.84	1.000		
8	−13.62	12	163.44			0	0
				349.22	0.964		
9	−83.08	11	913.89			0.964	12.06
				266.14	0.735		
10	−77.62	10	776.20			1.699	21.22
				188.52	0.520		
11	−107.43	9	966.87			2.219	27.75
				81.09	0.224		
12	−58.81	8	470.48			2.443	30.59
				22.28	0.061		
13	−53.69	7	375.83			2.504	31.35
				−31.41	−0.087		
14	−38.60	6	231.60			2.417	30.20
				−70.01	−0.193		
15	−33.88	5	169.40			2.224	27.81
				−103.89	−0.286		
16	−41.43	4	165.72			1.938	24.22
				−145.32	−0.401		
17	−26.28	3	78.84			1.537	19.21
				−171.60	−0.474		
18	−18.25	2	36.50			1.063	13.30
				−189.85	−0.524		
19	−4.99	1	4.99			0.539	6.75
				−194.84	−0.538		
20	+10.00	0	0			0	0
				−184.84	−0.509		
Total	−547.68	7.94	4 354.75	....	....	....	....

The foregoing values may be used directly in the slope deflection equations when they are written in the following general form:

$$M_{AB} = \frac{1}{1 - r_{AB}\, r_{BA}} \{E\, K_A\, [\theta_A + r_{AB}\, \theta_B - (r_{AB} + 1)\, R] + H_{AB} - r_{BA}\, H_{BA}\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

and,

$$M_{BA} = \frac{1}{1 - r_{AB}\, r_{BA}} \{E\, K_B\, [\theta_B + r_{BA}\, \theta_A - (r_{BA} + 1)\, R] - H_{BA} + r_{AB}\, H_{AB}\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

(The general formulas shown for the slope-deflection equations were developed from equations presented by Earle Russell, in "Analysis of Continuous Frames." His equations apply to constants for beams fixed at both ends; whereas, in this method, the constants are developed for one end fixed and the other free. Also, the letters designating the various terms have been changed to agree with those used in *Bulletin 108*, Engineering Experiment Station, University of Illinois.)

In using moment distribution,[6] the constants for stiffness and fixed-end moments must be calculated with both ends fixed for the intermediate spans.

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[6] "Analysis of Continuous Frames by Distributing Fixed-End Moments" by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

The stiffness obtained previously must be divided by $1 - r_{AB}\, r_{BA}$. The fixed-end moments, C_A and C_B, are determined by combining the values for H_A and H_B according to the following equations:

$$C_A = \frac{H_A - r_{BA}\, H_B}{1 - r_{AB}\, r_{BA}} \ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

and,

$$C_B = \frac{H_B - r_{AB}\, H_A}{1 - r_{AB}\, r_{BA}} \ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

For those who prefer to use graphic methods, it might be shown that all the stresses for Table 1 and the computations of Table 2 may be done graphically by methods found in any text on graphic statics. It should be noted that the elastic loads must be divided by the end reaction to obtain the slope of unity at the rotated end. In using the method of conjugate points,[7] the values of u, v, p, q, s, and t are necessary: $u\,l$ is the distance from the left end of the span to the center of gravity of the elastic loads with a unit rotation at the left end; $v\,l$ is a similar distance for the right end; $\frac{p\,l}{I}$ is the sum of the elastic weights for the left end, rotated; and $\frac{q\,l}{I}$ is a similar quantity for the right end. The values of s and t are found by the construction shown in Fig. 5.

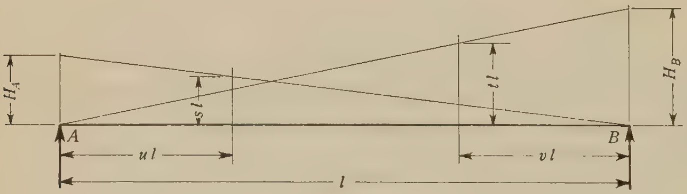

FIG. 5

In some cases it is desired to know the effect of the web system on the stresses in the truss. This can be done easily by omitting the elastic weights for the web members and carrying through the computations in Table 2.

The application of this method to beams of variable moment of inertia differs from the foregoing procedure only in the method of obtaining the elastic weights. The beam is divided into a number of sections, which need not be equal in length (although, if they are, the computations are simplified). Twelve equal sections will produce sufficiently accurate results. A moment is applied to one end of the span with the other end unrestrained and the value of $\frac{M\,dx}{I}$ is computed for each section. These values of $\frac{M\,dx}{I}$ are applied as elastic weights to the conjugate beam, as in Fig. 4. The method proceeds from this point in the same manner as explained previously.

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[7] Discussion of Walter Ruppel, Assoc. M. Am. Soc. C. E., on "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

CONCLUSIONS

Multiple-span, continuous truss or girder bridges may be analyzed by the method herein presented in much less time and with less work than is required by the method of redundant reactions. This method makes possible the use of slide-rule computations for continuous bridges, whereas the method of redundant reactions necessitates the use of a calculating machine.

DISCUSSION

C. W. Deans,[8] Esq. (by letter).—The writer favors the use of elastic weights for the determination of deflections of trusses, arches, and beams of variable section and finds Mr. Epps' paper most instructive and essentially useful for design office calculations.

While checking over the theory and detailed calculations of Mr. Epps' presentation it was felt that an actual demonstration of the proof of the theory by Maxwell's General Law of Reciprocal Energy Components might be helpful to those who desire to fix these ideas firmly in mind. In this law, if two states of stress for a structure are considered, the sum of the products of the forces for the first state (P_1) and the displacements, at corresponding points for the second state (y_2), is equal to the sum of the products of the forces of the second state (P_2) and the corresponding displacements for the first state (y_1). Written in equation form this is:

$$\Sigma\,(P_1\,y_2) = \Sigma\,(P_2\,y_1)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Beam $A\ B$, Fig. 6, is influenced by two states of stress and Equation (3) is written for the active forces only:

$$(-\,F_{AB})\,(\theta_A = +\,1) + (P = 1)\,y = M_{AB}\,(\theta_A = 0)\ldots\ldots\ldots(4)$$

Therefore, $y = F_{AB}$, which demonstrates that the deflections of a beam with unit rotation at one end are the influence ordinates for the fixed-end moment at the rotated end. In Fig. 7, Beam $A\ B$ is again shown under two states of stress. From Equation (3):

$$M_{AB}\,(\theta_A = 0) = m_{AB}\,(0_A = +\,1) + m_{BA}\,(-\,\theta_B)\ldots\ldots\ldots(5)$$

Therefore, $\theta_B = \frac{m_{AB}}{m_{BA}} = r_{BA}$, which demonstrates that the rotation at the far end of a beam is the carry-over factor (r_{BA}) for moments from the far end to the near end of the beam.

A practice problem illustrating the applications of Mr. Epps' method to a beam of variable section is afforded by a beam analyzed[9] previously by G. E. Large, Assoc. M. Am. Soc. C. E. The preliminary calculations are already made by Professor Large and the remainder necessary to arrive at the influence ordinates for fixed-end moments are quickly obtainable. This essential practice will help one to follow more readily the computations required for the truss analyzed by Mr. Epps.

The writer has favored the use of the column analogy,[10] or the elastic center method for the determination of beam constants; and thus he set to work to apply a moment, $M_0 = 1\,000$ ft-lb, to each end of Fig. 3 as shown in Fig. 8(*a*); thus: $\Delta_{M_0=1} = \Sigma\,g_0$; $\Delta_{V_0=1} = \Sigma\,(g_0\,x_0^2)$; $M_0 = \frac{\Delta_{M_0}}{\Delta_{M_0=1}}$; and, $V_0 = \frac{\Delta_{V_0}}{\Delta_{V_0=1}}$. Referring to Fig. 8(*b*): $\Delta_{M_0} = 1$; $\Delta_{V_0} = -6.41\ p$; $M_{AB} = M_0 + \bar{x}_A V_0$; $M_{BA} = M_0 + \bar{x}_B V_0$; and $r_{AB} = \dfrac{M_{BA}}{M_{AB}}$. Finally, referring to Fig. 8(*c*): $\Delta_{M_0} = +1$; $\Delta_{V_0} = +5.59\ p$; $M_{BA} = M_0 + \bar{x}_B V_0$; $M_{AB} = M_0 + \bar{x}_A V_0$; and, $r_{BA} = \dfrac{M_{AB}}{M_{BA}}$.

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[8] Estimator, Hamilton Bridge, Western, Ltd., Vancouver, B. C., Canada.

[9] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 105.

[10] "The Column Analogy," by Hardy Cross, M. Am. Soc. C. E., *Bulletin No. 115*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

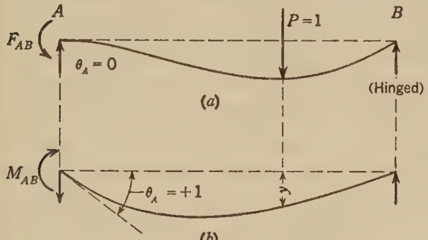
A
P=1
B
F_{AB}
$\theta_A = 0$
(a)
(Hinged)
M_{AB}
$\theta_A = +1$
(b)

FIG. 6

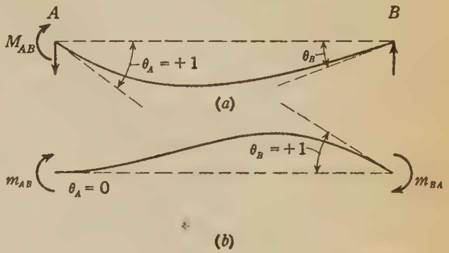
A
B
M_{AB}
$\theta_A = +1$
θ_B
(a)
$\theta_B = +1$
m_{AB}
$\theta_A = 0$
m_{BA}
(b)

FIG. 7

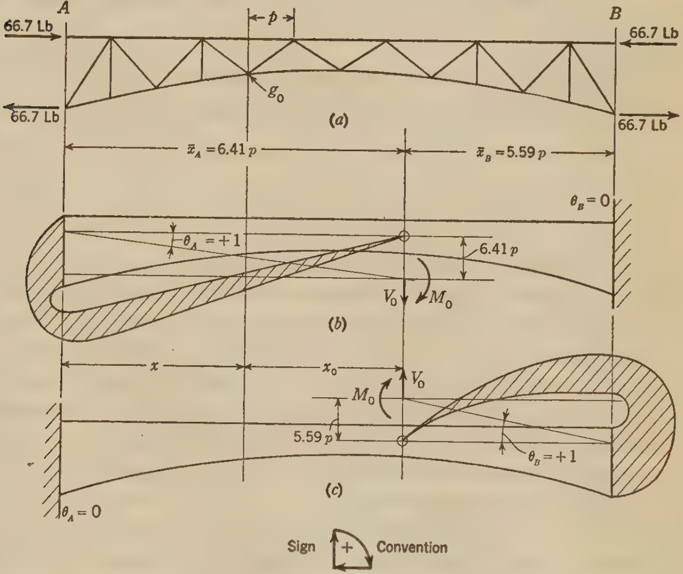
A
B
66.7 Lb
66.7 Lb
66.7 Lb
66.7 Lb
p
g_0
(a)
$\bar{x}_A = 6.41\ p$
$\bar{x}_B = 5.59\ p$
$\theta_B = 0$
$\theta_A = +1$
6.41 p
V_0
M_0
(b)
x
x_0
V_0
M_0
5.59 p
$\theta_B = +1$
(c)
$\theta_A = 0$
Sign
+
Convention

FIG. 8

Arrangements similar to Table 1 were made for $S_0 = \dfrac{M_0}{l}$ (in which l = the lever arm of the member), L, A, $\dfrac{L}{A}$, r, $\dfrac{L}{A\,r}$, and the partial elastic weights, $\dfrac{S_0\,L}{A\,r} = g_0'$. The elastic weights ($g_0$) were determined by summing g_0' for each panel point; and, thus it was found that the elastic center was 6.41 panel lengths from the left support at Panel Point 8. The next step is to compute $\Sigma\ g_0$ and $\Sigma\ (g_0\,x_0^2)$, which is the moment of inertia of the analogous column for the column analogy, or the vertical displacement of O, of the rigid bracket, $A\ O$ (with a vertical load $V_0 = +1$ at Point 0) for the elastic center method. These values in conjunction with V_0 and M_0 in Figs. 8(*b*) and 8(*c*) give stiffness factors and carry-over factors for both ends of the truss. The value of the carry-over factor from the far end (Point 20, Fig. 3) to the near end (Point 8, Fig. 3), thus determined, was 0.51, which is almost identical with that derived by Mr. Epps.

The influence lines for fixed-end moments are found from the values of V_0 and M_0 in Figs. 8(*b*) and 8(*c*) by calculating the cumulative displacements in the same manner as Mr. Epps in Table 2, Columns (6), (7), and (8). These values of stiffness factors, carry-over factors, and fixed-end moments can be used directly in the moment distribution and the slope-deflection methods.

The foregoing illustrates the fact that the moment transfer from span to span depends primarily on the proportions of the chord members and the depth variation of the trusses. The web members come into play mostly on account of the vertical shears caused by the loads and show up in the fixed-end moment calculations. It will save time in the calculation of Table 2 for both ends of the span to tabulate the values of $\frac{L}{A\,r}$ to be used with each of the two stress conditions.

With the elastic center method the fixed-end moments for beams having the far ends hinged are determined from the following equations:

$$H_B = C_B - r_{AB}\,C_A \qquad (6a)$$

and,

$$H_A = C_A - r_{BA}\,C_B \qquad (6b)$$

and the stiffness must be multiplied by $1 - r_{AB}\,r_{BA}$.

The author's method is very straightforward and even though the values found must be corrected on account of the far ends being considered hinged, only two sets of calculations (as in Tables 1 and 2) must be made—one for each end. In the suggestions offered by the writer three sets must be made, one for determining the position of the elastic center and two for the influence lines for fixed-end moments at each end. A full explanation has been made in this discussion because it was felt that a complete study would help in obtaining a clearer picture of the entire problem.

Mr. Epps is to be complimented on his choice of method, which is most efficient in that it saves design office time, and on his masterly, all-inclusive presentation. It would be interesting for Mr. Epps to comment on the feasibility of using multiple-tied, trussed arches on roller supports or multiple-trussed arches on flexible, trussed towers, and on the application of his method to their analysis.

George W. Lamb,[11] Assoc. M. Am. Soc. C. E. (by letter).—In making possible the computation of stresses in trusses by practically all the methods that are used for beams, this paper is especially commendable. Thus, it furthers the use of indeterminate structures. Although the writer does not agree that the method of analysis presented is more accurate or more convenient to use than any of several other methods, or combination of methods, the procedure recommended does have some distinct advantages on certain types of trussed structures, especially trussed frames, and similar structures that are commonly analyzed by use of moment distribution or slope deflection.

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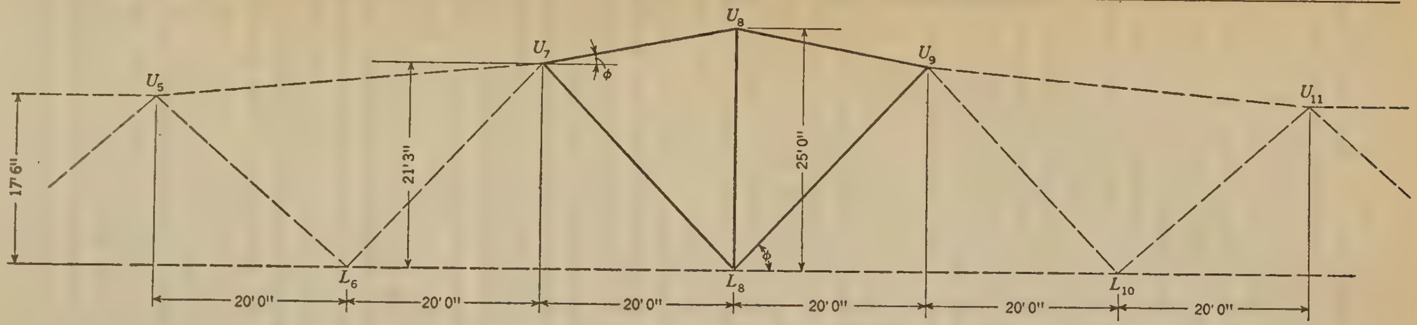
[11] Asst. Bridge Engr., State Highway Comm., Topeka, Kans.

In all methods of stress analysis and design, considerable value should be placed on the capability of any step in the analysis to lend itself to the calculation of other properties of the structure. In the method presented by Mr. Epps, this is true to some degree. The influence of the change in stress in the member over the pier on any other member in one span of the truss can readily be calculated. However, there are several other properties and conditions on continuous trusses that are always necessary to compute or investigate, to which the method does not adapt itself so readily. Some of the conditions and properties that this method does not handle easily are as follows: (1) Influence lines for all the members of the truss; (2) the effect of settlement of supports and the data necessary for adjustment of reactions; (3) the calculation of dead-load deflections; and (4) it is customary to camber continuous trusses by adding to, or subtracting from, the normal length of the chords and webs, the deformation caused by the stress produced by the load for which the structure is cambered. These deformations are small and are usually given on plans to the nearest $\frac{1}{16}$ in. The differences between the fabricated dimension to the nearest $\frac{1}{16}$ in. and the deformation caused by the load are small, but the total effect of these small differences on the total truss is sufficient in some cases to cause some trouble in erection closure, and the effect should be taken care of by raising or lowering the supports or by adjusting the length of some member with a high elastic constant. Therefore, in some cases it is necessary to calculate camber in the truss with the deformation in the members different from that which the loads on the truss would indicate.

The difficulty of handling the first and second conditions can be blamed directly on the method. That is, in the first condition the influence line for the member over the support is obtained easily, but in order to obtain the influence line for a member in the truss between supports, it is necessary to have influence lines for shears and moments at each support. This involves considerable work. The second condition, settlement of supports, is also rather tedious to calculate using the constants obtained, as it involves the elastic properties of all the spans in the unit.

The third and fourth conditions, the writer believes, can be handled more easily by using a different method of calculating the elastic weights and another form for tabulating them. The writer prefers to use the method of computing angle changes in obtaining the elastic weights. Table 3 shows the value of using one set of calculations for several different properties of the truss. The diagram is one panel of a continuous truss over the reaction. The elastic shear and moments are carried over from the dotted part of the truss. It will be noted that the values given in Column (6) are used for obtaining the influence-line ordinates for reactions, the dead-load deflections, and the camber ordinates. The values shown under Column (17), with small modifications, can be used in the calculation of secondary stresses for dead load. The values given in Columns (13), (14), (18), (19), (22), and (23) are relative, and are considerably larger than the true values. This was done to obtain numbers that are easier

TABLE 3.—SUGGESTION FOR TABULATING COMPUTATIONS



CONSTANTS						STRESSES				REACTIONS						DEAD-LOAD DEFLECTIONS				CAMBER			
Member	Area, A, in square inches	Length, L, in inches	Sec ϕ	Height of panel, in feet	$\frac{L \sec \phi^*}{A E h}$	Unit load at Point L_8	Unit load at Point $L_{8'}$	Dead-load stress, in kips†	Camber stress, in kips†	Joint	Elastic weight; unit load	Elastic shear	Elastic moment	Ordinate of Point L_8	Ordinate of Point $L_{8'}$	Elastic weights; dead load	Elastic shear	Elastic moment	Dead-load deflection, in inches	Elastic weight camber	Elastic shear	Elastic moment	Camber, in inches
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
$U_7 L_8$	27.04	350.2	1.46	25	0.756	−0.190	−0.084	−185	−290	L_6	+4.68	+34.6	302	+0.927	+0.163	+ 72.1	−563	825	0.550	− 71.9	+631	1 095	0.730
$U_7 U_8$	28.21	244.2	1.02	25	0.352	−4.51	−2.00	+298	+433	U_7	+6.53	+28.1	336	+0.984	+0.071	−301	−262	262	0.175	−265	+366	464	0.309
$U_8 U_9$	28.21	244.2	1.02	25	0.352	−4.51	−2.00	+298	+433	L_8	+3.14	+25.0	364	+1.00	+0	−535	+293	0	0	−837	−470	97	0.064
$U_8 L_8$	38.23	300.0	....	20	0.392	+1.66	+0.738	−110	−239	U_9	+6.67	+18.3	389	+0.992	−0.061	−311	+604	293	0.195	−265	−735	567	0.378
$L_8 U_9$	27.04	350.2	1.46	25	0.756	+0.718	−0.930	−210	−290	L_{10}	+5.79		407	+0.944	−0.101	+ 79.7		897	0.598	+ 56.6		1 301	0.868

* E = 1. † 1 kip = 1 "kilo-pound" or 1 000 lb.

to handle. To obtain the true deflection in inches, the elastic moments must be multiplied by $\frac{12 \times 1\,000 \times 20 \times 12}{144 \times 30\,000\,000} = \frac{1}{1\,500}$. The elastic weight given to Point L_8, Column (12), is obtained as follows: $-(+0.190 \times 756 - 4.51 \times 0.352 - 4.51 \times 0.352 - 1.66 \times 3.92 + 0.718 \times 0.756) = 3.14$ with due attention being paid to the sign of the stress and whether the member is a web or a chord member, the minus sign in front of the parenthesis shows that the angle at L_8 decreases under this condition of loading. The foregoing example is part of the calculation used in designing a three-span continuous truss by the method of redundant reactions; it shows a different method of tabulating values so that they can be used in calculating various properties. It is also different inasmuch as the distances to the intersection of the members do not have to be calculated, and the dimensions and stresses within the panel are sufficient to obtain the elastic weight at any point.

The accuracy of the method of redundant reactions, as presented by Mr. Pierce,[2] is questioned in this paper on the basis of the fact that in order to be accurate the large numbers must be extremely exact because of the small differences to be obtained. In the method of redundant reactions this objection to great exactness on this basis can be avoided as follows: Instead of taking the difference between two large deflections, the deflection of the structure is obtained at all points with a unit deflection at the support for which the influence line for reaction is desired. Consider, for example, a four-span continuous beam having a constant moment of inertia and having spans of 65 ft, 80 ft, 80 ft, and 65 ft. In order to obtain deflections easily, the total length of the unit is changed to a length of unity. Let $E = 1$; and $I = 1$. The beam shown in Fig. 9 is the result. To obtain the equation for the influence line for the reaction at Point (a), Fig. 9, a unit load will be placed at Point (a) and the equation for the influence line will be written

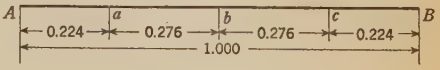


FIG. 9

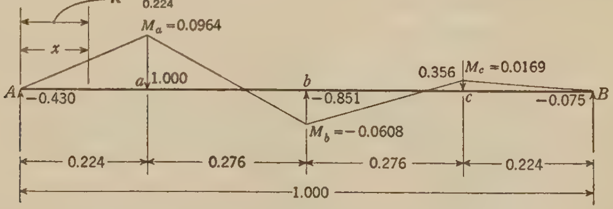


FIG. 10

Determine the reactions at Points A, b, c, and B with a unit load at Point (a). $y_{bb} = \frac{1}{48} = 0.0208$; $y_{bc} = 0.0131 = y_{ba} = y_{ab} = y_{cb}$; $y_{cc} = 0.0101$; $y_{ac} = 0.0075 = y_{ca}$; for example, y_{bc} = the deflection at b with a unit load at c, etc. Substituting:

$$0.0208\,R_b + 0.0131\,R_c + 0.0131 = 0 \quad \ldots\ldots\ldots\ldots\ldots (7a)$$

and

$$0.0131\ R_b + 0.0101\ R_c + 0.0075 = 0 \ldots\ldots\ldots\ldots (7b)$$

Solving Equations (7) simultaneously: $R_b = -0.851$; $R_c = +0.356$; $R_A = -0.430$; and $R_B = -0.075$. Applying these reactions to the beam a moment diagram is obtained as shown in Fig. 10.

From elastic shear, the slopes at the supports are as follows: $\phi_A = -0.0109$; $\phi_a = -0.00012$; $\phi_b = 0.00481$; $\phi_c = -0.00124$; and $\phi_B = +0.00062$. Use these slopes and write the equation of the elastic line in Span A–a as follows:

$$\frac{d^2y}{dx^2} = 0.430\ x \ldots\ldots\ldots\ldots (8a)$$

$$\frac{dy}{dx} = 0.215\ x^2 - 0.0109 \ldots\ldots\ldots\ldots (8b)$$

and

$$y = 0.0717\ x^3 - 0.0109\ x \ldots\ldots\ldots\ldots (8c)$$

When $x = 0.224$, $y = -0.00164$.

To obtain a unit deflection at Point (a) and to make Equation (8c) apply to any four-span continuous unit with the same ratios of span lengths, multiply this equation by $-\frac{1}{0.00164}$ and substitute, letting $x = 0.224\ K$, and letting O_{A-a} = ordinate for influence line for the reaction at Point (a) with a load in Span A–A: $O_{A-a} = -\frac{1}{0.00164}(0.0717 \times \overline{0.224}^3\ K^3 - 0.0109 \times 0.224\ K)$; or

$$O_{A-a} = 1.49\ K - 0.49\ K^3 \ldots\ldots\ldots\ldots (9)$$

and the area under this curve for uniform load equals 0.622. Using the foregoing reactions and slopes, the equation for the reaction at Point (a) with the load anywhere in the unit can be found; and a considerable portion of the foregoing information can be used in obtaining the influence lines for the other reactions. These calculations can be made with a slide-rule to the accuracy required for designing any structure. When applied generally, the foregoing procedure can be used for finding the influence line for reaction on trusses and beams of varying moments of inertia. Although it appears rather cumbersome this general method, when used by any one familiar with it, is as accurate and requires as little work for the complete analysis of a structure as any other that may be used, and also offers several checks on arithmetical errors. The complete analysis includes all shears, moments, deflections, camber, settlement of support, investigations, etc.

Any sound method of design, even when used in conjunction with relatively rough calculation, is far more accurate than the assumptions of dead load and live load applied to the structure. For instance, reinforced concrete is usually assumed to weigh 150 lb per cu ft. If this varies even 1.5 lb per cu ft, the dead load of the concrete would be inaccurate 1 per cent. Even structural steel will vary in weight from the theoretical, and live loads on most structures can and do vary widely from the assumed live load. This is not intended to imply that the calculations can be as erroneous as the assumption for load, but it is intended to show that the method used in the analysis is perhaps the most accurate part of the design even when handicapped with relatively rough calculations.

A. A. Eremin,[12] Assoc. M. Am. Soc. C. E. (by letter).—A method of analyzing stresses in continuous trusses by computing bending moments at the supports is presented in this paper. Mr. Epps has cited various advantages of the method over the conventional method in which the reaction forces must be computed. These advantages are even more numerous than those indicated. As long ago as 1930 Prof. G. G. Krivoshein showed that, by computing bending moments at the supports of continuous trusses, the results will be more accurate than by computing the redundant forces at the supports.[13] Professor Krivoshein developed various equations for computing bending moments at the supports.

The elastic properties of continuous trusses may be expressed by:

$$\frac{I}{I_0} = \left(\frac{d}{d_0}\right)^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which, I and d are moment of inertia and depth of truss at any section, respectively; and I_0 and d_0 are some arbitrarily selected moment of inertia and depth of truss, respectively.

In some trusses Professor Krivoshein stated that satisfactory results may be obtained with the exponent in Equation (10) equal to 2.5 instead of 3. Obviously, Equation (10) is exactly the same as that generally used in the analysis of stresses in continuous beams of solid cross-section and constant width. It is interesting to note that this formula may also be used in computing the constants of continuous trusses.

Mr. Epps computes the elastic weights of continuous trusses by means of the radii. However, a more direct computation may often be made by means of formulas previously cited by the writer,[14] in which the elastic weights are computed directly from the angular relation between truss members at each joint. Furthermore, the terms in the writer's equation[14] may also be used for analyzing secondary stresses in the trusses, thus saving time when additional computations for the complete analysis of stresses is required.

Leo Legens,[15] Esq. (by letter).—The author shows quite well the limitations of the method of redundant reactions as applied to continuous girders, and also the much greater simplicity of calculation in using the moments over the piers as the redundants. However, he does not show any relationship to the calculation of influence lines by Müller-Breslau's equations.[16] The writer would like to show the application of these equations to the three-span continuous beam of Fig. 2. The moment of inertia of this beam is constant for all spans, which simplifies the calculations as compared to a girder of variable section or truss.

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[12] Associate Bridge Engr., Bridge Dept., Div. of Highways. State Dept. of Public Works, Sacramento, Calif.

[13] "Simplified Calculations of Statically Indeterminate Bridges," by G. G. Krivoshein, Prague, Czechoslovakia.

[14] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 209.

[15] Engr., S. N. C. F. (French Railroads), Strasbourg, France.

[16] "Graphische Statik der Baukonstruktionen," by Müller-Breslau, Band II, Abteilung, 1908 Edition, p. 104 *et seq.*

The Müller-Breslau equation[16] states that:

$$X_a\,\delta_{aa} + X_b\,\delta_{ab} \cdots = \Sigma\,P_m\,\delta_{ma} \ldots\ldots\ldots\ldots (11)$$

The statically indeterminate parameters X_a, X_b, $\cdots$ are to be chosen so that the deflection curves may be obtained easily and so that the deflections $\delta_{aa}\delta_{ab}\cdots$ will vanish if possible.

Moment diagrams $M_a\,M_b\cdots$ produced by $X_a = -1$, $X_b = -1\cdots$ should be of simple forms (rectangles, triangles, or parabolas) for which Müller-Breslau[16] has indicated a rapid method of finding the deflections δ_{ma}, δ_{mb}, $\cdots$ with the values ω_R, ω_D, ω_P and ω_P''. These deflections are:

For Fig. 11:

$$E\,I\,y = \frac{z\,l^2}{2}\,\omega_R \ldots\ldots\ldots\ldots (12a)$$

in which $\omega_R = \frac{x}{l} - \frac{x^2}{l^2}$; and, for Fig. 12:

$$E\,I\,y = \frac{z\,l^2}{6}\,\omega_D \ldots\ldots\ldots\ldots (12b)$$

in which $\omega_D = \frac{x}{l} - \frac{x^3}{l^3}$; and, for Fig. 13:

$$\delta_{ii} = \int M_i^2\,dx = \frac{l}{3}(y_1^2 + y_1\,y_2 + y_2^2) \ldots\ldots\ldots\ldots (12c)$$

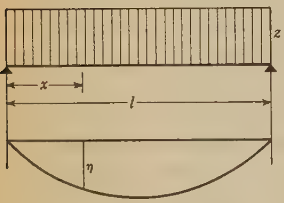

FIG. 11

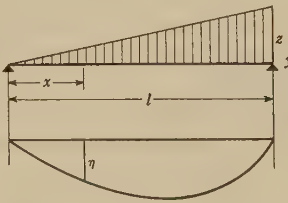

FIG. 12

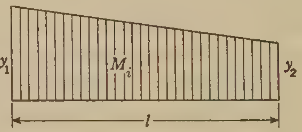

FIG. 13

In this problem only one indeterminate parameter, $X_a = R_1$, is introduced and the beam is treated as a two-span continuous girder (see Fig. 14(a)). Parameter $X_a = -1$ produces bending moments M_{1a} and M_{a2} at Supports 1 and 2 which may be calculated by the equation of three moments of Clapeyron:

$$M_{1a}\,l_2 + 2\,M_{2a}\,(l_2 + l_3) = 0 \ldots\ldots\ldots\ldots (13a)$$

and

$$M_{1a} = -1 \times l_1 = -60 \ldots\ldots\ldots\ldots (13b)$$

Therefore, $M_{2a} = \frac{l_1\,l_2}{2\,(l_2 + l_3)} = \frac{60 \times 72}{2 \times 132} = 16.36.$

The parameter X_a is equal to $\frac{\delta_{ma}}{\delta_{aa}}$ for a load of $P_{m=1}$. Since the deflection curve for a load $X_a = -1$ is the δ_{ma} line, the deflection curve for a load $\frac{1}{\delta_{aa}}$ is the influence line for $X_a = R_1$.

According to Fig. 13: $E I \delta_{aa} = \int M_a^2 \, dx = \frac{60}{3} \times 60^2 + \frac{72}{3} (60^2 - 60 \times 16.36 + 16.36^2) + \frac{60}{3} \times 16.36^2 = 146\,300$; $z_1 = - \frac{60}{146\,300} = - 0.000140$; and, $z_2 = + \frac{16.36}{146\,300} = + 0.000111$. Finally the ordinates of the X_a-line (see Fig. 14(d)) according to Figs. 11 and 12 are:

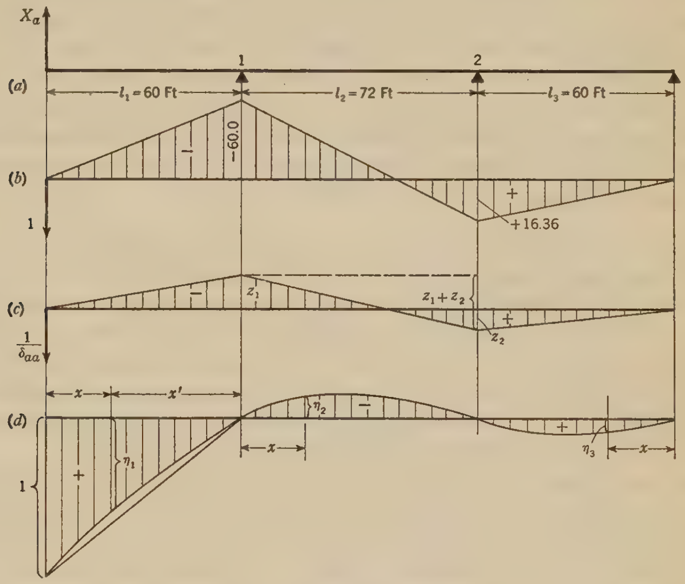
X_a
1
2
(a)
$l_1 = 60$ Ft
$l_2 = 72$ Ft
$l_3 = 60$ Ft
(b)
−
−60.0
+
+16.36
1
(c)
−
z_1
$z_1 + z_2$
+
z_2
$\frac{1}{\delta_{aa}}$
(d)
x
x'
η_2
−
+
x
η_1
+
1
η_3
x

FIG. 14

$$y_1 = \frac{x_1}{l_1} - \omega_D \times 0.000410 \frac{l_1^2}{6} = \frac{x_1}{l_1} - 0.246\, \omega_D \quad \text{(14a)}$$

$$y_2 = \omega_R \times 0.000410 \frac{l_2^2}{2} + \omega_D (0.000410 + 0.000111) \frac{l_2^2}{6}$$
$$= - 1.06\, \omega_R + 0.45\, \omega_D \quad \text{(14b)}$$

and

$$y_3 = \omega_D \times 0.000111 \frac{l_3^2}{6} = + 0.066\, \omega_D \quad \text{(14c)}$$

The values for ω_R and ω_D may be written rapidly, even without the aid of a table:

For $\frac{x}{l} = 0.2$

$\omega_R = 0.2 - 0.2^2 = 0.16$; and, $\omega_D = 0.2 - 0.2^3 = 0.192$

for $\frac{x}{l} = 0.5$

$\omega_R = 0.5 - 0.5^2 = 0.25$; and, $\omega_D = 0.5 - 0.5^3 = 0.375$

and for $\frac{x}{l} = 0.8$

$$\omega_R = 0.8 - 0.8^2 = 0.16; \quad \text{and,} \quad \omega_D = 0.8 - 0.8^3 = 0.288.$$

These values substituted in the equations for y_1, y_2 and y_3 give the same values for the influence line for R_1 as those shown in Fig. 2. This is a very simple and quick method of computing the influence line for a three-span continuous girder as it allows the use of a slide-rule for all computations.

For a multiple-span girder the writer recommends (as does the author) using the bending moments over the supports as redundants. Thus one can apply the equations of three moments, and the deflection curves are computed for each span separately with the values of ω_D, assuming a constant value of I. Müller-Breslau[16] presents a large number of examples demonstrating simple methods of computing such systems.

George L. Epps,[17] Assoc. M. Am. Soc. C. E. (by letter).—In view of the specialized and rather narrow field covered by this paper the writer is quite pleased with the discussion presented and appreciates the efforts put forth by the discussers. They have brought out some very helpful ideas and criticisms.

While the writer based his theoretical explanation on the Müller-Breslau equation, Mr. Deans shows that the procedure also satisfies Maxwell's law of reciprocal deflections. This undoubtedly will be of aid to some engineers who may be more familiar with the application of the latter principles.

In the second part of his discussion Mr. Deans applies the column analogy to continuous trusses. The writer worked from the column analogy in developing this method with the feeling that he was getting a more usable and also somewhat simpler tool of design. That is, the calculations proceed directly to the determination of the fixed-end moments, stiffness, and carry-over factors with one set of computations for each end of the beam, whereas in column analogy it is necessary to make a separate set of calculations for each of the stiffnesses, carry-over factors, and fixed-end moments for each end of the beam.

Mr. Deans requested that the writer comment upon the application of the method to multiple-trussed arches. The writer has had no occasion to make a careful study of this subject, but he sees no reason why it should not be possible to extend the method to include such problems.

Mr. Lamb, in his discussion, raises several objections to the method presented by the writer. First he states it is more difficult to determine influence lines for the individual members of the truss when using the pier moments as redundants than when using the reactions as redundants. There is no difference in the work after the reaction influence lines are obtained. Therefore, it may be enlightening to examine the procedure for obtaining the reaction influence lines.

Using the reactions as redundants, it is necessary to calculate the deflection of each panel point for the entire length of the bridge with a unit load at each of the intermediate piers in turn and no reaction at the other intermediate piers. In a four-span, symmetrical bridge this means that one set of deflections must be calculated for the entire bridge with a unit load at the first pier and another set for half the bridge with the unit load at the center pier. Thus deflections must be calculated for six spans. Using these deflections the reaction influence lines are obtained as outlined by Mr. Lamb in the last part of his discussion.

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[17] Senior Engr., Design Dept., State Highway Comm., Topeka, Kans.

Making the moments the redundants, it is necessary to calculate the deflections for the first span once and for the second span twice. This makes it necessary to calculate the deflections for only three spans, or half as many as for redundant reactions, as already mentioned. These deflections are the ordinates of the influence lines for fixed-end moments. The calculations necessary for the slope-deflection equations, pier moments, and reactions may be made easily in less time than is required to calculate the deflections of three spans. Thus it is evident that a good deal of time is saved using the pier moments as redundants.

Mr. Lamb's second objection deals with the subject of the settlement of supports. The effect upon the structure of raising or lowering any support is easily calculated, using the slope-deflection equations which give the moments at the piers.

Mr. Lamb's third and fourth objections deal with cambering of the truss and the calculations necessary for obtaining the proper camber. The writer agrees with Mr. Lamb that a little changing in the form of tabulation will simplify the calculation of elastic weights for camber.

Mr. Lamb states that he prefers to use the method of angle changes in computing elastic weights, whereas the writer used radii of the members. In Mr. Lamb's example, Table 3, Column (6), the term $\frac{h}{\sec \phi}$ is equal to the radius r, as used by the writer, thus showing that Mr. Lamb was using practically the same method as the writer in obtaining the elastic weights.

Mr. Lamb's method of calculating influence lines for reactions will remove the objection raised by the writer to the large numbers involved in the method of redundant reactions as presented by Mr. Pierce[2] for the case of uniform moment of inertia throughout the beam. However, for variable depth of beam or truss work, complications arise. After Equations ($7a$) and ($7b$) are solved for the reactions as shown in Fig. 10, it is necessary to find the deflected line of the beam which involves calculations similar to those shown in Tables 1 and 2, extended to include the entire girder from A to B. These calculations, while not difficult, consume almost as much time as is required for the complete analysis by distributing moments or slope deflection, to say nothing of the time required before solving Equations ($7a$) and ($7b$).

It is also possible to use Mr. Pierce's method to find the necessary influence ordinates. Of course, this again introduces the large numbers just mentioned. It should be noted that Mr. Lamb's method is essentially the same as Mr. Pierce's down to the solution of Equations ($7a$) and ($7b$).

Mr. Legens presents another variation of the method of redundant reactions for uniform moment of inertia. This also eliminates the large numbers and should be quite useful for the special case of beams of uniform cross-section. However, it becomes almost impossible for use with trusses or beams with haunches.

Mr. Eremin might well have enlarged upon his remarks. The work of Prof. G. G. Krivoshein is not familiar to the writer or his associates and could no doubt have been quoted more fully here. Equation (10), presented by Mr. Eremin, will undoubtedly be useful in preliminary design, but it would be well to use a more exact calculation for final designs.

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Paper No. 2046

SOLUTION OF EQUATIONS IN STRUCTURAL ANALYSIS BY CONVERGING INCREMENTS

By George H. Dell,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. A. Floris, L. E. Grinter, A. A. Eremin, and George H. Dell.

Synopsis

A method of solving equations in structural analysis by converging increments (which is related to the process of successive substitutions) is demonstrated in this paper. The method is also useful as a guide in developing alternate solutions, of the moment-distribution type, in cases where the proper mode of performing the distributions may not be otherwise readily apparent.

Introduction

The past decade has witnessed the widespread adoption of the masterly method of moment-distribution,[2] introduced to the Society by Hardy Cross, M. Am. Soc. C. E., in 1930, which has resulted in dispelling to an unprecedented extent the practical difficulties formerly associated with the subject of statically indeterminate structures. At the same time, the new "movement" has produced a stimulation of interest in the older, algebraic methods of analysis, such as three-moment equations, area moments, slope deflection, etc.

This interest appears to be justified. Many individuals have a natural preference for algebraic methods. Besides, the fact that algebraic procedure has played an important rôle in indeterminate analysis in the past would seem to provide grounds for supposing that these methods will continue to exercise a strong appeal to writers and students for a long time to come. However, algebraic analysis has been somewhat handicapped by a scarcity of practical methods of solving the simultaneous equations. Precise methods generally require too much time; hence, the process of successive substitutions, or iteration, has been used to a considerable extent. The latter method is open to the criticisms that it is lacking in form and is applicable to only the most rapidly converging types of equations.

Note.—Published in October, 1938, *Proceedings*.

[1] Associate in Civ. Eng., Univ. of Illinois, Urbana, Ill.

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

Herein, the writer will attempt to show that the method of converging increments provides an easy, rapid, orderly, and accurate means of solving the equations encountered in a large variety of structural problems; and, furthermore, that it can always be used as a guide to the development of alternate solutions of the distribution type, thereby extending the advantages of this form of solution to new problems in which the distribution ratios cannot be readily determined by inspection. (Applications of the method of converging increments to problems in least squares were described by the writer in 1935.)[3]

EXPLANATION AND APPLICATION OF THE METHOD

The method of converging increments, which is related to the process of iteration, is applicable to systems of simultaneous equations in which each of the several unknowns is strongly represented by a particular equation. As most persons who have worked with the iteration process have doubtless observed, after the completion of a few cycles of substitutions, such systems generally begin to converge with approximately equal rates in the separate equations. When this stage is reached, in the method of converging increments, the solution is brought to a conclusion. If the rate of convergence is rapid, the computations already made are added and the values of the unknowns immediately ascertained. If the rate of convergence is slow, the values of the remaining cycles of figures are computed by series summation, thus making possible a considerable saving in time, while preserving a relatively high degree of precision in the results.

A general derivation of the method as applied to diagonally symmetrical systems has been given elsewhere.[3] The derivation is equally valid for non-symmetrical systems.

The method is ideally adapted to the use of the slide-rule as each successive line in the solution is computed with a single setting of the slide, and the precision of the method as a whole is consistent with that of the slide-rule.

In order to illustrate the application of the method to problems in structural analysis, three examples are given. For Examples 1 and 2, a continuous beam is solved by means of three-moment equations and also by slope-deflection equations. These examples are used to represent the case of rapidly converging equations. Example 3, which deals with a crane-girder bent, is used to represent the case of slowly converging equations.

Example 1.—The equation of three moments for a continuous beam on unyielding supports and with modulus of elasticity constant throughout, may be written in the form:

$$M_0 K_2 + 2 M_1 (K_1 + K_2) + M_2 K_1 = -\frac{6 \bar{x}_1 A_1 K_2}{L_1^2} - \frac{6 \bar{x}_2 A_2 K_1}{L_2^2} \dots (1)$$

in which (as shown in Fig. 1): M_0, M_1, and M_2 = the bending moments at the left, center, and right supports, respectively; K_1 and K_2 = the relative $\frac{I}{L}$-values of the left and right spans, respectively; I = moment of inertia;

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[3] "The Adjustment of a Level Net," by George H. Dell, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 834.

L_1 and L_2 = the lengths of the left and right spans, respectively; $\bar{x}_1$ and $\bar{x}_2$ = the distances from the outer supports to the centroids of the two moment diagrams, the spans being considered as simple beams; and, A_1 and A_2 = the areas of the simple-beam moment diagrams on the left and right spans, respectively. Equation (1) may also be written:

$$M_0 K_2 + 2 M_1 (K_1 + K_2) + M_2 K_1 = K_2 (C_{01} + 2 C_{10}) + K_1 (2 C_{12} + C_{21}).$$

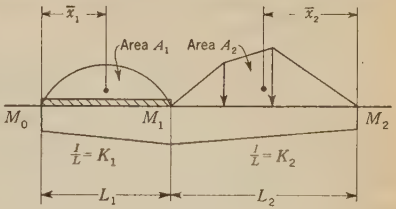

FIG. 1

in which C_{01}, C_{10}, etc., the fixed-end moments, are positive for tension in lower fibers.

For the continuous beam shown in Fig. 2, $\frac{\bar{x}_1}{L_1} = \frac{\bar{x}_2}{L_2} = \frac{1}{2}$, and Equation (1) may be written:

$$M_0 K_2 + 2 M_1 (K_1 + K_2) + M_2 K_1 = -\frac{3 A_1 K_2}{L_1} - \frac{3 A_2 K_1}{L_2} \quad \ldots\ldots(2)$$

The relative values of K for the various spans of the continuous beam are shown in parentheses in Fig. 2. Upon applying Equation (2) to successive pairs of adjacent spans in this continuous beam, the following system of equations is obtained:

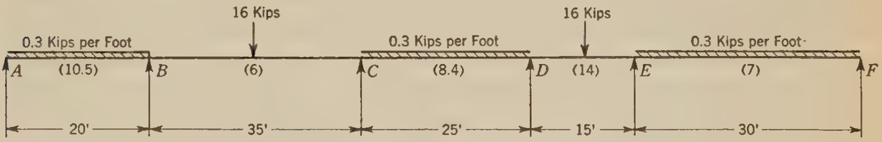

FIG. 2

$$\begin{aligned}
33\, M_B + 10.5\, M_C \qquad\qquad\qquad\qquad &= -2\,385 \ldots\ldots(B)\\
8.4\, M_B + 28.8\, M_C + \quad 6\, M_D \qquad\qquad &= -2\,045 \ldots\ldots(C)\\
14\, M_C + 44.8\, M_D + \ 8.4\, M_E &= -1\,412 \ldots\ldots(D)\\
7\, M_D + \quad 42\, M_E &= -1\,575 \ldots\ldots(E)
\end{aligned}$$

Preliminary to solving, Equations (B) to (E) are written in columns, as shown in Table 1(a). In the following description, Lines Nos. 1, 2, 3, and 4, Table 1, will be referred to as the "first line of the equations," the "second line of the equations," etc. It will be noted that the strong coefficient in each of the equations is underlined.

The arrangement of the solution is shown by the first set of calculations (see Table 2(a)), which (for a system of four equations) consists of the first four lines of the solution. In Line No. 1, Table 2(a), the underlined quantity, $-\underline{2\,385}$, is the value of the right-hand term of Equation (B). This quantity is extended horizontally in proportion to the coefficients in the first line of the equations, namely, $\underline{33}$, 8.4, 0, and 0. In Line No. 2, the underlined quantity is first computed by subtracting − 607 (in Line No. 1) from − 2 045 (the right-hand term of Equation (C)), giving − 1 438. This value is then extended horizontally in proportion to the coefficients of the second line of the equations, namely, 10.5, 28.8, 14, and 0. A similar procedure is followed in obtaining Lines Nos. 3 and 4.

In ensuing sets, the same order of operation is observed as in the first set. The underlined quantity in a given column is equal to the sum of the extensions entered in that column in the preceding lines of the solution below the last underscored extension, multiplied by − 1. The underlined quantities are always extended horizontally in proportion to the coefficients in the corresponding line of the equations. In this manner, the second and third sets of calculations are found to be as given in Tables 2(*b*) and 2(*c*).

Ordinarily, the foregoing process should be continued until the underlined quantities in the final set are found to be in a practically constant ratio with respect to the corresponding quantities of the preceding set (or until the extended items become negligible). Inspection of the computations in Table 2 shows that up to the third set such a condition has not been attained. Accordingly, a fourth set of calculations is made, as shown in Table 2(*d*).

It is now evident that in the second, third, and fourth columns of the calculations, the terms, − 8, 5, and − 1 are each about one-fifth of the corresponding quantities in the third set, namely, − 45, 32, and − 5, respectively. In the first column, this ratio does not hold; on the contrary, an apparent divergency exists. However, it is seen that if a fifth set of calculations were begun, the underlined quantity in the first column would be + 3, which, when compared to + 16, would give these values approximately the same ratio, or rate of convergence, as has been attained by the remaining columns. Consequently, the solution is terminated at the end of Set 4. The sums of the underlined quantities in the separate columns in Sets 1 to 4, inclusive, are:

Equation	Sum of underlined quantities in Table 2	Equation	Sum of underlined quantities in Table 2
(B)...........	− 1 832	(D)...........	− 366
(C)...........	− 1 528	(E)...........	− 1 518

The values of the unknowns are then found by dividing the foregoing sums by the underlined coefficients in the respective equations: $M_B = \frac{-1\,832}{33} = -55.5$; $M_C = \frac{-1\,528}{28.8} = -53.1$; $M_D = \frac{-366}{44.8} = -8.16$; and, $M_E = \frac{-1\,518}{42} = -36.1$.

Example 2.—In the slope-deflection method the expression for the bending moment at Point A in Member $A\,B$, in terms of the deformations, is given by Equation (3):[4]

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) + C \ldots\ldots\ldots\ldots\ldots (3)$$

in which E = the modulus of elasticity; K = the value of the ratio $\frac{I}{L}$ (moment

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[4] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, Members, Am. Soc. C. E., *Bulletin No. 108*, Section 7, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

TABLE 1.—ARRANGEMENT OF EQUATIONS IN EXAMPLES 1 AND 2

Line No.	(a) EXAMPLE 1				(b) EXAMPLE 2			
	(B)	(C)	(D)	(E)	(B)	(C)	(D)	(E)
1	33	8.4	....	....	27.75	6	....	....
2	10.5	28.8	14	....	6	28.8	8.4	....
3	....	6	44.8	7	....	8.4	44.8	14
4	....	....	8.4	42	....	....	14	38.5
Total	−2 385	−2 045	−1 412	−1 575	55.00	−54.38	14.38	3.75

TABLE 2.—SOLUTION OF EQUATIONS (*B*) TO (*E*), EXAMPLE 1

Line No.	(B)	(C)	(D)	(E)
	(a) FIRST SET			
1	−2 385	−607	....	....
2	−524	−1 438	−698	....
3	....	−96	−714	−112
4	....	....	−293	−1 463
	(b) SECOND SET			
1	524	133	....	....
2	−13	−37	−18	....
3	....	42	311	49
4	....	....	−10	−49
	(c) THIRD SET			
1	13	3	....	....
2	−16	−45	−22	....
3	....	4	32	5
4	....	....	−1	−5
	(d) FOURTH SET			
1	16	4	....	....
2	−3	−8	−4	....
3	....	1	5	1
4	....	....	0	−1

TABLE 3.—SOLUTION OF EQUATIONS (6), EXAMPLE 2

Line No.	(B)	(C)	(D)	(E)
	(a) FIRST SET			
1	55.00	11.90	....	....
2	−13.80	−66.28	−19.33	....
3	....	6.31	33.71	10.52
4	....	....	−2.46	−6.77
	(b) SECOND SET			
1	13.80	2.99	....	....
2	−1.94	−9.30	−2.71	....
3	....	0.97	5.17	1.61
4	....	....	−0.58	−1.61
	(c) THIRD SET			
1	1.94	0.42	....	....
2	−0.29	−1.39	−0.41	....
3	....	0.19	0.99	0.31
4	....	....	−0.11	−0.31
	TOTALS OF THE UNDERLINED QUANTITIES:			
	70.74	−76.97	39.87	−8.69
	DIVISORS (SEE TABLE 1(*b*)):			
	27.75	28.8	44.8	38.5
	VALUES OF THE UNKNOWNS:			
	2.55	−2.67	0.891	−0.226

of inertia divided by the length) for the member; θ_A, θ_B = the changes in slope of the elastic curve at Point A and at Point B, respectively, from its initial position; R = the ratio of deflection to length of member; and, C = the fixed-end moment at Point A due to external transverse forces acting on the member.

A somewhat more convenient form for Equation (3) is obtained by means of the substitutions: $A = 2\,E\,\theta_A$; $B = 2\,E\,\theta_B$; and, $\lambda = 6\,E\,R$. Equation (3) may then be written as follows:

$$M_{AB} = K\,(2\,A + B - \lambda) + C \qquad (4)$$

For a continuous beam on unyielding supports the quantity, λ, is equal to zero in each span. In accordance with Equation (4), the bending moments at the supports of the continuous beam in Fig. 2 may then be expressed as follows:

$$M_{AB} = 10.5\,(2\,A + B) - 10 \qquad (5a)$$

$$M_{BA} = 10.5\,(A + 2\,B) + 10 \qquad (5b)$$

$$M_{BC} = 6\,(2\,B + C) - 70 \qquad (5c)$$

$$M_{CB} = 6\,(B + 2\,C) + 70 \qquad (5d)$$

$$M_{CD} = 8.4\,(2\,C + D) - 15.62 \qquad (5e)$$

$$M_{DC} = 8.4\,(C + 2\,D) + 15.62 \qquad (5f)$$

$$M_{DE} = 14\,(2\,D + E) - 30 \qquad (5g)$$

$$M_{ED} = 14\,(D + 2\,E) + 30 \qquad (5h)$$

$$M_{EF} = 7\,(2\,E + F) - 22.5 \qquad (5i)$$

and,

$$M_{FE} = 7\,(E + 2\,F) + 22.5 \qquad (5j)$$

Upon putting $\Sigma M = 0$ at each of the joints, the following equations of static equilibrium (which may be written by inspection) are obtained:

$$21\,A + 10.5\,B = 10.00 \qquad (6a)$$

$$10.5\,A + 33\,B + 6\,C = 60.00 \qquad (6b)$$

$$6\,B + 28.8\,C + 8.4\,D = -54.38 \qquad (6c)$$

$$8.4\,C + 44.8\,D + 14\,E = 14.38 \qquad (6d)$$

$$14\,D + 42\,E + 7\,F = -7.50 \qquad (6e)$$

$$7\,E + 14\,F = -22.50 \qquad (6f)$$

By combining Equations (6a) and (6b), (6e) and (6f), the following system of equations in four unknowns results:

$$27.75\,B + 6\,C = 55.00 \qquad (6b')$$

$$6\,B + 28.8\,C + 8.4\,D = -54.38 \qquad (6c)$$

$$8.4\,C + 44.8\,D + 14\,E = 14.38 \qquad (6d)$$

$$14\,D + 38.5\,E = 3.75 \qquad (6e')$$

Using the procedure already described, the re-arrangement of the equations is shown in Table 1(*b*) and the solution in Table 3. At the end of the third set of computations, the solution shows the following rates of convergence: First column, $\frac{1.94}{13.80} = 0.14$; second column, $\frac{-1.39}{-9.30} = 0.15$; third column, $\frac{0.99}{5.17} = 0.19$; and, fourth column, $\frac{-0.31}{-1.61} = 0.19$.

The sums of the underlined quantities are given below the third set in Table 3, and the values of the unknowns are: $B = \frac{70.74}{27.75} = 2.55$; $C = \frac{-76.97}{28.8} = -2.67$; $D = \frac{39.87}{44.8} = 0.891$; and, $E = \frac{-8.69}{38.5} = -0.226$.

The values of the unknowns A and F may then be found from Equations (6*a*) and (6*f*), respectively. Substitution of the values of the unknowns in Equations (5*b*) to (5*i*), inclusive, gives the following values for the bending moments: $M_{BA} = 55.18$; $M_{BC} = -55.42$; $M_{CB} = 53.26$; $M_{CD} = -53.00$; $M_{DC} = 8.14$; $M_{DE} = -8.22$; $M_{ED} = 36.14$; and $M_{EF} = -36.12$.

The small discrepancies in the two solutions are due to slide-rule inaccuracies and to the neglect of small residuals in the calculations. The latter could have been taken into account, in the manner to be explained in connection with Example 3. Familiarity with the method of doing this enables one readily to determine that by neglecting these residuals the errors in the unknowns would be within 1 per cent.

Example 3.—The crane-girder bent shown in Fig. 3, with slight modification, is the same as that analyzed elsewhere[5] by Harold E. Wessman, M. Am. Soc. C. E. The roof truss is represented by Line $C\,C$; $B\,C$ and $B'\,C'$ are the roof columns; and $A\,B$ and $A'\,B'$ are the columns serving to support the combined roof and crane loads. The dotted line, $B\,B'$, may be considered to be a "dummy" member, with zero stiffness. The relative values of K are shown in parentheses in the diagram. In accordance with Equation (4), the bending moments are expressed by the following relations:

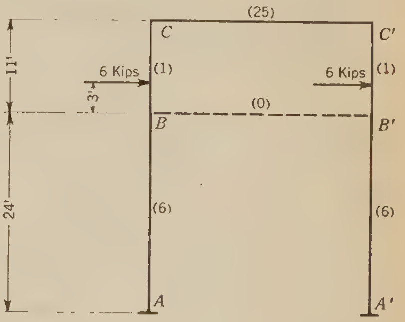

FIG. 3

$$M_{AB} = 6\,(B - \lambda_1) \quad \text{.......... (7a)}$$

$$M_{BA} = 6\,(2\,B - \lambda_1) \quad \text{.......... (7b)}$$

$$M_{BC} = 1\,(2\,B + C - \lambda_2) - 9.52 \quad \text{.......... (7c)}$$

$$M_{CB} = 1\,(B + 2\,C - \lambda_2) + 3.57 \quad \text{.......... (7d)}$$

and,

$$M_{CC'} = 25\,(3\,C) \quad \text{.......... (7e)}$$

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[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 22, p. 49.

Equations (8) may now be written to express the necessary conditions of equilibrium in the structure:

$$14\,B + C - 6\,\lambda_1 - \lambda_2 = 9.52 \qquad (8a)$$

$$B + 77\,C - \lambda_2 = -\,3.57 \qquad (8b)$$

$$-\,6\,B + 4\,\lambda_1 = 48.00 \qquad (8c)$$

and,

$$-\,B - C + 0.667\,\lambda_2 = 4.02 \qquad (8d)$$

The solution of Equations (8) proceeds in the same manner as in the previous examples, four sets of calculations being required to secure a uniform rate of convergence, as shown in Table 4.

TABLE 4.—SOLUTION OF EQUATIONS (8) IN EXAMPLE 3

Line No.	(a) Equations				(b) First Set				(c) Second Set			
	(B)	(C)	(λ_1)	(λ_2)	(B)	(C)	(λ_1)	(λ_2)	(B)	(C)	(λ_1)	(λ_2)
1.....	14	1	−6	−1	9.52	0.68	−4.08	−0.68	85.14	6.08	−36.50	−6.08
2.....	1	77	....	−1	−0.06	−4.25	....	0.06	0.01	0.88	....	−0.01
3.....	−6	....	4	....	−78.12	....	52.08	....	−54.75	....	36.50	....
4.....	−1	−1	....	0.667	−6.96	−6.96	....	4.64	−9.13	−9.13	....	6.09
....	9.52	−3.57	48.00	4.02	....	....	....	....	....	....	....	....

TABLE 4.—(*Continued*)

Line No.	(d) Third Set				(e) Fourth Set			
	(B)	(C)	(λ_1)	(λ_2)	(B)	(C)	(λ_1)	(λ_2)
1.....	63.87	4.57	−27.40	−4.57	47.98	3.43	−20.55	−3.43
2.....	0.05	4.56	....	−0.05	....	3.50	....	−0.05
3.....	−41.10	....	27.40	....	....	....	20.55	....
4.....	−6.93	−6.93	....	4.62	....	....	....	3.48

Upon the completion of the fourth set of calculations the rates of convergence in the separate columns are compared, to determine their degree of uniformity. They are found to be as follows: First column, $\frac{47.98}{63.87} = 0.75$; second column, $\frac{3.50}{4.56} = 0.75$; third column, $\frac{20.55}{27.40} = 0.75$; and, fourth column, $\frac{3.48}{4.62} = 0.75$.

Experience indicates that if the range of discrepancy in these ratios is within approximately 0.05, the remaining terms may generally be estimated by series summation without excessive error. If the desired agreement had not been attained at the end of the fourth set, it would have been necessary to calculate one or more additional sets.

In order to effect the summation by geometrical series, the average rate of convergence and the summation factor are required. In general, the average rate of convergence (denoted by the letter symbol, r) is best determined by dividing the sum of the underlined quantities in the last set of calculations by the sum of the corresponding quantities in the preceding set. In finding these two sums, the additions are performed without regard to sign; that is, the absolute sum is used. The summation factor is then given by the ratio, $\frac{r}{1 - r}$ (the common formula for the sum of the infinite convergent series, $ar + ar^2 + ar^3 + \cdots$ being $\frac{ar}{1 - r}$).

In Example 3 the sum of the underlined quantities in the fourth set is $47.98 + 3.50 + 20.55 + 3.48 = 75.51$; the sum of the underlined quantities in the third set is $63.87 + 4.56 + 27.40 + 4.62 = 100.45$; the average rate of convergence is $r = \frac{75.51}{100.45} = 0.75$; and the summation factor is $\frac{0.75}{0.25} = 3.00$.

The approximate sums of the quantities not included in the first four sets of calculations are then found by multiplying the underlined quantities of the final set by the summation factor, as follows: First column, $47.98 \times 3.00 = 143.94$; second column, $3.50 \times 3.00 = 10.50$; third column, $20.55 \times 3.00 = 61.65$; and, fourth column, $3.48 \times 3.00 = 10.44$.

The results of the series summations are then added to the underlined quantities that have been determined directly, and the values of the unknowns are found, as before, by dividing by the underlined coefficients in the respective equations, as shown in Table 5.

The solution of the problem is concluded by substituting the values of the unknowns in Equations (7), the following bending moments being thus found: $M_{AB} = -147.12$; $M_{BA} = 3.06$; $M_{BC} = -3.06$; $M_{CB} = -14.91$; and, $M_{CC'} = 14.70$.

TABLE 5.—DETERMINATION OF UNKNOWNS, EXAMPLE 3

Column:	First set	Second set	Third set	Fourth set	Series	Totals	Divisors	UNKNOWNS	
								Equation	Values
First. . . .	9.52	85.14	63.87	47.98	143.94	350.45	14	(B)	25.03
Second . .	−4.25	0.88	4.56	3.50	10.50	15.19	77	(C)	0.196
Third . . .	52.08	36.50	27.40	20.55	61.65	198.18	4	(λ_1)	49.55
Fourth . .	4.64	6.09	4.62	3.48	10.44	29.27	0.667	(λ_2)	43.90

In order that the relative accuracy of the method of converging increments in the present instance may be judged, the following results of a precise solution are given: $B = 25.043$; $C = 0.198$; $\lambda_1 = 49.564$; and $\lambda_2 = 43.893$.

The precision of the solution can be increased by computing the errors in the equations resulting from a first solution and using these, with reversed signs, as a new series of right-hand terms for making a few additional sets of calculations. In determining the errors in the equations at any stage of the solution, the appropriate quantities may be extended horizontally, as in the solution itself. If the utmost precision is desired, these extensions should be computed with accuracy.

In order to avoid mistakes in the solution it is desirable that the computations be checked frequently. One method of checking consists in making a careful review of each line before proceeding further. Another method makes use of an additional column of figures throughout. For the equations themselves this column contains the algebraic sum of the coefficients in a given horizontal line. In the solution, these sums are used to secure an additional extension in each line, which should agree with the algebraic sum of the remaining quantities in that line. Additions may be checked by inserting after each set a line of figures consisting of the sums of the quantities which follow the underlined item in each column of the given set; the aggregate of these sums provides a cross-check on the control column from set to set.

The Alternate Solution

If the calculations resulting from a solution of a given problem by the foregoing method are recorded, step by step, on a diagram of the structure in proximity to the joints or members to which they pertain, an alternate solution possessing a number of the characteristics of the method of moment-distribution will be obtained. A detailed comparative analysis of the two solutions then enables one readily to determine the correct manner of performing the distributions, so that additional problems of the same type may be solved by use of the alternate method alone. A procedure is thus provided for extending the method of distributions to various forms of problems in which the correct distribution ratios may not be initially obvious.

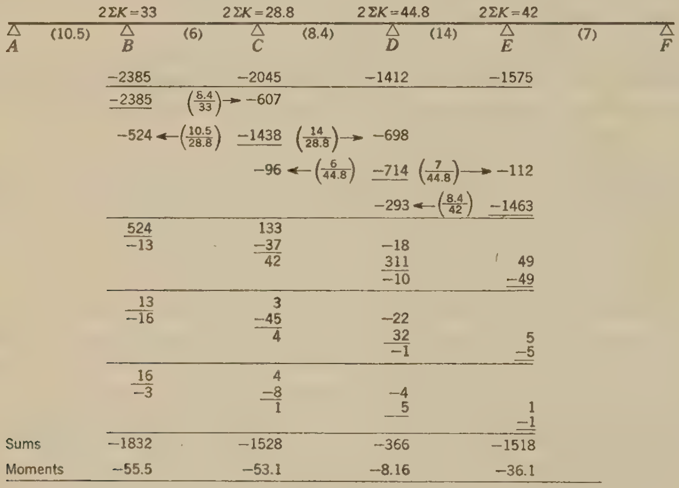

2ΣK=33
2ΣK=28.8
2ΣK=44.8
2ΣK=42
A
(10.5)
B
(6)
C
(8.4)
D
(14)
E
(7)
F
−2385
−2045
−1412
−1575
−2385
(8.4/33) → −607
−524 ← (10.5/28.8)
−1438
(14/28.8) → −698
−96 ← (6/44.8)
−714
(7/44.8) → −112
−293 ← (8.4/42)
−1463
524
−13
133
−37
42
−18
311
−10
49
−49
13
−16
3
−45
4
−22
32
−1
5
−5
16
−3
4
−8
1
−4
5
1
−1
Sums
−1832
−1528
−366
−1518
Moments
−55.5
−53.1
−8.16
−36.1

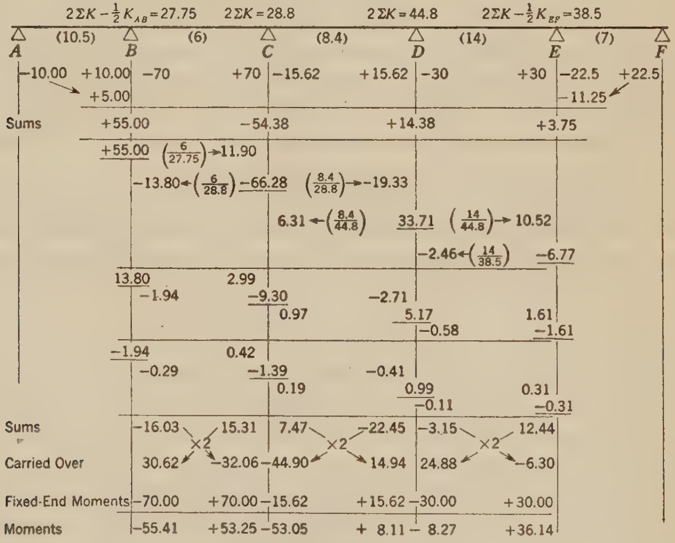
Fig. 4

In order to illustrate the foregoing statements, the solution of the equations of Example 1 is written below the various supports of the continuous beam in Fig. 4.

Comparison of the extended quantities with the coefficients of the equations in Example 1 (Table 1(*a*)) shows that the portion of the first numerical value appearing under Support B ($-$ 2 385) which is to be distributed to the adjacent support depends upon the ratio of the stiffness of Span $C\,D$ to the value of $2\,\Sigma\,K$ at Joint B. Similar ratios for performing the distributions at the remaining supports are indicated in parentheses in Fig. 4. In this example, the entire process of developing the alternate solution is readily followed by referring to the original solution (Table 2), and need not be described in further detail.

In order to illustrate the method of developing the alternate solution in problems in which the unknowns of the simultaneous equations are dependent upon auxiliary equations, Example 2 will be used.

The solution is begun by writing the fixed-end moments adjacent to the various supports in Fig. 5. In accordance with Equations (6*a*) and (6*b*), and (6*e*) and (6*f*), the fixed-end moments at the supports at the two extremes of the beam are divided by $-$ 2 and then brought over to the adjacent support. The moments at the various supports are then added, with change in sign (giving the right-hand terms of Equations (6*b*′), (6*c*), (6*d*), and (6*e*′), respectively), and the first set of calculations, as given by the original solution (Table 3), is written beneath the appropriate supports of the continuous beam in the diagram.



2ΣK − ½K_AB = 27.75
2ΣK = 28.8
2ΣK = 44.8
2ΣK − ½K_EF = 38.5
(10.5)
(6)
(8.4)
(14)
(7)
A
B
C
D
E
F
−10.00
+10.00
−70
+70
−15.62
+15.62
−30
+30
−22.5
+22.5
+5.00
−11.25
Sums
+55.00
−54.38
+14.38
+3.75
+55.00
(6/27.75) → 11.90
−13.80 ← (6/28.8)
−66.28
(8.4/28.8) → −19.33
6.31 ← (8.4/44.8)
33.71
(14/44.8) → 10.52
−2.46 ← (14/38.5)
−6.77
13.80
2.99
−1.94
−9.30
−2.71
0.97
5.17
1.61
−0.58
−1.61
−1.94
0.42
−0.29
−1.39
−0.41
0.19
0.99
0.31
−0.11
−0.31
Sums
−16.03
15.31
7.47
−22.45
−3.15
12.44
×2
×2
×2
Carried Over
30.62
−32.06
−44.90
14.94
24.88
−6.30
Fixed-End Moments
−70.00
+70.00
−15.62
+15.62
−30.00
+30.00
Moments
−55.41
+53.25
−53.05
+ 8.11
− 8.27
+36.14

FIG. 5

In starting the analysis of the extended items, it is seen upon examination of the original solution, that the part of the first moment (+ 55.00) which goes over to the right-hand end of Span $B\,C$ depends upon the ratio of the stiffness of that span to the quantity, 27.75. The latter quantity, according to Equations (6*a*), (6*b*), and (6*b*′), is equal to $2\,\Sigma\,K$ at Joint B, less 0.5 K_{AB}. In a similar manner, the form and significance of the remaining ratios are determined, as shown in parentheses in Fig. 5.

After the second and third sets of computations are entered on the diagram, the bending moments are ready to be calculated. This step could be done in the manner that was followed in the original solution—that is, by adding the underlined quantities, dividing by the underlined coefficients of the corresponding equations, and then substituting in the auxiliary formulas, Equations (5*c*) to (5*h*), inclusive.

However, at this point it is desirable to inquire into the significance of the extended quantities. Again referring to the original solution, it will be seen that the first extended item—namely, $11.90 \left(= \frac{6}{27.75} \times 55.00 \right)$—represents a part of the term, 6 B, in Equation (5*d*), and hence may be used in computing M_{CB}. Therefore, this quantity was written to the left of Support C. Similar reasoning governed the recording of the remaining extensions to the right or to the left of the supports, as shown in Fig. 5. Accordingly, in order to find the bending moments, the underlined quantities were ignored (and might well have been omitted from Fig. 5), and the extended items were added, giving: − 16.03, 15.31, 7.47, − 22.45, − 3.15, and 12.44 (see Fig. 5). Examination of the original solution shows that these quantities are equal to 6 C, 6 B, 8.4 D, 8.4 C, 14 E, and 14 D, respectively (in which the coefficients are the K-values of the various spans). By reference to Equations (5*c*) to (5*h*), inclusive, the remaining part of the calculation shown in Fig. 5 will be readily followed.

It is thus seen that, in problems of the type represented by Example 2, the method of converging increments not only solves the unknowns of the simultaneous equations, but actually evaluates the auxiliary equations at the same time.

CONCLUSION

Although the examples presented herein have been confined to structures involving only four equations in their analysis, the method that has been illustrated in the solution of the equations may be applied successfully to more complicated problems, provided, of course, that the analytical method on which the solution is based yields equations that are sufficiently convergent.

In many cases the rate of convergence can be improved materially by a simple re-arrangement of the equations. In determining bending moments produced by horizontal forces in multi storied structures an effective device consists in replacing each joint equation with a combined shear-and-joint equation. Doing this with Example 3, for instance, the rate of convergence is changed from 0.75 to 0.565, and the summation factor is reduced from 3.00 to 1.30.

DISCUSSION

A. Floris,[6] Esq. (by letter).—An ingenious method for solving a certain type of simultaneous linear equation by successive approximations is presented by the author. In these equations the major coefficients (that is, those belonging to the principal diagonal of the matrix) are appreciably greater than the other coefficients. A more refined criterion of these equations has been given by E. von Mises and H. Polaczek-Geiringer.[7]

The method of converging increments is in reality the mathematical counterpart of the statical analysis of moment distribution. It is exactly the same method as that developed by J. Morris in 1935.[8] The difference between the two is slight and consists mainly in the manner of presenting the subject and the arrangement of the equations. However, the speeding up of the calculations by series summation in case of slow converging equations is shown only by Professor Dell.

The method of converging increments is not an improvement over ordinary iteration.[9] The principal advantage in the use of iteration lies in the fact that, in each step or cycle, an approximate value of the unknowns is determined, which can be improved further by repeating the process. This important advantage can be realized in the author's method only by additional labor. For this purpose the underlined values must be added at the end of each cycle and then divided by the corresponding major coefficients in order to obtain the values of the unknowns. Unless these checks are made, errors that creep in during the distribution will remain unnoticed. Such intermediate calculations are not necessary in the ordinary method of iteration.

Inasmuch as the simultaneous equations encountered in structural analysis are usually very sensitive, the method of converging increments, in certain cases, may make the process extremely lengthy or divergent. The criterion for such cases is given by Mr. Morris.[8]

The assumed values of the unknowns in the ordinary iteration are corrected step by step, until the continuation of the process is rendered superfluous. Errors can easily be detected, and if they are overlooked they retard the solution to be sure, but they do not render the calculations worthless, as is the case with other methods. For equations arising in statics, the cycles of repetition will rarely exceed five or six. The method is simple and easy to apply; and, its range of application is wider.

With regard to the examples chosen by the author, it may be pointed out that the simultaneous equations of Example 1 could be solved equally well, if not better, by the method of successive elimination of unknowns, using an ordinary 10-in. slide-rule. Such cases are rather rare in practical problems.

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[6] Dipl.-Ing., Los Angeles, Calif.

[7] "Praktische Verfahren der Gleichungsauflösung," von E. von Mises und H. Polaczek-Geiringer, *Zeitschrift für angewandte Mathematik und Mechanik*, February, 1929, p. 58.

[8] "On a Simple Method for Solving Simultaneous Linear Equations by a Successive Approximation Process," by J. Morris, *Journal*, Royal Aeronautical Soc., April, 1935, p. 349.

[9] "Analysis by Moment Distribution Aided Through Use of Iteration," by A. Floris, *Engineering News-Record*, June 25, 1936, p. 922; or "Shear Deformation Included in Three-Moment Equation," by A. Floris, *Civil Engineering*, October, 1937, p. 711.

In conclusion, it may be stated that the method of converging increments is interesting no doubt, but it cannot replace ordinary iteration, as far as problems in statics are concerned.

L. E. GRINTER,[10] M. AM. SOC. C. E. (by letter).—This paper seems to bring into usable form a general method of solving simultaneous equations in structural analysis. Professor Dell sees very clearly the advantage of standardizing such solutions with the objective of eliminating errors and of shortening the labor involved.

The most interesting part of the paper, in the writer's opinion, is centered around the solution given in Fig. 4. The author likens this solution to the method of balancing moments, but actually the correspondence seems to be no deeper than a mere similarity of appearance. There is an interesting comparison of the basic conceptions involved that seems important enough to mention.

Within recent years there have been two quite different systems of analysis of indeterminate structures: (1) The classical methods dependent upon the mathematical solution of simultaneous equations; and (2) the modern methods dependent upon a physical picture of the action of the structure that can be expressed as a series convergence of moments and shears, or of angles and deflections. The two systems seem distinct and separate. Certainly the development of the modern methods, started by Hardy Cross, M. Am. Soc. C. E., occurred as a sharp cleavage from the mathematical procedure of the classical methods. Neither moment distribution nor the method of balancing angle changes was developed as a mathematical tool for solving a group of simultaneous equations. Each was developed as a physical procedure that gave rise to a solution of the problem without reference to simultaneous equations.

The calculations of Fig. 4 seem to fit in between the classical and the modern methods of analysis. It is strange that their presentation should follow rather than precede the presentation of the modern methods themselves. This merely illustrates what a distinct advance the procedure of balancing moments represented. In one tremendous sweep the classical methods were pushed aside and the modern methods were introduced. There was no intermediate step such as Fig. 4 might have represented.

Fig. 4, however, does not reveal a very clear physical picture for the computations recorded thereon. Equation (1) expresses as its right-hand member the important quantities used in Fig. 4. That is, the expression $\frac{6\,\bar{x}_1\,A_1\,K_2}{L_1^2} + \frac{6\,\bar{x}_2\,A_2\,K_1}{L_2^2}$ gives rise to the quantities recorded at the top of each column of numerals in Fig. 4; but this expression is actually equal to $6\,(\theta_1 + \theta_2)\,K_1\,K_2$, in which θ_1 and θ_2 are adjacent end slopes of the simple spans meeting at the joint considered. Thus, the quantity to be used in balancing is neither a moment nor an angle; it is the sum of two angles multiplied by the product of two stiffness factors. It does not seem hopeful to attempt to attach a very useful physical significance to such a quantity as this.

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[10] Dean, Graduate Div., and Director of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

The argument as to whether the desirable solution of a given problem is mathematical or physical can never be settled. A purely mathematical solution such as the author suggests is useful to persons who visualize with difficulty. A physical procedure, which is merely doing with arithmetic what could be done to a model of the structure in the laboratory, is advantageous primarily to the person with imagination.

The writer sees the physical concept in analysis as being most useful when there is need for extension of the method to new cases, new arrangements or new structures. One can always "fall back" upon the fact that the arithmetical process merely states, step by step, the physical treatment that could be accorded a model of the structure in the laboratory. There is a one-to-one correspondence between the recorded arithmetical steps and the physical adjustments visualized for the model.

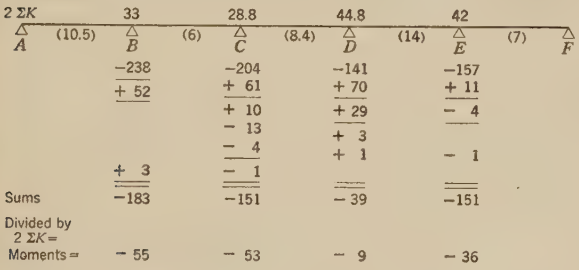

FIG. 6.—ABBREVIATED COMPUTATIONS OF $(\theta_1+\theta_2) K_1 K_2$

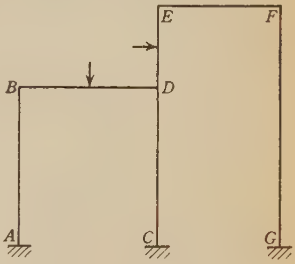

FIG. 7.—COMPLEX BENT

If the author is interested in saving time in analysis there is no reason why his calculations of Fig. 4 should not be reduced to those given in Fig. 6. In this form there will actually be fewer recorded intermediate results than would be used in balancing moments or angles, but an extension to a more complex case will not be self-evident. For example, if the problem is to analyze the bent of Fig. 7, what should be the initial factors at the joint *D*? Should the carry-over factors be the same as in Fig. 6, and how is the shear balance to be effected? These questions must be answered from a study of the simultaneous equations involved, and the answer is not self-evident.

The calculations of Fig. 6 have been simplified by a reduction of the number of significant figures and by a change of signs. Three significant figures obtainable from an ordinary slide-rule are, in the writer's experience, almost always adequate for design purposes. The results of Fig. 6 are expected to be in error by unity in the last figure, but the influence upon important moments is not significant. It is unlikely that the small moment of −9 at Point *D* would have any important influence upon the design. The change of sign for the carry-over values was adopted so that the column addition could be made for all recorded values and so that numbers need not be repeated. The result is a convenient and rapid automatic procedure for analyzing the special case of a continuous beam with simply supported ends. Whether it can be developed to be equally useful for other types of beams and for continuous frames is another matter. The author has performed a service by describing what might well have been an intermediate step between the classical and the modern methods of analysis although his actual procedure remains primarily a tool for use with the classical methods.

A. A. EREMIN,[11] ASSOC. M. AM. SOC. C. E. (by letter).—The solution of equations with more than three unknowns is a complicated task even with the aid of a computing machine. It is gratifying, therefore, to encounter a simplified method, such as that presented by Mr. Dell, based on the principle of converging approximations. The computations may be still further simplified as demonstrated in Fig. 8, as applied to Equations (*B*) to (*E*) of the paper.

Stiffness Factors	33		28.8		44.8		42
	10.5		8.5 | 6		14 | 8.4		7
Carry-Over Factors	*B*	0.255 → 0.347 ←	*C*	0.487 → 0.134 ←	*D*	0.166 → 0.20 ←	*E*
Fixed-End Moments	−2385		−2045		−1412		−1575
1st Distribution	−2385	→	−607				
	−524	←	−1438	→	−698		
			−96	←	−714	→	−112
					−293	←	−1463
2nd Distribution	524	→	133				
	−13	←	−37	→	−18		
			42	←	311	→	49
					−10	←	−49
3rd Distribution	13	→	3				
	−16	←	−45	→	−22		
			4	←	32	→	5
					−1	←	−5
4th Distribution	16	→	4				
	−3	←	−8	→	−4		
					5	→	1
					0	←	−1
Sums	−1832		−1528		−366		−1518
Moments	−55.3		−53.1		−8.2		−36.1

FIG. 8.—DISTRIBUTION OF BENDING MOMENTS

The stiffening factors in Fig. 8 are written in the same order as they appear in the equations. The carry-over factors were computed with stiffening factors at the joints and have logical directions. The distribution of fixed-end moments is made in a similar manner to that of the paper. The order of computations shown in Fig. 8 is easy to memorize and is in a convenient form, designed for greater assurance against errors of computation.

GEORGE H. DELL,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The method of converging increments, in a sense, is an iteration process. The term "successive substitutions" is more appropriate, perhaps, in describing the method preferred by Mr. Floris. It appears doubtful whether the occurrence of undetected errors is any less probable in that method than in the writer's. Regardless of the method used, all solutions should be checked, with a view to eliminating significant errors. If, in the method of converging increments, it is found that an error has occurred at some earlier stage of the solution, it may be eliminated conveniently by inserting the necessary correction at the point where the error occurred and carrying through the extended corrections, in red pencil, for a few lines.

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TABLE 6.—Partial Solution of Equations in Example 3 by Successive Substitutions

Line No.	(B)	(C)	(λ₁)	(λ₂)
	(a) Equations			
1	14	1	−6	−1
2	1	77	....	−1
3	−6	....	4	....
4	−1	−1	....	0.667
	9.52	−3.57	48.00	4.02
	(b) First Set			
1	9.52	0.68	−4.08	−0.68
2	−0.06	−4.25	....	0.06
3	−78.12	....	52.08	....
4	−6.96	−6.96	....	4.64
	(c) Second Set			
1	94.66	6.76	−40.58	−6.76
2	−0.05	−3.37	....	0.05
3	−132.87	....	88.58	....
4	−16.09	−16.09	....	10.73
	(d) Third Set			
1	158.53	11.33	−67.98	−11.33
2	0.01	1.19	....	−0.01
3	−173.97	....	115.98	....
4	−23.02	−23.02	....	15.36
	(e) Fourth Set			
1	206.50	14.76	−88.53	−14.76
2	0.06	4.69	....	−0.06
3	−204.79	....	136.53	....
4	−28.24	−28.24	....	18.84

TABLE 7.—Comparison of Method of Successive Substitutions and Method of Converging Increments, Example 3

Set No.	(a) Successive Substitutions			
	Calculated Values of Unknowns			
	(B)	(C)	(λ₁)	(λ₂)
1	0.68	−0.055	13.0	6.96
2	6.77	−0.044	22.2	16.1
3	11.3	0.016	29.0	23.0
4	14.8	0.061	34.1	28.3
5	17.3	0.096	38.0	32.2
6	19.2	0.122	40.9	35.1
7	20.7	0.141	43.1	37.3
8	21.8	0.155	44.7	38.9
9	22.6	0.165	45.9	40.2
10	23.2	0.174	46.8	41.1
11	23.6	0.181	47.4	41.7
12	23.9	0.185	47.8	42.1
	(b) Converging Increments			
4*	25.03	0.196	49.55	43.90
	(c) Results of Precise Solution			
....	25.043	0.198	49.564	43.893

* Including series summation at end of fourth set.

There is little or no advantage in finding the values of the unknowns before the solution has converged. The advantage in the method of successive substitutions lies, rather, in the fact that at each step the value of a given unknown is chosen in such a manner as to satisfy one of the original equations. However, unless the solution is thoroughly checked, errors will be just as likely and, frequently, as troublesome as in converging increments.

The computations in the method of successive substitutions may be arranged in almost precisely the same manner as in the method of converging increments. Those who prefer the former method will find this arrangement conducive to improved convenience, orderliness, and ease of checking. Table 6 shows such an arrangement of the first four sets of the solution of the equations in Example 3. The equations are written in vertical, or columnar, formation (Table 6(*a*)), as before (see Table 4). The first set of computations, Table 6(*b*), is exactly the same as in the method of converging increments. In the second set, Line 1, Table 6(*c*), the underlined term is $9.52 - (-0.06 - 78.12 - 6.96) = \underline{94.66}$, which is extended horizontally in the same manner as before. In Line 2(*c*), the underlined quantity is $-3.57 - (-6.96 + 6.76) = -\underline{3.37}$. In Lines 3(*c*) and 4(*c*), the underlined quantities are $48.00 - (-40.58) = \underline{88.58}$, and $4.02 - (-6.76 + 0.05) = \underline{10.73}$, respectively. The remaining sets are computed in a similar manner.

Mr. Floris states that the method of converging increments may result in a lengthy or divergent solution in some instances. The fact is that at any given stage in the solution the results are absolutely identical, whether by converging increments or by successive substitutions. He also states that the range of application of the latter method is greater. This is incorrect, for the reason just stated. On the contrary, the method of converging increments has a distinct advantage in the case of slowly converging equations which, after a few sets, acquire a uniform rate of convergence. To illustrate this fact, a comparison of the two methods of solution of the equations in Example 3 is given in Table 7. It will be observed that after twelve sets of successive substitutions (Table 7(*a*)), the errors in the unknowns are consistently in excess of 4%, while being of negligible magnitude after only four sets of converging increments and summation of series. It was precisely in order to gain this advantage that the writer chose to use increments, rather than the arrangement shown in Table 6.

In contrasting the mathematical processes of the older methods with the physical "pictures" of the Cross method, and variations thereof, Dean Grinter has presented a concise and extremely interesting discussion. His comparison is so complete and impartial as to make it difficult for one either to disagree with, or to add to, his observations. However, the writer believes that, fundamentally, the picture is the same in both classes of analysis, the principal difference being that, when equations are written, the physical conditions are built in, once and for all, whereas in the other type of solution, a single physical condition is satisfied at each step of the process. The solution is largely automatic in either case.

The form of solution used in Example 1 is readily extended to continuous beams with one or both of the end supports fixed against rotation. If, in Fig. 1, the left-hand support does not rotate, that fact is expressed by the equation

$$2\,M_0 + M_1 = -\frac{6\,A_1}{L_1}\left(1 - \frac{\bar{x}_1}{L_1}\right) \qquad \text{(8a)}$$

or, using fixed-end moments, with signs as previously defined,

$$2\,M_0 + M_1 = 2\,C_{01} + C_{10} \qquad \text{(8b)}$$

When included in a system composed largely of equations of the form shown by Equation (1), it is best to multiply both sides of Equation (8*a*) or (8*b*) by K_{01}. For the structure shown in Fig. 2, with Joints A and F fixed, the system of equations is as follows:

$$\begin{aligned}
21\,M_A + 10.5\,M_B &= -\ 315\\
6\,M_A + 33\,M_B + 10.5\,M_C &= -\ 2\,385\\
8.4\,M_B + 28.8\,M_C + 6\,M_D &= -\ 2\,045\\
14\,M_C + 44.8\,M_D + 8.4\,M_E &= -\ 1\,412\\
7\,M_D + 42\,M_E + 14\,M_F &= -\ 1\,575\\
7\,M_E + 14\,M_F &= -\ 472.5
\end{aligned}$$

The corresponding alternate solution is then much the same as in Fig. 4.

The simplications suggested by Dean Grinter and Mr. Eremin for the form of solution shown in Fig. 4 constitute worth-while improvements. The former consists in changing the signs of quantities carried over and in indicating with a bar the fact that a distribution has been made at a certain point. The change of signs is advantageous in the "alternate" type of solution, but is not recommended in the tabular form of solution of equations. Mr. Eremin's improvement, Fig. 8, consists in showing the ratios in decimal form at the beginning of the solution, accompanied with arrows written in such a position as to be most helpful in making the distributions.

An interesting variation of the solution in Example 1 is obtained by dividing each equation by $2\,\Sigma K$. The resulting alternate solution will then be found to resemble closely a variation of moment-distribution which involves the "skipping" of joints. In this form, the denominators of the distribution ratios will be different on the two sides of a joint, and hence it would be doubly desirable to use Mr. Eremin's suggestion.

It does not appear feasible to apply the method of Example 1 to the type of structure shown by Dean Grinter in Fig. 7. For such a case, the method of Example 3, involving the solution of a system of equations expressing the necessary statical and geometrical relations, offers a convenient means of analysis.

The writer is deeply appreciative of the interest shown by the discussers of the paper and has derived considerable satisfaction in learning their various reactions. He considers them to be mainly favorable to the method presented for the solution of certain types of equations frequently encountered in structural analysis.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2047

MOTOR TRANSPORTATION—A FORWARD VIEW
A SYMPOSIUM

Joint Responsibility of the Automotive and the Civil Engineer
By C. F. Kettering, M. Am. Soc. C. E. 1563

The Trend of Modern Highways
By Thomas H. MacDonald, Esq. 1581

Trunk-Line Highways in Metropolitan Areas
By Leroy C. Smith, M. Am. Soc. C. E. 1586

With Discussion by Messrs. F. Lavis, Edgar Dow Gilman, George Hartley, Robert Kingery, R. L. Morrison, Roy F. Bessey, Bruce D. Greenshields, Robert B. Brooks, H. George Altvater, W. W. Crosby, Thomas H. MacDonald, and Leroy C. Smith.

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JOINT RESPONSIBILITY OF THE AUTOMOTIVE AND THE CIVIL ENGINEER

By C. F. Kettering,[1] M. Am. Soc. C. E.

Synopsis

A number of years ago the writer was making a motor trip in Kentucky running some tests, and had gone far off the main road. The map did not show any roads leading away from the place it was so small. It was late in the afternoon and he wanted to return home before dark. Driving along a little stream bed (which was the only road there was) he overtook an old gentleman driving a donkey hooked to a stone boat loaded with milk cans. Question: "Neighbor, what is the best way to go to Cincinnati from here?" Answer: "Go right up to the forks in the road. Now, it don't make much difference which fork you take but, to tell you the truth, Stranger, if I was going to Cincinnati I wouldn't start from here." Sometimes people get to thinking like the old Kentuckian; but they must first recognize that, whatever their destination, they must start from where they are.

The leaders of the automobile industry have never been able to predict what the industry will offer the public two years in advance. If they knew what the car of ten or twenty-five years in the future was going to be like they would be making it now. To afford some idea of what the future might bring without taking all the responsibility upon his own shoulders the writer asked a number of men, holding responsible positions in the automobile and allied industries. On some questions they were almost unanimous in agreement; on others, they had entirely opposite ideas. All the various prophecies and predictions made in the past have been studied and, with these, the writer has tried to present, in some kind of complete picture, the requirements that will be demanded of those who will supply roads for the automobile of 1960.

All one can do now is to determine conditions as they are to-day, what they were yesterday, and, by projecting a line through the two known points, reach some idea of what the future may hold. It is an excellent idea for the road builders and automobile makers to get together and exchange ideas concerning their mutual problems. After all, the vehicle and the road must form a complete transportation system. Each one is dependent upon the other for the success of the whole. Civil engineers have the most difficult job because of the time factor. They must build a road to last twenty or twenty-five years for vehicles whose average life is only seven or eight years. Before a road is one-third used up, there is an entire new generation of motor vehicles with their new requirements; and before the road is finally ready for replacement the requirements have changed several times over. The aim of this paper is to predict what changes this time factor will demand so that civil engineers can build accordingly.

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INTRODUCTION

The greatest problem involved in all thinking is to determine the time rate of action. The time rate of gravitation is known. If a man jumps from a window the time it will take for him to hit the sidewalk can be computed. There are other natural laws which are just as fundamental as the law of gravitation. Each of them has an entirely different time rate of action, and these are not understood at all.

An interesting incident occurred in connection with the salmon fishing industry in one of the large rivers of the United States. The industry was persuaded that it would be a good thing to have a salmon fish hatchery and, therefore, two or three of them were built. Fish hatching was begun, and the product sent out, to return again after the proper cycle. One year, one of the hatcheries burned down, and there was an immediate demand for a new one. People went to the authorities to get an emergency appropriation for building a new fish hatchery; but the time rate of political action is not that fast. The subject had to be discussed and the hatchery was not built that year. Some of the opponents argued, "they had just as big a run of salmon the year after the fire as they had the year before." The result of the delay in building was that, when the proper year in the cycle came around, the salmon catch was small because of the absence of the product of the hatchery. This has happened in some other problems, such as road building. Eight or ten years ago engineers missed something; so "the run is not so good" this year. This time factor must be studied so that similar troubles will not recur. Maybe the place to start is with the politicians.

The writer belongs to a group of men who believe the world is not finished. Nothing is constant but change. Men work day after day, not to finish things, but to make the future better. They want the future just as nice a place as they can make it because they will spend the remainder of their lives in the future.

Many things need to be torn down and thrown away. The country needs to be rebuilt and made better than ever before. The economists and bankers say, "Where are you going to get the money?" The money will come from the same source as when the country was built this far. There was no money with which to start. All that the people did was to dig each building out of the ground. It has always been here in the country, only in a different form.

Man-hours can be used to convert material into anything that Man needs. An intelligent people will supply the work. When materials are converted into human utilities money is only used as conveyor to carry it from one point to another. Sometimes, people calculate entirely too much in dollars. The wealth of the nation is not in dollars; it is in useful products. The positive side of economics is the movement of useful material through the channels of trade. That must always precede the return flow of money through the counting houses.

In preparing this paper the writer sent a list of eighteen questions involving factors that might affect road requirements in 1950 or 1970 to many people who it was thought might have valuable information on at least some of them. Their opinions were requested on any particular one subject upon which they had information; or, if they wished, they could comment on all eighteen. The subjects upon which the questions were based were:

(1) Growth in registration of passenger cars.
(2) Growth in registration, use, etc., of trucks.
    (*a*) Will railroads pick up and deliver less than car-load lots?
(3) Growth in registration and use of buses.
    (*a*) Will buses supersede street cars in cities?
(4) Effect of the trailer on automobile design, roads, laws, traveling habits, trailer camps, home building, population movement, etc.
(5) Road speeds of passenger cars, trucks, and buses.
(6) Size and weight of road vehicles.
    (*a*) Will this make segregation of different types of traffic necessary?
(7) The future of other types of motive power, such as the Diesel engine and the problem of suitable fuels.
(8) Design changes in the automobile which will affect road-building requirements and make safer vehicles.
(9) Lighting on the vehicle and highway.
(10) The future of tires—changes in coefficient of friction, life, etc.
(11) Possibility of high-speed trunk highways and separate truck highways between cities.
(12) Will travel be overhead or underground in congested areas?
(13) Will the population live in larger cities or will the trend be back to smaller communities?
(14) What mechanical devices are practical to control traffic conditions and actions of the driver?
(15) How will parking in cities be solved?
(16) How will laws regulating traffic, vehicles, and the driver affect road requirements?
(17) Type, kind, and number of highways that should be provided for the future.
(18) Possibility of chemistry contributing better road materials or treatment.

Increase in Registrations

The probable growth in population in the United States for the next 25 yr is quite easy to predict and such predictions have proved reliable in the past.

In Fig. 1, Curve (1) shows the population growth from 1900 to 1930, as taken from reports of the United States Census Bureau. From 1930 to 1960 the probable future population is shown as estimated by the American Petroleum Institute. The data represent the averages of five estimates from life insurance companies and others. Passenger-car registrations are plotted as Curve (2) with the estimated increase per 5-yr period to 1960. The estimated number of persons per car is plotted as Curve (3). Registrations in trucks are estimated to increase to 6 000 000 by 1960 (see Curve (4)). Curve (5), Fig. 1, shows the total estimated motor-vehicle registration by 1960 as more than 37 000 000 vehicles. The registration values were estimated by the American Petroleum Institute.

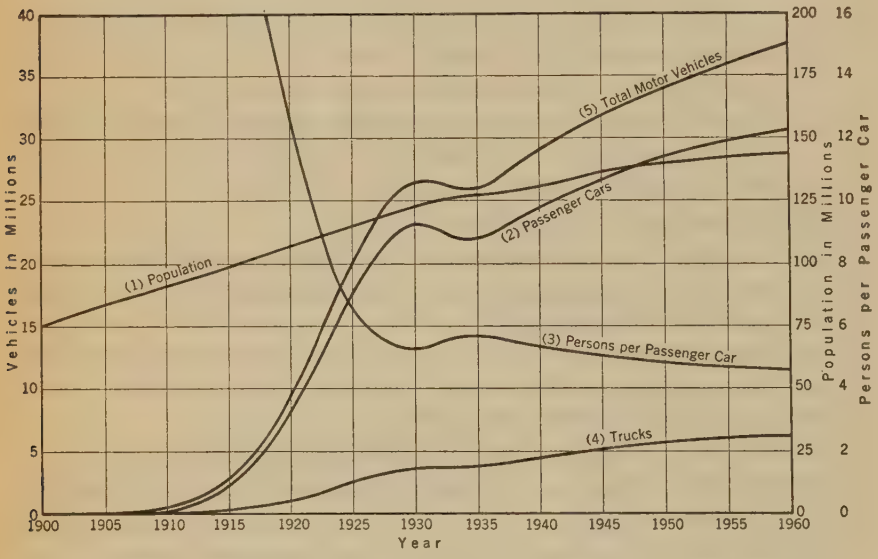

FIG. 1.—ESTIMATED MOTOR VEHICLE REGISTRATION TO 1960

From the foregoing it is apparent that engineers will have to provide roads for 50% more motor vehicles in the next 23 yr. Since many main roads are already inadequate to carry the traffic safely, it would seem that a tremendous task lies ahead. The area of the various countries per mile of road, and the number of cars per mile of highway, are as follows:

Nation	Area, in square miles per mile of road	Cars per mile of road
United States	1.0	8.2
Germany	0.8	4.4
France	0.5	4.9
Great Britain	0.5	10.3
Italy	1.1	3.6
Australia	6.3	1.3
Brazil	35.2	1.3

In comparison with most of the remainder of the world, highways in the United States are now congested. An increase in registration on the present system would almost prohibit much of the usual driving. What the optimum ratios should be is not known, but it is certain that more improved highways are badly needed.

Almost all those who answered the questionnaire believe that the population trend is away from congested areas. Railroads, unsupplemented by efficient highway transport, caused concentration of population at station points. Automobile transportation counters the railroad influence, and provides economic and social communication facilities, at greater distances from, and independent of, rails. Man has an inborn feeling for open spaces and a plot of ground, which he cannot lose entirely. There are a number of indications that the trend is away from cities and back to smaller communities. Several large manufacturers have definitely committed themselves to a policy of erecting new plants in small towns. One of the largest retail companies, with stores all over the country, is opening or building new stores only where large parking spaces can be obtained for the use of its customers. Traffic congestion and the problem of parking will have the greatest influence on this movement. A number of men think cities will be necessary for conducting many forms of business and government, but that a large percentage of the population can live more comfortably and find most of their needs cared for in small communities.

There are those who predict the "string" or "ribbon" city. This is an almost continuous population existing on both sides of a trunk highway. The ribbon city may be pictured as an elongated community like a long string with beads of different sizes strung upon it more or less erratically. The biggest beads are the large centers of population. They are certain to develop because of the need of centralizing large financial institutions, railway centers, ports, costly public facilities, such as water-works, gas-works, museums, libraries, and other agencies which have grown with large cities. Small "beads" will develop at strategic points and will consist of all gradations between the large city and the equivalent of small country cross-roads.

Others believe in the increase in importance and number of small communities which would be supported by a local industry. Each town would have at least one factory. Because of the greater amount of leisure which every one has, residents of small cities would be able to have plots of ground for cultivation and would raise part of their food requirements. This would make them less dependent upon manufacturing for their living. This type of rural life would make highways doubly important, both for pleasure and commercial use.

There are others who believe that faster means of transportation from outside into the center of large cities is coming. Workmen could live distances as great as fifty miles from their work. Homes would be situated on large plots in the open country. The city would be for work, business, and commerce. To accomplish this would require high-speed, limited-access trunk highways to the city limits. The city would have to have many elevated or depressed highways crossing it in every direction. Present level streets would be used for local travel. Local conditions will govern whether the thoroughfares are built underground or overhead.

PARKING

City parking is one of the most serious problems. It is inconceivable that motor-car owners will long continue to park cars almost bumper to bumper along busy city streets and suffer the damage and inconvenience which result. The traffic capacity of the present city street system could be almost doubled by eliminating parking and using the streets for travel only, as they should be used.

Experience seems to indicate that drivers and store owners have unconsciously, but effectively, accepted the idea that cities must provide the parking space required for the ordinary transactions of business and pleasurable pursuits of their citizens. The only regret in the mind of the average person seems to be that he has not been able to find any method of placing more than two curbs along a given street. The people in Detroit, Mich., have even found a way to do that on a few streets.

Accepting the idea that cities must provide space, it would seem far cheaper for them to provide parking lots than to supply expensively improved streets for parking purposes. If it is right for a city to go so far as to condemn and acquire strips from the front of existing lots, and even buildings, to convert these strips into additional highway widths, to pave them expensively, and then to have large parts occupied permanently for parking, why is it not legitimate and much more logical to condemn cheap but well-located property for conversion into city-operated parking lots?

In residential areas there is a grass plot between the sidewalk and the street. Why should not this space be used for parking instead of the traffic area of a street built for heavy duty? There are even city ordinances prohibiting such a practice.

Parking garages, either separate or located within the building they are meant to serve, are growing in number. Every large building should provide adequate parking space for its tenants and customers. Several well developed means of doing this are in use. Double-decked or triple-decked exposed parking lots may be necessary in some areas. However, the problem of parking is not one for either the automotive engineer or the civil engineer to solve directly.

PASSENGER CARS

What possible changes in the passenger automobile might affect road requirements? The car has, in no way, reached a state of standardization and there is no indication that it will. The car of ten or twenty-five years from now (1938) will be just as different and will have just as many improvements as they have had in the past ten or twenty-five years. The 1937 car will certainly seem as antiquated to those who see it in 1960 as the 1912 car does now. It is the policy of the automobile industry to improve its product constantly.

Sometimes an idea of the future may be gained by looking at the past. Suppose a buyer had picked out a brand new 1912 car just as it came from the production line. At that time every one agreed that it was the best car that could be obtained. Suppose it was sealed in an air-tight glass case and the price ($2 000) marked in gold letters where every one could see. As the years passed the buyer went back to look over his perfect car. The second year, its value had decreased to $1 800; and, as each year passed, its value decreased until in a short time (much less than the twenty-five years since it was sealed up) only the junk man would value it. Why? Remember it was sealed so that nothing could happen to it—no rust or corrosion, and no wear. The car remained exactly as on the day it was purchased; but what changes there have been since 1912 in the idea as to what an automobile should be! Cars now have self-starters, four-wheel brakes, better finishes, higher performance, improved appearance, fine riding qualities, and many more attributes than any one could mention. If a buyer were to do the same with a 1937 car now, he would surely have the same experience in 1960.

It is not possible to foretell exactly what the car of the future will be like. There are so many factors that might enter at any time to throw such prophecies out of line. It is feasible, however, to discuss some of the problems automobile engineers are talking about and experimenting with; questions they think important enough on which to put time and effort. The requirements for maneuvering and controlling the car, for example, are closely related to road conditions in many instances. These items are affected by such factors as steering, "hill climb," top speed, vision, lighting, riding quality, and ease of control, all of which are directly concerned with the safe use of the roads.

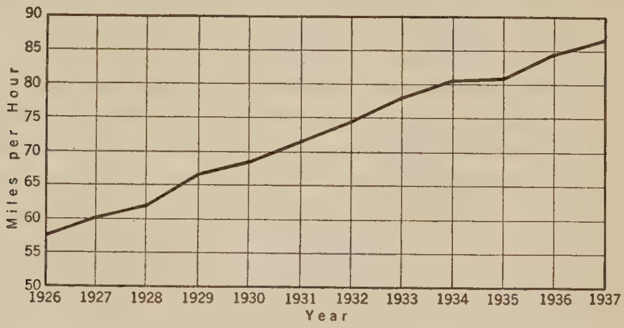

FIG. 2—CURVE OF MAXIMUM SPEED

Engineers do not always think of engine design as having anything to do directly with road building and safety and yet the power source may profoundly affect the type of surface upon which the car rides. It influences performance factors: Speed, acceleration, economy, flexibility, and "hill climb." From several sources of information one learns that there are indications that the practical usable maximum speed has been reached and that engineering emphasis will be placed on other factors in the future. Customer research surveys have shown that people, on the average, do not want to travel much faster than 60 miles per hr; but they want to go that fast without effort. Certainly, roads should not be designed for any less than 60 miles per hr. Other authoritative sources of information reveal that speed is an inherent reason for the widespread use of the automobile and that designers should do everything possible to maintain that advantage. As roads and cars become better, speeds may increase still further and with more safety than at present. Fig. 2 shows the trend in maximum speed of a representative group of cars for the eleven years, 1926–1937.

Although the question of maximum top speed may be controversial and may depend upon how safe the highways are made, every one insists that a higher average speed should be maintained between cities. This indicates the need for high-speed highways built to carry traffic safely. Elimination of grade crossings, limited access, separate lanes, and locations to pass around cities, are some of the requirements. These highways are necessary between main centers now, and their requirements will not depend upon changes in the vehicle that may be anticipated at the present time.

Acceleration and "hill climb" are closely akin. Accelerations will probably increase. This ability is important in "getting away" from a stop and increasing to traffic speed, to pass and return into line in a short distance, and to cross intersections without delay. It will affect the timing of traffic lights and regulating devices. It now appears that any passenger car will be able to negotiate a 10% grade without losing its speed. Undoubtedly, the trend is for the performance of trucks and buses to equal more nearly that of passenger cars. This will undoubtedly be true in the future.

The ability to stop is of major importance. Here is a problem that civil, as well as automotive, engineers can do much to solve. For its part the automobile industry will offer better and better brake equipment as it learns more and more how to do the job. However, the coefficient of friction between road and tire will always be the limiting factor in stopping distances. Although something may be done with the tire to increase the coefficient, the road surface itself seems to offer the greatest possibility for improvement. Everything possible should be done to increase this factor, and it might be well to include a minimum limit in road specifications. To show how much this may vary on different surfaces, the writer has computed stopping distances from 50 miles per hr for a number of coefficients, as follows:

Coefficients of friction	Distance, in feet, required to stop from a speed of 50 miles per hr
1.0	83
0.9	91
0.6	140
0.4	208
0.2	416

The large variations are clearly shown. Surfaces and materials vary the friction characteristics greatly and warrant important consideration in building new roads. The change with weather conditions also is of great importance. There is little that can be done with the tire to give higher values compared to the large variations encountered on present-day roads. At present, a coefficient of about 0.6 is all that can be relied upon in designing brakes.

The size of passenger cars will be determined largely by the space required to house the passengers. Probably this will not change very much. Road clearances of a little less than 8 in. have been common for a number of years and it would seem that nothing now being developed would reduce this value. Clearances are important in building alley intersections, intersection street grades, and driveways. Lower door clearances, apparently, are here to stay and must be taken into account in constructing curbs. The height of the center of gravity has a bearing on the capability of the car to "hold the road." For a number of years it has been about 24 in. above the road. Any lowering would only slightly affect roads. Turning radius is important only in parking. This value has averaged about 26 ft for some years.

What will be the effect of the engine in the rear? This subject seems to be much before the public at present. There are some good reasons why this design would be desirable and some, equally good, why it might not prove so feasible in production. A rear-engined car would provide better visibility with clear vision directly ahead. It would probably eliminate much engine noise, heat, and odors, and allow lower floor-boards. However, maneuvering, particularly around a corner, requires an almost equal distribution of weight on the front and rear wheels. To accomplish this in a rear-engined car of reasonable wheel-base will require an engine of about one-half the weight of the present units. The rear-engined car with acceptable handling characteristics must await power plant and drive developments. Even with a rear-engined car, road requirements would not be changed materially unless accompanied by radical changes in performance.

The rubber tire is one of the most important factors that made the present automobile possible. It is interesting to note that Mr. Dunlop was not thinking of automobiles at all when he developed the pneumatic tire. He undertook to make a better tire for his son's bicycle. He took a wheel and put a canvas over it, and inside inserted a rubber tube blown up by a football pump. That was the beginning of the pneumatic tire. Finally, an unknown bicycle racer beat a champion because he used pneumatic tires and the bicycle trade took them up. When the automobile came along the tire was ready.

Tire mileages have increased from less than 1 000 in the early days until now (1938) a tire will last for more than 20 000 or 30 000 miles and the owner objects if he does not get that many. One place where tires may be greatly improved is in blow-out resistance. At one proving ground tires have often been blown out with dynamite caps at speeds of 70 miles per hr or more without hurting any one. In fact, motion pictures taken by a following car and aimed at the tire show only a slight sidewise movement when the tire burst. Such procedure is not recommended, of course, as serious accidents can be caused by blow-outs, especially in front tires.

The trend in tire development is toward improvements in compounding technique that will result in more heat-resisting rubber compounds. A new tire is being produced which has a strong rayon cord that maintains its strength at a high temperature. More than 20 000 tires have been produced and 500 000 000 miles of wear imposed on them to be sure that the development is a sound one. The outstanding characteristic of this tire is its freedom from carcass failure and blow-outs, particularly where trucks and buses are operated at high speeds and under heavy loads. One company cited tests by which these new tires outlasted the specially constructed test cars themselves in runs at 120 miles per hr over the salt beds of Utah. It seems that the tire chemist has scored again and motorists will not be limited in road speed by tire weaknesses.

Mechanical Devices to Control Cars

It is not difficult to conceive of mechanical devices on roads and highways that might act to reduce the accident rate. Photo-electric cells, relays, and mechanical devices have been proposed for stopping cars automatically. This could be done in a practical manner just as is now done on railway trains. It is doubtful, however, whether engineers could justify the expense and difficulties of enforcement in the use of such devices.

For several years there has been a presistent clamor for governors on all motor vehicles. Speed is claimed to be the cause of most of the accidents. However, the public would not allow the designer to govern the speed to less than 50 miles per hr. Only a very small percentage of accidents happen at greater speeds. Limiting the speed to 50 miles per hr would not affect accidents in the city, where maximum speeds seldom reach this value; and it would only prevent a few accidents on the open highway. Many surveys have shown that the average speed is less than 45 miles per hr on good roads and only a small proportion of cars travel as fast as 60 miles per hr.

Automatic devices that limit speed or take the control away from the driver may be the cause of accidents. It is often necessary to accelerate rapidly to cross an intersection, pass around a car and back into line, or enter traffic and increase the speed to avoid trouble.

Lighting—Road or Vehicle?

Authorities agree on one fact: Night-driving conditions are far behind day-time conditions, and for one reason—lack of adequate lighting and glaring headlights. Many studies have been made on the relative accident rates between day and night driving. Most authorities agree that night driving is three or four times more hazardous than day-time driving. The future must avoid this condition. Here is a problem which both groups—the automobile industry and road builders—will have to solve individually. It seems logical that many miles of congested highways, especially between large centers of population, could be lighted economically. The first cost and the upkeep are high; and, with any electrical equipment now in prospect, these items make it impractical to light all highways. It has been estimated that the upkeep alone of the lighting for roads in the United States would cost more than the total annual expenditure for roads in the entire nation.

However, the writer would like to outline what has been done in the past generation in lights for other purposes. The present tungsten incandescent lamp is about eight times more efficient than the original Edison carbon lamps. The new sodium vapor lights are said to give nearly three times as much illumination as the best tungsten lamps. It does not take a very good prophet to guess that this progress will not stop. The future will certainly see vast improvements in methods of producing light with small expenditures of power. Entirely new methods will surely be found for producing light without the great heat loss which now must be accepted. The firefly, deep sea fish, and other of Nature's means of producing light can teach engineers an entirely new technique. Already, a few unrelated facts are known which may prove to be the starting point for large lighting industries. In a demonstration which the writer has often given, he pours two liquid chemical compounds together. Each is almost water-white and inert. When they are mixed, they create a pale blue light, strong enough to permit reading a newspaper in the dark, which persists for several hours. What this means for the future is difficult to say, but it is well known that many of the greatest industries are founded upon scientific facts as simple as this one.

For some time to come it seems likely that illumination on most roads will be supplied by the automobile itself. The problem is to obtain good vision without glare; and this the automobile engineer is working on all the time. From the crude oil lamps of twenty-five years ago the modern car has now progressed to fairly effective electric lights with several beams to suit varied conditions. This lighting can be improved still further, but how to do it is the problem. Since 1918, experimenters have tested thousands of means of eliminating glare, which would still provide adequate road illumination for the fast-driving pace most owners demand.

Only one solution has received widespread attention; that is, the use of polarized light. At first, this may seem to be the answer to the question, but there are still many problems to be solved before it can be utilized on a car. In this system a beam of polarized light is projected upon the road by a headlight which has a polarizing screen in front of the light bulb. The oncoming driver views the road through a screen in the windshield, or special glasses which have a polarizing screen set at right angles to that of the headlight of the oncoming car. Therefore, he sees only a small percentage of the light from the oncoming car, but the light from his own lights is little affected by the viewing screen. Glare is practically eliminated without much reduction in road illumination. The objection to the use of polarized light is that it requires about four to eight times the candle-power light of present systems to give the same visibility. To be effective, it would have to be fitted to all cars. The expense of the screens themselves is high, although this would undoubtedly be reduced with more experience in manufacturing.

Other systems have been proposed for providing adequate road illumination and, at the same time, eliminating glare from oncoming cars, such as a system of colored lights. Every car would be equipped with headlights capable of producing a beam of either one of two colors, together with viewing screens of these colors. Suppose the two colors were blue and orange-red. Cars traveling north and east, for instance, might be required to illuminate the road with the blue light and would use their blue viewing screen. Cars traveling south and west would use the orange-red light and screens. The colors would be selected so that the blue screen would transmit none of the orange-red and the orange-red screen would transmit none of the blue light. This would eliminate glare from oncoming cars although it appears that the efficiency of the system would be low.

A problem in road research closely akin to illumination is road color, and brightness, and their effect upon visibility, both day and night. This is probably one of the minor considerations in selecting road materials and yet it should be given some consideration. The related problem of visibility, front and rear, from the driver's seat is an important subject which has commanded increased interest, and should lead to much better visibility in the future. Narrower front pillars, better seat positions, glass angles, and locations to eliminate reflections, sun-glare shields, front-end shape, and many other considerations are involved. From the standpoint of the road, good highways signs, elimination of blind corners, location of signal lights, center lines and markers, and road color are problems of importance to the driver, especially when he drives over a new route.

Buses and Trucks

There will be a great expansion in the use of trucks and buses in the future. From the road-builders' standpoint the size and weight of the vehicle will be important. Almost every one agrees that this factor will be settled by law and that maximum sizes and weights will not exceed present limits. The trend seems to be toward many more smaller, faster, more mobile units.

There will be a tremendous growth in the use of buses, especially to replace street cars. Although buses cannot supplant street cars entirely, in all cities, it appears that there is a certain population below which buses can take their place. It has been estimated that this size is between 100 000 and 200 000. For large cities, and particularly in cities in which there are arterial highways, the street car will remain the backbone of the transportation system.

The records show how rapidly the public has accepted the motor bus in cities and, in the final analysis, their wants will always control. In 1922, only 18 cities depended exclusively on motor coaches for transportation. Within a period of 10 yr, 300 cities were using buses exclusively. To-day (1938) the number is well over 420. During the past five years 75% of equipment bought by city operators has been gasoline buses.

For inter-city use the bus has shown similar strides forward in the twenty-five years, 1913–1938. Passenger service is supplied by buses in 48 500 communities in America which are not served by rail lines and are totally dependent on highway transportation. These are the modern "lost Provinces" which have again been discovered by highway transportation. In several mid-western States, from 20% to 40% of the communities come under the heading of "Lost Provinces."

The public has "sold" itself on the value of buses, and their speed, economy, and convenient schedules. Comfort, appearance, speed, acceleration, "hill climb," and safety have shown marked improvement in the past few years and the limit has not been reached in these factors. The trend has been toward large units, seating more than thirty passengers, but of considerably lighter weight than those of the past. Aluminum and pressed steel reduced the weight of one large bus by 6 000 lb and with an increase in structural strength. The motor bus has definitely found a place in the transportation system of the United States and will have to be provided for in any future highway program.

Statements that the truck will eventually drive the railroads out of business can only come from those not acquainted with the problems of trucking. A "long haul" in truck language means a "short haul" in railroad language. At present, the truck is usually not economical for distances greater than 250 miles, or for other than the higher grades of freight. There is plenty of business for both the truck and the railroad.

Railroads, themselves, are using trucks in ever-increasing numbers. Door-to-door delivery has been tried sufficiently to show its value and this service will expand in the future. For short hauls there will be more trucks of every kind, their maximum size being limited by laws and regulations.

The possibility of separate highways for commercial vehicles presents many problems, most important of which, perhaps, is the cost. In most instances, the commercial development that has come in the wake of motor transportation has been along the present highways. Even if separate truck highways were constructed, it would still be necessary for trucks and buses to use present highways to serve the industries and business places along them. If separate highways are necessary, in some places, perhaps they should be constructed for passenger cars and leave trucks where they are. Trucks could be kept off the special highway for the passenger cars, but they cannot be kept off the present highways regardless of the construction of other truck highways.

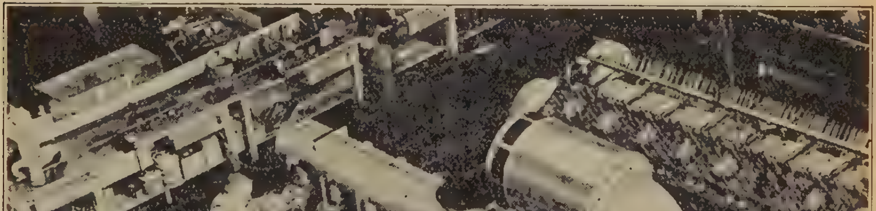

FIG. 3.—A MODERN TWO-CYCLE DIESEL ENGINE

DIESEL ENGINES FOR MOTOR VEHICLES

The Diesel engine is not new; it has been in use for more than forty years. Until given wide publicity by its use on the streamlined trains, it was little known by the public. Now, many car owners are talking Diesel engines as if they would displace the gasoline engine entirely in motor-vehicle transportation.

The Diesel is a remarkable engine for some purposes. It is the answer to many of the problems the railroad companies have been trying to solve for years. Passenger trains have been able to maintain faster schedules than they ever thought possible at a great saving in operation and fuel cost. Full-length trains have reached speeds exceeding 120 miles per hr. Time for upkeep in the roundhouse has been reduced one-half. Water is not a problem in dry, alkali areas because the loss of cooling water is negligible.

The Diesel engine is here to stay and will soon be common in large motor vehicles. Cross-country trucks and buses can now be obtained with Diesel engines and the future should show an increasing use of this economical form of power plant. The statement that they save one-half the fuel cost needs some explaining. One of the largest existing fleets of motor vehicles has a yearly gasoline bill of more than $2 000 000. To save one-half of that sum would be well worth while; but to do so, it would be necessary to spend more on the original equipment. It costs considerably more to make the Diesel engine, principally because of the expensive fuel injection system. By driving enough miles in a year, it may be possible to pay the extra first cost of the engine by savings in fuel, and still show a profit. This makes the Diesel engine possible in commercial vehicles, but not necessarily for passenger cars.

The main reason, however, why Diesel engines are not built in passenger cars is that no one knows how to build them. The writer has never seen a successful Diesel engine in automobile cylinder sizes. The larger the engine size (see Fig. 3) the easier it is to make a Diesel engine; and the smaller the size, the easier it is to make a gasoline engine. Truck and bus Diesels are still large engines as compared to passenger car engines. The use of Diesel engines should not materially affect road requirements in any event.

Trailers and Mobile Homes

A new factor that has entered the "picture" in the last few years is the house-trailer. Estimates vary as to the number now on the road, but it seems that there are several hundred thousand trailers, housing several million people. A conservative estimate is that 100 000 new ones were produced in 1937. These values are insignificant when compared to the 26 000 000 motor-vehicle registrations, but it must be remembered that trailer construction as an industry is just beginning. What the future holds, no one is sure. Opinions and prophecies vary from those by people who think that trailers are just a fad to those who predict that one-half the population of the country will be living in trailers within twenty years. There may be too much optimism in the trailer industry itself; and yet it has almost doubled itself every year for the past several years.

More than ten years ago (1926) the writer had some experience with living a trailer life. A stock bus was fitted out for living purposes and a party of seven took an 8 000-mile trip through the West. Traveling was surprisingly comfortable, and the accommodations, in many cases, were better than could be obtained in the country through which the party traveled. The present trailer is much better equipped and can find better parking places than it could ten years ago.

There are those who predict that the trailer will grow into a mobile modern home of several rooms, a home that can be moved when the head of the family changes locations of work. There are now several designs of expansible trailers which make three-room or four-room bungalows complete with all modern conveniences. They visualize a working population housed in these mobile homes, which can be moved from place to place as working conditions change. The owners would rent small plots of ground on the outskirts of industrial areas, where they could raise their own food, be out in the open country, and use the car for transportation to and from work. They would have all the benefits of owning their own home in the country, but would not be tied down because of high mortages or ownership of stationary property. Land could be rented from farmers and a new type of community life developed. After one studies some of the plans of these mobile homes, it becomes apparent that there are possibilities in this field, especially when it is realized that 80% to 85% of the present trailers are bought by working men for permanent homes.

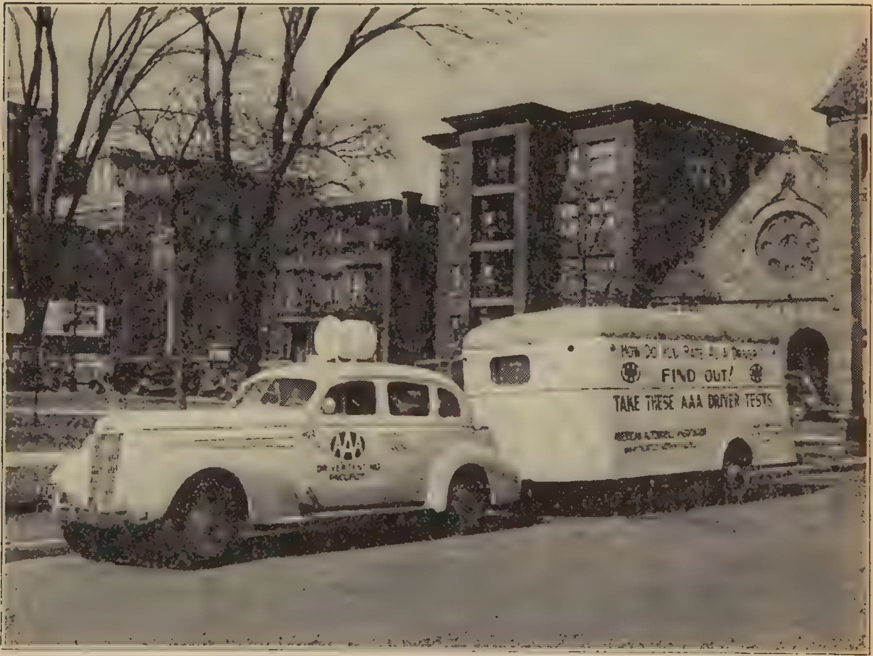

FIG. 4.—DISPLAY TRAILER USED TO PROMOTE HIGHWAY SAFETY

The small present type of trailer may serve two purposes. It makes a satisfactory temporary dwelling for vacations and it can be used as a sample display room for commercial products. With the trend definitely toward more leisure and earlier retirement of men in business, it is certain that some effect will be felt in the modern way of living. The trailer is one answer to those who retire on a modest income or who depend upon the Social Security Act for a living.

Already a number of businesses use trailers for commercial purposes. Almost every one can remember the old days when commercial salesmen, or "drummers," traveled their territories by horse and buggy or by train. Then, along came the automobile, and they could cover as much territory in a day as they could before in a week. Now another change is appearing in methods of selling—the display trailer. It is doubtful whether any one conversant with the problems of commercial selling will gainsay the advantages of display trailers as an effective tool in the hands of the salesman. To bring an alluring show window to the prospective buyer has been the ideal of many a sales manager. This is exactly what the display trailer is accomplishing. The adoption of the house-trailer to display use is certain to have some far-reaching effects on sales methods.

At present, the varied products being displayed in trailers range from medicines for pre-natal care, and obstetrical instruments, to caskets and religious instructions. Such vehicles have the advantage of full-line displays, privacy, mobility, and spot delivery of small articles. They may become mobile retail stores, selling a wide variety of products (see Fig. 4).

Rubber-tired transportation is an integral part of modern life and has completely changed many of the current ways of living. The future will hold as many changes in the use of this new means of transportation as the past. It has been the experience of the past that the second twenty-five years of an industry always shows the greatest progress. The motor-car industry is only well under way on its second twenty-five years. It will make more use of highway transportation as better, and more adequate, roads are provided.

Assuming that the trailer is here to stay and that its production is only in its infancy as an industry, road requirements will be changed to supply these new needs. Highways will have to be wider, with wider bridges. Many more turn-outs for parking, repairing, preparing roadside meals, and feeding the baby, will be necessary. The responsibility will have to be fixed for over-night parking facilities. Will there be private ventures promoted by gasoline stations and others, or will communities provide the space as they did for tourists using tents? Combined with these questions are problems of water supply, power outlets, and sanitation as well as social problems of education for children, taxation, effect upon the birth rate, police and fire protection, and a host of others.

It appears that trailer camps will fall into five general classes, according to the use to which they will be put. These uses can be listed as follows: (1) Stop-over camps; (2) vacation or recreational parks; (3) semi-permanent and permanent parks; (4) de luxe parks; and (5) display trailer marts.

Conclusion

The highway of the future must be as different from the present highway as the automobile of the future will be from the present automobile. One thing is certain; the future will demand change. As an automotive engineer, the writer does not know how the civil engineer will accomplish his results. There will be (in fact, there are now) certain fundamental requirements that have not always been incorporated in roads: The best road for the automobile will have the least obstructions to the steady flow of traffic. Whether this means limited-access "free ways," such as Germany is building in its Reichsautobahnen, is difficult to state. Neither can one state definitely whether the engineer should build elevated highways, longer curves, wider roads, underground highways, more grade separations, better surfaces, or create new road materials, to solve the problems. Civil engineers are in a much better position to make surveys and to design roads than automotive engineers could ever be, if they are given the opportunity.

Neither group is suffering from a lack of information on how to handle the problem. Any expert could design and build a suitable safe road to connect two cities, or to cross a large urban area, if he were given the job to do it as he really thought it should be done; but there are too many other than straight engineering problems with which to contend. Probably one of the greatest problems is to prevent diversion of tax money collected for roads into other channels. The money collected for roads should be used for that purpose and not for building city halls, jails, and other projects. To concentrate on the worst of problems: First, solve the means of handling traffic on the most congested routes; and then build the main streets to handle their traffic faster and with greater safety. Instead of trying to solve all problems at once, solve the most pressing ones first. If this is done in a systematic manner, the highway system in the United States will be the best in the world within a few years.

Men in the automobile industry believe that a highway system, should be based on traffic surveys to determine requirements for the types of roads designed to carry traffic safely. Such a plan would require a primary system of high-speed highways crossing the country in all directions. It is estimated that about 50 000 to 60 000 miles of such super-highways would be sufficient. Leading from these would be a secondary system of good highways serving small cities and towns. The third system would consist of land service highways serving rural areas. In addition, a similar system is needed, in miniature, to carry the traffic in large cities.

The best type of road should embody all the knowledge the highway engineer has at his command for building a safe roadway to carry motor vehicles at a reasonably high speed. Where it is necessary, it should be above or below the ground. In any event, it should be a limited-access "free way" designed for the one purpose of carrying traffic smoothly. Where roads do lead into it, they should be provided with accelerating lanes so that traffic will not be interrupted by new vehicles entering from the side. Everything should be done to make for a free, uninterrupted flow of traffic. Crossings should be eliminated, curves should be banked, long-sight distances should be provided, bridges should be wide, and all "bottle necks" eliminated. Traffic lanes should be separated to prevent "side swipes" and head-on collisions. Surfaces should be non-skid. Where it is necessary, adequate signals and signs should be installed. The entire system should be governed by a uniform set of traffic regulations made to fit the new conditions.

This entire highway system should be paid for by the motorist. However, all the money collected from him in taxes should be used for this purpose. The automobile owner pays more than $1 000 000 000 per yr in taxes without much complaint as long as he knows his money is going for good roads. In some years almost one-fourth of his money has been diverted, directly or indirectly, to other uses. This "rifling" of the road funds must stop if the United States is ever to have an adequate highway system.

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THE TREND OF MODERN HIGHWAYS

By Thomas H. MacDonald,[2] Esq.

Synopsis

Highway construction and use are undergoing changes that are relatively slow but none the less sure and continuous. Changes in motor vehicles, in the sources and amounts of highway revenue, and technical advances in road-building methods, are some of the more important factors that will influence future highway trends.

Scientific highway planning and administration are necessary to provide highways to serve urban and rural needs with the road-user taxes that can be made available. Factual data are being provided that will permit the scientific planning of State-wide and nation-wide systems. This is the outstanding trend in the highway field.

Stabilization of soils to produce foundations of predictable behavior, elimination of intersections at grade with railroads and other highways, and provision of facilities for free flow of traffic from congested to suburban areas, will be important parts of future work.

An important beginning has been made in construction of super-highways and such work will be continued. Present policies indicate that their location will be integrated with population centers, and that the layout will not be on the transcontinental basis.

Introduction

Any attempt to reach far into the future of highway development invades a speculative field if limited to isolated examples of the unusual. However, if taken as a whole, each of the major public undertakings changes slowly in character through definite causes. These changes, if relatively slow, are none the less sure and continuous, depending upon the rate of progress in science and invention and upon movements in the social structure.

The most important causes of change in highway utilization and improvement, viewed nationally and in the mass, have in themselves a variety of checks and balances which determine their actual course and influence. For example, the national market for large numbers of motor vehicles has resulted in the investment of so much capital, the growth of industrial plants of such large dimensions, and the establishment of such complicated routine of production, that year-to-year changes in the product are limited by the necessity of conserving the investment in plant, by the requirements of mass production, and by considerations of time. Thus, the change in the motor vehicle becomes definitely pronounced over a fairly long period, rather than from year to year.

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The most serious loss in the highway investment over the decade 1928–1938 is the obsolescence resulting from changes in the number and speed of motor vehicles operating on the highways.

The revenues accruing to the public treasuries from the special taxes levied upon the motor vehicle and its users in all the various forms have been increasing in their total over a long period except for the temporary recession during the worst of the depression years.

First intended and levied for highway improvement, the growth of income from these special taxes should permit highway officials to build more adequate and, consequently, more costly roads. However, because there have been large diversions of this income to other purposes, and because these funds have been spread over an ever-increasing mileage, many States are faced with a constantly growing financial dilemma.

Highway research is moving forward steadily on many "fronts," with definite objectives. The responsibility for carrying on intensive studies, in the physical and economic fields, has been accepted as a continuing obligation by highway engineers and scientists in allied fields. Concurrently, the necessity for greater street and highway safety has become a national emergency.

These are only a few of the factors that will determine the trend of modern highway development, but they are the most important.

Scientific Planning and Administration

Faced with the constantly increasing demand for the construction of new highways and the spreading of the income over a rapidly growing mileage, highway officials, State and Federal, realized the impossible situation which was developing and which, in a number of States, was actually current. Out of this situation was born the State-wide highway planning surveys which are now (1938) being conducted co-operatively in forty-four States and in the Territory of Hawaii. These surveys and supplemental studies will present a factual basis upon which to plan the complete administration of all the highways based upon sound principles and factual data. One of their important phases is the determination of the relation of the highways to other types of transportation and communication and to population distribution.

Beginning with water transportation and continued by railroad development, the influence of transportation in the past has been exerted to concentrate large populations in small areas. It is only necessary to examine maps of the United States, or any other country, to visualize clearly the concentrations of population in cities which have resulted inevitably from the characteristics of waterway and railway transportation.

Highway transportation by motor vehicle is the first great decentralizing transportation agency. The first notable effect of this dispersing influence is the formation of the wide bands of suburban development around the cities. Even cities of moderate size have developed, within a decade, a suburban band 5 to 10 miles in width (and depending upon topography) partly or completely enveloping the old city. The automobile, in conjunction with rail suburban service, has attracted city workers to make their homes in rural districts for distances easily as great as 50 miles from the city.

The next logical step is the break-down of over-sized industrial units into smaller units that will be free of many of the undesirable characteristics of over-concentration, yet will be of sufficient size to retain the economies of mass production. Sufficient progress in this direction has already occurred to indicate how inevitable must be this process even if slow. It is reasonably well established that economy of production can be secured in units of moderate size, and the national thought along social lines is becoming a potent force toward decentralization because of the opportunities presented for a more healthful and more desirable mode of life for the workers and their families.

It will be noted that no conjecture or uncertainty is involved in these statements, but simply a recognition of existing facts. The implications are clear that the scientific planning of highways and highway systems of State-wide and nation-wide dimensions will be the most characteristic trend in highway development.

Without such scientific administration there will be no possibility of providing adequate highways to serve both urban and rural needs, or to keep the cost of highway improvement, including maintenance, within the limits of revenues that can be raised by reasonable road-user taxes. This means a reversal of the present trend in many States where the maintenance cost of the rapidly growing mileage of local, poorly built roads is mortgaging far too large a portion of the highway budget. This situation has been produced largely by legal mandate and other public policies adopted and pushed into effect without any consideration for sound highway administration. The nation-wide movement, through the State and Federal highway departments, to place highway administration on a sound economic engineering basis, is not only the most important trend, but is also the cause of other trends which, on this account, may be predicted with some certainty. Out of the planning surveys will come definite specifications for the division of highways into groups classified by the service they are called upon to perform. By research and experience the details are being rapidly defined, that will determine the general type of highway design for each highway service group. In this field the principal items will be the alignment with limiting curves, sight distances, the number of traffic lanes, their widths, the shoulder widths, divided roadways for multiple-lane highways, and many other details.

Design Problems

The trend here is, first, to classify highways, based upon service, and then to design closely in accord with the classification—contrasting with the all too prevalent practice, in the past, of applying single standards to long mileages, and without change, to roads of widely varying traffic service requirements.

In the field of highway design, the most important single development is found in the possibilities of soil stabilization. The intensive research work of more than a decade has borne fruit in the understanding that now exists of the physical and chemical properties of soils, and methods of utilizing the knowledge no longer stop at mere superficial applications to the immediate sub-grade, but go further, to affect the entire graded road-bed in both cut and fill sections. Among the most important additions ever made to the highway organization are the soil technician and his specialized soils laboratory. The application on a broad front, of this new knowledge will come as rapidly as engineers may be given the specialized training, and as a result there will be, for the first time, the possibility of building really scientifically designed roadway sections since these will be placed upon foundations of predictable behavior.

With a large program of grade separation under way at railway and highway intersections, and with the changed public policy in paying construction costs largely from public funds, grade-crossing elimination has become a fixed policy that will continue until all important railway-highway grade crossings have been eliminated, and minor ones protected by adequate devices.

This same policy is being rapidly extended to the separation of important highways at intersections, and one of the important trends of highway design in the immediate future will be to rule out intersections at grade as a possibility in efficient highway design.

In this connection, incidentally, the widespread use of "stop" and "go" lights is not a solution for traffic movement, but has been a development of necessity imposed upon a system of highways designed and built before the present dimensions or speeds of highway traffic were considered possible. Obviously, the trend of highway improvement in the future must be to provide flow arteries in the congested-area traffic that will permit continuous flow of traffic from down-town areas well into the suburban areas. Although the cost will be high, it is only through such arteries that capital, invested now in land and buildings in the hearts of the business districts, can be even reasonably preserved.

The pioneer roadway, even on main traffic routes, was conceived as the single important objective. Now, with the recognition of values inherent in highway transportation beyond the bare utility, the roadway design has come to embrace the entire right of way. The trend of modern design is to provide landscaping of the roadsides, sidewalks, foot paths, bridle paths, and to stop, and protect against, soil erosion. The required additional attention and expense are paying large dividends through greater durability and through the recreational values inherent in attractive waysides.

SUPER-HIGHWAYS

There is more or less discussion in which the term "super-highways" is used without any adequate definition of what is intended by this term. Perhaps, it is more frequently used in connection with a very limited number of transcontinental highways designed for high speed and with multiple-lane roadways to carry traffic from coast to coast.

The German system of super-highways embodies this idea. In that country a system of about 4 500 miles of highways (which gives approximately three lines across the nation in each direction) is constructed on entirely new, wide rights of way without access from abutting lands, except at infrequent intervals. This design is for high-speed, motor-vehicle, through traffic. The travel section is composed of two roadways about 30 ft wide, separated by a parking. Both the horizontal and the vertical alignments are exceptionally good. All cross-traffic is directed over or under these highways. No detail that comes within the purview of highway engineering that will make a safer or more efficient highway has been omitted. The most advanced highway design technique has been embodied in this development. The economic utilization is not so clear.

In the United States there is need for a considerable mileage of highways having similar characteristics, but the disposition of this mileage, to be most efficient, must be planned on the basis of the careful studies now going forward. The system of German roads is being built in advance of, and to promote the development of, highway transport. In the United States the situation is just the reverse. Highway builders are proceeding on the principle that the utilization of the highways must produce directly the revenues with which to finance their construction. As long as the United States adheres to this method of financing, the building of super-highways must be limited to areas where the present and prospective traffic will justify it. As a trend of highway development, it is apparent, from the important beginnings already made, that a considerable mileage of motor super-highways will be developed, that their location will be carefully integrated with the population centers, and that the layout will not be on the transcontinental basis.

In France, where a system of national roads has been developed over a long period, the present construction is to take care of the traffic around the metropolitan districts, particularly the vicinity of Paris, by a system of circumferential and radial roads in combination. The detail of outstanding importance in this design is the separation of cross-traffic.

From the developments abroad and in the United States, one can conclude that super-highways will be created, but only in the vicinity of metropolitan areas, for relieving traffic congestion within these areas and for connecting those that are separated by relatively short distances. The first function has already been served to a considerable extent by parkways. It is logical that there will be further developments of the type of the Blue Ridge Parkway designed to connect the Shenandoah and the Great Smoky Mountain National Parks. The development of such parkways recognizes the large use of motor vehicles for recreational purposes.

The Influence on Population Changes

Finally, the power of highway improvement to accelerate the shift of population from areas of low productive potentials to those more favorably conditioned will be consciously used in the national policies developed for the long-term attack upon land-use problems. A definite start is already being made in this direction and will become more apparent in the layout of the system of secondary or feeder roads. This thought definitely emphasizes that the country has completed the pioneer stage of road development and every trend of highway development of the future must be an intelligent meeting of the particular service to be rendered.

TRUNK-LINE HIGHWAYS IN METROPOLITAN AREAS

By LeRoy C. Smith,[8] M. Am. Soc. C. E.

Synopsis

Elements essential to a master plan for trunk-line highways are discussed in this paper. The application of such a plan to the Metropolitan Area of Detroit, Mich., is offered as an illustration.

Introduction

Metropolitan areas in the United States are still in a condition of flux—the lines that might confine such an area to-day will, in general, be outgrown to-morrow; the uses made of any part of the area to-day may not be the uses of to-morrow; the pleasure route of to-day may become the industrial highway of to-morrow; the traffic and population of to-day will be greater to-morrow; the cross-roads at major highways of to-day may be a secondary center of to-morrow; and, unless the hub of the area of to-day is adequately served by arterial routes, it will not be, to the same extent, the hub of to-morrow.

Certainly, no one influence has been more responsible in giving impetus to the changes that are occurring in metropolitan areas, and to the uncertainties of the ultimate crystallization that may occur, than the automobile; and, certainly, there is no saturation point in evidence in the usage of automobiles or in their numbers. In the first place, the ratio of the number of automobiles to the number of people is still on the increase and, at the same time, the population is increasing. In the second place, it is becoming increasingly apparent that the sphere of use and influence of the truck and bus is enlarging, and it is dangerous to attempt to define their probable limitations.

This condition of flux and uncertainty as to the ultimate demands that may be made on trunk-line highways in metropolitan areas would present a much more complicated problem to the highway engineer in planning for the future, if it were not for the axiomatic fact that the essence of the solution lies in providing adequate right of way, and that such right of way is essential to any plan and to any use which may ultimately be made of any such highway.

Essential Elements of a Plan

The first step must be a plan providing for a system of trunk-line highways, so located and of such widths that the future highway requirements of the entire area can be met. Although, perhaps, it may be necessary to reinforce such a plan with layouts showing the ultimate developments requiring those widths, the engineer should remember that radically different details may be required to meet the results of changing conditions, but that with adequate right of way available the future can be viewed with confidence.

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Such a plan will provide for certain distinct locations and frequency of trunk-line highways to serve the various primary needs of the area, and will include:

(1) Arterial highways piercing to the hub of the area from the hinterland outside the area.

(2) Traffic headers or trunk-line loops which may serve several purposes, such as:

- (*a*) Permitting through traffic to by-pass congested areas and to avoid the hub of the metropolitan area.
- (*b*) Distributing traffic of the several arterial highways to the local street system and *vice versa.*
- (*c*) Meeting the demands of periods of peak loadings, such as Sundays and holidays in the case of pleasure traffic, or daily commercial peaks and seasonal conditions affecting industrial traffic.
- (*d*) Cross-town arteries for local traffic. Ultimately, in many cases, this may become the major function of certain such headers or loops.

(3) Boulevards and parkways for the exclusive use of pleasure traffic and to serve the recreational needs of the area.

(4) Secondary highways which are not quite trunk line in character but which are dispersed at frequent intervals as major local streets within the trunk-line network.

(5) Some of these trunk-line highways must meet the present and ultimate needs of mass transportation, whether on rails or rubber.

(6) Express roadways with separated opposite-direction free-traffic lanes may be provided in certain highways; or, the entire highway may become an express-way from which local traffic is excluded, except at long intervals and definite points; and, in still other cases, the entire highway may become a free-way without vehicular access from abutting property.

(7) Such trunk-line highways must almost invariably serve the needs of the area as trunk lines for public utilities, and inadequacy of right of way, in addition to cramping the highway possibilities and increasing the cost of the ultimate highway development, will affect the utilities in the same way. The public pays the cost of both. The minimum provision for such utilities may become a controlling factor in determining the width of the right of way in certain special cases, as, for example, a trunk line that becomes a subway route.

(8) Super-highways having sufficient right of way to permit all highway functions to be combined in one highway at grade, including provisions for mass transportation and local service to abutting property and complete grade separation and interchange at major intersections. In certain conditions such a super-highway may carry several highway functions on an elevated structure in a right of way sufficient to contain such elevated structure adequately, and the elevated structure may become a free-way without depriving abutting property and local traffic from the use of the highway at grade.

(9) Double-decked streets in which the entire right of way is used on two levels. This is a special case applicable generally only in extremely congested

areas where a built-up condition and right-of-way costs preclude other solutions. This subject will not be dealt with in this paper.

(10) Parking facilities. It is a moot point whether such facilities should be provided in highways.

Except in the case of a few of the major metropolitan areas of the United States, and except for the present need of widenings or other provisions to relieve "bottle-necks" and lack of capacity existing on arterial streets of most cities, there is usually no immediate necessity for making or deciding the ultimate structural development of the system of trunk-line highways. The immediate need is for a system of rights of way of adequate width and location and if that need is recognized, the future is reasonably insured. Structural development of such rights of way can be progressive as the need arises.

Right-of-Way Width

The Master Plan for the Metropolitan Area of Detroit fixes a minimum width of 120 ft for right of way of trunk-line highways. For secondary highways, as mentioned in Item (5), the plan calls for a minimum width of 86 ft and, for super-highways, a width of 204 ft.

Boulevards serve as something more than traffic arteries and the right of way must be sufficient not only for traffic needs but also to permit some beautification and park characteristics. A right of way 150 ft wide is a practical minimum and will permit two roadways each 36 ft wide, a center parkway strip, 38 ft wide, and 20 ft on each side for pedestrians, plantings, etc.

Parkways are essentially long narrow parks, served by an included roadway which is not directly accessible to local abutting property, and are generally planned to have grades separated at major intersections. The right of way is usually variable to conform to local topographic features; at minimum sections, however, it should not be less than 200 ft. Topographically, parkways often follow streams; they utilize low-lying stream valleys and, in such locations, can often be carried into and through comparatively well developed areas at relatively small cost, thereby replacing unsightly stream-valley dumps with a value that will enhance parkway facility.

Master Plan for the Metropolitan Area of Detroit, Michigan

In 1925, acting in conjunction with the Rapid Transit Commission of Detroit, the authorities controlling streets and roads in the metropolitan area adopted a Master Plan for the area. Although the plan dealt primarily with conditions within a 15-mile circle from the City Hall, the essential elements extend over a much larger area.

The plan provided that on each section line a right of way 120 ft wide would be available and that at every third section line the right of way would be 204 ft wide. The right-of-way width for roads on quarter-section lines is 86 ft. In addition, it provided that the right of way for the six major radial arterial highways (Fort Street, Michigan Avenue, Grand River Avenue, Northwestern Highway, Woodward Avenue, and Gratiot Avenue) would be 204 ft wide beyond the Six-Mile Circle (approximately) and 120 ft wide within that circle. Actually, these radial arteries have already been made 204 ft wide, and the system of 120-ft highways has been adhered to for many miles beyond the 15-mile limit of the original plan.

It may be well to explain that these radial arterial highways are of more than local significance and are, in fact, the major routes to the various sections of the State and to the neighboring States.

Five radial highways were established in 1825 under the direction of Governor Lewis Cass as the five great military highways for this section of the country. With a right-of-way width of 100 ft (which in many cases has since been encroached upon), they radiated in all directions and comprise the River Road, from Detroit to Perrysburg (Toledo), Ohio; Michigan Avenue, from Detroit to Fort Dearborn, in Chicago, Ill.; Grand River Road, from Detroit to the mouth of the Grand River, at Lake Michigan, on the west coast of the State; Woodward Avenue, from Detroit to Fort Saginaw, Mich., the major artery running north through the State; and Gratiot Avenue, from Detroit to Fort Gratiot north to Port Huron, Mich. (serving the "thumb" district of Michigan).

These five arteries are now super-highways under the plan, except River Road, the major portion of the traffic from which is locally taken by the Fort Superhighway. The sixth radial arterial highway, Northwestern Superhighway, is of recent origin and, although at present it terminates in the resort lake region of the adjacent county, it is ultimately projected to be extended northwesterly through the State to Ludington, on Lake Michigan.

Fig. 5 shows the Master Plan. It will be noted that more than 80% of the 204-ft rights of way for super-highways has already been secured as well as most of the land for all lesser widths. Furthermore, on the super-highway sections of the aforementioned six radial arteries, double pavement sufficient for present needs has been built, and the same is also true on seven other super-highways (Base Line, Mound, Kelly, Stephenson, Southfield, Schoolcraft, and, partly, on Telegraph).

In addition, 38 miles of the 42 miles of the Outer Drive have been constructed as a boulevard on a 150-ft right of way and many of the 120-ft trunk-line highways have been developed. Several parks have been acquired and developed, a plan for parks and parkways has been adopted, and a parkway more than 8 miles long has been completed on right of way varying from 250 ft to approximately ½ mile in width.

This progress since 1925 (when practically no right of way wider than 100 ft existed for any of these highways and only an inconsiderable part of that width) has been attained on a pay-as-you-go basis, without bond issues, and has been largely due to the following factors: (1) The unanimous adoption of a plan; (2) legislation enforcing the plan on sub-dividers of property; (3) adequate condemnation legislation; and (4) conserving automobile taxes to highway uses.

In respect to radial arterial highways, the Master Plan provided that they would be super-highways and the reduction from 204-ft width to 120-ft width at the Six Mile Circle (approximately) was predicated in layouts showing that, by carrying provisions for mass transportation in a 4-lane or 4-track subway

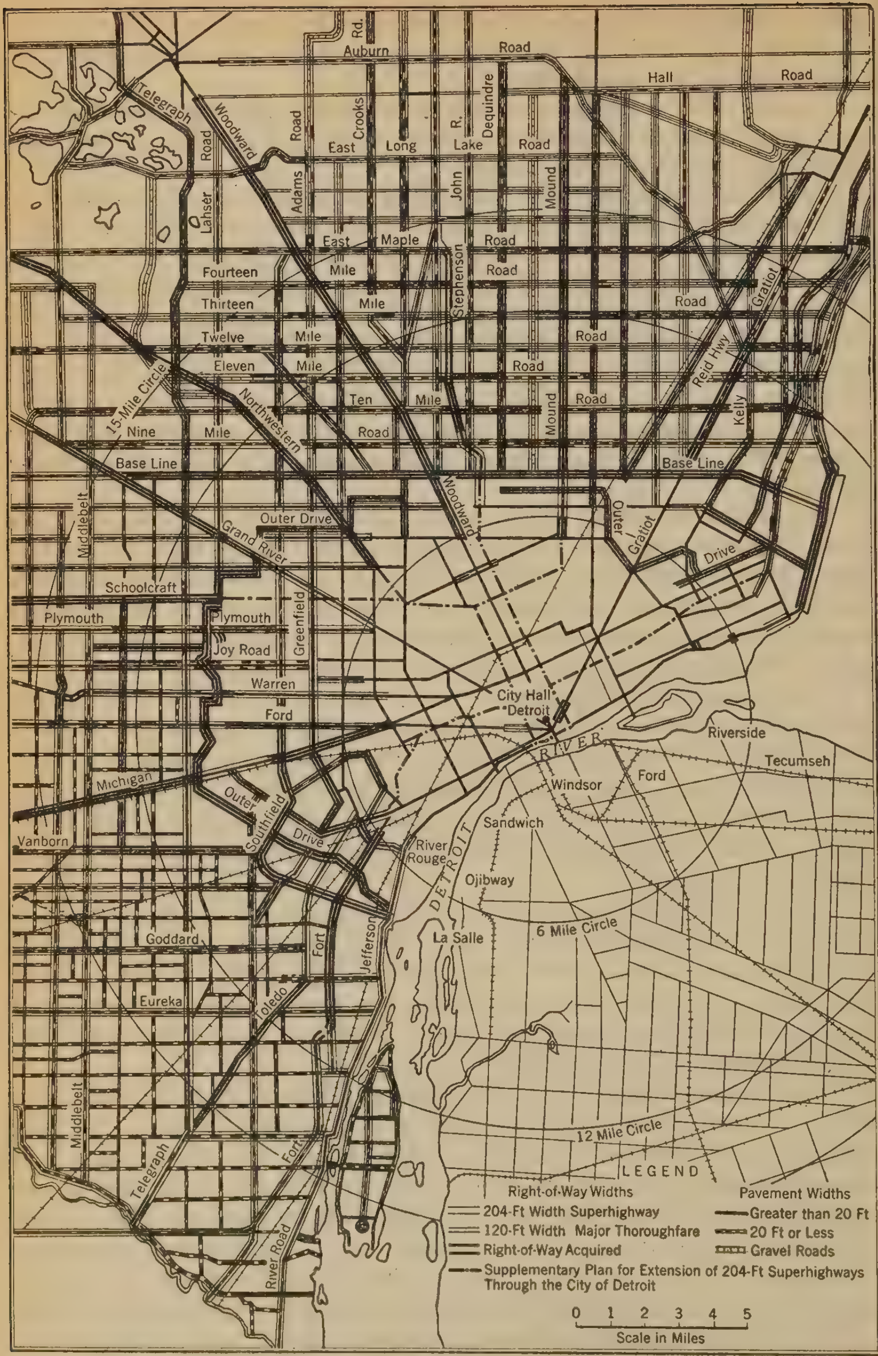
Rd.
Auburn
Road
Hall
Road
Telegraph
Woodward
Road
Road
Crooks
East
Long
R.
Lake
Dequindre
Road
Lahser
Adams
John
Mound
East
Maple
Road
Fourteen
Mile
Stephenson
Road
Thirteen
Mile
Road
Gratiot
Twelve
Mile
Road
Eleven
Mile
Road
Reid Hwy
15-Mile Circle
Northwestern
Ten
Mile
Mound
Road
Kelly
Nine
Mile
Road
Base Line
Base Line
Woodward
Middlebelt
Outer Drive
Outer
Gratiot
Grand River
Drive
Schoolcraft
Plymouth
Plymouth
Joy Road
Greenfield
Warren
City Hall
Detroit
Ford
Riverside
RIVER
Tecumseh
Michigan
Windsor
Ford
Outer
Southfield
Sandwich
Drive
River
Rouge
Vanborn
DETROIT
Ojibway
Goddard
Fort
Jefferson
La Salle
6 Mile Circle
Eureka
Toledo
Middlebelt
12 Mile Circle
Telegraph
Fort
River Road
LEGEND
Right-of-Way Widths
Pavement Widths
204-Ft Width Superhighway
Greater than 20 Ft
120-Ft Width Major Thoroughfare
20 Ft or Less
Right-of-Way Acquired
Gravel Roads
Supplementary Plan for Extension of 204-Ft Superhighways Through the City of Detroit
0 1 2 3 4 5
Scale in Miles

FIG. 5

below the 120-ft street, that street would have approximately the capacity of the 204-ft super-highway.

The congestion of surface traffic on these radial arterial streets within the Six Mile Circle had already reached a stage which demanded immediate widening, and the width adopted was 120 ft, with the aforesaid layout as a background. Since that time these widenings for Woodward, Gratiot, and Michigan Avenues have been almost completed, a start has been made on the others, and steady progress is under way toward ultimate completion of this element of the plan. Thus, the entire original Master Plan is almost a reality to-day (1938) in so far as ultimate widths of right of way with sufficient development for the needs of to-day are concerned.

SUPPLEMENTARY PLAN

With these several great super-highways converging toward down-town Detroit, the Wayne County Road Commission believed that they should be extended into, and connected across, the city proper on similar widths of right of way. Accordingly, a plan to accomplish this end was presented in a report to the County Board of Supervisors and adopted in principle. This plan calls for the widening of unimportant streets through the unprosperous area of old Detroit to provide new 204-ft thoroughfares. By means of 61 miles of such highways, all the outlying super-highways will be connected to a down-town loop approximately 1 mile square. This supplementary plan is shown in Fig. 5.

In presenting this plan the Road Commission was influenced by the following considerations:

(1) It is improbable that the few arterial streets that are being widened to 120 ft will prove adequate if the central section of the city is to be properly served in the long future and if the high values and concentrated uses of the down-town area are to be preserved.

(2) The cost of utilizing these 120-ft streets ultimately to accommodate the future capacity of the 204-ft super-highways feeding into them, will be so great that the excess cost in sub-surface works would finance a better and less expensive plan and additional and more adequate arteries in other locations.

(3) The supplementary plan would open blighted areas to renewed prosperity and would traverse sections of the city now inadequately served. The street widening in stagnant sections, where lots and buildings are of little value, will increase values along these new thoroughfares, the highest type of development will be attracted to these broad thoroughfares, and the central area of Detroit will be enabled to expand to meet future demands.

(4) The width of 204 ft can often be secured at a cost not appreciably greater than a much lesser width.

Having in mind the general street layout it is found that, in condemnation processes to widen a street to 120 ft, the taking line, whether the widening is on one or both sides of the street, passes through all buildings. The result is that the award, besides covering the land taken and damages to the remainder of the land parcel, also includes a value for the part of the building taken and damages to the remainder of such building as well as an allowance for business loss during alteration. These latter items approximate the reconstruction cost of the building.

On the other hand, if the widening is all on one side of the street and if the entire parcel is taken, only the present value of the property is involved, and that value may be less than the composite damages awarded. If the widening is so laid out, it will generally produce a street width of at least 186 ft, including the present street (say, 66 ft), the lot depth (say, 100 ft), and the alley width (say, 20 ft). In many cases it will produce even more, but in any event the condemnation of an additional strip, 18 ft or less in width, off the rear of the next and much cheaper series of lots, will produce the desired 204 ft.

Rapid Transit by Rail and Express Highways

Rapid transit, whether on rail or rubber, demands freedom from traffic lights and from local traffic interference. Although express highways and freeways built at grade with grades separated at major intersections may be possible for outlying districts, the permanence and adequacy of such a solution, as the future develops, is open to question. Exclusion of local traffic and of abutting property from use of the facility, if at grade, makes of it a Chinese Wall as the territory develops. To be effective rapid transit by rail and express highways must be above or below grade and local needs must be met at grade.

Layouts and perspectives prove the possibilities of the 204-ft right of way to permit facilities for mass and rapid transit on rail and on express highways, together with adequate surface facilities for local traffic. Furthermore, at intersections, such right of way will permit complete grade separation and, at the same time, complete local interchange. In the case where these rapid transit facilities are on an elevated structure, it will be noted that the area below the structure provides valuable parking space in addition to location for surface street cars. It will be noted, furthermore, that if in the future mass transportation on rubber rather than by rail becomes the rule, a much simpler structure can be adopted.

The usual objections to elevated structures are minimized or overcome if the street width is such that adequate roadways at grade flank the structure, and under such conditions the structure can become one of beauty rather than ugliness and, through proper design, noisiness can be reduced to a point where it becomes non-objectionable. In general, where such an elevated structure accommodates both rail and rubber express-ways the part of the width utilized at one point for express loading platforms will be used at another for local loading platforms of the rail facility, and at still other points will be utilized by the ramps to and from the motor express-way.

Another plan for express-ways or free-ways which is worthy of consideration to meet the needs of arterial highways in congested districts where property values are high, would entail the use of the alleys at the rear for single-direction elevated express-ways. This is particularly applicable if there is a legal method provided for condemning air rights.

Assuming a 20-ft alley, the construction of a 30-ft express-way would entail the use of approximately 8 ft of air rights on each side of the alley; a 40-ft express-way would require air rights over approximately 13 ft on each side.

In addition, unless it proved feasible to cantilever the part of the structure occupying the strip of air rights, it would be necessary to condemn occasional small areas (probably at the lot lines) to accommodate columns and footings.

At long intervals such express-ways would be served by ramps to and from the surface and, at those locations, additional condemnation will be unavoidable. The space under such a structure, on private property, could serve all present uses; or it could be enclosed to enhance those uses, or to provide new uses. The height of the structure would be governed by present uses and by minimum clearances at cross-streets or at crossings of similar elevated structures. The cost of increasing the height to meet local considerations is relatively small.

Given legal condemnation of air rights still another plan is applicable in certain instances. Predicated on the fact that railroad rights of way pierce most cities in a strategic manner, the use of air rights for elevated highways over such available routes would add express-ways or elevated highways for industrial uses where they would be most effective. However, consideration of such a plan must include weighing carefully certain opposing factors:

(*a*) It is important that the elasticity of the railroad facility to expand or to be altered to meet future needs be conserved (this is as important as any highway function).

(*b*) Such a plan may entail partial or complete electrification of the complicated terminals of the railroads at a cost that would preclude its further consideration.

Another general observation on the problem of rapid transit by rail or express-highways is that sub-surface structures are almost invariably much more costly than elevated structures. In many cases the excess cost of a facility of ample proportions below grade is so great that new and ample right of way in parallel locations can be secured and served by an elevated structure at less cost.

Furthermore, the use of sub-surface structures is much more inelastic; if designed for rails, it cannot (except with further additions at great expense) be used for highway traffic. On the other hand, the change from a rail facility to a highway facility, or *vice versa*, is easily accomplished in an elevated structure.

PARKING

It is difficult to foresee what the future may demand in the way of parking facilities as an adjunct of the highway.

In considering long-range highway and street plans the engineer should not pass the parking problem with a gesture and the assertion that parking is not a highway function. Perhaps the following comments may point to a future trend:

(1) The automobile owner now pays the cost of parking and at a high rate. The money comes from the same pockets as the highway money, and the expense is an unavoidable part of the cost of the highway transportation system.

(2) The present investments in covered parking facilities run into millions and, in addition, the public pays an enormous price for parking in open lots.

(3) Perhaps the cost of this element of highway transportation would be less if it were planned to be included in the highway system and, with adequate rights of way, it is conceivable that sub-surface regions, or elevated layouts, could be utilized for the purpose. In cases where ultimate development of the trunk-line highway in congested districts is through the use of elevated structures flanked by streets at grade, the space at grade below the structures may become a valuable parking area and its value in that regard may be a partial justification for the adoption of any elevated plan.

(4) If often occurs that all of a parcel of property or of contiguous parcels may be secured quite apparently at no greater cost than the pay needed for the highway width. Perhaps, if parking were recognized legally as a highway function, there would be justification for, and necessity could be proved for, taking such excess property for such use.

(5) The value of such facilities is recognized by business, particularly those businesses which are responsible for the growing trend toward decentralization through the establishment of smaller concentrated business centers in outlying districts. In many such cases the store or group of stores involved are providing free parking facilities (sometimes on a large scale) for patrons. This is perhaps a major factor in the growing success of such outlying business centers.

To a large extent such decentralization is at the expense of existing businesses and property values in the heart of the metropolitan area. To the extent that a plan aims to conserve such central values it should, perhaps, take cognizance of the parking problem as being a considerable factor.

Legislation to Make Plans Effective

A plan of a system of trunk-line routes with adequate right of way, adopted by all highway authorities of the metropolitan area, is essential to real and steady progress; but it is impotent and unproductive in the absence of legislation to enforce it, to preserve it from encroachments or abandonments, and to provide adequate methods for obtaining right of way by condemnation and otherwise.

Perhaps a review of the legislation of Michigan which has been found effective in creating, rapidly, a network of adequate rights of way in accordance with a definite plan for the Metropolitan Area of Detroit, may not be amiss even if the underlying constitutional provisions, highway laws, etc., may not be quite duplicated in other States.

The fundamental highway laws of Michigan set up jurisdictions, etc., as follows:

(1) Except as affected by home-rule constitutional provisions in the case of incorporated cities and villages, the State Highway Commissioner has jurisdiction over State trunk-line highways and is required to meet one-half the cost of maintenance, widening, and betterment thereof in metropolitan communities of the population of Detroit.[4]

(2) All other highways and sub-division streets and alleys, except in incorporated communities, are under the jurisdiction of each County Road Com-

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[4] PA 131 of 1931 amending Section 1 of PA 19 of 1919.

mission. With the consent of the incorporated community such commission may take jurisdiction over any street or part thereof within the community. Such commission is also empowered to take over, in whole or in part, the obligation of any such community arising out of a contract with the State Highway Commissioner in conection with the widening or improvement of a street which is a State trunk-line; and, in connection therewith, or as may be entailed on streets taken over by the commission, it may condemn property within the community.

Super-Highway Law.—Shortly after the Master Plan for the Metropolitan Area of Detroit was adopted by all municipalities and highway authorities, an effective legal instrument was made available in the super-highway law[5] which included the following:

(*a*) A contract may be entered into by boards of supervisors, and a super-highway commission can be formed by any two or more adjacent counties; and such commission includes the three members of the respective County Road Commissions and the State Highway Commissioner.

(*b*) Super-highways are defined as including highways of widths varying from 106 to 204 ft, or more.

(*c*) It is the duty of the super-highway commission to prepare and record a plan of the proposed highways and their widths, and where there are existing highways the plan must show on which side of the road private property is to be used for the widening. After recording of the plan no plat of land shall be accepted which does not conform thereto.

(*d*) The law sets up a limitation of 0.5 mill property tax in each county for the use of the commission. Except to cover the initial office expenses involved, this section of the law was never utilized and, in practice, each road commission has acquired right of way at its own expense from other funds (automobile tax revenues).

(*e*) The commission can accept donations of land and property and dedications of land, and may purchase, option, or condemn land.

The major effect of this legislation was to enforce the plan on sub-dividers of property and through that provision alone dedication of many miles of frontage in accordance with the plan was secured without cost, thereby saving millions of dollars.

Approval of Plats.—If a plat of property includes county roads or State trunk-line or Federal Aid roads, it must be submitted to the County Road Commission or the State Highway Commissioner for approval before being eligible for recording.[6]

Abandonments.—Although a Court may vacate a plat, it cannot abandon any part of a county road or State trunk-line highway. Such highway can be abandoned in whole or in part only by the County Road Commission or the State Highway Commissioner as the case may be. Abandonment by the County Road Commission must be recorded. Thus, the right of way, once secured, cannot be abandoned except by these authorities.[7]

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[5] PA 381 of 1925.

[6] Sections 31 to 37, PA 172 of 1929.

[7] Section 66 of PA 172 of 1929; Section 18 of Ch. 4, PA 283 of 1909 being Section 3993 of Compiled Laws (1929) as amended by PA 135 of 1935.

Encroachments.—Title to any part of the right of way of a highway cannot be secured by adverse possession.[8]

Condemnations.—County road commissions and the State Highway Commissioner have several methods of condemnation available: (1) By jury under two Acts;[9] (2) by a commission of three appointed by the Circuit or Probate Court;[10] and, (3) immediate possession after hearing of necessity and after tender of damages estimated by the County Road Commission. If tender is not accepted, condemnation must be done by processes provided to determine damages.[11]

Set-Backs Without Excess Condemnation.—If lots at the rear of the lots desired can be secured by agreement and if owners of front lots will accept a set-back by agreement, this process can be adopted. In outlying districts this provision has enabled the purchase of much frontage at the price of interior lots.[12]

Conveying Title.—By statute, County Road Commissions are incorporated bodies and are empowered to convey title to lands held by them and which are not a part of, or required to be used for, a road or park.[13]

May Condemn All of a Lot.—County Road Commissions can take all of a lot (if such part is required for the public improvement) as will destroy the value of the remainder as a single parcel or lot.[12]

Needed Legislation.—To meet the future, as the need presents itself, legislation may become necessary for condemnation of air rights, and for the exclusion of abutting property from access to free-ways. Free-way legislation has been enacted in the States of New York and Rhode Island.[14] Quoting from *Information Bulletin No. 36*, issued by the Regional Plan Association of New York:

> "This act [the Rhode Island act] is to be recommended for its directness and simplicity of treatment.
>
> "The adoption of freeway legislation by these two States may be hailed as the beginning of the most significant movement in several years in the progress of highway development. A look into the future would envision the idea extended to the other states of the Nation, expanded to include county and local highway departments and perhaps influencing the program of Federal highway aid."

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[8] PA 46 of 1907 and PA 314 of 1915.

[9] PA 149 of 1911 and PA 124 of 1883.

[10] PA 352 of 1925; PA 283 of 1909, as amended, being Sections 3986 to 3991 of Compiled Laws of 1929

[11] PA 352 of 1925.

[12] Section 1(*a*) of PA 124 of 1883, being Section 3785 of Compiled Laws of 1929.

[13] PA 283 of 1909, being Section 3984 of Compiled Laws of 1929.

[14] Chapter 2537 of the laws of 1937.

DISCUSSION

F. Lavis,[15] M. Am. Soc. C. E. (by letter).—The writer has been much interested in the paper by Mr. MacDonald, whose point of view is based on actual practice; but so much has been written during the past year or two about the "Highway of the Future" that the layman will expect this to develop at once into a clear, uninterrupted, wide pavement with cars "sailing along" at 100 miles per hr, or faster, with no stops for traffic lights or interference by traffic police, no grade crossings, no cars stopped on the pavement or traveled way, all traffic lanes separated, and perhaps (the Utopia) no noticeable expansion joints to mar the smoothness of the pavement.

With this ideal situation, pipes, sewers, and conduits will be laid outside the traveled way, so that pavements need never be disturbed and these latter will be so durable that they need never be relaid. There will be wide shoulders and rights of way, and the roadside will be beautifully landscaped.

In 1937, a speaker was reported by the Associated Press to have told the National Planning Conference in Detroit, Mich., that, "The American street and highway system must be scrapped and rebuilt unless the automobile is to become a malignant growth." "We cannot," he said "look with tolerance upon shackles which fetter this newest servant of mankind"; and then he also blamed the highway structure for the great toll of accidents on it.

Apparently, it is to be taken for granted that in this Utopia, the motor vehicles themselves will be of supreme design and construction, built of materials that cannot fail, that will have no mechanical defects, that will, perhaps, take their fuel out of the air so as to avoid stopping, and will be driven by supermen from Mars. The latter is the most essential because the frail humanity of this day and age has apparently not cultivated the skill and stamina to take advantage of these transcendental highways and vehicles, and as one reviews the slow progress in the development of the human machine it scarcely seems likely that succeeding generations will attain the ability and skill to operate these super-motor vehicles about which so much is being written.

The writer is entirely serious in attempting to create this vision because it must be given consideration. It is being preached in many quarters, not by laymen alone but by engineers—engineers who claim that most of their fellow craftsmen have no thought for the morrow, no vision of the future. These statements must be given consideration, but consideration based on some sanity.

The writer is not unmindful of the developments in both cars and highways of the past thirty years, since the time that he bought his first car in 1908 and thought a 100-mile drive in a day was a feat, a perilous adventure, and a difficult, grueling task. Without effort, he can compare that car with the car of to-day, which will go the 100 miles in a few hours with almost no apparent effort either on the part of the car or of the driver; and he has no doubt that similar or even greater progress will continue.

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[15] Cons. Engr., New York, N. Y.

After looking at these visions, however, engineers should look at the realities of this long-distance planning and attempted provision for the future and visualize the real status of their attitude toward the highway of the future (which is being built to-day), and think about these problems with sense and with their "feet on the ground."

It may be fairly claimed that highway engineers and highway authorities, without having indulged in any extravagant visions of the future, have contributed their share to this advance, and their progress to date has fully kept pace with that of the automotive engineer. There is no reason why they may not be expected to continue going along together.

The writer is not opposed to planning. For many years, while engaged in the planning and design of railroads, his work has been nothing but a long attempt to forecast the future properly and reasonably, and to build for the future, but always with the brake on his enthusiasm of having to "make a dollar earn the most interest." It seems to be a most desirable training for engineers to have been connected with commercial enterprises. Engineers building highways with public funds would do well to think, for some years to come, of the need of making public dollars earn the most interest rather than of providing special facilities for the few or of building highways for excessively high speeds.

It is trite to state that one should do the greatest good to the greatest number; but trite or not, it is one of the great duties of the engineer who spends public monies, and perhaps particularly that of the engineer who directs the spending of them for highways. This does not gainsay the fact that returns from the expenditures of public monies may reach the dollar stage, through health and happiness from recreation; but engineers should be reasonably certain that there is a real return some way or other, and that this return should be a return to the people at large and not to a special few.

Writers on highway subjects during the past year or two have very generally stressed two things: (1) The great number of highway accidents; and (2) the need of "building safety into highways." Strangely enough, many who have deplored the accidents and urged engineers to "do something about it," talk about designing highways for speeds of 100 miles per hr, or even more. They point to the fact that the motor vehicle is being improved every year and that when a highway is built it must remain unchanged for a long time. Consequently, one must build to-day the highway that will meet the requirements of 10, 15, 20, or 25 yr hence, and that, at first thought (and maybe on second thought) seems a very valid argument, particularly if the human element behind the wheel of the car is ignored.

The highest types of modern pavements may be expected to have a life of 20 or 25 yr, even under heavy traffic. There is no possible reason, therefore, for denying the responsibility of the engineer in laying them out so that the most use may be obtained from them during their life.

Accidents and accident prevention are primary topics of consideration today, and accidents on highways are a growing menace to the entire body politic. Furthermore, the prevalence of accidents is now tending to be greater in the open country, in the rural districts, than in cities and congested areas. British experience in this respect, coincides with that of the United States, and a reasonable inference from this information is that excessive speeds are a predominant cause.

It has been quite generally stated that 85% of all accidents are caused by the failure of the human element, the driver of the car. On the other hand, it is claimed that accident reports do not show the extent or the occasions in which the highway structure itself is the cause. Admitting that this may be true, to some extent, and that, in some cases, faults of the highway structure may contribute to, or aggravate, accidents and possibly increase their severity, it still remains a fact that highway accidents are very largely the result of incompetent and careless driving and excessive speed. Here, again, British statistics and British observers are in accord with those in the United States. A New York State report[16] published in 1937 states: "So long as the motor vehicle is employed as a means of transportation the driver will remain the chief factor in the accident problem."

When writers refer so glibly, therefore, to designing highways for speeds of 100 miles per hr, they not only ignore this very definite fact of human fallibility but they probably are definitely increasing the accident risk. The writer does not believe that design for high speed provides a margin of safety for drivers at lower speeds. Few drivers are capable of driving faster than 40 to 50 miles per hr, with safety to themselves and to the other users of the highways.

This question of speed as it affects the design of highways and their cost is a most important one. Designs based on speeds greater than 50 to 60 miles per hr are not warranted to-day, or by any reasonable consideration of the future.

Most engineers visualize the highway of the future as something in the nature of the trunk-line military "autobahen" created in Germany, the so-called "freeways," or the parkways near New York City, N. Y., in Westchester County, and on Long Island. These are typical of high-capacity highways in open country. There are also the great arterial highways through congested areas such as New Jersey Route 25, from the Holland Tunnel to Elizabeth, of which the Pulaski Skyway is a part, the great West Side Highway in New York City, and others of a similar type.

In general, such highways have good alignment, gradients that do not require trucks to reduce speed rates, say, of less than 5%, and four to six traffic lanes with traffic in opposing directions definitely separated by a raised curbed strip, perhaps 3 to 5 ft wide.

All these very expensive roads, however, have been built to meet heavy traffic demands, and they carry large numbers of vehicles. It is well known, of course, that maximum traffic capacity is attained at comparatively low speeds, probably less than 30 to 35 miles per hr. There seems to be little reason, therefore, for assuming speeds much greater than this, certainly not greater than 50 to 60 miles per hr, in designing such roads.

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[16] Annual Rept., New York State Bureau of Taxation and Finance, New York *Herald-Tribune,* July 18, 1937.

Probably the most important task that confronts highway authorities and highway engineers to-day is the provision of sufficient highway capacity to meet traffic demands. This means that new highways, built to-day, should have capacity for meeting the traffic demands of some years to come. Ability to estimate such demands with reasonable accuracy is a necessary qualification for all responsible highway engineers.

Such engineers also should have some reasonable appreciation of the economic relation of expenditures to highway capacity and highway use. Expenditures in this case mean total annual cost—that is, interest on investment, plus annual cost of upkeep. These factors should have a definite relation to traffic volume and operating costs. Of course, highway engineering cannot be entirely evaluated in dollars and cents but the nearer highway engineers can come to such evaluation, the nearer they will approach reasonable economic justification for the large expenditures they make.

The writer is inclined to believe that where there are legitimate criticisms of highway design and highway structures this is due to incompetent engineering rather than to any lack of engineering knowledge by those who keep abreast of the developments of the times. This, however, is not a failing confined to highway engineering alone.

Because of the general nature of highway work as part of a public works program, by which the appointment of engineers and the methods of their functioning may, in some cases, is dictated by political needs and exigencies, there is probably some incompetence in road design and construction and this is especially true in the smaller governmental units and rural districts. Such defects as result from this source cannot be enumerated or specified herein but the writer suspects that these are the defects that cause writers and commentators to refer to certain roads as obsolete before they are completed and to place the blame for accidents on such defects. The remedy is better engineering by properly qualified engineers.

Most important highway officials and engineers are fully alive to the necessity of meeting future needs, and know how to do it; but they evaluate these needs with some sanity and common sense. The writer, however, would like to mention, very briefly, some of the features of modern road construction that should receive general attention.

Most of the States to-day have reasonably well conceived plans of main and secondary highways and are fully alive to the need of adequate surveys both of terrain and traffic. The importance of securing ample rights of way is fairly generally recognized, but there should be a reasonable balance between providing for the future and the permanent alienation of taxable values. It is probably difficult to evolve any formulas that will guide the engineer in adjusting alignment and gradients to the topography. The old-time railroad locating engineer had his Wellington and values for curvature, distance, rise and fall, and rates of gradient. The highway locating engineer of to-day has little to guide him, but experience and judgment. It is here that communities and governments would be wise and well advised in procuring the widest experience and best judgment. Those responsible for highway location should probably err (if they err at all) in laying out the best alignment and gradients they can, to fit the topography and terrain reasonably well. It is suggested that sight-distance diagrams should be developed, at least in all cases of doubt.

The present practice of separating the traffic on four-lane roads by a center strip may well be extended. There is some tendency to build traffic lanes wider than 10 ft. This is a question that should receive careful consideration as to its economic justification. Probably one of the most serious defects of some modern roads is their obstruction by stopped vehicles. Roadside filling stations and eating places probably serve a public need and help taxable property, but provision should be made for stopping vehicles off the main traveled way instead of on it. It is undoubtedly desirable that consideration be given to the provision of sidewalks, at least on some highways.

The provision of reasonably wide hard shoulders and proper forms of side ditches is now generally recognized both as a necessary safety measure and as an aid to increased capacity. It would seem advisable to have all roadside structures set back a reasonable distance from the pavement or traveled way.

It should not be necessary to refer to narrow bridges, except for the fact that so many exist as interruptions to the free flow of traffic on otherwise good roads. It should be a recognized necessary attribute of road improvement that the general newly established character of such roads should be maintained throughout the length of the improvement.

It seems generally desirable that new roads should be designed so that they maintain certain general characteristics of alignment, gradients, width of pavement, etc., for considerable distances; but there seems to be no valid reason, why if proper signs and signals are displayed, changes in these characteristics may not be made if adequate considerations exist to warrant them and drivers are properly notified. A certain standardization of warning signs and the method of their display is most desirable.

All these features are well within the purview of the competent, informed highway engineer of to-day. In view of the needs of the vast network of roads all over the United States, it is doubtful whether highway planners can afford to be too visionary. Grade crossings with other highways and with railways can only be eliminated in special cases. The construction of expensive "freeways" to facilitate certain traffic must be weighed against the needs of the entire highway net and particularly their effect on taxable values of adjacent lands.

Perhaps the time has come when the construction of toll highways for trucks may be considered, but this involves consideration of the economic effect of maintaining the railway system, which seems to be gradually drifting toward Government control and possibly support by the taxpayers. Both railways and highways are tending to become part of one general network of land transportation.

Viewing the entire problem as a whole, it seems that engineers cannot go much further to-day than to adopt the best current practice and to continue improving it as the need arises. What is really needed is to make certain that highway problems are placed and kept in the hands of engineers competent to solve them.

No one expects to stand still, because standing still is really going backward; but, on the other hand, highway engineers are not to be stampeded into attempting to build highways for super-cars, driven by super-men who do not exist.

Engineers (that is, competent engineers) have always been as forward looking and alert as other people—perhaps more so. They should continue to justify that tradition, but should continue trying to make their dollars earn the most interest.

EDGAR DOW GILMAN,[17] M. AM. SOC. C. E. (by letter).—The Symposium entitled "Increasing the Traffic Capacity and Safety of Thoroughfares"[18] raised many of the same questions that the present Symposium now raises.

First, one wonders if, in giving consideration to traffic problems, the word, "traffic," does not mean the movement of all people and not just the movement of motor vehicles. Sidney J. Williams, M. Am. Soc. C. E., in his comment[19] on the Symposium, stated that "much attention has been given to the time lost by automobilists on account of pedestrians, but little attention has been paid to the time lost by pedestrians on account of automobilists." This is a "ray of light" that should be brightened. Traffic regulation should be for masses of people and not for individuals.

Second, although the writer's natural instinct would be to emphasize the importance of the Engineering Profession in traffic problems, studies of hundreds of accidents prove quite conclusively that only a small proportion of accidents are traceable to conditions in which pure engineering is a factor. In fact, there are data to indicate that as engineering improves conditions at certain points, the accidents increase rather than decrease. Some of the points in the city that are obviously the most dangerous have the fewest accidents. The obviousness of the danger seems to be one of the best deterrents to accidents that has been found. As roads are straightened, automobile drivers take more chances and are not as attentive. As a matter of fact, the situation can be summed up in this statement: "If people drove as they should drive, they could drive perfectly safely even in unsafe places."

Studies of events leading to an accident show that the vast majority of accidents are caused by influences that no act of engineering, short of complete separation and isolation of traffic lanes, could avoid. They are caused primarily by foolishness and carelessness, selfishness, lack of courtesy, and lack of consideration on the part of drivers of cars. No mechanical devices or physical rearrangement of streets can be substituted for the manner (or lack of manners) in which people handle automobiles.

The full question of engineering, although important, has been emphasized to the point where automobile drivers use the theory as an alibi for their own actions. This does not mean that the engineering should not be made as beneficial as possible, but engineers are leading themselves into "the hole" of building up a point of view on the part of the public that accidents are the result of something they have done wrong or something they have failed to do, and that drivers themselves cannot be held responsible unless and until the engineering is something highly idealistic and hypothetical and beyond all practical means of attainment. The engineering should go ahead, but the emphasis should be placed on the drivers and not on the engineering.

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[17] Director, Dept. of Public Utilities, Cincinnati, Ohio.

[18] Transactions, Am. Soc. C. E., Vol. 104 (1939), p. 440.

[19] *Loc. cit.*, p. 504.

Mr. Kettering states (see heading,' 'Lighting—Road or Vehicle?"), "authorities agree on one fact: Night-driving conditions are far behind day-time conditions, and for one reason—lack of adequate lighting and glaring headlights." Many data have been published showing that accidents at night are more numerous than accidents in the day time and in many of the papers including such data the definite conclusion is drawn that this is due to the absence of light, without any consideration of the many other factors in night driving. Of late, a few writers have attempted to proportion these factors properly. Certainly there are enough other factors to justify some question as to whether lack of adequate light on streets and highways is the one reason for a greater number of accidents at night than in the day time.

Nearly all day-time driving is done by people who are soberly intent on the particular accomplishment of a business in hand. It is after the day's work is done that social functions and parties are held. Practically all the driving under the influence of liquor is done at night. Accident records show that this is a big factor. The element of fatigue enters into night driving to a considerably greater extent that it does in day-time driving. Many police records of accidents at night have the notation "Driver fell asleep." All the young men working, and in most cases those not working, use the evening to drive with their "best girls," and their attention is not always on their driving under these conditions.

A mere tabulation of the time of day that accidents occur does not present a justification for the conclusion that night-driving hazards can be reduced at least to day-time driving hazards by lighting the highways. Furthermore, some serious thought should be given to the question as to what would be the effect of illuminating certain highways and not others. It is admitted that it would be impracticable to light all. What will be the effect of lighting some to an extent that will make it safe to go at certain speeds, upon the speed of drivers on other highways where it would not be safe to go at anywhere near those speeds?

If the mental attitude of the mass of drivers could be "built up" to a point where good judgment could be counted upon, this would not be a problem; but in view of records of the immediate past is one justified in assuming that the drivers of cars in mass will exercise good judgment in the manner in which they drive? It is a strange commentary on the entire situation to say that if the streets were studded with cast-iron posts set in the road, requiring automobiles to be driven a tortuous and difficult path, there would probably be no accidents and no traffic deaths. Of course, there would be no speed and no mass movement in short times, and this is not desirable.

It is good engineering to pull out these cast-iron posts from the middle of highways so that automobiles can travel in straight lines, and engineers, very properly, study and eliminate these physical obstructions to movement; but it is a fair question to ask whether they are not improperly designating such things by saying that they are eliminating "hazards" when actually, due to the psychology of the situation, they may many times be adding an intangible type of hazard when they are removing a physical obstacle.

Sometimes, it seems that the public has placed the engineer in a very unfavorable position, that in which engineers are taking the responsibility even where it is not an engineering problem. Coincident with the study of engineering in traffic, engineers should constantly emphasize the fact that the effect of engineering can be but minor on accident records and that deaths and injuries and accidents are largely a result of the action of the people themselves. Although engineers will work to try to get greater utilization of motor vehicles, there is a responsibility on the part of the people who use motor vehicles over which the engineer can have no control.

This feeling that the engineer is responsible for the accidents has progressed to such an extent that there is a one-page article in a popular magazine of recent date rather strongly setting forth the claim that it is the duty of the engineer to separate roadways and crossings and make it impossible for children and pedestrians to get in the path of automobiles. It is a strange mind that can take such a position that this is the sole duty of the engineer. The engineering involved would present no great difficulties. The big problem in the accomplishment of such plans is that of financing, and this means taxes. Engineers can submit billions of dollars worth of plans for eliminating intersectional highway crossings in high-sounding terms of medial and other frictions. The plans will involve a multitude of details, but none that presents insurmountable engineering obstacles. The obstacle to such plans is in the financing and the payment by the public for such idealistic construction. The public has not been able to provide the necessary money with which to eliminate the relatively few railroad grade crossings, and yet in the article referred to the statement is made that it is "up to the engineer" to provide road separations and other tremendously expensive structures.

Engineers should not complacently permit themselves to be thrust into a false position in the traffic situation. They can do much to facilitate traffic movement if people will use their plans as they are supposed to be used. They can do something to add to traffic safety. They cannot do anything that will be a substitute for good judgment, common sense, and decency on the part of the man who has his hands on the wheel and his foot on the accelerator. The attitude of the public itself is the major problem in the safety angle of traffic. The public says it wants safety, but it resents enforcement and there has actually grown up a predominating belief that the problem of traffic safety is a contest in which the police and other enforcement agencies are on one side and the public itself is on the opposite side, fighting against each other. The problem of traffic safety is a problem of mass psychology—the establishment in the minds of the masses of people of the true factors that cause accidents. Engineers are too readily letting this fact sink into oblivion.

George Hartley,[20] Esq. (by letter).—In the group of papers comprising this Symposium a valuable contribution to the art of planning future highways is ably presented. In his interesting résumé, Mr. Kettering has illustrated, splendidly, the possible future of highway travel. The entire Symposium would have been rendered more complete by the inclusion of a few additional details.

The major part of Mr. Kettering's paper evidently represents careful editing of replies to the questionnaire (Items (1) to (18) under "Introduction") by the various contributors listed under "Acknowledgments." It would have been of interest had it been possible to include typical replies or edited replies to show the range of opinion expressed by the various contributors. Views not directly in accord with those of the authors might have been discarded or considered insignificant.

Each of the authors attacks the problem from a theoretical engineering standpoint with only a slight reference to the importance of motor vehicle laws and their relation to engineering design. What good will a highway or motor vehicle, designed for, say, 80 miles per hr, be if some provision is not made to bring the law into such form as to allow speeds approaching this value. Engineers are constantly thinking in terms of increasing speed but, to-day, even on well traveled secondary highways, posted speed limits of 15 and 20 miles per hr are not uncommon and the main express highways, rigorously patrolled, keep travel at a speed representing only a fraction of that for which the highway was designed. Safety proponents are always demanding slower speeds and these bodies are constantly increasing in number and importance. Any serious attempt to revise present highway laws so as to allow increased speeds will surely meet organized opposition.

Some mention should also have been made in regard to some form of standardized financing. Funds for any one structure may come from a number of sources with the subsequent duplication of services. Surely, in a properly planned highway, the financing should be arranged in such form as to permit steady growth and permanence.

Messrs. Smith and Kettering discuss the parking problem in terms of a future Utopia without referring to the parking-meter system. Intended originally as an added tax-gathering system these meters are being installed at a rapid rate in crowded business districts. They create convenient short-time parking in shopping districts where once it was practically impossible to park.

Mr. MacDonald condemns the design practice of the past without due consideration of the fact that these highways were built along single standards, because a certain minimum of width and construction were required regardless of service requirements. That many of these structures are still in service after a score or more years is truly a tribute to their designers. The older bridges in the United States (some of them more than fifty years old) also carry modern traffic. Fifty years ago when the horse and buggy represented the acme of transportation on the highways, who might have forecast or planned future traffic requirements? With their planned programs and improved materials, can engineers to-day design structures that will serve throughout the life of a planned program? May not the entire viewpoint be completely revised in only a few years by new inventions or even the technique of planning itself? If so, the present generation of engineers must, in turn, await the criticism of its successors. The development of express highways to by-pass crowded centers of population may be one of the major factors in causing a revision of planning methods in Germany where the express highway, established on a larger scale than in the United States, is already attracting commercial establishments with the attendant growth of adjacent residential districts.

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[20] With Robinson & Steinman, New York, N. Y.

ROBERT KINGERY,[21] ESQ. (by letter).—As Mr. Smith states (see heading, "Introduction") no identical pattern or design of a road facility can be transplanted directly from one metropolitan area to another. However, the good designs and successful operation of a given type of highway facility may be adapted to conditions in most metropolitan centers. Without question the original boulevard system in Chicago, Ill., laid out and acquired in 1869 and 1870, has been imitated in many parts of the world. Perhaps the original was an imitation of some other system. Members of the Regional Planning Association of Chicago have profited not only from the Wayne County Highway, and the Detroit Rapid Transit policies, but have studied and have incorporated, in major thoroughfares and super-highway designs, some of the parkway features from Westchester County, New York, and from the Mount Vernon Parkway, near Washington, D. C.

In the ten years since, in 1928, the Chicago Regional Planning Association originally laid out a program of major highways, super-highways, boulevards, or parkways, almost exactly one-fifth of that program has been completed. It is now (1938) advancing at a much more rapid rate. Of the 603-mile system in the 15-county area, 127 miles of right of way from 160 ft to 240 ft in width have been acquired, and 114 miles of pavement have been built, all outside the City of Chicago itself.

Within the city about 14 miles of major trunk traffic artery of the parkway type have been completed along the lake shore from a point 6.5 miles north of the center of the city to a point 7.5 miles south. Entrance to the city from other directions is more difficult, there being no existing broad rights of way. Many studies have been made by the Chicago Plan Commission and by other official and unofficial bodies and now it seems more hopeful than ever before that a decision upon a comprehensive plan will soon be agreed upon, and construction begun by the co-operation of several highway building agencies.

However, one note of caution might be injected into the subject of Mr. Smith's paper. At many meetings of city planners and in many papers about metropolitan highway planning and about zoning one will hear or read of the conception of the dreamer who pictures cities with buildings and towers a thousand feet high, with miles of two-decked, three-decked, and multi-decked highways in centers of population concentration. To some planners this type of thinking is in the wrong direction. There is adequate evidence of the result of excess concentration of population and of commerce in small centers. One example among many is the fact that closely surrounding such an intensive concentration of high buildings is almost always a broad belt in which disintegration is far advanced. The City of Chicago has such a belt, which is 10 miles long and more than 3 miles wide. It covers 36 sq miles, or one-sixth the area of the city; and more than 260 000 people have moved from it during the past twenty years. Many of the tax delinquencies are in this area; and it includes a substantial number of both juvenile and adult delinquencies which form the principal police problem; as well as a substantial majority of fire-protection problems. Tax receipts from this area do not nearly support the governmental services to it. In slightly modified form, the same can be said of every large metropolitan area.

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[21] Secy., Chicago Regional Planning Assoc., Chicago, Ill.

Although high buildings with multi-decked highways—with masts for the anchorage of dirigibles, with landing fields for the clouds of aeroplanes that are seen in photographs of the "City-of-the-Future"—are all stirring to the imagination, by no means do they involve solvency of the municipal pocket-book.

Construction of highway facilities, of course, will not solve all these problems, but it can come nearer to it. Penetrating the centers of these large cities and surrounding the business centers of them in the so-called blighted areas, the development of exceptionally broad rights of way or parkways with adequate space for pavements—on the ground level, elevated, or depressed—with adequate space for grass and trees will, in the judgment of many, do more to reconstruct the city at its core than almost any other single influence. With the construction of such radial or belt line boulevards (or both), and with the addition of adequate parks and playgrounds in such a zone, and with courageous and intelligent zoning or re-zoning of the neighborhood, property values will be restored for residential occupancy of much of that area.

For some decades too much attention has been given to the construction of buildings in the development of cities in the United States, and too little to their setting and their surroundings. The introduction of more open space in the form of boulevards and parkways is one of the greatest services that the highway officials in metropolitan areas can perform, not only toward facilitating traffic movement, but toward salvaging and even enhancing the value of the metropolitan area as a whole. There would follow the further introduction, by park authorities, of the parks and playgrounds with their trees and grass, and the intelligent, unselfish and courageous zoning or re-zoning of large units of the metropolis. The result will be a better rounded community and sounder and safer distribution of traffic volumes.

R. L. MORRISON,[22] M. AM. SOC. C. E. (by letter).—One of the most outstanding items in Mr. Kettering's interesting paper is the constantly increasing maximum speed of automobiles from about 58 miles per hr in 1926 to about 87 miles per hr in 1937. This is an increase of 50% in 11 yr, and the curve is almost a straight line, with practically no indication that a top limit is even being approached.

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[22] Prof. of Highway Eng. and Highway Transport, Univ. of Michigan, Ann Arbor, Mich.

Mr. Kettering states that "people, on the average, do not want to travel much faster than 60 miles per hr," and perhaps that is a reasonably safe speed on a good clear road in the daytime. Investigations made by the Michigan State Highway Department in 1936 showed average speeds of less than 50 miles per hr, but the fastest 15% of vehicles were driven more than 65 miles per hr and, on one route, 70 miles per hr.[23] This means that many of the drivers must have been using the entire speed capacity of their cars. Darkness slowed down this fastest 15% only 2 or 3 miles per hr although, even with good brakes and lights, they could not possibly stop before hitting a dark object on the road.

While the maximum speed of vehicles was increasing 50% between 1930 and 1937, the fatalities on rural roads (the only ones on which maximum speeds can be used) increased about 37%, and the rural night accidents increased 60 per cent.[24] The increase in automobile registrations during the same period was less than 5 per cent. A close relationship between increased speed capacity of cars and increased rural night fatalities is evident.

Mr. Kettering states that the roads "where it is necessary should be above or below the ground. * * * Crossings should be eliminated * * *," and other safety features, which he lists, should be provided. Such roads would certainly cost at least $300 000 per mile. If 10% of all motor taxes could be spent entirely on the construction of these superhighways (which is very unlikely), it would be possible to build about 300 miles per yr. Therefore, at the present rate of revenues, it would take nearly 200 yr to build the 50 000 or 60 000 miles of superhighways which are assumed to be needed. It is impossible for the writer to see how superhighways can ever make more than "a small dent" in the accident rate, assuming that they do reduce accidents where they are built.

The paper discusses two items, the vehicle and the road, both of which are constantly being improved. The cars go faster each year and the roads are gradually being changed to facilitate fast driving; but, unfortunately, there is a third factor, and a very important one. It is very doubtful whether this third factor—the driver—had any more inherent ability to handle cars at high speeds in 1937 than he had in 1926. He doubtless developed a little additional skill during that time, but most accidents are caused by lack of judgment rather than by lack of skill, and certainly there is little evidence that the judgment of the average driver has improved. In fact, the less judgment he has the faster he goes. In other words, the automotive engineers and civil engineers are constantly producing cars and roads with speed capacities more and more beyond the ability of average drivers to use them safely. Of course, this is done to meet the desires of the motorists whom they both serve.

To state the problem is much easier than to offer a solution. It can be said, however, that regardless of the responsibilities of the automotive and civil engineers, public officials have a very definite responsibility to use all reasonable methods to reduce accidents, and this is a responsibility that is very far from being met in many localities. For instance, there are scarcely more than 100 traffic engineers in the entire United States, and when a State or city does decide to have one the usual procedure is not to find and appoint an experienced traffic engineer, but to appoint to that office some engineer who has no knowledge of traffic engineering. Residence in the State or city concerned is usually considered of much greater importance than experience in the work which he is expected to do. A number of States still do not require standard driver's licenses, and, in many places, both traffic laws and their enforcement are far below what they should be.

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[23] "Graduated Speed Regulations," by D. Grant Mickle, *Proceedings*, 1937 Michigan Highway Conference, pp. 14–25.

[24] "Accident Facts," 1938 Edition, pp. 28 and 36.

Although Mr. MacDonald does not make the exact statement in his paper, it seems to be his guiding principle, that highways should be built so as to be of maximum service to the motorists who use and pay for them. Maximum service to the motorist through highway improvement includes decreasing his operating costs, saving his time, and increasing his safety, comfort, and general enjoyment of travel. It also includes making new areas available to travel for recreation or for other purposes. Of course, there are many other "benefits of good roads" of a more or less incidental nature. The main problem before highway administrators is to determine how available funds should be expended in order to render maximum service to the greatest number of highway users.

Unfortunately, it is the spectacular that is likely to appeal most to the general public and to politicians in high places. One school bus is struck by a train and immediately almost a billion dollars is appropriated for railway grade separations, although the same sum of money spent for such things as crossing gates, better street-lighting, the flood-lighting of street obstructions, and more efficient policing would probably save a vastly greater number of lives. Other billions are proposed for building four or six-lane roads across the deserts to accommodate a largely imaginary coast-to-coast or border-to-border traffic although millions of dollars are wasted annually by motorists traveling over untreated gravel roads and poor earth roads, and in crawling through congested city streets for the lack of by-passes around the cities.

Broad vision of future needs is required for real progress in any field, but it is possible that sound economics is more needed than dreams and visions in planning highway improvements for the next few years. Perhaps the greatest need of all is a determination of what constitutes sound highway economics. Even in that field there seems to be a great temptation to pile up vast imaginary savings. This is done by taking the difference in operating costs over pavements and over earth roads and assuming that every mile operated on a pavement results in this saving. This involves the assumption, of course, that all the millions of vehicles now using pavements would be traveling on earth roads if the pavements were not there, which seems palpably absurd. If these imaginary savings could be used to finance the super-highways on the desert all would be well.

The very real savings that can be made to present highway users through sound improvements justify the kind of road-building program outlined by Mr. MacDonald.

Roy F. Bessey,[25] M. Am. Soc. C. E. (by letter).—This Symposium is thought-provoking, timely, and valuable. The flattening out of motor vehicle curves shown by Mr. Kettering does not indicate the end of the great national task of modern highway building, but rather the transition from the era of pioneer construction to the era of reconstruction, readjustment, and improvement for greater efficiency, safety, and beauty.

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Those responsible for highway design and construction have done an excellent job under difficult conditions, trying to keep pace with very rapidly developing or changing needs—with the growth of population, cities, industry, commerce, distribution, and technics, and, particularly, with the mushroom expansion of the automotive industry in both technics and production. In their thinking, many of those responsible have done much better than keep pace with public opinion in appreciation of many important requirements—and, consequently, have been far ahead of adequate public financial support in meeting conditions.

Although a great national network of good roads has been created, it may be concluded that transition from the pioneer stage of motor highway development is still under way. Advances in technology generally, and particularly in the field of automotive equipment; increasing density, diversity, volume and speed of traffic; and growing needs in safety, convenience, comfort, time-saving, and recreation—all call for further, probably radical, improvements in highway design and construction.

The pressures upon highway design and construction have been so great until recent years that there has been little time and means for refinement; but the present is none too soon to complete a new estimate of the situation—to re-evaluate needs and practices and to overhaul standards and plans. For this work the current nation-wide highway-planning survey is providing the essential data.

A number of desirable changes and adjustments are indicated in the Symposium. More will be disclosed by the analyses of the planning survey. Others will be found in city, metropolitan, county, district, State, regional, and national planning studies. Obviously, highway plans will have to be adjusted: (*a*) To the shifts of population and industries out of the centers of cities; (*b*) to other changes in population and industrial patterns; (*c*) to changing land use; (*d*) to activities relating to the conservation or exploitation of natural and economic resources (including flood and erosion control); (*e*) to other means of transportation (including rapid transit); and (*f*) to various other physical, economic, and social changes.

Obviously, too, it will be desirable to transfer some of the emphasis in highway design and construction from questions, such as line, grade, and pavement, in which the art is relatively far advanced, to others, such as surface, lighting, parking, rights of way, and roadsides, which (perhaps unavoidably) have had less attention in the past. All the latter features are increasingly becoming of importance to highway efficiency and safety. Wide rights of way and suitable roadside control will be particularly important factors in delaying absolescense.

From a regional planning viewpoint, the new highway era seems very definitely to involve three outstanding problems and programs:

(1) Trunk highway systems—modernization;

(2) Secondary road systems—more complete co-ordination with such resources and facilities as forests, recreational areas, mineral areas, present and probable future land use, schools, etc.; and,

(3) Co-ordinated transportation—establishment of more adequate relationships of one means of transportation with another, and with the entire system.

Requirements for trunk highway modernization will vary considerably (with geographical location, density of population, kind of traffic, etc.); but there are many requirements common to various conditions, such, for example, as location, alignment, grade, roadway and pavement standards to permit fast and continuous travel; freedom from the threats to movement; the safety and convenience involved in too-frequent crossings and points of ingress and egress, in roadway or roadside parking, in improper or excessive roadside use and obstruction, in conflicts with city and town congestions; provision for pedestrians; and provision for parking on sites away from the roadway. Fortunately, an outstanding requirement of recreational highways—the control of roadsides from the aesthetic standpoint—harmonizes generally with the requirements of commercial traffic and involves broad economic values as well.

The "freeway" or "limited motorway," with its wide rights of way, separation of opposing traffic lanes, grade separations, rotary intersections, border controls, limited points of access, etc., seems to provide the general solution and to indicate the directions in which the most effective improvements can be made in trunk highways.

A very intimate connection between highway-planning and land-use inventories and plans would seem to be the key to the most rational and efficient development of secondary road systems.

The advantages of co-ordinated transportation need not be elaborated. Not the least is elimination of much of uneconomic or needless duplication and competition. Some of the directions for study and planning of co-ordinating features in the highway field might be exemplified: Feeder lines for railroads, rapid transit lines, waterways, ports; integrated terminal facilities; interchangeable freight bodies or containers; roadside emergency airplane landing fields; and arterial express highways co-ordinated with airport locations.

In passing into the new era of automotive transport, a higher degree of co-operation is essential. Now that some of the urgencies of the pioneer period have diminished, it is to be assumed that close co-operation is more practicable. Of course, this co-operation must be based upon a mutual willingness to strive to keep design in automotive equipment and design in highways in close harmony. A closer co-operation is also required as to highway location, design, construction, and operation between various levels of government, between adjoining governmental jurisdictions, and between those responsible for highways and those concerned with other means of transportation.

Finally, if broad highway planning is to reach maximum practicable rationality and effectiveness, quickly, it should be very closely related to national, regional, State and local planning programs, and to functional planning in such fields as land use, zoning, industrial location, transportation, resource development, and recreation. Of course, the comprehensive highway planning work will be reciprocally valuable in the general planning programs.

Bruce D. Greenshields,[26] Assoc. M. Am. Soc. C. E. (by letter).—The least understood but perhaps the most important problem of motor transportation is that of harmonizing highway construction and motor-vehicle design with human behavior.

The engineer has been building highways for hundreds of years. He has learned how to deal with inanimate materials and forces that behave in a definite manner under given conditions. Human nature seems perverse and is so uncertain that the engineer, having correctly designed a highway from a mechanical standpoint, is inclined to let the "fools" kill themselves if they insist upon it.

Whatever his inclination, however, the engineer must accept the fact that the people who use the highways pay for them and, therefore, have a right to demand that they be safe. Each driver should be held to account for his own mistakes, but he must be protected from the mistakes of his neighbors on the road.

Mr. Kettering states, in the "Synopsis," that "the vehicle and the road must form a complete transportation system" and that "each one is dependent upon the other for the success of the whole." The writer would add that the whole must include the driver. Not only is highway safety dependent upon the driver's ability, but its utility is limited by his whims and desires. One loitering motorist may hold up a long line of vehicles.

There is ample evidence that drivers are foolhardy. Recently the daily press,[27] quoting from a pamphlet issued by the Board of Railway Commissioners of Canada, asks why so many people risk their lives by trying to beat the train to the crossing:

> "The latest safety devices and cautionary signals may be in operation, yet to a strange element of humanity these appear to present only a challenge.
>
> "A summary of the dangerous practices at railway crossings only adds to the wonderment as to why many drivers behave as they do: of all the accidents and near accidents reported (350 drivers and pedestrians in 6 months) 22.3 per cent of offenders ran into lowered gates, 18 per cent crossed in front of approaching trains, 11.7 per cent drove under gates while being lowered, while the remainder resorted to utterly foolish ways of risking their lives."

The people who deliberately commit suicide on the highway are few indeed. The explanation lies in another direction. One reason is that the driver or the pedestrian does not know the risk he is taking. Statistics show that a large percentage of injured pedestrians do not drive cars. They put undue trust in the ability of the driver to see a pedestrian and to bring a car under control. If one's attention is momentarily distracted he may not see something that is otherwise perfectly obvious. Every one has had such an experience.

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[26] Associate Prof. of Civ. Eng., School of Technology, Coll. of the City of New York, New York, N. Y.

[27] *The Kansas City Times*, August 8, 1938.

The driver has had little more than a generation within which to learn to avoid the unnatural hazards of high-speed cars and to adjust himself to the unnatural experience of driving.

Professor Yandell Henderson speaking before the National Academy of Science, at Charlottesville, Va., in November, 1935, stated that:

> "'The car went out of control' is a usual explanation of automobile fatalities, but in reality, it is the driver who goes out of control. He acts in the only manner that his nature permits, the manner in which every human being always instinctively acts and always will act under the circumstances."

Any motorist, no matter how experienced, according to Professor Henderson, when suddenly and severely jolted, instinctively re-acts to steady himself in his seat, and in doing so, presses his foot down on the accelerator pedal. The more he bumps along the harder his foot presses and the more rigidly he grasps the wheel. People cannot be changed but motor cars can. Professor Henderson would have a pedal under the left foot which would counteract the effect of the right by shutting off the power.

There are many phases of behavior—physical, mental, and emotional—that need to be considered, but obviously only a few can be considered in a short discussion. One of the natural attributes most discussed in connection with automobile driving is that of "reaction-time." This is the brief interval between the happening of something in one's environment and the time when a certain response is made to it. This interval may be, for example, that between the time a pedestrian steps into the street and the time the brake is applied to avoid hitting him.

In general, the speed of all forms of reaction varies from one person to another and from time to time in the same person. Extreme fatigue decreases the ability to respond. Emotions such as fear or anger interfere with the driver's reactions. The effect of alcohol varies with the individual—it does not necessarily lengthen reaction-time, but an excess amount makes the reaction-time more uncertain. The reaction to hearing is quicker than to sight.[28]

Certain responses are made more quickly than others. The hand is quicker than the foot. Perhaps a hand-operated brake would be better in an emergency.

Although reaction-time has been considered mainly in connection with highway safety, a means for reducing it is probably more important in highway design than in safety work because it has not been demonstrated that reducing reaction-time would lessen accidents. On the other hand, it would speed up traffic and increase highway capacity.

Persons who react more quickly drive at higher speeds. They thus void whatever advantage their quickness gives them and have just as many accidents as slower drivers.[29] The average minimum spacing between vehicles depends upon the reaction-time of the individual drivers. The spacing between vehicles fixes the number that can occupy any length of roadway and, hence, is one of the factors determining highway capacity.[30]

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[28] "Reaction-Time and Traffic Behavior," by Bruce D. Greenshields, *Civil Engineering*, June, 1937, p. 384.

[29] "A Study of Factors Influenced by Automobile Brake-Reaction Time," by Frank R. Olmstead, *Proceedings*, 22d Annual Highway Conference, Ann Arbor, Mich.

Earl Allgaier, Safety and Traffic Engineer of the American Automobile Association, after subjecting approximately 22 000 drivers to a battery of 15 tests, states that driving ability reaches a maximum at about 25 yr of age.[31] The decline begins between 40 and 50 yr and grows rapidly from that point on. The tests included field of vision, eye dominance, excitability, blood pressure, hearing, and steering ability. Mr. Allgaier states that the falling off in the test scores made by drivers more than 40 to 50 yr of age need not necessarily indicate that these drivers are actually poorer or less safe than younger drivers. They are more experienced and likely to be more cautious.

Dr. H. M. Johnson, referring to such tests, states that he has found no published report of a comparison which warrants the conclusion that test performance is closely associated with accident rate.[32]

Dr. A. R. Lauer, of Iowa State College, who has developed many of the tests, states[33] that after having analyzed several sets of data it is his guess that "when our final analysis is made we will find no high validity coefficient." He adds as a reason the fact that "human nature is unstable and highway conditions are highly variable." There is significance for the highway designer in the last statement. Human nature cannot be changed but highway conditions can be made less variable.

To claim that human nature cannot be changed does not mean that one cannot learn to do things better or that natural handicaps cannot be overcome. The quickness with which one can move his foot in a certain way in response to a given stimulus cannot be reduced by practice beyond a certain point; but by simplifying the movement required to control the vehicle the car manufacturer can, in effect, shorten the reaction time. Although there is a natural limitation to one's ability to see, cars can be made easier to see out of and highway signs can be improved by an increase in size or by illumination.

The experienced driver has learned to make the proper responses to a large number of situations. He is less likely to be surprised. Experience enables one to make quicker judgments. Judgment is more complex than perception. For example, in taking a laboratory test of brake reaction-time, a light is flashed and the subject presses the brake pedal. He perceives the signal and acts. No judgment is involved. He has decided what to do beforehand. In the case of passing a vehicle on the road, he first sees the vehicle approaching on the opposite lane. He then decides whether he can pass the car ahead. This deciding is a matter of judgment and is a much more complex process than "sensation." It may take minutes instead of seconds.

The young driver cannot be given experience but experience can be made less necessary by simplifying the highways and by informing the motorist of approaching conditions so that he has time to decide what to do.

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[30] "Studying Traffic Capacity by New Methods," by Bruce D. Greenshields, *Civil Engineering*, May, 1935, p. 301.

[31] *Public Safety*, May, 1938.

[32] "The Detection of Accident-Prone Drivers," *Proceedings*, Highway Research Board, 1937, p. 445.

[33] *Loc. cit.*, p. 452.

Consider highway curves, for example; in most States all curves are heralded by the same type of sign. The driver does not know whether the curve is sharp or flat or whether he can drive it at 50 miles per hr or whether he should slow to 20 miles per hr. Why not give the motorist the information he needs?

Common arguments against such a practice are that the safe speed for one car is not that for another car and that the State would be to blame if a defective or old-model car had an accident while traveling at the announced speed. The curves should be designated by numbers as 1, 2, 3, etc., to indicate riding qualities. The motorist would soon learn at what speed his car would take a No. 1 or a No. 2 curve.

Adequate highway signs might have prevented two fatal accidents that occurred at Fidelity Corner, six miles south of Carthage, Mo., in August, 1938. In the writer's opinion the rather inconspicuous "Slow" sign, followed by another indicating the intersection, should be replaced by (1) a prominent intersection sign, and (2) a danger sign; and, to make sure, a "blinker" should be added. Note that, first, the driver would be informed of what to expect; and, second, would be warned of danger.

Highways must not be deceptive. Many a new highway has failed to reduce accidents. Its deceptive appearance of safety has induced motorists to step up their speed to a point at which they did not have time to react to the road's surprises.

Speaking on the subject of "Highway Design and Safety" at the Highway Safety Conference, Yale University, on April 4, 1938, Guy Kelsey, of the Signal Service Corporation, Elizabeth, N. J., described the surprise hazards confronting drivers operating on 5.4 miles of a "very modern highway" selected at random. Low visibility, head-on-intersection-of-vehicle hazards, as well as several traffic flow inefficiencies were among the deficiencies found in the short stretch.

The statement is made by Mr. A. R. Nichols[34] that even if the safety and utilitarian requirements have been met, the road that does not closely follow the lay of the land results often in glaring artificiality of ugly cuts, high fills, and an increased area of destruction of natural vegetation. Such unnecessary artificiality makes a road more dangerous. Its unnaturalness leads to surprises.

Although the unexpectedness in human nature cannot be removed like the perplexities of the highway there is the gratifying fact that human nature *en masse* is not so variable after all. Insurance companies cannot tell how long any individual will live, but they have built up a thriving business on predicting how long the individuals in a large group will live.

From his studies of behavior on the highway (such as the spacings maintained between vehicles, the distance required for passing, and the point at which drivers slow down because there is a vehicle ahead) the writer has arrived at the definite conclusion that drivers do behave in a definite manner under certain conditions. The curves resulting from the plotting of such data are surprisingly smooth.

The traffic engineer must study human nature as the design engineer has studied soils and concrete and steel. Road intersections that are unsafe must be compared with those that are safe to find out how they differ. The "specifics" of driving behavior must be determined by careful analysis. How prominent must a sign be for the driver with poor eyesight to see it? Advertising signs are much more prominent than highway signs. At what distance can one determine whether or not an object is moving? These are the facts the motorist needs to know, and does not.

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[34] "Highway Design: Its Relation to Landscape Objectives," by A. R. Nichols, *Proceedings,* Highway Research Board, 1937, pp. 269–278.

The highway of the future must expedite traffic and it must do it safely. It must be honest. If it invites speed it must have no dangerous surprises. As sight distances are increased and vehicles are made even more responsive to control, speeds will go up in direct proportion. If 50 miles is a high average speed now it is because drivers feel comfortable and safe at that speed. If average speeds of 100 miles per hour are adopted in the future it will be because the driver feels safe at that speed. When speeds on limited ways go high enough they will be restricted to the best drivers.

It is hoped that these remarks have added something to the excellent papers of the Symposium by emphasizing the importance of studying the highway user.

ROBERT B. BROOKS,[35] M. AM. SOC. C. E. (by letter).—In the final analysis, the place of the express highway in a modern road system must be determined by its economic justification.

The most important economic phase of the paper by Mr. MacDonald is that in relation to the continued construction of super-highways. His idea is that present policies indicate that the location of super-highways will be integrated with population centers and that the layout will not be on the transcontinental basis.

It is undoubtedly true that in Germany and Italy the express highways are built for protection and aggression as well as for peace-time traffic. In the United States express highways are built to satisfy the demand of peace-time traffic, as well as to provide highways that can be used for handling men and equipment in time of war. This protection did not exist twenty years ago, as was shown definitely, time after time, by the wrecking of highways by even limited troop movements during the World War. There can be no doubt that there is a definite place for the express highway in a modern road system.

In January, 1932, the United States had a total of 3 790 miles of extra-wide highways in its State systems and not included in the mileage of city streets and parkways.[36] By extra-wide highways are meant those having more than two lanes. There were 2 230 miles of three-lane width, 1 385 miles of four-lane width, and 175 miles of five-lane width and greater.

Five years later, and bringing this date up to July 1, 1937, there is a total of 7 999 miles of important highways exceeding two-lane width, of which 4 703 are three-lane, 3 096 miles are four lane, and 201 miles are five and six-lane.[37] This increase in width is significant of the trend in highway construction. Within that same time the genuine planning of modern highway systems has been given exhaustive study, under the basic assumption that they should be planned to do the greatest good for the greatest number.

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[35] Cons. Engr., St. Louis, Mo.

[36] *Public Roads*, November, 1932.

[37] *American Highways*, July, 1937.

State traffic surveys should reveal not only the traffic volumes, and other physical information, but should include a description of financial status and explanations of how the citizens are to pay for highways in the future. Consider Missouri, for example. The total mileage of all the roads in the State is 104 850. The State highway system, as now designated, contains a total of 17 193 miles. Of this total there are 1 651 miles in the primary system, 5 877 miles in the secondary system, and the remainder in the supplementary system with its traffic relief, park connections, etc. In other words, the State highway system in Missouri specifically consists of 16.4% of the total road mileage in the State which, however, does carry more than three-fourths of all the traffic when measured in vehicle-miles per year. Carrying this computation a step further, the several systems of highways in Missouri, in the survey report of 1934, showed the following percentages of traffic:

System	Highway mileage	Vehicle mileage
United States highways	3.6	48.1
State highways	4.6	18.5
Supplementary	4.1	8.0
County roads	87.7	25.4
Total	100.0	100.0

Although proper consideration must be given to the farm-to-market roads and to the secondary road system, it is readily apparent that the main highways carry by far the greater volumes of traffic. This is as true in other States as it is in Missouri.

The era of toll roads in the United States is passed. Although individual experiences may warrant the impression that the user would be willing to pay an extra sum for using express highways between centers of population, the cold facts are that there are not enough motorists to amortize the cost of building an express highway in the United States. This is proved by the fact that most motorists will drive many miles out of their way to avoid paying even a small toll on a bridge which might otherwise save them considerable time as well as the price of the bridge fare.

If the cost of the express highway is to come from gasoline tax and vehicle license fees, with added grants from the Federal Government, engineers must "cut the coat to fit the cloth," and in so doing build express highways in such a way that they will handle those traffic volumes in, around, and near metropolitan areas where the expense of construction can be justified, not only from an engineering standpoint but from that of public opinion.

With the proceeds of most of the State bond issues for road purposes already spent, road programs seem to be on a "pay-as-you-go" basis. This means a further economic justification rather than a lavish expenditure to take care of anticipated needs.

Highways are often called traffic arteries through which streams of automobiles flow. The comparison, of course, is to that most perfect system of veins and arteries which takes care of the circulation of the blood. Comparing traffic flow to the distribution of water, the largest pipes are next to the source of supply, or reservoir. As the water is carried to various sections, the sizes of the pipes are lessened in deference to the demand for water by the consumers. The system of arteries and veins in the human body shows very definitely that the need for the larger arteries and the larger veins is near the heart, and the size of each is smaller and smaller as it gets away from those parts of the body that need circulation least.

It is self-evident that the pressure of traffic is from the centers of congestion in metropolitan areas outwardly, just as water is distributed from a reservoir, and just as blood is circulated in the body of an individual.

"Flow" maps invariably show that traffic demands wider traffic-ways near centers of population. The same maps also show that the two-lane highways are sufficient to take care of traffic needs a reasonable number of miles away.

Although the out-of-State farmer can see no reason why his gas tax money should go for the building of an express highway from Chicago, Ill., to another locality, neither does the urban dweller see any reason why his tax money and that of his neighbors in the large city, which may pay three-fourths of the entire cost of constructing highways in the State, must be used in building farm-to-market roads which he, the urban dweller, may never use.

The opinion has been expressed that large cities must construct express highways in order to exist. The writer believes that various State traffic surveys will supply the factual data which will educate every one, whether urban or rural dwellers, as to the mutuality of traffic needs so that gas taxes and vehicle license fees can be spent on a network of traffic arteries, which will take all classes of roads into consideration.

During the eight years, 1925–1933, a definite system of streets was laid out in the City of St. Louis, Mo. These streets were divided roughly into three classes: 60-ft streets, 80-ft streets, and 100-ft streets. A progressive system of construction each year saw various sections completed, so that to-day with the street plan practically completed, traffic flow maps show, definitely, the value of the various widths of streets. Consider the planned design of what is known as US 40TR, in Missouri, which extends from St. Louis westward about 40 miles to Wentzville, Mo. It is a two-lane highway from Wentzville to the Missouri River; a three-lane highway from that point to Chesterfield; a four-lane contiguous highway from Chesterfield to Bellefontaine; and then 2 two-lane highways divided by a 30-ft parkway to a point close to the City of St. Louis. At this point it is necessary to converge into a subway running three miles inside the city limits to Vandeventer Avenue, where traffic is then poured into two or three streams for distribution.

Through bitter experience highway engineers in Missouri have already learned how costly it is to obtain right of way sufficiently large for an express highway. The longer the acquisition of land is delayed, the more it will cost. Therefore, on express highways they have, and are, securing a 200-ft right of way which will take care of future traffic needs, although the present-day construction of extra-wide highways may not be warranted at the present time.

Although many engineers would like to see the general acceptance of four-lane highways instead of two lanes, it seems self-evident that there is not economic justification for such an expenditure in many cases, and, although there may be some accidents caused by a third lane, the three-lane traffic is a transition from a two-lane highway to the express highway. Any one who doubts this point should note the immediate relief afforded when driving in holiday traffic on a two-lane highway almost at the moment it passes into a three-lane highway.

State traffic surveys now being completed and analyzed will show where express highways should be located and, although there may not be economic justification for the construction of an express highway now, it certainly is good planning to acquire land for the subsequent building of these roads when, if, and as, the need arises.

At its meeting in July, 1937, the members of the Missouri State Highway Commission were sufficiently impressed with this idea on U. S. Route 54, between Jefferson City, Mo., and the Lake of the Ozarks, that they instructed the Engineering Department to arrange for the acquisition of sufficient right of way to provide for the building of an express highway of extra width sometime in the future. At present, only a two-lane highway, leading south from Jefferson City for a few miles, will be constructed. Missouri needs express highways to take care of the outward pressure of traffic from its large cities. The writer cannot agree that there is a need for express highways up and down the land where the traffic itself cannot show an economic justification. He does agree most emphatically that engineers must plan wisely to secure sufficient right of way in order to add more lanes to take care of future traffic demands.

Mr. R. E. Toms, Chief, Division of Design, U. S. Bureau of Public Roads, recently presented data to demonstrate that from 95% to 97% of the State highway mileage in the United States may never progress in improvement beyond a two-lane highway.[38] If it is desirable to improve such a small percentage of the highway system as express highways, then let it be realized from the standpoint of economic justification, that the function of an express highway in a modern road system is to take care of the outward pressure of traffic from metropolitan areas to the extent that this extra pressure exists, and not as a continuous super-highway connecting centers of population.

If any further evidence is needed to show that the location of super-highways should be integrated with respect to population centers and that their layout should not be on a transcontinental basis, it is only necessary to observe the restricted use of the State and Federal highways immediately outside the orbit of metropolitan areas as ably shown by Mr. MacDonald.

H. GEORGE ALTVATER,[39] ASSOC. M. AM. SOC. C. E. (by letter).—Referring to "Trunk-Line Highways in Metropolitan Areas," Mr. Smith states that highways must serve the area as trunk lines for public utilities and that inadequacy of right of way will affect the utilities. The practise of laying pipes beneath the traveled roadways of streets was inaugurated in the days of horse-drawn wagons on dirt roads, and then it caused little inconvenience in digging up the streets. With the carefully designed pavement of to-day, however, there is no reason why each block of it must be repeatedly trenched through and patched numerous times during its life.

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[38] *American Highways,* January, 1937, p. 12.

[39] Cons. Engr., Denver, Colo.

At the time of the World War, the United States found itself so thoroughly unprepared to supply munitions that the only experienced engineering organization in the nation that was qualified to build explosives plants could undertake to build only about one-half the necessary capacity, and the remainder was built by entirely new inexperienced organizations. Under these circumstances, the matter of utilities in streets was a serious cause of delay. The main plant transportation, at Nitro, near Charleston, W. Va., where one of the two American smokeless powder plants was constructed, consisted of 45 miles of narrow-gage railroad, which was responsible for the results obtained. In the deep mud the tractors and even horses became bogged down, so that oxen were resorted to for moving heavy machinery. It was not possible to build roads first because the streets were torn up for utilities.

There were untreated industrial water lines, treated domestic water lines, high pressure fire-lines, steam lines, industrial sewer lines, domestic sewer lines, storm sewer lines, gas lines, electric power lines, telephone and telegraph lines, sulfuric acid lines, nitric acid lines, caustic soda lines, ether lines, and alcohol lines. Their construction in the streets obstructed the delivery of essential machinery. Time was the only consideration, because in July, 1918, the Allied Armies were using explosives about six times as fast as they could be produced by all plants. The separation of rights of way for utilities would have permitted road construction to be done previously or concurrently. The writer recommends to industrial city planning engineers that an attempt be made to provide separate rights of way for utilities on new developments.

This is in no way a reflection upon the excellent work of those who were responsible for the miraculous speed in the construction of the plant at Nitro, because they produced their $75 000 000 job in about the same time as the slightly larger Nashville (Tenn.) job was done by the explosives specialists.

Although the presence of utilities in streets contributes greatly to the engineering difficulty and cost of subways in the large cities there does not seem to be any feasible way to avoid it; but in new developments where land is relatively cheaper a separation of the utilities in the rights of way should be possible.

W. W. CROSBY,[40] M. AM. SOC. C. E. (by letter).—The conclusion of Mr. Kettering's "salmon" story is good, and his remarks on "where to get the money," should be regarded as fundamental. Although the writer could applaud most, if not all, the remainder of Mr. Kettering's remarks, it is from the ideas just mentioned that he wishes to propose an hypothesis which may account for many of the defects in the highways of the United States as well as suggest an answer to an engineering question that has been widely discussed for years.

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[40] Cons. Engr., Coronado, Calif.

Before so doing let the writer note that he has carefully read the paper by Mr. MacDonald, which seems to express, fairly, the attitude of the U. S. Bureau of Public Roads for the past fifteen years. A significant quotation (regardless of the question of fact) in this case is "Highway builders [in the United States] are proceeding on the principle that the utilization of the highways must produce directly the revenues with which to finance their construction" (see heading "Super-Highways").

The third paper of the Symposium, although limited to a narrower field, seems commendably broad in conception. Mr. Smith remarks several times on the fundamental importance of adequate rights of way, which the writer has been emphasizing for years. Mr. MacDonald barely suggests anything on this point although its prompt recognition is even more important in his work outside the cities. In some of its recent publications[41] the writer believes that the Bureau of Public Roads has admitted the need for "elbow room"[42] in the future.

The writer believes with Mr. Kettering that "Sometimes, people calculate entirely too much in dollars," particularly in setting up ideals—that is, in planning. Often it is necessary for the material approach to ideals and standards to be gradual as funds may be found; but to establish standards on the basis of funds in sight only results in regrets.

This brings the writer to the hypothesis mentioned:

May it not be that an answer to the questions discussed for so many years—"What is the matter with engineering?" and "Why does it not rank with the learned professions?"—lies in the fact that too many of its more prominent representatives cling to the definition of an engineer as one who "can make one dollar do the work of two," and are not imbued with the spirit of one familiar to the Society: "An engineer is one who adapts the forces of nature (including Human Nature) to the benefit of mankind," or perhaps the latest one:[43]

> "An Engineer is one who, through application of his knowledge of mathematics, the physical and biological sciences and economics and with aid, further, from results obtained through observation, experience, scientific discovery and invention, so utilizes the materials and directs the forces of nature that they are made to operate to the benefit of society. An Engineer differs from the technologist in that he must concern himself with the organizational, economic, and managerial aspects as well as the technical aspects of his work."

THOMAS H. MACDONALD,[44] Esq. (by letter).—The last paragraph of the paper emphasizes the fact "that the country has completed the pioneer stage of road development and every trend of highway development of the future must be an intelligent meeting of the particular service to be rendered." In order that there may be an intelligent meeting of the particular service to be rendered in the highway development of the future, it is essential that all pertinent factual information be available to highway engineers and administrators. In the paper reference was made to the State-wide Highway Planning Surveys. These are now (1939) being conducted co-operatively between the Bureau of Public Roads and 46 States and the Territory of Hawaii. Already these surveys have produced the factual basis for the solution of specific highway problems within a single State. They have also formed the basis for a comprehensive report from the U. S. Bureau of Public Roads, submitted to the Congress of the United States on April 27, 1939, by Special Message from the President of the United States. This report[45] was made in compliance with the specific direction of the Congress and consisted of two parts—Part I covering the "Feasibility of a System of Transcontinental Toll Roads," and Part II covering "Master Plans for Free Highway Development."

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[41] *Roads and Streets*, February, 1939, p. 53; also *Public Roads*, January, 1939, p. 209.

[42] *Proceedings*, Am. Soc. C. E., May, 1938, p. 973.

[43] *Scientific Monthly*, February, 1939, p. 176.

[44] Chf., U. S. Bureau of Public Roads, Washington, D. C.

From the factual information developed in the State-wide Planning Surveys and other specific studies developed by the Bureau of Public Roads and the State highway departments, the definite conclusion was reached, and the recommendation made to the Congress, that it was not feasible to construct a system of transcontinental toll roads—three east and west, and three north and south—with any expectation of such a system serving the genuine traffic needs of the United States, or of being self-liquidating. The facts and considerations leading up to this conclusion were fully discussed in the report.

Further analysis and study of the Planning Survey data clearly indicate the problems confronting highway engineers and administrators. Some of them are of long standing and have become acute through the tremendous increase in highway traffic. Not only increased numbers but also increase in normal speeds critically affects the various problems.

Much of the yearly obsolescence of the rural highways built in the past has been due to the restrictions imposed upon the design by inadequate rights of way. Another outstanding problem centers around the metropolitan areas and involves the provision of means for getting into, around, and through such areas without the delay that is involved in using present inadequate facilities. Traffic-flow maps indicate the natural flow lines between the various regions of the United States. A limited system of inter-regional highways approximating 1% of the total highway mileage constructed to modern standards is needed to provide adequately for the ever-growing traffic. Further study is needed to determine the location and approximate mileage in each State of such a system.

From a full discussion of these problems, there emerges the general outline of a master highway plan for the entire nation. The prosecution of the plan in all its parts calls for appropriate action by the Federal and State Governments and all county and municipal subdivisions. As desirable joint contributions of the Federal and State Governments, the report lists several undertakings as follows:

(1) The construction of a special, tentatively defined system of direct inter-regional highways, with all necessary connections through and around cities, designed to meet the requirements of the national defense in time of war and the needs of a growing peace-time traffic of longer range;

(2) The modernization of the Federal-aid highway system;

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[45] H. R. Doc. No. 272, 76th Cong., 1st Session.

(3) The elimination of hazards at railroad grade crossings;

(4) An improvement of secondary and feeder roads, properly integrated with land-use programs;

(5) The creation of a Federal Land Authority empowered to acquire, hold, sell, and lease lands needed for public purposes and to acquire and sell excess lands for the purpose of recoupment.

The report emphasizes the difficulties encountered in the acquisition of adequate rights of way; and, in view of the fundamental necessity of such rights of way, proposes definite measures by which the United States could aid in the acquisition of suitable rights of way and simultaneously contribute helpfully to the solution of other urgent problems, especially certain problems confronting the larger cities.

LEROY C. SMITH,[46] M. AM. SOC. C. E. (by letter).—With the ever increasing use of the passenger automobile, together with the increased use of motor trucks for industrial transportation, the necessity for adequate highway systems in and about the metropolitan areas is becoming more and more acute. The problem has reached such proportions that the Federal Government has taken definite steps to develop a system of inter-city super-highways, connecting the major metropolitan centers, and are contemplating acquiring such widths of right of way that even the most enthusiastic planners of the past had not ventured to advocate. The Government plans, discussed for connecting major metropolitan centers, provide for right-of-way widths ranging from 200 ft to $\frac{1}{4}$ mile, many of them being parallel to railroad lines. Although these roads will alleviate the condition between the centers, there must be a system of express highways within the metropolitan centers to be connected with the inter-city roads.

Industrial Super-Highway Development in the Detroit Area.—Many of the civic and industrial leaders of Metropolitan Detroit and Wayne County are proposing a system of cross-town highways to permit the free flow, not only of the passenger motor traffic but also the large volume of industrial traffic. Factories of all types and descriptions are scattered over Wayne County and they depend upon co-ordination with one another in the process of their manufacturing operations. This is particularly true of the automobile industry where parts, bodies, etc. must be brought to the assembly plants. Narrow streets and roads not only delay transportation to the plants but create a dangerous situation with respect to other vehicles. These conditions could be eliminated by a system of wide thoroughfares. The "Drive Away" system of transporting new automobiles is another factor that creates congestion on the highways in the metropolitan areas as well as on the inter-city rural highways. Adequate facilities for delivering new automobiles would result in savings in transportation costs, thereby reducing the cost of new vehicles and encouraging new car buying.

Public Reaction.—Public reaction continues to be toward rapid transportation on rubber. A large majority of people are pleased to use their automobiles in lieu of buses or street cars in spite of the fact that transportation costs may be a little greater. This fact must be recognized and encouraged by the public agencies—whether local, State, or Federal—who are in charge of highway development. In 1938, the people of the State of Michigan adopted, by the vote of a great majority, a constitutional amendment which specifically provides that highway revenues be used for highway purposes only. This is another indication of public reaction to highways. Wayne County, in which Detroit is situated, has no property tax for highways. The only funds accruing to the County for highway construction and development are from the motor-vehicle gas and weight tax revenues. This is general throughout the State of Michigan. The constitutional amendment, however, guarantees that there shall be no diversion of highway funds for purposes other than highway construction and development.

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[46] Chf. County Highway Engr., Wayne County Road Commrs., Detroit, Mich.

Huron-Clinton Parkway.—The Legislature of the State of Michigan has enacted a law providing for the incorporation of the Huron-Clinton Metropolitan Authority. The purpose of this act is to permit the planning, development, and construction of parkways and limited access highways through two to five contiguous counties in the valley of the Huron and Clinton rivers. The act is comprehensive and is well worth detailed study.

Conclusion.—All State agencies are in accord with the Federal plans for cross-country highways. As in all cases of development, however, the question of adequate funds for such development always seems to be the chief obstacle. With a program of feasible and economical financing, there is no reason why these plans cannot be effected in the near future. The United States needs highways and, in line with the provisions of the Federal Constitution, there should be free intercourse between the cities and States of the country.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2048

RECONSTRUCTION OF A PIER IN BOSTON, MASSACHUSETTS, HARBOR

By Charles M. Spofford,[1] M. Am. Soc. C. E.

With Discussion by Messrs. William G. Atwood, John N. Ferguson, John Ayer, and Charles M. Spofford.

Synopsis

The reconstruction of the outer parts of a large ocean pier, the wooden pile foundations of which had been seriously damaged by marine borers, is described in this paper. The wooden piles were removed and steel bearing piles many of which had a length in excess of 100 ft were substituted. These are believed to be the longest steel bearing piles ever used. They were driven by steam hammer through fill and blue clay to a bearing on bed-rock, or the hard material immediately overlying it. The piles were placed in groups of four with the upper ends protected by steel cylindrical caissons filled with concrete.

For the most part, the concrete deck of the old structure and its supporting concrete beams were removed and replaced by a new concrete deck supported on steel beams and girders. A test was made on a pile having a length of 137 ft, and the complete data of the test are given. As far as the writer is aware no other test on a pile of this length has been reported previously.

The use, in certain parts of the pier, of a light-weight concrete to protect the wooden piles from further damage is also described.

General

The control of the water-front of Massachusetts is under the direction of the Massachusetts Department of Public Works which is administered by one commissioner and two associate commissioners. Among the duties of the Department are the construction, maintenance, and operation of State piers located at various points along the Massachusetts seacoast, the largest of which is Commonwealth Pier No. 5, in South Boston.

Note.—Presented at the meeting of the Waterways Div., Boston, Mass., October 7, 1937, and published in October, 1938, *Proceedings*.

[1] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

Boston is one of the important ports in the United States, its standing on the Atlantic seaboard being exceeded (as measured by the value of its foreign and domestic commerce) only by the Ports of New York, N. Y., Philadelphia, Pa., and Baltimore, Md. Commonwealth Pier No. 5, with its transit sheds and other facilities, is the principal pier for passenger ships on the southerly side of the harbor and is used by several foreign lines, including the Italian, French, and Holland-American Lines. It is also used by the American Hawaiian and the Luckenbach Lines furnishing service to the Pacific Coast, and by the Canadian National Steamship Company operating to the West Indies. Much freight is received at Pier No. 5, wool being one of the principal products. Boston is the second largest wool port of the world, exceeded only by the Port of London, England.

The pier provides berths in excess of 1 100 ft in length, with a depth of water of at least 40 ft at mean low tide. It is sufficient in size and draft to accommodate the largest ships now in existence. Fig. 1 shows the location of this and other State piers in Boston.

HISTORICAL

The construction of the pier was begun in 1897, under the direction of the State Board of Harbor and Land Commissioners, by the building of a granite sea-wall around a site, 300 ft wide by 1 150 ft long, and the filling of the interior with material dredged from docking spaces alongside the pier. This wall was supported on spruce piles capped with spruce planks and timber. The spaces between the piles and in front of them were filled with small stone chips and gravel, except that the face of the slope in front of the wall was protected by large stones weighing 100 to 300 lb each. After the solid part of the pier had been completed, it was surrounded by a 50-ft platform of hard pine supported on oak piles. The heads of the piles were covered with coal tar applied at a temperature of not less than 120° F, but no other preservative treatment was used. The total cost of this part of the pier was not quite $400 000.

The then comparative inaccessibility of the pier by land delayed the making of satisfactory arrangements for its use. In July, 1907, a part of it was seriously damaged by fire. The pier was finally leased on November 1, 1910, together with certain land in its rear, to the Old Colony Railroad Company at the annual rate of $75 000, the lessee to pay the city taxes. This lease was not of long duration as on June 28, 1911, the Massachusetts Legislature created a board entitled "The Directors of the Port of Boston" and transferred to this board all the rights, powers, and duties of the former Board of Harbor and Land Commissioners pertaining to the Port of Boston. It also authorized the expenditure of $9 000 000. The Directors decided to build a terminal to serve large vessels and reached the conclusion that Commonwealth Pier No. 5 afforded the best opportunity for the development of such a terminal.

After months of negotiations the Directors secured from the Old Colony Railroad Company, on November 6, 1912, the control of the pier and at once set aside $2 500 000 for improvement, and work was started two days later. This improvement included dredging the berths alongside to 40 ft below mean low water; cutting off burned piles on the easterly side to Elevation + 10

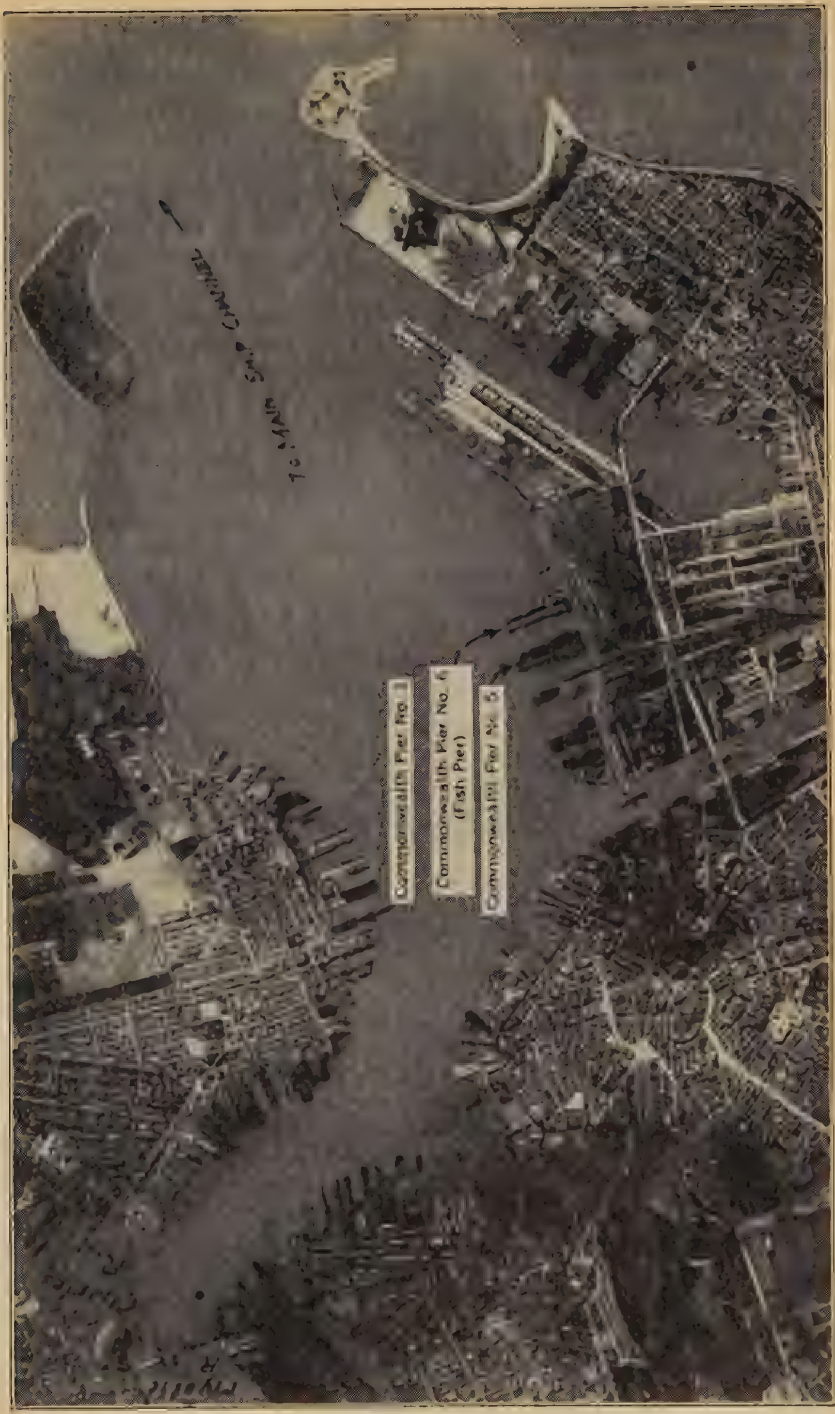
To MAIN SHIP CHANNEL
Commonwealth Pier No. 1
Commonwealth Pier No. 6
(Fish Pier)
Commonwealth Pier No. 5

FIG. 1.—AIRPLANE VIEW OF BOSTON HARBOR, SHOWING COMMONWEALTH PIERS

and the remainder of the piles to Elevation + 14, and supporting the deck structure by concrete beams resting on the original oak piles; replacing the timber platform with a concrete deck, covering the entire pier except the outer parts of the platforms, and the outer end, with two-story non-combustible sheds supplied with facilities for handling both freight and passengers; and constructing railroad and highway connections. The pier was opened for business on May 31, 1913, when a new service of the Hamburg-American Line was instituted, the western half of the pier being assigned to this line. The Directors also agreed to furnish the White Star Line with service on the east side.

Pier improvements, as originally planned, were estimated to cost $1 250 000, but through amplifications as a pier and as a monument to advertise Boston the total sum finally expended was approximately $4 000 000.

Attacks by Marine Borers

In 1925, the attention of the Public Works Department was called to the fact that marine borers were becoming active in Boston Harbor, and it was decided to investigate a few of the foundation piles under the platforms at Commonwealth Pier No. 5 to determine the extent of the destruction by these borers. Later examinations were made in 1932 and in 1934, and the results of all these examinations are shown in Table 1.

TABLE 1.—Effect of Marine Borers, Commonwealth Pier No. 5

Condition	(*a*) 1925 Investigation				(*b*) 1932 Investigation					(*c*) 1934 Investigation					
	East side	West side	Total*	Percentage of total	East side	West side	North end	Total*	Percentage of total	East side	West side	North end	Clusters	Total*	Percentage of total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
One-eighth gone	1	0	1	0.9	16	26	9	51	10.5	56	53	10	13	132	13.9
One-fourth gone	0	9	9	7.6	34	46	19	99	20.4	46	103	37	24	210	22.2
One-half gone	2	0	2	1.7	12	60	22	94	19.4	7	38	20	4	69	7.3
Three-fourths gone	..	..	..	..	4	5	4	13	2.7	9	15	2	0	26	2.7
Slightly touched	28	25	53	44.9	91	76	15	182	37.6	175	114	12	134	435	46.0
Uninjured	23	30	53	44.9	25	18	2	45	9.4	38	25	7	5	75	7.9
Total	54	64	118	100.0	182	231	71	484	100.0	331	348	88	180	947	100.0

* Number of piles.

It will be seen that, in the 1925 examination, approximately 55% of the piles examined were found to be affected by marine borers, whereas in the last examination about 92% were affected. The action of the borers had been constantly increasing and, based on the results of the last examination, about 10% of the piles were one-half or more destroyed. During the last examination (Table 1(*c*)), six of the piles were found to have a diameter of not more than 3 in. at the mud line, instead of their original diameter of about 10 in.

As a result of these examinations, the Engineers of the Department recommended that studies be made of means to prevent further action by marine borers and a tentative plan for placing solid filling under the pier platforms, retained by a steel bulkhead around the pier, was developed at an estimated cost of approximately $437 000. This method had to be rejected after the nature of the subsoil was thoroughly investigated.

It is interesting to note that during reconstruction most of the piles that were examined by the divers were pulled out by the contractor, and it was possible to get a complete view of the damage. In general, the reports of the divers were found to be correct except that piles reported three-fourths gone were found to be in a worse condition than reported. Piles upon which the original bark remained in place were found to be in good condition, but wherever the bark was missing, the pile had been badly attacked by the worms. The major part of the damage was caused by *Limnoria;* some *Teredo* were found, but not in sufficient number to affect the piling seriously.

METHODS OF RECONSTRUCTION

The reconstruction may be treated conveniently under two divisions: (1) The repairs of the platforms and pile foundations of the outer parts of the pier; and (2) the repairs of the foundations of the head-house at the shoreward end of the pier.

Platforms and Pile Foundations.—Three different types of bulkhead construction were considered for this work: (*a*) An enclosure of steel sheet-piles filled with sand and gravel; (*b*) built-up "master-piles" with intermediate sheet-piles; and (*c*) the cellular type. Type (*a*) consists of driving a row of steel sheet-piles immediately outside the present face of the deck, holding the pile tops back by wales and tie-rods, with sand and gravel fill around the timber piles, thus preventing further damage to them from marine borers. With the data available from the borings and laboratory analyses described elsewhere, it was possible to make studies to determine the size and length of the steel sheet-piles necessary to withstand the pressure from the fill to be placed around them to the elevation necessary to protect them from further damage from marine borers. From these studies, it was found that due to the soft underlying clay, the piles would have to be driven to an excessive length, in order to obtain sufficient pressure at the toe to hold them in place. This was determined by computing the active earth pressure on the bulkhead by the application of the Rankine equations and equating this pressure with the resistance of the blue clay at the face of the pile corresponding to its shearing resistance. The latter was determined by tests on undisturbed samples.

The width of the wharf platform was too small to permit the sheeting to be tied back efficiently to the cross-bents supporting the wharf and the only feasible method of restraining the upper ends of the sheeting seemed to be to build a series of concrete "dead men" at a considerable distance back of the sea-wall and to tie the ends of the sheeting to these "dead men" by heavy steel tie-rods. This process would have necessitated the drilling of holes in the existing walls, a laborious, time-consuming, and expensive process.

Type (*b*) consists of the construction of "master piles" composed of heavy built-up sections placed at intervals along the front of the wharf face, and the

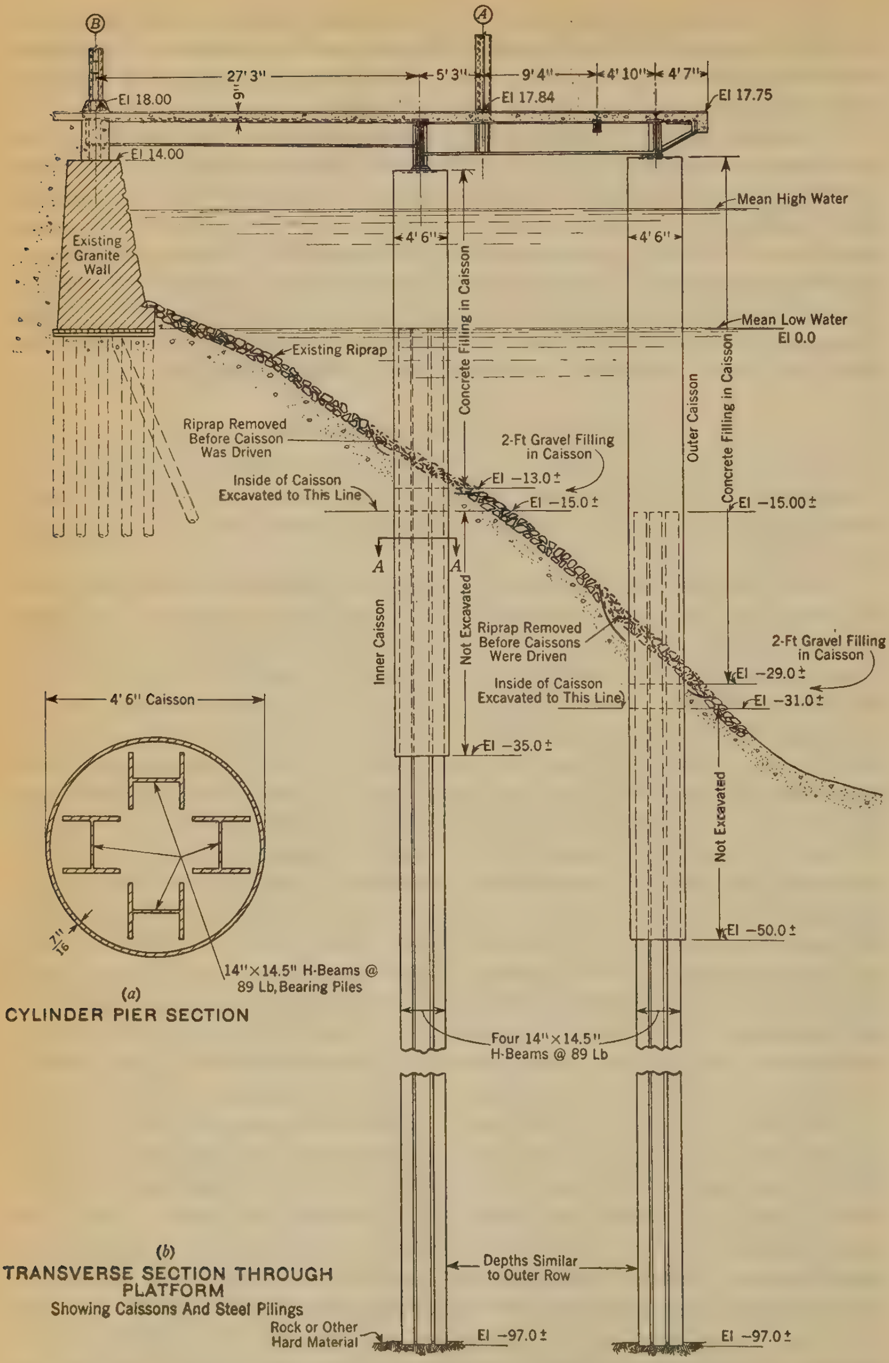
B
A
27' 3"
5' 3"
9' 4"
4' 10"
4' 7"
El 18.00
9"
El 17.84
El 17.75
El 14.00
Mean High Water
4' 6"
4' 6"
Existing Granite Wall
Concrete Filling in Caisson
Mean Low Water El 0.0
Existing Riprap
Outer Caisson
Concrete Filling in Caisson
Riprap Removed Before Caisson Was Driven
2-Ft Gravel Filling in Caisson
El −13.0±
Inside of Caisson Excavated to This Line
El −15.0±
El −15.00±
A
A
Inner Caisson
Not Excavated
Riprap Removed Before Caissons Were Driven
2-Ft Gravel Filling in Caisson
El −29.0±
Inside of Caisson Excavated to This Line
El −31.0±
El −35.0±
4' 6" Caisson
Not Excavated
El −50.0±
7/16"
14"×14.5" H-Beams @ 89 Lb, Bearing Piles
(a)
CYLINDER PIER SECTION
Four 14"×14.5" H-Beams @ 89 Lb
(b)
TRANSVERSE SECTION THROUGH PLATFORM
Showing Caissons And Steel Pilings
Depths Similar to Outer Row
Rock or Other Hard Material
El −97.0±
El −97.0±

Fig. 2.—Transverse Section of Platform, Showing Caissons and Steel Piling

driving of intermediate sheet-piles between, and supported by, these built-up members. For this type of construction the master piles would have to be held at the bottom by being driven a considerable distance into the underlying clay and at the top by tie-rods fastened to concrete "dead men" as proposed for Type (*a*). The same difficulties were found in supporting this type of bulkhead at its toe as were found for the simple steel sheet-piles.

The third type (Type (*c*)) of bulkhead considered was a cellular type but, among other difficulties, it was found that the available distance between the sea-wall and the face of the existing wharf platform was insufficient to provide the necessary stability for this type of construction which would have to be built on a steep slope.

In view of the foregoing difficulties with bulkhead construction and its excessive cost, it became necessary to develop another method for this part of the work. After considerable study it was decided to use steel bearing piles driven either to bed-rock or into the hard overlying strata and to support the reinforced concrete slab by a framework of steel beams.

The following description covers this method of construction: Steel bearing piles, 14 in. by 14.5 in., weighing 89 lb per ft were driven, in clusters of four, to practical refusal into bed-rock or into hard overlying material. During driving, they were held in position laterally by timber frames. The upper parts of each group were surrounded by a structural steel cylinder filled with concrete, and concrete was deposited in the caisson above the tops of the steel piles which, in no case, were above mean low water. The piles and caissons are shown in Fig. 2.[2] For most of the work the cylinder piers thus formed were spaced 40 ft, center to center, longitudinally and 21 ft 5 in., center to center, transversely. The various steps in the construction were as follows:

(1) The rip-rap and a portion of the silt in the area in which a group of piles was to be driven were excavated to as great a depth as feasible;

(2) The steel caisson for the group of piles was generally driven by a steam hammer to a minimum depth of 5 ft in the clay subsoil with its top at the proposed level of the bottom of the new steel and concrete deck;

(3) The material within the caisson was excavated to a depth of 15 ft below the anticipated level of the tops of the piles;

(4) The four steel piles in each caisson were then driven, one by one, by a steam hammer. (Except in a few cases, all piles had to be spliced once. The minimum length of pile was 98 ft. In certain cases the piles twisted in driving and in certain cases the driving of one pile carried down another pile. Possibly these effects would have been avoided by increasing the spacing, but this would have increased the cost and added to the load carried by individual piles);

(5) A 2-ft layer of sand and gravel was deposited within the caisson around the piles and over the overlying soil to form the bottom for the concrete to be placed in the caisson; and,

(6) Concrete was placed in the caisson around the steel piles by means of a chute, the caisson proving to be water-tight, so that it was unnecessary to place an under-water concrete seal at the bottom.

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[2] See, also, *Civil Engineering*, December, 1937, p. 843.

Prior to driving the piles, borings were made to determine the required lengths. A tolerance of 6 in. in the location of the center of the caisson caps was permitted, and no caisson moved more than this distance from its correct position.

In the process of construction, it was necessary to cut holes in the roof and second floor of the pier shed in order to place the piles and the steel cylinders in position. In driving, the tops of the piles were protected, somewhat ineffectually, by various types of cushions. Some of the rivets in the splices were loosened during driving. However, the ends of the abutting sections at the splices were faced and enough rivets used to insure that the abutting piles would be held in line. Fig. 3 shows the splices used. Each pile was designed for 100 tons and, as seen in Fig. 3, thirty-six shop rivets were used above the splice and forty-four field rivets below it. In the worst case, 25% of the shop rivets and 32% of the field rivets were loosened in the lowermost splice during the first stage of the driving. Assuming that further loosening of the rivets in this splice would occur in the second (final) stage of the driving in proportion to those already loosened, twenty tight shop rivets and twenty tight field rivets would still be left. This number is more than ample to comply with the specifications of the American Railway Engineering Association for splicing bridge chords.

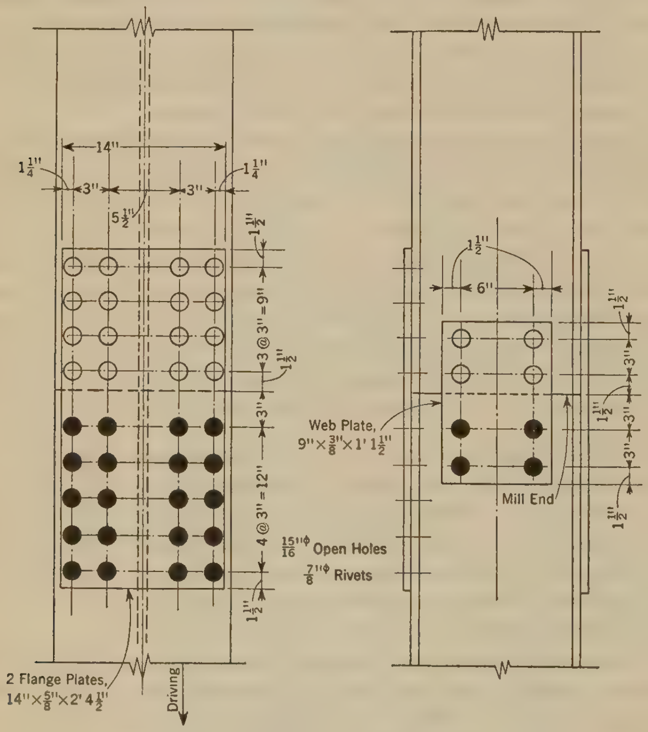

FIG. 3.—SPLICES IN STEEL PILES

After the cylinder piers were constructed, the platform shown in Fig. 4 was built. The wharf-shed columns were supported temporarily and the steel framework forming the support for the concrete deck was installed. The steel of this framework consisted chiefly of 30-in. and 36-in. H-beams of silicon steel placed longitudinally, and supporting transverse H-beams, also of silicon steel. The longitudinal members were usually continuous over two spans. Transverse expansion joints were provided at 80-ft intervals.

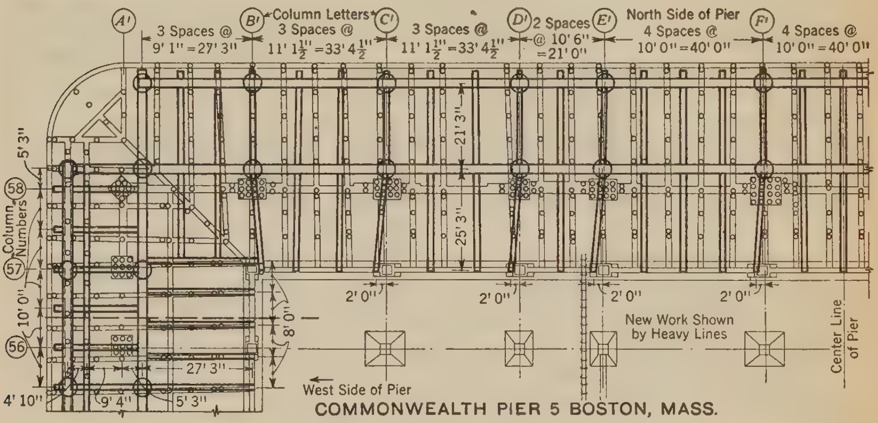
A'
B'
Column Letters
C'
D'
E'
F'
3 Spaces @ 9' 1" = 27' 3"
3 Spaces @ 11' 1½" = 33' 4½"
3 Spaces @ 11' 1½" = 33' 4½"
2 Spaces @ 10' 6" = 21' 0"
North Side of Pier
4 Spaces @ 10' 0" = 40' 0"
4 Spaces @ 10' 0" = 40' 0"
21' 3"
25' 3"
5' 3"
58
Column Numbers
57
10' 0"
56
4' 10"
9' 4"
5' 3"
27' 3"
8' 0"
2' 0"
2' 0"
2' 0"
2' 0"
New Work Shown by Heavy Lines
Center Line of Pier
West Side of Pier
COMMONWEALTH PIER 5 BOSTON, MASS.

Fig. 4.—Arrangement of Steel Beams in Platform (Symmetrical About Center Line)

Head-House Foundations.—The method of reconstructing the wharf platform could not be used at the shoreward end of the pier under the head-house (which is a massive brick building extending out to within 20 ft of the face of the wharf platform), on account of the difficulty in supporting, temporarily, the heavy columns and brick walls of the head-house structure. Various methods were proposed for this part of the work and that finally adopted consisted of driving, to a limited depth, a light-weight steel sheet-pile bulkhead just outside the foundations of the building walls and filling the space between it and the existing sea-wall, to approximately low-water level, with a light-weight concrete. This concrete was placed in comparatively small quantities so that it would exert a minimum pressure against the steel bulkhead.

It was originally proposed to use fine and coarse coke aggregate for the concrete, and experiments were made to determine the feasibility of placing this class of concrete under water. These tests, on a small scale, were successful; but, when the coke concrete was deposited in large quantities through tremies, it was found that there was a tendency for the large pieces of coke to float. Therefore, cinders were substituted for the coke and the piles were protected up to low water back of the bulkhead by placing this concrete in place with a tremie. To protect the part of the piles above low water up to the bottom of the beam and column footings, forms were built around the pile clusters and on either side of the pile bents, using creosoted timber, and the spaces within these forms were filled with the cinder concrete. In this manner, it was possible to provide an efficient protection of all piles within the building site itself without the necessity of removing any of the structure. The adequacy of this method of protection was referred to William F. Clapp, Consulting Biologist, who reported as follows:

> "(1) Effect on Marine Borers Now Present. All destructive marine organisms require a constant flow of salt water containing a sufficient amount of oxygen to support life in the case of *Limnoria* and *Chelura*, and to supply food as well in the case of *Teredo*. The proposed protection would completely prevent the flow of salt water to the protected piles. This will result in the immediate extermination of all species of marine borers on or in the protected wood. There will be therefore no possibility of further pile deterioration from the marine borers now present. This has been demonstrated in numerous instances where protection of similar type has been used.
>
> "(2) Efficacy against Future Attack. Since the protection shown on the plans effectually prevents the free and continuous flow of salt water to the protected parts, under these conditions, no future invasion by marine borers of any type can take place. Consequently, in my opinion, the piles thus protected will be thoroughly safeguarded against future attack by any species of marine borer."

The platforms extending from the building outward to the face of the dock were removed, and new creosoted piles were driven to support a new concrete deck. The dock opposite the head-house had not been dredged to the full depth required alongside the pier structure itself, and when the platform piles were pulled in this part of the pier, it was found that the damage to them by marine borers had been less than farther out into the harbor where the water was materially deeper.

SUBSOIL INVESTIGATIONS

Preliminary investigations of the subsoil conditions were made by the usual wash-boring methods. Undisturbed samples of the clay were also obtained by a method developed under the supervision of Glennon Gilboy, Assoc. M. Am. Soc. C. E., in the Soil Mechanics Laboratory of the Massachusetts Institute of Technology, at Cambridge, Mass., in co-operation with the former Committee of the Society on Earths and Foundations. This process consists of driving a 6-in. casing into the soil, pumping and bailing out the water, and removing the soil by an auger to the elevation where the sample is to be secured.

A 2-in. pipe, enlarged to a 4-in. spoon, is then lowered through the casing into the soil and the sample cut off at the bottom of the spoon by a piano wire which extends to the top of the pipe. A vacuum is applied to the 2-in. pipe while raising, to help hold the sample in place. After it was finally decided to use steel piles driven to bed-rock, a series of core-borings was also made. The underlying bed-rock was found to consist of seamy slate rock varying, in elevation referred to mean low tide, from − 94.00 to − 142.67. Core-borings were also made to determine the elevation and character of the bed-rock.

PILE TEST

In view of the unusual length of the steel piles under consideration for the construction of the platforms and pier foundations, a test was made on one pile, consisting of a 14-in. by 14.5-in. by 89-lb steel bearing pile having a cross-section area of 26.19 sq in. and a total length of 137 ft and consisting of three sections which were spliced in the field during driving. The pile was driven through overlying materials, mostly blue clay, with a steam hammer, and to refusal on bed-rock. Refusal occurred 113 ft below mean low water, and ten blows were struck by the hammer without further penetration. At the conclusion of the test 54 ft of the pile was unsupported and 83 ft was in the blue clay. A steel loading platform and steel rails were used for the applied load, as shown in Fig. 5. The basic data applying to the test were as follows:

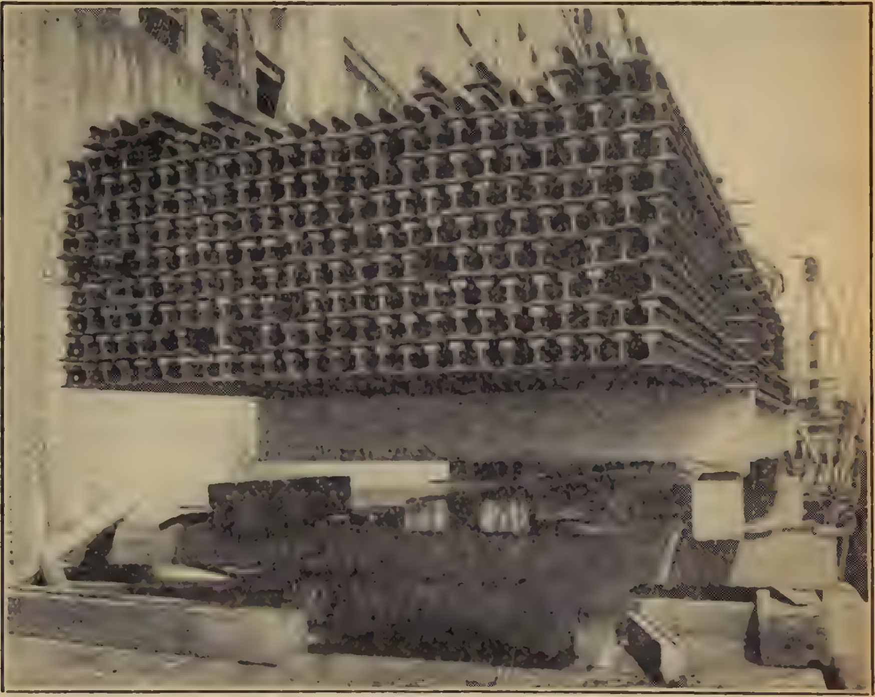

Fig. 5.—Loading for Test Pile

Description	Loads, in pounds
Weight of platform	10 552
Weight of cap	2 674
244 rails @ 880	214 720
Weight of hammer	14 000
Total weight (120.97 tons)	241 946

The deformation was measured at a point approximately 3 ft below the top of the pile and was observed from an initial load of approximately 40 tons to a maximum load of 121 tons at intervals of about 10 tons. The maximum load was allowed to remain on the pile for 24 hr, the total schedule being as follows:

Began loading April 14, 1936; loaded 63.6 tons; finished loading April 15, 1936, 11:30 A.M.; total load, 121 tons; began unloading April 16, 1936; 11:40 A.M.; and finished unloading April 16; 3:45 P.M. During unloading, observations were taken at intervals of about 10 tons from 86 tons until the pile was completely unloaded. The log of the pile-driving is given in Table 2 and a diagram of the deformation of the pile for various loads during loading and unloading is shown in Fig. 6 which also shows lines giving the theoretical deformation for lengths of 51 ft and 134 ft assuming the full load to be distributed over these respective lengths—that is, assuming no load to be transferred to the surrounding material.

TABLE 2.—LOADING LOG, INDIVIDUAL PILE TEST

(a) Loading			(b) Unloading		
Weight, in Tons		Deformation, in feet	Weight, in Tons		Deformation, in feet
From	To		From	To	
0	40.8	0.002	121.0	86.4	0.004
40.8	52.2	0.003	86.4	75.0	0.005
52.2	63.6	0.005	75.0	63.6	0.004
63.6	75.0	0.003	63.6	52.2	0.003
75.0	86.4	0.006	52.2	40.8	0.002
86.4	97.8	0.003	40.8	29.4	0.002
97.8	109.2	0.002	29.4	18.0	0.003
109.2	121.0	0.003	18.0	0	0.002
Total....	....	0.027	....	....	0.025

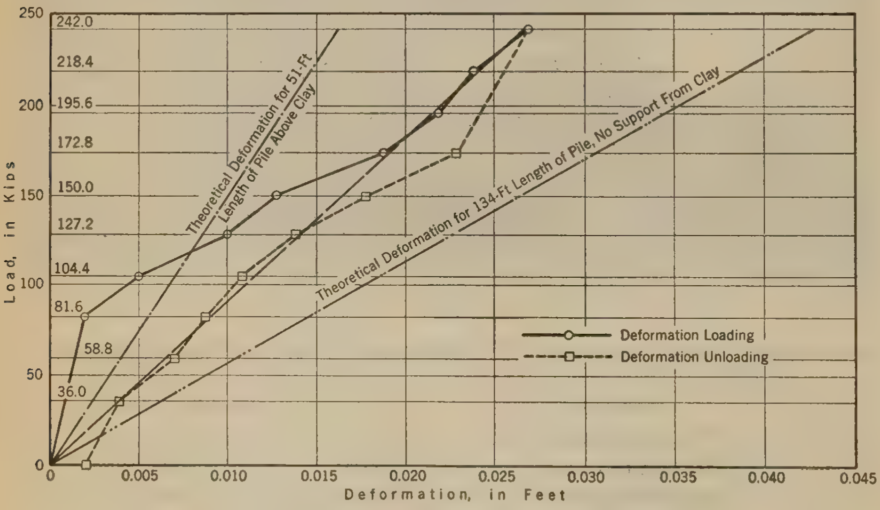

FIG. 6.—DEFORMATION DIAGRAM FOR TEST PILE

It will be noted that the deformation up to the load of 81 600 lb equals the permanent set of 0.002 ft as shown by the unloading diagram, this probably being equal to the penetration of the pile into bed-rock. From 81 600 lb to

195 600 lb, the deformation agrees very closely with that which would occur for the total length of 134 ft, which would seem to indicate that deformation was taking place over the full length of the pile. From a load of 195 600 to the full load of 242 000 lb, the deformation at all points was less than the theoretical deformation for a 134-ft length, indicating that during this period the load was being carried partly by the clay and partly on the rock.

When the pile was unloaded, the part above the clay first expanded at the theoretical rate for a 51-ft length of pile, the clay apparently holding the lower part of the pile firmly in position. From a load of 172 800 lb to a load of 127 200 lb the deformation of the pile was approximately equal to that for its entire length, namely, 134 ft. From the latter load to a load of 36 000 lb the pile expanded at a rate intermediate between that which would have occurred had the load been applied over its entire length and that which would have occurred had it been applied over only 51 ft of pile. It is impossible to discern from the diagram how much of the load was carried by the clay and how much directly on rock, but in the judgment of the writer, a considerable part of the load was carried by the clay.

Special Contract Requirements

Substitution of Steel Cylinders for Steel Piles.—After the contract for the work on the outer section of the pier had been signed and the work begun, the contractor requested permission to substitute steel cylinders driven into bed-rock by rotation for the steel piles included in the contract, and proposed a considerable reduction in the contract price if this change was approved. After careful consideration by the State Engineers, the Engineers of the Public Works Administration and the Consulting Engineers, permission was given for this change provided two test caissons could be driven successfully. Thereupon, the contractor undertook the construction of a test caisson. The experiment was not successful, however, as the material overlying the clay, consisting in part of rip-rap, offered such resistance that the trial cylinder was irrevocably damaged long before it reached bed-rock. Further efforts along this line were not made, and the construction was completed in the manner originally contemplated and described in this paper.

Removal of Existing Piles and Bracing.—The conditions of the contract required the eventual removal, in whole or in part, of all main piles, spur shores, and cross-bracing under the existing platform. Piles that interfered with the driving of steel piles or caissons were pulled, and other piles were cut off just above the ground surface. The existing fender piles and capping and the pile and timber corners were also removed and replaced by new fenders.

Avoidance of Interference with Business of Pier.—In order to avoid interruption of business at the pier as far as was reasonably possible, the pier was divided into five sections; the contractor was allotted one section at a time and was required to complete the work in that section before proceeding with work in another unless the pile and caisson work in another section could be done simultaneously without interfering with the business at the pier.

Settlement of Piles.—No settlement, either in the piles or in the concrete deck, has been discovered since the completion of the pier.

Bid Prices.—The following are some of the unit prices offered by the low bidder:

Caissons for the inner row, in place, per caisson	$ 790.00
Caissons for the outer row, in place, per caisson	1 050.00
Steel piles per pound	0.037
Structural steel per pound	0.050
Concrete per cubic yard	18.00
Yellow pine timber creosoted per 1 000 B. M.	120.00

CONCLUSIONS

The method of construction described in this paper, which involves the use of long steel piles driven to bed-rock and supporting a steel and concrete deck, is satisfactory for conditions such as existed at this pier. Furthermore, the use of a light-weight cinder concrete surrounding the wooden piles is a satisfactory method of protecting such piles from further damage by marine borers.

ACKNOWLEDGMENTS

The thanks of the writer are due to the Hon. William F. Callahan, Commissioner, Richard K. Hale, M. Am. Soc. C. E., Associate Commissioner, G. H. Delano, Chief Engineer, John W. Ferguson, M. Am. Soc. C. E., District Engineer in Charge of the Boston District, and Charles J. O'Donnell, Assistant Civil Engineer, Massachusetts Department of Public Works. These officials co-operated generously in furnishing information and giving other assistance in the preparation of the paper. John Ayer, M. Am. Soc. C. E., with Mr. O'Donnell, assisted in writing it. Fig. 1 was supplied by the United States Engineer Department (Boston, Mass., District); Glennon Gilboy, Assoc. M. Am. Soc. C. E., made the tests on undisturbed soil samples in the Soil Mechanics Laboratory of the Massachusetts Institute of Technology, and, the description of the single pile test presented herein was written by B. A. Rich, M. Am. Soc. C. E.

DISCUSSION

William G. Atwood,[3] M. Am. Soc. C. E. (by letter).—One possible method of reconstruction, used in some other piers in Boston, Mass., is not mentioned by Professor Spofford, and it would be interesting to know whether it was considered. Reference is made to the use of treated timber piles to replace the damaged piles. This would have made it necessary to remove the rip-rap but that might not have been costly enough to make this method uneconomical as compared to that used. By the use of timber-pile replacements the entire reconstruction of the deck might have been avoided.

The successful driving of piles of such great length is a tribute to the care and preparation by the engineers and contractors.

It may seem strange that the original designers of the pier did not protect the piles against the attack of marine borers, but the reason is clear to those who know the conditions in Boston Harbor. The investigation of 1922–1923 did not disclose the presence of borers in sufficient numbers to indicate that there was any danger from them. There were piers in the harbor that dated back to Colonial days, and there was no indication that they had ever been attacked; and there was no record of serious damage to any structures. Except for some of the railroad piers there was practically no treated timber in the harbor. The conditions in New York (N. Y.) Harbor are practically the same to-day, except that there is a history of attack by borers in the past.

The cause or causes of the invasion of borers into Boston Harbor is not known since there seems to have been little change in the water conditions. The cause of the heavy attack in the Harbor of Lynn, Mass., on the other hand, followed the removal of the sewage from the harbor and the consequent return of the normal oxygen content to the water.

There seems to be little doubt but that the protection of the substructures from attack by marine borers is cheap insurance. The cost of reconstructing two wharves in Boston Harbor, one of which[2] is described in this paper, would probably pay for the protection of at least 150 000 piles of normal length. Many other wharves have been rebuilt as a result of this attack and most of the remainder will have to be rebuilt eventually.

John N. Ferguson,[4] M. Am. Soc. C. E. (by letter).—The reconstruction of the platforms at Commonwealth Pier No. 5, Boston Harbor, was due to the action of marine borers on wooden pile foundations which made it necessary for a more permanent type of foundation to be provided. The unique and unusual features of the type of construction adopted are ably and concisely described in the paper by Professor Spofford. However, there are a few other items which may be of interest.

In addition to the creosoted piles used in the new foundations for the platforms at the head-house, creosoted timber piles and timbers for bracing were used for fenders along the outside edge of all platforms, and as fender clumps at the corners of the pier.

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[3] Cons. Engr., Ithaca, N. Y.

[4] Dist. Waterways Engr., State Dept. of Public Works, Boston, Mass.

The specifications for creosoting read, in part, as follows:

> "All new piles driven in the fender system and platforms shall be southern yellow pine * * * . The piles shall be treated by the full cell process with 16 lb per cu ft of Grade 1 Creosote Oil. All timber below high-water line shall be treated by the full cell process with 12 lb of Grade 1 Creosote Oil * * * . The fender caps and the top planking of the corners may be treated with preservative salts if the Contractor so desires * * * ."

The fender system along the edge of the platforms consists of two plumb piles and a spur pile in sets spaced 10 ft on centers. The plumb piles are placed one behind the other, and the spur pile is bolted to the inner plumb pile just below a wale which is about 5.5 ft above low water. The top of the inner plumb pile is fitted with girder caps which are bolted to the platforms at intervals. The outer plumb pile is bolted to the inner plumb pile through a continuous longitudinal wale which extends between the plumb piles about 2 ft above low water. By this arrangement full flexibility of the upper 16 ft of the outer plumb pile is utilized.

The structural frame for the platform on the piles abreast of the head-house consists of reinforced concrete girders and beams instead of the silicon steel frame work that was used abreast of the freight sheds. To fasten the girders and beams to the piles, an inverted U-bolt was placed in the head of each timber pile.

The placing of the additional load of light-weight concrete on the pile clumps supporting the columns of the head-house has presented no apparent settlement, as might be evidenced by cracks in the plastering and artificial stone of the face of the building.

To protect the platform slab, a part of which was placed during cold weather, the contractor provided some trackmen's railroad hand-cars on which was built a frame of wood extending over two bays of the platform. This frame was covered with canvas, and heat to prevent freezing was supplied from the boilers of the contractors' floating equipment. As the work progressed the weather protection was moved along to a new section on the standard-gage track which extended the full length of the side platforms.

The concrete used in the platforms was all transit mix of the following proportion by volume: 1 part cement, 2 parts sand, 3.5 parts coarse aggregate, and 5.75 gallons of water.

JOHN AYER,[5] M. AM. SOC. C. E. (by letter).—In gathering data for this paper, copies of the specifications for the original construction of Commonwealth Pier No. 5 were found, and certain parts of them were of interest in determining the method to use in this work.

The piles and timber for the foundations of the retaining wall were of spruce. The piles for the wharf area outside the solid filled part of the pier were oak and were purchased under the following specifications:

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[5] Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

> "All piles are to be new white or red oak sticks, with the bark on, sound, straight, and free from large knots or other imperfections which would materially affect the strength of the timber; not less than 50% of the piles to be white oak. They are to be not less than 14 in. in diameter 5 ft from the butt, and not less than 7 in. in diameter at point, when ready for driving, including the bark in both cases.
>
> "All piles from south of the fortieth degree of North latitude must be cut between the first day of September and the first day of March of the following year. North of that line they may be cut one month later.
>
> "When the heads of the piles have been prepared to receive the caps and before the caps are put on, they are to be thoroughly covered with a coat of the dead oil of coal tar, containing not less than 40% of naphthaline, applied at a temperature of not less than 120° F, or with such similar preparation as may be approved by the Engineer."

A similar specification as to the cutting of oak piles had been followed by the writer's firm until about 1930, when the question arose as to why piles should be so specified and whether winter-cut piles were not as susceptible to decay as summer-cut piles. Various persons were asked as to their opinion on this matter, and it seemed to be the general idea that summer-cut piles contained more sap and for this reason were more susceptible to decay.

To obtain a definite answer, the U. S. Forest Products Laboratory was asked this question, and replied in the form of its Technical Note No. F–15, which states as follows:

> "As a matter of fact, the time of cutting has very little effect upon the durability or other properties if the timber is properly cared for after it is cut * * *.
>
> "Timber, cut in late fall and winter, seasons more slowly and with less checking than during the warmer months. When proper storage or handling is impracticable, winter cutting is best. Fungi and insects do not attack wood out of doors in cold weather, and by the time warm weather arrives the wood is partly seasoned and somewhat less susceptible to attack. It is for this reason that winter cutting is advantageous and not on account of a smaller amount of moisture or sap in the wood in winter as the popular belief has it. There is practically no difference in moisture content of green wood in winter and summer."

This opinion is borne out by the Forest Products Laboratory of Canada[6] whose engineers also discussed the question as to whether winter cutting was more desirable than summer cutting, owing to difference in chemical composition.

Certain experiments conducted in Switzerland[7] on spruce and fir showed that "the wood is most susceptible to attack by fungi if felled during the period of greatest interchange of material in the tree." Furthermore, one year's seasoning under cover tends to obliterate differences shown by wood felled at different seasons.

It is generally recognized that if wood is cut in the spring or early summer and allowed to stay in the woods, or if it is not properly stored, it is extremely susceptible to attack by fungi; but if cut in the winter it has a chance to season somewhat and is less easily attacked by fungi.

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[6] "Canadian Woods—Their Properties and Uses," published by Forest Products Laboratory of Canada, 1935, p. 190.

[7] *Loc. cit.*, p. 191.

The report of the Forest Products Laboratory of Canada states[7] further that:

> "The unfavorable environmental conditions to which summer-cut wood is exposed can, however, be overcome by proper handling of the felled trees, since they do not affect the inherent durability of the wood at the time of cutting."

Oak piles are always cut on order and shipped as cut. If oak piles are scheduled to be used during summer months, the piles will be cut and shipped as required on the work. They are not left exposed to attack by fungi. This question of winter-cut *versus* summer-cut piles was discussed in one report made by a certain engineer concerning a pier built some years previously. In this report it was stated that the oak fender piles which were then showing decay were not winter-cut as specified, and therefore such rot was to be expected in these summer-cut piles. It is probable that the specifications for this pier were written, in part, with the scissors instead of a pen, and that the time-honored specification for winter-cut piles was included without full knowledge as to why the piles should be so cut.

Provided that oak piles are cared for properly between the time they are cut and the time they are used, the time of cutting makes no difference in the durability of the piles so far as decay is concerned.

In the light of present knowledge, however, the provision in the specifications that the oak piles at Commonwealth Pier No. 5 be winter cut was well taken, not from the standpoint of resistance to decay, but rather from the standpoint of the bark remaining on them, thereby resisting the attack of *Limnoria* materially better than would peeled piles. As Mr. William F. Clapp has shown,[8] oak piles removed from Pier No. 5 on which the bark remained had been attacked but little by *Limnoria* and then only through the knot-holes or where the bark had been knocked off. The bark remains on winter-cut piles much longer than on piles cut during late spring or early summer. Observations at various points along the New England coast agree with those of Mr. Clapp that oak piles, where the bark remains, show little attack from *Limnoria.*

The fender piles for the reconstruction of Commonwealth Pier No. 5 are of creosoted yellow pine. Since about 1920 it has been extremely difficult to obtain long oak piles with tips of a proper size, and the butts of oak piles, where used for fender piles, showed signs of decay within a comparatively few years after installation. For this reason, it was decided to use creosoted yellow pine fenders and at the same time to provide floating fenders. This procedure has been followed in the reconstruction of another wharf in Boston Harbor during 1937. Creosoted yellow pine fender piles will show some wear from the floating fenders rubbing against them, but the creosoted pine fender piles, even under these conditions, will outlast the oak fender piles that it has been possible to obtain within recent years.

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[8] "Recent Increases in Marine-Borer Activity," by William F. Clapp; paper delivered on October 7, 1937, before the Waterways Division of the Society at its Fall Meeting in Boston, Mass. (Abstracted in *Civil Engineering,* December, 1937, p. 836.)

Charles M. Spofford,[9] M. Am. Soc. C. E.—Mr. Atwood has raised the question of why treated timber piles were not used in the reconstruction of this pier instead of steel piles. In reply the writer wishes to state that, before deciding to use steel piles, careful estimates of cost were made for a scheme involving the use of treated timber bearing piles, but the steel-pile scheme actually adopted was found to cost about the same and was believed to be much more permanent.

The discussion by Mr. Ferguson includes additional information relative to certain features of the reconstruction not touched upon by the writer, and that by Mr. Ayer should be of interest and value to engineers preparing specifications for timber to be used in salt water.

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[9] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2049

ENERGY MASS DIAGRAMS FOR POWER STUDIES

By John W. Hackney,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Edgar E. Foster, Lyle A. Whitsit, and John W. Hackney.

Synopsis

A method of applying the mass-diagram principle to complex hydro-electric power generation systems is developed and illustrated in this paper. This application involves the computation of potential energy inflow to the system, the segregation of energy lost due to friction, leakage, lack of capacity, evaporation, etc., and the study of the storage, waste, and use of the remainder by the mass-diagram method. That part of the energy which is storable is segregated by using two diagrams concurrently, one of total energy, the other of unstorable energy only, this device being necessary to insure against considering power developed at run-of-river stations to be storable.

General Problem

The mass diagram as a "tool" for engineering analysis is familiar to the profession. In hydro-electric work it has been commonly used to compute the effect of storage on stream flow, and when used in this manner is simply the integral of the stream-flow record, or hydrograph, with respect to time. The outflow in terms of cubic feet per second, as determined from the study of this integral curve, is converted into kilowatts of power output by the use of a power-outflow diagram or a mean power constant. (This mention of the standard mass-diagram method is necessarily abbreviated. A complete discussion is presented elsewhere.[2]) This method of procedure is applicable to a single plant or to a storage plant above one or more run-of-river plants. If other storage plants are added, more complex systems develop. Hydro plants with little or no storage capacity are tied to high-head storage plants and the systems thus formed are tied into other systems on other water-sheds, so that a system may have plants of various storage characteristics, heads, and capacities using the water of several different streams.

Note.—Published in October, 1938, *Proceedings*.

[1] Asst. Hydr. Engr., Aluminum Co. of America, Pittsburgh, Pa.

[2] "Hydro-Electric Handbook," by William P. Creager and Joel D. Justin, Members, Am. Soc. C. E., p. 99.

It is desirable, if possible, to combine the power characteristics of such a system into a single diagram. This diagram should be adaptable to studies of power output under various load and water conditions, and to studies of various alterations or additions to the system. Such a diagram may be obtained by reducing the various stream-flow mass diagrams of the separate plants by the common denominator, power, to what has been termed an energy mass diagram.[3]

BASIC THEORY

The energy mass diagram measures the cumulative inflow of usable energy to the system with respect to time. When a cubic foot of water has entered a pond or a reservoir, it has a potential energy equivalent to its own weight falling from the height at which it entered the reservoir through the developed head of all plants through which it may subsequently pass. This potential energy may be stored, wasted due to lack of load or capacity, lost in friction, leakage, and evaporation, or converted into useful work. If the mean energy losses are segregated from the initial potential energy, and the remainder is plotted as an energy mass diagram, this diagram may be applied to the study of the storage, waste, and use of the available energy.

An energy mass diagram is the cumulative sum of the energy inflow (corrected for losses) to the hydro-electric system. It has the ordinary properties of a mass diagram. Being the integral of energy inflow, its slope is the rate of inflow and its ordinates are quantities of energy, or storage. In the usual hydro system, it is made up of two kinds of energy—storable and unstorable. Flow from uncontrolled parts of the water-shed represents unstorable energy and must be used as it comes. Flow from areas up stream from reservoirs may be stored to the extent that reservoir space is available. These classes of energy may be segregated by plotting not one but, two, energy mass diagrams. The first is the complete mass diagram, the cumulative sum of all energy inflow to the system. The second, an auxiliary curve, is the cumulative sum of the unstorable energy only.

The application of this basic theory is best explained by an illustrative example. The system chosen for the purpose is a simple one, consisting of one run-of-river plant and one head-water storage plant. It is believed, however, that the discussion of this limited system will reveal the salient features of more complex studies.

Hydro-electric power studies may be divided into three parts, the determination of stream flow available, the determination of load requirements, and the correlation of stream flow to load. The energy mass diagram is designed to be an aid in the study of the last-mentioned phase; therefore, the first two phases are not emphasized in the illustrative problem. This problem has been designed to indicate some of the uses of the energy mass diagram, given the stream flow and the load requirements.

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[3] See, also, the "Mass Curve of Equivalent Flow," described by Don Johnstone, Jun. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 1294–1295.

ILLUSTRATIVE PROBLEM

Fig. 1 shows the water-shed of "Typical River"[2] in plan and profile. This river has the flow characteristics of a large stream of the South. "Type Creek," one of its tributaries, drains a fairly high plateau region, and, therefore, has quite a different run-off. The average monthly flows, listed in Table 1, were computed from the actual stream-flow records of corresponding streams.

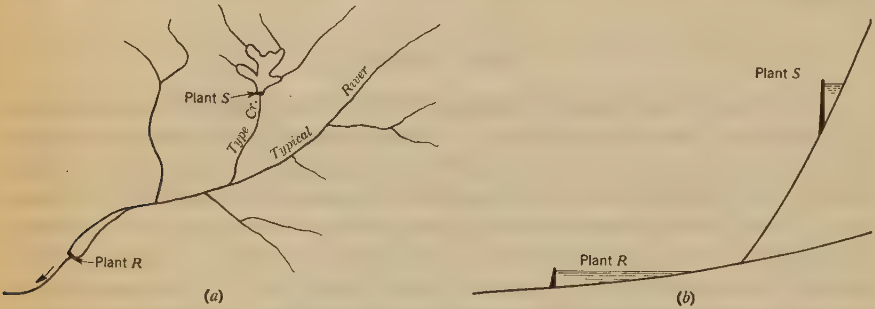

FIG. 1.—"TYPICAL" VALLEY PLANTS

TABLE 1.—AVERAGE MONTHLY FLOWS COMPUTED FROM ACTUAL STREAM-FLOW RECORDS

Month (1913)	NATURAL FLOW, IN CUBIC FEET PER SECOND*			AVAILABLE ENERGY INFLOW, IN THOUSANDS OF KILOWATTS		SUMMATIONS,† IN THOUSANDS OF KILOWATT-MONTHS	
	Main river, Plant *R*	Tributary, Plant *S*	Uncontrolled flow at Plant *R*; Column (2) minus Column (3)	From Plant *S*, acting through Plants *R* and *S*	From Plant *R*	Column (6) minus 240	Column (5) plus Column (6) minus 240
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
......	......	......	......	......	......	2 000.0‡	2 000.0‡
January	40 300	2 760	37 540	138.0	280.0	2 040.0	2 178.0
February	31 900	3 490	28 410	174.5	280.0	2 080.0	2 392.5
March	49 400	6 390	43 010	319.5	280.0	2 120.0	2 752.0
April	26 500	3 450	23 050	172.5	230.5	2 110.5	2 915.0
May	14 600	2 560	12 040	128.0	120.4	1 990.9	2 923.4
June	13 600	1 960	11 640	98.0	116.4	1 867.3	2 897.8
July	5 420	1 380	4 040	69.0	40.4	1 667.7	2 767.2
August	5 000	1 380	3 620	69.0	36.2	1 463.9	2 632.4
September	3 730	1 120	2 610	56.0	26.1	1 250.0	2 474.5
October	3 570	1 180	2 390	59.0	23.9	......	2 317.4
November	3 900	1 140	2 760	57.0	27.6	......	2 162.0
December	5 290	1 460	3 830	73.0	38.3	......	2 033.3

* Corrected for evaporation. † Summations are for the end of the month.
‡ Arbitrary initial value on January 1.

On the main stream, well below the outlet of Type Creek, is a fairly large run-of-river plant, Plant *R*, with adequate pondage, but no storage. Provision has been made here for the installation of additional units when and if they are required. Now, in the course of events, another company acquires a reservoir site on Type Creek and plans a storage project to be known as Plant *S*.

The fundamental data concerning the two plants are as follows:

Description	Plant *R*	Plant *S*
Drainage Area, in Square Miles:		
Controlled	1 000	1 000
Not controlled	9 000	0
Total	10 000	1 000
Gross head, in feet	145	620
Capacity, in thousands of kilowatts	280	190
Storage, in thousands of acre-feet (pondage only)		603

The general problem is to determine the total increase in useful power output that will be produced by operating the new storage plant in conjunction with Plant *R*.

COMPUTATION OF ILLUSTRATIVE ENERGY MASS DIAGRAM

In order to construct energy mass diagrams for the "Typical Valley" plants, it will first be necessary to estimate the losses that should be deducted from the initial quantity of potential energy. Considering the storage plant, *S* (Fig. 1), the following major losses should be taken into account: (*a*) Evaporation; (*b*) loss of head due to draw-down; (*c*) pipe-line losses; (*d*) turbine losses; and (*e*) electrical losses. These items are variable with reservoir elevation, output, and other factors too numerous to mention. It is necessary to assume values for these losses that either are constants or that will vary only with natural phenomena, such as stream flow and time of year, which are not affected by the operation of the system. For Plant *S* the following assumptions have been used:

(1) Evaporation from the reservoir is a fixed quantity for any given month of the year;

(2) Loss of head due to draw-down is a mean quantity equivalent to the head lost when four-tenths of the volume of the reservoir is drawn (in this case, 30 ft);

(3) Pipe-line losses have a mean value of 20 ft;

(4) Turbine efficiency is 90%; and,

(5) Generation efficiency is 95 per cent.

Considering the foregoing losses the power output at Plant *S* may be determined. Let P = electrical power output; e = conversion efficiency; H = mean net head; Q = stream flow (corrected for evaporation); and, k = the power constant of the plant. Then,

$$P = 0.0846\, H\, Q\, e = k\, Q \qquad (1)$$

By Equation (1), $P = 0.0846 \times (620 - 20 - 30)\, Q \times 90\% \times 95\% = 40\, Q$ kw; and $k = 40$ kw per cu ft per sec. Similar assumptions as to losses have been made for the run-of-river plant (*R*, Fig. 1): (1) Evaporation is a fixed quantity for any given month; (2) the mean loss of head due to draw-down is 1 ft; (3) the mean conduit loss is 1 ft; (4) turbine efficiency is 87%; and (5) generation efficiency is 95 per cent.

Therefore, by Equation (1): $P = 0.0846 \times (145 - 1 - 1)\, Q \times 87\% \times 95\% = 10\, Q$ kw; and $k = 10$ kw per cu ft per sec. This factor, k, applies only to normal conditions of stream flow. If the flow is such that the capacity of the turbines is reached, or the head is reduced by high tail-water, k is no longer valid. For these conditions a curve of stream flow *versus* power is developed in three steps:

(*A*) Construct a curve of head *vs* capacity from turbine and generator characteristic curves;

(*B*) Construct a curve of head *vs* total stream flow by using head-water and tail-water rating curves; and,

(*C*) Construct a curve of stream flow *vs* capacity, using the curves developed in Steps (*A*) and (*B*).

In Step (*B*), since capacity limits are being determined the total stream flow is used with the tail-water curve; but this is reduced by the approximate turbine discharge in determining the head-water elevation from the rating curve of the dam.

Using the aforementioned constants, the energy mass diagram is developed using a computation table similar to Table 1, in which Column (5) gives the available energy from the inflow to Plant *S* acting through Plants *R* and *S*; that is, Column (3) times the sum of k-values for the two plants $(40 + 10 = 50)$. Column (6) shows the available energy from the uncontrolled inflow to Plant *R* (Column (4) times $k = 10$). The effects of capacity limitations have been approximated. Column (7) is the summation for the auxiliary diagram; that is, the cumulative summation of Column (6) less the sum of the means of Column (5) and Column (6); and Column (8) is the summation for the main diagram, or the cumulative summation of Column (5) and Column (6), less the sum of the means of these columns.

Columns (7) and (8), Table 1, require some further explanation. It is convenient to plot energy mass diagrams in terms of cumulative excess over the mean, or an even value close to the mean. This produces a mass curve which moves in a generally horizontal direction and makes it possible to enlarge the vertical scale greatly without throwing the curve off the paper. It also produces more definite points of tangency and a more readable curve. For this reason, in Columns (7) and (8) the mean monthly energy inflow for the entire period of record is subtracted from the energy inflow for each particular month before it is added to the cumulative sum. This device usually makes it necessary to assign an arbitrary initial value to the cumulation in order to avoid negative quantities. The auxiliary diagram is plotted to the same mean as the main diagram in order that one slope scale may be used for both. Since the auxiliary curve is normally used only in wet seasons, it may be broken conveniently in the dry season each year to keep it from running off the paper.

COMPUTATION OF OUTPUT OF ILLUSTRATIVE SYSTEM

With the energy mass diagrams computed, the next step in the solution of the problem is to determine, by means of these diagrams, the maximum useful output that would have been produced by the two plants if they had been operated in conjunction for the period of stream-flow record. This requires a thorough-going investigation to determine the most efficient method of correlating stream flow and load. A discussion of the various methods of reservoir control and system operation is beyond the scope of this paper There appear to be as many "criteria" of operation as there are men interested in their development and use, but one and all may be tested and tried in a manner similar to that used for the trial of Typical River Operation Criterion, as outlined subsequently herein.

It is assumed that the Typical Valley plants have two types of load available: (1) The public utility load of two fairly large towns; and (2) a large electro-chemical industrial load. Continuity of service is the prime requisite of the public utility load. This power must be available at all times, and especially in the dryest years every effort must be made to protect the prime output required for this portion of the load. On the other hand, the electro-chemical industry finds it possible to use power which, although normally continuous, is nevertheless interrupted at long intervals by periods of sub-normal stream flow. Demand for the product during these dry periods will be satisfied from stock piles accumulated during the wet periods.

With such loads available for the system, it is proposed that a two-rate plan of operation be used. The higher, or normal rate, will be carried continuously except during abnormally dry years. In these dry years the prime, or utility, load only will be carried. It will be necessary, then, to develop some form of criterion that will indicate the proper times to drop and resume the secondary load. The drops from the normal rate must be timed so that the prime rates may be guaranteed through any sub-normal flow period; but, on the other hand, both drops and resumptions must be made in such a manner that the greatest possible total value of output may be obtained.

TYPICAL RIVER OPERATION CRITERION

The following criterion, although not the most efficient available, was chosen for its simplicity:

(1) There shall be designated a "normal rate" and a "prime rate" of power production;

(2) Power in excess of the designated "normal rate" shall be produced only from water which it is impossible to store; and

(3) Stored water may be used in generating power at any rate which does not exceed the normal, except when estimates show that, with the available stored water and assumed continuance of the prevailing stream flow conditions, the normal rate cannot be continued: For three months after July 1; for three months after August 1; for three months after September 1; or for one month after the first of any month.

When estimates have shown such a condition to exist, no storable water shall be used to produce power in excess of the prime rate until, at the end of a 30-day period in which the average stream flow has been greater than that necessary to produce the normal power, the reservoirs of the system have been completely filled. Then, only, shall the normal rate of power production be resumed.

Fig. 2(*a*) is the energy mass diagram for the Typical River system plotted from Table 1 for 1913 and from similar computations not included herein for the years 1914–1917. (In an actual problem the entire period of record should be considered, together with any additional information available concerning minimum flows.) The upper or main curve is the total usable energy inflow to the system, plotted in terms of excess over the mean. The lower, auxiliary, curve in Fig. 2(*a*) is the summation of unstorable energy only. It is plotted in terms of excess over the same mean. To the right of the curves is a slope diagram which is for the purpose of determining the rates of energy inflow

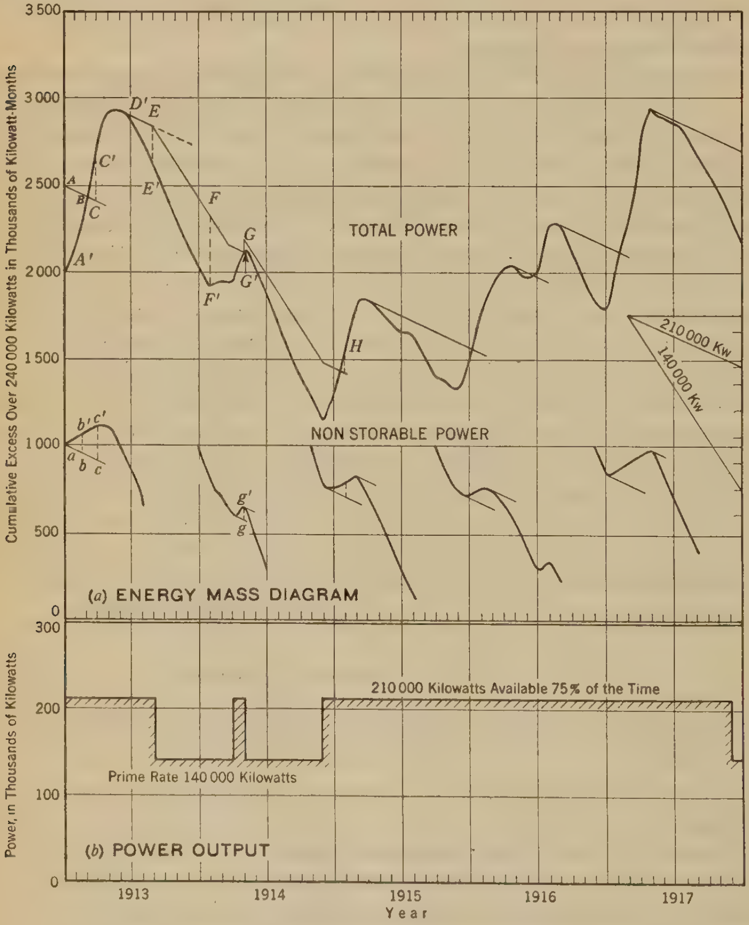

Fig. 2.—Storage Valuation for "Typical" River Plants

and use corresponding to slopes of the mass curves. The vertical scale on the extreme left is used in plotting the curves and measuring storages.

The total amount of energy storage available at Plant *S* is determined by multiplying the storage volume of the reservoir in terms of month-second-feet by the sum of the *k*-values for the stations through which this storage will act. For Plant *S* this will be 603 000 acre-ft, or 10 000 month-sec-ft, times 50 = 500 000 kw-months of storage. With this storage, a determination of the output of the system will be made, applying the operation criterion, with a normal rate of 210 000 kw and a prime rate of 140 000 kw, to the mass diagrams of Fig. 2(*a*).

Assume that on January 1, 1913, the reservoir is at its point of maximum draw-down, and the system is operating at its normal rate. Ordinate *A A′*, Fig. 2(*a*), represents the reservoir depletion (500 000 kw-months), and the slope of Line *A C* represents the normal output rate (210 000 kw). Since the slope is less than that of the mass curve, inflow exceeds outflow and water is being stored at Plant *S*. If all the energy inflow to the system were storable, Point *B* would indicate the time at which the reservoir would be filled. However, if Line *a c*, which has a slope equivalent to the normal output, is drawn on the auxiliary diagram of unstorable energy, it will be found that the unstorable flow at Plant *R* is more than sufficient to provide the requisite 210 000 kw and some of this flow must be wasted. Ordinate *b b′* indicates that at this time the unstorable energy in excess of that used in carrying the normal rate has amounted to 170 000 kw-months since the first of the year; that is, 170 000 kw-months have been wasted or used as dump power, and, therefore, the reservoir is not filled by this amount. The reservoir will be filled only when the rate line, *A C*, has fallen below the mass curve a distance equal to the Ordinate *c c′* between the auxiliary curve and the rate line, *a c*.

From Point *C′* to Point *D′* Fig. 2(*a*), the energy inflow to the system still exceeds the normal outflow and since no more water can be stored a quantity of dump power is produced. The exact amount may be most easily determined from the tabulation of power inflows, making corrections for capacity limits at both plants. From Point *D′* to Point *E* outflow exceeds inflow and the normal rate is being produced by draft from storage, the storage depletion at any particular time being the ordinate between the rate line and the mass curve. At Point *E′* it will be found that, projecting Line *D′ E* and the slope at Point *E′* for three months, the ordinate at that point would exceed the storage available. This means that, if the present stream flow were to continue for three months, the normal output rate would exhaust the available storage. According to the criterion operation, this means that power production should be reduced immediately to the prime rate, represented by the slope, *E F*. Continuing on this new rate, the maximum storage used is *F F′* —about 420 000 kw-months.

The prime rate is used until April, 1914, when the run-of-river power at Plant *R* alone is sufficient to produce the normal rate. The question then arises at *g′* whether to continue at this rate or to drop again to the prime. On the auxiliary curve two tangents are drawn at a slope representing the normal rate. One is at the point where the slope of the auxiliary curve exceeds this rate, and the other where the normal rate exceeds that of the auxiliary curve. At Point G' on the mass curve the rate line has fallen below the curve, but subtracting the energy ($g\,g'$) produced at Plant R in excess of the normal rate by laying off this ordinate above Point G' it may be seen that the reservoir is not full and, therefore, stored water should not, according to the criterion, be used for the production of power in excess of the prime rate. Therefore, the normal rate is not resumed until the first part of December, when the run-of-river power is again sufficient to produce the normal rate. Stored water may again be used in the production of the normal rate after the reservoir is filled, at Point H, Fig. 2(*a*).

In like manner the remainder of the power graph (Fig. 2(*b*)) has been developed. To illustrate the methods used, only one determination of power output has been made. For a single actual problem the writer has made thirty-seven such determinations, using various criteria and different normal and prime rates. These results were analyzed graphically in order to select the best operating method. Assuming that this has been done for the Typical River plants, and that the criterion presented herein with a normal rate of 210 000 kw and a prime rate of 140 000 kw has been selected, it is possible to determine the increase in useful power output produced by operating storage Plant S in conjunction with Plant R.

Without the storage plant, but using available pondage at Plant R, the prime output is limited to the average output of the lowest week, which occurred in October, 1913, when the output was 23 000 kw. Duration curve studies indicate that a rate of 54 000 kw was available 75% of the time. Tabulating these data and the data from Fig. 2(*b*):

Description	Thousands of kilowatts of prime power available 100% of the time	Thousands of kilowatts of power available 75% of the time
With Plant S	140	210 − 140 = 70
Without Plant S	23	54 − 23 = 29
Gain	117	41

These data represent the power gain produced by building Plant S and operating it in conjunction with Plant R. Attention should be called to the fact that the actual gain is slightly greater, since the power available 75% of the time from Plant R alone is of poor quality, being interrupted at frequent intervals, as compared to the power available from the combined plants.

CONCLUSION

The illustrative problem has indicated a few of the uses of the energy mass diagram. It is not suggested as a cure-all for power studies. It is another "tool" to be used with analytical methods, the energy draw-down diagram, the hydrograph, the duration curve, and the area-duration curve. Each of these methods is useful in its own field, and the water power engineer should at least be familiar enough with all of them to select the one best suited to his own needs.

The chief advantage of the mass diagram is the great rapidity with which power determinations can be made, once the original computations have been completed. Every method of operation can be given a fair trial, and the engineer is encouraged to originate new methods by the ease with which they can be tested.

For a strict solution, every new plant or change in head would require a new diagram. This objection may be overcome by the use of proportion. The shape of the diagram is assumed to be typical of the water-shed, the addition of plants and changing of heads being represented by changes in scale. Used in this manner, the diagram becomes available for preliminary investigations, the determination of economic development limits, and other less refined calculations.

Limitations which must be borne in mind are the use of: (1) The graphical solution; (2) mean values for losses; and (3) weekly or monthly averages. Limitation (1) may introduce some errors due to paper shrinkage, inaccuracies in slope measurements, etc. However, the graphics may be checked analytically if necessary, using the summation columns for this purpose. Limitation (2) is a necessary feature of the method and may require special investigation of critical periods, such as the weeks of minimum stream flow. With judicious selection of these mean values, this feature will cause little trouble. The errors from Limitation (3) are common to all methods and have been thoroughly discussed elsewhere.[2]

In this presentation of the energy mass diagram no pretense is made of covering all the uses of the method. Ingenious minds will discover other devices and technical improvements which will alter it to fit their particular problems. The energy mass diagram has evolved from the ordinary mass diagram, and it may be hoped that even better methods of power computation will grow from the energy mass diagram method as herein set forth.

ACKNOWLEDGMENTS

The writer wishes to acknowledge his indebtedness to James P. Growdon, Robert F. Ewald, and Harold A. Thomas, Members Am. Soc. C. E., and also to the great number of other engineers whose ideas and methods have been consciously or unconsciously incorporated in this paper.

DISCUSSION

EDGAR E. FOSTER,[4] ASSOC. M. AM. SOC. C. E. (by letter).—The usefulness of the energy mass diagram for solving problems of power supply is well presented in this paper. As the author states, the idea of the energy mass diagram is derived by analogy from the mass flow diagram or curve; but, by substituting rates of energy or power for the stream flow the utility of the diagram is extended considerably. It may then be applied to any form of power generation, steam and pumped storage, as well as to hydro-electric power.

The basis of the extended utility may be easily inferred from a simple equation expressing the mass flow diagram, as follows:

$$B = \int_0^t Q \, dt \quad \ldots\ldots (2)$$

in which B is the volume of water obtained at a given point in time, t, and Q is the rate of flow. For Q and B, similar quantities of other elements may be substituted. As the author has done in effect, E may be used for the total output of energy from a power system and e for the rate. Then,

$$E = \int_0^t e \, dt \quad \ldots\ldots (3)$$

Integrating, $E = e\,t$, from which $e = \frac{E}{t}$, which is the average rate over the time, t. Expressing the equation in another form,

$$\frac{dE}{dt} = e \quad \ldots\ldots (4)$$

that is, the tangent at any point equals the rate.

The writer used this type of diagram several years ago in a study to calculate the maximum prime power in a co-ordinated system of several reservoirs and run-of-river plants. Although the answer sought was only an approximation, the quantities and graph will serve very well to describe the process.

Thirteen proposed and existing plants were included in the system. There were five reservoirs with a total capacity of 75.4 billion cu ft of useful storage. On the assumption that the uppermost reservoir would be drawn down first and the procedure repeated for each successive reservoir down stream, and that the water was utilized at full head on the plants below, the energy equivalent of the storage was 0.843×10^9 kw-hr.

From previous study it was known that the critical period of the stream flow was that from January 1, 1930, to December 31, 1931. The problem, therefore, consisted in the determination of the maximum rate of prime power obtainable through that period with the given system of power plants.

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The energy mass curve used in solving this problem is shown in Fig. 3 as Line *O A E* for the period from January 1, 1930, to December 31, 1931. Following its construction, a part of the same curve was plotted parallel to itself at a distance of 0.843 billion kw-hr, as shown at Point *E′*. The entire curve was not plotted parallel because, as will readily be seen, only the critical parts need be repeated. A tangent, *A E′*, was then drawn from Point *A* on the main curve (at which point the reservoirs were assumed to be full) to Point *E′* on the upper segment. This completed the construction of curves.

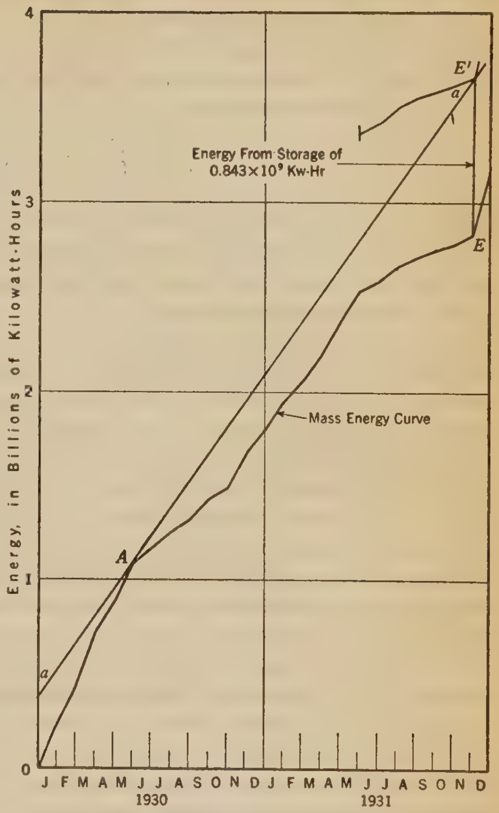

FIG. 3.—COMMON TANGENT ON PARALLEL MASS DIAGRAMS

The slope of this tangent, *A E′*, is the maximum prime power obtainable through that period of record. Point *E′* is the sum of the total power from natural run-off plus the regulated flow from storage. At Point *E′* all the reservoirs are empty and the maximum energy, therefore, has been obtained for the period. The time of *E* and *E′* is identical. Then,

$$E' = e' t \ldots\ldots\ldots (5)$$

in which e' is the rate of output from the time of Point *A* to the time of Point *E′*, or,

$$e' = \frac{E'}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

which equals the maximum uniform rate of prime power. With Equation (6), e' is found from the slope of Tangent *A E′*. From Point *a* to Point *a′*, the time is approximately 25.2 months, or 766 hr, and the total energy output is 3.6×10^9 kw-hr; or $e = \frac{3.6 \times 10^9}{766} = 197\,000$ kw, which is the maximum rate of prime power.

The author has constructed his energy mass diagram by using the differences between the monthly output of power and the mean for the period. The construction of the mass diagram from residuals does have an advantage (as the author has stated) in bringing into sharper prominence the critical points of a mass curve. The usual depressions caused by deficiencies of natural stream flow are filled in to some extent by the energy equivalent of storage, when a mass curve of energy is constructed from the output of a system of power plants, including reservoirs.

This method of construction, however, does not change the fundamental principles. If $\bar{e}$ is the mean rate for the period, the mass diagram may be represented by the equation,

$$E = \int_0^t e\,dt - \int_0^t \bar{e}\,dt \dots\dots\dots\dots\dots\dots\dots (7)$$

Since this is true, the mass curve of residuals (or only the critical segments) can be plotted above the parallel to itself at a distance denoting the energy equivalent of the storage, and a common tangent can then be drawn to determine the maximum prime power.

By constructing two complete curves, at a distance representing the energy equivalent of stored water, the studies of operation can be readily conducted by means of the tangents. Any rate of power output can be introduced by simply changing the slope of the tangent. All that is necessary is that the tangent stay within the parallel mass curves.

Referring now to Fig. 2, it appears to the writer that the auxiliary diagram could be eliminated by simply limiting, in the calculations, the power output of Plant *R* to the normal rate—namely, 210 000 kw. By crediting Plant *R* with power below this rate instead of 280 000 kw as a maximum, the "dump" power or waste would not be included in the upper diagram and, therefore, no correction would be necessary as at Point *G*.

Errors of this method of power analysis are not serious enough to be objectionable. In the event that the errors arising from monthly means are too great, shorter periods of time, as a week or even a day, can be taken. The critical periods are relatively short and excessive work is not entailed by the use of shorter increments of time through such periods to refine the calculations as much as is consistent with the data. The errors of the process can be reduced by this means to limits within the errors of the original stream-flow and power-load data.

LYLE A. WHITSIT,[5] M. AM. SOC. C. E. (by letter).—That a mass curve of energy may be used to analyze a power supply in a manner similar to the use of a mass curve of flow to analyze a water supply, has been demonstrated in this paper. When the water supply is first determined by a mass curve, the result is then combined with the net head for power determination. If the power-generating system consists of a series of reservoirs and power houses on one stream and its tributaries, the use of water is in a concatenated sequence. The water availabilities at one plant depend upon the use of water from the reservoirs and in the plants up stream. The analysis by means of a mass curve of stream flow becomes complicated and cumbersome to the *n*th degree. As a result, the writer, like the author, proposed[6] the use of a mass curve of energy in 1913. However, even the use of the mass curve of energy is an unsatisfactory method.

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[5] Cons. Hydr. Engr., Ebasco Services, Inc., International Div., New York, N. Y.

[6] *Engineering News*, September 11, 1913, p. 496.

In the investigations and reports prepared since his earlier work, the writer has never found the use of the mass curve of energy advisable because of its complicated character and the precautions, adjustments, and corrections that are required. Graphs accompanying reports must necessarily be easily read, and their determination must be understandable to executives and other readers of a report. For this reason the writer believes that the use of duration curves of flow, or hydrographs, in combination with duration curves of load, are preferable.

Analysis by several duration curves or power graphs can be incorporated into one diagram. It is a method used in other analysis, such as the superposition of the hourly output of two or more generation stations, to produce a combined daily load curve of a power system. Such graphs can be made to show clearly the source of stored energy for later use, such as night storage for day-time or week-time peak-load use in the analysis of resources possessing pondage; or such as storage during high-water period for any rate of use during low-water periods.

At present (1939) there are very few power companies with generating systems made up entirely of hydro prime movers. Practically all have a complement of steam or other thermal generated capacity. In fact, it is very seldom that a hydro-electric project by itself can compete with a steam-electric project by itself. The proper position of hydro capacity, especially when it is equipped with storage regulation, is primarily one furnishing capacity, large amount of kilowatts for a small consumption of energy. Energy can be produced usually at much less cost by steam.

The economical arrangement results in the bulk of energy being produced from thermal plants because of lower cost; but hydro plants with pondage and storage can compete effectively in the production of capacity, that is, kilowatts of power in the peak of the load at a small energy consumption. Quite often an additional installation may be provided in such hydro plants to serve as the standby capacity for the system. When this portion of the installation is serving as standby for the other hydro capacity it is doing so at no extra consumption of energy, but when serving as standby for a base thermal capacity its energy use is at the detriment of the hydro peak carrying capacity. Perhaps the economical arrangement is to split the standby provision into hydro for the hydro load capacity and steam for the steam portion.

For this reason other forms of graph analysis, such as the duration curve or hydrograph, are more convenient and more commonly used in preliminary and final steps of the determination of a "load-carrying capacity" of a proposed project. In the determination of the load-carrying capacity, the plant output with unregulated water supply up stream from reservoirs is allocated to base load. The plant output of an unregulated water supply close to an up-stream storage plant is essentially a part of the storage plant and is allocated to peak load. The output of a plant with an unregulated water supply below a storage plant but with its own large tributary drainage area is probably allocated to base load.

JOHN W. HACKNEY,[7] JUN. AM. SOC. C. E. (by letter).—Mr. Foster suggests the use of two complete mass diagrams at a distance representing the energy equivalent of stored water. The writer believes that this is an excellent device for making the necessary operations more understandable, especially when the computer is inexperienced or when the computation must be explained to persons unfamiliar with the method. After using mass diagrams for a time and becoming familiar with them, this parallel curve will be found to be unnecessary.

Mr. Foster's suggestion that the auxiliary diagram may be eliminated by limiting the power output of Plant *R* to the normal rate is rarely practicable, since the normal rate is usually one of the unknowns of the problem.

Mr. Whitsit favors the "use of duration curves of flow, or hydrographs, in combination with duration curves of load," especially for graphs accompanying reports to executives and other non-technical readers. The writer agrees that mass curves are difficult to explain to such readers. However, duration curves are quite as difficult to explain and are often subject to misinterpretation. The best solution appears to be to solve the problem by duration curves, mass diagrams, or whatever other method is best suited, and then report the results, stripped of all non-essentials, in the manner most easily understood.

The writer also agrees with Mr. Whitsit that the energy mass diagram is not particularly well adapted to a study of the distribution of load between steam and hydro plants in a system operated at low load factor, or other problems of a similar nature. On the other hand, the energy mass diagram is well adapted to the solution of problems concerning storage in a system composed largely of hydro plants. As stated in the paper, the energy mass diagram is advanced, not as a method of solution for every problem, but as a method which the writer has found to be often useful and sometimes indispensable. To the person familiar with its use, the energy mass diagram is not complicated; on the contrary, it presents a picture of the inflow and outflow of power not easily grasped in a duration curve or hydrograph. As for the "precautions, adjustments and corrections" required, most of these are involved in the use of other methods and certainly all of them apply to the duration curve method.

The writer wishes to thank Mr. Foster for his explanation of the energy mass diagram from the viewpoint of the calculus involved.

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Paper No. 2050

MECHANICAL STRUCTURAL ANALYSIS BY THE MOMENT INDICATOR

By Arthur C. Ruge,[1] Assoc. M. Am. Soc. C. E.,
and Ernst O. Schmidt,[2] Esq.

With Discussion by Messrs. John B. Wilbur, William J. Eney, Otto Gottschalk, and Arthur C. Ruge, and Ernst O. Schmidt.

Synopsis

A new method for the mechanical analysis of models of indeterminate structures is described. This method involves the use of a simple and inexpensive instrument known as the "moment indicator." A suitable technique for the application of the method to models made of semi-plastic materials like celluloid is described, and the results of measurements on a Vierendeel structure are given. Other applications are suggested.

The writers conclude that the technique discussed is capable of giving results accurate to 1%, and that an accuracy of 2% or 3% is easily obtained with considerable economy of time over other available methods of stress determination. It is suggested that the technique is suitable for use in practical engineering offices as a design tool.

Introduction

The purpose of the paper is to describe a simple and direct method of determining the bending moments in models of structural frames due to external loads. The method is particularly well adapted to the study of structures having a high degree of statical indeterminancy, whether the loading is static or dynamic in character.

For statical load conditions, the moment indicator is a simple and inexpensive instrument, capable of yielding very accurate results. The technique is of such simplicity and speed that it could well be adapted for routine design work in engineering offices. Less manipulative skill is required than for other available methods of mechanical analysis capable of giving comparable accuracy.

As to the merits of the model method for the analysis of complex structural problems, very little needs to be said beyond making reference to the excellent

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Note.—Published in October, 1938, *Proceedings*.

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[2] Asst., Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

paper by R. L. Templin, M. Am. Soc. C. E., published in 1937.[3] Several effective techniques have been developed for determining the stresses in structural models, the outstanding one being the deformeter method introduced by George E. Beggs, M. Am. Soc. C. E.[4]

No attempt is made herein to compare one model method with another, since each has its own particular applications for which it is especially suited. Indeed, the efficiency of any method is too closely associated with the skill and preferences of the investigator to make general comparisons possible. In so far as correctness of results is concerned, there is little choice between methods known to be based upon sound principles of mechanics.

After giving attention to the principle of the moment indicator, the writers confine themselves primarily to a description of its application to the particular case of celluloid models of two-dimensional indeterminate frameworks. The behavior of celluloid as a model material is discussed, and an adequate technique for dealing with it and with similar semi-plastic materials is described. Results of measurements and calculations for a four-panel Vierendeel structure are given in sufficient detail to establish the probable accuracy of the moment-indicator method. Other applications of the method are discussed briefly.

PRINCIPLE OF THE MOMENT INDICATOR

Consider a straight bar, subjected to bending moments and shear, and having no external loads, on a uniform section of length, L (see Fig. 1(a)). The moment indicator consists of a set of rigid levers attached at the ends of this section. The levers are provided with indices at one-third and two-thirds the length, L.

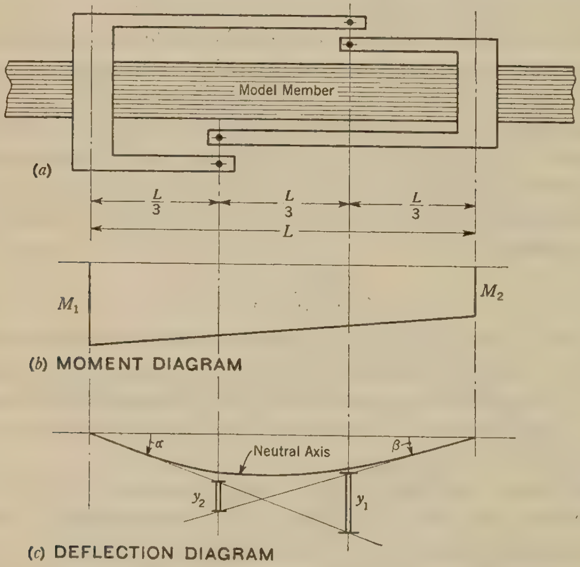

FIG. 1.—DIAGRAM ILLUSTRATING PRINCIPLE OF MOMENT INDICATOR

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[3] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1211.

[4] *Loc. cit.*, Vol. 88 (1925), p. 1208.

In Fig. 1(*c*), Deflections y_1 and y_2 represent the relative motions of the indices due to the applied bending moments of Fig. 1(*b*). The relation of y to M is easily found: $\alpha = \frac{M_1 L}{3 E I} + \frac{M_2 L}{6 E I}$, and $\beta = \frac{M_1 L}{6 E I} + \frac{M_2 L}{3 E I}$. Therefore:

$$y_1 = \frac{2}{3} L \alpha - \frac{1}{3} L \beta = \frac{M_1 L^2}{6 E I} \dots\dots (1)$$

The measurement of y_1 and y_2 thus gives the bending moments directly:

$$M_1 = \frac{6 E I}{L^2} y_1 \dots\dots (2a)$$

and,

$$M_2 = \frac{6 E I}{L^2} y_2 \dots\dots (2b)$$

At either point, motion of the indices toward each other corresponds to a positive bending moment. (In Fig. 1, M_1, M_2, α, β, y_1, and y_2, are shown positive.)

Equations (2) hold only if the shear deflections can be neglected. In short and deep bars the influence of shear must be taken into account, the correction to y being deduced as follows: The shear in the section equals,

$$s_s = \frac{M_1 - M_2}{L} \dots\dots (3)$$

The contribution of shear deflection to indicator reading is,

$$\Delta y_1 = - \Delta y_2 = \frac{s_s}{G A'} \dots\dots (4)$$

in which G is the shear modulus of the material, and A' is the effective area of cross-section in shear. Since Δy will be a small percentage of y (rarely more than 5%), it is sufficient to introduce the approximate values of M from Equations (2) into Equation (3); and to substitute for G, its value, $\frac{E}{2\left(1 + \frac{1}{m}\right)}$, in which m = Poisson's number. Then, Equation (4) gives:

$$\Delta y_1 = \frac{12 I}{L^2 A'} \left(1 + \frac{1}{m}\right) (y_1 - y_2) \dots\dots (5)$$

In the case of a bar of rectangular cross-section having a depth, d, $A' = \frac{A}{1.2}$ (approximately). Furthermore, for celluloid (and many other model materials), $\frac{1}{m} = 0.4$. There result, then, the corrections:

$$\Delta y_1 = 1.68 \left(\frac{d}{L}\right)^2 (y_1 - y_2) \dots\dots (6a)$$

and,

$$\Delta y_2 = 1.68 \left(\frac{d}{L}\right)^2 (y_2 - y_1) \ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

Therefore, the true bending moments are:

$$M_1 = \frac{6\,E\,I}{L^2}(y_1 - \Delta y_1) \ldots\ldots\ldots\ldots\ldots\ldots (7a)$$

and,

$$M_2 = \frac{6\,E\,I}{L^2}(y_2 - \Delta y_2) \ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

In many practical design problems, the correction represents an unnecessary refinement. It is so easily calculated, however, that it may as well be taken into account.

By obvious extension of the foregoing analysis, the effect of loads applied within Section L can be included. Curved bars and bars of non-uniform cross-section can likewise be treated by determining the appropriate slope-deflection coefficients, either experimentally or by calculation. In nearly all haunched-beam problems the moment indicator can be used on the straight uniform section between haunches. In certain special cases the indices may be placed profitably at points other than the third points, but the arrangement shown in Fig. 1 will be found most satisfactory for general use.

DESCRIPTION OF THE INSTRUMENT

In the practical design of the indicator, it is advantageous to deviate from the arrangement shown schematically in Fig. 1 to the extent of placing all the indicating levers on the same side of the bar. Fig. 2 illustrates a construction that was found very convenient and satisfactory for problems involving static loads. Except as shown, the parts are of aluminum.

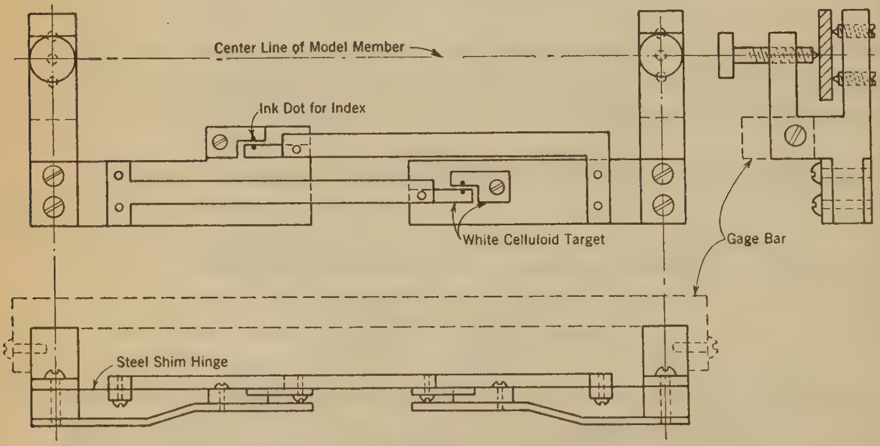


FIG. 2.—DETAIL OF MOMENT INDICATOR

The indicator is fixed on the model bar by two clamps (see Fig. 3), each provided with three sharply pointed screws. This clamping arrangement has the important advantage that no restraint is imposed upon the elastic deflections of the bar. The long levers are attached to the clamps by means of thin spring hinges, their free ends sliding on an extension of the short levers in order to keep the indices always in the same focal plane for microscope reading. It will be seen that the levers are placed so that the indices are approximately in the middle plane of the model bar; thus, twisting of the bar will not produce error in the reading.

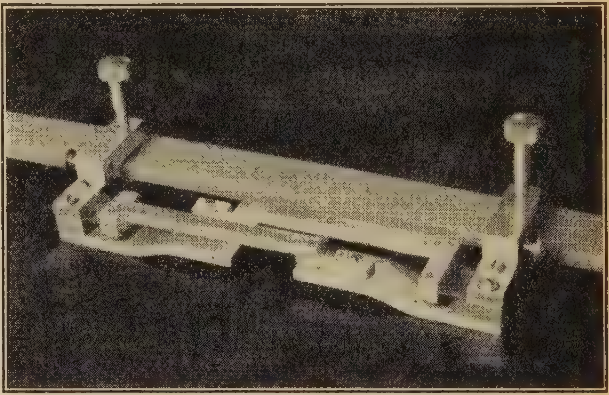


Fig. 3.—Moment Indicator Mounted on Bar: Gage Removed

The indices are small India ink dots on white celluloid. A gage bar, shown dotted in Fig. 2, conveniently sets the exact positions of the clamps and indices, the gage being removed after the clamp screws are tightened.

It will be evident to the reader that many interesting variations in design are possible. For example, one instrument was built with a gage length variable from 4 to 12 in. Furthermore, the sensitivity may be increased ten or more times by means of a magnifying lever, or by the use of measuring devices more sensitive than the ordinary measuring microscope. Extension of the principle to three-dimensional frameworks does not appear to be difficult, but has not yet been attempted.

Application to Celluloid Models

Celluloid is particularly well adapted to structural model work. It is isotropic and has a low modulus of elasticity which is sensibly constant up to fairly high stresses. It is very easy to work with either hand tools or machine tools, and it can be cemented perfectly with acetone, in which it dissolves.

Questions of similarity to prototype will not be discussed in this paper. It will only be observed that a celluloid model can be built to represent a given indeterminate structural framework with greater fidelity than a purely analytical treatment can represent it. Even when rectangular model sections are used to represent the members of a rigid frame structure, it is probable that the model will behave more nearly like the prototype than the hypothetical structure used as a basis of calculation.

Physical Properties of Celluloid

Samples of celluloid of different quality, age, and manufacture show marked differences in their mechanical properties. Moduli of elasticity ranging from

175 000 to 375 000 lb per sq in. will be encountered among commercial brands. Over the limits of any one sheet, however, the material is quite uniform. The following data may be considered as typical but cannot be applied quantitatively to any given celluloid. They were taken on samples of green celluloid, $\frac{3}{16}$ in. thick, known as "toothbrush stock."

For obtaining the data given herein, cantilever beams about 6 in. long and $\frac{3}{4}$ in. wide were used. The beams were tested at constant temperature in a water bath, ample time being given to allow the entire piece to come to temperature for each test.

The most striking peculiarity of the material is its creeping under load. After loading, the deflection continues to increase perceptibly for a considerable time. This fact makes it difficult to work with loaded celluloid models by conventional means. Fortunately, however, the creep-rate of celluloid is found to be independent of the magnitude of the load applied, within wide limits of stress. Stated in another way, the modulus of elasticity of celluloid is a function of time but is independent of stress up to the proportional limit. Were it not for this fact, semi-plastic materials like celluloid could not be used for accurate quantitative model studies by any known method.

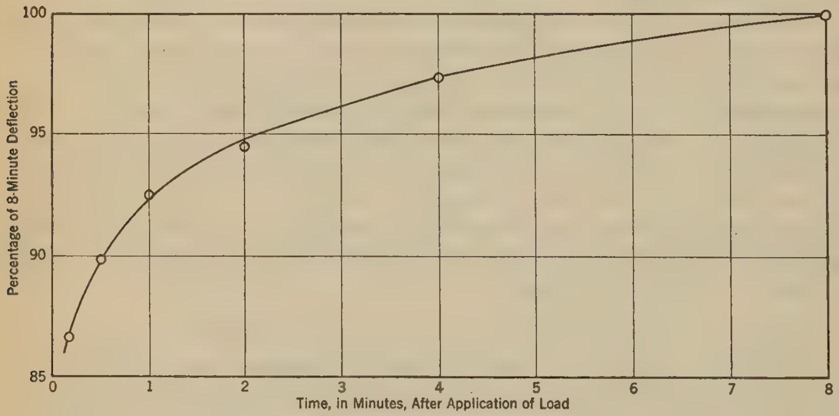


FIG. 4.—TYPICAL CREEP CURVE FOR CELLULOID (TEST TEMPERATURE, — 71° F)

A typical creep curve for stresses below the proportional limit is shown in Fig. 4. The data are plotted in terms of the percentage of the 8-min deflection reached at various times up to 8 min. Even after 8 min, the deflection had not reached a maximum. The curve was found to be the same for all stresses below the proportional limit, both for loading and for unloading. The proportional limit occurs at a stress of about 500 lb per sq in. for this material. At 1 000 lb per sq in., the secant modulus of elasticity is about 10% lower.

The modulus of elasticity is very sensitive to changes of temperature. Fig. 5 shows the behavior of the material under varying temperatures. The curve was taken for increasing temperature; after the material has been heated to somewhat more than 100° F, the curve cannot be reproduced. The sensitivity of the modulus at room temperature is nearly 1% per degree F. The coefficient of linear expansion was found to be 6.5×10^{-5} per degree F; that is, about ten times that of steel.

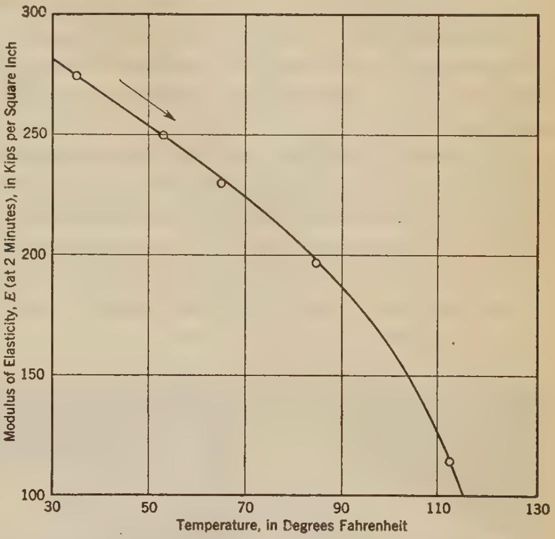


Fig. 5.—Typical Variation of E with Temperature

These data indicate the difficulties that have to be overcome in dealing with celluloid as a model material if results of high accuracy are to be obtained. Room temperature variations from day to day will often give considerable variation in modulus. In addition, temperature variations produce model movements due to direct expansion and contraction of the material. The effects of creep may be allowed for to some extent by timing readings by stopwatch, but this procedure is awkward and time-consuming. The effect of aging may sometimes be of importance.

The technique developed by the writers for overcoming the aforementioned difficulties is described herein (see heading "Test Technique with the Moment Indicator"). It was found that by using ordinary loading methods and exceptional care in timing, the average accuracy obtainable was about 10%, due to temperature and creep effects. The technique described is capable of giving results accurate to within 1%, in addition to being more convenient and more economical of time.

Test Technique with the Moment Indicator

Since the principal difficulties with celluloid as a model material have to do with its uncertain modulus of elasticity, the writers chose a test technique that does not depend upon the knowledge of that quality. The method used consists of imposing a constant deflection at the load point, instead of imposing a constant load. A celluloid model thus loaded is free from creep, since the load applied to it follows precisely the time variation of the modulus of elasticity.

The constant-deflection method, by avoiding difficulties with creep, makes the taking of readings on the indicator an easy matter. Of course, the indicator readings taken at various points on the structure give only the relative magnitudes of the moments. The absolute values are easily determined by the application of simple statics. Thus, it is not necessary to know the value of the modulus of the celluloid; nor is it necessary to know the magnitude of the deflection imposed upon the model, both quantities disappearing from the problem entirely.

The model to be studied is fixed upon the table in such a way as to simulate the support conditions of the prototype. Fig. 6 shows the arrangement used for the Vierendeel truss discussed in this paper. A fixed hinge support is simulated at the left end; the right end is supported by the equivalent of a hinge on rollers. The model members are supported on steel balls rolling on glass plates.

The deflector is shown attached at the first panel point to the right of the left support. It is a simple stiff lever hinged at the left end, connecting to the load point by a thin steel strap. Two nails driven into the table at the free end of the deflector are used for fixing the magnitude of the deflection imposed. A counterweight, opposing the deflector, allows the deflection to be imposed first upward and then downward, thus making the net deflection twice as large as could be imposed in one direction.

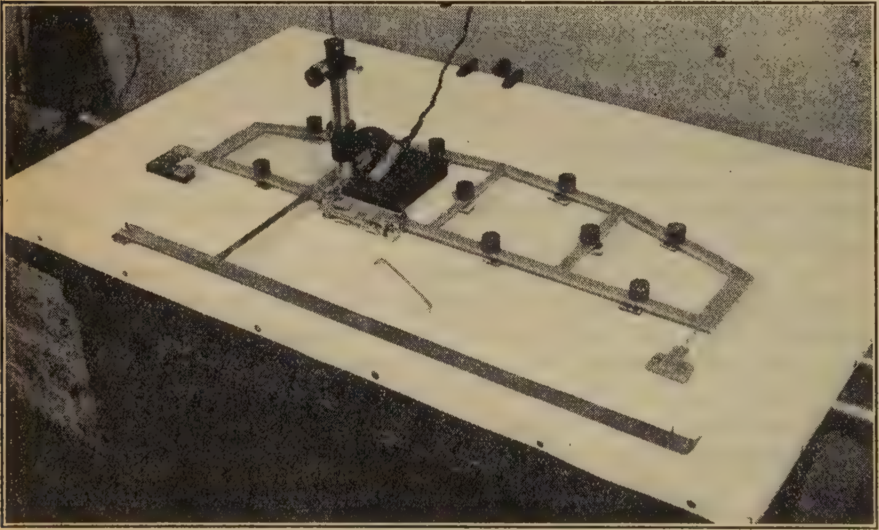


Fig. 6.—Vierendeel Truss Model with Deflector and Indicator in Position

In Fig. 6 the moment indicator is shown attached to a bottom chord member, and the microscope is in position for observing. The microscope is provided with a small spotlight which illuminates its field. This feature avoids the necessity of using a large lamp, the heat of which can cause annoying model movements.

It will be obvious that a number of indicator readings equal to the degree of statical indeterminancy, N, plus one is sufficient for the absolute stress determination throughout the structure. In the studies described in this paper readings were taken on all members of the structure on which it was possible to mount the indicator; that is, all except the end verticals, which were too short to accommodate the instrument being used. The extra readings were taken for the purpose of investigating the fidelity of the method.

The procedure is as follows: With the indicator fixed on any member, microscope readings of the relative index movements are made at both indices. Thus, the relative bending moments at the two ends of the gage length are determined from one instrument setting. From these two moments the moment distribution over the entire member is known at once. It was found very convenient to plot the readings (after being reduced by Equations (7)), directly on a line diagram of the truss, positive and negative moments being plotted above and below the line. The moment diagram is then constructed by drawing straight lines through the plotted points. The only unknown, E, is eliminated by applying simple statics to a section cut anywhere through the structure.

In order to secure the maximum accuracy, many more measurements were taken than would have been made in practice. Five successive measurements of the distance between index points were taken, with deflection applied alternately upward and downward. The indicator was read in two positions on each bar, with the levers first on one side and then the other. Such refinements are of value only in the investigation of the method itself, or when it is desired to work to accuracies greater than 1 per cent.

STRESSES IN VIERENDEEL TRUSS, OBSERVED AND CALCULATED

The dimensions of the Vierendeel model discussed in this paper are given in Fig. 7, and the relative moments of inertia of the members are indicated. The model was made of $\frac{3}{16}$-in., green celluloid. Butt-joints (cemented by means of acetone) were used for connecting the members, which were milled to the proper depth, since it was desired to eliminate the small errors always present in hand work. (Incidentally, it was found that when two or more pieces of the same depth were to be made, the pieces could be milled faster than they could be finished by hand, and the accuracy, of course, was much greater.)

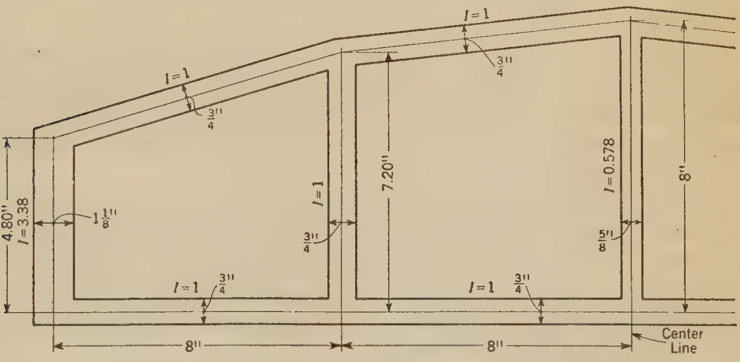
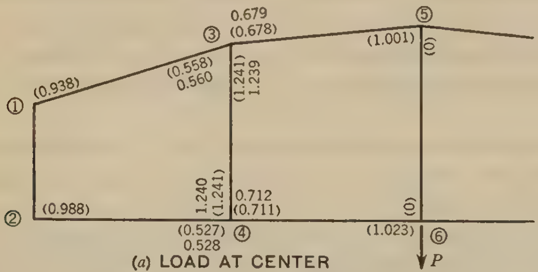
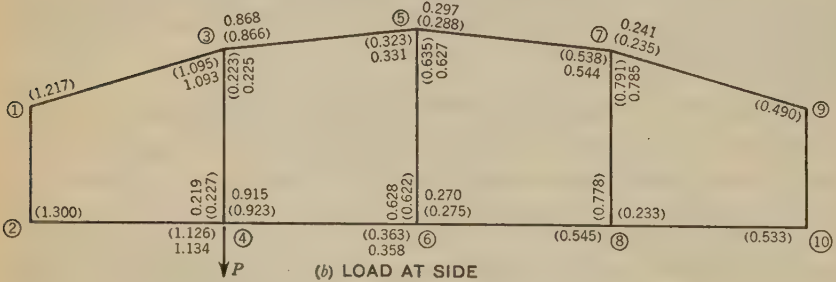


Fig. 7.—Model Dimensions

In Figs. 8(*a*) and 8(*b*) all results are given in terms of the bending moments at the joints, in inch-pounds, due to a 1-lb load applied as indicated. The observed moments due, respectively, to a load at the center and at the side panel, are shown. The values in parentheses are the joint moments obtained by averaging all readings. The others are the joint moments as adjusted to make the summation of moments balance to zero at each joint. The joint closures are good, the maximum correction applied to produce balance at any joint being less than 1% of the largest moment in the structure.


0.679
③
(0.678)
⑤
(1.001)
(0)
(0.558)
0.560
(1.241)
1.239
(0.938)
①
1.240
(1.241)
0.712
(0.711)
(0)
②
(0.988)
(0.527)
④
0.528
(1.023)
⑥
P

(*a*) LOAD AT CENTER


0.868
③
(0.866)
⑤
0.297
(0.288)
⑦
0.241
(0.235)
(0.323)
0.331
(0.538)
0.544
(1.095)
1.093
(0.223)
0.225
(0.635)
0.627
(0.791)
0.785
(1.217)
①
(0.490)
⑨
0.219
(0.227)
0.915
(0.923)
0.628
(0.622)
0.270
(0.275)
(0.778)
(0.233)
②
(1.300)
(1.126)
1.134
④
(0.363)
0.358
⑥
(0.545)
⑧
(0.533)
⑩
P

(*b*) LOAD AT SIDE

FIG. 8.—OBSERVED AND BALANCED MOMENTS

Perhaps it should be mentioned that both the indicator and the deflector used in these tests were of home-made quality, such as can be achieved by any one who is reasonably handy with tools. Aside from the measuring microscope (reading to ± 0.0001 in.), the equipment needed for the indicator method can be made at negligible cost.

The symmetry of the model was found to be so good that only averages are given in Fig. 8, since the differences due to non-symmetry were of the same order as the differences between the individual readings. In a model made entirely by hand work, such symmetry would be highly unlikely, but for practical work, a hand-made model will be sufficiently accurate if reasonable care is exercised in finishing the members.

Upon comparing the results obtained by using only the first reading taken at any point with the result obtained by using the average of all readings taken there, it was found that the maximum difference amounted to 3.5%, with an average difference of less than 2 per cent. Thus, it may be concluded that one or, at most, two readings at each point will give sufficient accuracy for all practical work. The greatest errors due to using only the first readings are much less than the greatest errors in the theoretically calculated stresses, which are shown in Table 1.

The theoretical bending moments (see Columns (3), Table 1) were calculated with the usual assumptions for such structures. Member lengths were taken to be from center to center of joints. The effects of direct stress and shear were considered, but were found to have a negligible effect upon the results. The two sets of six simultaneous equations involved in the calculation were computed on the "simultaneous equation machine"[5] which eliminates most of the labor usually encountered in obtaining sufficiently accurate results.

TABLE 1.—Comparison of Measured and Computed Bending Moments (in Inch-Pounds) Due to a Load of One Pound

(*a*) Load at the Center				(*b*) Load at the Panel to the Left of the Center							
Point* (1)	Measured (2)	Computed (3)	Percentage, error (4)	Point* (1)	Measured (2)	Computed (3)	Percentage, error (4)	Point* (1)	Measured (2)	Computed (3)	Percentage, error (4)
1	0.938	0.967	+3.1	1	1.217	1.247	+2.5	6- 4	0.358	0.332	−7.3
2	0.988	1.044	+5.6	2	1.300	1.286	−1.1	6- 5	0.628	0.599	−4.6
3-1	0.560	0.489	−12.7	3-1	1.093	1.088	−0.5	6- 8	0.270	0.267	−1.1
3-4	1.239	1.189	−4.0	3-4	0.225	0.194	−13.8	7- 5	0.544	0.560	+2.9
3-5	0.679	0.700	+3.1	3-5	0.868	0.894	+3.0	7- 8	0.785	0.778	−0.9
4-2	0.528	0.506	−4.2	4-2	1.134	1.114	−1.8	7- 9	0.241	0.218	−9.5
4-3	1.240	1.212	−2.3	4-3	0.219	0.200	−8.7	8- 6	0.545	0.556	+2.0
4-6	0.712	0.706	−0.8	4-6	0.915	0.914	−0.1	8- 7	0.778	0.779	+0.1
5-3	1.001	1.007	+0.6	5-3	0.331	0.326	−1.5	8-10	0.233	0.223	−4.3
6-4	1.023	1.004	−1.9	5-6	0.627	0.596	−4.9	9	0.490	0.523	+6.7
....	....	....	....	5-7	0.297	0.270	−9.1	10	0.533	0.517	−3.0

* See Fig. 8.

The percentage differences between calculated and measured moments are designated as "error" (Columns (4), Table 1). On the basis of the demonstrated accuracy of the moment-indicator method, these values may properly be regarded as a good measure of the errors in the theoretical results for the structure studied. The truss discussed is fairly ideal from the standpoint of theoretical analysis; it was not the intention of the writers to select a structure showing large differences between actual and calculated stresses.

Further Considerations

One marked advantage of the method described lies in the fact that the stresses are measured by a device wholly external to the structure. In dynamical models such an approach is almost indispensable.

By arranging the instrument for photographic recording of the relative index motions, the indicator (or "moment recorder" as it is then called) has been used for studying the bending stresses in a model of a building frame vibrating as a result of earthquake motions. In problems of this nature it is clearly not practical to determine the bending stresses from records of the deflections of the model, on account of the tremendous labor involved when the model has several degrees of freedom.

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[5] "The Mechanical Solution of Simultaneous Equations," by John B. Wilbur, Assoc. M. Am. Soc. C. E., *Journal*, Franklin Inst., Vol. 222, No. 6, December, 1936.

Perhaps the most valuable application of the method may be to the study of the so-called secondary stresses in trusses. In certain types of trusses, it is well known that the conventional analytical secondary stress determination leads to serious errors, and a truly accurate analysis requires such a mass of work as to make its application practically impossible. With the sensitivity of the indicator increased several-fold, it should be ideal for measuring secondary stresses in models.

The indicator may also be found useful in certain full-sized tests. With two sticks clamped to a beam or girder, the bending moments acting could be measured accurately with less effort than by the customary strain-gage measurements. A gage length of several feet gives large deflections, the deflection increasing with the square of the gage length, as seen from Equations (2).

For routine model work, it would be desirable to have several moment indicators, in order to effect a further saving of time by avoiding moving the instrument from member to member.

In some instances, it may be desired to ascertain the absolute magnitude of the bending moment at a point in the model, without having to measure the $N + 1$ moments previously mentioned. This can be accomplished through the simple expedient of a "spring balance" made of the same material as the model, as shown diagrammatically in Fig. 9.

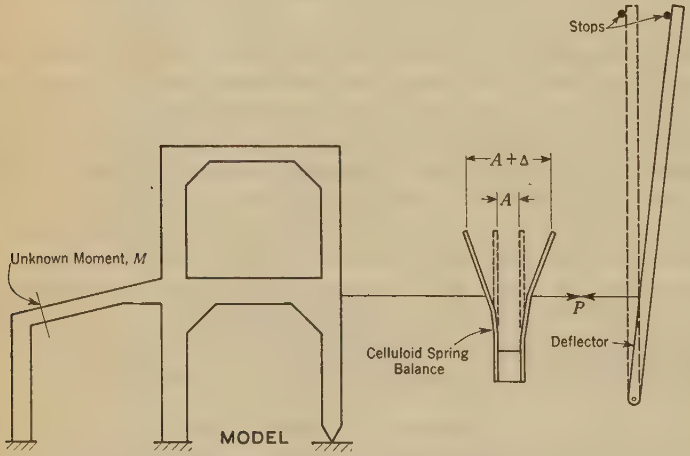

Fig. 9.—Direct Elimination of Modulus of Elasticity, E

The "spring balance" is made of two bars so supported as to be statically determinate. Any other calculable or calibratable system will serve the purpose equally well. The purpose of the balance is to weigh the load imposed upon the model in terms of the modulus of elasticity of the material.

Let P be the load applied due to the deflection imposed by the deflector, and let Δ be the resulting relative motion of the indices of the balance. Then, if the moment indicator reading for the unknown moment, M, is y, the following relations may be written:

$$M = K_1 E y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8a)$$

and,

$$P = K_2 E \Delta \qquad \text{(8}b\text{)}$$

in which K_1 and K_2 are known constants. Consequently,

$$M = \left(\frac{K_1}{K_2} \times \frac{y}{\Delta}\right) P \qquad (9)$$

Thus, the unknown, E, which is a function of time and temperature, is made to disappear directly.

CONCLUSIONS

The moment indicator offers a simple and inexpensive means of analyzing indeterminate structural models. The technique involved is so easy and straightforward that a minimum of skill is required. Results accurate to 2% or 3% are obtainable with an average time consumption of less than 2 min for each moment measured. By taking a few more measurements than are required, the method is made self-checking.

The writers believe that the method described in this paper is sufficiently practical to commend itself for use in engineering offices as a design tool. Any one who can read a measuring microscope can become proficient in the use of the device within an hour. The construction of celluloid models of most frames met in practice is a simple matter, requiring few tools and little skill. An office assistant under the guidance of an engineer could be entrusted with all the detail model work and observations.

DISCUSSION

JOHN B. WILBUR,[6] ASSOC. M. AM. SOC. C. E. (by letter).—By the development of the moment indicator, the authors have not only made an important contribution to the available instruments for model analysis, but have opened the way for further development along similar lines. In the past, many different types of strain-gages have been developed for the measurement of linear strains, but the measurement of angular strains has not received as much attention. The moment indicator measures angular strains in such a manner that bending moments are easily obtained.

After describing the application of the moment indicator to the determination of moments in a Vierendeel truss model, the authors discuss several possible future developments of the instrument (see heading, "Further Considerations"), stating that a "valuable application of the method may be to the study of the so-called secondary stresses in trusses." They declare also that " * * * in some instances, it may be desired to ascertain the absolute magnitude of the bending moment at a point in the model, without having to measure the $N+1$ moments previously mentioned," and suggest that "this can be accomplished through the simple expedient of a 'spring balance' made of the same material as the model."

Subsequent to the presentation of the paper, further research under the direction of the writer has led to an investigation of these suggested applications. Since this investigation was based directly on the tentative suggestions made in the paper, a synopsis of the results obtained should be of interest.

The primary purpose of the later investigations was to determine, by means of the moment indicator and celluloid models, secondary bending moments in trusses with rigid joints. Three basic problems were present: First, it was necessary to devise a method of attaching the moment indicator to model members so that its readings would not be influenced by shear distortions, because in truss models used for this purpose it is often necessary to use I-beams or their equivalent for members, the shear deflections of which are rather large, and impossible to determine accurately; second, it was necessary to magnify the readings of the moment indicator by a lever system, in order to insure sufficient precision of the microscope readings; and third, it was necessary to build and use a celluloid "spring balance," as illustrated in Fig. 9. Although the theory of the spring balance had been developed previously by Mr. Schmidt, it had not been tested by actual use. Equations of statics which are usually available for the conversion of relative values of moments (as measured by the moment indicator without the use of the spring balance) to actual values of moments due to the applied loads, are not present in dealing with the determination of secondary moments in trusses, so that it was essential that the spring balance be used.

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[6] Associate Prof., Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

An investigation by Mr. Johann Meier[7] showed that, for a rectangular beam, a transverse section which is plane before bending will, if not adjacent to points where external loads are applied to the beam, deform due to shear so that it takes the shape of a cubic parabola. Mr. Meier not only reached this conclusion from analytical investigations, but obtained an excellent experimental check working with a celluloid model. Near points of applied loads, however, longitudinal linear strains are affected by shear distortions, and the resulting variation from linear distribution in stresses normal to transverse sections causes shear distributions that are no longer parabolic, and shear distortions of transverse sections that are no longer in the shape of a cubic parabola. Because of the effect of Poisson's ratio, transverse linear stresses due to applied forces also affect strains normal to transverse sections, and, in turn, shearing distortions. Because of the uncertainty as to the shape which transverse sections will take as a result of shear distortion under varying circumstances, it was concluded that corrections to moment indicators, to allow for shear distortion effect, could at best be only approximate if the arms of the indicator were each attached to two points on a transverse section. Under no circumstances can the shear corrections be computed and proper corrections applied, when the moment indicator is attached to a member within a distance from the member support equal to the width of the member. When shear distortion is small, the errors due to this source are likely to be negligible, but in model studies for secondary moments in trusses, this may not be the case. It was believed highly desirable, therefore, to devise a method of attaching the moment indicator arms to members so that readings would not be influenced by shear.

After an extended investigation, it was shown by a model study that this could be accomplished for a moment indicator of the same dimensions as that described by the authors, by attaching an arm of the moment indicator to a member at two points 0.25 in. apart, both lying in the neutral axis of the member. Attached in this manner, it was found that readings were not only uninfluenced by shear distortions, but that small errors due to other resulting sources were negligible. These negligible errors result from two causes: (1) The partial restriction of linear strains along the neutral axis between the two points of attachment; and (2) the fact that the arm of the indicator now is parallel to the chord joining the two points of connection to the elastic curve of the member, rather than tangent at a point half-way between these two points. The moment indicator designed for tests for secondary moments was built with this means of attachment, and it is believed that future indicators of this type should adopt this improvement. Simpler procedure as well as more accurate results may be obtained.

The rate of change of slope in members of a truss is not as large as it is in members of rigid frames without diagonals, so that a more sensitive moment indicator is required than would otherwise be the case. For this reason, a special moment indicator was built, which differed in two respects from that described in the paper. A lever system was inserted in the indicator which magnified the readings five times as compared to those which would otherwise have been obtained. Due to the presence of this lever system, it was desirable to omit the feature of the moment indicator which enables one to determine moments at both sections of attachment. In order to simplify the construction of the instrument, provision was made to determine moments at only one end of the indicator for a given setting on a member.

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[7] "An Experimental Check of the Theory of Shear Deformation and Shear Deflection," by Johann H. Meier. Presented in partial fulfillment of the requirements for the degree of Master of Science, at Mass. Inst. Tech., Cambridge, Mass., 1938.

The practicability of the celluloid spring balance was first tested by measuring moments in a rigid frame without diagonals, using a celluloid spring balance in the line of loading. From a series of tests conducted in this manner, it was demonstrated that the idea could be used successfully, and that the accuracy of the moments thus obtained was comparable to that resulting from converting relative values of moments obtained without the spring balance, into actual values of moments, by the application of equations of statics. In the tests on celluloid trusses which followed, it was necessary to design the celluloid spring balances carefully, so as to make certain that they would be flexible enough to permit accurate readings, but at the same time strong enough so that they would not be over-stressed.

In a rigid frame without diagonals, where the loads are carried largely in bending, errors in model construction due to slight eccentricities in the members meeting at joints are not important, because the moments due to these eccentricities are small in comparison to the moments in the members meeting at the joint. Under these circumstances the moments in the ends of the members meeting at a given joint, as determined by the moment indicator, will be found to be nearly in static equilibrium. In trussed structures, however, eccentricity moments due to unavoidable, slight errors in model construction are relatively large in comparison with measured secondary moments. For this reason the moments measured at any joint may fail to be in static equilibrium by an appreciable amount. Therefore, a correction to the measured moments is necessary. This may be accomplished by distributing the unbalanced moments, as measured by the moment indicator, by the method of moment distribution, and combining the results of this analysis with the previously measured moments. An alternate procedure might be followed on the model by applying moments at different joints, successively, and measuring the resulting moments set up in the members of the model, thus obtaining data to be used as a basis for corrections due to the eccentricity of members.

In comparing secondary moments as determined by model studies with analytical values for corresponding moments, an excellent agreement was found, with the exception of isolated cases in which, due to improper model design, the secondary moments were so large that the celluloid of the model was apparently stressed beyond its elastic limit. At such points, measured moments were appreciably lower than computed values.

It is believed that model studies of secondary stresses with the moment indicator are thoroughly practical, and that they offer an excellent approach to further investigations in the field of secondary stresses.

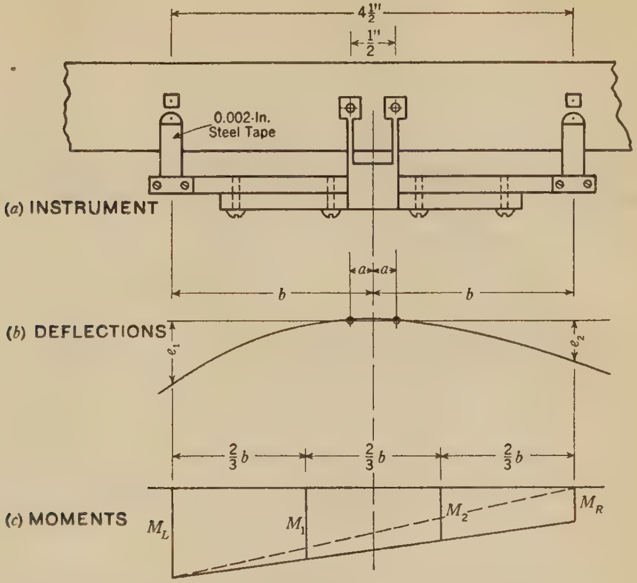

FIG. 10

It is of interest to consider types of moment indicators that differ in principle from that described in the paper. Perhaps the most direct method of determining bending moments in models is to measure the change of slope between two sections by means of two arms, normal to the member, each rigidly attached at points equidistant from the section where moment is to be determined. The measurement of the relative movements of points on the ends of the arms makes it possible to compute the bending moment by the moment-area theorem. This method was probably first devised by Professor Beggs, and has been used successfully under a variety of conditions. For use in celluloid models, however, it has been found that moment indicators, similar to that described by the authors, offer certain advantages, due mainly to simpler technique of application, and freedom from direct stress indication superimposed upon the moment indication.

Several other types of moment indicators which differ in principle from that of the paper were designed in the Structural Analysis Laboratory in 1937. One of these designs, which will be referred to as Type II, holds considerable promise, and was actually constructed and tested. The theory of this type of moment indicator may be explained by reference to Fig. 10. In Fig. 10(*a*), e_1 and e_2 are the deflections of the ends of the instrument, measured relative to the chord through the two points of attachment to the beam, which are near the center of the instrument.

Deflections e_1 and e_2, in terms of the moments at the ends of the instrument, can easily be calculated, using, for example, the method of virtual work. They are:

$$e_1 = \frac{b^2 - a^2}{6\,E\,I}\,(2\,M_L + M_R) \qquad \text{(10a)}$$

and,

$$e_2 = \frac{b^2 - a^2}{6\,E\,I}\,(2\,M_R + M_L) \qquad \text{(10b)}$$

From the moment diagrams (Fig. 10(*c*)) it follows immediately, for the moments at one-third and two-thirds of the length of the instrument, that:

$$M_1 = \tfrac{2}{3}\,M_L + \tfrac{1}{3}\,M_R = \tfrac{1}{3}\,(2\,M_L + M_R) \qquad \text{(11a)}$$

and,

$$M_2 = \tfrac{2}{3}\,M_R + \tfrac{1}{3}\,M_L = \tfrac{1}{3}\,(2\,M_R + M_L) \qquad \text{(11b)}$$

Introducing these expressions in Equations (10):

$$e_1 = \frac{b^2 - a^2}{2\,E\,I}\,M_1 \qquad \text{(12a)}$$

and,

$$e_2 = \frac{b^2 - a^2}{2\,E\,I}\,M_2 \qquad \text{(12b)}$$

from which,

$$M_1 = \frac{2\,E\,I}{b^2 - a^2}\,e_1 \qquad \text{(13a)}$$

and,

$$M_2 = \frac{2\,E\,I}{b^2 - a^2}\,e_2 \qquad \text{(13b)}$$

Therefore, readings at the ends of the instrument are proportional to the moments at the one-third point and at the two-thirds point of its length.

Type II has not been as thoroughly investigated as the instrument described by the authors, but it is believed that it will give about the same accuracy of results. Its readings are not affected by shear distortions, and the labor involved in using the instrument may be somewhat less. Clamping the instrument to the center of a member rather than nearer its ends may be advantageous because of the effect of uncertain stress distribution in members near joints.

Type II is not mentioned for the purpose of comparing it with the moment indicator in the paper, but rather to illustrate the wide variety of instruments of this type that may be devised, and in this manner to bear out the thought that Professor Ruge and Mr. Schmidt have opened the way for important developments.

WILLIAM J. ENEY,[8] ESQ. (by letter).—In developing the moment indicator, the authors have made a very worth while contribution to the art of model analysis. As with all fundamental things, it is simple; and, as stated by the authors, it has many possibilities.

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[8] Asst. Prof., Civ. Eng., Lehigh Univ., Bethlehem, Pa.

A structure composed of members with constant cross-section was wisely selected to serve as a demonstration of the method. The authors show that if the moment indicator is attached to a member of variable section, the appropriate slope deflection coefficients must be determined, either experimentally or by computations, before the moments are known. Since a large number of structures include variable section or curved members, the writer wishes to demonstrate a simple experimental procedure for treating such cases.

For determining these coefficients, use may be made of the apparatus shown in Fig. 11. This was originally designed to serve both in the determination of the elastic constants for the Cross moment-distribution method and as a deformeter gage for mechanical stress analysis. It is more elaborate than necessary for the purpose at hand. A large-scale model is made of that part of the frame model on which the moment indicator readings are taken and is attached to the gages, which are adjustable along a fixed aligning bar. End moments are applied by measured longitudinal pulls on the gage arms until the rotations of the arms are of the proper value. The effect of shear is automatically taken into account so that the true moments are obtained. In detail, the logic and procedure are as follows: In Fig. 12(*a*), the moment indicator is shown attached to a member with variable cross-section. Due to external loads on the structure, acting outside the gage length, the member deflects and the relative motions of the indices at the third points, y_1 and y_2, Fig. 12(*d*), are measured. The rotations of the elastic tangents at Points *A* and *B* relative to the chord, *A B*, shown as Angles α and β on Fig. 12(*d*), are:

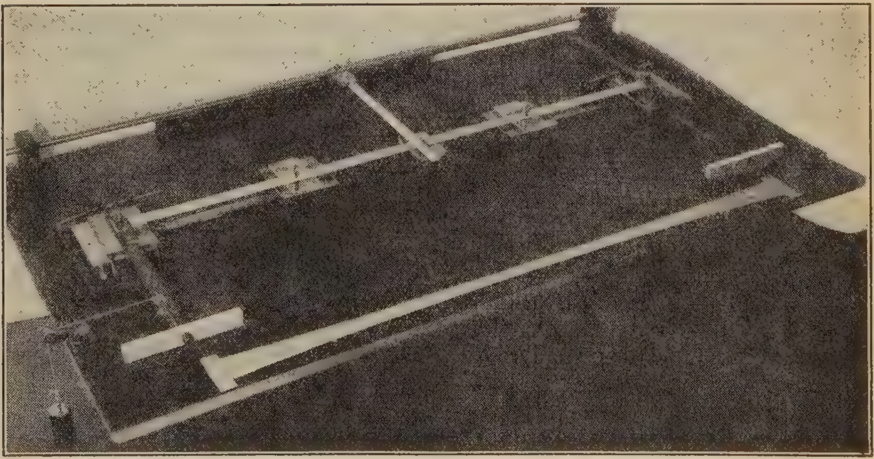

FIG. 11.—APPARATUS USED TO TEST MOMENT MODEL

$$\alpha = \frac{y_4}{L} = \frac{2\,y_1 + y_2}{L} \qquad (14a)$$

and,

$$\beta = \frac{y_3}{L} = \frac{2\,y_2 + y_1}{L} \qquad (14b)$$

The distances, y_3 and y_4, are relatively small, compared with the gage length, L, and, therefore, the angle, in radians, and its tangent can be taken as equal.

The problem, then, is to determine what moments applied at Points A and B (Fig. 12(a)) of the frame model produce end rotations of α and β, respectively. It is not convenient to ascertain these values by applying moments to the frame model, nor to remove this section for testing and, later, to replace it by cementing with acetone. With such a small model, the application of the moments to produce the exact end rotations, α and β, would be tedious, and precision would be difficult to obtain. However, if this section of the frame model is reproduced to a larger scale such that it is 30 in. or 36 in. long, the end rotations can then be greatly increased, and the testing is a simple and practical matter.

Assume that this second model, which may be designated as the moment model, is proportioned so that the width scale is the same as in the frame model; but that the longitudinal scale is increased so that m-units of length in the moment model represent one unit of length in the frame model. The moment and frame models are cut from the same sheet material and, therefore, have the same modulus of elasticity. The $\frac{M}{I}$-diagrams in Fig. 12(c), for the two models subjected to numerically equal end moments, M_1 and M_2, would be similar in every respect since the moment of inertia at corresponding sections is equal.

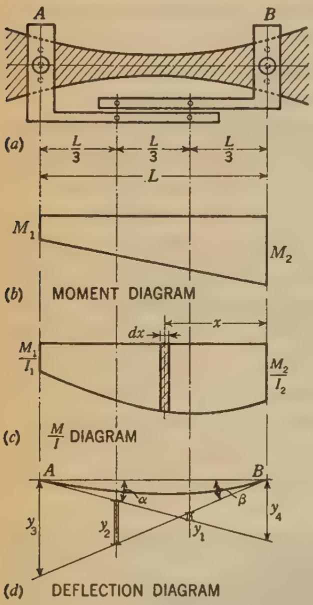

FIG. 12.—DIAGRAM ILLUSTRATING PRINCIPLE OF MOMENT INDICATOR.

The end rotation, α, for the frame model is the moment of the area under the $\frac{M}{I}$-diagram about Point B, divided by $E\,L$; that is,

$$\alpha = \frac{\int_B^A \frac{M}{I}\,dx\,x}{E\,L} \qquad (15a)$$

Likewise, the rotation, α', for the moment model for the same end moments, M_1 and M_2, is:

$$\alpha' = \frac{\int_B^A \frac{M}{I}\,(m\,dx)\,(m\,x)}{E\,(L\,m)} \qquad (15b)$$

or the end rotation, α', of the moment model is m times larger than the rotation, α, of the frame model.

Sometimes it is desirable to use a different width scale for the moment model. For instance, if the moment of inertia at corresponding sections is represented by a width of 0.500 in. in the frame model and 0.250 in. in the moment model, the moment of inertia of the frame model is then eight times larger than that of the moment model at corresponding sections; hence, from Equations (15), $\alpha' = 8\, m\, \alpha$.

The precision can be increased if larger rotations are induced in the moment model; for instance, if α' and β' are both doubled, the frame moments, M_1 and M_2, will be one-half the moments applied to the moment model. The following example will further illustrate the method: The moment indicator of 4-in. gage length is attached to a member with a straight axis (see Fig. 12(*a*)). The width of the member varies but is symmetrical within the gage length. The relative moments, y_1 and y_2, of the indices at the third points (Fig. 12(*d*)) are zero and 0.0050 in., respectively. By Equations (14), the rotations, α and β, of the frame model are: $\alpha = \frac{0 + 0.0050}{4} = 0.00125$ radian; and, $\beta = \frac{2(0.0050) + 0}{4} = 0.00250$ radian.

The moment model is made 32 in. long, or eight times longer than the gage length of the frame model, and the width scale is unchanged. The corresponding end rotations to be induced in the model moment are: $\alpha' = m\, \alpha = 8 \times 0.00125 = 0.010$ radian; and, $\beta' = m\, \beta = 8 \times 0.00250 = 0.020$ radian. To increase the precision, rotations of $\alpha' = 0.020$ radian and $\beta' = 0.040$ radian will be used so that the frame moments will be one-half those of the moment model.

The moment model is rigidly clamped to the gage arms, as shown in Fig. 11. These arms pivot about a pin or ball-bearing and carry a pointer 10 in. from the pivot. The rotation is thereby magnified so that it is easily read on a scale of 100 divisions per in. The normal readings of both scales for the unstressed position of the moment model are recorded (see Fig. 13(*a*)) and then with thumb-screws the left arm at Point A is rotated clockwise 0.020 radian so that its pointer deflects 0.200 in. The right arm at Point B rotates freely 0.0135 radian (see Fig. 13(*b*)). A longitudinal pull must now be applied to the right arm to increase the counter-clockwise rotation, β', to 0.040 radian.

Instead of adjusting the pull until this exact rotation is secured, longitudinal pulls of a 1.54-lb weight acting 8 in. from the pivot, first on the pointer end of the right arm and then on the opposite end of the right arm, were applied. The right arm rotated 0.0235 radian counter-clockwise and then 0.0230 radian clockwise. (Fig. 13(*c*)), or an average rotation per inch-pound of moment of $0.0465 \div 24.62 = 0.00189$ radian. Therefore, to increase the rotation of the right arm from 0.0135 to 0.040 radian a counter-clockwise moment of 14.00 in-lb is required (Fig. 13(*f*)). Consequently, a counter-clockwise moment of 7.00 in-lb acts on the frame model at the right end of the gage length.

The process is now reversed. The right arm at Point B is rotated 0.040 radian with thumb-screws and the left arm at Point A freely rotates 0.0278 radian (Fig. 13(*d*)). A longitudinal pull must now be applied so as to reduce the left arm rotation from 0.0278 radian to 0.0200 radian. As before, a longitudinal pull of 12.32 in-lb of moment acting first clockwise and then counter-clockwise causes a rotation of 0.0460 radian, or 0.0187 radian per in-lb of moment (Fig. 13(*e*)). Then, by proportion, a counter-clockwise moment of 4.17 in-lb decreases the rotation to 0.020 radian (Fig. 13(*f*)). The corresponding moment, M_1, on the left end of the frame-model gage length is 2.08 in-lb, counter-clockwise.

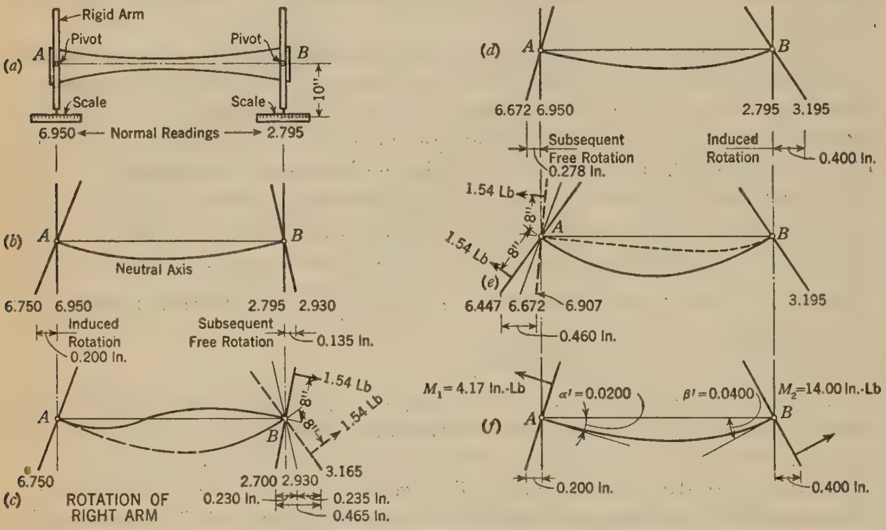
Rigid Arm
Pivot
Pivot
(a)
A
B
10"
Scale
Scale
6.950
Normal Readings
2.795
(b)
A
B
Neutral Axis
6.750
6.950
2.795
2.930
Induced Rotation 0.200 In.
Subsequent Free Rotation
0.135 In.
A
B
1.54 Lb
8"
8"
1.54 Lb
6.750
(c)
ROTATION OF RIGHT ARM
2.700
2.930
3.165
0.230 In.
0.235 In.
0.465 In.
(d)
A
B
6.672
6.950
2.795
3.195
Subsequent Free Rotation 0.278 In.
Induced Rotation
0.400 In.
1.54 Lb
8"
8"
1.54 Lb
A
B
(e)
6.447
6.672
6.907
3.195
0.460 In.
M_1 = 4.17 In.-Lb
α' = 0.0200
β' = 0.0400
M_2 = 14.00 In.-Lb
(f)
A
B
0.200 In.
0.400 In.

FIG. 13

The entire procedure is quite similar to the experimental determination of the elastic constants described elsewhere by the writer.[9] Note that no attempt is made to adjust the moments until the exact rotations, α′ and β′, are secured. In this way the tedious balancing of the weights simultaneously on each arm is eliminated. If the moment model has a curved axis the procedure followed in the foregoing example must be modified because the rotations due to the longitudinal pulls are dependent upon the kind of stress set up in the moment model.

The pivoting of the moment model so that in effect it is a hinged beam is a departure from perfect similitude. However, the writer has found that the model acts as if it were simply supported, and the end rotations are proportional to the longitudinal pulls over a large range. If one is concerned with friction about the pivot or in the ball-bearing pulleys used for the longitudinal pulls, a constant-section moment model could be tested to calibrate the apparatus for all time.

By an obvious extension of the foregoing method, the moments can be determined for the case of loads acting within the gage length of the moment indicator. Some engineers will prefer the calculation of the moments, M_1 and M_2, to this simple experimental test. A summation process treating the diagrams as trapezoids, similar to the method proposed by George E. Large,[10] Assoc. M. Am. Soc. C. E., will expedite this work.

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[9] "Fixed End Moments by Cardboard Models," by W. J. Eney, *Engineering News-Record*, December 12, 1935.

OTTO GOTTSCHALK,[11] ESQ. (by letter).—As its name implies, the moment indicator permits the abstract analysis of stresses, but, without proper and direct reference to actual structures. In dealing with abstract stresses, the same as with abstract mathematics, the investigator frequently derives beautiful and harmonious results, which then require mechanical interpretation. The moment indicator is such a device, and the author's optimism is understandable. However, the writer feels a certain skepticism regarding a few aspects of the problem. In 1937,[12] the writer emphasized the inconsistency and confusion inherent in the practice of analyzing structures mathematically, in terms of abstract stresses, as if those stresses, themselves, were part of the structure. Unfortunately, this fundamental fallacy is being extended to mechanical analysis with equally fallacious results.

In mechanical stress analysis, the model generally is loaded and the bending moments are computed from the measured rotation of the tangent to the deflection curve. In mechanical structural analysis (as distinct from stress analysis), the model is subjected to a unit displacement or rotation at the section being studied. The deformation of the model results in a physical graph of the influence line on a scale, such that the ordinates can be read at the places where the loads are acting. The latter is the correct approach and is the basis of the Beggs deformeter gage mentioned by the authors.[4]

The authors have developed valuable formulas for correcting errors in determining tangents. Rigid exactness of individual tangents, however, depends so much upon perfect homogeneity that the slightest microscopic imperfection in the material or the device clamped upon it will produce errors which are magnified still more by levers and light rays. The use of metallic devices and a relatively soft model material (the true homogeneity of which can be only surmised) would certainly require a degree of precision control that cannot be maintained outside of permanent and special laboratories.

On the other hand, ordinates can be made relatively large by mechanical devices, and quite in accord with abstract theories. They may be considered as the sums of individual deformation, in which local errors or variations are counterbalanced. Perhaps that is the mechanical explanation for the fact that tangent methods of analysis have failed to compete with mechanical methods involving ordinate reading. Although it certainly seems to be a favorable characteristic of the moment indicator that it does not involve the reading of tangent changes, but merely the relation between such changes, this in itself does not guarantee that errors in observing two tangents, relatively, may not be cumulated. The usefulness of any instrument for mechanical analysis is impaired if the instrument requires microscopic reading. This fact in itself tends to take the instrument out of the hands of the analyst and place it into the care of some subordinate or assistant. In the analysis of a special structure of some magnitude it might be justifiable to spend the time cutting and investigating five or ten separate models in order to compare the consistency of results; but for daily routine, it will generally be considered an unjustified burden on the designing staff. The statement in the paper to the effect that the moment indicator requires less manipulative skill than other available methods of mechanical analysis, capable of giving comparable accuracy, is a sweeping one to which much can be said in refutation.

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[10] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 101.

[11] Buenos Aires, Argentine Republic.

[12] "Structural Analysis Based Upon Principles Pertaining to Unloaded Models," by Otto Gottschalk, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1019.

For the past one hundred years or more, the entire general theory of statics has been submerged in a veil of mystery by a mass of artificial, intricate mathematical functions of stress. The effect has been to obscure the simple geometrical relation that can be observed by eye. Similarly, in mechanical analysis, the present tendency is to solve structural problems by microscopic examinations which can be visualized macroscopically, more easily and more efficiently. To demonstrate his point, the writer must refer to a device first described in 1926.[13] Similar devices for studying the effect of vertical loads,[14] and for wind stresses,[15] have been described by the writer subsequently. These devices have demonstrated how the theory of statics may be simplified fundamentally by a geometrical interpretation of the visible natural behavior of the given structure.[12] A third fundamental application of the splines and metal clamps recommended by the writer is to the analysis of the structural element with which the designer deals every day. The basic principles underlying the writer's views on this subject have been demonstrated a number of times. The fundamental operations for a structural analysis may be demonstrated briefly by reference to Fig. 14, which is a continuous beam of constant cross-section. With the load shown in Fig. 14(*a*), only a single unit displacement is required to supply the influence line of the reaction R_c in Fig. 14(*b*) and the shear effect, F_E, shown in Fig. 14(*c*). To the left of Point *A*, in Fig. 14(*a*), a load P_1 is acting on a cantilever for which the influence line is the tangent of the curve at Point *A*. When Beams *A B C D* in Fig. 14(*a*) are fixed at Point *A*, the influence line is found by simply lifting the extended end beyond Point *A*, until the tangent at Point *A* is horizontal, as shown by the dotted line in Fig. 14(*a*). Fig. 14(*d*) demonstrates how the influence line for the bending M_F, at Point *F*, is obtained by joining two similar splines by a standard moment clamp and running these lines through the points of support until the apex of one clamp rests below Point *F*. Without using any special degree of precision, and independent of local conditions, the observed result of this analysis is well within 3% of those found by theoretical computations. The same degree of accuracy will be obtained for reactions and shears (Figs. 14(*b*) and 14(*c*)) by applying two transverse displacements equal to one-half unit, without moving the splines horizontally; that is, the support at Point *A*, or at Points *A* and *B*, respectively, is first lifted and then lowered through one-half unit displacement. It should be evident that results obtained in this manner will require less time than would be necessary even to adjust microscopes. When the section of a beam varies in adjacent spans, splines may be added or the spans of the model may be reduced accordingly.

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[13] *Journal*, Franklin Inst., July, 1926, p. 61, and February, 1929, p. 245.

[14] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), Fig. 30, p. 1067.

[15] *Proceedings*, Am. Soc. C. E., December, 1938, Fig. 5, p. 2030.

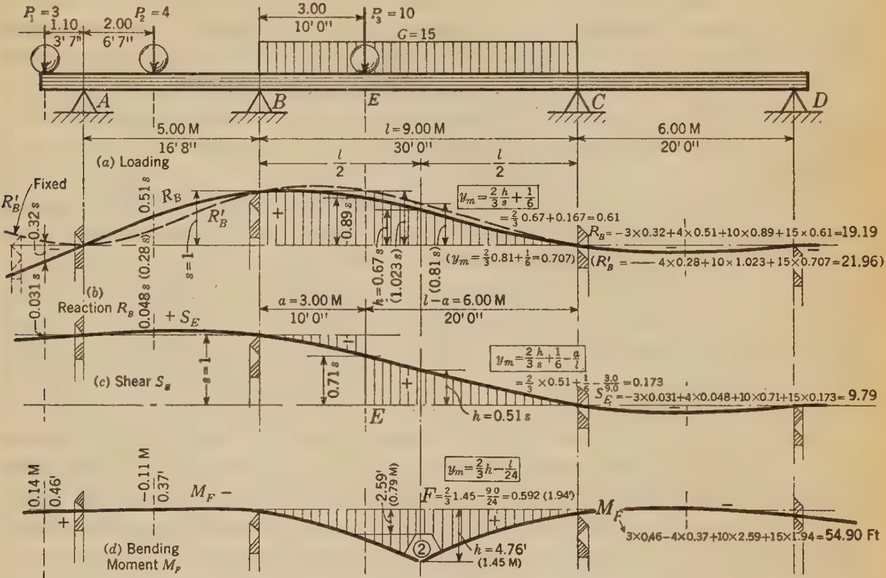
$P_1=3$
1.10
3' 7"
2.00
6' 7"
$P_2=4$
3.00
10' 0"
$P_3=10$
G = 15
A
B
E
C
D
5.00 M
16' 8"
l = 9.00 M
30' 0"
6.00 M
20' 0"
(a) Loading
$\frac{l}{2}$
$\frac{l}{2}$
Fixed
R'_B
R_B
R'_B
0.32 s
0.51 s
0.89 s
0.031 s
0.048 s (0.28 s)
s = 1
(b) Reaction R_B
h = 0.67 s
(1.023 s)
(0.81 s)
$y_m=\frac{2}{3}\frac{h}{s}+\frac{1}{6}$
$=\frac{2}{3}0.67+0.167=0.61$
$R_B=-3\times0.32+4\times0.51+10\times0.89+15\times0.61=19.19$
$(y_m=\frac{2}{3}0.81+\frac{1}{6}=0.707)$
$(R'_B=$ —— $4\times0.28+10\times1.023+15\times0.707=21.96)$
a = 3.00 M
10' 0"
l − a = 6.00 M
20' 0"
$+S_E$
(c) Shear S_E
s = 1
0.71 s
E
h = 0.51 s
$y_m=\frac{2}{3}\frac{h}{s}+\frac{1}{6}-\frac{a}{l}$
$=\frac{2}{3}\times0.51+\frac{1}{6}-\frac{3.0}{9.0}=0.173$
$S_E=-3\times0.031+4\times0.048+10\times0.71+15\times0.173=9.79$
0.14 M
0.46'
−0.11 M
0.37'
M_F −
2.59'
(0.79 M)
$y_m=\frac{2}{3}h-\frac{l}{24}$
$F=\frac{2}{3}1.45-\frac{9.0}{24}=0.592\ (1.94')$
M_F
$3\times0.46-4\times0.37+10\times2.59+15\times1.94=54.90$ Ft
(d) Bending Moment M_F
②
h = 4.76'
(1.45 M)
+
−

FIG. 14.—STRUCTURAL ANALYSIS AND STRESS CALCULATION OF A CONTINUOUS BEAM

The system is applicable equally to complex frames in steel or reinforced concrete. It can be applied to typical tunnel cross-sections, and to other common elements such as are encountered daily in the modern structural engineering office. No promoters of mechanical equipment for structural analysis should lay claim to merit unless their device can be made to show this range of practical usefulness.

In general, the authors are to be commended for their thorough investigation of model material, and for the formulas introduced to determine necessary corrections. Furthermore, by fastening the moment indicator to the members of the model instead of incorporating it as a part of the member, the authors have, in principle, solved the complex problems of handling. The writer feels, however, that the authors have not maintained a sufficiently clear distinction between the fundamental concept of a mechanical structural analysis, and a mechanical stress analysis as defined roughly in this discussion. The authors have confined themselves to the latter, whereas the writer, by experience, is an advocate of the former.

ARTHUR C. RUGE,[16] ASSOC. M. AM. SOC. C. E., AND ERNST O. SCHMIDT,[17] ESQ. (by letter).—The writers are grateful for the valuable discussions contributed by Messrs. Wilbur, Eney, and Gottschalk. It is particularly gratifying to note that the original study has led to further research and to new instrumental approaches to some of the problems involved.

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Professor Wilbur's discussion develops a number of important points, particularly with regard to secondary stress measurement. The improvement in attaching the levers so as to avoid the shear correction is excellent, and the introduction of a magnifying lever should make for increased efficiency in working with rigid frames as well as in secondary stress applications. The successful use of the spring balance arrangement in conjunction with the moment indicator for secondary stress investigation should encourage others to follow in this interesting and important field of model study; the writers are not aware of any other practical technique for measuring secondary stresses in models with accuracy.

In his discussion of the method of handling the unbalanced joint moments in secondary stress models, Professor Wilbur presents an excellent solution to this problem which is, of course, bound to arise in all such studies, regardless of the method used for measuring moments. The new type of moment indicator, shown in Fig. 10, is an interesting and clever development of the general concept and has obvious points in its favor which would seem to justify a careful study of its possibilities.

Professor Eney contributes an ingenious and elegantly simple method of dealing with the application of the moment indicator to members of variable section. It seems to the writers that observation of the two end rotations with both ends pinned, for a moment applied first at one end and then at the other, might be somewhat faster, in spite of the two simultaneous equations involved. However, Professor Eney has probably found good reasons for adopting his method of observation.

Mr. Gottschalk unfortunately errs in assuming that the authors are "promoters of mechanical equipment for structural analysis." The writers are glad of the opportunity to make it clear that their interest here is solely in the research and engineering aspects of model study and not in the commercialization of their device. The moment indicator is purposely not patented, though it is doubtless patentable, the aim of the writers being to make this easily built and inexpensive tool available to the profession and to encourage thought in a new field of model instrumentation. Most of Mr. Gottschalk's objections were answered in advance in the fourth paragraph of the "Introduction," which the authors here wish to emphasize.

The argument advanced by Mr. Gottschalk to the effect that the basis of the Beggs and Gottschalk methods is "the" correct approach could be argued at length, but the writers prefer to think of it as "a" correct approach—one of many possible approaches. Incidentally, the suggestion of the advisability of building "five or ten separate models in order to compare the consistency of results" is obviously no more applicable to moment indicator work than to analysis by any other method.

Under the heading "Further Considerations," the writers expressed their belief that the most valuable application of the moment indicator may be to secondary stress study. With the increased magnification suggested in the original paper and discussed by Professor Wilbur, the indicator opens a field of model analysis which has long lain dormant for want of effective instrumentation.

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LATERAL EARTH AND CONCRETE PRESSURES

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With Discussion by Messrs. A. E. Cummings, Robert G. Hennes, Robert F. Legget, Charles Terzaghi, Jacob Feld, M. G. Spangler, Raymond D. Mindlin, D. P. Krynine, David A. Molitor, Ralph W. Stewart, and Lazarus White and George Paaswell.

Synopsis

The former Committee of the Society on Earths and Foundations, in conjunction with its studies of the distribution of vertical pressures beneath foundations, assigned to the writers, as a Sub-Committee, the study of lateral earth pressures. This paper is a result of such a study. It has long been recognized that the classic methods of analyzing lateral pressures as developed by Coulomb and Rankine were extremely limited in practical application and many attempts have been made to reshape the classic analysis to bring the theory into better consonance with practice. The writers feel that it is better to discard the classic method entirely, and start from the fundamental laws of the theory of elasticity, as had been done with recognized success in the study of the distribution of vertical pressures. The mathematical work and resulting equations are not as simple as the writers would desire, but it is hoped that further study and application of the methods outlined herein will lead to simplifications without loss of rigor. The labor of using the somewhat involved equations is lightened by the tables presented herewith. It is believed that not only are more correct values determined, but a much better understanding may be had of the phenomena attendant upon the three phases of the problem of lateral earth pressures.

Introduction

The science of soil mechanics has made rapid strides in the decade, 1928 to 1938, in the analysis of foundation problems. The formulas developed by

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Note.—Published in September, 1938, *Proceedings*.

[1] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[2] Secy. and Treas., Spencer & Ross, Inc., Detroit, Mich.

Boussinesq[3] have proved to be extremely valuable in this connection and the writers have used them in the study of lateral pressures upon earth-sustaining structures. For more than a century many attempts have been made to reconcile actual problems to formulas based upon artificial and non-existent soil types. The writers feel that these attempts are futile and that the engineering terminology identified with them is no longer of real significance.

Soils possess sufficient elastic properties to permit the application of the theory of elasticity in solids to lateral earth pressures. There are three fundamental phases of the analysis of lateral pressure: (I) The lateral pressures induced upon a sustaining structure of sufficient rigidity to prevent fracture of the bank; (II) the lateral pressures induced upon a wall when movement of the wall is sufficient to permit the formation of fracture surfaces in the bank; and (III) the lateral pressures induced upon a wall due to a surface loading. It is seen that only for Phase II have the classic theories of Coulomb and Rankine had any coherence and practical application. Charles Terzaghi, M. Am. Soc. C. E., has given an interesting discussion on this aspect of the problem.[4]

The writers feel that the ever-important question of the pressures developed in forms due to concrete may find a solution along the lines developed for lateral pressures. Although the mathematical treatment may seem complex to those who seek a simple "rule-of-thumb" expression, the formulas as derived are comparatively simple.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix.

PHASE I

In Phase I it is assumed that the wall is sufficiently rigid so that the sustained earth bank will not develop fracture surfaces. When an earth mass exerts a lateral pressure upon a sustaining structure this structure is distorted. Since the earth mass itself possesses elasticity, the yielding of the sustaining structure permits a lateral yielding of the earth mass toward the wall or sheeting. This lateral yielding is accompanied by a reduction in the lateral pressures. If the wall is a composite one, comprising members of varied rigidity, the less rigid will yield more, with release of pressure upon them, the more rigid ones finally taking the greater part of the pressure. The distortion of the structure may be small, and yet the re-arrangement and reduction in pressures may be very marked. In the illuminating series of experiments conducted by Professor Terzaghi, at the Massachusetts Institute of Technology, it was found that a movement of the experimental wall as little as 1 mm caused a reduction of 75% in the pressures induced upon the wall. If the movement of the wall persists so that fracture surfaces are developed, it has been found that the general shape of such fracture is as shown in Fig. 1. The surface of fracture is generally recognized as a shear failure surface. At some future time the writers hope to present the development of the pressures due to this shear failure based upon the assumption that the failure surface is a cosine curve. It is important to remember that this characteristic failure surface with the emergent break at the surface, approximately distant from the wall one-half the depth of wall, is independent of the type of ground, the only requisite for its appearance being a sufficiently flexible wall, or a yielding foundation under the wall. The general ground movement due to a shear failure in saturated soils such as clay and silts is a separate phenomenon not discussed in this paper.

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[3] See Progress Rept., Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 779.

[4] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., *Paper J-3*, Vol. 1, Harvard Univ., June 22, 1936.

Sheeting and bracing, when properly designed and installed, come within Phase I. The system is one of varied rigidity—the sheeting being the most flexible, the wales next, and, finally, the braces the most rigid. The relative yielding of these component parts of the sustaining structure produces an accumulation of stress zones at the more rigid points. The pressures induced upon these component parts may be viewed as derived from soils of three characteristic properties each producing a system of lateral pressures of increasing intensity going from the sheeting to the wales and finally to the braces or tie-rods.

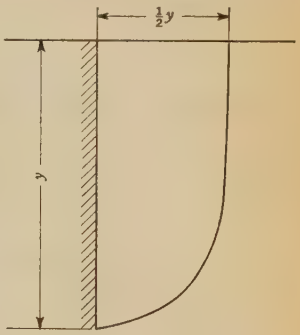

Fig. 1.—Typical Section of a Fracture Surface

Theory of Design—Sheeting and Bracing

For an elastic solid subject to its own weight alone, the total lateral pressure at any depth, y, below the surface is given by,

$$F = -w\,y\,\frac{\mu}{1-\mu} \qquad (1)$$

in which μ is the Poisson ratio. The negative sign in Equation (1) indicates a compression. In accord with elastic theory, μ may have any value between 0.5 and -1. If it were greater than 0.5, the material would not resist distortion; and if it were less than -1, it would not resist compression. As a matter of fact it is positive for all known structural materials. For a true liquid, $\mu = 0.5$, and Equation (1) reduces to the hydrostatic law, $F = -w\,y$.

As the theory of lateral pressures involves Poisson's ratio, it may be well to outline, briefly, its relation to the other elastic constants. Assume that a load, P, is applied along the X-axis of a unit cube and is the only force acting upon this solid. Let the resulting strains induced in this solid by the load along the X, Y, and Z-axes be ϵ_e, ϵ_f, and ϵ_g, respectively. If m and n are two, at present, undefined constants, the following equations exist:

$$P = (m + 2\,n)\,\epsilon_e + n\,(\epsilon_f + \epsilon_g) \qquad (2)$$

$$0 = (m + 2\,n)\,\epsilon_f + n\,(\epsilon_g + \epsilon_e) \qquad (3)$$

and,

$$0 = (m + 2\,n)\,\epsilon_g + n\,(\epsilon_e + \epsilon_f) \qquad (4)$$

From these three equations of equilibrium,

$$P = \frac{n\,(3\,m + 2\,n)}{m + n}\,\epsilon_e \qquad (5)$$

and,

$$\epsilon_f = \epsilon_g = - \frac{m\,\epsilon_e}{2\,(m + n)} \quad \text{.....................} \quad (6)$$

The modulus of elasticity is defined as $E = \frac{P}{\epsilon_e}$, giving a relation between the constants, m and n, and the modulus, E, equal to:

$$E = \frac{n\,(3\,m + 2\,n)}{m + n} \quad \text{.....................} \quad (7)$$

The Poisson ratio is defined by the ratio between axial and lateral strain,

$$\epsilon_f = \epsilon_g = - \mu\,\epsilon_e \quad \text{.....................} \quad (8)$$

so that,

$$\mu = \frac{0.5\,m}{m + n} \quad \text{.....................} \quad (9)$$

The modulus of rigidity or shear (usually denoted by the symbol, G) is the same as the coefficient, n. The three elastic constants, E, G, and μ, are related by:

$$E = 2\,G\,(1 + \mu) \quad \text{.....................} \quad (10)$$

For soils it has been found that μ may vary between $\frac{3}{8}$ and $\frac{1}{6}$, the lower values applying to loose-textured soils and the higher values to clays and, possibly, dense sands. It should be noted that a variation in the value of μ changes only the scale, and not the manner, of the distribution of lateral pressure. A general form of expression for lateral pressure against a sheeted cut may be given as,

$$F = - K\,w\,y \quad \text{.....................} \quad (11)$$

Referring to Equation (1) K may receive a maximum value of 0.6 and a minimum value of 0.2. A rational method of designing sheeting and bracing would be to use the maximum value of K for the more rigid members of the supporting system and a much smaller one for the flexible sheeting.

Permanent structures, such as retaining walls, sub-surface structures, abutments, etc., require careful study as to possible movement before determining the maximum design pressures. Walls of uniform rigidity will move under the initial pressure. A drop in pressure ensues as the material supported by the wall expands elastically behind the wall. The pressure may again build up slowly, causing a further movement of the wall with another cycle of elastic expansion, until sufficient movement has occurred to cause fracture surfaces and possible slides. An analysis of these movements with their possible limitations by design of wall and foundation may indicate economies of design by using lower pressure coefficients. A further complication may enter should the material be saturated and should the foundation itself be wet, particularly if such hydraulic conditions may permit a loss of sustaining power due to the upward flow of water beneath the foundation.

Lateral Pressures Due to Surface Loads

The characteristic distribution of lateral pressures along a vertical surface due to a surface load is shown in Fig. 2. The maximum intensity is at the surface, diminishing rapidly with the depth. Any effect of the introduction of a free vertical surface upon the analysis is ignored. It is assumed that the neglect of this factor introduces no serious error.

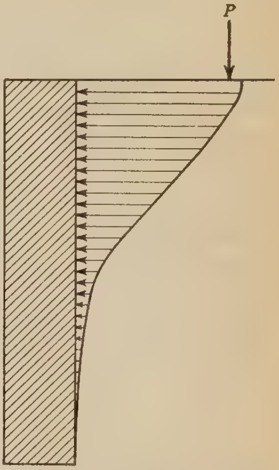

FIG. 2.—TYPICAL DISTRIBUTION OF LATERAL PRESSURES AGAINST A VERTICAL FACE DUE TO A SURFACE LOAD

The simplest problem is that of a single load, analyzed by Boussinesq.[5] (The formulas usually attributed to Boussinesq are more properly termed the Boussinesq-Hertz formulas.) As loads are generally spread over large areas it becomes necessary to partition the area into smaller ones, treating each as a single load and adding the total to secure the composite effect. However, it is possible, by an analysis developed by A. E. H. Love,[6] to obtain, directly, the effect of a rectangularly distributed surface load. The formulas are more cumbersome, but are in a usable form entailing less work than the partitioning process; and they remove the uncertainty involved in a failure to partition the area into small enough units. When the loaded area is long and narrow, the strip method may be used—that is, the method of plane sections. Fairly simple formulas result for this case. To complete the analysis for strip loading, equations developed by Professor N. M. Guersevanoff for irregular types are added.[7]

Point Load.—Referring to Fig. 3(a),

$$F = -k\frac{P}{\rho^2} \qquad (12)$$

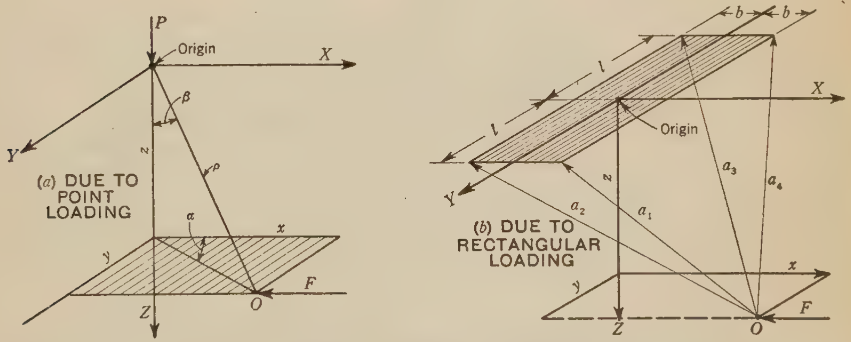

FIG. 3.—CO-ORDINATE SYSTEM FOR LATERAL PRESSURES

in which P is the intensity of surface loads, and k is a constant, expressed by:

$$k = \frac{1}{2\pi}\left[3\sin^2\beta\cos\beta\cos^2\alpha - (1 - 2\mu)(\cos\beta - U)\right] \qquad (13a)$$

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[5] See "Applications des Potentials," by Boussinesq.

[6] *Philosophical Transactions*, Series A, Vol. 228, 1929.

[7] These equations developed by Prof. Guersevanoff were taken from a brochure by him sent to the writers. The brochure was lost in a fire and the writers are unable to refer to the exact title and date of publication.

and, U is a substitution factor:

$$U = \frac{1}{1 + \cos\beta} - \frac{\sin^2\beta\,(2 + \cos\beta)}{(1 + \cos\beta)^2}\cos^2\alpha \ldots\ldots\ldots\ldots (13b)$$

Table 1 gives the value of k for a range of values of the co-ordinate angles, α and β, and for several values of the Poisson ratio, μ. Values of k above the upper, and below the lower, heavy lines denote tensile stresses.

TABLE 1.—VALUES OF k FOR SOLVING EQUATIONS (13)*

Values of Angle β (see Fig. 3(a))	VALUES OF k FOR THE FOLLOWING VALUES OF ANGLE α (SEE FIG. 3(a))									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
					$\mu = 0.1$					
0°	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
10°	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06
20°	0.02	0.01	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.06
30°	0.04	0.04	0.03	0.02	0	0.02	0.02	0.04	0.04	0.04
40°	0.09	0.08	0.07	0.05	0.03	0.01	0	0.01	0.01	0.02
50°	0.10	0.09	0.09	0.08	0.06	0.04	0.03	0	0.01	0.01
60°	0.09	0.09	0.09	0.08	0.06	0.05	0.04	0.03	0.03	0.02
70°	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
80°	0.03	0.02	0.02	0	0.03	0.04	0.06	0.07	0.08	0.09
90°	0.12	0.12	0.10	0.06	0.01	0.03	0.06	0.10	0.12	0.13
					$\mu = 0.2$					
0°	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
10°	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05
20°	0.01	0	0	0.01	0.02	0.03	0.03	0.03	0.04	0.04
30°	0.05	0.05	0.04	0.03	0.02	0	0.01	0.03	0.03	0.03
40°	0.10	0.10	0.08	0.06	0.05	0.02	0.01	0	0.01	0.02
50°	0.12	0.12	0.11	0.10	0.07	0.05	0.03	0.01	0.01	0.01
60°	0.12	0.11	0.11	0.10	0.07	0.06	0.04	0.03	0.02	0.02
70°	0.07	0.07	0.07	0.07	0.06	0.05	0.05	0.04	0.04	0.04
80°	0	0	0	0.02	0.03	0.03	0.05	0.06	0.06	0.07
90°	0.09	0.09	0.07	0.05	0.01	0.02	0.05	0.07	0.09	0.10
					$\mu = 0.3$					
0°	0.03	0.03	0.03	0.03	0.03	0.03	0.03	−0.03	0.03	0.03
10°	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03
20°	0.02	0.02	0.02	0.01	0	0.01	0.02	0.02	0.03	0.03
30°	0.07	0.07	0.05	0.05	0.03	0.01	0.01	0.01	0.02	0.02
40°	0.11	0.11	0.10	0.08	0.06	0.04	0.02	0	0.01	0.01
50°	0.14	0.13	0.12	0.11	0.09	0.05	0.04	0.01	0.01	0
60°	0.14	0.13	0.12	0.11	0.08	0.06	0.04	0.02	0.02	0.01
70°	0.10	0.10	0.09	0.08	0.06	0.06	0.05	0.03	0.02	0.02
80°	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
90°	0.06	0.06	0.05	0.03	0	0.01	0.03	0.05	0.06	0.06

* Values above and below the two heavy lines denote tensile stresses.

TABLE 1.—*Continued*

Values of Angle β (see Fig. 3(*a*))	Values of k for the Following Values of Angle α (see Fig. 3(*a*))									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
					$\mu = 0.4$					
0°	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10°	0	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02
20°	0.04	0.03	0.03	0.02	0.01	0	0	0	0.01	0.01
30°	0.09	0.08	0.07	0.06	0.05	0.03	0.02	0	0.01	0.01
40°	0.13	0.13	0.11	0.09	0.08	0.05	0.03	0.01	0	0.01
50°	0.16	0.15	0.14	0.13	0.10	0.06	0.04	0.02	0.01	0
60°	0.16	0.15	0.14	0.13	0.09	0.07	0.04	0.02	0.01	0.01
70°	0.12	0.12	0.11	0.10	0.07	0.06	0.04	0.02	0.01	0.01
80°	0.05	0.05	0.05	0.05	0.04	0.03	0.03	0.02	0.02	0.02
90°	0.03	0.03	0.02	0.02	0	0.01	0.02	0.02	0.03	0.03
					$\mu = 0.5$					
0°	0	0	0	0	0	0	0	0	0	0
10°	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0	0
20°	0.05	0.05	0.05	0.04	0.03	0.02	0.01	0	0	0
30°	0.10	0.10	0.09	0.08	0.06	0.04	0.03	0.01	0	0
40°	0.15	0.15	0.13	0.11	0.09	0.06	0.04	0.02	0.01	0
50°	0.18	0.17	0.16	0.14	0.11	0.08	0.05	0.02	0.01	0
60°	0.18	0.17	0.16	0.14	0.10	0.07	0.04	0.02	0.01	0
70°	0.14	0.14	0.13	0.11	0.08	0.06	0.04	0.02	0	0
80°	0.08	0.08	0.07	0.06	0.05	0.03	0.02	0.01	0	0
90°	0	0	0	0	0	0	0	0	0	0

Rectangular Loading.—Referring to Fig. 3(*b*),

$$F = \frac{P}{2\,\pi}\,[2\,\mu\,A - (1 - 2\,\mu)\,B - z\,C] \ldots\ldots\ldots\ldots\ldots (14)$$

in which,

$$A = -\,[2\,\pi - (q_1 + q_2 + q_3 + q_4)] \ldots\ldots\ldots\ldots\ldots (15a)$$

$$B = (m_1 + m_2 + m_3 + m_4) - (n_1 + n_2 + n_3 + n_4) \ldots\ldots (15b)$$

and,

$$C = -\left\{\frac{x_1}{k_1{}^2}\left[\frac{y_1}{a_1} + \frac{y_2}{a_4}\right] + \frac{x_2}{k_4{}^2}\left[\frac{y_1}{a_2} + \frac{y_2}{a_3}\right]\right. \ldots\ldots\ldots\ldots (15c)$$

$$x_1 = b - x \ldots\ldots (16a) \qquad y_1 = l - y \ldots\ldots (16c)$$

$$x_2 = b + x \ldots\ldots (16b) \qquad y_2 = l + y \ldots\ldots (16d)$$

and,

$$k_1{}^2 = x_1{}^2 + z^2 \ldots\ldots (17a) \qquad k_3{}^2 = y_2{}^2 + z^2 \ldots\ldots (17c)$$

$$k_2{}^2 = y_1{}^2 + z^2 \ldots\ldots (17b) \qquad k_4{}^2 = x_2{}^2 + z^2 \ldots\ldots (17d)$$

and,

$$\cos q_1 = \frac{x_1\,y_1}{k_1\,k_2} \ldots\ldots (18a) \qquad \cos q_3 = \frac{x_2\,y_1}{k_2\,k_4} \ldots\ldots (18c)$$

$$\cos q_2 = \frac{x_1\,y_2}{k_1\,k_3} \ldots\ldots (18b) \qquad \cos q_4 = \frac{x_2\,y_2}{k_3\,k_4} \ldots\ldots (18d)$$

If x or y is greater than b or l so that $\cos q$ is negative, substitute for q, in Equation (15a), the value, $\pi - q$. All angular values are in circular measure:

$$\tan m_1 = \frac{y_1}{x_1} \ldots\ldots (19a) \qquad \tan m_3 = \frac{y_1}{x_2} \ldots\ldots (19c)$$

$$\tan m_2 = \frac{y_2}{x_1} \ldots\ldots (19b) \qquad \tan m_4 = \frac{y_2}{x_2} \ldots\ldots (19d)$$

and,

$$a_1^2 = k_1^2 + y_1^2 \ldots\ldots (20a) \qquad a_3^2 = k_4^2 + y_2^2 \ldots\ldots (20c)$$

$$a_2^2 = k_4^2 + y_1^2 \ldots\ldots (20b) \qquad a_4^2 = k_1^2 + y_2^2 \ldots\ldots (20d)$$

and,

$$\beta_1 = \frac{z}{a_1} \ldots\ldots (21a) \qquad \beta_3 = \frac{z}{a_3} \ldots\ldots (21c)$$

$$\beta_2 = \frac{z}{a_2} \ldots\ldots (21b) \qquad \beta_4 = \frac{z}{a_4} \ldots\ldots (21d)$$

and,

$$\tan n_1 = \beta_1 \tan m_1 \ldots (22a) \qquad \tan n_3 = \beta_2 \tan m_3 \ldots (22c)$$

$$\tan n_2 = \beta_4 \tan m_2 \ldots (22b) \qquad \tan n_4 = \beta_3 \tan m_4 \ldots (22d)$$

If $\tan m$ or $\tan n$ appears negative, use negative values for these angles as derived from Equations (22).

This analysis for rectangular loads fails for points directly below the corners of the load. Determinate values may be found, however, for neighboring points and should serve the purpose.

TABLE 2.—LATERAL PRESSURES AT VARIOUS POINTS BELOW A LOADED AREA

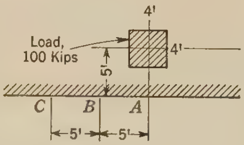
($P = 1$; $b = 1$; $l = 2$; and the area $= 8$)

Values of Poisson ratio, μ	(a) $x=0$; $y=0$; and $z=1$				(b) $x=0$; $y=0$; and $z=0.5$			(c) $x=2$; $y=2$; and $z=1$			
	Rectangular values, Equation (14)	Values by Equation (12) for the following number of sub-divisions:			Rectangular values	Values by Equation (12) for the following number of sub-divisions:		Rectangular values	Values by Equation (12) for the following number of sub-divisions:		
		2	4	16		4	16		1	4	16
0.1	0.04	−0.16	0.0	0.03	0.26	0.28	0.24	0.06	0.04	0.06	0.06
0.2	0.08	−0.13	0.10	0.08	0.31	0.20	0.32	0.07	0.04	0.08	0.07
0.3	0.11	−0.06	0.16	0.11	0.35	0.20	0.36	0.08	0.05	0.09	0.08
0.4	0.14	−0.03	0.26	0.12	0.40	0.16	0.40	0.09	0.06	0.09	0.09
0.5	0.18	0	0.32	0.16	0.44	0.16	0.40	0.10	0.06	0.11	0.10

In applying the Boussinesq equations to spread footings, it is not easy, for points close to the loaded surface, to arrive at a proper sub-division for a fairly accurate result. Equations (14) to (22) give the correct theoretical result. For comparative purposes, the data in Table 2 show the values of the lateral pressures at certain points below a loaded area as derived by Equation (14) and then show the derived pressures as obtained by using the Boussinesq equations for single and subdivided loads for the same area. The applied loading is taken as unit intensity.

Table 3 shows the distribution of lateral pressures under a pier footing. The total load on the pier is 100 kips, giving a load of 6.250 kips per sq ft.

TABLE 3.—DISTRIBUTION OF LATERAL PRESSURES, IN KIPS* PER SQUARE FOOT UNDER A PIER FOOTING



Distance below base of footing, in feet	Interval, in feet	Point *A*		Point *B*		Point *C*	
		Pressures for the Following Values of Poisson's Ratio, μ:					
		0.3	0.4	0.3	0.4	0.3	0.4
0		....	....	....	....	....	....
	2						
2		0.38	0.45	0.09	0.11	0.03	0.02
	3						
5		0.26	0.30	0.09	0.12	0.03	0.02
	3						
8		0.04	0.06	0.05	0.06	0.01	0.01
	5						
13		0.01	0.01	0	0.02	0	0
	5						
18		0	0	0	0.01	0	0

* 1 kip = 1 000 lb.

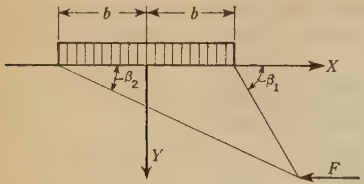
The lateral pressures have been found on the assumption of a single concentration at the center of the pier. As a check, using Equation (14) for Point $x = 5$; $y = 0$; $z = 2$, with $\mu = 0.3$; and $F = 0.38$. For $\mu = 0.4$, $F = 0.47$, which shows that the assumption of a single concentration is accurate.

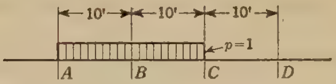
Strip Loading.—For a continuous strip of loaded area, such as a wall, railroad track, etc., a typical section may be taken, and the lateral pressure from this section is given by:

$$F = -\frac{p}{2\pi}\left[2(\beta_1 - \beta_2) + \sin 2\beta_1 - \sin 2\beta_2\right] \dots\dots\dots\dots (23)$$

The angles are in circular measure. Table 4 shows the distribution of pressures under the center, edge, and a distance of 10 ft away from a strip, 20 ft wide, loaded with unit loading. This type of loading is important in the analysis of the effect of neighboring foundation loads upon sub-surface structures and the use of Equation (23) obviates the application of "rule-of-thumb" methods of determining the zone of disturbance and calculation of lateral pressures from equivalent "surcharge" effects.

TABLE 4.—DISTRIBUTION OF LATERAL PRESSURES, IN KIPS* PER SQUARE FOOT, UNDER STRIP LOADING





Distance below base of footing (1)	Interval, in feet (2)	Point B (3)	Point C (4)	Point D (5)	Distance below base of footing (1)	Interval, in feet (2)	Point B (3)	Point C (4)	Point D (5)	Distance below base of footing (1)	Interval, in feet (2)	Point B (3)	Point C (4)	Point D (5)
0		....	....	....	5		0.45	0.35	0.17	30		0.01	0.04	0.09
	1					5					10			
1		0.88	0.47	0.04	10		0.18	0.23	0.22	40		0	0.02	0.05
	1					5					10			
2		0.75	0.44	0.08	15		0.08	0.14	0.18	50		0	0.01	0.03
	1					5					10			
3		0.64	0.41	0.12	20		0.04	0.09	0.15	60		0	0	0.02
	1					5					10			
4		0.52	0.37	0.15	25		0.02	0.06	0.11	70		0	0	0
	1					5					10			
5		0.45	0.35	0.17	30		0.01	0.04	0.09	80		....	....	....

* 1 kip = 1 000 lb.

Point Load.—If the strip is narrow, or points are selected at a distance from the strip (see Fig. 4(*a*)) a simple formula gives the lateral pressure,

$$F = \frac{2\,P\,y\,x^2}{\pi\,\rho^4} \qquad (24)$$

Trapezoidal Loading.—For the loading shown in Fig. 4(*b*):

$$F = \frac{2\,y}{\pi}\,(b_1\,M + b_2\,N) + \frac{1}{\pi}\,(b_1\,x + p_0)\,\tan^{-1}\frac{2\,b_2\,y}{u^2} \qquad (25a)$$

$$p_0 = \frac{1}{2}\,(p_a + p_b) \qquad (25b)$$

$$b_1 = \frac{1}{2\,b_2}\,(p_b - p_a) \qquad (25c)$$

$$M = \log_e \frac{\rho_1}{\rho_2} \qquad (25d)$$

$$u^2 = x^2 - b_2{}^2 - y^2 \qquad (25e)$$

and,

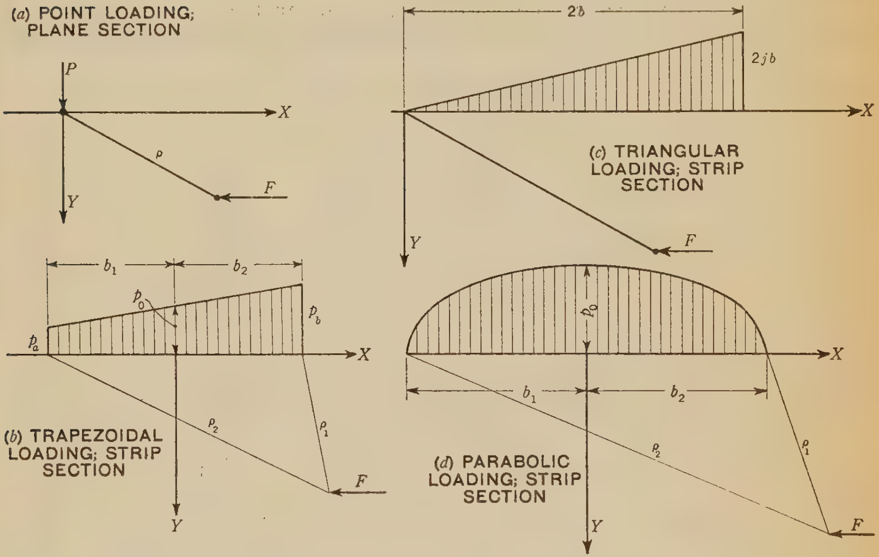
$$N = \frac{2\,b_1\,x\,y^2 + b_1\,x + p_0)\,u^2}{u^4 + 4\,x^2\,y^2} \qquad (25f)$$

Triangular loading, a special case of trapezoidal loading (see Fig. 4(*c*)), is expressed by,

$$F = -\frac{j}{\pi}\left[y\,Q + x\left(\tan^{-1}\frac{y}{x-t} - \tan^{-1}\frac{y}{x}\right) + y\,t\,\frac{x-t}{(x-t)^2+y^2}\right] \quad \text{. . (26}a\text{)}$$

and,

$$Q = \log_e \frac{(x-t)^2+y^2}{x^2+y^2} \quad \text{. . . . . (26}b\text{)}$$



(*a*) POINT LOADING; PLANE SECTION
P
X
p
F
Y
2b
2jb
X
(*c*) TRIANGULAR LOADING; STRIP SECTION
F
Y
b1
b2
p0
pb
pa
X
(*b*) TRAPEZOIDAL LOADING; STRIP SECTION
p2
p1
F
Y
p0
X
b1
b2
(*d*) PARABOLIC LOADING; STRIP SECTION
p2
p1
F
Y

FIG. 4.—LATERAL PRESSURES: VARIOUS TYPES OF LOADING

Parabolic Loading.—For the loading shown in Fig. 4(*d*):

$$F = -\frac{p_0}{\pi\,b_2^2}\left[(3\,y^2 + b_2^2 - x^2)\tan^{-1}\frac{2\,b_2\,y}{u^2} - 4\,x\,y\,M - 2\,b_2\,y\right] \quad \text{. . (27)}$$

in which u^2 and M are as defined by Equations (25*d*) and (25*e*).

The lateral pressures, as given by Equations (23) to (27), are assumed to be distributed along rigid supports yielding no more than the elastic material which it supports. Should the supporting structure be a composite one, such as temporary sheeting and bracing, the same cycles of elastic expansion and loading take place as described herein under the heading, "Theory of Sheeting and Bracing," and the more rigid parts of the sustaining system gradually take up the bulk of the loading.

CONCRETE PRESSURES

The construction of high concrete piers for the Alton (Ill.) Dam, on the Mississippi River, afforded an opportunity to make a series of readings to ascertain pressures exerted by green concrete on forms. The pressures were computed from extensometer readings taken on tie-rods supporting the forms. The tie-rods were free to slip inside boiler-tubes which acted as separators for the form panels. Two pressure curves, derived from the extensometer readings, are shown in Fig. 5. The translation of extensometer readings to pressures is not given herein and is based upon an assumed distribution of pressure from sheeting panel to stud and wales and, in turn, to tie-rod.

The experimental curves have suggested to the writers that the methods outlined previously for the determination of lateral pressures may prove serviceable in analyzing concrete pressures.

A form of fairly standard construction is shown in Fig. 6. The problem is to trace the concrete pressures at some elevation, Plane *AA*, as the newly poured concrete rises in the form, above the level, *AA*. From Point *A* to some height, *h*, the pressures at Point *A* are due to the weight of the concrete mass as expressed by Equation (6). The value of the Poisson ratio, μ, decreases from that of a pure liquid at Point *A* to the value of concrete when set. At the height, *h*, it is assumed that the concrete at Point *A* has taken a set and the pressures at that point are then due to a superimposed load upon a free surface which itself rises as the concrete takes a set above Level *A*. As the pressures gradually decrease in accordance with this condition, the theoretical curves should follow that shown in Fig. 5. Actually, however, the setting of the concrete prevents the tie-rod from returning to its initial length; it can simply recover the distortion it has undergone since the concrete passed Level *h*. The curves of pressure obtained experimentally take this shape and the foregoing analysis is an attempt to explain the result.

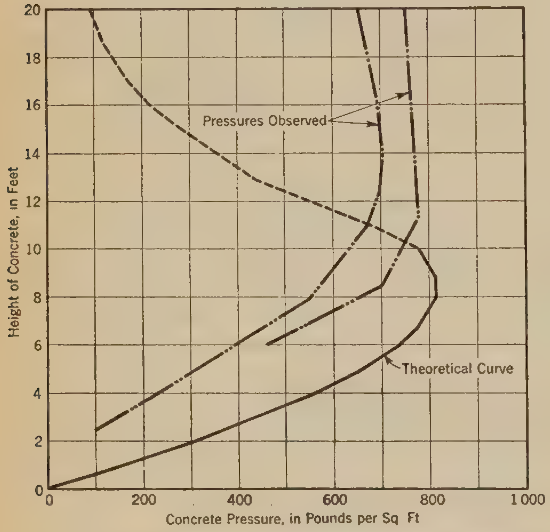


FIG. 5.—PRESSURE EXERTED BY NEWLY FORMED CONCRETE

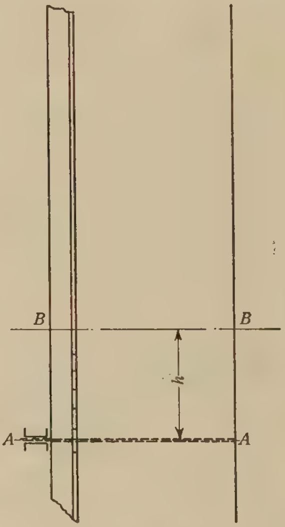


FIG. 6

To give mathematical form to the foregoing reasoning it may be assumed that the change in the Poisson ratio occurring in the first stage of the concrete pressures is given by an empirical relation,

$$\mu = \mu_0 + c\,t^2 \qquad (28)$$

in which t is the time the concrete has been in the form at Elevation AA (Fig. 6). If μ is assumed to have the liquid value at time, $t = 0$; and if it finally attains a value of 0.25 when the set has occurred (or, for time, $t = t_0$), the relation becomes,

$$\mu = \frac{1}{2} - \frac{t^2}{6\,t_0^2} \qquad (29)$$

and Equation (6) may be written in the form,

$$F = w\,h\,\frac{3 - U\,h^2}{3 + U\,h^2} \qquad (30)$$

in which,

$$U = \frac{1}{H^2\,t_0^2} \qquad (31)$$

since $h = H\,t$. The maximum pressure occurs for a value of $h = 0.84\,H\,t_0$ and, at this point, the maximum pressure becomes,

$$F = 0.52\,w\,H\,t_0 \qquad (32)$$

The determination of the pressures when the concrete at Elevation AA has set is taken from the formula for strip loading (Equation (23)). Since the condition is expressed by Fig. 7, the equation may be simplified for this case so that,

$$F = \frac{P_B}{2\,\pi}\,(2\,\beta_0 - \sin 2\,\beta_0) \qquad (33)$$

in which P_B is the weight of the concrete above Plane BB (Fig. 6). Angle β_0 is determined by the width of the form and the distance, z. The weight of concrete, P, will be constant and the variable will be the distance, z. In Fig. 5 the shape of this curve of decreasing pressures is shown dotted.

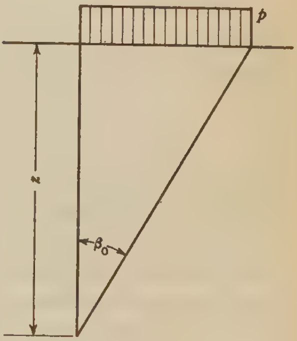


FIG. 7

In the experiments at Alton the weight of the concrete was about 155 lb per cu ft and the level rose in the form at a rate of 2 ft per hr. The time of set was about 5 hr. In the foregoing analysis, therefore, $H = 2$; and, $X_0\,t_0 = 5$. The height for maximum pressure is then 8.4 ft and the maximum pressure, 806 lb. To complete the curve (not of practical significance herein) take $B = 1\,550$ lb and use Equation (33).

Obviously, better fit may be made to the actual observed curve of pressure by "juggling" the empirical relation given by Equation (28), but this mathematical manipulation is not warranted by the conditions of the problem, and the similarity of shape of the curves indicates that the theory possesses some validity and, therefore, seems a better guide to the determination of concrete pressures than the usual assumptions of modified hydrostatic relations.

Conclusions

The analysis of lateral earth pressures resolves itself into a study of three phases of this problem. The first phase is that of the distribution of pressures along a wall sufficiently rigid or unyielding so that no fracture surfaces appear in the ground. In this case the pressures cause a slight yielding of the wall and a redistribution of pressures appears, tending to concentrate around the more unyielding portions of the supporting system. The pressure systems may be then separated for each system of support using a smaller intensity of pressures for the more yielding portion of the system such as the sheeting and to some extent the wales, and the maximum intensity of pressures for the braces or tie-rods. To accomplish this analysis it is suggested that the soil be assumed to possess a variable Poisson's ratio, the smaller for the sheeting and the highest for the braces. Although this may seem an arbitrary assumption as to physical constants of the soil, it seems at present the most suitable device to use.

The second phase, that of pressures developed when a fracture surface is permitted to develop is discussed very briefly. Only in this phase has the analysis of Rankine and Coulomb any significance. The writers hope to present some numerical examples of pressures developed in actual work in this phase at some later date.

The third phase, that of the distribution of pressures due to a surface load, is covered in extensive mathematical detail. It is hoped that the results given will supersede the usual rule-of-thumb method for determining pressures due to surcharges. It is important to note that the intensity of pressures due to a surface load is a maximum quite near the surface and diminishes rapidly in intensity, a fact quite in contradiction to the usual method of analyzing surcharge loads.

The writers have purposely avoided introducing the terms, "coherent," "granular," "non-coherent," and, finally, "angle-of-repose" or "internal friction." For a long period they have recognized that these terms are misleading and of no significance in discussing lateral pressures. The successful application of the principles of the theory of elasticity to soils, both in theory and in their own practice has convinced them that the road to correct determination of soil phenomena lay in such application and not in futile attempts to extend the classic formulas by modifications of coefficients, and the like.

Finally, it should be stated that almost ten years have elapsed since the writers started the preparation of this paper. Experiments have been performed and observations made on lateral pressures in this period which the writers trust will be brought forth in the course of discussion which may clarify or even refute some of the theory presented.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[8] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

A = a substitution factor defined by Equation (15*a*);
a = a distance; a_1, a_2, a_3, and a_4 (see Fig. 3(*b*)) are distances to the corners of a rectangular load;
B = a substitution factor defined by Equation (15*b*);
b = breadth; one-half the width of a rectangular load; b_1 and b_2 are defined where they are introduced;
C = a substitution factor defined by Equation (15*c*);
E = modulus of elasticity;
e = base of Naperian logarithms;
F = total lateral pressure per unit of area;
G = modulus of elasticity in shear; modulus of rigidity;
g = acceleration due to gravity;
H = head;
h = height;
I = rectangular moment of inertia;
j = a ratio in Fig. 4(*c*);
K = hydrostatic pressure ratio;
k = a substitution coefficient defined by Equation (13*a*);
L = length;
l = one-half the length of a rectangular load;
M = a substitution factor defined by Equation (25*d*);
m = a constant; m_1, m_2, m_3, and m_4 refer to the corners of a rectangular load;
N = a substitution factor defined by Equation (25*f*);
n = a constant (equal to G); n_1, n_2, n_3, and n_4 refer to the corners of a rectangular load;
P = a load; P_B = the weight of the concrete above Plane BB;
p = pressure per unit area; p_H = intensity of lateral pressure; p_V = intensity of vertical pressure;
Q = a substitution factor defined by Equation (26*b*);
q = a substitution factor defined by Equations (18);
t = setting time for concrete;
U = a substitution factor defined by Equation (31);
u = a substitution factor defined by Equation (25*e*);
w = load per unit volume;
x = distances parallel to the X-axis; x_1, x_2, x_3, and x_4 are distances, x, to the corners of a rectangular load;

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[8] A.S.A.—*Z* 10*A*—1932.

y = deflection; also distances measured parallel to the Y-axis; y_1, y_2, y_3, and y_4 are distances, y, to the corners of a rectangular load;

z = distances parallel to the Z-axis measured from the base of a footing; z_1, z_2, z_3, and z_4 are distances, z, to the corners of a rectangular load;

α = angular distance to the point of application of Force F, in the XY-plane;

β = half-angle formed under a load, P, by the radius, ρ (Fig. 3(a));

δ = unit elongation;

ϵ = strain; for example, ϵ_e, ϵ_f, and ϵ_g in Equations (2) to (6);

θ = angular distance in polar co-ordinates;

μ = Poisson's ratio;

ρ = radial distance in polar co-ordinates.

DISCUSSION

A. E. Cummings,[9] M. Am. Soc. C. E. (by letter).—An interesting analysis of the problem of lateral pressures is contained in this paper. However, from Fig. 2 and the supporting remarks, it appears that the authors have misinterpreted the nature of the horizontal stresses due to surface loads.

Referring to Fig. 3(*a*), the complete representation of stress at the point, *O*, would include three normal stresses and three shearing stresses. For the purpose of this discussion, it is not necessary to consider the shearing stresses. The three normal stresses are:

$$p_z = -\frac{3\,P}{2\,\pi}\,\frac{z^3}{\rho^5} \quad \text{(34a)}$$

$$p_y = \frac{P}{2\,\pi}\left[-\frac{3\,y^2\,z}{\rho^5} + (1 - 2\,\mu)\left\{\frac{z}{\rho^3} - \frac{(\rho^2 - y^2)\,(z + \rho) - \rho\,y^2}{\rho^3\,(z + \rho)^2}\right\}\right] \quad \text{(34b)}$$

and,

$$p_x = \frac{P}{2\,\pi}\left[-\frac{3\,x^2\,z}{\rho^5} + (1 - 2\,\mu)\left\{\frac{z}{\rho^3} - \frac{(\rho^2 - x^2)\,(z + \rho) - \rho\,x^2}{\rho^3\,(z + \rho)^2}\right\}\right] \quad \text{(34c)}$$

These stresses are written in rectangular co-ordinates. The plus sign represents tension and the minus sign compression. The subscripts, x, y, and z, refer to the co-ordinate axis with which the stress is parallel. This discussion will concern itself with the stress, p_x, which corresponds to the authors' stress, *F*, in Equation (12).

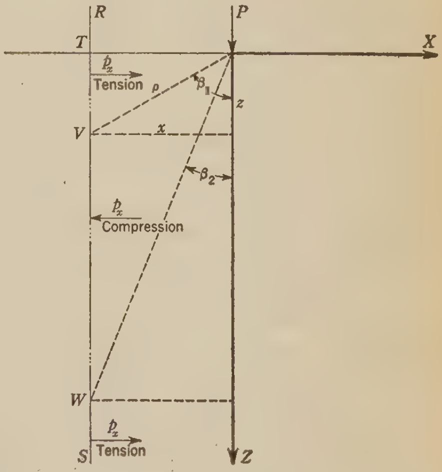

Fig. 8

Fig. 8 represents the *X–Z* plane of Fig. 3(*a*). Line *R S* is the trace on the *X–Z* plane of an imaginary plane which is parallel to the *Y–Z* plane. The unit horizontal stress, p_x, at any point on the line, *R S*, is given by Equation (34*c*). Inspection of this equation does not show whether the stress is tension or compression. However, it is a simple matter to determine the nature of this stress at the point, *T*, which is on the surface of the solid. Substitution of $z = 0$ and $\rho = x$ in Equation (34*c*) gives:

$$p_x = (1 - 2\,\mu)\,\frac{P}{2\,\pi\,\rho^2} \quad \text{(35a)}$$

In a similar manner the nature of this stress at any point on the *Z*-axis may be determined. Substitution of $x = 0$ and $\rho = z$ in Equation (34*c*) gives:

$$p_x = (1 - 2\,\mu)\,\frac{P}{4\,\pi\,\rho^2} \quad \text{(35b)}$$

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[9] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

Since μ is never numerically greater than 0.5 in any elastic solid, the right-hand sides of Equations (35) are positive for all elastic solids. Stress p_x, therefore, is a tension at the point, T, and at all points on the Z-axis. For an incompressible material ($\mu = 0.5$) Stress p_x becomes zero at the point, T, and at all points on the Z-axis. In elastic materials with μ-values less than 0.5, the stress, p_x, on the line, $R\ S$, is a tension from the surface down to some point, V. At the point, V, the stress changes sign and becomes a compression. It is a compression down to the point, W, where it again changes sign and becomes a tension.

The location of the points, V and W, can be determined from Equation (34c) by setting the expression in square brackets equal to zero. In the new equation thus obtained, it is necessary to substitute $x^2 = \rho^2 - z^2$ and $\frac{z}{\rho} = \cos\beta$. Reduction of the equation then leads to,

$$\cos\beta + \cos^2\beta - \cos^3\beta - \cos^4\beta = \frac{1 - 2\mu}{3} \ldots\ldots\ldots\ldots (36)$$

It is seen from Equation (36) that the positions of the points, V and W, are determined by the elastic properties of the material. For a given numerical value of μ, the angle, β, can be found by solving the fourth-order equation in $\cos\beta$. When $\mu = 0.2$, Equation (36) has two real roots which are (approximately) $\cos\beta_1 = 0.175$ and $\cos\beta_2 = 0.945$. The other two roots are a pair of conjugate complex numbers which have no physical meaning in this problem. Therefore, in an elastic solid having $\mu = 0.2$, the angle, β_1, is about 80° and the angle, β_2, is about 19 degrees.

It should be noted that Equation (36) applies only in the X–Z plane. In order to locate the regions of tension and compression throughout the three-dimensional space, it is necessary to use Equation (34c). The numerical value of μ, together with the numerical values of the co-ordinates of a given point, must be substituted in the equation. If the right-hand side of the equation is positive, the stress is a tension at the given point. When the right-hand side of the equation is negative, the stress is compression. Numerical calculation with Equation (34c) is rather laborious although it involves less labor than is required to calculate with Equations (12), (13a), and (13b).

In a practical problem, the given dimensions would be the rectangular co-ordinates of the point at which the stress was to be determined. These dimensions could be substituted directly in Equation (34c). In order to use Equations (12), (13a), and (13b) it would be necessary to transform the rectangular co-ordinates, x, y, and z, into spherical co-ordinates α, β, and ρ, and then to substitute the spherical co-ordinates in the equations. The writer is unable to understand why the authors chose to handle this rectangular problem in spherical co-ordinates since the method does not simplify the problem in any way.

Furthermore, if it is considered desirable to use angles instead of rectangular co-ordinates, it is much easier to use the direction cosines of the radius vector, ρ, than it is to transform the entire problem to spherical co-ordinates. If β is the angle between ρ and the Z-axis and if ϕ is the angle between ρ and the X-axis, Equation (34c) can be expressed in terms of functions of these angles as follows:

$$p_x = \frac{P}{2\,\pi\,\rho^2}\left[-3\cos^2\phi\cos\beta + (1 - 2\,\mu)\left\{\cos\beta - \frac{\sin^2\phi}{1+\cos\beta}\right\}\right]..(37)$$

It is easy to see that Equation (37) is much less complicated than Equations (12), (13a), and (13b).

In connection with Fig. 2, the authors state that they are ignoring any effect that may be caused by the introduction of the free vertical surface. In effect, this means that the wall in Fig. 2 represents nothing more than the imaginary plane, $R\,S$, in Fig. 8. However, the horizontal stress on the line, $R\,S$, is a tension near the surface, and it changes to a compression at some point, V, below the surface. At the point, W, the stress changes again from a compression back to a tension. Even with the simplifying assumption made by the authors, Fig. 2 does not represent the typical distribution of lateral stress on a vertical plane due to a point load on the surface of the semi-infinite elastic solid.

As far as the actual effect of the free vertical surface is concerned, this problem has been studied both theoretically and experimentally. A theoretical analysis has been published by Raymond D. Mindlin,[10] Assoc. M. Am. Soc. C. E. The same problem has been the subject of considerable experimental investigation conducted by M. G. Spangler,[11] Assoc. M. Am. Soc. C. E. Mr. Mindlin's analysis and Mr. Spangler's experiments indicate that the actual pressure on the back of the wall due to a load at the surface is considerably different from Fig. 2. In other words, it has already been demonstrated that the free vertical surface has a decided effect on the stress distribution so that the authors' assumption about this question was not justified.

It is interesting to note that the authors' equations themselves, disprove the validity of Fig. 2. Table 1 shows that there are regions in the semi-infinite elastic solid where the horizontal stress, F, is a tension. This table also shows that the value of μ determines the amount of the tension and the region in which the tension occurs. Throughout Table 1, the X–Z plane is represented by $\alpha = 0$. In that part of the table which is calculated for $\mu = 0.2$, the stress for $\alpha = 0$ is tension when β lies between zero and 20° and it is also tension when β lies between 80 and 90 degrees. This agrees exactly with the results obtained from Equation (36).

It can also be seen quite easily that Fig. 2 does not represent the characteristic lateral pressure distribution due to the types of loading shown in Fig. 4. Throughout Fig. 4, the ground surface is represented by $y = 0$. By inspection, it can be seen that the substitution of $y = 0$ in Equation (24) will make $F = 0$. In other words, the stress, F, in Fig. 4(a) is zero at the ground surface. Substitution of $y = 0$ in Equations (25a), (26a), and (27) shows that the stress, F, is zero at the ground surface for all the types of loading shown in Fig. 4.

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[10] "Pressure Distribution on Retaining Walls," by Raymond D. Mindlin, *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Harvard Univ., Cambridge, Mass., Vol. III, 1936, p. 155.

[11] "Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads," by M. G. Spangler, *Bulletin No. 140*, Iowa Eng. Experiment Station, 1938.

In conclusion, the writer wishes to call attention to an important point in connection with problems of this kind. Aside from the fact that Fig. 2 does not represent the typical distribution of lateral stress on a vertical plane for any kind of surface loading on a semi-infinite elastic solid, there is the question of ignoring the free vertical surface on the assumption that the neglect of this factor introduces no serious error. Most problems in the theory of elasticity can be described as boundary value problems. The problem is usually set up as a differential equation for which a particular solution is determined by the use of certain boundary conditions. With a different set of boundary conditions, the solution of the differential equation may be entirely different. For all the various types of surface loads mentioned in this paper, the authors' equations for F were derived on the assumption that the elastic solid was infinitely deep and that its plane horizontal boundary extended to infinity in both horizontal directions. The solutions of the differential equations were required to satisfy certain conditions at infinity and certain other conditions on the free horizontal boundary. Equations derived for these boundary conditions could not be expected to be valid when a new boundary condition is introduced in the form of a free vertical surface not far from the load. Mr. Spangler's experiments[11] show that the maximum horizontal pressures on the back of the wall could be at least twice as great as those calculated with Equation (12) even in a material having $\mu = 0.5$. For smaller values of μ, Equation (12) gives tensions in certain parts of the stress field but tension could not exist unless the soil were somehow fastened to the back of the wall. These are conditions that should be kept in mind by any one attempting to calculate pressures on retaining walls with the equations in this paper.

ROBERT G. HENNES,[12] ASSOC. M. AM. SOC. C. E. (by letter).—It is important to realize that, as the authors indicate, Equations (1) to (11) are primarily intended for the design of sheeting and bracing. When applied to permanent rigid structures, such as underground tanks and other sub-surface construction, in soils subject to elastic expansion and contraction with seasonal variations in water content, the value of K in Equation (11) is not necessarily within the range of 0.6 to 0.2, but may reach much higher values.

When gravity is the sole body force, F, in Equation (1), becomes equal to s_x, the stress in the soil. If a sufficiently prolonged drought causes shrinkage, the surface tension of the capillary water creates a capillary pressure of intensity, p_c, throughout the soil adjacent to the substructure, and the horizontal earth pressure is,

$$F' = -s_x + p_c \qquad \text{(38a)}$$

in which $s_x = (p_c + w\,y)\left(\frac{\mu}{1-\mu}\right)$; or,

$$F' = p_c - (p_c + w\,y)\left(\frac{\mu}{1-\mu}\right) \qquad \text{(38b)}$$

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[12] Asst. Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

If F' falls below the magnitude of the minimum earth pressure, F_{II}, acting on a yielding wall, as determined by methods applicable to Phase II, then the soil mass behind the wall will slip until $F' = F_{II}$ as a possible minimum value. When the water-table is again raised by rainfall the tendency of the soil to expand with falling capillary pressure is limited by the fact that the soil as a whole has slipped into closer contact with the rigid structure. The ultimate pressure in this case is not the value obtained from Equation (1), but:

$$F = -s_x = -\left[(p_c + w\,y)\left(\frac{\mu}{1-\mu}\right)\right] - w\,y\left(\frac{\mu}{1-\mu}\right) + F_{II} \quad \text{..(39a)}$$

or,

$$F = F_{II} - K\,(p_c + 2\,w\,y) \quad \text{..................(39b)}$$

The value of F, as thus determined, cannot rise above the value of the passive earth pressure, as it would then be relieved by the rupture of the soil mass. However, this latter criterion probably imposes a limitation seldom attained in Nature, as the wet-dry cycles are not generally of sufficient amplitude to make the actual capillary pressure in cohesive soils a maximum; that is, the subsoil is not commonly dried to its shrinkage limit for any appreciable depth.

Equation (39*b*) is introduced mainly to emphasize the distinction between temporary and permanent rigid structures. Its application in a specific case would be handicapped by the need to determine the proper value of p_c. Where circumstances warrant the procedure, this obstacle could be overcome by measuring the natural moisture content of the soil at various times of the year, and then finding the corresponding pressure from a consolidation test on an undisturbed sample.

If it is important to distinguish between a permanent, rigid, underground structure and a temporary structure (such as sheeting and bracing) in choosing a basis for design, it is also important to realize that neither method would ordinarily be chosen for the design of retaining walls, which fall within the boundaries of Phase II of the authors' "Introduction." For reasons of economy most engineers will probably continue to use the classical theories in those cases where a slight yielding of the wall is of no great consequence. Such a procedure is subject to the restriction that the wall must be actually free to slide or otherwise deflect the necessary amount. Where this rule is violated (as often happens in the case of right-angle turns in the low retaining walls on residential property), the wall will crack unless Equation (39*b*) has been satisfied.

It is possible, of course, to use Equation (11) for the design of flexible retaining walls, by choosing an arbitrary value for K, lower than that furnished by Equation (1). When so used, Equation (11) is no longer based upon the theory of elasticity, but becomes a statement of the equivalent fluid method. The limitations of this and other methods with respect to the position and direction of the resultant pressure make it desirable to consult the reference[4] given by the authors for cases involving Phase II.

ROBERT F. LEGGET,[13] ASSOC. M. AM. SOC. C. E. (by letter).—It is refreshing to encounter so forthright a paper on so old and contentious a subject as lateral earth pressure. The background of the ten years of work which the authors mention might be thought to have provided practical evidence in support of the conclusions advanced. Fig. 5 appears to be the only such record included in the paper. Of the fourteen pages, nine are devoted to an interesting mathematical discussion of the effect of surface loads on lateral pressure, and only two to the general problem of design encountered in the vast majority of lateral earth-pressure problems. The "Theory of Design" appears to be contained in Equation (11), with the accompanying qualification, which gives the equivalent hydrostatic pressure for some arbitrary value of K, as now commonly used.

These features of the paper are mentioned since they contrast strangely with statements regarding fundamentals such as: (*a*) "* * * it has been found that the general shape of such [soil] fracture is as shown in Fig. 1. * * * [which] is independent of the type of ground * * *." (See heading, "Phase I.") And (*b*) "[The authors] have recognized that these terms are misleading and of no significance in discussing lateral pressures. * * * [and] that the road to correct determination of soil phenomena lay in such application and not in futile attempts to extend the classic formulas * * *." (See heading, "Conclusions"; the "terms" include coherent, non-coherent, internal friction, etc.). These two statements defy discussion; they prompt many questions, typical of which the following are submitted:

(1) What proof can the authors offer that the general shape of earth fracture behind a retaining wall is always curved and independent of the type of ground?

(2) Professor C. F. Jenkin[14] has conducted an investigation on the lateral pressure exerted by sand, in which theory and experience were correlated accurately. Are investigations of this caliber included in the general group of "futile atempts to extend classic formulas," and if not, how can they be considered in relation to the statement referred to in the first question?

CHARLES TERZAGHI,[15] M. AM. SOC. C. E. (by letter).—In their paper the authors reject the application of the classical earth-pressure theories to the computation of the pressure of earth on the timbering of cuts. They also propose the application of the formula of Boussinesq to the computation of the lateral pressure due to surcharges.

During the past few years the classical earth-pressure theories have been superseded by much broader concepts, involving the elimination of the traditional assumptions of hydrostatic pressure distribution and of plane surfaces of sliding. As a consequence, most of the contradictions between theory and experience have disappeared. This "general wedge theory" explains both the high location of the center of the pressure on the timbering of cuts and the concentration of the pressures in the vicinity of the supports of flexible members. The only requirement for the successful application of the general wedge theory consists in almost full mobilization of the shearing resistance of the soil along the potential surface of sliding.

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[13] Asst. Prof., Dept. of Civ. Eng., The Univ. of Toronto, Toronto, Ont., Canada.

[14] "The Pressure on Retaining Walls," by C. F. Jenkin, *Minutes of Proceedings*, Inst. C. E., Vol. 234, 1932, pp. 103–223.

[15] Dr. Ing.; Visiting Lecturer, Harvard Univ., Cambridge, Mass.

In the earth-pressure tests on clean sand, reported by the writer[16] in 1934, it was found that the minimum lateral pressure developed before the wall yielded, exceeded one-fifth of the yield required for inducing a slip in the back-fill. No minimum pressure is conceivable unless the shearing resistance of the back-fill is mobilized. Hence, the fundamental requirement of the general wedge theory was satisfied long before a noticeable slip in the back-fill occurred. For the same reason one is entitled to expect that the shearing resistance of the banks on both sides of a cut through fairly dense, sandy soil can be almost fully mobilized, although the banks are still far from failing along a visible shearing surface. In such a case the application of the general wedge theory would also be justified. Quite recently this conclusion was confirmed by the results of reliable measurements made by the Siemens Bau-union in Berlin, Germany, in a cut through sand and fine gravel. On the other hand, the lateral pressure of a plastic clay on the timbering of a cut will undoubtedly be greater than the minimum pressure computed by means of the general wedge theory. For dealing with cases of this type, the authors recommend the theory of elasticity. In order to accept their suggestion, the investigator would need to know: (*a*) How to determine the value of the required soil constants by experiment; (*b*) how to compute the lateral pressure from the soil constants; and (*c*) how the results thus obtained compare with experience. No answer to these vital questions can be found in the paper. Hence, it would be premature to discuss the practical value of the suggestion.

The proposed application of Boussinesq's formulas to the computation of the lateral pressure, due to surcharges, is so tempting that the writer succumbed to this temptation twenty years ago, at the time when he started to invade his present field of research.[17] In the reference the authors can even find their Fig. 3 and the pertinent computations. However, the writer never recommended his theory for practical application because he realized at the very outset that the actual pressure values can either be much smaller or considerably greater than the theoretical ones, depending on the conditions of lateral support for the surcharged bank of earth. Thus, for instance, if the surcharge consists of a mass of earth rising at a slope from the crest of a retaining wall, the lateral pressures computed by means of Boussinesq's formulas were found to be far in excess of those which were measured by Audé and many others. On the other hand, if one applies the theory to the computation of the lateral pressure produced by concentrated surcharges whose weight is small compared to that of the "sliding wedge," the real values are approximately twice as high as the theoretical ones. It may suffice to call attention to the results of pertinent experimental investigations by M. G. Spangler,[18] Assoc. M. Am. Soc. C. E., and to the interpretation of the test results by R. D. Mindlin,[19] Jun. Am. Soc. C. E

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[16] "Large Retaining Wall Tests," by Charles Terzaghi, *Engineering News-Record*, February 1, 1934.

[17] "Die Erddruckerscheinungen in örtlich beanspruchten Schüttungen," von Karl Terzaghi, *Öst. Wochenschrift für den öff. Baudienst*, 1919.

[18] "Earth Pressures on a Retaining Wall Due to a Concentrated Surface Load," by M. G. Spangler, *Proceedings*, First International Conference on Soil Mechanics, Vol. I, Cambridge, Mass., 1936, p. 200.

In 1929, E. Gerber published the results of direct measurements of the lateral pressure produced by circular and rectangular surcharges resting on the surface of a back-fill with a depth of 80 cm.[20] The measurements were made by means of very sensitive pressure cells. The writer suggests that the authors should check their theory against Mr. Gerber's test results, but he is afraid that the outcome will be rather disappointing.

The lateral surcharge pressure as computed by means of Boussinesq's formulas can be intolerably far from the truth, therefore, and the error can either be positive or negative, depending on the degree of rigidity of the lateral support and on the ratio between the weight of the surcharge and that of the "sliding wedge." On account of the difficulties connected with pertinent theoretical investigations, the subject still offers a promising field for experimental research. It would be particularly useful to measure the influence of the weight of spoil-banks and of other surcharges on the pressure in the struts of cuts at different depths below the surface.

If results of actual measurements and observations in the field are available, it is to be hoped that the authors will include them in the closing discussion.

Jacob Feld,[21] M. Am. Soc. C. E. (by letter).—A careful analysis of this paper fails to disclose very much new material or reasonably reliable methods for the determination of lateral earth or concrete pressures. It is sincerely hoped that any attempted use of the formulas or recommendations announced by the authors will be made only after a careful study of the limitations on such results. These limitations are stated in the paper, but not in such manner as to warn, sufficiently, the reader who may attempt to apply the results to future design problems.

In the "Introduction," three "phases" of the analysis of lateral pressures are enumerated, the first two dealing with lateral pressures of unfractured and fractured banks, the third being a study of the effect of surface loading. The third case should be discussed as an additional loading to each of the first two and also the possibility that such addition to the first case, unfractured bank without surcharge, will bring it into the second class. In general, if a wall, whether part of a retaining wall, bin, sheathing, or concrete form, has no elastic, plastic, or displacement motion, no pressure can be measured. If the motion is purely an elastic strain, the corresponding elastic strain in the earth can be measured by the elastic theory if the moduli of elasticity of wall and earth are fairly similar.

In the discussion of Phase I, the assumption that there are no fractured surfaces (both internal and external) is equivalent to an assumption that the internal stresses in the earth are of such nature that at no point is the tensile strength greater than the elastic limit in tension. The total distortion, elastic and displacement, must not exceed the strain at the point of elastic limit in tension. In an elastic make-up of elastic members, the less rigid will take less than its share of the load, as determined from static computation. However, as stated in the paper, the experimental work of Professor Terzaghi indicates that extremely minute displacement, far less than the smallest deformation of any economically designed sheathing or retaining wall, will result in a reduction of the measured pressure to amounts indicated by the classic earth-pressure theories; and this case is Phase II, as stated by the authors in the "Introduction."

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[19] "Pressure Distribution on Retaining Walls" (Discussion), by R. D. Mindlin, *Proceedings*, International Conference on Soil Mechanics, Vol. III, Cambridge, Mass., 1936, p. 155.

[20] "Untersuchungen über die Druckverteilung in örtliche belastetem Sand," von E. Gerber; pub. Doctor's Thesis, Zürich, Switzerland, 1929.

[21] Cons. Engr., New York, N. Y.

The typical section of a fracture surface, Fig. 1, has been found by several previous writers on the subject, notably E. G. Haines[22] and H. G. Moulton,[23] Members, Am. Soc. C. E. This type of fracture is usual in slides.

In suggesting the assumption of different characteristics for soil pressure computations for members in the same sheeting and bracing unit, the authors should explain that the requirements of statics must not be forgotten. The total load on the wales can only come from the sheeting, and the total load on the braces or ties can only come from the wales. However, in computing reactions, the effect of continuity as well as the deformation of supports should be considered. The total load in all the braces or ties cannot be more or less than the total load on the sheeting.

In the discussion of the "Theory of Design—Sheeting and Bracing," it should be emphasized that Equation (1) applies only when the elastic modulus for tension as well as compression is not exceeded, and the strain and stress relationship is linear. In the design of retaining walls, the magnitude of the lateral pressure is relatively less important than its direction and the location of the point of application; these last two items are not considered in the paper.

The typical distribution of lateral pressures against a vertical face due to a surface load as shown in Fig. 2 is a fiction. No such distribution has ever been found; nor can any material be imagined in which the maximum effect is found at the top of the wall. The discontinuity of the earth surface must be considered even in the case where elastic equations may be used. Experimental results on this problem have been published by M. G. Spangler,[24] Assoc. M. Am. Soc. C. E.

A complete paper on the use and limitations of elastic theories on the determination of earth pressures was published by the late J. H. Griffith,[25] M. Am. Soc. C. E.

The limitations to the use of Equations (23) to (27),[26] evaluating lateral pressure due to various types of loading, are given as follows: "The lateral pressures, as given by Equations (23) to (27), are assumed to be distributed along rigid supports yielding no more than the elastic material which it supports." If any case of retaining wall, bin, or sheeting, can be found to comply with that limitation, the equations and tables are of value.

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[22] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1513; also, *Engineering News-Record*, Vol. 86 (1921), p. 89.

[23] *Loc. cit.*, Vol. LXXXIII (1919–1920), p. 299; also, Am. Inst. Min. and Met. Engrs. (1920), Section 13, Paper No. 158.

[24] "Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads," by M. G. Spangler, *Bulletin No. 140*, Iowa Eng. Experiment Station, 1938.

[25] "Dynamics of Earth and Other Macroscopic Matter," by J. H. Griffith, *Bulletin No. 117*, Iowa Eng. Experiment Station.

[26] *Proceedings*, Am. Soc. C. E., May, 1938, p. 859.

The theory developed for concrete pressures is not in agreement with the results of the experimental works by F. R. Shunk[27] and by E. B. Smith.[28] As the authors state (see heading, "Concrete Pressures"), pressures were obtained by translating extensometer readings based on an assumed distribution of pressure from sheeting panel to stud and wales and, in turn, to tie-rod. If the effect of continuity of studs and wales on reactions is not considered, errors in the assumed pressures will result.

For those who may be interested in a further study of the elastic theories as applied to earth pressures, reference should be made to the reports of the Special Committee of the Society to Codify Present Practice on the Bearing Value of Soils for Foundations.[29]

With the limitations included in the requirement of elasticity taken into consideration, the authors' paper is a general solution of problem. The principal objection to its applicability is the non-existence of earth materials which may be classified as elastic.

M. G. Spangler,[30] Assoc. M. Am. Soc. C. E. (by letter).—Although retaining walls and allied types of construction are among the oldest and simplest kinds of structures with which engineers are concerned, there are many phenomena, related to the overturning and translating forces which retaining walls must be built to withstand, that are only dimly understood. It is to the credit of the Engineering Profession that its members are continually seeking further knowledge in regard to these phenomena, as is attested by the very extensive bibliography of this subject which is to be found in the literature. This paper is a valuable contribution to retaining wall literature and is especially noteworthy in that the authors have discarded the classical wedge theories and have based their argument upon the fundamental laws of the theory of elasticity, an approach with which the writer is wholly in sympathy.

The authors very properly divide their subject into three fundamental phases: (I) Lateral pressures when the wall is rigid enough to prevent fracture of the back-fill mass; (II) lateral pressures when the wall yields enough to permit fracture planes within the back-fill; and (III) lateral pressures induced upon the wall by surface loading. This discussion will be confined mainly to the last of these phases.

In 1936, rather extensive experiments were reported by the writer[31] in which the lateral pressures on retaining walls caused by the application of concentrated loads on the surface of the back-fill were measured. Typical results of these experiments are shown in Fig. 9. Fig. 9(*a*) shows the results of some similar experiments[32] performed at Erdgenössichen Technischen Hochschule at Zurich, Switzerland, in 1929 which are in substantial agreement with those conducted at Ames, Iowa.

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[27] "Pressure of Concrete in Forms," by F. R. Shunk, *Professional Memoirs*, Corps of Engrs., U. S. Army, Vol. 1, 1909, pp. 247–260.

[28] "Pressure of Concrete in Forms," by E. B. Smith, *Proceedings*, Am. Concrete Inst., February 15, 1920.

[29] *Proceedings*, Am. Soc. C. E., 1915, 1916, 1917, 1920, 1921, 1922, 1923, and 1925.

[30] Research Associate Prof., Civ. Eng., Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

[31] "Horizontal Pressures on Retaining Walls Due to Concentrated Surface Loads," by M. G. Spangler, *Bulletin No. 140*, Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa, April, 1938; also, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass., Vol. 1, 1936, pp. 200–207.

This experimental evidence confirms the applicability of the Boussinesq type of equation to the qualitative distribution of pressure on a retaining wall due to a concentrated load (such as a truck wheel) acting on the surface of the back-fill. Quantitatively, however, the experimental pressures were much greater than those indicated by the Boussinesq theory when the strain-interrupting effect of an interposed retaining wall is ignored, and the authors' assumption that this effect can be neglected without serious error is not confirmed. The average pressures measured in the experiments were about 2.1 times as large as the Boussinesq pressures calculated with Poisson's ratio = 0.5 when a wheel load was placed 1.5 ft from the wall and this factor ranged to about 1.7 with the load 3.0 ft from the wall. These greater pressures resulted from the fact

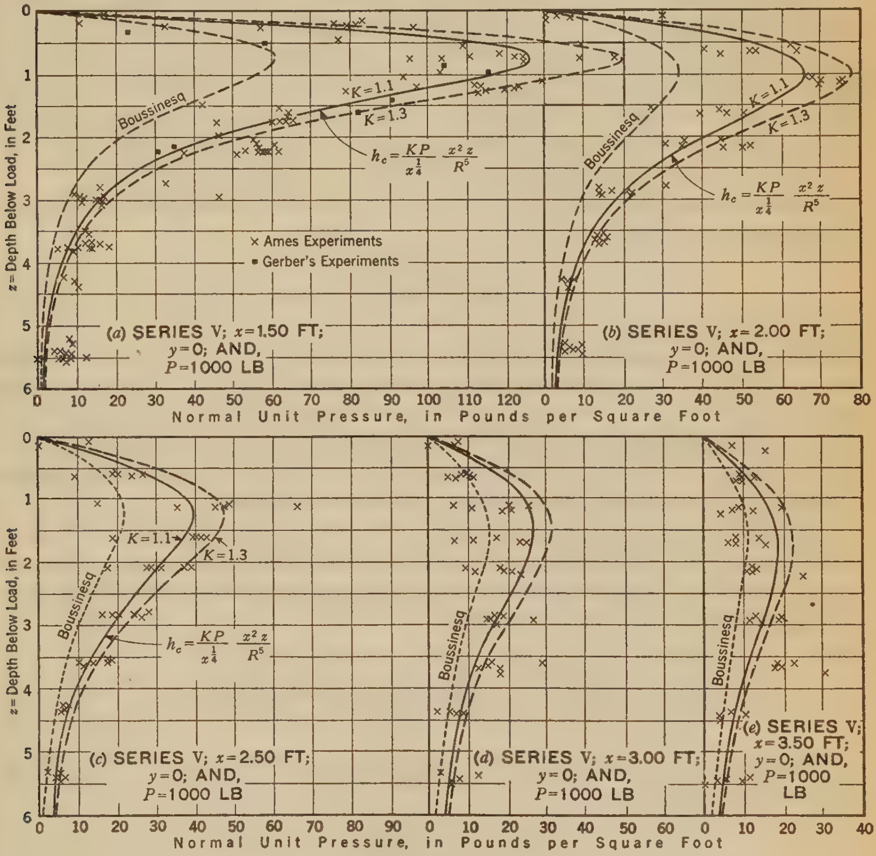

FIG. 9

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[32] "Untersuchungen über die Druckverteilung im örtlich belasteten Sand," by Emil Gerber, Diss. a-g., gebr., Leemann, Zurich, 1929.

that the relatively rigid retaining wall suddenly interrupted the lateral strains in the gravel back-fill mass at the plane of the back face of the wall and caused an accumulation of stress over and above that which would have existed at this same plane if the wall had not been present and the gravel mass had been indefinite in extent. Furthermore, this accumulation of stress was greater when the load was nearer the wall, because the lateral strains at this plane were greater for this load position, and their interruption caused a relatively greater effect.

The experiments also show that the position of the point of maximum pressure on a wall is a function of the distance from the wall to the load and that it is independent of the height of the wall. The maximum pressure occurs at a point whose distance below the plane at which the load is applied is about one-half the distance from the wall to the load, rather than at the top of the wall as shown in Fig. 2 of the paper. This phenomenon is in accord with the Boussinesq theory, as discussed by D. P. Krynine,[33] M. Am. Soc. C. E.

An empirical formula of the Boussinesq type which seems to fit the data of these experiments is:

$$p_w = \frac{\kappa P x^2 z}{x^n \rho^5} \quad \text{.........................} (40)$$

in which p_w = normal unit pressure on the wall at any point; P = applied wheel load; x = distance from load to back face of wall; y = lateral distance from any point on the wall to the normal vertical plane containing the load; z = vertical distance from any point on the wall to the horizontal plane containing the load; ρ = radius vector = $\sqrt{x^2 + y^2 + z^2}$; and, κ and n = empirical constants = 1.1 and 0.25, respectively, in these experiments.

The co-ordinate system for Equation (40) is the same as given in Fig. 3(*a*) of the paper. By considering a uniformly distributed line load parallel to the wall to be a series of closely spaced equal concentrated loads, this formula may be integrated in the *Y*-direction to yield:

$$p_l = \frac{2 \kappa P x^{(2-n)} z}{\rho_1^4} \left[\frac{\rho_1^2 y_0}{3 (\rho_1^2 + y_0^2)^{1.5}} + \frac{2 y_0}{3 \sqrt{\rho_1^2 + y_0^2}} \right] \quad \text{.......} (41)$$

in which: y_0 = one-half the length of the line load; and $\rho_1 = \sqrt{x^2 + z^2}$. When y_0 is equal to infinity,

$$p_l = \frac{1.33 \kappa P x^{2-n} z}{\rho_1^4} \quad \text{......................} (42)$$

In one of the experiments, a uniformly distributed line load was applied 2 ft back of, and parallel to, the retaining wall. The lateral pressures measured on the back face of the wall at a vertical element opposite the mid-point of the load are shown in Fig. 10, together with the pressures indicated by Equations (41) and (42), and by the Boussinesq theory. The close agreement between the measured values of pressure and those calculated by the integrated empirical equation proves the validity of the principle of superposition and the premise that a line load acts as a series of closely spaced concentrated loads.

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[33] Discussion of Paper No. *J-1*, by D. P. Krynine ("The Distribution of Normal Pressure on a Retaining Wall Due to a Concentrated Surface Load," by M. G. Spangler, Vol. 1, pp. 200–207), *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. 3, pp. 159–160.

No experiments have been conducted as yet with area loads applied at the back-fill surface. In the meantime, with the line load experience as a background, it seems tenable to treat an area load as a series of parallel line loads. However, the integration of Equation (41) or (42) in the X-direction is a difficult process and has not yet been accomplished; but, by dividing an area load into a series of parallel strip loads about 1 ft wide, the pressure on a wall can be determined by arithmetical summation without an unreasonable amount of labor. It is the writer's opinion that strip elements 1 ft wide will be small enough to yield results of sufficient accuracy when dealing with stress transmission through materials having the degree of heterogeneity which most retaining wall back-fills possess.

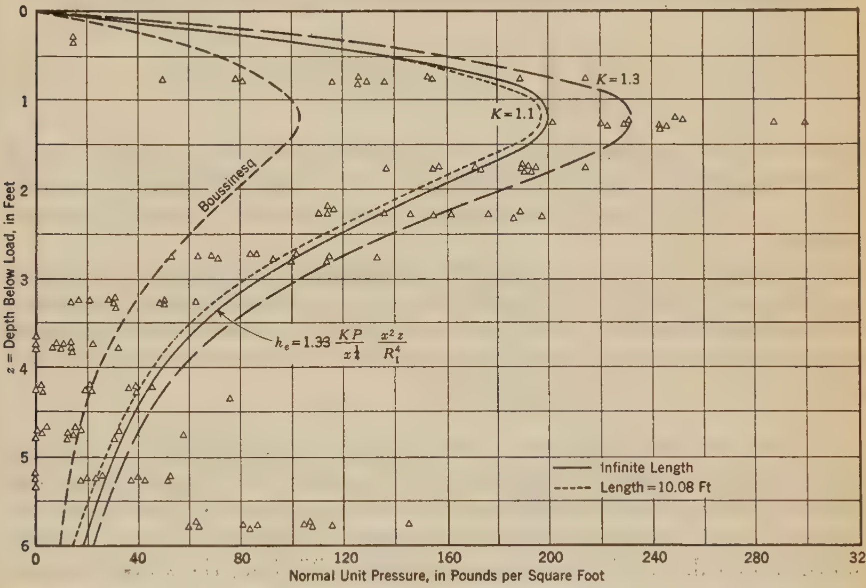

FIG. 10.—SERIES III; LINE LOAD PARALLEL TO WALL, $x = 2.00$ FEET AND $P = 1\,000$ POUNDS PER LINEAR FOOT

Furthermore, the fact of heterogeneity of back-fill materials raises the question of the feasibility of introducing Poisson's ratio into any lateral pressure formulas, whether empirical or rational, because of the difficulty of determining suitable values of this constant. An impracticable number of field determinations of the ratio would be required for a specific back-fill to obtain a suitable average value for use in formulas. Rather, the effect of Poisson's ratio should be included in the empirical disposable κ in Equations (40), (41), and (42), and working values of this factor should be determined by extensive experiments on various kinds of back-fills and retaining walls. The experiments mentioned herein are fairly adequate for the one kind of back-fill and a light wall, but they need to be duplicated for many other combinations of back-fills and walls.

Finally, the writer wishes to close this discussion by suggesting that a new approach to the problem of retaining wall pressures due to earth back-fill might be made in the light of the remarkable qualitative resemblance between the measured pressures due to surface loads and those calculated by the classical Boussinesq equations for stress distribution in an elastic solid. In such an hypothesis, the weight of each small increment of back-fill material might be considered to be a concentrated load which would transmit a horizontal pressure to a retaining wall through the mass of fill material lying below the increment. The sum of the horizontal pressures due to all such increments of volume above any point on a retaining wall would be the pressure on the wall at that point.

The general form of a mathematical expression of this idea would be:

$$p_z = \kappa\, w \int_0^z \int_{-\infty}^{+\infty} \int_0^{\infty} \frac{z_i\, x^{(2-n)}}{(x^2 + y^2 + z_i^2)^{2\cdot 5}}\, dx\, dy\, dz_i \;\ldots\ldots\ldots (43)$$

in which: p_z = horizontal pressure on a retaining wall at any depth, z, below the surface, due to back-fill material; w = unit weight of back-fill material; and z_i = vertical distance from any incremental volume of back-fill down to a depth, z.

For restricted volumes of back-fill material other appropriate limits than those shown in Equation (43) should be used. A consideration of this proposition indicates that the pressure on the wall may be affected by the distance that the back-fill extends back of the wall to a much greater extent than is indicated by the orthodox wedge theories.

RAYMOND D. MINDLIN,[34] ASSOC. M. AM. SOC. C. E. (by letter).—In applying the mathematical theory of elasticity to the study of a complex stress distribution, there are certain fundamental rules which must be observed. For the purposes of this discussion, it will suffice to consider only three of these, known technically as the stress equations of equilibrium, the strain compatibility conditions, and the boundary conditions.

The equilibrium equations represent a set of laws which control the variation of stress from point to point in the solid in such a manner as to assure the existence of a state of equilibrium between each element of the solid and the surrounding elements.

Proper observance of the compatibility conditions assures geometric continuity of the deformed solid; that is, there will be no overlapping or separation of adjacent elements if these conditions are fulfilled.

The foregoing requirements are satisfied by infinite numbers of stress distributions. Which of these distributions will actually exist in the solid depends upon the shape of the body and the forces acting on it. The "boundary conditions" constitute a formulation of the latter conditions.

A simple example will illustrate these principles. Consider a beam that is subjected, successively, to the following actions: (1) Simply supported at its ends and carrying a concentrated load at its center; (2) fixed at its ends and carrying a load uniformly distributed over its length; and (3) constrained along a certain part so that it will remain rigidly undeformed while loads are applied to other parts. It is clear that each of these cases will result in a different stress distribution but all of them may satisfy the equilibrium and compatibility conditions. No designer could properly maintain that the stresses calculated for one case would apply to the others.

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[34] Instr., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

An analogous situation to the foregoing presents itself in the paper. The authors have taken several well-known solutions of the elasticity equations, valid for the particular boundaries and boundary conditions for which they were intended, and have applied them to very different boundaries and boundary conditions. The stresses, of course, still satisfy the equilibrium and compatibility conditions but, without more supporting evidence than is given in the paper, they can no more be said to describe the states of stress that exist in the problems presented than do the stresses in a uniformly loaded, fixed-end beam coincide with the stresses in a simply supported beam carrying a concentrated load. In fact, certain key solutions are noted subsequently which indicate that the conclusions of the authors may involve under-estimates as great as 100%, and almost unlimited over-estimates.

Gravitational Loading.—Turning to the equations offered in the paper, it is observed that the authors state that "for an elastic solid subject to its own weight alone, the total lateral pressure at any depth, y, below the surface is given by [Equation (1)]." In the light of the foregoing discussion, it is necessary to investigate the boundary conditions to which such a pressure might conform.

First, a body acted upon by its own weight alone would move with gravitational acceleration and would be entirely unstressed. If the body is to remain stationary, it must be supported. One method of support is by means of forces applied to the surface of the body. This implies specification of the shape of the solid—that is, a description of its boundaries. Furthermore, it implies specification of the conditions of stress or displacement on these boundaries. A sphere resting on a horizontal plane support will not have the same stress distribution as a cube held in a similar manner; and a sphere resting on three point supports will have still a different stress distribution.

In writing Equation (1), the authors perhaps had in mind the semi-infinite solid. In order to study the validity of the equation in this case, it is necessary first to describe, completely, the state of stress of which Equation (1) is only a part.

The semi-infinite solid is one having as its upper boundary a horizontal plane of indefinite extent, the solid itself extending indefinitely downward from the plane. Consider a system of rectangular co-ordinates, x, y, z, oriented so that the $X-Y$-plane coincides with the plane boundary, and the Z-axis penetrates vertically downward into the solid. Let σ_x, σ_y, σ_z, τ_{yz}, τ_{zx}, τ_{xy} represent the six components of stress at any point in the solid, in accordance with an accepted notation.[35] Then, the complete specification of stress, partially represented by Equation (1), is:

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[35] "Theory of Elasticity," by S. Timoshenko, N. Y., 1934.

$$\sigma_x = -\, w\, z \frac{\mu}{1 - \mu} \tag{44a}$$

$$\sigma_y = -\, w\, z \frac{\mu}{1 - \mu} \tag{44b}$$

$$\sigma_z = -\, w\, z \tag{44c}$$

and,

$$\tau_{yz} = \tau_{zx} = \tau_{xy} = 0 \tag{44d}$$

The strains, ϵ_x, ϵ_y, ϵ_z, γ_{yz}, γ_{zx}, γ_{zy}, corresponding with the six components of stress, are found by using Hooke's law for an isotropic material:

$$\epsilon_x = \frac{1}{E} [\sigma_x - \mu(\sigma_y + \sigma_z)] \tag{45a}$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \mu(\sigma_z + \sigma_x)] \tag{45b}$$

$$\epsilon_z = \frac{1}{E} [\sigma_z - \mu(\sigma_x + \sigma_y)] \tag{45c}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \tag{45d}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \tag{45e}$$

and,

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \tag{45f}$$

Substituting in Equations (45) the values of the stress components given in Equations (44), it is found that,

$$\epsilon_x = \epsilon_y = 0 \tag{46}$$

Hence the lateral pressure given by Equations (44) (and Equation (1)) results from the assumption that there is no lateral displacement or deformation of any vertical plane in the solid. In a semi-infinite solid this may be interpreted as setting up a boundary condition. Since many other types of lateral restraint may be postulated, it is apparent that Equation (1) cannot be accepted as general even for the semi-infinite solid.

The state of stress in a large earth mass, approximating the specifications of a semi-infinite solid, and subjected to gravitational loading only, cannot be determined definitely from the theory of elasticity alone. The stress situation that exists in a particular earth mass depends upon the past history of the body. Experiments, interpreted by means of elasticity theory, might give some indications, but not theory alone.

Poisson's Ratio and Hooke's Law.—Following Equation (1) the authors make certain statements concerning Poisson's ratio which are not quite accurate. They state, "if it were greater than 0.5, the material would not resist distortion; and if it were less than − 1, it would not resist compression." These limits are determined from a consideration of the principle of conservation of energy.

Concerning the properties of a material for which Poisson's ratio is 0.5, two possibilities exist: (1) If the shear modulus is zero; and (2) if the shear modulus is not zero. In the first case the medium resembles a perfect liquid; in the second case, the material is an incompressible solid. The authors further state that Equations (2), (3), and (4) are equations of equilibrium. These equations represent a partial statement of Hooke's law for an isotropic material.

Lateral Pressures Due to Surface Loads.—Under this heading the authors discuss the Boussinesq solution and certain subsidiary solutions obtained from Boussinesq's problem by integration. All these solutions satisfy the boundary conditions for the semi-infinite solid, and were intended by their originators to apply only to such a solid. The authors, however, propose to use these results to describe the pressure distributions against retaining walls and sheeting.

The presence of an obstructing wall in the material completely changes the boundary conditions and, therefore, will change the state of stress in the solid. To visualize the causes of the change, consider, in the semi-infinite solid, a section, *A B C* (Fig. 11), which is a vertical plane before the application of the surface load. Then, apply a load on the surface at some distance from the trace, *A C*, of the vertical plane in the surface. Plane *A B C* then deforms into a curved surface and there will be certain stresses acting on this surface. Suppose, now, that the material to the left of *A B C* is removed. To retain the original state of stress in the material on the right side of *A B C*, the deformation of *A B C* must be maintained by applying to this surface, from the left, the same distribution of stress that was originally supplied by the material removed. If either the stress or the strain on *A B C* is changed, the Boussinesq distribution will no longer be maintained in the remaining material. If a wall is placed against *A B C* instead of the stresses applied from the left, the deformation of Surface *A B C* will be changed and, hence, the stress distribution both in the entire body and against *A B C* will also change. The magnitude of these changes will depend upon the shape and elastic properties of the wall and its surface roughness. By the principle of superposition, the same conclusions will be reached if the wall were present from the beginning and the surface load applied later. That these considerations are not pure speculation is verified by the remarkable experiments of M. G. Spangler,[11]

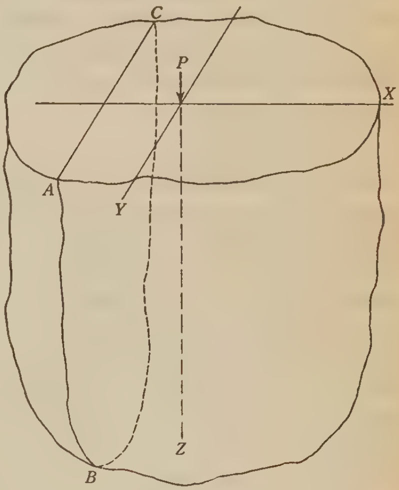

FIG. 11

Assoc. M. Am. Soc. C. E., which show that the actual pressures are about double those predicted by the authors.

It must be concluded, therefore, that Equations (12) to (27) do not apply to the problem under consideration. A sound study of the problem should include consideration of the interaction between the wall and the hypothetical elastic soil. A general solution of the problem involves difficulties which have not yet been overcome by students of elasticity. However, limiting cases have been treated and these throw light on the situation. For example, if the wall is relatively rigid in comparison with the soil, it may be assumed, as a first approximation, that the wall is perfectly rigid. Solutions obtained on this assumption[10] show rather good agreement with Professor Spangler's experiments. They do not, however, agree with the stress distribution depicted by the authors in Fig. 2. In fact, the authors' own proposals do not agree with Fig. 2.

"Rigid wall" solutions are also available for a variety of other boundary conditions. For example, the back face of the wall may be inclined, or two walls may form a right-angle corner. Consideration can also be given to the finite height of the walls and to the roughness or smoothness of some of the contact surfaces.

D. P. Krynine,[36] M. Am. Soc. C. E. (by letter).—Much has been written in recent years concerning the determination of the vertical pressure in earth masses and very little about the horizontal pressure. This is the reason why the paper by Messrs. White and Paaswell is especially welcome. Their contribution offers an excellent opportunity to clarify the question of when the mathematical theory of elasticity may or may not be applied to the determination of the horizontal pressure in soils. Analytical procedures applicable to that science have now been developed to such an extent that average engineers feel themselves overwhelmed with complicated operations and page-long formulas; and some of the students in that field stop to ponder, and begin to ask themselves if they have gone too far with all this mathematics. After all, mathematics is nothing more than a simple engineering tool like a shovel or a pick and is to be used only to the extent necessary to serve its purpose. Perhaps, the time has come when investigators must simplify existing analytical procedures and search for some new and more direct methods of engineering analysis.

Continuity of Strains.—In order to apply the theory of elasticity to the determination of stresses within a body the strains must be continuous, and deformations must be extremely small. With these conditions in mind, the horizontal pressure against either a wall or a sheeting and bracing can be determined by using the theory of elasticity, but only if these particular structures or parts of structures do not yield under the action of that horizontal pressure. If they yield, the upper surface of the back-fill will settle, and there will be no continuity of strains.

Professor Terzaghi[37] has shown that when a retaining wall begins to move, the horizontal pressure decreases and reaches a minimum (which is practically the value given by the Coulomb formula) at the moment of failure. Hence, the horizontal pressure determined by using elastic formulas must be greater than the pressure computed using Coulomb's formula. A rather rigid retaining wall, completely prevented from yielding, may fail, whereas a rather weak wall, but one that is permitted to move, may stand the strain quite satisfactorily. It may be concluded from this apparently paradoxical statement, that structures acted upon by the horizontal pressure should not be fixed against a slight lateral movement; and this restricts the possible field of application of the theory of elasticity to such structures.

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[36] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[37] "A Fundamental Fallacy in Earth Pressure Computation," by Charles Terzaghi, M. Am. Soc. C. E., *Journal*, Boston Soc. C. E., Vol. 23, No. 2, April, 1936.

The pressure distribution shown in Fig. 2 never occurs. This statement can also be proved by the principle of continuity of strains. In fact, strains in a sand mass can be continuous only if isolated particles are so pressed against each other that no mutual displacement of grains is possible, and if only elastic deformations occur. Mass acquires this condition when isolated particles are entirely restrained against movement. It is evident that particles at the surface of a sand mass are not so restrained. They can be moved freely, and, therefore, do not form a body in the sense used in mechanics. They are merely loose grains which can produce no horizontal pressure. Experiments by M. G. Spangler,[38] Assoc. M. Am. Soc. C. E., have revealed zero horizontal pressure at the top of a sand-gravel mass, which is quite consistent from the theoretical point of view.

When the back-fill is composed of clay (which may be considered as an elastically isotropic body) the situation is somewhat different. The horizontal pressure computed according to elastic theories is first negative (directed from the wall), the computations progressing from the surface of the ground, A, down to a certain point, B; then it is positive (toward the wall) down to a certain point, C; and negative again from Point C downward, as shown in Fig. 12 which is a simple free-hand sketch. Pressure measurements would show positive pressure along Distance $B\ C$ only; and the appearance of a fissure at the top of the wall along Distance $A\ B$ may be expected. In any event, Fig. 12 is not the same as Fig. 2, even in the case of a clay back-fill.

Poisson's Ratio.—Elastic formulas for determining stresses contain Poisson's ratio, μ, or its reciprocal, $m = \frac{1}{\mu}$. As used in elastic formulas, m is a constant. The authors propose using two different values of Poisson's ratio—the smallest for the sheeting and the highest for the braces. In other words, it has been suggested to design the sheeting for one kind of soil, and the bracing for another, both kinds of soil being completely hypothetical. Such a suggestion would be entirely acceptable if it were certain that Poisson's ratio maintains a constant value during the process of loading; but there is no such certainty since experimental data to support such a statement are completely lacking.

Investigators in soil mechanics agree that the modulus of elasticity of a soil is not a constant value; it increases with the depth. It is only logical, then, to think that Poisson's ratio also changes its value with the depth since, due to compression by the over-burden and to the formation of a "disturbed zone" under the loaded part of the earth surface, physical properties of an earth mass change with the depth. This is still another argument against the use of elastic formulas in the given case.

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[38] "The Distribution of Normal Pressure on a Retaining Wall Due to a Concentrated Surface Load," by M. G. Spangler, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 1 (1936), pp. 200–207; also, *Bulletin No. 140*, Iowa Engineering Experiment Station (1938), Iowa State Coll., Ames, Iowa.

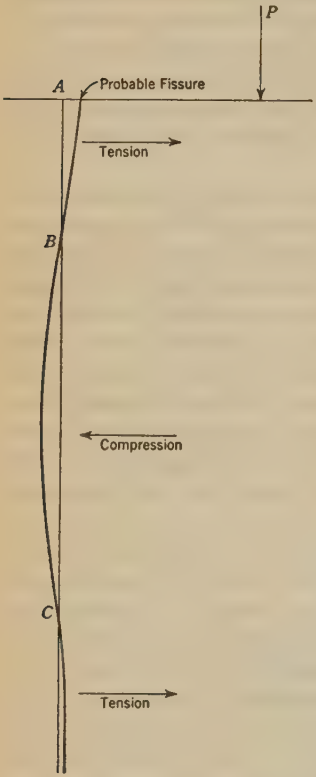

FIG. 12.—POSITIVE AND NEGATIVE HORIZONTAL PRESSURE

In general, the use of the term "Poisson's ratio" as applied to soils is questionable. The term may be applied to an elastically isotropic "continuum" in which there are no pores or voids; it represents the ratio of lateral strain to the axial strain in such a continuum. In a porous material the lateral deformation that appears as the result of an axial stress, changes the shape of individual grains and gradually fills the voids with matter, while the surface of the mass settles. In this case, the term "Poisson's ratio" loses its original meaning. German engineers prefer the use of a conventional term, *Querzahl*, or "transverse number," instead of the classical term, "Poisson's ratio."

Horizontal Pressure Due to the Weight of the Earth Mass.—The authors suggest that Equation (11) be used to compute the horizontal pressure due to the weight of the earth; and, that the coefficient, K, in this equation, "may receive a maximum value of 0.6 and a minimum value of 0.2. A rational method of designing sheeting and bracing would be to use the maximum value of K for the more rigid members of the supporting system and a much smaller one for the flexible sheeting." The writer understands that the authors advance this rule as an empirical one based on practical experience. From a strictly theoretical viewpoint, however, Equation (1), to which the authors refer after introducing Equation (11), was developed for a semi-infinite, elastically isotropic, mass and cannot be applied when retaining walls, sheeting, and bracing are included in that semi-infinite mass.

Live Load.—The situation involving a live load acting at the boundary of the back-fill, at a certain distance from the wall, is different from that corresponding to earth pressure. The live load is applied when the wall has already moved and is in a rather steady position. Due to repeated applications of the live load the earth mass acquires elasticity. Furthermore, the weight of a live load is generally small in comparison with that of the back-fill. Thus it may be assumed that in the case of a live load elastic formulas hold. Obviously, this is true only so far as stresses at a certain depth are concerned since the horizontal pressure at the ground surface equals zero, as previously stated.

Spherical Co-ordinates.—Equation (12) representing the lateral pressure, F, is given in spherical co-ordinates, α, β and ρ. This is an excellent way to describe the stressed condition in the case of a single concentrated force, P, acting vertically at the boundary of the mass, since the entire problem is perfectly symmetrical about the vertical axis, $O\,Z$ (Fig. 3(*a*)); but, in the case of two or more loads acting at the boundary, there are difficulties in using spherical co-ordinates since the results given by Equation (12), for each of the given loads, cannot be added vectorially. To clarify this statement, Fig. 13 has been prepared. The maximum horizontal pressure caused by each of the two loads, P_1 and P_2, acting at the ground surface, is confined in the vertical plane passing through the vertical line of action of the respective load and the given point, O. The two horizontal pressures intersect as shown in the plan (Fig. 13(*a*)) and are to be added geometrically.

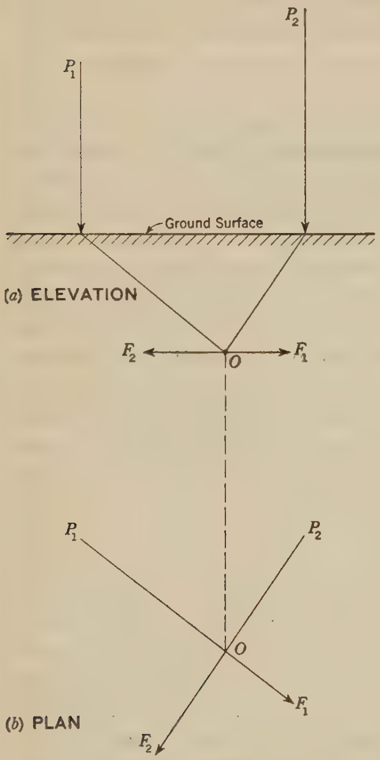

FIG. 13.—HORIZONTAL PRESSURE FROM TWO CONCENTRATED LOADS

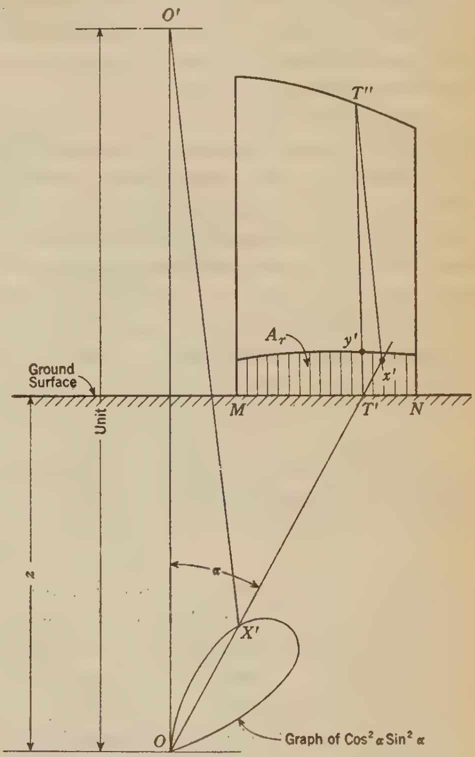

FIG. 14.—HORIZONTAL PRESSURE UNDER A STRIP LOAD

Rectangular Loading.—It is to be borne in mind that Equation (14), and the text to Equation (22*d*), refer to the case of a rectangular loading, one of the sides of the latter being parallel to the retaining wall in question or, more accurately, to the plane normal to which the horizontal pressure in question acts (see drawing in Table 3). The authors rightly state that "the assumption of a single concentration is accurate" in the case of a rather small pier footing;

but, in the general case of a long rectangular loading, as shown in Fig. 3(*b*), this statement is no longer valid. It may be concluded from the paper that the authors themselves are of this opinion, but this fact should be emphasized. Mr. Spangler's experiments[38] destroyed the wrong assumption made by designers as to uniform distribution of pressure along the height of a wall if that pressure is caused by a surcharge. Messrs. White and Paaswell take a theoretical step in the same direction; and the writer appreciates greatly their plausible intention to contribute to the elimination from the engineering practice of an evidently erroneous assumption.

Strip Loading.—Instead of using analytical formulas as developed by the authors, graphical methods of solving this problem have been proposed.[39] In the general case of a non-uniform strip loading (Fig. 14) covering a section, $M\,N$, at the horizontal surface, of a semi-infinite elastically isotropic body, the horizontal pressure at Point O may be determined by tracing a vertical line, $O\,O' = 1$, and plotting along radial vectors, drawn from O, values of $\cos^2\alpha \sin^2\alpha = \frac{1}{4}\sin^2 2\,\alpha$. By joining the ends of the radii vectors, Locus $O\,X'\,O$ is obtained. An ordinate, $T'T''$, of the area, $M\,T''N$, showing the non-uniformly distributed load, is "reduced" by drawing a straight line through points O and T', and joining the point, X', of its intersection with the locus referred to, with Point O'. Drawing $T''\,x'$ (Fig. 14) parallel to $O'X'$, the value of the ordinate, $T'\,y' = T'\,x'$, of the "reduced" area, A_r, is obtained. The reduced area is then measured, by planimeter, or in some other way, and the value of the horizontal pressure is:

$$F = \frac{2}{\pi}\frac{A_r}{z} \qquad \text{(47)}$$

in which z is the depth of Point O, under the ground surface. If the scale of the reduced area, A_r, in Fig. 14, is such that 1 in. represents a tons per sq ft vertically, and b ft horizontally, each square inch of that area would represent $a\,b$ tons per ft. Then the stress intensity, F, would be measured in tons per sq ft. A somewhat different procedure proposed by the writer may be also used.[40]

In the interest of a more accurate terminology, the "point load" in Fig. 4(*a*) should be termed "line load," since this is an infinitely long straight line at the ground surface loaded with $\overline{p}$ units of weight per unit of length.

Horizontal Pressure and Pressure on a Retaining Wall.—Attention should be called to the fact that Pressures F, computed according to elastic theories, are stresses within the mass itself. Using the method of images, R. D. Mindlin,[41] Jun. Am. Soc. C. E., has shown that the pressure on a retaining wall is not F, but $2\,F$.

Conclusions.—The main principle of this paper, in that it defends the application of the theory of elasticity to the determination of the horizontal pressure against sheeting and bracing, is questionable. The paper itself, however, is to be welcomed as a contribution to the field in which practically no mathematical study has been done in recent years.

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[39] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 834–835.

[40] "The Stress Function and Photo-Elasticity Applied to Dams," by John H. A. Brahtz, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 1288–1295.

[41] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 3 (1936), p. 71.

David A. Molitor,[42] M. Am. Soc. C. E. (by letter).—After reviewing this paper, wherein it is recommended that the classic theories of earth pressure be discarded without supplying any adequate substitute, one wonders what method should be used in future. Actually, only the special case of lateral pressures due to single or strip-surface loads is treated. The title, therefore, is too inclusive for the subject matter presented.

The authors mention three phases of retaining wall problems, as follows:

(1) Pressures distributed along a wall sufficiently rigid or unyielding so that no fracture surfaces appear in the ground or bank;

(2) Pressures developed as a result of a fracture surface or surface of rupture caused by a slight movement of the wall; and,

(3) The distribution of pressures due to a surface loading. Phase (1) is not treated in detail; Phase (2) is the basis for Coulomb's theory, which the authors recommend be discarded; and Phase (3) is the case analyzed by an application of the Boussinesq-Hertz formulas and comprises most of the paper. The writer has never seen a case that could properly be classed as belonging under Phase (1). Even if the wall structure is perfectly rigid (which condition was more likely to exist in the days when only gravity type walls were in use) the elasticity of the foundation is always sufficient to cause a decided movement in the back-fill, not to mention the ever-occurring settlement.

No wall can be regarded as safe and enduring unless it is so designed that it will stand at least 1 yr without undergoing any serious displacement, either of overturning or sliding on the base. Under such circumstances would any designer be warranted in pursuing any other than a safe course by providing adequately for the possible maximum earth pressure? Hence, whether he likes it or not, the engineer's design problems will continue to be solved according to Coulomb, Poncelet, Scheffler, Rankine, Winkler, etc., on the basis of the prism of maximum pressure, Phase (2), and nothing better has been proposed as yet.

As for Phase (3) (which is merely a special case of surcharge superimposed on Phase (2)) the lengthy mathematical treatment in the paper is interesting but of comparatively little value until an entirely new set of empiric coefficients is determined. This is equivalent to reminding the designer that he must first conduct some elaborate test experiments prior to designing a wall.

The authors state that they hope to present the development of the pressures due to shear failure on the assumption that the failure surface is a cosine curve. Furthermore, a typical section of fracture surface is shown in Fig. 1, and it is stated that this surface is independent of the type of ground. Others have assumed the fracture surface to be of circular cross-section.

The writer contends that the surface of rupture is of hyperbolic section, and that its particular shape and location is purely a characteristic of the type of ground and its moisture content, as described in a paper[43] giving a full account of actual measurements of such failure surfaces for clay embankments as high as 65 ft, with a possible application to the earth-pressure problem.

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[42] Cons. Engr., Harlingen, Tex.

[43] "The Present Status of Engineering Knowledge Respecting Masonry Construction," by David Molitor, *Journal*, Association of Eng. Societies, 1900.

In 1937 the writer presented in brief form[44] the most usable features of the many theories evolved in the past, and explained how these theories may be applied in a practical manner to retaining-wall and sheet-piling problems. To those designers who still feel that the classic earth-pressure theories are not yet doomed to oblivion, this information should prove valuable.

It is regrettable that the term "equivalent fluid pressure" has crept into modern textbooks. The term could apply only to the case of level-back fill. As most wall problems involve surcharge in some form, the designation is not fitting.

In conclusion it should be emphasized that the design of a large, reinforced retaining wall, with or without surcharged back-fill, requires more than mere textbook knowledge and should be undertaken only after considerable experience and mature judgment are acquired.

Predicated on these qualifications, the classic theories will no doubt endure for quite some time to come, or at least until something much better can be evolved. When one considers the immense number of walls and structures, designed and built during the past, with a relatively small percentage of failures traceable to poor judgment, it must be acknowledged that the classic theories have served well in practice.

Ralph W. Stewart,[45] M. Am. Soc. C. E. (by letter).—Lateral pressures on retaining walls produced by concentrated loads on the ground surface behind the walls, which is investigated by the authors, was also studied both experimentally and analytically by M. G. Spangler, Assoc. M. Am. Soc. C. E., at the Iowa Engineering Experiment Station and the results were published in April, 1938.[46] One rather important difference is noted between the authors' conception of the distribution of pressure and the findings by Professor Spangler. In Fig. 2 the lateral pressure due to a surface load which is set back some distance from the back of the wall is indicated as maximum at the top of the wall. The Iowa tests show this pressure as zero. The latter is obviously correct as a thin stratum of granular earth along the surface could not act as a strut to transmit horizontal pressure from P (in Fig. 2) to the top of the wall. If called upon to perform this duty it would fail by buckling—that is, the surface of the ground would bulge upward. As illustrated in Fig. 15, the actual pressure distribution curve, according to the numerous observations reported in Iowa, forms a bulb whose maximum ordinate is somewhat below the top of the wall, depending on how far back the load is.

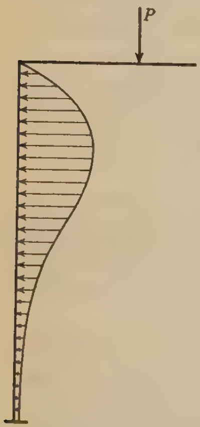

Fig. 15.—Typical Distribution of Lateral Pressures Against a Vertical Face Due to a Surface Load

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[44] "Structural Engineering Problems," by David A. Molitor, The Peters Co., Detroit, Mich., 1937, Chapters 9 and 10.

[45] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

[46] Iowa State College Bulletin No. 140, Ames, Iowa, 1938.

LAZARUS WHITE,[47] AND GEORGE PAASWELL,[48] MEMBERS, AM. SOC. C. E. (by letter).—In May, 1933, Hardy Cross, M. Am. Soc. C. E., addressing the New York Metropolitan Section of the Society, stated that by nature the engineer is not primarily a scientist, a mathematician, an experimental physicist nor a laboratory man. Formulas and experiments are "mechanical brains" which cannot take the place of the human brain when applied to the control of the forces of nature for the benefit of Man.

For the best description of the nature of the engineer, Professor Cross quoted from Robert Louis Stevenson's book "Records of a Family of Engineers," which records the history of Stevenson's father and grandfather, both famous engineers. For the benefit of the discussers of the writers' paper, foundation engineers are in a field where there is, in the words of Stevenson, "but a restricted use for formulas. In this sort of practice, the engineer has need of some transcendental sense. Smeaton, the pioneer, bade him obey his 'feelings'; my father, that 'power of estimating obscure forces which supplies a coefficient of its own to every rule.' The rules must be everywhere indeed; * * * everywhere bent to the impression of the trained eye and the *feelings* of the engineer." Stevenson calls attention to "the insignificance of much Engineering literature." "So far as the science can be reduced to formulas or diagrams, the [engineers] book is to the point; so far as the art depends on intimate study of the ways of nature, the author's words will too often be found vapid."

The writers believe that their observations were based upon an "intimate study of the ways of nature" and even if the formulas they attempted to advance are deficient (as they are certain to be) they should be of aid to any engineer who wishes to comprehend the processes of nature in the field discussed; and they will help to supply, to obscure forces, that "transcendental sense" which distinguishes the engineer. As Stevenson observes, he should have "a sentiment of physical laws and of the scale of nature, which shall have been strong in the beginning and progressively fortified by observation, and must be his guide in the last recourse."

It is to be hoped that genuine engineers are not as rare as Smeaton, of whom Stevenson recites this anecdote: "Duke of Argyle to Smeaton (who declined a commission), 'You can recommend some other fit person?' asked the Duke. 'No, I am sorry I cannot,' replied Smeaton. The Duke: 'What a profession with only one engineer in it. Pray, who taught you?' 'Why,' said Smeaton, 'I believe I may say I was self taught, may it please your grace.'"

The writers, instead of being self-taught like Smeaton, had the benefit of competent teachers of engineering and imbibed the classical theories of Coulomb and Rankine who were of quite a respectable age even in Smeaton's time. For quite a while the writers believed in them thoroughly. However, being brought up to check theories as against nature, constantly, their experience quite flatly indicated the classical theories, and as far back as 1908 came to observe with the late J. C. Meem, M. Am. Soc. C. E., and E. G. Haines, M. Am. Soc. C. E., that the pressures of an earth bank should be held by braces placed at the top—that the fracture of banks was conchoidal instead of plane. By a more correct observation of actual pressures, great economies in systems of bracing and sheeting were realized without a sacrifice of safety—on the contrary, with an increase of safety that should always be realized when dealing with a correct understanding of nature's working. Later, about 1915, in connection with underpinning, the Pretest method was invented which depended upon the elastic properties of earth. This was given mathematical expression by Professor Terzaghi, in the application of Boussinesq, in computing the vertical pressures beneath a footing; the plottings of such pressures in isopressure lines by experiment or calculation gives the "bulb of pressure," which affords a valuable graphic representation of the variation in vertical pressures beneath a footing, or can be applied also to pressures in a horizontal direction opposite wales or braces in a timber system; photo-elastic observation presented a strikingly visual confirmation of the "bulb of pressure."

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[47] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[48] Secy. and Treas., Spencer & Ross, Inc., Detroit, Mich.

In his experiments on retaining walls, Professor Terzaghi demonstrated the great effect of very small movements on the lateral pressure.[16] Such effects, to the writers' minds, could only be explained properly by the elastic properties of earth.

The elastic theory was also applied successfully to a study of concrete pressures, with great economy. The writers are surprised that no discussions of this part of the paper were submitted because great sums are spent on forms which are designed with some modification of liquid pressure, or by "rule-of-thumb," with consequent crudeness or extravagance. Considerable experimentation could be done in this field as actual measurements of pressures in forms are much easier than in earth banks.

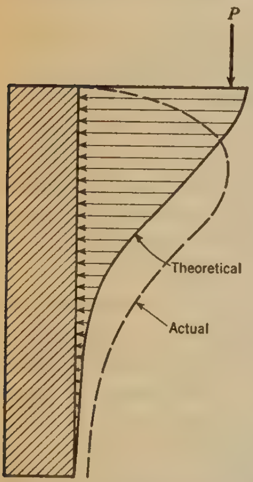

FIG. 16.—TYPICAL DISTRIBUTION OF LATERAL PRESSURES AGAINST A VERTICAL FACE DUE TO A SURFACE LOAD

An undue amount of attention was given to Fig. 2 of the paper; the effect of a free surface was ignored but a slight modification such as that in Fig. 16 would satisfy the criticism. From time to time the writers obtained measurements on braces which, in general, corroborated Fig. 16, but no favorable opportunity was afforded until the last section for the Sixth Avenue subway in New York, N. Y., was being excavated. Here a long section of cut was supported by steel wales and braces rigidly wedged to the masonry walls of the Hudson and Manhattan Railroad, affording an almost ideal opportunity to measure the actual lateral pressures. The cross-section of the cut and method of steel support in open cut is shown in Fig. 17. Where the cut was clear of buildings and adjacent underpinning, vertical steel beams were driven and directly supported by steel braces. Horizontal sheeting was used between verticals and the ground was carefully drained by ditches and sumps during excavation.

By means of the apparatus shown in Fig. 18 the pressures on the braces could be ascertained within 2 or 3 tons. The apparatus was that ordinarily used in underpinning—a hand pump, capable of exerting a pressure of 5 000 to 6 000 lb per sq in.; two 4.5-in. hydraulic jacks of about 40 tons capacity; the wooden wedges expanded against the flanges of the 10-in. H-beam and after a slight slippage gripped sufficiently to hold 10 to 20 tons for each pair; the end of the beam, when tapped with a steel hammer, while holding its load, emitted a clear ringing note like a tuning fork; when the beam was unloaded between the two pairs of wooden wedges and the wale, the note changed so that its unloaded condition could be detected before any space between beam and wale was visible to the eye. Thus the loading conditions were practically unchanged. The apparatus described is simple and accurate when a calibrated hydraulic gage is used. (In Fig. 18 note the copper tubes leading to the jacks.)

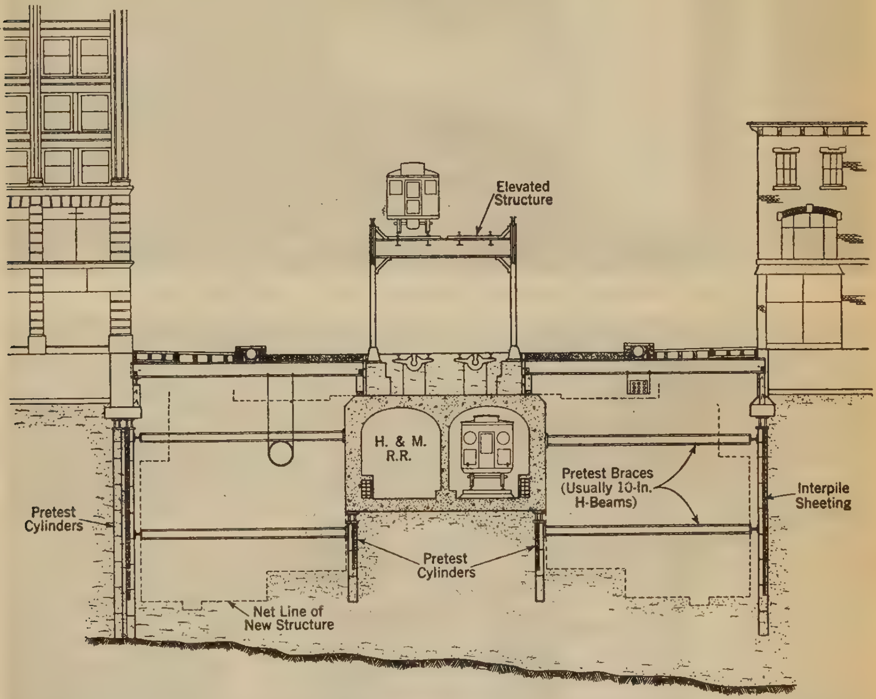

FIG. 17.—TYPICAL CROSS-SECTION OF STEEL BRACING AND UNDERPINNING FOR SUBWAY CUT, SIXTH AVENUE, FROM FOURTEENTH TO SEVENTEENTH STREETS, NEW YORK, N. Y.

For several months a trained gang of three men accumulated considerable data which were used in the designs of steel bracing, with resulting economies. It was easy to check the actual loads at any time—in no case were dangerous pressures observed—and yet the bracing (see Fig. 19) seemed light when compared to that ordinarily used; particularly the lower braces were more widely spaced, which required the use of heavier wales. The best system was to omit wales and brace the vertical beams directly. One surprising result was that despite a great variation of ground—quicksand, "bull's liver," fine to coarse sand and gravel in the natural drained ground—the pressures as mea-

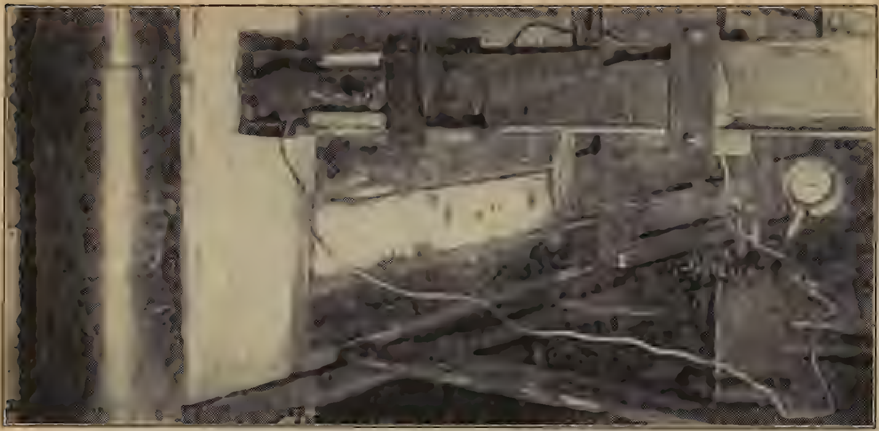

FIG. 18.—VIEW OF HYDRAULIC TESTING DEVICE FOR STEEL BRACES

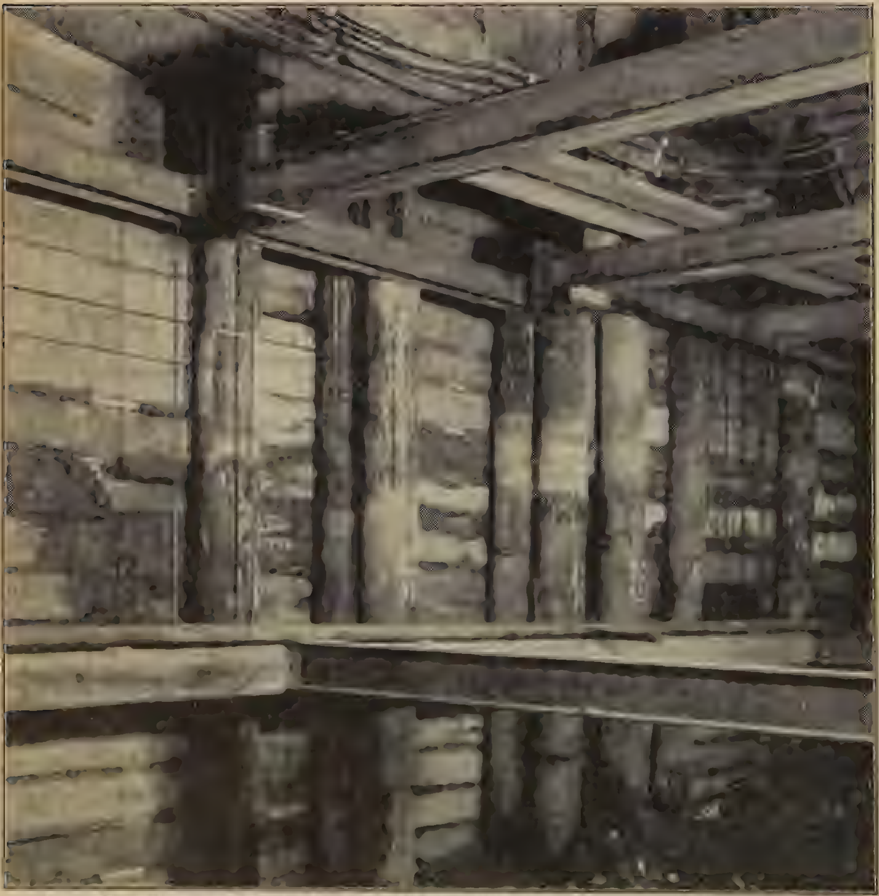

FIG. 19.—VIEW OF STEEL BRACING, UNDERPINNING AND SHEETING, SUBWAY CUT, NEW YORK, N. Y.

sured seemed to be quite uniform and not responsive to variations in strata. The results of measurements are given in Fig. 20. The curves conform in general to those of the elastic theory. Note that in one case where a new foreman neglected his drainage and allowed the bottom to become quick, the bank broke, and settlement occurred in the adjacent underpinning, with no lateral movement because of the steel bracing against the adjacent subway; yet the pressure after the break was less than the pressure before, indicating that the "elastic" pressure is greater than the "wedge" pressure. Mr. Bedrich Fruhauf computed pressures by the formulas given in the paper and found that it is necessary to make a modifying assumption to obtain values similar to those of Fig. 20. The formula assumes perfect elasticity to infinity which produces a straight-line increase of pressure with depth. This does not conform with observation, so it seems reasonable to limit the assumption to a

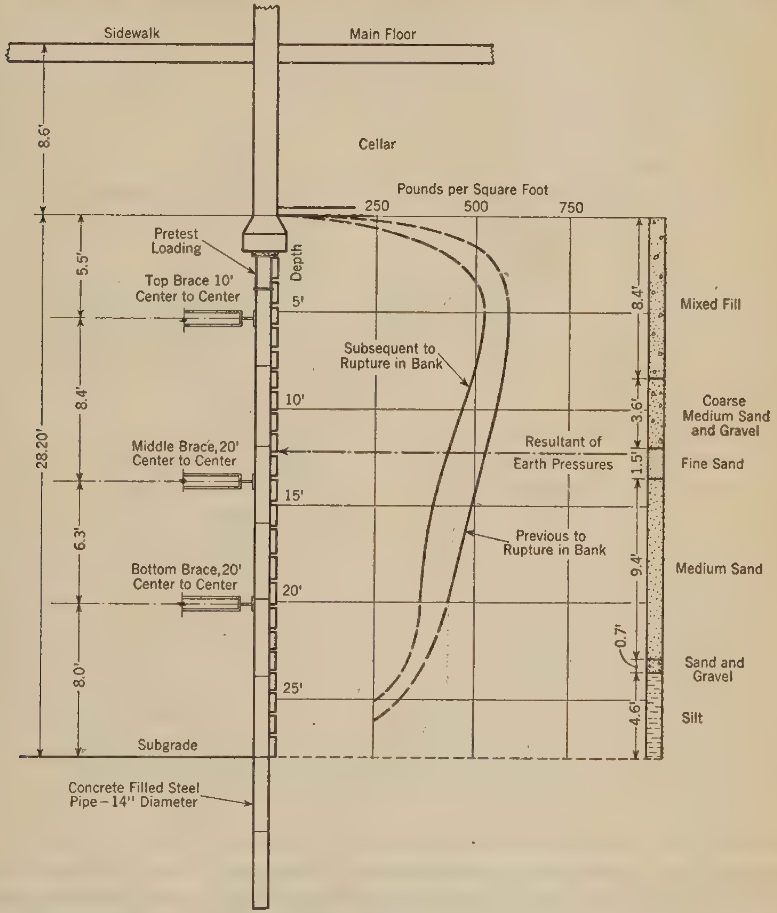
Sidewalk
Main Floor
8.6'
Cellar
Pounds per Square Foot
250
500
750
Pretest Loading
Depth
5.5'
Top Brace 10' Center to Center
5'
8.4'
Mixed Fill
Subsequent to Rupture in Bank
10'
8.4'
3.6'
Coarse Medium Sand and Gravel
28.20'
Middle Brace, 20' Center to Center
Resultant of Earth Pressures
1.5'
Fine Sand
15'
6.3'
Previous to Rupture in Bank
9.4'
Medium Sand
Bottom Brace, 20' Center to Center
20'
0.7'
Sand and Gravel
8.0'
25'
4.6'
Silt
Subgrade
Concrete Filled Steel Pipe – 14" Diameter

FIG. 20.—HYDRAULICALLY MEASURED LATERAL EARTH PRESSURES AT SIXTEENTH STREET AND SIXTH AVENUE, NEW YORK, N. Y., FOR CUT 120 FEET LONG. AVERAGE DEPTH IS 28 FEET

vertical plane a reasonable distance back. In ordinary materials (well drained) a distance equal to the depth of cut or wall would seem to be ample. Calculations on different assumptions are given in Fig. 21. The bank was divided into horizontal layers 1 ft thick, and vertical slices 20 ft, 40 ft, and 100 ft wide. Each horizontal layer was considered as a surcharge to the material below. Poisson's ratio, μ, was assumed at 0.2. Different values of μ affect the total pressure but not the form of pressure curve.

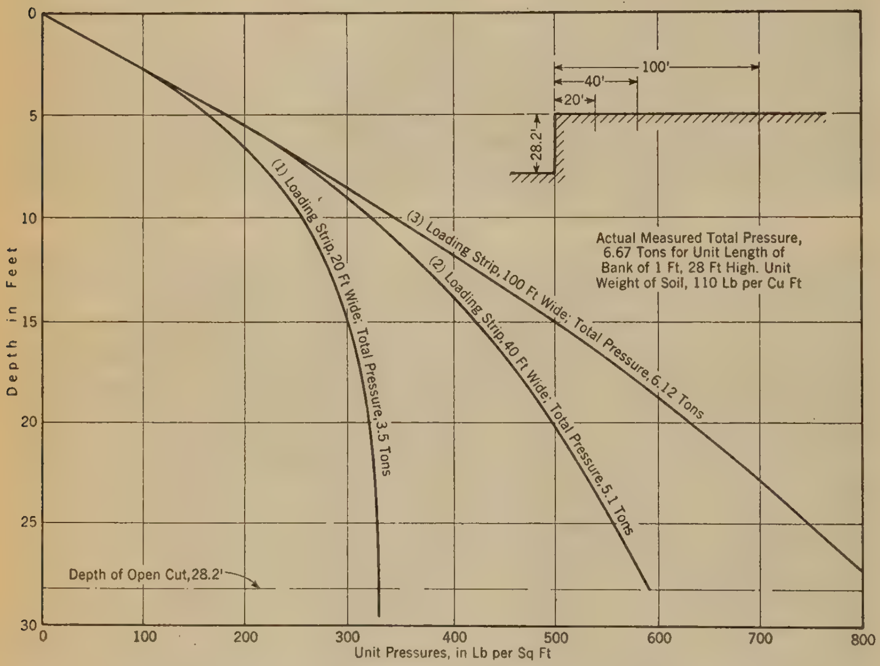

FIG. 21.—COMPUTED LATERAL EARTH PRESSURE FOR LENGTH OF BANK OF 1 FOOT, USING EQUATION (14) AND THE RECTANGULAR LOADING IN FIG. 3(*b*)

In comparing the calculated value with measured values, allowance must be made for the modifying effect of wedging the upper braces, which tends to throw the lateral load high. It will be seen that some of the curves approximate quite closely the measured results of Fig. 20 with reasonable values of μ, Poisson's ratio.

It may be objected that the writers are modifying their equations in two ways by limiting the elastic effect to a material between the sheeting and an arbitrary vertical plane and also by selecting a favorable value of μ. To this they must plead that they are applying Stevenson's "transcendental sense," or applying a coefficient of their own in estimating obscure forces.

The writers believe that their method of approach conforms to nature much more closely than others; and that, by simple and rational assumptions which they often have made, an economical and safe system of bracing can be devised. They have done this repeatedly and on a large scale without untoward results. However, this must be accompanied by good workmanship, thorough drainage, by the use of open sheeting, by thorough packing of the sheeting, and by the entire elimination of quick or boiling ground in every stage of the work.

An erroneous conception of earth and concrete pressures is given by the analogy of fluid pressures, particularly by the use of the coefficient K where earth or concrete material is considered as a fluid, with pressures reduced in accordance with the values of K. Although the total pressure by choosing a proper K may be approximately correct, this gives a totally erroneous concept of the increase of pressure with depth. The Coulomb or Rankine wedge is only a special case and does not explain, adequately, the phenomena of earth pressure; nor does it lead to proper design of supporting sheeting or bracing or retaining walls.

Although the total pressure of a bank or fill may range between fairly wide limits, its manner of application or points of application are profoundly modified by the various rigidities of the supporting members, either sheeting or bracing or retaining walls. The vertical members of a system of bracing act as a foundation, and as the cut deepens they are subjected to increasing loads. These loads may cause serious distortion of the entire bracing system if they do not provide an adequate foundation. The writers do not claim any final word on the subject of lateral earth or concrete pressures, but merely wish to set their successors on the correct path.

However, they must close with some words of warning: It seems simple to build a box and fill it with earth and measure the resultant pressures; and this has been done repeatedly with results varying in value with the experience or "transcendental sense" of the observer. Compare the material deposited in a box, however, with nature's materials. It is disturbed, drained, and consolidated; it has a concrete or wooden floor, a rigid foundation; the material is restrained on the sides and bottom; in practice the wall or sheeting has a toe hold, it is braced arbitrarily, and has flexible sheeting and wales. It may be saturated at times (if not thoroughly drained) and subject to hydrostatic pressure added to dry earth pressures; the foundation may become quick and yielding due to imperfect drainage; and, the wall may move seasonably. The materials and wall are often subject to vibrations from railroad and traffic, compacting the material which follows up every slight movement of wall. It thus happens that retaining walls have the worst record of failures of any engineering structures, and such failures may occur long after they are built; but close observation would probably have shown progressive movements. A retaining wall cannot be designed arbitrarily from any formula but requires that "transcendental sense" necessary to the engineer in the design of all structures, which cannot be safely "picked" from handbooks.

It seems strange that a century and a half after Coulomb, engineers are only on the threshold of knowledge in the field of lateral earth pressures. Still more difficult of analysis are the pressure effects of breaks in materials like silt and soft clays which are difficult to drain and which break in wide and deep spherical masses, creating tremendous disturbances.

The writers wish to comment on the concrete pressures given in Fig. 5. A very rigid form with steel wales and bolts was filled with vibrated concrete of low slump; the bolts were evenly spread and about 5 ft apart; the form was filled to the height of about 40 ft; in the bolts whose tension was measured by a strain gage, a maximum tension was built up corresponding to about 750 lb per sq ft. This tension was retained long after the concrete had set and until the nuts were slackened. Certainly this demonstrated the fallacy of considering concrete as a fluid.

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A THEORY OF SILT TRANSPORTATION

BY W. M. GRIFFITH,[1] ESQ.

WITH DISCUSSION BY MESSRS. JOE W. JOHNSON, G. W. HOWARD, HARRY F. BLANEY, E. W. LANE, O. A. FARIS, J. E. CHRISTIANSEN, SAMUEL SHULITS, GERALD LACEY, GLENN W. HOLMES, HUNTER ROUSE, S. G. BAUER, C. C. INGLIS, AND W. M. GRIFFITH.

SYNOPSIS

A theory of silt transportation is outlined in this paper, and equilibrium equations are presented which it is claimed are applicable to channels of all sizes and shapes, provided the silt load and the bed consist of "loose granular material," and provided certain hydraulic conditions are satisfied. The equations are of special value in problems relating to river-control works and tidal river outfalls; for example, they can be used to estimate the change in bed level that will result from widening or "tightening" a river section, or to determine whether a proposed dredge cut can be expected to maintain itself.

The study of silt transportation originated in the efforts of irrigation engineers to design artificial channels that would carry the heavily silt-laden waters of the rivers which supplied them, without silting their beds or scouring their banks. Attention is still mainly directed to this aspect of silt transportation, but the principles are actually of wider application. The same problem, in general, is involved in river-control works and in the improvement of the outfall of tidal rivers for navigation and drainage purposes.

THE GENERAL THEORY

In a paper[2] published in 1927, the writer advanced a theory of silt transportation, in which it was suggested that changes which occur in the cross-section of rivers and canals after artificial alteration of the existing conditions can be mathematically calculated by reference to a single elementary law. The theory was based on two assumptions: (1) That the transportation of silt is effected by the reaction of vertical eddies or the vertical resultant of other

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NOTE.—Published in May, 1938, *Proceedings*.

[1] Cambridge, England.

[2] "A Theory of Silt and Scour," by W. M. Griffith, *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927).

eddies; and (2) that the level of the bed at every point in a cross-section is governed by the law of silt transportation.

This theory is applicable to any channel section in which: (1) The bed of the channel is composed of loose granular material incapable of substantial shear resistance; (2) the silt load is of the same material; and (3) the flow is turbulent and follows the boundary surfaces.

The elementary law of silt transportation may be briefly stated thus: In a given cross-section of a stream, the condition for equilibrium at any point in the bed is given by the relation,

$$v = c\, d^n \qquad (1)$$

in which v is the mean velocity in the vertical through the point in question; d is the depth at that point; c is a coefficient varying with the amount of silt load carried; and n is an exponent the numerical value of which must slightly exceed 0.5.

Consider a naturally formed river section in which flow is taking place under the conditions stated (see Fig. 1). This may be segmental, semi-elliptical, parabolic, or of similar shape, but for simplicity's sake it will be designated parabolic. It is claimed that from Chezy's basic equations of flow for unerodible perimeters, the average velocities in the vertical planes through the points, A, B, C, etc., across the section will vary as the square root of the depths at those points; that is, as $d_1^{0.5}$, $d_2^{0.5}$, $d_3^{0.5}$, etc.[3] Now, if the silt load is uniformly distributed across the section, the velocities required for equilibrium under the law of silt transportation will vary as d^n. Hence, if the true value of n is 0.5, all points in the cross-section will be in equilibrium whatever its shape may be—deep and narrow, or broad and shallow; but if the value of n is either greater or less than 0.5, there can be only one value of d at which equilibrium is attained.

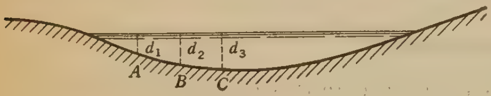

FIG. 1

Assume this value to occur at the mean depth of the parabolic section. Then, if the true value of n is less than 0.5, in the deep center the actual velocity will exceed that required for equilibrium by Equation (1) and in the shallow sides it will be less. The section, therefore, will tend to scour in the center and shoal near the sides and thus become deeper and narrower. If the true value of n is greater than 0.5, the reverse will be the case and the channel will tend to flatten out into a broad, shallow cross-section.

As channel sections carrying heavy silt loads do tend to flatten out, it is argued that the value of n must be greater than 0.5. However, observation shows that parabolic sections flowing under the conditions stated do have some stability; hence, the value of $(n - 0.5)$ cannot be great.

A parabolic section can be in equilibrium at all points if the silt load varies across the section directly as $\frac{v}{d^n}$, or, in other words, as $\frac{1}{d^{n-0.5}}$, since $\frac{v}{d^{0.5}}$ is constant across the section by the hydraulic law. If the value of $(n - 0.5)$ is small, this required variation in silt load across the section will not be great. By depositing silt where the actual velocity is in deficit, and picking it up by scour where the velocity is in excess, the stream will tend to adjust its silt load at different points across the section to that distribution which will give stability.

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[3] *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), pp. 245, 246, and 279.

Cross-eddies redistributing the silt charge will tend to upset this state of equilibrium. Hence, a channel of parabolic section, with a bed of sand and a silt charge of the same material, can have no permanent stability, and its temporary stability will depend on such absence of cross-eddies as can be gained by the channel being straight and of uniform section.

If a channel with an erodible bed is provided with smooth, unerodible sides, it can achieve stability only by flowing at one depth—that is, by forming a level bed across the section—and it invariably does so. A common example is found in the beds of irrigation canals which have to carry silt loads of sand in their upper reaches. An example will also be found in the beds of tidal rivers, either near their outfall, where the sides are lined, or at the outfall, where unerodible sides are provided by training walls.

Beyond the training walls, where no unerodible sides exist, any channel forming in the sand bed must of necessity have a parabolic section, but such channels have only a temporary stability. This stability is increased to some extent by the deposit, at the sides, of colloids, which produce a material having some shear resistance.

DEVELOPMENT OF EQUATIONS

It follows from Equation (1) that,

$$V_m = f\,c\,(d_m)^n \qquad (2)$$

in which V_m is the mean velocity of the whole cross-section and d_m is the mean depth; f is a factor depending on the shape of the cross-section. $f = \frac{V_m}{v_m}$, in which v_m is the mean velocity in the vertical through the point where the depth is d_m.

The value of f can be mathematically determined for any known type of cross-section. Thus, for a rectangular cross-section, $f = 1.0$; for a triangular cross-section, $f = 1.13$; and for a natural river cross-section,[4] its value will differ little from 1.1.

If B = the width at water surface, and Q = the discharge, then, $Q = V_m\,B\,d_m$, or,

$$V_m = \frac{Q}{B\,d_m} \qquad (3)$$

Substituting in Equation (3) the value of V_m from Equation (2),

$$\frac{Q}{f\,c} = B\,(d_m)^{n+1} \qquad (4)$$

Consider any two cross-sections of a channel, sufficiently close to each other to insure the value of Q and c to be unchanged. Let d_{m1} and d_{m2} be the mean depth of these two sections, and B_1 and B_2 their surface widths. Since for the same type of cross-section the value of f is constant, and since $\frac{Q}{f\,c}$ is constant, from Equation (4),

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[4] "A Theory of Silt and Scour," by W. M. Griffith, *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), p. 253.

$$B_1\,(d_{m1})^{n+1} = B_2\,(d_{m2})^{n+1}$$

Therefore,

$$\frac{B_1}{B_2} = \left(\frac{d_{m2}}{d_{m1}}\right)^{n+1} \tag{5}$$

or,

$$d_{m2} = d_{m1}\left(\frac{B_1}{B_2}\right)^{\frac{1}{n+1}} \tag{6a}$$

Equation (6*a*) gives the values of changes in depths resulting from widening or tightening a river section when the types of the two cross-sections are similar.

Where the type of cross-section is changed, as for example from the parabolic to a rectangular type, as will occur at the site of a weir or bridge, the value of f will not be constant and Equation (6*a*) becomes,

$$d_{m2} = d_{m1}\left(\frac{f_1\,B_1}{f_2\,B_2}\right)^{\frac{1}{n+1}} \tag{6b}$$

in which f_1 and f_2 are the shape factors of the cross-sections at d_1 and d_2, respectively.

For a change from a natural river cross-section to a rectangular cross-section, the value of $\frac{f_1}{f_2}$ will approximate to 1.1.

In the fundamental equation (Equation (1)) since c varies with the silt load, it can be expressed in terms thereof. Let c_1 be the silt load of loose granular material (excluding colloids), expressed in terms of parts per 10 000 by weight, that is carried in the vertical above a point on the bed of a channel. Then,

$$c_1 = z\,\frac{v}{d^n} \tag{7}$$

in which z is a new coefficient and,

$$c_1 = z\,c \tag{8}$$

The term, "loose granular material," is intended to include shingle or sand down to an order of fineness of about $\frac{1}{500}$ in. in diameter, but it does not include light materials carried in suspension by all eddies—such as clay, organic matter, and sand of an extreme order of fineness. All such light materials, for the purpose of this paper, are classed as colloids because they do not fulfill the primary condition that their transportation is dependent on the vertical eddies alone.

If the value of $\frac{v}{d^n}$ is sufficiently high to transport the loose granular material, the colloids will remain in suspension and cannot be deposited, so they do not affect the bed. For this reason the beds of channels in which loose granular material is being transported will not be found to contain colloids in any quantity. Some quantity of fine sand, which by comparison with the loose granular material is of a second order of smallness, will be found within the interstices of the latter, where it finds shelter. However, the quantity will not exceed the natural volume of the voids, and so will not affect the bulk of the bed material, or, consequently, the level of the bed at any point.

If the values of n and z are known, Equations (1), (2), (5), (6a), and (7) offer a mathematical solution to many river and canal problems.

The Value of n.—The value of n can be determined by a graphic analysis of the values of V_m and d_m of a series of channels of different depths, all carrying the same quantity of silt load and all having beds in equilibrium under this silt load.

This method was adopted by R. G. Kennedy in his classical paper.[5] However, Kennedy analyzed the data in terms of the maximum depth of the section, instead of the mean depth, because he made no fundamental assumption, and his analysis was purely for the purpose of discovering an empirical relationship applicable to a number of irrigation canals, all having more or less the same type of cross-section. A number of other observers followed the same process, but although the channels studied by any one observer may have been of a single type of cross-section, they were not necessarily similar to those studied by Kennedy or by any of the others.

Kennedy's value for the exponent, n, was 0.64, and other observers have arrived at values[6] varying from 0.44 to 0.727. As the beds of all the channels studied appear to have been of sand, such a large variation in the value of the exponent is illogical, because model experiments indicate that normal differences in the sizes of grains in the wetted perimeter do not materially affect the coefficient of roughness, or the conditions of flow.

Of course, if boulders are present in a river bed, they form definite obstructions to flow, of a different order of size, and the hydraulic conditions are altered. Their effect can be observed, for example, in time of flood, when the water surface will be found to be irregular, having the appearance of a series of waves, and indicating flow of a different order of turbulence. Hence, for shingle and boulder beds, a different value for n might be anticipated.

In his paper of 1927,[7] the writer found the value of n in Equation (2) to be equal to 0.59, approximately, by a graphic analysis of Kennedy's data in terms of d_m instead of d. In examples given in that paper, relating to sites on the Jumna and Ganges Rivers, which have beds of boulders and shingle, the writer assumed a value for n of 0.64. In 1930, a similar analysis of both Kennedy's data and the Madras[8] data showed that a value of n of 0.57 fitted both sets of data with reasonable accuracy.[9] Further investigation has led to the conclusion that this value may be accepted as approximately correct for beds of medium to fine sand. With regard to beds of shingle and boulders, a rough analysis of some flood discharges of the Jumna River gave a value of 0.65.

The Value of z.—Having now obtained a provisional value for n, the value of z in Equation (7) can be found from observations of silt content, velocities, and depths.

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[5] "Hydraulic Diagram for Canals in Earth," by R. G. Kennedy, *Minutes of Proceedings*, Inst. C. E., Vol. 119 (1895), p. 281.

[6] "Stable Channels in Erodible Material," by E. W. Lane, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 127.

[7] *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), p. 279.

[8] "Critical Velocity Observations," Public Works Dept., Irrigation Branch, Madras, India, 1912.

[9] *Minutes of Proceedings*, Inst. C. E., Vol. 229 (1927), pp. 321, and 322.

An examination of the very complete collection of silt analysis and hydraulic data in a valuable paper by Harry F. Blaney, M. Am. Soc. C. E., and the late Samuel Fortier, M. Am. Soc. C. E.,[10] together with certain silt-sample analyses from the waters of the Wash at the tidal outfall of the River Great Ouse (which have been made under the writer's supervision), has led to the conclusion that a value of z of 7.5 in Equation (7) fits both sets of data with reasonable accuracy. This value is, therefore, provisionally recommended for graded sand (colloids being omitted); however, further investigation is required to determine how this value is affected by the size of the sand carried, and how far, if at all, it is affected by colloids if present in large quantities.

Substituting these values for n and z, Equations (1), (2), (6a), (6b), and (7) become, respectively,

$$v = c\, d^{0.57} \qquad (9)$$

$$V_m = f\, c\, (d_m)^{0.57} \qquad (10)$$

For similar types of section,

$$d_{m2} = d_{m1} \left(\frac{B_1}{B_2}\right)^{0.637} \qquad (11)$$

For dissimilar types of section,

$$d_{m2} = d_{m1} \left(\frac{f_1\, B_1}{f_2\, B_2}\right)^{0.637} \qquad (12)$$

and,

$$c_1 = \frac{7.5\, v}{d^{0.57}} \qquad (13)$$

CHECK OF EQUATIONS WITH OBSERVATIONAL DATA

Canals.—The examination of silt analyses just referred to is given in Tables 1 to 4. Table 1 shows an analysis of the silt load in a cross-section of the Central Main Canal (Imperial Valley Canals, California). Columns (1), (2), (3), (4), and (6) are taken from Table 21 of the paper by Messrs. Fortier and Blaney,[10] and Columns (5), (7), (8), and (9) have been computed by the writer.

Column (7), Table 1, gives the arithmetic means of the four silt-load observations at each station. However, owing to the relatively high concentration of silt load near the bottom of the channel at certain stations, this arithmetic mean does not always give an accurate value of the true mean silt load. Hence, the observations at each station were plotted, as in Fig. 2, and the mean silt loads determined graphically from these plots are given in Column (8). It will be seen that where the relative value of the bottom silt load is high, the value of the arithmetic mean is much in excess of the mean value found graphically.

In Column (9), the values of $\frac{7.5\, v}{d^{0.57}}$ are entered, and a comparison of Columns (8) and (9) shows that a close agreement exists between the actual silt load and the load calculated from the hydraulic conditions by Equation (13).

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[10] "Silt in the Colorado River and Its Relation to Irrigation," *Technical Bulletin No. 67* (1928), U. S. Dept. of Agriculture.

TABLE 1.—DISTRIBUTION OF SILT IN A CROSS-SECTION OF CENTRAL MAIN CANAL, IMPERIAL VALLEY CANALS

Distance between station and bank, in feet	DEPTHS, IN FEET		VELOCITIES, IN FEET PER SECOND		PROPORTION OF SILT BY WEIGHT RETAINED BY SIEVE NO. 200, EXPRESSED IN PARTS PER 10 000			Values of $\frac{7.5\,v}{d^{0.57}}$
	Total (d)	To sampling point	At sampling point	v	Sample	Arithmetic mean	Graphic mean	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
6	5.3	0.5	2.57	2.36	2.2	6.02	5.8	7.0
		2.0	2.81		5.4			
		3.5	2.27		5.9			
		5.0	1.79		10.6			
10	5.8	0.5	3.01	3.11	1.6	7.6	8.6	8.6
		2.0	3.30		8.6			
		3.5	3.11		6.5			
		5.0	3.01		13.7			
14	5.1	0.5	3.50	3.29	1.7	8.25	7.1	9.8
		2.0	3.35		2.8			
		3.5	3.35		8.0			
		5.0	2.67		20.5			
22	4.7	0.5	3.55	3.18	2.6	12.1	10.3	9.8
		2.0	3.60		13.3			
		3.5	3.21		8.6			
		4.6	2.38		23.9			
26	4.8	0.5	3.35	3.21	1.8	16.7	9.1	9.94
		2.0	3.50		3.5			
		3.5	3.16		8.5			
		4.7	2.82		52.9			
30	4.8	0.5	2.82	2.92	1.1	14.4	8.8	8.9
		2.0	3.26		6.3			
		3.5	3.01		8.8			
		4.7	2.58		41.4			

Table 2 gives similar data for the Dahlia Canal, the observed data being taken from Table 22 of the Fortier and Blaney paper.[10] In this case, owing to the fact that only three silt samples were taken in the total depth, the graphic method of arriving at the mean silt content is not possible. Where the bottom values are relatively high, the arithmetic mean must be in excess of the true mean, which would account for the fact that the higher values in Column (7) are in excess of those in Column (8).

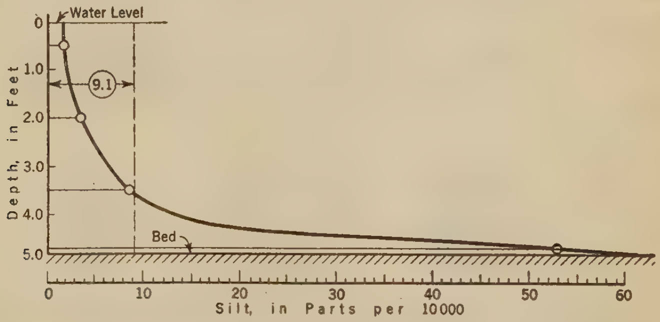

FIG. 2.—TYPICAL GRAPHIC DETERMINATION OF MEAN SILT LOAD

TABLE 2.—DISTRIBUTION OF SILT IN CROSS-SECTION OF DAHLIA CANAL

Distance between station and bank, in feet	DEPTHS, IN FEET		VELOCITIES, IN FEET PER SECOND		PROPORTION OF SILT BY WEIGHT RETAINED BY SIEVE NO. 200, EXPRESSED IN PARTS PER 10 000		Values of $\frac{7.5\,v}{d^{0.67}}$
	Total (d)	To sampling point	At sampling point	v	Sample	Arithmetic mean	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.5	1.6	0.20	1.34	1.26	1.1	9.2	7.3
		0.96	1.39		5.3		
		1.40	1.05		21.1		
4.5	1.2	0.20	1.74	1.75	1.3	21.5	11.9
		0.72	1.98		13.3		
		1.00	1.54		50.0		
7.5	1.2	0.20	2.13	1.93	1.5	27.8	13.2
		0.70	2.08		9.1		
		1.00	1.59		72.8		
10.5	1.2	0.20	1.64	1.77	0.7	17.3	12.0
		0.72	1.98		11.1		
		1.00	1.69		40.2		
13.5	1.0	0.20	1.44	1.42	1.2	12.2	10.65
		0.60	1.49		4.5		
		0.80	1.34		30.8		

Table 3 gives similar figures for other canals and for the Colorado River. In these cases the values of the bottom samples are not relatively high and the silt contents given by the arithmetic mean (Column (7)) would be more nearly correct. The very close agreement between the entries in Columns (6), covering a fairly wide range of depths and velocities, is somewhat remarkable.

TABLE 3.—ANALYSIS OF SILT DISTRIBUTION IN VARIOUS CANALS AND IN THE COLORADO RIVER AT YUMA, ARIZONA

(Observations made on or near center vertical of cross-section)

Serial No.	REFERENCE TO *Technical Bulletin**		Name of canal or river	Total depth, d, in feet	v, in feet per second	Proportion of silt by weight retained by Sieve No. 300, expressed in parts per 10 000 (arithmetic mean)	Values of $\frac{7.5\,v}{d^{0.67}}$
	Table No.	Page					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	26	50	Ash Canal	3.2	3.22	12.1	12.38
2	27	51	Birch Canal	1.8	2.21	11.0	11.93
3	28	51	Alamo Canal	10.0	4.1	10.1	8.3
4	54	74	Colorado River	11.0	5.71	16.74	11.0
5	55	75	Alamo Canal	10.0	4.1	8.64	8.26
6	56	75	Brawley Canal	4.4	2.68	8.44	8.63

* "Silt in the Colorado River and Its Relation to Irrigation," *Technical Bulletin No. 67* (1928), U. S. Dept. of Agriculture.

Tidal River.—The Wash, at the outfall of the River Great Ouse, is shown in Fig. 3; Points *A*, *B*, and *C* indicate the stations at which observations were made. Series of observations at each of these points, covering various parts of the tidal cycle, are recorded in Table 4.

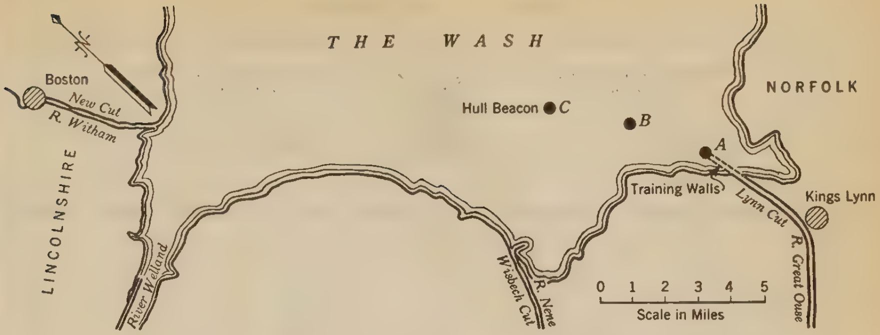
THE WASH
Boston
New Cut
R. Witham
LINCOLNSHIRE
River Welland
Hull Beacon C
B
A
Training Walls
Lynn Cut
NORFOLK
Kings Lynn
R. Great Ouse
Wisbech Cut
R. Nene
0 1 2 3 4 5
Scale in Miles

FIG. 3.—OUTFALL OF THE RIVER GREAT OUSE, SHOWING SAMPLING STATIONS

TABLE 4.—STUDY OF SILT DISTRIBUTION AT TIDAL OUTFALL OF RIVER GREAT OUSE

Sample No.	DEPTHS, IN FEET AND INCHES		Surface velocity, v_s, in feet per second	SILT CONTENT, IN PARTS PER 10 000		Value of $\frac{7.5\, v_s}{d^{0.57}}$
	To sampling point	Total		Sample	Mean	
(a) OBSERVATIONS AT SITE *B*, TAKEN DURING AN AVERAGE NEAP TIDE, FROM LOW WATER TO HIGH WATER						
1	3–0	10–0	0	1.23	1.18	0
2	7–0			1.14		
3	3–0	9–0	0.31	1.05	1.21	0.663
4	6–0			1.37		
5	3–0	9–6	1.5	0.88	1.18	3.12
6	6–0			1.48		
7	3–0	9–0	1.7	2.72	3.32	3.64
8	6–0			3.92		
9	7–6	11–3	1.9	3.54	3.51	3.53
10	3–6			3.48		
11	7–6	11–6	2.23	4.22	3.92	4.14
12	4–9			3.62		
13	4–8	11–0	2.33	3.06	4.01	4.43
14	8–0			5.0		
15	4–0	12–0	2.3	3.95	4.26	4.18
16	8–0			4.58		
17	5–6	16–0	3.0	3.89	3.73	4.58
18	11–0			3.58		
19	12–0	17–6	1.88	3.24	2.86	2.74
20	6–0			2.48		
21	7–0	21–0	1.47	2.35	2.48	1.93
22	14–0			2.61		
23	7–6	22–0	1.30	1.43	1.5	1.66
24	14–9			1.575		
25	8–0	24–0	0.70	1.43	1.31	0.855
26	16–0			1.19		
27	7–10	23–6	0.40	....	1.33	0.494
28	15–10			1.13		

TABLE 4.—(*Continued*)

Sample No.	Depths, in Feet and Inches		Surface velocity, v_s, in feet per second	Silt Content, in Parts per 10 000		Value of $\frac{7.5\, v_s}{d^{0.57}}$
	To sampling point	Total		Sample	Mean	
(*b*) Observations at Site *B*, Taken During a Low-Spring Tide, on the Latter Part of the Ebb						
61	5–0	15–0	2.21	2.4	5.0	4.95
62	10–0			7.6		
63	3–8	11–0	2.80	2.6	6.65	7.7
64	7–4			10.7		
65	3–0	9–0	2.6	8.32	6.63	6.8
66	6–0			4.94		
69	Surface	6–6	1.8	4.65	6.28	7.5
70	3–0			6.30		
71	6–0			10.90		
72	Surface	7–0	1.1	4.43	4.48	4.68
73	3–6			4.67		
74	6–6			4.45		
75	Surface	7–6	0.8	5.28	5.26	3.66
76	3–6			5.58		
77	7–0			4.93		
78	Surface	7–6	Nil	2.66	3.62	Nil
79	3–9			3.42		
80	7–0			4.78		
(*c*) Observations at Site *A*, Taken During a Low Spring Tide, on the Ebb (Rough)						
81	8–0	25–0	2.93	3.39	2.41	3.51
82	16–6			1.93		
83	24–0			1.92		
84	Surface	22–0	2.65	1.70	3.74	3.40
85	21–6			3.60		
86	14–0			3.58		
87	7–0			2.35		
88	Surface	22–0	3.22	2.31	3.3	4.10
89	21–6			4.20		
90	14–6			3.59		
91	7–3			3.10		
92	17–6	81–0	3.20	3.57	3.66	4.45
93	Surface			3.24		
94	6–0			3.80		
95	12–0			4.02		
96	Surface	16–0	3.56	3.80	4.87	5.44
97	15–6			5.85		
98	5–3			4.92		
99	10–6			4.92		
100	4–8		4.16	4.87	6.0	6.94
101	13–6			6.92		
102	10–4			6.20		

The velocities in Table 4 are surface velocities. Observation indicates that the relationship between the surface velocity, v_s, and the mean velocity in a vertical plane, v, agrees roughly with Bazin's relationship between the maximum surface velocity and the mean velocity of the entire section; that is, the ratio of v_s to v is on the order of 1.28 to 1. On the other hand, the silt contents given

TABLE 4.—(*Continued*)

Sample No.	Depths, in Feet and Inches		Surface velocity, v_s, in feet per second	Silt Content, in Parts per 10 000		Value of $\frac{7.5\, v_s}{d^{0.57}}$
	To sampling point	Total		Sample	Mean	
(*d*) Observations at Site *C*, Taken During a Neap Tide, from Low Water to High Water						
30	3–0	6–0	0	1.63	1.63	0
31 32	2–2 4–4	6–6	1.55	1.93 1.55	1.74	0.26
33 34	2–9 5–6	7–3	0.33	2.75 2.80	2.77	0.8
35 36	2–10 5–8	8–6	0.43	2.23 2.00	2.11	0.96
37 38	5–2 2–7	7–9	0.533	2.60 2.58	2.59	0.776
39 40	5–6 2–9	8–3	0.46	2.61 2.45	2.53	1.03
41 42	6–6 3–3	10–0	0.87	2.20 1.82	2.01	1.78
43 44	3–8 7–4	11–0	1.43	1.93 2.83	2.38	2.72
45 46	4–8 9–4	14–0	1.66	2.56 3.03	2.80	2.77
47 48	5–8 11–4	17–0	1.7	2.67 3.55	2.86	2.5
49 50	6–0 12–0	18–0	1.6	(?) 3.04	....	2.05
51 52	6–8 13–4	20–0	1.13	2.60 2.85	2.72	1.52
55 54	7–0 14–0	21–0	0.7	2.68 2.73	2.70	0.916
55 56	7–0 14–0	21–0	0.51	2.73 2.76	2.74	0.665
57 58	6–10 13–3	20–6	0	1.45 2.27	1.86	0
59 60	6–10 13–8	20–6	0	2.37 2.29	2.33	0

in Table 4 include colloids, because apparatus for the separation of the colloid portion was not available.

Owing to the tendency of salt water to deposit colloids, the colloid content in these samples was not great. However, it must be greater on the ebb tide, or outflow, than on the flood tide, or inflow. It will be less, the farther out from Point *A* (Fig. 3), the observation point is and will be small at Site *C*.

Fig. 4 shows the variation in silt content at Site *B* on a neap tide, during the period from low water to high water, throughout which the velocities and depths were changing. (These observations were taken from a boat at anchor and the length of anchor chain permitted of some change of position under the varying flow, so the samples were not all taken at exactly the same spot.)

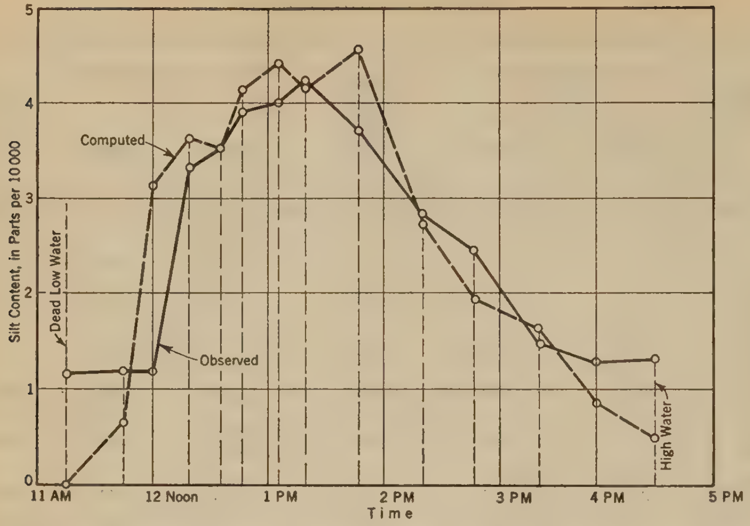

FIG. 4.—SILT CONTENT AT SITE *B*; AVERAGE NEAP TIDE, LOW WATER TO HIGH WATER

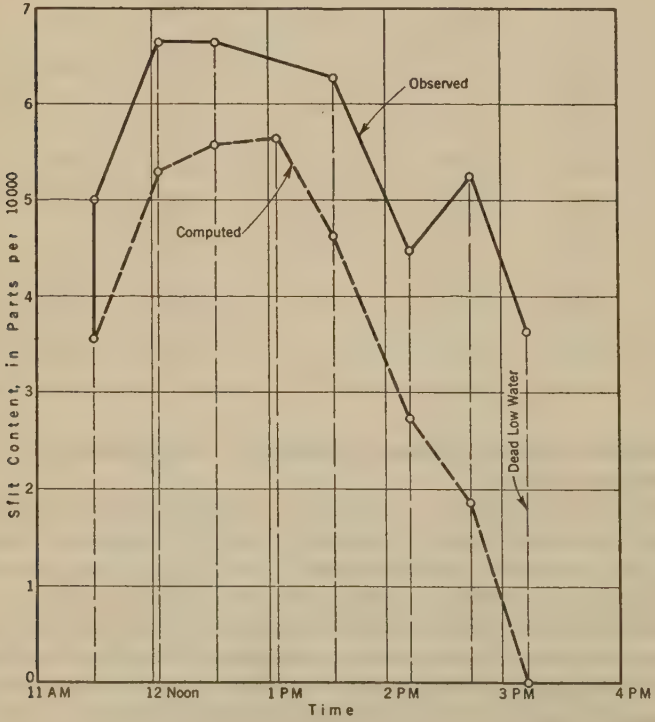

FIG. 5.—SILT CONTENT AT SITE *B*; LOW SPRING TIDE, EBB

The curve of values of $\frac{7.5\, v_s}{d^{0.57}}$ is also plotted. It will be noticed that the two curves show a close agreement, but that there is a time lag of about 30 min between them, as might be expected. Details of these observations are given in Table 4(*a*).

Fig. 5 gives similar curves at the same site for an ebb tide. Again, the curves are similar, but in this case the observed silt content is continuously in excess of that computed by formula. This fact is presumably due to the increased quantity of colloids on the ebb flow. The values are tabulated in Table 4(*b*).

Fig. 6 and Table 4(*c*) give observations taken for part of the ebb flow at Site *A*. In this case, despite the fact that owing to the ebb flow the colloid content would be relatively great, the computed curve lies above the curve of observed silt content. In explanation it may be stated that at Site *A* conditions are changing so rapidly that the actual silt content lags behind that computed from the hydraulic conditions.

Fig. 7 and Table 4(*d*) give similar curves for a neap tide, from low water to high water at Site *C*. At that site, velocities are lower and depths greater, and there is a considerable increase in wave action. This wave action, in the shallow waters of the Wash, creates a turbulence sufficient to maintain a considerable silt content even when the velocity is zero. Under such conditions, the hydraulic factors have little effect on the silt content.

The turbulence created by wave action depends on the height of the waves, which, in turn, depends on the area of the water surface. This area is greater at high tide than at low tide on account of the area of sand banks then exposed, and explains why the silt content is greater at high water than at low.

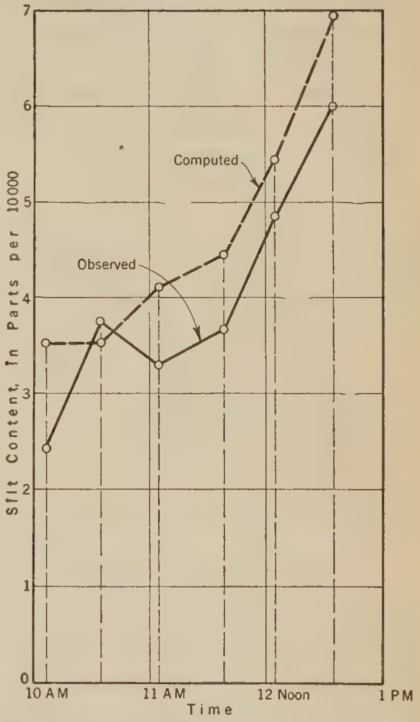

FIG. 6.—SILT CONTENT AT SITE *A*; LOW SPRING TIDE, EBB (PART ONLY)

APPLICATION OF THEORY

Canals.—In canal systems required to carry quantities of graded sand together with colloids as silt loads, the conditions named apply in the upper reaches. In the lower reaches much of this graded sand may have passed out of the system and a different condition may pertain.

The difference between these two conditions may be seen from the difference in the type of cross-section. In the upper reaches the section will be nearly rectangular, the bed being approximately level except close to the sides, where the reduction of velocity resulting from the horizontal eddies from the sides will cause a slight rise of bed. In the lower reaches, if the silt load consists of light material only, the bed will be curved.

In turbulent flow within stream-lined boundaries the eddies in general are a function of R, the hydraulic radius. However, the vertical eddies (or the vertical resultants of the eddies) are a function[11] of d_m. Therefore, if the silt content is largely colloidal, the silt transporting power will be a function of R^n instead of $(d_m)^n$, and the lighter the material the more closely will the value of n approximate to 0.5. Under such conditions the theory outlined in this paper is not applicable. However, the light material found in the lower reaches of canal systems usually deposits slowly, and when excavated forms a useful material for repairing banks; hence, it is often more beneficial than harmful, and the problem of its transportation is usually not a serious one. The presence of clay insures binding, and the very fineness of the sand appears to give it a shear resistance not present in coarser sand.

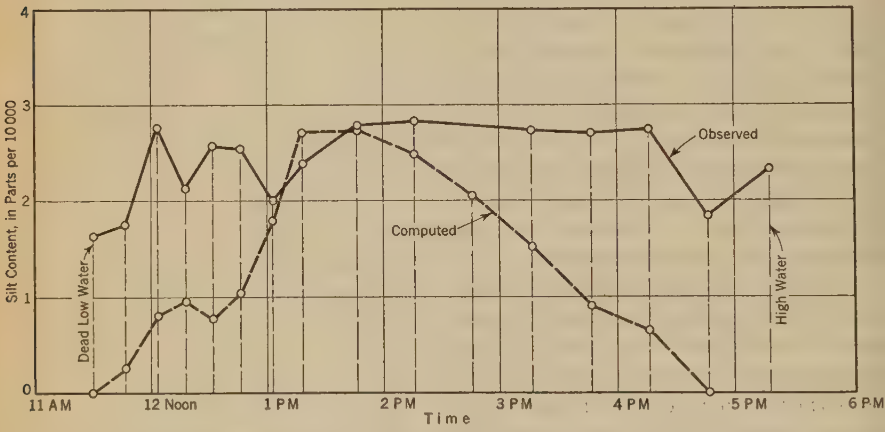

FIG. 7.—SILT CONTENT AT SITE *C*; AVERAGE NEAP TIDE, LOW WATER TO HIGH WATER

Where the present theory is applicable, the condition required to insure that the full silt load of the parent stream will be carried by the offtake canal is that the value of $\frac{V_m}{(d_m)^n}\left(\text{or, for ordinary silt, } \frac{V_m}{(d_m)^{0.57}}\right)$ of this offtake canal shall not be less than the value of this ratio in the parent channel. The question of whether the inflow to the canal actually has a higher or a lower silt concentration than the parent channel itself, depends on the hydraulic conditions at the offtake. By scientific design of the offtake, it is possible to reduce the silt load greatly in the offtake channel.

Again, where the silt charge is heavy, and the surface slope limited, it is desirable to provide a canal section having the maximum silt-carrying capacity. Such a section is secured if the ratio $\frac{V_m}{(d_m)^n}$ has a maximum value, and the width and depth of bed giving the maximum value for this ratio can be found by plotting its values for different types of sections.[12]

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[11] *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), pp. 276 and 277.

In the foregoing it has been assumed that the most suitable side slopes for the channel have been predetermined. In practice, the correct value for the side slopes is found by experience, and will depend on such factors as: (1) Vegetation at the sides; (2) amount of colloids in the silt content; (3) angle of repose of the natural material; (4) velocity; and (5) ruling radius of curvature.

It is not claimed that this or any theory of silt transportation can solve all silt difficulties of canals. The writer's experience indicates that most cases of silt trouble are most easily and cheaply dealt with by reducing the silt content at the head of the canal by a more scientific design of the intake works. The writer considers that Kennedy's original theory, by concentrating so much attention on the detail of the channel design, has diverted attention from the study of the silt distribution at the offtake, where the true solution to the silting problem may usually be found. The Imperial Valley Canals appear to be a case in point.

River Control Works and Tidal River Outfalls.—As already stated, Equations (6*a*), (6*b*), or Equations (11) or (12), provide a means of determining changes in bed level which may be anticipated from widening or "tightening" existing river sections. An example is afforded by the three cross-sections of the Hundred Foot River given in Fig. 8. Approximately $\frac{5}{8}$ mile of this river, between Cross-Section No. 108 and Cross-Section No. 114, was widened ill advisedly. Cross-Section No. 111 is a typical cross-section of this widening. The elevation of the water surface varies, but the mean water level is about as shown in Fig. 8. The necessary hydraulic data for testing Equation (11) are given in Table 5.

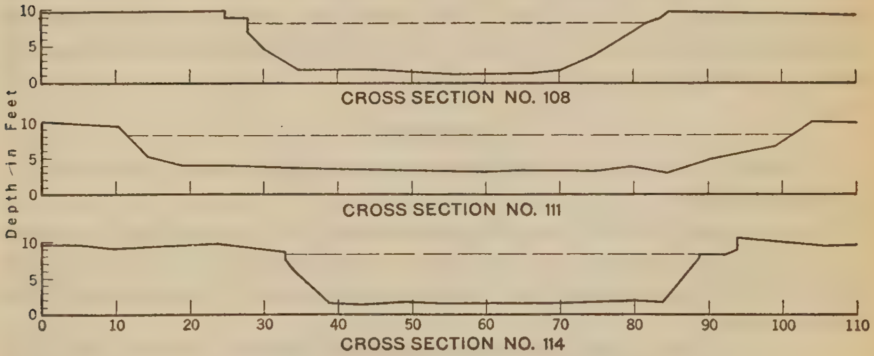

FIG. 8.—CROSS-SECTIONS OF HUNDRED FOOT RIVER

In Equation (11), let B_1 and d_{m1} be the mean values of B and d_m, respectively, for the unwidened sections, and let B_2 be the width of the widened section. Then, $d_{m2} = d_{m1}\left(\frac{B_1}{B_2}\right)^{0.637} = 5.725\left(\frac{54.9}{89.8}\right)^{0.637} = 4.18$ ft, the calculated value of the mean depth of the widened section. This value compares with an observed mean depth of 4.08 ft, a difference of less than 3 per cent.

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[12] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 184 and 185.

As a general example of the use of this theory in tidal river outfall problems, mention again may be made of the River Great Ouse. In recent years the tidal section of that river has silted sufficiently to threaten the safety of the low-lying fen country which it drains. Several types of remedial measures have been proposed, but opinions have differed as to their respective merits.

TABLE 5.—Hydraulic Data for Hundred Foot River

Cross-Section No.	B, in feet	Area, in square feet	d_m, in feet	Remarks
108	53.8	304	5.65	Unwidened sections
114	56.0	325	5.80	Unwidened sections
...	54.9	...	5.725	Mean value of unwidened section
111	89.8	366	4.08	Widened section

By the use of the present theory, a study of the hydraulic conditions of the non-tidal and tidal sections of the River Great Ouse and its tributaries enables one to trace the true cause of recent changes in bed level and the actual reason for the present silting trouble. Further, the theory enables one to calculate, within limits, the changes in bed which may be anticipated to result from each of the various outfall improvement projects that have been proposed. Finally, it shows the inadvisability of dredging a deeper channel without altering the hydraulic conditions to insure that the greater depth would be hydraulically maintained; and, conversely, that if the hydraulic conditions were properly altered, the dredging would appear to be unnecessary, as the improved hydraulic conditions would insure the required alteration in bed level within the range of strata consisting of loose granular material.

Conclusions

In summary, the following conclusions regarding this theory of silt transportation are noted:

(1) In a channel carrying a silt load consisting of loose granular material, such as sand, and flowing over a bed of the same material, equilibrium at any point in the bed is governed by the velocity and depth at that point, and by the silt load carried.

(2) If any two of these factors are known, the third one can be computed.

(3) Statements (1) and (2) are true for the channel section as a whole, equilibrium being governed by the mean velocity, mean depth, and average silt load across the section.

(4) A channel flowing under these conditions will tend to flow at one uniform depth across the section, the depth which gives equilibrium independent of the volume of flow.

(5) In the case of a channel carrying only very light material (capable of being carried by all eddies, and classifiable as "colloids") and flowing over a bed of the same material, different conditions pertain and the type of cross-section is then different.

DISCUSSION

Joe W. Johnson,[13] Assoc. M. Am. Soc. C. E. (by letter).—The author extends his previous theory of silt transportation and presents supporting data which, in most cases, show excellent agreement between observed and calculated values of silt load. The theory set forth presents a method which, with certain limitations, appears to permit the silt load of a stream to be computed from easily observed factors. The method should be a valuable aid in many studies concerned with the transportation of sediment.

The various examples given in the paper cover, rather completely, the various conditions that may prevail in practice. Other data which represent conditions slightly different from those presented by the author are given in Table 6. These data were computed from observations made in Italy by Mario Giandotti[14] on the Po River and Francesco Sensidoni[15] on the Reno River, and, in Texas, by O. A. Faris,[16] M. Am. Soc. C. E., on the Brazos River. Unfortunately, only the data of Mr. Faris include a mechanical analysis of the suspended material, thereby permitting colloids to be excluded from consideration.

The observations on the Po River are interesting in that the two gagings were taken in the autumn and in the spring at practically the same stage; yet the total silt load in the latter gaging was about four times that in the former. As mentioned by Professor Giandotti, in the spring gaging the stream carried a large quantity of fine material—material which the author would class as colloids. Referring to Table 6(*a*) it is noted that with the exception of one vertical the calculated value of silt content for the autumn gaging is greater than the observed silt content, although the observed value is for total silt content and, of course, includes colloids. On the other hand, in the spring gaging (Table 6(*b*)) the calculated silt contents are without exception much smaller than the observed silt content, a fact due no doubt to a large quantity of colloidal matter.

Table 6(*c*) gives an analysis of silt distribution in the Reno River, in Italy. In most verticals, very close agreement between observed and calculated values of silt content are noted, although the observed silt load, as in the Po River, includes colloids.

Tables 6(*d*), 6(*e*), and 6(*f*) give calculated and observed values of silt content for the Brazos River, in Texas. In addition to the observed silt content which was retained by a No. 200 sieve, the silt content including colloids is also given. Agreement between observed and calculated mean silt content is good in the verticals near one stream bank but becomes poor as

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[13] Asst. Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[14] "Les Déplacements du Lit du Po," by Mario Giandotti, Le Service Hydrographique Italien, XV Congrès International de Navigation, Venice, September, 1931, p. 297.

[15] "Rilievi e Determinazioni sul Trasporto Solido del Fiume Reno a Malalbergo," by Francesco Sensidoni, *Annali* dei Lavori Pubblici, Anno LXXVI–Fascicolo, N. 1, January, 1938, A. XVI, Rome, Italy.

[16] "The Silt Load of Texas Streams," by O. A. Faris, *Technical Bulletin No. 382*, U. S. Dept. of Agri., September, 1933.

TABLE 6.—Analysis of Silt Distribution

Distance between station and bank, in feet	Total depth, d, in feet	Velocity, V, in feet per second	Silt Content, in Parts per 10 000 (Colloids Included)		Value of $\frac{7.5\,v}{d^{0.57}}$
			Arithmetic mean	Graphic mean	
(1)	(2)	(3)	(4)	(5)	(7)
(*a*) Po River near Roncocorrente, Italy, December 9, 1924 (Width of Water Surface, 754.4 Feet; and Discharge, 61 510 Cubic Feet per Second)					
32.8	11.5	2.17	2.8	2.5	4.1
98.4	12.1	2.87	2.9	2.7	5.2
164.0	14.1	3.05	2.3	2.2	5.1
229.6	17.2	3.14	2.6	2.3	4.7
295.2	20.3	3.61	5.1	3.3	4.9
360.8	24.3	3.68	4.1	2.9	4.5
426.4	27.9	3.99	6.4	4.5	4.5
492.0	31.7	4.27	6.9	4.7	4.5
557.6	33.3	4.35	7.3	4.4	4.4
623.2	33.1	4.63	4.7	3.1	4.7
688.8	33.1	4.36	3.4	2.4	4.5
(*b*) Po River near Roncocorrente, Italy, March 4, 1925 (Width of Water Surface, 767.5 Feet; and, Discharge, 75 850 Cubic Feet per Second)					
78.7	14.8	2.97	12.0	10.6	4.8
144.3	17.7	3.39	14.1	11.8	5.0
209.9	21.2	3.42	13.8	12.6	4.5
275.5	23.6	3.69	11.0	9.2	4.6
341.1	24.9	4.23	12.3	9.9	5.1
406.7	28.5	4.56	14.1	10.1	5.1
472.3	31.5	4.61	16.4	12.2	4.8
537.9	34.4	5.20	17.8	13.0	5.2
603.5	34.1	5.38	17.3	13.3	5.4
669.1	33.1	4.18	12.2	8.8	4.3
734.7	31.1	3.43	7.7	6.2	3.6
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Distance between station and bank, in feet	Total detph, d, in feet	Velocity, V, in feet per second	Silt Content, in Parts per 10 000			Value of $\frac{7.5\,v}{d^{0.57}}$
			Arithmetic mean (colloids included)	Graphic mean (colloids included)	Proportion retained by Sieve No. 200; graphic mean	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(*c*) River Reno, near Malalbergo, Italy. (Width of Water Surface, 142.8 Feet)						
4.9	5.1	0.49	7.78	7.86	....	1.45
13.1	9.8	2.36	7.54	7.55	....	4.81
32.8	8.6	3.12	8.24	7.77	....	6.88
42.6	9.3	3.14	8.12	7.56	....	6.63
52.5	9.5	3.23	8.38	8.06	....	6.71
72.2	8.9	3.17	8.18	7.42	....	6.83
82.0	9.8	3.23	7.49	7.13	....	6.86
91.8	10.0	3.20	6.78	6.75	....	6.45
111.5	10.0	3.05	7.44	6.98	....	6.17
121.4	9.7	3.14	6.58	6.34	....	6.45
131.2	10.3	2.15	6.02	5.82	....	4.27
(*d*) Brazos River, at Rosenberg, Tex., April 16, 1929						
63	18.2	4.39	....	80.1	7.9	6.3
98	20.0	4.42	....	80.9	7.1	6.0
138	21.3	4.65	....	79.9	6.8	6.1
178	24.3	5.98	....	75.4	4.6	7.3
218	23.5	6.77	....	73.7	3.6	8.4
258	25.0	6.44	....	73.1	3.4	7.7
298	12.3	5.76	....	72.1	3.1	10.3
338	3.4	5.13	....	68.5	2.9	19.1
(*e*) Brazos River, at Rosenberg, Tex., May 31, 1929						
68	41.5	5.03	....	72.0	8.1	4.54
(*f*) Brazos River, at Rosenberg, Tex., June 1, 1929						
68	42.2	5.03	....	56.9	8.6	4.47

the opposite bank is approached. Such a condition is probably due to a local distortion in the velocity distribution and will be discussed subsequently.

A factor of more practical importance than whether there is good agreement between calculated and observed mean silt content in the various verticals appears to be whether the total calculated silt load of the stream agrees with the total observed load. Discharges and the observed and calculated silt loads, determined by ordinary methods, for the various canals and rivers for which data were summarized in Tables 1, 2, and 6, are shown in Table 7(*a*). Data from flume studies[17] conducted at the U. S. Waterways Experiment Station, Vicksburg, Miss., are included in Table 7(*b*). As mentioned previously the observed silt loads in the Po and Reno Rivers include colloids, and the difference between observed and calculated values are relatively large; however, except for the March, 1925, gaging in the Po, the Imperial Canals, and Brazos River data do not show agreement appreciably better than the Italian gagings.

TABLE 7.—SUMMARY OF OBSERVED AND COMPUTED SILT LOAD IN VARIOUS RIVERS AND CANALS

(*a*) DATA FROM CANALS AND RIVERS					(*b*) FLUME TESTS			
Canal or river	Discharge *Q*, in cubic feet per second	Silt Load, in Tons per Day			Run	Discharge *Q*, in cubic feet per second	Observed silt load, in tons per day	Table No. in U. S. Waterways Experiment Station *Paper No. 17*
		Observed	Computed	Table No. in this paper				
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)
Dahlia Canal	27	122	74	2	24	1.5	0.30	32
Central Main Canal	486	1 005	950	1	25	2.2	0.99	32
Brazos River	31 000	34 850	64 850	6(*d*)*	16	1.7	0.46	33
Reno River	3 665	6 830	5 680	6(*c*)	17	2.2	0.74	33
Po River, December 9, 1924	61 510	51 500	72 130	6(*a*)	13	1.8	0.58	34
Po River, March 4, 1925	75 850	204 770	90 785	6(*b*)	14	2.2	1.01	34

* Also Tables 6(*e*) and 6(*f*).

An interesting result may be observed by plotting the data in Table 7, logarithmically, so as to obtain a plot of points representing the silt load calculated according to the author's theory. The Brazos River data failed to plot according to that of the other streams, possibly because of the relatively high value of 1.4 for the velocity-head coefficient,[18] α, at this cross-section. Such a high value for α in an open channel is usually found only where the velocity distribution is greatly distorted by obstructions, bends, etc.

For the given data, which cover a wide range of discharge and location of streams, the silt load of loose, granular material (consisting of material larger than $\frac{1}{500}$ in. in diameter and excluding colloids) expressed, in tons per day, is given by the equation,

$$L = 4.0\,Q^{0.89} \qquad (14)$$

in which Q = discharge, in cubic feet per second. That the silt load (including colloids) of a stream may be many times the load expressed by Equation (14) is evident upon examination of Tables 6(*d*), 6(*e*), and 6(*f*), wherein it is observed that 90% of the suspended silt in the Brazos River consisted of particles small enough to pass the No. 200 sieve and thereby can be classed as colloids. Due to the fact that vertical eddies are perhaps not important in the transportation of very fine material, it is doubtful whether a simple relationship between discharge and the silt load of a stream (including colloids) can be formulated. A viewpoint on this subject, as stated by C. P. Vetter,[19] M. Am. Soc. C. E., is:

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[17] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, 1935.

[18] "Velocity-Head Correction for Hydraulic Flow," by Morrough P. O'Brien, M. Am. Soc. C. E., and Joe W. Johnson, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, Vol. 113, No. 7, August 16, 1934, p. 214.

> "As long as the discharge and the slope of the river are constant the intensity of the boundary layer, and the concentration of suspended matter maintained by it, is undoubtedly also constant. The fine material, not available in the bed of the stream for many miles up stream from the point of observation, must originate elsewhere on the river or on its tributaries and it cannot be expected that any fixed relationship should exist between the discharge and the load of the finer material. Bearing in mind that the term, 'saturation,' should probably be used with caution in this connection, the phenomenon can, nevertheless, be illustrated by saying that the stream is saturated with the material freely available in the bed but under-saturated with material finer than that found in the bed.
>
> "Yet another conclusion may be drawn from the above reasoning. If the part of the silt load which lies within the range of sizes abundantly available in the bed bears a fixed relationship to the discharge there is no reason to believe that this relationship will be materially changed if the finer part of the load is partly or entirely eliminated."

In the latter conclusion it might also be noted with reference to the Brazos River data that the load of very fine material may be ten times that of the "loose granular" material and still the load-discharge relationship is not changed provided the abnormal velocity distribution as discussed herein is local in character and responsible for the relatively high value of the calculated load.

The ultimate solution of the problem of transportation of suspended silt appears to be from the approach of fundamental mechanics according to a theory such as that developed by W. Schmidt,[20] enlarged by Professor O'Brien,[21] and applied by J. E. Christiansen,[22] Assoc. M. Am. Soc. C. E. Although the author's theory of silt transportation has little scientific foundation, the important factor is that it apparently "works" and does so under a large variety of conditions. In fact, it describes silt transportation, within certain limits, approximately as well as the Chezy formula describes the flow of water in open channels.

G. W. Howard,[23] Jun. Am. Soc. C. E. (by letter).—Any stimulation of thought with regard to a subject as important as silt transportation is indeed welcome. The data presented as a result of the observations on the River Great Ouse and the Hundred Mile River are both interesting and enlightening.

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[19] "Why Desilting Works for the All-American Canal?" by C. P. Vetter, *Engineering News-Record*, Vol. 118, No. 8, March 4, 1937, p. 321.

[20] "Die Massenaustauch in freien Luft und verwandte Erscheinungen," von W. Schmidt, H. Grand, Hamburg, 1925.

[21] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Hydrology Section, 1933.

[22] "Distribution of Silt in Open Channels," by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Hydrology Section, 1935, p. 478.

[23] Junior Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

In considering the application of the author's theory to river-control works, however, it is believed advisable that the term, "river," as used in this paper, be clarified. That the theory which is suitable for the Hundred Mile River may be applied to any other river is questionable. The Hundred Mile River, with its cross-sectional area of about 300 sq ft, is small when compared with the Lower Mississippi, which has a bank-full cross-sectional area of about 200 000 sq ft. The differences in hydraulic elements between the Hundred Mile River and most of the rivers in the United States are so great that this hypothesis of silt transportation should be used cautiously.

Of course, in the development of any theory, it is necessary to make certain basic assumptions. Some type of distribution of grain sizes is essential in formulating an equation. On the other hand, actual observations are of great value when one considers the characteristics of any stream. The remarks by Mr. H. S. Kahai[24] in connection with a silt survey of the channels at the Makhi Weir are pertinent. Mr. Kahai states:

> "The following conclusions were drawn:
>
> "(*A*) That it was not possible to determine a definite position on a vertical in any of the channels under observation to represent the mean velocity, silt charge, and weighted mean diameter for the cross-section."
>
> * * * * *
>
> "(*C*) That the behavior of the quality of silt was so erratic throughout the cross-section that nothing could be said about its position of occurrence."

The only illustration in this report which referred to the study of silt, was a cross-section (approximately 750 sq ft in area) of the Mithrao Canal. The distribution for mean grain size and velocity in the canal were as shown in Fig. 9. The silt, however, had a mean grain diameter of 0.030 mm, which is smaller than that used in developing the author's theory, and falls into the class referred to as "light materials carried in suspension by all eddies." These results show the indefinite tendencies for the movement of silt. This also seems a logical conclusion for practically every stream of appreciable size that draws its silt load from tributaries and is of sufficient cross-sectional area not to be made-to-order.

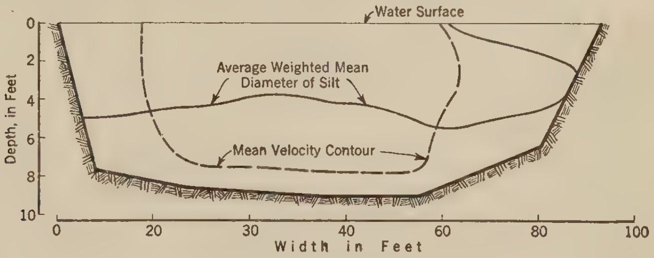

FIG. 9

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[24] "Report on the Work Done in the Development and Research Division, Sind, for the Year Ending May, 1938," by H. S. Kahai, Executive Engr.

Satisfactory formulas have been derived for various canals in certain localities, but the paucity of data from observations has not permitted the proper approach to the river problem. The magnitude of this problem can be seen from the recent study of variations of suspended silt along the Colorado River.[25] The publication of complete silt surveys and bed-load observations should enlighten the profession on the subject of larger streams to a greater degree within the next few years.

Harry F. Blaney,[26] M. Am. Soc. C. E. (by letter).—The theory of silt transportation advanced by Mr. Griffith is of special interest to engineers in the southwestern part of the United States where many of the rivers carry large quantities of silt. In this section one of the most important problems to be solved in designing irrigation and flood-control works properly is that of estimating the silt load carried by the stream. In most instances long-period records of silt measurements are not available, and estimates are based on fragmentary or short-period records. However, long-period stream-flow records may be available and, consequently, the author's method would be valuable in estimating silt loads.

It is noted that Mr. Griffith uses some of the silt studies made by the writer in Imperial Valley, California, and reported[10] in 1928, to illustrate his theory of silt transportation. In reviewing his original field notes the writer has found some unpublished observational data and has used these to check Equation (13). The results are shown in Table 8.

TABLE 8.—Analysis of Silt Distribution in No. 5 Main Canal and East Highline Canal, Imperial Valley, California

(Observations made on or near center vertical of cross-section)

Name of canal	Width, in feet	Total depth, d, in feet	Velocity, v, in feet per second	Proportion of silt by weight retained by Sieve No. 200, expressed in parts per 10 000 (arithmetic mean)	Values of c_1 in Equation (13)
No. 5 Main	32	3.3	3.48	10.3	13.2
No. 5 Main	25	3.9	3.79	18.7	13.1
East Highline	40	4.6	2.85	8.5	8.9
East Highline	40	4.6	1.92	6.9	6.0

Perhaps some of the results[27] of the investigation made by Orville A. Faris, M. Am. Soc. C. E., for the U. S. Bureau of Agricultural Engineering, in Texas, could be used to check Mr. Griffith. The writer does not have access to the original data on Texas streams but, in one instance, for observations on the Brazos River, the computed value for $\frac{7.5\,v}{d^{0.57}}$ amounts to 7.9 as compared with observed value, 9.6 parts per 10 000. However, in another instance, the agreement is not so close.

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[25] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 1104–1151.

[26] Irrig. Engr., Div. of Irrig., U. S. Dept. of Agriculture, Los Angeles, Calif.

[27] "The Silt Load of Texas Streams," *Technical Bulletin No. 382* (1933), U. S. Dept. of Agriculture.

E. W. LANE,[28] M. AM. SOC. C. E. (by letter).—The author's theory regarding the shape of a channel in erodible material is based on the hypothesis that the fundamental relation between depth and velocity is of the type:

$$B = c\,d^n \qquad (15)$$

There can be little doubt that some such rough relation holds as is shown by the many results obtained by observing stable channels under a wide range of conditions.[29] However, this relation is believed to be an empirical one which gives only approximate results.

The author has used the Chezy equation which has long been known to be unreliable when applied to open channels. If the more reliable Manning's relation is assumed, Mr. Griffith's reasoning indicates that the value of n should be close to 0.67 rather than 0.50.

Mr. Griffith believes that the size of the material of which the banks and bed of a channel are composed does not affect the shape that they will assume, as long as they are of granular material of a fineness between 0.002 in. and shingle. The writer does not believe that any formula can be found covering such a wide range of materials unless the material size is included as a factor. The writer[29] has expressed the belief that the size of the material forming the bed and both the quantity and the size of the load carried by the stream were factors in determining the channel shape. In that paper, emphasis was placed on the importance of the quantity of the material carried since the Lacey formula (which was then under active discussion in India) neglected that factor. Mr. Griffith's relations consider the quantity of material carried, but within the aforementioned very wide range of sizes, ignore the factor of material size and the writer wishes again to urge that both size and quantity are essential factors.

The resistance of granular material to motion by flowing water is closely related to the velocity of settlement in water of the particles of which it is composed. The settling velocity of a particle 0.002 in. in diameter is approximately 0.2 cm per sec, whereas particles 1 in. in diameter would fall about 50 cm per sec, or more than 250 times as fast, and the particles composing a shingle bed might be several times larger than 1 in. in diameter. It does not seem reasonable that beds of such widely different material would act in exactly the same manner.

The writer does not believe that any formula can be found, even for a single stream, that will give the relation between the velocity and depth in any vertical and the total silt load at that point of a size greater than 0.002 in. It is well known that most streams carry widely different suspended loads at the same discharge. For example, at one time, a stream may carry a given load at a certain discharge; and, at another time, it may carry twenty times as much with the same flow.

The quantity of silt carried in an ordinary river channel is composed of two parts, one of which is of the sizes found in the bed, which the river is constantly picking up from the bed and which are also constantly settling to the bottom again. The second part is composed of material brought in by the tributaries which is so fine that it is kept in suspension almost continuously. The river is usually able to carry many times the load of this fine material which the tributaries bring to it. The quantity of this material in suspension at any time, therefore, is usually independent of the nature of the main channel, and depends solely on what the tributaries provide.

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[28] Associate Director in Chg., Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[29] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 126.

For a given river, there is a definite relation between the first part of the load as described herein and the discharge, but since the total load is composed of both parts, and since the second part bears no relation to the discharge, the sum of the two is almost independent of the discharge. If the lower limit of the size of bed material were 0.002 in., the material carried of a size greater than that, being all obtainable from the bed, would have a definite relation to the discharge. However, in most rivers having granular beds, the minimum size of bed material is about 0.005 in. and a large part of the suspended load lies between this size and 0.002 in. In most rivers, therefore, the quantity of this material bears no relation to the discharge or velocity, and the total greater than 0.002 in., since it includes this range, would also bear no relation to them.

In the present state of the knowledge of silt transportation, with the tremendous range of silt concentrations encountered and the inaccuracies that probably exist even in the best silt measurements, differences between observed and computed results which would be intolerable in other branches of engineering science, in this field, may represent important contributions to that science. The relatively excellent agreement of the observed data with the values computed by Mr. Griffith's formulas, however, is believed to be fortuitous.

For example, in Tables 1 and 2 the data are expressed in proportions of silt by weight retained on the 200-(0.0029-in.) mesh sieve, but in Table 3 the percentage retained on the 300-(0.0018-in.) mesh sieve was used. This is the way the data were presented in the original source. A computation from the original data on which Table 3 is based, however, shows that if the 200-mesh sieve had been used as the limit, the results would have been only 60% as great. This indicates that if the division point of Tables 1 and 2 had been at 0.002 in. instead of 0.0029 in., the results would have been roughly 55% larger than those given. Data published elsewhere in the *Bulletin* from which Tables 1, 2, and 3 were taken[10] do not substantiate Mr. Griffith's results. Using the data for the Elder Canal (Table 23),[10] results are obtained giving silt concentrations averaging only 4% of the values given by Mr. Griffith's formula, and, for the Rositas Canal,[10] results averaging only 38 per cent.

O. A. Faris,[30] M. Am. Soc. C. E. (by letter).—The theory of silt transportation suggested by Mr. Griffith may be applicable under the limitations and conditions which he proscribes; however, it is believed that its application to most of the rivers of Texas will ordinarily lead to serious error. Investigations,[31] under the direction of the writer, include the collection and determination of the silt content of thousands of samples of water from various rivers of Texas, over a 9-yr period. The velocity of the water at the point of sampling was determined in many instances.

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[30] Engr.-Appraiser, Federal Land Bank of Houston, Houston, Tex.

[31] "The Silt Load of Texas Streams," by Orville A. Faris, *Technical Bulletin No. 382*, U. S. Dept. of Agriculture, 1933.

In sampling a flood originating in the upper regions of a water-shed, at a station farther down stream and out of the storm area which produced the rise, it was found that the first water reaching the station was comparatively free from suspended material. This was due to the arrival of clear water forced out of pools, along the river, by the heavier silt-laden water of the flood. As the stage built up, the silt percentage increased to a certain point and then decreased although the river stage continued to rise. During the falling stage, the percentage of silt again increased to some extent. This secondary increase in silt percentage is believed to be due to the sliding of recently deposited silt, into the stream, from the sloping banks where trees and brush retarded the velocity to the extent that deposition resulted. In some reaches of the streams, the secondary increase in silt percentage is due largely to the caving of banks undercut by the current. The greater part of the silt load, of the streams under observation, is made up in advance by the process of weathering. After a long dry period the first water that runs off picks up the fine weathered material and carries it into the stream. After the first flushing, run-off from the wet area must depend on erosion for its silt load, which is ordinarily light since the part of a large water-shed where excessive erosion occurs is usually small in comparison with the entire area. This accounts for the decrease in silt percentage while the river stage continues to rise.

The character of the part of the water-shed on which a flood originates affects the quantity of silt reaching a down-stream station at equal stages of the river. At Waco, Tex., on the Brazos River, it has been observed that floods from the Bosque River which joins the Brazos about 3 miles up stream from the station, contain a lower percentage of silt, at the same stage, than floods originating on the upper part of the Brazos water-shed.

These investigations led to the conclusion that the total quantity of silt passing a station in these streams, at a given time, is a function of loading rather than velocity and depth of water in the section. It is true that the higher the velocity the greater is the carrying capacity; but, since the capacity load was not even approximately reached and the silt content at a single station varied for equal stages of different floods, the magnitude of the silt charge carried appears to be a function of the loading and not of capacity to carry.

J. E. CHRISTIANSEN,[32] ASSOC. M. AM. SOC. C. E. (by letter).—Following Equation (1) the author states,

> "It is claimed that from Chezy's basic equations of flow for unerodible perimeters, the average velocities in the vertical planes through the points, *A*, *B*, *C*, etc. [Fig. 1], across the section will vary as the square root of the depths at those points; that is, as $d_1^{0.5}$, $D_2^{0.5}$, $d_3^{0.5}$, etc."[3]

This is only a rough approximation at best, and cannot be considered a basic law. Mean velocities in vertical planes of channels with flat bottoms often vary considerably from the center of the channel to sections near the sides. In the Chezy formula,

$$V = C\sqrt{R\,S} \qquad (16)$$

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[32] Asst. Irrig. Engr., Coll. of Agriculture, Univ. of California, Davis, Calif.

C is not constant; it is a function of S, R, and Kutter's coefficient of rugosity, n, assuming that Kutter's formula represents the relationship correctly. For a given value of Kutter's n, and between limits of R and S, the Kutter-Chezy formula can be approximated closely by simple exponential formulas; for example, for Kutter's $n = 0.0225$, and for slopes greater than 0.0001, the approximate equations are:

For $0.1 < R < 1.0$,

$$V = 66\ R^{0.81}\ S^{0.51} \qquad (17a)$$

for $1.0 < R < 10$,

$$V = 66\ R^{0.69}\ S^{0.51} \qquad (17b)$$

for $10 < R < 100$,

$$V = 74\ R^{0.58}\ S^{0.49} \qquad (17c)$$

Compare Equations (17) with Manning's formula, which for Kutter's $n = 0.0225$ becomes,

$$V = 66\ R^{0.667}\ S^{0.5} \qquad (18)$$

The author's theory of silt transportation is based fundamentally on a formula of the Kennedy type, Equation (1), in which he assumes that c is primarily a function of the quantity of silt carried. If his reasoning from Equation (1) is correct, the value of n should exceed the value of the exponent of R in Equations (17) and (18). Most of the formulas of the Kennedy type[6] involve the use of exponents of d or R less than the values of the exponents of R in these equations, except Equation (17c), which apply up to values of $R < 10$.

The writer believes that undue importance has been attached to the Kennedy formula. In 1927, discussing the prior paper cited by the author,[4] Mr. F. M. Woods stated[33] that Mr. Griffith had relied greatly on the Kennedy formula and had also extended its application to conditions of great rivers in unstable regime, involving every kind of irregularity of flow. The formula was stated to have been consistent with hydraulic data observed by Mr. Kennedy on a limited range of artificial earth canals; but, to extend its application as described would be questionable even if it were correct for canals in stable regime. According to Mr. Wood, Mr. Kennedy had enunciated no original theory, having merely deduced, from observed data, a useful empirical formula in support of theories current in India, which had been enunciated thirty years earlier by Mr. Thomas Login, an engineer on the Ganges Canal. The latter, in turn, was merely confirming, from his own experiences in the Ganges Canal, theories suggested by Mr. J. Dupuit in 1848. Mr. Woods then proceeds to warn that the Kennedy formula, based on "observations of certain Punjab irrigation-canals," has marked limitations despite the fact that it had been a useful guide to the design of canals for thirty years. For example, Mr. Kennedy did not measure the water-surface slope of any of his sample channels; nor did he compute the corresponding values of Kutter's coefficient of rugosity. He evaluated the discharges from discharge tables and the mean velocities by dividing this discharge by the area of cross-section. According to Mr. Woods, even Mr. Kennedy's determination of area of waterway was not quite satisfactory, being expressed in terms of bed width and depth, without reference to the inclination of the side slopes.

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[33] "A Theory of Silt and Scour," by W. M. Griffith, *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), p. 268.

Since the values of the exponents of d or R in both the silt formulas and general flow-of-water formulas are of approximately the same magnitude, the most logical conclusion would appear to be that the slope, for stable channels, should be primarily a function of the type and quantity of silt to be carried. Equating Kennedy's and Manning's formula:

$$C\, d^{0.64} = \frac{1.486\, R^{0.667}\, S^{0.5}}{n} \qquad (19)$$

from which,

$$S = \frac{C^2\, n^2}{2.21} \left(\frac{d^{0.64}}{R^{0.667}} \right)^2 \qquad (20)$$

Because of the inexactness of the exponents of d and R the ratio, $\frac{d^{0.64}}{R^{0.667}}$, may be assumed to be constant. For a given value of Kutter's n, then:

$$S = K\, C^2 \qquad (21)$$

This leads to the conclusion that a channel may be in stable equilibrium at any depth providing the slope is correct, a conclusion very different from that reached by the author.

The theory of silt distribution developed by W. Schmidt[34] and M. P. O'Brien, Assoc. M. Am. Soc. C. E.,[35] has been checked against some of the same data[36] used by the author, with a correlation equally as good. This theory does not account for the concentration of silt in suspension, or that which is moved along the bottom; but it does show that the distribution of silt within the section is a function of the size of the silt particles and the turbulence of the stream. From this theory, it would appear questionable to attempt to divide silt arbitrarily into "colloids" which are "carried in suspension by all eddies" and "loose granular material" that is "dependent on the vertical eddies alone." What may be "colloids" in one case may be "loose granular material" in another case, with a different degree of turbulence.

E. W. Lane,[37] M. Am. Soc. C. E., has called attention to the correlation between the quantity of coarse material in suspension and the discharge of a given stream, and the apparent lack of any correlation between the very fine material and the discharge. He attributes this condition to a balance between the kind of material in the bed of the channel and that in suspension. The percentage of very fine material in suspension, of which there is little in the bed material, is governed primarily by the source of such material. The writer came to essentially this same conclusion when studying silt data from the

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[34] "Die Massenaustauch in freien Luft und verwandte Erscheinungen," von W. Schmidt, H. Grand, Hamburg, 1925.

[35] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Hydrology Section, 1933, p. 487.

[36] "Distribution of Silt in Open Channels," by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Hydrology Section, 1935, p. 478.

[37] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 193.

Colorado River where high concentrations of fine material generally occur during the fall of year when the discharge of the Lower Colorado is fairly low. These high concentrations of fine material coincide, however, with the period of thunder-storms in desert areas, and with the flashy peak discharges of some of the lower tributaries that bring this material into the Colorado River. The concentration of the coarser material coincides fairly well with the discharge, but appears also to depend upon whether the river stage is falling or rising. With a sustained high discharge, the concentration of coarse material generally decreases.

SAMUEL SHULITS,[38] ASSOC. M. AM. SOC. C. E. (by letter).—An empirical approach to the problem of the transportation of solids by streams is presented in this excellent paper. The value of the method outlined lies in its simplicity and its seeming success in fitting existing data. The writer will attempt to subject certain parts of it to a critical analysis, in the hope that more light will be thrown on the fundamental bases of Mr. Griffith's formula.

The statement that Equation (1) is the elementary law of silt transportation seems to lack rigorous foundation. This equation can be regarded as scarcely more than an empirical formula derived from measured data of stable channels by the principles of mathematical curve-fitting. Accordingly, it cannot be vouchsafed the position of a fundamental law. By varying the exponent, n, the equation can be made to fit any set of data. Comparative compilations of such equations by E. W. Lane,[39] M. Am. Soc. C. E., have revealed such wide disagreement between existing formulas of this type that their practical reliability is rather uncertain. The thoughts expressed in this paragraph also apply to Equation (2).

When it is remembered that Chezy's velocity formula is based on the arbitrary assumption that the tractive force is porportional to the square of the mean velocity, it would appear that the criterion of $n = 0.5$ for equilibrium (or for channel stability) may not be well founded. Furthermore, since the tractive force is equal to $w\,R\,S$ (in which w is the unit weight of water; R, the hydraulic radius; and S, the energy gradient), considerable error may be incurred by substituting the mean depth, d_m, for the hydraulic radius, especially in small channels.

In natural channels the movement of material on the bed is limited to a strip approximately in the center, no movement occurring near the sides. This has been shown by field measurements made by S. Kurzmann[40] and explained experimentally by A Schoklitsch [41, 42] These results are at variance with Mr. Griffith's deductions that bed movement exists only where his depth criterion based on $n=0.5$ is exceeded. Actual measurements to substantiate the hypothesis would be extremely valuable.

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[38] Asst. Prof., Mechanics and Hydraulics, in Chg., Fluid Mechanics Laboratory, Colorado School of Mines, Golden, Colo.

[39] "Stable Channels in Erodible Materials," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 126–127.

[40] "Beobachtungen ueber Geschiebefuehrung," by S. Kurzmann, Munich, A. Huber, 1919.

[41] "Der Geschiebetrieb und die Geschiebefracht," by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, 1934, No. 4.

[42] Stauraumverlandung und Kolkabwehr," by A. Schoklitsch, Vienna, Julius Springer, 1935.

It is interesting to consider the significance of the stated possibility of mathematical calculation of f, the ratio of the cross-sectional mean velocity to the mean velocity in a particular vertical. This connotes that the mean velocity in any vertical of a cross-section can be predicted accurately.

Tables 1 to 4 merely show that Equation (13) fits the particular data well. One wonders if measurements in a tidal channel with its complex flow conditions should be used as the basis of a general formula, particularly when agreement was achieved by using the surface velocity instead of the mean velocity, a procedure amounting to an arbitrary 25% correction.

It is difficult to conceive a formula for solids transportation that does not include the effect of size of particles carried. The variation with discharge of the particle size for channel stability has been demonstrated elsewhere[43] to be of appreciable magnitude.

The practical application of the Griffith formula (Equation (13)), necessitates ultimately a knowledge of the mean velocity in various verticals—an exceedingly difficult physical quantity to predict with reasonable accuracy.

Although the evidence is scientifically meager, the method presented in the paper seems to yield useful results. The approach is representative of the time-honored hydraulic method and stands in contradistinction to the contemporary technique of fluid mechanics as incorporated in the works of Messrs. W. Schmidt,[34] M. P. O'Brien,[35] and J. E. Christiansen,[36] on the subject of solids transportation. The writer leans toward the latter concepts in the belief that, in the end, they will be more fruitful. The contemporary experiments of fluid mechanics are broadening the language of hydraulics. Then, after a long period of development this new language, this new body of concepts, will be such a natural part of an engineer's intellectual equipment that he should be able to create and not merely conjecture with it.

GERALD LACEY,[44] ESQ. (by letter).—Canal engineers in India who have made a study of silt transportation will read this paper with great interest. The writer was rather disappointed to find that the author had not modified his opinions expressed in 1927.[2] The writer had hoped that two contributions of his own on this subject[45] might have induced Mr. Griffith to reconsider his views. The part of Mr. Griffith's paper dealing with silt charge is of value as an interesting and original attempt at a quantitative solution.

When the author broadly refers to changes in the cross-sections of rivers and canals that could be computed mathematically by a "single elementary law" (see heading, "The General Theory") he is possibly representing the subject as being simpler than it really is. Furthermore, when Mr. Griffith proceeds to argue by reference to "Chezy's basic equations of flow" without expressing them mathematically, he is on debatable ground.

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[43] "Bed-Load Transportation and the Stable-Channel Problem," by Samuel Shulits, Assoc. M. Am. Soc. C. E., and W. E. Corfitzen, Jun. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 18th Annual Meeting, 1937, p. 465.

[44] Superintending Engr., Irrigation Branch Secretariat, Lucknow, U. P., India.

[45] "Stable Channels in Alluvium," by Gerald Lacy, *Minutes of Proceedings*, Inst. C. E., Vol. 229; and "Uniform Flow in Alluvial Rivers and Canals," *Loc. cit.*, Vol. 237.

It is of interest to trace the subject historically. The classic equation is that proposed by Kennedy[5] in 1895,

$$V = 0.84\, d^{0.64} \qquad (22)$$

Kennedy was of the opinion that in silt-transporting channels the bed was practically horizontal and the sides practically vertical. In Equation (22), therefore, V represents the mean velocity of the entire cross-section and d the average vertical depth as measured on the horizontal part of the bed. Kennedy described his equation as an empirical relationship, and although some attempt was made in his paper to deduce expressions having reference to silt charge, the equation was a practical engineer's empirical formula and of most value when used with discretion and applied to channels in India within the range of Kennedy's observations. Kennedy's paper was published without discussion by the Institution of Civil Engineers, London and, by the irony of fate, has been discussed almost without intermission ever since.

In 1912, Mr. W. J. Howley,[46] taking into account the fact that the side slopes in Madras, India, were not always vertical, preferred to deal with the mean vertical depth (the cross-sectional area divided by the water width) rather than the vertical depth, and produced for Madras the new equation,

$$V = 0.72\, d_m{}^{0.55} \qquad (23)$$

Both these equations are of importance inasmuch as they deal (very sensibly, in the writer's opinion) with the mean velocity, throughout a cross-section, and no attempt was made to draw inferences from such a generalized equation, as to the mean velocities in any given vertical.

In 1926, the writer[47] found that by treating the hydraulic mean depth, $\mathbf{R}$, as a variable that the Kennedy equation could be replaced by the expression,

$$V = 1.16\, \mathbf{R}^{0.5} \qquad (24)$$

and, in 1930, demonstrated that both the Madras and the Kennedy data could be represented with accuracy by the general equation,

$$V = c\, \mathbf{R}^{0.5} \qquad (25)$$

In 1930 Mr. Griffith,[48] adopting the Kennedy equation as modified by Howley, and following the writer's expedient of reconciling the Kennedy and Madras data, obtained the expression for the mean velocity that is introduced in this paper as Equation (9).

From the practical engineer's viewpoint it makes little difference whether the writer's equation or that by Mr. Griffith is used. All the results that Mr. Griffith claims for his expression can be obtained as accurately and more readily by using the equation involving the hydraulic mean depth. B. A. Etcheverry, M. Am. Soc. C. E., has dealt with the subject competently and fully.[49]

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[46] *Minutes of Proceedings*, Inst. C. E., Vol. 229, p. 261; "Critical Velocity Observations," Madras Public Works Dept., 1912.

[47] *Loc. cit.*, Vol. 223, p. 292.

[48] *Loc. cit.*, Vol. 229, p. 321.

[49] "Land Drainage and Flood Protection," by B. A. Etcheverry, 1931, see heading, "Problems of Flow in River Channels."

The arguments advanced for using the mean vertical depth, d_m, rather than the hydraulic mean depth, **R**, as a variable, are shared by other engineers of the Kennedy school. Mr. Griffith, however, presents an original argument as to why the power of d_m should be greater than 0.5 in his equation.

The Chezy equation can be written in the form,

$$V = C\sqrt{R\,S} \qquad (26)$$

Experience has shown that the expression is incorrect. For regime channels there is now a great mass of evidence to show that for alluvial material of constant grade the equation,

$$V = C\,R^{0.75}\,S^{0.50} \qquad (27)$$

is approximately correct. That the Manning equation,

$$V = C\,R^{2/3}\,S^{1/2} \qquad (28)$$

is superior to the Chezy equation as originally presented, is conceded by every one.

Attempts to apply the Chezy general equation to calculating the mean velocities over verticals appear to the writer an unjustified step. There exists a certain mental confusion between equations of different types. Thus, when the writer first published his equation,[50]

$$V = C\,R^{0.5} \qquad (29)$$

for regime channels, it was argued[51] in India that the writer's equation was a backward step to Chezy. It was contended that the power in Chezy's equation (following Manning) was not $\frac{1}{2}$ but $\frac{2}{3}$ and, therefore, the power in the writer's equation was wrong. As a matter of fact there are two relationships, each one correct; namely: $V \propto R^{0.50}$; and, $V \propto R^{0.75}\,S^{0.50}$.

It follows, therefore, that in regime channels the slope varies inversely as the square root of the hydraulic mean depth, as the writer has demonstrated statistically.[52]

Mr. Griffith claims from Chezy's "basic equations of flow" that the average velocities in vertical planes above points on the bed should vary as the square roots of the vertical depths. If the original Chezy equation is used, it is wrong; if, on the other hand, Chezy as corrected (Manning, for example) is used, the power $\frac{2}{3}$ is greater than 0.57 and, therefore, Mr. Griffith's channels should perform in a manner opposite to that which he predicts. The point, however, is of little importance as the propriety of the entire line of argument is in doubt. Whether or not channels in incoherent alluvium widen depends on the simple relationship connecting the wetted perimeter, P, and the discharge,

$$P = 2.67\,Q^{0.50} \qquad (30)$$

a relationship first evolved by the writer[53] in 1928 and confirmed by thorough statistical analysis of data analyzed in Sind and the Punjab.[54] The value of the constant varies from 2.5 to 2.8, the power is invariably 0.50.

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[50] "Stable Channels in Alluvium," *Minutes of Proceedings*, Inst. C. E., Vol. 229.

[51] *Indian Engineering*, Vol. LXXXIX, February 7, 1931, p. 111.

[52] *Minutes of Proceedings*, Inst. C. E., Vol. 237, p. 425.

[53] *Loc. cit.*, Vol. 229, p. 273.

[54] See Annual Repts., Central Irrig. Board Library, Simla.

There appears little doubt that the correct variable to use is the hydraulic mean depth, **R**, rather than the mean depth. It is true that silt is suspended by vertical components of eddies; but the section is generated by forces normal at all points to the wetted perimeter, and there is every reason why the hydraulic mean depth should be retained unless some other variable, without recourse to argumentation, can be proved statistically preferable.

Instead of Equation (11), the writer would use the equation,

$$\mathbf{R}_2 = \mathbf{R}_1 \left(\frac{P_1}{P_2}\right)^{2/3} \qquad (31)$$

Unfortunately, Mr. Griffith failed to quote (see "Application of Theory") the fundamental wetted perimeter, P, or the mean hydraulic depth, **R**. The writer would be grateful if Mr. Griffith would supply the missing values of P and **R** and also compare and quote the accuracy of the result obtained by using Equation (31).

The theory offered by Mr. Griffith in respect of silt charge is original and valuable. To the writer it appears that the theory would be improved by using his own simple equation: Silt factor $\infty \frac{V^2}{\mathbf{R}}$.

The silt in suspension is a function of the turbulence, or as V. V. Tchikoff, M. Am. Soc. C. E., terms[55] it, "kineticity." For this turbulence Mr. Griffith presents the ratio, $\frac{V}{d_m{}^{0.57}}$, instead of the writer's $\frac{V}{\mathbf{R}^{0.50}}$.

The ratio, $\frac{V^2}{\mathbf{R}}$, has the dimensions of an acceleration and has a physical meaning associated with turbulence, or "vorticity," whereas the expression, $\frac{V}{d_m{}^{0.57}}$, is dimensionally unsatisfactory and, from a practical standpoint, somewhat unmanageable.

With much of Mr. Griffith's general views the writer is in agreement, but he feels that little progress can be made in fundamental theory by retaining the mean depth as a variable or by relying too implicitly on a doubtful interpretation of Chezy's equation.

Glenn W. Holmes,[56] Jun. Am. Soc. C. E. (by letter).—Under the conditions established and on the assumptions made, the author's theory of silt transportation seems both logical and reasonable. In practice, however, the writer believes the conditions would rarely be met and the assumptions in general would not be found valid.

If the velocity in the center of a section exceeds that required for equilibrium as defined by the author, it will necessarily incline to scour. It does not follow, however, that the sides will shoal as the author claims. The scoured material is carried down stream and not directly to the sides, and the correspondingly reduced marginal velocity may not be slow enough to cause deposition of material coming from up stream.

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[55] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 167.

[56] Associate Hydr. Engr., Soil Conservation Service, Washington, D. C.

In 1925, Fred C. Scobey and the late Samuel Fortier, Members, Am. Soc. C. E., wrote:[57]

> "* * *. It has been considered that for each depth of water there is a certain velocity below which silt will be deposited and above which silt will be eroded from the bed of the channel. In the opinion of the writers, two phenomena only slightly related are thus confused. The power of flowing water to maintain a movement of separate soil, sand, or gravel particles, has been confused with the power to break the bedding of a canal bottom and ravel off particles of that bed and transport them to places of lesser velocities where they may be deposited. In their [Messrs. Fortier and Scobey] opinion there is no sharp line of demarcation between the velocities that can no longer maintain silt in movement and those that will scour a canal bed. It is believed that there is a broad belt of velocities between these two 'critical' velocities, within which silt already loosened or brought in through a head-gate will remain in suspension while the bed nevertheless will remain undisturbed as regards scour. It is easy to show the absurdity of accepting the laws of silting as giving the immediate answer to the laws of scour."

The inertia of particles in bed material, the cohesion, adhesion, mechanical interlocking, and other properties all contribute to a resisting force opposed to scouring—such resistance not having a counterpart in the forces opposed to transportation once the particles are in motion.

It would seem to the writer that clear streams and mud flows both deny the existence of definite critical velocities above which scouring occurs abruptly and below which sedimentation suddenly takes place.

Whether or not the author is correct in assuming such critical velocities or conditions of equilibrium, it is believed he attempts too great a step in applying his theory, based on a particular size range, to the measurement of the entire silt load of a stream. To be applicable to such a problem it is necessary to assume that there is a dependable relationship between total load and the part comprised of particles of sizes greater than 0.002 in. Table 9 shows the results of mechanical analyses made of silt samples collected by the U. S. Geological Survey for the Soil Conservation Service at four stream-gaging and silt-sampling stations. It is clearly shown that there was wide variation in the proportion of particles greater than 0.002 in., not only for different streams but also for different observations of the same stream. Although the samples were taken to represent the cross-sections at the time of observation, they were in other respects, without going into detail, essentially random samples. It should be noted also that the larger silt particles made up only a small part of total loads.

The author states that the particles finer than 0.002 in. will not affect the bulk of the bed lining of channels in which loose granular material is being transported. This may or may not be true for channels or rivers in general; but if the theory is to be used to measure entire silt loads that may endanger the life of reservoirs or other slack-water sections, the particles larger than 0.002 in. are often found to play minor rôles in damages as may be seen in Table 10 prepared from information presented by the late Henry M. Eakin[58] in 1936. Mr. Eakin wrote:

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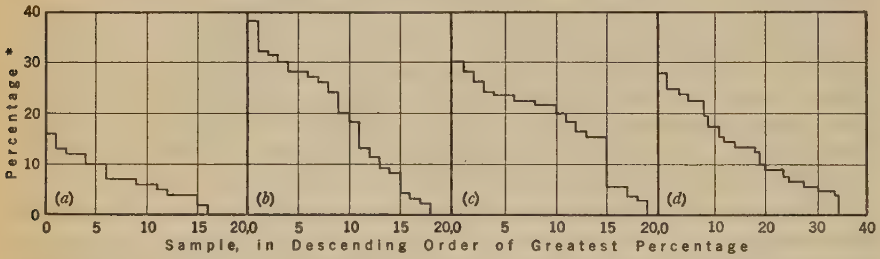
[57] "Permissible Canal Velocities," by Samuel Fortier and Fred C. Scobey, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 941.

[58] "Silting of Reservoirs," U. S. Dept. of Agriculture, *Technical Bulletin No. 524*, July, 1936, p. 7.

"It is of signal importance to systematic silt studies in that the volume of finer grained bottom-set beds, more frequently than otherwise, has been found to exceed the total volume of delta deposits and that, contrary to customary thought, the depletion of deeper reservoir storage space in the vicinity of the dam may not await the gradual approach of growing deltas but may, and in most cases does, begin at selective rates from the very beginning of storage."

TABLE 9.—PROPORTION OF PARTICLES LARGER THAN 0.002 INCH IN SUSPENDED MATTER OF RANDOM SAMPLES OF THREE SMALL STREAMS



Curve	Description	Number of samples	Percentages of Total Load		
			Average	Maximum	Minimum
(a)	Coon Creek, at Coon Valley, Wis.	16	8	16	2
(b)	Coon Creek, at Stoddard, Wis.	18	19	38	1
(c)	Little La Crosse River, near Leon, Wis.	19	17	30	2
(d)	West Fork of Deep River, near High Point, N. C.	34	13	27	3

* Percentages of total suspended loads composed of particles larger than 0.002 in., arranged in order of magnitude.

TABLE 10.—SEDIMENT DISTRIBUTION IN LAKES AND RESERVOIRS

Lake, or reservoir	Location	*Bulletin No. 524,** page:	Sedimentation, in Acre-feet		
			Delta deposits	Bottom-set beds	Total sediment
Lake Michie	Durham, N. C.	32	Not pronounced	395	395
Greensboro Municipal Reservoir	Greensboro, N. C.	39	123	137	260
High Point Reservoir	High Point, N. C.	42	105	142	247
Concord Lake	Concord, N. C.	45	7.48	71.72	79.20
Spartanburg Reservoir	Spartanburg, S. C.	49	174	289	463
Lake Waco	Waco, Tex.	67	....	4 766	4 766
White Rock Lake	Dallas, Tex.	71	1 256	2 626	3 882
Guthrie Reservoir	Guthrie, Okla.	74	77	379	456
Boomer Lake	Stillwater, Okla.	77	0	170.93	170.93
San Carlos Reservoir	Coolidge Dam, Arizona	97	11 896	25 000	36 896

* "Silting of Reservoirs," by Henry M. Eakin, U. S. Dept. of Agriculture, *Technical Bulletin No. 524*, July, 1936.

It seems probable to the writer that no simple formula involving such factors as velocity and depth will be found to define silt loads of any particular size or range of sizes, because in addition to reasons already stated, those factors do not, and cannot, regulate or indicate the rate of débris supply, largely soil erosion, which is known to be a complicated process in itself.

HUNTER ROUSE,[59] ASSOC. M. AM. SOC. C. E. (by letter).—"Theory" is a word which has come to possess several distinct connotations. Among scientists, theories are generally based upon reasonably sound physical analysis; many such theories of past decades have become the natural laws of present-day science, after having stood the test of thorough experimental observation. Although many engineers adhere to the scientific meaning of the word, in general practice numerous so-called theories vary from sheer hypotheses with neither analytical nor experimental foundation to concepts based upon either imperfect or totally erroneous methods of reasoning; as a result, more than one practical engineer has come to regard the word "theoretical" as the antonym of "actual." What is meant by "theory" in the present paper must be judged from the fact that it is based upon "two assumptions" and an "elementary law," the latter apparently having no more foundation than the two assumptions.

If Equation (1), the "elementary law," was developed on the basis of experimental data, it is a true empirical formula; if it was first formulated and subjected later to experimental check, it apparently began as a pure hypothesis. In neither case could it be considered to embody a scientific theory, and to regard it as a basic law is to presume that every possible experimental test had confirmed its validity. Since the author gives no reasonable physical justification for this initial statement, it would appear to be a purely arbitrary formula, which can be made to fit experimental data by adjustment of a variable coefficient and a variable exponent—as could other relationships of different mathematical form. The expression is really not as simple as it appears, for it embodies implicitly these two additional unknown functions.

Equation (2), again, may "follow from Equation (1)," but only if f bears the brunt of the change. This factor depends not only upon the shape of the cross-section, as the author states, but also upon the relative roughness of the bed and the mean velocity of the flow. For instance, f is surely not unity for a rectangular cross-section unless the width-depth ratio is large, as may be seen from any measured pattern of isovels. Perhaps f may be estimated empirically with sufficient accuracy for practical purposes; but if this factor can be "mathematically determined for any known type of cross-section," it is by a method as yet unknown to the writer. Equations (3) to (9) are subject to the same criticisms as Equations (1) and (2).

It would seem to the writer that the author's initial relationship could be classed with the dozen or more empirical formulas for bed-load transport, if used in the form

$$G = c_1 v = \frac{z\, v^2}{d^n} \qquad (32)$$

to predict the amount of material carried per unit time and per unit width of flow section. It may be written in other forms, of course, by substituting from the equation for tractive force

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[59] Associate Hydr. Engr., Soil Conservation Service; Asst. Prof. of Fluid Mechanics, California Inst. of Technology, Pasadena, Calif.

$$T = w\,d\,S \text{.........................} (33)$$

and from the Chézy relationship

$$v = C\sqrt{d\,S} \text{.........................} (34)$$

This presumes, to be sure, that the flow is uniform and steady and that the channel is very wide, v and d then being mean values in accordance with the true significance of the Chézy equation. The author, however, recommends its use for individual elements of a cross-section of irregular form, for cases of non-uniform and unsteady flow, and for the transportation of suspended load as well as bed load—indicating, moreover, that z and n will remain essentially constant under all these conditions. No bed-load investigator, to the writer's knowledge, has dared predict that his own formula would include even one of these departures from the normally assumed conditions of motion.

The writer does not wish to be interpreted as being against the empirical method of attack. The method is of extremely great importance to engineering practice, and in specific cases is even more likely to yield immediately useful results than methods which are purely rational. The problem is far more complex than the author's treatment would indicate, however, and the very complexity which prevents a rational solution also stands in the way of a general empirical treatment. With proper adjustment of coefficient and exponent, the equation in question will undoubtedly yield satisfactory results in many instances, and the author is only to be encouraged in studying the variation of these factors with boundary, flow, and sediment parameters. What the writer does protest is the acceptance of an apparently unfounded relationship as a cure-all for the thousand and one ills of the sediment problem, without thorough check. If the theory really has a sound physical basis, it is unfortunate that this was not demonstrated in the present paper.

S. G. Bauer,[60] Esq. (by letter).—As an attempt to discover the influences determining the shape of a stable channel by studying equilibrium considerations, this paper is exceedingly interesting. No doubt it would be quite impossible to take into account all the factors that might influence the cross-section of a channel so that simplifying assumptions must be made. The author's deductions from his assumptions are certainly most interesting and show that the principle is a step in the right direction. His assumptions, however, cannot be viewed with quite the same confidence.

In a channel of infinite width and constant depth it would be quite justifiable to consider the vertical components of the turbulence eddies only for the upward movement of the silt; but when considering the shape of a finite channel, with sloping walls, the horizontal components of the eddies should also be taken into account, responsible as they are for the horizontal velocity distribution across the channel. This raises another point. If the mean velocity in any particular vertical plane were proportional to the square root of the depth in that plane the horizontal velocity distribution would be determined by the bed contour only. As the horizontal components of the eddies provide a constant interchange of energy from the faster to the slower flowing water, this appears a very unlikely assumption.

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[60] Eng. Laboratory, Univ. of Cambridge, Cambridge, England.

Another point is that the mean velocity of the silt charge of a channel is quite different from the mean velocity of the water, because most of the silt is carried near the bottom of the channel where the average forward velocity of the water is only a fraction of its all-round mean velocity. Moreover, the finer silt moves much faster than the coarser silt in the same channel. In consequence, the distribution of the silt charge along a vertical section has a profound influence on the silt-carrying capacity of a channel. The author assumes a silt distribution varying with the square of the depth, irrespective of the nature of the silt charge. This may be the case, approximately, with some types of fine sand on a very smooth bed. The coarser grade of silt is lifted from the ground by a large velocity gradient which results in a higher velocity of flow above a particle than below it, thereby imparting to it a "lift" similar to that of an airplane wing in flight. The type of silt carried must therefore enter into the law of its distribution. The writer's experience with fine sand in a tidal river in North Wales, Great Britain, flowing in its own silt bed, indicates that even with fine sand the silt charge is much heavier near the bottom of the channel than would correspond to the square law. That gravel and boulders are carried along the bed of a river or channel only is common experience.

Apart from that, the silt distribution is strongly influenced by the presence, or otherwise, of large eddies of the type which produce an unevenness of the water surface. There seems to be some critical rate of flow above which such large-scale eddies appear, sometimes covering the entire depth of the channel, and thus changing completely the entire velocity and silt distribution. At a rate of flow at which the water still appears quite smooth, but which is near that critical rate of flow, this large-scale turbulence can often be released by a small disturbance, such as can be made by throwing a stone into the water or disturbing the evenness of the bed with a stick.

This critical condition has been observed by many, no doubt, and should be taken into account in a manner similar to the critical Reynolds number for the flow in pipes. Dimensional analysis may ultimately lead to a solution. Where these large eddies appear, which are of the same order of magnitude as the width of the channel, it is obviously difficult to derive any consistent laws, but it may well be that this condition is actually a limiting one for the stability of the channel itself.

C. C. Inglis, Esq.[61] (by letter).—This paper may be divided into two parts: (1) That dealing with quantity of silt transported; and (2) the theory on which the formulas are based.

The part dealing with quantity of silt transported is interesting and likely to be valuable in many ways; but "grade of silt" is normally more important than "quantity of silt being transported," because it is an independent variable, whereas silt charge is a dependent variable, varying with V, d, and time. For this reason, grade cannot be ignored and the coefficient C_1 must be a variable. The effect of temporal changes in velocity must also be taken into account. In the case of the River Indus, silt charge depends on conditions during the previous eight days, during which the silt charge continues to increase on a rising flood. Similarly, on a falling flood the heavy silt charge tends to persist, and all of the excess silt will not deposit for eight days. This effect is shown clearly in Table 4.

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[61] Director, Central Irrigation and Hydrodynamic Research Station, Poona, India.

The part of the paper dealing with the theory on which the author's formulas are based is not nearly so "happy." In this paper, as in his earlier paper[7] based on Kennedy's formula

$$V = C\,0.84\,d^{0.64} \qquad (35)$$

the author has assumed that there is a simple, experimental, fundamental formula, applicable to the relation between depth and velocity on all verticals across a channel section. This, definitely, is not the case. As a rough approximation, it is a valuable aid, which has been applied widely in India for many years; but since Mr. Gerald Lacey wrote his first paper in 1928 on "Stable Channels in Alluvium"[62] it has been clear that the Kennedy formula was only an approximation, and that velocity and dimensions of channels were related, simply and directly, to hydraulic mean depth, and only indirectly to depth.

The main objection to the Kennedy formula has been that it ignored shape—that is, according to the formula, a channel with a normal silt value expressed by Equation (35) might be 1 000 ft wide and 1 ft deep or 23 ft wide and 10 ft deep.

The author has evolved a method of determining "the section having the maximum silt carrying capacity" by trial and error from his approximate formulas. This shows that he too considers that the shape of a channel may be varied. Where the material is inerodible this is true; but in erodible material, especially where the silt charge is considerable, the channel (as pointed out by E. S. Lindley, M. Am. Soc. C. E., in 1919[63]) is uniquely determined. Mr. Lacey produced the following formulas giving dimensions for regime channels in 1928:[62]

$$P = 2.668\,Q^{1/2} \qquad (36)$$

$$V = 1.17\sqrt{f\,R} \qquad (37)$$

$$V = \left(\frac{Q\,f^2}{3.8}\right)^{1/6} \qquad (38)$$

$$R = \frac{0.7305\,V^{1/2}}{f} \qquad (39)$$

and,

$$S = \frac{f^{3/2}}{2\,587\,Q^{1/9}} \qquad (40)$$

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[62] *Minutes of Proceedings*, Inst. C. E., Vol. 229 (1929–30), p. 259.

[63] "Regime Channels," *Proceedings*, Punjab Eng. Congress, Vol. VII, 1919.

in which f was a silt factor depending on silt grade, but affected by silt charge. From Equations (38) and (39)

$$R = 0.468 \left(\frac{Q}{f} \right)^{1/3} \qquad (41)$$

Mr. Lacey wrote his second paper "Uniform Flow in Alluvial Rivers and Canals"[64] in 1933 and since then he has improved his formulas dimensionally by introducing kinematic viscosity and gravity. Subsequently he modified the exponents of his original, simple formulas, very slightly. It is interesting to compare these results with the formulas derived statistically by the Punjab Irrigation Research Institute from a mass of Punjab data for silts of less than 0.6 mm diameter (see Table 11). The principal difference in the Punjab formulas is the omission of any silt factor. This does not mean, of course, that a silt factor should not enter the formulas—silt must affect the last three formulas and in the writer's opinion also the P–Q relationship—but merely that owing to the small range of silt found in the Punjab Canals and the low powers in which the silt factor enters the equation, it has not been possible to obtain a significant correlation.

TABLE 11.—COMPARISON OF FORMULAS.

Lacey	Punjab Irrigation Research Institute
$P = 2.667\, Q^{1/2}$	$P = 2.80\, Q^{1/2}$
$R = 0.473 \left(\frac{Q}{f} \right)^{1/3}$	$R = 0.47\, Q^{1/3}$
$V = 1.1512\, (f\, R)^{1/2}$	$V = 1.11\, R^{1/2}$
$S = \frac{f^{5/3}}{1\,858.6\, Q^{1/6}}$	....

The Punjab Irrigation Research Institute have never accepted the Lacey formulas and have produced formulas of their own, which, until recently, have differed considerably from the Lacey formulas. The fact that they have now produced almost similar formulas, after five years of exhaustive and painstaking work, shows that the regime relationships derived by Mr. Lacey are fundamental. They are not, however, directly applicable to variable, non-regime conditions; and it is for non-regime conditions that approximate formulas of the type evolved by the author are valuable.

W. M. GRIFFITH,[65] ESQ. (by letter).—It is to be regretted that the abbreviated exposition given of this theory has lead to a misunderstanding of its physical basis and of the method by which it was derived. It was not, as has been assumed, based on Kennedy's empirical equation for the Barrey Doale Canals, or any subsequent equations of the same type; it was derived independently from the writer's observations during many years of experience in the training of rivers and in dealing with the problems of some of the heavier silt-laden canals of the United Provinces (India). The writer appreciates the suggestion of Professors Shulits and Rouse that the arbitrary value of the exponent "n" in Equation (1) prevents definite scientific test of the correctness of the elementary law on which this theory is based. To meet the general misunderstanding that has been created it is necessary to give a more complete explanation of this theory and the evidence on which it rests. Furthermore, to enable the theory to be fully tested the writer has added one equation not hitherto published, which definitely connects the value of "n" with the hydrallic conditions pertaining, to enable the mathematical deductions from his elementary law to be checked against observed natural phenomena over a wide range of conditions. This should answer most of the questions raised in discussion.

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[64] *Minutes of Proceedings*, Inst. C. E., Vol. 237 (1933–34), Pt. 1, p. 421.

[65] Cambridge, England.

Mr. Christiansen demonstrates that, accepting the Kutter-Chézy formulas as giving correct results in formulas of the Manning type, the exponent of R will vary materially with the value of R itself. This does not appear to support the contention that Manning formulas should supersede Chézy's equation as a basic equation of flow, or Mr. Lacey's suggestion that the exponent of R should be $\frac{3}{4}$ and not $\frac{2}{3}$. As the question of what is the basic equation of flow appears to be controversial, the writer's claim that the average velocities in the vertical of streamlines d_1, d_2, d_3, etc., across an unerodible section (Fig. 1), will tend to vary as $\sqrt{d_1}$, $\sqrt{d_2}$, $\sqrt{d_3}$, etc. will perhaps be better appreciated if it is considered that these streamlines are all models of one streamline but plotted to different vertical scales. These vertical scales then will be proportional to the depths d_1, d_2, d_3, etc. As it is accepted that, in models, the velocities will vary as the square root of the vertical scale, it is reasonable to argue that the velocities across the section will tend to vary as $\sqrt{d_1}$, $\sqrt{d_2}$, $\sqrt{d_3}$, etc.

Professor Rouse was not correct in suggesting that the theory was advanced for non-uniform and unsteady flow. Condition (3) stipulates that "the flow is turbulent and follows the boundary surfaces." This was meant to specify sinuous flow within streamline surfaces. To make this point clear one may divide sinuous or turbulent flow into two classes:

(1) Streamline flow, or the lines of flow following the boundary surfaces, flow being steady and uniform; and,

(2) Non-streamline flow, a condition occurring at a sudden enlargement, or other deviation of the boundary surface from the straight line, too rapid for the lines of flow to follow. In this condition, loss of head, by what is termed "shock," occurs, the energy being dissipated by vortices at the point of deviation.

The flow in a canal is usually designed to be streamlined, and therefore Equation (1) will normally hold; but sometimes there is a sudden expansion, perhaps below a tight bridge, and flow will not follow the boundary surface. At this point a scour hole will be found in the bed, the depth of which will not be found to agree with Equation (1). What has happened is that at this point the release of energy, due to the head lost by shock, provides additional turbulence, and the values of c and d are increased temporarily. Beyond the area where this energy is dissipated, the value of $\frac{v}{d^n}$ will fail to support this additional silt charge so that the excess is deposited, causing a hump in the bed.

Beyond this latter area, in turn, the value of c having become normal, Equation (1) will again hold.

However, if the expansion is sufficiently gradual to insure streamlined flow, instead of bed scour, it will be found that, as the sides expand, the bed rises, in accordance with values found by Equation (6a), as occurred in the example given of the Hundred Foot River, and *vice versa*. As the radius of curvature of a bend in an artificial channel is usually sufficient to insure streamlined flow, the cross-section at a canal bend generally will be found to be as sketched in Fig. 10.

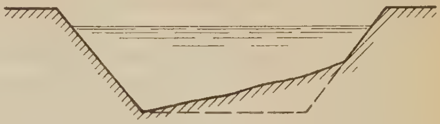

FIG. 10.

Allowing for the decreasing value of the velocity from the outer to the inner margins of the bend, Equation (1) will be found to hold across the section. The writer believes this to be the true explanation of the greater depth observable at the outer margin, and the shoaling along the inner margin, of a bend.

In a river, the radius of curvature of the bends are often insufficient so that the lines of flow cease to follow the inner margin of the bend. This results in loss of head by shock and scour, and then the creation of a hump and an irregular bed generally.

However, the fact that river channels are badly streamlined does not prevent the application of this theory to the general solution of silting problems any more than it prevents the general application of Chézy's formulas to problems of flow. In general, as turbulence for the same velocity will be greater in a river channel than in a streamlined canal, a river should carry a heavier silt charge than a canal for the same values of V and d. From Table 3 it will be seen that the silt charge carried by the Colorado River is greater than that for the Imperial Valley canals for the same values of V and d.

Professor Lane points out that in most rivers the minimum size of bed material is about 0.005 in. Elsewhere, he[66] has shown that at about this size of particle there is a change in the laws governing the rate of settlement of particles in still water. The writer suggested 0.002 in. as the limiting size for classification as loose granular material for rivers, because this appeared to be the limiting size of the actual bed material in the Colorado River and Imperial Valley canals, which carry very fine silt. This was not meant to imply that 0.002 in. was the limiting size for material classifiable as "loose granular" in all rivers. In any river or canal the limiting size that can classify as "loose granular," under the writer's theory, is the limiting size of the actual bed material of that river or canal.

The silt samples of the Central Main and Dahlia canals given in Tables 1 and 2, screened to a 200-mesh sieve, had a much smaller percentage of material passing that sieve than the canals of Table 3, for which the silt samples were screened to a 300-mesh sieve. This is presumably the reason why it was not considered necessary to screen these below the 200-mesh sieve, and why the results of both sets agreed reasonably well with Equation (13).

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[66] *Transactions*, Am. Geophysical Union, 1938.

Mr. Faris refers to the investigation of silt loads of the Texas streams, concerning which many valuable data have been obtained. Elsewhere, he has presented a comprehensive silt analysis of the cross-section of Brazos River at Rosenberg, Tex., together with data of velocities and depths.[67] These data are summarized in Table 12. In Column (5), Table 12, is given the loose granular contents retained by Sieve No. 300, and in Column (7), the total silt load, both being expressed in parts per 10 000. Column (6), Table 12, contains the loose granular contents computed by Equation (13). It will be seen from this table that, of a total silt content of 76.85 parts per 10 000, the mean observed loose granular content was 7.9 parts as against a computed value, by Equation (13), of 7.3 parts. This agreement appears to be close and justifies the application of this theory to the rivers of Texas.

TABLE 12.—DISTRIBUTION OF SILT IN THE CROSS-SECTION OF THE BRAZOS RIVER, AT ROSENBERG, TEXAS

Serial No. (1)	Station No. (2)	Total depth, in feet (3)	Mean velocity, v, in feet per second (4)	Silt retained, in parts per 10 000† (5)	Values of $\frac{7.5\,v}{d^{0.57}}$ (6)	Total silt contents, in parts per 10 000‡ (7)
1	142	18.2	4.16	12.3	5.9	81.6
2	240	20.0	4.31	10.5	5.9	81.3
3	280	21.3	4.59	9.9	6.0	80.5
4	320	24.3	5.86	6.1	7.1	75.2
5	360	23.5	6.62	5.8	8.1	73.7
6	400	25.0	6.41	5.6	7.7	73.3
7	440	12.3	5.63	5.1	10.1	72.3
8	480	....*	....*	....*	....*	....*
Total				53.3	50.8	537.9
Mean silt load				7.9	7.3	76.85

* Data incomplete. † Proportions of silt by weight, retained by a No. 300 sieve, expressed in parts per 10 000 (arithmetic mean). ‡ Including colloids.

Mr. Christiansen equates Kennedy's empirical equation and Manning's formulas and argues from this equation that a channel may be in stable equilibrium at any depth, provided the slope is correct. This is not contrary to the writer's conclusions. It will be shown subsequently, however, that there is only one depth of flow which will give the maximum silt-carrying capacity for a channel.

The writer's observation does not agree with Professor Shulits' contention that in natural channels the movement of material on the bed is limited to a strip approximately in the center of the bed, and that no movement occurs near the sides. If this were true, the sides of natural channels must always remain stable, because when there is no movement of material there is no scour or silting. The writer's experience has always been that natural channels in loose granular materials are not stable and have unstable sides.

Mr. Lacey was not correct in assuming that the writer had aopted Kennedy's equation as modified by Howley. Equation (2) was derived mathematically from his elementary law, as explained in his previous paper published in 1927.[2] He then analyzed Kennedy's data in terms of d_m to evaluate n, and again in 1930, when Howley's data were published, evaluated n from both. This evaluation ($n = 0.57$) was only true for a limited set of hydraulic conditions. It will be shown subsequently that Equation (43*b*) gives an evaluation that covers the widest range of conditions.

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[67] "The Silt Load of Texas Streams," by Orville A. Faris, *Technical Bulletin No. 382*, U. S. Dept. of Agriculture, 1933, Table 5.

The difference between Equation (2) and Mr. Lacey's Equation (29) was fundamental because it could be shown mathematically[68] that the behavior of eddies created by the friction of flow past the wetted perimeter was a function of R, the hydraulic mean radius; but the vertical components of the eddies were functions of d_m, the mean depth, and not of R. Mr. Lacey now agrees that silt was suspended by the vertical eddies. Equation (29) appeared to postulate a transportation of silt by all eddies and Equation (2) by the vertical eddies only.

Again, Mr. Lacey's theory,[50] as advanced in 1930, postulated an elliptical section with a perimeter of silt in permanent regime whereas, according to the writer's theory for loose granular matter, permanent equilibrium was possible only with a bed level across the section.

Observations showed that with a perimeter of loose granular material no permanent regime was ever attained, and where sides existed of a material having different physical properties, regime was attained only if the bed was level across the section and not curved or elliptical.

In consequence the writer had seen no reason to modify his original equation in light of Mr. Lacey's later equations. He was indebted to Mr. Lacey's valuable work, however, for a clearer understanding of the conditions to which his theory was applicable and these have been stated in this paper. In fact, the writer was led to the conclusion that there were two different sets of conditions governing silt transportation—one for the heavier class (loose granular matter), the critical velocity for equilibrium depending on d_m^n, and one for light material, which he classed as colloidal held in suspension by all eddies, the critical velocity for equilibrium depending on $R^{0.5}$. He considered that these two conditions were exemplified by the different types of cross-section: (*a*) Rectangular at the head of the canal where loose granular matter governed the conditions; and (*b*), the curved, possibly semi-elliptical, section found at the tail, where the heavy material had passed from the system.

He could not agree that it made little difference whether d_m^n or $R^{0.5}$ were used because, as will be shown subsequently, it is this difference in value between d_m and R of a flowing section which actually determines the most efficient depth for a channel to carry heavy silt.

Mr. Lacey suggests Equation (31) as an alternate to Equation (11); but, he gives no proof of this equation, which appears to be a purely empirical one, and there would appear to be no advantage in substituting an empirical equation for one reasoned mathematically from a natural law.

Mr. Holmes suggests that there is a wide difference between the velocity which will cause accretion and that which will scour. This is not borne out by the writer's observation of loose granular beds. As the writer's theory is not applicable to the calculation of the total silt load, but only the loose granular contents, it is not applicable to problems dealing with the silting of reservoirs. The mathematical equations for determining the value of f in Equation (2) $\left(\text{that is, the ratio } \frac{V_m}{v_m}\right)$ were presented[2] by the writer in 1927.

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[68] "A Theory of Silt and Scour," by W. M. Griffith, *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), pp. 276–277.

The writer agrees with Mr. Inglis that channels with a loose granular perimeter can have no permanent regime. The Punjab formulas, like Mr. Lacey's equations, are based on the function of R and not d_m and, apparently, are purely empirical. As they provide for no variation in silt factor, they appear to be incorrect in form, and to be applicable only to a limited range of conditions. They can have no general application.

The writer agrees that the value of z in Equation (7) might be affected by the grading of the loose granular matter. His value advanced for z (7.5) was not intended to be taken as an absolute value for all grades; it was advanced provisionally for fine sand. Further investigation was necessary to determine whether the value was suitable for coarse sand, and whether it was affected by an over-load of colloidal matter, or other factors.

Mr. Bauer raises the point that the horizontal components of eddies will affect the velocities in the vertical planes. Theoretically, no doubt, this is so, but in practice—especially in canals—this effect is small, except in the vicinity of the sides. In model experiments, where the vertical scale is distorted, the effect is greater, and in model experiments in a flume with a sand bed having gradually expanding or contracting sides, the writer has found that the value of d_m in Equation (6a), found for the different widths, has been somewhat affected by this factor. In practice the effect is so small that it can be neglected in applying these equations for silt transportation.

It is not clear why Mr. Bauer contends that this theory postulates a silt distribution in the vertical varying with the square of the depth. With a bed of sand and some boulders, two classes of an entirely different order in size, the boulders may roll along the bed; but this would be an exceptional case. Even under such circumstances, small boulders, after a flood, have been found wedged in the lattice girder of a bridge, showing they were definitely not traveling along the bed at the time. That large vertical eddies appear to be released periodically in a flowing section is a matter of common observation. The effect, however, does not appear to affect the general theory, as will be seen from the agreement of the data, submitted by the writer and others, of observed silt loads, with those computed by Equation (13).

Basis of the Theory Advanced.—The writer based his theory on his own observations of the following phenomena during a long period of training rivers with beds of loose granular matter:

(*a*) In all cases, in a flowing section charged with this loose granular material, if the velocity at any point in the cross-section was reduced artifically by weeds, brushwood, or other means, the depth at the point was reduced until equilibrium was again attained, this reduced depth, in some way, corresponding to the reduced velocity;

(*b*) If the velocity was increased, on the other hand, the depth increased; and,

(*c*) If the velocity remained the same but the silt charge was increased, the depth was reduced, and *vice versa*.

Phenomena (*a*), (*b*), and (*c*) lead to the conclusion that at any point in the cross-section, equilibrium of the bed could be expressed by the equation

$$V = f(c) \times f(d) \dots\dots\dots\dots\dots\dots\dots\dots (42)$$

in which V is the mean velocity in the vertical for equilibrium, d the depth, and c a factor representing the silt charge in magnitude. In consequence the writer was led, as early as 1912, to the conclusion that Equation (1) was a natural law, expressing the relationship between velocities and depths at any point in a flowing cross-section. This led also to the second of the writer's assumptions: namely, that the level of the bed at every point is governed by the law of silt transportation.

The first of the assumptions (that the transportation of silt is effected by the vertical resultant of eddies) was the result of his conception of the method in which loose granular matter is transported. It differed from the generally accepted theory that the heavier silt is rolled or dragged along the bed. In 1927 he demonstrated that true rolling of particles could occur only in one layer.[2] If the complete layer were rolled, the quantity would be only a small fraction of the heavy silt in motion.

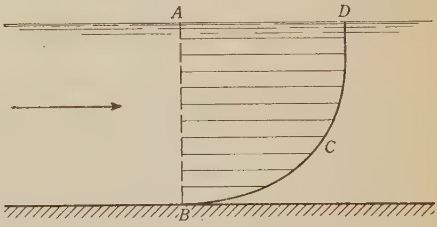

FIG. 11.

The writer's conception of the transportation of the loose granular matter is that, in general, it is the same as that in which air currents provide transportation for heavier-than-air particles, as exemplified by the glider. This idea is quite different from the conception of bodies being either rolled or dragged along a surface and is one depending mainly on the vertical eddies or the vertical resultant of eddies.

Consider a section along some streamline depicted in Fig. 11, *A B* being the depth and *B C D* representing the velocity curve from the bed, *B*, to the surface at Point *D*. Imagine the fluid divided into horizontal laminæ of equal thickness, all moving at different velocities. This difference in velocities of the laminæ creates eddies normal to their surface—that is, vertical eddies, since horizontal laminæ are being considered. The intensity of these eddies will vary with the rate of change of the velocities of the laminæ.

As the rate of change of velocities of the laminæ is a maximum at the bed and thereafter reduces to the point of maximum velocity, and as the quantity of loose granular matter which the eddies can support will vary with the intensity of these vertical eddies, it is natural to expect a silt distribution in the vertical plane, in general, in accordance with Fig. 2.

All eddies must have a return current and where the maximum velocity is materially below the surface, it is reasonable to assume that these eddies create two sets of rotation—one clockwise, to the line of maximum velocity, and one counter-clockwise, from this point to the surface; and this condition would account for some increase in the intensity of silt charge at the point of maximum velocity where the two sets of eddies meet. This increase in intensity of loose granular matter carried at the line of maximum velocity will be noticed in Table 1 at Station 10 ft and Station 22 ft from the bank.

The Value of n.—As an analysis of the hydraulic data of the boulder-strewn cross-sections of the Jumna River at Kalsi $\left(\text{having a bed slope of } \frac{1}{140}\right)$ gives a value $n = 0.65$,[69] as against a value of from 0.57 to 0.59 given by the analysis of the Madras and Kennedy canal data—and, as the limiting value of n for a minimum turbulence is $n = 0.5$—it is clear that the value of n will increase with the turbulence. This is true also of the value of Kutter's rugosity factor, N, whose limiting value for no turbulence is zero; for the very high turbulence of a boulder bed and very steep bed slope it would be of the order of 0.04 or 0.045.

The foregoing considerations lead the writer to the conclusion that an equation could be obtained connecting the value of n with Kutter's rugosity factor N. Assuming that this is a linear equation, it will be of the form

$$n = 0.5 + A\,N \qquad \text{(43a)}$$

From an analysis of data the value of A appears to be 3.33; so Equation (43*a*) becomes:

$$n = 0.5 + 3.33\,N \qquad \text{(43b)}$$

For canals the value of Kutter's N is usually accepted as varying between 0.02 and 0.025, and, from Equation (43*b*), this yields values for n equal to from 0.567 to 0.583. These values agree with those found by analysis of the Madras and Kennedy's data. Equation (43*b*) is important because it enables the elementary law, expressed by Equation (1) and other equations mathematically derived from it, to be tested against observed phenomena over the widest range of conditions.

First Test.—Consider first the case of Kennedy's and the Madras data, the hydraulic data of two sets of channels, each set stated to be in regime, carrying approximately the same silt load, but the silt load being different in each set of channels. Values of Kutter's N are not given for these channels, but presumably they will not differ materially and may be assumed to be $N = 0.0225$, giving a value, from Equation (43*b*), of $n = 0.575$. As these canal sections are approximately rectangular, and in any case are all of the same type, f can be neglected, and Equation (2) becomes $V = c(d_m)^n$ or $\left(\text{substituting } \frac{1}{n} = e\right)$;

$$d_m = \left(\frac{V_m}{c}\right)^e \qquad \text{(44)}$$

If B = the width at the surface, and Q = the discharge,

$$Q = V_m\,B\,d_m \qquad \text{(45)}$$

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[69] *Minutes of Proceedings*, Inst. C. E., Vol. 229 (1930), p. 322.

From Equations (44) and (45)

$$B = \frac{c^e Q}{(V_m)^{1+e}} \qquad \text{.........................(46)}$$

From Equation (44) it will be seen that, for equilibrium, the mean depth of a channel must vary directly as the velocity and inversely as the silt charge, all to the power of $\frac{1}{n}$, and from Equation (46) that the width of the water surface

TABLE 13.—Comparison of Observed and Computed Values

Serial No. (1)	Flow, Q, in cubic feet per second (2)	Mean velocity, V_m, in feet per second (3)	$\frac{V_m}{(d_m)^{0.575}} = C$ (4)	Observed mean depth, d_m, in feet (5)	Computed value of $d_m = 1.068(V_m)^{1.74}$ (6)	Observed width at water surface, B, in feet (7)	Computed value of B by Equation (47b) (8)
			(a) Kennedy's Original Data				
1	1 700	2.86	0.956	6.73	6.60	88.5	90.5
2	1 700	2.91	0.983	6.61	6.84	87.45	85.2
3	1 700	2.90	0.987	6.55	6.80	89.4	86.2
4	1 250	2.75	0.948	6.38	6.20	71.35	73.2
5	1 500	2.83	0.977	6.35	6.50	83.3	81.4
6	1 250	2.81	0.984	6.20	6.42	73.25	69.0
7	924	2.59	0.913	6.14	5.60	58.25	63.6
8	940	2.55	0.963	5.45	5.46	68.85	67.8
9	940	2.55	0.981	5.28	5.46	68.75	67.8
10	650	2.40	0.930	5.21	4.90	50.75	55.2
11	633	2.52	1.007	4.93	5.34	52.6	47.3
12	700	2.33	0.946	4.80	4.65	63.5	64.6
13	390	2.25	0.947	4.50	4.38	38.4	39.0
14	220	2.15	0.956	4.10	4.03	24.25	25.3
15	120	2.00	0.970	3.52	3.57	16.0	16.8
16	142	1.90	0.923	3.52	3.26	19.95	22.8
17	138	2.04	1.032	3.26	3.69	19.80	18.35
18	85	1.70	0.953	2.74	2.68	17.50	18.6
19	75	1.70	0.958	2.72	2.68	15.50	16.4
20	70	1.80	1.018	2.69	2.97	13.50	13.1
21	65	1.60	0.934	2.55	2.42	15.40	16.8
22	65	1.60	0.966	2.40	2.42	16.30	16.8
23	26	1.40	0.938	2.01	1.92	9.15	9.7
	Total		= 22.170	....	....	....	....
	Mean value of c		= 0.963	....	....	....	....
			(b) Madras Data; Godavari Western Delta Canals				
1	6 100	2.64	0.680	10.48	10.50	220	219
2	2 882	2.54	0.703	9.3	9.85	122	115
3	2 716	2.45	0.685	9.2	9.26	121	119.5
4	2 917	2.28	0.662	8.6	8.32	149	156.7
5	3 137	2.45	0.711	8.6	9.26	149	138.2
6	2 020	2.07	0.661	7.28	7.00	134	141.8
7	1 865	1.98	0.642	7.1	6.40	133	147.5
8	1 921	2.08	0.689	6.86	6.98	134.5	132.2
9	1 026	1.71	0.646	5.44	4.96	110.0	121.0
10	158.7	1.44	0.651	4.01	3.68	27.5	30.0
11	57.10	1.33	0.727	2.86	3.20	15.0	13.4
12	57.20	1.15	0.700	2.375	2.48	21.0	19.9
13	36.26	1.04	0.682	2.08	2.08	16.7	16.67
14	33.0	1.05	0.699	2.03	2.12	15.5	14.66
15	51.06	1.09	0.697	2.18	2.26	21.5	20.72
	Total		= 10.235	....	....	....	....
	Mean value of c		= 0.682	....	....	....	....

will vary directly as the discharge and as the silt charge to the power of $\frac{1}{n} (= e)$, and inversely as the velocity to the power of $\frac{n+1}{n} (= 1 + e)$.

The data, as they apply to Kennedy's channels, are given in Table 13(*a*) and the Madras channels in Table 13(*b*). From Column (4), Table 13, it will be seen that with a value of $n = 0.575$ the value of c varies in Kennedy's channels from 0.913 to 1.032 and the mean value is 0.963 (corresponding to a silt charge of loose granular matter of 7.2 parts per 10 000 by weight, from Equation (13), accepting $z = 7.5$ as found for the silt in the canals of the Imperial Valley). In Table 13(*a*) the width at the water surface (see Column (7)) has been taken as the mean depth, plus one-half the depth given by Kennedy, on the assumption that the side slopes were 1 on $\frac{1}{2}$. The mean depth d_m (Column (5), Table 13(*a*)) has been computed also from the discharge velocities and depth, as given by Kennedy, on the same assumption. For the Madras channels the value of c varies from 0.642 to 0.71 and has a mean value of 0.682 (corresponding to a silt charge of 5.1 parts of loose granular matter per 10 000 parts by weight)—a much lighter silt charge.

Substituting values of $n = 0.575$ and the values of $c = 0.963$ and 0.682 in Equations (44) and (46):

For Kennedy's channels—

$$d_m = 1.068\ V_m^{1.74} \qquad \text{(47a)}$$

and,

$$B = \frac{Q}{1.068\ V^{2.74}} = \frac{0.94\ Q}{(V_m)^{2.74}} \qquad \text{(47b)}$$

and, for the Madras channels—

$$d_m = 1.95\ V_m^{1.74} \qquad \text{(48a)}$$

and,

$$B = \frac{Q}{1.95\ (V_m)^{2.74}} = \frac{0.513\ Q}{(V_m)^{2.74}} \qquad \text{(48b)}$$

In Columns (5) and (7), Table 13, the observed values of d_m and B are given; and in Columns (6) and (8) the computed values derived from Equations (47) and (48) are given for comparison. In general this comparison will be found to support the theory advanced.

Second Test.—Consider next the case of the ratio of depth to width which has been found by practice to give the best results for channels carrying heavy silt loads.

It follows from the basic law that the channel section which has dimensions that give a maximum value for $\frac{V_m}{(d_m)^n}$ will provide the maximum silt-carrying capacity. To test this deduction it is necessary to examine particular cases. Consider three different discharges—100, 1 000, and 5 000 cu ft per sec. It is required to find the type of cross-section that gives the maximum silt-carrying capacity for these discharges. Assume a surface slope of $\frac{1}{4\ 000}$ (a sufficient slope to carry a heavy silt load) and side slopes of 1 on 0.5 Assume a value of Kutter's $N = 0.225$, which, by Equation (43*b*), gives a value of $n = 0.575$.

Table 14 contains the hydraulic data of a number of different sections giving these discharges, computed by the Kutter-Chézy formulas, the values of $\frac{V_m}{(d_m)^n}$ being given in Column (8). By plotting Column (3) against Column (8), it will be seen that: For a discharge of 100 cu ft per sec, a channel 2.5 ft deep has the highest silt-carrying efficiency; for 1 000 cu ft per sec, a 4.5-ft channel has the highest efficiency; and for 5 000 cu ft per sec, a 7-ft to 8-ft channel has the highest efficiency. These results agree with the writer's observations of the heavier silt-laden canals of the United Provinces.

TABLE 14.—HYDRAULIC DATA PERTAINING TO DIFFERENT CHANNEL SECTIONS

No. (1)	Bed width, in feet (2)	Depth, in feet (3)	Area, in square feet (4)	Mean velocity, V_m, in feet per second (5)	Water surface, B, in feet (6)	Mean depth, d_m, in feet (7)	$\frac{V_m}{(d_m)^{0.575}}$ (8)
(*a*) DISCHARGE = 100 CUBIC FEET PER SECOND; $\frac{d_m}{R} = 1.12$							
1	47.0	1.6	76.98	1.30	48.6	1.64	0.979
2	43.0	1.7	74.45	1.34	44.7	1.66	0.984
3	32.5	2.0	68.0	1.47	35.0	1.94	1.002
4	22.8	2.5	60.1	1.664	25.3	2.376	1.011
5	17.0	3.0	55.5	1.81	20.0	2.775	1.006
6	13.5	3.5	53.4	1.88	17.0	3.14	0.974
7	10.9	4.0	51.6	1.94	14.9	3.46	0.950
8	8.8	4.5	49.6	2.01	13.3	3.77	0.935
(*b*) DISCHARGE = 1 000 CUBIC FEET PER SECOND; $\frac{d_m}{R} = 1.07$							
1	155	3.0	469.5	2.13	158	2.97	1.132
2	120	3.5	426.0	2.35	121.5	3.51	1.142
3	108	3.75	414.0	2.415	111.75	3.705	1.145
4	97	4.0	396.0	2.525	101.0	3.92	1.149
5	80	4.5	370.0	2.70	84.5	4.38	1.152
6	68	5.0	352.5	2.84	73.0	4.83	1.149
7	58	5.5	335.0	2.985	63.5	5.28	1.147
8	51	6.0	324.0	3.085	57.0	5.68	1.139
9	32.5	8.0	292.0	3.42	40.5	7.21	1.100
(*c*) DISCHARGE = 5 000 CUBIC FEET PER SECOND; $\frac{d_m}{R} = 1.046$							
1	325	5	1 637	3.055	330	4.96	1.212
2	240	6	1 460	3.423	246	5.94	1.237
3	188	7	1 340	3.760	195	6.87	1.240
4	152	8	1 250	4.0	160	7.82	1.240
6	126	9	1 174	4.262	135	8.70	1.230
7	106.5	10	1 115	4.48	116.5	9.57	1.223
8	92.5	11	1 078	4.64	103.5	10.41	1.210
9	82	12	1 060	4.73	94	11.27	1.187

Third Test.—Values of $\frac{d_m}{R}$ in the sub-captions of Table 14 show that the greater the volume of discharge, the nearer this ratio of the most efficient section approaches unity. In the limit, for a section carrying an infinite discharge, the mean depth d_m must equal the hydraulic mean depth, R, and the most efficient depth for a channel carrying an infinite volume will then be that for which $\frac{V_m}{R^n}$ has a maximum value.

Accepting the Kutter-Chézy formulas for flow, it is possible to calculate the mean depth of a channel having the maximum value of $\frac{V_m}{R^n}$ as follows: Kutter's formulas may be written

$$V_m = \frac{a\,R\,\sqrt{S}}{\sqrt{R} + b} \quad \text{(49)}$$

in which: S is the surface slope;

$$a = 41.6 + \frac{1.811}{N} + \frac{0.00281}{S} \quad \text{(50a)}$$

and

$$b = \left(41.6 + \frac{0.00281}{S}\right) N \quad \text{(50b)}$$

Therefore

$$\frac{V_m}{R^n} = \frac{a\,R^{1-n}\,\sqrt{S}}{\sqrt{R} + b} \quad \text{(51)}$$

Equation (51) has a maximum value when

$$R = b^2 \left(\frac{1 - n}{n - 0.5}\right)^2 \quad \text{(52)}$$

that is, when

$$R = \left(41.6 + \frac{0.00281}{S}\right)^2 N^2 \left(\frac{1 - n}{n - 0.5}\right)^2 \quad \text{(53)}$$

Equation (53), then, gives the most efficient depth for carrying its silt charge for a channel having an infinite volume of discharge. It is reasonable to suppose that rivers of which the perimeter is governed by the silt charge will adopt the most efficient section. Table 15 gives the hydraulic data of some of the largest rivers with surface slopes ranging from $\frac{1}{140}$ to $\frac{1}{15\,700}$ the depths of which will be seen to support Equation (53). For slopes flatter than $\frac{1}{15\,700}$ Equation (53) tends to give too high results, presumably because for very flat slopes the velocities are not sufficient to support the loose granular material and, therefore, the writer's theory cannot hold.

Equation (53) will explain why the depth of a river having an unrestricted perimeter governed by its loose granular silt charge will: (*a*) Tend to increase as the volume of discharge increases; and (*b*), for the same volume of discharge, will increase as the slope reduces.

Mr. Lacey, in analyzing a number of channels that were claimed to be in permanent regime, found that the dimensions of these channels conformed to Equation (36). Mr. Inglis quoted the result of the Punjab investigation[63] which supports this equation, the Punjab equation being $P = 2.80\,Q^{0.5}$ (see Table 11). These equations are purely empirical but they supply strong evidence that some such empirical relationship holds at least within certain limits of hydraulic conditions.

Referring to Table 14, it will be seen that, for a flow of 100 cu ft per sec, the width at the water surface, B, of the channel section having the maximum silt-carrying capacity, is approximately 25.3 ft, for a flow of 1 000 cu ft per sec, approximately 84.5 ft and, for a flow of 5 000 cu ft per sec, approximately $\frac{195 + 160}{2} = 177.5$ ft. Consequently, the following relations must hold:

TABLE 15.—Hydraulic Data of Sundry Large Rivers

Site of observation station	Approximate discharge, in cubic feet per second	1 000 S	Kutter's coefficient, N, of rugosity	Exponent n	Observed Depths, in Feet		Depths, in feet computed by Equation (53)
					R or d_m	Maximum	
(*a*) River Jumna, in India							
Kalsi, India*	165 300	7.14	0.045	0.65	6 to 10	..	19.35
Fizebad, India†	....	3.33	0.035	0.616	9.7	20 to 26	23.6
(*b*) Mississippi River, in the United States							
Vicksburg, Miss.‡	1 215 000	0.06379	0.0308	0.6025	64.1	100	104.5
Columbus, Ky.‡	1 014 000	0.068	0.0327	0.609	65.88	88	82.4
New Madrid, Mo.§	480 000	0.124	0.0434	0.6445	24.06	...	46.6
New Madrid, Mo.§	680 000	0.128	0.0469	0.656	29.53	...	43.1
New Madrid, Mo.§	530 000	0.1133	0.0463	0.654	31.72	...	47.7
New Madrid, Mo.§	744 000	0.1218	0.0460	0.6534	37.96	...	38.8
(*c*) Irrawaddy River, in India§							
Saiktha, Burma	742 000	0.06456	0.0325	0.6084	33.57	61	101.8
Saiktha, Burma	1 000 000	0.07748	0.0337	0.612	39.16	67	83.0
Saiktha, Burma	1 240 000	0.08608	0.0327	0.609	42.82	71	75.7
Saiktha, Burma	1 382 000	0.09039	0.0314	0.6045	44.47	73	75.0

* *Minutes of Proceedings*, Inst. C. E., Vol. 229, p. 327. † Loc. cit., Vol. 223, pp. 256 and 257. ‡ "A General Formula for the Uniform Flow of Water in Rivers and Other Channels," by Ganguillet and Kutter; in English measure by Rudolph Henry and John C. Tunturne, p. 223. § *Loc. cit.*, p. 220.

Channel flow, in cubic feet per second	Values of B
100	$2.53\ Q^{0.5}$
1 000	$2.67\ Q^{0.5}$
5 000	$2.54\ Q^{0.5}$

This result is important as it appears to establish the law that permanent regime in a channel can be attained only for the sectional dimensions which provide the maximum silt-carrying efficiency. This result is in accordance with nature's tendency for efficiency.

It also gives reason to question whether this relationship should not be some relationship, such as $B = 2.6\ Q^{0.5}$, instead of $P = 2.67\ Q^{0.5}$; and, whether this is a purely empirical relationship or in itself represents a natural law.

Equating $B = 2.6\ Q^{0.5}$ with Equation (46):

$$2.6\ Q^{0.5} = \frac{C^e\ Q}{(V_m)^{1+e}} \quad \text{(54)}$$

Therefore,

$$Q^{0.5} = \frac{2.6\ (V_m)^{1+e}}{C^e} \quad \text{(55)}$$

At first sight this does not appear likely to be more than an accidental relationship holding approximately within a limited range of hydraulic conditions; but the point offers scope for further investigation.

For the last test it will perhaps not be out of place to consider the application of this theory to the silt problem of the All-American Canal.

Some years ago the Hoover Dam was constructed on the Colorado River. It de-silts the river waters completely of both its loose granular and colloidal contents. According to the writer's theory, on leaving this reservoir, the river discharge, to attain equilibrium, must again pick up its full loose granular silt load, to satisfy Equations (2), (7), and (8), provided such content is available in the bed material. Given sufficient length of river-bed to attain this equilibrium, therefore, the discharge should be found to be carrying the same silt load of loose granular material again, for any given velocity and depth of flow, as obtained before the building of this reservoir. A report on the working of this project, dated 1937, indicates that this has been found to be the case.[70]

Consider next the dimensions of the new All-American Canal cross-section given by Professor Lane.[71] The corresponding values of $\frac{V_m}{(d_m)^n}$ (assuming a values of Kutter's $N = 0.0225$, giving a value of $n = 0.575$) are given in Column (6), Table 16, and the values of $\frac{7.5\ V_m}{(d_m)^n}$, the calculated silt load of loose granular matter which these different cross-sections can carry are listed in Column (7).

TABLE 16.—CROSS-SECTIONS, ALL-AMERICAN CANAL

Serial No. (1)	Discharge, in cubic feet per second (2)	Sectional area, in square feet (3)	V_m, in feet per second (4)	d_m, in feet (5)	$\frac{V_m}{(d_m)^{0.575}}$ (6)	Load of loose granular matter, $\frac{7.5\ V_m}{(d_m)^{0.575}}$, expressed in parts per 10 000 (7)
1	15 155	4 041.3	3.75	17.4	0.725	5.44
2	13 155	3 508.0	3.75	16.15	0.758	5.68
3	10 155	2 708.0	3.75	13.77	0.831	6.23
4	7 600	2 171.0	3.50	12.26	0.825	6.18
5	7 400	2 114.0	3.50	12.13	0.834	6.26
6	7 100	2 029.0	3.50	11.86	0.844	6.34
7	6 800	1 943.0	3.50	11.90	0.843	6.33
8	5 100	1 569.0	3.25	10.82	0.824	6.18

It will be seen that in Reaches No. 3 to No. 8 the silt-carrying capacity of the different sections is substantially the same; but the silt-carrying capacity of the head reaches (Nos. 1 and 2) is rather less. Furthermore, the silt-carrying capacity of all sections varies from 5.44 to 6.34 parts per 10 000 of loose granular matter by weight, as against a silt-carrying capacity of the present Imperial Valley Canals of from 9.0 to 12.0 parts per 10 000 by weight (see Column (9) Table 1 and Column (8) Tables 2 and 3).

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[70] *Engineering News-Record*, March 4, 1937.

[71] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Table 14, p. 194.

Until such time as the erosion of the river-bed between the Hoover Dam and the head-works of the All-American Canal has materially reduced the slope of the river in this reach, it would appear to be necessary to rely on de-silting works near the head to reduce the loose granular silt load in the river materially, if this canal is to maintain its designed bed without heavy dredging.

For ready reference the principle equations of this theory are given in the Appendix and the writer hopes that they may be of some use to American engineers who have such vast problems of river and canal engineering to solve.

Acknowledgments.—The writer's thanks are due to those who have contributed to the discussion of this paper, especially Mr. Johnson, Professor Lane, and Mr. Blaney, for supplying new and unpublished data and scientific information against which the theory can be checked. Prof. J. Gould, of Cambridge University, Cambridge, England, aided greatly in checking the mathematics. Others, who discussed the paper, cited references of important works by authorities in this field, many of which were new to the writer. It is a source of satisfaction to note that the new data support the theory advanced.

APPENDIX

A summary of the pertinent notation in this paper is as follows: d = depth; d_m = mean depth of the cross-section; v = mean velocity in the vertical, above a point in the bed; v_m = mean velocity in the vertical above the point of mean depth; V_m = mean velocity of the cross-section; $f = \frac{V_m}{v_m}$; Q = the discharge; N = Kutter's rugosity coefficient; n = an exponent (for value see Equation (43*b*); B = width at water surface; c = general silt factor; c_1 = loose granular silt load expressed in terms of parts per 10 000 by weight; z = silt coefficient value = 7.5 for fine sand); S = surface slope; and $e = \frac{1}{n}$.

(*1*) General equations connecting velocity, depth, and silt charge at any point in a cross-section, and for the entire cross-section:

$$v = c\, d^n \qquad (1)$$

$$V_m = f\, c (d_m)^n \qquad (2)$$

(*2*) Equation to obtain the value of Exponent n:

$$n = 0.5 + 3.33\, N \qquad (43b)$$

(*3*) Equation connecting values of widths at water surface and mean depths of two adjoining cross-sections:

(*a*) Similar types of cross-sections—

$$d_{m2} = d_{m1}\left(\frac{B_1}{B_2}\right)^{\frac{1}{n+1}} \quad \text{......................} (6a)$$

(*b*) Dissimilar types of cross-sections—

$$d_{m2} = d_{m1}\left(\frac{f_1 B_1}{f_2 B_2}\right)^{\frac{1}{n+1}} \quad \text{......................} (6b)$$

(*4*) Equation connecting silt coefficients:

$$c_1 = Z\,\frac{v}{d^n} = Z\,c \text{......................} (7) \text{ and } (8)$$

(*5*) Equations for the mean depth and surface width of canals in equilibrium:

$$d_m = \left(\frac{V_m}{c}\right)^e \quad \text{......................} (44)$$

$$B = \frac{c^e}{(V_m)^{1+e}} \text{......................} (46)$$

(*6*) Equation giving the most efficient depth of flow to carry the silt charge for an infinite discharge:

$$R = \left(41.6 + \frac{0.00281}{S}\right)^2 \times N^2 \times \left(\frac{1-n}{n-0.5}\right)^2 \text{..........} (53)$$

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SIPHONS AS WATER-LEVEL REGULATORS

By J. C. Stevens,[1] M. Am. Soc. C. E.

With Discussion by Messrs. A. Griffin, H. P. Currin and D. M. Umphrey, T. J. Corwin, Jr., Henry R. King, and J. C. Stevens.

Synopsis

No novelty attaches to siphons used as wasteways, but the design of siphons for close regulation of water levels, for example, in the forebay of power plants, by discharging continuously through a wide range of flow, marks a distinct advance in such structures and offers opportunities for putting siphons to uses not heretofore contemplated.

This paper describes tests made on two new siphons designed as regulators of the forebay level at the Walterville Hydro-Electric Plant, a unit of the municipal power system of Eugene, Ore. Each of these siphons is capable of discharging continuously any rate between 100% and 3% of its maximum capacity. Working in unison the pair has a maximum discharging capacity of 1 050 cu ft per sec and a minimum of 11 cu ft per sec, so that, considered as a unit, they have a discharging range of from 1% to 100% of their maximum.

The Walterville Siphons

These siphons, two in number, with capacities of 350 and 700 cu ft per sec, respectively, were put into service in the fall of 1936. They replace an old gate and flume waste from the forebay of the Walterville Plant. A part of an overflow spillway was retained by which ice and drift could by-pass the siphons. The discharge from the spillway is carried in an open flume made by extending the sides of the siphon barrel block 4 ft above its top surface. General dimensions of these siphons, in feet, are as follows:

Description	Siphon No. 1	Siphon No. 2
Length of barrel..............	127.8	123.5
Elevation of overflow crest at summit................	595.1	595.0
Elevation of crown at outlet....	550.0	550.0
Elevation of top of primer orifice	595.3	595.2

Note.—Published in October, 1938, *Proceedings*.

[1] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

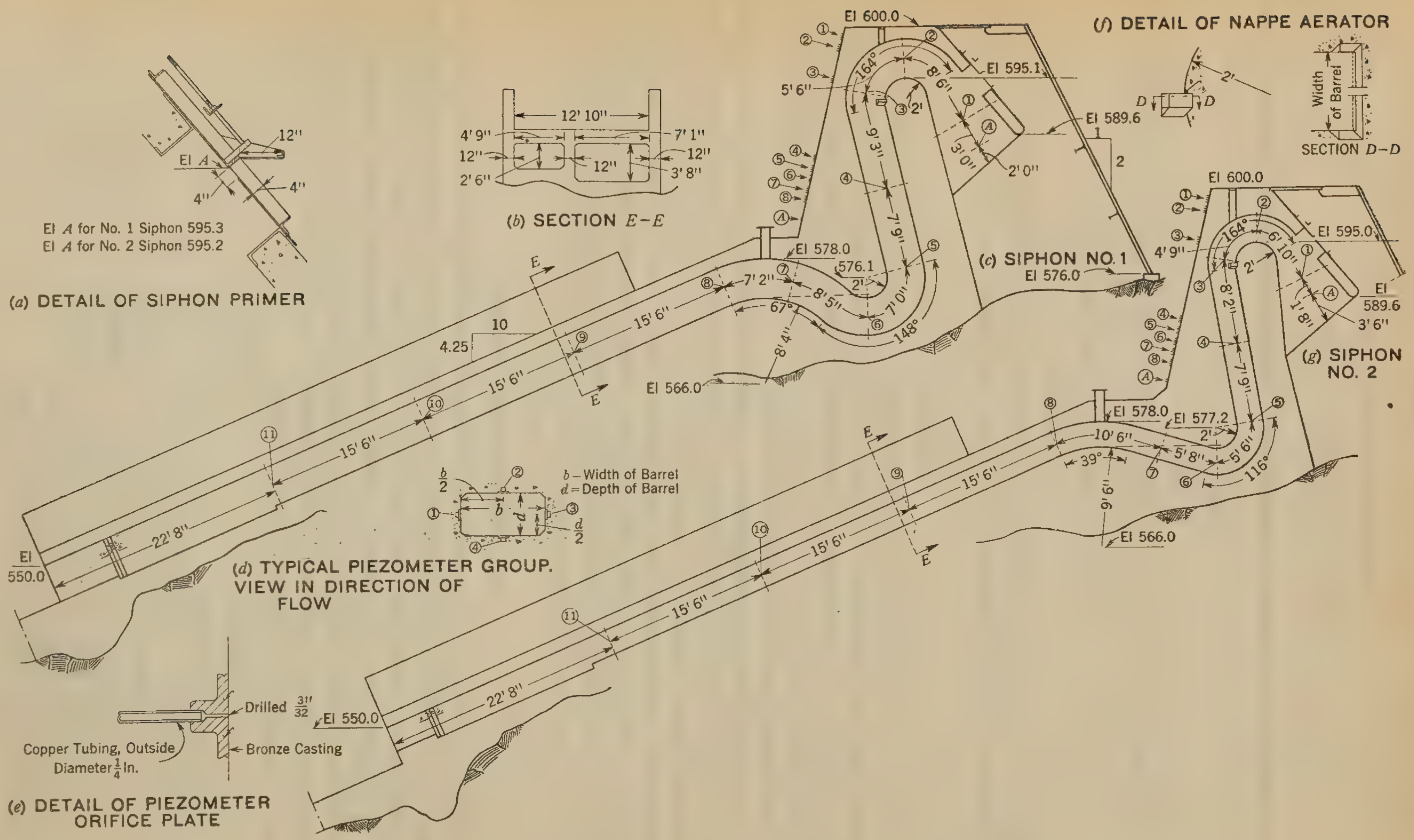

(a) DETAIL OF SIPHON PRIMER
12"
El A
4"
4"
El A for No. 1 Siphon 595.3
El A for No. 2 Siphon 595.2
(b) SECTION E–E
12' 10"
4' 9"
7' 1"
12"
12"
12"
2' 6"
3' 8"
(c) SIPHON NO. 1
El 600.0
El 595.1
El 589.6
El 578.0
576.1
El 576.0
El 566.0
5' 6"
164°
8' 6"
2'
2'
9' 3"
7' 9"
3' 0"
2' 0"
7' 0"
148°
7' 2"
8' 5"
67°
8' 4"
15' 6"
15' 6"
15' 6"
22' 8"
10
4.25
1
2
E
E
El 550.0
(d) TYPICAL PIEZOMETER GROUP. VIEW IN DIRECTION OF FLOW
b – Width of Barrel
d = Depth of Barrel
b/2
d/2
b
d
(e) DETAIL OF PIEZOMETER ORIFICE PLATE
Drilled 3/32"
Bronze Casting
Copper Tubing, Outside Diameter 1/4 In.
(f) DETAIL OF NAPPE AERATOR
2'
D
D
Width of Barrel
SECTION D–D
(g) SIPHON NO. 2
El 600.0
El 595.0
El 589.6
El 578.0
El 577.2
El 566.0
El 550.0
4' 9"
164°
6' 10"
2'
1' 8"
3' 6"
8' 2"
7' 9"
2'
5' 6"
116°
10' 6"
5' 8"
39°
9' 6"
15' 6"
15' 6"
15' 6"
22' 8"
E
E
①
②
③
④
⑤
⑥
⑦
⑧
⑨
⑩
⑪
Ⓐ

Fig. 1.—Essential Dimensions of Siphons Nos. 1 and 2, Walterville Siphons

Description	Siphon No. 1	Siphon No. 2
Nominal Dimensions of Barrel:		
At inlet	8.35 by 7.08	8.50 by 4.75
Section *A* (Fig. 1)	7.43 by 7.08	5.80 by 4.75
Section 1 (Fig. 1)	5.95 by 7.08	4.75 by 4.75
Section 2 (Fig. 1), to outlet.	3.67 by 7.08	2.50 by 4.75

Other essential data are presented in Table 1.

TABLE 1.—WALTERVILLE SIPHONS

Section (see Fig. 1)	SIPHON NO. 1			SIPHON NO. 2		
	Distance, in feet, along the center line of barrel	Area, in square feet, at right angles to the center line	Elevation of center line, in feet	Distance, in feet, along the center line of barrel	Area, in square feet, at right angles to the center line	Elevation of center line, in feet
Inlet	0	59.0	586.7	0	40.4	586.8
A	2.00	52.5	88.5	3.50	27.5	90.0
1	5.00	42.0	91.0	5.17	22.6	91.4
2	13.50	25.90	96.9	12.00	11.84	96.2
3	19.00	25.90	93.7	16.75	11.84	93.4
4	28.25	25.90	84.6	24.92	11.84	85.4
5	36.00	25.90	77.0	32.67	11.84	77.9
6	43.00	25.90	72.2	38.17	11.84	74.0
7	51.50	25.90	75.9	43.48	11.84	75.5
8	58.67	25.90	75.4	54.34	11.84	75.9
9	74.17	25.90	69.3	69.84	11.84	69.9
10	89.67	25.90	63.2	85.34	11.84	63.9
11	105.17	25.90	57.2	100.84	11.84	57.8
Outlet	127.80	25.90	548.3	123.47	11.84	548.8

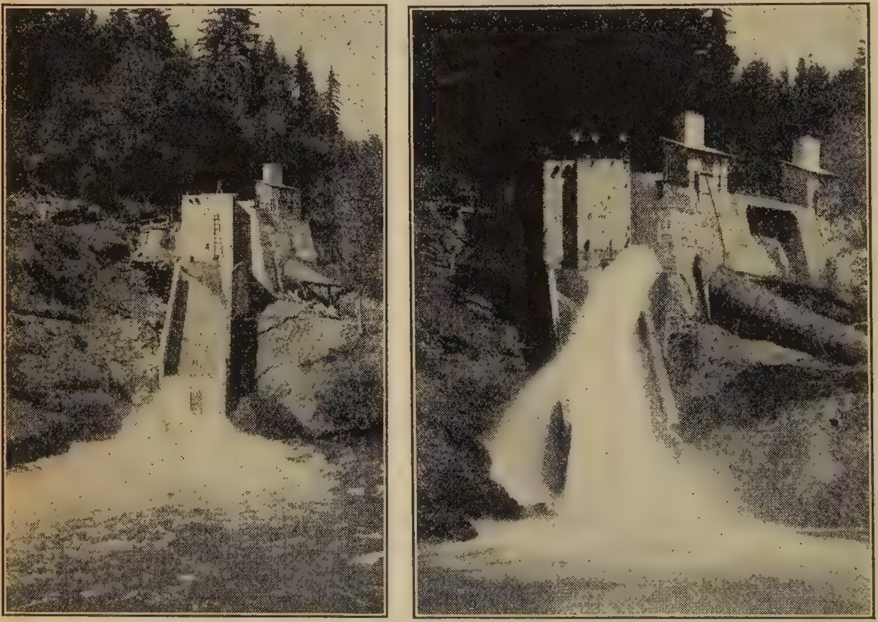
Fig. 1 gives the longitudinal profile of the siphons and essential details of the primer, nappe aerator, and piezometers. There was some question as to whether or not the long lower leg would run full. Therefore, a depression was left in the invert near the end of each barrel in which later a baffle could be built, that would deflect the water to the crown and thus keep the air exhausted from the barrel. Experience indicates that such a device is not necessary, and later these depressions were filled. As long as the siphon does not inhale air the barrel runs full to the end.

The primer is a special feature of this design. It constitutes a discharge regulator since it permits the siphon to inhale air whenever the forebay level drops below the horizontal lip of the primer. As the siphon inhales air the discharge diminishes, until the siphon discharge equals the surplus flow into the forebay. The priming orifices are adjustable both as to depth of opening and as to elevation, and each is provided with a horizontal lip above the orifice to act as a regulator of air supply, and to eliminate the drop-down curve that otherwise would appear as the water accelerates into the priming orifice.

The lower leg is effective in reducing discharge because as flow diminishes this leg runs partly full, the head on the siphon then being from the forebay level to that point in this leg where the water leaves the crown of the barrel.

The nappe aerator was provided inside the barrels just down stream of the summit crest in order to admit air behind the nappe and thus permit it to leave the face of the barrel and fall in a curtain across it, to exhaust the air more quickly. A somewhat similar aerator was included in the Leaburg Siphons.[2]

The piezometers were installed in order to determine the behavior of the siphons for use in future designs. Each piezometer has a copper tube sloping upward from the tap to a manifold on the down-stream face of the siphon block, with a cock in each tube and a connection on the manifold for a vacuum hose. The locations of these manifolds are shown in Figs. 1 and 2(*b*).



(*a*) General View (*b*) Sluice-Gate Discharging Under Full Head

Fig. 2.—Walterville Siphons

One 6-in. pipe with removable cap was provided in Siphon No. 2 and two such pipes in Siphon No. 1 in the summit sections for the purpose of venting the siphons. There is also a vent at the summit of the lower sealing curve to be used for any contingency that might arise.

At Section *A* there are two taps in each siphon in the center of the sides of the barrel. The copper tubing slopes downward from these taps to the point marked *A* on the siphon block (see Fig. 1) where they connect to a header. From this header a rubber hose extends up to a vertical glass tube laid alongside another tube connected to the forebay water. From a scale behind the glass tubes the water level in the forebay and that at Section *A* could be read simultaneously.

Siphon Discharge

Several expedients were adopted to determine the discharge through the siphons during tests. For the higher discharges two methods were used: (1)

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[2] "On the Behavior of Siphons," by J. C. Stevens, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986.

By means of current meter measurements made in the canal about 3 miles up stream on days when the power plant was shut down; and (2), by means of a Pitot tube in the outlet end of each siphon barrel.

Flow from the forebay into the open wasteway is controlled by stop-planks, the head on which is an index of flow. This measuring device was designated the "stop-log weir." There is also a 30-in. by 30-in. sluice-gate discharging from the forebay. Knowing the forebay level and gate-opening the discharge through it was calculated as for an orifice. The stop-log weir is shown to the left of the siphon block in Fig. 2(*a*), and Fig. 2(*b*) shows the sluice-gate discharging under a full head.

On days when the current meter was used three separate measurements were made about 2 hr apart while a steady canal regimen was maintained. Water in excess of that through the siphon being tested was wasted over the stop-log weir or through the sluice-gate, or both, and was thus measured. Water was also flowing freely over the crest of the unprimed siphon which was calculated as a weir with a rounded crest-ogee type. In addition, there was a small quantity of leakage through the closed gates to the three penstocks which had to be estimated. The discharge through the siphon under test was taken as the average of the three current meter measurements made during that day, less that wasted *via* the sluice-gate, the stop-log weir, the unprimed siphon, and the penstocks. Figs. 3 and 4 show the discharges for the sluice-gate and also for the unprimed siphons. The discharge over the stop-log weir was calculated from the Bazin formula for a crest length of 6.29 ft and a height of 2 ft. A correction of 0.15 h was made in the length to allow for suppressed contraction at one end.

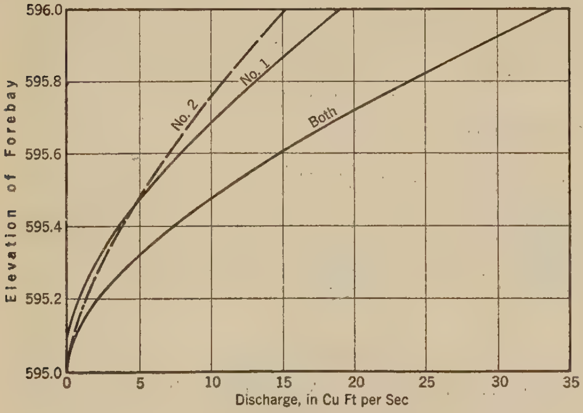


FIG. 3.—FREE DISCHARGE OVER SIPHON CRESTS

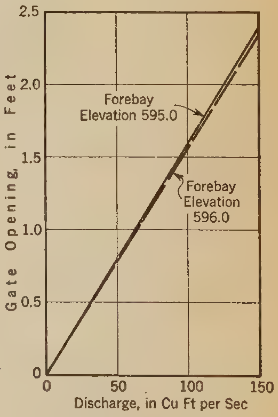


FIG. 4.—DISCHARGE THROUGH SLUICE-GATE

For the smaller discharges, current meter measurements were impracticable, and tests were made while the plant was running. Water not taken through the turbines was passed first through the siphon and then changed to the sluice-gate or stop-log weir for measurement. This manipulation involved occasional small variations in forebay level so that some slight canal and forebay storage was involved. For the low discharges that could be taken entirely by the sluice-gate this effect did not exist since the change from sluice-gate to siphon and *vice versa* could be made without affecting the forebay level.

In order to make a check on the canal measurements a frame was set up by which a Pitot tube could be held in any desired position in the outlet with the orifice perpendicular to the direction of flow. The Pitot tube measurements, of course, could be made only for siphons under full discharge with water completely filling the barrel at the outlet. These measurements are more accurate than those by the other methods since the discharge of the siphon under test could be measured directly.

There is a drop in the hydraulic grade line from the forebay level to Section *A* in which positive pressures exist for all flows. The decrease in area from the Inlet to Section *A* constitutes an effective Venturi section that was calibrated by observing this drop for each measured discharge. This drop was called the "meter-head" and was designated as H_a; the quantity, $\sqrt{H_a}$, was used as the index of flow. Table 2 gives the results of the various discharge measurements made, including all data required for plotting. A correction of + 0.01 ft was made to low heads, H_a, as it was observed that the piezometer at Section *A* stood that much above the forebay level with no flow through the siphon.

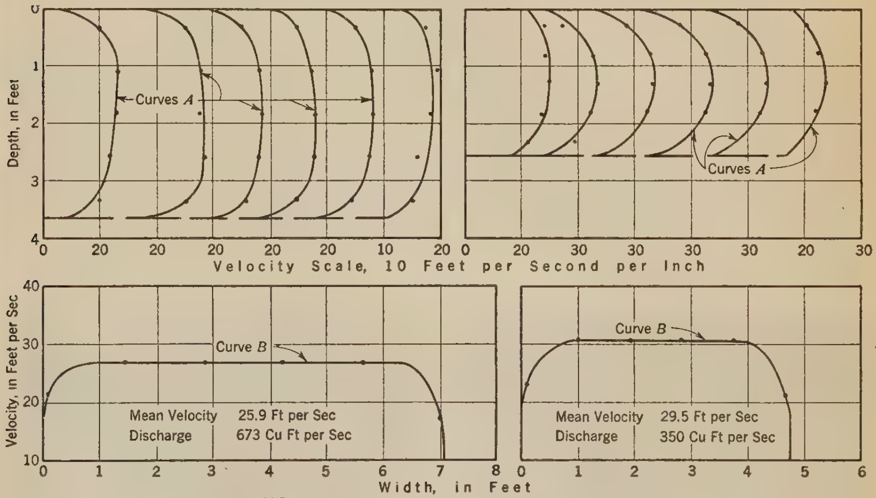
TABLE 2.—MEASUREMENTS OF DISCHARGE THROUGH SIPHONS

Date	SIPHON NO. 1					SIPHON NO. 2				
	Discharge, in cubic feet per second	Elevations, in Feet		Meter Head, in Feet		Discharge, in cubic feet per second	Elevations, in Feet		Meter Head, in Feet	
		Fore-bay	Section *A*	H_a	$\sqrt{H_a}$		Fore-bay	Section *A*	H_a	$\sqrt{H_a}$
October 14, 1936*	688	595.75	591.65	4.10	2.02	....	....	....	....	....
October 15, 1936*	594	95.58	92.35	3.23	1.80	....	....	....	....	....
October 16, 1936*	346	95.37	94.25	1.12	1.06	....	....	....	....	....
October 22, 1936†	116	95.24	95.02	0.23**	0.48	117	95.24	94.65	0.59	0.77
November 5, 1936‡	720	95.86	91.55	4.31	2.08	....	....	....	....	....
November 17, 1936†	45	95.28	95.27	0.02**	0.14	46	95.16	95.05	0.12**	0.35
March 5, 1937‡	22	95.27	95.27	0.01**	0.10	11	95.20	95.20	0.01**	0.10
April 19, 1937§	673	95.94	91.59	4.35	2.08	350	95.76	93.19	2.57	1.61

* Current meter method. † Stop-log weir method. ‡ Sluice-gate method. § Pitot tube method.
** Correction, + 0.01 ft.

For the measurements of April 19, 1937, the Pitot tube was held at five depths in six vertical sections in the mouth of each barrel. The tube itself had been rated previously, obtaining a coefficient of 0.98. It was screwed into the end of an extra strong ½-in. pipe. A ¼-in. copper nipple was soldered into this pipe just below the upper end for vacuum hose connection to the mercury manometer. The end was closed with a cork which could be removed to "bleed" the pipe between readings.

Fig. 5(*a*) gives the results of the Pitot tube measurements for Siphon No. 1, Curve *A* showing the vertical velocity curves. The mean velocity in each vertical was determined by planimeter; these mean velocities, in turn, determine the plotted points in the horizontal velocity curve (Curve *B*), and the planimetered area of this curve divided by the width gives the mean velocity in the barrel. In similar manner Fig. 5(*b*) gives the results for Siphon No. 2. Comparison with similar curves for the Leaburg siphons[3] shows how the long lower leg has "ironed out" the effect of the up-stream curves.



(*a*) SIPHON NO. 1 (*b*) SIPHON NO. 2

FIG. 5.—RESULTS OF PITOT TUBE MEASUREMENTS

Fig. 6 shows the outlet of the barrels with Siphon No. 1 discharging 700 cu ft per sec (Fig. 6(*a*)), and Siphon No. 2 discharging 350 cu ft per sec (see Fig. 6(*b*)). The flow in the open flume is from the spillway in the forebay and that in the unfilled barrel is from the free flow over its crest. The issuing jets look white and foamy but they are really crystal clear at the exit, under the curtain of water.

Range of Discharge.—The measurements of March 5, 1937, were made using a small quantity of waste water, all of which could be measured by passing it through the sluice-gate. From the rating curve (plotted from Table 2) and from these measurements the range of discharges, in cubic feet per second, was found to be:

Description	Siphon No. 1	Siphon No. 2
Maximum discharge for minimum forebay level without inhaling air............	700	350
Minimum discharge......................	22	11
Minimum discharge expressed as a percentage of the maximum................	3.1	3.1

Priming Head.—The head at which a siphon will prime is influenced by the rapidity of the rise of the forebay. When the forebay was raised very slowly

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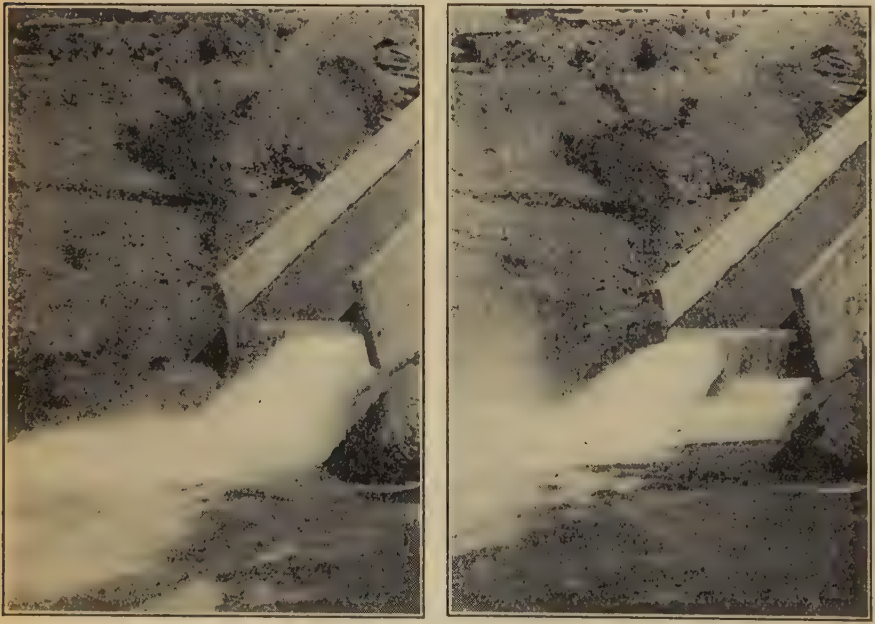
[3] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 991.

(say, 0.5 ft per hr, or less) the priming heads, in feet, were:

Description	Siphon No. 1	Siphon No. 2
Elevation of crest	595.10	595.00
Forebay level at time of priming	595.40	595.23
Priming head	0.30	0.23

TESTS BY PIEZOMETERS

As shown in Fig. 1 there are twelve sets of piezometer taps in each siphon. In making a test it was necessary first to establish a uniform flow with a stable canal and forebay regimen. By opening each cock in turn the pressure at each piezometer was indicated by the depression of the mercury (or water) column of a U-tube manometer. Oscillations were virtually eliminated by a damper attached to the manometer. Since all readings, except for the forebay and for Section *A*, were negative and since the tubes contained no water and the weight of the air column could be neglected, the elevation of the manometer was of no consequence.



(*a*) SIPHON NO. 1 (*b*) SIPHON NO. 2

FIG. 6.—CHARACTERISTICS OF FLOW WITH MAXIMUM DISCHARGE

Table 3 gives the list of piezometer tests made. The discharges indicated were taken from the rating curves as plotted from Table 2.

Fig. 7(*a*) gives the essential results of the tests for Siphon No. 1 running full. The vertical lines at the bottom give the negative pressure head, in feet of water. Each bar is the average of the pressure heads at the corresponding piezometer tap from Tests Nos. 4 and 11 in Table 3. The relation of the negative pressure head to that of one atmosphere is readily seen. The pressure heads are in the order of the numbering of the taps given in Fig. 1; that is, No. 1 in the left face, No. 2 in the crown, No. 3 in the right face, and No. 4 in the invert of the barrel, while facing the direction of flow.

TABLE 3.—LIST OF PIEZOMETER TESTS

Test No.*	(a) SIPHON No. 1					Test No.*	(b) SIPHON No. 2				
	Average Water Levels, in Feet		Meter Head, in Feet		Discharge, in cubic feet per second		Average Water Levels, in Feet		Meter Head, in Feet		Discharge, in cubic feet per second
	Forebay	Section *A*	H_a	$\sqrt{H_a}$			Forebay	Section *A*	H_a	$\sqrt{H_a}$	
4	595.91	591.57	4.34	2.08	700	1‡	595.72	593.20	2.52	1.59	342
5	595.30	594.34	0.96	0.98	300	2	595.80	593.22	2.58	1.61	350
6	595.24	595.01	0.23	0.48	135	3	595.22	594.55	0.67	0.82	120
9	595.28	595.27	0.01	0.10	22	10‡	595.76	593.16	2.60	1.61	350
11†	595.97	591.62	4.36	2.09	700	....	....	....	....	....	....

* Except as noted, all tests were run on March 3, 4, and 5, 1937. † April 13, 1937. ‡ October 23, 1936.

Similarly Fig. 7(*b*) gives the results of the tests for Siphon No. 2 running full. The negative pressure heads, in feet of water, are the average of those for each tap in Tests Nos. 1, 2, and 10. The negative pressure heads from each tap in a section were averaged to obtain the average pressure head for that section. The average heads were then subtracted from the elevation of the center line of the barrel and 33 ft added to obtain absolute pressure heads from sea-level datum. These absolute pressure heads for each section are shown as circles on the drawings.

The elevation of the center line of the barrel has been plotted against its distance from the inlet, giving a developed profile of the siphon. The numbers indicate sections at which piezometer sets were installed.

Energy Line.—The discharge divided by the area gives the nominal mean velocity at each section, the velocity head of which, added to the absolute pressure, gives the points indicated by triangles. If the barrel were filled at all points by down-stream moving water, and if the piezometers correctly indicated the true pressures, and the average of all in one set indicated the correct pressure in that section, the triangles would define a line of energy for that particular discharge. That they do not truly define such a line is due to the barrel being partly occupied by eddies, by accelerating water, by entrained air greatly expanded under the nearly complete vacuum that obtains, and the inherent inaccuracies in determining mean pressures in a section by means of the four piezometers. The absolute pressure line has been drawn to fit the points as nearly as is consistent with fundamental principles. In general, it fits the observations.

It was impossible for the energy line to conform to the triangles throughout in view of the following principles that must obtain:

(*a*) It can never rise above its initial elevation but must continuously slope down stream because energy is being lost throughout the entire length of the barrel;

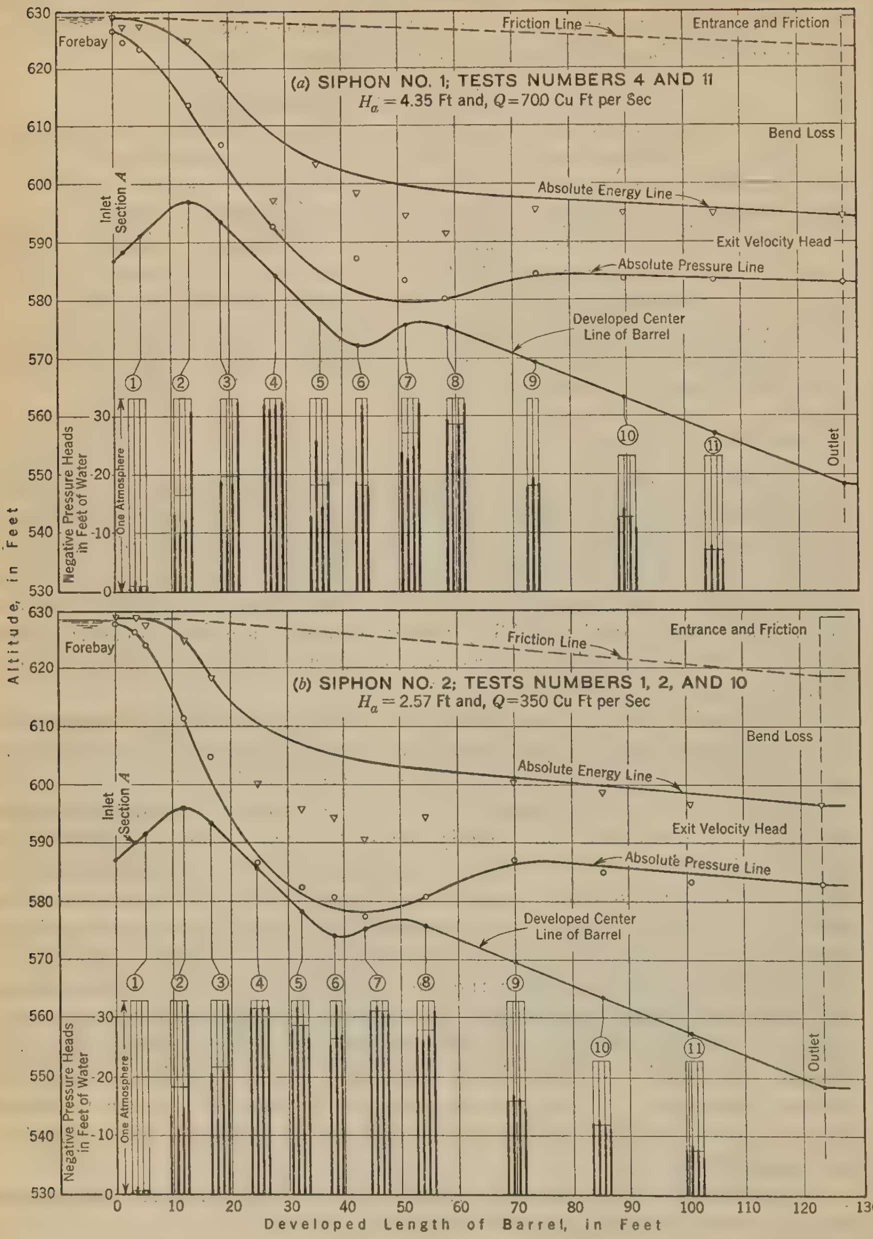


FIG. 7.—PRESSURE AND ENERGY GRADIENTS

(b) Its slope must never be less than that corresponding to the hydraulic friction for that discharge; and,

(c) Its distance from the pressure line must never be less than the velocity head corresponding to the mean velocity obtained by dividing discharge by barrel area.

Energy Losses.—The hydraulic losses are primarily of two kinds—hydraulic friction, and eddy and impact losses due to bends. The nominal friction losses were computed by the Kutter formula with $n = 0.013$, and by using the mean velocities obtained by dividing discharge of area.

The difference between the total loss and that due to entrance and friction is assigned to bends. There seems to be no doubt that the bend losses increase with the size of the barrel as there is more room in the larger barrels for eddying and spiral motion. They also are greatly affected by the distribution of velocities in the approach to the bends. Normally, the highest velocity and lowest pressure are on the inside of the bend. If the inside velocity is highest in the approach, however, the loss will be far greater than if the approach velocities are reversed.[4] Heretofore, all the experiments of flow around bends have been made with small conduits and under positive pressures. Large conduits under less than atmospheric pressures present an entirely new field of research that has scarcely been touched.

In the present case it will be noted that the difference in pressures between the concave and the convex sides of the summit bend amounted to almost 20 ft of water. (See Sections 2 and 3, Fig. 7.) Great inequality of velocities must result from the summit bend so that the approach conditions for the next bend become very complex and uncertain. To the loss in the summit bend must be added the effect of the discharge through the priming orifice joining the main stream at an angle.

In the present state of knowledge it is impossible to calculate or define the individual bend losses in these siphons. The total bend losses, however, can be determined approximately by the difference between total loss and calculated entrance and friction losses.

Table 4 gives the results of the tests for siphons running full, that is, not inhaling air. Explanation of the columns follows: Column (2) gives the average of the observed pressure heads at the four taps in each section (where the taps were plugged—Sections 6 and 9, Siphon No. 1—the average for Taps 1 and 3—at the sides—was used); Columns (3) and (5) were taken from Table 1; Column (4) is the elevation of center line of the barrel, minus (or plus) the pressure head, plus 1 atmosphere (33 ft); Column (6) is the discharge divided by the area; Column (7) is the velocity head for velocities in Column (6); Column (8) contains the absolute pressure heads plus the velocity heads; Columns (9) and (10) are the adjusted values of the absolute pressure, and energy lines, read from the curves of Fig. 7; Column (11) is the difference between adjusted values of absolute pressure and energy lines; Column (12) gives the velocities corresponding to velocity heads of Column (11); Column (13) is the discharge divided by velocities of Column (12). (This represents the portion of the barrel filled by down-stream moving water, the remainder of the area being occupied by eddies); and Column (14) is the percentage of the total area occupied by eddies.

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[4] "Flow of Water Around Bends in Pipes," by the late David L. Yarnell and the late Floyd A. Nagler, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1018.

TABLE 4.—Energy Losses with Siphons Running Full

<table>
<tr><th rowspan="3">Section</th><th rowspan="3">Pressure head, in feet</th><th colspan="2">Elevation, in Feet</th><th rowspan="3">Area, in square feet</th><th rowspan="3">Mean velocity, in feet per second</th><th rowspan="3">Velocity head, in feet</th><th rowspan="3">Elevation, in feet, energy line of mean velocity</th><th colspan="4">Adjusted Values</th><th rowspan="3">Useful area, in square feet</th><th rowspan="3">Eddy area (percentage)</th></tr>
<tr><th rowspan="2">Center line of barrel</th><th rowspan="2">Absolute pressure line</th><th colspan="2">Elevations, in Feet</th><th rowspan="2">Velocity head, in feet</th><th rowspan="2">Velocity, in feet per second</th></tr>
<tr><th>Pressure line</th><th>Energy line</th></tr>
<tr><th>(1)</th><th>(2)</th><th>(3)</th><th>(4)</th><th>(5)</th><th>(6)</th><th>(7)</th><th>(8)</th><th>(9)</th><th>(10)</th><th>(11)</th><th>(12)</th><th>(13)</th><th>(14)</th></tr>
<tr><td colspan="14">(a) Siphon No. 1; Average of Tests Nos. 4 and 11, March 4 and April 13, 1937; Discharge, 700 Cubic Feet per Second (26 Cubic Feet per Second Through Primer); Average Forebay Elevation, 595.94; and, Average Meter Head 4.35 Feet</td></tr>
<tr><td>*</td><td>....</td><td>....</td><td>628.9</td><td>....</td><td>0</td><td>....</td><td>628.9</td><td>628.9</td><td>628.9</td><td>0</td><td>....</td><td>....</td><td>....</td></tr>
<tr><td>†</td><td>+ 6.8</td><td>586.7</td><td>626.5</td><td>59.0</td><td>11.4</td><td>2.0</td><td>628.5</td><td>626.5</td><td>628.5</td><td>2.0</td><td>11.4</td><td>59.0</td><td>0</td></tr>
<tr><td>A</td><td>+ 3.1</td><td>588.5</td><td>624.6</td><td>52.5</td><td>12.8</td><td>2.5</td><td>627.1</td><td>625.7</td><td>628.2</td><td>2.5</td><td>12.8</td><td>52.5</td><td>0</td></tr>
<tr><td>1</td><td>− 0.6</td><td>591.0</td><td>623.4</td><td>42.0</td><td>16.0</td><td>4.0</td><td>627.4</td><td>624.0</td><td>628.0</td><td>4.0</td><td>16.0</td><td>42.0</td><td>0</td></tr>
<tr><td>2</td><td>−16.3</td><td>596.9</td><td>613.6</td><td>25.9</td><td>27.0</td><td>11.3</td><td>624.9</td><td>613.0</td><td>624.3</td><td>11.3</td><td>27.0</td><td>25.9</td><td>0</td></tr>
<tr><td>3</td><td>−19.8</td><td>593.7</td><td>606.9</td><td>25.9</td><td>27.0</td><td>11.3</td><td>618.2</td><td>605.2</td><td>618.2</td><td>13.0</td><td>28.9</td><td>24.2</td><td>7</td></tr>
<tr><td>4</td><td>−31.7</td><td>584.6</td><td>585.9</td><td>25.9</td><td>27.0</td><td>11.3</td><td>597.2</td><td>592.6</td><td>608.5</td><td>15.9</td><td>32.0</td><td>21.8</td><td>15</td></tr>
<tr><td>5</td><td>−18.0</td><td>577.0</td><td>592.0</td><td>25.9</td><td>27.0</td><td>11.3</td><td>603.3</td><td>585.0</td><td>604.0</td><td>19.0</td><td>35.0</td><td>20.0</td><td>22</td></tr>
<tr><td>6</td><td>−18.1</td><td>572.2</td><td>587.1</td><td>25.9</td><td>27.0</td><td>11.3</td><td>598.4</td><td>581.4</td><td>601.5</td><td>20.1</td><td>36.0</td><td>19.4</td><td>24</td></tr>
<tr><td>7</td><td>−25.9</td><td>575.9</td><td>583.2</td><td>25.9</td><td>27.0</td><td>11.3</td><td>594.5</td><td>580.0</td><td>599.8</td><td>19.8</td><td>35.7</td><td>19.6</td><td>23</td></tr>
<tr><td>8</td><td>−28.4</td><td>575.4</td><td>580.0</td><td>25.9</td><td>27.0</td><td>11.3</td><td>591.3</td><td>580.2</td><td>598.4</td><td>18.2</td><td>34.2</td><td>20.5</td><td>20</td></tr>
<tr><td>9</td><td>−18.1</td><td>569.3</td><td>584.2</td><td>25.9</td><td>27.0</td><td>11.3</td><td>595.5</td><td>583.8</td><td>597.2</td><td>13.4</td><td>29.4</td><td>23.8</td><td>8</td></tr>
<tr><td>10</td><td>−12.6</td><td>563.2</td><td>583.6</td><td>25.9</td><td>27.0</td><td>11.3</td><td>594.9</td><td>584.0</td><td>596.4</td><td>12.4</td><td>28.2</td><td>24.8</td><td>4</td></tr>
<tr><td>11</td><td>− 6.9</td><td>557.2</td><td>583.4</td><td>25.9</td><td>27.0</td><td>11.3</td><td>594.7</td><td>583.8</td><td>595.6</td><td>11.8</td><td>27.6</td><td>25.4</td><td>2</td></tr>
<tr><td>‡</td><td>+ 1.7</td><td>548.3</td><td>583.0</td><td>25.9</td><td>27.0</td><td>11.3</td><td>594.3</td><td>583.0</td><td>594.3</td><td>11.3</td><td>27.0</td><td>25.9</td><td>0</td></tr>
<tr><td colspan="14">(b) Siphon No. 2; Tests Nos. 1, 2, and 4, October 23, 1936, and March 3, and April 13, 1937; Discharge, 350 Cubic Feet per Second (17 Cubic Feet per Second Through Primer); Average Forebay Level, 595.76; and, Average Meter Head, 2.57</td></tr>
<tr><td>*</td><td>....</td><td>....</td><td>628.7</td><td>....</td><td>0</td><td>0</td><td>628.7</td><td>628.7</td><td>628.7</td><td>....</td><td>0</td><td>....</td><td>....</td></tr>
<tr><td>†</td><td>+ 7.6</td><td>586.8</td><td>627.4</td><td>40.4</td><td>8.25</td><td>1.1</td><td>628.5</td><td>627.4</td><td>628.5</td><td>1.1</td><td>8.25</td><td>40.4</td><td>0</td></tr>
<tr><td>A</td><td>+ 3.5</td><td>590.0</td><td>626.1</td><td>27.5</td><td>12.1</td><td>2.3</td><td>628.4</td><td>626.1</td><td>628.4</td><td>2.3</td><td>12.1</td><td>27.5</td><td>0</td></tr>
<tr><td>1</td><td>− 0.5</td><td>591.4</td><td>623.9</td><td>22.6</td><td>14.7</td><td>3.4</td><td>627.3</td><td>624.5</td><td>627.9</td><td>3.4</td><td>14.7</td><td>22.6</td><td>0</td></tr>
<tr><td>2</td><td>−18.0</td><td>596.2</td><td>611.2</td><td>11.84</td><td>29.5</td><td>13.5</td><td>624.9</td><td>611.4</td><td>624.9</td><td>13.5</td><td>29.5</td><td>11.84</td><td>0</td></tr>
<tr><td>3</td><td>−21.7</td><td>593.4</td><td>604.7</td><td>11.84</td><td>29.5</td><td>13.5</td><td>618.2</td><td>600.5</td><td>618.2</td><td>17.7</td><td>33.7</td><td>10.4</td><td>12</td></tr>
<tr><td>4</td><td>−31.8</td><td>585.4</td><td>586.6</td><td>11.84</td><td>29.5</td><td>13.5</td><td>600.1</td><td>588.0</td><td>610.4</td><td>22.4</td><td>38.0</td><td>9.21</td><td>22</td></tr>
<tr><td>5</td><td>−28.7</td><td>577.9</td><td>582.2</td><td>11.84</td><td>29.5</td><td>13.5</td><td>595.7</td><td>581.4</td><td>607.0</td><td>25.6</td><td>40.6</td><td>8.63</td><td>27</td></tr>
<tr><td>6</td><td>−26.4</td><td>574.0</td><td>580.6</td><td>11.84</td><td>29.5</td><td>13.5</td><td>594.1</td><td>578.8</td><td>605.2</td><td>26.4</td><td>41.2</td><td>8.50</td><td>28</td></tr>
<tr><td>7</td><td>−31.2</td><td>575.5</td><td>577.3</td><td>11.84</td><td>29.5</td><td>13.5</td><td>590.8</td><td>578.0</td><td>604.0</td><td>26.0</td><td>40.9</td><td>8.57</td><td>28</td></tr>
<tr><td>8</td><td>−28.0</td><td>575.9</td><td>580.9</td><td>11.84</td><td>29.5</td><td>13.5</td><td>594.4</td><td>580.6</td><td>602.6</td><td>22.0</td><td>37.6</td><td>9.32</td><td>21</td></tr>
<tr><td>9</td><td>−15.9</td><td>569.9</td><td>587.0</td><td>11.84</td><td>29.5</td><td>13.5</td><td>600.5</td><td>586.2</td><td>601.2</td><td>15.0</td><td>31.1</td><td>11.3</td><td>5</td></tr>
<tr><td>10</td><td>−11.9</td><td>563.9</td><td>585.0</td><td>11.84</td><td>29.5</td><td>13.5</td><td>598.5</td><td>586.2</td><td>599.9</td><td>13.7</td><td>29.7</td><td>11.79</td><td>0.5</td></tr>
<tr><td>11</td><td>− 7.6</td><td>557.8</td><td>583.2</td><td>11.84</td><td>29.5</td><td>13.5</td><td>596.7</td><td>585.0</td><td>598.5</td><td>13.5</td><td>29.5</td><td>11.84</td><td>0</td></tr>
<tr><td>‡</td><td>+ 1.2</td><td>548.8</td><td>583.0</td><td>11.84</td><td>29.5</td><td>13.5</td><td>596.5</td><td>583.0</td><td>596.5</td><td>13.5</td><td>29.5</td><td>11.84</td><td>0</td></tr>
</table>

* Forebay. † Inlet. ‡ Outlet.

The discharges through the primer orifices were determined as illustrated for Siphon No. 1 as follows:

Forebay level	595.9	
Pressure line at Section 1	590.4	
Head on orifice (h), in feet		5.5
Velocity ($= 0.6 \sqrt{2gh}$), in feet per second		11.2
Area of orifice, in square feet		2.35
Discharge, in cubic feet per second		26

The discharge through the Inlet, and Sections *A* and 1, therefore, was 674 cu ft per sec and, through the remainder of the barrel, 700 cu ft per sec.

Table 5 is a summary of the results for siphons when running full. It is to be noted that the total losses are greater in the larger barrel. The friction losses in the large siphon are only about one-half those of the smaller one, but the bend losses are 32% greater. The entrance losses were taken at two-tenths the velocity head at the Inlet Section.

TABLE 5.—SUMMARY OF TESTS FOR SIPHONS RUNNING FULL

Siphon No.:	Average of Tests Nos.: (1)	HEAD LOSS DATA, IN FEET							VELOCITY, IN FEET PER SECOND				PERCENTAGE	
		Discharge, in cubic feet per second (2)	Siphon head (3)	Outlet velocity head (4)	Total energy head (5)	Entrance head (6)	Friction head (7)	Bend loss (8)	Corresponding to siphon head (9)	Actual mean outlet velocity (10)	Summit mean velocity, Section 2 (11)	Corresponding to atmospheric head (33 ft) (12)	Coefficient of discharge, outlet section (13)	Efficiency of siphon (14)
1	4 and 11	700	45.9	11.3	34.6	0.4	5.1	29.1	54.3	27.0	27.0	46.0	50	59
2	1, 2, and 10	350	45.7	13.5	32.2	0.2	9.8	22.2	54.2	29.5	29.5	46.0	54	64

The coefficient of flow (Column (13), Table 5), is the ratio of the actual mean velocity at the outlet to the velocity corresponding to the siphon head. For these siphons running full this coefficient is 50% for the larger, and 54% for the smaller, barrel; that is, they actually discharge those percentages of the theoretical discharge for these particular siphons.

The efficiency of the siphon (see Column (14), Table 5) has been defined as the ratio of the actual mean velocity at the summit section to the velocity corresponding to a head of 1 atmosphere.[5] For these siphons, Siphon No. 1 has an efficiency of 59% and Siphon No. 2, an efficiency of 64%; that is, when running full they will discharge those percentages of the theoretical maximum that any siphon whatever could discharge.

Partial Discharges.—When the forebay level is too low to keep the priming orifice completely submerged, the siphons will inhale air, the volume depending on the forebay level. As they inhale air the discharge diminishes until it just equals the surplus inflow to the forebay. The pressures that exist within the barrels for partial discharges, were also determined.

Fig. 8(*a*) gives the result of Test No. 5 on Siphon No. 1 while discharging 300 cu ft per sec, or 43% of its maximum. The negative heads obtained from the piezometers are shown in the same manner as in Fig. 7. The pressure line shown is the normal pressure line (not absolute pressures). It coincides with the barrel at Section 9 showing that the barrel was aerated from the outlet to that point.

In similar manner Figs. 8(*b*) and 8(*c*) show the results for Tests Nos. 6 and 9 on Siphon No. 1 for 130 and 22 cu ft per sec, respectively. In these tests the

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[5] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1004.

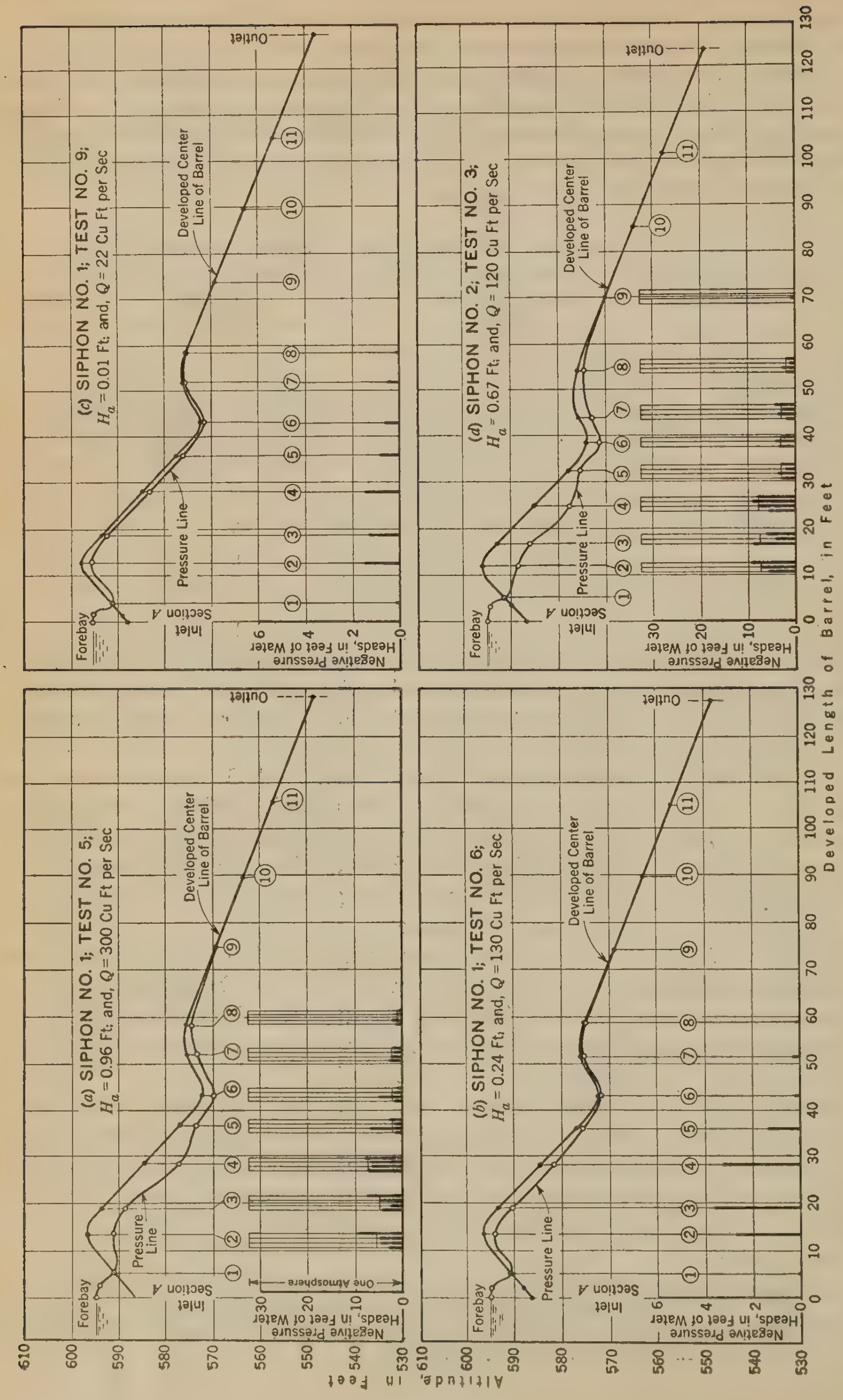

FIG. 8.—PRESSURE GRADIENTS

pressures in the piezometer manifolds, only, were observed. Negative pressures are indicated as far as Section 8 for both tests. The water flowed over the lower crest between Sections 7 and 8 without filling the barrel. This was equivalent to moving the siphon outlet to some point between Sections 7 and 8, materially reducing the siphon head below that for full flow.

Likewise, Fig. 8(*d*) shows the results of Test No. 3 on Siphon No. 2. As in the case of Siphon No. 1 the barrel was aerated from the outlet to Section 9.

CONCLUSIONS

The conclusion seems justified that the larger the barrel the greater will be the eddy losses and the less the efficiency. Efficiency, however, is only a minor consideration in the design of such siphons. The most important function is sensitiveness and range of discharge, features in which the Walterville siphons excel any others known to the writer.

One important function of the lower leg is to reduce the siphon head under partial discharges. As the discharge diminishes below the lower limit, Siphon No. 1 will cease first, and if the available discharge does not become less than 11 cu ft per sec, Siphon No. 2 will take it continuously. Thus, considered as a unit, the pair has a discharge range between 1 050 and 11 cu ft per sec, or 100% to 1 per cent. Siphons with such a range of discharge for continuous operation are truly remarkable and are believed to be unique in the history of such structures.

ACKNOWLEDGMENTS

Special credit is due to the Eugene (Ore.) Water Board, of which Mr. J. W. McArthur is Superintendent, for furnishing the facilities and assistance in making these tests. Their value looms large in the field of engineering research.

The details of the preliminary tests and of the design and construction were handled by Don Umphrey, Assistant Engineer, under the direction of Hugh P. Currin, Chief Electrical Engineer. The writer acted in an advisory capacity throughout and had charge of the final tests.

DISCUSSION

A. Griffin,[6] M. Am. Soc. C. E. (by letter).—Siphon regulators can be designed and built in many different ways and still perform their intended functions usefully. However, much remains to be learned as to the various factors affecting efficient design and most useful adaptation, and the author's paper is a valuable contribution to the subject.

It has been a debated subject whether a head greater than one atmosphere can be used effectively in a siphon regulator. W. P. Creager,[7] M. Am. Soc. C. E., has given a mathematical analysis to show that this is possible. The author indicates the same conclusion but does not specifically state it. His tests show that the siphon heads of nearly 46 ft are effectively used in the Walterville siphons. They closely approach the limit of head at Point 4. With a greater extension of the lower leg there would probably be a break in the water column near Point 4. With a shortening of the lower leg to give a head of one atmosphere, or less, there would be a reduction in coefficients of flow and siphon efficiency. The additional head greater than one atmosphere that can be used effectively is related to the energy losses and velocity changes below any point in the outlet leg selected for analysis.

The study of siphon behavior is important to the improvement of design and particularly to confident predetermination of capacity and performance. Violently destructive effects are not likely to be encountered, and there is usually ample opportunity to repair minor injury such as results from cavitation and wear. Nevertheless, it is desirable to reduce the destructive effects of turbulence and high velocities, and to gain the advantages of improved discharge efficiencies. This is especially true where large, or critical, heads are utilized in large-capacity siphons.

In any particular design siphon efficiency may properly be sacrificed in favor of some one or more desired characteristics, or to accommodate construction methods. Coefficient of flow and sensitivity or other desired characteristics are likely to be important points in the mind of a designer. Coefficients of flow will tend to be greater for smaller heads. Keeping in mind, also, that velocity varies as the square root of the head, it is evident that siphons utilizing comparatively low heads (even as low as 1 ft or 2 ft) may be employed usefully. A siphon with a head of 6 ft to 8 ft will probably discharge half as much as one with the maximum usable head.

A siphon is not likely to be clogged by any quantity of drift in pieces short enough to pass through it readily. The writer has seen a 4-ft by 4-ft siphon passing tangled masses of Russian thistles at an almost unbelievable rate, and has never seen a siphon clog. Two adjacent siphons, operating simultaneously, would be more likely to clog because of drift catching on the partition wall between them. This is a source of risk in any multiple-opening structure that is passing drift. It is very difficult to clog a single-opening structure.

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[6] Chf. Engr., Dept. of Natural Resources, C. P. Ry., Calgary, Alberta, Canada.

[7] *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 1122.

Perhaps the greatest and most common service that can be performed by a siphon regulator is as a strictly automatic regulator in which rôle it is almost perfect. For such use the writer suggests that it is most desirable to eliminate every form of manual control or adjustment. Of course, a siphon can also perform services in which manual control or adjustment is included as an essential feature.

It is desirable to emphasize to engineers the ease of design and construction and the wide variety of useful applications of the siphon regulator. In one case the writer built two inside forms, fixed the reinforcing in place, and then applied gunite. The completed siphons were hauled to location and hung in place over the top of a previously constructed concrete wall. The summit vertical dimension was 6 in., the width 3 ft, and the head about 9 ft. One siphon was given a sealing basin, the other discharged freely, and both gave the same satisfactory service. In 1938 a siphon of about 30 cu ft per sec capacity, utilizing about 6 ft of head, was welded together in the shop in sheet metal. It was installed by hanging it over a flash board in an irrigation-canal structure where it performs a very useful service. It is also hung to a screw lift so that the control elevation can be altered.

For most cases the siphon is effective from zero discharge to 100% of its capacity. When the flow is too small for the siphon to operate continuously it can still operate as an overflow spillway and intermittently as a siphon.

The writer likes the author's designation "siphon regulator." This is more adequately descriptive of its field of usefulness than the more limited term "siphon spillway." He disagrees with the term "primer." This opening is an air inlet to break siphonic action and thus would be better described as a "de-primer," or simply as an "air-inlet." At times it would be useful to have more information than is now available on the factors determining the minimum dimensions of the air-inlet necessary to interrupt siphonic action. It would be interesting to know whether any tests were made of the capped pipes at the summits, with the air-inlets ("primers") sealed.

The writer has never used a nappe aerator and would be interested in comments by the author as to conditions under which it is either necessary or desirable.

Engineers interested in siphon regulators should re-read the paper and discussions on this subject by the late George F. Stickney,[8] M. Am. Soc. C. E., published in 1922, and that by the author[9] published in 1934.

H. P. Currin,[10] Esq., and D. M. Umphrey,[11] Esq. (by letter).—Experience with the Leaburg siphons mentioned by the author indicated that these siphons would maintain flow down to about 25% of their capacity before breaking. At Leaburg there are five siphons rated at 500 cu ft per sec each, one rated at 250 cu ft per sec, and one at 100 cu ft per sec. It was decided that the Walterville siphons could be designed with fewer units of higher capacity. This would reduce the cost materially. A study was made at Leaburg[12] to determine the factors favoring the lower partial flows so as to take advantage of these points in the Walterville design. The backward sloping vertical legs at Leaburg are about 28 ft long. The lower leg is terminated about 25 ft from the end of the sealing pool section as contrasted with about 68 ft finally adopted for the Walterville installation.

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[8] *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 1098.

[9] *Loc. cit.*, Vol. 99 (1934), p. 988.

[10] Supt., Elec. Dept., Eugene Water Board, Eugene, Ore.

[11] Asst. Elec. Engr., Eugene Water Board, Eugene, Ore.

With the Leaburg siphons running at full flow the water completely fills the outlet. With a small air admission at the primer the water falls away from the top of the outlet, venting back up the barrel and thus reducing the head on the siphon. The Walterville siphons were designed with a long lower leg to take advantage of this condition. Fig. 8 shows that this feature was justified. The shorter length above the sealing pool allows the siphon to prime faster, as less air must be removed to start siphonic action. These design features were favored by the configuration of the forebay wall and lower hillside at Walterville.

At Leaburg air is admitted under the primer into a narrow space. From that point it is drawn upward into a horizontal slot and discharged into the highest point of the upper crown where there is a very low absolute pressure when the siphon is running full and pressure is still considerably below that of the atmosphere at low partial flows. This results in a high velocity through the constricted passage behind the primer. Water is drawn several inches above the surface of the forebay into this passage, and the hydraulic conditions are such that there is an alternate passage of water and air through the priming passage in a series of gulps. When the siphon is running nearly full these gulps occur several times a second; but as the forebay falls and more air is admitted the period lengthens until the gulps occur about once a second when the siphon finally breaks at about 25% flow.

It was felt that this surging action had a tendency to break the siphon sooner than if the air were admitted steadily, and special attention was paid to eliminating this surge at Walterville, as far as possible. Fig. 1 shows how this was done. Air is admitted into each barrel as directly as possible from just above the forebay level. Although it was not possible to do away with the gulps entirely, they occur more frequently and are less violent than at Leaburg.

The horizontal primer lips shown in Fig. 1(*a*) were installed after the siphons had been in service long enough to show how far the surface drop-down curve, mentioned by the author, extended from the primer opening. Later these 12 in. lips were added.

Observations before and after adding the lips indicated that these devices caused Siphon No. 1 to run full 0.27 ft, and Siphon No. 2 to run 0.23 ft, below the forebay level necessary to shut off regulating air without them. Breaking and priming levels were affected very slightly, if at all.

An important element in the regulation of these siphons to low flows is the difference in elevation between the upper crest and the priming air intake. In each of the Walterville siphons, the top edge of the intake opening is set 0.2 ft above the corresponding crest.

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[12] For illustrations, see *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 989.

In Table 2 it is shown that the lowest measured flows in both barrels occurred with the forebay at approximately 0.2 ft above the corresponding crest or at about the level of the lower edge of the priming lip. Although this head over the crest is not sufficient to re-establish siphonic action after the siphon has broken, it does tend to stabilize the lower flows which otherwise might be interrupted by surging in the siphon barrels. When these openings were lowered below their present position for testing the siphons were less stable at the lower flows and broke at a higher minimum discharge.

A 12-in. pipe was installed as a vent in the summit of the lower sealing curve in Siphon No. 1 and a 10-in. pipe in Siphon No. 2. It was thought that there might be some tendency of the water to cling to the tops of the long lower sections of the barrels at lower flows, and make it necessary to admit air through these vents by means of auxiliary regulating hoods in the forebay to vent the lower sections. Experience showed that this was not necessary, and the vents were closed by bolted flanges.

There has been some trouble with ice in the primers of the Leaburg siphons.[13] The high velocity and consequent sharp reduction of temperature of the admitted air in the constricted passages caused the primers to choke up with ice in a place very difficult to reach for clearing. Finally, 6-in. iron pipe nipples were cemented into holes drilled into the crowns of these siphons so that air could be admitted to break the siphons or to regulate their flow when the primers were closed by ice. This feature was also used at Walterville.

In the winter of 1936–1937, ice occurred at both Walterville and Leaburg. At Walterville some ice formed beneath the primer lips on both siphons, but it was easily removed. In a locality where ice is common it would probably be advantageous to install some heating device on the primer lips. These venting openings were very convenient while testing.

The author gives the priming level for both siphons with the forebay rising slowly. In 1936 a test was made to determine the effect, on the priming level, of raising the water quickly. The plant was shut down as fast as possible under normal operation. The forebay water raised at the rate of 0.38 ft per min. Both siphons primed together at Elevation 595.6 which is 0.2 ft above the normal priming level for Siphon No. 1, and 0.37 ft for Siphon No. 2.

In the Walterville design special care was taken to avoid air leakage around the adjustable primers. Smooth steel faces were used for all sliding joints, and jack bolts were provided to hold these joints together. It was found necessary to calk these joints to make them tight. Some form of gasket should have been included in this design.

The Walterville siphons vibrate to a certain extent at full flows. This is similar to the effect at Leaburg mentioned by the author[14] and is probably due to the same cause; that is, the water column separates and rejoins in the upper part of the vertical section.

Inspection of the Walterville siphons late in 1938 showed some slight erosion on the outside of the curves where water velocity is highest. Some erosion had also occurred in Siphon No. 2 on the inside of the upper curve, extending downward to the nappe aerator, or from Section 2 to Section 3 (see Fig. 9).

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[13] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1011.

[14] *Loc. cit.*, p. 998.

The wear from Section 2 to Section 3 in Siphon No. 2 is in a place where absolute pressure is very low when the siphon is discharging the higher flow (see Fig. 7). This is also at a location where it was found very difficult to place concrete properly. A gravel pocket was found in this area after the inside form was removed during construction, and was filled by a surface patch. This patch shows very little wear.

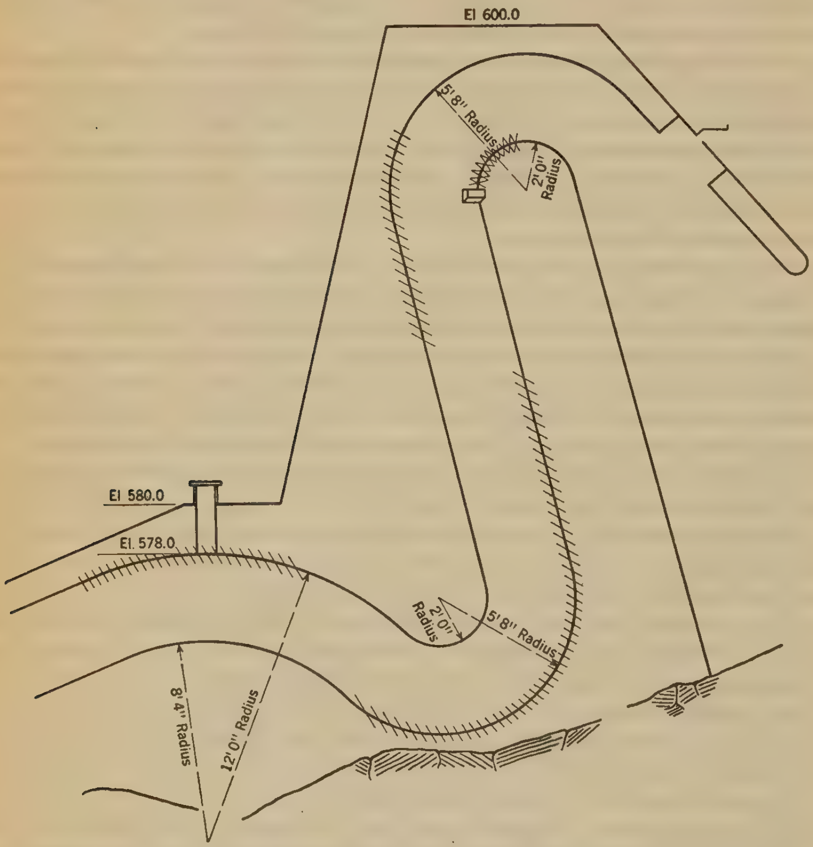

Fig. 9.—Wear in the Walterville (Ore.) Siphon Spillway as Shown by Inspection of November 22, 1938 (Single Hatching Indicates Points of Wear on Both Siphons; and Double Hatching Indicates Wear in Siphon No. 2 Only)

Under normal operation Siphon No. 2 runs continuously with partial flow, and Siphon No. 1 primes only after Siphon No. 2 has built up a considerable discharge. Evidently, Siphon No. 2 is subjected to more severe conditions of low absolute pressure inside the barrel and should show more deterioration due to this condition than Siphon No. 1. The erosion in Siphon No. 2, Section 2 to Section 3, is no doubt due to cavitation.

Low absolute pressure occurs at all four piezometers at Section 4 at the higher flows. This section is in good condition in both siphons. Dense concrete is essential under conditions of this nature.

After eight years of experience with the Leaburg siphons and two years with those at Walterville, the writers feel that they would not hesitate to build other siphons for a similar purpose. It is felt that they are superior to any mechanical gate for this service. To quote from a report from the man in charge of operation "they are 'darn' good spillways."

T. J. CORWIN, JR.,[15] M. AM. SOC. C. E. (by letter).—The use of siphons for the control of water level, or for spillways, is of particular interest to the hydraulic engineer since a close regulation of water surface is possible. Mr. Stevens has presented a paper that represents a most thorough study on the action of siphons.

The design, arranged to have both a low head or high head on the siphon, is most unusual and can be used to advantage where enough head is available to make this layout possible. The adjustable primer can be used where unusual operating conditions will not permit a constant maximum water surface elevation.

Siphon spillways were selected to discharge excess flows occurring in the Tiger Creek conduit of the Mokelumne River power development of the Pacific Gas and Electric Company, in California, on the basis of reliability, close regulation of water surface, low cost, and freedom from maintenance. The conduit with a capacity of 550 cu ft per sec, completed in 1931, is about 20 miles long, of which 16 miles is concrete flume 14 ft wide and 7 ft deep. At several points along the conduit, feeder flumes or pipe lines pick up the flow of tributary streams. During certain seasons, the run-off from heavy rain or melting snow increases the flow from these feeder flumes to such an extent that the main flume would be overtopped unless protected by automatic spillways. Seven siphons were installed at points where feeders enter the conduit line, and tests were made on four, to check performance and to effect possible improvements in the design.

Fred C. Scobey, M. Am. Soc. C. E., A. W. Kidder, Assoc. M. Am. Soc. C. E., and the writer, made these tests in April, 1932.[16]

In so far as they bear upon the paper by Mr. Stevens the following conclusions from this test may be cited:

(1) The lower end of an inclined barrel should be either sealed or should have some vertical bends to cause the water to jet across the barrel;

(2) An auxiliary priming weir seems justified where the rise in water surface is limited (the depth required for priming was reduced about 2 in. when the auxiliary device was operating);

(3) Use of the lowest throat height consistent with good proportions, for the barrel, tends to decrease the head required for priming;

(4) The vacuum breaker pipes could be placed directly in front of the inlet rather than at the side since there is no appreciable surface disturbance when an enlarged inlet section is used;

(5) The air vent, which has an area of about 6% of the throat area, should be placed about 2 in. above the desired water surface elevation at which the

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[15] Asst. Engr., Pacific Gas & Elec. Co., San Francisco, Calif.

[16] "Results of Tests on Siphon Spillways for Large Flumes," by A. W. Kidder and T. J. Corwin, *Engineering News-Record*, May 5, 1932, p. 649.

discharge should stop (the reason for this is that the siphon continues to operate at a partial discharge, sucking air and water through the vent until the water depth in the flume is correct for the setting of the throat of the siphon); and

(6) The use of more piezometer connections should be made for a complete study and the oscillation of the mercury columns should be damped. The rate of rise of the water surface should be controlled.

The operation of these siphons was free from objectionable vibration at all times, although the addition of sealing pools had been blamed for pulsations of some siphons. The pool had no apparent effect upon the discharge capacity.

Any designer contemplating the location of a siphon spillway in a conduit or small forebay must be sure that the water is free from entrapped air. The writer has had experience with a siphon located in a structure with two sources of water. It would prime with the water diverted from the stream but would fail to function when the excess water entered the structure in a turbulent manner from a pipe line.

Without a doubt, Mr. Stevens' test is the most thorough ever made and published. The energy line is of particular interest and shows where the major losses occur in the barrel, and the designer might be able to improve the coefficient of discharge by reducing the radius of curvation of the bends in the barrel.

It is interesting to note that the heads required for priming are about the same for the Tiger Creek and Walterville systems, and that the rate of rise in the water surface was found to influence the priming head in both cases. The apparent installation of trash-rack bars in front of the siphon shown in the Walterville structure might reduce the effectiveness of the siphon unless automatic rakes are provided to keep the racks clean.

HENRY R. KING,[17] ESQ. (by letter).—In 1933, Mr. Stevens called attention to the fact that "once it has started, the flow through any siphon may * * * diminish to a small part of its normal capacity without breaking of its siphonic action." Other writers have also discussed this phenomenon.

In its Southwest Sewage Treatment Works, the Sanitary District of Chicago, Ill., has three large siphon spillways which serve as excess flow and emergency by-passes. The spillways are designed to prime with water surfaces within 18 in. of the tops of channels and open tanks and to operate at any rate from zero to full discharge. Therefore, they may be classified as water-level regulators. The design was based on data obtained from small model tests made in 1935 to determine whether a siphon was suitable for this kind of service. The type of spillway tested (see Table 6) was found to be quite satisfactory. The overflow crest functioned as a weir until the flow was sufficient to raise the water surface to the bottom edge of the primer plate, at which elevation the level remained constant, the siphon inhaling air for the larger flows up to full discharge.

Table 6 gives the capacities, number of siphons, and dimensions for the large spillways at the Southwest Sewage Treatment Works. Each consists of a number of identical siphons in which all primer plates are set at the same elevation. The overflow crest is made of castings that can be raised for the installation of other castings under them—an essential provision in order to keep water levels down until after the addition of possible future stages in the sewage treatment process. The accessibility of the overflow crests makes this elevation adjustment possible. The overhang of the overflow castings is intended to serve as a nappe aerator.

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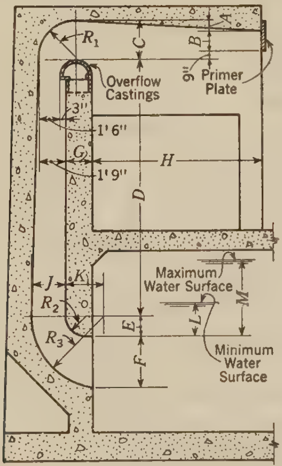
[17] Senior Civ. Engr., The Sanitary District of Chicago, Chicago, Ill.

TABLE. 6—SCHEDULE OF DIMENSIONS, SIPHON SPILLWAYS, SOUTHWEST SEWAGE TREATMENT WORKS, SANITARY DISTRICT OF CHICAGO, ILL.

Typical Section	Dimension	SIPHON SPILLWAY:		
		No. 1	No. 2	No. 3
Capacity (highest discharge water elevation), in cubic feet per second		1 000	2 200	2 500
Number of siphons		5	7	8
	Width	5 ft 6 in.	8 ft 0 in.	7 ft 10½ in.
	A	6 in.	6 in.	12½ in.
	B	1 ft 3 in.	1 ft 6 in.	11½ in.
	C	2 ft 6 in.	2 ft 9 in.	2 ft 9 in.
	D	14 ft 9 in.	23 ft 0 in.	26 ft 0 in.
	E	12 in.	1 ft 3 in.	1 ft 6 in.
	F	4 ft 10½ in.	3 ft 2¼ in.	3 ft 8½ in.
	G	1 ft 9 in.	1 ft 9 in.	2 ft 0 in.
	H	6 ft 0 in.	13 ft 0 in.	14 ft 9 in.
	J	3 ft 0 in.	2 ft 0 in.	1 ft 9 in.
	K	3 ft 0 in.	2 ft 6 in.	3 ft 9 in.
	L	1 ft 0 in.	1 ft 0 in.	1 ft 0 in.
	M	8 ft 0 in.	8 ft 0 in.	8 ft 0 in.
	R_1	2 ft 6 in.	2 ft 6 in.	2 ft 9½ in.
	R_2	1 ft 3 in.	1 ft 3 in.	1 ft 6 in.
	R_3	6 ft 0 in.	6 ft 0 in.	5 ft 6 in.

The wide variation of water depth in the discharge conduits of these spillways (see Table 6) will sometimes cause considerable depth of seal on the siphon outlets. Therefore, appreciable priming heads are required, and these determine the maximum water levels in the forebays. The lower edges of the primer plates have been set at approximately the priming head elevations. This means that when the water level is just below the primer plate, the free weir discharge over the siphon crest will be sufficient to provide a minimum water velocity, probably about 0.8 ft per sec, which will carry the entrained air down to the siphon outlet. For these spillways the discharge will reach about 5% of capacity before the water level rises to the primer plates, which means a lower relative range of siphonic discharge than was obtained for the Walterville siphons, with a minimum discharge of about 3% of capacity.

If the depth of seal on a siphon outlet is considerable, the ratio of the downward flow area to the width at the crest should not be too great if the priming head is to be limited; but if the depth of seal is only a few inches, as in the case of the Walterville siphons, priming heads are small and the ratio of flow area to crest width is probably not of much importance. The siphonic action, however, broke on the Walterville siphons at a water rate of about 0.9 cu ft per sec per sq ft of downward flow area.

When a siphon is inhaling air, it is functioning as an hydraulic air compressor. For this reason, experiments made by the writer on a small hydraulic air compressor, using a more elaborate type of water-level regulator, are of considerable interest in this connection.

The experimental set-up is shown in Fig. 10. Air entered the down-draft tube just below the siphon crest from several pipes, with perforated inverts, extending across the flow section. These pipes were supplied from an air chamber, under the crest, which was connected by pipe to the top of the water-level regulator, a chamber with open bottom partly submerged in the water in the headbox. The restricted area between the perforated pipe produced downward nozzle-like discharges into the surface of the water below, in the down-draft tube, entraining air which the water carried down into the separator chamber. When the perforated air pipes were supplied from the atmosphere, the water level in the headbox rose as the flow increased. To maintain a constant level in the headbox it was necessary to have, in the down-draft tube above the water surface, a partial vacuum varying with the flow. The water-level regulator produced this effect. The chamber inhaled air through a horizontal slot on one side. The water surface in the headbox remained level with the top of this air inlet for the full range of discharge used in the tests. The rise of the water inside the level-regulator chamber, due to the negative

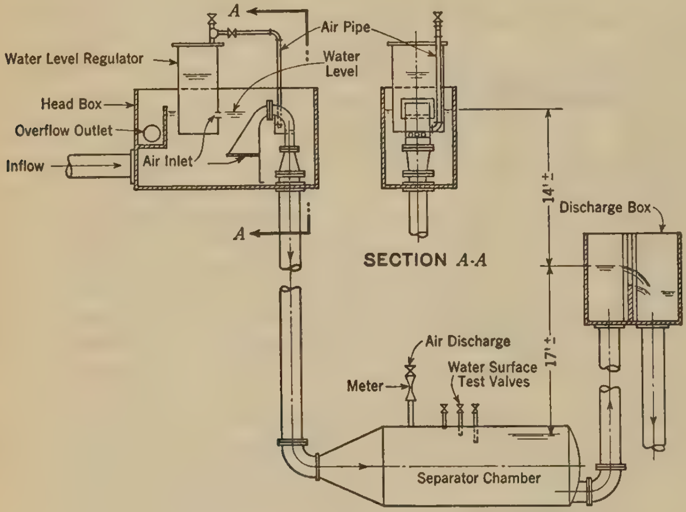


Fig. 10.—Experimental Hydraulic Air Compressor

head transmitted from the down-draft tube of the compressor, could be limited by throttling the valve in the air line.

The tests indicated that, in order to avoid surging, the air inlet should be above the invert of the siphon crest and, also, high enough so that the rate of flow in the down-draft tube would increase to about 1.0 cu ft per sec per sq ft of area before the water level in the headbox rose to the top of the air inlet. Then, for the higher flows the water level remained unchanged. The maintenance of a constant water level at the entrance to an hydraulic air compressor is important, particularly if there is a varying flow and a relatively small working head.

The author's paper has genuine value. Presentation of the results of such careful and complete tests as those made on the large Walterville siphons is highly desirable. Much experimental work remains to be done before the engineer can compute with certainty the capacity of a siphon or similar conduit which includes bends of varying degree of curvature, closely spaced and, in some cases, with the final bend right at the outlet.

J. C. STEVENS,[18] M. AM. SOC. C. E. (by letter).—Mr. Griffin raises the question of the total head under which a siphon may operate. The fundamental principles of siphon behavior are fully set forth in the writer's paper[19] on the Leaburg siphons, wherein it was shown that a head considerably in excess of one atmosphere may be utilized. A straight pipe between forebay and tail-water will discharge more than a siphon of the same size, but its action is not automatic without the use of automatic gates. By placing a part of the pipe above the normal hydraulic grade (not the absolute pressure line) it becomes a siphon, and no gates or valves are required. The advantage of a siphon over an ordinary connecting pipe lies principally, if not solely, in its automatic functioning without moving parts.

The maximum possible discharge that any siphon can have will occur when the velocity head at the summit equals one atmosphere. This is an ideal condition, however, that cannot be realized for two reasons: First, some of the atmospheric head is consumed in entrance and friction losses from the inlet to the summit; and second, the continuity equation $Q = AV$ (in which A is the cross sectional area of the siphon barrel) does not hold for such high velocities around bends. The result is that the maximum velocity occurs at some point below the summit where a part of the siphon barrel is occupied by eddies. In the Walterville and Leaburg siphons, under full discharge, the points of maximum velocity were down stream of the summit, as shown in Table 7.

Mr. Creager's discussion of Mr. Stickney's paper, to which Mr. Griffin refers,[7] presents the analysis of siphon behavior current at that time but which is now known to be faulty in some particulars. Mr. Creager states "The pipe will always remain full if it is tight and the hydraulic grade line, as used in Fig. 1 [see Fig. 11(*a*)], is above the top of the pipe at all points." Presumably, Mr. Creager meant "full of down stream flowing water." The experiments on the Leaburg and Walterville siphons have demonstrated that this is not necessarily true. Mr. Creager's proof that the siphon can discharge under heads greater than one atmosphere is also faulty as will be seen readily from Fig. 11(*b*).[20] Suppose the siphon did not discharge horizontally but continued sloping downward to a pool. Both hydraulic grade lines shown in Fig. 11(*b*) would intersect the lower leg throughout its entire length. Now suppose that the horizontal outlet were turned through 180°, as actually occurs in the siphon of the Southwest Sewage Treatment Works, illustrated in Mr. King's discussion (see Table 6), wherein the tail-water is vertically beneath the head-water. The hydraulic grade line would not touch the siphon barrel no matter how widely the head and tail-water were separated, and according to Mr. Creager's criterion the siphon could then operate under an infinite head.

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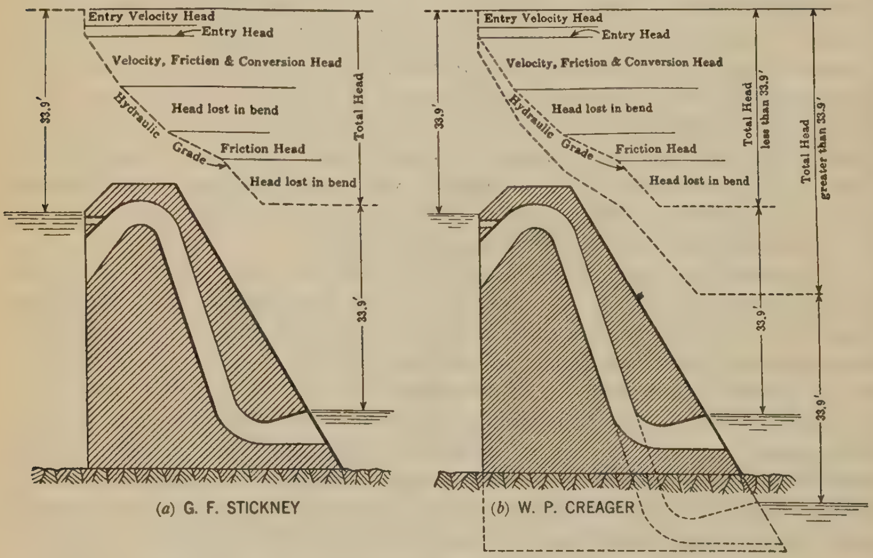
[18] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[19] "On the Behavior of Siphons," by J. C. Stevens, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 999.

TABLE 7.—POINTS OF MAXIMUM VELOCITY IN WALTERVILLE AND LEABURG SIPHONS

Description	LEABURG, SIPHON NUMBERS:			WALTERVILLE, SIPHON NUMBERS:	
	5	6	7	1	2
Discharge, in cubic feet per second	470	250	119	700	350
Maximum mean velocity, in feet per second	43.0	44.0	42.6	36.0	41.2
Distance below summit, in feet	8.8	7.2	7.3	29.5	26.2


Entry Velocity Head
Entry Head
Velocity, Friction & Conversion Head
Hydraulic Grade
Head lost in bend
Friction Head
Head lost in bend
Total Head
33.9'
33.9'
Entry Velocity Head
Entry Head
Velocity, Friction & Conversion Head
Hydraulic Grade
Head lost in bend
Friction Head
Head lost in bend
33.9'
Total Head less than 33.9'
Total Head greater than 33.9'
33.9'
33.9'

(*a*) G. F. STICKNEY (*b*) W. P. CREAGER

FIG. 11.—DISCUSSION OF SIPHON SPILLWAYS

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[20] *Transactions*, Am. Soc. C. E., Vol. 85 (1922), Fig. 16, p. 1123.

The fault lies in not developing the siphon barrel as was done by the writer in Figs. 7 and 8 wherein the elevation of the siphon barrel is plotted against its length along the barrel from the inlet. When this is done Mr. Creager's criterion holds. Of course, it is conceivably possible to build a siphon with any head whatsoever—say, several hundred feet; but, wherever the absolute pressure line intersects the siphon barrel, developed as previously indicated, the absolute pressure in the space between such intersections will be equivalent to the vapor pressure of the water—virtually a vacuum—and that part of the barrel will be useless as a siphon. This condition is illustrated in the writer's paper "On the Behavior of Siphons."[21]

Mr. Griffin objects to the term "primer" and suggests "air inlet." The facts are that when siphonic action starts it is caused by shutting off the air to this "air inlet"; the device, therefore, does act as a true "primer." He also asks for comments on the "nappe aerator," and under what conditions it is necessary. Whenever the sheet of water flowing over the crest is to fall free from the invert across the siphon barrel to facilitate exhaustion of the air, such an aerator will aid in freeing the nappe. In Fig. 1 note that the lower legs of the siphons slope toward the head-water to aid in air exhaustion by this method.

Messrs. Currin and Umphrey have mentioned the advantages of the backward sloping short lower leg in quick priming. They have also added some valuable data on cavitation within the barrel. The pitting of concrete is minor and is readily repaired at intervals of 3 to 5 yr.

Mr. Corwin mentions the question of racks. This is an important feature in siphon design. At Walterville and Leaburg the racks were spaced 6 in. clear, the object being merely to prevent anything from sticking in the primers. Racks spaced closer than this are not necessary and would give considerable trouble with frazil ice.

The large-capacity siphons described by Mr. King are quite interesting. He does not give the priming head but intimates that it is considerable. The forebay must rise 0.75 ft above the crest before it closes the primer. The writer believes that if greater sensitiveness is desired it can be secured by reconstructing the primer. Doubtless the design of the primer adopted was based on the model tests mentioned by Mr. King. The writer seriously questions the value of model tests for this purpose. The laws of similitude do not cover the case of less than atmospheric pressures unless the model tests are conducted within a chamber in which the atmospheric pressure can be reduced to correspond to the model scale. Moreover, surface tension becomes important in small models but may be safely neglected in the prototype.

The provision for raising the crest at some future time is a novel and practicable idea. Apparently the crest could be raised at least a foot without throttling the flow.

In indicating the range of discharge for the Walterville siphons, the free flow over the crests before priming was not considered. When that is taken into account the siphon practically discharges at any point from 0 to 100% of its flowing-full capacity, as Mr. King states is the case with the siphons illustrated by him.

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[21] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), left-hand sketch of Fig. 10, p. 1000.

Fig. 12 shows a discharge curve for Walterville Siphon No. 1 in terms of forebay elevation. The free discharge over the crest is shown to a more extended scale than for the remaining flows. As the forebay rises and falls the discharge will follow the cycle indicated by the arrows.

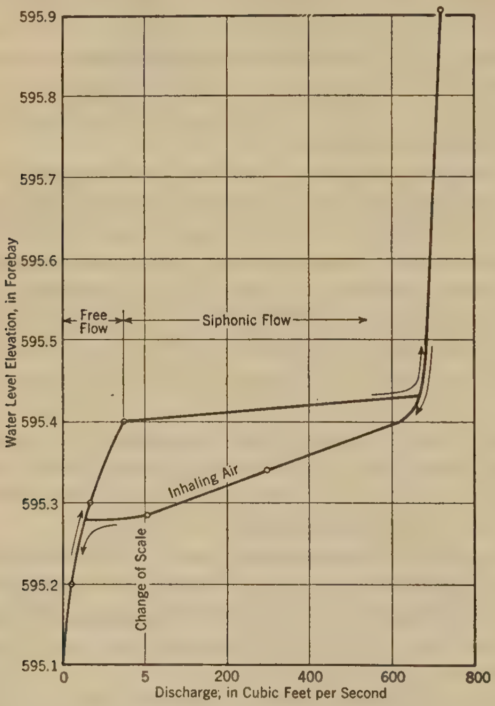

FIG. 12

None of those who discussed the paper mentioned the use of an energy coefficient in determining the energy line of Figs. 7 and 8. Where the mean velocity, Q/A, is used the energy head may be expressed correctly by

$$h_e = \frac{C_e V^2}{2g} \quad \ldots\ldots\ldots (1)$$

as has been shown by Morrough P. O'Brien, M. Am. Soc. C. E., and Joe W. Johnson, Assoc. M. Am. Soc. C. E.[22] The coefficient C_e is a measure of the velocity distribution within the siphon barrel—the greater the velocity distortion the higher the coefficient. It can never be negative. Since the velocity distribution was unknown this coefficient could not be computed in advance. Through the writer's method of adjusting the energy line, however, the value of these coefficients can be determined from Table 4. It is the ratio of the adjusted velocity heads of Column (11) to the velocity heads in Column (7). The values of this coefficient are thus seen to vary between 1.00 and 1.78 for Siphon No. 1 and between 1.00 and 1.95 for Siphon No. 2. These high values are not surprising when the disturbances of the bends are taken into account.

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[22] "Velocity Head Correction for Hydraulic Flow," by Morrough P. O'Brien and Joe W. Johnson, *Engineering News-Record*, August 16, 1934, p. 214.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2054

FRONTIERS OF ENGINEERING

ADDRESS AT THE ANNUAL CONVENTION, SAN FRANCISCO, CALIFORNIA, JULY 26, 1939

BY DONALD H. SAWYER,[1] PRESIDENT, AM. SOC. C. E.

Progress, although viewed in some circles of thought as a glorious symbol of more speed and bigger machines and in others as a covering for inhuman materialism, is in reality a conception embodying the most profound and the most potent ideas at work in the Modern Age. It is, at the same time, an interpretation of the long history of mankind and a philosophy of action in this world of bewildering choices. It gives a clue of meaning to the rise of civilization out of the crudities of primitive barbarism and offers a guide to the immense, impending future.

PROGRESS OR STAGNATION?

In essence, the idea of progress belongs to present times for it was not known to the ancients and to the thinkers of the Middle Ages. It has become more than a theory. It has achievements to its credit on every hand—disease and pain alleviated, the span of life lengthened, famine made obsolete, comforts and conveniences established, and improvements developed in every line of trade that points toward the goal of increasing perfection. Of course, progress is born of faith—faith that the world, as Emerson stated, "is all gates, all opportunities, strings of tension to be struck." This faith confronts obstacles with assurance; where the pessimists face the worst, it proposes a search for the best. To suffering, ignorance, and fear, which drive the timid to the Nirvana of doubt and oblivion, this faith calls to action, to research, to planning, to conquest, to a naked avowal in these United States that the earth is to be made submissive to the secure welfare and delight of them that dwell thereon.

About 1787 Mr. Tench Coxe of Philadelphia, Pa., opined as follows:

> "The population of Pennsylvania appears to have increased in 23 years, nearly in the proportion of 39 to 91, though the whole term of a revolutionary and invasive war of seven years was included. This considerably exceeds Dr. Franklin's estimate of doubling in twenty years. Now that all New England is full, except Maine and Vermont, the contiguous states

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[1] Chf., Section of Space Control; U. S. Treasury Dept., Procurement Div., Washington, D. C.

> of New Jersey and Delaware are overstocked, and Maryland nearly so; and above all, now that Europe is full and much disturbed, a curious rapidity of population is to be expected in a state with so much unimproved land, disposition and capacity for manufacturers, wealth, foreign intercourse, energy and enterprise as Pennsylvania."

In 1843 a report of the United States Patent Office contained the statement that "the advancement of the arts from year to year taxes our credulity and seems to presage the arrival of that period when human improvements must end." These incidents occurred fifty-six years apart and in the interim there probably were many other forecasts by reputable characters voicing their beliefs that the incipient American spirit of creative genius had about run its course. After the interval of a century the same foreboding is abroad in the land—in very recent years we have listened to a new slogan to the effect that there are no more frontiers in America.

When trite phrases that catch the public fancy emanate from authoritative sources, it is quite natural that credence be given to the philosophy of the remarks. But examples such as these throw a "wet blanket" over the enthusiasm of those whose part in American advancement has been the production of better things for life and industry.

Amazing Advances in Transportation

It is quite true that migration from the Eastern shore lines of America to the Pacific Coast has been completed, that the intervening spaces have been adapted to agriculture and industry in large part. But this does not imply that all of the open spaces are productively at work, and especially it does not mean that Americans, most favored people in the world, are satisfied with their present surroundings and facilities.

The discovery of gold in California emphasized the acute need for railroads to cross the continent. Enormous distances and natural resources spurred the inventors and engineers to develop the railway—and to-day the United States boasts the greatest network of railroads of any country in the world.

Where once it took months for a slow-moving animal-drawn caravan to make the journey from the Mississippi River to the West Coast, there followed railroad trains which cut the time of travel across the country to two weeks, and more recently to two and one-half days. Airplanes have reduced their schedules to fifteen hours, while passenger automobiles can make the journey comfortably in a week.

Tremendous strides have been made since the gold rush to California. Years ago it took a vessel months to circle the Horn, whereas to-day fast steamships complete the journey from coast to coast, through the Panama Canal, in two weeks. The engineer has justly taken credit for all these advancements in science.

If it be true that the territorial frontiers have been surmounted, can it be said that the frontiers of technical advancement have likewise reached their highest culmination? If the answer to this question is affirmative or is couched in doubtful language, it must be concluded that America must accommodate itself to tremendous changes. But the answer is "no"; and although fundamental things have been provided for a higher civilization there will be no pause in the thinking and accomplishments of the technicians of this nation for years to come.

The engineer has not yet reached his frontier nor has he even come in sight of it. Habits of mind, clarity of thinking, trust in his own self-assurance, opportunity for public service—all these constitute mental and physical resources which show no indication of exhaustion but rather have provided him with better tools that permit him to perform even greater deeds for mankind.

COMBINATION OF EFFORTS

It is well, in contemplating the equipment of the engineer, to recognize the mental and physical forces which join hands with him in developing his enterprises. Roughly his colleagues are men and materials. These may be grouped into the categories of finance, design and supervision, and construction.

In finance are included those elements that contribute money in its various forms in order that labor and materials may be joined into lasting works.

By design and supervision is meant the process wherein the engineer, the architect, and all other technical men whose ingenuity permits them to foresee what is needed, reduce their conceptions to drawings and specifications; and later watch the unfolding of their plans in the field, to the end that their creations may be honestly and efficiently brought into being.

Finally, construction comprehends the manipulation of men and materials. Millions of people have given the best that is in them to this great productive development. They not only labor at the site of the undertaking but are engaged in producing raw materials, in following these through the fabrication stages in the factories, and in expediting their transportation to the location of the job. Through a long sequence of complex operations, they trace the ore and the timber from its natural habitat to its final placement in a completed structure.

This picture of the construction industry, sketchily drawn, is so commonplace in American life that it fails to receive the credit which is its due. This business adds to the permanent wealth of the country and is second only to agriculture in its vast volume. It gives a livelihood to millions and constitutes the greatest single force that has made the United States the leader of the world in providing necessities, conveniences, and luxuries for its people.

SETBACK OF INDUSTRY CREATES UNTOLD HARDSHIP

This great army of men has been steadily advancing for a hundred years. Its progress continued until the early 1930's, when something occurred that interrupted its forward movement. Since then this great part of American business and life has staggered in an effort to recover its lost ground.

Many tonics, many medicines and panaceas, and great profusions of words have been administered in valiant attempts toward resuscitation. In spite of all these efforts the industry is to-day far below its legitimate level. Graphically, in Fig. 1, a diagram is displayed[2] which indicates more forcefully than words the historical rise and decline of this great industry.

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[2] From "Construction Activity in the U. S., 1915–37"; publication of the U. S. Department of Commerce.

While credit must be given to the engineer for the benefits he has conferred on modern life, it must be realized at the same time that in gaining the high position which he enjoys to-day he has carried along with him a section of humanity for which he must stand sponsor if he remains true to his faith. No one can accurately estimate how many men look to the construction industry for their livelihood but the best available figures range from 5 000 000 to 8 000 000 human beings. The tragic drop in the volume of construction has turned a considerable percentage of these people out on the streets and put them on relief rolls. Possibly the greatest portion of public charges to-day (1939) were formerly engaged in the pursuits of the engineer. To his responsibility must also be added a part of the increased population year after year, together with men displaced due to the introduction of labor saving devices.

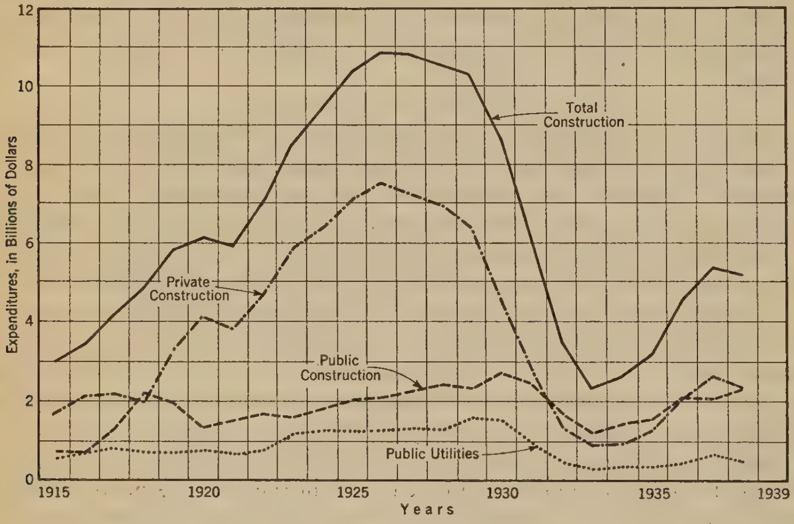

Fig. 1

Challenge to Engineering Ability

And so the technical man—qualified as he is and distinguished in his trustworthiness, his thinking, and his zeal for public service—must pick up this load if the United States is to have substantial recovery. So long had he been ingrained with the idea of a market for his creations that he was not prepared for the shock of the depression. He floundered, became discouraged, accepted the fallacy that America had no more frontiers, and somewhat succumbed to an undefined bewilderment. He must shake off his apathy, and his recourse to temporary alternatives. He must revive his clear thinking, reassert his courage and inventive genius, and again take up the burden that properly belongs to him as his full share of American industry.

To do so involves no great departure from the engineer's former habits of mind. The problem constitutes a challenge to his talents that promises a greater reward for public service than has ever before presented itself to him. If he will not reassert himself, he and his industry must continue to suffer; but if he will go forward he will do for America greater things than have been accomplished in the past and will play his full part in bringing the nation out of its difficulties.

But, why pick on the engineer and charge him with leadership in such a program? The answer is that he has always been in the key position of the construction industry. His have been the ideas, his the means by which these plans can be brought to reality including, of course, financing and construction. He has been the motivating force of the industry, the man who has been sought by those who desire to create physical things. He has been the captain of the construction team and it is his function to call the signals, to direct the plays, along lines that give assurance of success.

Presumably most people will agree with these observations regarding the desperate condition of the construction industry, the welfare of the men for whom the industry stands responsible, and the faith of the engineer in his calling. If these things be taken as true it remains to discuss what means should be employed to revive the business that has engaged his talents.

Creative Improvements

There seem to be no short cuts to a substantial return of the former volume of construction. No miracle man is in sight who can lead America out of the present wilderness. The road back must be along conventional lines where experience has demonstrated assurance of success, and must include hard work, good judgment, and the revitalization of creative genius. Nor is the struggle a one-man job for the Civil Engineer; rather it must be attained by team-work among all elements of the construction business. Leadership may well be, and should be, exercised by the technical forces. All those others whose livelihoods depend largely on construction must collaborate.

Eternal change has been the life blood of the industry, whose slogan has been, "Don't do to-day what you did yesterday; do something better." Indicative of the striking changes of the last half century are many available illustrations, all of which emphasize the trite saying, "Off with the old, on with the new." Few facilities and improvements in America disappear because their usefulness has passed; most of them are displaced because something better has been introduced that promises higher service.

The construction industry has been energetic during the last few years of the depression in producing machinery, equipment, and facilities to lower the cost of production—labor saving devices. The type of inventive mind that has brought these things into being apparently has been insensible to the proper planning and creating of things that the public will buy.

Success in Other Fields

Of course, there are bright spots in American industrial life where creative genius has maintained the pace of former years and has not faltered. A prime example of this is the chemical industry. This sector of American business has been alive in its pursuit of new products that a waiting public might acquire. Millions of dollars have been invested every year in research, other millions have been invested in new factories and improved equipment that bring into being the ideas which have been germinated. Brand new creations have appeared in recent years that were never thought of before and eager buyers have absorbed these products. The public, however, would not know of these productions were it not for the good judgment and funds which have been invested in promoting and marketing them. This experience indicates that genius has not disappeared, that money is available, and that people will buy something new when it is better than the old.

The thrilling story of the advancement of chemistry has its parallel in the by-gone days of the construction industry. But vastly greater than was the case with chemistry are the phenomenal opportunities for construction that lie ahead. The creation of these opportunities calls for greater emphasis on investigation, discovery, and invention than has been devoted to these purposes in former years.

A few developments of contemporary significance can be mentioned in the engineering field. Air-conditioning of our buildings was launched as an adventure a few years ago and to-day its application is found in a wide variety of structures and railway accommodations. What its ultimate future may be cannot be predicted, but with money and human talent allocated to its development it gives promise of becoming a means of permanent and expanding service to mankind.

The latter-day adaptation of internal combustion motors to railway transportation with lighter and more modernized equipment forecasts a trend that with the greater improvements bound to follow is entirely revolutionizing travel by rail. These examples may be largely multiplied as showing the creative contributions to American technical life.

A Program

Greater sums must be set aside for research; and capable staffs must be formed to develop new ideas which show promise for commercial use. This money should be allocated to laboratories and research institutions, as well as to manufacturers and technicians, wherever the assurance seems justified that its distribution will lead to betterments. There are countless numbers of men who have within their minds convictions which, to bring into being, would require funds. Some efforts will succeed and others will fail but in the aggregate the investment will prove its value. Manufacturers and many other institutions of great variety foresee innovations which require expenditures for commercialization. All of these budding ideas should be supported, when seemingly of worth, to the end that the resources latent in the American mind may be reawakened. For such a program a sizable sum of money would be needed and there is doubt if it could be accumulated in a short time within the construction industry, where the units are moderately small and the collection of a large amount presents many difficulties. Since this is true, the Federal Government, if properly advised of the purposes to which an appropriation would be devoted and of the reliable auspices under which the spending would be conducted, might give consideration to legislation that would motivate a program. The unified support of the industry should be behind a program of this nature.

The creation of new facilities is the target at which to shoot. Americans have an appetite for the best that life affords. As a companion movement, therefore, the industry must devise all proper means to acquaint the public with that which it has to offer and must do so by more thoroughgoing practices than have been in vogue in the past. When this great machine is in working order, with money, man-power and materials, and with a public eager for the best, the sale of its wares whenever properly presented can be expected.

The American Society of Civil Engineers is so organized as to give at least initial momentum to such a procedure. Its twelve technical divisions speak for specialized engineering, embracing city planning, construction, engineering economics, highways, hydraulics, irrigation, power, sanitation, soil mechanics and foundations, structures, surveying and mapping, and water-ways. These groups can well examine and explore this problem and they can become the motivating forces in this program to restore construction. From such centers the ideas and recommendations can be communicated to other elements of the industry, which might bring about a larger conference with a concord of views. Out of a meeting that speaks for all elements might come a highly qualified committee to give direction to the formulation of future plans.

Construction Problems and Construction Opportunities

Let us restate the condition and future of the construction industry:

This business has assimilated a very significant portion of the population in its advancement in the last century;

It has won its way by the introduction and execution of improvements startling in their benefits to humanity;

The depression has pushed the volume of construction down to a fraction of its former level and has forced its men into idleness and on relief;

The industry cannot escape its responsibility in taking the leadership and overcoming the situation as it exists to-day—to do this it must revert to the methods which gave it success in the past, namely, the display of creative and organizing ability;

The time has come for construction to reassert itself, to shoulder its responsibility—this means a co-operative movement founded on hard work and sane thinking of its component groups.

If this business will shake off its lethargy, re-establish its belief in itself, and work industriously, engineers can face the future with a revived confidence and with convictions; they can pick up their load and quickly start American business on the up-grade.

American ingenuity has not finished its course.

The fertility of the technical mind is not exhausted.

The frontiers of engineering have not been reached!

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2055

INTERSTATE WATER PROBLEMS

FINAL REPORT OF THE COMMITTEE OF THE IRRIGATION DIVISION ON INTERSTATE WATER RIGHTS

INTRODUCTION

The Committee published progress reports in 1927, 1928, and 1930.[1] Much of the interstate litigation and negotiations for interstate compacts, covering the use of waters of interstate streams, has occurred since the latter date. Vital questions of Federal *versus* State jurisdiction have arisen, largely because of increased activities of various Federal agencies as a result of legislation enacted by the Congress since 1932, in the interest of industrial recovery and conservation of the natural resources of the nation.

In connection with the important subject of interstate water with which this Committee is concerned, U. S. Federal and Supreme Court decisions, and papers written by various Federal, State, legal, and engineering authorities, have been reviewed by this Committee and freely drawn upon in the preparation of this report, with the view of bringing up to date a compilation of the history covering compacts, litigation, and Court decisions concerning the use of the waters of interstate streams.

In its desire to make this report as inclusive as possible, certain phases of the work were allocated to various members of the Committee, the special topics involving review and research of current engineering literature, Federal Court digests, reports by various Federal agencies, such as the Water Resources Committee of the National Resources Committee, the U. S. Bureau of Reclamation, Federal Power Commission, American Bar Association,[2] Federal legislation affecting interstate and international streams, the Rocky Mountain Law Review, Harvard Law Review,[3] Yale Law Journal, New Jersey Law Review, and University of Pennsylvania Law Review. In addition, questionnaires were sent to the Attorneys General and State Engineers, or other water officials, of all the forty-eight States, requesting data concerning interstate litigation pending or concluded, or negotiations for, or compacts which have been concluded between States, covering the use of the waters of interstate streams. The Committee acknowledges the sources of much of the material used herein and desires to express its thanks and appreciation for such use, and for the valuable assistance rendered by these various agencies. The Chairman also desires especially to thank J. B. McCormick, Dean of the Law School, University of Arizona, Tucson, Ariz.; William R. Kelly, Attorney at Law, Greeley, Colo.; and Wells Hutchins, Irrigation Economist, Bureau of Agricultural Engineering, Berkeley, Calif., for reviewing the report of the Committee, and the Honorable Ralph L. Carr, who prepared that part of the report covering the litigation over the La Plata River Interstate Compact.

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[1] *Proceedings*, Am. Soc. C. E.: Society Affairs, March, 1927, p. 106; Society Affairs, March, 1928, p. 120; and, Papers and Discussions, March, 1930, p. 595.

[2] "A River is a Treasure," by William R. Kelly, Greeley, Colo., 20 A. B. A. J. 559 (1934).

[3] "Interstate Rights in Interstate Streams in the Arid West," by L. Ward Bannister, Denver, Colo., 36 *Harvard Law Review*, 960 (1923).

The Committee does not conceive it to be its function to express any particular view, or to draw conclusions with respect to any of the matters discussed herein, with which various individuals may disagree. It has attempted to limit its report to a compilation of the history of interstate litigation and compacts covering the important subject of interstate waters, together with a brief digest thereof, and the views of various commentators thereon.

Problems Related to Interstate Waters.—Interstate water problems exist on almost every interstate stream of the nation, or are potential sources of interstate controversies. They are becoming more complicated and more difficult to solve as the need and demand for water increases. In recent years, public attention has been directed as never before toward the water problems of the nation. Withering droughts and devastating dust storms in the arid regions of the West, wide-spread soil erosion, record-breaking floods, and receding water-tables—with increasing needs for ample and stabilized water supplies for great centers of population and industrial areas, and the need for mitigating the dire effects of stream pollution—have caused grave alarm throughout the nation. No doubt, these conditions arose because there has been a protracted cycle of deficient precipitation that culminated, generally throughout the West, in the extreme low of 1934. The gradual increase in population and expansion of irrigated areas in the arid West have likewise made heavy and increasing demands upon the limited water supplies available to those areas, the future development of which must of necessity be restricted to the ultimate consumptive uses of such supplies.

In a humid region, except for comparatively limited demands for domestic, industrial, and sanitary uses, waters are more often an agent of destruction than a blessing, and place a heavy financial drain upon the resources of communities, States, and the nation, for their regulation and disposal. Such demands, however, have not raised problems of an interstate nature, as have the more vital needs for domestic and sanitary purposes, and for the production of food supplies in the arid West. These latter problems may be considered, in general, to be of more local than national concern. They have forced communities, centers of population, and even States, to search beyond their confines for water supplies to meet their growing necessities. As a result, the people dependent upon the waters of a stream view with distrust any attempts to divert a portion of its waters to another water-shed or basin, and a State is even more jealous of the water supplies that originate within its boundaries. Its people are inclined to insist upon their right to the unrestricted use of its waters, not only for present, but for all future requirements, irrespective of the needs of the citizens of another State.

In the arid West, practically all development and progress is dependent upon a consumptive use of its water supplies. Based upon the doctrine of prior beneficial use, earlier development in a lower State has tended to place heavy restrictions upon uses of water from an interstate stream by an upper State. This condition has brought on long and costly litigation between States in their efforts to protect what they conceive to be the legitimate interests and the rights of their citizens.

The Constitution of the United States, as drafted in 1787, represented a bold and original effort to create a political system in which the cumulative advantages of a large National State could be established, with full opportunity for political development and expression, through the autonomy of member States. That these member States were conscientious and jealous of their individual personalities and interests is amply demonstrated, not only by colonial history, but also by that of the Confederation. The Constitution of the United States is in the nature of a contract between the States, whereby specified and limited functions of Government are delegated by the States to the United States of America by assignment of certain inherent sovereign powers and duties to the Congress, the President, and the Supreme Court.

Article I of the Constitution, in which powers are conferred upon Congress, also includes Article X of the Amendments, which places certain restrictions upon the exercise of these powers. Further reservations were attached in earlier years by Articles I to X, in the form of amendments to the original Constitution. Article I, in which powers are conferred and restrictions placed upon the Congress, also places certain restrictions upon the several States, among which is the following: "No State shall, without the consent of Congress, * * * enter into agreement or compact with another State, or with a foreign power * * *." It has been assumed that this part of Section 10 of Article I of the Constitution is to be interpreted as actually permitting or authorizing agreements between two or more States with the consent of Congress.

Under the general conception, the Federal Government's authority in regard to waters is limited to that required for the regulation and control of the waters of an interstate stream, in the interest of navigation and national defense, and also to the consummation of international treaties. It is also possible that, with respect to certain of the natural water supplies of the Western States, the Federal Government may have rights, superior to those claimed by the States, under its powers to enter into treaty obligations with Indian tribes, during the territorial status of the land, and with bordering foreign countries. In the Western States, however, there is a disposition to question this purported authority of the Federal Government with respect to Indian tribes. In general, when this question has arisen between Federal and State authorities, the conflicting viewpoints have been adjusted amicably without the necessity for a final determination of the principle by the Courts. There has been a tendency for various agencies of the Federal Government to extend their authority over the waters of navigable streams, or even tributaries thereof, in the administration of the asserted functions of their respective offices, as a result of various Federal Laws based upon the constitutional power of the Federal Government to control navigable waters.

Doctrines of Water Law.—The doctrines or principles governing the uses of water vary in different sections of the United States, as do also the problems involved. In the arid States of the West, where the principal use of water is for irrigation, the doctrine of "prior appropriation" obtains, whereas in the humid States, where needs for domestic water supplies, sanitation, power development, and navigation are of primary concern, the doctrine of "riparian ownership" is recognized. In areas lying within the so-called semi-arid regions, no little conflict has arisen between these two doctrines, which it has been difficult to reconcile and adjust. In general, however, laws and Court decisions within the States in which these regions occur have made it possible, in a measure, for an appropriator, or user of water, to secure recognition of, and reasonable protection in, such uses.

In addition to the interstate decisions, the United States Supreme Court, in repeated decisions, down to the California Oregon Power Company case in 1935, has recognized the right of each State to adopt its own system of water law. The appropriation doctrine is an approved system of water law. In 1905 the Supreme Court stated[4] concerning alteration of the common law doctrine in Utah: "This court must recognize the difference of soil and climate, which renders necessary these different laws in the states so situated." In the California Oregon Power Company case, also, it was stated that the desert land legislation does not bind or purport to bind the States to any policy, but simply recognizes the doctrine of appropriation "and seeks to remove what otherwise might be an impediment to its full and successful operation."

The trend of recent decisions by the Supreme Court of the United States indicates a tendency to modify both of these fundamental doctrines so as to permit an "equitable apportionment," as between States, of the available water furnished by an interstate stream.

The uses and pollution of the waters of interstate streams, and the settlement of controversies between States, resulting from such uses or claims, constitute at present (1939) one of the most urgent problems with which the States and the Federal Government are confronted. A conflict of interests inevitably arises sooner or later on nearly every interstate stream. This fact becomes highly significant when it is realized that no general comprehensive plan, with respect to the development of an interstate stream, can be effective or satisfactory without co-operation between the States, and at times between the States and the Federal Government.

The riparian doctrine of the common law of England was adopted in the United States at an early date, and always has been the controlling principle of law governing the use of water in the humid regions and in parts of the semi-arid States. Under this doctrine, each riparian proprietor is entitled to have the stream come to his land as it would naturally flow, substantially undiminished in quantity, and unimpaired in quality. All riparian owners have an equal right to make reasonable use of the waters of a common stream consistent with their necessities. This right does not mean, however, that all users are entitled to equal quantities of water from the same source. This doctrine worked no particular hardship so long as the competitive demand for water was relatively low in relation to the available supply. As the demands for water increased, however, a rule so absolute and inflexible became increasingly impossible of strict application. Force of circumstances and necessity for continued existence compelled a modification, if not a theoretical abandonment, of the strict riparian doctrine.

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[4] Clark *vs.* Nash, 198 U. S. 361.

Four decisions of the Supreme Court of the United States have laid down the principle of "Equitable apportionment" of the waters of an interstate stream, based on the "equality of right" existing between States. In the case of Kansas *vs.* Colorado[5] the controversy involved the use of the waters of the Arkansas River, which rises in the State of Colorado and flows into Kansas. The Government of the State of Kansas alleged that the depletion of the flow of the river by the irrigation of a large acreage in Colorado had injured its lands, institutions, and citizens. Recognizing, at the time only, the riparian doctrine, Kansas claimed the right to the entire flow of the river. Colorado, where the exclusive appropriation doctrine had been adopted in its Constitution, claimed the right to the use of all the waters arising within its boundaries, on the basis of its sovereign right of ownership in said waters. Notwithstanding the fact that Kansas as a State had suffered some damage from the depleted flow of the river, the Court decided: That, since Colorado lies entirely within the arid region, and due to the exigencies of its people as a result of such natural conditions, and the large development that had accrued in that State as a result of the theretofore large use of water by its citizens, "it would seem that equality of right, and equity between the two states, forbid any interference with the present use of water in Colorado"; but that further depletion of the water by Colorado might result in a situation in which "Kansas may justly say there is no longer an equitable division of benefits, and may rightfully call for relief." The decision in this case has been interpreted by many as establishing the principle of "equitable division" or "equitable apportionment" of the waters of an interstate stream.

In the case of Wyoming *vs.* Colorado,[6] involving the use of the waters of the Laramie River, which rises in Colorado and flows into Wyoming, the latter State alleged that a proposed diversion of water in Colorado from the Laramie River Basin, for the irrigation of lands outside that basin in the State of Colorado, by means of the so-called Laramie-Poudre Tunnel, would injure prior appropriators of water out of the Laramie River in Wyoming. Wyoming contended that priority of use, regardless of State lines, should prevail, and therefore that, since it had prior appropriations of a sufficient volume to consume the firm water supply of the river, it was entitled to protection for such appropriations. Colorado claimed the right to the use of all the waters originating within its boundaries, by reason of her sovereignty of Statehood. The Constitutions of both States recognize only the doctrine of prior appropriation and beneficial use.

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[5] Kansas *vs.* Colorado, 185 U. S. 208; 206 U. S. 46 (1901–1907).

[6] Wyoming *vs.* Colorado, 259 U. S. 419–496; 260 U. S. 1 (1922).

Colorado claimed (*a*) wasteful use by Wyoming, (*b*) that Colorado uses would be more economical and more productive, (*c*) that the proposed tunnel diversion was an appropriation whose priority dated from 1902, and (*d*) that Colorado uses would not deplete the river at the State line beyond the demands of Wyoming priorities, which were earlier. There was an issue as to the seniority of appropriations in Wyoming, as compared with the tunnel appropriations, and it was not alone an issue of State sovereignty claim to the water.

The Court held that the doctrine of prior appropriation, recognized by both States, was applicable to a determination of the equities claimed by both States in the waters of the Laramie River and furnished "the only basis which is consonant with the principles of right and equitable application to such a controversy as this." Many attorneys have concluded that, in this decision, the Court decided the case strictly upon the principles of "priority of appropriation and use." Other authorities, however, are firmly of the opinion (*a*) that the principle underlying the decision was that of "equality of right" between the States and that of "equitable apportionment" of the benefits provided by the river; and (*b*) that a determination of the controversy, founded upon priority of appropriation and beneficial use in the two States under the doctrine announced in their respective Constitutions, would conform to the fundamental principle of equitable apportionment.

In this connection, the Court re-affirmed the principle laid down in the Kansas-Colorado[5] decision of equitable apportionment between the two States, adopting as a basis of equitable apportionment the doctrine of appropriation, recognized in both States, regardless of State lines.

In the case of Connecticut *vs.* Massachusetts[7] concerning a proposed diversion from the drainage area of the Connecticut River to provide water for the metropolitan area of Boston, Mass., Connecticut set up its right under the riparian doctrine, which both States recognized, to the undiminished flow of the river, and alleged that, to take water entirely out of the Connecticut River Basin, would impair the navigability of the Connecticut River, increase danger from pollution, and take away flood rights. In rendering its opinion, the Court clearly indicated abandonment of the riparian doctrine as applicable to interstate questions, and again enunciated the principle of "equitable apportionment" in the following language:

> "* * * but the laws in respect of riparian rights that happen to be effective for the time being in both states do not necessarily constitute a dependable guide or just basis for the decision of controversies, such as that herein presented. Such disputes are to be settled on the basis of equality of right, but this is not to say that there must be an equal division of the waters of an interstate stream among the states through which it flows. It means that the principles of right and equity shall be applied, having regard to the 'equal level or plane on which all the states stand, in point of power and right, under our constitutional system,' and that upon a consideration of the pertinent laws of the contending states, and all other relevant facts, this court will determine what is equitable apportionment of the use of such waters."

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[7] Connecticut *vs.* Massachusetts, 282 U. S. 660 (decided February 24, 1931).

The case of New Jersey *vs.* New York,[8] concerning the Delaware River, involved a set of facts similar to those in the case of Connecticut *vs.* Massachusetts.[7] The Court again applied the doctrine of equitable apportionment by announcing "the effort is always to secure an equitable apportionment without quibbling over formulas," and decided that "removal of water to a different water shed obviously must be allowed at times unless states are to be deprived of the most beneficial use on formal grounds."

Under the appropriation doctrine, the individual or group that first makes a beneficial use of water thereby acquires a prior right to its continued beneficial use. As heretofore stated, practically all of the States of the arid and semi-arid West have adopted this doctrine. It was borrowed from foreign countries and first applied in New Mexico and Arizona, which, upon acquiring Statehood, adopted the exclusive doctrine of priority. In about 1849, this doctrine came into practice extensively in California, which has always recognized the riparian doctrine, or a modified form thereof. This early adoption of the appropriation doctrine in California resulted from the necessities of the situations obtaining in the old California mining days, when the rapid development of placer mining frequently created a demand for all available water furnished by a stream.

The community developed a system of "first come, first served," the claim dating from the time work was initiated. These activities all occurred on public lands. Congress later recognized such local laws and customs (and thereby the doctrine of appropriation) by an Act[9] of 1866, and confirmed this policy by subsequent legislation.[10] This doctrine appears to be firmly established, not only as a result of legislation to which reference has been made, and by such further legislation as that of the U. S. Reclamation Act and the Federal Power Act, but by decisions of the Supreme Courts in the various Western States and of the Supreme Court of the United States, notably that of Kansas *vs.* Colorado,[5] and Wyoming *vs.* Colorado.[6]

Federal Control over Water Resources.—The right of the Federal Government to exercise control over the waters of the nation is based primarily upon the constitutional power of Congress to regulate navigable waters. Congress has also exercised jurisdictional authority in connection with flood control, treaty obligations, and to a certain extent in interstate compacts and the use of the public domain.

This power is not granted expressly by the Constitution, but is incidental to, and based solely on, its power "to regulate commerce with foreign nations, and among the several States * * *" (Article I, Section 8, clause 3 of the Constitution). The power of Congress over navigable waters has been exercised from the outset and has been upheld by repeated decisions of the Supreme Court of the United States. The exercise of this power has not always been limited to control over streams and tributaries navigable in fact, but apparently has been extended to some streams in which navigability is really a fiction.

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[8] New Jersey *vs.* New York *et al.*, 283 U. S. 336 (1931).

[9] Act of 1866, 14 Stat. 253.

[10] Act of July 9, 1870, 16 Stat. 217.

Federal control of water power is based primarily on the power of the Government to control navigable waters, but in certain instances also results from the use of public lands and reservations. The Acts of March 3, 1899, and June 23, 1910, forbid the construction of dams in navigable waters except upon affirmative authorization by Congress.[11] The numerous requests for special permission to construct dams for power purposes, and the work involved in determining whether or not the interests of the United States would be affected adversely by the proposed constructions, led to the creation of the Federal Power Commission[12] to relieve Congress of this burden, and to afford a more general determination of policy in such matters. The Act is the culmination of a steadily increasing assertion of Federal authority over navigable waters, by legislative construction of the constitutional power. Under this Act, control has been extended even to tributary streams which affect the navigable capacity of navigable streams. Perhaps a greater degree of authority is exercised over some phases of the development and use of power than strictly is required for navigation purposes. The lower Courts have upheld the Act, however, on the ground that its principal purpose is the protection of navigation.

Flood control projects of the Federal Government usually have involved the protection or promotion of navigation, which affords unquestionable constitutional authority for this type of work. The Federal Government, doubtless, also has power to build flood-control projects to protect interstate commerce over highways and railroads, the postal service, and the lives and safety of its citizens. The Act of March 1, 1917, authorized examinations, surveys, and reports on flood-control projects generally, and flood-control development work on the Mississippi and Sacramento Rivers.[13] Under the Act of January 21, 1927, the War Department has prepared about 185 Reports (known as the 308 Reports), covering investigations and surveys of many of the principal streams of the country with respect to navigation, flood control, water power, irrigation, and related features.[14] Prior to 1936, however, Congress based its approval of flood-control projects upon the theory that they were in the interest of navigation.

The Federal Government, through its authority in the matter of rights of way over the public domain, has been able to exercise considerable control over the use of water in the public-land States, especially where its interests, due to reclamation or forestation, were involved. The reclamation of Federal lands was really the basis of the Reclamation Act of 1902 under which the Bureau of Reclamation operates.[15] As a result of its construction activities, this Bureau has exercised, indirectly, a certain degree of control over the development and administration of the waters of interstate streams on which some of its projects are located.

The question of State *vs.* Federal ownership of the unappropriated waters of the public-land States has long been debated by proponents of both theories, who, in support of their views, have cited numerous decisions of the Supreme Court touching upon this important matter.

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[11] Act of March 3, 1899, 30 Stat. 1151; and, Act of June 23, 1910, 36 Stat. 593.

[12] Act of June 10, 1920, 41 Stat. 1077.

[13] Act of March 1, 1917, 39 Stat. 950.

[14] Act of January 21, 1927, 44 Stat. 1010; H. R. Doc. 308, 69th Cong., 1st Session (see also, Flood Control Acts of 1936 and 1938).

[15] Reclamation Act of 1902, 32 Stat. 388.

One theory, known as the "California Doctrine," holds that, when the Federal Government originally acquired the territory which now constitutes the State of California, it became not only the owner of the lands, but also the owner, in a strict and proprietary sense, of the right to the use of the waters appurtenant to these lands; and further that the conference of Statehood passed political powers and not proprietary rights. The leading case in support of this theory is that of Lux *vs.* Haggen.[16]

Under the opposing theory, generally known as the "Colorado Doctrine," the Federal proprietary title to water is denied, and all rights to water within the confines of the State are held to be vested in the State for the use of the public. Under the theory of State sovereignty and State law, this doctrine is written into the Constitution of the State of Colorado, as well as in those of several other States of the arid West. The proponents of this doctrine contend that the Congress of the United States recognized and accepted such doctrine in the Enabling Acts under which the States came into the Union.

Although, on numerous occasions (notably in Kansas *vs.* Colorado[5]), the Federal Government has asserted its claim to the ownership of all of the unappropriated waters of the public-land States of the West, the question as to whether the Supreme Court of the United States has definitely spoken in regard to this important matter is probably not open to debate. The United States Supreme Court permitted the Government to file a petition in intervention of Nebraska *vs.* Wyoming, in which Colorado was an impleaded defendant; such action would not constitute a final adjudication of the rights of the Federal Government to remain in the suit.

On Wyoming's motion to dismiss in this case,[17] the U. S. Supreme Court, in 1935, expressly declared that the water belongs to the State:

> "The motion asserts that the Secretary of the Interior is an indispensable party. The bill alleges, and we know as matter of law, that the Secretary and his agents, acting by authority of the Reclamation Act and supplementary legislation, must obtain permits and priorities for the use of water from the State of Wyoming in the same manner as a private appropriator or an irrigation district formed under the state law. His rights can rise no higher than those of Wyoming, and an adjudication of the defendant's rights will necessarily bind him."

In discussing the effect of the Reclamation Act, the Court, in 1935, came to the same decision in California-Oregon Power Company *vs.* Beaver Portland Cement Company[18]—namely, that the Government, as owner of the public domain, had the power to dispose of the water separately and did dispose of it separately to the States. In 1937, this was followed by the U. S. Supreme Court in Ickes *vs.* Fox[19] where it is declared:

> "The Federal Government, as owner of the public domain, had the power to dispose of the land and water composing it together or separately;

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[16] Lux *vs.* Haggen, 69 California 255 (1886).

[17] Nebraska *vs.* Wyoming, 295 U. S. 40; 79 L. ed. 1289 at p. 1291.

[18] California-Oregon Power Company *vs.* Beaver Portland Cement Company; 295 U. S. 142, 79 L. ed. 1356.

[19] Ickes *vs.* Fox, 300 U. S. 82, 81 L. ed. 525, 530.

> and by the Desert Land Act of (March 3) 1877,[20] if not before, Congress had severed the land and waters constituting the public domain and established the rule that for the future the lands should be patented separately. Acquisition of the government title to a parcel of land was not to carry with it a water-right; but all non-navigable waters were reserved for the use of the public under the laws of the various arid-land states."

The Court there held that the United States was not an indispensable party defendant, even if it had built a reclamation project which used water from the stream; and, that the ownership of the water right is in the land-owners and not in the United States.

The final decision of the Court in the suit over the waters of the North Platte River should definitely dispose of this long-disputed question.

The Federal Government unquestionably has control over the waters of an international stream, in so far as such authority may be needful in the negotiation and ratification of an international treaty concerning use of such waters. It is also probable that the Federal Government has similar authority in carrying out any specific obligation that it incurred in connection with its Indian charges with respect to public waters prior to Statehood.

State Control over Water Resources.—As heretofore stated, it is the theory and contention of the States in the arid region that control of the waters within their boundaries lies exclusively with such States, and most of them have declared, by constitutional provision, or by legislative enactment, that such waters are the property of the State or of the public. Most of these States have legislative codes providing a complete system by which rights to the public waters of the State may be recognized and determined by adjudication proceedings. The latter are usually by Court action, and for the administration of the rights of the appropriators, if such rights have been determined, as required by statute. Such system provides quite complete control over the appropriation, administration, and use of the public water supplies of these States. In some public-land States, where the doctrine of appropriation does not apply, and especially in the States east of the Mississippi River, there has never been any question as to the power of a State to control the waters within its boundaries, there being no Federal interests, except in connection with navigation, or diversions from one drainage basin to another. The extent of State control varies from practically no control in some of the Southern States to complete control over all water uses and developments, as in the States of New York, Maryland, and Pennsylvania.

Present Methods of Settling Interstate Water Problems.—Interstate water controversies may be settled by: (*a*) Original suits between States in the Supreme Court of the United States; (*b*) review of decisions in the lower Federal Courts between individual or corporate interests; and (*c*) interstate compacts or treaties. The Supreme Court is the only agency having jurisdiction over interstate water controversies between States with authority to effect a division of the waters of an interstate stream. In Missouri *vs.* Illinois, in connection with its authority, the Court has announced[7, 21] that "the governing rule is that this Court will not exert its extraordinary power to control the conduct of one state at the suit of another, unless the threatened invasion of rights is of serious magnitude and established by clear and convincing evidence."

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[20] Desert Land Act of March 3, 1877, Chapter 107, 19 Stat. at L. 377, 43 U. S. C. A., Section 321.

[21] Missouri *vs.* Illinois, 200 U. S. 496 (1906).

Suits between private parties in different States concerning the waters of interstate streams merely settle individual controversies and afford little, if anything, in the solution of the larger interstate problems. Where the findings of the lower Federal Courts, however, are reviewed by the Supreme Court, the results of such suits have all the force and effect upon the litigants, and possibly also upon such States in which the waters flow, as if the findings were the result of an original suit between such States.[22]

Voluntary, informal interstate agreements have been resorted to in an attempt to adjust or settle interstate problems, aside from recourse to the Courts. A notable example is the Ohio River Interstate Stream Conservation Agreement, entered into in 1924 between the Health Departments of Ohio, West Virginia, Pennsylvania, and subsequently by eight other interested States, for the purpose of conserving the waters of interstate streams and preventing undue pollution. The co-operative activities of the States under this agreement still continue, but a formal compact has been drafted. Such agreements afford some opportunity for the development and administration of the waters of those streams, but because of their *quasi* legal character, have no power of apportionment of the waters thereof between the States.

Fusion of public, State, and administrative agencies, by concurrent legislation of the States, has also been employed as a means of settling interstate problems. The New York-New Jersey Port Authority was thus created for purposes of planning and financing important undertakings, but has no authority to enforce its activities against either State.

Basic Principles Enunciated by the Supreme Court of the United States with Respect to Interstate Water Controversies

In the fifteen decisions heretofore rendered by the Supreme Court of the United States, involving the waters of interstate streams, the Court has announced and applied a number of fundamental or underlying principles applicable to such controversies. These fundamental principles are reviewed in the case of Connecticut *vs.* Massachusetts,[7] decided February 24, 1931, and are as follows:

(*a*) That of "equality of right" between States;[5, 7]

(*b*) That each State has full jurisdiction over the lands within its borders, including the beds of streams and other waters;[5]

(*c*) That a State may determine for itself whether the riparian or appropriation doctrine shall control as regards the waters within its boundaries, and is free to change its system;[4, 7, 23, 24]

(*d*) That "whenever the action of one state reaches through the agency of natural laws into the territory of another state, the question of the extent and limitations of the rights of the two states becomes a matter of justiceable dispute between them";[5, 8]

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[22] Pioneer Irrigation Co. *vs.* John E. Field and A. A. Weiland as State Engineers of the State of Colorado, 259 U. S. 498.

[23] United States *vs.* Rio Grande Irrigation Co., 174 U. S. 690–702.

[24] California-Oregon Power Co. *vs.* Beaver Portland Cement Co., *et al.*, 73 Fed. 2d 555.

(*e*) That no State may claim the exclusive right to the use of all the waters within its boundaries; that there must be an equitable division or apportionment of the benefits of an interstate stream between the States affected;[5, 6, 7, 8, 21, 25]

(*f*) That, as between two or more States which recognize exclusively the doctrine of appropriation and beneficial use, such doctrine may properly be made the basis in a determination of equitable apportionment of the common water supply[26] (priority of appropriation, however, does not necessarily create superiority of right in the waters of an interstate stream;[8] see also Item (*e*));

(*g*) That water may be taken from one drainage basin for use in another drainage basin, although such diversions may result in some injury to the basin in which such waters originate;[6, 8]

(*h*) That the burden of constructing storage reservoirs for conserving the surplus waters of an interstate stream, which otherwise would run to waste, rests upon the lower State, and an upper State is not required to give up waters, which it may need, to a lower State, when such practice would result in waste of the common supply;[6]

(*i*) That the appropriators and users of water in a State are represented by the State under which their claims arise, and are bound by the limitations which may be imposed upon that State, either by Supreme Court decree or by interstate compact;[17, 27, 28]

(*j*) That the United States may perform its functions without conforming to the police regulations of a State; that, where such authority is involved (such, for instance, as the constitutional right to control navigable waters), it is immaterial if other benefits such as flood control, reclamation, conservation, apportionment of the waters between States, or the performance of an international obligation may result from the exercise of such constitutional power;[29, 30]

(*k*) That the "Court cannot issue declaratory decrees"[8, 30] (since the decisions affecting Item (*k*) were rendered,[8, 30] Congress has empowered Federal Courts to render declaratory decrees. However, this authority may or may not be considered as applying only to Federal District Courts);

(*l*) That "to appropriate water means to take and divert a specified quantity thereof and put it to beneficial use in accordance with the laws of the State where such water is found and, by so doing, to acquire under such laws a vested right to take and divert from the same source, and to use and consume the same quantity of water annually forever, subject only to the right of prior appropriators";[30]

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[25] Georgia *vs.* Tennessee Copper Co., 206 U. S. 230–237.

[26] Wyoming *vs.* Colorado, 259 U. S. 419; 260 U. S. 1; 286 U. S. 494; 298 U. S. 573.

[27] Wyoming *vs.* Colorado, 286 U. S. 494 (1932).

[28] Hinderlider *et al. vs.* La Plata River and Cherry Creek Ditch Company, 58 S. Ct. Rept. 803 (decided April 25, 1938).

[29] Wisconsin *vs.* Illinois *et al.*, 278 U. S. 395 (1929).

[30] Arizona *vs.* California *et al.*, 283 U. S. 423 (1931).

(*m*) That an upper State is not relieved of its responsibility for damages inflicted upon a lower State as the result of an increase of flow in an interstate stream by the drainage of lands in the upper State:

> "Right of a state as parens patriae to bring suit to protect the general comfort, health and prosperity of its inhabitants, threatened by the proposed or continued action of another state by prayer or injunction is to be differentiated from its lost power as a sovereign to prevent and impose individual claims of its citizens as their trustees against a sister state."

The Eleventh Amendment to the Constitution of the United States provides:

> "The judicial power of the United States shall not be construed to extend to any suit in law or equity, commenced or prosecuted against one of the United States, by citizens of another state, or by citizens or subjects of any foreign state."

Neither may a citizen of a State prosecute a suit in law or equity against another State;[31]

(*n*) That "the governing rule is that this court will not assert its extraordinary power to control the conduct of one state in the suit of another, unless the threatened invasion of rights is of such serious magnitude and established by clear and convincing evidence";[21, 32]

(*o*) "Injunction issues, to prevent existing or presently threatened injuries, will not be granted against something merely feared as liable to occur at some indefinite time in the future."[7, 8, 30, 32] That "the burden on the complaining state of sustaining its allegations is much greater than that generally imposed upon a complainant in an ordinary suit between private parties";[31]

(*p*) The laws in respect to riparian rights that happen to be effective for the time being in both States do not necessarily constitute a dependable guide or just basis for the decision of controversies such as that presented here. The rules of the common law on that subject do not obtain in all of the States of the Union, and there are variations in their application. The doctrine of appropriation prevails in some States, and every State is free to change its laws covering riparian ownership and to permit the appropriation of flowing waters for such purposes as it may deem wise;[7, 23]

(*q*) That, in a given case, a division of the waters of an interstate stream may not necessarily create a prior appropriation or superiority of right over another State in the enjoyment and use of the waters of an interstate stream;[8]

(*r*) That, where the Court has by decision equitably divided the waters of an interstate stream between two States, the apportionment or quantum received by each State may be administered for the benefit of its appropriators, in accordance with its own laws, without regard to the interests of the other State, so long as the aggregate quantity so allocated is not exceeded;[27]

(*s*) That the appropriators or water users under a Federal reclamation project obtain whatever rights they may have to the use of water from the States in which such projects are located, and the continued enjoyment of such right is subject to the laws of such State;[5, 33]

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[31] North Dakota *vs.* Minnesota, 263 U. S. 365 (1923).

[32] Washington *vs.* Oregon, 297 U. S. 517 (decided March 2, 1936).

[33] Nebraska *vs.* Wyoming and Colorado, Impleaded Defendant, 295 U. S. 40 (decided Apr. 1, 1935).

(*t*) That States have the unquestioned authority to agree upon the division and use of the waters of an interstate stream, even if such division and use may have the effect of disturbing or destroying the rights of individual appropriators which had theretofore been recognized by the laws of either State;[28]

(*u*) That the highest use of interstate waters is for drinking and other domestic purposes;[7]

(*v*) That States may select sources of water supply, provided no substantial damage is done to sister riparian States;[8]

(*w*) That release of compensatory water depends upon down-stream river conditions and needs;[8]

(*x*) That judicial allowance of diversions affecting navigation or navigable capacity is conditioned upon Federal approval.[7, 8, 34]

Original Cases Between States Before the Supreme Court of the United States

Arkansas River Suit.—Litigation between Colorado and Kansas over the waters of the Arkansas River has been practically continuous for the thirty-six years, 1904–1939. In the noted suit brought by Kansas against Colorado in 1901,[35] the U. S. Supreme Court decided that, although certain irrigation interests in western Kansas had been injured by diversions of water from the Arkansas River in Colorado, such diversions by Colorado had not exceeded its equitable share of the waters of this interstate stream, and dismissed the case without prejudice to Kansas to reinstate a suit at a later date, if continued diversions in her judgment justified such action. Up until 1929, Kansas failed to institute action for any relief to which that State might have felt it was entitled since the former decision. Almost immediately after the decision of the former suit in 1907, administrators of certain ditches in Kansas instituted suit in 1910 in the Federal Court for the District of Colorado against a large number of agencies that controlled ditches and reservoirs in Colorado, impleaded in the former suit, seeking to restrain the latter from diverting water in Colorado to their alleged injury. This case is known[36] as the United States Irrigating Company *vs.* Graham Ditch Company suit. This case was settled out of Court in 1916, but a later suit was instituted in the same Court in 1917 by one of the Kansas ditch companies not a party to the settlement against the same ditch companies in Colorado mentioned in the former suit[37] and later amended to include ditches and reservoirs on the tributaries of the Arkansas in Colorado (Coler Ditch and Reservoir). This condition resulted in Colorado, in her sovereign capacity, filing suit in the Supreme Court of the United States in 1929 against the State of Kansas for the purpose of requiring the latter State to enforce upon its water users the provisions of the former Supreme Court decree, and to quiet further litigation by her agents against appropriators in Colorado. The Supreme Court of the United States assumed jurisdiction of the subject matter and permitted Colorado to file an original suit against

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[34] Wisconsin *vs.* Illinois *et al.*, 278 U. S. 395 (1929); 281 U. S. 179 (1930); 289 U. S. 395 (1933).

[35] Kansas *vs.* Colorado, 206 U. S. 46 (1906); 51 L. ed. 956; 27 S. Ct. 665; same case on demurrer, 185 U. S. 125; 46 L. ed 838; 22 S. Ct. 552.

[36] United States Irrigating Company *vs.* Graham Ditch Company (case was compromised and did not go to decision).

[37] Finney Co. Water Users Association *vs.* Graham Ditch Co. *et al.*, 1934 Colorado, 1 Fed. 2d, 650.

the State of Kansas. In its answer, Kansas filed a cross bill of complaint against Colorado in which, among other things, Kansas alleged increased depletion of the flow of the river, since the original case had been decided, and also its right to one-half of the undepleted flow of the river at the Colorado-Kansas State line. Voluminous testimony, covering a period of nearly ten years, has been taken in this second suit before a stenographic commissioner, appointed by the Court.

In the meantime, the representatives of the two States entered into a stipulation to maintain the *status quo* of the use of the waters of the Arkansas River pending: (*a*) The construction of a large storage and flood-control reservoir, which is known as the Caddoa Reservoir, to be located on the Arkansas River about 40 miles west of the Colorado-Kansas State line and which has been approved for construction by the United States Corps of Engineers in the Omnibus Flood Control Act of 1936; or (*b*), until the present suit has been decided by the Court.

The *status quo* provided by the stipulation is described as permitting Colorado to continue to divert substantially 160 000 acre-ft of water per yr for the irrigation of 72 000 acres of land below the Caddoa Reservoir in Colorado, and that Colorado will not deplete the flow at the interstate line below 25 000 acre-ft, in order to permit storage in reservoirs in western Kansas between October 1 and April 1, and not below 52 000 acre-ft for direct application in Kansas between April 1 and October 1; that when the Caddoa Reservoir has been constructed, both States will share equally in the surplus water which the reservoir will capture and make available under the administration of the two State Engineers. The stipulation is without prejudice to the claims of either party, as set out in the pending interstate suit, and is subject to the decision of the Supreme Court of the United States as to the respective rights of the States to the water furnished by the Arkansas River, or stored in the proposed reservoir. The stipulation provides that its terms shall not be filed in the present action between the States until the construction of said dam and reservoir by the United States Government has been assured. The two States agreed to co-operate in obtaining the construction of the Caddoa Dam. In this connection, the Legislature of Colorado created the Caddoa Reservoir and Arkansas River Basin Conservancy and Improvement District to enable the water uses to meet any obligations imposed by the Federal Government in connection with the construction of the Caddoa Dam.

Laramie River Suit.—This case[6] involved the use of the waters of the Laramie River which rise in Colorado and flow into Wyoming. The Government of the State of Wyoming alleged that a proposed diversion, by means of a tunnel from the Laramie River to the Cache la Poudre River in Colorado, would injure prior appropriators in Wyoming. In their Constitutions both Wyoming and Colorado recognize only the doctrine of appropriation, but Colorado claimed the right to the use of all of the waters within her borders by reason of sovereignty or statehood. Wyoming contended that priority, regardless of State lines, should apply, and that Wyoming, having prior appropriations of sufficient volume to use practically all of the waters of the river, was entitled to full protection in such uses.

The U. S. Supreme Court, however, did not accept the priority date (1902) given the tunnel by the Colorado State Court, but set it back 7 yr to 1909, which put it behind many large intervening priorities in Wyoming and thereby reduced the remaining flow available for the Laramie-Poudre Tunnel. It imposed an acre-foot per acre of use limit on Colorado meadows and rejected the Colorado greater-productivity consideration.

The Court held as follows:

(*a*) That the doctrine of prior appropriation, recognized by both States, was applicable to interstate streams and controversies, and furnished "the only basis which is consonant with the principles of right and equity applicable to such a controversy as this";

(*b*) That Colorado's claim to all the waters within its boundaries was untenable;

(*c*) That water-shed trans-basin diversions are permissible where both States recognize such diversions within their respective boundaries;

(*d*) That the burden of conserving, by storage reservoirs, the surplus waters of a stream is upon a lower State, which has the effect of relieving the upper State of the obligation to pass water to a lower State when such action may result in waste; and

(*e*) That there was no need to pass on the contention of the United States that all unappropriated waters of the Western States belong to the United States and, therefore, are already removed from State control.

One of the interesting results of this suit was the fact that, in allocating the entire water supply furnished by the Big Laramie River in Colorado and Wyoming, the Court arrived at its decision by first determining the total water supply furnished by the stream. It recognized the then existing uses in both States, based upon their relative orders of priority, as determined by State laws. This sum total was deducted from the total water supply, which left a residue of 15 500 acre-ft of water per yr, which the Court allocated to the Laramie-Poudre Tunnel, a Colorado appropriator. As determined by the Court, this tunnel was the most junior appropriator on the stream in both States, but the decision of the Court made its appropriation, to the extent of 15 500 acre-ft annually, actually senior to any appropriation out of the river in Wyoming. As a result of later years of subnormal stream flow, many senior appropriations in Wyoming have lacked water, whereas the tunnel each year diverts all that it is physically possible to divert up to 15 500 acre-ft.

The second suit over the waters of this river[38] was filed by Wyoming against Colorado on October 6, 1930. This suit involved the interpretation of the decree of the U. S. Supreme Court entered in the former suit. The decision of the Court, announced June 1, 1936, sustained the contentions of Colorado that the quantities of water, theretofore recognized by the Supreme Court in the former case as having been diverted out of the Laramie River by agencies in Colorado, were in the nature of an allocation of waters to the State of Colorado, and not to its individual water users, and that Colorado could rightfully administer such *quantum* under the provisions of its own laws without violating the interests of Wyoming, so long as such *quantum* (which was fixed at 39 750 acre-ft in any one year) was not exceeded.

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[38] Wyoming *vs.* Colorado, 298 U. S. 573.

The decision, however, had the effect of discontinuing two small trans-mountain diversions, which were not mentioned in the former proceeding, out of the Laramie River to the Cache la Poudre River. Colorado's interpretation of the words "divert and take" in the former decree, with reference to the meadowlands, meant a consumptive use and not head-gate diversions. The Court held, however, that previous head-gate diversions of these meadowland ditches resulted in an unnecessary use of water and that the actual consumptive use of water on wild meadowlands in Colorado, which was found equal to 1 acre-ft per acre actually irrigated, should be measured at the head-gate of the ditches.

Because of the practice of flooding peculiar to this type of crop, enforcement of the decree will have the effect of destroying the greater part of about 4 250 acres of hay lands in Colorado. The Court gave the State of Colorado until September 15, 1938, to file application for a rehearing in respect to that part of the decision relating to the meadowlands. The Court denied Wyoming's demand that an interstate bailiff be designated by the Court to administer the waters of this stream, and left the administration of the stream in the hands of the State Engineers of the two States; but it retained jurisdiction of this phase of the case for the purpose of affording Wyoming relief in the event that future difficulties of administration between the States justified such relief.

Chicago Lakes Drainage Cases.—This suit[39] was instituted by the State of Wisconsin and other States against the State of Illinois and the Sanitary District of Chicago, Ill., to enjoin defendants from withdrawing 8 500 cu ft of water per sec from Lake Michigan. The Sanitary District of Chicago increased diversions from Lake Michigan through the Chicago drainage canal from 4 167 to 8 500 cu ft per sec in violation of a permit theretofore issued by the War Department. Later, the War Department authorized this increase for purposes of sewage dilution. The Court held that, in so far as the diversion was not for the purpose of navigation in the Chicago River, it was without legal basis and could be enjoined by the States bordering on the Great Lakes and having riparian rights that were being injured by the lowering of the level of the lakes.

The matter was referred by the Court to a Master to take testimony and make findings of facts. Following the report of the Master, the Court entered a decree in accordance with his opinion and recommendation and required the Sanitary District of Chicago to provide proper treatment facilities to reduce the diversions through the drainage canal.[40]

In its reply, the Chicago Sanitary District pleaded its inability to finance the obligations imposed by the Court. Upon recommendation of the Master, permission was given to divert an enlarged quantity of water and for extension of time for the State of Illinois to finance the cost of necessary sewage disposal works, which the Court required that State to meet.

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[39] Wisconsin *vs.* Illinois, *et al.*, 278 U. S. 367; 49 S. Ct. 163 (1929).

[40] Wisconsin *vs.* Illinois, *et al.*, 50 S. Ct. 256 (1930).

Connecticut River Suit.—This original suit[7] was brought by Connecticut to enjoin Massachusetts and the Boston, Mass., Metropolitan Water District from diverting water from the Swift and Ware rivers, tributaries of the Connecticut River in Massachusetts, for use in the Boston Metropolitan area, which had been authorized by the Legislature of Massachusetts and permitted by the Secretary of War, under certain limitations and requirements. Connecticut established its right, under the doctrine of riparian ownership which both States recognize, to the undiminished flow of the river, and alleged, among other things, that the riparian doctrine forbade such diversions out of the water-shed, and that to take water entirely out of the Connecticut River Basin would impair the navigability of the Connecticut River, the fishing industry, agriculture, power development, increase dangers from pollution, and take away flood rights.

The Special Master, appointed by the Court to take the testimony, upheld the right of the City of Boston to construct the proposed project, and held that the damage that would result to navigation, agriculture, and to the sanitary condition of the Connecticut River by the proposed project would be inconsequential.

In rendering its opinion, the Court apparently abandoned the exclusive riparian doctrine as applicable to interstate questions, and announced another principle by the statement:

> "* * * but the laws in respect to riparian rights that happen to be effective for the time being in both states do not necessarily constitute a dependable guide or just basis for the decision of controversies such as that here presented. Such disputes are to be settled on the basis of equality of right, but this is not to say that there must be an equal division of the waters of an interstate stream among the states through which it flows. It means that the principles of right and equity shall be applied, having regard to the 'equal level or plane on which all the states stand in point of power and right under our constitutional system,' and that upon consideration of the pertinent laws of the contending states, and all other relevant facts, this Court will determine what is an equitable apportionment of the use of such waters."

The Court also established the principle that an injunction will not be granted against something merely feared as likely to occur at some indefinite time in the future.

The Court also held that a diversion of a State's equitable share of the water to another water-shed was not in itself ground for injunctive relief, and that a State had the right to select its own source of water supply, provided that such action would not result in serious damage to other States. In its ruling, the Court recognized the limitation of diversions by the Boston Metropolitan area, of the waters of the Swift and Ware rivers, imposed by the Secretary of War in the interest of navigation.

Delaware River Suit.—This action[8] was brought by the State of New Jersey to prevent the State of New York from diverting waters from certain tributaries of the Delaware River, an interstate stream common to the States of New Jersey, New York, Pennsylvania, and Delaware, and from the water-shed of the Hudson River, for the purpose of increasing the water supply of the City of New York.

In 1923, the Legislatures of New Jersey, New York, and Pennsylvania authorized appointment of Commissioners who negotiated a Compact allocating the water resources of the Delaware River between the three States. The Compact was ratified by the Legislature of New York only. A second Compact, negotiated in 1927, provided for the diversion of 600 million gal of water daily from the Delaware River and its tributaries by New York, 600 million gal daily by New Jersey, and 900 million gal daily by Pennsylvania, with provisions for released flows. This second Compact was not ratified by New Jersey or Pennsylvania.

New York then proceeded with its plan for diverting 600 million gal per day from tributaries of the Delaware River, and for releasing water in accordance with the terms of the last Compact, which plan was approved by the Water and Power Control Commission of New York State. New Jersey filed its bill in the U. S. Supreme Court to enjoin the State and City of New York from continuing with this plan, alleging among other things that it would cause substantial damage to navigation, power development, sanitary conditions, industrial use, fisheries, and other uses, and that it would violate riparian rights, particularly by diversions out of the water-shed.

The Court permitted Pennsylvania to intervene in behalf of its own interests in the river. The decision of the Court was handed down on May 4, 1931. The decision permitted New York to divert 440 million gal per day, a quantity ess than claimed by that State, subject to certain reservations.

It is interesting to note that, in recognizing the right of New York to make such diversions, it enunciated the important principle that, in the absence of a compact between the States, such diversion by New York does not constitute a prior appropriation, and shall not give the State of New York, and the City of New York, any superior right over New Jersey and Pennsylvania in the enjoyment and use of the Delaware River and its tributaries.

In this case, as in the case of Arizona *vs.* California *et al.*,[30] the Court held that it was without power to render a declaratory judgment to prevent the consummation of that which is merely feared as likely to occur. The Court dismissed the bill of complaint of New Jersey without prejudice to the equities of New Jersey and Pennsylvania in the waters of that stream basin.

The Court held that,

> "* * * although one state has the physical power to cut off all water within its jurisdiction from flowing into a river constituting a boundary between states, the exercise of such power to the destruction of the interests of the lower state, could not be tolerated, and likewise that neither will a lower state be permitted to require an upper state to give up its power altogether in order that the river may flow to the lower state undiminished; that both states have relative and substantial interests in an interstate river, which must be reconciled as best they may be."

This case involved a set of facts similar to those in Connecticut *vs.* Massachusetts.[7] In this case, the Court again applied the doctrine of equitable apportionment, and stated that "* * * the effort is always to secure an equitable apportionment without quibbling over formulas"; also that "* * * the removal of water to a different watershed obviously must be allowed at times unless states are to be deprived of the most beneficial use on formal grounds."

Colorado River Suit.—In this case,[30] Arizona filed its bill of complaint against the Secretary of the Interior, the State of California, and the other States of the Colorado River Basin, alleging, among other complaints, that the construction of a dam in the Colorado River at Black Canyon would invade the sovereign rights of the State of Arizona by diverting a part of the waters of the Colorado River for consumptive use elsewhere, and thus would interfere with the beneficial consumptive use by the State of Arizona of the unappropriated waters of the river flowing in that State. The Court held: (*a*) That the authority of the Secretary to construct the dam (later named the "Hoover Dam") was a valid exercise of the power of Congress, since the purposes enumerated included that of improving navigation; (*b*) that there was then no present threat of any act by the defendants which would interfere with the enjoyment by Arizona of any present or future appropriation of water out of the Colorado River by that State; and (*c*) that Arizona had not constructed any diversion works to divert the waters claimed. In this connection it is of interest to note that the Court clearly defined the status of an "appropriator of water" as follows:

> "* * * to appropriate water means to take and divert a specified quantity thereof and put it to beneficial use in accordance with the laws of the state where such water is found, and by so doing, to acquire under such laws a vested right to take and divert from the same source, and to use and consume the same quantity of water annually forever, subject only to the right of prior appropriators."

The bill was dismissed without prejudice to Arizona to make application for relief in case the water stored in Lake Mead was used in any way such as to interfere with the enjoyment by Arizona of any right already perfected, or with the right of Arizona to make additional legal appropriations. All the citations affecting this case[41, 42, 43] involve the Colorado River Compact and Boulder Canyon Project Act.

Walla Walla River Suit.—The purpose of the suit,[32] as disclosed by the allegations of the complaint and the relief sought, was to have the Court determine the relative rights of the State of Washington and Oregon to the use of the waters of the Walla Walla River and its tributaries, flowing within both States, and to enjoin the State of Oregon from continued interference with the rights of the State of Washington. The State of Washington alleged: (*a*) That the continued diversions of water from the tributaries of the Walla Walla River in Oregon would work irreparable injury to the State of Washington and its land owners unless restrained; (*b*) that both States, by their statutes and decisions of their Courts, recognize the doctrine of riparian ownership, and that the owner of land riparian to a stream is entitled to his proportionate share of the waters of such stream for the purpose of irrigation, subject to the requirement that such riparian user shall beneficially apply such water within

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[41] Arizona *vs.* California *et al.*, 283 U. S. 423 (1931), 298 U. S. 558 (1936).

[42] Greeson *vs.* Imperial Irrigation District, 59 Fed. 2d, 529.

[43] United States *vs.* California, 295 U. S. 174 (1935).

a reasonable time; and (*c*) that each of said States further recognizes the doctrine of appropriation whereby the first appropriator in time is first in right, subject only to vested riparian rights.

In the same term in which it rendered the decision in the second Wyoming *vs.* Colorado suit,[6] the Court illuminated the principles which would actuate it in equitable apportionment. In Washington *vs.* Oregon,[32] decided March 2, 1936, both States stipulated for the purposes of that case that the individual rights of the respective landowners and water owners concerned in both States are governed by the doctrine of prior appropriation, so that question was not in issue.

In deciding against Washington, the Court related as impelling considerations that, due to the porous and gravelly formation in the river-bed, only a small portion of water would go by even if the dams of Oregon users were removed. "Fifty years of long established possession and uninterrupted use by the Oregon appropriators" were repeatedly commented on and given weight by the Court, since, said Justice Cardozo, to restrain the diversion

> "* * * would bring distress and even ruin to a long-established settlement of tillers of the soil for no other or better purpose than to vindicate a barren right. This is not the high equity that moves the conscience of the court in giving judgment between states * * *. In default of reasons for removal more urgent and compelling, the tillers of the soil will be left where they have settled.
>
> "In such circumstances, an injunction would not issue if the contest were between private parties at odds about a boundary. Still less will it issue here in a contest between states, a contest to be dealt with in the large and ample way that alone becomes the dignity of the litigants concerned."

A priority decree as of 1892 by Washington Court for the Gardena Ditch claim was held "of no force against Oregon or Oregon appropriators not parties to the suit," and the appropriation lost by laches and abandonment while Oregon rights were being matured.

The Court held that the State of Washington was not entitled to injunctive relief against Oregon under the evidence showing diversions by Oregon water users for domestic and agricultural purposes for 50 yr, and upon demonstration that, at periods of shortage of stream flow, only a small quantity of water would escape, even if not diverted, and would be lost in deep gravel formations beneath the stream channels.

The Court held that the priority dates and quantities awarded an appropriator by the Courts of his own State were not necessarily binding on the appropriators from the same stream in another State.

Rio Grande Suit.—Charging that rights to water from the Rio Grande for irrigation, dating back to 1861, were being violated by the State of New Mexico, and the Middle Rio Grande Conservancy District and its officials, the State of Texas commenced an original action[44] before the United States Supreme Court at the October term of that Court in 1935 to enjoin them from making any increased storage or diversions beyond those in existence prior to January 23, 1906.

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[44] Texas *vs.* New Mexico and the Middle Rio Grande Conservancy District; disposed of by Rio Grande Compact now (1939) in force.

The complaint detailed the circumstances which caused the Republic of Mexico, in the early 1890's, to make complaints to the Federal Government that diversions of water on the upper stream had deprived Mexican lands of similar irrigation water supplies. This was followed by the establishment of an International Boundary Commission and a treaty which assured to Mexico a certain flow perpetually. Thereafter, the Elephant Butte Dam was constructed near Engle, N. Mex., at a cost of about $14 000 000.

Texas charged that New Mexico had violated that clause of the Rio Grande Compact which had been entered into by those two States with Colorado and which had been approved by Act of Congress dated June 17, 1930, and extended by a similar Act on June 5, 1935. The section in question provided:

> "New Mexico agrees with Texas, with the understanding that prior vested rights above and below Elephant Butte Reservoir shall never be impaired hereby, that she will not cause or suffer the water supply of the Elephant Butte Reservoir to be impaired by new or increased diversion or storage within the limits of New Mexico unless and until such depletion is offset by increase of drainage return."

New Mexico, and particularly the Middle Rio Grande Conservancy District, was accused of having constructed various permanent diversion dams, including the El Vado Reservoir with a capacity of 200 000 acre-ft, and with making greatly increased diversions of water upon increased acreages since the execution of the Compact.

The complainant's right to this water was based upon two filings made by the Reclamation Service in the office of the State Engineer of New Mexico on January 23, 1906, for 730 000 acre-ft near Engle, and on April 8, 1908, for all of the unappropriated water of the Rio Grande for storage in the Elephant Butte Reservoir. It was likewise charged that New Mexico had not offset such increased storage and diversions by drainage, and that the supply of the Elephant Butte Reservoir had been greatly impaired, both in quantity and quality.

New Mexico joined issue on the allegations of the bill, and the matter was referred to a Special Master by the U. S. Supreme Court. Evidence was taken at a series of hearings held in the two States over a long period of time, during which Texas presented extended testimony as to the impairment of the quality of water because of the presence of great quantities of salts deleterious to plant life.

In the Summer of 1937, the hearings were temporarily suspended until the Fall of 1938. In the meantime, the representatives of the three States, acting upon the facts disclosed by the joint investigation of the Rio Grande Basin by the National Resources Committee, signed the Rio Grande Compact on March 18, 1938. The compact has been ratified by the Legislatures of the three States and has been approved by the Congress, which actions settle all questions of dispute between the States and their agencies and disposes of this Interstate suit.

By the terms of this agreement the waters of the Rio Grande are equitably apportioned between the three States, with the rights of the Republic of Mexico fully protected and considered, and "all controversies between said three states relative to the quantity or quality of the water of the Rio Grande are composed and settled." The ratification of the Compact will undoubtedly result in the dismissal of the pending litigation, and another victory will have been "chalked up" for the exponents of the Compact method of composing differences between States.

North Platte River Suit.—On October 15, 1934, the State of Nebraska filed its bill of complaint[45] in the Supreme Court of the United States against the State of Wyoming, in which Nebraska asked the Court:

> (1) "To require that Wyoming, in the administration of waters of the North Platte River, should deny water to her direct-flow water users having junior priorities when water is needed by senior Nebraska appropriators;
>
> (2) "To require that Wyoming prevent her appropriators for storage from taking water for such purposes when the water is needed by senior Nebraska appropriators;
>
> (3) "To prevent Wyoming from allocating to a new irrigation project, known as the Casper-Alcova Project, a 1904 priority, when, as Nebraska claims, it is only entitled to a 1934 priority, and many Nebraska projects of 1904 priority and later would be deprived of water in the administration of the stream with a 1904 priority of the Casper-Alcova Project;
>
> (4) "As an incident to said direct relief, and in order to provide an exact basis for a decree covering the administration of the stream in the future to fix and determine the respective priorities on the stream of Nebraska and Wyoming appropriators."

In its demurrer and motion to dismiss, Wyoming claimed that Colorado, and the Secretary of the Interior, because of the North Platte Federal Reclamation Project, are necessary parties to this action.

The State of Nebraska, in its answer to Wyoming's motion, asserted that, since no relief is asked by complainant against the State of Colorado, and since the State of Colorado has no interest in the relief asked, against the State of Wyoming, or in the controversy between the State of Nebraska and the State of Wyoming, the State of Colorado is not a necessary party to the action because the States of Nebraska and Wyoming represent the interests of their respective water users under the Federal Reclamation Project; but upon Wyoming's amended complaint, the Court required that Colorado be made a party to the suit. On May 4, 1936, Colorado filed its answer and cross bill, in which issue was taken with the claims of both Nebraska and Wyoming, and in which Colorado establishes its own claims to an equitable division of the waters of the North Platte River.

The fundamental contentions with respect to the ownership and use of the waters of this interstate stream are as follows:

Nebraska contends that priority of appropriation shall prevail regardless of State lines. Wyoming contends that such a doctrine in the present case, due to the peculiarities of the river, would inevitably result in an unwarranted and wasteful use of the water supplies of the river, while Colorado bases its contentions upon "equitable apportionment" between the three States, regardless of priority of appropriation or riparian rights.

The Court appointed a Master to hear testimony, which began in the Summer of 1936. Nebraska submitted voluminous testimony, which has been subjected to cross-examination. It is now (1939) anticipated that several years will elapse before Colorado and Wyoming have concluded their testimony and the case is presented to the Court for final determination.

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[45] Wyoming *vs.* Colorado, 295 U. S. 40.

Federal Intervention in Nebraska vs. Wyoming and Colorado, North Platte River Interstate Suit.—On June 15, 1938, the Department of Justice filed its petition for intervention in the pending interstate river suit in the case of Nebraska *vs.* Wyoming, and Colorado an impleaded defendant, over the waters of the North Platte River. In the brief, the petition makes two distinct claims with reference to North Platte River water:

(1) Federal ownership of all the waters of the North Platte River (subject to vested rights), under the reserve powers of the Federal Government (this of course includes all unappropriated water); and,

(2) Federal ownership of such water as has been acquired under State laws and in conformity with Section 8 of the Reclamation Act.

It is inconceivable that both contentions can be maintained in the pending suit, and it is believed that the United States will have to elect "which horse it will ride."

In the ruling earlier in this case,[17] on Wyoming's motion to dismiss on the ground that the Secretary of the Interior had not been made a party defendant, although a holder of appropriations from the North Platte River, the United States Supreme Court had said,

> "The motion asserts that the Secretary of the Interior is an indispensable party. The bill alleges, and we know as a matter of law,that the Secretary and his agents, acting by authority of the Reclamation Act and supplementary legislation, must obtain permits and priorities for the use of water from the State of Wyoming in the same manner as a private appropriator or an irrigation district formed under the state law. His rights can rise no higher than those of Wyoming, and an adjudication of the defendants' rights will necessarily bind him. Wyoming will stand in judgment for him as for any other appropriator in that state. He is not a necessary party."

Since the Court declined to allow individuals as appropriators to intervene in that suit and had also declined to allow individuals as appropriators to intervene as parties defendant in the pending Colorado *vs.* Kansas suit in that Court, it would seem that the United States has no greater standing as an appropriator or party in interstate litigation. Answers to the petition were filed by Nebraska and Wyoming on July 14, 1938, and by Colorado on August 29, the Court having extended the date for filing to September 1.

Suit over Return Waters from the North Platte Government Project on the North Platte River Between Whalen Dam and Bridgeport, Nebr.—In February, 1938, the Attorney General of the United States filed suit in the Federal District Court in the North Platte Division, claiming all the return flow from the lands of the North Platte Government Project, consisting of about 225 000 acres, served by the Interstate, North Port, and Fort Laramie Canals, of which about 200 000 acres are now irrigated. The suit is recorded as "United States *vs.* Tilley, Willis, and the Farmers Irrigation District."[46] The Farmers

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[46] United States *vs.* Tilley, Willis, and the Farmers Irrigation District; in the U. S. Dist. Court of Nebraska (not yet reported).

Irrigation District is a mutual organization in Nebraska, which owns and operates the Tri-state Canal, diverting water from the North Platte River just below the Wyoming-Nebraska State line. This District was made a party to the proceeding for the reason that it has a Warren Act contract with the U. S. Bureau of Reclamation for the delivery of as much as 100 000 acre-ft of storage water per yr out of the Government-owned Pathfinder Reservoir located on the North Platte River in Wyoming. This contract contains a clause whereby the Bureau of Reclamation may substitute return flow water from the North Platte Government Project lands in lieu of storage water from the Pathfinder Reservoir.

The suit was brought against water officials, the State Engineer, and the Chief of the Bureau of Irrigation, Water Power and Drainage, of Nebraska, on the question of the ownership of return flow from the Government Project lands which reaches and is diverted by the Tri-state Canal for the North Port unit of the North Platte Government Project.

State officials of Nebraska have always taken the position that return flow waters resulting from irrigation belonged to the public streams of the State into which they would naturally flow, and are subject to distribution to appropriators along the stream in their relative orders of priority. The suit in question is based upon conditions as of May 8 to May 20, 1936, during which period there was insufficient natural river flow to supply appropriators in Nebraska with priorities senior to December 6, 1904, the date of the priority of the Government canals. This condition caused the Nebraska water officials to deny water from the natural flow of the river to the North Port unit of the Government project. The quantity of return flow available to the Tri-state Canal during the irrigation season amounts to about 75 000 acre-ft. Should the Bureau of Reclamation get a favorable decision in the pending suit, the Bureau would then be in a position to claim control of all the return flow from the North Platte Government Project lands, which amounts to approximately 500 000 acre-ft per yr, of which 250 000 acre-ft returns to the North Platte River during the irrigation season.

In its bill of complaint, the United States contends for all return flow from the North Platte Government Project.

Should the United States, through the Bureau of Reclamation, secure control and ownership of the return flow to the North Platte River, resulting from the irrigation of the lands under the Government Project, it is probable that the decision would be cited in the future as establishing a legal basis applicable to return waters from other Government projects. For this reason, the decision is awaited with much interest and concern.

The return flows from the lands under the North Platte Government Project, plus return flows from privately owned canals between the Whalen Diversion Dam in eastern Wyoming, owned by the Government, and Bridgeport, are equal to about 850 000 acre-ft per yr. That portion of this return flow which reaches the river during the irrigation period relieves the upper river from the necessity of supplying a large quantity of water for canals and storage units of the North Platte Government Project, approximating 200 000 acre-ft per yr. It is alleged, therefore, that the North Platte Government

Project is a direct beneficiary of the return flows to the North Platte River, not only from the North Platte Government Project lands, but also from privately owned canals. If control of the return flow is secured by the Federal Government, it is alleged that senior Nebraska appropriators will be deprived of a portion of the waters they are now receiving, which in turn will have an adverse effect upon the natural water supplies in the upper reaches of the river.

Nebraska alleges that, in the late summer months, the upper river flow diminishes to such an extent that Nebraska canals will receive less water during the late summer period than under the present conditions of stream flow. It is also alleged that the United States has never acquired, through State permits, right to the use of the return flow waters in controversy, but in lieu thereof has asserted claim to the same by posting notices in 1912 on various tributaries of the North Platte River, where the return flow waters have accumulated.

Cases Decided by Federal District Courts

Pioneer Irrigation Company vs. Weiland, State Engineer of Colorado, et al.—In 1922, water users in Nebraska under the Pioneer Ditch (which heads on the North Fork of the Republican River in Colorado a few miles above the Colorado-Nebraska State line) brought suit[47] in the Federal District Court for the District of Colorado against water users under the same ditch in Colorado, including State Engineer John E. Field, M. Am. Soc. C. E., who was later succeeded by A. A. Weiland, M. Am. Soc. C. E. Substantially one-third of the water diverted by this ditch is used in Colorado and two-thirds in Nebraska. The allegations in this case were to the effect that the water users in Nebraska under the Pioneer Ditch were being deprived of their rightful share of water by users in Colorado.

The Federal District Court found that the water users in Nebraska must receive the same treatment as if the canal and the irrigators thereunder were located entirely within the State of Colorado, and enjoined the water users in Colorado and its State officials from any action which would not be consonant with such requirement. The decision of the Federal District Court was appealed to the U. S. Circuit Court of Appeals, which upheld the decision of the District Court. From this decision, an appeal was taken to the U. S. Supreme Court, which confirmed the decision of the lower Federal Courts on June 5, 1922, and the waters diverted by the Pioneer Ditch have been distributed in accordance with the mandate of the Court since that date.

Action Brought to Restrain the City of New York, N. Y., from Maintaining a Dam on the West Branch of the Byram River.—It was held[48] that the State of New York, in its exercise of power of eminent domain, could not enable the City of New York to erect, within the limits of the State, a dam across an unnavigable river, having its source in New York, and divert the water for use in the City of New York, thus diminishing the flow and injuring riparian rights in Connecticut. The Court further held that right of complainants to use of water in its natural flow was not a mere easement, but was a right inseparably annexed to the soil itself.

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[47] Pioneer Irrigation Co. *vs.* John E. Field and A. A. Weiland as State Engineers of the State of Colorado, 238 Fed. 519; 259 U. S. 498 (1922); 42 S. Ct. 568.

[48] Pine *vs.* New York City, 112 Fed. 98; 185 U. S. 93; 22 S. Ct. Rept. 592 (1902).

Action Brought by Citizens of Wyoming against Citizens of Colorado to Enjoin Diversion of Water from Sand Creek within the State of Colorado.—It was held[49] that complainants who were in possession of the water of an interstate stream for irrigation purposes in Wyoming were entitled to an injunction restraining the diversion of water of a stream flowing from Colorado into Wyoming by subsequent appropriators, who diverted one-half of the water into another stream, entirely different in its course, for irrigation of lands in Colorado.

Action Brought by Citizens of State of Nevada to Enjoin Diversions of Waters of West Fork of Carson River by Citizens of State of California.—In this case[50] it appeared that the doctrine of appropriation prevailed in Nevada whereas common law governing riparian rights was in effect in California. It was held that riparian owners in California and appropriators in Nevada of water of streams flowing from California into Nevada each had a right to have a reasonable apportionment of the water during the dry season, and a decree was made giving each party alternately the right to use the water for a certain number of days during the dry season, ordering the up-stream owners to return the surplus to the river.

Howell vs. Johnson[51] *and Morris vs. Bean.*[52]—In these cases the Court held to the following effect; One who has acquired a right to the water of an interstate stream by prior appropriation, in accordance with the laws of the State where it was made, is protected in such right against subsequent appropriations, although the latter withdrew the water within the limits of a different State.

Rickey Land and Cattle Company vs. Miller and Lux.[53]—Controversy arose in the case of Rickey Land and Cattle Company *vs.* Miller and Lux as to respective rights of the parties to the waters of the Walker River in the States of Nevada and California. It was held that the Federal Court in Nevada, and the California State Courts, had concurrent jurisdiction in the matter, and whichever Court first acquired jurisdiction was entitled to proceed to final determination without interference from the other, this action being *in personam* and affecting private rights only.

Morris vs. Bean.—This case[54] involved respective rights of the parties to the waters of Sage Creek in Montana and Wyoming. It was held that, subject to such rights as the Supreme Court might decide that the States have, and to vested rights, each State had full legislative power over Sage Creek while it flows within such State. In the absence of legislation, it will be assumed that the States were willing to ignore boundaries and allowed the same rights to be acquired from outside the State that could be acquired from within.

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[49] Hoge *vs.* Eaton, 135 Fed. 411 (C. C. D. Colorado, 1905).

[50] Anderson *vs.* Bassaman, 140 Fed. 14 (C. C. N. D. California, 1905).

[51] Howell *vs.* Johnson, 89 Fed. 558 (1899).

[52] Morris *vs.* Bean, 146 Fed. 423 (1906).

[53] Rickey Land and Cattle Company *vs.* Miller and Lux, 152 Fed. 11; 218 U. S. 258, 54 L. ed. 1032; 31 S. Ct. 11 (1910).

[54] Morris *vs.* Bean, 123 Fed. 618; 146 Fed. 423; Affirmed in 159 Fed. 651; 221 U. S. 485; 55 L. ed. 821; 31 S. Ct. 703 (1911).

Right of States to Settle Interstate Differences by Compacts

The right of States to divide the waters of interstate streams by the compact method was definitely determined by the U. S. Supreme Court in a decision handed down on April 25, 1938. The validity of an interstate compact, which allocated the equitable portion of the river flow to which each of the compacting parties and its respective citizens were entitled, had been drawn directly in question.

In the decision written by Mr. Justice Brandeis, the Supreme Court held that a compact approved by Congress, which apportioned the waters of an interstate stream equitably between the two States, was binding upon the States and upon the citizens and officials of the States, even when neither of the States was a party to the suit.[28]

The La Plata River and Cherry Creek Ditch Company appropriated irrigation waters out of the La Plata River in southwestern Colorado, and thereafter in a water adjudication proceeding in the District Court of La Plata County, Colorado, received a decree for 41.5 cu ft per sec of water with date as of June 2, 1890. Under the Colorado constitution and statutes, relative dates of appropriation determine the rights of water users. The ditch company in this case was given the sixth priority in Water District No. 33. The five prior rights on the stream had been decreed a total of 19 cu ft per sec of water.

Residents of New Mexico with lands capable of being irrigated from the same river had also applied the waters of that stream to a beneficial use, and in 1921 when the citizens of New Mexico protested that Colorado water users were taking so much of the stream flow that the lands in the lower State were deprived of their necessary supply, the two States determined to apportion the water equitably by exercising the power to compact, reserved to the States under the provisions of the Constitution of the United States. Acts were passed by the Legislatures of the two States, authorizing the appointment of commissioners to apportion the waters of the La Plata River equitably between them, and specifically providing that such agreement should become binding only when ratified and approved by the Legislatures of the two States and consented to by Act of Congress.

Thereafter, the Commissioners for Colorado and New Mexico executed the La Plata River Compact, and it was duly ratified by the Legislatures of the two States and consented to by Act of Congress.[55]

In the Spring of 1928, during a severe water shortage in the river, the flow became so scant that the two State Engineers agreed that the most beneficial use of the water could be obtained by permitting each State to enjoy the entire flow in alternating periods of 10 days. On a day in June, 1928, a Deputy State Engineer, under direction of the State Engineer, closed the head-gate of the La Plata River and Cherry Creek Ditch, as well as the head-gates of all other ditches in the State of Colorado, in order to permit the entire flow to go across the interstate line into New Mexico. At the time this action was taken, there were only 57 cu ft per sec of water in the stream in Colorado.

In order to give the La Plata River and Cherry Creek Ditch Company the water to which it claimed to be entitled under its Colorado decree, it would have been necessary to curtail deliveries to New Mexico greatly, due to losses in transit.

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[55] Hinderlider *et al. vs.* La Plata River and Cherry Creek Ditch Company (see Compact, p. 64).

The ditch company immediately brought suit to enjoin the State Engineer of Colorado and his subordinates from closing its head-gate, charging that there was water in the stream to which it was entitled under its decree, and that his administration of the stream was contrary to the laws of Colorado and the Court decrees under which the ditch company had acquired its rights.

The defense of the water officials was based upon the proposition that the head-gate was closed in enforcement of the provisions of the La Plata River Compact. The ditch company replied that the State water-administrative officials had ignored Court decrees entirely and were claiming the right to distribute, prorate, and rotate the waters without regard to the decrees of the laws of Colorado.

The case was first tried in the local District Court in May, 1929, although the decision was not rendered until June, 1930. The trial Court dismissed the complaint and held that the State water-administrative officials were distributing the river flow in conformity with the Compact, and that no rights of the ditch company had been violated. The ditch company prosecuted a writ of error to the Colorado State Supreme Court, which reversed the lower Court for the reason that a vested property right of the ditch company had been taken without due process of law and without just compensation, contrary to the provisions of the State and Federal Constitutions.[56]

In discussing the right to rotate the use of the entire flow in alternate periods, the Court said that "if private rights may be stripped from the citizen by state 'compacts,' by legislative fiat, by commissioners, by the uncontrolled discretion of state engineers, then 'due process' is dead in Colorado." Again the decision says: "It [the Compact] is a mere compromise of presumably conflicting claims, a trading therein, in which the property of citizens is bartered without notice or hearing, and with no regard to vested rights."

The Attorney General of Colorado then took an appeal from the decision of the Colorado State Supreme Court to the U. S. Supreme Court, but the latter tribunal refused to take jurisdiction on the ground that at that time there was no final judgment in the trial Court from which the appeal could be taken.[57]

Thereafter the matter was returned to the trial Court, where a new judge was presiding and, upon a stipulation that the evidence in the first trial should be considered as the record in the second hearing, a judgment was entered in harmony with the views expressed by the State Supreme Court. This commanded the water-administrative officials to regulate the head-gate of the ditch company so as to permit it to receive the water, to which it was entitled under its decree from the State Court, at all times, without reference to the provisions of the Compact.

The water-administrative officials then prosecuted a writ of error to the Supreme Court of the State of Colorado which reaffirmed its former decision, with certain explanatory observations and statements tending to strengthen and support its original position.[58] The case then went to the Supreme Court of the United States, on appeal a second time, and was argued before that Court on February 10 and February 11, 1938.

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[56] Hinderlider *et al. vs.* La Plata River Cherry Creek Ditch Company, 93 Colorado 128.

[57] Hinderlider *et al. vs.* La Plata River Cherry Creek Ditch Company, 291 U. S. 650, 54 S. Ct. Rept. 557.

Colorado's Constitution dedicates the use of all unappropriated waters in the natural streams of the State to the people, and expressly declares that the water is the property of the public. Under this doctrine, Colorado had always claimed the right to divert the entire flow of any stream rising within its borders, regardless of the claims of sister States and their residents. In the cases of Kansas *vs.* Colorado[59] and Wyoming *vs.* Colorado,[60] the Supreme Court of the United States held that this constitutional provision did not give to Colorado the ownership of all the water flowing within its borders, but expressly announced the doctrine that every State upon an interstate stream is the owner of an equitable portion of the flow of the stream, which the other States are bound to recognize and respect. This doctrine has been announced by the Supreme Court on numerous occasions since that time.

The defendant water-administrative officials took the position that the equitable apportionment which had been effected by the La Plata Compact determined the limit of the water to which Colorado and its citizens were entitled, and that the title of any water users in Colorado was necessarily limited by the rights of New Mexico. In other words, it was contended that no vested right of any Colorado water user was invaded when New Mexico received only the quantity to which it was entitled by the terms of the Compact. Of necessity, water delivered to New Mexico under the terms of the Compact was water which no Colorado appropriator might acquire under a decree from a Colorado Court.

The ditch company contended, and the State Supreme Court took the position, that the entry of a decree in a water-adjudication proceeding by a Colorado Court vested in the claimant a property right which could be taken away only after an action in Court with the payment of just compensation therefor. The United States Supreme Court held that no property of the ditch company had been taken, and no vested right had been violated.

Mr. Justice Brandeis said:

> "As Colorado possessed the right only to an equitable share of the water in the stream, the decree of January 12, 1898, in the Colorado water proceeding did not award to the ditch company any right greater than the equitable share. Hence the apportionment made by the compact cannot have taken from the ditch company any vested right unless there was in the proceedings leading up to the compact or in its application some vitiating infirmity. No such infirmity or illegality has been shown."

In answer to the charge that rotation constituted a taking of a vested right, the water-administrative officials asserted that, in time of scarcity when evaporation and transportation losses are heavy, greater benefits attend the diversion of a larger body of water over a given period than the diversion of half the quantity over twice that period. On this proposition, the U. S. Supreme Court held that the delegation to the State Engineers of the authority to determine when the water should be rotated was a matter of detail clearly within the constitutional power.

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[58] Hinderlider *et al.* *vs.* La Plata River Cherry Creek Ditch Company, 101 Colorado 73.

[59] Kansas *vs.* Colorado, 206 U. S. 100.

[60] Wyoming *vs.* Colorado, 259 U. S. 494.

The Supreme Court re-announced the doctrine that no State may divert all the water of a stream without regard to the injury or prejudice it may work upon the down-stream State. It said also that the compact adapts the age-old treaty-making power of independent sovereign nations to the union of sovereign States, and although the equitable rights of the States might have been determined in an original suit in that Court, the result of the exercise of the compacting power is equally effective.

The Supreme Court held that a division of the water of a stream, whether by compact with the consent of Congress, or by decree of the Supreme Court, is binding upon the citizens of each State. This part of the decision was based upon a number of early decisions in which questions arose over the location of the boundaries between the original States and had been settled by compacts or boundary conventions. Thereafter, when private parties claimed title to the same land under patents from different States, the Supreme Court upheld the boundary conventions and said that the title claimed under a patent from a State which had no title failed because of an intrinsic defect in the title of the State.

An interesting side-light on the legal phases of the question arose from the fact that the water officials had claimed that the Compact, when consented to by an Act of Congress, became a Federal statute, and that under Section 237 (*a*) of the Judicial Code they had the right to take the case to the Supreme Court on appeal. The U. S. Supreme Court denied this claim and dismissed the appeal, but held that the injunction from administering the Compact denied an important claim under the Constitution which may be reviewed by certiorari under Section 237 (*b*). The Court held that jurisdiction over controversies concerning rights in interstate streams is not different from those concerning boundaries, and whether the water of an interstate stream must be apportioned between the two States is a question of Federal common law upon which neither the statutes nor the decisions of either State can be conclusive. Likewise the Court held that it had jurisdiction to determine the validity and effect of the Compact even if New Mexico and Colorado were not parties to the suit and could not be made parties.

The State of Colorado was the pioneer in the movement to settle disputes over irrigation waters between States by the Compact method. The La Plata is a small stream with a comparatively short run-off, but all of the questions involved in the determination of the validity and effect of such a compact were presented in this case. For nearly ten years the matter was litigated in the Courts, while the validity of all of Colorado's interstate compacts hung in the balance.

Now (1939) it has been judicially determined that the States, through commissioners conversant with the problems of irrigation, may divide the waters of western rivers equitably and intelligently, and thus long drawn-out and expensive litigation possibly may be avoided. With the relevant facts at hand, any group of fair-minded men may make a compact that will protect and preserve the interests of the people fairly and adequately.

Interstate Compacts

Settlement of controversies between States, in regard to the use of waters of interstate streams, has been effected in a number of cases, notably in the arid region, by compacts between the States involved. Such compacts are permitted under the Constitution, but must be ratified by the Legislatures of the compacting States and approved by the Congress of the United States before becoming effective.

There appears to be a disagreement between authorities concerning the necessity for the compacting States first to obtain the consent of Congress before entering into negotiations. Many authorities take the position that prior consent of the Congress is not essential when the compact is later approved by Congress. Congress may enforce any conditions it deems desirable for conserving the interests of the Federal Government prior to approving any interstate compact. In each case involving the Western States, the President has designated a representative of the United States to participate in the negotiations between States, and this representative is usually designated by the representatives of the States to act as chairman of the Interstate Commission.

The Weeks Act of March 1, 1911, provides[61]

> "Be it enacted by the Senate and the House of Representatives of the United States of America, in Congress assembled, that the consent of the Congress of the United States is hereby given to each of the several states of the Union to enter into any agreement or compact, not in conflict with any law of the United States, with any other state or states, for the purpose of conserving the forests and water supply of the states entering into such agreement or compact."

Likewise, the Omnibus Flood Control Act of 1936 authorized the States to enter into compacts under certain conditions. At times the Supreme Court of the United States has recommended solution of interstate problems by means of interstate treaties or compacts. However, comparatively few of the States have availed themselves of this very effective method for the settlement of their differences with respect to the waters of an interstate stream. A survey of all of the forty-eight States discloses that those which have no compacts or interstate agreements concerning the waters of an interstate stream system, nor litigation over such waters, are Alabama, Arkansas, Florida, Georgia, Iowa, Maine, Mississippi, Missouri, and Rhode Island. In all the other thirty-nine States, there has been litigation between one or more States.

According to the report of the National Resources Committee of December, 1935, fifty-seven interstate compacts have been authorized, of which thirty-four have finally become effective through State ratification. In addition, thirteen compacts have been authorized by one or more States without Congressional authority or approval. The larger number of these compacts, however, had to do with such matters as State boundaries, cessions of territory, jurisdiction over boundary waters, uniformity of legislation, interstate accounting, utility regulation, and prevention of crime.

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[61] Weeks Act of March 1, 1911, 36 Stat. L. 961.

Of the forty-four compacts to which Congress has given its approval since 1900, thirty have been effected within the period since 1918. In this connection, the Federal Government has not been content with a constitutional rôle of passive approbation, as disclosed by the Weeks Act. Congress may take the rôle of stimulator of interstate compacts in a particular field by a general authorization for the States to enter into such agreements, but even in the case of more specific authorization (as in the statute for the apportionment of the waters of the Columbia River between Washington, Idaho, Oregon, and Montana) Congress has established the principle of Federal participation in the negotiations. There has been no State action as yet (1939) under the provisions of the Weeks Act, and apparently Congress has been willing to approve in advance, and without serious delay, the negotiation of compacts relating to a particular question or region. As heretofore stated, it appears that the participation by Congress may precede or follow the stage of negotiations. There are examples of both types of procedure.

Some students of the compact method, as a device for the adjustment of disputes over interstate streams, have declared what they conceive as uncertainties associated with its use as a means for securing State co-operation in the development of streams, in view of the existing conflicts of law, among the States, relating to water rights. These uncertainties, they contend, are increased because of the fact that there is no clear line that determines the ultimate application of Federal authority over navigable streams, or the regulation of the source of supply of such streams. The question of the authority of the Federal Power Commission to pass upon the rights of States to enter into compacts concerning the use of certain streams in the Eastern States has now been injected into the problem (Connecticut, Merrimac, and Delaware River Compacts).

Colorado River Compact.—This Compact, providing for the equitable division of the waters of the Colorado River system in the United States, was signed by the commissioners representing the States of Arizona, California, Colorado, Nevada, New Mexico, Utah, and Wyoming at Santa Fe, N. Mex., on November 24, 1922, and was thereafter ratified by Wyoming on February 25, 1925, by Colorado on February 26, 1925, by Utah on March 13, 1925, by New Mexico on March 17, 1925, by Nevada on March 18, 1925, and by California on April 8, 1925, and promulgated by the President on July 25, 1925. Arizona has declined to ratify the Compact to date (1939) or, by self-limitation through legislative action, to subordinate itself to the provisions of the Compact, and has asserted that the assured and planned projects in California would absorb all of the additional water made available to the Lower Basin States by the Compact.

The Compact apportions in perpetuity, from the Colorado River System to the Upper Basin and to the Lower Basin, respectively, the exclusive beneficial consumptive use of 7 500 000 acre-ft of water per yr. The dividing line between the basins is at Lee Ferry, Ariz., 1 mile below the mouth of the Paria River. An additional 1 000 000 acre-ft per yr are apportioned to the Lower Basin for beneficial consumptive use when the first 7 500 000 acre-ft have been consumed.

If the United States should hereafter recognize in Mexico, as a matter of international comity, any right to the use of the waters of the Colorado River system, such waters shall be supplied from surplus waters over and above the aforementioned apportionments. If such surplus waters are insufficient for this purpose, the deficiency shall be borne equally by the Upper and Lower Basins.

The Compact provides that the States of the Upper Division (Colorado, New Mexico, Utah, and Wyoming—not identical with "Upper Basin") will not cause the flow of the river at Lee Ferry to be depleted below an aggregate of 75 000 000 acre-ft for any period of ten consecutive years reckoned in continuing progressive series. Waters not thus apportioned may be apportioned equitably for beneficial use any time after October 1, 1963.

The ratification of this Compact was followed by the passage in 1928 of the Boulder Canyon Project Act which provided for the construction of the Hoover Dam and the All-American Canal. It also requires California to limit, by statute, her portion of the beneficial consumptive use of the 7 500 000 acre-ft, apportioned to the Lower Basin, to 4 400 000 acre-ft. This was done later by legislative enactment.[62]

The Compact makes the use of water throughout the Colorado River Basin for the generation of power subservient to that for domestic and agricultural purposes.

Further comment concerning litigation between Arizona and California, and the other States of the Colorado River Basin, relating to the fulfilling of the provisions of the Boulder Canyon Project Act, has been made in greater detail herein under the heading "Interstate Litigation."

La Plata River Compact.—This Compact between Colorado and New Mexico, dividing the waters of the La Plata River in southwestern Colorado and northwestern New Mexico, was signed at Santa Fe by the commissioners representing the two States on November 27, 1922, following which Colorado ratified the Compact on April 13, 1923, and New Mexico on February 7, 1923. The Compact was approved by the Congress of the United States on January 25, 1925, and promulgated by the President on January 29, 1925.

In the negotiation of this Compact, no regard was given to the relative dates of priority and decreed quantities of water in the two States, the constitutions of which specifically adopt the doctrine of appropriation rather than the doctrine of riparian ownership, the attempt and intention being to apportion the waters of the La Plata River equitably between the two States. It so happens that practically all of the firm water produced by the river during the critical period of the irrigation season is claimed under Court decrees in New Mexico. Due to their relative geographical locations on the stream, later appropriations of water in Colorado exhausted practically the entire dependable stream flow, which resulted in a threat on the part of New Mexico to ask for relief against Colorado in the Supreme Court of the United States, which was the basis for the consummation of the Compact.

The Compact provides (*a*) that at all times between the first day of December and the fifteenth day of the succeeding February, each State shall have the unrestricted right to the use of all the waters originating in such State, or flowing within its borders, (*b*) that at all other times each State shall have the unrestricted right to the use of all the waters within its boundaries on each day when the daily flow at the Colorado-New Mexico State line is 100 cu ft per sec or more, (*c*) that on all other days Colorado shall deliver at the Colorado-New Mexico State line, for beneficial consumptive uses in New Mexico upon demand of New Mexico, a quantity of water equivalent to one-half of the mean daily flow on the preceding day at the Hesperus gaging station, situated about 33 miles above the interstate line, but not to exceed 100 cu ft per sec.

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[62] Assembly Bill No. 1070, California Legislature, passed March 4, 1929.

From the foregoing, it will be noted that Colorado must bear the burden of the loss of water in transit when delivering one-half of the flow of the river down to the State line. This requirement has proved to be a serious burden at times since the initiation of the administration of the Compact in 1925. The Compact also provides that the total quantity of water in the river may be rotated between the States for predetermined periods of time by the State Engineers of the two States for the more efficient use of the common supply. This practice has been in effect each year during periods of low stream flow since 1924. As a result of this practice of rotation, the La Plata River and Cherry Creek Ditch, with priority No. 6 on the La Plata River in Colorado, sought to enjoin the water officials of Colorado in making such rotations, which had the effect of depriving that ditch of all water during such periods of rotation, alleging that such action on the part of the water officials was unconstitutional, since it had the effect of taking away property rights without due process of law. This suit (discussed at greater length under the subject of "Interstate River Litigation") resulted in the noted case of Hinderlider *vs.* La Plata River and Cherry Creek Ditch Company, just decided April 25, 1938, by the Supreme Court of the United States.

South Platte River Compact.—This Compact between Colorado and Nebraska was signed by the commissioners representing the two States on April 27, 1923. Thereafter it was ratified by Nebraska on May 23, 1923, and by Colorado on February 26, 1925, and was approved by the Congress of the United States on March 8, 1926, and promulgated by the President on March 9, 1926.

This Compact provides that, between the first day of April and the fifteenth day of October of each year, Colorado shall have the use of the entire flow of the South Platte River, with the provision for full recognition of the right of the Western Ditch, which heads on the South Platte River 4 miles east of the Colorado-Nebraska line, to divert as much as 120 cu ft of water per sec under its date of priority of June 14, 1897.

The Compact also provides that at all times between the 15th day of October and the 1st day of the succeeding April, Colorado shall have the full and uninterrupted use of the water of the river flowing within the borders of the State, provided, however, that Nebraska and its agents shall have the right to construct a canal, heading on the river near Ovid, Colo., with the recognized date of priority as of December 17, 1921, to divert as much as 500 cu ft per sec during the period of October 15th to April 1st, for the irrigation of lands in Nebraska. This provision is subject, however, to a prior reservation of 35 000 acre-ft of water per yr for future uses in Water District No. 64, which embraces the lower section of the river in Colorado.

The Compact also covers several minor features, such as the use of the waters of Lodgepole Creek (a small tributary of the South Platte River which rises in southern Nebraska and flows into Colorado), together with matters of administration, which are placed in the hands of the State Engineers of the two States. This Compact does not change the former administration of the waters of the South Platte River in Colorado, but does remove the shadow of future claims by the State of Nebraska against existing uses of water from the South Platte River in Colorado, and possible litigation in this connection.

This Compact has been administered since the Spring of 1926, under rules promulgated by the State Engineers of the two States, without the least friction, and may be considered to be one of the outstanding examples of this character on record.

It may be of interest, in passing, to note that, in an interstate suit between Nebraska and Wyoming [17] over the waters of the North Platte River (in which suit Colorado is made an impleaded defendant), the Attorney General of Nebraska has acknowledged the validity of the South Platte River Compact between Colorado and Nebraska, and that Nebraska fully intends to maintain its obligations faithfully under the Compact. Although it does not attack the validity of the Compact, Wyoming has expressed the view that, had Nebraska not entered into the Compact with Colorado (by the terms of which Nebraska foregoes the right to the use of a larger share of the waters of the South Platte River than that to which she may have been entitled), any burden which may fall upon Wyoming as a result of an adverse decision in the pending North Platte River suit would be of less magnitude than it would have been except for the South Platte River Compact between Colorado and Nebraska.

Rio Grande Compact.—The States of Colorado, New Mexico, and Texas have been conducting negotiations since before 1929 with a view to adjusting their respective claims on the waters of the Rio Grande above Fort Quitman. This interstate problem has "hung like a pall" over the water users of the upper Rio Grande basin in the three States for more than 40 yr. As a result of previous negotiations, the three States entered into a temporary Compact designed as a truce to maintain the *status quo* on the upper river on February 12, 1929. This Compact was ratified by Colorado on April 9, 1929, by New Mexico on March 9, 1929, and by Texas on May 2, 1929. Thereafter it was approved by the Congress of the United States on June 1, 1930, and promulgated by the President on June 17, 1930.

This temporary Compact, which covered a period of 5 yr, provided for the equitable apportionment, among the three States, of the waters of the Rio Grande above Fort Quitman. It also declared that, since the United States Government had disposed of 60 000 acre-ft of water per yr in perpetuity to the Republic of Mexico, in the settlement of claims preferred by that nation against the United States, it follows as a corollary that the United States should defray the cost of the discharge of any obligation thus assumed; as, for example, the repayment to the three States for the loss of water taken from these States in settlement of the Federal Government's assumed obligation to the Republic of Mexico. For the purpose of meeting this obligation, at least in part, and for the economic development and conservation of the waters of the Rio Grande Basin, and to aid in the settlement of the interstate controversy, the States agreed in this temporary compact that: (*a*) It is of primary importance that the area in Colorado, known as the Closed Basin, be drained; (*b*) that the water thus recovered be added to the flow of the river; and that, in this connection, a reservoir for equating the flow be constructed near the Colorado-New Mexico State line—all at the expense of the Federal Government.

The temporary Compact also provided that, until the construction of the aforementioned Closed Basin Drain and the State line reservoir, but not subsequent to June 1, 1935, the date fixed for the expiration of the Compact, Colorado would not cause, or permit, the water supply at the Colorado-New Mexico State line to be impaired by new or increased diversions or storage within the State of Colorado, unless and until such depletion was offset by increased drainage return.

The temporary Compact also provided for the maintenance of the then *status quo* on the Rio Grande in the three States above Fort Quitman, and that no development in either State during the life of the Compact should have the effect of establishing any claim upon the river or its tributaries as against the rights, or to the prejudice, of any of the signatory States.

Negotiations were continued from time to time during the 5-yr life of this temporary Compact, looking to the consummation of a permanent Compact, without tangible results. The Legislatures of the three States, by unanimous agreement, extended the life of the Compact to June 1, 1937, and again until October 1, 1937, when the temporary Compact was allowed to expire. Prior to the latter date, however, the Commissioners duly appointed by the Governors of the three States (for Colorado, M. C. Hinderlider, M. Am. Soc. C. E.; for New Mexico, Thomas M. McClure, M. Am. Soc. C. E.; and for Texas, Mr. Frank B. Clayton), and a representative of the United States (S. O. Harper, M. Am. Soc. C. E.), continued with their negotiations for a permanent compact. On September 23, 1936, however, the President, by executive order, instructed the appropriate Federal agencies "* * * not to approve any application for a project involving the use of Rio Grande waters without securing from the National Resources Committee a prompt opinion on it from all relevant points of view." This executive order was based upon the assumption that further developments in the Rio Grande Basin above Fort Quitman might impair existing Federal investments, and also violate the terms of the temporary Rio Grande Compact, to which the Federal Government was a party. Following the issuance of the foregoing order, the three Interstate River Commissioners entered into an agreement with the National Resources Committee for a comprehensive and detailed survey of the land and water resources of the Upper Rio Grande Basin, present uses, deficiencies, surpluses, and future requirements, together with a determination of the feasibility of supplementing existing water resources of the basin through the importation of waters from the Colorado River Basin.

The investigation was conducted by the Water Resources Committee under the general supervision of the National Resources Committee, and under the immediate supervision of Harlan H. Barrows and Prof. Frank Adams, representing the Water Resources Committee, and N. C. Grover, M. Am. Soc. C. E., Chief Engineer of the United States Geological Survey, representing that Department. Separate phases of this investigation were allocated to the United States Bureau of Agricultural Engineering, the United States Geological Survey, United States Bureau of Reclamation, and United States Bureau of Plant Industry, the three States co-operating through their Interstate River Commissioners and State Engineers, and through financial assistance to the sum of $55 000. The major cost of the investigation, amounting to about $400 000, was borne by the National Resources Committee and various agencies of the Federal Government. This investigation, it is believed, was the most complete ever undertaken by the Federal Government in connection with an interstate stream. The investigation covered a period of about 1½ yr, and the results have been compiled in two volumes by the National Resources Committee.[63]

Following the conclusions of this investigation in the Summer of 1937, the Interstate River Commissioners and their advisers, after six different conferences at Santa Fe, finally concluded and signed, on March 18, 1938, a permanent Compact which divides the waters of the Rio Grande, equitably, above Fort Quitman, provides for their better regulation and conservation, and for the importation of waters to the basin from other drainage areas, and also for the recovery of waters from the Closed Basin area in Colorado, which are not now tributary to the Rio Grande. It is believed that this document —consummated after more than ten years of strenuous endeavor, under the most difficult conditions resulting from long and bitter opposition between the three States, and as a result of an international treaty and previous huge Federal investments in the lower basin—is an outstanding example of the ability of States and the Federal Government to adjust the most complicated problems arising out of the use of the waters of an interstate stream.

Briefly, the Rio Grande Compact accomplishes two major purposes:

(*a*) It protects the present uses of water in the three sections of the Rio Grande in Colorado, New Mexico, and Texas by setting up schedules of required delivery of water by Colorado at the Colorado-New Mexico State line, and by New Mexico at San Marcial, which is just above the Elephant Butte Reservoir, and by fixing the average annual releases from the Elephant Butte Reservoir for the irrigation of lands below that reservoir, and for use in Mexico, under the provisions of the international treaty.

(*b*) It permits the construction and operation of additional reservoirs in the basin above Elephant Butte Reservoir, for the purpose of regulating stream flow used at the present time (1939), and for capturing and making usable the water in the Upper Rio Grande Basin, which otherwise would spill from the Elephant Butte Reservoir and be lost.

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[63] Regional Planning, Part VI, Upper Rio Grande, February, 1938, National Resources Committee.

The schedules of water deliveries are based upon the relation found to exist between the annual inflow into, and the outflow from, the San Luis Valley in Colorado for the years 1928 to 1937, both inclusive, and the relation between the flow at Otowi Bridge in Northern New Mexico and the flow at San Marcial for all years of record prior to 1930 (pre-middle Rio Grande Conservancy District period).

The Colorado-New Mexico State-line schedule of water deliveries is divided into two parts, one applying to the Conejos stream system, which is the largest tributary to the Rio Grande in Colorado, and the other to the Rio Grande proper above the Colorado-New Mexico State line. Required deliveries by the State of Colorado are taken as the sum of the required deliveries by the Conejos River System and by the Rio Grande proper, less 10 000 acre-ft annually. This schedule of deliveries will permit the use of water in Colorado which has been in force since the early Eighties, and will also permit the construction of large regulatory reservoirs in Colorado to make the seasonal stream flow more nearly parallel to the seasonal crop requirements and the elimination of distortion which has existed between these two natural factors in past years. Deliveries of water in exact accordance with the schedules are not required on an annual basis.

The Compact provides for a system of accounting whereby deviations from the deliveries required by the schedules are set up as debits or credits to the State making the deliveries. Such credits and debits are allowed to accumulate, subject to certain conditions. In the case of Colorado, departures of 100 000 acre-ft from the scheduled deliveries on the debit side in any one year, or an accumulated departure in the same amount, due to natural causes, are allowable. Deliveries of water in excess of the required scheduled deliveries are used to reduce debits and, in the absence of debits, are allowed to accumulate as credits. The credits may be reduced either by spill from Elephant Butte Reservoir, above El Paso, or by departures on the debit side in the delivery of water from the required scheduled deliveries. The Compact provides that debits may accumulate in any amount, so long as an equivalent quantity of water is held in storage in reservoirs to be constructed in New Mexico and Colorado in the future, provided that any portion of that water which otherwise would have spilled from Elephant Butte Reservoir becomes the property of the owners of the new reservoirs, and may be used in any way they see fit.

Deliveries of water by Colorado and New Mexico in excess of the quantities required by the schedule are accumulated in the Elephant Butte Reservoir as credits to the upper States, so long as the water remains in storage in Elephant Butte Reservoir. Should the stage of storage in Elephant Butte Reservoir cause an unusable spill of water from that reservoir, the water which is in storage to the credit of the upper States is reduced by the amount of such spill. In order that the area above Elephant Butte Reservoir shall have practically exclusive right to the consumption of the water which would otherwise spill from Elephant Butte Reservoir, the average annual release from that reservoir for beneficial, consumptive use is fixed at 790 000 acre-ft, which includes the 60 000 acre-ft required to be delivered to Mexico at El Paso under the existing international treaty. For the same reason, project storage in that reservoir is fixed by the Compact at 2 638 860 acre-ft, which was the original capacity of Elephant Butte Reservoir.

The Compact places the administration in the hands of the Rio Grande Compact Commission, consisting of the State Engineers of Colorado and New Mexico, and a representative to be appointed by the Governor of Texas. This Commission is authorized and required to maintain a number of gaging stations along the river system, and to compile and exchange the records obtained therefrom. Within certain limitations, the Commissioners, by unanimous action, may make minor modifications in the provisions of the Compact which later years may show to be desirable. Such modifications, however, must be made within the fundamental provisions of the Compact; and they become operative only when unanimous action has been ratified by the Legislatures of the States and consented to by the Congress of the United States. It was the intent of the drafters of the Compact that the actual administration of the waters of the river, to which each State is entitled, should be administered by the water officials of such State without the interference of a super-authority. The Compact also makes provisions for a representative of the United States Government on the Commission. This representative, however, will be without voting authority.

The Compact has been ratified by the Legislatures of the three States and by the Congress and is now (1939) in full force and effect.

Red River Compact Between Minnesota, North Dakota, and South Dakota (Signed June 23, 1937).—In the valley of the Red River of the North, a critical situation is being met through interstate co-operation between the States of Minnesota, North Dakota, and South Dakota. The problem of the Red River of the North is primarily a question of water. In the period from 1916 to 1919, the problem was one of floods, and from 1930 to 1935, one of drought, which threatened the water supplies of cities and farms. The successful meeting of these problems required interstate co-operation and Federal assistance. This assistance has been in the nature of a complete study of the problems by the Water Resources Committee of the National Resources Committee, in co-operation with the United States Corps of Engineers under a co-operative agreement between the Federal agencies and the Planning Boards of the respective States. The major problem was the conservation of water in lakes and reservoirs, and its release to stream channels which will provide a dependable water supply for the principal cities and other users. The plans include: (*a*) An increase in the effective storage in the lakes of the Otter Tail River System; (*b*) plans for a somewhat similar improvement in the use of Red Lake storage; and (*c*) plans for the construction of a large dam on the Cheyenne River and possibly dams on other streams on the Dakota side of the basin. In this connection, small dams are also proposed on various normally dry channels for the impoundment of stock water for rural needs. Another important phase of the plan is the improvement of stream channels for reducing losses of water in transit, which are now reported to be extremely high in this basin. The Compact provides for an interstate authority, with power to control the water resources of the area, and to levy limited assessments on the beneficiaries. However, the Minnesota Act that ratifies the Compact, excludes the Otter Tail Basin from the provisions of the Compact. The comprehensive report of the investigations leading up to the consummation of this Compact is included in the report of the National Resources Committee of August, 1937, on "Regional Planning."[64]

NEGOTIATIONS FOR INTERSTATE RIVER COMPACTS

Columbia River.—Negotiations concerning the use of the waters of the Columbia River were conducted for a number of years between the States of Montana, Oregon, and Washington, but it is understood that nothing tangible has resulted from such negotiations.

Yellowstone River.—Under authorization of the Congress, dated June 14, 1932, a Compact for the equitable apportionment of the waters of this river was signed by the representatives of Montana and Wyoming on February 6, 1935, which, however, was not acted upon by the Legislatures of the two States within the time limit fixed by Congress. It has been reported that the terms of this Compact would have been opposed by the United States Indian Service as a result of certain treaties with Indians involving the question of priority of use of the waters of this stream. Congress again authorized these two States to attempt to agree upon a compact by the Act of August 2, 1937.[65]

Control of the South Canadian River.—This Compact was signed on December 31, 1926, by the Commissioners representing the States of New Mexico, Oklahoma, and Texas. It was approved by New Mexico and Oklahoma, but never approved by Texas. By Act of the Legislature, New Mexico later withdrew its approval. The object of the Compact was to further the construction of the Conchas Flood Control Reservoir.

Congress has also authorized Compacts between New Mexico, Oklahoma, and Texas concerning the waters of the Canadian, Cimarron, and Rio Grande rivers. Under such authorization representatives of the three States agreed upon a Compact[66] concerning the waters of the Canadian River which was later ratified by the Legislatures of the States, but was vetoed by the Governor of Texas.

Congress also authorized a Compact between New Mexico and Oklahoma, covering the waters of the Cimarron River and other streams;[67] and Compacts between Colorado, Oklahoma, and Kansas, concerning the waters of the Arkansas River and other streams.[68] It is understood that controversial questions exist between Oklahoma and New Mexico, concerning the waters of the South Canadian and Cimarron rivers. Another interstate question in which the State of Oklahoma is interested concerns pollution from salt brines in the Kansas oil fields. It is understood that producers are co-operating in methods for correcting this trouble by returning brine extracted from oil wells back to salt-water strata outside the producing area.

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[64] Regional Planning, Part V, "Red River of the North."

[65] Publication L. No. 237.

[66] Compact between New Mexico, Oklahoma, and Texas, 45 Stat. 1502.

[67] *Loc. cit.* 45 Stat. 1503.

[68] *Loc. cit.* 45 Stat. 1517.

Problems of an interstate nature between Oklahoma, Arkansas, and Louisiana concern retardation measures for the prevention of the flooding of both the Arkansas and Red rivers. Similar interstate problems between Texas and Oklahoma concern the flood waters of the Red River, which originates in Oklahoma. It appears that the solution of this problem may rest upon the construction of the large Denison Flood Control Reservoir on the Red River.

Compact to Divide Waters of the Snake River.—Congress authorized[69] the States of Idaho and Wyoming to enter into a Compact not later than January 1, 1934, providing for an equitable division and apportionment, between these States, of the waters of the Snake River and its tributaries. Pursuant to such authorization, the two States and the Federal Government appointed Commissioners who agreed upon a Compact. It was approved by the Legislature of Idaho in 1933, and later by the Lower House of the Legislature of Wyoming; but it failed to be approved in the Senate of that State, and later was withdrawn by its sponsors.

Authorization was given to these States by Congress,[70] and approved by the President May 16, 1938, to negotiate a new compact not later than January 1, 1940. Commissioners have not been appointed to date (1939).

Compact for Flood Control in the Connecticut and Merrimac River Basins.—Pursuant to the provisions of the Flood Control Act of 1936, establishing a national policy with respect to flood control throughout the United States, the States of Connecticut, Massachusetts, Vermont, and New Hampshire negotiated a Compact for the control of flood waters in the Connecticut River Basin, and the States of Massachusetts and New Hampshire negotiated a Compact for the control of floods in the Merrimac River Basin. Under the provisions of the Omnibus Flood Control Act of 1936, the Connecticut River Compact contemplates the construction, over a period of years, of a series of flood-control reservoirs upon tributaries of the Connecticut River to control approximately 21% of the drainage area. Of these, two could be constructed in Massachusetts, three in Vermont, and three in New Hampshire, at specified sites. The Compact provides for the creation of the Connecticut River Valley Flood Control Commission, consisting of twelve Commissioners, three to be chosen from each State. It is understood that the four States involved have made appropriations of funds required by the Compact for the States' obligations. Furthermore, the Compact had a favorable report by the Senate Committee on Commerce, by the United States Senate and by the Flood Control Committee of the House; but thus far (1939) it has failed to receive the approval of the Congress. Opposition to the approval of these Compacts by the Congress is reported to have arisen from the Federal Power Commission, on the grounds that the Compact violates the long-established policy of the Federal Government regarding the development of hydro-electric energy, and that the Attorney-General of the United States has taken the position that the Compact was defective in that it did not provide for the transfer of title to the Federal Government of all lands and easements purchased by the States for flood-control purposes.

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[69] *Public Resolution No. 36*, 72d Cong.; and Senate Joint Resolution 148.

[70] *Public Resolution No. 94*, 75th Cong., 3rd Session; also House Joint Resolution No. 150.

The position of the Federal Power Commission has been expressed[71] by its general counsel, Oswald Ryan, who states that the Commission's position can be properly evaluated only when examined in the light of the legal questions which the Compacts have raised. In this connection, Mr. Ryan states that the purpose of his paper[71] "is to discuss not the questions of policy, but the questions of law which must inevitably be faced in considering not only the Connecticut and Merrimac Compacts, but all other flood control compacts which may hereafter be proposed in connection with the program of flood control, authorized by the Flood Control Act of 1936." The position of the attorney for the Federal Power Commission has been resisted strenuously by the Governors and Attorney-Generals of the compacting States.

Compact Covering the Waters of the Delaware River Between the States of New York, Pennsylvania, and New Jersey.—Interstate Compacts between the States of New York, Pennsylvania, and New Jersey were negotiated in 1925 and 1927, but these Compacts were ratified only by the Legislature of the State of New York. The failure of ratification resulted in the litigation between New Jersey and New York (in which the State of Pennsylvania intervened for the protection of its equities in that stream), in the Supreme Court of the United States, which action is discussed herein under "Interstate Litigation."

Interstate Negotiations Concerning Pollution of the Delaware River.—The States of Delaware, New Jersey, New York, and Pennsylvania have heretofore created an Interstate Commission for the consideration of the control of pollution of the waters of the Delaware River. Similar conferences have been conducted by representatives of the States of New York, Pennsylvania, West Virginia, Ohio, Kentucky, Indiana, Illinois, and Tennessee, looking to the consummation of a compact for the control of pollution of the Ohio River.

Compact for Potomac River.—Pursuant to Public Resolution 74, Seventy-fifth Congress, the States of Pennsylvania, Maryland, Virginia, and West Virginia, and the District of Columbia are authorized to enter into a compact for the abatement of the pollution of the Potomac River. For this purpose, the four States and the District of Columbia have organized the Potomac Valley Pollution Conference, the purpose being to draft a compact to cover the problems involved.

Compact Between the States of Ohio and Pennsylvania Concerning the Waters of Pymatuning Lake.—This Compact relates to the development, use, and regulation of Pymatuning Lake, and the State-owned land surrounding it, for fishing, hunting, recreational, and park purposes. It was ratified by the Legislature of Ohio on May 18, 1937; by Pennsylvania on June 5, 1937; and approved by the Congress of the United States on August 28, 1937.

Compact Between the States of New York, New Jersey, and Connecticut Concerning Abatement of Pollution of Coastal and Tidal Waters.—On August 27, 1935, the Congress approved a Compact between the States of New York, New Jersey, and Connecticut, covering control and abatement of pollution in the harbor, coastal, and tidal waters of these States.

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[71] *Public Utilities Fortnightly*, January 20, 1938.

Negotiations for Compact Between Colorado, Wyoming, and Nebraska, Covering the Waters of the North Platte River.—Negotiations between representatives of these three States, looking to a compact on the North Platte River, which rises in Colorado and flows thence into Wyoming and Nebraska, have been conducted for many years. It is probable that the States of Colorado and Wyoming could have reached an agreement concerning their relative equities in the waters of this stream, in so far as such uses in the North Platte Basin in Colorado and a nominal trans-mountain diversion of water out of the North Platte River into the Basin of the South Platte River in Colorado were concerned. Colorado, however, insisted upon a larger trans-mountain diversion out of the North Platte Basin than Wyoming would concede. As a result, negotiations between the two States were ended abruptly on December 4, 1933. Since that time little attempt has been made to reach an understanding.

Throughout these negotiations, Nebraska took the position that it could not negotiate a compact which might adversely affect uses of water in that State. As a result of failure to reach an agreement, Nebraska filed its original bill of complaint against the State of Wyoming in December, 1935, in which Colorado was later made an impleaded defendant (see heading "Interstate Litigation"[17]).

Negotiations Between Colorado and Nebraska, Covering the Waters of the North and South Forks of the Republican and the Arikaree Rivers.—Negotiations between Commissioners representing the States of Colorado and Nebraska have been conducted intermittently since 1934, with a view to a compact covering the waters of the aforementioned streams, which head in Colorado and flow into Nebraska. It appeared at one time that an agreement would be reached; but, upon advice of the Attorney-General of Nebraska that the Commissioner for that State could not enter into a compact, the terms of which would invalidate or limit any decreed, existing, appropriation of water in Nebraska, that State declined to continue the negotiations, and nothing further has been accomplished. Nebraska also took the position that the State of Kansas (through the northeast corner of which the South Fork of the Republican River flows) was a necessary party to any interstate agreement. The Commissioner for Kansas advised that he was agreeable to participating in negotiations, but, due to the afore-assigned reasons, the negotiations were allowed to lapse. It is thought that the decision of the Supreme Court, in the case of Hinderlider *vs.* La Plata River and Cherry Creek Ditch Company,[28] which recognizes the authority of States to negotiate interstate compacts irrespective of their effect upon private vested rights, removes any valid objection heretofore raised by Nebraska in this connection.

Negotiations Between Colorado and Wyoming Concerning the Waters of the Little Snake River.—Authorization has been given the State Engineers of Colorado and Wyoming to enter into negotiations concerning the division of the waters of the Little Snake River, which is an interstate stream common to Colorado and Wyoming. This stream which rises in Colorado crosses the Wyoming-Colorado State line four times. Considerable irrigation development has occurred along this stream, and decrees and permits have been issued for the use of water in both States. Ditches decreed in Colorado irrigate lands both in Colorado and Wyoming, whereas ditches decreed in Wyoming likewise irrigate lands in both States. As a result of this condition, rather important questions of administration have arisen between the administrative officials of the two States. The laws of Colorado and Wyoming prohibit the diversion of water arising in either State, for use in another State, which makes the administration of these interstate ditches a subject of interstate dispute. Thus far (1939), however, no active negotiations for a compact have been entered into. Field surveys and studies of land and water resources in the two States needful for a full understanding of the needs of both States are now being made by the U. S. Bureau of Reclamation. Further negotiations for a compact suspended pending results of investigations by the Bureau.

Negotiations for an Interstate Compact Between Colorado, New Mexico, Utah, and Wyoming, Covering the Waters of the Colorado River.—Soon after the approval of the Boulder Canyon Project Act by the Congress, a conference of the representatives of the four upper States of the Colorado River Basin was convened to consider the desirability of a four-State compact to divide between these States the 7 500 000 acre-ft of water allocated to the upper basin States by the compact. This conference disclosed that the representatives of Colorado, New Mexico, and Wyoming were ready to proceed with negotiations looking to a four-State compact. Utah, however, declined to undertake negotiations pending the completion of the comprehensive survey and study of the land resources of the four upper-basin States of the Colorado River Basin, which the Secretary of the Interior, under the provisions of the Boulder Canyon Project Act, was directed to make. These investigations have been in progress since 1929, but will not be completed until possibly 1942.

Further negotiations for a four-State compact are not contemplated at this time (1939).

Respectfully submitted,

H. K. Barrows
C. J. Bartholet
C. S. Clark
Edward Hyatt
George S. Knapp
G. E. P. Smith
J. C. Stevens
R. J. Tipton*
Abel Wolman
M. C. Hinderlider, *Chairman*

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* Was unable to participate in preparation of the final report.

MEMOIRS OF DECEASED MEMBERS

HARRISON PRESCOTT EDDY, Past-President, Am. Soc. C. E.[1]

Died June 15, 1937

Harrison Prescott Eddy was born in Millbury, Mass., on April 29, 1870, the son of William Justus and Martha Augusta (Prescott) Eddy. He was a descendant of John Eddy, born in March, 1597, in Boxted, Suffolk County, England, who came to Plymouth, Mass., in 1630.

Mr. Eddy obtained his early education in the public schools of Worcester, Mass. In 1891 he was graduated from the Worcester Polytechnic Institute with the degree of Bachelor of Science in Chemistry.

Finding immediate employment in the Sewer Department of the City of Worcester, Mr. Eddy served for one year in charge of the disposal works, and, thereafter, for fifteen years as Superintendent of the Department. During this period he constructed by day labor approximately one hundred miles of sewers and drains, enlarged the chemical treatment plant, installed sludge-drying equipment, and added forty-three acres of intermittent sand filters to the disposal system. As Superintendent, he also had charge of all departmental activities, including the investigation of sewage treatment processes and of stream pollution, in which he was fortunate in a stimulating association with the late Leonard P. Kinnicutt, an internationally recognized authority in sanitary chemistry. Thus, coincidentally with the classic experiments in sewage treatment of the Massachusetts State Board of Health at Lawrence, Mass., Mr. Eddy obtained in the operation of the Worcester plant an unusually valuable experience in the early development in the United States of the art of sewage disposal.

In 1906, as a result of his work in Worcester, Mr. Eddy was recommended by the late Samuel M. Gray, M. Am. Soc. C. E., as a member of a board of engineers to develop a new system of sewers for Louisville, Ky. This was Mr. Eddy's first engagement as a Consulting Engineer on an important project.

In 1907, he formed a partnership with the late Leonard Metcalf, M. Am. Soc. C. E., under the firm name of Metcalf and Eddy, which continued until the death of Mr. Metcalf early in 1926, when a new partnership which included a number of previously associated partners and assistant engineers was formed under the original name. The association of Harrison Eddy and Leonard Metcalf was most fortunate. Both foresaw an opportunity for a broad-scale, liberally planned practice; both recognized an obligation to do their parts in the advancement of their chosen profession by active participation in the work of various technical societies. Since its organization the firm of Metcalf and Eddy has been accorded a foremost place in its field of work.

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[1] Memoir prepared by Frank A. Barbour and J. H. Kimball, Members, Am. Soc. C. E.

During his thirty-one years of practice as a Consultant, Mr. Eddy rendered service to more than 125 cities and towns in problems of water supply, water purification, sewerage, sewage disposal, industrial waste treatment, and municipal refuse disposal. Reference can here be made only to a few of these engagements.

In 1906, he prepared the general plan for the improved sewerage of Louisville; in 1910, he designed and supervised the construction of one of the earlier Imhoff tank-trickling filter plants at Fitchburg, Mass.; in 1911, he investigated for the Metropolitan Sewerage Board of New York City the effect of the discharge of sewage into local waters; in 1912, he reported to the City of Cincinnati, Ohio, on the pollution of the Ohio River in relation to its use for water supply. He prepared general plans of sewerage and detailed plans of sewage treatment plants for Akron and Dayton, Ohio, and for Allentown, Pa. In 1918, he investigated and reported to the City of Cleveland, Ohio, on sewage disposal. As a member of a Board of Engineers he participated in 1920 in a general plan for interception and sewage treatment at Milwaukee, Wis.; in 1924, he was a member of the Chicago Engineering Board of Review appointed to investigate and report on the lake lowering controversy, including a study of water supply and purification for the City of Chicago, Ill.; in 1925, he was a member of the Board which developed for the City of Detroit, Mich., a general plan of sewerage interception and sewage treatment; from 1926 to 1928, he was a member of a Board which reported on additional water supply for the City of New York; from 1934 to 1936, he participated in the investigation of sewage disposal for the District of Columbia and prepared construction plans for the treatment works recommended; in 1935, for the Metropolitan District of Southern California, he investigated the problem of softening the Colorado River supply and made general plans of the plant recommended.

Among the other municipalities to which Mr. Eddy rendered service were included: New Orleans, La.; Pittsburgh, Pa.; San Francisco, Calif.; Portland, Ore.; San Antonio, Tex.; Boston, Mass.; and Toronto, Ont., Canada. He was a member of the Technical Board of Review appointed by the Federal Public Works Administration in 1935 to pass upon public works projects, also of a Board of Consulting Engineers appointed in 1936 by the Tennessee Valley Authority to advise in problems of flood relief in the Tennessee River Valley. At the time of his death, Mr. Eddy was serving as one of the Consulting Engineers to the New York Board of Water Supply on the Delaware River project and also as a Consultant to the Water Resources Service of the Department of Forests and Waters of the State of Pennsylvania, relative to the water supply of Pittsburgh, Harrisburg, and Philadelphia, Pa. Briefly summarized, Mr. Eddy's engagements may be said to have involved participation in a large proportion of the major water supply and sewerage problems which have required investigation during the three decades of his private practice.

It is significant that although he was trained as a chemist—a line of study that was largely theoretical—he turned his attention shortly to the very practical details of supervising a large amount of construction. To this combination those who knew him well attributed his keen capacity for solving technical problems having various difficult phases, while at the same time keeping a very sound view of the practical features involved. His early experience as an executive in the management of a city department was reflected in the promptness and dispatch with which he disposed of matters before him.

As a speaker, Mr. Eddy was convincing; and his unusual gift for simple, logical presentation of his thoughts made him a valuable witness in numerous important litigations, among which may be mentioned the inter-State cases of New York *vs.* New Jersey, relative to the proposed discharge of Passaic Valley sewage into New York Harbor; Wisconsin *vs.* Illinois and the Sanitary District of Chicago; New Jersey *vs.* New York relative to the taking of the Delaware River for additional water supply for the City of New York; Connecticut *vs.* Massachusetts relative to the taking of water from the Ware and Swift Rivers for the additional supply to the Boston Metropolitan District.

Mr. Eddy contributed numerous valuable papers to various technical organizations. He was joint author of the generally accepted, standard, three-volume treatise on "American Sewerage Practice," and of the textbook entitled "Sewerage and Sewage Disposal." In 1920, the Boston Society of Civil Engineers awarded him the Desmond Fitzgerald Medal for his paper[2] on "Maximum Rates of Precipitation at Boston for Various Frequencies of Occurrence." In 1925 he was awarded the Norman Medal of the American Society of Civil Engineers, for his paper[3] on "Imhoff Tanks—Reasons for Differences in Behavior"; and in 1927, the Rudolph Hering Medal was given to him by the Sanitary Engineering Division of the Society for this same paper. All his published material is characterized by careful, time-consuming development of the controlling facts.

He was a member of numerous technical organizations, including the American Institute of Consulting Engineers, the Institution of Civil Engineers of Great Britain, the Institution of Sanitary Engineers of Great Britain, the Engineering Institute of Canada, the American Society for Testing Materials, the Boston Society of Civil Engineers, the American Water Works Association, the New England Water Works Association, the American Public Health Association, the American Society for Municipal Improvements, and the American Chemical Society. His attainments were recognized by many of these organizations by election to office and honorary membership.

In 1914 he was made President of the Boston Society of Civil Engineers, and in 1925–1926, Vice-President of the American Institute of Consulting Engineers. Mr. Eddy was awarded Honorary Membership by the Boston Society of Civil Engineers, New England Water Works Association, and the Institution of Sanitary Engineers of Great Britain. His last journey to Montreal was made to receive Honorary Membership in the Engineering Institute of Canada, but his death intervened in the early morning of June 15, 1937, only a few hours before the award was scheduled to take place. In 1930 he was made Doctor of Engineering by the Worcester Polytechnic

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[2] *Journal*, Boston Soc. of Civ. Engrs., Vol. VII (1920), p. 47.
[3] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 465.

Institute, and, in 1931, Northeastern University of Boston awarded him the same degree.

Throughout his professional career, Mr. Eddy was deeply interested in the Society and in its Sanitary Engineering Division; but it was not until 1927, when he was officially nominated for Director from District No. 2, that he was brought prominently into the work of the organization as a whole. From then on until his death, he was continuously in this service, either as an officer, as an official representative, or as a member of some important committee.

During his term as Director (1928–1930), he made a deep impression on his associates by his ability and conscientious service. This was recognized in the fall of 1933 when he was enthusiastically nominated for the office of President. His work as President, and during the following two years as Past-President was of great value to the Board and the Society. Many important committees felt the impress of his judgment, among which may be mentioned the Technical Procedure Committee, the Committee on Public Works, and especially the Executive and the Publications Committees. From its inception he was deeply interested in the ideals and progress of the Engineers Council for Professional Development, to which he gave generously of his time and thought, as with all other of his professional duties.

It was during the depth of the economic depression that Mr. Eddy served as President of the Society. This was hardly a time in which a spectacular showing could be made but it was a period of very strenuous personal work. Important committees, such as those on Hydraulic Research, and on Flood Protection Data, were appointed; practice was established for the defense of Society members unjustly accused; the relations of Federal relief agencies to engineers were investigated; a code of fair practice for the Engineering Division of the Construction Industry was studied intensively; and a liberal policy was instituted in favor of those members who had suffered severe financial loss in the depression. All this work for the Society demanded a great sacrifice of time and effort, which Mr. Eddy cheerfully gave. He even assigned one of his own office staff to assist him in his studies of Society duties and opportunities. Many trips to New York and Washington were required. He estimated that during the first six months of his incumbency he visited Washington, in the interest of the Engineering Profession, at the rate of one day a week for the twenty-six weeks.

In May, 1932, between his terms as Director and President, he was appointed on the Society's Committee on Public Works, which entailed extensive activity including many interviews in Washington with Congressional leaders and others. Through this Committee, the Society was enabled to share generously in efforts for the enactment of the Emergency and Relief Act of 1933 whereby, under the Public Works Administration, many engineering projects were constructed. His influence with the administrators of Title II of this Act was great and many of its features had his close attention. The passage of this Act resulted in the employment of thousands of engineers at a time when there was a great hardship among members of the profession due to unemployment.

One of Mr. Eddy's great services came during his term as Director (1928–1930) when he served continuously on the Committee on Publications, being Chairman in 1930. This Committee had to consider the vital question of the feasibility and propriety of establishing an illustrated Society monthly. In April, 1930, the Committee made a report to the Board of Direction, suggesting the publication of "a journal under the title *CIVIL ENGINEERING* to contain those papers of a more popular character, items relating to Society activities, and other material of general appeal to the membership." It is typical of Mr. Eddy that his decision on this matter was made only after many conferences with officers and staff and a careful study of the functions and possibilities of the project. After the decision to undertake the new publication was reached, he gave unstintedly of his time in helping to develop the character of *Civil Engineering*.

At the time of his death Mr. Eddy was generally recognized as the Dean of American Sanitary Engineers. His success was due to an unlimited capacity for work, to power of concentration, to a mental discipline—which enabled him in the later years of his life to triumph over serious physical disability, and to a character and intellect which engendered an enduring confidence in his opinion and advice. Dignified but possessed of a keen sense of humor, he was a delightful companion and a firm friend. He was a considerate employer, and his helpful interest in the work of young engineers was notable.

It is not too much to say that Mr. Eddy was outstanding in every human and professional relationship. His status in the minds of his fellow engineers was thus epitomized in the July, 1937, *Civil Engineering:* "His character was noble, his counsel wise, his friendships true. Few engineers will leave as deep an impression on their generation as Harrison P. Eddy."

He was married on June 1, 1892, to Minnie Locke Jones, of Worcester, who, with three children, Harrison Prescott, Randolph Locke, and Mrs. Charles W. Eaton, survives him.

Mr. Eddy was elected an Associate Member of the American Society of Civil Engineers on May 7, 1902, and a Member on January 3, 1907. He served as a Director of the Society from 1928 to 1930 and as President in 1934.

ALGERNON BROWN ALDERSON, M. Am. Soc. C. E.[1]

Died May 13, 1939

Algernon Brown Alderson, the son of Thomas Brown and Eleanor (Sutton) Alderson, was born in London, England, on November 6, 1867. He was educated in the Taplow Grammar School near London, the Bristol Grammar School and Cliftonville College at Brighton, England. After coming to America he studied in the Ottawa Collegiate Institute at Ottawa, Ont., Canada, and decided on civil engineering as his life work.

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[1] Memoir prepared by F. B. Chamberlin, Esq., Town Engineer, West Hartford, Conn.

In his early career, from 1887 to 1897, Mr. Alderson served in the following positions: Chainman and Rodman with the Pennsylvania, Slatington and New England Railroad Company; Draftsman for the Pennsylvania Railroad Company; Draftsman for the Inter-Continental Railway Commission, Ecuador and Peru; Draftsman and Assistant Engineer for the Baltimore Traction Company; Draftsman for the New York, New Haven and Hartford Railroad Company; Draftsman and Assistant Engineer on design and construction of sanitary sewers for Bristol, Conn.; private practice in Bristol; and Assistant Engineer in charge of surveys for additional water supply for the City of New Britain, Conn.

In 1898 Mr. Alderson moved to West Hartford, Conn., where he engaged in private practice and at the same time acted as Engineer for the town whenever his services in that capacity were required. When the town set up an engineering department in 1915 he became the first Town Engineer and served in that capacity until 1918. During these twenty years with the Town of West Hartford he saw it grow from a small farming community of a few thousand to a sizable residential suburb; and, in connection with this growth he contributed much in the design and construction of several sanitary sewerage systems, improved pavements, sidewalks, drainage systems, and various other improvements required by such expansion.

From 1918 until his death Mr. Alderson was engaged in private practice as an Engineer and Surveyor with the exception of short periods when he was employed by the Town of West Hartford and by the Connecticut State Highway Department.

He was born to the engineering profession as his father was a member of the Institute of Mining Engineers of England and had extensive experience in the management of properties in Spain, Norway, and his native country. A brother in England is also a civil engineer and served as a Major in the Royal Engineering Corps during the World War.

Mr. Alderson was a life member of the Connecticut Society of Civil Engineers and a member of Wyllys Lodge, A. F. and A. M., of West Hartford. He was always a highly respected citizen of the community and gave considerably more than his professional services toward the welfare of the town.

On October 26, 1897, he was married to Anne Geneva Gladden of New Britain. He is survived by his widow and two daughters, Mrs. Ralph L. Dame and Mrs. Alderson Magee.

Mr. Alderson was elected an Associate Member of the American Society of Civil Engineers on October 2, 1907, and a Member on October 29, 1912.

WALTER HINDS ALLEN, M. Am. Soc. C. E.[1]

DIED APRIL 19, 1938

Walter Hinds Allen was born in Newton Highlands, Mass., on January 9, 1875. His father, Walter Allen, was born in Worcester, Mass., and his mother, Grace Mason (Weston) Allen, was born in New Braintree, Mass. Both parents came of Massachusetts Colonial stock. His father's first American ancestor settled in the vicinity of Boston, Mass., in 1640. His mother was of direct *Mayflower* descent. Successive generations were reared in the New England atmosphere and tradition.

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[1] Memoir prepared by Captain Ralph Whitman (CEC), U. S. Navy, M. Am. Soc. C. E.

Walter Hinds Allen received his elementary education in the Public Schools of Newton Highlands and, later, attended High School in New Haven, Conn., from which he was graduated in 1891. He then entered Yale University, at New Haven, Conn., and was graduated with the degree of Bachelor of Arts in 1895. This education was supplemented by a course as Larned Scholar in the Yale Graduate School from 1895 to 1898, during which period (in 1896) he received the degree of Bachelor of Philosophy from the Sheffield Scientific School. During this period also, he was employed at times as a member of the Instructing Staff in engineering subjects.

The outbreak of the Spanish-American War occurred toward the end of his college courses. Mr. Allen had been for some time a member of the Connecticut Naval Militia. In June, 1898, he enlisted in the United States Volunteer Navy, being given initially the rating of seaman. His qualifications and natural ability obviously indicated employment in a different capacity, but the standard procedure of the Navy provided no regular means for this unusual case and the natural preoccupation of the naval authorities with the conduct of the War made an alteration in his status irregular and somewhat difficult. His lack of sea training barred him from a commission in the line of the Navy, and legal obstacles prevented his appointment as a midshipman or naval cadet, titles which were reserved for young men appointed to the Naval Academy in regular course. However, a way was found. He was appointed an Acting Naval Cadet, a matter in which he always took much pride. The writer has heard him refer to it as the only case of its kind.

Being honorably discharged from the Navy in September, 1898, Mr. Allen began his active civil engineering career as Rodman in the service of the New York, New Haven and Hartford Railroad Company, going shortly afterward to the Mexican International Railway Company as Transitman. In quick succession he moved to the position of Leveler and Topographer with the Kansas City, Mexico and Orient Railway Company and back to the New York, New Haven and Hartford Railroad Company as Transitman and Assistant Division Engineer.

Leaving railroad work in 1903, he was commissioned in June of that year as Civil Engineer, United States Navy, with the rank of Lieutenant, Junior Grade. He was advanced to the rank of Lieutenant in 1906, and to that of Lieutenant-Commander in 1914. Incident to the expansion of the commissioned naval personnel in the World War, he was commissioned as Commander (temporary), Civil Engineer Corps, U. S. Navy, in July, 1918. This commission was made permanent in 1921. In 1924, he attained the rank of Captain in the Civil Engineer Corps, and retained this rank until the time of his death.

Captain Allen's duties in the Civil Engineer Corps of the Navy took him to many stations. Successively, he served at the Navy Yards at New York,

N. Y., and at Charleston, S. C.; at the Naval Ammunition Depot, Hingham, Mass.; Bureau of Yards and Docks, Navy Department, Washington, D. C.; again, at the Navy Yard, New York; then at the U. S. Naval Station, Olongapo, Philippine Islands; a third time at the New York Navy Yard; then at the U. S. Naval Training Station, Great Lakes, Ill.; Navy Yard, Philadelphia, Pa.; Navy Yard, Puget Sound (Bremerton), Wash.; U. S. Naval Operating Base, Hampton Roads (Norfolk), Va.; a second time at the Bureau of Yards and Docks; and, lastly, at Headquarters, 12th Naval District, San Francisco, Calif. In each of these field duties, except his first upon induction into the Naval Service, he was Senior Officer of his Corps at his station.

In so diversified a career, and with the variety of duties entailed, a list of projects with which Captain Allen was connected cannot be recorded in a memoir of reasonable length. Mention may be made of the large dry dock at Charleston built under his supervision when he was only thirty-one years old. In his last duty in Washington, he served four years (1930–1934) as Design Manager of the Bureau of Yards and Docks in charge of design of the Navy's shore station structures involving many millions of dollars.

It is characteristic of Captain Allen that in later years his conversation turned frequently upon his World War service at the U. S. Naval Training Station, Great Lakes, and not so much upon the technical aspects of it as upon the human relations involved. In addition to supervising the great quantity of war construction necessary for this station—one of the nation's largest training stations—he took a leading part in organizing and directing the activities of the Twelfth Regiment, the "Public Works Regiment" composed of raw recruits sent to the Station for initial training for Naval Service. In this regiment, they were trained for the design, construction, and maintenance of shore station activities in continental United States and abroad. From it, more than 18 000 men were sent to the Atlantic seaboard, to the Fleet, and to Europe. Among those who went to Europe were the Railroad Force to assemble and operate the locomotives and cars for the 14-in. railroad guns on the Argonne front. These were naval guns adapted for the new purpose. The ésprit of this body of young men, from many parts of the country, and with a vast diversity of origin and personal background, became notable even in the short time they were together. It is significant that upon Captain Allen's separation from this organization he was presented with the "regimental colors," a handsome flag about 4 ft hoist by 6 ft fly, of heavy silk richly embroidered and with gold fringe. This flag was one of his prized possessions. He had it with him wherever he went, and it was displayed in the interment services at the National Cemetery at Arlington, Va. These services were conducted with full military honors on May 3, 1938, following more intimate services at his residence in San Francisco, Calif., on April 21.

The same qualities that emphasized his relations with his regiment at the Great Lakes Station manifested themselves later in his activities with and in behalf of the Civil Engineer Corps of the United States Naval Reserve. In 1935, while he was on duty in the Navy Department as War Plans Officer of the Bureau of Yards and Docks, his personal efforts stimulated interest in this important branch of national defense among young engineers throughout the country. Upon being transferred to San Francisco in the fall of 1936 Captain Allen carried with him the same infectious enthusiasm. This trait is well described by Commander Frank M. Harris, U. S. Naval Reserve, who has written of Captain Allen that: "He possessed the peculiar ability to attach young men to himself and to encourage them to give their best efforts to any cause in which he was leading them." In such way he had impressed himself upon the community of a large city in only a year and a half of residence following his arrival there as a stranger.

The writer has preferred to dwell upon human attributes more than on professional qualities and attainments, and to write of the man rather than of his work. The work was shared by others. His nature was exclusively his own. Pre-eminently it was Captain Allen's social instincts that endeared him to those who knew him. He was essentially social and companionable, pleasantly aggressive in his loyalty to those with whom he was associated, whether superiors or sub-ordinates, and very lovable. Combining balance with youthful enthusiasm and sympathy, at sixty-three he was still a young man. This nature left its imprint upon all who became associated with him in any way, and increasingly with the length of the association.

Captain Allen was a member of the Society of American Military Engineers, and a Companion of the Military Order of the Loyal Legion. He also belonged to the Graduates Club, in New Haven; the Army and Navy Club, of Washington, D. C.; the Army-Navy Country Club (Washington); the Mountain View Country Club, Greensboro, Vt.; and during his residence in the respective localities had belonged to the Yale Clubs of Boston, Mass., Washington, D. C., Chicago, Ill., Philadelphia, Pa., Seattle, Wash.; and also to the Presidio Golf Club and the Engineers Club of San Francisco.

In January, 1906, he was married to Edith Stark Hancock, who, with a daughter, Edith Isadora Allen, survives him. He is also survived by a brother, the Hon. J. Weston Allen; and by three sisters, Mrs. F. S. Hollis, Mrs. J. N. Eaton, and Mrs. Alice A. Bonham.

Captain Allen was elected a Junior of the American Society of Civil Engineers on May 1, 1900; an Associate Member on November 4, 1903; and a Member on February 4, 1908.

JOSEPH CHESTER ALLISON, M. Am. Soc. C. E.[1]

Died May 29, 1936

With the passing of Joseph Chester Allison, the Southwest lost not only a brilliant engineer, but also a successful colonizer and financier. It is not usual to find such a combination of talent in civil engineers who are prone to limit their activities and thought along strictly professional lines. In the case of Mr. Allison, even the absence of a complete college training was no obstacle to a successful engineering career and the consummation of large enterprises.

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[1] Memoir prepared by A. L. Sonderegger and Ray S. Carberry, Members Am. Soc. C. E., and H. M. Rouse, Assoc. M. Am. Soc. C. E.

Born in San Diego, Calif., on July 12, 1884, young Allison attended the Grammar School and the first few years of High School in his native city. He was graduated from the Los Angeles (Calif.) High School in 1901, and was enrolled for the engineering course in Stanford University, at Stanford University, Calif. Ill health, however, forced him to stop university training and to take up outdoor work as an aid physically. Thus, he went to work in 1902 with the California Development Company in Calexico, Calif., as a helper on a surveying crew. This Company at that time was promoting the reclamation of the desert of Imperial Valley by the diversion of the Colorado River water, near Yuma, Ariz. Mr. Allison soon advanced to the position of Assistant Engineer under the leadership of Mr. C. R. Rockwood, and the late C. N. Perry, H. T. Cory, and F. C. Herrmann, Members, Am. Soc. C. E., all of whom successively occupied the position of Chief Engineer. It was Mr. Allison's privilege to be entrusted with a position of responsibility during the break of the Colorado River in 1906–1907, which threatened the permanent inundation of Imperial Valley. Subsequently, when the Company was confronted with the problem of reconstructing the canal structures which had been installed during the first years of the Imperial Valley enterprise and, of necessity, were of more or less temporary type, Mr. Allison proved himself to be an adept superintendent of construction. Climatic, geological, and physiographical conditions in the Imperial Valley, which confront the engineer, are unusual and demand an instinctively clear conception of the natural phenomena indigenous to that country. Mr. Allison, who ate and slept on the job for years, began to know the River and the soil.

The disastrous events of 1905, 1906, and 1907 finally forced the California Development Company into receivership, in January 1910. The receiver, William H. Holabird, soon recognized young Allison's ability and promoted him to the position of Chief Engineer; and in 1912 he became Chief Engineer and Assistant General Manager, which position he held until the termination of the receivership, in 1916.

Mr. Allison's first major undertaking was the reconstruction of Sharp's Heading, in Mexican territory, which is still the main point of diversion for the lateral canal system of Imperial Valley. In the meantime, the irrigation enterprise was expanding rapidly, and in 1911 the northern part of Imperial Valley was organized into Imperial Water Company No. 3. Mr. Allison, as Consulting Engineer for this water company, designed and supervised the construction of its canal system which, laid out along new lines, proved eminently satisfactory.

As Consulting Engineer for the Receiver, from 1912 to 1915, one of the writers had an opportunity to know Mr. Allison and his capacity as an organizer, construction engineer, and executive. He was particularly impressed with the resourcefulness of the young engineer and his exceptional understanding of the problems presented by the Colorado River and the delta formation which baffled more than one engineer. Thus, Mr. Allison's early training and the long fight with the River served, successfully, in later years, to meet the problems that continually confronted his Company, not only at the Colorado River Heading, but also at every point where structures and local conditions required the solution of questions of unusual character.

There was one dual problem which presented itself year after year: First, around the Fourth of July, the threatened invasion of the flood waters of the Colorado River along the levees of the Alamo Canal and Volcano Lake; and second, about two months later, a threatening water famine due to the combination of a low-water stage of the River with a river-bed which previous floods had scoured away below the sill of the Alamo Canal Intake at Hanlon, on the west bank of the river below Yuma. Year after year, these baffling conditions were met successfully. During a year of extremely low run-off (1913), Mr. Allison succeeded in building a diversion weir across the River, by the hydraulic process.

However, the spirit of the pioneer asserted itself also along commercial lines. While he was Chief Engineer of the California Development Company, Mr. Allison, in co-operation with Mr. Sydney McHarg, his brother-in-law, and others, became identified with the organization of the First National Bank in Calexico, with a branch office in Mexicali, on the Mexican side of the international boundary line. Mr. Allison became a Director, in 1912.

In 1916, when the assets of the California Development Company were acquired by the Imperial Irrigation District, Mr. Allison entered the field of private practice in Calexico and shortly thereafter became associated with the late Paul M. Entenman, Assoc. M. Am. Soc. C. E. Mr. Allison acted in consulting capacity for the Imperial Irrigation District, Imperial Mutual Water Companies, Imperial Valley Cities, Palo Verde Mutual Water Company, etc.

The opportunity presented itself in those years for colonization on the Mexican side of the Colorado River Delta. Mr. Allison and his associates saw possibilities in land development on a large scale in the delta lands of the Colorado River in Baja California, Mexico. The first enterprise undertaken in partnership with Messrs. Sydney McHarg and Tom Daley was the Alamo Mocho Plantation Company, which included the improvement of a large tract of virgin land. Irrigation water was supplied from the Alamo Canal of the Imperial Irrigation District, which enters Mexico a short distance below the point of diversion. The land was planted to cotton, under individual tenancy of small tracts by Mexicans and Chinese. Mr. Allison was the guiding spirit of the engineering features, as well as of the financial and commercial aspects. This project was completed successfully.

Mr. Allison then organized the Delta Land Company and leased 102 000 acres of virgin delta land from the Colorado River Land Company. He also organized the Delta Canal Company. This latter company installed its own irrigation systems and the cropping was subleased to Mexican, Japanese, and Chinese ranchers. These lands were developed by two processes: One by means of a large pumping plant on the Alamo Canal, together with three large deep wells; and the other by a large gravity diversion from the Alamo Canal. Later the Alamo Canal gravity diversion was augmented by a direct diversion from the Colorado River by the installation of siphons on an exceptionally large scale. This second enterprise also proved highly successful, Mr. Allison brilliantly justifying his reputation as an engineer, promotor, and financier.

In the well-known controversy about the All-American Canal Project of Imperial Valley, Mr. Allison used his pen as well as his personal influence in favor of the Mexican route, as he was doubtful of the success of the project, both from a technical and financial point of view.

In 1924, he opened offices in Los Angeles, Calif., and established his residence in La Jolla, San Diego County, in 1925.

Cotton being the principal crop of the plantations in Baja California, Mr. Allison conceived the project of warehousing the crop at the port of San Diego instead of Los Angeles. To this end, he incorporated, in 1928, the San Diego Municipal Warehousing Corporation. The authorities of the City of San Diego lent their assistance to this venture by giving the Company a lease on 9½ acres of tideland. A spacious warehouse was built and the operation of a general warehousing business initiated. This business was successful through the years of the depression. In order to further shipping through the Port of San Diego, Mr. Allison undertook a trip to Mexico, and Central America in 1935, establishing promising business connections.

Mr. Allison died suddenly on May 29, 1936.

Mr. Allison was first married to Gladys Mitchell, who bore him two children; she died in 1921. In 1922 he was married to Carlotta Snow, who, with his five children, survives him. Always a loving and devoted husband, and greatly attached to his children, his memory will be held very dear to those he left.

The passing on, in the prime of life, of a man like Joseph Chester Allison is always a great loss to his country. A man of exceptional ability, he combined the qualities of an engineer with the sound judgment and foresight that make the successful business man.

Mr. Allison was elected an Associate Member of the American Society of Civil Engineers on September 3, 1912, and a Member on June 12, 1917.

EDWIN PERCIVAL ARNESON, M. Am. Soc. C. E.[1]

Died December 7, 1938

Edwin Percival Arneson was born in Fort Worth, Tex., on July 25, 1888, the son of A. and Emma (Grimland) Arneson. His grandparents were truly pioneers in Texas in a day when it took character and tenacity to settle and make a living in that State. His maternal grandfather arrived in Texas in 1848 and the paternal grandfather in 1868, both coming to the New World from Norway. Edwin attended the public schools of Fort Worth and the Agricultural and Mechanical College of Texas at College Station, Tex. He received the degree of Bachelor of Science in Civil Engineering in 1910, and the degree of Civil Engineer in 1929, from the latter school.

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[1] Memoir prepared by a Committee consisting of E. W. Robinson, and A. P. Rollins, Members, Am. Soc. C. E., and P. S. Park, Esq., and Harry P. Drought, Esq., both of San Antonio, Tex.

His first position after leaving college was with the Texas Company at Houston, Tex., on the design of warehouses, wharves, and similar structures. From 1911 to 1913 he was engaged as Office Engineer for the Medina Irrigation Company of San Antonio, Tex., on the canal system in connection with Medina Dam project. From 1913 until the work was stopped by the outbreak of the World War in 1914 Mr. Arneson was Resident Engineer on various large projects in Spain for the Ebro Irrigation and Power Company of Barcelona.

From 1914 to 1915 he was in immediate charge of surveys and studies for a drainage project at Waco, Tex., and from 1915 to 1916 was Chief Draftsman in the office of the City Engineer of San Antonio, supervising the design of extensive municipal improvements. From December, 1916, to March, 1930, with the exception of a brief period in Washington, D. C., he was a member of the firm of Walton and Arneson, Civil Engineers of San Antonio, and engaged in topographic surveying, irrigation, highway projects, and structural work. During the World War he served in the Navy Department in Washington in the capacity of Structural Draftsman on the design of ship-building ways, crane runways, and water-front improvements.

From March, 1930, until his death, Mr. Arneson maintained his own office in San Antonio for the general practice of Civil Engineering. Besides the usual run of general practice, he was frequently called into consultation on, and engaged to do, highway location by the Texas State Highway Commission. Because of the fine reputation he had acquired as a locating engineer he was employed by several of the more populous counties of Texas to prepare a master-highway plan for the larger cities in those counties, and it was in this line of work that he became recognized as an outstanding engineer. Dallas, Fort Worth, Waco, San Antonio, and Corpus Christi, Tex., have master plans designed by him. Nueces Valley Authorities' Flood Control and Irrigation Enterprise, and the San Antonio River Beautification Project (his last work) bear further testimony to his engineering skill and vision. To quote an editorial in the *San Antonio Express:*

> "As those enterprises shall take shape through the years, countless motorists who never heard of Mr. Arneson's name, and many a farmer tilling his acres watered by a harnessed river, will benefit from his far-sighted service."

In July, 1935, during the organization of the Works Progress Administration in Texas, it was realized that the selection of the first District Director would set the standard for the others who were to be appointed. Therefore, the State Administrator, recognizing Mr. Arneson's outstanding ability, both as an engineer and an executive, and knowing of the high place he had won for himself in his profession and among his fellow citizens, determined to induce him to take the first appointment as a District Director and to serve in that capacity for the San Antonio District. Although it meant a considerable financial sacrifice to Mr. Arneson, he was led by patriotic motives to accept the appointment, and he took the Oath of Office as District Director on July 10, 1935. The San Antonio District Office under his direction was one of the best in the State and the Works Program which he supervised did credit to the efforts of the Federal Government to relieve unemployment.

A number of times he asked the State Administrator to relieve him of his duties in order to enable him to return to his engineering practice, but he continued in this position until the planned consolidation of Works Progress Districts in Texas became effective, thereby making available another District Director who could take his place. Upon his insistence, the State Administrator reluctantly accepted his resignation effective July 15, 1937. However, this did not end his connection with the Works Progress Administration for, thereafter, his counsel and advice were often asked and gladly given.

Ed Arneson was well grounded in the theory and practice of Civil Engineering. Undoubtedly, had he lived the normal span of life, his work would have attracted greater attention, and his reputation as an engineer would have spread further over the country. But, to his intimates, it was in the rôle of a citizen that he made his greatest contribution to society. He had an inquiring mind, was ever a student and a gentleman, and his interests were broad, covering literature, art, music, and general culture. He had mastered the Spanish Language, was fast becoming familiar with French, and was a student and lover of Greek culture.

His thoughts and opinions were sound, and he was the type of man with whom one could differ radically and yet highly respect his position. While he adhered with fidelity to his opinions in fundamental matters, he always had an open mind for opposite arguments and would not hesitate to compromise on non-essentials or even change his opinions when argument convinced him he was wrong. He was a man who worked intently and played joyously, and he drew the deepest of devotion from the hearts of his numerous friends. In addition to the many other sides of his well-rounded personality Mr. Arneson was a student of history. His mental equipment was replete with facts and relationships covering the salient features of Texas development and the influence of Spanish culture on the southwestern part of the United States. Several of his papers on this subject have been published in the *Southwestern Historical Quarterly.*

He was always ready and willing to assume his full share of civic duty, even at personal cost. Mr. Arneson was of that type—all too few in any community—on which opposing political and civic factions were willing to rest their case and agree with his decisions as an arbitrator. His character can best be summarized by stating that he was the kind of a man that any one could point out to the young engineer as a safe guide to emulate.

Mr. Arneson was a member of the Rotary Club, Scientific Society, Technical Club, Yanaguana Society, and the First Presbyterian Church, all of San Antonio. He served the Rotary Club as Director for two years and was President in 1933. In 1935 he was elected District Governor of the 47th District of Rotary International. He was ever active in the affairs of the Texas Section of the American Society of Civil Engineers and was President for the year 1933–1934. His election as Director of the Society District No. 15 followed in 1936, which office he held at the time of his death.

As a Society officer he was conscientious in his work and faithful to his obligations. His breadth of vision enabled him to see the problems of the whole organization in a large way. Petty considerations had no place in his make-up. His ability to do real work was early recognized; for his entire term he took his full share of the arduous responsibilities as one of the Committee on membership qualifications. He served as member and finally as Chairman of the Board's Committee on Districts and Zones.

This capacity for seeing that a job was done, and done well, earned him the respect of his fellow Board members, while his delightful disposition and personality gained their affection. On those occasions when he took part in open discussion his grasp of the question at issue was always evident. No Board member had a more respectful hearing. His standing was evidenced in the tribute after his death adopted by the Board, as follows:

> "*Whereas* the Board of Directors of the American Society of Civil Engineers has been shocked to learn of the death, on December 7, 1938, of its fellow Director, Edwin P. Arneson; and
>
> "*Whereas* Mr. Arneson's death while still in the prime of life has removed one who had ahead of him many years of usefulness to the Society and to the profession at large; and
>
> "*Whereas* the Board has found him to be a most active and valued co-worker in all its affairs, with the interests of the Society always at heart; and
>
> "*Whereas* his character and friendliness have earned for him the esteem and affection of his fellow members of the Board,
>
> "*Now, therefore, be it resolved* by this Board, assembled in meeting at New York City this day, January 16, 1939, that it records its warm regard for its late associate, and its deep sorrow for his loss; that it extends its sympathy to his family, and directs that this resolution be spread on the minutes of the Board, and that a copy be forwarded to Mr. Arneson's family."

He was married to Anita Park at Bryan, Tex., on April 26, 1916, and this union was blessed with three children, Edwin Park, Lily Anita, and Richard Owen.

Mr. Arneson was elected an Associate Member of the American Society of Civil Engineers on May 12, 1919, and a Member on March 28, 1932.

CYRUS CATES BABB, M. Am. Soc. C. E.[1]

Died October 2, 1937

Cyrus Cates Babb was born June 18, 1867, at Portland, Me., the son of Cyrus Knap and Mary Lucretia (Judkins) Babb. He was of English and

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[1] Memoir prepared by a Committee of the North Carolina Section, consisting of C. L. Mann, George H. Maurice, and W. M. Piatt, Members, Am. Soc. C. E.

Welsh descent. On the English side he was descended from Governor Joseph Dudley, of Massachusetts, and from Lieutenant James Peabody who came over in the *Planter* in 1635. It is not certain when his Welsh forebears arrived in this country, but it is known that James Babb bought land and settled in Westbrook, Me., in 1735.

After a High School education obtained at the English High School, at Boston, Mass., he was graduated from the Massachusetts Institute of Technology, in 1890, with the degree of Bachelor of Science.

During his college course, in 1887, Mr. Babb had employment on a preliminary railway survey between Rockland and Rockport, Me., and, during 1888, he served as Transitman under Mr. Gilbert Hodges on street railway work.

After his graduation in 1890, he obtained a position with the United States Geological Survey, and thus entered into his life work on hydrographic, reclamation, and water-power projects. With the exception of the summer of 1894 which was spent traveling in Europe, Mr. Babb remained with the Survey until 1904, as Hydrographer and Topographer, and traveled throughout the entire United States establishing gaging stations and enlisting the interest of local engineers, under the direction of the late Frederick Haynes Newell, M. Am. Soc. C. E., then Head of the Hydrographic Division. He established more gaging stations in this country than any other man in the Hydrographic Division. One—Babb, Mont., on Saint Mary's River—was named after him and is still in operation. During the fall and winter of 1898–1899, he made a daring reconnaissance survey of the Gila River Canyon. This was accomplished with a small pack-train outfit which struggled for days through the boulder-strewn canyon bottom, alternating with quicksands, the only human beings encountered being a few Apache Indians. It was on this trip that Mr. Babb made the preliminary survey for the San Carlos Dam site, on which, twenty years later, was built the Coolidge Dam.

Following this work he was in charge of surveys and investigations in Utah and Montana for water supply and irrigation projects, so that when the long-range planning of Mr. Newell bore fruit in 1902 as the Reclamation Act, Mr. Babb was transferred to the United States Reclamation Service as Project Engineer in charge of all designs for reservoir dams, outlet gates, canal sections, and estimates of cost. He had charge of the St. Mary-Milk River (Montana) Project, including extensive field surveys, borings, and all designs for the irrigation of 150 000 acres, involving the expenditure of upward of $6 000 000. While on this work he joined the Masonic Malta Lodge during 1904.

From 1909 to 1911, he was engaged as District Engineer of the U. S. Geological Survey, on detail to the Storage Commission of the State of Maine, and, in the latter year, became Chief Engineer of this Commission, continuing in this capacity through 1914. His duties included the investigation of the water resources of the State, examinations and estimates of the water powers and storage possibilities, and the preparation of the Annual Reports of the Commission. This work increased his interest in water power and resources so much that they became his chief interest during the future years of his life. He was a member of the Maine Association of Engineers, and served as its President in 1911, later becoming an Honorary Member.

In 1915 Mr. Babb opened an office in Augusta, Me., and began private practice as a Consulting Hydraulic Engineer. His activities included Consultant on water supply of towns in Maine; valuation of water-works property for a rate revision contest; detailed reports on a number of large power and storage prospects for certain New York financial interests; Court Expert in cases involving water-power questions; a noteworthy design for a hollow concrete dam on a glacial drift foundation, etc.

In 1917, he moved to Granite Falls, N. C., and continued his hydraulic and general engineering practice. He served as Consulting and Designing Engineer for a number of hydraulic and power developments, manufacturing companies, and municipal water supply and sewerage systems. He was Consulting Engineer for the Rhodhiss Manufacturing Company, and the E. A. Smith Manufacturing Company, Rhodhiss, N. C., manufacturers of cotton goods, on one of the earlier modern hydraulic power and electrification developments in the South. He designed and superintended the construction of the water supply and sewerage systems for Blowing Rock, N. C., and the 18-hole golf course and subdivisions for the Green Park Company, in Blowing Rock.

In January, 1929, Mr. Babb returned to service with the Government in the United States Engineer Office, at Charleston, S. C., and, later, at Norfolk, Va., as Senior Hydraulic Engineer, United States Corps of Engineers, in charge of water-power and flood-control investigations. He covered the resources of the Santee River, in North Carolina and South Carolina; the Yadkin-Pee Dee River, in North Carolina and South Carolina; the James River, in Virginia; the Roanoke River, in North Carolina and Virginia; and the Susquehanna River, in New York State and Pennsylvania.[2]

After the completion of this assignment, he was transferred to the Baltimore, Md., Engineer District, and after the appointment of the National Resources Committee, worked for that Committee as Associate Consultant on the Drainage Basin Studies, first at Durham, N. C., and, later, at Columbia, S. C., under William M. Piatt, M. Am. Soc. C. E., Regional Consultant. After the completion of the field work in this District, he was transferred to Washington, D. C., to assist in the preparation of the final text of that part of the report of the Water Resources Committee under the title "Drainage Basin Problems and Programs."

In December, 1936, Mr. Babb resigned to take up his consulting engineering practice, being retained by the Rhodhiss Manufacturing Company for enlargements to its plant. He was also engaged in extensive investigations and estimates for possible hydraulic power developments in the Piedmont Region of North Carolina.

Gerard H. Matthes, M. Am. Soc. C. E., one of Mr. Babb's associates in the U. S. Geological Survey and Reclamation Service, writes as follows:

"It was always a pleasure to meet Babb for he was a royal good fellow to be with out in the wilds. He was a 'good sport' and had the knack of adjusting himself to circumstances and making the most of any situation. His sunny disposition and true-blue character always managed to win a way for him even in difficult situations. * * * To my mind, the outstanding traits of Babb's character were faithfulness and dependability. He truly possessed a 'New England Conscience' and those who roughed it with him in the Far West learned to appreciate those sterling qualities."

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[2] Under the provisions of H. R. Doc. No. 308, 69th Cong., 1st Session.

During the World War, Mr. Babb was an Associate Member of the Legal Advisory Board, and Chairman of the Red Cross and War Savings drives for Caldwell County, North Carolina. He was a member of the Granite Falls School Board from 1921 to 1925, serving as Secretary-Treasurer during the building of the new schoolhouse.

Tennis had been Mr. Babb's chief sport until of late years, when gardening and the reading of contemporary authors became his chief recreation.

He was married, at Lutherville, Md., on March 7, 1906, to Mary Grace Crowther, who died in July, 1938. Mrs. Babb, with two children, Mrs. John P. (Katharine) Rabb, and Dr. Dudley C. Babb, survived him.

At the time of his death, he was an Honorary Member, Maine Association of Engineers (Past-President, 1915) and a Member of the American Geophysical Union. He had contributed papers[3] and discussions to the publications of the Society.

Mr. Babb was elected a Junior of the American Society of Civil Engineers, on February 2, 1892; an Associate Member on February 3, 1897; and a Member on March 1, 1904.

CHARLES LELAND BATCHELDER, M. Am. Soc. C. E.[1]

Died January 18, 1939

Charles Leland Batchelder was born in North Reading, Mass., on January 1, 1887. He was the son of Leland D. Batchelder and Mary (Putnam) Batchelder who were of old New England stock, their forefathers having landed at Salem, Mass. Both parents were greatly interested in educational and church work, and the Leland D. Batchelder School of North Reading was named in honor of the father in recognition of his outstanding interest in public education.

Mr. Batchelder's boyhood was spent in North Reading. He attended the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated with a Bachelor of Science degree in 1909.

Upon graduation, he at once secured employment under the City Engineer of Lowell, Mass., where he served from 1909 to 1912 as Instrumentman engaged in the design and construction of sewers, water mains, water-supply structures, and various bridges built by the city.

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[3] *Transactions*, Am. Soc. C. E., Vol. XXVII (1892), p. 21; and, also, Vol. XXVIII (1893), p. 323.

[1] Memoir prepared by E. V. Willard and C. G. Paulsen, Members, Am. Soc. C. E., and C. H. Prior, Assoc. M. Am. Soc. C. E.

From 1912 to 1918, Mr. Batchelder was employed by the United States Geological Survey. Starting as Junior Engineer, he was assigned in the central office in Washington, D. C., where he spent one year. He was next assigned in the Survey's district office in Salt Lake City, Utah, and then in the Oregon district with headquarters in Portland.

In recognition of the high quality of his work, he was soon promoted to Assistant Engineer in which position he took an active part in the design and construction of structures and equipment for stream-flow measurements, as well as in the computation and compilation of stream-flow records in the areas covered by his assignments.

In 1918 he resigned from the Geological Survey to accept a position as Auditor and Chief Cost Accountant for the Warm Springs Irrigation District at Vale, Ore., where he was responsible for auditing the construction and operation costs of this district, the construction of which cost more than one and one-quarter million dollars. Because of his outstanding ability to make friends and reconcile conflicting view-points of opposing factions within the district, as well as his recognized engineering ability, he was soon appointed its Secretary and Manager. This position not only carried the responsibility of managing the district, but also included responsibility for the design and construction of the many irrigation head-gates and diversion dams needed for this project, and the task of instructing water users as to the proper and economical use of water. His friendly and tolerant attitude toward those with whom he came in contact, and his fair and impartial administration, brought to him the respect of all who were interested in the project and made for him a host of personal friends.

In 1928 Mr. Batchelder returned to the United States Geological Survey as Hydraulic Engineer in charge of field investigations in connection with international flood control problems on the Roseau River in Minnesota. Soon thereafter a district office of the Geological Survey was established in St. Paul to conduct the above international work and the general surface water investigations in Minnesota in co-operation with the State, and because of his excellent organizing and administrative ability, Mr. Batchelder was designated district engineer in charge of the work. In addition, he supervised similar co-operative activities in parts of North Dakota, South Dakota, Wisconsin, and Iowa.

Mr. Batchelder was unstinting in his efforts in helping to solve the many complex water problems of the district. Laying the ground work for a long-time study of stream flow in his district, he proceeded in a most efficient and thorough manner to carry on this program of work so as to produce results of the greatest value in connection with the water problems of the State. He became an authority on problems of stream flow, and his willingness to co-operate with various local and Federal agencies was very effective in producing vital data for the solution of these water problems. These data were used in preparing plans to alleviate water shortage in the Red River of the North during drought periods, and water-conservation programs throughout the district.

Mr. Batchelder was greatly interested in the improvement of stream-gaging equipment, and various improvements in the design and assembly of equipment for making discharge measurements from boats were made under his direction. Many of his suggestions were incorporated in the designs for other types of stream-gaging equipment.

His friendly and understanding attitude toward his associates was appreciated by all who worked with him, and he made many contributions to engineering and hydrology. His untimely passing is a distinct loss to the profession.

Mr. Batchelder was married on December 4, 1912, to Esther Emerson who, with a daughter, Mrs. Charles Bodey, and a son, Charles L. Batchelder, Jr., survives him. He is also survived by a brother, Stewart P. Batchelder, and two sisters, Sarah and Elizabeth Batchelder.

Mr. Batchelder was elected a Member of the American Society of Civil Engineers on March 16, 1936.

RICHARD BIRBECK, M. Am. Soc. C. E.[1]

Died December 6, 1937

Richard Birbeck, the son of John and Katherine Birbeck, was born at Lawrence, Kans., on June 2, 1861. His parents, both of whom were born in England, were among the early pioneer farmers who went to Kansas prior to the Civil War. They settled on a farm near Great Bend in the south central part of the State in 1866, and there Richard Birbeck spent his hard-working youth in comparative poverty. Educational facilities were very limited, but he entered the Preparatory Department of the University of Kansas in 1880 and continued his preparatory and college work without interruption. He was graduated in the Class of 1886 with the degree of Bachelor of Science in Civil Engineering.

The story of his hard fought battle to secure an education is an interesting one. Mr. Birbeck found a room over the largest livery stable in town and, with a small stipend for taking care of livery service night calls, he secured the job of lighting the gas street lamps, and for six years was a familiar figure running miles around the town with his short ladder on his shoulder.

He was a hard-working, conscientious student who did good work, made many friends, but who had no social life other than that of the class-room and a few college extra-curricular activities. He was always genial, helpful, and ambitious, and was well liked by his classmates, although his financial condition was such that his studies and his work occupied him fully.

After his graduation, Mr. Birbeck was employed by the Santa Fé Railroad Company and remained in that service until December, 1887. From January to July, 1888, he was County Surveyor of Stafford County, Kansas, which position he left to enter the service of Baxter L. Brown, M. Am. Soc. C. E., who writes of him as follows:

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[1] Memoir prepared by A. C. Reed, Esq., Port Orange, Fla., Henry Earle Riggs, President, Am. Soc. C. E., and Baxter L. Brown, M. Am. Soc. C. E.

> "In August, 1888, I was organizing a party to make a preliminary and location survey for the Kansas City, Wyandotte & Northwestern Ry. from Seneca, Kansas, to Beatrice, Nebraska, and the Superintendent urged me to give a job to a young man who had recently graduated from Kansas University, and who used to take care of his horse when in Lawrence. I consented, and the next day Dick Birbeck showed up.
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> "It did not take me very long to learn that Dick was well qualified to be pushed ahead as fast as circumstances would permit, and it was always my pleasure to have him working for me whenever possible."

According to the files kept by Mr. Brown, the following is a record of Mr. Birbeck's work from August, 1888, to March, 1901, when he left for Cuba: August, 1888, to February, 1890, Topographer and Draftsman, Kansas City, Wyandotte and Northwestern Railway Company; April to June, 1890, Assistant Engineer, Bridge and Building Department, Missouri Pacific Railroad Company; June, 1890, to May, 1892, Surveyor and Draftsman in Southeastern Kentucky for the Kentucky Land and Improvement Company (later, known as the Central Appalachian Company, Limited); June, 1892, to August, 1895, with the Watts Steel and Iron Syndicate, in Tennessee; also Engineer of mine work, etc., and Superintendent of mine for six months; September, 1895, to May, 1896, Transitman and Draftsman on the relocation and reconstruction of the St. Louis, Chicago, and St. Paul Railroad, from Alton to Grafton, Ill. (this road later became a part of the Chicago, Peoria and St. Louis Railroad); May, 1896, to January, 1897, Draftsman and Topographer, Shelbyville Southern Railway from Shelbyville, Ill., to Altamont, Ill. (when this work was completed, it became a part of the Chicago and Eastern Illinois Railway); and January, 1897, to March, 1901, with the St. Louis, Iron Mountain and Southern Railway Company, on office engineering work.

In March, 1901, Mr. Birbeck left the United States and entered the service of the Cuba Company and the Cuba Railroad Company. Mr. A. C. Reed, of the Cuba Company, tells of these twenty-two years of his life:

> "From March, 1901, to September, 1923, he was with the Cuba Co., and the Cuba R. R., except a short period in 1919 and 1920 when he secured leave to go to the Atlantic Fruit Co. as Chief Engineer in charge of design and construction of railroad lines and dock facilities. During the period that he was with the Cuba Company and the Cuba R. R., he was Assistant Chief Draftsman, Construction Engineer, Office Engineer, and Chief Engineer.
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> "For many years he took all of his vacations by returning to the field on location or construction of branch lines or other projects either for his own company or for the many sugar companies. His services were in great demand by the various sugar companies for their yard and mill track-layouts, and for the location of railway lines coming in from the fields to the mill.
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> "In September, 1923, he retired, moved to New York, and did only such work as the Cuba Company requested until his death on December 6, 1937."

Mr. Birbeck was married on August 6, 1896, to Ellora Landoun, of Missouri, who died on April 7, 1900. On his return to the United States from

Cuba he was married to Mrs. Jeannette Deppe, a sister of his first wife. He is survived by his widow and by a son, Gratz Birbeck, a sister, Mrs. Jennie Umphrey, and two nieces, Catherine (Umphrey) Shepard and Patrica Jane Shepard.

He was a member of the Masonic Order and of the Society of American Military Engineers.

Those who had been associated with Richard Birbeck at any time during his long life, remember him as a hard-working, thoroughly reliable engineer who was intensely loyal to his employers and to his friends. He found great pleasure in helping young engineers to get a start in the profession and for this he will be long remembered by engineers who are resident in many countries.

Mr. Birbeck was elected a Member of the American Society of Civil Engineers on December 3, 1928.

HENRY ROBINSON BUCK, M. Am. Soc. C. E.[1]

Died August 11, 1934

Henry Robinson Buck was born on September 14, 1876, in Wethersfield, Conn. He was descended from distinguished colonial stock, and his ancestors were among the earliest settlers in Connecticut. A graduate of Yale University, at New Haven, Conn., in the Class of 1896, Sheffield Scientific School, he was throughout his life a loyal alumnus, taking a special interest in all the activities and reunions of the University.

As might be expected from one of his historic background, Mr. Buck was intensely patriotic. An opportunity to exhibit this loyalty occurred early in his professional career; he served in the United States Navy during the Spanish-American War, being one of the crew that made a memorable voyage in one of the old Monitors from Boston Harbor to Philadelphia, Pa. He was an Associate member of the Naval Consulting Board in 1915, and State Director of the Industrial Preparedness Board in 1916.

His life was devoted to work throughout his native State in the fields of general and sanitary engineering. As Chief Engineer for the State of Connecticut, from 1905 to 1923, he remonumented the New York and Massachusetts boundaries of the State, and served as Engineer for the State Armory Board at Hartford, Conn. During the period from 1923 until his death he was Chief Engineer for the Emergency Conservation Work in Connecticut, having charge of all construction work undertaken in the C. C. C. Camps. He was the engineer member of the State Factory Waste Commission. He was a member of the Park Board in the City of Hartford, where he lived and carried on his business. These and many other public responsibilities were discharged in addition to his own work as a Consulting Engineer.

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[1] Memoir prepared by Edward Eastman Minor, M. Am. Soc. C. E.

Some of his outstanding work on sewage treatment was done for the Connecticut towns of Watertown, Wethersfield, New Canaan, and Putnam. In the design of sewage systems, the following projects were designed and carried out by him—sewage systems for Wethersfield, Hamden, Watertown, West Hartford and Windsor, all in Connecticut. In connection with State Institutions, both sewage and water systems were designed and constructed by him throughout the State.

An engineer serves his profession also in the various technical societies covering his field. Mr. Buck gave generously of his time for this purpose. He was President and later an Honorary Member of the Connecticut Society of Civil Engineers; a charter member of the American Institute of Consulting Engineers and on its Executive Committee; Vice-President of the New England Water Works Association; and one of the founders of the Hartford Engineers Club.

One of the most distinguished of such services was given on behalf of the Society. District No. 2, comprising practically the entire New England States, had had many eminent representatives on the official Board of the Society, but it had been some years since such representation had been filled by a member from Connecticut. Mr. Buck's many professional friends advanced his name for this service, thus indicating their high estimate of his character and professional standing. Accordingly, he was elected in January, 1931, as Director, serving for the following term of three years.

He soon gained for himself a position of prominence and respect in the Board deliberations. Recognizing his ability, he was called upon to serve on a number of important committees, including that on Technical Procedure for two years. In particular, however, his great contribution was in connection with the Society's publications. For all three years of his term as Director, he served on this important Committee, being a chairman in his last year, 1933.

On this Committee he was permitted to be of unusual service to his profession. The publication, " Civil Engineering," had just been started and many problems of procedure and policy were actively before the Committee. On such his judgment was sound; and he had the strength of his convictions. During the time of financial stringency, he was a tower of strength. His fairness and judicious temperament made him an ideal chairman of the Committee. His tact in handling difficult situations made for the smooth handling of Society publications during this critical period.

The fine record that he made on this work was recognized by a tribute to his memory by his associates on the Publications Committee, who spread on the Minutes of the Board the following " Appreciation of Henry R. Buck:

> " All activities of the Society, but particularly the publication work, suffered a severe loss in his death. During all his term of office on the Board of Direction, Mr. Buck was a member of the Publications Committee, being Chairman in 1933. He made an ideal chairman, being faithful in every detail of his work and vitally interested in all Society publications. An enthusiastic supporter of all the best interests of the Society, he endeared himself especially to the members of the Publications Committee,

> who are a unit in feeling for him personally the greatest of respect and for his loss the deepest of sorrow. Every aspect of the Society work is the richer for the sincere efforts of Henry R. Buck."

An engineer who combined technical skill with executive ability, whatever task he undertook was carried through with marked ability, his enthusiasm and confidence inspiring those associated with him. In addition to his engineering proficiency, his integrity of character, broadmindedness, and sympathetic understanding made his services of exceptional value both to the State, his clients, and his friends.

Mr. Buck found relaxation in all outdoor activities. He was an ardent mountain climber and a member of the Appalachian Mountain Club. The writer recalls being in camp at the foot of Katahdin in Maine with a group of Appalachian Club members when Mr. Buck arrived. He immediately became the leader and driving force of the party. This enthusiasm and leadership was always in evidence in whatever activity he was engaged.

Among his social activities, Mr. Buck was a Thirty-second degree Mason, and Past Master of Hartford Lodge AF and AM No. 88. He was also a member of the Hartford Rotary Club, the Twentieth Century Club, the University Club of Hartford, the Graduates Club of New Haven, Appalachian Mountain Club of which he was Vice-President in 1927, the Connecticut Forest and Park Association as Vice-President and Scout Commissioner, Charter Oak Council, B. S. A. 1924–1928.

Mr. Buck was married on December 3, 1901, to Mary Ladoyette Wolcott. He is survived by his widow; two sons, Henry Wolcott and Robinson Dudley Buck; and one daughter, Elizabeth Roswell (Mrs. Dallas W. Dort). He was killed in an automobile accident on Avon Mountain, just west of Hartford, on August 11, 1934, and was buried in Cedar Hill Cemetery at Hartford.

Henry Robinson Buck was elected a Junior of the American Society of Civil Engineers on June 5, 1900; an Associate Member on October 4, 1905; and a Member on February 4, 1908.

EDWARD ABRAHAM BYRNE, M. Am. Soc. C. E.[1]

DIED DECEMBER 6, 1938

Edward Abraham Byrne was born in New York, N. Y., on January 27, 1864, the son of Frederick John and Susan Mary (Power) Byrne, and the descendant of an old New York family of Irish heritage. Educated in the Public Schools of the city, he was graduated from the College of the City of New York in 1884. Thus, by ancestry, birth, and education he was preeminently a New Yorker, and the love he bore for his city was revealed throughout his strenuous life.

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[1] Memoir prepared by Isidore Delson, Assoc. M. Am. Soc. C. E.

Mr. Byrne began his engineering career in 1886 with the New York Aqueduct Commission on the construction of the Croton Water Supply System. It is of interest that on this project he met the late Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E., who also was destined to become a distinguished engineer and an outstanding civil servant.

From 1889 to the close of 1897, Mr. Byrne worked on highways and bridges for the old Department of Public Works of New York City.

On January 1, 1898, he joined the Department of Bridges and began a striking and noteworthy service which ended in November, 1933, with his resignation from the position of Chief Engineer of the Department of Plant and Structures (the successor of the Bridge Department), in order to assume the duties of Chief Engineer of the Triborough Bridge. His thirty-six years of service in the Department of Bridges, and its successor, the Department of Plant and Structures, may be divided into two periods. From 1898 to 1912 he was in charge of bridge construction and maintenance. During this period he supervised the construction of the Willis Avenue Bridge over the Harlem River, Vernon Avenue Bridge over Newton Creek, Borden Avenue Bridge and Hunters Point Bridge over Dutch Kills, and the old Flushing Bridge over Flushing River. The second period began with his assumption of the office of Deputy Chief Engineer in 1912, followed by his appointment as Chief Engineer in 1915. At the close of the World War, the Department of Plant and Structures began a period of intense activity which was cut short only by the depression of 1930. Not less than fifteen new bridges of major importance, of which ten were of the bascule type, were built in this period. In addition, under Mr. Byrne's direction, the Department designed and constructed a number of large incinerator plants and many other structures for the several departments of the City. He increased the capacity of the Manhattan and Queensboro Bridges over the East River by the construction of additional vehicular roadways on their upper decks.

Mr. Byrne was greatly preoccupied with the study of traffic facilities. As far back as 1916 he conceived the idea of the Triborough Bridge to link the Boroughs of Manhattan, Bronx, and Queens. He fostered this idea until he thought the time was ripe for its materialization. The actual work on the foundations and anchorages was begun in 1929 under his direction, while Chief Engineer of the Department of Plant and Structures.

Then, in November, 1933, upon the organization of the Triborough Bridge Authority, he became its first Chief Engineer. There is no doubt that Mr. Byrne looked upon the construction of the Triborough Bridge as the culmination of his career, and there is pathos in the fact that he was forced to relinquish his post in February, 1934, only several months after he had entered upon it with such high anticipation.

Primarily Mr. Byrne was an engineer and executive, and because of character, integrity, and devotion, held his position, independent of politics, throughout successive administrations. The administration headed by Mayor LaGuardia assumed office on January 1, 1934, with a mandate for a "clean sweep," and the complete reorganization of the Triborough Bridge Authority was among the changes which followed. It is fitting that credit for the initiation of the Triborough Bridge should be accorded Mr. Byrne.

Another instance of Mr. Byrne's active concern with traffic facilities is his advocacy of a vehicular tunnel from the Battery to Hamilton Avenue, in Brooklyn, N. Y., by way of Governors Island. He prepared preliminary plans for this project which was an exceedingly live story in 1939. In 1929, also, he prepared preliminary plans and estimates for a vehicular tunnel under the East River from Manhattan Island at Thirty-eighth Street to the Borough of Queens.

Mr. Byrne was a Consulting Engineer of the New York State Bridge and Tunnel Commission, and of the Camden-Philadelphia Bridge. He was a Past-President of the Society of Municipal Engineers of New York; a Past-President of the Kings County Chapter of the New York State Society of Professional Engineers; a member of the Brooklyn Engineers Club, and of the Catholic Club. He was a trustee of the Greenpoint Savings Bank.

Mr. Byrne was married to Elizabeth Mary Dillon in 1903. She died in 1912, leaving him two young sons. Mr. Byrne did not remarry. His heart and soul was with his children, and he participated in their lives until they were safely through college.

Those associated with Mr. Byrne realized that he had an unusual personality. He possessed power, energy, and a driving force which overcame apparently unsurmountable obstacles. The uncompromising integrity with which he conducted his responsible office was felt throughout his department. Beneath an exterior which years of command and responsibility had rendered somewhat stern, he fostered a kindly, generous, and charitable disposition —always ready with help and sympathy for fellow humans in distress. His engineers will remember the pleasure he derived from the genial luncheon intermissions when out on some job. No matter how serious the business might have been, Mr. Byrne would relax completely and, with a generosity which became proverbial, play the host to his party.

He was far from being a recluse. He was fond of the theater, particularly musical comedy. He delighted in attending football games and other sports. When his sons were at the University of Notre Dame, in Notre Dame, Ind., he attended religiously all the Notre Dame games and was as enthusiastic about them as any under-graduate.

Mr. Byrne was deeply religious and loyally attached to the Roman Catholic Church. Withal, he possessed the gift of social, political, and religious tolerance, characteristic of the best type of New Yorker who has grown up in contact with the populous amalgam of the astounding city.

Mr. Byrne died in Rye, N. Y., on December 6, 1938, at the home of his son Edward. He was buried in Calvary Cemetery, in the Borough of Queens, Long Island, and thus his last ride was over the bridge which played so great a part in the drama of his last years.

He is survived by two sons, Edward and George Dillon; and by two granddaughters, Elizabeth and Barbara Byrne, the children of Edward.

Mr. Byrne was elected a Member of the American Society of Civil Engineers on April 16, 1918.

FRED EDWARD CALDWELL, M. Am. Soc. C. E.[1]

DIED JANUARY 2, 1939

Fred Edward Caldwell was born at Heuvelton, N. Y., on August 9, 1879, the son of Thomas and Ruth (Smithers) Caldwell. He attended the Public School in Heuvelton, and later, from 1897 until 1900, he attended the State Normal School at Potsdam, N. Y. In 1901 he enrolled as a Civil Engineering student in the Clarkson College of Technology at Potsdam, where he studied until 1904. Each summer vacation he was employed as Transitman and Recorder with the United States Lake Survey, first on the St. Lawrence River and later at Green Bay, Wis., on Lake Michigan.

In 1904 Mr. Caldwell was employed by the firm of Williams, Proctor and Potts as Resident Engineer on the construction of sewerage systems and sewage disposal plants at various places, the more important of which were at Havre de Grace, Md., Newton, N. J., and Morristown, N. J. He also assisted in the preparation of plans for sewerage systems for Waverly, N. Y., Leetonia, Ohio, and Chatham, Ridgewood, and Washington, N. J. In 1910 Mr. Caldwell became associated with the Ansonia Manufacturing Company of New York, N. Y., as Designing Engineer on automatic sewage control mechanisms for disposal plants.

In September, 1911, he returned to the practice of sanitary engineering as Chief Engineer to Clyde Potts, M. Am. Soc. C. E., Consulting Engineer of New York City. He remained in this position for eight years, acquiring an intimate knowledge of all projects that passed through the office. Sewerage systems were built at Salem, N. J., West Haven, Conn., Verona, N. J., and Nazareth, Pa. He assisted in the preparation of many reports on proposed projects and prepared plans for sewerage systems for several New Jersey towns in the valley of the Rockaway River above the water-supply reservoir of Jersey City at Boonton, N. J. These included Dover, Rockaway, and Boonton. A trunk sewer was designed to collect the sewage of these towns and convey it to a point below the Jersey City Reservoir where it would receive complete treatment. Another similar project upon which Mr. Caldwell did the early development work and made preliminary plans was a trunk sewer to convey the sewage from the towns along the Rahway River to a disposal plant at tide-water. These towns were Rahway, Cranford, Westfield, and South Orange. This project, with some change in the towns participating, was constructed in the late 1920's.

From March until December, 1918, he was engaged on war work as Sanitary Engineer at the Hammonton, N. J., shell loading plant of the United States Ordnance Department. A complete system of sewers and sewage disposal was constructed, as was also a water-supply and fire-protection system. At the conclusion of the war Mr. Caldwell was made Promotion Engineer on concrete products with the Portland Cement Association which was then developing from a back-yard enterprise, producing a product of questionable merit, to the modern concrete products plant of to-day. Mr. Caldwell remained with the Association until 1922 and it was through his efforts that the first high production automatic machines for the production of both concrete building units and concrete pipe were introduced in the New York area. Until his death all of Mr. Caldwell's efforts were directly or indirectly toward better concrete, new uses for concrete, and the securing of recognition of the merits of concrete products by the trade.

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[1] Memoir prepared by Benjamin F. Batchelder, Assoc. M. Am. Soc. C. E.

In 1922 he acquired an interest in the Springfield Sand and Tile Company, of Springfield, Mass., and became Vice-President and Manager. Through his efforts the use of concrete units was greatly increased in this section by the production of a dependable product, and building codes were soon opened to their use in all classes of construction. Severing his connection with this Company in 1928 he became Sales Engineer in the New York area for the Keystone Portland Cement Company which had just started production of one of the first high early-strength Portland Cements. He remained in this position for two years and secured a wide use of this cement in the concrete products industry.

In 1932 Mr. Caldwell acquired an interest in the Haller Testing Laboratory at Plainfield, N. J., and as Vice-President of the Company continued his interest until a short time before his death. In 1936 the Company organized the New England Testing Bureau with offices at Boston, Mass., to accommodate their rapidly expanding business in that section, and Mr. Caldwell became Vice-President of this subsidiary.

He was married to Bessie M. Harris at Newton, N. J., on February 26, 1908. They had one child who died in infancy.

Mr. Caldwell was a Mason, an Elk, and a member of the Engineers Club of New York City. During his college days he was active in athletics and particularly excelled in football and baseball. He had a wide circle of friends and business associates, all of whom admired and respected him.

Mr. Caldwell was elected a Junior of the American Society of Civil Engineers on April 3, 1906; an Associate Member on February 1, 1910; and a Member on May 12, 1919.

JAMES EDWARD CARROLL, M. Am. Soc. C. E.[1]

Died January 25, 1939

James Edward Carroll was born in Austin, Minn., on August 11, 1870, the son of Patrick Bernard and Margaret (MacMahon) Carroll. He received his High School education in Minneapolis, Minn., and attended the University of Minnesota, in Minneapolis, from 1886 to 1891, completing the Civil Engineering course with the exception of one term.

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[1] Memoir prepared by George M. Shepard, M. Am. Soc. C. E.

Mr. Carroll began his engineering career in 1885 as a Rodman and Draftsman with the late George W. Cooley, who was later State Highway Engineer.

During his university course he was employed as Chainman and Assistant Transitman by Hennepin County, Minnesota; as Timekeeper and Rodman for the Great Northern Railway Company; as Levelman for the Chicago, Milwaukee and St. Paul Railway Company; and as Chief of resurvey crew for the Northern Pacific Railway Company on its Dakota, Montana, and Yellowstone Divisions.

Upon leaving school in 1891 he became Scaler and Assistant Yard Foreman for the N. P. Clarke Lumber Company in Minneapolis. In 1893 he joined the engineering staff of the City Engineer of Minneapolis and began the long and creditable career of municipal engineering, both in public service and private practise, which terminated only a few weeks before his death. Beginning as a Draftsman and Levelman in the Bridge Department, he advanced to the position of Street Paving Engineer.

Mr. Carroll was appointed City Engineer of Crookston, Minn., in 1903, and served in this capacity, as well as in the capacity of Consulting Engineer to adjacent cities, until 1912. In the latter year he was appointed Assistant Commissioner of Public Works of St. Paul, Minn., and served as such until 1914 when the position was abolished. He was then appointed Assistant Chief Engineer of the Department of Public Works of St. Paul and Superintendent of Construction and Repair. He continued in this position until 1923, except for his absence during the World War, and again served in the same position from 1932 to 1934.

Following his resignation from the city service in 1923, he formed a partnership with the late Oscar Claussen, M. Am. Soc. C. E. This partnership, under the name of the Claussen and Carroll Engineering Company, with offices in St. Paul, engaged in a general consulting practise, specializing in municipal engineering matters. This partnership continued until the death of Mr. Claussen in 1932. In 1934 Mr. Carroll formed the James E. Carroll Engineering Company and continued his consulting practise up to within three or four weeks of his death.

In 1917 he entered the Engineer Corps of the United States Army and was commissioned a Major, serving 1 yr in France. He spent the greater part of this time as commanding officer of the 527th Engineer Service Battalion, taking part in the St. Mihiel and Meuse-Argonne offensives.

Major Carroll was a student of municipal engineering and allied subjects, and was the author of articles on " Cost Data—Drainage Engineering and Contracting " and " Tree Planting in St. Paul."

He was interested in the welfare and activities of World War Veterans and was one of the organizers and first commanders of Argonne Post, American Legion, in St. Paul. At the time of his death at the local Veterans Bureau Hospital, he was active in affairs of the Veterans of Foreign Wars.

Major Carroll was extremely fond of hunting, which formed his principal recreation. In the Fall of 1938, only two and one-half months before his death, he made his regular deer hunting trip and as usual " bagged his deer." His activity in public office for so many years and his long, successful practise give evidence of his personal integrity and professional ability. An engineer whose education was interrupted by the necessity of work, and whose early training was obtained by hard experience, Major Carroll through his years of experience acquired a knowledge of municipal problems which made his service invaluable to the communities he served. He was a member of the Catholic Church, and Delta Upsilon Fraternity.

In 1893 he was married to Frances Bayley of Minneapolis. She died in 1917. He is survived by three daughters, Frances, Adelaide, and Agatha.

Major Carroll was elected a Member of the American Society of Civil Engineers on June 23, 1916.

GEORGE B. CHRISTIE, M. Am. Soc. C. E.[1]

Died August 1, 1938

George B. Christie was born in Atlanta, Ill., on September 7, 1862, the son of James Rodney Christie, of Dover, N. H., and Miriam Hunt (Deaver) Christie, of Baltimore, Md. His father was of Scotch ancestry, and his maternal ancestors were among the early settlers of Maryland, who served in the War of the Revolution and the War of 1812. His father organized the 106th Company of Illinois Volunteers and served as its Captain during the Civil War. He was christened George B. McClellan Christie in honor of General George B. McClellan.

Mr. Christie received his early education in the public schools of Atlanta. Later, he entered the University of Illinois, at Urbana, Ill., and finished his Sophomore year in June, 1881. At that time he entered the service of the St. Louis, Fort Scott, and Wichita Railroad Company, in Kansas City, Mo., as Instrumentman on location. He followed railroad location and construction as Engineer in Charge until January, 1884.

He then became Assistant City Engineer of Omaha, Nebr. From February, 1886, to July, 1888, he was a member of the engineering firm of Rosewater and Christie, of Omaha. Victor Rosewater, the Senior Partner, later became a famous newspaper editor. This firm designed and superintended the construction of sewerage systems in Lincoln and Grand Island, Nebr., in Wichita and Atchison, Kans., and in other cities. Its members also made reports on many other enterprises.

In July, 1888, Mr. Christie and the late Jesse Lowe, M. Am. Soc. C. E., organized the firm of Christie and Lowe, Civil Engineers and Contractors, maintaining offices and residence in Chicago, Ill. The members of this firm constructed fifty miles of cable road in Denver, Colo., and in Brooklyn and New York, N. Y. In 1892 and 1893, they served as Engineers and Contractors for an underground electric railroad in Washington, D. C. Afterward, they built the masonry piers of the Bellefontaine Bridge for the Wabash Railroad Company, across the Missouri River near Bellefontaine, Mo. They also built

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[1] Memoir prepared by A. D. Millard, Esq., Beardstown, Ill.

Sections I and K of the Chicago Drainage Canal, this being an exceptionally heavy job of rock cutting for which Mr. Christie designed special machinery and appliances.

For a number of years the firm of Christie and Lowe also included the late James O. Heyworth, M. Am. Soc. C. E., of Chicago (who, later, became a Contractor in his own right and was highly successful), and the late George A. Lederle, M. Am. Soc. C. E. Among the works performed by Christie, Lowe and Heyworth were the harbor works at Sabine Pass, Tex., and the Bear Trap Dam, on the Chicago Sanitary District Canal, at Lockport, Ill., which, however, was a part of the original contract of Christie and Lowe with the Sanitary District.

The work at Sabine Pass consisted of building a sea-wall at the mouth of the Sabine River where it empties into the Gulf of Mexico. This contract amounted to more than $1 000 000. The firm of Christie, Lowe and Heyworth also built four locks on the Black Warrior River, near Tuscaloosa, Ala. After the Sabine Pass contract was completed, the plant was shipped to Fernandina, Fla., where a contract was executed to build jetties at the mouth of the St. Marys River, for the United States Government. This was a contract for $1 500 000. The Sabine Pass work was finished in September, 1900, and the Fernandina work in the summer of 1904. Offices were established in 1904 at New Orleans, La., and during the succeeding period of eight years the firm built the sea-walls at Fort Gaines and Fort Morgan, in Alabama. It also built the jetties at the Southwest Pass, at the mouth of the Mississippi River, the latter work amounting to about $6 000 000. This particular piece of work is a monument to the skill and ability of Mr. Christie and his associates.

Mr. Lederle died during the period that the firm's offices were in New Orleans, and Mr. Heyworth withdrew from the firm when the Fernandina work was finished, thus leaving the partnership of Christie and Lowe as it was originally. The jetty work at the mouth of the Mississippi River, at Southwest Pass, was the last large work performed by Christie and Lowe, after which the office was moved to Beardstown, Ill., to develop the firm's large land holdings there. After the death of Mr. Lowe, in 1918, Mr. Christie continued to reside in Beardstown giving his time and attention to these interests until about 1932, when he moved to Ottawa, Ill., and from there to Mobile, Ala., where his death occurred.

Mr. Christie was a delightful companion, an exceptionally capable engineer, and a man of much executive ability. He was widely read not only in engineering but also in the literary Classics, particularly in the English essays. His range of knowledge was wide and accurate. It was a rare pleasure to spend an evening with him and listen to his descriptions of men and affairs. His humor was keen and his characterizations were accurate. The work he performed as Engineer and Contractor was well done.

He was married to Myrthé Baquié, on January 14, 1908. He is survived by his widow, two brothers, W. H. Christie and James Kent Christie, and a sister, Caroline Christie.

Mr. Christie was elected a Member of the American Society of Civil Engineers on October 2, 1895.

BERNARD LABROQUERE CROZIER, M. Am. Soc. C. E.[1]

Died July 23, 1938

Bernard Labroquere Crozier was born in Baltimore County, Maryland, on February 8, 1887. He attended the public schools in Baltimore, Md., including the Baltimore City College. In 1906, he completed the third year in the Alabama Polytechnic Institute, Auburn, Ala. Mr. Crozier received the degree of Bachelor of Laws from the Baltimore Law School in 1911, and was admitted to the Maryland State Bar in the same year. He attended the night classes at the Johns Hopkins University, Baltimore, in 1916, 1917, 1919, and 1920.

Mr. Crozier's first employment was with the Baltimore and Ohio Railroad Company, as Rodman, from March 7 to April 7, 1906. From September, 1906, to January, 1909, he served as Rodman and Instrumentman with the United Railways and Electric Company, of Baltimore; and, from August 12, 1909, to July 17, 1912, he was employed as an Inspector with the Maryland State Roads Commission.

Mr. Crozier was an Assistant Engineer with the Paving Commission of Baltimore, from July 18, 1912, to February 20, 1918, and, during the World War in 1918, he was Assistant Superintendent of Shops with the Bartlett-Hayward Company of Baltimore, which Company was engaged in making munitions of war.

When the Paving Commission of Baltimore resumed its paving program on February 17, 1919, he was appointed Office Engineer, and held that position until June 12, 1923, when he was appointed Highways Engineer of Baltimore. On July 1, 1925, when the Department of Public Works was created, he was promoted to the position of Chief Engineer. By virtue of this office, he was a member of the Board of Estimates and a member of the Public Improvement Commission of the City of Baltimore.

Due to a change in administration in Baltimore, on October 1, 1927, a new Chief Engineer was appointed, and Mr. Crozier then opened an office as Consulting Engineer. As such he designed and supervised the construction of several highway bridges for the City of Baltimore.

On April 11, 1929, he was appointed Chief Engineer of the Metropolitan District of Baltimore County, Maryland, which position he held until he was again, on June 15, 1931, appointed Chief Engineer of the Department of Public Works, which position he held at the time of his death. This Department embraces eleven bureaus, covering all the engineering work of the City, except that of the Park Board.

During Mr. Crozier's terms as Chief Engineer, many large public improvements were planned and constructed. He was highly esteemed as an efficient public servant, and his loss is deeply regretted by his many friends.

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[1] Memoir prepared by Frank K. Duncan, M. Am. Soc. C. E.

On December 6, 1913, he was married to India J. Yewell, who, with a sister, Mrs. James Arthur, of Houston, Tex., survives him.

He was a member of the Engineers Club of Baltimore and President of the Maryland Association of Engineers at the time of his death. He was a York and Scottish Rite Mason and a Noble of the Mystic Shrine.

Mr. Crozier was elected an Associate Member of the American Society of Civil Engineers on January 19, 1920, and a Member on March 15, 1926.

EDGAR DUDLEY CRUISE, M. Am. Soc. C. E.[1]

DIED JUNE 5, 1938

Edgar Dudley Cruise was born on February 22, 1867, at Wyandotte (later Kansas City), Kans. He was educated in the Public Schools of that city and matriculated at the University of Kansas, at Lawrence, Kans., in September, 1883, at which time the writer's lifelong friendship with him began.

He left college in the Spring of 1887, after the completion of more than three years' work in the College of Engineering, and entered the service of the Missouri Pacific Railroad Company, expecting to return to the University for his final semester, but as promotion followed promotion, he never did receive his diploma.

He was employed steadily from May, 1887, until March, 1900. Rodman, Instrumentman, Resident Engineer, Engineer for Contractors on bridge construction, and several years of service for Armour and Company in various capacities, including the position of branch manager—these engagements constituted the various steps in his post-college training for his life work of location and construction.

From March, 1900, to March, 1909, he was on railroad location, continuously, for the Choctaw, Oklahoma and Gulf Railroad Company in Oklahoma, the Kansas City, Mexico and Orient Railway Company of Texas, and for nearly seven years of this period with the Southern Pacific Railroad Company in Arizona and Mexico. He was in full charge of location of the lines of the latter company in Mexico.

From March, 1909, to November, 1916, he was back on construction work as Division Engineer on the Kansas City, Mexico and Orient Railway in Texas. During this period he was also in charge of an irrigation project and dam in Southwest Texas, and served as construction engineer of the Teton Basin Branch of the Oregon Short Line Railway and of 70 miles of the Salt Lake and Utah Railway.

In November, 1916, Mr. Cruise became principal locating engineer of the Union Pacific Railroad Company with headquarters at Omaha, Nebr. He retained that position until November, 1918, when he turned from railway work to the location and construction of the rapidly developing highway system of the United States.

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[1] Memoir prepared by Henry E. Riggs, Past-President, Am. Soc. C. E.

For four years he was Resident and District Engineer for the Tennessee Highway Department; then for three years he was Principal Locating Engineer for the North Carolina Highway Commission, after which he was called back to Mexico as Principal Locating Engineer and Chief Engineer for the Comision Nacional de Caminos. From October, 1925, to January, 1927, he was Principal Locating Engineer and Chief Engineer on construction of 600 miles of highway between Laredo, Tex., and Puebla, in Mexico, and for the next two years was engaged on highway studies in Central America.

On the completion of this work he returned to the United States and for about two years was at his boyhood home in Kansas City. In February, 1931, he entered the service of the New Mexico State Highway Department and was with that organization continuously for eight years until his death, caused by an automobile accident, on June 5, 1938.

The foregoing sketch of fifty busy years shows clearly the fact that his work was largely pioneering in new and partly developed areas, and that he was an out-of-doors man—literally, an " engineer at the front " during his entire life.

The writer's friendship with him extended over the more than half a century from his entrance in the freshman class of the University of Kansas and his joining Phi Gamma Delta Fraternity until his death.

Two months before Mr. Cruise's death, the writer attended a meeting of the New Mexico Section at Albuquerque. Edgar Cruise, Life Member, with his wife, drove 260 miles to the meeting, and he and the writer spent a most enjoyable half-day together. In 1936 it was the writer's privilege to spend a week in New Mexico and to go with him over his work then in progress. Charles M. Upham, his chief on the North Carolina work, has stated that " his work was always of outstanding quality."

Mr. Cruise was a man of few words, loyal to his friends, loyal to his employer and to the men under him, and faithful to the job, whatever or wherever it was. He made many good friends wherever he worked and was esteemed by all who knew him as an excellent engineer and a man of fine personality and high ideals.

He was married June 6, 1894, at Topeka, Kans., to Sarah Corinna Wharry. They had one son, John D. Cruise, M. Am. Soc. C. E., a graduate of the Engineering College of the University of Michigan, at Ann Arbor, Mich.

The writer will always carry the picture of Edgar Cruise as he was in April of 1938, a handsome, upstanding man whose outdoor life had given him the bronze of perfect health, and whose more than seventy years was indicated only by his snow-white hair; a man who still had the modesty and apparent shyness of the sixteen-year-old boy of years gone by, and with the same friendliness and hearty hand-clasp. That last picture of the dear old friend will never fade. It was his good fortune to be " in harness " until the final call came. In perfect health, and with joy in his work, he could look back on long years of splendid service in the task of making America a better place in which to live.

Mr. Cruise was elected a Member of the American Society of Civil Engineers on June 4, 1913.

BENJAMIN HERMAN DAVIS, M. Am. Soc. C. E.[1]

Died August 17, 1927

Benjamin Herman Davis, the son of Dr. Benjamin Byron Davis and Sophie (Shultz) Davis, was born in Palmyra, Ohio, on October 15, 1882. He received his early education in the public schools of Palmyra, and his engineering education at the Case School of Applied Science, at Cleveland, Ohio, which, in 1905, conferred upon him the degree of Bachelor of Science in Civil Engineering. He was a member of the Alpha Tau Omega Fraternity.

After his graduation Mr. Davis served for a short time as Laboratory Assistant in the Department of Testing Materials of the Osborn Engineering Company, at Cleveland. From September, 1905, to March, 1906, he was employed as Draftsman on the design and detail of steel and reinforced concrete structures in the Bridge Department of the Lake Shore and Michigan Southern Railway Company. In this latter position, he laid the groundwork for his future career.

In March, 1906, Mr. Davis left the employment of the Lake Shore and Michigan Southern Railway Company to become Engineer of Concrete Structures for the Delaware, Lackawanna and Western Railroad Company, at Hoboken, N. J., where he continued to serve until April, 1910, when he resigned his position to enter private practice. It was while employed by the Lackawanna Railroad Company that Mr. Davis became an enthusiast in the promotion, design, and construction of concrete structures. It was also at this point in his career that he made close friendships with Lincoln Bush, Past-President and Hon. M. Am. Soc. C. E., George T. Hand, M. Am. Soc. C. E., and others, who were to be of great assistance to him in his private practice as a Consulting Engineer in concrete construction. During Mr. Davis' employment with the Lackawanna Railroad Company he had charge of the construction of more than one hundred concrete bridges, the most important of which were the Paulin's Kill Viaduct, at Hainesburg, N. J., and the Delaware River Bridge, at Slateford, Pa.

From May, 1910, to the time of his death, Mr. Davis continued in private practice as a Consulting Engineer on the design and construction of all types of concrete structures. To enumerate all the structures with which he was connected during this period would require too much space. Therefore, only a few of the more important ones will be mentioned, such as the Penn Street Bridge over the Schuylkill River, at Reading, Pa., for Berks County, Pennsylvania; the Market Street Bridge over the Tennessee River, at Chattanooga, Tenn., for Hamilton County, Tennessee; Central Bridge over the Merrimac River and Essex County Canals, for the City of Lawrence, Mass.; Seventh Street Bridge over the James River, Norfolk and Western, Chesapeake and Ohio, and Southern Railroad tracks for the Norfolk Western Railway Company, at Lynchburg, Va.; the Washington Memorial Bridge over

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[1] Memoir prepared by George J. Ray and George L. Dresser, Members, Am. Soc. C. E.

Brandywine River, at Wilmington, Del., for New Castle County, Delaware; the Great Western Gateway Bridge over the Mohawk River, at Schenectady, N. Y., for the State of New York; and the Market Street Bridge over the Susquehanna River, at Wilkes-Barre, Pa., for Luzerne County, Pennsylvania.

During the World War, Mr. Davis became interested in the design and construction of concrete ships for the United States Government. He devoted considerable time and energy toward the development of this type of carrier. After the war, he became interested in the design and erection of lofty, reinforced concrete memorials to World War heroes.

During his career, he also acted as Consulting Engineer and Designer in the construction of several trainsheds for railroad companies and as a Consulting Engineer and Designer of several reinforced concrete buildings.

Mr. Davis had a most pleasing personality. He possessed an enthusiasm and knowledge in the design and construction of concrete structures, which was infectious and inspired confidence, not only among his associates, but among all with whom he came in contact. He was a tremendous worker, and never during his career as a Consulting Engineer did he permit a design or specification to leave his office until he had checked it carefully, and made himself entirely familiar with every detail. To every job under Mr. Davis' supervision, whether large or small, he gave of his great enthusiasm and knowledge of detail. He had a rare combination of engineering ability and salesmanship. His technical opinions and designs were always sound, and his integrity was always above question.

On July 28, 1918, Mr. Davis was married to Mabel DeWees, at New York, N. Y., and to them were born the following children: Benjamin Herman, Marjorie Angela, Dorothy Jeanne, and Joanne Louise, who, with his widow and mother, survive him.

He was a member of the American Association of Engineers, the American Concrete Institute, the Albany Society of Engineers, the American Society for Testing Materials, and the Engineers Society of Pennsylvania.

Mr. Davis was elected an Associate Member of the American Society of Civil Engineers on November 30, 1909, and a Member on October 15, 1923.

RALPH GRAY DAVIS, M. Am. Soc. C. E.[1]

DIED FEBRUARY 21, 1939

Ralph Gray Davis, the son of Alfred and Ada (Gray) Davis, was born in Elkton, Md., on July 23, 1888. He received his elementary education in the Elkton Public Schools and then entered the University of Delaware at Newark, Del., from which he was graduated in 1911 with the degree of Bachelor of Science in Civil Engineering. In 1930 he was awarded an honorary degree in Civil Engineering by the same university.

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[1] Memoir prepared by Forrest E. Byrns, Assoc. M. Am. Soc. C. E.

Immediately after his graduation, Mr. Davis accepted a position as Recorder and Surveyman in the United States Engineer Office at Wilmington, Del. He was promoted to Inspector the following year and continued in that capacity until December 28, 1917, when he resigned to accept a commission as Second Lieutenant in the 57th Engineers, with which organization he served his country overseas from June 30, 1918 to July 30, 1919.

After his return from France, Mr. Davis again entered the Civil Service in the Wilmington Engineer District, with the grade of Junior Engineer. His principal work in the several years following was in connection with the reconstruction of the Chesapeake and Delaware Canal, a $10 500 000 project, for which he had charge of all surveys and later the design and preparation of construction drawings.

He was promoted to Assistant Engineer in 1926, to Associate Engineer in 1929, and to the grade of Engineer in 1930. At that time he was assigned as Principal Civilian Assistant to the District Engineer and placed in responsible charge of all engineering work in the Wilmington District.

In the River and Harbor Act of 1930, Congress provided for the organization of the Beach Erosion Board to make special studies throughout the various States and territories with a view to devising effective means of preventing shore erosion. It was the opinion of Earl I. Brown, M. Am. Soc. C. E., a recognized authority on the subject, that Mr. Davis' work on the Chesapeake and Delaware Canal and his interest in studies pertaining to its hydraulic characteristics and in studies relating to other coastal improvements, placed him foremost among those engineers available for duty with the Board.

Accordingly, Mr. Davis was prevailed upon to accept a position on the Board's staff, in the office of the Chief of Engineers, Washington, D. C. On July 6, 1931, he assumed duties as Senior Assistant to the Resident Member and was placed in charge of all field work of the Board and the preparation of its technical reports. In 1935 he was promoted to the grade of Senior Engineer. While on official duty, starting on an inspection trip in April, 1934, he suffered severe injuries in an automobile accident from which he never fully recovered and which ultimately caused his death.

Mr. Davis was a close student of tidal hydraulics and of shore processes and shore-line development. His experience and sound judgment on problems dealing with the protection and preservation of beaches were of inestimable value to the Beach Erosion Board. His quiet, unassuming manner and cheerful personality endeared him to all of his associates. His principal interest outside of his family and his work was in the science of photography which, to him, was a fine art as well as an absorbing hobby.

On January 26, 1916, Mr. Davis was married at Elkton to Margery E. Marshbank of that city. He is survived by his widow and their two children, Margery M., and Ralph Gray Davis, Jr., and a brother, A. Victor Davis.

He was a member of the Society of American Military Engineers, the American Legion, and several organizations of the Masonic Order, including St. John's Commandery No. 1, Knights Templar, of Wilmington, and the Boumi Temple Shrine.

Mr. Davis was elected a Member of the American Society of Civil Engineers on February 10, 1930.

JAMES HERVEY DINGLE, M. Am. Soc. C. E.[1]

Died June 3, 1938

James Hervey Dingle, the son of Gabriel Wesley Dingle and Josephine (von Siebold) Dingle, was born in Charleston, S. C., on September 9, 1869. He was educated in private schools and at the High School of that city; and, in 1888, he received the degree of Bachelor of Arts from the College of Charleston. Later, he entered Cornell University, at Ithaca, N. Y., from which he was graduated in 1892 with the degree of Civil Engineer.

After his graduation Mr. Dingle returned to Charleston, and, for six months, was engaged in the private practice of his profession. In January, 1893, he was made Assistant City Civil Engineer, which office he held until January 1, 1894, when the title of office was changed and he was made City Surveyor, which position he filled from 1894 to 1905, when the title of office was again changed to that of City Engineer. On January 1, 1905, he was elected City Engineer, which position he held until his death.

During the forty-five years that Mr. Dingle served the City of Charleston, he had charge of the repaving of the entire city, substituting modern paving for dirt, clay, oyster shells, and cobble-stones; the design and installation of a modern sanitary sewerage system; the enlargement of the storm-water drainage system; building of the Murray Boulevard and sea-wall; construction of a modern incinerator; and, more recently, the construction of an aviation field, and during 1937 the construction of a sea-plane base in anticipation of Trans-Atlantic Service. These engineering accomplishments will be lasting memorials to his efficiency, fidelity, engineering ability, and service to his fellow men and to the City.

Mr. Dingle's duties as Head of the Department of Public Service of the City of Charleston included the supervision, construction, improvement, and maintenance of all streets, and the supervision of garbage collection and disposal. He was also Engineer for the Zoning Administration.

With but one exception (a member of the Fire Department), Mr. Dingle, in point of service, was the oldest employee of the City of Charleston, and had the honor and distinction of having served continuously under seven Mayors of the city.

In addition to his professional work, Mr. Dingle found time to devote to many public interests. He was an officer in the Washington Light Infantry, rising to the rank of Captain. He was Past-Master of Pythagorean Lodge, A. F. M., and a Noble of Omar Temple, Imperial Council, A. A. O. N. of the M. S., for North America. He was a member of the Knights of Pythias, the German Friendly Society, the Alumni Association of the College of Charleston, the Cornell Association of Civil Engineers, and the American Society of Municipal Engineers. This latter Society, in 1931, awarded him its medal for continuous service with one employer—45 yr with the Municipal Government of Charleston. He was also a member of the Public Works Association, and the American Society for Testing Materials. He was one of the Charter Members of the South Carolina Society of Engineers, and the South Carolina Section of the American Society of Civil Engineers, and was Past-President of both these organizations.

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[1] Memoir prepared by a Committee of the South Carolina Section, consisting of J. E. Gibson and L. S. LeTellier, Members, Am. Soc. C. E.

He was a member of St. John's Lutheran Church and served as a member of its Board of Vestry and Wardens from 1894 to 1898, and again from 1917 to his death. In 1931, he was made President of the Congregation, which office he held at the time of his death. A resolution passed by the Board of Vestry and Wardens and the Congregation upon his death, reads as follows:

> "Mr. Dingle, by his Christian faith and fine devotion; by his gentlemanly culture and tactful courtesy; by his clear interpretation of parliamentary law; by his keen appreciation of the splendid traditions of St. John's Church; and by his impartial decisions at our Board and Congregational meetings, made justice and good will to prevail, and the memory of our personal and official association with him shall long be cherished as one of life's choicest privileges."

Mr. Dingle also served on The Board of Education of The United Lutheran Church in America from 1926 to 1932.

In 1899, he was married to Olive Bartlett Olney of Charleston, who, with one daughter, Josephine von Siebold Dingle, and a sister, Mrs. John Rutledge Dingle, survives him. His only son, James H. Dingle, Jr., died in early youth.

Mr. Dingle was elected a Junior of the American Society of Civil Engineers on April 30, 1895; an Associate Member on October 4, 1899; and a Member on November 28, 1916. He became a Life Member on January 1, 1934.

VICTOR GELINEAU, M. Am. Soc. C. E.[1]

Died January 21, 1938

Victor Gelineau was born in Lowell, Mass., on January 21, 1886, the son of Cesaire and Mary (Lemieux) Gelineau. His technical training was obtained in the College of Civil Engineering at Cornell University, Ithaca, N. Y. He also held a Law degree which he received in 1922 from New York University, at New York, N. Y.

Mr. Gelineau began the active practice of engineering at Atlantic City, N. J., as Draftsman, Transitman, and Computer on general surveying and municipal work; then, shortly thereafter, as First Assistant in charge of all field work and design for the municipal work of Hammonton, Egg Harbor City, and South Atlantic City, in New Jersey, including drainage systems, roads, pavements, bulkheads, bridges, and ocean-front board walks; hydraulic dredging and land reclamation; and water power and water supply surveys. His work also included surveys and the sub-division of large tracts of land; water power, and water supply surveys; plans and estimates for Mays Landing (N. J.) Power Company, and plans for a channel and bridge connecting Atlantic City and Brigantine Beach, in New Jersey. By this early experience in work on the South Jersey waterways, he developed a keen interest in the waterway and coastal problems and the possibilities for the development of these areas of the State for recreational and residential uses.

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[1] Memoir prepared by a Committee consisting of Charles W. Staniford, M. Am. Soc. C. E., *Chairman*, J. Spencer Smith, Pres., Board of Commerce and Nav., State of New Jersey, Tenafly, N. J., and Robert Simon, Office Engr., Board of Commerce and Nav., State of New Jersey, Newark, N. J.

In 1910 and 1911, Mr. Gelineau was Assistant Chief Engineer in charge of field work and design of sub-division, road location and construction, surveys, and designs for bridges, and bulkheads and board walks, for an extensive land development at Tangier, Long Island, N. Y.

In 1912, he entered the service of the State of New Jersey, in the Department of Inland Waterways, as Draftsman, Computer, and Field Assistant on topographic and hydrographic surveys, hydraulic channel dredging work in connection with the construction of the Inland Waterway Channel extending along the New Jersey coast from Cape May to Bay Head, and the construction of the Barnegat Bay-Manasquan River Canal.

From 1916 to 1918 he was engaged on surveys for the proposed New Jersey Ship Canal from Delaware River to Raritan Canal and in charge of surveys and investigations of unauthorized occupations of the State's lands under water for the Board of Commerce and Navigation of New Jersey.

Mr. Gelineau was appointed Acting Director of the Board of Commerce and Navigation in May, 1918, and Director and Chief Engineer in November, 1918, which office he held until the time of his death.

As Chief Administrator of the Board he devoted his talents and ability to the best interests of the State, and gave conscientious service to the public which won for him the undeniable respect of all those with whom he came in contact. He had an abiding interest in his work and pursued the details of the many functions of the Board from both the engineering and legal standpoints. As Director of the Board, Mr. Gelineau prepared many of its reports, notably the several reports on the erosion and protection of the New Jersey beaches and the preliminary report on the reclamation of the Hackensack (New Jersey) Meadows.

He made a thorough study of riparian law and its application. He was regarded as an expert in this complex and specialized subject, and his counsel on riparian questions was sought and accepted by engineers engaged on waterfront structures, and leading legal authorities. He was the author of the monograph, " Riparian Rights—Certain Principles of the Law and Its Application."

Perhaps his outstanding contribution to his profession is his work on, and study of, beach erosion and protection. In this field he was considered the ranking expert in the United States, if not in the world. Under his leadership, the State of New Jersey was the first State to recognize the need for a comprehensive plan for beach protection to be undertaken by the community and the State, rather than the individual shore owners. His designs of sea walls, jetties, groins, and bulkheads, have withstood the test of time and ravages of the sea, and have proved their effectiveness in stabilizing and building up the beach. He devised the successful methods for preventing the erosion that had threatened the destruction of Barnegat Light.

In 1930, Mr. Gelineau was appointed one of the three civilian members of the United States Beach Erosion Board. As a member of this Board, he inspected and familiarized himself with the problems of shore protection along the entire Atlantic and Pacific seaboards. He was consultant on beach protection projects in Florida and in Long Island, New York.

Mr. Gelineau made frequent trips to Washington, D. C., representing the State of New Jersey, to urge the improvement of its waterways. He had attended the Thirty-Third Annual Convention of the National Rivers and Harbors Congress, at Washington, and was returning home when sudden death, on his fifty-second birthway, cut short a remarkable career.

Mr. Gelineau was Secretary and Treasurer of the American Shore and Beach Preservation Association, a member of the National Society of Professional Engineers, New Jersey State League of Municipalities, American Planning and Civic Association, The Canadian Society of New York, Cornell Club of New York, and the American Power Boat Association. He was a Past-Secretary of the American Association of Port Authorities and a Past-President of the Society of Terminal Engineers. On January 15, 1938, he was inducted into office as President of the New Jersey Association of Professional Engineers and Land Surveyors. He was also a member of the Masonic Order.

Victor Gelineau was fully conscious of the obligation of citizenship, and participated in community and State activities. He was liberal in his thinking and the giving of his time for worthy causes. He believed sincerely that public office was a public trust, and acted accordingly. That he carried into his daily life his beliefs is attested by the unusually large number of tributes paid to his memory by those who came in contact with him, whether officials in high places, governing bodies, or individuals. A great public servant and a real gentleman has passed on.

The following is a resolution adopted by the Metropolitan Section of the American Society of Civil Engineers:

> " *Whereas,* the late Victor Gelineau was for many years an active member of the Metropolitan Section of the American Society of Civil Engineers and at the time of his death was a valued member of its Board of Directors, be it
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> "*Resolved,* that the Board of Directors of the Metropolitan Section, American Society of Civil Engineers, hereby expresses its deep sorrow at the loss of so eminent a member, and be it
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> "*Further Resolved,* that these resolutions be entered in the minutes of the Board and a copy thereof transmitted to Mrs. Gelineau, with the heartfelt sympathy of his former associates."

A memorial service was held on April 19, 1938, in the auditorium of the Department of the Interior Building, in Washington, D. C., in memory of the several Directors of the American Shore and Beach Preservation Association who had died. Richard K. Hale, Assoc. M. Am. Soc. C. E., of Massachusetts, paid a glowing tribute to the achievements of Victor Gelineau. Arthur S. Tuttle, Past-President, Am. Soc. C. E., presided. Captain Robert

W. Workman, Chaplain, U. S. Navy, offered the Invocation and pronounced the Benediction.

Many resolutions were adopted on the death of Mr. Gelineau by various Government, State, city and civic bodies, including the following: U. S. Beach Erosion Board, National Rivers and Harbors Congress, Harbor Control Board of Atlantic Highlands, N. J., New Jersey State Board of Commerce and Navigation, New Jersey Association of Professional Engineers and Land Surveyors, Society of Terminal Engineers, City of North Wildwood, N. J., South Jersey Port Commission, and American Shore and Beach Preservation Association; and many personal expressions of sympathy were received from various parts of the United States.

Mr. Gelineau wrote numerous reports and technical papers relating to coast protection problems and the waterways of New Jersey, among which are the following: "Particulars of the Administrative Powers and Functions of the New Jersey Board of Commerce and Navigation"; "Hudson's Matchless Waterfront and the State's Great Waterways"; "Port Planning and Improvement—Certain Engineering and Legal Considerations"; "Coast Protection with Related Principles"; "Recent Construction and Improvement of Activities on the New Jersey Waterfront"; "Coast Protection and the Public Interest"; "Save the Band of Golden Beaches"; "Economics of the Coast"; "Barnegat Lighthouse"; "The Work of the United States Beach Erosion Board and Study of Erosion"; "Principles and Present Practice of Sandy Seacoast Protection"; "New Jersey Coast Protection Problems"; "Surveys on the Coast of New Jersey"[2]; "Coast Protection Problems in the Metropolitan Regions"; "The Accomplishment of Beach Preservation in New Jersey"; "Hampton Problems" (Hampton Beach, N. H.); "State's Interest in the Preservation and Improvement of Sea Beaches"; "Report on Reclamation of the Meadows and Improvement of Adjacent Waterways within the New Jersey Portion of the Port of New York"; "Damage to New Jersey Coast Resulting from the Storm of 1932"; and, "Special Report of New Jersey State Board of Commerce and Navigation to Governor and Legislature of New Jersey * * * Summary of Riparian Rights, Power Vessels on Inland Lakes, Inland Waterways, New Jersey Ship Canal, Delaware and Raritan Canal Abandonment, Coast Protection, Public Parks."

Mr. Gelineau was elected a Member of the American Society of Civil Engineers on September 10, 1923.

WILLIAM TILLOTSON GOULD, M. Am. Soc. C. E.[1]

Died February 11, 1939

William Tillotson Gould was born at Paducah, Ky., on December 5, 1858, the son of Captain John Vinson Gould, Civil Engineer (born at Stonington,

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[2] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1501.

[1] Memoir prepared by W. H. Kirkbride, M. Am. Soc. C. E.

Conn.), and Mira Rebecca (Bronson) Gould, of Woodford County, Kentucky. William Gould was graduated from the High School at Paducah in June, 1876, having also studied under his father in the field and in the office from 1872 to 1876.

For four years immediately after graduation, he served as City Engineer for Paducah, engaged on engineering surveys, construction, and drainage work. During this period he furthered his education by self study.

From 1880 to 1885 he was engaged in various engineering capacities from Leveler to Resident Engineer for the Northern Pacific Railroad Company, on railroad location and construction work on various parts of the system.

From June, 1886, to April, 1902, he was engaged, principally as Resident Engineer, on location, construction, and maintenance, by several railroad companies in the Central and Southern States. For eight months of this period he was employed by the Northern Pacific Railroad Company as Resident Engineer on the construction of a branch railroad in the Cascade Mountains in Washington.

From April, 1902, to June, 1904, Mr. Gould was engaged as Assistant Engineer with the Brooklyn Rapid Transit Company in connection with surveys, plans, and estimates for elevated railroad and tunnel connections on the East River Bridge and under the East River, the elimination of grade-crossings on the Brighton Beach Railroad, and special work on surface lines in Brooklyn, N. Y.

From July, 1904, to April, 1911, he was engaged as Assistant Engineer by the Pennsylvania Railroad Company in connection with the construction of railroad terminals at New York, N. Y., and environs. From that time until June, 1913, he served as Expert Aide with the United States Navy in connection with the construction of Dry Dock No. 4, New York Navy Yard, until its completion; thereafter he was in charge of the erection of large fuel oil, gasoline, and water station facilities at the United States Naval Station at Guantanamo, Cuba. The next three years he was Chief Engineer, and also Director, of the Nevada-California-Oregon Railway Company.

In May, 1917, Mr. Gould entered as a student at the United States Training Camps at Madison Barracks, New York, and on June 28, 1917, he was commissioned Captain, U. S. Army, Engineer Officers Reserve Corps. He went overseas in November, 1917, with the American Expeditionary Forces, and was in charge of the erection of large steel gasoline storage tanks, handling and distributing plants at Blaye, Furt, and St. Loubes, France.

Mr. Gould returned to America early in 1919 and in April resumed his former position as Chief Engineer of the Nevada-California-Oregon Railway —which railway later was made a part of the Southern Pacific Lines—and on March 31, 1930, having reached the age limit, he was retired and pensioned by the latter company.

Mr. Gould had a high sense of responsibility and devotion to duty and all work entrusted to him was completed with painstaking care. He was a skillful and accomplished engineer and a man of high principles and integrity.

He was a member of The Society of American Military Engineers, Companion: Military Order of the World War (life member). He was also an active member of the American Legion; and of the Engineers' Club, of St. Louis, Mo. (life member).

Mr. Gould died on February 11, 1939, at Hastings-on-Hudson, N. Y. He is survived by his widow, Mrs. Malvina Sassarans Gould, to whom he was married on January 1, 1895; and by a son, Mr. William T. Gould, Jr.; and a daughter, Mrs. E. F. Howrey.

Mr. Gould was elected a Member of the American Society of Civil Engineers on January 2, 1890.

AMBROSE GOULET GRANDPRÉ, M. Am. Soc. C. E.[1]

Died April 24, 1938

Ambrose Goulet Grandpré was born on July 31, 1883, at Kankakee, Ill., the son of Victor and Ida (Goulet) Grandpré. His father was born in Canada but grew up in Bourbonnais, Ill. The family later moved to Chicago, Ill., and Ambrose attended the Public and High Schools there and was graduated from West Division High School. He attended the University of Illinois, at Urbana, Ill., and was graduated in 1906 with the degree of Bachelor of Science in Civil Engineering.

After his graduation he was employed by the Chicago, Milwaukee and St. Paul Railroad Company in Montana and Washington. In 1909 he entered the employ of Marshall and Fox, Architects, in Chicago, as a Superintendent of construction. In this capacity he supervised construction of the Blackstone Hotel, Morrison Hotel, Chicago Burlington and Quincy Railroad Company Office Building, Federal Life Insurance Company Building, Woods Theater and Office Building, Lyon and Healy Building, and others.

In 1917 he became Construction Engineer for Longacre Engineering and Construction Company, in which capacity he had charge of construction of State-Lake, Apollo, Selwyn-Harris Theaters, Trianon Ball Room in Chicago, and Capital Theater in Detroit, Mich.

In 1923 he became a Vice-President of the firm of Bierd, Lydon and Grandpré, and in 1926 he was made President of the company. The company specialized in railway and industrial construction.

Some of the representative work done by this company under Mr. Grandpré's direction consisted of a foundry for Pettibone-Mulliken Company, at Chicago; a Terminal for the Missouri, Kansas and Texas Railroad Company at Denison, Tex.; Roundhouse for the Nickle Plate Railroad Company at Frankfort, Ind.; Roundhouse and Freight Terminal, at Battle Creek, Mich.; Sewage Disposal Plant, at Springfield, Ill., and Elyria, Ohio; Water Filtration Plant at Kenilworth, Ill.; and the Arlington Hotel at Binghamton, N. Y.

The Bierd, Lydon and Grandpré Company ceased operation in 1930, and in 1933 Mr. Grandpré went into business for himself, specializing in the installation of pipe work on dams, sewage and filtration plants, and power houses. He continued in this work until his death on April 24, 1938.

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[1] Memoir prepared by Meyer Fridstein, M. Am. Soc. C. E.

He was married in Tekoa, Wash., on April 14, 1909, to Eleanor Harris who, with three children, Lorraine, Anonarose, and Ambrose Robert, survives him.

Mr. Grandpré was elected an Associate Member of the American Society of Civil Engineers on December 3, 1912, and a Member on January 19, 1925.

JOHN HENRY GRAY, M. Am. Soc. C. E.[1]

Died August 5, 1938

John Henry Gray, the son of James and Cordelia (Lyons) Gray, was born on May 26, 1865, at Watkins Glen, N. Y. His father was born in Scotland, and his mother was of Dutch descent. He attended the Cook Academy at Montour Falls, N. Y., and in 1883 he entered the Rensselaer Polytechnic Institute at Troy, N. Y., and was graduated in June, 1887, with the degree of Civil Engineer.

After graduation Mr. Gray served, for a time, with a firm of consulting engineers on sewage disposal plants and water works. In November, 1887, he entered the services of G. W. G. Ferris and Company of Pittsburgh, Pa., designing and inspecting steel for bridges and buildings. He continued in this type of work for the remainder of his professional life. In 1890 he was First Assistant Engineer for the Ferris Company. At the end of that year he was engaged by E. Duque Estrada and Edward H. Kenyon to form the engineering firm of Estrada, Kenyon and Gray with offices in Chicago, Ill., Pittsburgh, and Philadelphia, Pa.

It was at this time that he patented the building column which bears his name. It is an open section that can be painted inside and outside. The design results in economy of material, and is such that pipes and conduits can readily be installed within its area. It was used in a number of large buildings at that time, although it has since been almost completely displaced by other sections.

From 1893 to 1895 Mr. Gray was engaged in consulting practice in Chicago, designing, making shop drawings for, and inspecting, steel work of various framed structures. He designed two conveyor bridges of novel design for the Chicago Drainage Canal, with spans of 180 and 210 ft. They traveled in a direction transverse to their length and carried dump cars for excavated material, running longitudinally. Material was dug from the canal section with steam shovels, loaded on to the cars and dumped beneath the bridges when the car had been properly "spotted." When one allotted space had been filled the bridge was moved transversely and the operation repeated. This is probably one of the earliest cases of material being conveyed in large quantities, in this manner. After the excavation work was completed the towers were removed and the bridges were used as highway spans. Mr. Gray also designed an ornamental 140-ft arch highway span for Cyrus H. McCormick at Lake

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[1] Memoir prepared by Thomas Earle, M. Am. Soc. C. E.

Forest, Ill. Such structures were unusual at that time and aroused considerable interest.

Mr. Gray was connected with the engineering firm of Davis and Sax of Philadelphia from 1895 to 1899, and was a member of the firm of Gray and Groneau of New York, N. Y., during some of that time.

In 1899, with Charles H. Davis and P. M. Sax, he formed the engineering firm of Gray, Davis and Sax in Philadelphia. This firm was dissolved in 1906 and Mr. Gray then formed the firm of J. H. Gray and Company in New York City. The latter firm was engaged primarily in general engineering practice until 1913, when it entered the contracting field. In 1918 Mr. Gray's health began to fail and he was forced to retire gradually from active work; his firm was dissolved in 1920.

Mr. Gray was particularly interested in the various engineering problems that arose in connection with the design and erection of steel structures.

From 1915 to 1917 he was Contractor's Engineer in connection with the replacement of the existing highway crossing over the Hudson River, at Troy, parallel to an older bridge. The old bridge consisted of a swing span and several approach spans. The swing spans in each structure were opposite each other. Traffic had to be maintained at all times over the old bridge and in the river. The bridges were so close to each other that the swing span in neither structure could be opened fully while both of them were in position. A temporary lift span was used to permit the passage of both river and highway traffic. Another serious difficulty arose from the fact that the center pier of the old span was immediately adjacent to the pivot pier of the new span; and the new foundation went down to rock whereas the pivot pier of the old span was built on piles, its base being about 20 ft higher than the base of the new pier. Therefore, there was great danger that excavating so low for the new span would undermine the old foundation. Mr. Gray solved this problem in a most satisfactory manner, and under unusual circumstances. A flood occurred in the Hudson River while work was in progress, at a season when the river stage is usually quite low.

Mr. Gray had wide experience in his field of work and was both ingenious and capable. He was an employer of many engineers and his loss has been felt widely.

On May 16, 1892, he was married to Thena G. Kirk, who survives him.

Mr. Gray was elected a Member of the American Society of Civil Engineers on October 2, 1895, becoming a life member in 1930.

JOHN HOWELL GRIFFITH, M. Am. Soc. C. E.[1]

Died July 14, 1938

John Howell Griffith was born in Detroit, Mich., on June 16, 1868. His parents were John Howell and Helen (Bristol) Griffith. He was the first born and only survivor of that union.

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[1] Memoir prepared by A. H. Fuller, M. Am. Soc. C. E., and W. M. Dunagan, Esq., Ames, Iowa.

Mr. Griffith was graduated from the Department of Civil Engineering at the University of Wisconsin, in Madison, Wis., in 1893. He received the degree of Master of Science in Mathematics from the same institution in 1898. Throughout his career he alternated between teaching and engineering practice with an increasing interest in certain unsolved problems; research leading to the solution of these problems occupied the full time of his later years.

Mr. Griffith worked for railroads for three summers before graduation, and then had short-time engagements with the Brown Ketchum Iron Works, Gillett Hertzog Iron Works, Pittsburgh Bridge Company, Union Bridge Company, American Bridge Company, the Canadian Bridge Company, the Canadian Pacific Railway Company, and the Chicago and Northwestern Railway Company; he also did designing and estimating for William G. Fargo, M. Am. Soc. C. E., and was Designing Engineer under J. C. Bland, M. Am. Soc. C. E., of the Pennsylvania Railroad Company. He was Chief Draftsman for the James Stewart Contracting Company, Chief Draftsman for the American Structural Steel Company, and Chief Engineer for the Metallurgical Company of America. In 1893 he began an extended experience in structures with the Massillon Bridge Company where his most important work was on the Toledo Court House. In 1901 he was Assistant Engineer for the Cambria Steel Company with responsibility for the design of many buildings, bridges, and industrial plants. Mr. Ralph V. Sage, formerly of the Cambria Steel Company, states, "I respected and admired Mr. Griffith for his engineering knowledge and his desire to take responsibility. I also value highly his loyalty and friendship. His resignation was accepted with regrets by all of us."

His teaching experience began with the acceptance of a fellowship at the University of Wisconsin in 1897—first fellowship award in engineering of that University. After a year of graduate work, he was called to the Clarkson College of Technology at Potsdam, N. Y., where he organized a course in civil engineering and installed a hydraulic laboratory. He then returned to engineering work but was lured back into teaching by the University of Michigan at Ann Arbor, Mich., in 1907. Dean Mortimer Elwyn Cooley, Hon. M. Am. Soc. C. E., writes,

> "I liked Professor Griffith very much indeed. My own recollections are that he was a gentle-man, most considerate of others, thorough in teaching a subject rather difficult for students and one requiring much patience. The fact that his students respected him and did well is a fine testimonial. He taught applied mechanics and our late Professor Ziwet, with whom he was associated, frequently spoke of him in most complimentary terms. He was highly esteemed by all his colleagues. Would we could have more men of Professor Griffith's type in this world of ours which today is so upset and going we know not where."

A noteworthy epoch in his professional career started in 1911 when he began work for the U. S. Bureau of Standards. As engineer in charge of physical tests and equipment, he planned and superintended the construction of the physical testing laboratory at Pittsburgh, Pa. The outstanding piece of equipment here was the 10 000 000-lb testing machine built for the purpose of securing more reliable information concerning the behavior of large columns following the Quebec bridge failure. Bureau of Standards *Technologic Paper No. 101* reports his tests of eighteen large bridge columns; it is recognized as an outstanding contribution to the understanding of the behavior of heavy structural compression members. Researches in brick, terra cotta, and concrete columns of large dimensions served as the bases for other bulletins, each of which is noteworthy in its field. Mr. Griffith was also author of Bureau of Standards *Technologic Paper No. 121* on Strength and Other Properties of Wire Rope.

Mr. Griffith was unyielding in his reverence for the majesty of truth and virtue. He was a thorough scholar, a student of engineering and of Emerson and the philosophers. His private papers include many computations, results of investigations, and written discussions. He has submitted for publication only those which satisfied him. His "Theory of Secondary Stresses in Trusses"[2] was among the early publications in the United States on the subject. It was possibly the first one in which the principle of area moments was used as a basis for the computation of secondary stresses.

In 1919, Mr. Griffith responded to a call from Iowa State College at Ames, Iowa, as Professor of Experimental Engineering. For many years he carried a full schedule of teaching in the field of engineering materials. In 1934 he was transferred to full time as Research Engineer with the Engineering Experiment Station. At Iowa State College he was author of several bulletins for the Engineering Experiment Station: No. 101, Physical Properties of Earths, 1931; No. 117, Dynamics of Earth and other Macroscopic Matter; No. 128, Thermal Expansion of Typical Rocks, 1936; and, No. 131, Physical Properties of Typical American Rocks, 1937.

In Bulletin No. 117, Professor Griffith discusses and uses his formula[3] for intensity of the vertical pressure produced in earth masses by concentrated loads on the surface:

$$Z_z = -\frac{(n-2)}{2\pi} P \frac{\cos^n \theta}{Z^2}$$

In a letter to Mr. Griffith, Dr. Otto Karl Fröhlich says, "It was of special interest to me to learn that this equation was established by you as far back as 1929, because I considered my formula as new when I found it in 1931."

His research interest and aptitudes were evident in each employment; structures detailed under his supervision frequently contained innovations and some of his methods of computations, although not in formal publications, will long remain the property of his students and his colleagues. Mr. Griffith has contributed many articles to the engineering press on various subjects, including clays, soils, soil mechanics, foundations, cement, reinforced concrete, and structural theory. His writings reflect a comprehensive grasp of mechanics, mathematics, physics, chemistry, and geology, and their relation to engineering theory and practice. His works, like the works of many eminent scientists, are replete with a philosophy of unity which does not exclude even the poets from the family of builders.

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[2] *Michigan Technic*, February, 1909.

[3] *Bulletin No. 117*, Iowa State College Eng. Experiment Station, Ames, Iowa, p. 56, Equation 45.

Mr. Griffith was married on April 17, 1895, to Ida Catherine Dengler of Madison. He is survived by one daughter, Mrs. Louis Stewart Louvan.

Mr. Griffith was elected an Associate Member of the American Society of Civil Engineers on April 3, 1907, and a Member on June 4, 1913.

EDWIN LINCOLN GRIMES, M. Am. Soc. C. E.[1]

DIED MARCH 4, 1939

Edwin Lincoln Grimes was born at Hubbardston, Mass., on November 8, 1864, the son of Edwin Grimes and Hannah Elizabeth (Ferguson) Grimes. He was the second of four sons. One of his brothers, Maurice W. Grimes, is a Member of the Society. Edwin received his early education in the common schools at Princeton, Mass., and later entered Worcester Polytechnic Institute, at Worcester, Mass., from which institution he was graduated with honors in 1887 with the degree of Bachelor of Science.

Following his graduation Mr. Grimes entered the employ of the Chicago, California and Santa Fe Railroad Company in Chicago, Ill., as a Draftsman. In February, 1888, he returned to Worcester, where he held the position of Assistant Engineer with Woods and Rugg, Consulting Engineers of that city. During this engagement he supervised the construction of the laboratory at Clark University in Worcester.

From May, 1889, to March, 1899, Mr. Grimes was associated with Chappell and Burke, Chappell and Smith and Thomas F. Chappell, of Rutland, Vt., as Assistant Engineer. During this engagement he was in charge of the re-survey of 66 miles of the Rutland Railroad, construction of 6 miles of the Bristol Railroad, and construction of the Ausable Branch of the Delaware and Hudson Canal Company Railroad (later the Delaware and Hudson Railroad Corporation). He also designed and supervised the construction of a 110 000-gal storage reservoir for Rochester, Vt., and the water supply for the J. and J. Rogers Company's pulp mill at Ausable Forks, N. Y.

In March, 1899, Mr. Grimes opened a consulting engineering office in his own name at Rutland, and maintained this office until August, 1902. In this capacity he was retained on a number of important and interesting projects, chief of which were the design and supervision of construction of the Boardman Memorial Arch Bridge, a 46-ft masonry span, at Pittsford, Vt.; the design and supervision of construction of a 150 000-gal, covered, storage reservoir at Pittsford; the design and supervision of construction of a 500-hp water-power development (consisting of a masonry dam, wooden penstock, and power-house), at Blackbrook, N. Y., for the J. and J. Rogers Company; preliminary and location surveys for the Whitehall and Granville Electric

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[1] Memoir prepared by Donald Mills, Assoc. M. Am. Soc. C. E.

Railroad Company, about 25 miles in length; remeasuring quantities of the Rutland Canadian Railroad for valuation purposes; and designing the sewerage system for a part of the Village of Pittsford.

In August, 1902, Mr. Grimes closed his consulting engineering office and assumed the position of Principal Assistant Engineer, Bureau of Water Supply, Troy, N. Y. In September, 1903, he was promoted to Chief Engineer, in which capacity he was in charge of the completion of the additional water supply for the city, a project which ultimately cost $1 500 000. In January, 1908, he became City Engineer of Troy, and held this position until January, 1913, when he again opened his consulting engineering office in Troy. He maintained this office until April, 1919, when he moved to Atlanta, Ga., to become designing engineer for the J. B. McCrary Engineering Corporation of that city.

Mr. Grimes was employed by this company continuously for the remainder of his life. In the capacity of Designing Engineer he was responsible for the design of a large number of municipal water-works and sewerage systems, one of the most noteworthy of which was the Utoy Creek Disposal Plant, one of the units of the Atlanta Metropolitan Sewerage System. At the time of his death Mr. Grimes was in charge of the construction of a $500 000 gravity water-supply project for Washington County, Virginia.

In 1937 he was presented with a gold plaque commemorating the golden anniversary of his graduation from Worcester Polytechnic Institute. In November, 1937, he received his certificate of life membership from the American Society of Civil Engineers. Mr. Grimes was also an active member of the American Waterworks Association and of the New England Waterworks Association.

He was held in high esteem by his many friends, both in the profession and in private life, and his kindly and lovable manner made a lasting impression upon all with whom he came in contact. His entire life was devoted to the civil engineering profession, and his engineering contributions to the well being of mankind constitute his greatest epitaph.

Mr. Grimes was married to Elizabeth Shangrau of Pittsford on March 10, 1892. His wife preceded him in death on December 1, 1926.

Mr. Grimes was elected a Member of the American Society of Civil Engineers on March 2, 1904.

CARL EWALD GRUNSKY, JR., M. Am. Soc. C. E.[1]

Died March 22, 1938

Carl Ewald Grunsky, Jr., added luster to a famous engineering name. Son of the late Carl Ewald Grunsky, Past-President, Am. Soc. C. E., and Martha (Powers) Grunsky, he was born at Sacramento, Calif., on December 12, 1884. After progressing through the public schools and the Polytechnic

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[1] Memoir prepared by Frank E. Bonner, M. Am. Soc. C. E.

High School, of San Francisco, Calif., he took up his technical education at the University of California, at Berkeley, Calif., from which he was graduated in 1905 with the degree of Bachelor of Science in Mining Engineering. He then went on with his engineering training at Columbia University, at New York, N. Y., where he obtained the degree of Engineer of Mines in 1907.

Through the following ten years Mr. Grunsky's work was mostly in the mining field. He started with the Bunker Hill and Sullivan Mine in Northern Idaho and then went to South America for a year, where he served as Resident Engineer with the Rinconada Mining Company, at Poto, Peru. Returning to the United States at the beginning of 1910, Mr. Grunsky was engaged for several months as Engineer and Mill Foreman by the United Colorado Mines Company at Dorchester, Colo., and, afterward, for two years as Assistant Engineer by the American Engineering Corporation of San Francisco. In June, 1912, he assumed the position of General Superintendent for the Standard Consolidated Mining Company at Bodie, Calif. About a year later he rejoined the staff of the American Engineering Corporation with which he continued as Consulting Engineer for the Standard Consolidated Mining Company and, at the same time, specialized in mine examinations and valuations of many other properties.

Soon after the entrance of the United States into the World War in 1917, Mr. Grunsky was commissioned as Captain in the Engineer Corps of the United States Army. His first assignment was on the construction and operation of Camp Kearney, at San Diego, Calif. He went overseas in July, 1918, and served as Personnel Adjutant on the Staff of Brig.-Gen. (then Colonel) G. B. Pillsbury, Corps of Engineers, U. S. Army (*Retired*), M. Am. Soc. C. E., 115th Engineers. During January and February, 1919, Captain Grunsky was assigned to detached service with the Engineering Department, of the United States Peace Commission, at Paris, France, on the assessment of damages done in France. At the conclusion of his military service in July, 1919, Mr. Grunsky joined his father as a partner in the consulting engineering firm of C. E. Grunsky Company, at San Francisco. During the succeeding five years, he was engaged on many projects involving the design and construction of highways, irrigation, and water supply works, and in the valuation of gas, water, and electric utilities.

Upon the organization of the East Bay Municipal Utility District, in 1924, which had for its purpose the development of the Mokelumne River water supply for Oakland, Berkeley, and the seven other California cities along the east shore of San Francisco Bay, Mr. Grunsky was selected by the Chief Engineer, the late Arthur Powell Davis, Past-President, Am. Soc. C. E., to be Resident Engineer on the construction of Pardee Dam. Under Mr. Grunsky's capable supervision the construction operations on this great structure, which rises 360 ft above the foundations and impounds 222 000 acre-ft of water, were accomplished with complete success. When the Mokelumne Project was finished and the District assumed operation of the water service in the East Bay cities, Mr. Grunsky was appointed General Superintendent for the District and placed in charge of the operation and maintenance of the Mokelumne System. He continued in this position until his death.

Diversion of a large part of the Mokelumne flow to the distant seacoast cities inevitably led to serious litigation over water rights between the District and the water users along the river. Complicated technical questions arose concerning the relation of the river discharge to adjacent ground-water supplies. Mr. Grunsky directed the comprehensive engineering investigations and studies conducted by the District throughout the entire 8-yr period, and it was largely due to his diligence, clear judgment, and passion for fair dealing that a successful compromise of the controverted issues was finally achieved just two months before his death. This settlement confirmed the rights of the District to conserve and divert enough water ultimately for the needs of 2 000 000 population. In all the varied history of water-right litigation in California, no engineer had attained a more conspicuous success in arbitrating an important water controversy. The ample water supply now assured to support future expansion of the East Bay cities will stand as an enduring monument to his memory.

Mr. Grunsky's sudden passing after a major operation, while in the prime of life and accomplishment, was a shock to his host of friends and co-workers. His exceptional professional attainments, outstanding integrity, and conscientious application to his work, commanded the respect of his fellow engineers; and the intense loyalty of his subordinates offers ample evidence of his friendly, human qualities.

The high esteem in which Mr. Grunsky was held by his employers is well expressed in the following resolution adopted by the Board of Directors of the East Bay Municipal Utility District on March 23, 1938:

> "On March twenty-second, nineteen hundred and thirty-eight, Carl Ewald Grunsky, Jr., was called to his final rest. His place in his profession was high, his record of accomplishment great. One of the pioneer professional men with the East Bay Municipal Utility District since its inception, his talents and achievement helped fashion the District into the real constructive agency it has become.
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> "His work was first the planning of a new distribution system for this area; then, Division Engineer on the District's great dam across the Mokelumne River; later, expert witness and leader of the District's engineering staff in important and far-reaching litigation; as General Superintendent he had charge of the Mokelumne water supply and the responsibility of bringing that water to the East Bay cities. His ability contributed greatly to the success of the District's undertaking.
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> "Loyal, lovable and kind, in his passing the District has lost an able, conscientious official, his fellow workers a sincere comrade and friend, his family a devoted husband and father.
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> "*Be It Resolved,* that to his family the Board of Directors of the East Bay Municipal Utility District extends its deep and heartfelt sympathy; and be it further resolved that a copy of this resolution be spread upon the minutes of this Board as a tribute to his memory and worth, and an engrossed copy thereof sent to his family."

Mr. Grunsky was married on January 9, 1913, to Hazel Kathryn Palmanteer, of Oakland. He is survived by his widow and two children, Kathryn Elizabeth Grunsky and Carl Ewald Grunsky III, and by his younger brother, Eugene L. Grunsky, M. Am. Soc. C. E.

He was a member of the Alpha Tau Omega Fraternity, the Commonwealth Club, and the Engineers Club of San Francisco.

Mr. Grunsky was elected a Member of the American Society of Civil Engineers on June 19, 1922.

HANS JORGEN HANSEN, M. Am. Soc. C. E.[1]

Died November 4, 1938

Hans Jorgen Hansen was born in Aarslev, Fyn Island, Denmark, on February 14, 1871, the son of Rasmus and Mette Kathrine (Knudsdatter) Hansen. His parents died when he was a young boy, and at the age of fifteen, his older brother, realizing that Hans' chances for securing an education and obtaining employment were much better in the United States, arranged passage for his brother and himself. Hans was ambitious, found employment, and earned the money to defray the expenses of his early education and to prepare himself for college.

He attended Valparaiso University at Valparaiso, Ind., the Y. M. C. A. evening schools at Hartford, Conn., and in 1903 received the degree of Bachelor of Science in Civil Engineering from Armour Institute of Technology, at Chicago, Ill.

Following his graduation he was employed by the Hall Bascule Bridge Company as a Draftsman; the Strauss Bascule Bridge Company as a Draftsman and on field work; by Vierling McDowell and Company on shop detailing; and by the late W. M. Hughes, M. Am. Soc. C. E., as Detailer and Designer. His experience with these firms gave him a thorough training in the detailing and designing of bridges and other concrete and steel structures.

On May 6, 1906, he entered the service of the Chicago, Milwaukee, and St. Paul Railroad Company as a Draftsman; was promoted to Squad Foreman on July 1, 1915; and on February 1, 1917, was made Office Engineer in charge of the designing, detailing, and estimating of steel, wood, and concrete structures, which position he held until his death.

His work was of a highly diversified character, and included the design of some very large structures. He was an able designer. Thoroughly versed in precedents and standard practice, he also possessed a high order of initiative and resourcefulness which gave him marked advantage in dealing with situations that presented unusual difficulties. He examined with great care each factor of a problem, however complicated, and visualized, with accurate perspective, the project as a whole; but he was always open for suggestions consistent with the problem in mind.

In expressing his views orally, or in writing, Mr. Hansen was deliberate, clear, and precise, and was highly esteemed for his ability to draw sound conclusions. He held the respect of his fellow men and was genuinely liked by all with whom he came in contact. He was modest and retiring, always considerate of all those under his direction, helpful to his associates, just and generous in his dealings with every one.

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[1] Memoir prepared by W. H. Penfield, M. Am. Soc. C. E., in collaboration with W. S. Lacher, Assoc. M. Am. Soc. C. E.

Among the many structures designed under his direction were the pontoon spans in the bridges across the Mississippi River at Prairie du Chien, Wis., and Wabasha, Minn. Very few bridges of that type for conveying heavy railroad traffic have been built. The application of the principle of the pontoon to heavy railroad loadings imposed difficulties that entailed the adoption of many unusual expedients. One of these pontoon spans was described in a paper that Mr. Hansen presented before the Western Society of Engineers on November 8, 1915.

He became a member of the American Railway Engineering Association in 1920, and took an active interest in its work.

In September, 1924, he asked for a leave of absence from the Railroad in order to take up work that would enable him to become more intimately acquainted with construction methods, and he returned to service in September, 1925.

He was a man of excellent habits, an indefatigable worker, devoted to his duty; and he encouraged and stimulated the efforts of others.

In the death of Hans Jorgen Hansen the Chicago, Milwaukee, St. Paul and Pacific Railroad Company has lost an employee whose loyal service has left a deep impression on all with whom he worked.

He was married at Waupaca, Wis., on September 26, 1907, to Louise Sorensen of Neenah, Wis., who survives him.

In 1912 he became a member of St. Ansgars Lutheran Church, which in 1927 was absorbed by the Danish Trinity Lutheran Church. He served both churches as a member of the Board of Trustees.

Mr. Hansen was elected a Member of the American Society of Civil Engineers on March 11, 1919.

ALFRED HANSON HARTMAN, M. Am. Soc. C. E.[1]

Died September 24, 1938

Alfred Hanson Hartman was born at Wilmington, Del., on February 26, 1879, the son of John Robinson and Laura V. (Hanson) Hartman. He was a direct descendant of Major Peter Hartman who served as Aide-de-Camp to General George Washington at Valley Forge.

Mr. Hartman was educated in the Public Schools of Wilmington and attended Delaware College from which he received the degrees of Bachelor of Civil Engineering in 1900, and of Civil Engineer in 1905. Strong and well built, he was interested in all branches of athletics while at college.

During the Spanish-American War, he was a Corporal in Company M, First Delaware Infantry, but he did not see service in Cuba.

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[1] Memoir prepared by Gustav J. Requardt, M. Am. Soc. C. E.

Mr. Hartman began his professional career in the summer of 1899. He became a Rodman on a survey party for the New York Shipbuilding Company at Camden, N. J. During the summer of 1900 he was Transitman on track work for the Central Railroad Company of New Jersey. In 1900 and 1901 he was with the Kansas City Southern Railroad Company as Engineer-Inspector on a girder and drawbridge at Lake Charles, La., and on the foundations and abutments of a five-span bridge across the Illinois River.

In June, 1901, Mr. Hartman returned to Delaware to act as Assistant Engineer and Superintendent of construction for the late T. Chalkley Hatton, M. Am. Soc. C. E., on extensions to the sewerage systems of Wilmington and Milford, Del., and on the construction of a complete water-works for Seaford, Del. From December, 1901, to March, 1903, he was Assistant Engineer of the Maryland Division of the Pennsylvania Railroad Company, in charge of many preliminary and location surveys and on the construction of track elevation and bridge work through the City of Chester, Pa.

Mr. Hartman was employed by the Du Pont Company from 1903 to 1906 as Engineer of Construction in responsible charge of surveys, design, and construction of a dam across Brandywine Creek, Pa.; a slow sand-filter plant; several sewage-treatment plants; and a powder mill. He was also in charge of sanitary improvements along Brandywine Creek, and he made surveys and studies with a view to the improvement of the company's water-power properties.

In 1905 the officials of the City of Baltimore, Md., began organizing the forces of the Baltimore Sewerage Commission to undertake the design and construction of complete sanitary and storm-water systems, interceptors, outfalls, and a sewage treatment plant for the city. Mr. Hartman entered the employ of the Commission as an Assistant Engineer in 1906, shortly after construction work had started. He remained with the Commission until the completion of its program in December, 1915.

When its sewerage work started, Baltimore had few private sewers and drains and the streets were mostly paved with cobbles. Kitchen, bath, and laundry wastes were permitted to flow in the street gutters, and as there were few street inlets, this water flowed for great distances. At the completion of its work, the Commission had built 626 miles of sanitary sewers and 134 miles of storm-water drains and had expended about $23 000 000; it had connected most of the existing houses to the separate sanitary system, had built complete storm-water drains to remove storm flows from the street gutters, and had intercepted sanitary sewage and conducted it to a treatment plant located some 6 miles beyond the city limits at Back River in Baltimore County, Maryland.

Mr. Hartman rose rapidly in the personnel of the Commission, being assigned to surveys, design, and supervision of construction of the storm-water system. He was appointed Division Engineer of the Storm Water Division on August 15, 1910, which position he held until the completion of the work. The works designed and constructed under Mr. Hartman's direction included tunnels, drains through deep cuts and in soft ground supported on piles, and many types of street inlets, several of them of original design. Many of the standards of construction used at the present time by the City of Baltimore are those originally proposed by his Division of the Sewerage Commission.

The Jones Falls Diversion Tunnel and Conduits were built under Mr. Hartman's direction. Jones Falls is a stream of about 35 sq miles water-shed and flows through Baltimore to empty into tide-water in the harbor. Storm flows had topped the side walls many times, flooding streets for blocks on each side of the stream. The stream was completely covered from Mt. Royal Avenue to near the harbor, and no flooding has occurred since this construction. This was a major improvement of the City, and its features have been studied by many engineers.

In 1916 Mr. Hartman was retained as Consulting Engineer by the Consolidated Engineering Company of Baltimore, in connection with tunnel work the Company was doing for the Southern Railroad in North Carolina. In the early part of 1917, Mr. Hartman came with the Company, devoting his entire time to it. On February 9, 1918, Mr. Hartman was elected a Director and continued as a Director of the Company until his death. On the same date he was made General Manager of the Engineering Department, which position he held until February 19, 1927, at which time he was elected Third Vice-President, and continued in that position until February, 1938. He was then elected Second Vice-President, which office he held at the time of his death.

Under Mr. Hartman's direction, the Consolidated Engineering Company rehabilitated the Asheville Division of the Southern Railway, which was made necessary by the floods of 1916 and 1917. This work included the restoration and building of a number of bridges and tunnels; the sedimentation basins at Monroe, Va., the under-pass at Greensboro, N. C., and the 17½-mile cut-off in Tennessee from Bullsgap to Leadvale for the Southern Railway Company; bridges at Thurmont, Md., for the Western Maryland Railway Company and the State Roads Commission, and the Gay Street Bridge in Baltimore for the Pennsylvania Railroad Company; new track work and bridges for the Baltimore and Ohio Railroad Company at Romney, W. Va., caused by damages to the system by the 1936 flood; the dam and reservoir for the Frederick, Md., water supply, the filter plant at York, Pa., the Rodney Street Reservoir at Wilmington, and the Ashburton Pumping Station for the Baltimore Water Department; the foundations for the new Department of Commerce Building in Washington, D. C., and many other structures. He was also associated in an advisory capacity with many other projects upon which the Company was engaged.

Mr. Hartman was a member and Past-President of the Engineers Club of Baltimore, and a member and Vice-President of the Maryland Section of the American Society of Civil Engineers. He took an active interest in the affairs of these organizations during the whole of his membership. He was a member of the Civitan Club of Baltimore, the Baltimore Country Club, and the Emmanuel Episcopal Church of Baltimore. He had lived in Balitmore continuously since 1906.

Mr. Hartman was an authority on foundations, tunnels, drainage, and railroad track work, and on the planning of construction methods and equipment. During his career he trained many young engineers. He will be remembered by them because of his kindly, retiring nature, and his vast fund of technical experience.

On June 15, 1904, Mr. Hartman was married to Annie G. W. Summers at Elkton, Md. He died in Oakland, Calif., while on a visit with his family. He is survived by his widow; his children—Elizabeth Hartman Ward, Nancy G. W. Rumsey, and Wilmon Whilldin Hartman; and by his grandchildren—Nancy G. W. Ward, Alfred Gilfry Ward, and Elizabeth Hamilton Rumsey.

Mr. Hartman was elected a Junior of the American Society of Civil Engineers on December 6, 1904; an Associate Member on December 5, 1906; and a Member on October 1, 1912.

JAMES NOBLE HATCH, M. Am. Soc. C. E.[1]

Died November 11, 1938

James Noble Hatch was born on January 3, 1868, at Saint Helena, Napa County, California, the son of Lucius Lucian and Sarilda Berry (Hornbeck) Hatch. After attending Grammar School and High School in his native State of California, he entered the University of Michigan, at Ann Arbor, Mich., in 1888, and was graduated with the degree of Bachelor of Science in Civil Engineering in 1892. In 1898 he was granted the degree of Civil Engineer and in 1909 was given the Masters degree of Mechanical Engineer.

His professional record was long and impressive, combining work in both civil and mechanical lines, in which he was equally proficient. After a year of preliminary engineering experience he became Instructor of Engineering at the Michigan Agricultural and Mechanical College, at East Lansing, Mich., and from then on engaged in active engineering work.

In 1894 and 1895 he was Assistant Engineer in the Maintenance of Way Department of the Big Four Railroad Company. He left the railroad to become Structural Engineer and Designer for the Carnegie Steel Company, the Pittsburgh Steel Company, and the Brown Hoisting Machinery Company, of Cleveland, Ohio, respectively, from 1896 to 1900. During this period, Mr. Hatch had charge of the design of tall steel buildings and coal handling plants, which were then coming into general use, both in the United States and abroad. He added considerably to their development along the best, and most practical, engineering lines.

In 1900 he passed a Civil Service examination, was appointed Superintendent of Construction and was assigned in this capacity to many important buildings for the United States Government. He remained in this position until 1902, at which time he joined the Engineering Staff of the American Bridge Company at Chicago, Ill., for one year. In 1903 he became associated with the consulting firm of Sargent and Lundy, of Chicago, in charge of the

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[1] Memoir prepared by I. F. Stern, M. Am. Soc. C. E.

Structural Department. He remained in this position for more than eleven years in the design, construction, and operation of large, central, electric power stations throughout the United States. This work involved many difficult civil and mechanical engineering problems relating to the design of machinery, intake tunnels, bridges, foundations, structural work, chimneys, and other details. During this period, Mr. Hatch assisted greatly in the development of a type of self-supported steel stacks, on which he became a recognized authority.

In 1914 Mr. Hatch resigned his position with Sargent and Lundy and opened his own offices in Chicago as Consulting Engineer, on bridges, buildings and other structures, power plants, valuations, and general engineering service. Many clients availed themselves of his services and he maintained his offices until 1928, at which time he yielded to a long-cherished desire to return to his native State of California. In California, he devoted himself to literary work and to his orchards, but the lure of his profession, in which he had been so successful, was too great to be withstood, and in 1935 he accepted an appointment as Engineering Examiner for the Public Works Administration in California, which he retained as long as his health permitted.

Mr. Hatch combined with a great engineering ability, a natural inclination toward literary work. He was a frequent contributor to technical magazines, and was also the author of numerous sketches, essays, and poems, usually in a humorous vein, but exhibiting a deep philosophy.

During a busy life, Mr. Hatch was dearly beloved by his associates, not only for his sterling character, kindness and loyalty to his associates, but also for his quaint sense of humor which found expression in his whimsical sketches and poems. These have been collected and published under the title of "Mirthful Musings by Truthful James." Many of these poems were originally read at meetings of the Chicago Engineers Club, to whom Mr. Hatch was fondly known as its "Poet Laureate."

Mr. Hatch was a member of the Chicago Association of Commerce, the Western Society of Engineers, the Chicago Engineers Club, the National Electric Light Association, and the Pasadena Engineering Society.

In 1893, he was married to Lufanna Barber of Ann Arbor. He is survived by his widow; two daughters, Marion and Esther Lou; one son, Hadley; and a sister, Mrs. Anna Croall.

Mr. Hatch was elected a Member of the American Society of Civil Engineers on December 7, 1904.

CHARLES WILLIAM HAZELTON, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 29, 1937

In recording the life story of any man who has completed a full and active career in his community, one finds that there is always some feature or some phase of that life which has left its lasting impression. With such a remembrance whether faint or vivid, whether it be a spoken word, an ideal, or a structure of brick and stone, there is preserved the character and spirit that guided his life's endeavors.

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[1] Memoir prepared by C. E. Hazelton, Esq., Turners Falls, Mass.

For Charles William Hazelton there was one absorbing interest during his most active and creative years. The control and development for human service of the Connecticut River at Turners Falls, Mass., was a project with which he was intimately associated at the beginning and which was basically modified and greatly expanded under his direct supervision. After relinquishing the responsibilities of active management, he had the rare distinction of seeing created such a wider and greater field of use and service as was literally inconceivable to the originators, but of which he had had a vision and had prepared the way.

Although time and change have so greatly altered the scene and design of those earlier years of application, there still remain in the traditions and annals of the organization that Mr. Hazelton served, in the facilities and aspects of a community which he helped to bring into being, many mute but characteristic evidences of a spirit and personality which mingled inspiration with achievement during more than forty years of leadership.

In a valley indelibly associated for so many years with the struggles and hardships of gaining an established settlement; and at the very spot where occurred one of the most destructive conflicts of that period, there arrived soon after the Civil War, Alvah Crocker, of Fitchburg, Mass., a genius of industrial promotion and development. Disregarding the rights of picnic parties and sightseers at the brink of the Great Falls and dislodging the ancient gate-keeper at the canal locks, he commanded a new era of work instead of lassitude for the river which had for so many centuries been its own master—and still is at times. With the construction of a dam, head-gates, and canal, for harnessing the river's power, there appeared, almost overnight, factories, stores, a hotel, apartment blocks, schools, and churches—in fact, a city had been set down in a scene so recently a hill and woodland retreat. Although it is true that many of the aspects of civic attainment were wholly on the surveyor's plans, or in a beautifully lithographed prospectus, there had been brought into being the Town of Turners Falls, new born among ancient neighbors with the lusty cries and rough behavior typical of a Western mining bonanza.

Charles William Hazelton, the son of Charles and Hannah (Sargent) Hazelton, was born at Plymouth, N. H., on October 31, 1848 When a boy, the new railroad—the Boston, Concord, and Montreal—had claimed his early interest. When work on his father's farm on the outskirts of the village or freedom from school would permit—and sometimes without such permission—he would be found at the railroad station located in the old Pemigewasset Hotel, or around the roundhouse which housed the glittering iron horses of that day. In fact, his first gainful employment at the age of twelve was as a train boy selling apples from the farm orchard on the run between Plymouth, and Concord, N. H. With the help of the night operator he learned telegraphy and unofficially was allowed to send and receive some of the more unimportant railroad orders. Although there were some parental objections to his "hanging around the depot," this interest brought him what turned out to be his opportunity, as it was in the railroad yard at Plymouth that he met William P. Crocker, brother of Alvah Crocker, then a Consulting Engineer for the Concord and Montreal Railroad Company. At the age of seventeen years Mr. Hazelton was employed by Mr. Crocker as a Rodman on the surveys for the Whitefield, Woodstock, and Northern New Hampshire, and the Boston, Concord, and Montreal Railways—the first rung in the ladder of a civil engineering career. He was fortunate in his association during formative years with so eminent a civil engineer of that period.

As an Assistant to the Engineer in charge of surveys made during 1867 and 1868, preliminary to the actual construction, Charles Hazelton, at the age of nineteen years, was climbing over the rocks and cliffs about the Great Falls and cutting his way through the brush and forest surrounding it with measuring chain and sighting rod, and with probably no thought that the primitive scene before him was to be his future life's environment. Apparently, at this time, civil engineering had already gained his interest and allegiance as he continued with William P. Crocker, the Engineer in charge of the new project, as Assistant on the same work that he had started a few years earlier, and on various railroad surveys and reconnaissances in Northern New Hampshire.

With the completion of surveys and detailed plans, work on the dam, head-gates, and canal, was begun during the following year—1868—and for two years Mr. Hazelton was engaged in supervising their construction from the plans which he helped create. This was a period of exciting interest in the growth of the project. With the influx of laborers, masons, and carpenters, temporarily living in the two dwelling houses and the home of the former gate-tender, and with the promised location of the first mills for using the new water power, added emphasis was laid on the prospective attractions of the new City of Turners Falls. The wide avenues and cross-streets which had been shown in lithograph with such a geometrical and metropolitan design, were staked out and roughly graded and then with free railroad fares and dinners, the public was invited to participate, through the purchase of building lots, in the immediate realization of a vision. During this period, the young Assistant Engineer had many duties in addition to overseeing the construction work on the dam and head-gates. A water system for the new community, sewers, and even police and fire protection, had to be provided; all these required facilities came within his province. There had been no four or six years of technical school education for this young engineer, still in his minority, and problems of design and construction were only solved through the help of older and experienced men, the use of a few texts, and a native intelligence and aptitude for the work.

It was at this time that there came to Turners Falls a man of striking personality who was to have a profound influence on the course of Mr. Hazelton's life. As general contractor for the enlargement of the old canal and for the construction of many of the new factories and buildings, the late Bernard N. Farren, Fellow, Am. Soc. C. E., desired to secure a Resident Superintendent, and by amicable arrangement, Mr. Hazleton severed his connection with the Turners Falls Company and started to learn the details of contract construction. For three years he remained in Turners Falls for the completion of various contracts and then for the next four years, 1872 to 1876, his duties took him to different parts of New England where "Barney" Farren, as he was then called, had secured work for his organization. The most important of these contracts, all of which had their problems as well as many interesting features, included the construction of a railroad north from Worcester, Mass.; the removal of Fort Hill with corresponding filling of Back Bay, in Boston, Mass.; and the installation of a brick lining in the Hoosac Tunnel. During all the seven years of this association, Mr. Hazelton had his home with the family of Mr. Farren and between these two men of quite dissimilar ages, there resulted a close and enduring friendship.

In 1877, at the age of twenty-eight, Charles Hazelton gave up his position with Mr. Farren and formed a partnership with William Campbell, under the name of Campbell and Hazelton, for the purpose of conducting a contracting business on his own account. The firm was reasonably successful and constructed several buildings in Turners Falls, one of the first halls for the Moody Schools at Northfield, Mass., and as its largest and most important project, relocated a section of the railroad a short distance east of Millers Falls, Mass., with a corresponding change in the bed of the Millers River at that point. However, the partnership was dissolved by mutual consent about two years after its formation.

At this time, progress of the water-power and industrial development at Turners Falls had reached the stage where problems of maintenance, operation, and general policy predominated, with the consequent necessity for an active and progressive resident management. It was not coincidence that Charles Hazelton was selected, in 1880, by the officials of the Turners Falls Company as Engineer for the properties; his earlier connection with the Company had demonstrated an ability and character that made the choice a natural one.

Thus, at the age of thirty-two years, he assumed the rôle of guardian for one of Nature's greatest resources—hydraulic power—with a responsibility for continued service to human needs that knows no day of rest. The river was in harness, but not subdued.

In the period that had elapsed since its establishment, Turners Falls had given up the hopes of immediate expansion to civic proportions and had settled down to normal growth as a typical New England industrial town. Self-centered, as were most small communities at that time, but with a spirit of aggressiveness sponsored by its unique background, the town thrived. Churches, schools, banks, and stores were established, an extensive sewerage system and a highly satisfactory water supply were installed—all in less than ten years from the time the first prospectus was printed. In all these enterprises Charles Hazelton had his part, not only because he had in his official position a partial responsibility for their success, but because he possessed, in addition, an inherent and characteristic desire to be of service as a citizen and as a friend.

He had already formed two personal associations of major importance in his career—one with William Crocker, and one with Bernard Farren—and now a third contact was to bring him not only an intimate friendship and a community of interest, but financial independence as well. In Pelham, Mass., Eugene Bartlett had established a small factory for the manufacture of fishing rods, and with a prosperous business he sought new capital and associates for expansion. Possibly through a mutual liking for the piscatorial pastime, the contact was made and a new corporation was organized with a factory near Turners Falls. The Company continued to be successful for many years although the activity of Mr. Hazelton in this business was always incidental to the larger responsibilities which he had previously assumed.

Although from 1880, when he had returned to the Water Power Company, there had ensued a continuing period of more gradual expansion both in the utilization of the energy created and in the community that had been fostered, it was also a period of adjustment and revision with many problems of policy and administration to be worked out by both Company and townspeople. The original indentures for the sale of water power had provided for a rental of $5 per yr per hp, with use permitted for 24 hr per day and extending for a term of 999 yr. In the light of present-day conditions, this contract seems astonishing in its low cost and privileges. However, the mill owners, in an eagerness to expand their plants and increase production, were careless in observing the limitations of the contract, and it became a major part of Mr. Hazelton's duty to regulate and apportion the use of water both as to legal quantities and under conditions of diminished river flow.

In 1890, he was made Treasurer and Clerk of the Water Power Corporation, in addition to his position as Engineer for the properties, and with these increased responsibilities there came opportunities to prepare the way for the larger development that few visioned at that time.

Although the Company had benefited materially through supplementing the old indentures for permanent rights with new contracts for the sale of surplus water, Mr. Hazelton had realized that the most efficient use of the natural drop of the Connecticut River in the vicinity of Turners Falls necessitated the eventual abandonment of the original plan and the substitution of a radically different and much more extensive design.

The first step was the acquisition of various tracts of land and flowage rights, a long and difficult process requiring considerable tact and diplomacy. Again, came the period of preliminary surveys, of plans and computations, all with a definite goal in mind. The vast strides being made by the electrical industry at this time, especially the long-distance transmission of power, had a vital effect on these new considerations, and as a step toward a still distant horizon, the Directors of the Turners Falls Company, at Mr. Hazelton's suggestion, authorized, in 1904, the widening, deepening, and extension of the original power canal, together with the construction of a hydro-electric generating plant at Turners Falls.

With this additional power capacity there came new problems of operation and management and although a relatively small expansion, it represented an entirely different type of use and service. No longer would it be necessary to locate factories on the river bank in order to take advantage of its energy, for with electricity the water power could be carried to distant localities for manufacturing, lighting, heating, and a multitude of future uses unthought of at that time. The effect was far-reaching. The Company's field was broadened, and the former surveyor and hydraulic engineer now in the full maturity of his career was faced with the new technical problems of electrical generation and distribution.

During the next eight years, Charles Hazelton as the Managing Executive was to be responsible for the second rebirth of a century-old organization. The navigation canal and locks around the Great Falls of 1791 had been the basis of Alvah Crocker's realized dream of an industrial center and now the Water Power Company which he founded was on the threshold of an even greater service to human needs—a true public utility was envisioned. Fortunately, at this period of change and adaptation, there had entered the employ of the Company a young man named Fred C. Abercrombie whose help as Mr. Hazelton's Assistant during these trying years was to prove invaluable.

While the first small electrical development was being ushered into the world with attendant problems of refinance, public relations, and a market appetite, the larger plan based on Mr. Hazelton's earlier studies and recommendations, was gaining wider support and more general interest.

As its extent and possibilities would necessitate new resources of capital and a revised managerial organization and with the feeling that the first step in the hydro-electric development at Turners Falls was in successful operation, Charles Hazelton, at the age of sixty-four, and after forty-four years of active engagement in the Company's affairs, requested that the responsibilities of the expanded program be transferred to others, and, in 1912, he retired as Treasurer and Engineer of the Company. However, for the next twenty years, he acted as a Director and in an advisory capacity for the Turners Falls Power and Electric Company and Associated Companies. His service to this interest thus covered a period of sixty-four years.

Although his retirement had marked the close of active participation in what had been his life's major theme and should have entitled him to a well-earned rest, it was in character that Mr. Hazelton now felt it necessary to devote more time and attention to his interest in the fish-rod business—the Montague City Rod Company—and to assume, when requested, various responsibilities of a semi-public nature. He served as President of the Connecticut Valley Street Railway Company from 1912 to 1922; as Trustee of the Western Massachusetts Companies from 1927 to 1934, and as President of the Crocker National Bank, at Turners Falls, from 1920 to 1930.

Fortunately, too, his release permitted more indulgence in various avocations which had been kept in the background during previous years. Chief among these was a deep interest and keen enjoyment in flowers, ornamental plants, and even vegetables, as evidenced by the greenhouse and garden that he maintained at his home. Foreign travel had always been a dream which now he was able to realize, and it is interesting to note that his inclinations preceded the present trend toward Southern travel. On numerous different trips, he visited most of the Caribbean Islands, all the countries of Central America, and spent a most enjoyable two months in Brazil and the Argentine Republic. Incidentally, and again self-taught, he acquired a reading knowledge of the Spanish language.

Although he had been denied the benefits of much more than a grammar school education, he seemed to have a natural inclination to history and languages. His own lifetime covered an industrial and economic transition in the Connecticut Valley, and the events and personalities of that period were vivid in his recollection and descriptions. The geological formations of the Valley were of great interest to him, this latter study undoubtedly being influenced by his profession.

While still with the Turners Falls Company he started an intensive study of the German language and, with a system of his own and with lessons from native Germans in Turners Falls, he acquired a reading and speaking command of that language.

Although in his early years there had been little time for the recreational relaxations now considered so essential, he did take pleasure in ice skating and fresh-water fishing both of which, either by coincidence or design, were associated with the major interests in his life.

Mr. Hazelton was active in political and social life. He was a Representative in the Massachusetts State Legislature, a Commissioner in Franklin County, a member of the School Committee in the Town of Montague, a Delegate of the National Republican Convention, a Senator of the Massachusetts State Legislature, and a member of Pocomptuck Valley Memorial Association. He was also very active in the Masonic Order, being a charter member of the Mechanics Lodge of Masons, and a member of the Franklin Arch Chapter, the Titus Strong Council, and the Connecticut Valley Commandery. He was a member of the Aleppo Temple of the Mystic Shrine in Boston, and a member of the First Congregational Church in Turners Falls.

On May 12, 1870, he was married to Ella Wilder, of New Bedford, Mass., who predeceased him. On December 1, 1917, he was married to Grace Frensz, of Houston, Tex., who survives him, together with their adopted daughter, Mary Margaret; also one son and two daughters by his first wife—Charles Edward, Ella Macomber, and Helen Wilder Hazelton.

In summation, the life story of Charles William Hazelton, of Turners Falls, is the history of an institution and of a town, as physically, economically, and socially the three are as one. With the release from all responsibilities on September 29, 1937, at the age of eighty-nine years, there passed on a man whose life had been well spent.

Charles William Hazelton was elected a Member of the American Society of Civil Engineers on January 7, 1891.

THOMAS HEATLEY, M. Am. Soc. C. E.[1]

Died October 27, 1938

Thomas Heatley, Jr., was born at Birkenhead, England, on October 6, 1865, the son of Thomas Heatley of Pettigo, County Donegal, and Elizabeth (Sproule) Heatley of Kesh, County Fermanagh, Ireland. About 1870 the family emigrated to New York, N. Y., where Thomas, Jr. attended the Public Schools. Shortly after graduation, he began a long and interesting career in the building art, starting as an apprentice bricklayer for his father (who was a mason building contractor), and serving successively through the ranks as mechanic and eventually professional engineer. These occupations are attested by Life Membership No. 80 964 in the Bricklayers', Masons', and Plasterers' International Union.

After some fifteen years spent in the practical art as Apprentice, Journeyman, Foreman Mason, and eventually in partnership with William Bailey, as mason contractor (during which time he laid brick in many of the then prominent building structures, including the original Bloomingdale Asylum at White Plains, N. Y.), Tom Heatley began an active career in the field of public building supervision. In 1895, he was appointed Special Inspector in the then Department of Buildings of New York City, serving on the same staff with Rudolph P. Miller, M. Am. Soc. C. E.—who later became Superintendent of Buildings—and Thomas Lamb, who as Architect later designed and constructed many of the now prominent theater structures in New York City.

From 1895 until his retirement from the city service on June 30, 1934, Tom Heatley worked up through the grades of Civil Service, serving successively as Inspector of Iron and Steel, Engineer Inspector, General Inspector in Charge of Field Force, Plan Examiner, Engineer in Charge of tests of materials, Assistant Engineer, and finally acting Chief Engineer under Superintendent Patrick J. Reville. The latter part of this career, subsequent to 1902, was spent in the then Bureau of Buildings, Borough of the Bronx, New York City.

Supplementing his practical experience with a strong latent desire for theoretical knowledge, Tom Heatley attended evening technical courses in the Brooklyn Polytechnic Institute, Brooklyn, N. Y., and with the aid of his many professional friends, devoted much time to study supplemented by technical committee work for the American Society for Testing Materials, in which connections his practical recommendations proved of lasting value.

During this decade of the early Twentieth Century, rapid gains and advances were made in the art of building, involving the use of protected iron and steel and reinforced concrete in the development of skyscraper construction. Through his official connection with the Bureau of Buildings, Borough of the Bronx, in charge of tests of new materials and methods of construction, Mr. Heatley took an active part in this pioneering stage. He was closely associated with Mr. R. P. Miller and Prof. Ira H. Woolson in the conduct of their many fire and strength resistance tests of fireproof construction, and interested himself actively in the research and experimental work of the epoch. Subsequently, in his official capacity, he represented the City of New York in the investigation and testing of European materials of construction for importation into the United States, and made trips of inspection to Belgium, France, Holland, and Germany for purposes of study and research.

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[1] Memoir prepared by George E. Strehan, Assoc. M. Am. Soc. C. E.

During his forty years of active building supervision with the City of New York, he acquired a wealth of human, as well as technical, experience. With an inimitable skill in verbal expression and narrative, inherited from Irish Ancestry, he entertained friends and associates with astounding tales of building and engineering art as it was practiced in this somewhat hectic period of speculative growth. A keen sense of humor and thorough human understanding eased the official path in the enforcement of law and endeared him to his many friends, associates, and public alike.

On June 16, 1894, Mr. Heatley was married to Adele W. Engle at Long Branch, N. J., who survives him.

Mr. Heatley was elected an Associate Member of the American Society of Civil Engineers on January 15, 1917, and a Member on April 21, 1930.

IRA GRANT HEDRICK, M. Am. Soc. C. E.[1]

DIED DECEMBER 28, 1937

Ira Grant Hedrick was born at West Salem, Ill., on April 6, 1868, the son of Henderson and Mary Ann (Bryan) Hedrick. His parents moved to Fayetteville, Ark., and there he attended the University of Arkansas (then called Arkansas Industrial University) from March, 1890, until he was graduated in December, 1892, with the degree of Bachelor of Civil Engineering. He was married at the time he entered this school, and before his graduation he was the father of two children. He supported his family and made his way through school by working during his spare time at anything he could find to do. Mrs. Hedrick nobly assisted him by keeping boarders. The community was small and jobs were scarce. Mr. Hedrick's principal employment was teaching during vacation periods, which he prolonged as much as practicable by leaving before the close of the school year and entering as late as he could after the beginning of the next session. In spite of these handicaps, he completed the four-year engineering course in less than three years and always stood at the head of his class. For a long time after his graduation, faculty members were wont to cite his scholastic record as being the best in the history of the school.

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[1] Memoir prepared by Victor H. Cochrane, M. Am. Soc. C. E.

Immediately after his graduation Mr. Hedrick entered the employ of the late J. A. L. Waddell, Hon. M. Am. Soc. C. E., at Kansas City, Mo., and was associated with him until 1907, except for a period of service with the Kansas City, Pittsburg and Gulf Railroad Company (later, the Kansas City Southern), in 1898 and 1899, as Assistant Chief Engineer. Mr. Waddell's practice was chiefly in the field of bridge design and construction, and here Mr. Hedrick found work suitable to his talents. He soon became Mr. Waddell's Principal Assistant. His design calculation sheets were models of neatness and accuracy. In those days the art of bridge design was developing rapidly, and Mr. Hedrick was of great assistance to Mr. Waddell in the preparation of the latter's pioneer book, "De Pontibus." In 1899, Mr. Hedrick entered into partnership with Mr. Waddell in the firm of Waddell and Hedrick. Structures designed by this firm included a bridge over the Missouri River, at Omaha, Nebr., another over the same stream, at St. Charles, Mo., the Fraser River Bridge, at Westminster, B. C., Canada (having an unusual "spread" span), bridges for the Vera Cruz and Pacific Railroad Company, and for other railroad companies, a bridge over the Red River for the Kansas City Southern Railway Company, and the Sixth Street Viaduct, in Kansas City (later, called the Intercity Viaduct).

Mr. Hedrick's resourcefulness was demonstrated during and after the Kaw River flood of 1903. One span of the bridge carrying the main flow line of the Kansas City Water-Works was washed out, and the city was entirely without a supply of water. The firm of Waddell and Hedrick was commissioned to construct a temporary bridge to restore service, and Mr. Hedrick took charge of the construction work. Using old street railway cables and such odds and ends of materials as could be procured, and working continuously day and night, the bridge was completed in about eight days. In this same flood, two 200-ft riveted spans of the Kansas City Southern Railway Bridge were carried away, and one span landed about 600 ft down stream. Mr. Hedrick devised ingenious means for righting these spans and restoring them to position intact.

At the turn of the Twentieth Century, pin-connected trusses were in general use for all but the shortest spans. Mr. Hedrick early saw the advantages of riveted trusses and was a pioneer in their development. In 1904, he designed 300-ft riveted spans for the Intercity Viaduct crossing of the Kaw River, at Kansas City.

In 1907 Mr. Hedrick withdrew from the firm of Waddell and Hedrick and opened an office of his own in Kansas City. At the same time he took over the management of the affairs of the Company which owned and operated the Intercity Viaduct. Three years later the writer joined Mr. Hedrick in the firm of Hedrick and Cochrane. Among the structures designed by this firm were the Dallas-Oak Cliff Viaduct; bridges over the Kaw River, at Lawrence, Kans., and Kansas City; the Arkansas River Bridge, at Pine Bluff, Ark.; two bridges and a dam for the City of Cedar Rapids, Iowa; the Red River Bridge, at Shreveport, La.; and several bridges for about six other railroad companies.

Upon Mr. Cochrane's withdrawal from the firm in 1916, Mr. Hedrick and his son, Arly, continued the business as Hedrick and Hedrick. Mr. Arly Hedrick volunteered for service in the 110th Engineers, was commissioned a Captain, and served with distinction in France, being decorated for bravery in action. In March, 1919, while in Brest, France, awaiting transportation to America, Captain Hedrick contracted influenza and died. Ira Grant Hedrick never recovered from this blow, and thereafter did not try to maintain a large engineering organization. He was content to accept only one or two major engagements at a time, and he removed his office several times, in order to give personal attention to the work in hand.

After closing his Kansas City Office in 1923, Mr. Hedrick removed to Portland, Ore., and there was first associated with Robert E. Kremers, and, later, with William Durkee Smith and Lloyd Garner Frost, Assoc. M. Am. Soc. C. E. Mr. Hedrick and Mr. Frost, under the name of Hedrick and Frost, conducted an engineering business in New Orleans and Shreveport, La., for about three years (1925–1927). Mr. Hedrick removed to Arkansas and stayed there during the remainder of his life, except for a brief period of residence in Oak Point, Wash. In Arkansas, Mr. Hedrick's office was situated in Hot Springs National Park.

Among the structures designed by Mr. Hedrick and his associates during the period 1922–1928 were two bridges over the Arkansas River, one at Little Rock, Ark., and the other at Fort Smith, Ark.; two bridges over the Willamette River, at Portland; a bridge over the Atchafalaya River, at Simmesport, La., for the Louisiana Railway and Navigation Company; the Lake Worth and Royal Street Bridges, at Fort Worth, Tex.; the Back Bay Bridge, at Biloxi, Miss.; and the Main Street Viaduct, in North Little Rock, Ark. For the Arkansas State Highway Department Mr. Hedrick designed two bridges over the Red River, three over the White River, one over the Ouachita River, and three over Lake Hamilton. His last engagement was that of Examining Engineer in the Little Rock Office of the United States Public Works Administration.

The period of forty-five years during which Mr. Hedrick carried on his professional activities was characterized by great advances in the art of bridge design. He kept himself informed concerning the latest developments, and contributed no little to the progress in this field. He was not inclined to write technical papers but encouraged his associates to do so. His engineering designs were brilliantly conceived and were always developed with a view to economy and practicability in construction. His advice was frequently sought by contractors in regard to difficulties in pier sinking and other problems of construction.

In recognition of his professional attainments he received from his Alma Mater the degree of Civil Engineer in 1901, and from McGill University, at Montreal, Que., Canada, the following degrees: Bachelor of Applied Science, 1898; Master of Science, 1899; and Doctor of Science, 1905. He was elected to Tau Beta Pi Fraternity in 1915. He was a life member of the Masonic Order, and a Shriner. He was also a life member of the Benevolent and Protective Order of Elks.

Mr. Hedrick possessed a forceful and magnetic personality and had a great capacity for making friends. He was always fair to his associates and just in his professional decisions. His great ability was quickly perceived by those with whom he came in contact, and he ever retained the loyalty of his associates.

He is survived by his widow, the former Mary E. Bryan, to whom he was married at Kansas City, on September 22, 1926. In 1889, he was married to Louise Luther, of Fayetteville, Ark. To this union three children were born: Arly Luther Hedrick, the son who died in France; Marie (Mrs. Morton Steiger), and Louise (Mrs. Russell Cowgill). In 1909, after the death of Mrs. Hedrick, Mr. Hedrick married her sister, Addie M. Luther. Two children of this union survive: Ira Grant Hedrick, Jr., Jun. Am. Soc. C. E., and Betsy Carolyn. In 1918, at Liberty, Mo., Mr. Hedrick was married to Della M. Threlkeld. Mr. Hedrick is also survived by his mother, three brothers, Dennis, John, and Gideon; and two sisters, Della, and Mrs. C. A. Wofford.

Mr. Hedrick was elected a Junior of the American Society of Civil Engineers on October 3, 1893; an Associate Member on June 2, 1897; and a Member on November 7, 1900.

VICTOR KING HENDRICKS, M. Am. Soc. C. E.[1]

DIED MARCH 19, 1938

Victor King Hendricks died at his home in Oak Park, Ill., on March 19, 1938; the world was better for his having passed this way and a loser by his going. He was a man peculiarly gifted with those innate qualities of courtesy and kindliness to his fellow men, and all who knew him loved him for what he was.

Victor King Hendricks came from distinguished parentage and carried to a high degree the heritage he bore. He was born on May 28, 1869, at Indianapolis, Ind., the son of Abram W. and Sarah (Butler) Hendricks. In 1839 his father moved from Westmoreland County, Pennsylvania, to Madison, Ind., to study law with his uncle, the Hon. William Hendricks, Governor of Indiana, from 1822 to 1825, and, in 1866, he went to Indianapolis to join his cousin, the Hon. Thomas A. Hendricks, Governor of Indiana from 1873 to 1877, and Vice-President of the United States with President Grover Cleveland, from 1885 to 1889, one of the most distinguished sons of Indiana. His law firm was Hendricks, Hord, and Hendricks.

Victor Hendricks was educated in the Indianapolis public and private schools and was graduated from Rose Polytechnic Institute, at Terre Haute, Ind., with the degree of Bachelor of Science in Civil Engineering in 1889. He received the degree of Master of Science in 1900. He was very popular as a student, and entered into athletic and social activities. He was also an accomplished master of the banjorine.

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[1] Memoir prepared by a Committee, consisting of Theodore L. Condron, M. Am. Soc. C. E., *Chairman*, and Alonzo J. Hammond, Past-President, Am. Soc. C. E., and F. G. Jonah, M. Am. Soc. C. E.

Immediately following his graduation, Mr. Hendricks spent some time with the Edgemoor Bridge Works, at Wilmington, Del.; but shortly thereafter he followed the urgent invitation of a classmate and went to the State of Washington, where he was engaged for several years in railroad construction by the Fairhaven and Southern Railway Company and the Bellingham Bay and Eastern Railway Company, experiencing throughout this period some of the trials of a locating engineer in a mountainous terrain.

In 1893, he returned to Indiana with the La Fayette and Indiana Bridge Companies, and then engaged in railroad engineering which was to be his principal life work, as Assistant Engineer on the Terre Haute and Logansport Division of the Vandalia Railroad. He was promptly advanced to the position of Engineer, Maintenance of Way, and Principal Assistant Engineer, retaining his connection with the Vandalia Railway Company until 1902. During this period he displayed a keen interest in the advancement of the art of railway engineering in all its phases.

Leaving the Vandalia Railway Company, Mr. Hendricks became Assistant to the Engineer, Maintenance of Way, of the Baltimore and Ohio Railway (System) and, later, Division Engineer. In 1907, he joined the staff of the St. Louis-San Francisco Railroad (System) as Assistant Engineer, Maintenance of Way, and he remained with that Company in various capacities for about thirteen years; as Office Engineer (1909–1911); Principal Assistant Engineer (1911–1913); Assistant Chief Engineer (1913–1918); and Chief Engineer under the United States Railroad Administration (1918–1919). During the same period he was also with the Missouri Kansas and Texas Railway Company. When his former chief came back from Army Service in France, in 1919, Mr. Hendricks returned to the position of Assistant Chief Engineer. In the succeeding years of 1920–1921, he was Assistant Manager and Field Engineer in the Department of Liquidation Claims, of the U. S. Railroad Administration at St. Louis, Mo., and, later, he became Regional Engineer for the Southwestern Region until that office was abolished in 1921.

Moving then to Chicago, Ill., Mr. Hendricks was first with the Illinois Appraisal Company, and, later, became associated with Rufus W. Putnam, M. Am. Soc. C. E., as Railroad Engineer in the studies for the Harbor Plan of Chicago for the Commercial Club of that city.

During 1928, to and including 1931, he was associated with Walter A. Shaw, M. Am. Soc. C. E., in the preparation of plans for the grade separation of the Chicago and Northwestern Railway and the Chicago North Shore and Milwaukee Railroad (Electric), where they pass through the municipalities of Kenilworth, Winnetka, Glencoe, and Highland Park, in Illinois. This work included detailed estimates and the preparation of the case for presentation to the Illinois Commerce Commission.

After this engagement, Mr. Hendricks retired from active service except for occasional brief engagements.

He was married, on May 28, 1908, to Sue Ross, of Terre Haute, a very charming girl whom he had known during his college days. He is survived by his widow and two children, Victor K., Jr., and Alan Ross.

One of the writers emphasizes strongly some of Mr. Hendricks' personal characteristics, his industry and courtesy, and his thorough analysis of problems before reaching conclusions. According to Colonel Jonah: "These were some of the characteristics which brought him to my favorable notice while he was with the 'Frisco.'"

He was a life member of the American Railway Engineering Association. He was also a member of the Western Society of Engineers.

Mr. Hendricks was elected an Associate Member of the American Society of Civil Engineers on May 1, 1895, and a Member on April 4, 1911.

EDWIN WESLEY HESS, M. Am. Soc. C. E.[1]

Died May 27, 1938

Edwin Wesley Hess, the son of Harmon and Sarah Hess, was born at Fishing Creek, Pa., on September 5, 1868, on a farm in Columbia County. His ancestors were from the "Rhine Provinces," and his forebears came to America in 1745 to escape the oppression of Frederick the Great. They settled near Bethlehem, Pa., but, in 1790, some of the family moved northward to Columbia County.

Edwin Wesley Hess received a common school education, and attended a country "academy" for one year. After teaching for two terms, he decided to turn to engineering, which had been his ideal since boyhood.

His engineering career was begun in 1887 when he was employed as Rodman by the Louisville and Nashville Railroad Company, near Cumberland Gap, Va. Then followed a series of railroad jobs for the Norfolk and Western Railroad Company; the New York, Ontario and Western Railroad Company; the Pennsylvania Lines West; the Choctaw, Oklahoma and Gulf Railroad Company; and many others. In 1899, he became Assistant Engineer for the New York Central Railroad Company, on the "River Line" east of Clearfield, Pa.

Upon the completion of this work, Mr. Hess opened a general engineering office in Clearfield for private practice, and it was here that he spent most of the remainder of his life. In 1906, after the building of the United Traction Company trolley line between Du Bois and Sykesville, Pa., on which he was Chief Engineer of Construction, he opened a branch office in Du Bois under the management of his brother, Mr. Oliver P. Hess. In both these offices, Mr. Hess built up a complete record of dealings which forms an interesting and authentic chronology of much of the county's history.

Work in these offices has been carried on with as many as twenty-five assistants, and included much of the coal and fire-clay developments of this region, railroads, trolley lines, and land surveys, with some of the operations extending as far as New York, West Virginia, Kentucky, and Tennessee.

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[1] Memoir compiled from information supplied by the family and from data on file at the Headquarters of the Society.

Paving of streets, and presentation of water supply and dam cases before the State Public Utility Commission were also problems handled by this firm.

In later years, Mr. Hess made the boundary surveys for tracts bought for the State of Pennsylvania by the Forestry Department. He was one of the last of the older men who practiced the art of land surveying.

He was Clearfield County's first Chairman of the County Relief Board established under the Pennsylvania Emergency Relief Board. He was appointed to this position in September, 1932, and served in that capacity for eighteen months.

Mr. Hess was a member of the Presbyterian Church; the Clearfield Lodge of Free Masons, Clearfield Chapter; Williamsport, Pa., Consistory, Jaffa Temple of the Shrine; Fraternity Club of Clearfield; and the Acorn Club of Du Bois. He was a former member of the Rotary Club of Clearfield, Acacia Club of Williamsport, the Pennsylvania Society of New York, the Pennsylvania Forestry Association, and the American Geographical Society. Among the technical Societies, he was a member of the American Water Works Association, the American Institute of Mining and Metallurgical Engineers, The Engineers Society of both Pittsburgh, Pa., and Harrisburg, Pa., and the American Association of Engineers of Chicago, Ill.

As an Engineer, the variety of railroad surveys and construction, the numerous clay and bituminous coal properties and mines with which Mr. Hess had to do, and the municipal work which he planned and executed will attest. As a man his relations with those with whom he came in contact were such as to create many friendships of long standing, and to win for him the respect of all with whom he associated. In his university of " hard knocks," Mr. Hess taught, and proved to be an inspiration to, many a young man in the field of engineering. As a citizen, he developed many ideas for the common good not only of his own community, but of many others.

He was married in 1899 to Maude D. Bouton, who died in 1916. There were no children by this marriage. In 1918, he was married to Bessie M. Rhinard, who survives him, together with their three children, Ruth Marie, Sara Agnes, and John Wesley. Also surviving are four brothers, Bruce W., George W., Robert F., and Oliver P. Hess.

Mr. Hess was elected a Member of the American Society of Civil Engineers on May 7, 1913.

FRAZER CROSWELL HILDER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 7, 1937

Frazer Croswell Hilder was born in St. Louis, Mo., on January 16, 1881, the son of Frank Frederick Hilder and Jeanne (Croswell) Hilder.

He received his Grammar School education and his High School training in St. Louis. He was graduated from the University of Missouri, at Columbia, Mo., in 1904 with the degree of Bachelor of Science in Civil Engineering, and, in 1907, he received the degree of Civil Engineer from the same University.

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[1] Memoir prepared by William S. Anderson, Assoc. M. Am. Soc. C. E.

During his undergraduate days, Mr. Hilder spent his summers with survey parties employed in the Water Department in Washington, D. C. After his graduation he worked as Instrumentman and Draftsman in the District of Columbia Engineering Department from July, 1904, to December, 1905. From January to May, 1906, he was employed as Instructor in charge of Engineering at the Intercontinental Correspondence University, at Washington, D. C. In this connection he also prepared engineering courses and supervised instruction. From June to July, 1906, he served as Instrumentman in charge of sewer construction at Spencer, N. C., for the firm of Pressey and Weller.

Mr. Hilder was with the Chesapeake and Ohio Canal Company as Draftsman in charge of right-of-way mapping from August to September, 1906, after which, until January, 1907, he was engaged as Leveler for the Mississippi River Commission. From February to May, 1907, he served as Surveyor in charge of control survey on the Mexican border with the Southwest Division, United States Army. From June to September, 1907, he was engaged as Leveler, Draftsman, and Topographer on location by the Missouri River and Gulf Railroad Company, after which, until February, 1909, he was again with the Mississippi River Commission, serving as Junior Engineer in charge of levels, Permanent Marks Party, being responsible for two groups engaged on ordinary levels of high accuracy, also tertiary triangulation and base lines.

In March, 1909, Mr. Hilder entered the service of the United States Indian Office, Washington, D. C., as Draftsman in charge of mapping Indian Reservations, and from July, 1910, to September, 1916, he served as Office Engineer in charge of all engineering work of the Service. In this capacity Mr. Hilder had the distinction of having designed the first highway bridge over the Colorado River at Yuma, Ariz. He also acted in advisory capacity for the design of a three-hinged arch bridge over the Colorado River near Needles, Calif. From October, 1916, to April, 1917, he was with the Bureau of Yards and Docks, United States Navy Department, as Design Draftsman on Structural Steel and Concrete Design.

In January, 1917, Mr. Hilder was appointed a Captain of Engineers in the Officers Reserve Corps, and in May of the same year he was assigned to active duty at the Reserve Officers Training Camp, Fort Myer, Virginia. Later, he was transferred to the 317th Engineers, at Camp Sherman, Ohio, and left the United States in December, 1917, for service overseas. He was assigned to duty as Assistant Construction Officer with Headquarters, Base Section No. 3, Line of Communication, American Expeditionary Forces. In April, 1919, he was promoted to the rank of Major (Engineers), and assigned to duties as Construction Officer. He returned to the United States on August 19, 1919, and was mustered out of service on October 31, 1919. He accepted appointment as Major, Engineers Reserve, in March, 1920, and was a very active and valuable member of this organization until his death.

From November, 1919, to September, 1926, Major Hilder was engaged as Supervising Draftsman, Bureau of Yards and Docks, Navy Department, on a wide range of structural design; he investigated possibilities of reinforced concrete for radio towers, and designed deferrization plants for the Naval Air Station, at Pensacola, Fla., and at the Naval Academy, at Annapolis, Md. Later, he was in charge of the design of plants for storage and handling of fuel oil, lubricating oil, and gasoline, including pumping plants and fire protection, notably of the plant at Portsmouth, N. H., costing $1 000 000.

From October, 1926, until his death in September, 1937, he served in the office of the Supervising Architect in the Treasury Department, at Washington, D. C., as Associate Structural Engineer on the structural design of public buildings.

Major Hilder well typified a large number of engineers who devote a lifetime to the service of the Government. He will be remembered by his many friends and associates for his thoughtful and kindly concern for all with whom he came in contact; his devotion to his highest ideals, both in his professional and daily life, was unfailing.

He was a member of the Society of American Military Engineers, the National Reserves Officers Association, and the National Society of Professional Engineers.

He was married on December 7, 1910, to Janie Frost, of Washington, D. C., who, with his two sons, Frazer and Peter, survives him. He was buried with full military honors in Arlington National Cemetery, Virginia.

Mr. Hilder was elected a Junior of the American Society of Civil Engineers on September 6, 1904; an Associate Member on April 30, 1912; and a Member on January 14, 1924.

EDWARD LOVERING INGRAM, M. Am. Soc. C. E.[1]

DIED JULY 25, 1938

Edward Lovering Ingram, for many years Professor of Railroad Engineering and Geodesy in the Civil Engineering Department of the University of Pennsylvania, Philadelphia, Pa., died on July 25, 1938, at his home in Grand View-on-Hudson, N. Y.

Edward Lovering Ingram was born in Philadelphia, on September 9, 1862, the son of John and Ella F. (Guier) Ingram. He received his early education in the public schools of Philadelphia, and in 1885 was graduated from Columbia University, in New York, N. Y., with the degree of Civil Engineer.

He was employed in 1885 by the United States War Department as Assistant Engineer on river and harbor improvement and remained in this work for six years. Mr. Ingram spent the next four years as Assistant Engineer with the United States State Department on an international boundary survey between the United States and Mexico. From 1895 to 1901, he was engaged in private practice. For a part of the year 1901 he was employed by the New York Central and Hudson River Railroad Company, and from 1901 to 1906 he served as Civil Engineering Expert Aid at the New York Navy Yard, in Brooklyn, N. Y.

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[1] Memoir prepared by Francis P. Witmer, M. Am. Soc. C. E.

In 1906 Mr. Ingram was appointed to the Faculty of the Civil Engineering Department of the University of Pennsylvania, in Philadelphia, and continued in that connection until his retirement in 1932. From 1906 to 1911 he was Assistant Professor of Civil Engineering and from 1911 until his retirement, he was Professor of Railroad Engineering and Geodesy.

Professor Ingram was the author of a book entitled "Geodetic Surveying," published in 1911 by the McGraw-Hill Book Company; and for a number of years he aided the Pennsylvania State Registration Board for Professional Engineers in the conduct of qualifying examinations.

During his twenty-six years on the Faculty of the Civil Engineering Department of the University of Pennsylvania, Professor Ingram administered many important courses, particularly in the fields of Railroad Engineering and Geodesy.

A course in which he took particular interest was the Senior Seminar during which he developed a high degree of enthusiasm on the part of the students. His invariably kindly and courteous personality endeared him to both students and Faculty and his passing is a matter of keen regret.

He was a member of Tau Beta Pi and Sigma Xi Fraternities, the American Association for the Advancement of Science, the American Railway Engineering Association, the Alumni Association of the Schools of Science of Columbia University, and the Alumni Society of the University of Pennsylvania.

Professor Ingram was elected an Associate Member of the American Society of Civil Engineers on September 4, 1895; and a Member on October 12, 1925. Having paid dues as a Corporate Member for thirty-five years, Professor Ingram became a Life Member of the Society on January 1, 1930.

HURLBUT SMITH JACOBY, M. Am. Soc. C. E.[1]

Died November 16, 1938

Hurlbut Smith Jacoby was born on February 7, 1886, at Memphis, Tenn. He was the second son of Henry S. Jacoby, M. Am. Soc. C. E., and Laura

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[1] Memoir prepared by a Committee of the Central Ohio Section consisting of J. R. Shank, M. Am. Soc. C. E., as Chairman, and R. B. Jennings and C. V. Youngquist, Assoc. Members, Am. Soc. C. E.

(Saylor) Jacoby. His father is Professor Emeritus of Bridge Engineering of Cornell University, Ithaca, N. Y., and the author of a number of textbooks and papers on structural engineering. His ancestors were among the group of Pennsylvania Germans who settled southeastern Pennsylvania during the middle third of the Eighteenth Century. These people were truly American in spirit, a characteristic which was born of their sense of escape from the devastating horrors of European wars, and which was demonstrated by their activities in providing munitions and sustenance, as well as men, for the American army during the most perilous times of the Revolution.

Mr. Jacoby's early childhood years were spent at Bethlehem, Pa., where his father taught Civil Engineering at Lehigh University from 1886 to 1890. The family then went to Ithaca, where the father continued his teaching at Cornell University. Mr. Jacoby received his early education in the Public Schools at Ithaca, and was graduated from Ithaca High School in 1904. His parents, who had inherited the fervent religiousness of his ancestors, were devout Methodists, and were intensely interested in religious education. He was graduated from Cornell University in 1908 with the degree of Bachelor of Arts, and in 1910 received the degree of Civil Engineer from the same University. For two summers preceding his graduation, Mr. Jacoby worked as a templet maker for the McClintic Marshall Company. When he finished school he continued working for the same company until March, 1913, as Draftsman, Designer, Inspector, and Contract Engineer. During this time he was Inspector on the Panama Canal Lock Gates.

From 1913 to 1915 Mr. Jacoby was Chief Engineer for the newly organized Standard Steel Construction Company of Welland, Ont., Canada. While in this position he designed the power plant for the Canadian Kodak Company at Toronto, Ont., Canada, and the grain elevators for the Harbor Commissioners at Montreal, Que., Canada. After Canada entered the war in 1914, the control of the company passed into other hands, and Mr. Jacoby accepted a position in March, 1915, as Assistant Professor of Structural Engineering at the Pennsylvania State College, State College, Pa.

Although Professor Jacoby and his wife were settling down to a life in academic circles, the call of the building industry was too strong, and he accepted a position as District Engineer with The Austin Company of Cleveland, Ohio, in August, 1915. While in this work he became interested in the idea of standard building designs, originated by Mr. H. K. Ferguson; he developed it and became engineer in charge of this phase of the work of The Austin Company. He became an expert on industrial buildings and designed many large industrial plants, of which one of the most important was the Hooker-Electric-Chemical Company at Niagara Falls, N. Y.

In 1918 Mr. Jacoby joined with H. K. Ferguson in founding the H. K. Ferguson Company, of Cleveland, Engineers and Builders. He became its Secretary and Chief Engineer, which office he held until 1923, when he became Vice-President in Charge of Sales. As Chief Engineer he designed many large industrial plants, including some for the National Cash Register Company, the General Electric Company, and the plant for the Homer-

Laughlin China Company at East Liverpool, Ohio, which was original and outstanding for plants in this industry. This work took him to many parts of the world, including a trip to Germany in 1921. He also went to Mexico where he investigated and reported on the conditions of monumental buildings that were settling.

Mr. Jacoby was a particularly able engineer. He was at all times interested in developing new types of structural design, including early experiments on welded structures. He was engaged in conjunction with H. K. Ferguson in the preparation of a book on the history and development of industrial structures, beginning with the early home craftsmen in England and on the Continent, through the Guild days, and up to the present time. Unfortunately this work was not finished at the time of his death. He was the author of a number of magazine articles on industrial building construction.

In October, 1933, while visiting his mother in Washington, D. C., Mr. Jacoby was discovered by one of the officers of the National Industrial Recovery Act (N. R. A.) and immediately pressed into service as Assistant Administrator in the machinery and allied products division. This position demanded a variety of types of work, among which was the developing of specifications for materials and construction.

He remained with the N. R. A. until January, 1935, when the Ohio State Research Foundation, just being developed, called him to be its first Director. He worked in a preliminary capacity as organizer until the incorporation of the Foundation was announced at the First Annual Conference on Industrial Research in November, 1936, when he officially became Research Director and Secretary.

Mr. Jacoby's background of experience and personality peculiarly fitted him for this task. The broad knowledge of industrial processes and problems which he obtained while working with The Austin Company and the H. K. Ferguson Company, together with his ability to make and retain friends, his energetic and affable presence and good nature, and his sound attitude toward the major problems of life, were apparent to every one with whom he came in contact.

Mr. Jacoby laid the groundwork for a comprehensive program of research to the mutual advantage of the Ohio State University and the industries of Ohio and of the Nation. Co-operative arrangements in existence were extended. Graduate fellowships, full-time research positions, and arrangements for faculty members to direct research, testify to his activities in many departments. The Foundation has suffered a severe loss but his work will go on.

On June 26, 1912, he was married to Emma Betsey Farley of Oneonta, N. Y., who survives him. Their children are Hurlbut Saylor, Mary Vaughn, and Betsey Ann. Both of his parents also survive him.

He was a member of the Engineers' Club of Columbus, Ohio, and of the Faculty Club of Ohio State University.

Mr. Jacoby was elected an Associate Member of the American Society of Civil Engineers on November 3, 1915, and a Member on June 20. 1922.

ALFREDO CARLO JANNI, M. Am. Soc. C. E.[1]

Died February 26, 1938

Alfredo Carlo Janni was born in Caserta, Italy, on November 2, 1869, of an old Italian family of noble lineage. He bore the title of Count, but preferred not to use it after he became an American citizen. He was graduated from the University of Naples, at Naples, Italy, in 1893, with the degree of Civil Engineer, and remained at the University as Assistant Professor of Steel Construction and Strength of Materials for two years.

He then engaged in professional practice, first as Assistant to the Chief Engineer of "Ferriere Meridionali," in which capacity he designed all bridges and steel structures for the Cairo Awan Railway, in Egypt. He also served as Engineer on the construction of a reinforced concrete building for the Egyptian Museum in Cairo, for the English Government, and as Construction Engineer for Zaafrau Palace for the Khedival Family, in Cairo.

From 1896 to 1899, he was Assistant in the Chief Engineer's Office of D'Alessandro and Company, which Company built the several tunnels on the railroad line from Florence to Bologna, Italy. He worked for the same Company on the first Simplon Tunnel.

Mr. Janni came to the United States in 1904 and his first job was the erection of a distillery for the Southern States Alcohol Manufacturing Company, in New Orleans, La. In 1905, he went to St. Louis, Mo., as Assistant Bridge Engineer for the Southern Railway Company, designing most of the reinforced concrete structures on the French Lick Line.

In 1909, he became Assistant Bridge Engineer for the City of St. Louis, in which capacity he served until October, 1912, designing and supervising the construction of many of the steel and reinforced concrete structures built by the City, including the Carondelet Park Bridge and the Kingshighway Viaduct. The Kingshighway Viaduct was noteworthy at the time as containing the flattest reinforced concrete arches of this size constructed to that date. The three main spans were 140 ft, 170 ft, and 140 ft, and the rise of each was one-tenth the span. Considerable difficulty was encountered in the foundations due to soft clay, and Mr. Janni solved this problem by means of an ingenious reinforced concrete spread footing.

On leaving the service of the City of St. Louis, Mr. Janni entered private practice, and soon, thereafter, moved to New York, N. Y., where he practiced until his death.

In 1915, he won a competition for a monumental reinforced concrete bridge design at Petersburg, Va. In 1918, at the request of the United States Shipping Board, he developed a theory and method of design for reinforced concrete ships which were accepted and used. In 1926, he designed and supervised the construction of the large memorial concrete bridge at Lynchburg, Va.

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[1] Memoir prepared by Clyde T. Morris, M. Am. Soc. C. E.

Mr. Janni was much interested in the field of theoretical mechanics and introduced many European methods of analysis into the United States. In 1922 he wrote a letter to the Board of Direction of the Society, suggesting that tests be made to verify the theory of elasticity when applied to the design of reinforced concrete arches, especially when supported on elastic piers. This letter resulted in the appointment, by the Board, of the Committee on Concrete and Reinforced Concrete Arches, in May, 1923, and Mr. Janni was made a member of that Committee. He gave freely of his time and energy to the work of the Committee until the completion of its Final Report in November, 1934.[2]

He was the author of numerous articles on structural subjects in the technical press and in the *Proceedings* and *Transactions*[3] of the Society.

On March 2, 1908, he was married to Marie Isabel Willcox, and is survived by his widow and a brother, General Vincenzo Janni.

Mr. Janni was elected a Member of the American Society of Civil Engineers on December 3, 1913.

EDWIN SETON JARRETT, M. Am. Soc. C. E.[1]

Died December 26, 1938

Edwin Seton Jarrett was born in Brooklyn, N. Y., on March 7, 1862. His parents were James M. and Sarah Olivia (Heather) Jarrett. The family moved to Buffalo, N. Y., and Edwin was graduated from the Central High School of Buffalo in 1881.

His further education was interrupted until 1885, when he entered the Rensselaer Polytechnic Institute at Troy, N. Y. He was graduated from that institution in 1889 with the degree of Civil Engineer. This period of his life made a profound impression upon him and its influence is seen throughout his subsequent career.

He began his engineering career in the employ of the late Charles Sooysmith, M. Am. Soc. C. E., a son of General William Sooy Smith, M. Am. Soc. C. E. The latter was largely instrumental in the application, in this country, of the pneumatic method for the construction of substructures for bridges. The son was associated with the father in this work but later established an independent company, and it was this company which first employed Mr. Jarrett and, at the same time, another young engineer, the late Daniel E. Moran, M. Am. Soc. C. E.,[2] with whom he was long associated.

Until nearly the end of the Nineteenth Century there were no tall buildings in New York City. South of Tenth Street, on Manhattan Island, the surface of bed-rock disappears below ground level and lies at greater and greater depths toward the south, down to Duane Street. From that point it rises toward the surface, more or less gradually, until, at the Battery, it again emerges. The rock in this region usually is overlaid, first by a comparatively thin stratum of glacial till, or hardpan, and then by sand of various degrees of fineness. Below the ground-water level, the sand is usually very fine and contains mica scales so that, when not confined, it is very unstable.

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[2] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1427.

[3] *Loc. cit.*, Vol. LXXX (1916), p. 1609; also, Vol. 88 (1925), p. 1142.

[1] Memoir prepared by Ralph H. Chambers, Philip W. Henry, and H. C. Baird, Members, Am. Soc. C. E.

[2] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1840.

The buildings which, up to that time, had been erected in Lower Manhattan, were usually founded on the sand, with footings spread out above the ground-water level. In some cases wooden piles had been used but, except for these, no means had been found to carry excavations down, through the saturated fine sand, to the bed-rock, without danger to adjoining structures.

In 1893, the concentration of business in Lower Manhattan and the rise in the value of real estate made it imperative to increase the height of buildings, and the construction of the Manhattan Life Building at No. 66 Broadway was planned. The architects, Messrs. Francis H. Kimball and George Kramer Thompson, discovered, however, that even by the use of wooden piles, a foundation of sufficient capacity could not be obtained. The area of the lot was not large enough to permit the driving of the required number of piles. The only solution was to reach the bed-rock and this lay 54 ft below the surface.

At this juncture Charles Sooysmith suggested to the architects the use of pneumatic caissons and subsequently built the foundations for this first skyscraper by this method. Later the Sooysmith Company built, by the pneumatic method, the foundations for the American Surety Building at Broadway and Pine Street, the Washington Life Building at Broadway and Liberty Street, and the Empire Building at Broadway and Rector Street.

It was into this work—the building of bridge foundations and light-houses, the sinking of shafts for coal mines and the construction of the foundations for the Cathedral of St. John the Divine in New York, and the building of deep foundations for buildings in Lower Manhattan—that Mr. Jarrett entered as a young engineer.

In 1898 Charles Sooysmith retired from the construction business and Mr. Jarrett, with some of his associates in the Sooysmith Company, organized the Engineering Contract Company. This company was unfortunate, and in 1900 Mr. Jarrett opened an office in New York as a Consulting Engineer. During this period, he was, for some time, engaged in mining phosphate rock in Florida.

In 1902, with Mr. Franklin Remington and Mr. Moran, Mr. Jarrett organized The Foundation Company. This Company, beginning with very modest financial resources, has completed engineering works in many different countries. It has built the foundations for a great number of the large buildings in Lower Manhattan, among which are the Singer, the United States Realty, the Irving Trust, the J. P. Morgan, the Equitable Trust, the Westinghouse, the Chase Bank, the American Telephone and Telegraph, the Guaranty Trust, the Woolworth, and the Municipal Buildings.

In 1914 Mr. Jarrett resigned from The Foundation Company and, with Ralph H. Chambers, M. Am. Soc. C. E., organized the Jarrett Chambers Company which continued for thirteen years in the construction of foundations and other engineering works, including the foundations for the main office building of the Standard Oil Company of New York at No. 26 Broadway, New York, the shore-protection works at Sandy Hook, the Battle-ship Launching Ways No. 2 at the Brooklyn Navy Yard, the 10 middle launching ways at the Hog Island (Pennsylvania) shipyard, the foundations for the Standard Milling Company Building in Jersey City, N. J., the foundations for the Oswego Elevator of the New York State Barge Canal, and many others.

In his professional work Mr. Jarrett was particularly interested in the construction of mine shafts in soft ground, and 48 such shafts were sunk by companies with which he was associated. He had a firm faith in the final success of the freezing method which was advocated by his first employer, Charles Sooysmith, and which has, of late years, been so successfully employed in the sinking of very deep shafts in northern France, Belgium, and Germany. That it has not come into use in the United States is probably due to the fact that shafts in soft ground, of comparable depths to those sunk in Europe, have not yet been needed here.

Mr. Jarrett's great store of experience, calm judgment, and steadfastness of purpose were always recognized by his associates, and many times he was called upon to assume the entire direction of work which had proved to be particularly difficult. On one of these occasions he spent many months in Montana in the closing of the Hauser Lake Dam. In another, he remained for months in New Orleans, La., on professional work during the yellow-fever epidemic.

Mr. Jarrett's interest in the welfare and development of the Rensselaer Polytechnic Institute and in its students has been lifelong. When he was a student there, he was a member of a students' local society which was founded about 1864 as the Pi Eta Society, but later was changed to the Rensselaer Society of Engineers. Notwithstanding its original name, it was not a Greek letter fraternity, but it came to have a similar function in the student body. It draws its membership from the more earnest and serious-minded of the students. Mr. Jarrett's interest in this society and in its members became, in the last 20 years, one to which he devoted a large part of his time and efforts. In 1923 the late Mr. John M. Lockhart gave a large sum of money to Mr. Jarrett, as trustee, to be used for the building of a house, with living quarters, for the members of this society. Mr. Lockhart was graduated from the Institute, and while he was a student he had been a member of this society. He was graduated two years before Mr. Jarrett, and it is an evidence of the esteem in which Mr. Jarrett was held that this sum was placed in his hands without any specific limitations. The building was planned, built, and furnished under Mr. Jarrett's direction.

Mr. Jarrett's interest in the Institute was not confined to this society alone. In 1924 he was elected an Alumni Trustee of the Institute and served one year, but resigned because of ill health. In December, 1926, he was elected a Life Trustee and served as such until his death. In December, 1931, he was elected a member of the Prudential Committee, the executive committee of the

Trustees, and served until February, 1938. He was also a member of the Finance Committee from November, 1935, until December, 1938.

Because of these several offices, he became thoroughly conversant with the affairs and problems of the Institute. When the President of the Institute, the late Palmer C. Ricketts,[3] Hon. M. Am. Soc. C. E., died, there was no one so thoroughly familiar with them, and in December, 1934, Mr. Jarrett was elected Vice-President, with all the duties of President. He served in this office until September, 1935, when William O. Hotchkiss, M. Am. Soc. C. E., was elected President. Following this, he continued to serve the Institute in an advisory capacity and in charge of the enlargement and reconstruction of its gymnasium building, until within about a year of his death. His conscientiousness, which was one of his outstanding characteristics, was shown in February, 1938, when he insisted that his resignation as Vice-President be accepted because he felt that the state of his health prevented his filling the position properly.

Mr. Jarrett had traveled widely. During one of his vacations, in his student days at the Institute, he made a trip to Australia and around the world. At a later time, he spent one of his vacations in climbing the mountains of Wales. He made several trips to Europe, and in 1926 spent a winter with Mrs. Jarrett on the Island of Majorca. His work carried him many times to many parts of the United States.

The outstanding quality of Mr. Jarrett's character was his high sense of honor in his personal and business relationships. As an instance of this, he repaid, over a period of more than ten years, investments which friends had made and lost in the Engineering Contract Company.. These were in no sense loans to him, or to the Company, but were investments in the stock of the Company, made in the belief that it would be successful.

Mr. Jarrett's early school days in Buffalo brought him into an environment which awakened in his mind a love for the use of perfect English, and in his later life his letters and other writings were excellent examples of English composition.

He was fond of people and, particularly, of his many friends in the Century Club of New York. He was ever ready to help younger men, professionally, where a little unsolicited action on his part would further their interests. He was very thoughtful of the welfare of his personal friends, frequently doing favors for them which entailed considerable effort on his part.

Mr. Jarrett was also greatly interested in public affairs, although he never took an active part in politics. He was fond of music and of fishing but not of outdoor games. However, he was fond of country life, and in 1911 he purchased Wild Goose Farm in the Shenandoah Valley, near Shepherdstown, W. Va. The house was very old and he modernized it completely without altering its mellow attractiveness. It was here that his children were brought up, and it remained in his possession until his death.

On June 26, 1909, Mr. Jarrett was married to Cora Hardy of Norfolk, Va., who survives him. He is also survived by his two sons, Edwin Seton, Jr. and William Armstead, and by his daughter, Mrs. Olivia Jarrett Fowler.

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[3] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1497.

He was a member of the Louisiana Engineering Society, the Rensselaer Society of Engineers, and St. George's Society; also the University and Century Clubs of New York, the Nassau Club of Princeton, the Cosmos Club of Washington, D. C., and the Engineers Club of London, England.

Mr. Jarrett was elected an Associate Member of the American Society of Civil Engineers on May 1, 1895, and a Member on April 6, 1909.

JENKS BUFFUM JENKINS, M. Am. Soc. C. E.[1]

Died February 1, 1939

Jenks Buffum Jenkins was born at Allegheny, later a part of Pittsburgh, Pa., on January 28, 1869, the son of Friend William and Seloma Wood (Buffum) Jenkins. On the maternal side his American Ancestry ascended through nearly three centuries to the early days of Plymouth Colony. His only brother, Lieut. Friend W. Jenkins, Jr., U. S. Navy, was one of those who perished when the Battle-ship *Maine* was destroyed by explosion in the harbor of Havana, Cuba, on February 15, 1898.

His formal education was begun in the Public Schools of Pittsburgh, and was concluded by a 4-yr engineering course in the Western University of Pennsylvania, later the University of Pittsburgh, from which he was graduated in June, 1888, with the degree of Civil Engineer.

Mr. Jenkins' first practical experience was acquired during the summer vacation of 1887, when he served from June to September as Assistant on the Engineering Corps of Edeburn and Cooper, Civil and Mining Engineers of Pittsburgh. Upon graduation he began his long and useful engineering career in the capacity of assistant in the office of the City Engineer of Allegheny.

His railroad service began on June 24, 1889, when he entered the organization of the Maryland Central Railroad Company, later the Maryland and Pennsylvania Railroad Company. He remained with this company until November 15, 1889, serving as Transitman and Assistant Engineer.

On November 18, 1889, he entered the service of the Baltimore and Ohio Railroad Company as Assistant Engineer, and the association thus begun was continued until March 31, 1893. Within this period he served as Resident Engineer on the construction of the classification yard at Brunswick, Md., made extensive property surveys for the Real Estate Department, and supervised the construction of the Staten Island Terminal and South Beach Extension of the Staten Island Rapid Transit Railway.

His work during the succeeding seven years comprised a wide variety of duties and service in several organizations. From June to August, 1893, he was employed as Structural Steel Draftsman by Riter and Conley, of Pittsburgh; during the succeeding year he served as Topographer for the Baltimore and Washington Traction and Tramway Company, and as Transitman in charge of a party on preliminary and location for the Washington and Great Falls Electric Railway. In July, 1894, he entered private practice in general engineering, including electric railway construction, at Pittsburgh, which he continued until March, 1896. He then entered the Department of Street Improvements, New York, N. Y., as Topographical Draftsman, and, in June, 1898, became Assistant Engineer for the Board of Public Improvements of that municipality, in charge of street openings for the Borough of Queens, in which capacity he remained until May, 1900.

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[1] Memoir prepared by Philip G. Lang, Jr., M. Am. Soc. C. E.

On May 1, 1900, he re-entered the service of the Baltimore and Ohio Railroad Company, and, with the exception of the World War period, the remainder of his life was devoted to the interests of that Company. Upon appointment he was assigned to responsible charge of the construction of the coal pier at Curtis Bay, Md.; he then, successively, handled the construction of the Crisp Yard at Curtis Bay; line improvements on the Curtis Bay Branch; the sixth Street Bridge at Wilmington, Del.; Jackson Street coal pier, at Philadelphia, Pa.; improvements at Glenwood, Pa.; the development of a low-grade line through the Pittsburgh District; and third and fourth track work between Wheeling Junction and McKeesport, Pa.

Between 1905 and 1914 Mr. Jenkins undertook an extensive program of surveys and studies for improvements and changes on various parts of the Baltimore and Ohio System, and compiled numerous statistical reports embodying the results of his investigations. Although some of these projects were not actually completed, his analysis was, in each case, characterized by extraordinary clarity and wealth of detail, which developed every phase of the subject, and omitted nothing which might be of value in guiding the management to an intelligent decision.

Mr. Jenkins' mental powers were of a keenly analytical character, whether concentrated upon abstract scientific fact or questions of railway economics. Credit is due him for the establishment of rules to govern the use of the railway transition curve and the development of the ten-chord spiral, adopted by the American Railway Engineering Association. In the field of railway organization he has a claim to distinction as the originator of the movement which resulted in making the railroad fiscal year coincident with the calendar year—that is, January 1 to December 31, instead of July 1 to June 30; this arrangement has been productive of vast economic advantages in the allocation of maintenance appropriations.

When, early in 1914, it became necessary to consider the appointment of a properly qualified man to supervise the valuation of the Baltimore and Ohio properties, the diversified experience and peculiar intellectual gifts of Mr. Jenkins marked him as one eminently fitted for this post. Accordingly, on March 1, 1914, he became Valuation Engineer for the Baltimore and Ohio System. The magnitude of this task, and the challenge that it offered, even to Mr. Jenkins' capacity for sustained and intensive research, can be appreciated, in some degree, when it is realized that the Baltimore and Ohio is the world's oldest trunk-line railroad, with a continuous history dating from 1827, and that many priceless records of its earlier days had been destroyed in the great Baltimore fire of 1904, lost, or otherwise dissipated through the vicissitudes of this long period.

During the succeeding four years, Mr. Jenkins applied himself assiduously to the difficult task confronting him. Under his guidance this great and complicated undertaking progressed steadily, and the results of systematic investigation and research began to take definite and intelligible form.

Mr. Jenkins had been a vigorous proponent of the National Defense Act of 1916, by which the Reserve Officers Training Corps was created, and had observed with profound interest the developments of the World War, since its beginning, in August, 1914. As the final crisis of that conflict approached, and the participation of the United States became inevitable, the martial and patriotic tradition of his family asserted itself, although he remained at his post in the Baltimore and Ohio organization until the end of 1917.

Effective January 2, 1918, he obtained a furlough for the purpose of entering the National organization, and was appointed Constructing Quartermaster, in charge of hospital erection, with the rank of Captain. In that capacity he supervised the work on U. S. Hospitals No. 18, Azalea, N. C., No. 12, Biltmore, N. C., No. 19, Waynesville, N. C., and No. 23, Hot Springs, N. C. The nature of Colonel Jenkins' work and responsibilities detained him in the military establishment for several months after the actual conclusion of hostilities. He was honorably discharged from the National service on August 16, 1919, and, two days later, on August 18, 1919, resumed his duties as Valuation Engineer of the Baltimore and Ohio Railroad Company, to the administration of which he devoted the remainder of his life. His major accomplishment during this concluding phase of his career was the completion of the Baltimore and Ohio Railroad valuation, which he pushed with accelerated speed, after the interruption of the war period.

Through these mature and studious years, a portion of his time was devoted to the interests of various patriotic and military organizations, with the special object of maintaining the Nation in a state of readiness to defend its interests. In 1920 he was made Lieutenant Colonel of Engineers, and, three years later, became Colonel of Engineers in the Officers Reserve, which distinction he bore throughout the remainder of his life.

He was a member of the American Railway Engineering Association, for which he served as Chairman of the Track Committee for five years. He also held membership in the Society of American Military Engineers, American Legion, Military Order of the World War, and the Engineers Club of Baltimore, Md.

On March 24, 1897, Mr. Jenkins was married to Margaret Bailey Newman, at Baltimore, who, with two daughters, Mrs. Clyde E. Loose, and Mrs. Gwynn Kenneth Crowther, and a granddaughter, Dean Jenkins Loose, survives him.

Mr. Jenkins was elected a Member of the American Society of Civil Engineers on April 18, 1916.

FRANK WASHBURN JENNINGS, M. Am. Soc. C. E.[1]

Died September 28, 1938

Frank Washburn Jennings was born at Logan, Ohio, on March 11, 1868 His parents were William Henry Jennings and Caroline Frances (Allen) Jennings.

His father who was also an engineer and member of the Society began his career on location and construction of the Hocking Valley Railroad, which he later served for twenty-two years in the capacity of Chief Engineer.

His mother, a member of a pioneer family that emigrated from England in the early part of the Seventeenth Century, was a direct descendant of John and Priscilla Alden who crossed on the Mayflower.

Mr. Jennings' family moved from Logan to Columbus, Ohio, when he was five years old, and the greater part of the remainder of his life was spent in this city. He was graduated from Central High School in Columbus in 1886, and attended Ohio State University for two years, leaving to take a position under his father with the Hocking Valley Railroad Company.

The early years of Mr. Jennings' professional life were devoted exclusively to railroad work. He served as Rodman and Transitman for the Hocking Valley, Kanawha and Michigan, and Pittsburgh and Lake Erie Railroad Companies in the period from January, 1885, to January, 1893. After a short period with the Toledo and Ohio Central Railroad Company as Resident Engineer, and with the Kanawha and Michigan Railway Company as Assistant Engineer, he rejoined the engineering staff of the Hocking Valley Railroad Company in the capacity of Assistant Chief Engineer, in charge of preliminary and location surveys, occupying this position from October, 1894, to November, 1897. Following this engagement he served four years as Assistant Chief Engineer for The Columbus Northwestern and the Detroit and Toledo Shore Line Railway Companies, and from 1901 to 1903 he occupied the position of Assistant Chief Engineer of the West Virginia and Kentucky Railroad Company.

In November, 1903, Mr. Jennings left the railroad work to engage in private practice in Columbus. He organized the firm of Jennings and Lindsay, engaging in general engineering and specializing in railroad and municipal engineering and surveying. Later the firm name was changed to Jennings, Lawrence and Lindsay, and subsequently to The Jennings-Lawrence Company. Mr. Jennings held the position of President of this firm until his death.

This firm under Mr. Jennings' leadership became one of the leading engineering organizations of Central Ohio and served as consulting engineers for many towns throughout Ohio, designing and supervising the construction of streets and highways, water-works, water treatment plants, sewer systems, and sewage treatment works.

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[1] Memoir prepared by a Committee of the Central Ohio Section, consisting of C. W. Ozias and F. D. Stewart, Members, Am Soc. C. E., and R. B. Jennings, Assoc. M. Am. Soc. C. E.

During the later part of Mr. Jennings' life he became well known as an authority on street and highway construction, and was frequently called upon to serve in the capacity of Master Comissioner in litigation relative to highway contracts.

At the time of the trouble on the Mexican border in 1916, Mr. Jennings served as Chief Engineer at Camp Willis at Columbus, and during the World War he was Chief Engineer in charge of surveys at Camp Sherman, located at Chillicothe, Ohio, remaining there until the signing of the Armistice.

On April 28, 1900, he was married to May Bond of Columbus, who survives him. He leaves also three children—Robert B. Jennings, Assoc. M. Am. Soc. C. E., Alice C. (Jennings) Gotter, wife of Robert J. Gotter, and Dr. Frederick A. Jennings—a sister, Mrs. Carl D. Postle, and four granddaughters.

Mr. Jennings was active in a number of engineering and civic organizations. He was a Past-President and member for many years of the Ohio Engineering Society, later the Ohio Society of Professional Engineers. He was also a member of the National Society of Professional Engineers. He was always most active in the affairs of the Central Ohio Section of the American Society of Civil Engineers and was a Past-President of this organization.

Another organization dear to his heart was the Engineers Club of Columbus, of which he, as well as his father, was a Charter Member, and both were Past-Presidents of this organization.

He was affiliated with the Columbus Lions Club, was a member of Sigma Chi Fraternity, and a member of York Lodge, Free and Accepted Masons. He belonged to the Methodist Church.

Mr. Jennings was elected a Member of the American Society of Civil Engineers on April 16, 1918.

FRANCIS DENIS HUBERT LAWLOR, M. Am. Soc. C. E.[1]

Died January 25, 1939

Francis Denis Hubert Lawlor was born in St. John, New Brunswick, Canada, on January 30, 1857, a son of James J. and Mary (Ford) Lawlor. His father was a watchmaker and jeweler. Francis attended a Christian Brothers School.

In 1873 he entered the office of the City Engineer in his home town as a student without pay. At that time the English system required students to remain four years without pay. After two years he secured a place on the staff of the Superintending Engineer for the Maritime Provinces of the Department of Public Works in Canada. From 1875 to 1880 he was engaged in the Maritime Provinces on surveys for river and harbor improvements, dredging bars, constructing public piers and breakwaters, surveying St. John Harbor, in the extension of the Intercolonial Railway, later the Canadian

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[1] Memoir prepared by Edward Bartow, M. Am. Soc. C. E.

National Railway, in the construction of a terminal, and enlargement of St. Peter's Canal and the Negro Town Point Breakwater.

In 1880 Mr. Lawlor came to the United States and secured a position in the engineering department of the Chicago, Burlington and Quincy Railroad Company. He was placed in charge of the first division of the branch line toward Bethany, Mo., under S. R. Ballard. In 1881 he was transferred to Chicago, Ill., where he was engaged in the work of reorganization and extension of the terminal facilities of the Burlington Lines. He supervised the construction of the old union depot which has since been replaced by the present new Union Station.

In February, 1883, Mr. Lawlor was transferred to Burlington, Iowa, and was the first District Engineer of the Iowa Lines. He held this position until December, 1889, when with several other Burlington Route employees, he went to Montevideo, Uruguay, South America, as Assistant Engineer for the "Ferrol Carril del Oeste," to build railroads for the government of that country. Owing to financial difficulties in the Argentine Republic, prior to the panic of 1893 in the United States and the failure of the National Bank of Uruguay, all work was suspended.

After his return to the United States, he made surveys in the Black Hills, S. Dak., for the Burlington Railroad Company, and later entered the contracting firm of Lawlor, Connor and Company, St. John, to build the Tobique Valley Railroad in the northern part of the province. His next position was that of Assistant Engineer on the enlargement of the Galops Canal at Cardinal, Ont., Canada on the St. Lawrence River, about 9 miles below Ogdensburg, N. Y. Later he was transferred to Port Colborne, Ont., the upper end of the Welland Canal, and was in charge of the contract work of the Department of Railways and Canals at that place.

In 1903 Mr. Lawlor accepted the position of Superintendent of the Citizens Water Company in Burlington, a position which he held for thirty-six years, until his death.

He was recognized as one of the outstanding water-works engineers of the upper Mississippi Valley. He was a member of the Illinois Water Supply Association, a life member of the American Water Works Association, and a member of the Canadian Society of Civil Engineers.

He was keenly interested in his profession and was ready at any time to promote experiments and adopt modifications which would improve the operation of his plant and the purity of the product. He regularly attended meetings and conferences of the numerous engineering societies of which he was a member and had an active interest in the Illinois and Missouri Valley sections of the American Water Works Association.

He was married on January 17, 1900, to Lila W. Foote, daughter of Mark S. Foote of Burlington. Mrs. Lawlor died in 1933. One daughter, Mrs. Albert Schulte, formerly Mary Josephine Lawlor, and three grandchildren, Elizabeth Lucile, Carol, and Francis Lawlor Schulte survive. Mr. Lawlor was the last of his family; two sisters, Mother Clara of Montreal, Que., Canada, and Mother Josephine of Boston, Mass., both members of the order of the

Ladies of the Sacred Heart, and a brother, James, of St. John, preceded him in death.

Mr. Lawlor was active in community affairs. He was a member of the Rotary Club, Knights of Columbus, an honorary member of the St. Vincent De Paul Society, and for some years served as a member of the board of trustees of the Free Public Library of Burlington. He is reported by his fellow members to have been just as conscientious in his public work as in handling the business of the water company and his associates on the library board had high commendation for the services he rendered. He was a member of St. Paul Catholic Church.

Mr. Lawlor was elected a Junior of the American Society of Civil Engineers on October 6, 1886, and a Member on March 6, 1895. He was made a life member of the Society in January, 1928.

ALLAN BENJAMIN LEA, M. Am. Soc. C. E.[1]

Died July 17, 1938

Allan Benjamin Lea was born on May 15, 1866, in Buenos Aires, Argentine Republic, the son of Samuel and Margaret Isabella (Rankin) Lea. He received his early education in the public schools of Buenos Aires. At the age of 12, he entered the National University of Buenos Aires and was graduated in 1883. He maintained a high scholastic record and was rated as one of the three leading students in his Class. Later, he returned to the University and received the degree of "Agrimensor" in 1894, and that of Civil Engineer in 1902.

His engineering work was begun in 1884, at the age of 18, in the Drafting-Room of John F. Latrobe Bateman, Engineer, on the sewerage and water-works system of Buenos Aires. The following year Mr. Lea was made Junior Assistant on construction of the sewerage works.

In March, 1886, Mr. Lea was employed by the Buenos Aires Great Southern Railway Company with which he remained for three years. He spent the first year in the Drafting-Room; the second year making surveys for new stations; and the third year as Leveler and Instrumentman on surveys for new lines.

From December, 1888, to January, 1890, he acted as Chief Assistant to the Resident Engineer on the construction of South of Santa Fé and Cordoba Railway. From January, 1890, to May, 1894, he was Manager and Chief Engineer with the firm of Hopkins, Gardom, and Tetley, Contractors, as follows: January, 1890, to December, 1891, on the construction of a railway line from Belgrano to Las Conchas (a branch of the Buenos Aires and Rosario Railway), including a 140-ft skew bridge over the Central Argentine Railway and a retaining wall a mile long, on the shore of the River Plate; 1892, on the Punta Piedras Division; and 1893, to May, 1894, on the line from Magdalena to Atalaya.

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[1] Memoir compiled from data on file at the Headquarters of the Society and information supplied by James C. Lea, Esq., Buenos Aires, Argentine Republic.

In November, 1894, he received his degree as "Agrimensor Publico," or Surveyor, in the Province of Buenos Aires, and for the following seven years he was engaged in private practice and in studying for his degree of Civil Engineer which he received in 1902.

From December, 1902, to December, 1906, Mr. Lea was Chief Engineer of a Commission appointed to explore and survey, by triangulation, 12 600 sq miles in Chubut and Rio Negro Territories.

Mr. Lea returned to private practice in 1906, his work including surveys, valuations for Law Courts, and advice on irrigation and drainage work.

Mr. Lea took great interest in the study of mathematics, and was considered one of the leading mathematicians in the Argentine Republic. From childhood he was fond of reading and was thoroughly versed in philosophy, logic, political economy, history, and the Classics. Being endowed with an excellent memory, he could converse freely on all these subjects.

Modest by nature and possessing high moral principles he was much esteemed and beloved by his friends. He was respected by his assistants and workmen for his considerate kindness, altruism, and knowledge, and by his employers for the conscientious and able manner in which he carried out all work entrusted to him.

Mr. Lea was not married. He is survived by three brothers—John S., Norman E., and James C. Lea.

He was a member of the Centro Nacional de Ingenieros, Sociedad Cientifica Argentina, and of the Association of Members of American National Engineering Societies in Argentina. He was the author of several engineering articles published in *La Nacion* and *La Ingenieria.*

Mr. Lea was elected a Member of the American Society of Civil Engineers on March 4, 1908.

JOHN WALTER LEDOUX, M. Am. Soc. C. E.[1]

Died November 7, 1932

John Walter Ledoux, the son of John and Almina (Knox) Ledoux, was born near St. Croix Falls, Wis., on August 28, 1860. His ancestry on his father's side was French-Canadian; on his mother's side, he was of Scotch descent.

His father was a farmer and, later, an explorer in the Lake Superior mining regions, and the boy's earlier education was obtained in the public schools of villages in the Lake Superior District, of Michigan. He worked in the iron mines mines of this District from the age of nine (when his mother died) until he was twenty-one, leaving school at the age of thirteen, upon the death of his father. His duties in this work were connected with engines and boilers and other mine machinery.

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[1] Memoir prepared from information on file at the Headquarters of the Society.

While working in the mines, his spare time and nights were spent in study and, at the age of twenty-one, he had devised a method of finding the direction of a mine drift; this was practically his first invention. Its success and the recognition of its value in mining operations attracted attention, and led to advice which resulted in Mr. Ledoux preparing for a higher education. For a short time he attended Adams Collegiate Institute, at Adams, N. Y., and in 1883, he entered Lehigh University, at South Bethlehem, Pa., from which he was graduated with honors in 1887, receiving the degree of Civil Engineer.

Technical practice for Mr. Ledoux began during his student days through employment on engineering work at spare times and during vacations. Upon his graduation from Lehigh University, he became Borough Engineer of West Bethlehem, Pa., in order to carry to completion work upon which he had been engaged. This work included a complete survey of the Borough, establishing grades, and setting monuments. Next, he entered the employ of the Stanwix Engineering Company, of Rome, N. Y., of which firm he became a member in 1889, continuing the association until he was appointed Chief Engineer of the American Pipe Company, of Philadelphia, Pa., in 1891. He served in this capacity until 1920 (when the Company's expansion activities practically ended) after which he continued as its Consultant, and entered into private practice as a Consulting Engineer.

The engineering activities of Mr. Ledoux were many and diversified. Although specializing in hydraulics and making investigations and reports on water supply and water-power projects, together with the design of plants and systems, he also invented and manufactured extensive scientific apparatus used in connection with water supply systems and filtration plants for regulation, control, and measurement. He also testified before Public Service Commissions and other tribunals in condemnation proceedings, rate cases, valuations, and other legal matters, and wrote many papers for publication and presentation on such engineering subjects as hydraulics and water supply and treatment, valuations, and rates. Mr. Ledoux visited and studied plants in Europe and in South America. He was among the early investigators on the scientific appraisal of utility properties for valuation and rate-making, on which he was a recognized authority; he was also one of the pioneers and an authority on water treatment and on rapid sand filtration in the United States.

During his career as an engineer, investigations were made of many projects with reports thereon for water supplies and power installations, followed by the construction of some of them. Mr. Ledoux designed and constructed gravity supplies, pumping plants, filter plants, wells, more than 100 dams and reservoirs, pipe lines of all classes, conduits, power plants and transmission lines, electric railways, and gas plants.

While with the Stanwix Engineering Company, he was engaged in water supply investigations, mainly in the Allegheny Mountain regions, measuring stream flow, studying rainfall, and investigating water rights; he also served as Resident Engineer on surveys and the construction of a reservoir at Horse Shoe Curve on the Pennsylvania Railroad, with pipe lines to Altoona, Pa.; he constructed reservoirs and pipe lines for the Westmoreland Water Company, supplying Greensburg, Pa.; and investigated and reported on water supply projects in other localities.

As Chief Engineer of the American Pipe Company (later, the American Pipe and Construction Company), his duties were broad and diversified as the Company was engaged in initiating, financing, constructing, owning, and operating water-works properties, and through a subsidiary, the National Gas and Electric Construction Company, extended its activities to other utilities, such as gas-works, electric plants, and street railways.

Among the water-works designed and constructed by Mr. Ledoux were those for: Paris Mountain Water Company, Greenville, S. C., with a filter plant built in 1891, one of the earliest in the United States; Sumter, S. C.; LaGrange, and Milledgeville, Ga.; Opelika, and Selma, Ala.; Asbury Park, Clayton, Glassboro, Millville, Westfield, and Newbold, in New Jersey; Norfolk, Va.; Waukesha, Wis.; Texarkana, Tex.; Syracuse, and Cortland, N. Y.; Rochester and Lake Ontario Water Company; Buffalo and Niagara Falls Water Company; Depew and Lake Erie Water Company; Western New York Water Company; New York Suburban Water Company; New York Interurban Water Company; and the Springfield Consolidated Water Company for the suburban areas surrounding Philadelphia, Pa. Mr. Ledoux developed the water supply system for the Pennsylvania Railroad Company, supplying trunk and branch lines from Philadelphia to Pittsburgh, Pa., from Philadelphia to New York, N. Y., and from Philadelphia to Baltimore, Md., with developed gravity or pumped supplies at: Christiana Creek, and Winters Run, in Maryland; Octararo Creek; in Maryland and Pennsylvania; Stony Creek, near Dauphin; Powell Creek and Clark Creek, near Duncannon; Wild Cat Creek, near Newport; Licking Creek, near Mifflin; Tipton Run, near Tipton; Bell Creek, near Bellwood; Blair Gap, above Hollidaysburg; the South Fork of the Conemaugh; the North Fork of the Conemaugh; Quemahoning Creek; Salt Lick Creek; Bens Creek; Trout Run; Beaver Run, above Johnstown; Tub Mill Creek, near Bolivar; Sugar Run, near New Florence; and Indian Creek, near Connellsville, all in Pennsylvania. He designed and built this system and operated it for two years before turning it over to the owners; the system also furnishes water to some towns.

Mr. Ledoux designed and constructed plants for the Norfolk Light and Power Company, Norfolk, Va.; the Greenville Light, Power, and Railway Company, Greenville, S. C.; a hydro-electric plant on the Schuylkill River, at Conshohocken, Pa.; and a hydro-electric plant on the Lamoille River, at Fairfax Falls for the City of St. Albans, Vt. The latter was built at the beginning of the Twentieth Century, with transmission at 2 300 volts, which was high at that time. He also made reports for power on the Androscoggin River, at Lewiston, Me., and the Salmon River, near Syracuse, N. Y.; and he handled sixteen power and water rights on the Nine-Mile Creek outlet of Otisco Lake. He reported on rights and power of the Susequehanna River, at Wilkes-Barre, Pa.; above and below Harrisburg, Pa.; at Turkey Hill and at Conowingo. He also reported on the Wallenpaupack Creek, above Hawley; Neshaminy Creek, at Neshaminy Falls; Niaugua River, in Missouri; various streams in Arizona, California, and British Columbia, and the Clarion River and Beaver Creek, in Pennsylvania.

In addition to serving as Chief Engineer for the American Pipe Company many outside engagements were filled for water companies and municipalities including, Norfolk, Pittsburgh, Harrisburg, Lancaster, El Paso, Lehigh Coal and Navigation Company, Clear Springs Water Company, Citizens Water Company, of Scottdale, Bethlehem City Water Company, Bristol, Tenn., Water Company, Washington County Water Company, of Hagerstown, Md., New Castle, Del., Water Company, Camden, N. J., Water Department, Wyoming Valley Water Company, and the Girard Water Company, in Pennsylvania.

In 1920 Mr. Ledoux entered private practice as a Consulting Hydraulic Engineer and continued in this status until his death. Among his engagements during this period were those with the Norfolk County Water Company, Western New York Water Company, City of Jamestown, N. Y., Lehigh Valley Coal Company, Wyoming Valley Water Company, Lehigh Coal and Navigation Company, City of Bethlehem, Pa., Clear Springs Water Company of Pennsylvania, Manheim, Pa., Water Supply Company, Mountain Water Supply Company, City of Philadelphia (as Chairman of a Commission for a future Water Supply), City of Norfolk, Pennsylvania Railroad Company, Long Beach Water Company (arbitration case), Lock Joint Pipe Company, City of Harrisburg, City of Camden, Lehigh Water Company, of Easton, Pa., and others.

He was the organizer of the Simplex Valve and Meter Company for the manufacture under his patents of scientific apparatus used in connection with water supply and other liquids, for measurement and regulation.

Through a long and intensely active career Mr. Ledoux wrote many engineering reports on various subjects for commercial purposes, together with papers presented before technical societies and published in technical journals He had had charge, also, of the design and construction of more than one hundred engineering works. He had received more than forty United States patents for his inventions. The Longstreth Medal of Award of the Franklin Institute was awarded Mr. Ledoux in 1919 for meter inventions for use with Venturi tubes, and, at the time of his death, a second award had been reported on favorably for another invention.

In addition to engineering reports and papers, a number of theses were written and works of fiction composed. He was a student of classical literature and music, and an accomplished violinist. As a student, he had been a member of the college track team and of the first intercollegiate football team of his Alma Mater, and, in later years, he took up gymnasium work and lawn tennis.

He was a member of the American Society of Mechanical Engineers, American Institute of Consulting Engineers, New England Water Works Association, American Water Works Association, Pennsylvania Water Works Association, Franklin Institute, Philadelphia Engineers Club, University Club, and Tau Beta Pi Society.

Although he was enjoying excellent health and perfect mentality at the age of seventy-two, Mr. Ledoux was seriously injured by an automobile while crossing a street on a stormy night, November 6, 1932, at Media, Pa. He died the following morning without regaining consciousness.

He was married to Laura A. Uberroth, of South Bethlehem, Pa., on July 9, 1888, and is survived by his widow, a daughter, Mrs. Stephen Gibbs, a son Leonard K. Ledoux, and a grandson John Walter Ledoux.

Mr. Ledoux was elected a Member of the American Society of Civil Engineers on June 5, 1895.

HENRY READ LEONARD, M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1939

Henry Read Leonard was born in Boston, Mass., on April 13, 1858, the eighth generation in direct descent from James Leonard, who emigrated from Pontypool, England, in 1652, and who founded the first Iron Forge in the New World, at Taunton, Mass.

On March 1, 1874, he entered the employ of the Boston City Engineer's Department, where he was engaged in general construction work and bridge design. He resigned from this position in March, 1883, to become an engineer for Cofrode and Saylor (The Philadelphia Bridge Works), remaining with that firm until 1896. During this time Mr. Leonard was intimately connected with the work of designing and constructing many bridges and other structures for the Pennsylvania Railroad Company. In 1897, he was a member of the firm of Leonard-Foley Construction Company.

During 1899 and 1900, he was engaged by the Southern Railway Company to make a general survey and inspection of all metal bridges on the railroad. From 1900 until March 1, 1902, he was connected with the American Bridge Company as Contracting Engineer.

On March 1, 1902, Mr. Leonard was appointed Engineer of Bridges of the Pennsylvania Railroad Company, and served in this capacity until 1906, when he was made Engineer of Bridges and Buildings. On April 1, 1923, he was appointed to the new position of Chief Engineer of Bridges and Buildings, having jurisdiction over the entire Pennsylvania System. This position he held until his retirement, on May 1, 1928.

During Mr. Leonard's service with the Pennsylvania Railroad Company he built a large number of important structures, of both structural steel and reinforced concrete. He was a tireless worker and very accurate in everything he did. He had a host of friends, and they will long remember his keen wit and remarkable sense of humor. He is survived by his widow, three sons, and two daughters.

Mr. Leonard was an enthusiastic member of the American Society of Civil Engineers, of which he served as Director from 1913 to 1915. He was elected a Member of the Society on April 4, 1894.

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[1] Memoir prepared by Robert Farnham, M. Am. Soc. C. E.

WASHINGTON IRVING LEX, M. Am. Soc. C. E.[1]

Died December 21, 1938

Washington Irving Lex was born on January 19, 1874, at Philadelphia, Pa., the son of Henry W. and Kate C. Lex. He received his early education in the Public Schools of Philadelphia. In 1896 he was graduated from the Civil Engineering Department of the University of Pennsylvania at Philadelphia, Pa., with the degree of Bachelor of Science in Civil Engineering. Later, in 1925, he received the degree of Civil Engineer from that institution.

After graduation, Mr. Lex was in charge of a party making a field survey of about 200 acres of land on the outskirts of Brooklyn, N. Y. He also prepared plans and specifications, with estimated costs, for the development of this work.

In February, 1897, he entered the employ of the Pencoyd Iron Works at Pencoyd, Pa. He served successively as tracer, detailer, and checker of detail shop drawings for all types of steel bridge and building construction. Among the notable structures on which he worked at this time were the Atbara Bridge over the Nile River in Egypt for the British Government, the Clifton Arch Span over the Niagara River at Niagara Falls, N. Y., and the Land, Title, and Trust Building in Philadelphia.

The American Bridge Company was organized in 1900 and took over the Pencoyd Iron Works. At that time Mr. Lex entered the employ of the American Bridge Company and was with that company continuously until the date of his death. He first served as Squad Foreman, in charge of 50 men in the Pencoyd drawing room. Among the larger contracts that he directed at this time were a series of railroad viaducts for the Uganda Railway Company in Africa for the British Government, a series of railroad bridges for the Chinese and Japanese Governments, and the Senate and House of Representatives Office Buildings at Washington, D. C.

In 1903 Mr. Lex was appointed Assistant Engineer in the Estimating and Designing Department of the Eastern Division of the American Bridge Company at Philadelphia. He was in direct and responsible charge of making designs and estimates for all types of steel railroad and highway bridges.

From 1904 to 1908, he served as Assistant to the Plant Engineer of the Pencoyd Plant of the American Bridge Company in charge of 100 to 150 men in the drawing room. Among the notable contracts which he handled at this time were those for the Boston Elevated Railroad Company, the Subway and Elevated Railway on Market Street in Philadelphia, the Wanamaker Store Building in Philadelphia, the first subway construction with elevated extensions on Broadway, New York, N. Y., the Hudson and Manhattan Terminal Buildings, and the Grand Central Terminal Buildings in New York City.

In 1908, Mr. Lex was transferred to the New York Office of the American Bridge Company, where he was made Assistant Engineer in the Estimating and Designing Department. Here he prepared designs and estimates on all types of bridge and building work.

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[1] Memoir prepared by William H. Chorlton, M. Am. Soc. C. E.

For the past twenty-five years, Mr. Lex acted as a co-ordinator and "expediter" of work, and for this purpose he was a contact man between the American Bridge Company and the customer. In a number of cases he designed and prepared estimates on the work. In this capacity he would follow a contract through the drawing room, shop, inspection, shipment, and erection. Among the important contracts which he handled in this way were the steel for the third tracking of the Third, Sixth, and Ninth Avenue Elevated structures in New York City, the Elevated Railroad Construction for the East New York and Coney Island Districts, Frankford Elevated Railway in Philadelphia, Subway Construction for the Interborough Rapid Transit Company and the New York Municipal Railway Corporation in New York City, the Delaware River Suspension Bridge between Philadelphia and Camden, N. J., the Broad Street Subway in Philadelphia, the Elverson Building for the Philadelphia Inquirer Company in Philadelphia, and the Richmond Power Station for the Philadelphia Electric Company in Philadelphia.

Mr. Lex was stationed in the Philadelphia Office of the American Bridge Company from 1922 to 1928. With the exception of this time, his headquarters were the New York Office of the company from the year 1908 to the time of his death. He always made his home in Philadelphia and commuted daily between the two cities.

In all his interests he was critically exact and as definitely trustworthy and responsible. He shunned no trust to which he had subscribed any accountability. By these qualities he endeared himself to his friends and associates.

On April 18, 1901, he was married to Inez Estella Apgar of Philadelphia. His widow and three children, Evelyn, W. Irving, Jr. (an electrical engineer), and Dorothea, survive him.

Mr. Lex was a member of, and for the past fifteen years the Treasurer of, the Spring Garden Street Methodist Episcopal Church of Philadelphia. He was a member of several branches of the Masonic Order. He was a registered Professional Engineer in the State of Pennsylvania.

Mr. Lex was elected an Associate Member of the American Society of Civil Engineers on May 2, 1906, and a Member on April 1, 1914.

JOHN MACDONALD, M. Am. Soc. C. E.[1]

DIED JANUARY 28, 1938

John Macdonald was born in Glasgow, Scotland, on October 18, 1878. He was the son of James Macdonald, a builder, and Catherine Macdonald.

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[1] Memoir prepared by Warren C. Taylor, M. Am. Soc. C. E.

He came to the United States when he was seven years old and lived in New York, N. Y., where he received his education and was graduated from the Woodbridge School, and New York University.

In September, 1895, Mr. Macdonald began his technical career, as an Apprentice for C. T. Wills, Incorporated, of New York City, becoming an Assistant Foreman in 1902. In September of the same year he entered New York University, in New York City, and in June, 1906, received the degree of Bachelor of Science in Civil Engineering.. In college he took an active interest in his Class and was Chairman of the Class Day Committee. During his Junior and Senior years he acted as Instructor in Drawing at the University.

Soon after his graduation, Mr. Macdonald was employed by the Bliss Griffiths Company as Construction Superintendent, which position he held until November, 1908. During the next year, he was Superintendent for Deisler and Stephenson on the construction of large residences. In October, 1909, Mr. Macdonald again joined the staff of C. T. Wills, Incorporated, and served as Construction Superintendent for six years.. He then went with the Thompson-Starrett Company, with which he remained until September, 1924.

His work with these construction companies gave him varied experience in building construction, which included supervision of such buildings as residences for James Speyer, Adolph Lewisohn, and Otto Kahn; the station of the New York, New Haven and Hartford Railroad Company, at New Haven, Conn.; Sterling Laboratory, at Yale University, at New Haven; Steinway Building, in New York City; and the International General Electric Building, at Schenectady, N. Y.

During his residence in New York City Mr. Macdonald found time to be an Instructor in Plan Reading and Estimating at Cooper Union from 1912 to 1915.

He was sought to take charge of the course in Building Construction established by the Thompson Starrett Foundation, at Union College, in Schenectady, and became Assistant Professor of Civil Engineering in 1924. This gave him opportunity to give to the students the benefits of his years of experience in this field of work. He also taught courses in Structural Design. He was Associate Professor from 1927 until his resignation in 1933.

At the time of his death Professor Macdonald was engaged in private engineering, and resided at Dumont, N. J.

He was a patient, energetic worker and a man of sterling character. In his younger days, he took an interest in the activities of the Presbyterian Church, and from 1900 to 1910 was Superintendent of the Sunday School at the Rutgers Presbyterian Church, in New York City.

Professor Macdonald's fraternal interests centered in the Chancellor Walworth Lodge No. 271, F. and A. M., of New York City.

During his residence in New Haven, he was married on January 4, 1922, to Lillian Hargrave, who survives him.

John Macdonald was elected a Member of the American Society of Civil Engineers on April 25, 1932.

FRANCIS CHARLES McMATH, M. Am. Soc. G. E.[1]

Died February 13, 1938

Francis Charles McMath was born on January 29, 1867, at St. Louis, Mo. He was the son of Robert Emmet and Frances (Brodie) McMath. His father, a distinguished engineer of the past generation and also a Member of the Society,[2] was born in New York State where he attained manhood, but seeking a career farther west, he moved to St. Louis. His mother, until her marriage, lived in Detroit, Mich.

Francis Charles McMath was educated at Washington University, at St. Louis, from which he was graduated in 1887 with the degree of Bachelor of Engineering. Fifty years later, in 1937, Wayne University, at Detroit, conferred on him the Honorary Degree of Doctor of Engineering. In 1931 he was made an Honorary Alumnus of the University of Michigan, at Ann Arbor, Mich.

Following his graduation, Mr. McMath became a Draftsman with the old Detroit Bridge and Iron Works. A year later, he was promoted to the position of Assistant Engineer, and designed many important structures. In another seven years, he was advanced to full responsibility as Chief Engineer. He continued to hold this position until 1900 when he severed his connection with the Company. Although he designed many railway and highway bridges during these years, including the rebuilding of the Victoria Jubilee Bridge, over the St. Lawrence River at Montreal, Que., Canada, this period might well be considered one of preparation for even more important work.

In 1900, together with Willard Pope, M. Am. Soc. C. E., and Mr. B. S. Colburn, Mr. McMath organized the Canadian Bridge Company, Limited, at Walkerville, Ont., Canada. For twenty-one years he was President of this Company, during which time many bridges, both large and small, from the Atlantic to the Pacific, were built for the Grand Trunk, Grand Trunk Pacific, Canadian Pacific, and other Canadian Railway Companies.

In 1911, the St. Lawrence Bridge Company, Limited, of Canada, was organized for the purpose of constructing the cantilever bridge, with a span of 1 800 ft, over the St. Lawrence River near Quebec. Mr. McMath became a Director and Consulting Engineer for the new Company, which was owned jointly by the Canadian Bridge Company, Limited, and the Dominion Bridge Company, of Montreal. The successful completion, in 1918, of this record-making cantilever span was due in no small part to the energy and engineering skill of Mr. McMath. It stands as a symbol of his career.

He was also actively interested in other work. In 1906, for instance, he organized the Essex Terminal Railroad Company, of Walkerville, Ont., serving as its President and Director until 1921. He was also a Director in the Motors Metal Manufacturing Company and the Grosse Isle Bridge Company.

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[1] Memoir prepared by Willard Pope, M. Am. Soc. C. E., and L. C. Wilcoxen, Assoc. M. Am. Soc. C. E.

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 2268.

As is so often the case with busy men, Mr. McMath found time to devote to civic affairs. In 1920, he became a Trustee of the Detroit Bureau of Governmental Research, a work in which he remained actively interested to the end of his career. He served as President for the years 1926 to 1928, inclusive. The Bureau, which is devoted to improving the City Government, accomplished much during this period. Mr. McMath, having previously served the City as Street Railway Commissioner, gave particular attention to the organization and management of the Detroit Street Railway. He also gave considerable study to the collection and disposal of municipal refuse and the City's sewage disposal problem, as well as many other municipal activities.

He served, too, on the Bell Isle Bridge Commission and the Sewage Disposal Commission. His service with the former resulted in the beautiful multiple-arch bridge which connects Belle Isle to Detroit, while the latter connection is bearing fruit in the construction of a treatment plant.

Mr. McMath's zest for knowledge led him into the realm of astronomy. In order to carry through original researches, he helped to build the McMath-Hulbert Observatory, at Lake Angelus, Michigan (later given to the University of Michigan). He was a member of its staff from 1930 to 1938, and assisted in the production of some remarkable motion pictures of celestial bodies. This work has been recognized by the Franklin Institute, at Philadelphia, Pa. In 1933 he, his son, Robert R. McMath, and Judge Henry S. Hulbert were the joint recipients of the John Price Wetherill Medal awarded by the Institute for " their design and construction of novel apparatus for making motion pictures of astronomical subjects."

In 1890, Mr. McMath was married to Josephine Cook, of Detroit, who died in 1914. The three children of this marriage, Robert R., Niel C., and Marion McMath Edwards, survive him. In 1916, he was married to Madeline Davenport King who also survives him. This was the family about which his life revolved. He was primarily a home-lover, a loyal husband, and a devoted father. They, in turn, furnished him the inspiration for his breadth of living.

Mr. McMath was a Presbyterian and a Republican. He was a member of the Country Club of Detroit, the Detroit Club, Grosse Pointe Club, the University Club (Detroit), and the Engineering Society of Detroit. He was a Fellow Member of the American Association for the Advancement of Science; a member of the American Astronomical Society; Honorary Curator, University of Michigan Observatory (1928–1938); and a Faculty Member of Sigma Xi Fraternity (Michigan Chapter) in 1933. His love of the out-of-doors found expression through his membership in the Huron Mountain Club, and the Turtle Lake Club, an old hunting and fishing club.

It is not easy to pick out the most important of Mr. McMath's many striking characteristics, but certainly a most outstanding one was his absolute and unfailing intellectual integrity. He simply did not know what a dishonest thought was, either toward himself or any one else. He had a way of penetrating almost instantly to the central essence of things, never being led astray by non-essentials, but instinctively discarding them like so much chaff.

The unusual quality of his fine, rare intelligence and power of initiative is shown by his many achievements, not only in the engineering field, but in civic matters, and also of real importance, with the collaboration of his son, in astronomy. These achievements speak for themselves. There was far more to the man than his remarkable mind, however, as his countless friends would testify. He had a rare gift of friendship, never speaking or thinking ill of any one—other than having a righteous scorn for any dishonesty—and accepted people as they are, always seeing the best in them. His never-failing sense of humor was not only a delight to every one with a grain of appreciation, but many times would ease off a tense situation and prevent things from being taken too seriously. He never was over-subtle, but had a certain essential simplicity and directness which was like a breath of fresh air. Beneath all, as a deep undercurrent of his nature, was a high courage and sound optimism, which kept him from admitting defeat until the very last "card had been played," but let him accept the absolute inevitable without the slightest repining.

His zest for life was remarkable and he got the greatest joy and satisfaction not only from his work, but from his play, always being a real lover of the out-of-doors. It is difficult to imagine a finer companion than he was, either as a business associate or as a fellow sportsman, and he lived his life to the full to the very end.

Mr. McMath was elected a Member of the American Society of Civil Engineers on February 25, 1924.

WALTER HUNTLEY MANSFIELD, M. Am. Soc. C. E.[1]

Died September 10, 1938

Walter Huntley Mansfield was born in Camden, Me., on June 28, 1863, the son of Alfred D. and Sophia Josephine Mansfield. His mother was born in London, England. The boy received his education in the public schools of Boston, Mass., and was tutored seven years while at work for the Boston and Providence Railroad Company, under Mr. C. D. Miner, who was Chief Engineer of the railroad.

From 1879 to 1885, Mr. Mansfield acted as Rodman, Instrumentman, and Assistant Engineer on surveys and plans for terminal improvements and double-track work for the Boston and Providence Railroad Company. He then went with the West End Street Railway Company, later, the Boston Elevated Company, as Assistant Roadmaster, and on surveys and general work until 1890. For the next eleven years Mr. Mansfield was a member of the firm of H. Gore and Company, of Boston.

In 1901 he was made Manager of the Boston Asphalt Company and Superintendent of the National Construction Company. This work included the paving of streets, building sewers, etc. Mr. Mansfield was with this firm until 1902 when he went into private practice as an engineering member of Powers and Mansfield Company, of Troy, N. Y. His first work in this capacity was to build and line with concrete the Tomhannock Tunnel for the Troy Water-Works. This firm converted the Au Sable-Keesville Railroad from steam to electricity and also rebuilt the Corning and Painted Post Street Railway, and constructed the Elmira-Corning and Waverly Interurban Railway.

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[1] Memoir prepared by James M. MacMartin, M. Am. Soc. C. E.

Mr. Mansfield preferred building street railways and he was very successful in this line. He did this type of work in Revere, Malden, Medford, Fall River, and Lowell, Mass.; Pawtucket, R. I.; New Haven, Conn.; Burlington, Vt.; and Plattsburg, and Corning, N. Y. In 1912 Mr. Mansfield and Mr. L. B. Harrison, of New York, N. Y., built seventeen miles of canal in Demarest, N. J.

In 1913, Mr. Mansfield entered the Engineering Department of The Delaware and Hudson Railroad Company and, later, was made Valuation Engineer, remaining in this position until 1921 when he became Engineer for Industrial Development, a position he occupied until his illness in November, 1926, at which time he retired to his farm in Colchester, Vt., where he lived for twelve years. He died there on September 10, 1938. His interest in engineering affairs was maintained until his death.

He is survived by his widow, Frances Rachel (Macrae) Mansfield, and one brother, the Hon. Edward C. Mansfield, of Boston.

He was a member of the Ticket and Freight Association of The Delaware and Hudson Railroad Corporation, the Engineers Club of New York City, the Troy Club, and the Albany Society of Civil Engineers, of which he had served as President in 1923.

Mr. Mansfield was elected a Member of the American Society of Civil Engineers on September 3, 1913.

WALLACE NELSON MAYHEW, M. Am. Soc. C. E.[1]

DIED JANUARY 1, 1939

Wallace Nelson Mayhew was born in Philadelphia, Pa., on March 14, 1880, the son of Wallace and Sadie E. (MacGinley) Mayhew.

His early education was received in the Public Schools of Philadelphia, followed by the Central Manual Training High School. He entered the University of Pennsylvania, at West Philadelphia, Pa., as a member of the Class of 1901, but left in the spring of 1900, in his Junior year, to enter the active practice of structural engineering. He received the degree of Bachelor of Science from the University in 1921 and the degree of Civil Engineer in 1927.

From 1900 to 1902 Mr. Mayhew was actively engaged in structural design. In 1902 he became a partner and Chief Engineer of Stewart and Stevens Iron

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[1] Memoir prepared by a Committee of the Philadelphia Section consisting of Reginald C. Scott, and Charles A. Howland, Associate Members, Am. Soc. C. E.; and Berthold F. Hastings, M. Am. Soc. C. E .

Works, a steel fabricating firm in Philadelphia. Stewart and Stevens Iron Works merged with and became the Montgomery Iron and Steel Company in 1912, Mr. Mayhew continuing his connection and partnership. At his death he was President and owner of the Montgomery Iron and Steel Company.

In addition to his executive work, he maintained an active contact with the engineering activities of his firm. He also engaged in considerable research work in structural engineering and metallurgy.

Mr. Mayhew was very active in sponsoring legislation of benefit to the construction industry, and it was largely due to his untiring efforts that uniform Bond Laws were passed by the State of Pennsylvania in 1931, and by the Federal Government in 1935. In addition, he was very active in sponsoring "Buy American," particularly with regard to steel.

He served as a Director or an Executive Officer in many organizations, among them being the Structural Steel Board of Trade of Philadelphia, Philadelphia Builders Exchange and Philadelphia Federation of the Construction Industry. He was President of the Northeast Chamber of Commerce.

He is remembered by his friends and associates as a man of unusually broad activities and attainments and a man of great kindness in all his contacts.

Mr. Mayhew was elected a Member of the American Society of Civil Engineers on March 5, 1928.

BRADLEY REVERE METCALF, M. Am. Soc. C. E.[1]

Died May 6, 1936

Bradley Revere Metcalf was born at Elgin, Ill., on August 8, 1884, of English parents, James and Isadora (Chapman) Metcalf. He was educated as a Civil Engineer at the University of California, at Berkeley, Calif., and followed this profession throughout his life.

Mr. Metcalf began his engineering career in 1909, devoting most of his time to construction and to design. He worked, successively, with the City Architect of San Francisco, Calif., on plans; with the Southern Pacific Railway Company on general engineering and revaluation; and with Heller and Wilson, on the construction of a building for use as a temporary City Hall by the City of San Francisco. In 1912, he became Superintendent of Construction for J. E. Krafft and Sons on the Mt. Zion Hospital, in San Francisco. When this job was finished he became Office Engineer and concrete designer for the City of Sacramento, Calif. He then supervised the construction of a four-story concrete building for Heller and Wilson, after which he had direct supervision of Huntington Terrace Bridge over Arroyo Seco, at Pasadena, Calif.

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[1] Memoir prepared by John C. Park, Assoc. M. Am. Soc. C. E.

In 1914, Mr. Metcalf began work for the Los Angeles, Calif., County Board of Flood Control on the design of a 145-ft dam for Big Tujunga Canyon and remained with this organization, engaged in design, construction, and research, until 1915, at which time he began work on the design of small structures for the Los Angeles County Road Department. In 1916, he became Engineer for the Los Angeles Flood Control District in charge of flood-control measures and dam design. A notable structure which he designed while on this job was the Devil's Gate Dam, a combination of dam and bridge over the Arroyo Seco, at Flint Ridge. In 1917, he went to Bisbee, Ariz., and began work for Cochise County, spending two years in the design and construction of pavements and bridges. He was employed on similar work for the Pima County Highway Department for two years, after an intermission of a year with the Truscon Steel Company.

In 1922, Mr. Metcalf was engaged by the City of Tucson, Ariz., on street improvement plans; and, in 1923, he helped design and construct a large high school building in the same city. In 1924, after designing a reinforced concrete power house for the California Edison Company, he was again employed by the Los Angeles County Flood Control District and was quickly promoted to the position of Senior Engineer in charge of design of storm drains and preliminary plans for dams. He was then made Resident Engineer and had full charge of the construction of the Santa Anita, Sierra Madre, and Big Dalton Dams. Following this engagement, he had joint charge of work at San Gabriel Dam No. 1, where he remained until 1930. At this time, Mr. Metcalf became Senior Designing Engineer for the City of Tucson, and had charge of the construction of the Broadway and Sixth Avenue Subways. On May 1, 1932, he became Assistant City Engineer and on May 1, 1935, he became City Engineer, which position he held at the time of his death.

He enjoyed the confidence and respect of citizens and engineers alike, so that his death occasioned editorial comment. City officials reposed unfailing confidence in his advice and direction. He was noted for his loyalty to his job and to his friends.

Mr. Metcalf was a man of many interests and possessed a well balanced culture. He was a Vestryman of Grace Protestant Episcopal Church, a member of the Masonic Lodge, and a member of Phi Sigma Kappa, the American Association of Engineers, and the Tucson Engineers' Club.

Creation was his forte, also his hobby. He loved design and construction, as witness the many monuments which he left in stone, steel, and concrete. For his own amusement Mr. Metcalf wrote stories and scenarios, some of which were published. In his idle moments he made models, designs, and wrote articles, just for the joy of creating things. He left several unpublished papers. He was an indefatigable worker and student. Another hobby was athletics, and he was an expert tennis player.

He was married to Hazel Gulley on September 24, 1914, and is survived by his widow and a daughter, Jeanne.

Mr. Metcalf was elected a Member of the American Society of Civil Engineers on June 9, 1930.

RALPH MILLIS, M. Am. Soc. C. E.[1]

DIED NOVEMBER 11, 1938

Ralph Millis was born in Atlanta, Ga., on August 26, 1894, the son of Col. John Millis, of the U. S. Corps of Engineers, and Mary (Raoul) Millis.

After attending the University School in Cleveland, Ohio, Ralph Millis entered the Massachusetts Institute of Technology, in Boston, Mass., in 1912, graduating with the degree of Bachelor of Science in Civil Engineering in June, 1916. At school he stood high in scholastic ratings and took a prominent part in under-graduate activities, in which his cheerful enthusiasm and willingness to work for causes won him a large circle of lifetime friends.

After graduation, he was employed in sales and acoustical construction by the H. W. Johns-Manville Company in New York, N. Y. He was appointed a Second Lieutenant in the Engineer Section, Officers Reserve Corps, on April 2, 1917, and was assigned to active duty on May 8, 1917. After serving in the training camps at Fort Oglethorpe, Georgia, Belvoir, Virginia, and American University in the District of Columbia, he was assigned on August 27, 1917, to the 306th Engineers at Camp Jackson, South Carolina. Lieutenant Millis was promoted to First Lieutenant on March 29, 1918, and on entering the Corps of Engineers of the Regular Army to rank from July 8, 1918, was later promoted to Captain (temporary) to rank from that date.

On July 30, 1918, he sailed for France, serving with his regiment and at headquarters of the 81st Division in action in the Meuse-Argonne Offensive and in the occupation of the St. Dié Defensive Sector. From April to June, 1919, he served in France and Poland with General Haller's Polish forces. He returned to the United States on July 5, 1919.

After temporary duty in Washington, D. C., and Wilmington, Del., he became a student at the Engineer School, Camp A. A. Humphreys (later Fort Belvoir), Virginia, in October, 1919, graduating the following year. He continued there as an Instructor until August, 1922, except for the period from July to September, 1921, during which time he was detailed to the field artillery school at Camp Knox, Kentucky. On July 1, 1920, he was promoted to the grade of Captain in the Corps of Engineers. From September, 1922, to December, 1923, he was in charge of the Supply Division and later of the lock construction at Wilson Dam in Alabama. Returning to duty with troops in January, 1924, he served until November, 1926, as a Company Commander and Staff Officer with the 2d Engineers at Fort Sam Houston, Texas. In December, 1926, he joined the 11th Engineers in the Canal Zone on garrison and military mapping duties and continued this work until September, 1929.

From November, 1929, to June, 1933, Captain Millis served in Washington, D. C., in the office of the Chief of Engineers in charge of the map reproduction plant at Washington Barracks. His next station was at Fort McIntosh,

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[1] Memoir prepared by Maj. A. C. Lieber, Jr., Corps of Engrs., U. S. Army, M. Am. Soc. C. E.

Texas, where he served with the 8th Engineers Squadron until April, 1934, with the position of Post Commander for a part of that period. He next served as Assistant to the District Engineer at Savannah, Ga., on river and harbor works until July 5, 1935, when he was appointed District Engineer at Wilmington, N. C. On August 1, 1935, he was promoted to the grade of Major in the Corps of Engineers. In September, 1938, he was transferred to military duty as Instructor of Engineers, New Jersey National Guard, at Teaneck, N. J., where he remained on duty until his death on November 11, 1938.

On June 28, 1930, he was married to Elizabeth Myers, of Atlanta, Ga. They had two daughters, Catherine McDonald, born in 1934, and Elizabeth Manigault, born in 1935. In addition to his wife and children, he is survived by his father, Col. John Millis, M. Am. Soc. C. E., U. S. Army (*Retired*), his mother, Mrs. John Millis, and his brother, Walter Millis.

Major Millis was a member of the Society of American Military Engineers, the Army and Navy Club of Washington, the University Club of New York, Phi Beta Epsilon Fraternity of Massachusetts Institute of Technology, and various civil and social clubs. His diversions included sailing and tennis, in which he maintained a keen interest.

He was a qualified and capable troop commander, staff officer, and engineer. His service was efficient and conscientious, and he possessed the confidence and admiration of his colleagues. The associates of his youth and maturity will long remember the gay spirit and enthusiasm he put into all of his activities, and his untimely death is deeply regretted.

Major Millis was elected a Member of the American Society of Civil Engineers on November 9, 1936.

GEORGE PILLSBURY MORRILL, M. Am. Soc. C. E.[1]

Died September 29, 1938

George Pillsbury Morrill was born in Feeding Hills, Mass., on October 27, 1874. He was the son of Julius Abbot Morrill and Sarah Chamberlain (Bell) Morrill, both of whom came from lines of old New England ancestry. He attended the Public Schools at Bellows Falls, Vt., and the Springfield (Mass.) High School, entered Yale University, at New Haven, Conn., where he studied civil engineering, and was graduated from the Sheffield Scientific School of Yale with the degree of Bachelor of Philosophy in the Class of 1897.

He taught school for a year at Gardner, Mass., where he was Assistant Principal of the High School. Then he was employed until 1901 as Chainman, Rodman, and Transitman on cadastral and topographic surveys in the

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[1] Memoir prepared by Howard E. Hyde and Alexander F. Armstrong, Members, Am. Soc. C. E.

City Engineer's office at Springfield. He was also Junior Engineer on street and sewer construction.

The lure of the Orient, the quest for adventure, and his decided interest in English and literature then induced him to go to the Philippine Islands as a teacher of English. He was among the first teachers sent by the U. S. Government to the Philippines and crossed the Pacific on the U. S. Army Transport *Thomas* in 1901. There he became Superintendent of Schools of the Province of Batangas, Island of Luzon, riding the rough roads and trails of the Province to install, or superintend, its schools. He liked the life, the people, and the beautiful country, but engineering seemed to be in his blood, and he turned to it again, accepting an engineering appointment in Manila, P. I.

From January, 1903, to July, 1907, he served as Instrumentman and later as Assistant Engineer in the office of the City Engineer of Manila. Then he became Engineer in Charge of Construction of dam, head-works, and 42-in. pipe line, near Montalban, Rizal Province, for the new water-supply system of Manila. In those days Manila was a combination of an old, Spanish-type city and of a tropical Oriental city, with all the architectural interest and charm that that implies, and also with all the foul odors and lack of sanitation that were a part of it. A picturesque and historical stone wall of massive proportions, that was some centuries old, and a system of moats, filled with stagnant water which harbored immense snakes, surrounded the oldest portion of the city, known as Intramuras. Tidal lagoons, known as esteros, were used both for drainage and transportation. These meandered in all directions throughout the remainder of the city, and were more than generous with their foul odors, particularly at low tide. Many of the streets were narrow, lacked grading and drainage, and meandered almost as much as the esteros. Under these conditions there was much engineering to be done, and Mr. Morrill undertook his part in the work with vigor and zeal—surveying, mapping, and planning for new street lines, bridges, water and sewer mains, and other city projects.

This was one of the most interesting periods of his life. He made many friends, both American and Filipino, who became greatly attached to him. His young Filipino boy, Quintin Cueto, who attended to his personal wants, looked upon him as a father. It was at this time that Mr. Morrill explored the crater of Taal Volcano on Luzon Island, which was semi-active at the time, and which erupted violently a few years later, causing great loss of life. He traveled in China and Japan and among the southern Islands of the Philippine Archipelago. Perhaps his most adventurous trip was a journey on horseback across a part of the Island of Mindanao. Intermittent fighting was occurring between the Moros of that territory, who had not yet been subdued, and the U. S. Army forces, and army permission to make the trip and an army escort had to be secured for himself and his companions.

In July, 1907, he left the Philippines and spent several months traveling in the Orient, Africa, and Europe, and then returned to the United States.

In December, 1907, Mr. Morrill received an appointment as Engineer in the Department of Public Works, Republic of Cuba. Until December, 1908, he was in charge of work at Santa Clara and Camajuani, in the Province of Santa Clara, making surveys and designs for dams, pipes, and distribution systems for those two cities. From then until December, 1914, he was in charge of the construction of flood diversion works at Sagua la Grande, Santa Clara Province, and also located and constructed 30 km of new highways with culverts and bridges, including the Sitiecito Bridge which had seven 150-ft Pratt trusses.

From January, 1915, to March, 1916, he took postgraduate work at Yale University, taught in the Yale Summer School of Engineering, and served a short engagement with the Westinghouse, Church, Kerr Company on construction of a powder plant in Drummondville, Que., Canada.

From March, 1916, to July, 1931, Mr. Morrill was with the Raymond Concrete Pile Company. For four years he was Engineer and Construction Superintendent on work in various locations in Northeastern United States; for the next four years he was District Manager of the New England Territory; he was then sent to Tokyo, Japan, for a year, as Manager of the Oriental Territory; after that he served for six years as Manager of the Florida Territory, with headquarters at Miami.

From 1932 until his death he represented The American Bitumuls Company in the sale and application of highway products in the State of Connecticut.

In 1916, Mr. Morrill was married to Alice Maude Gray, at Springfield, who, with three children, Janet, Margaret, and George, survives him. Since 1932 he and his family had lived at Westbrook, Conn.

Mr. Morrill had many friends. A fine, honest, upstanding man, he was unusually sincere in his friendships, and all who knew him well loved him. His honesty, loyalty, and belief in fair play, and his humor and wit were outstanding characteristics. He abhorred deceit and insincerity in any form and was always outspoken against injustices of any kind. His generosity and willingness to help a friend in need and his kindliness toward those less fortunate than himself were among his finest traits. He took great pride in being a civil engineer and was a credit to his profession.

He was also well versed in history and literature, had a classical as well as an engineering education, and took a keen interest in educational subjects of any nature. He was a devoted husband and father and his home was very dear to him. His untimely death in an automobile accident at Bridgeport, Conn., on September 29, 1938, cut short a life of much usefulness and many accomplishments, and his passing will be deeply mourned.

He was a member of the The Cuban Society of Engineers, the Boston Society of Civil Engineers, the Royal Society of Arts of England, the American Concrete Institute, the Associated General Contractors of America, and the Connecticut Society of Civil Engineers. He was the President of the Miami Section of the American Society of Civil Engineers while in Florida.

Mr. Morrill was elected an Associate Member of the American Society of Civil Engineers on January 2, 1907, and a Member on June 6, 1911.

HARRISON CURTIS MOWER, M. Am. Soc. C. E.[1]

Died April 16, 1939

Harrison Curtis Mower was born at Grand Haven, Mich., on December 29, 1874. After attending St. John's Military Academy, at Delafield, Wis., he studied Civil Engineering at the University of Michigan, at Ann Arbor, Mich.

He was commissioned Captain in the Detroit (Mich.) Light Infantry in 1896, but at the outbreak of the Spanish American War, he resigned this commission and enlisted as a Private in the 31st Michigan Volunteers in order to see active service at an earlier date. After serving in Cuba, with promotion to the grade of Sergeant, he was mustered out when hostilities ceased.

He then returned to Civil Engineering practice, and from September, 1899, to March, 1902, was with the Michigan-Lake Superior Power Company, at Sault Ste. Marie, Mich. During the latter part of this engagement, he was in responsible charge of contractor's work Sections 2 and 3 of the canal.

From June, 1902, until June, 1917, he was continuously in the civilian service of the United States Engineer Office, being engaged most of this time as Superintendent or Assistant Engineer. He was also occupied with the design and, later, the construction, of many of the locks and dams constructed on the waterway system of the Black Warrior and Tombigbee rivers in Alabama.

His activities as a civil engineer were climaxed by construction of Lock and Dam No. 17 on the Black Warrior River which was completed in 1915. He personally directed the design, and, later, the construction of this project, which provided a 63-ft lift in two flights, at the time the highest lift lock in the United States, and still one of the outstanding structures of its kind.

When the United States entered the World War he again volunteered his services, and, in June, 1917, was commissioned Captain in the Engineer Reserves. In August of that year he was promoted to the rank of Major, and to that of Lieutenant Colonel in April, 1918. He served in France as executive officer of the 307th Engineers, 82d Division, participating in the Meuse-Argonne offensive, and commanded the regiment for a time during the absence of the then commanding officer (Col. Julian L. Schley, later Gen. Julian L. Schley, Chief of Engineers, U. S. Army, M. Am. Soc. C. E.) on other duty.

Colonel Mower returned to the United States and was discharged in May, 1919. He then engaged in business in Los Angeles, Calif. until his retirement in 1930.

He was assigned as commanding officer to the 349th Engineers in July, 1921, and immediately proceeded with the organization and training of his command. Shortly after this assignment he was promoted to the rank of Colonel, Engineer Reserves. At the training camp held at Fort Scott, California, in 1928, this regiment won first place among the units in attendance.

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[1] Memoir prepared by the regimental staff of the 349th Engineers, under command of Col. Leroy Foster.

Having brought the regiment to this high state of efficiency, and feeling the need of relief from duty, by his own request he was transferred to the inactive list on February 15, 1930, by Special Orders No. 31, Headquarters First Reserve Area.

Major William H. Lanagan, Corps of Engineers, at the time was unit instructor, and made this comment:

> " Colonel Mower represents the highest type of reserve officer and deserves great credit for the excellent work he has accomplished in building up his Regiment. I am loath to see him go, but he has had this step in mind for several months and feels that it is a necessary one."

On February 25, 1930, Colonel Mower was elected Member Emeritus of the Los Angeles Post, The Society of American Military Engineers, in view of the long service, splendid enthusiasm, and special ability in building up the Reserve Corps in general and the 349th Engineers in particular.

Colonel Mower was a member of the Sons of the Revolution, Society of Colonial Wars, the Ancient and Honorable Artillery of Boston, Mass., and other organizations.

After relinquishing command, Colonel Mower continued to take a deep interest in the regiment. Each year he has presented a saber to the junior officer making the greatest progress in preparing himslf for service in time of emergency. Throughout his career he has displayed qualities of loyalty, friendship, and devotion to the highest ideals of the military service, which have been an inspiration to all officers with whom he has served.

Colonel Mower is survived by his wife, Mrs. Mabel Smith Mower, daughter of Dr. and Mrs. Eugene Smith of Detroit, whom he married in 1901.

Colonel Mower was elected a Member of the American Society of Civil Engineers on July 9, 1912.

RICHARD AUSTIN MYERS, M. Am., Soc. C. E.[1]

Died December 28, 1938

Richard Austin Myers was born at Charlotte, N. C., on April 14, 1884, the eighth of eleven children. It may be said that he inherited all the sterling qualities of his forbears; the Myers of North Carolina, the Moores, Rawlinsons, and Springs of South Carolina.

His father, John Springs Myers, born near York, S. C., was a planter of the old school—loved, admired, and respected for his piety, character, and splendid quality of citizenship. He devoted himself to what would now be called scientific agriculture. Because of the proximity of his property holdings to the fast growing city and his far-sighted vision, he became a pioneer in suburban development of beauty, and the first citizen to set aside tracts of land for park purposes. He was active in church work and city affairs, and served in the State Legislature.

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[1] Memoir prepared by C. A. Mees, M. Am. Soc. C. E.

His grandfather, Colonel W. R. Myers, served in the Mexican War and as a Captain in the Confederate Army. One of his gifts was ten acres of land in Charlotte for the erection of a school for colored youths, later Johnston C. Smith University.

His mother, Mary (Rawlinson) Myers, had an ancestry of equal distinction. Her father served as aide-de-camp to Governor Pickens of South Carolina through the turbulent years of the Civil War. She was a member of the Colonial Dames and the Daughters of the American Revolution, and was active in the management of St. Peter's Hospital and the Thompson Orphanage.

Since succession to the planters' life of his family had become impracticable because of the changes wrought by time, Richard Austin Myers elected a career as an engineer. After graduation from the Public Schools and attendance at Horner Military School, at Oxford, N. C., he matriculated at the Virginia Polytechnic Institute, at Blacksburg, Va., from which he was graduated with distinction in 1905 with the degree of Bachelor of Science in Civil Engineering. Immediately thereafter he joined the field organization of the Southern Power Company (later the Duke Power Company) which was then engaged in the development of hydro-electric plants on the Catawba River at Great Falls, S. C.

After completion of the second, or Rocky Creek Plant, in the late spring of 1909, this organization was dissolved. On June 1, 1909, Mr. Myers joined the Engineer Corps of the United States Army with headquarters at Charleston, S. C. This organization was at that time making preliminary surveys for the Intra-Coastal Waterway System from Wilmington, N. C., to Savannah, Ga.

From his work as surveyor, he advanced rapidly to full charge of this important undertaking, which, by its very nature, made demands on men that are little realized by those who now pass through this waterway. An infected hand called for the utmost in will power and perseverance for continuous effort in the conduct of such work, and these qualities of character, with which Mr. Myers was so richly endowed, made him stay on the job until he was compelled to seek a furlough on February 10, 1912. He returned to his home in Charlotte for treatment, but on October 2, 1912, he again succumbed to the lure of the work and rejoined the Corps as Junior Engineer in charge of river and harbor improvement, which consisted of dredging for channel alterations through Winyah Bay to Georgetown, S. C.

On January 29, 1913, he was married to Marguerite Springs, of Charlotte, and a year later he resigned from the Corps to return to Charlotte where, in rather leisurely occupation and in the continued search for health, he practiced his profession privately until July, 1916.

Following the disastrous floods of July, 1916, as Resident Engineer for C. A. Mees, M. Am. Soc. C. E., the Consulting Engineer, he agreed to undertake direction of the hurried reconstruction of the dam of the South Carolina Light, Power, and Railways Company on the Broad River of the Carolinas, near Gaffney, S. C. (later absorbed in the Duke Power Company system). An outstanding feature of this undertaking was the construction of a 42-ft coffer-dam in the 225-ft wash-out within a period of forty days. This expedient work enabled the operation of the power plant to be maintained during the period of construction of the new masonry spillway.

After completion of this dam, he retained his connection with Mr. Mees until October, 1918, supervising the construction of steel-tower transmission lines from a hydro-electric plant near Saluda, N. C. into Spartanburg, S. C., for the Manufacturers Power Company of Spartanburg.

On November 20, 1918, Mr. Myers joined the Engineering Staff of the Southeastern Underwriters Association, with headquarters, first in Atlanta, Ga., and later in Charlotte. During this last period of professional practice, he was made Hydraulics Expert in the municipal engineering division, and as such he graded and made recommendations for the improvements in a great number of municipal fire defenses throughout the Southeastern States.

He remained in this position until April 30, 1927, when he resigned, and retired from active practice of his profession to devote his entire time to affairs connected with the management of his realty holdings.

In a letter written to the writer, W. S. Fitzsimons, Assoc. M. Am. Soc. C. E., states:

> "He was a man of sterling character and high attainments as an engineer. It was characteristic of him to devote his abilities unstintingly to bring to a successful conclusion any task that he undertook. His fine personality and willingness to help others endeared him to his many associates in the Engineer Department."

Mr. Myers is survived by his mother, his widow, a son, John Springs (a graduate of Virginia Military Institute in the Class of 1938), a younger son, Brevard Springs, a daughter, Caroline Clarkson, and two sisters, Mrs. George Stephens and Mrs. Harold Dwelle.

Mr. Myers was elected an Associate Member of the American Society of Civil Engineers on November 25, 1919, and a Member on July 12, 1926.

EDWIN JAY NICHOLS, M. Am. Soc C. E.[1]

DIED DECEMBER 11, 1938

Edwin Jay Nichols was born at Clear Lake, Iowa, on February 26, 1858, the son of Edwin Nichols and Elizabeth (Hendrickson) Nichols.

He was graduated from Iowa State College, at Ames, Iowa, receiving the degree of Bachelor of Civil Engineering on November 12, 1884.

In March, 1886, he entered the employ of the Mason City and Fort Dodge Railway Company, serving as Chainman, later as Instrumentman, and finally as Resident Engineer on 40 miles of construction. From December, 1886, to February, 1889, he was Resident Engineer in charge of construction, bridging, track laying, and town site work for the St. Louis, Arkansas and Texas Railway Company. From September, 1889, to January, 1890, he was Assistant City Engineer for Dallas, Tex., having charge of construction of street grading, paving, and sewers built under contract.

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[1] Memoir prepared by W. S. Hanley, Assoc. M. Am. Soc. C. E.

He was with the St. Louis Southwestern Railway Company as Assistant Engineer and Resident Engineer from January, 1890, to August, 1903.

From 1903 to 1912, Mr. Nichols served in various capacities in the Engineering Departments of the Texas and New Orleans Railroad Company and Galveston, Harrisburg and San Antonio Railroad Company, with the exception of the period from September, 1909, to April, 1910, when he was Superintendent for Horton and Horton, constructing street railways at Houston, Tex. From January, 1912, to August, 1912, he was employed as Division Engineer and Roadmaster on the Stephenville North and South Texas Railway. From August, 1912, he was in continuous employment in various capacities in the Engineering Department of the St. Louis-Southwestern Railway Lines until his retirement on January 16, 1933. His most outstanding work was the relocation of the main line from Mount Pleasant, Tex., to Corsicana, Tex., a distance of 142 miles.

Mr. Nichols was a man of sterling and rugged character and one of the last of the early railroad engineers who helped develop our railroads from the pioneer state. His work always reflected the scrupulous and conscientious effort that he put into it. He took a kindly interest in, and gave encouragement to, the younger engineers with whom he was associated. To give an insight into this man's character—he has been known to get up in the middle of the night, when one of his party was suffering with a cold, and administer to his relief. He never lay down to sleep at night until the last youngster in the location party had retired.

During his retirement, Mr. Nichols traveled extensively and kept up his contacts with his former associates. He remained in good health until a short time before his death from pneumonia. He was laid to rest in Oakwood Cemetery, in Tyler, Tex., on December 13, 1938.

He was married on May 10, 1893, to Mrs. A. J. Phillips at Camden, Ark. Although blessed with no children, his home life was ideally happy. He was very much interested in the cultivation of flowers, which he raised with great success, and which added to the beauty of his home. Mrs. Nichols preceded him in death on June 16, 1935. He was a member of the Baptist Church, and Delta Tau Delta College Fraternity.

Mr. Nichols was elected a Member of the American Society of Civil Engineers on September 2, 1896, receiving a life certificate of membership in October, 1937.

CHARLES HAROLD OLMSTEAD, M. Am. Soc. C. E.[1]

Died February 1, 1939

Charles Harold Olmstead, the son of Charles Wesley and Ella S. (Blanchard) Olmstead, was born at Sandlake, Rensselaer County, N. Y., on September 22, 1891.

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[1] Memoir prepared by George E. Martin, Assoc. M. Am. Soc. C. E.

He received his early education at Troy, N. Y., and was graduated from Troy High School. He took a competitive examination and was awarded a scholarship at Cornell University, at Ithaca, N. Y. He then attended Cornell University from which he was graduated in 1916 with the degree of Civil Engineer. At Cornell he became a member of Pi Kappa Alpha Fraternity, and retained his active interest in that organization throughout his life. At one time he was the Princeps of the Birmingham District, and was also National Chairman of the Endowment Fund.

After graduation, Mr. Olmstead was a Junior Assistant Engineer with the New York State Highway Department at Buffalo, N. Y. He left that position in August, 1917, to enter the Fort Niagara Military Training Camp. On November 27, 1917, he was commissioned a Second Lieutenant in the Field Artillery and later was transferred to the Air Service. He served the major part of his Army career at Kelly Field, San Antonio, Tex., and Vancouver Barracks, Washington. He was discharged from the Army in December, 1918.

After leaving the Army Mr. Olmstead worked for about six months on highway surveys and design with the Illinois State Highway Department at East St. Louis.

In 1919 he took the civil service examination for employment as an Engineer with the newly organized Tennessee State Highway Department and was at the top of the list of those taking the examination. He joined the Tennessee State Highway Department as Assistant Engineer in June, 1919; in August, 1920, he was promoted to District Engineer, and in 1921 was promoted to Division Engineer in charge of the Nashville Division. He served in this position until 1928. A part of this time Mr. Olmstead was Editor of *Tennessee Highways*, the official publication of the Tennessee Highway Department. During this period Tennessee was establishing a State highway system, and Mr. Olmstead took an active part in solving the early problems of organization, design, construction, and maintenance. Many highway projects serve as monuments to him and commemorate his ability as a highway engineer and executive.

In August, 1928, Mr. Olmstead left the service of the State of Tennessee to accept a position as State Representative of The Barrett Company, with headquarters at Nashville, Tenn. In 1930 he was made District Sales Manager with offices at Birmingham, Ala. He was advanced to the position of Consulting Engineer in 1937, with headquarters at Chicago, Ill., and a year later was transferred to Washington, D. C., in the same capacity. In this work he utilized his highway experience by preparing specifications, writing articles, lecturing to engineering students, and advising engineers on various phases of the use of bituminous materials in highway construction and maintenance.

Mr. Olmstead was married to Agnes I. Demers, at Troy, on June 21, 1916. His widow and two children, Alice E. and Charles Harold, Jr., survive him. He is also survived by his father, Mr. Charles W. Olmstead, a brother, Dr. Albert Olmstead (a professor at the University of Chicago), and a sister, Mrs. Jessie Simmons.

Because of his wide knowledge of a variety of subjects outside of engineering, Mr. Olmstead was an entertaining conversationalist and a delightful companion. He was deeply interested in his family and was a devoted husband and father. He had a host of friends inside and outside the engineering fraternity.

Mr. Olmstead was elected an Associate Member of the American Society of Civil Engineers on November 21, 1921, and a Member on April 13, 1936.

JOHN EDWIN OSTRANDER, M. Am. Soc. C. E.[1]

DIED OCTOBER 19, 1938

John Edwin Ostrander was born in Slingerlands, N. Y., on March 20, 1865, the son of John and Catherine (Van Den Bergh) Ostrander. He was graduated from Union College at Schenectady, N. Y., in 1886, with both the degrees of Bachelor of Arts and Civil Engineer. He was one of the Honor Students of his Class. In 1889 he was awarded the degree of Master of Arts by his Alma Mater.

His first engineering work was with the town of West Troy, N. Y., as Levelman on sewer construction. Later the same year he became associated with the New York State Survey as a computer. During 1887 he was first associated with the Chicago, St. Paul and Kansas City Railroad Company as Assistant on Construction, and then as Draftsman for the Phoenix Bridge Company.

Mr. Ostrander's next professional work was with the Engineering Department of the New York State Canals. He was in charge of the lengthening of Locks Nos. 60 and 61 on the Erie Canal. Later he made studies for, and supervised the construction of, the water supply for Clinton Prison at Dannemora, N. Y. In 1889 he was engaged in construction of locks at Amsterdam and Fort Hunter, and later at Rotterdam Junction and Mohawk, all in New York State. In 1890 he was in charge of a field party engaged in making surveys for the improvement of the Champlain Canal. He also worked on the final estimates for the Hudson River improvement.

After these varied and interesting experiences in engineering practice, which were to serve him in good stead in later years, Mr. Ostrander accepted an appointment as Instructor in Civil Engineering at Lehigh University, Bethlehem, Pa., in September, 1891. There he worked under the late Mansfield Merriman, M. Am. Soc. C. E., from whom he apparently received his inspiration to continue in the teaching profession.

During the summer of 1892 Mr. Ostrander was Engineer in charge of construction for L. M. Loss, contractor for the substructure of the railroad bridge across the Mississippi River at Alton, Ill.

In October, 1892, Mr. Ostrander began a five-year period as Professor of Civil Engineering and Mechanical Arts at the University of Idaho, Moscow, Idaho. Professor Ostrander was the first instructor engaged by the University. In this position he organized the Civil and Mining Engineering Departments. He also had charge of construction of the main University building. Professor Ostrander also served as Irrigation Engineer for the Experiment Station at the University, and made surveys and reported on irrigation projects for sub-stations at Idaho Falls and Nampa, in Idaho.

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[1] Memoir prepared by George A. Marston, Jun. Am. Soc. C. E.

In July, 1897, Professor Ostrander was appointed Professor of Mathematics and Civil Engineering at the Massachusetts Agricultural College, later the Massachusetts State College, at Amherst, Mass. At the same time he took over the duties of Meteorologist for the Experiment Station at the College. As Professor Ostrander used to tell, during the first month of his new duties Mother Nature celebrated his appointment by establishing a new monthly July rainfall record at Amherst.

As head of the Department of Mathematics and Civil Engineering at Massachusetts State College, Professor Ostrander saw many generations of students sit before him. Exacting as a man of science, practical as an engineer, "Johnny O," as he was affectionately known to his students, was an interesting and sympathetic teacher. He was a good judge of character, both of his students and of his colleagues.

Professor Ostrander served as Acting Professor of Astronomy at Amherst College during 1900, 1901, and 1907. He designed and supervised the construction of the sewerage system at Sunderland, Mass., during 1902 and 1903. He contributed to Johnson's Encyclopedia during 1893 and to Webster's New International Dictionary in 1907. He had been a member of the Society for the Promotion of Engineering Education, the Mathematics Teachers of New England, Phi Kappa Phi honorary scholastic fraternity, the Holland Society of New York, and the American Institute of Mining and Metallurgical Engineers, and was also a Mason.

In 1928 Professor Ostrander gave up his duties as Meteorologist at the College and in 1935 he retired from active teaching, having completed thirty-eight years of continuous work at the Massachusetts State College. He was appointed Professor Emeritus of Mathematics upon his retirement.

He was married to Sarah C. Cowan of Youngstown, N. Y., on September 26, 1888. He is survived by his widow; a son, Commander John E. Ostrander, Jr., U. S. Navy; and a daughter, Mrs. Milton Colvin.

Professor Ostrander was elected a Junior of the American Society of Civil Engineers on May 2, 1888; an Associate Member on September 2, 1891; and a Member on April 4, 1905.

JOHN NELSON OSTROM, M. Am. Soc. C. E.[1]

DIED JANUARY 30, 1938

John Nelson Ostrom was born in Corinth, N. Y., on June 6, 1851, the son of Thomas Legitt and Louisa (Pope) Ostrom. It is interesting to note that Mr. Ostrom's father was born before George Washington's first inauguration and was ten years and nine months old when George Washington died. In 1856, the family moved to East Randolph, N. Y., and it was here that John attended school. Following his graduation from Chamberlain Institute in Randolph he won a scholarship from Cornell University at Ithaca, N. Y., which he attended from 1873 to 1876. He was an exceptional athlete and famous oarsman, being a member of crew from 1873 to 1874, and Captain, Coach, and Stroke in 1875 and 1876, as well as a member of other athletic teams.

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[1] Memoir prepared by G. A. Haggander, Assistant Chf. Engr., C. B. & Q. R. R. Co., Chicago, Ill.

After completing his course at Cornell he secured employment with the Detroit Bridge and Iron Works, and in 1882 he took a position with the Chicago, Burlington and Quincy Railroad Company.

From 1886 to 1888 he directed the building of bridges on the Gulf Colorado and Santa Fe Railroad in Oklahoma and Texas, later part of the Atchison, Topeka and Santa Fe Railway. The following year he established his own office, specializing in inspection of bridges and serving as Consulting Engineer. For many years he was Consulting Bridge Engineer of the Chicago, Burlington and Quincy Railroad Company. He first maintained an Office in New York, N. Y., then in Pittsburgh, Pa., and about 1915 opened an office in Chicago, Ill., which he maintained until 1932 when he returned to East Randolph to live.

Mr. Ostrom specialized in shop and mill inspection of steel bridges and handled this work on many notable structures, including the Beaver Bridge, at Beaver, Pa., the bridge over the Ohio River, at Metropolis, Ill., and the Chicago, Burlington, and Quincy Railroad Bridge over the Missouri River at Kansas City, Mo. He also made frequent inspections of the Eads and Merchants bridges of the Terminal Railroad Association of St. Louis, Mo., spanning the Mississippi River, and supervised repair work on these structures. One of his last large inspection jobs was that on the Wabash Railway Bridge over the Missouri River at St. Charles, Mo., which has since been replaced.

Mr. Ostrom was very fond of outdoor sports and also kept an active interest in them. His wife, Caroline Deming Ostrom, whom he married in 1903, died in 1913. Mr. Ostrom is survived by a son, John N. Ostrom, Jr.

He was a member of the Western Society of Engineers, and Sons of the Revolution, the East Randolph Methodist Church, and he donated the Ostrom Memorial Building.

Mr. Ostrom was elected a Member of the American Society of Civil Engineers on November 5, 1890.

CHARLES PENROSE PERKINS, M. Am. Soc. C. E.[1]

Died August 19, 1934

Charles Penrose Perkins was born in Philadelphia, Pa., on October 2, 1844, the son of Abraham R. and Margaret R. (Penrose) Perkins. At an age early even for modern times, he entered the University of Pennsylvania, then at its old location on Ninth Street, Philadelphia, and took the collegiate course, including the classics. Graduating in 1863 with the degree of Bachelor of Arts, he entered the Rensselaer Polytechnic Institute, Troy, N. Y., for a course in technical instruction, and received the degree of Master of Arts in 1866.

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[1] Memoir prepared by Charles P. Keith, Esq., Philadelphia, Pa.

Mr. Perkins began his work at the Bessemer Steel Plant, of Winslow, Griswold, and Holly, at Wyandotte, Mich. There, he remained about a year. With the exception of this first position and his superintendency of the McHaffie Steel Casting Company, at Lamokin, near Chester, Pa., he spent twenty-six years in the service of the Pennsylvania Railroad Company and its Affiliated Companies.

Beginning as Assistant Engineer on the location and construction of the Queen Anne and Kent Railroad and its branch connection with the Delaware Railroad, he was advanced, in 1869, to the position of Assistant in the Office of the Resident Engineer and was engaged on maintenance of way on the Middle Division of the Pennsylvania Railroad. Moving then to the Western Pennsylvania Division, Mr. Perkins was made Assistant Engineer and, later, held the same position on the Pittsburgh Division. In 1873 and 1874, he was engaged on the construction of the Pennsylvania Railroad Station, in Jersey City, N. J. For the next ten years he was, successively, Assistant Engineer in charge of maintenance of way on the Elmira and Canandaigua Divisions of the Northern Central Railroad and the Philadelphia and Erie Railroad. In 1894, he was appointed Assistant to the Real Estate Agent of the Pennsylvania Railroad System east of Pittsburgh, Pa., and moved to Philadelphia. He remained in this work until his resignation in 1892.

Mr. Perkins was a member of the Historical Society of Pennsylvania, the Genealogical Society, Society of Colonial Wars, Art Club, Merion Cricket Club, and a Manager of Hahnemann Hospital and, for years, served as its Treasurer. An Episcopalian, Mr. Perkins was a Vestryman of the Church of the Good Shepherd, at Rosemont, Pa., where he died in his ninetieth year.

On November 13, 1878, he was married to Helen Armstrong Wright, the daughter of Col. John A. Wright. They had three children, Emma M., Rowan Penrose, and Mary Hathorn. His wife and children died before him.

Mr. Perkins was elected a Junior of the American Society of Civil Engineers on February 3, 1875, and a Member on April 5, 1882.

CHARLES STOCKTON POPE, M. Am. Soc. C. E.[1]

DIED MARCH 16, 1938

Charles Stockton Pope was born in Fort Stockton, Texas, on August 10, 1874, the son of Benjamin Franklin and Sara Lee (Poston) Pope. He came of an interesting and accomplished family. His father served as a Surgeon in the United States Army during the Civil War; later, as Chief of the Pension Bureau; Chief Medical Officer in the Santiago Campaign of the

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[1] Memoir prepared by a Committee of the Sacramento Section, consisting of E. N. Bryan, Thomas E. Stanton, Jr., Fred J. Grumm, and Albert Givan, Members, Am. Soc. C. E.

Spanish-American War; and as Surgeon General of the Philippine Islands. One brother—Saxton T. Pope—was Instructor in Surgery at the University of California, at Berkeley, Calif., and was rated as one of the greatest surgeons of his time in the United States.

Charles Stockton Pope received his early education in Army, private, and public schools. He was graduated in 1897 from Stanford University, at Stanford University, Calif., with the degree of Bachelor of Arts in Civil Engineering. He was a member of the Chi Psi Fraternity, and served on the University track team in 1896 and 1897. In the second semester of his Senior year, he was elected Perpetual Class President.

Immediately after his graduation, Mr. Pope went to work as a laborer on bridge repairs on the lines of the Southern Pacific Railroad Company. During the following two years, he was engaged as Chainman, Rodman, Water Gager, and Sub-Foreman on irrigation surveys and ditch work.

In 1900, he entered the employ of the Standard Electric Company, making surveys for power lines, power-houses, and dams; and in the following year he was engaged in the construction of these facilities. Mr. Pope was engaged in similar work in 1903, after having spent the previous year on the construction of ferry slips and power-houses for the North Shore Railroad Company.

Following a year as a Mine Surveyor in Arizona, Mr. Pope entered private practice in Los Angeles, Calif., in 1905, working on land surveys and on the breakwater, at Venice, Calif. From 1907 to 1915, he was employed by the City of Los Angeles, rising from Surveyor on all classes of municipal work to Assistant Engineer in charge of pavement construction, and, finally, to Highway Engineer in charge of all construction except sanitary sewers and bridges.

His experience in street work led to his acceptance of employment with Warren Brothers Company in 1916 for which Company he acted as District Engineer of California and Nevada, on consulting and promotional work on roads and streets, until 1921. This specialization in street work, particularly of the asphaltic type, furnished the basis for his later achievements in advancing the design and quality of pavements.

The State Highway Department of California was fortunate to secure the services of Mr. Pope in 1922, because of his large, practical experience with asphalt pavements and his knowledge of the California asphalts. For three years he had direct charge of asphalt work on the State Highway System and was appointed Construction Engineer for the Department in 1924, supervising all construction work except bridges. During the years, 1924 to 1928, he was also in charge of the Testing and Research Laboratory of the State Division of Highways. He continued in his capacity as Construction Engineer until his death.

Mr. Pope's contributions to improved practice on highway construction were numerous. He was primarily responsible for the development of the improved asphalt concrete specifications used in California. He developed and patented certain details of the process of machine finishing of asphaltic pavements. His research and studies led to improved Portland cement concrete design. He did much in developing and promoting soil studies and investigation as they apply to highway work.

Technical papers and discussions written by Mr. Pope were many, and appeared in various publications, or were presented at engineering conferences, meetings, and conventions. He prepared a brief and was Chief Reporter on Bituminous Roads at the International Road Congress, held at Milan, Italy, in August, 1926.

Mr. Pope was a member of the Project Committee on Distortion of Concrete Pavements, Highway Research Board, American Association of State Highway Officials Committee on Road Design, American Road Builders Association, and American Asphalt Association. At one time he was a Director of the California Museum Association; a member of the Engineers and Architects Club of Southern California, American Association of Engineers, and the Engineers Club of San Francisco. Membership in fraternal and social organizations included: Sacramento Lodge No. 40 F. and A. M.; Isaac Davis Lodge of Perfection; Masonic Luncheon Club; Sutter Club of Sacramento, Calif., and Del Paso Country Club.

Unfailing courtesy, dignified cordiality, tolerance, generosity, and kindliness were Charles Stockton Pope's marked characteristics. He created and maintained cordial relations with hundreds of contractors with whom he came in contact in his work. His keen sense of humor helped him to maintain a constantly cheerful aspect and behavior. An analytical mind, firmness, and rare judgment made his counsel and advice valuable and sought after. These fine qualities and his lovable nature endeared him to his many friends and associates. He was always, and under all circumstances, a gentleman in the truest and best sense of the word. He rendered distinguished service to the Engineering Profession, ever striving for the higher and finer ideals and by his own example inspiring his brother engineers toward achieving such higher standards. Charles Stockton Pope was a highway engineer. It can fittingly be said of him that: "He always traveled the upper road."

He was married in San Francisco, Calif., on September 19, 1903, to Marian S. Holden, of that city, a painter and etcher of note. She and his two sons, Charles St. George and Paul Francis Pope, survive him.

Mr. Pope was elected a Member of the American Society of Civil Engineers on October 1, 1913. He was a Charter Member of the Sacramento Section of the Society and its third President.

HENRY WITBECK POTTER, M. Am. Soc. C. E.[1]

Died October 9, 1938

Henry Witbeck Potter was born at Whitehall, N. Y., on August 26, 1859, the third son of the Hon. Joseph Potter (Judge of the State Court of Appeals) and Catherine (Eights-Boies) Potter. He received his early education in the public schools of his native town, and later, at the age of seventeen, he entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1879.

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[1] Memoir prepared from data supplied by the family and on file at the Headquarters of the Society.

In August, 1879, Mr. Potter received his first engineering position as Rodman on surveys of the Buffalo, Bradford and Pittsburgh Extension Railroad. Shortly afterward, he was made Assistant Engineer on the same surveys under the late Oliver Weldon Barnes, M. Am. Soc. C. E., as Chief Engineer. He remained in this position until September 1, 1880, when the work was finished.

In November, 1880, Mr. Potter went to Mexico as Assistant Engineer for the Construction Company that built the Mexican National Railway, and remained in this position until November, 1881. From this time until January, 1883, he was in charge of a subdivision of the same road.

In May, 1883, he entered the service of the Trojès Mining and Smelting Company as Engineer in charge of tunnels and surveys of mines. He left this position in March, 1884, and became Chief Engineer of the firm of Bowes Scott Read, Campbell and Company, Engineers and Contractors of London, England, and City of Mexico, D. F., Mexico. In this position, he assisted in such important works as the Drainage Canal in the Valley of Mexico and the pumping plant at Chapultepec, Mexico.

In 1892, he had charge of the Cerro Colorado Mines, in the State of Chihuahua, Mexico, and when these mines were closed, he went to Guatemala where he had charge of the electric light plant and system of telephones, at Quezaltenango. In 1898, Mr. Potter received an offer from the Westinghouse Electric and Manufacturing Company of a position in its Spanish-American Department. He accepted this offer and traveled extensively through South America and Central America, until 1905, when he was sent to Torreon, Durango, Mexico, to open a Northern Branch Office. Shortly afterward the Westinghouse Company went into the hands of a Receiver, and Mr. Potter was called to New York, N. Y. However, he resigned from the Westinghouse Company and accepted a position as Assistant Manager of the Tlahualilo Agricultural Company, in the State of Durango, Mexico, and remained in this position until ill health necessitated his retirement in 1936.

Mr. Potter was a member (Veteran) of the New York Athletic Club, Camera Club of New York, Rensselaer Polytechnic Alumni Association, Troy, N. Y., and Mexican Pilgrim Society.

He was married on September 29, 1891, to Jennie P. Pratt, of New York City, who survives him.

Mr. Potter was elected a Member of the American Society of Civil Engineers on December 5, 1888.

SAMUEL R. PROBASCO, M. Am. Soc. C. E.[1]

Died January 19, 1910

Samuel R. Probasco was born in the City of New York, N. Y., on September 13, 1833, and died at his home in the City of Burlington, N. J.

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[1] Information for memoir furnished by Theodore Belzner, Affiliate, Am. Soc. C. E. (from Municipal Engineers of the City of New York, 1910).

Mr. Probasco was educated in the Public Schools of his native city. In 1854, during the California gold fever, he went to San Francisco, Calif., by way of Cape Horn. While there he became a member of the Second Vigilance Committee.

In 1856 he left San Francisco and returned to New York and took up the study of engineering. Mr. Probasco worked on the construction of the old Albany Bridge, under the late Julius Walker Adams, Hon. M. Am. Soc. C. E. Subsequently, from 1871 to 1875, he was Principal Assistant Engineer on the Water Board of the City of Brooklyn. At this time he was in charge of the construction of the Storage Reservoir at Hempstead, L. I. and as Deputy was in charge of all details of the work on the water-shed. He was also in charge of the paving and repaving, and for some time was in charge of sewer construction in Brooklyn.

In 1868 Mr. Probasco made preliminary surveys for the New York and Brooklyn Bridge across the East River and in 1875 he became one of the Assistant Engineers for that Bridge. Mr. Probasco remained with the Department of Bridges, the successor to The Trustees of the New York and Brooklyn Bridge, until the time of his death.

In 1898, after the consolidation of New York City and the City of Brooklyn, Mr. Probasco was appointed Chief Engineer of the Department by the Commissioner of Bridges, and he remained in that position until 1901 when he resigned and was transferred to the New York and Brooklyn Bridge. He was appointed one of the Consulting Engineers by the Bridge Commissioner, the late Gustav Lindenthal, Hon. M. Am. Soc. C. E., during 1902 and 1903.

While associated with the Brooklyn Water Works Mr. Probasco invented the modern process of casting curved specials for water pipe.

In 1858 he was married to Anna M. Phillips, of Lumberton, N. J. They had two children, Selden R. Probasco, an engineer, and Samuel K. Probasco, a lawyer.

Mr. Probasco was a member of The Municipal Engineers of the City of New York, The Brooklyn Club, and the Long Island Historical Society, of which he was one of the regents.

He contributed a paper to the Society entitled "An Account of the Operation of the Gunpowder Pile-Driver"[2] which was published in *Transactions*.

Mr. Probasco was elected a Member of the American Society of Civil Engineers on November 18, 1868.

ROBERT REIMANN, M. Am. Soc. C. E.[1]

DIED APRIL 12, 1938

Robert Reimann, the son of Louis Albert Hansen and Christiane Euphrosyne Eugenie Reimann-Hansen, was born in Copenhagen, Denmark, on September 28, 1881. He was christened Robert Louis Reimann-Hansen, but shortly after coming to America, he elected to drop the more cumbersome double name and thereafter used the shorter name of Robert Reimann.

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[2] *Transactions*, Am. Soc. C. E. Vol. II (1874), p. 403.

[1] Memoir prepared by H. O. Arnurius, Field Engr., The B. & O. R.R., and D. W. Fry, Assoc. M. Am. Soc. C. E.

Mr. Reimann's father, Louis Albert Hansen, was one of the leading wholesale merchants of Copenhagen. His maternal grandfather, Arnold Eugen Reimann, filled a prominent place in Copenhagen's business and civil life and was a co-founder of Handelsbank. A grand uncle, Frederick Jacobsen, was the first to analyze cryolite in Greenland, spending the winter of 1858–59 there. He also was the first to manufacture cement in Denmark and his work in developing a better formula was considered notable at his time.

During his youth Mr. Reimann lived in the old part of Copenhagen bordering on the harbor and opposite the Royal Christiansborg Castle. Here the dignified residences and the state buildings stand side by side overlooking the picturesque waters of that port, while near-by is the cultural center of the city.

Robert Reimann attended the Gammelholms Preparatory Latin and High School from which he was graduated with distinction in September, 1899. He then attended the University of Copenhagen for four and one-half years where he took courses in Philosophy and Engineering, and was graduated with honors in January, 1904, with the degree of Civil Engineer. His formal education was supplemented by courses in languages with a private tutor, under whose guidance he became an accomplished linguist, writing and speaking Danish, French, German, and English, as well as having a thorough knowledge of Latin. In later life, he mastered Spanish and Italian. Following his graduation he rounded out his education by extensive travel in Europe.

Mr. Reimann came to the United States in April, 1904. In view of his travels abroad, his impressions of the splendor of the New York of 1904 are of interest. An excerpt from his notes reads:

> "My first impression of the skyscrapers was, that they are among the most beautiful buildings in the world. * * * Such a magnificent row of houses as line Fifth Avenue can hardly be found elsewhere. Adjacent is the vast Central Park. * * * Fine carriages in never ending rows file by, especially on Sundays, the one more elegant than the other; there are all types of carriages, often drawn by four horses, with coachman and footmen. And there are fine bridle paths, which are much used. Even those who walk are much more elegantly dressed than in Europe."

In May, 1904, Mr. Reimann secured a position as Draftsman under the late Francis Lee Stuart, Past-President and Hon. M. Am. Soc. C. E., then Engineer of Surveys of the Baltimore and Ohio Railroad Company. Mr. Reimann's first assignment was at Cumberland, Md., under the late Harry R. Talcott, M. Am. Soc. C. E., and, later, at the same place under G. D. Brooke, M. Am. Soc. C. E., who is now (1938) President of the Chesapeake and Ohio Railway Company. Both of these men left memoranda of the young engineer's brilliance and of his quick mastery of the principles of railway engineering.

In 1907 Mr. Reimann secured a leave of absence and traveled in Europe for several months, returning in September of the same year to re-enter the

Surveys Department of the Baltimore and Ohio Railroad Company. From that time until February, 1908, he served as Transitman; and from then until October, 1910, as Draftsman in charge of the field office of the Assistant Engineer. During the latter period, Mr. Reimann made studies to determine the location of the revision of line and reduction of grade on the Cumberland Division in West Virginia between Blaser and West End, which included the present double-track Kingwood Tunnel having a length of 4 202 ft.

In October, 1910, he was transferred to the Headquarters Office in Baltimore where he made his home for the remainder of his life. He was assigned as Head Draftsman in charge of office work for the Surveys Department. Serving in this capacity until July, 1915, he had charge of making studies of the improvements proposed on the Baltimore and Ohio Railroad. Of special interest were his studies and determination of the location for the Magnolia Cut-Off, which is east of Cumberland, Md., in the Potomac River Valley, where the river is deeply entrenched on a meandering course through the eastern front of the Allegheny Mountains. The cut-off is 13.2 miles in length, has four tunnels aggregating 6 958 ft, two crossings of the Potomac River, eliminates a helper grade, and effects a saving of 5.7 miles in main-line distance.

A notable part of his work was the exceptionally clear and accurate maps made under his direction. In many cases the maps not only served the immediate purpose for which they were made, but have continued to be used for general purposes. Also, during this period, Mr. Reimann prepared a very comprehensive set of tables for estimating, arranged in such a clear and concise form that they have been in wide use on the Baltimore and Ohio Railroad ever since.

From July, 1915, to the end of his service with the Baltimore and Ohio Railroad Company in May, 1932, Mr. Reimann was Assistant Engineer in charge of surveys, supervising the work of the office force and field parties. The surveys and studies made under his supervision embraced a wide variety of projects, ranging from minor improvements to the proposed reconstruction of major parts of an entire Division, including proposed extensions of the line.

He had a broad grasp of the factors involved in the proposed improvements and with it the ability to plan and conduct his work in an orderly and competent manner. Whenever possible the surveys and studies made under his supervision were carried to a conclusion and the results were recorded and reported in a concise and logical manner so that the merits of the projects could readily be considered at any time.

The general features of a number of his studies were written as separate professional papers and are of lasting value. As an illustration, after making a survey to determine whether an uncontaminated supply of water could be obtained in a coal territory, Mr. Reimann prepared a paper describing the general manner in which the problem was solved. The paper was printed in the *Railway Age* on July 23, 1920. In connection with surveys to determine the liability for flood damage, he made an investigation of stream flow and prepared an outline of a general method of procedure based upon the determination of back-water curves. Also, in connection with his work, Mr.

Reimann made an investigation of several types of retaining walls, bridge abutments, and arch culverts, and, on the basis of the Coulomb theory, prepared an outline of a simple graphic method of procedure for investigating these structures when required for special conditions.

In handling the men under his supervision, Mr. Reimann maintained a firm policy, tempered with impartial justice. He expected of each man a high character, a keen mind, and good work. He felt very strongly his obligation to reward the meritorious worker with further opportunity to develop, and was markedly successful in training his men and in fostering in them a high degree of pride and interest in their work. In return, he derived a very real pleasure from their development and was accorded their high esteem.

Although Mr. Reimann's primary interest was engineering, he also had an intense interest in many subjects outside his profession. As a young man, he studied music and obtained a good general knowledge of the art, as well as becoming a proficient pianist. In later life, he seldom played, but retained a genuine love of music and regularly attended the concerts and the opera. Among a wide circle of musical friends in Baltimore his judgment of music was held in high regard. He was a student of classical literature and was especially interested in, and had a deep insight into, the past civilizations.

On Saturday evenings he held open house for a group of young intellectual friends who were encouraged to participate in discussions covering a wide range of topics. He thoroughly enjoyed the stimulation of their interest, and, being a man of strong personality and decided convictions, he dearly loved and encouraged a spirited argument. These evenings at home, where the events of the past and present, and where their significance with respect to history and to present-day happenings, were discussed, were probably among the most enjoyable occasions of Mr. Reimann's life and are treasured memories among his friends.

During the last twenty years of his life Mr. Reimann traveled extensively, visiting all parts of the United States, every country of Western Europe, and parts of Canada, Mexico, and Africa. It was characteristic that his desire for first-hand knowledge of other civilizations, past and present, was insatiable. Throughout his life he was an assiduous student, keeping fresh the knowledge he had gained and constantly adding to it. Prior to each of his journeys, he carefully planned an unhurried schedule and adhered to it. His were pleasure trips, but with a mind eager to perceive and an industry that impelled him to make clear and concise notes of all his important travels. His notes have been bound and have been enjoyed by his friends, but have never been published.

Mr. Reimann was an accomplished engineer and a cultured gentleman—an extremely interesting companion to his friends and a kind and just preceptor to his subordinates.

He is survived by his widow, Sarah Genevieve (Becker) Reimann, residing in Baltimore, and by two brothers, Albert Reimann-Hansen, Captain Carsten Reimann-Hansen, and a sister, Mrs. Nanna Grønlund, all residing in Denmark.

He was a member of the American Railway Engineering Association; and the Charcoal Club, the Edgar Allan Poe Society, the Engineers Club, the Maryland Academy of Science, and the University Club, all of Baltimore.

Mr. Reimann was elected a Junior of the American Society of Civil Engineers on October 3, 1905; an Associate Member, on February 4, 1913; and a Member on April 17, 1918.

JAMES HERBERT RICHARDSON, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 1, 1936

Whether it was in the calculation of intricate earthquake stresses or in the detailing of important and involved layouts James Herbert Richardson always acted on the principle that deeds speak louder than words, that an engineer should be seen and not heard and that when he is seen he should be hard at work. He lived out these precepts efficiently and with an equipoise that was an example to others. His associates (and the writer was one of these) have missed him greatly; individually we have at divers times paid tribute to his cherished memory in short periods of silence. During his life he was fairly difficult really to know; but once he became your friend he was a friend completely for life; perhaps that is the reason his memory lingers.

Mr. Richardson was born October 3, 1876, at Lawrence, Mass. Thus, at the date of his death, he lacked a month of being 60 years old. His early schooling was obtained at Lawrence until he was 14 years of age, when he entered High School at Newtonville, Mass. His father was a master car builder and was in charge of the shops at Lawrence. James Herbert was the youngest of seven children, Brother Will being the oldest—twenty-two years older than Herbert. His sister Lucy was nine years older. The four other children died in infancy; and their four little graves at Andover, Mass., were always affectionately tended to by Herbert. Indeed these graves were a point of pilgrimage later when he and his wife were on their honeymoon. His mother died when Herbert was eight years old; thus he became the affectionate "apple of his father's eye"; and the boy had the complete run of the shops at Lawrence. At an early age he developed those traits of acceptance of responsibility and other engineering characteristics which he learned at his father's elbow and which later in his career made him invaluable as a checker and as an independent thinker. He was meticulous in his verification no matter who had been over the ground before. His father had become a noted pool player; therefore, in his later years it was but natural for Herbert to develop a fondness for billiards and pool. He inherited his father's keen eyesight and never had to wear glasses.

From September, 1895, to June, 1900, Mr. Richardson was a student of civil engineering and hydraulics at the Massachusetts Institute of Technology, at Boston, Mass., after which he received a certificate of honorable dismissal in 1900, having completed 150 semester hours of the 160 required for graduation. During his college course, he attended the Institute summer school in surveying, geodesy, stream-flow measurement, topography, and geology. During one summer vacation he was employed as Transitman in charge of party by the Massachusetts Harbor and Land Commissioner on surveys giving line and grade for street construction, drainage and granite block paving of new streets of South Boston. Along with his engineering, as a side hobby, Mr. Richardson took many related courses in biology. Probably he had a yearning to be a physician, but under the influence of his brother Will he turned wholeheartedly to civil engineering.

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[1] Memoir prepared by Thomas K. A. Hendrick, Assoc. M. Am. Soc. C. E.

From 1900 to 1902, Mr. Richardson was employed by the Eastern Bridge and Structural Company of Worcester, Mass. For this company, he designed, estimated, and detailed structural steel, masonry and timber; and he inspected steel work in the shop and field. This work included railroad and highway bridges, industrial, store, and loft buildings, heavy girders and trusses, and coal handling machinery and other mechanisms.

From 1902 to 1909, Mr. Richardson was employed by the American Bridge Company of New York at East Berlin, Conn., where he designed, estimated, and detailed a great variety of structures including railroad and highway bridges. In 1903, this plant office was closed and Mr. Richardson was one of the few engineers transferred to the design and estimating offices in New York, N. Y., where his duties were similar to those at East Berlin but included checking. In 1905, he was transferred to the office of O. E. Hovey, Hon. M. Am. Soc. C. E., where he designed and estimated heavier steel structures. He designed, detailed, and checked steel work on the Belmont Park Grand Stand, the New York Central Power-Houses at Port Morris, N. Y., and Yonkers, N. Y. He also worked on locomotive turntables. Much of his work was for foreign countries and involved familiarity with foreign specifications.

On November 4, 1904, at Orange, N. J., Mr. Richardson was married to Doctor Anna Mann, a graduate physician, daughter of the Rev. Charles H. Mann, minister of the Swedenborgian Church.

As a young couple, Mr. Richardson and his wife were keenly interested in the work of their church. In later years, they grew to feel that they could better express their religion outside of the organized church. When he was first introduced to his future wife, Mr. Richardson was in the act of placating a small child and this sympathetic understanding of children remained with him throughout his life. Mr. and Mrs. Richardson had two sons, David and Charles Holbrook, both of whom were graduated from The Engineering School of the University of Cincinnati, at Cincinnati, Ohio. Doctor Richardson was college physician at Smith College, Northampton, Mass., for eleven years; and from her home at Gateway House, son David was married to Marjorie Kendall, a California girl. Son Charles Holbrook married Virginia Henkel, a Cincinnati girl. David's sons are David Kendall and Gunnar Drake; Charles' sons are John Henkel and Philip Clark. To his sons, Mr. Richardson always maintained that stimulation of a steady, quiet enthusiasm that kept him perennially youthful in spirit.

In 1909, Mr. Richardson went to the Erie Railroad, Department of Bridges and Buildings, in New York City, where he designed the passenger tunnel and underground station beneath the Erie Railroad tracks at Jersey City, N. J. This work was below the North River water level and for that reason entailed difficult foundation problems; but the work was accomplished without delay to train traffic in the terminal station. He investigated the stresses in many existing railroad bridges and trestles—both steel and timber—under increased loading conditions caused by modern locomotives. He also designed new railroad and highway bridges and viaducts as well as the reinforced arches over Bergen Hill cut.

During the period 1910–1916, Mr. Richardson served with the Boston and Albany Railroad Company in Boston. For them he checked, for litigation, the reinforced concrete of the new Worcester passenger station which was an "all reinforced concrete building." He designed, detailed and checked, and was in charge of the work in the office and field on the Worcester Grade Crossing Elimination. He supervised the construction in the field and did the shop inspection on an important steel two-hinged arch. He solved many difficult foundation problems in the designing of heavy girder, truss and arch bridges, retaining walls, and abutments. During his tenure with the Boston and Albany Bridge Department about 150 spans were installed of which Mr. Richardson was given the most interesting structures to design such as steel arches, concrete arches, and special skew bridge work. Practically all of these spans were erected in a manner to avoid delays in train schedules. Many of the bridges were three-track or four-track structures; on some the old bridge was rolled out and the new one rolled in simultaneously. Besides supervising the actual construction of work that he had already designed, on frequent trips to the work of installation Mr. Richardson often made the necessary surveys for changes required in track alignment, grade and curvature.

Upon leaving the Boston and Albany Railroad Company, Mr. Richardson was engaged by Ford, Bacon and Davis, Engineers, of New York City, during the time that this firm was making an appraisal of the Rhode Island Company's properties from their offices in Providence, R. I. The properties consisted of about 400 miles of traction lines, most of which were electric trolley lines although some of the mileage was steam with the usual power plants, repair shops, bridges, and trestles. Mr. Richardson was in charge of party, had the use of a special car, and reported on all structural work including a drawbridge and large city bridges.

In 1917 Mr. Richardson was given a leave of absence from the firm of Ford, Bacon and Davis to accept a commission as Captain, Engineer-Officers Reserve Corps, in which he served for about two years with the American Expeditionary Forces in France. Upon his return to the United States and his honorable discharge from the U. S. Army, Captain Richardson returned to the firm of Ford, Bacon and Davis for whom he designed and estimated the steel and masonry for the Cheswick, Pa., plant of the Duquesne Power and Light Company and checked the water-work masonry and foundations. He also designed the structures for a briquet plant for coal on the Carolina, Clinchfield and Ohio Railroad.

During the period of 1920 to 1923, Mr. Richardson was employed by Perin and Marshall, McClellan and Junkersfield, and the Central Railroad Company of New Jersey on special problems.

In 1923 and 1924, Mr. Richardson served the firm of Westcott and Mapes, Engineers, of New Haven, Conn., for whom he designed and detailed supports for steam turbo-generators, condensers, and other power plant equipment. He designed various extensions to existing power plants and parts of the new Bridgeport plant of the United Illuminating Company. This work included steel supports for brick chimneys, boiler supports, coal handling machinery, gate houses, and bulkheads on the water-front. He designed a boiler house for the New Haven Gas Company. He designed and checked a reinforced concrete storage warehouse for storing paper which presented difficult foundation problems. In this work, he controlled field operations by means of frequent trips to the site.

During part of 1924, Mr. Richardson was employed by the Foundation Company of New York to design bridges, culverts, trestles and locomotive repair shops for a proposed railroad yard of the Southern Railway Company.

During the period 1924 to 1927, Mr. Richardson joined the forces constructing the Holland Tunnel for which organization he was appointed Assistant Engineer on August 20, 1924. He checked the design of the floor in the tubes and did detail work on the inner lining of the tubes and shafts in which there was a great amount of intricate steel and concrete work. His work also consisted of designing and laying out details of the ventilating buildings which housed the fans and electrical equipment, including piping, air-ducts and conduit system and in checking the detail and shop drawings. He also worked on the "final estimate" until August, 1927, at which time he was recommended to the Board of Water Supply, City of New York, by the then Chief Engineer of the Holland Tunnel, Ole Singstad, M. Am. Soc. C. E.

On November 1, 1927, Mr. Richardson was appointed Assistant Engineer Designer, Board of Water Supply, City of New York, and started to work for that well-known organization. For a short time his work consisted of the design of reinforced concrete on valve chambers and of steel trusses for the support of the ordinary chambers, City Tunnel No. 2. He was with the Board of Water Supply until April 15, 1934. After his initial work, his time was spent on the design and detail work of the drainage shaft No. 9A, its upper and lower chambers, and the masonry and structural steel building which are carried on the upper chamber walls and trusses. This work involved forces of great magnitude and the many studies, layouts, and working drawings bearing the initials "J. H. R." attest the enormous amount of work, ability, and industry of Mr. Richardson. He was in charge of a squad of engineers and draftsmen.

From March, 1935, to May, 1935, Mr. Richardson served with the Department of Parks, New York City, on the Henry Hudson Bridge project, Project No. 16 of the Temporary (State) Emergency Relief Administration. As of May 21, 1935, he was transferred to the Works Division of the Emergency Relief Bureau, City of New York, for work on reports in the Engineering Section. From February, 1936, to August 31, 1936, he served with the Department of Sanitation on the Tallman's Island Sewage Project during which period he was stricken with the illness (appendicitis) to which eventually he succumbed. Mr. Richardson was scheduled to return to the Board of Water Supply to resume his important work for that body on the day that he died, September 1, 1936.

Mr. Richardson's hobbies besides pool and billiards were walking and photography. He had a large album of pictures and snapshots of bridges and locomotives. On a trip to Europe in 1900, he took many pictures of European bridges and engineering works. Although he was not particularly athletic, he was fond of swimming and hiking. At one time, he was a member of the Appalachian Club and enjoyed long walks. Also he liked to study, scientifically, systems of diet, trying them out and adapting his own variations. He particularly delighted in his sons' accomplishments during their college careers. It was a never-ending pleasure for him to describe David's exploit in setting up the firm of "David Richardson—Dogfish." David had an establishment at Bailey Island, Me., for procuring dogfish. He injected the dogfish with colored gelatine and sold them to the biology laboratories of colleges and universities.

Mr. Richardson was a member of the Technology Club of New York and participated actively in its recreational and social programs. On account of his skill at pool and billiards, he won many prizes and tournaments; but he seemed to take more delight in explaining difficult shots to the younger members. He was always ready to help younger members with that kindly advice born of wide experience. The members of the club attest his princely character as a friend and counselor particularly to the younger members as told to the writer by the senior member, Dr. James Duff; and Arthur Hamilton, one of the younger members. In every way he had the cordial regard of his fellow-members and was much beloved for his colorful personality. Without exception, all admired him for his clean living and his confidence as a friend. At any time, he would demonstrate his willingness to go out of his way to do a helpful turn or perform a kindly act for a member. Slow to get acquainted with, once a member knew him, the breadth and depth of his friendship more than compensated for an apparent initial indifference. His friendship conscientiously and steadfastly given was the key to the charm of his personality.

Mr. Richardson at one time was a member of the American Railway Engineering Association and of the American Concrete Institute. He was a licensed Professional Engineer of the State of New York.

Mr. Richardson was elected an Associate Member of the American Society of Civil Engineers on January 2, 1907, and a Member on February 10, 1930.

WILLIAM JACKSON ROBERTS, M. Am. Soc. C. E.[1]

Died April 6, 1938

William Jackson Roberts was born in the Caroline Islands (an extensive group in the Pacific Ocean east of the Philippine Islands), on May 13, 1860, the son of Ephraim P. and Myra (Farrington) Roberts, Congregational Missionaries. In 1861, his parents resolved to carry on their missionary work in the United States. They landed in San Francisco, Calif., where they took the old sailing vessel, *Jane A. Falkenburg,* for Portland, Ore., reaching there on November 9, 1861. The next spring they moved to White Salmon, in Washington Territory, an early settlement on the Columbia River, where they remained for three years. In 1865, they located permanently at The Dalles, Ore., then one of the most thriving frontier posts.

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[1] Memoir prepared by Harold J. Doolittle, M. Am. Soc. C. E.

Mr. Roberts received his early education in the pioneer schools of those days. He was prepared for college, however, by his father and mother. His father was a graduate of Williams College, at Williamstown, Mass., and devoted to the Classics. This part of his son's preparation he took upon himself. From his mother, Mr. Roberts received his very thorough training in basic mathematics. In 1886, he was graduated from the University of Oregon, at Eugene, Ore., with the degree of Bachelor of Arts. In 1891 he received his degree of Bachelor of Science in Civil Engineering at the Massachusetts Institute of Technology, at Boston, Mass. Subsequently, he received his degree as Master of Arts from the University of Oregon.

Mr. Roberts began his engineering work in Portland, Ore., being employed on topographic surveys for that city. He soon became Engineer for the Hood River (Oregon) Irrigation Company and thus began a life-long interest in hydraulic engineering. In 1892, he was appointed the first City Engineer at Colfax, Wash. Then, in 1895, he turned his attention to teaching. For thirteen years he was Associate Professor of Mathematics and Civil Engineering at the State College of Washington, Pullman, Wash. During those years more than 1 300 young men came under his influence and training. Mr. Roberts gave tirelessly of his time and experience to "his boys," as he always called them, holding up by precept and example ideals of thorough scholarship and integrity of character, without which, he held, no man can become a real engineer.

During those thirteen years, Mr. Roberts served as Consulting Engineer for many cities and districts in the Pacific Northwest, interesting himself especially in water supplies for municipalities and irrigation projects. For ten years of this period he served on the Washington State Board of Health. He was also instrumental in organizing the Pullman Building and Loan Association, which was one of the first institutions of its kind in the State of Washington.

In 1908, Mr. Roberts decided to devote his entire time to private practice. His first project was to build a modern water system for Medford, Ore., the second in size in the State. In 1911, he was appointed State Highway Commissioner for the State of Washington by Governor M. E. Hay. He laid out and established the primary highway system for the entire State, which was one of his major contributions during his term of office. In December, 1913, Mr. Roberts returned to his field of greatest interest—hydraulic engineering. King and Pierce Counties, in Washington, were confronted with a serious flood-control problem. Under a joint agreement, they formed the Inter-County River Improvement, with headquarters in Tacoma, and Mr. Roberts became Chief Engineer. During the next nine years he directed the expenditure of $2 000 000, straightening, widening, and deepening the Puyallup River and permanently diverting the waters of the White and Stuck Rivers into it. This project was of a pioneering type, and it attracted the attention of engineers from many sections of the United States. Flood conditions have been successfully eliminated within the area improved.

In 1917, when the United States entered the World War, Mr. Roberts turned aside from this flood-control work long enough to lay out and supervise the construction of the water and sewerage systems for the Army Cantonment at Camp Lewis, Washington, accommodating 55 000 troops. These utilities, consisting of 40 miles of water mains and 30 miles of sewers, were completed in the record-breaking time of 78 days, and, although completed under the pressure of war-time preparations, they have proved satisfactory and adequate through the years during which Fort Lewis has become a permanent Army Post.

Mr. Roberts built more than forty water and sewerage systems throughout the States of Washington, Oregon, and Idaho. His later services in the engineering field were usually those of Consulting Engineer. However, he was frequently called upon as an expert witness in flood damage suits, or as a trustworthy appraiser in evaluating water systems. His integrity was as unquestioned as his engineering ability.

In December, 1936, the Tacoma Engineers' Club of Tacoma, Wash., unanimously voted to bestow an Honorary Life Membership upon Mr. Roberts. He had been active in organizing the Engineers' Club, had served as its first President, and his interest in its growth and activity had been sustained through the years. He was also a member of the American Waterworks Association.

He was for many years a member of the Tacoma Chamber of Commerce and also of the Rotary Club. In 1936, he donated a large part of his engineering library to the Tacoma Engineers' Club, and that Club placed it in the library of the College of Puget Sound, at Tacoma. He was always greatly interested in the betterment of the community wherever he lived. He found time to be an active worker in the First Church of Christ, Scientist, of Tacoma, serving as First Reader for a period of three years.

William Jackson Roberts was not only an engineer, he was a gentleman, loved and esteemed by all who knew him, especially by his "1 300 boys," of whom the writer is one.

He was married to Harriet Kitchel James, of Middlebury, Vt. To them were born three children: Arthur Kitchel, Margaret, and James Donald. Following the death of his first wife, he was married to Caroline Odell Wilmore, to whom three children were born: William Jackson, Jr., Mary Jane, and Joan.

At its meeting on April 12, 1938, the Tacoma Section of the Society passed unanimously the following resolution:

"*Whereas,* William Jackson Roberts, a beloved fellow member, having achieved works of outstanding merit and having lived a conscientious and useful Christian life, has been called to his reward; and,

"*Whereas,* the engineering profession has been uplifted by his faithful services and the whole Pacific Northwest has been benefitted by his earnestness, friendliness, efficiency, and wisdom; and,

"*Whereas,* his professional associates mourn his loss and in so doing keenly realize that Mrs. Roberts and the remainder of his family have sustained a still greater loss: Now, therefore, be it

"*Resolved,* that we, members of the Tacoma Section, American Society of Civil Engineers, bow to the Divine Will with a spirit of thankfulness for the inspiration gained by long association with William Jackson Roberts; and,

"*Resolved,* that we express deep appreciation of his professional integrity, his loyalty to ideals, his earnest efforts in behalf of his profession, and his helpfulness to younger engineers by both precept and example; and,

"*Resolved,* that our heartfelt sympathy be extended to Mrs. Roberts and others members of the family."

Mr. Roberts was elected an Associate Member of the American Society of Civil Engineers on June 6, 1900, and a Member on September 3, 1907. He became a Life Member of the Society on January 1, 1931.

HARRY EDGAR SAWTELL, M. Am. Soc. C. E.[1]

Died January 3, 1939

Harry Edgar Sawtell was born in Keene, N. H., on September 6, 1872, the son of Albert and Abby Sawtell.

His engineering experience began about 1894 with Edward A. Buss, a mill engineer of Boston, Mass. In 1900, he took a position with the Boston Steel and Iron Company as Draftsman and Structural Designer, and in 1902 he was made Chief Engineer of the company, a position which he held for three years.

In 1905 Mr. Sawtell joined the force of Dean and Main of Boston, designers of industrial plants. In 1907 the firm was dissolved, and Mr. Main reorganized on a broader basis under his own name and placed Mr. Sawtell in charge of the Structural Department. In 1926 the organization was incorporated and Mr. Sawtell became a member and director of the firm and continued so until his death.

As head of the Structural Department of Charles T. Main, and, later Chas. T. Main, Inc., Mr. Sawtell was employed either as Consultant or Designing Engineer on a large number of important structures. Among the more notable are the foundations of practically all the buildings of the Massachusetts Institute of Technology, at Cambridge, Mass., from the first main group begun in 1913 to the Architectural Building completed in 1938.

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[1] Memoir prepared by Charles T. Main, M. Am. Soc. C. E., and Charles W. Robinson, Esq., for joint publication by the Society and the Boston Soc. of Civ. Engrs.

Other large projects upon which Mr. Sawtell served as designers or consultant were the foundations for a group of manufacturing buildings for the Revere Sugar Refinery in Charlestown, Mass.; a group, including foundations, for the American Sugar Refining Company, in Baltimore, Md.; a group for the Lever Brothers Company in Hammond, Ind.; the principal buildings of the Gillette Safety Razor Company, the foundations of the Boston and Maine Railroad Station, and the foundations for the U. S. Parcel Post Station at the South Terminal, all in Boston. The latter involved the use of open caissons to bed-rock, extending in some cases 125 ft below the surface.

Mr. Sawtell's main interest was in foundation work, and the more difficult the problem was, the more he enjoyed it. He was a pioneer in the investigation of the physical and mechanical properties of soils and a firm believer in research, both in field and laboratory. He was widely known as an authority on foundations, and was the author of a large number of papers on engineering subjects, mainly on foundation design, which have been published in engineering journals.

In 1908 Mr. Sawtell became a member of the Boston Society of Civil Engineers. In 1926 he was made a Director; in 1929, Vice-President; and in 1931 he was elected President of that Society. His presidential address was on "Society Research" and contained valuable material on timber and soils.

For more than ten years, Mr. Sawtell was Chairman of the Boston Society's "Committee on Boston Sub-soils," and much useful material was collected, under his direction, concerning the geological formations and distribution of soils under Boston and vicinity. He devised a scheme of mapping and card tabulation, and the maps and cards were placed on file at the headquarters of the Boston Society of Civil Engineers for general use.

He was Chairman for a year and a half of a sub-committee to revise the section of the Boston Building Laws covering "Excavations and Foundations." He was Chairman also of a committee of citizens of the town of Winthrop, Mass., where he resided, to revise the Building Laws of that town.

Mr. Sawtell was a member of the Boston City Club, the Boston Yacht Club, and the Rotary Club of Boston. He was a member of the Board of Directors of the Rotary Club in 1930–1931.

Motoring and boating were his principal recreations. He was the owner of a 40-ft, 2-cabin cruiser on which, with his family, he spent considerable time every summer cruising along the coast.

He was a member of the Union Congregational Church of Winthrop, and served for many years on the Prudential Committee.

Mr. Sawtell had a keen sense of humor, was sympathetic and kindly by nature, and always had an encouraging word for his assistants and associates. His loss will be deeply felt.

On May 3, 1899, he was married to Cora Pearson, of Melrose, Mass. He is survived by his widow, one daughter, and three grandchildren.

Mr. Sawtell was elected a Member of the American Society of Civil Engineers on September 2, 1914.

WILLIAM ROBERT SCANLIN, M. Am. Soc. C. E.[1]

Died March 30, 1938

William Robert Scanlin was born in Philadelphia, Pa., on September 26, 1885, the son of Charles P. and Louisa M. Scanlin. He was educated in the Public Schools and was graduated from the Northeast Manual Training School in June, 1902. He later attended classes in engineering subjects at Drexel Institute of Technology in Philadelphia.

In 1903 he entered the employ of the Philadelphia Rapid Transit Company, where for five years he was a Junior Engineer and Draftsman in connection with the planning and construction of the Market Street Subway and Elevated Passenger Railway.

Later he was employed for six months by the New York Shipbuilding Company of Camden, N. J., as a Structural Draftsman. He left this company to take a position in the Chief Engineer's office of the Philadelphia and Reading Railway Company as Assistant Computer and Designer on an important project of grade-crossing removal in Philadelphia. He was with the company from June, 1909, to June, 1912.

At that time Mr. Scanlin entered the employ of the Philadelphia Rapid Transit Company as Structural Designer, engaged in work on buildings and the Subway and Elevated Railway System.

In 1915 he was engaged as the Engineer in charge of the design and construction of a large warehouse for the W. J. McCahan Sugar Refining Company of Philadelphia.

In 1916 he entered the employ of the City of Philadelphia as an Assistant Engineer (Squad Leader) in the Department of City Transit. He was engaged for two years on the design of sections of the projected Broad Street Subway System.

In April, 1918, he returned again to the employ of the Philadelphia Rapid Transit Company, as Structural Engineer in the Department of Rolling Stock and Buildings. In December, 1922, he was advanced to the position of Assistant to the Chief Engineer; later he was made Assistant to the Vice-President in charge of engineering, and in July, 1927, he was made Chief Engineer of the Company. He continued in this position until his death.

Mr. Scanlin's career in the engineering profession was marked by untiring attention to the details of his work, careful study of the problems before him, loyalty to his employers and associates, and a broad spirit of fairness to all who came in contact with him through business relationship.

Mr. Scanlin took an active interest in the work of the Philadelphia Section and contributed in a large measure to making the Section an influential body. He served as Director from 1926 to 1928, as Vice-President from 1929 to 1930, and as President from 1930 to 1931.

Mr. Scanlin was a member of the Engineers Club of Philadelphia, the American Society for Testing Materials, and the American Transit Association. He was also a member of the Thomas R. Patton Lodge, Free and Accepted Masons, a Past Master of Union Assembly No. 30, Artisans Order of Mutual Protection, and a member of Diligent Council No. 4, Fraternal Patriotic Americans. He was an active member of the Lutheran Church.

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[1] Memoir prepared by Charles H. Stevens, M. Am. Soc. C. E., aided by J. H. M. Andrews and Lemuel F. Parlette, Members, Am. Soc. C. E., and William F. Scanlin, Esq.

He was married on June 27, 1906, to Ella R. Fox of Philadelphia, who, with a son, William F. Scanlin, survives him.

Mr. Scanlin was elected an Associate Member of the American Society of Civil Engineers on April 19, 1920, and a Member on October 14, 1930.

ERNEST GEORGE SCHURIG, M. Am. Soc. C. E.[1]

DIED FEBRUARY 12, 1939

Ernest George Schurig was born in New Haven, Conn., on February 18, 1874. He attended Yale University in New Haven, and was graduated in 1894 with the degree of Bachelor of Philosophy in Engineering.

Soon after graduation he was employed by the Union Bridge Company of Athens, Pa., where he remained until 1900. In 1898 he was engaged in the erection of wharves, coaling stations, etc., for the Bridge Company at Key West, Fla., and on the Florida Keys.

At the outbreak of the yellow fever in 1900 he came north and was employed in the Bridge Engineering Department of the Illinois Central Railroad Company. From 1902 to 1904 he was with the Navy Department in the Bureau of Yards and Docks at the Philadelphia Yard, and also at New London, Conn. He prepared plans and designed structural work and checked other work for many projects, including sewer systems, coal pockets, retaining walls, and other items of architectural engineering. In 1904 he was appointed a Superintendent of Construction in the office of the Supervising Architect, U. S. Treasury Department, and was assigned to supervise the construction of the U. S. Post Office Building at Logansport, Ind. In 1906 he was transferred to the Washington office as Structural Engineer, remaining there until December, 1907, at which time he was reappointed a Superintendent of Construction for the remodeling and extension of the U. S. Post Office at Peoria, Ill. From there he was assigned to various public buildings at Rock Island, Ill.; Lincoln, Nebr.; Marshall, Mo.; and Lebanon, Pa.

In 1917 Mr. Schurig was appointed a Supervising Superintendent of Construction, with headquarters at New Haven, and he continued in that position and location until 1933, when the office was reorganized and he was assigned to Boston (Mass.), as a District Engineer, succeeding Mr. E. H. Pearson, Supervising Engineer, who was retired about that time.

In September, 1938, he retired because of disability. Until his death, his friends and family had hoped for his ultimate recovery from what started, about a year previously, as a nervous breakdown.

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[1] Memoir prepared by Levi R. Whitted, Assoc. M. Am. Soc. C. E.

Mr. Schurig was a man of unusual ability and was highly respected by all with whom he came in contact. He was given many difficult assignments during his long service in the Treasury Department, and he completed them most successfully, and to the satisfaction of his superiors.

Mr. Schurig died of pneumonia on February 12, 1939, in New Haven. His rather sudden, although not unexpected, death is a serious loss to the Treasury Department; and his genial, generous, and cordial nature will be missed by his numerous friends.

Mr. Schurig was elected a Member of the American Society of Civil Engineers on September 12, 1916.

FRANK FORREST SINKS, M. Am. Soc. C. E.[1]

Died April 14, 1938

Frank Forrest Sinks, the son of Davis W. Sinks and Margaret (Pearson) Sinks, was born on May 12, 1873, at West Milton, Ohio. His ancestors, Prussian and English, arrived in America in 1750.

His early education was obtained in the grade and High Schools of West Milton and immediately thereafter he began his engineering career as Chainman and Rodman on irrigation surveys in Southern California.

In September, 1892, Mr. Sinks entered Rose Polytechnic Institute, at Terre Haute, Ind., from which he was graduated in 1896, with the degree of Bachelor of Science in Civil Engineering. He was a Charter Member at the Institute of the Alpha Tau Omega Fraternity.

After his graduation, he was employed as Rodman by the Chicago, Hammond and Western Railway Company and was advanced to Resident Engineer. In 1898 he became Office Engineer of the Chicago Junction Railway Company, and from 1899 to 1903 he was with the Pittsburgh Testing Laboratory as Inspector in mills, foundries, and fabricating shops, under the Resident Engineer, T. L. Condron, M. Am. Soc. C. E.

In 1903, after Mr. Condron began his private engineering practice, Mr. Sinks entered his employ as an Engineering Assistant. His work included inspections, computations of stresses, and reports on existing railway and highway bridges, as well as the designing and supervision of the erection of new structures. Later, the Condron and Sinks organization was formed, and included among its clients the Chicago and Eastern Illinois Railroad Company, the Mobile and Ohio Railroad Company, the Missouri, Kansas, and Texas Railway System, and numerous private individuals and corporations.

During the years from 1903 to 1906, when reinforced concrete was in its infancy, the firm of Condron and Sinks contributed constructive work in the design and development of reinforced concrete structures, and Mr. Sinks was regarded as one of the leaders in the field. He originated and patented a type of flat-slab floor design which, later, became known as the "Akme System." The special characteristic of this type of design was the placing of all reinforcing bars in but two directions, instead of four directions, in such a manner as to make the design " structurally determinate "; also, in this two-way system, the percentage and distribution of reinforcement for different spans and loads could be proportioned in accordance with generally accepted rules or formulas for designing girders and slabs. The Akme system was extensively used in the construction of many buildings erected in at least thirty-five cities of the United States, and in Canada.

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[1] Memoir prepared by John Arrington, Esq., Seattle, Wash., and T. L. Condron, M. Am. Soc. C. E.

In August, 1908, Mr. Sinks was temporarily engaged by the Corrugated Bar Company, of St. Louis, Mo., to go to the Pacific Coast, and assist its agents there in the promotion and sale of its products.

The firm of Condron and Sinks continued until 1911, when Mr. Sinks withdrew and returned to Seattle, Wash., and the Pacific Coast where he had been very successful in promoting the corrugated bar and had made many friends. During the summer of 1911 he induced the Pacific Coast Steel Company (later, a Pacific Coast subsidiary of the Bethlehem Steel Company) to roll corrugated bars from old railroad rails for reinforcing concrete. This character of steel bar was at first not favored by engineers of the Pacific Northwest for use in reinforced concrete structures, but Mr. Sinks, together with the organization which he had formed, succeeded in establishing re-rolled corrugated bars as an acceptable type of reinforcing material in the Pacific Northwest.

In January, 1914, he was engaged by the Pacific Coast Steel Company, as Sales Engineer in charge of reinforcing steel sales, and, later, was made Sales Manager in charge of domestic and foreign sales of the Seattle plant.

In 1918, after America had entered the World War, Mr. Sinks resigned as Sales Manager of the Pacific Coast Steel Company, to accept the position of General Manager of D. W. Hartzell, Incorporated. This Corporation had leased some poorly arranged existing shipyards at Winslow, Wash., and had secured contracts from the United States Shipping Corporation for installing machinery and equipment in ten wooden hulls that had been launched to help win the war. This Corporation was also engaged in dry docking and marine repair work. Again, his natural ability asserted itself as General Manager. He re-arranged and remodeled these yards, installed new equipment, and perfected an organization for their operation which was as complete as any in the Pacific Northwest.

In 1921, he returned to the private practice of engineering and established himself, in Seattle, as a Consulting and Contracting Structural Engineer. He was retained to go to New York, N. Y., to revise the structural plans of the Hotel Olympic, a 1 000-room hotel to be constructed in Seattle. The plans had been prepared by New York architects and engineers for the use of structural steel, but the building was built of reinforced concrete in accordance with Mr. Sinks' recommendations.

In 1930, he designed and supervised the construction of the Reinforcing Bar Fabricating Plant and Stock Shed, in Seattle, for the Pioneer Sand and Gravel Company, and became Engineer and Manager of that Department. He remained with that Company and in that capacity until 1935, when he returned to the private practice of engineering.

Mr. Sinks was married on February 25, 1911, to Laura Castle Nichols, of Chicago, Ill., who survives him. They had no children.

He was possessed of a winning genial personality and made friends wherever he went. He was very sincere, quiet, and unassuming, and was not only admired for his technical ability but for his fine character, his always sound common sense, and his keen kindly humor. He not only made friends with all with whom he came in contact, but he retained his many friends throughout his life, even from his college days and early associations, when associations were intimate, free from the competition of commercial life, when likes and dislikes were frank and unfettered.

The following quotation is taken from a letter received by Mrs. Sinks from A. M. Young, M. Am. Soc. C. E., at the time of her husband's death. This letter is quoted as it is typical of the many letters which she received and shows the esteem in which her husband was held:

> "I want to express to you my very great sorrow at the passing of your husband. I had known him for a very great many years. I met him as early as 1914. I had known him by reputation for many years before that. At the Cement Show at the Coliseum at Chicago, about 1906, I remember the models of his patents. I used these patents and paid him royalties. I never thought that I would come to know him as well as I did, for in engineering he stood on a pedestal which few of us will ever reach. It has been one of the high lights of my experience to know and enjoy the friendship of Frank.
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> "Frank's assistance on our work was of the greatest value to us. He was absolutely dependable and he got things done. We will miss him very greatly.
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> "Frank's contribution to the art of building is so outstanding as to need no comment. No concrete engineer has made a larger. It is not often that a man lives to see his conceptions generally adopted and acclaimed, and it must have been a very great satisfaction to him to have participated so largely and effectively in improved building methods."

Mr. Sinks was interested in mining and loved Nature and the great outdoors, especially the mountains. His engineering took him into all parts of the United States, but his vacations were spent with his wife, accompanied by a guide, pack and saddle horses, and hunting dogs, traveling through the Cascade Mountains, the Olympic Mountains, and the mountains of British Columbia, hunting and fishing, and studying bird and plant life.

Mr. Sinks was elected an Associate Member of the American Society of Civil Engineers on May 2, 1906, and a Member on June 4, 1907. He was also a member of the Western Society of Engineers.

WILSON FITCH SMITH, M. Am. Soc. C. E.[1]

DIED NOVEMBER 29, 1937

Wilson Fitch Smith, the son of Fitch W. Smith and Isabel C. (Eagle) Smith, was born in New York, N. Y., on August 2, 1871.

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[1] Memoir prepared by George Perrine, M. Am. Soc. C. E.

"Wilson Fitch," as his close friends referred to him, attended Berkeley School in New York City, where he prepared for entrance into the School of Mines at Columbia University, when that institution was situated at Forty-Ninth Street and Madison Avenue. While attending the School of Mines, he excelled in his studies and was also proficient in athletics; he took part in inter-class and inter-collegiate games—his favorite events being sprints and hurdling. His fraternity was Alpha Delta Phi. The degree of Civil Engineer was conferred upon him in June, 1894.

Immediately after graduation Mr. Smith was appointed Assistant Instructor in Surveying at the Summer School conducted by Columbia University at Bantam, Litchfield County, Conn., on a farm owned by the University. Engineering students attended this school for three summers from June to September. This summer course afforded these students more practical instruction than a corresponding course at any other engineering school.

In October, 1894, Mr. Smith joined the Engineering Staff of the New York Central Railroad Company as Volunteer Assistant, which means, that one in that position worked without stipend until he proved his worth. The first position held by him from October, 1894, to April, 1895, was Rodman.

From April, 1895, until October, 1896, he was Assistant Engineer in charge of a Field Party. It is quite likely that the meager pay given to beginners in the railroad field is discouraging to many of the more ambitious. If Mr. Smith had continued with the Railroad Company, he would, without a doubt, have developed into a valuable member of the Executive Staff.

After severing his connection with the New York Central Railroad Company, Mr. Smith entered the employ of R. W. Hildreth and Company, as Shop Inspector. The more important structures upon which he was engaged were the inspection of the structural steel for the Commercial Cable Building and the Queens Insurance Buildings, in New York City.

While in Berkeley School, Wilson Smith attended military drills. In later years, and about this time, he joined Squadron A, N. Y. N. G.

From January, 1897, to June, 1898, he was in the employ of the late William Barclay Parsons, Hon. M. Am. Soc. C. E., Chief Engineer of the Rapid Transit Commission of New York City. His first duty in this position was to prepare a perspective design for a viaduct to carry a proposed extension of the Amsterdam Avenue Line of the Third Avenue Railway Company from the high ground at 190th Street, north to Kingsbridge. The structure was very attractive in design, but, unfortunately, the Railway Company did not construct the viaduct. Upon the completion of this design, Mr. Smith was put in charge of a certain portion of contract drawings for the Rapid Transit Underground Railway. The original detail and alignment, in existence when Mr. Parsons was promoted from Assistant Engineer to Chief Engineer of the Rapid Transit Commission, was very typical and sketchy, because of a scarcity of funds.

At that time, and for a little while afterward, the Rapid Transit Subway was considered a great venture, and capitalists were hard to convince of its possibilities and great value as an investment. Mr. Parsons and Mr. John B.

McDonald, the most prominent contractor at that time, convinced Mr. August Belmont, banker, that the subway was feasible and could be built at reasonable costs, and from that time rapid transit was destined to become a fact.

The typical plans mentioned previously were immediately elaborated upon, and from them the first contract drawings for the first subway were developed. Mr. Smith had charge of preparing a certain portion of these plans.

In 1898, progress in the preparation for the receiving of bids for the construction of the subway was delayed. Mr. Parsons was engaged by certain financial interests to go to China with a party of engineers to make a survey for a railroad from Canton to Hankow.

During the lull in subway progress, Mr. Smith accepted a position with the late F. Stuart Williamson, M. Am. Soc. C. E., Consulting Engineer, whom he met through his association with Mr. Parsons. His duties in this position were in connection with the design and preparation of plans for the Riverside Drive Viaduct at 129th Street, New York City. The period covered by this engagement was from June, to August, 1898, inclusive. From August to November, 1898, he was in the service of the Laflin Rand Powder Company, at Pompton, N. J., making a complete topographical survey of its properties.

From November, 1898, until June, 1900, Mr. Smith was in the employ of the late George Shattuck Morison, Past-President and Hon. M. Am. Soc. C. E., Consulting Engineer. While in this engagement, he obtained valuable experience in the design of bridges of the most elaborate types and magnitude. Two of the bridges for which he assisted in preparing plans were the Atchison Bridge, Competitive Design for Memorial Bridge across the Potomac River at Washington, D. C., which comprised five 180-ft span masonry arches; and the new superstructure for the Columbus Bridge, for the Mobile and Ohio Railroad Company.

Mr. Smith returned to subway work in June, 1900, as Assistant Engineer of the Rapid Transit Commission. Because of his former connection with the staff of the Chief Engineer, the Civil Service Commission approved his application without formal examination, upon the request of the Chief Engineer. This fact is probably recorded in the Minutes of the Board of Transportation. Mr. Smith was assigned as Section Engineer of that part of the Broadway route from West 103d Street to West 125th Street. His assignment on this section terminated in June, 1903, when he was put in charge of the reinforced concrete tunnel under Lenox Avenue from 145th Street to 148th Street, and also the tunnel under the Harlem River. The reinforced concrete subway was called the "bird cage" type. This type of construction was discontinued after a short period. The river tunnel was of a novel and original design, devised by the contractors for whom the late William H. Burr, M. Am. Soc. C. E., was Consulting Engineer.

After this work was completed, Mr. Smith was appointed Division Engineer of the Kensico Division, Board of Water Supply of New York City. This appointment was gained through the recommendation to the Chief Engineer, the late J. Waldo Smith, Hon. M. Am. Soc. C. E., by his close friend and former comrade in arms in Squadron A, the late Merritt H. Smith, M. Am. Soc. C. E., Deputy Chief Engineer of the Board.

The Division Engineer's Staff made all the surveys, both topographical and lines and grades for the construction of the dam and appurtenant structures. There was included in this Division, 4 300 ft of aqueduct in tunnel, gate chambers, aerating basin, and other structures. This assignment covered a period of eleven years from May, 1906, to October, 1917.

The Kensico Dam was of cyclopean masonry, with a rough rock face, the total content being 900 000 cu yd. The length of the dam is 1 850 ft, with a total height at center of 307 ft. The top is 28 ft wide under the coping.

After the completion of this term of service, and from October, 1917, to May, 1918, Mr. Smith was Resident Engineer of the Division of Shipyard Plant, of the Emergency Fleet Corporation, stationed with the Merchants Shipbuilding Corporation, at Bristol, Pa. This engagement covered the supervision and construction of shipyard and housing at this plant. The cost of this work was about $15 000 000, and all expenditures were under the control and approval of Mr. Smith.

This connection with the Merchants Shipbuilding Corporation continued until September, 1918, when Mr. Smith was Manager in charge of a housing development of the industrial Town of Harriman, Penna. Later, he was Special Assistant to the President and Vice-President of the Corporation, engaged in investigation and reports. During 1921 and 1922, Mr. Smith was Manager in charge of properties at Harriman Yard.

After this, he was Chief Engineer of the Bear Mountain Hudson River Bridge Company, in charge of the construction of the Bear Mountain Bridge, crossing the Hudson River north of Peekskill, N. Y.

From November, 1925, to the date of his death, Mr. Smith was Vice-President and Director of the Dutchess Bleachery, Incorporated, and the Rockland Finishing Company, Incorporated, at Poughkeepsie, N. Y. He was a member of the University Club of New York City and the Quill and Arkwright Clubs, also of New York City.

Mr. Smith is survived by his mother, Mrs. Fitch W. Smith, a sister, and three children, one of whom is Dr. Wilson Fitch Smith, Jr. Mrs. Wilson Fitch Smith, who died about two years before her husband, was Alice Mary Stimson, daughter of the Rev. Henry Albert Stimson and niece of the Hon. Henry L. Stimson, former Secretary of State.

Wilson Fitch Smith was one who had a charming personality. Some thought him one with whom it was difficult to become acquainted. To those who knew him well, however, he was a man with a keen sense of humor, very pleasant company, and a faithful friend. In business, he was very painstaking, accurate, and conscientious. His assistants respected him on account of his thorough technical knowledge and skill as a Construction Engineer, for his fairness toward them, and for his appreciation of their faithful services.

Mr. Smith was elected a Junior of the American Society of Civil Engineers on January 3, 1895; an Associate Member on May 1, 1901; and a Member on February 6, 1912.

GEORGE PUTNAM STOWITTS, M. Am. Soc. C. E.[1]

DIED MARCH 5, 1938

George Putnam Stowitts was born on March 19, 1879, in Westfield, N. Y., the son of Cornelius S. and Maria (Putnam) Stowitts. He attended Hamilton College, at Clinton, N. Y., during the years 1896 and 1897, and Rensselaer Polytechnic Institute, at Troy, N. Y., from 1898 to 1900.

Upon leaving Rensselaer he entered the Engineering Department of the New York Central Railroad Company, where he remained until 1920, occupying, successively, the positions of Chainman, Rodman, Draftsman, Assistant Engineer, Chief Draftsman, and Assistant Designing Engineer, on the Lines East of Buffalo. In 1910 he became Designing Engineer and in the next eight years prepared plans, specifications, and estimates for a great variety of work, including the Castleton Cut-Off across the Hudson River south of Albany, N. Y.; the Gardenville Yard at Buffalo, N. Y.; piers at Weehawken, N. J.; extensive four-tracking and double-tracking; and new engine-houses. During this period he revised many old standards and established new ones in many important fields.

In 1918 he was appointed District Engineer of the Eastern District of the New York Central Railroad Company (embracing the Hudson, Harlem, Putnam, River, and Pennsylvania Divisions) and had charge of the contract construction work, including notably some important grade revisions, four-tracking, station buildings, and terminals.

In 1920 Mr. Stowitts resigned from the New York Central Railroad Company to take charge of the construction of a ship repair plant at West New York, N. J., as Chief Engineer of the Lord Dry Dock Company.

From 1922 to 1925 he engaged in consulting practice at New York, N. Y. He was associated with the late George F. Swain, Past-President and Hon. M. Am. Soc. C. E., of Boston, Mass., in a report and project for passenger terminals at Toronto, Ont., Canada. He was retained by the City of New York to report on valuation of surface, elevated, and subway lines. He was associated with the late Francis Lee Stuart, Past-President, Am. Soc. C. E., under a committee of railroad presidents to make a complete study and plan for improving the railroad problem in the Metropolitan District in connection with the City of New York and harbor affairs. He did consulting work for the Foundation Company of New York on its important railroad problems.

In 1925 Mr. Stowitts became Engineer of Construction of the Seaboard Air Line Railway and the Seaboard All-Florida Railway Companies, having charge of the location and construction of their new line from West Palm Beach to Miami, Fla. This was a 75-mile extension with a 25-mile branch. It included 35 bridges (7 of which were movable) and 20 passenger stations.

When the Cincinnati Union Terminal Company was organized in 1927 to build a single terminal to replace the scattered passenger stations of the numerous railroads entering Cincinnati, Ohio, Mr. Stowitts was made Engineer of Design; and in 1929 when construction began, he accepted the position of Engineer of Construction, to bring all the railroads to one terminal. He worked out studies and estimates, developed the adopted scheme, and had charge of the design and construction of the whole enterprise, involving 6 500 000 yd of grading, 25 000 tons of steel, 30 000 yd of concrete, 15 000 piles, and a total cost of about $41 000 000. The success of his solution of the many difficult problems involved, such as topography, foundations, and operation, is attested by its performance during subsequent floods.

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[1] Memoir prepared by E. F. Kriegsman and H. M. Waite, Members, Am. Soc. C. E.

Upon the completion of the Cincinnati Terminal in 1933, Mr. Stowitts engaged in consulting practice in Cincinnati. Later in that year he was appointed State Engineer Inspector for Iowa, of the Federal Public Works Administration. In 1935 he was made Chief Engineer of the Inspection Division. When the PWA was re-organized on a regional plan basis in November, 1937, Mr. Stowitts became Engineer of the New York Region.

His success in engineering work was due to his organizing ability in training personnel and securing their loyal co-operation. His ability to study out the solution to a difficult railroad problem was outstanding.

Mr. Stowitts was a member of the Sigma Phi fraternity, the Society of Terminal Engineers, and the Georgetown Presbyterian Church of Washington, D. C.

Mr. Stowitts greatly loved out-door occupations. He was an expert in gardening, kept bees, and his sport was upland hunting of quail and partridge. The making of stocks for shotguns was a hobby which gave him great pleasure. His taste in reading was catholic, with especial emphasis on the value and beauty of Greek literature. He was of the opinion that a man widely read, with a broad, general as well as technical education, and a vital interest in nature, made the best sort of engineer.

He was married to Edith Aylworth Gillespie on October 3, 1906, at Albany, N. Y. Three daughters, Emily, Mary Gardner, and Rhoda Putnam, were born to them. Mrs. Stowitts and one daughter, Mrs. Gaylord Marsh Merriman (Emily), and two sisters, Mrs. G. A. Hulbert and Mrs. H. Frank Paddock, survive.

On March 5, 1938, Mr. Stowitts died suddenly of a heart attack, at the terminal of the Lackawanna Railroad, in Montclair, N. J., where he had lived since the previous December. Interment was at Kensico Cemetery.

Mr. Stowitts was elected an Associate Member of the American Society of Civil Engineers on June 3, 1908, and a Member on September 3, 1912.

EMORY SUDLER, M. Am. Soc. C. E.[1]

DIED APRIL 21, 1938

Emory Sudler was born in Anne Arundel County, Maryland, on February 14, 1870, the son of Colonel and Mrs. John (Emory) Sudler. He attended the public schools and the Baltimore (Md.) City College, from which he was graduated in 1888.

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[1] Memoir prepared by Guy Bonney, M. Am. Soc. C. E.

After his graduation he obtained his first engineering position as Draftsman and Instrumentman on town-site development at Buena Vista, Va., where he remained for about three years.

In 1891, Mr. Sudler returned to Baltimore, and with the exception of a period of two years—during which time he was employed by the Baltimore and Ohio Railroad Company on location in West Virginia, and by the Louisville and Nashville Railroad Company in Kentucky—he remained continuously in that city until 1914. He served for three years as Transitman and Assistant Engineer on river and harbor surveys relative to improvements for Baltimore's water shipping; one year as Draftsman for the Electrical Subway Commission of Baltimore; and for fifteen years he was with the Baltimore City Water Department.

At the time that he entered the service of the Water Department many improvements were necessary, due to the rapid growth of the city, and because of the outstanding ability displayed by Mr. Sudler, the development of a large part of these improvements, from preliminary to finished stages, was given over to his direction. The most outstanding of these improvements were the Roland Park stand-pipe, preliminary surveys and studies for a 20 000 000 000-gal impounding reservoir on the Gunpowder River, and investigations of storage reservoir sites, culminating in the design and construction of Lake Ashburton, a 210 000 000-gal reservoir.

In 1910, when the Gunpowder River improvements for the water supply of Baltimore were made possible by appropriations obtained for that work, Mr. Sudler was made Division Engineer in charge, and under his direction these improvements were designed, and to a large extent completed by the time of his resignation in 1914. In addition to the impounding dam, gatehouse, and conduit connecting to an existing feeder tunnel, several miles of highways and two long highway bridges replacing flooded roads were constructed. At the same time he also designed and constructed distributing mains of steel and concrete within the city limits, which, too, were a part of this construction program. In 1914, he resigned from the Water Department and spent a year or more of leisure in his native county, in Maryland.

From 1916 until 1919, Mr. Sudler was with the United States Government as Engineer on design and construction of wharves, docks, bulkheads, and in charge of dredging operations at the Norfolk, Va., Army Supply Base and Terminal.

He was employed from 1919 until 1929, by the Ambursen Construction Company as Chief Engineer and played an active part in the large number of dams and power-houses designed and constructed during this period by that Company.

From 1930 until his death, Mr. Sudler was more or less inactive in engineering work. He was employed for a short time on the preparation of data to be used in the legal battle between New Jersey and New York, N. Y., concerning the diversion of water from the Delaware River; and spent some time with the Westchester County Park Commission and the Department of Parks, in New York City.

Although throughout his career there is exhibited a high degree of talent and thoroughness, his outstanding work, and that which appealed to Mr. Sudler most, was the design of dams and other hydraulic structures. While with the Baltimore City Water Department he directed the design of the Loch Raven Dam and the earth dam at Ashburton Reservoir. During the ten years he was with the Ambursen Company, he designed and supervised the installation of automatic crest gates for the Patillas Dam, in Puerto Rico; created the design in an engineering competition, which was selected by a Committee of Engineers, covering the reconstruction of the Oklahoma City, Okla., Water-Works Reservoir; and made the preliminary design for the Rodriguez Dam, in Mexico. This dam, completed in 1938, is 250 ft in height and is the highest dam of the Ambursen type in the world. Besides designing many other dams that were built by this Company during this period, he patented the Sudler Radial Gate, an under-sluice of the Tainter gate type, suitable for large openings under head.

Mr. Sudler was highly educated, and this education, obtained through the years by hours of study and reading, extended far beyond the field of engineering. A gentleman, an engineer, and a Christian in the true sense of the words—his record bears out this statement.

He was unmarried, and is survived by his mother, Mrs. John Emory Sudler, a brother, Ralph Eric Sudler, and a sister, Elsa deTalma Sudler.

Mr. Sudler was elected an Associate Member of the American Society of Civil Engineers, on September 5, 1911, and a Member on October 12, 1925.

ALFRED FRANCIS THÉARD, M. Am. Soc. C. E.[1]

DIED JANUARY 2, 1939

Alfred Francis Théard was born in New Orleans, La., January 29, 1865. His ancestors, on both sides, were members of old and distinguished Louisiana families. His father, Judge Paul E. Théard, and his mother, Athenais (Pilie) Théard, were representative of that ante-bellum culture and refinement for which the old South was noted.

Mr. Théard's early education was obtained at the Jesuit College in New Orleans, supplemented by more extended studies at Spring Hill College at Spring Hill, Ala., from which institution he was graduated in 1881. His first activities were in commercial pursuits, but his aptitude for scientific study led him to seek a connection in the Engineering Department of the City of New Orleans, in 1893. He remained there until 1896, when he became identified with the office of the New Orleans Drainage Commission—first, as Draftsman, and later, as Assistant Engineer. In 1903 the New Orleans

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[1] Memoir prepared by a Committee of the Louisiana Section composed of John Klorer, J. H. O'Neil, and G. G. Earle, Members, Am. Soc. C. E., assisted by M. T. Ducros, Esq.

Drainage Commission was consolidated with the Sewerage and Water Board of New Orleans. In the transfer of forces, Mr. Théard was retained and later, in 1913, became Principal Assistant Engineer in Charge of Drainage. He held this post until January 1, 1934, when he was elevated to the position of General Superintendent of the Sewerage and Water Board of New Orleans.

His was the unusual privilege of participating actively, step by step, in the development of the unique system of artificial drainage in New Orleans from the time that the total capacity of the City's drainage pumping stations was rated at approximately 1 000 cu ft per sec, to the present high degree of efficiency represented by ten drainage pumping stations with an aggregate capacity of 25 000 cu ft per sec.

In addition to his engineering qualifications, Mr. Théard possessed talents as an architect which were recognized in the engagement of his services in that capacity by the U. S. Engineer Department to prepare plans for the completion of the Chalmette Monument in New Orleans on the site of the memorable and decisive victory of General Andrew Jackson over the British Forces on January 8, 1815. Subsequently, in 1919, he was engaged by the Louisiana-Vicksburg Park Memorial Commission to prepare plans for the design and erection of the Louisiana Memorial Monument in the National Military Park at Vicksburg, Miss.

He was one of the organizers and charter members of the Louisiana Engineering Society and served as President of the organization during 1902. He was also a member of the Governing Board of the Southwest Section of the American Water Works Association in 1938.

In 1908 Mr. Théard engaged in the successful effort to have the State Legislature pass a law regulating the practice of Civil Engineering and Surveying in Louisiana, the first State in the Union to have such a law.

In 1937 he was the recipient of the Veterans' Plaque from the American Public Works Association—"Awarded for long and faithful services, to Mr. Alfred F. Théard, who has served New Orleans continuously since 1893."

Of commanding presence and bearing, and apparently the picture of health, in spite of advancing years, his unexpected death came as a surprise and shock to his friends and associates. He had been at his desk, actively engaged in his official duties to within a few weeks of his death, dispensing those with a geniality and agreeableness for which he was noted, when a sudden heart attack, upon leaving the office after the day's toil was over, made him request to be taken home. He rallied sufficiently to return to his office for about a week, when a second similar attack ended fatally.

His long and useful career with the Sewerage and Water Board of New Orleans is testified to in the following tribute paid him by that Board:

> "In the death of Alfred Francis Théard the Sewerage and Water Board has suffered a loss of an outstanding official and citizen, whose value and worth cannot be measured in words.
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> "In the various relations of life, and especially in the great cause of the Sewerage and Water Board, and Civic Affairs, he brought to the performance of every duty, whether allotted to him, or voluntarily as-

> sumed, an earnestness of purpose which never wavered, and, at all times, the measure of his strength was the limit of his willingness.
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> "He was a true and sincere man in his intercourse with his fellowman. He knew of one rule of conduct and his even and uniform application was never affected by any influence of station or condition or mutation of fortune and was also marked and distinguished by that species of kindly courtesy, which seems born of a desire to serve.
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> "As we pay to his endearing memory this tribute of our love, our heartfelt sympathy goes out to those across whose path has fallen the heavy shadow of this great grief, and we pray that it may please a merciful Providence to lift up the light of His Countenance upon them and send to them that peace and comfort and consolation which passeth all understanding and which no human agency, however loving and sympathetic, can give."

He was married on October 17, 1887, to Marie Delvaille, of New Orleans, who, with two sons, Delvaille and Albert Théard, and a daughter, Mrs. Marie Théard Moses, survives him. He also left, as immediate members of his family, an elder brother and two sisters.

Mr. Théard was elected a Member of the American Society of Civil Engineers on March 2, 1915.

THOMAS HENRY TUTWILER, M. Am. Soc. C. E.[1]

Died September 3, 1938

Thomas Henry Tutwiler was born on September 22, 1866, in Palmyra, Va., the son of Thomas Henry and Caroline (Sloan) Tutwiler. He received his early education in the public schools and High School of Palmyra, and was tutored privately in Engineering.

He began his engineering career in 1882 when he was employed as Rodman by the Richmond and Alleghany Railroad Company, the Georgia Pacific Railroad Company, and several smaller companies, until 1886. The next two years he was with the Georgia Pacific Railroad Company (later, the Southern Railway Company) acting first as Assistant Resident Engineer on ten miles of construction in mountainous country; then as Transitman with a party that made more than 100 miles of finished railway location (much of the time in complete charge of the party); and then as Resident Engineer in charge of 10 miles of construction, including the Sunflower River Draw-Bridge.

In 1889, Mr. Tutwiler was employed by the Louisville, New Orleans and Texas Railroad Company (later, the Yazoo and Mississippi Valley Railroad Company) as Assistant Engineer in charge of 25 miles of location and construction, reporting to the Division Engineer. Following this engagement he served as Division Engineer in charge of 40 miles of construction, including track-laying, ballasting, depot buildings, and 150 miles of location.

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[1] Memoir prepared by James H. Haylow and W. F. Schulz, Members, Am. Soc. C. E., and Harry N. Howe, Assoc. M. Am. Soc. C. E.

In 1892, he became Assistant Engineer on the Yazoo and Mississippi Delta Levee Board, in charge of levee construction.

From 1893 to 1900, Mr. Tutwiler was with the New Orleans (La.) Traction Company, starting as First Assistant Engineer in charge of track construction, design of special track work, bridges, and other structures, and then as Chief Engineer in charge of construction and maintenance, both track and overhead, directing all designs.

In 1901, he was Superintendent of street railway construction at Birmingham, Ala., for Ford, Bacon and Davis, Engineers, of New York, N. Y. The following year he was Engineer at Kansas City, Mo., including work on the cable system for the same firm; and, in 1902, he was in charge of rehabilitating the Nashville Street Railway, at Nashville, Tenn.

Mr. Tutwiler was elected Vice-President and General Manager, and then President, of the Memphis (Tenn.) Street Railway Company, in 1905, during which time the entire system was reconstructed and brought to a high state of efficiency, which was continued throughout his tenure of office.

In 1923, he was elected President and Director of the Memphis (Tenn.) Power and Light Company. This system was in a run-down condition but he completely rebuilt it so as to provide sufficient capacity and good dependable service. One of his most outstanding achievements was the perfecting of the entire organization, every person of which held him in the highest respect and gave him full co-operation. He retired in 1930. The following tribute is quoted from the *Memphis Commercial Appeal*, of September 4, 1938: "His interest in community growth and welfare has been keen, intense, and sustained. No citizen stands higher in the community."

When the project of moving Southwestern University from Clarksville, Tenn., to Memphis took form, Mr. Tutwiler was one of its originators. In the initial campaign to raise the $500 000 needed to engineer the move, he was a tireless worker and executive, as he was in subsequent drives for funds for the institution. After his retirement from business, the affairs of Southwestern were his principal interest. Dr. Charles E. Diehl, President of Southwestern University, writes of him:

> "The day was never too long nor the night too dark for him to do something that might benefit Southwestern. Efforts in its behalf were the one thing he never ceased to attempt, even during his last illness. Whenever the need came, he was eager to drop everything else and devote himself to the school's interest."

Mr. Tutwiler was a member of the Tennessee Club, the Memphis Country Club, the Louisiana Engineering Society, and The Engineers Club of Memphis.

He was married, on December 19, 1894, to Mary E. Goodloe, who survives him. He also leaves two daughters, Mrs. Mary T. Langmead and Mrs. Zena Land, and one son, Thomas H. Tutwiler, Jr.

Mr. Tutwiler was elected a Member of the American Society of Civil Engineers on June 4, 1902.

CHARLES COURTLANDT WHITAKER, M. Am. Soc. C. E.[1]

Died August 3, 1938

Charles Courtlandt Whitaker was born in Brooklyn, N. Y., on August 19, 1877, the son of Charles Courtlandt Whitaker and Etta (Myers) Whitaker. He received his general and technical education in Brooklyn, and started in the engineering profession, holding increasingly important positions with such firms as Thompson-Starrett Company, Levering and Garrigues Company, Post and McCord, and Frank B. Gilbreth, all in New York, N. Y. His work consisted primarily of drafting, detailing, design, and supervision of construction.

From 1909 to 1923, Mr. Whitaker was employed by the Truscon Steel Company of Youngstown, Ohio. During his first year and a half with this company, he held the position of Sales Engineer, and for the remainder of his fourteen years with the company, he was Manager of Sales for the Atlanta, Ga., office, covering the Southeastern territory. During this time he designed, or supervised the design, of many of the important reinforced concrete structures in the Southeast, including schools, hotels, court houses, bridges, factory buildings, and hospitals.

From 1923 to 1938, Mr. Whitaker operated his own business as a Consulting and Designing Engineer, preparing plans, and specifications and supervising construction from his own office in Atlanta. Among his best-known works are the following buildings: 9-story Hicks Hotel and Carnegie Library in Savannah, Ga.; Wells Theater in Augusta, Ga.; 18 bridges for Hillsboro County, Florida; 172-ft Booker Creek Bridge and Don ce Sar Hotel (500 rooms) in St. Petersburg, Fla.; Supreme Court Building in Tallahassee, Fla.; National Paper Company and Scripto Manufacturing Company plants, and buildings for Montag Brothers, Inc., in Atlanta.

He was married on March 16, 1911, to Anna May Gregory of Windsor, Ont., Canada, who survives him. They had two sons, Charles Courtlandt and John, both of whom survive Mr. Whitaker.

His principal hobby was his business, which he thoroughly enjoyed, although in his youth he was quite an enthusiast in sail-boating, and also won a number of prizes in bicycle racing in and around New York. In his later life he was very fond of golf and motoring.

During his lifetime, he served on all committees of the Georgia Section of the American Society of Civil Engineers, and also held all offices, including Secretary, Vice-President, and President. He was active in the Atlanta Athletic Club and the Atlanta Lions Club, and was a director of the Atlanta Motor Club, at which time he served on the traffic survey of Atlanta, later being elected to a membership on the National Commission of Traffic Survey. He was an honored and respected citizen of Atlanta, where he had many professional and personal friends.

Mr. Whitaker was elected a Member of the American Society of Civil Engineers on January 14, 1924.

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[1] Memoir prepared by Robert S. Fiske, Atlanta, Ga.

FRIEND PITTS WILLIAMS, M. Am. Soc. C. E.[1]

Died February 20, 1939

Friend Pitts Williams was born at Richmond Mills, N. Y., on December 18, 1875, the son of Allan Irving Williams and Emily (Barton) Williams. His early education was received at the Olean High School at Olean, N. Y. He entered Cornell University, at Ithaca, N. Y., in 1895, and was graduated from that institution in 1899 with the degree of Civil Engineer.

For four years, beginning June, 1899, Mr. Williams was employed in the Engineering Department of the Pittsburgh, Shawmut and Northern Railroad Company as Assistant and Resident Engineer on design, location, and construction. His work covered 50 miles of road in New York and Pennsylvania.

In June, 1904, he was appointed Assistant Engineer in the Department of the State Engineer and Surveyor of the State of New York. In this capacity he was engaged on highway work in Westchester County and in the designing department of the New York State Barge Canal.

Mr. Williams was promoted to Resident Engineer in June, 1906, and had charge of location surveys at Lyons, N. Y., construction of five locks and dams on the Mohawk River near Amsterdam, N. Y., and the dredging and construction of various works along the 30 miles of Hudson River near Mechanicsville, N. Y. In April, 1914, he was appointed Division Engineer, in charge of the Western Division of the department, with headquarters in Rochester, N. Y. There he had charge of the work on some 140 miles of canal, including the design and construction of the Bushnell Basin Trough at Rochester Harbor.

Mr. Williams was appointed Special Deputy State Engineer in January, 1919, with headquarters at Albany, N. Y. In this capacity he had charge of the work on the entire canal system. The principal construction during his administration was in connection with Barge Canal terminals and particularly with the grain elevator at Gowanus Bay, in New York, N. Y. On completion of the canal work, in 1921, this position was abolished.

In May, 1921, Mr. Williams was appointed Engineer-Secretary of the newly created Water Power Commission and served with that Commission and its successor, the Water Power and Control Commission of the State of New York, for the remainder of his active life. In that capacity he organized the work of the Water Power Commission and, in connection with the activities of the Commission, made extensive studies of water-power matters connected with the Niagara and St. Lawrence rivers and the effect of the Chicago Drainage Canal diversion on these rivers. He collected and presented material for important litigation undertaken during that period. Ill health compelled his retirement from the State service in November, 1938.

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[1] Memoir prepared by R. Suter and Ernest D. Hendricks, Members, Am. Soc. C. E., and John P. Newton, Assoc. M. Am. Soc. C. E.

In 1906 Mr. Williams was married to Alma Horton. She died in 1927. Two daughters, Grace and Emily L. Williams, survive.

Mr. Williams was a member of the Albany Section of the New York State Society of Professional Engineers, which he assisted in organizing, and of the Albany Society of Civil Engineers of which he was President in 1927. He was a member and Director of the University Club of Albany; a member of Ondawa Lodge, F. and A. M., Mechanicsville; of Beta Theta Pi Fraternity; and of the First Presbyterian Church of Albany.

He possessed the attributes of a true gentleman in the strictest sense. He was a conscientious and capable engineer. Ever loyal to his superiors and considerate of his subordinates, it may be said truly that all with whom he had professional and social contacts were his friends and held him in unusually deep respect and esteem.

Mr. Williams was elected a Member of the American Society of Civil Engineers on July 1, 1909.

CHARLES BEATTY WILSON, M. Am. Soc. C. E.[1]

Died April 8, 1938

Charles Beatty Wilson was born in York County, South Carolina, on September 6, 1871. He was the son of James and Mary (Williamson) Wilson. His father, the son of Presbyterian missionaries, was born in Allahabad, India. Mr. Wilson was brought to Nashville, Tenn., when he was a very young boy and lived there until three years before his death, when he moved to Honolulu, Hawaii, where he passed away on April 8, 1938. He attended Montgomery Bell Academy, and, later, Vanderbilt University, at Nashville, where he specialized in Civil Engineering.

In 1889, he was employed by the Nashville, Chattanooga, and St. Louis Railroad Company, and was with that line for eighteen years. During this time he was engaged in maintenance and construction work—one project being a bridge over the Tennessee River, at Johnsville, Tenn.—and rose to the position of Assistant Chief Engineer.

In 1907, Mr. Wilson was made Assistant Engineer for Davidson County, Tennessee, during the construction of the Sparkman Street Bridge over the Cumberland River. In 1909, he entered the contracting construction field with the Foster-Creighton-Gould Company. While with that Company he handled many large bridge projects, including the substructure work on the bridge across the Ohio River, between Louisville, Ky., and New Albany, Ind.; The Louisville and Nashville Railroad Bridge over the Cumberland River, at Nashville; and the Apalachicola and Choctawatchee draw-bridge in Northern Florida.

Mr. Wilson became Vice-President of the Gould Contracting Company in 1915, and in 1917 he was elected President, which office he held until his retirement from active business in 1934. This Company was one of the outstanding builders of the South, and, under Mr. Wilson's supervision, constructed sixteen bridges over the Cumberland River, one of the most important of which was the $800 000 highway bridge between Nashville and Old Hickory, Tenn. The Company also built bridges for the Louisville and Nashville; the Nashville, Chattanooga, and St. Louis; and the Pennsylvania Railroad Companies. In addition, it constructed power plants, dams, and reinforced concrete buildings.

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[1] Memoir prepared by Arthur J. Dyer, M. Am. Soc. C. E.

In 1903, Mr. Wilson was married to Maiella Byrns, of Nashville, sister of the late Speaker of the House of Representatives, Joseph W. Byrns. His widow and one daughter, Mary Elizabeth, survive him.

He was active in religious, fraternal, and civic circles in Nashville, having been an Elder in the Presbyterian Church, a Mason, a Shriner, a Son of the American Revolution, a member of the Rotary Club, and a member of the Chamber of Commerce. One honor of which Mr. Wilson was particularly proud was his life membership, attained by virtue of long membership, in the Society.

He was known everywhere as a man of high ideals and unswerving integrity. Never strong physically, he rose through his own efforts and ability to a place of highest respect in his profession.

Mr. Wilson was elected a Member of the American Society of Civil Engineers on December 2, 1903.

GEORGE LEVERETT WILSON, M. Am. Soc. C. E.[1]

Died April 1, 1939

The death of George Leverett Wilson on April 1, 1939, terminated an unusually long and useful engineering career. For fifty-six years he had been actively associated with the growth and development of the Northwest, and particularly with the development of transportation facilities in St. Paul, Minn., and Minneapolis, Minn., and the territory surrounding these two cities. At first he was associated with the building of railroads in Iowa, Minnesota, and the Dakotas, next with the construction of paved streets in St. Paul in the horse-and-buggy days, then with the development of many miles of bicycle paths in St. Paul in the nineties, and finally, for more than a third of a century, with the electric railway trackage in the Twin Cities and their suburbs. At the time of his death, Mr. Wilson was the oldest living past-president of both the Engineers Club of Minneapolis and the Engineering Society of St. Paul.

George Leverett Wilson was born at West Killingly, Conn., on February 14, 1855. He was the son of Seth A. and Ellen M. (Lee) Wilson. He was graduated from Sheffield Scientific School of Yale University, in New Haven,

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[1] Memoir prepared by Frank S. Altman, Assoc. M. Am. Soc. C. E., and O. F. Moore, Engr., Maintenance of Way Dept., Twin City Lines, Minneapolis, Minn.

Conn., in 1877, taught school for two years, and spent one year in post-graduate study at Yale. He went west in 1880, and was employed as a Transitman and Resident Engineer in the Chicago, Rock Island and Pacific, the Chicago and Northwestern, and the Minneapolis and St. Louis Railway Companies. From 1881 to 1883, he was in charge of construction of the Devils Lake extension of the St. Paul, Minneapolis and Manitoba Railway (Great Northern). In 1884 he began his service with the City of St. Paul, first as an engineer with the water department, making surveys and constructing a masonry conduit for the new water supply for that city. The following year he was appointed Assistant City Engineer under the late L. W. Rundlett, M. Am. Soc. C. E., for many years City Engineer of St. Paul. He continued with the city engineering staff until 1903, holding the position of Chief Sewer Engineer during a great part of this period. Thus he had much to do with the engineering development of the city during its period of greatest growth.

In June, 1903, Mr. Wilson started work with the Twin City Rapid Transit Company as Engineer of Maintenance of Way, and remained with the street railway companies until his death. His active service ended June 1, 1936. During this time of service he supervised the construction and reconstruction of many miles of street railway tracks in Minnesota, including St. Paul, Minneapolis, South St. Paul, Stillwater, Lake Minnetonka, and White Bear Lake. Of the more permanent structures built under his supervision, the most notable is the Selby Hill tunnel in St. Paul, built in 1907. During Mr. Wilson's service with the company, the street railway system grew from 250 miles in 1903 to 524 miles of single track at the peak in 1931. This has been reduced since to 460 miles of single track.

Mr. Wilson was married to Fannie M. Scranton at Danielson, Conn., in 1884. One daughter, Ruth E. Wilson, was born to them.

Mr. Wilson was a thirty-second degree Scottish Rite Mason, a Knight Templar, a member of the Minneapolis Rotary Club, the Six O'Clock Club, the Engineers Club of Minneapolis, and the Engineers Society of St. Paul.

He was one of those rare individuals whom you meet far too seldom in the engineering profession—always friendly and sympathetic with younger members of the profession and held in the highest regard by associates and older fellow engineers.

Mr. Wilson was elected a Member of the American Society of Civil Engineers on February 5, 1890.

JOSEPH FRANKLIN WITMER, M. Am. Soc. C. E.[1]

Died May 23, 1938

Joseph Franklin Witmer was born in Williamsville, N. Y., on November 25, 1859, the son of Tobias Witmer and Anna (Frick) Witmer. He received his early education in the public schools of Buffalo, N. Y., and was graduated from the New York State Normal School, at Buffalo. He then entered Bethany College, at Bethany, W. Va., from which he was graduated in 1884, with the degree of Bachelor of Arts. He received his engineering education at Rensselaer Polytechnic Institute, at Troy, N. Y., and was graduated in 1887, with the degree of Civil Engineer.

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[1] Memoir prepared by C. W. Mower, Esq., Akron, Ohio.

From 1887 to 1890, Mr. Witmer engaged in the construction of water-works at Elgin and Sycamore, Ill., Mount Clemens, Mich., Rogers Park and La Grange, Ill., and an intake into Lake Michigan for the water-works of Escanaba, Mich. In this period he designed during the winter and superintended the construction in summer. He designed the water systems of Washington Heights, Ill., Ingersoll, Kincardine, London South, and Galt, Ont., Canada, and assisted in the preparation of the plans for the Peoria, Ill., Water-Works. In 1890, he was appointed Principal Assistant to the Chief Engineer, on the construction of water-works, at Seattle, Wash.; and, in 1891, he was made Resident Engineer in charge of construction of the water-works.

From 1892 to 1895, Mr. Witmer was a member of the firm of Vorhees and Witmer, Civil Engineers, with offices at Buffalo. During that period he served for a time as engineering member of the Civil Service Commission of the City of Buffalo, by appointment of the Mayor.

During the fifty-one years of his professional life, Mr. Witmer was identified with the design and construction of the water-works and sewerage systems serving four nations, nine States, and the Philippine Islands, and more than one hundred cities and communities. He designed and supervised the construction of water-works systems at Morris and Pana, Ill.; Greenfield, Tipton, Rochester, Columbia City, and La Grange, Ind.; Celina, Montpelier, and Ironton, Ohio; Dansville, Windsor, De Ruyter, Quogue (Long Island), Lima, Prattsburg, Clifton Springs, Suffern, Horseheads, Warsaw, Mexico, Naples, Perry, Craig, Epileptic Colony at Sonyea, the New York State Custodial Asylum, at Newark, N. Y.; Williamsville, Alden, Akron, Bolivar, Angelica, Silver Creek, Livonia, Caledonia, Allegany, Belfast, Cherry Creek, and Argola, N. Y.; and, at Cambridge Springs, Edenborn, Grove City, and Saegerstown, Pa.

His foreign engagements led Mr. Witmer to the Philippine Islands to make an estimate of the cost of the construction of a water-works and sewerage system for the City of Manila, and to Russia, for an English firm, to investigate and report on a proposed water-works system for Vladivostok, Siberia. In 1910, he was engaged by a London and Paris syndicate, to investigate and report on a project for supplying water to the City of Taltal, Chile, and for supplying the nitrate fields of Central Chile, with water from the Andes Mountains, through a conduit more than a hundred miles long. He investigated and made a comprehensive report for a New York Company on the sewerage project for Edmonton, Alberta, Canada.

Mr. Witmer was prominently identified with the engineering connected with the construction of the Buffalo and Williamsville Electric Railroad.

In evaluation and appraisal work, he established a national reputation, having appraised water-works systems for Jamestown, Salamanca, Corning, Medina, Suffern, Bolivar, Mount Morris, and Depew, N. Y.; Billings, Mont.; Meadville, Pa.; and the Huntington Water-Works, Huntington, W. Va.

Mr. Witmer was a member of the American Water Works Association. He was also a member of the following fraternities; Beta Theta Pi and Sigma Xi, and Ivy Lodge F. and A. M.

He was liberal in his views and noted for his fairness and ethical dealings in his engineering contact with others. He made and held many intimate associates throughout his life.

On October 8, 1924, Mr. Witmer was married to Minnie M. McCollom. He died in Elmira, N. Y., on May 23, 1938, and is survived by his widow.

Mr. Witmer was elected a Member of the American Society of Civil Engineers on May 4, 1904.

FRANK CHARLES WOLFE, M. Am. Soc. C. E.[1]

Died April 2, 1939

Frank Charles Wolfe was born at Union Bridge, Md., on December 7, 1870, the son of Daniel and Susan E. (Haines) Wolfe.

His education was begun in the public schools of Maryland, and was terminated by an engineering course at Cornell University, at Ithaca, N. Y., from which he was graduated in June, 1895, with the degree of Civil Engineer. For the years 1891 to 1895, inclusive, he was the winner of the Sibley Scholarship, an award made annually under a gift of the late Hiram Sibley for excellence of scholarship in the Engineering Schools of Cornell.

Shortly after his graduation, in 1895, Mr. Wolfe entered the employ of the American Bridge Company, at Chicago, Ill., as draftsman, and remained in that position until 1899, when he resigned, and entered the service of the Scherzer Rolling Lift Bridge Company, as designer, at Chicago. While thus employed he designed the foundations and superstructures for about one hundred movable lift bridges, with clear spans ranging in width from 20 ft to 160 ft.

In 1903 he became Assistant Bridge Engineer of the Chicago, Rock Island and Pacific Railway Company. During his four years of service with this line his duties comprised responsibility for the design of superstructure and substructures of all bridges built within the period concerned. In 1907 he was appointed Assistant Bridge Engineer of the Northern Pacific Railway Company, charged with the design of substructures and superstructures for all bridges. In 1909 he was offered the position of Chief Engineer, Maintenance of Way Department, for the Northern Pacific Railway Company, with headquarters at Denver, Colo., but, due to the death of Mrs. Wolfe and the special arrangements which he was forced to make with respect to his family, he was unable to accept this appointment. During 1909 he became associated with Mr. D. B. Banks, as structural expert, and, in that capacity, designed the power house for high pressure fire line for the City of Baltimore, Md.

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[1] Memoir prepared by P. G. Lang, Jr., M. Am. Soc. C. E.

In 1911 Mr. Wolfe was appointed Bridge Engineer, Western Maryland Railway Company, with headquarters at Baltimore, and, during his two years' administration of that post, exercised responsible charge of the design and construction of all bridges involved in the project then under way between Cumberland, Md., and Connellsville, Pa. His duties also included supervision of bridge maintenance on the entire Western Maryland System, and water-front work, comprising piers, bulkheads, and transfer bridges.

He was appointed Chief Engineer of the McLean Contracting Company, of Baltimore, during 1913, and assumed charge of construction projects totaling about $1 000 000, including piers, bridges, dredging operations, and railroad construction. In 1915 he left the service of that firm to become Resident Engineer on the Stratford Avenue Bridge, Bridgeport, Conn., as representative of The J. E. Greiner Company, Consulting Engineers. In 1919 he resumed his post as Chief Engineer of the McLean Contracting Company, which he continued to administer until 1924. During this post-war period of five years he supervised projects totaling about $8 000 000, involving the Long Beach Bridge on Long Island, New York; the Western Maryland Railway Export Coal pier, at Baltimore; the Schuylkill River Bulkhead, Chestnut Street Pier, Substructure for Girard Pier 5, and Concrete Piling for Pier 84, Philadelphia, Pa.; and the South Street Bridge at Wilkes-Barre, Pa.

In 1924 Mr. Wolfe became Secretary-Treasurer and Chief Engineer for B. F. Johnson Company. After four years in this position he resigned and entered private practice as a Consulting Engineer, with headquarters at Baltimore. In that capacity his services were retained by H. K. Ferguson and Company, The Empire Construction Company of Baltimore, the Western Maryland Railway Company, the County of Nassau, New York, and others. For H. K. Ferguson and Company he designed and supervised the construction of the bulkhead at the plant of the Western Electric Company, at Baltimore; and for Nassau County, New York, he designed and supervised the prosecution of extensive repairs to the Long Beach Bridge and the construction of the Bayville Bridge. The termination of the latter project was nearly coincident with that of Mr. Wolfe's long and useful career, since the final papers were signed February 27, 1939, and his death occurred April 2 of that year.

On May 12, 1898, Mr. Wolfe was married to Wealthy Ann Burr, who died on February 1, 1909. The surviving children are Mrs. C. E. Whitmore, and Mrs. K. B. Pittinger. A son, Charles, was a Junior in the Engineering School at Cornell University, when he was drowned in the Delaware River, Philadelphia, in January, 1923.

Mr. Wolfe was a member of the Engineers Club and the Press Club of Baltimore, and of the Cornell Club, of New York, N. Y.

Mr. Wolfe was elected a Member of the American Society of Civil Engineers on January 31, 1911.

CARMI IRVING BAUSHER, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 5, 1938

Carmi Irving Bausher was born in Reading, Pa., on November 14, 1869, a son of DeWalt P. Bausher and Ellen M. Bausher. He was graduated from the Reading High School in June, 1888, and from that time until 1896 was employed as a Structural Draftsman by Coffrode and Saylor, of Pottstown, Pa.; Phoenix Bridge Company, of Phoenixville, Pa.; the Reading Rolling Mill Company; and the Elmira (N. Y.) Bridge Company.

From 1896 until his retirement on August 1, 1932, he was employed by the Pennsylvania Steel Company, in the Bridge and Construction Department, at Steelton, Pa., and finally with the Fabricated Steel Construction Department of the Bethlehem Steel Company, Bethlehem, Pa.

He became a skilled draftsman and engineer of wide experience in structural steel work, his services having been employed in such capacity in connection with buildings and bridges of all types, among which the following structures are particularly outstanding: Double-Track, Double-Leaf Scherzer Rolling Lift Bridge over Taylor Street, Chicago, Ill.; the Williamsburg and the Queensboro bridges over the East River, New York, N. Y.; the Harahan Bridge over the Mississippi River at Memphis, Tenn.; and the Lakehurst Hangar, Lakehurst, N. J.

Mr. Bausher possessed the ability to conceive the requirements from the engineering standpoint, and to apply thereto the knowledge obtained from years of close association with shop practice. He was interested to a marked degree in the training of the younger draftsmen and engineers who knew him as a friend to whom they could turn at all times for counsel and guidance in engineering matters with the confidence that his advice would be given cheerfully in a most helpful and constructive manner.

He was married to Bertha Virginia Lindecukle on June 28, 1894, and is survived by his widow and two sons, George Irving and Richard Vastine.

Mr. Bausher was elected an Associate Member of the American Society of Civil Engineers on December 7, 1898.

JOHN ALLEYNE CALHOUN BOGART, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 1, 1939

John Alleyne Calhoun Bogart was born in Savannah, Ga., on July 15, 1904. His Canadian father, a sea captain running into Savannah, retired soon after John's birth on account of his wife's health, and the family moved to New Brunswick, Canada. The father took part in the development of the western provinces of Canada, at one time raising wheat in Saskatchewan. As a high school student in Edmonton, Alberta, John was active as a Boy Scout, reaching the rank of Rover. His family removed to Victoria, British Columbia, about 1928, where his mother, Henrietta (Calhoun) Bogart, still lives (1939).

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[1] Memoir prepared by Charles H Mercer, M. Am. Soc. C. E.

[1] Memoir prepared by Charles Carroll Brown, M. Am. Soc. C. E.

The presence of an uncle, Captain William D. Bogart, doubtless aided John to make Florida his home. In Tampa, Fla., his employment by an engineer in laying out a large subdivision was the persuasive factor in his becoming a civil engineer, and he entered the University of Florida, at Gainesville, as a special student until he completed his high school work. With some aid from his uncle during the first two years of college work he sustained himself as working students do, usually, and by acting as an undertaker's assistant.

On December 29, 1929, he was married to Ruby Knight, of Florida, who was of great assistance to him during his undergraduate days and his brief practice of Civil Engineering.

He received the degree of Bachelor of Science in Civil Engineering in 1931. With the aid of a position as graduate assistant, and with employment in research work on the new sewage treatment plant at the University, and on the construction and early operation of the new sewage treatment plant of Gainesville, he was able to continue his studies and practical application of them and to earn the University degree of Master of Science in Civil Engineering in 1933, and that of Civil Engineer in 1934.

In 1933 and 1934, as a member of the firm of Brown and Bogart, he was engaged in developing methods of operation of the new sewage treatment plant at Gainesville, in the design of a new sewage treatment plant for Tallahassee, Fla., and in formulating satisfactory methods of operating the three plants mentioned. The firm of Brown and Bogart also prepared several plans for water supplies for small cities and towns in Florida, which extended his practice into that field. The financial abilities of the municipalities did not prove equal to the requirements of Public Works Administration (PWA), and Mr. Bogart took a position as Director of Operation in Bradford County, Florida, under Emergency Relief Administration (ERA) and Civil Works Administration (CWA), and later in Hillsborough and Walton Counties. In 1935 he designed water and sewage collection and distribution systems for Key West, Fla., under relief agencies, followed by supervision of the work until it was intermitted. In May, 1936, he received an appointment as Lieutenant (jg) C.E., USNR. He worked as Assistant Engineer at the Navy Yard in Charleston, S. C., in charge of extensive mosquito control work and as hydraulic engineer compiling data regarding the feasibility of the Santee-Cooper River canal.

On October 1, 1937, he was appointed City Manager, Auditor, and Clerk of the City of Fernandina, Fla. During his term of office municipal highway improvements amounting to $15 000 were completed, an $80 000 municipal dock was constructed under PWA, and a PWA mosquito control project was begun. Many changes in municipal procedure and policy were instituted under his administration.

Mr. Bogart's illness developed during this period and culminated on March 1, 1939, at the residence of his aunt, Mrs. Ida Bogart, in Fernandina.

John A. C. Bogart was a keen student, a careful worker, an enthusiastic research worker, and an honest man, constant in upholding the ethical principles of the engineering profession. Notwithstanding the broken nature of his early education, he developed rapidly when he made his choice of a profession, acquiring the methods of study, observation, and self-education in the University of Florida which showed his innate ability to find the important ideas behind any work in which he was engaged, and to develop effective methods of gathering information and using it in making practical solutions of problems confronting him. His most promising career was cut short in its beginning with undoubted loss to the engineering profession and its practice.

Mr. Bogart was christened in the Protestant Episcopal Church, and was a member of the Christian Science Society, at Gainesville.

His immediate relatives at his death were his wife, Ruby Knight Bogart, his mother, Mrs. Frank C. Bogart, a brother, Charles F. Bogart, and two sisters, Mrs. Ian McGavin and Miss Rachael Bogart.

John A. C. Bogart was a member of the Student Chapter, American Society of Civil Engineers, in the University of Florida, and was admitted as a Junior after his graduation on October 26, 1931. He was elected an Associate Member on January 14, 1936.

FORREST McKENDREE CASEY, Assoc. M. Am. Soc. C. E.[1]

Died March 19, 1938

Forrest McKendree Casey was born on October 27, 1884, in Oak Harbor, Ohio. His parents were McKendree A. and Clara E. (Rowe) Casey, of Toledo, Ohio. He attended the Grammar and High Schools in Toledo, and the John B. Stetson University, at DeLand, Fla.

From 1906 to 1909, Mr. Casey was employed as Draftsman in the Office of the City Engineer of Seattle, Wash., where he designed plans for paving, sewers, and street extensions. From 1910 to 1918 (except as noted subsequently), he was employed as Draftsman, and after May 16, 1916, as Assistant Engineer, by the City Engineer of Los Angeles, Calif., in designing storm drains, making paving and street plans, as well as studies and reports on drainage.

On July 1, 1916, Mr. Casey was one of the organizers of a new regiment of Coast Artillery in the National Guard of California, and enlisted in that regiment as a Private in the 15th Company. Six days after war was declared with Germany, in 1917, this regiment was called into Federal service and entrained immediately for Fort Rosecrans, San Diego, Calif. On July 1,

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[1] Memoir prepared by W. T. Knowlton, M. Am. Soc. C. E.

1917, Mr. Casey's hearing was impaired, and he was given a discharge from the United States Army for disability.

From June, 1918, to June, 1921, he was employed by the General Petroleum Corporation of Los Angeles, at its Vernon, Calif., refinery, in the design of apparatus, structures, and pump-houses, and in the standardizing of field equipment. From July, 1921, to June, 1923, he served as Junior Designer in the Engineering Department of the Southern California Edison Corporation, working in its hydro-electric power plant on general hydraulic and structural design, also plant assembly and penstock calculations, and studies of surge tanks and tunnels, as well as the design of special tunneling machines and equipment for testing materials.

From June, 1923, until his death on March 19, 1938, Mr. Casey was again employed by the City Engineer of Los Angeles as Junior Civil Engineer on storm-drain design, and general engineering computations and estimates. From August 16, 1924, he served as Assistant Engineer in charge of storm drain design and was responsible for the solution of construction difficulties and changes in design and detail.

He was a member of the Sunshine Post No. 251 of the American Legion from the time of its organization, and at his death he was a member of the Executive Committee and Historian of this Post. He was also a member of the Clarence F. Smith Masonic Lodge.

Mr. Casey was always interested in helping young engineers by passing on to them those things which he had learned by experience and hard work, even if, later, it might prove to his own disadvantage. He was happy to have lived to see the Engineering Profession take its rightful place in the business and professional world. Although never rugged in physique, he had a clear sense of humor and had many interests in life. Among his several hobbies he enjoyed fencing most of all.

He was married on June 29, 1907, to Helen Fahs, in Renton, Wash., who survives him, together with his brother, Cranston A. Casey.

Mr. Casey was elected an Associate Member of the American Society of Civil Engineers on May 25, 1931.

JOHN ANTHONY CROOK, Assoc. M. Am. Soc. C. E.[1]

Died August 9, 1937

John Anthony Crook was born on April 9, 1879, at Falls City, Nebr., and was the eldest child of William Hampton Crook and Lydia (Worley) Crook.

His earlier education was received in the public schools at Falls City. After graduating from the high school, he attended the University of Nebraska at Lincoln, Nebr., where he studied engineering.

He began engineering work in 1897 as estimator, designer, and contracting engineer in the Omaha (Nebraska) Office of the Canton Bridge Company, which was engaged in the building of highway and railway bridges. He remained in this position until 1900 when he became associated with the John Gilligan Company of Falls City as Chief Engineer and Secretary of the Company, which was engaged in general contracting and engineering work in connection with the design and construction of public works.

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[1] Memoir prepared by Oliver W. Childs, M. Am. Soc. C. E.

Concurrently with other work, Mr. Crook was County Engineer of Richardson County, Nebraska, from 1903 to 1910, and was City Engineer of Falls City from 1908 to 1912.

In 1908 he organized the Monarch Engineering Company of Falls City and engaged in the practice of civil engineering and in general contracting. This business was developed and extended, with branch offices in Kansas City, Mo., Oklahoma City, Okla., and Denver, Colo., and included the designing and construction of bridges, buildings, roads, pavements, sewers, water-works, and public improvement in general. As the business developed mainly along the lines of bridges and buildings, structural steel was an important item, and in 1916 Mr. Crook acquired a structural steel shop in Denver, and started the operation of the Denver Steel and Iron Works Company as a structural steel fabricating plant. At this time he moved to Denver and continued to reside there and to operate the two associated companies until his death.

When reinforced concrete came into use, Mr. Crook spent considerable time investigating the design and use of concrete as a structural material and studying the comparative value of concrete and steel construction. He became a firm believer in the superiority of steel construction and was very active in advocating and promoting its use in preference to concrete. He was among the first to use steel piles for foundations and made use of them in a large number of highway bridges before they were generally recognized by engineers as legitimate and economical construction.

In addition to his work in engineering and contracting, Mr. Crook had many and varied interests, among which were livestock raising and farming. He owned a number of farms, among which was the Pine Bar Ranch near Denver, to which he paid particular attention. He installed all modern improvements necessary to make it a model ranch, and raised pedigreed stock according to the most approved practice. He also owned or was interested in the operation of a number of mines. He took an active interest in public affairs, but was not a political partisan.

Mr. Crook had unusual ability as a public speaker, and could be depended on to make an interesting address on any subject in which he was interested, or on any subject of current public interest, whenever there was occasion to do so.

His various business interests did not prevent him from being a reader and student. He studied carefully any subject that interested him, analyzing it before making conclusions, and was able to give reasons in support of his ideas. He was a Mason, a Shriner, an Elk, and a member of the Denver Athletic Club. His religious affiliations were with the Protestant Episcopal Church.

In addition to his engineering training and experience, he had a proper balance of other information, which enabled him to consider matters from their various viewpoints and qualified him as a successful financier and business executive.

He was married on June 4, 1899, to Minnie Dalbey of Coin, Iowa, at Chillicothe, Mo., and is survived by his widow and a daughter, Constance Crook Ambrose.

Mr. Crook was elected a Junior of the American Society of Civil Engineers on February 3, 1903, and an Associate Member on December 1, 1908.

NOAH CUMMINGS, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 13, 1939

Noah Cummings, the son of James Daniel Cummings and Nancy Ransom (Hubbell) Cummings, was born in Middlefield, Otsego County, New York, on January 14, 1874. On his mother's side he traced his ancestry to Richard Hubbell, born in 1628 in Great Britain, who emigrated to Connecticut in 1648, and to Robert Ransom who emigrated from Kent, England, to Massachusetts in 1654. Thomas Ransom, fifth in line from Robert, served his country in the Revolutionary War. Mr. Cummings was one of the few remaining Americans to be born in a log cabin. He was the fourth of six children. His parents were poor struggling farmers, and early in life he helped in the simpler duties on the farm. At the age of six he learned to milk cows and at eight he was driving horses.

He began his schooling in a one-room, one-teacher school and during that period, at the age of seven, his seventeen year old teacher told him: "You know more arithmetic than I do."

When he was ten years old his parents were living on a farm 3 miles from Schenevus, N. Y., and Noah had the opportunity to enter the High School in that village. He walked the 3 miles to and from the school in all kinds of weather. In that school he completed his grade school as well as his high school subjects and made such progress that he graduated at the age of fourteen.

He had decided on an engineering course at Cornell University, at Ithaca, N. Y., but was confronted with a minimum 16-yr age requirement. He spent the next two years taking post-graduate work at the Schenevus High School to such good advantage that he not only secured a State Scholarship, which provided his tuition at Cornell, but also passed a highly competitive examination at the University which entitled him to the John Stanton Gould Scholarship in mathematics, which he held during his four years at the University. It is evidence of high scholarship that he was elected to the Sigma Xi Society in his Senior year.

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[1] Memoir prepared by a Committee of the Metropolitan Section consisting of Thomas S. Clark, Reginald H. Keays, John A. Knighton, Members, Am. Soc. C. E., and Isidore Delson, Assoc. M. Am. Soc. C. E.

Although he had a keen sense of humor and was an excellent companion, he always held to the serious things of life. During his college days he was a member of the Cornell University Christian Association, Secretary of Section B of the Mathematical Club, Class Treasurer during his Junior year, and Chief Engineer of the Senior and Junior classes engaged in geodetic surveys in his Senior year. He was graduated in 1894 with the degree of Civil Engineer.

One of his classmates says of him: "At college every one looked up to him, not only for his ability, clear brain and brilliancy in his engineering courses, but for his really lovable nature."

Like most young engineers, Mr. Cummings found employment in several minor capacities before entering upon what was to prove his final life work.

From July, 1894, to April, 1895, he was employed by the Aqueduct Commission of the City of New York, first under the title of Laborer and then as Axeman. It was while thus engaged that he met the late Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E., and there began a friendship that lasted until Mr. Ridgway's death.

From April, 1895, to April, 1896, Mr. Cummings was employed by J. W. Hoffman and Company, Contractors, on the foundations for a steel highway bridge over the Delaware River between Easton, Pa., and Phillipsburg, N. J., and on a dam for the water supply of Cold Spring, N. Y.

In April, 1896, he began working for the City of New York and continued in such employ until his death. From the time he entered the City's service until August, 1901, he was employed as Transitman and Computer and later as Assistant Engineer in the Topographical Bureau of the Board of Public Improvements in the Bronx.

Although his connections with that Board were not severed, Mr. Cummings was absent during the Spanish-American War. In June, 1898, he enlisted as a Private in the First United States Volunteer Engineers, Company C, and saw service in Puerto Rico. The peace protocol was signed while the regiment was en route to Puerto Rico so that he had little actual military service. One of his comrades relates that about one week after arrival, tents were pitched alongside the Rio Portuguesa near Ponce and he and Noah were assigned to take cross-sections of the river opposite the camp. The site of the camp was unusual in that the alluvial plain alongside the river was somewhat higher than the camp ground back of it and some of the officers conceived the idea of cutting drainage ditches perpendicular to the river to drain the ground back of the camp. This was done. The river was a mere trickle and the sun was shining, so the surveyors discarded much of their clothing and were wading in the river-bed when word came from up stream that a flood was on its way. Soon a wave came foaming down the river like a wall of water. The channel was immediately filled to the top of the banks, also the lateral ditches, flooding the back of the camp. The ditches were then stopped up as well as possible with bales of hay. Those who knew Noah Cummings can almost hear the characteristic, "Oh, my gosh," and the familiar laugh, as the waters came rushing down.

Private Cummings received his honorable discharge in 1899, but like so many others he returned to the United States with health impaired and it was some months before he was able to resume his duties with the Board of Public Improvements in the Bronx.

On August 1, 1901, Mr. Cummings entered the Department of Bridges as Bridge Draftsman. At that time plans were being developed for Bridges No. 3 and No. 4, later known as the Manhattan and Queensboro Bridges. Mr. Cummings' first work was on the plans of Bridge No. 4. He advanced to the grade of Assistant Engineer and for several years was in charge of surveys for the Department.

In 1916 the Department of Bridges became the Department of Plant and Structures. Some time before that Mr. Cummings was brought into the Chief Engineer's office where he was to remain so many years. There, in his contacts with other departments, he acquired a knowledge of city government surpassed by few.

In 1935 he became Deputy Chief Engineer of the Department of Plant and Structures with increased responsibilities. Much of the official correspondence of the Department went through his hands. In preparing a reply to an unfair or a bitter communication his answer would be courteous. Often he would say: "A soft answer turneth away wrath." On January 1, 1938, when the revised New York City Charter became effective, the functions of the Department of Plant and Structures, other than the ferry service, were taken over by the new Department of Public Works. As the Department was finally organized on July 1, 1938, the Bureau of Bridges was established which virtually embraced the functions of the old Department of Plant and Structures.

The merits of Mr. Cummings, his character and his personality, had deeply impressed the Commissioner and the Chief Engineer of the new department and accordingly he was appointed Director of the Bureau of Bridges, a choice which was received with the utmost gratification by his associates in the service.

With characteristic modesty, tact, and kindliness, Mr. Cummings assumed his new responsibilities. It was not long, however, before sickness attacked him. The conscientious and faithful civil servant chafed under inaction and could not reconcile himself to illness and prolonged absence from duty. Always, he had been on the job. His associates who loved him, anxiously waited and devoutly hoped, but the end finally came, swiftly and mercifully.

Mr. Cummings was a Charter Member of the Municipal Engineers of the City of New York. He was successively, Member of the Finance Committee, Director, Vice-President, and in 1928 was President of that Society. He was also a member of Bunting Lodge, Free and Accepted Masons of New York City.

In his religion, Mr. Cummings was a Unitarian. He was a member and a Trustee of the First Liberal Church of Mount Vernon, N. Y. He took an active part in that church, and supported it generously.

Mr. Cummings was married on January 9, 1907, to Louise Lerch who died on December 16, 1935. The passing of the "Madam" as he was wont to call her, deeply affected him, and saddened his later days.

He is survived by his mother, a son George, a grandson Gifford, and by two brothers, Lee Herbert and Howard Raymond.

Mr. Cummings was elected a Junior of the American Society of Civil Engineers on March 3, 1896, and an Associate Member on January 6, 1904.

CLARENCE DERRICK, Assoc. M. Am. Soc. C. E.[1]

Died March 20, 1938

Clarence Derrick was born on September 7, 1881, at Halifax, Va., the son of Henry Clay and Martha (Cosby) Derrick. His father, the late Henry Clay Derrick, M. Am. Soc. C. E., was prominently identified with railroad construction in the South between the years 1851 and 1888. He also had a distinguished military career as Captain of Engineers in the Confederate Army and, later, as Lieutenant Colonel of Engineers in the service of the Khedive of Egypt.

Clarence Derrick was graduated from the Virginia Polytechnic Institute, at Blacksburg, Va., in 1901, with the degree of Bachelor of Science in Mechanical Engineering. Later, he spent a year, 1904–1905, at Lehigh University, at Bethlehem, Pa., under the late Mansfield Merriman, M. Am. Soc. C. E., in the study of structural design.

His first engineering work was with the American Bridge Company, at Trenton, N. J. Until 1908, he had a varied engineering experience, principally in railway bridge design, having been associated, successively, with the Rock Island Railroad Company, in Chicago, Ill.; the Massillon Bridge Company, in Massillon, Ohio; the Carolina, Clinchfield and Ohio Railroad Company, at Bristol, Tenn.; and the Virginian Railway Company, at Norfolk, Va. In 1907, he was employed by the Pulaski Mining Company, on surveys of its extensive properties in Southwestern Virginia.

In 1908, Mr. Derrick became associated with the Isthmian Canal Commission in connection with the design of the emergency dams for the Panama Canal. This work was under the general direction of Henry Goldmark, M. Am. Soc. C. E., and Mr. Derrick was associated at Washington, D. C., with David A. Molitor, M. Am. Soc. C. E., and, later, at the Canal Zone with T. B. Mönniche, M. Am. Soc. C. E., in developing the highly complicated design for these structures.

In 1911 Mr. Derrick accepted a position as Engineer in Structural Steel Design with The Berlin Construction Company, Incorporated, at Berlin, Conn., and retained this connection until his death. He made his home in the near-by city of New Britain, Conn. He brought to this organization a well-grounded experience, and many New England structures bear silent testimony to the skill of this son of the South.

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[1] Memoir prepared by Charles F. Chase, M. Am. Soc. C. E., and G. B. Taylor, Assoc. M. Am. Soc. C. E.

In the days when the cities and towns, State Highway Departments and electrical railways, and even the railroads, sought assistance in their bridge problems, the construction company's engineer enjoyed a wide range of experience, and, in this field, Mr. Derrick labored with both technical skill and common sense, rarely, however, with personal recognition.

In later years, he encountered the variety of design inherent in modern industrial operations, solving his problems with unfailing ingenuity until ill health forced his partial retirement in 1935.

He was rather shy in manner at first acquaintance, not given to self-assertion, content to let his work speak. When one enjoyed his confidence, he proved a capable and humorous story-teller, drawing on his Military School life, his Canal Zone experience, and his keen faculty for detailed observation. He had musical talent, with the flute for its outlet, and a bent for caricature that was difficult to suppress, making the victims among his associates assert that an artist was lost in this engineer.

Mr. Derrick was married to Eva Reid, of Trenton, N. J., who died in 1934. He is survived by two sons, H. Reid Derrick and Clarence Derrick, Jr.; a sister, Cornelia C. Derrick; and a brother, G. H. Derrick, M. Am. Soc. C. E.

He died on March 20, 1938, after a rather long illness, at the Gaylord Farm Sanatorium, at Wallingford, Conn., and was buried in Trenton, N. J.

On the top of Talcott Mountain, at Avon, Conn., west of Hartford, Conn., stands the beautiful Heublein Tower, overlooking the valley of the Farmington River. The tower is visible for miles from the College Highway running north from New Haven, Conn., to Northampton, Mass. The invisible foundations and framework which enable this structure to stand steadfast in its exposed situation were designed by Clarence Derrick. His friends in Connecticut will remember him as they pass this tower. It typifies his hidden strength.

Mr. Derrick was elected an Associate Member of the American Society of Civil Engineers on October 4, 1910.

JOHN LESLIE DOBBINS, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 25, 1938

John Leslie Dobbins was born on September 6, 1883, at Calistoga, Calif. He was the son of the Reverend Hugh Hillis Dobbins, a Congregational Minister, and Mary Eloise (Trowbridge) Dobbins. He was the youngest of the family, and had several older sisters.

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[1] Memoir prepared by Ned D. Baker, M. Am. Soc. C. E.

His father was one of the early pioneer missionaries of the West, and young Dobbins was reared in the religious atmosphere of a minister's home. He became affiliated with the Presbyterian Church as a young lad, and was a member until the time of his death.

He was graduated from the University of California, at Berkeley, Calif., with the degree of Bachelor of Arts in Letters in 1905. Not content with an academic education, he continued his studies and received his Bachelor of Science degree in Civil Engineering in 1907.

Mr. Dobbins then spent two years in railroad and mining work in Nevada. In 1909 he went to China to become Professor of Railway Engineering at the Imperial Pei Yang University at Tientsin, a position which he held for three and one-half years.

It was during this time in China that Mr. Dobbins first recognized the commercial possibilities of oriental trade, which led him later to organize several companies for that purpose.

In 1913 he returned to the States, and for a year engaged in engineering work in San Francisco, Calif., principally on flood control and irrigation work.

About this time he was married to Natalia Potter, who survives him together with their two sons, John Potter and David Trowbridge.

Mr. Dobbins took his bride back to China where he resumed his teaching, for another three years, as head of the Engineering Department of Pei Yang University at Tientsin. For a time he was also Chief Engineer of the Yunnan Szechuan and Tengyueh Railway Company.

While in China Mr. Dobbins organized and operated the American Machinery and Export Company, carrying on trade between America and China and other Far Eastern countries. Returning to San Francisco early in 1917, he remained in this company as Vice-President and General Manager.

Mr. Dobbins also widened his interest in the oriental trade by becoming a Vice-President and Member of the Board of Directors of the Koster Company of San Francisco. In this capacity, his engineering training and intimate knowledge of China were of great value in solving the many problems in packaging and shipping unusual commodities.

From 1921 to 1928, he conducted offices as Consulting Engineer in New York City and San Francisco, specializing in development, organization, and operation of hydro-electric and mining projects.

The following years Mr. Dobbins served as official and director of numerous successful enterprises. One such, of more than ordinary interest, was the Anchorage Light and Power Company, of Anchorage, Alaska, said to be the most northerly hydro-electric plant on the American Continent. This project was conceived, organized, financed, and constructed under Mr. Dobbins' guidance while he was engaged in mining developments around Anchorage.

For several years prior to his death, Mr. Dobbins had maintained his office in San Francisco, but he was not very active in the practice of Engineering. His interests were largely in connection with companies that he had been instrumental in organizing, and in which he had served as officer or on the Board of Directors.

His chief recreations were golf, chess, and tennis, in all of which he was better than the average. He traveled extensively, both in connection with his profession, and for pleasure.

He was actively engaged in enlisting sympathy and means for relief for the unfortunate victims of the undeclared war in China. At the time of his death he was a member of the East Bay Committee of the Associated Boards for Christian Colleges in China.

He was a member of the Sigma Xi and Tau Beta Pi Societies, the American Railway Engineers Association, the Olympic Club of San Francisco, the Faculty Club of the University of California, and the St. John's Presbyterian Church of Berkeley.

Mr. Dobbins was elected an Associate Member of the American Society of Civil Engineers on May 28, 1912.

CHARLES HENRY FARNHAM, Assoc. M. Am. Soc. C. E.[1]

Died June 11, 1938

Charles Henry Farnham was born in Newburg, Me., on January 16, 1875. He was the son of Walter and Mary (Kilham) Farnham. He received his early education in the public schools of Beverly, Mass., and, later, attended the University of Maine, at Orono, Me., from which he was graduated in 1897, with the degree of Bachelor of Civil Engineering. During his college years he worked as Rodman and Leveler on different land surveys, and also on the construction of the Bangor, Orono, and Old Town Electric Railway.

In 1898, at the outbreak of the Spanish-American War, Mr. Farnham was appointed a First Lieutenant of the 8th Massachusetts Volunteer Infantry, and was detailed as Assistant to the Division Engineer, 3d Division, First Army Corps. In that year he had charge of road construction at Chickamauga Park, Georgia, and camp construction at Lexington, Ky., and Americus, Ga.

At the close of the Spanish-American War, he was appointed Resident Engineer of the Massachusetts Highway Commission, engaged in construction work at Haverhill and Attleboro, Mass., and in office work at Boston, Mass.

In 1900 and 1901, Mr. Farnham was with the Isthmian Canal Commission, as Assistant Engineer in charge of the Test Boring Division, in Nicaragua. The work included test borings for estimates and foundations for locks and dams; and office work in Washington, D. C.

He was engaged as Provincial Supervisor for the Philippine Islands, in 1901, and was assigned to the Province of Bulacan in charge of road and bridge construction and repair. In 1902 he served as Assistant Engineer in the Department of Public Works, of Manila, in charge of streets, and the building of a new stone-crushing plant at Talim Island, Laguna de Bay, Philippine Islands.

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[1] Memoir prepared from data supplied by the family and on file at the Headquarters of the Society.

Mr. Farnham went to China in 1902, to work on the Canton-Hankow Railway as Resident Engineer on the Sam-shui Division; later, he was in charge of the final location and field work between Canton and Fat Shan; and afterward became Division Engineer of the Sam-shui Division. This Division included twenty-two steel bridges, two concrete arch culverts, four concrete box culverts, thirty-two iron pipe culverts, and the work was completed at an estimated cost of $1 000 000. He returned to the Philippine Islands in 1906, and was employed as Division Engineer by the Philippine Railway Company. He had entire charge of the construction and operation of the railroad, including bridges and stations, on the Island of Cebu.

After the completion of the railroad in Cebu Mr. Farnham had charge of the construction of an electric street railway in Birmingham, Ala., and also worked on an electric tramway in Pernambuco, Brazil. When this work was completed he gave up foreign engagements and went into the constructing business in Boston. In 1919, he became Assistant Superintendent of a Boys Industrial School, in Shirley, Mass., and Supervisor of Construction of the three Industrial Schools of Massachusetts. He retired to private life in 1935.

Mr. Farnham was a man of sterling character, quiet, conscientious, modest, and of the highest integrity, with an outstanding quality of loyalty.

He was a member of Alpha Tau Omega Fraternity. While at the University of Maine, he, with two other students, originated an honor society called Lambda Sigma Eta. In 1900, when it became a National Society, this name was changed to Phi Kappa Phi. It has now more than fifty Chapters, and is established in some of the leading universities of the United States.

He is survived by his widow, Florence (Merrill) Farnham, to whom he was married in Manila, in 1906.

Mr. Farnham was elected an Associate Member of the American Society of Civil Engineers on June 1, 1904.

MILO CLINTON HALSEY, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 12, 1939

Milo Clinton Halsey was born at Lewis, Iowa, on August 24, 1881. His father was George Clinton Halsey, and his mother, Grace Ellen (Morphy) Halsey, both of whom were natives of New York, but whose families moved to Iowa at an early date. George Clinton Halsey was a general building contractor, and it was this fact which largely influenced his son in deciding upon structural engineering as his life's work.

Mr. Halsey's parents were among the Iowans who elected to settle in Southern California, which they did when Milo was but two years of age;

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[1] Memoir prepared by C. K. Bowen, Assistant to President, Pacific Electric Ry., Los Angeles, Calif.

and, since practically all his life was spent in or near Los Angeles, he could almost qualify as a "Native Son." He received his early education in the public schools, and was graduated from the Los Angeles High School.

He began his practical engineering experience as Rodman and Chainman for the East Whittier Land and Water Company in March, 1902, and by June, 1903, had been advanced to Foreman in charge of construction. Determined to obtain a scientific approach to what he had now decided upon as a career, young Halsey entered Stanford University, at Stanford University, Calif., in August, 1904, as a special student in Civil Engineering, and so continued until April, 1906, spending his summer vacation periods in building construction jobs; thus he put into practice the theories he had acquired in the class room.

The period from May, 1906, until August, 1908, was spent by Mr. Halsey in the service of various building contractors and architects in the design and erection of steel and reinforced concrete buildings, and, since the latter type was just beginning to prove a serious rival of the older type of steel frame construction, many problems, new to the building engineer, had to be considered and overcome. It was in this phase of engineering that this rising young engineer first showed the aptitude, courage, and resourcefulness which at a later period was so dominant a characteristic of his life and of his life's work.

In June, 1909, Mr. Halsey entered Massachusetts Institute of Technology, at Boston, Mass., for a year of advanced work in structural engineering, and upon completion of the course, returned to California where he re-entered the field of reinforced concrete construction, serving as Supervising Engineer in the design and erection of various buildings for the firm of Richards-Neustadt Construction Company, as well as for other contractors in the same field.

It was in December, 1910, that Mr. Halsey was appointed to the position of Chief Structural Draftsman, in the office of the Chief Engineer, Pacific Electric Railway Company, and thus formed a connection that was to endure to the end of his life. During this entire time, it was the writer's privilege to have been his close personal friend and professional associate.

An early recognition of his exceptional engineering ability led to his promotion in April, 1911, to the position of Structural Engineer of this rapidly expanding electric interurban railway system, and this position he held until his death. Various duties were added to his position, so that for many years, he not only supervised the design and construction of all buildings, bridges, elevated structures, subways and the like, but also had charge of their maintenance, reporting directly to the Chief Engineer.

A complete list of the various structures which Mr. Halsey designed and constructed for this railroad during his twenty-eight years of service would be too lengthy to include herein. A few of the more important (the cost of which totals millions of dollars) would include the Company's main shops, covering 125 acres at Torrance, Calif.; the Los Angeles-Glendale-Hollywood Subway; elevated tracts serving the main Los Angeles Station; Subway Terminal Station; grade-separation structures at Pico Boulevard, Roosevelt Highway, La Cienega Boulevard, Firestone Boulevard, Mission Boulevard, and at many other points, and dozens of sub-stations, passenger and freight stations, bridges, drainage structures, etc.

Personally, Milo Halsey was a kindly, generous, considerate gentleman—a man who merited, won, and retained the love of family, friends, and professional associates. Uncompromisingly honest, he was at all times just, and he never asked a subordinate to do that which he, himself, was not willing and ready to do twice over.

In 1907, Mr. Halsey was married to Mary Cosette Dorland, daughter of the Reverend Chester Paul Dorland, a prominent Congregational minister. Mr. Halsey was, himself, a member of that Church. He is survived by his widow, two daughters, a son, and two grandchildren.

He was also a member of the Structural Engineers Association of Southern California, and in 1936–1937 was Treasurer of the Los Angeles Section, American Society of Civil Engineers.

Mr. Halsey was elected a Junior of the American Society of Civil Engineers on June 30, 1910, and an Associate Member on June 4, 1913.

SAMUEL ALEXANDER HART, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 20, 1938

Early on Sunday morning, March 20, 1938, Samuel Alexander Hart succumbed to the ravages of a long illness at his home in Sacramento, Calif. Although in poor health for a number of years, he had continued his duties in the Office of the State Engineer as Senior Engineer, Supervision of Dams, until less than three weeks before his death.

The son of Henry Eldredge and Anzolette (Hayward) Hart, Samuel Alexander Hart was born at Stockbridge, Wis., on December 14, 1885, where he received his early education. In 1903, he went to California where he completed his preparation for college, entering the University of California, at Berkeley, Calif., with the Class of 1910, but the necessities of self-support prevented his graduation until 1911, when he received his Bachelor of Science degree.

Although he was graduated in mining, Mr. Hart chose to follow irrigation and hydraulic engineering, and devoted the major part of his professional career to these two branches. The first year after leaving college he spent in various capacities on surveys and construction in California and Arizona. From June, 1912, until May, 1915, he served as Chief Engineer of the Waterford Irrigation District, at Waterford, Calif., leaving that work to assume the position of Assistant Engineer with the South San Joaquin (Calif.) Irrigation District where he was employed on the investigation of a mountain water supply and on the construction of the Woodward Dam, a large hydraulic-fill structure.

Mr. Hart served in the World War from August, 1917, until June, 1919, and during the following year he was engaged in private business. From

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[1] Memoir prepared by a Committee of the Sacramento Section consisting of W. A. Perkins, M. Am. Soc. C. E., and G. F. Engle, Assoc. M. Am. Soc. C. E.

July, 1920, until October, 1921, he was in the employ of Joseph W. Gross, M. Am. Soc. C. E., Consulting Engineer of Sacramento, as Engineer in charge of irrigation investigations, structural design, and construction. Upon the completion of this work, he was appointed Assistant Hydraulic Engineer in the California State Division of Engineering and Irrigation and assigned to reconnaissance surveys of reservoir sites. After four months he resigned to accept a position as Assistant Engineer with the City of Sacramento where he was engaged in water supply and flood-control studies and in charge of street paving. In June, 1925, he was promoted to the position of Assistant City Engineer with administrative duties. Mr. Hart served in this capacity until March, 1928, when he was made City Engineer in charge of a construction program totaling $3 000 000 annually. In November, 1928, he resigned to accept the position of Sanitary Engineer for the City of Berkeley in charge of the design and construction of a storm sewer system. This work was completed in April, 1930, and he was appointed to the position of Senior Engineer of Hydraulic Structure Design in the California State Division of Water Resources, Department of Supervision of Dams, a position he held at the time of his death.

Mr. Hart joined the military forces of the United States in August, 1917, and was commissioned First Lieutenant of Company H, 23d Engineers. The regiment went overseas on March 30, 1918, and was engaged for the greater part of the time in building and maintaining communications to the front lines. The regiment took an active part in the battles of San Mihiel and the Meuse-Argonne, returned to the United States, and was disbanded in June, 1919, Mr. Hart receiving his honorable discharge on June 17.

On August 5, 1921, he was married to Elsie A. Silman who, with their two sons, Samuel A., Jr., and William E., survives him.

Mr. Hart was an able and conscientious engineer, loyal to those with whom and under whom he was working and considerate of his subordinates. He was home loving and keenly interested in movements looking to the upbuilding of the youth of the country, and energetic in his support of constructive civic endeavors. His friendly greeting and genial smile will be sorely missed by his numerous friends and associates. He was a member of the Masonic Fraternity and of the American Legion.

Mr. Hart was elected an Associate Member of the American Society of Civil Engineers on October 10, 1916.

HERBERT McDONALD HINCKLEY, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 27, 1938

Herbert McDonald Hinckley was born in Dallas, Tex., on July 6, 1897, the son of Frank D. Hinckley, Sr., and Sarah Celeste (Chandler) Hinckley. His father was a Sheet Metal Contractor and most of his relatives were builders.

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[1] Memoir prepared by Wyatt C. Hedrick, Esq., Fort Worth, Tex., in collaboration with Robert J. Cummins, and R. O. Jameson, Members, Am. Soc. C. E.

He received his education in the public schools of Dallas, and was graduated from the Dallas High School in 1915. He was self-taught in calculus, and from 1920 to 1924 he took an Extension Course in Structural Engineering and Reinforced Concrete from the University of Wisconsin, at Madison, Wis.

During the years of 1915 to 1920, Mr. Hinckley was associated with the Mosher Steel and Machinery Company, of Dallas, as a Draftsman and Checker. From 1920 to 1922, he served as Draftsman and Estimator for the W. C. Hedrick Construction Company, on framing plans for concrete structures and estimating quantities for buildings. From 1922 to 1924, he was again associated with the Mosher Steel and Machinery Company, of Dallas.

Mr. Hinckley was engaged as a Structural Engineer with R. O. Jameson, M. Am. Soc. C. E., Consulting Engineer, of Dallas, from 1924 to 1931. In this position he had a responsible part in the design of many large building frames handled by Mr. Jameson, among which were the Dallas National Bank (16 stories); Tower Petroleum (22 stories); Central Young Men's Christian Association (16 stories); Medical Arts Building Annex (18 stories); and many other buildings.

In 1931 and 1932 he was a Structural Engineer for Robert J. Cummins, M. Am. Soc. C. E., Consulting Engineer, of Houston, Tex. During this time he handled the design of such structures as the City County Hospital and Nurses' Home, of Houston, of which Mr. A. C. Finn was the Architect.

From 1932 to 1934 he was in private practice as Consulting Structural Engineer. During this time he was also in the employ of W. G. Clarkson and Company, Architects, of Fort Worth, Tex., as Structural Engineer. In 1934 and 1935 he was Bridge Engineer for the Forth Worth and Denver City Railway Company, of Fort Worth.

In 1935 Mr. Hinckley was the Structural Engineer for Wyatt C. Hedrick, Architect and Engineer, of Forth Worth, during which time he handled various assignments, including the Library Building of the North Texas State Teachers College, at Denton, Tex.; and, later, in 1935 and 1936, he designed the Coliseum and Memorial Tower, erected by the City of Fort Worth at a cost of approximately $733 000 as a part of its Centennial program, on which Wyatt C. Hedrick and Elmer G. Withers Architectural Company, Incorporated, were Associated Architects.

After Mr. Hinckley finished the above assignments he went to New Orleans, La., where he was employed by George P. Rice, M. Am. Soc. C. E., Consulting Engineer, on the design of various structures, during 1936 and 1937. In 1937, he went to the Virginia Bridge Company of Roanoke, Va., where he was employed as Designing Engineer until the time of his death.

He entered the Army on October 28, 1918, and was honorably discharged as a Private on January 1, 1919. He entered Artillery Central Officers Training School, at Camp Zachary Taylor, Kentucky, and was commissioned Second Lieutenant, Field Artillery Reserve, on February 1, 1919. At the time of his death, Mr. Hinckley held a commission as Major in the Field Artillery Reserve. He devised an ingenious training board for the use of students in their study of artillery problems, and took an active part in lecturing and teaching in this branch of the Service.

Mr. Hinckley was a gentleman in all his dealings with fellow engineers and was thought of always as a conscientious, hard-working engineer whose greatest delight seemed to come from the doing of a difficult task and doing it well.

In regard to Mr. Hinckley's work, Mr. Jameson states:

> "I found him to be very thorough and capable as an Engineer; his designs were economical and straightforward; and he worked constantly to improve his efficiency * * * . Mr. Hinckley was self-taught in engineering and his rise to the top as a designer is proof of his courage and determination. The Engineering Profession has lost an outstanding member in his passing."

Mr. Rice writes as follows: "Mr. Hinckley proved himself a gentleman of the first order, and I considered him one of the ablest men with whom I ever had the pleasure of working."

On September 17, 1921, Mr. Hinckley was married to Sadie Martha Burton, who survives him, together with two sons, Herbert M., Jr., and Conrad Hinckley.

Mr. Hinckley was elected an Associate Member of the American Society of Civil Engineers on November 11, 1929.

LUCIUS SANFORD HULBURD, Assoc. M. Soc. C. E.[1]

DIED SEPTEMBER 6, 1938

Lucius Sanford Hulburd was born in Brasher Falls, N. Y., on June 2, 1879. He was the son of Sanford Whittemore and Julia Amanda (Chittenden) Hulburd, and a direct descendant of William Hulburd who came from Wales, Great Britain, to New Haven, Conn., in 1630.

His preparation for entrance into the Civil Engineering College of Cornell University, at Ithaca, N. Y., included graduation from high school and three years of teaching in the public schools—he began teaching at the age of seventeen.

The summer vacations from his studies at Cornell were spent in practical engineering work, one season in the employ of the late John Bogart, M. Am. Soc. C. E., as Chainman on the construction corps supervising the hydro-electric developments by the St. Lawrence River Power Company at Massena, N. Y., and one season as Rodman in the Department of New York State Engineer and Surveyor.

During the year following his graduation from Cornell in 1903, Mr. Hulburd was employed by the S. Morgan Smith Company of York, Pa., on the construction of a 15 000-hp hydro-electric development on the Chattahoochee River near Atlanta, Ga. In June, 1904, he returned to New York State under a Civil Service appointment as Leveler in the Department of State Engineer and Surveyor. Employment by the State, with a short interruption for military service, continued until his death.

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[1] Memoir prepared by H. O. Schermerhorn, M. Am. Soc. C. E.

The continuous State service began with four years of design and estimate work for the improved State Canal System which was then just being undertaken. His major work in this connection was computing slopes for canalized river sections of the canal system.

In March, 1908, Mr. Hulburd, as Assistant Engineer, was placed in charge of one of the first barge canal construction contracts to be awarded in the $101 000 000 canal improvement project.

In June, 1909, with a background of five years in canal design and construction, he was selected to conduct surveys and make studies for a canal to connect the Finger Lakes with the Improved Erie Canal. He was assigned the construction of the Cayuga and Seneca Canals, and for nearly seven years he was located at Seneca Falls, N. Y., first as Assistant Engineer and later as Resident Engineer.

In January, 1916, he was transferred to Residency No. 8 on the Erie Canal, and until entering military service in August, 1918, he had supervision over 38 miles of canal improvement construction through Wayne County, New York. During this interval, the military training which he received while a student at Cornell was supplemented by attendance at the Plattsburg Training Camp.

On August 25, 1918, he was commissioned Captain of Engineers and reported to Camp A. A. Humphreys, in Virginia. Orders to proceed to the port of embarkation were received on November 9, but the signing of the Armistice permitted cancellation of those orders before execution. On November 30 he was discharged, and he returned to his work in the Engineering Department of the State, supervising canal improvement construction in the vicinity of Rochester, N. Y., retracing surveys of old canal lands and investigating water supplies for State institutions.

In August, 1921, he was transferred to the main office in Albany, N. Y., to compile records of the canal improvement work. He also made inspections and reports on the construction of dams within the State.

With the canal improvement work (on which he had been employed for nineteen years under the State Engineer) brought practically to a conclusion, Mr. Hulburd became associated with the Highway Division of the State Department of Public Works and was assigned to the project for the construction of the vehicular bridge over the Hudson River at Poughkeepsie, N. Y. This work began in June, 1923, with preliminary surveys, and he continued as the principal State representative throughout construction. Foundation problems made this construction especially interesting, the two main piers being located in water 55 ft deep with an additional 60 ft and 80 ft of depth, respectively, to the rock foundation. The work was done in open caissons. Upon the opening of the bridge for traffic in August, 1930, he was appointed Superintendent of Maintenance and Operation. This supervision was extended in July, 1933, to include the Rip Van Winkle Bridge spanning the Hudson River at Catskill, N. Y., which was completed during that year. Both of these are toll bridges under a plan of operation to make them self-liquidating and eventually free.

A heart affliction from which he never recovered developed during his training at Camp Humphreys. However, he did not permit this to interfere with his activities and interest in his work. There was a sterling quality about Mr. Hulburd's character and abilities which was recognized by everybody who knew him. He possessed a fine moral fiber and so despised pretension as to appear at times modest to the extent of diffidence. Behind this reserve a vein of humor was revealed to his intimate friends to whom he was loyalty itself. As a career man in the State service, he felt a high sense of responsibility, and to this service he made no small contribution of honest and efficient accomplishment.

Mr. Hulburd never married. He attended the Presbyterian Church. He was a member of Gamma Alpha Fraternity, the Amrita Club of Poughkeepsie, the Cornell Club of Dutchess County, and Lafayette Post American Legion.

Mr. Hulburd was elected an Associate Member of the American Society of Civil Engineers on October 31, 1911.

JOHN FRANKLIN JACKSON, Assoc. M. Am. Soc. C. E.[1]

Died August 22, 1936

John Franklin Jackson was born in Scipio Township, Allen County, Indiana, on April 24, 1864. He is descended from Thomas Jackson who was a silk weaver in Mackelsfield, Cheshire County, England, as early as 1820. The silk mill of J. and F. Jackson is still in operation in that city. His grandmother was Martha Upton of Upton Hall in the vicinity of Mackelsfield. Thomas Jackson migrated to America in 1828 with his three children, Hannah, Abraham, and Adam, his wife having been killed in an accident a year previous. The family settled at Marietta, Ohio, and moved to Scipio Township, Indiana, about 1840.

John Franklin Jackson was the son of Adam Jackson and Mary Jane (Murphy) Jackson, the daughter of John Murphy, a farmer near Cincinnati, Ohio. He spent his early life on the farm, attended the country school, and was graduated from the High School at Hicksville, Ohio. He then taught school and attended the Ada Normal University, later called Ohio Northern University, at Ada, Ohio, where he was graduated from the Civil Engineering course in June, 1886.

From September, 1886, to October, 1887, Mr. Jackson served as Deputy County Surveyor of Hodgeman County, Kansas. From March, 1888, to July, 1889, he was engaged in private practice at Sioux Falls, S. Dak., where he platted several thousand lots, published a large city map, built an electric railway from Sioux Falls to East Sioux Falls, and made several preliminary surveys for the Great Northern Railway Company. In July, 1889, he was appointed City Engineer of Sioux Falls and served until April, 1890, when he resigned to engage in private work.

In September, 1890, Mr. Jackson located at Des Moines, Iowa, to travel for the Wisconsin Bridge and Iron Company of Milwaukee, Wis., with which he was associated for more than forty-five years. He started in the Erection Department and supervised the construction of bridges and other structures for the Canadian Pacific Railroad Company. In 1895 he went to Houghton, Mich., as Contracting Engineer for the Wisconsin Bridge and Iron Company and was responsible for the introduction and development of steel shaft houses for numerous mining companies in that district. He was a pioneer in this field and replaced many timber structures with steel. He also designed and constructed the first steel dam in that locality in 1901 at Redridge, Mich., which was the second steel dam ever built in the United States.

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[1] Memoir prepared by Raleigh W. Gamble, M. Am. Soc. C. E.

While on the Upper Peninsula Mr. Jackson designed and constructed a large ore dock for the Lake Superior and Ishpeming Railway Company at Marquette, Mich., and also two large ore handling bridges for Darrow, Mann and Company. Among his more important works there were also twenty-four ore crushing plants and an equal number of large concentrating mills.

In 1906 Mr. Jackson came to Milwaukee and erected the first large plant of the A. O. Smith Corporation. He later constructed a cantilever bridge across the Mississippi River at Burlington, Iowa, and a similar structure across the Ohio River at Ironton, Ohio, as well as many other bridges in the Middle West. He also constructed wholesale coal unloading docks at Boston, Mass., and Providence, R. I., and was appointed by the Bethlehem Steel Company to inspect ore handling facilities in Chili, South America. His outstanding work was in the field of mine construction which won for him nationwide recognition.

In 1910 Mr. Jackson was made Vice-President and Consulting Engineer for the Wisconsin Bridge and Iron Company, in which capacity he served until 1926. In 1915 he suffered a heart attack which necessitated his taking occasional rests, and in 1926, owing to failing health, he was obliged to retire; but he remained a Director of the company and served in an advisory capacity until his death.

From 1925 until 1929 Mr. Jackson acted as a Consulting Engineer with the Engineering Department of the City of Milwaukee on the construction of new bridges and the rebuilding of old ones. In 1917 and 1918 he served as a member of the Board of Examiners of engineers entering the military training camps of the United States Army. He was a member of the Western Society of Civil Engineers, the Franklin Institute, and was the first President of the Engineering Society of Milwaukee, holding that office in 1920–1921. In the fall of 1935 he was honored with a dinner by the Milwaukee Section of the American Society of Civil Engineers, of which he was then the oldest living member. He was also active in the affairs of the Rotary Club and the Milwaukee Athletic Club. His home was known to a great many " Milwaukeeans " because of its extensive gardens, stretching to the banks of the Milwaukee River and constituting his principal hobby.

Mr. Jackson was first married in 1888 to Margaret Eberl, by whom he had three children, all of whom died in infancy. On December 26, 1911, he was married to Elizabeth Tillson, a daughter of Pembrook S. and Carrie Louise (Emerson) Tillson of Manitowoc, Wis. He had one son by this marriage, John Tillson Jackson.

Mr. Jackson was elected an Associate Member of the American Society of Civil Engineers on May 2, 1894.

JOHN YOUNG JEWETT, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 1, 1938

John Young Jewett was born at Gilford, N. H., on February 16, 1870. He was the son of Benjamin Quincy Jewett and Huldah Maria (Brown) Jewett, descendants of old New England families.

He received his early education in the public schools of Gilford, and from 1887 to 1890 he attended the New Hampshire College of Agriculture and Mechanical Arts (later, the University of New Hampshire), at Durham, N. H., and was graduated with the degree of Bachelor of Science. In 1892–93 and 1894–95, he attended the Thayer School of Civil Engineering, Dartmouth College, at Hanover, N. H., and was graduated with the degree of Civil Engineer.

In June, 1896, Mr. Jewett was appointed Cement Inspector of the Metropolitan Water Works of Boston, Mass., in connection with the construction of Wachusett Dam and related works at Clinton, Mass. He was engaged in experimental work on permeability of cement mortars, setting time, effect of freezing, etc. The work included both natural and Portland cements, and foreign and domestic varieties of the latter class. This was in the early days of cement when comparatively little was known of the various qualities. From January to March, 1901, he took a special course in Chemistry of Cements at the Massachusetts Institute of Technology, in Boston, Mass.

In July, 1904, Mr. Jewett became the Cement Expert of the United States Reclamation Service which was then in its formative stage. He organized and conducted the laboratory for testing of cements and related materials at Denver, Colo., and reported directly to the Director's Office, at Washington, D. C., until the opening of the Chief Engineer's Office, at Denver, in 1915. Mr. Jewett had general supervision of matters relating to purchase, test, and shipment of cement for all United States Reclamation Service projects. Auxiliary work included the examination of concrete aggregates from various projects of the Service. He made an extensive series of long-time tests on cement and aggregates, running to 10-yr periods. The study of alkali action on cement, as well as many other tests, was made under his direction.

The laboratory was taken over by the National Bureau of Standards in July, 1917, and Mr. Jewett continued doing work in his chosen field as Associate Engineer Physicist.

In October, 1918, he went to San Diego, Calif., to open a new laboratory in which work was to be done in connection with the building of concrete ships and other activities of the World War period.

He made an investigation and report for the Salt River Valley Water Users Association, in Phoenix, Ariz., in August, 1919, regarding the feasibility of cement manufacture by the Association.

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[1] Memoir prepared by Fred D. Pyle, M. Am. Soc. C. E., and Paul Beermann, Assoc. M. Am. Soc. C. E.

The laboratory of the Bureau of Standards was taken over by the City of San Diego, in August, 1919, and Mr. Jewett became Testing Engineer for the City, in charge of all laboratory testing work.

In June, 1923, he made an examination and report on disintegration of concrete at Gem Lake Dam, in Mono County, California, and in March and April, 1928, he made an examination and report on cement and concrete problems for the Mexican National Irrigation Commission, under F. E. Weymouth, M. Am. Soc. C. E., who was Chief Engineer.

He served as a member of a Committee organized by the United States Naval Public Works Office, 11th Naval District, to investigate the condition of concrete structures exposed to action of sea water in San Diego Harbor and vicinity. His service with the City of San Diego included work on major dam construction, water development, street improvements, and building inspection. During the last eight years of his service for that city, he also directed the work in the Water Bacteriological Laboratory, in addition to his other duties.

Mr. Jewett was a member of the American Society for Testing Materials for thirty-seven years. He was a lifelong member of the Congregational Church and served for many years as an Officer of Plymouth Church, in San Diego. He also held membership in the American Concrete Institute and National Geographic Society. He was the author of several technical articles and discussions published in the *Transactions* of the Society.

Few engineers gain the reputation of dependability enjoyed by Mr. Jewett, and he could always be counted on to give his best efforts and report true unbiased results of his investigations. His ideas were always constructive and helpful to those seeking aid, and he was always willing to do any amount of work to investigate a problem thoroughly. Those who knew him will miss his kindly disposition and pleasant association. He was unusually free from animosity and enjoyed doing his part.

Mr. Jewett was married to Annie Wallace Smith, of Glasgow, Scotland, on September 3, 1896, in Malden, Mass.

On June 30, 1937, he retired from active life and moved to Paso Robles, Calif., where he passed away. He is survived by his widow and a son, J. Q. Jewett, Assoc. M. Am. Soc. C. E.

Mr. Jewett was elected an Associate Member of the American Society of Civil Engineers on September 6, 1910. In 1926, he served as President of the San Diego Section of the Society.

HENRY GORTON OPDYCKE, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 30, 1938

Henry Gorton Opdycke was born on December 23, 1870, in Philadelphia, Pa., the son of Doctor Washington Opdycke, a descendant of the original

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[1] Memoir prepared by T. Kennard Thomson, M. Am. Soc. C. E.

Dutch Settlers in New York. His Uncle George Opdycke was Mayor of New York City from 1862 to 1864.

Henry was graduated from the High School in Jersey City, N. J. He then started work in Mechanical Engineering in the employ of, and under the instruction of, General Dudley S. Steele, pursuing the study and manufacture of iron and steel products in the Tatan Iron Works, manufacturers of railroad, dredging, and ice machinery.

When about 21 years old, Mr. Opdycke became an assistant in the employ of The Lawyers Engineering and Surveying Company, of which organization Mr. W. M. K. Olcott was Secretary, with offices at 170 Broadway, New York, N. Y.

For more than six years Mr. Opdycke was Principal Assistant to the Chief Engineer of that company, practically in charge of the office and field forces. He was frequently employed in Court as an expert witness in cases requiring unusual engineering ability. In 1895, after an examination by The Consulting Engineer of The City of New York, he was appointed a City Surveyor.

In 1897 Mr. Opdycke left The Lawyers Engineering and Surveying Company to accept a commission as Signal Officer in the United States Army, in which position he served for nearly a year, principally in Cuba, making military surveys in addition to his other duties.

In 1899 Mr. Opdycke opened his own office in New York City for the practice of engineering, surveying, etc. His work then consisted of municipal engineering, planning many systems of streets, grades, sewers, etc.

He was employed by the late Elmer Lawrence Corthell, Past-President, Am. Soc. C. E., to make maps, breakwater plans, etc., for the Cape Cod Canal. The late Alfred Noble, Past-President Am. Soc. C. E., also retained him to establish a system of co-ordinates for the Pennsylvania, New York and Long Island Railroad Terminal between 31st and 33rd Streets, Seventh and Ninth Avenues, New York City.

Warren and Wetmore, Architects, were among the enormous list of prominent concerns which availed themselves of his abilities. He planned many connections with the Rapid Transit Subways for his clients and developed large properties in New York City, such as Morris Park, for the Astor estate and others. He also had charge of the construction of the Foundations for many large buildings in New York.

Mr. Opdycke was often retained as an expert witness and as such earned a great reputation in the Courts where his natural engineering ability and careful, thorough, and exact studies and preparations for the case were of great benefit to his clients. Once, after he had won an important case for his clients, the writer was stopped on the street by a man who introduced himself as having been one of the jurymen on that case, and then told him that Mr. Opdycke and his associates were all young men of whom the jury had never heard, while the other side had a galaxy of engineers of international reputation. He said that the jury had commented on the fact that the young men, in that case, knew every detail of their work, while the older men, of great reputations, only seemed to know what the contractors had told them about the subject.

Mr. Opdycke for many years was probably the leading specialist on the vibration of buildings, and other structures, due to the subway construction, etc., having designed and constructed machines for recording such vibrations in the Grand Central Station (due to trains) and in other buildings.

One of his machines recorded movements, or vibrations, of as much as one-eighth of an inch—both vertically and horizontally—in a building every time a big engine started or stopped running in another building more than 700 ft away from the recording machine.

During the construction of the Seventh Avenue Subway, Mr. Opdycke directed the underpinning of the Metropolitan Opera House, the New York Produce Exchange, and other difficult underpinning jobs.

In 1925, as Consulting Engineer of the Eighth Avenue Association, later the West Side Chamber of Commerce, he developed his plan of submerged by-passing of gas mains during subway construction. Previously gas mains had been carried on over-head structures during subway construction, which resulted in losses to merchants, annoyance to pedestrians, and fire hazards.

Mr. Opdycke served as Managing Director of the Broadway Association and was instrumental in forming the Sixth Avenue Association with which he was connected for many years. It is of interest that later his son, Barkley, was also connected with the development of Sixth Avenue, being in charge of work for one of the contractors on the subway. Among the Major's many clients were R. H. Macy and Co., John Wanamaker, The Chemical National Bank, Hotel Imperial, Hotel Ansonia, 1270 Broadway, etc.

He organized the Signal Corps of the National Guard of New Jersey, and, during the Spanish-American War, served as Signal Officer of the Department of Mananzas and Santa Clara, Cuba.

Major Edward Van Winkle writes:

> "He grew up in Jersey City with me, married a sister of a classmate of mine. He organized the first Signal Corps in New Jersey National Guard in the nineties and commanded the battalion for the first years. He was a very nice fellow, thorough in his work and had a natural bent for history."

During the World War, Major Opdycke spent 14 months in France as Chief Signal Officer of the Intermediate Section of the American Expeditionary Forces. For this service he was made a Chevalier of the Legion of Honor (a distinguished American Colonel once stated that he would cheerfully have given 10 years of his life if he could have earned that honor). Major Opdycke was also cited by the United States Government.

In 1921 and 1922 he managed the Veterans Bureau of the Second District, which embraced New York, Connecticut, and New Jersey, a clearing house for Federal agencies helping Ex-Service men.

Major Henry G. Opdycke was a born engineer with unusual ability and foresight, and industry. He was tireless in looking after the interests of his clients and friends. He was an authority on acoustics.

He had an intense pride in, and affection for, his family, loyalty for his friends, and "carried on" to the last in spite of the increasing infirmities due to his work in the World War.

Surviving are his widow, the former Ida Louise Stryker, a descendant of General Stryker; a son Barkley Opdycke, Civil Engineer and Surveyor; and three grandchildren, Elizabeth Louise Opdycke, James Barkley Opdycke, Jr., and Jan Opdycke.

He was a member of many Military Organizations. For thirty years before his death he had all the qualifications for full membership in the Society and was often urged to transfer to that grade. It is a profound tribute to his innate modesty that he was content with the grade of Associate Member.

Major Opdycke was elected an Associate Member of the American Society of Civil Engineers on September 7, 1904.

OTTO COFFEEN FITZ RANDOLPH, Assoc. M. Am. Soc. C. E.[1]

Died May 8, 1938

Otto Coffeen Fitz Randolph was born in Arkansas City, Kans., on November 27, 1890, the son of Jackson Fitz and Lacie (Coffeen) Randolph. He received his primary and High School education in the public schools of Chicago, Ill., and his university education at the University of Illinois, at Urbana, Ill., where he enrolled in 1908. After a year's absence to earn school money, he was graduated from that University with the Class of 1913, with the degree of Bachelor of Science in Civil Engineering.

During his year of absence from college and during vacation periods, Mr. Randolph worked as Timekeeper and Laborer for George Hinchliff Company; as Draftsman for the Robbins Conveying Belt Company; and as Masonry Inspector for the Illinois Central Railroad Company. After his graduation, he entered the employ of the Michigan Central Railroad Company as Bridge Inspector and Investigator of stresses in bridges east of Jackson, Mich. Later, he was made Inspector in charge of the bridge being built over the Lake Shore Yards, in Detroit, Mich.

From July, to December, 1915, Mr. Randolph was Engineer in charge of building and structural appraisal of the Detroit properties of the Michigan State Telephone Company. From December, 1915, to September, 1916, he was Construction Engineer for the Timpkin Detroit Axle Company in charge of the construction of two 4-story factory buildings and of the design of all factory additions. From September, 1916, to June, 1917, he was Superintendent of The Austin Company in charge of the construction of the plant for the Michigan Steel Tube Products Company, the design of a power house for the same Company at Detroit, and the construction of buildings for the Dow Chemical Company, of Midland, Mich.

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[1] Memoir prepared by W. L. Abbott, Esq., Chicago, Ill., assisted by Edward Haupt, M. Am. Soc. C. E.

On June 6, 1917, Mr. Randolph joined the 16th Railway Engineers, U. S. Army, in Detroit, and on July 28, with the rank of Second Lieutenant, he sailed with his regiment for France. There he was engaged in the construction of main advance post warehouses at Is-sur-Tille, the construction of narrow and standard-gage railroads on the British front north of Arras, and of a 40 000-bed hospital at Mesves sur Bulcy. In addition to this work, the 16th Combat Engineers, in March, 1918, took its place at the front, with the British on the Aisne River where the great German drive was halted and turned back.

Because of exceptional ability shown in expediting construction work, Mr. Randolph was advanced in rank to First Lieutenant and then to Captain, and on September 19, 1918, he was sent back to the United States, stationed at Camp Humphreys, Virginia, promoted to the rank of Major, and ordered to form a regiment of sappers of which regiment he was to be the Colonel.

With the declaration of the Armistice, all organizing work was stopped, and soon, thereafter, Major Randolph resigned his commission and entered the employ of the H. K. Ferguson Company, General Contractors, as District Superintendent in charge of the Chicago office. As such, he was engaged on the construction of a furniture plant for Showers Brothers, of Bloomington, Ind.; a power plant and factory for Nordyke and Marmon, at Indianapolis, Ind.; and the design and construction of a straw-board paper mill for The Hinde and Dauch Paper Company, at Fort Madison, Iowa.

After two years with the Ferguson Company, Mr. Randolph resigned, and, in 1922, with some of his Army associates, organized the Engineering and Contracting Company—Otto Randolph, Inc.—of which he was President and Treasurer. The Company was engaged in a variety of construction work—residences, hotels, factories, sub-stations, power plants, roads, and bridges, in which work it prospered all through the depression of 1929–1935.

Not only was the Company prosperous in the contracting business but Mr. Randolph, personally, was prominent in contractors' organizations, having served as President of the Landis Award Association and President of the Mason Contractors' Association for negotiations with union labor. Although somewhat brusk in outward manner, he was of a genial nature and of the type known as a " square shooter "—one whose ethics and business integrity never wavered for a temporary advantage, however great, in his dealings with competitors, clients, or labor officials.

Mr. Randolph was a man of unusual energy and physical strength, which were contributing causes to his death, at the age of forty-seven, resulting from a too strenuous participation, one evening, in the events of a fathers' and sons' athletic competition. At two o'clock the next morning, he had a paralytic stroke that rendered him helpless. He was taken to a hospital and was making slow progress toward recovery, when pneumonia set in, causing his death a few days later. He was buried, May 11, 1938, in Rosehill Cemetery, Chicago, with impressive military honors under the direction of Castle Post No. 151 of the American Legion of Illinois.

Mr. Randolph was a member and Past-President of the Illini Club of Chicago, a member of the Union League Club, of Chicago, the University

Club, of Chicago and the Chicago Engineers' Club, of which he was Past-President. He was also a member of the Evanston (Ill.) Golf Club. He was a registered Structural Engineer in several States, and a member of the American Society of Military Engineers, the American Legion, Phi Gamma Delta Fraternity, the Masonic Order, the Democratic party, and the Presbyterian Church.

On June 16, 1917, ten days after he had enrolled in the Army, Mr. Randolph was married to Helen Abbott, a college classmate. This happy union lasted fourteen years, when Mrs. Randolph died November 3, 1932, leaving four children, William, Otto, Josephine, and Burleigh. On February 20, 1935, Mr. Randolph was married to Wilma Hall, who bore him a son, Michael.

Mr. Randolph was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919.

WILLIAM SHEARER, Assoc. M. Am. Soc. C. E.[1]

Died November 23, 1938

William Shearer was born at Burson, Calaveras County, California, on October 20, 1886, the son of Willow Spring and Bertha E. (Gaylord) Shearer. His preliminary education was received in the Public Schools of Marysville, Calif., and he was graduated from the High School in that city in June, 1908.

Shortly after leaving High School he accepted a job with the Pacific Gas and Electric Company in Marysville as a sub-station operator. In January, 1910, he accepted a position with Leslie B. Crooks, private civil engineer and land surveyor, engaged in business in Yuba and Sutter Counties, California, this being his first contact with the civil engineering field. His association with Mr. Crooks lasted until August, 1915, interrupted only by a 2-yr assignment with the Pacific Gas and Electric Company where he served as Construction Foreman on power-line, sub-station, and power-house construction. While with Mr. Crooks, he was engaged on irrigation and drainage projects, highway and bridge construction, and general land surveying.

Realizing that advancement in the profession demanded further educational qualifications, Mr. Shearer entered the College of Engineering at the University of Nevada, at Reno, Nev., in August, 1915. His undergraduate work, however, was interrupted in October, 1917, when he enlisted in the United States Navy. His war service with the Navy lasted until June, 1919, and during his enlistment he was assigned to ships in both home and foreign waters, serving as an Electrician and Ensign Engineer Officer.

After his discharge from the Navy, he served a short assignment with the Nevada State Highway Department as Resident Engineer on highway construction, and re-entered the University of Nevada in January, 1920. He completed his course in May, 1920, receiving a Bachelor of Science in Civil Engineering. At the University he distinguished himself in scholarship and extra-curricular activities. He was affiliated with Sigma Alpha Epsilon Fraternity, was elected to Phi Kappa Phi (Honorary) Fraternity, and was a member of Coffin and Keys, a local honorary society.

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[1] Memoir prepared by Committee of the Sacramento Section, consisting of Owen G. Stanley, Chairman, M. Am. Soc. C. E., Henderson E. McGee, Assoc. M. Am. Soc. C. E., and O. Haven Hart, Jun. Am. Soc. C. E.

From July, 1920, until April, 1921, he was again employed by the Pacific Gas and Electric Company, in the vicinity of Marysville, as Foreman on the construction of transformer stations, hydro-electric plants, and power lines. During 1921 he served a short assignment with the U. S. Engineer Office, Sacramento District, as Chief of Survey Party on hydrographic and topographic surveys along the Sacramento River.

In April, 1922, Mr. Shearer was made Assistant City Engineer of the City of Marysville, and served in that capacity until January, 1923, when he was elected County Surveyor of Sutter County, California. He served in this capacity until April, 1934, and was responsible for the construction of many highway and other county improvements. During his tenure of office all bridges and highways were constructed on designs made by Mr. Shearer, and built under his supervision. The above contacts led Mr. Shearer to an interest in the fields of City Planning and Management as applied to smaller units, in which he became quite active.

In April, 1934, he returned to the U. S. Engineer Office, Sacramento, and remained on this assignment until the accident which caused his untimely death in November, 1938. He was engaged on various projects connected with flood control, river and harbor improvements, and the control of hydraulic mining débris.

Mr. Shearer was a registered Civil Engineer and Land Surveyor in the State of California. He was a member of the Masonic Order, Knights Templar, Scottish Rite, and Mystic Shrine, and also a member of the Order of the Eastern Star. He was a conscientious and efficient engineer of unquestioned integrity, a loyal friend, and a worthy citizen.

On September 8, 1928, he was married to Eva Kennedy, who, with two children, William Kennedy and Margaret Sue, survives him.

Mr. Shearer was elected an Associate Member of the American Society of Civil Engineers on July 11, 1921.

JOSEPH WARREN SILLIMAN, Assoc. M. Am. Soc. C. E.[1]

Died September 22, 1938

Joseph Warren Silliman was born in Ashtabula, Ohio, on August 25, 1865, and was graduated from Allegheny College, at Meadville, Pa., in June, 1889, with the Degree of Civil Engineer.

Immediately after his graduation, Mr. Silliman obtained a position as Rodman with the firm of Cole, Alvord, and Shields, Engineers, of Chicago,

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[1] Memoir prepared by Benjamin Franklin, M. Am. Soc. C. E.

Ill. He resigned from this firm in February, 1890, to accept a position as Instructor at his Alma Mater, Allegheny College.

Evidently teaching, as a profession, did not appeal to him, for he resigned at the end of the college year to become an Assistant in the firm (well known at that time) of Patton and Frank, Civil Engineers of Duluth, Minn., remaining with them for more than five years.

On severing his connection with Messrs. Patton and Frank in October, 1895, he went to Philadelphia, Pa., where he spent the remainder of his professional life, with the exception of one year spent in Long Island as Chief Engineer of the New York and Queen's County Railway Company, and another year as Superintendent of Construction for the Wheeling and Western Railway Company of Bridgeport, Ohio.

Among the important positions held by Mr. Silliman while in Philadelphia was his connection with the Peoples and Union Traction Companies in charge of construction and maintenance of power station and track. Later he became Assistant Engineer for the Philadelphia Rapid Transit Company in charge of maintenance of way, covering construction of underground conduits, track, and roadway.

At the conclusion of this work, he entered the employ of the Highway Department of the City of Philadelphia, passing through various grades as Bridge Inspector; in charge of sewer construction; and finally reaching the position of District Engineer in charge of Highway Construction and Maintenance.

Mr. Silliman was noted for his thorough technical knowledge—which was ripened by a valuable and varied experience—and for reaching his objectives through a very genial and pleasant personality.

He had many warm and constant friends in Philadelphia who respected his high ideals, his sincerity, and his genuine kindliness. Mr. Silliman never married.

His membership in the following organizations indicated his deep interest in his profession: Member of the Engineers' Club of Philadelphia since 1896—a period of 43 years; Member of the Forestry Association of Pennsylvania. He was also active in the Philadelphia Section of the American Society of Civil Engineers.

Mr. Silliman was elected an Associate Member of the American Society of Civil Engineers on September 2, 1914.

HARVEY ALFORD TUTEWILER, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 8, 1938

Harvey Alford Tutewiler was born in Indianapolis, Ind., on June 9, 1890, the son of Charles W. and Caroline (Alford) Tutewiler. He was graduated from High School in June, 1906, and shortly afterward entered the Engineering Department of the City of Indianapolis. From June, 1906, until October, 1919, he was continuously employed in this department, with the exception of nine months of service in the United States Army during the World War.

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[1] Memoir prepared by E. B. Rayburn, Jr., Assoc. M. Am. Soc. C. E.

He served his apprenticeship in the Engineering Department of the City of Indianapolis from 1906 to 1910, and acquired an engineering foundation in the various capacities of Rodman, Levelman, Inspector, and Instrumentman. In 1910 he entered the Engineer's Office as Draftsman and Estimator. His new duties introduced him to the fundamentals of street and bridge design and layout. Quite a clamor also had arisen for grade-crossing eliminations in the southern part of the city and it was necessary to prepare many preliminary plans and estimates for the purpose. July, 1912, saw him further advanced to the position of Chief Draftsman and Office Engineer. About this time the tempo of street building quickened appreciably as the demand for hard-surfaced streets increased with the mounting number of motor cars in use. Construction technology was also advancing rapidly, and the young engineer without a formal engineering education had to keep continually on his toes in his successful effort to remain abreast of this advance.

His ability to get his job done was rewarded in 1915 by advancement to the position of Assistant City Engineer. As such he was directly responsible for the planning, construction, and maintenance of the city's continually growing street system. During this period the number of miles of new hard-surfaced streets showed a tremendous increase, which was in keeping with the general trend throughout the country. It was necessary to expand the Department's Engineering and Inspection divisions to handle this increased volume adequately, and part of the system he set up to supervise and expedite construction still remains in use to-day.

The World War interrupted his engineering career for nine months beginning in October, 1918. He was unable to escape from construction, however, as he enlisted in the Army's Construction Division at Fort Knox, Kentucky, and as a First Lieutenant he continued his rôle of road and street builder.

On his return from the Army he resumed his post in the Engineering Department, but his Army experience had left him with a restlessness which the duties of his job could not overcome. Accordingly, in October, 1919, he joined The Mansfield Engineering Company as Superintendent and General Manager. The company was engaged in heavy engineering and building construction and one of his first assignments was the supervision of the construction of a hydro-electric plant at Monticello, Ind. He could not, however, forsake his old love—street building—and upon completion of the job at Monticello the company forsook heavy construction to specialize in the building of concrete and asphalt streets. Under his direction the paving business increased yearly and even to-day the product of those early years bears evidence of the quality of his workmanship.

The company also branched out into the manufacture of a dragline bucket, but the death of the head of the firm in 1925 brought about the dissolution of The Mansfield Engineering Company, and in its stead was formed the Standard Paving Company, with Mr. Tutewiler as part-owner and President. The activities of the company followed closely those of its predecessor, but much of the volume and allure had vanished from street building by the end of the 20's and he was again on the lookout for a business which would absorb more of his boundless energy.

The search ended in 1931 with the founding of the Ready Mixed Concrete Corporation. As Principal Owner and his own General Manager, Mr. Tutewiler had ample opportunity to exercise his talents in the promotion of the use of his product—pre-mixed concrete. In a city where concrete had previously been mixed on the site of the work, pre-mixed concrete was still regarded as somewhat of a novelty, although its use had already become general in most large population centers. For several years, therefore, he had his hands full in the building up and expansion of a market which would absorb the facilities of his plant. Until his death in an automobile accident he had seen each year's volume exceed that of the preceding year, and the growth of his plant and equipment had increased accordingly.

Aside from his business interests, he loved hunting and fishing and would never overlook an opportunity to engage in either of them. He was active in the social and fraternal life of his city and belonged to numerous organizations of this nature. During the year preceding his death he had been able to devote some of his time to affairs of a strictly civic nature, and undoubtedly he would have occupied himself more and more with things of this sort as his business became more routine, and demanded less of the constant attention he had found necessary heretofore.

He was a licensed Professional Engineer, a member of the Indiana Section of the American Society of Civil Engineers, and of the Indiana Engineering Council.

In 1910 he was married to Ora Harcourt. His widow, their one child, Martha Tutewiler Simpson, and a sister, Mrs. Fred L. Evans, survive him.

Mr. Tutewiler was elected an Associate Member of the American Society of Civil Engineers on October 29, 1934.

WILLIAM NOEL ALLISON, Jun. Am. Soc. C. E.[1]

DIED JULY 29, 1938

William Noel Allison was born in Golden, Colo., on January 1, 1909, the second child of William F. and Mabel (Hegeman) Allison. He attended Grammar School, High School, and College in Seattle, Wash., where his father was Professor of Municipal and Highway Engineering at the University of Washington. Mr. Allison received his technical training at the University of Washington from which he was graduated in December, 1931, with the degree of Bachelor of Science in Civil Engineering. He continued his training during the following year and received the additional degree of Master of Science in Civil Engineering, also from the University of Washington, in June, 1932. His was an enviable college record; he was an excellent student, and he combined admirably his technical training and balanced social and student activities to endear his friendship to both his classmates and the Faculty. He was a member of Alpha Kappa Lambda Fraternity.

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[1] Memoir prepared by L. Vaughn Downs, Jun. Am. Soc. C. E.

During the year, 1927–1928, Mr. Allison was engaged as a Draftsman for the Wallace Bridge and Structural Steel Company, in Seattle. Later, during summer vacations from College, he was employed on surveying assignments by the State of Washington, and by C. W. Harris, M. Am. Soc. C. E., of Seattle. After the completion of his college training he was employed by the Boeing Airplane Company, and the United States Coast and Geodetic Survey (both of Seattle), and the Pacific Car and Foundry Company, of Renton, Wash. In May, 1934, he again entered the service of the United States Government, and was employed by the Bureau of Reclamation on the construction of the Grand Coulee Dam. He continued with this organization as an Inspector and as a Junior Engineer, until his untimely death.

Mr. Allison was married on July 16, 1938, to Margaret Prosser, of Veradale, Wash. He died in Amarillo, Tex., on July 29, 1938, after a short illness while en route to the City of Mexico, Mexico, on his honeymoon. He is survived by his widow, his mother, and two sisters.

His passing cut short a career of much promise for he was diligent in his efforts, strong in character, and keen in intellect. He was possessed of a good sense of humor, a kindly regard for his fellow men, and an intense interest in social and economic problems.

Mr. Allison was elected a Junior of the American Society of Civil Engineers on May 23, 1932.

ROBERT EDWARD AUSTIN, Jun. Am. Soc. C. E.[1]

Died November 17, 1937

Robert Edward Austin was born on January 2, 1914, the son of Seneca B. Austin and Elizabeth M. Austin, at Jacksonville, Fla.

He was graduated from the University of Florida at Gainesville, Fla., and was awarded the prize from the Florida Section of the American Society of Civil Engineers for his outstanding work as a student.

Mr. Austin was an exceptionally able young engineer. At the time of his death he was employed by The Weber Company, Inc., of Jacksonville, as Supervising Engineer for the construction of a tall reinforced concrete chimney for the new pulp and paper mill of the National Container Corporation. Previous to this he was employed by the Shore-line Builders, Inc.

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[1] Memoir prepared by Carl Weber, Esq., Jacksonville, Fla.

of Jacksonville, as Field Engineer in the construction of the water-supply intake tunnels of the municipal electric plant.

He died of pneumonia, after a short illness, on November 17, 1937, in Jacksonville, at the age of 23 years. He is survived by his parents, one brother, Seneca B. Austin, Jr., and two sisters, Mrs. A. V. McDowell and Ruth Austin.

Although Mr. Austin was a young man and consequently his record of accomplishments a brief one, he was nevertheless one of the outstanding young men in the engineering profession. He took his work seriously and proved that he possessed engineering skill and knowledge far beyond that which could be expected from a man of his age.

He was a member of the Florida Engineering Society and the Jacksonville Professional Engineers Club.

Mr. Austin was elected a Junior of the American Society of Civil Engineers on November 9, 1936.

ARTHUR SAMUEL BENT, Affiliate, Am. Soc. C. E.[1]

Died February 17, 1939

Arthur Samuel Bent was born in Downieville, Calif., on April 25, 1863, the son of Henry Kirke White Bent and Jessie (Crawford) Bent, who were of staunch New England heritage. Arthur went to Los Angeles, Calif., in his early boyhood, and was educated in the Los Angeles Public Schools.

In his young manhood he came to believe that great development lay ahead for Southern California and that the distribution and development of water would be conspicuous in that growth. Accordingly, he became interested in the manufacture and installation of concrete pipe, and was one of the few pioneers in this industry, starting in 1883 with crude hand methods and little knowledge of concrete. For many years he was an authority on this class of work throughout the United States. As far back as 1891 he constructed the main supply water-line for the Temescal Water Company of Riverside County, California. In later years he took personal charge of a large gravity line near Boulder, Colo., for the Central Colorado Power Company, and a large water-supply line for the City of Pendleton, Ore.

Seeking the improvement of conditions of which he was a part characterized Mr. Bent's entire life, and he did much, over a period of thirty years, to improve the quality, methods of manufacture, and specifications of concrete pipe. He laid many miles of this pipe throughout Southern California. After his brother, H. Stanley Bent, became associated with his construction activities, the interest of their firm gradually extended into larger engineering construction works, ultimately under the name of Bent Bros., Inc.

Mr. Bent was President of Bent Bros., Inc., engineering contractors. This firm constructed numerous dams, irrigation systems, and other large projects throughout Western America. For many years, based on integrity, industry, and accomplishment, it has enjoyed a place of unique confidence in various phases of engineering construction—engineers, financiers, and material suppliers. Between the years of 1920 and 1925, the firm constructed for the large oil companies in California about forty oil-storage reservoirs, having a capacity of from 500 000 barrels to 3 000 000 barrels each.

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[1] Memoir prepared by J. B. Lippincott, Hon. M. Am. Soc. C. E.

From the years 1920 to 1939 the firm also constructed eighteen dams of various types, all of which were concrete except one—the Henshaw Dam in San Diego County, California, which was an earth-fill structure. The better known of these dams were: Bull Run for the City of Portland, Ore., for water supply; Exchequer Dam built for the Merced Irrigation District for irrigation purposes; Pacoima Dam built for the Los Angeles County Flood Control; Morris Dam built for the City of Pasadena, Calif., for the city water supply; and Conchas Dam, New Mexico, built under the Army Corps of Engineers for flood control.

Mr. Bent's most notable characteristic was his great versatility and his interest in many types of public-welfare enterprises. He contributed greatly to the public life of the community in which he lived. One of his outstanding ambitions was to elevate and establish the general standing of the contractor in the community and in the construction industry, throughout the United States. His life was an outstanding example of the practice of high ethics in contracting. The company of which he was president so lived up to these standards that contracts were often awarded to it on slight formality.

The ideals and labor of Arthur Bent had a profound effect on the organization of the Associated General Contractors of America, of which he became national president in 1924, following his activity in the organization and presidency of the Southern California Chapter. These ideals were expressed in an address that he made before that organization in 1922, in which he stated:

> "In a little while other men will take the places of us all. It is an inspiring thought to me that we are not only blazing the way for them and straightening the path for their feet, but we are establishing practices and ideals and standards that will draw better and better men into the contracting game and all down through the years will inspire them with a zeal for worthy service; and that, I think, is the finest definition of Americanism that I know."

It is stated in the contractors' magazine, *The Contractor,* that "It is to the ideals of Arthur Bent and his co-workers in the early days of the A.G.C. that the association owes the sound position and high esteem it holds today."

During his active construction years, he was never too busy to be human. Many of his employees have remembered for years his kindly word, courtesy, and real interest in them.

The following positions which he has held are illustrative of Mr. Bent's versatility:

He was City Editor of the Los Angeles Evening Express at the age of eighteen. In 1926 he was President of the Los Angeles Chamber of Commerce. On the occasion of his inauguration to this office, he delivered an address entitled "Lorenzo the Magnificent," in which he held up ideals to the City of Los Angeles similar to those which prevailed in Florence, Italy, under

Lorenzo. Elected as the head of a large commercial organization, he had the courage, on taking office, to stress the value of culture in a community and the fact that mere size could not make a city great. His address is still remembered in Los Angeles for its high ideals and eloquence. He was a Director of the U. S. Chamber of Commerce from 1923 to 1929, and a Director of the California State Chamber of Commerce from 1926 until his death.

As a further illustration of the varied culture of the man, he was on the advisory board of the Los Angeles Philharmonic Orchestra, and served a term as a member of the Los Angeles Board of Education. He held the position of Trustee of Pomona College, Claremont, Calif., continuously from 1921 until his death. Both as a member of the Board of Education of Los Angeles and as a Trustee of Pomona College, Mr. Bent followed in the footsteps of his father. In 1927 he was presented with a watch by the Los Angeles Realty Board as the City's most useful citizen during 1926. As a great patriot, he was President of the distinguished Lincoln Club of Los Angeles in 1938, and was re-elected for 1939.

Abounding in ready wit and a sense of humor, Mr. Bent also had a charming personality and was greatly beloved by his associates. The following excerpts from quotations made at the time illustrate the genuineness of this sentiment:

> "He was one of the rarest and most charming men I have ever known. Los Angeles will never be the same without him. There is a lonesome place against the sky."

* * *

> "He was not only an inspiration to all who knew him intimately but he always had the highest esteem and respect of those who knew him slightly."

* * *

> "He gave much to the world in friendship in kindness by example, in good deeds, in upholding the right always. I shall think of him for the light he gave others. And never did I hear him say an unkind much less an unjust word."

* * *

> "He is not only remembered by men of his station and ideas, but by the humble man in the street also."

* * *

> "Few are gifted with the sympathetic understanding of humanity that was his—few endowed with the tolerance and the kindly but just appraisal of men and their efforts."

* * *

> "His splendid character, his uprightness, his love for truth and sincerity together with his personal charm will be an inspiration to all who knew and loved him. He has left in memory a streak of gold in the lives of all his friends who remain."

* * *

> "To me his most outstanding characteristics were his love of humanity and an earnest and unfailing desire to be of service to his fellow man. He had a rare capacity for friendship."

Mr. Bent was elected an Affiliate of the American Society of Civil Engineers on November 28, 1916.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 104

1939

SUBJECT INDEX, PAGE 2060

AUTHOR INDEX, PAGE 2073

Titles of papers are in quotation marks when given with the author's name.

VOLUME 104

SUBJECT INDEX

ACCIDENTS.

Improvements to reduce traffic accidents. 474.

ADDRESSES.

Presidential Address at the Annual Convention at San Francisco, California, July 26, 1939. Donald H. Sawyer. 1815.

AERODYNAMICS.

"Aerodynamics of the Perisphere and Trylon at World's Fair." Alexander Klemin, Everett B. Schaefer, and J. G. Beerer, Jr. 1449. Discussion: Elliott G. Reid, Karl Arnstein, and W. B. Klemperer. 1469.

AUTOMOBILE LIGHTING.

Lighting—road or vehicle. 1572.

AUTOMOBILE PARKING.

Joint responsibility of the automotive and the civil engineer. 1563.

Parking facilities as an adjunct of the highway. 1568, 1593.

AUTOMOBILES.

"Motor Transportation—A Forward View": A Symposium. C. F. Kettering, Thomas H. MacDonald, and Leroy C. Smith. (With Discussion.) 1562.

BEAMS.

"Natural Periods of Uniform Cantilever Beams." Lydik S. Jacobsen. 402. Discussion: K. Bert Hirashima, and Merit P. White. 432.

See also **CONTINUOUS BEAMS.**

BRIDGE FLOORS.

"Structural Behavior of Battle-Deck Floor Systems." Inge Lyse and Ingvald E. Madsen. 244. Discussion: H. N. Hill and R. L. Moore, and Jonathan Jones. 267.

BRIDGES.

"Preliminary Design of Suspension Bridges." Shortridge Hardesty and Harold E. Wessman. 579. Discussion: A. W. Fischer, Jacob Karol, Glenn B. Woodruff and Norman C. Raab, Hardy Cross, A. A. Eremin, A. Fraser Rose and William A. Rose, Leon S. Moisseiff, O. H. Ammann, and Leon Blog. 609.

CANALS.

Florida ship canal. 693.

See also **IRRIGATION CANALS.**

CERAMIC MATERIALS.

"Principles Applying to Highway Road-Beds." Ira B. Mullis. (With Discussion.) 1381.

CITY PLANNING.

"Practical Methods of Re-Zoning Urban Areas." Hugh E. Young. (With Discussion.) 904.

CIVIL ENGINEERS.

Joint responsibility of the automotive and the civil engineer. 1563.

CONCRETE.

"Lateral Earth and Concrete Pressures." Lazarus White and George Paaswell. (With Discussion.) 1685.

CONTINUOUS BEAMS.

"Beam Constants for Continuous Trusses and Beams." George L. Epps. 1522. Discussion: C. W. Deans, George W. Lamb, A. A. Eremin, and Leo Legens. 1530.

"Relative Flexure Factors for Analyzing Continuous Structures." Ralph W. Stewart. 521. Discussion: Frederick Shapiro, Dean F. Peterson, Jr., George W. Housner, Homer M. Hadley, Adolphus Mitchell, John B. Wilbur, Leon Blog, A. Floris, D. B. Hall, Ralph W. Hutchinson, Arthur B. McGee, and E. Neil W. Lane. 531.

COSTS OF WORK.

Cost in terms of pipe diameter. 191.

"Economics of Sewage Treatment." George J. Schroepfer. (With Discussion.) 1210.

Estimated cost of seven projects, Tennessee Valley Authority. 729.

Florida Ship Canal. 693.

Hazards of uneconomical construction of public works. 668.

"Multiple-Stage Sewage Sludge Digestion." A.M. Rawn, A. Perry Banta, and Richard Pomeroy. (With Discussion.) 93.

"Relation of Rainfall and Run-Off to Cost of Sewers." John A. Rousculp. 1473.

DAMS.

Brief description of Fort Peck Dam. 673.

"Earthquake Stresses in an Arch Dam." Ivan M. Nelidov and Harold E. von Bergen. 133. Discussion: A. Floris, Cecil E. Pearce, F. W. Hanna, A. W. Fischer, and Ray L. Allin. 143.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. (With Discussion.) 275.

Possible effects of earthquakes on rock-fill dams. 84.

"The Design of Rock-Fill Dams." J. D. Galloway. 1. Discussion: Cecil E. Pearce, H. B. Muckleston, Harold K. Fox, Charles H. Paul, A. Floris, Howard F. Peckworth, Oren Reed, Walter L. Huber, Samuel B. Morris, L. F. Harza, Paul Baumann, O. W. Peterson, George W. Howson, John E. Field, John H. Wilson, Frederick H. Fowler, I. C. Steele and Walter Dreyer, F. Knapp, Ralph J. Reed, F. J. Sanger, C. S. Jarvis, L. R. East, and Francisco Gomez-Perez and Miguel Jinich. 25.

See also **RESERVOIRS.**

DEPRECIATION.

See **VALUATION.**

DIESEL ENGINES.

Diesel engines for motor vehicles. 1575.

EARTH PRESSURES.

"Lateral Earth and Concrete Pressures." Lazarus White and George Paaswell. 1685. Discussion: A. E. Cummings, Robert G. Hennes, Robert F. Legget, Charles Terzaghi, Jacob Feld, M. G. Spangler, Raymond D. Mindlin, D. P. Krynine, David A. Molitor, and Ralph W. Stewart. 1701.

EARTHQUAKES.

"Earthquake Stresses in an Arch Dam." Ivan M. Nelidov and Harold E. von Bergen. (With Discussion.) 133.

"Natural Periods of Uniform Cantilever Beams." Lydik S. Jacobsen. (With Discussion.) 402.

Possible effects of earthquakes on rock-fill dams. 84.

ECONOMICS.

"Cost of Energy Generation": Second Symposium on Power Costs. John C. Page, C. F. Hirshfeld and R. M. Van Duzer, Jr., H. K. Barrows, Ezra B. Whitman, Maurice R. Scharff, and W. F. Uhl. (With Discussion.) 1050.

"Economic Aspects of Energy Generation": A Symposium. Geo. A. Orrok, Frank H. Rogers, Joel D. Justin, Philip Sporn, Ralph E. Freeman, and B. A. Thresher. (With Discussion.) 942.

"Economics of Sewage Treatment." George J. Schroepfer. (With Discussion.) 1210.

"Engineering Economics and Public Works": A Symposium. Frederic H. Fay, Daniel W. Mead, Henry Earle Riggs, and William J. Wilgus. (With Discussion.) 635.

See also **VALUATION.**

ELASTICITY.

"Lateral Earth and Concrete Pressures." Lazarus White and George Paaswell. (With Discussion.) 1685.

ELECTRIC POWER.

Cost of generation of electric energy. 984.

Electric power in economic perspective. 1001.

Improvements in the utilization of energy. 971.

See also **POWER TRANSMISSION.**

ENGINEERS AND ENGINEERING.

"Frontiers of Engineering": Address at the Annual Convention, San Francisco, California, July 26, 1939. Donald H. Sawyer. 1815.

See also **CIVIL ENGINEERS.**

ENGINES.

See **DIESEL ENGINES.**

EQUATIONS.

"Solution of Equations in Structural Analysis by Converging Increments." George H. Dell. (With Discussion.) 1543.

EXPANSION JOINTS.

"The Design of Rock-Fill Dams." J. D. Galloway. (With Discussion.) 1.

FILTRATION.

"Economics of Sewage Treatment." George J. Schroepfer. (With Discussion.) 1210.

FLOODS.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. 275. Discussion: Cecil S. Camp, R. D. Goodrich, E. L. Myers, Ralph W. Powell, William T. Collins, George E. Townsend, and Waldo E. Smith. 295.

Sanitation problems incidental to floods. 844.

"Stream Pollution in the Ohio River Basin": A Symposium. George E. Barnes, Harold W. Streeter, Charles E. Ryder, W. L. Stevenson, E. S. Tisdale, D. E. Davis, J. E. Root, and Abel Wolman. (With Discussion.) 812.

FLOORS.

See **BRIDGE FLOORS.**

FLUMES.

"Laboratory Investigation of Flume Traction and Transportation." Y. L. Chang. 1246. Discussion: Hans Kramer, E. W. Lane, Joe W. Johnson, J. E. Christiansen, W. H. Huang, Lorenz G. Straub, and Hunter Rouse. 1285.

FOUNDATIONS.

"Design of Pile Foundations." C. P. Vetter. 758. Discussion: Hibbert M. Hill, Odd Albert, August E. Niederhoff, A. A. Eremin, Jacob Feld, John M. Coan, Jr., A. Agatz, and D. P. Krynine. 779.

"Lateral Earth and Concrete Pressures." Lazarus White and George Paaswell. (With Discussion.) 1685.

"Reconstruction of a Pier in Boston, Massachusetts, Harbor." Charles M. Spofford. (With Discussion.) 1625.

GOVERNMENT.

The record of government in business. 652.

GRAPHICAL CHARTS.

"Energy Mass Diagrams for Power Studies." John W. Hackney. (With Discussion.) 1644.

"Graphical Representation of the Mechanical Analyses of Soils." Frank B. Campbell. (With Discussion.) 150.

GRAVEL.

"Transportation of Sand and Gravel in a Four-Inch Pipe." G. W. Howard. (With Discussion.) 1334.

GRIT CHAMBERS.

"Economics of Sewage Treatment." George J. Schroepfer. (With Discussion.) 1210.

"Grit Chamber Model Tests for Detroit, Michigan, Sewage Treatment Project." George E. Hubbell. 214. Discussion: G. M. Ridenour, Thomas R. Camp, and W. E. Howland. 230.

HIGHWAYS AND ROADS.

Economies of alignment, grade and width. 485.

"Increasing the Traffic Capacity and Safety of Thoroughfares": A Symposium. Lewis W. McIntyre, Donald M. McNeil, Arnold H. Vey, and E. L. Schmidt. 440. Discussion: John A. Oakey, L. W. Mahone, Fred Lavis, Hugh A. Kelly, Sidney J. Williams, Harold Nevin Carey, W. W. Crosby, Roger L. Morrison, W. R. Flack, Hawley S. Simpson, and Charles M. Noble. 497.

Lighting—road or vehicle? 1572.

"Motor Transportation—A Forward View": A Symposium. C. F. Kettering, Thomas H. MacDonald, and Leroy C. Smith. (With Discussion.) 1562.

"Principles Applying to Highway Road-Beds." Ira B. Mullis. 1381. Discussion: A. F. Greaves-Walker, H. Z. Schofield, W. H. Campen, Bernard E. Gray, W. J. Turnbull, Bert Myers, and E. Neil W. Lane. 1402.

HYDRO-ELECTRIC POWER.

See WATER POWER.

INDUSTRIAL WASTE.

See TRADE WASTE.

INSTRUMENTS.

"Mechanical Structural Analysis by the Moment Indicator." Arthur C. Ruge and Ernst O. Schmidt. (With Discussion.) 1659.

See also METERS.

IRRIGATION.

"Interstate Water Problems." Final Report of the Committee of the Irrigation Division on Interstate Water Rights. 1822.

IRRIGATION CANALS.

"A Theory of Silt Transportation." W. M. Griffith. (With Discussion.) 1733.

JOINTS.

See EXPANSION JOINTS.

LAW.

Legislation to make highway plans effective. 1594.

LIGHTING.

See HIGHWAYS AND ROADS.

MARINE BORERS.

"Reconstruction of a Pier in Boston, Massachusetts, Harbor." Charles M. Spofford. (With Discussion.) 1625.

MATERIALS OF CONSTRUCTION.
"The Design of Rock-Fill Dams." J. D. Galloway. (With Discussion.) 1.
See also **GRAVEL: SAND.**

MEASURING INSTRUMENTS.
See **INSTRUMENTS.**

MEMOIRS OF DECEASED MEMBERS.
See name of member in Author Index.

METERS.
"Siphons as Water-Level Regulators." J. C. Stevens. 1787. Discussion: A. Griffin, H. P. Currin and D. M. Umphrey, T. J. Corwin, Jr., and Henry R. King. 1802.

MODELS.
Bibliography on geometric distortion in hydraulic models. 1508.
"Grit Chamber Model Tests for Detroit, Michigan, Sewage Treatment Project." George E. Hubbell. (With Discussion.) 214.
"Laboratory Investigation of Flume Traction and Transportation." Y. L. Chang. (With Discussion.) 1246.
"Mechanical Structural Analysis by the Moment Indicator." Arthur C. Ruge and Ernst O. Schmidt. (With Discussion.) 1659.
"Observed Effects of Geometric Distortion in Hydraulic Models." Kenneth D. Nichols. 1488. Discussion: Herbert W. Ehrgott, E. H. Taylor, and Herbert D. Vogel. 1510.
"Structural Behavior of Battle-Deck Floor Systems." Inge Lyse and Ingvald E. Madsen. (With Discussion.) 244.

MOMENTS.
"Mechanical Structural Analysis by the Moment Indicator." Arthur C. Ruge and Ernst O. Schmidt. (With Discussion.) 1659.

MOTOR BUSES.
Joint responsibility of the automotive and the civil engineer. 1563.

MOTOR TRUCKS.
Joint responsibility of the automotive and the civil engineer. 1563.

PARKING REGULATIONS.
See **AUTOMOBILE PARKING.**

PIERS.
See **WHARVES.**

PILES.
"Design of Pile Foundations." C. P. Vetter. (With Discussion.) 758.
"Lateral Earth and Concrete Pressures." Lazarus White and George Paaswell. (With Discussion.) 1685.
"Reconstruction of a Pier in Boston, Massachusetts, Harbor." Charles M. Spofford. (With Discussion.) 1625.

PIPE LINES.

"Economic Pipe Sizes for Water Distribution Systems." Thomas R. Camp. 190. Discussion: Clinton L. Bogert and J. M. M. Greig, Charles M. Mower, Jr., A. C. Michael, Ellwood H. Aldrich, M. H. Klegerman, and F. Knapp. 203.

"Transportation of Sand and Gravel in a Four-Inch Pipe." G. W. Howard. 1334. Discussion: Fred R. Brown, Josef E. Montgomery, Elliott J. Dent, David L. Neuman, Morrough P. O'Brien and R. G. Folsom, R. L. Vaughn, M. P. Durepaire, Pierre F. Danel, and H. S. Gladfelter. 1349.

PLASTIC MATERIALS.

Effect of plasticity index on resistance to earth failure. 1423.

PLATES, STEEL.

See STEEL PLATES.

POWER COST.

"Cost of Energy Generation": Second Symposium on Power Costs. John C. Page, C. F. Hirshfeld and R. M. Van Duzer, Jr., H. K. Barrows, Ezra B. Whitman, Maurice R. Scharff, and W. F. Uhl. 1050. Discussion: Joel D. Justin, Frank H. Mason, W. V. Burnell, Daniel W. Mead, A. G. Christie, William E. Rudolph, Louis Elliott, H. G. Gerdes, Leverett S. Lyon, John T. Madden, R. L. Thomas, Theodore B. Parker, Philip Sporn, L. N. McClellan, James C. Bonbright, R. Robinson Rowe, V. B. Libbey, Edwin D. Dreyfus, R. H. Parsons, and F. Knapp. 1149.

"Economic Aspects of Energy Generation": A Symposium. Geo. A. Orrok, Frank H. Rogers, Joel D. Justin, Philip Sporn, Ralph E. Freeman, and B. A. Thresher. 942. Discussion: W. S. Finlay, Jr., R. M. Riegel, Ralph Bennett, J. M. Mousson, Ben C. Sprague, V. M. Marquis, R. L. Sackett, D. J. McCormack, I. E. Moultrop, M. M. Samuels, E. W. Kramer, Paul E. Gisiger, F. A. Dale, Ray S. Quick, F. Knapp, and D. S. Jacobus. 1009.

POWER INDUSTRY.

Cost of power, showing growth of power industry in the United States. 1147.

Electric power in economic perspective. 1001.

Social implication of technological trends in the power industry. 995.

POWER-PLANTS.

"Cost of Energy Generation": Second Symposium on Power Costs. John C. Page, C. F. Hirshfeld and R. M. Van Duzer, Jr., H. K. Barrows, Ezra B. Whitman, Maurice R. Scharff, and W. F. Uhl. (With Discussion.) 1050.

"Economic Aspects of Energy Generation": A. Symposium. Geo. A. Orrok, Frank H. Rogers, Joel D. Justin, Philip Sporn, Ralph E. Freeman, and B. A. Thresher. (With Discussion.) 942.

"Energy Mass Diagrams for Power Studies." John W. Hackney. (With Discussion.) 1644.

POWER TRANSMISSION.

Elements of cost of energy generation. 1051.

PUBLIC WORKS.

"Engineering Economics and Public Works": A Symposium. Frederic H. Fay, Daniel W. Mead, Henry Earle Riggs, and William J. Wilgus. 635. Discussion: J. K. Finch, Elliott J. Dent, C. Frank Allen, Bradley G. Seitz, Alfred Allen Stuart, Pierce P. Furber, R. F. Bessey, Donald M. Baker, Philip W. Henry, H. K. Barrows, Harry A. Wiersema, J. D. Galloway, E. S. Martin, K. Bert Hirashima, Louis E. Ayres, John H. Meursinge, and Samuel B. Folk. 701.

RAINFALL.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. (With Discussion.) 275.

"Relation of Rainfall and Run-Off to Cost of Sewers." John A. Rousculp. 1473.

REPORTS OF COMMITTEES.

"Interstate Water Problems." Final Report of the Committee of the Irrigation Division on Interstate Water Rights. H. K. Barrows, C. J. Bartholet, C. S. Clark, Edward Hyatt, George S. Knapp, G. E. P. Smith, J. C. Stevens, R. J. Tipton, Abel Wolman, and M. C. Hinderlider. 1822.

RESERVOIRS.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. (With Discussion.) 275.

Low-flow regulation by Pymatuning Reservoir. 832.

RIVERS.

"A Theory of Silt Transportation." W. M. Griffith. (With Discussion.) 1733.

"Energy Mass Diagrams for Power Studies." John W. Hackney. (With Discussion.) 1644.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. (With Discussion.) 275.

Improvements for navigation on the Missouri River. 672.

"Observed Effects of Geometric Distortion in Hydraulic Models." Kenneth D. Nichols. (With Discussion.) 1488.

"Stream Pollution in the Ohio River Basin": A Symposium. George E. Barnes, Harold W. Streeter, Charles E. Ryder, W. L. Stevenson, E. S. Tisdale, D. E. Davis, J. E. Root, and Abel Wolman. (With Discussion.) 812.

The Columbia Basin project. 676.

See also **WATER, FLOW OF, IN OPEN CHANNELS.**

RUN-OFF.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. (With Discussion.) 275.

"Relation of Rainfall and Run-Off to Cost of Sewers." John A. Rousculp. 1473.

SAFETY DEVICES AND MEASURES.

"Increasing the Traffic Capacity and Safety of Thoroughfares": A Symposium. Lewis W. McIntyre, Donald M. McNeil, Arnold H. Vey, and E. L. Schmidt. (With Discussion.) 440.

"Motor Transportation—A Forward View": A Symposium. C. F. Kettering, Thomas H. MacDonald, and Leroy C. Smith. (With Discussion.) 1562.

SAND.

"Transportation of Sand and Gravel in a Four-Inch Pipe." G. W. Howard. (With Discussion.) 1334.

SANITATION.

Sanitation problems incidental to floods. 844.

SEPTIC TANKS.

"Multiple-Stage Sewage Sludge Digestion." A. M. Rawn, A. Perry Banta, and Richard Pomeroy. (With Discussion.) 93.

SEWAGE DISPOSAL.

"Deoxygenation and Reoxygenation." C. J. Velz. 560. Discussion: Earle B. Phelps. 573.

"Economics of Sewage Treatment." George J. Schroepfer. 1210. Discussion: Charles Gilman Hyde, and Frederic Bass. 1239.

"Grit Chamber Model Tests for Detroit, Michigan, Sewage Treatment Project." George E. Hubbell. (With Discussion.) 214.

"Multiple-Stage Sewage Sludge Digestion." A. M. Rawn, A. Perry Banta, and Richard Pomeroy. 93. Discussion: Edward W. Moore, Willem Rudolfs and H. Heukelekian, Hermann Bach, and C. E. Keefer. 120.

SEWERAGE.

"Relation of Rainfall and Run-Off to Cost of Sewers." John A. Rousculp. 1473.

SHEET PILING.

See **PILES.**

SILT.

"A Theory of Silt Transportation." W. M. Griffith. 1733. Discussion: Joe W. Johnson, G. W. Howard, Harry F. Blaney, E. W. Lane, O. A. Faris, J. E. Christiansen, Samuel Shulits, Gerald Lacey, Glenn W. Holmes, Hunter Rouse, S. G. Bauer, and C. C. Inglis. 1749.

SLABS.

"The Design of Rock-Fill Dams." J. D. Galloway. (With Discussion.) 1.

SOILS.

Characteristics of soil types. 1413.

"Graphical Representation of the Mechanical Analyses of Soils." Frank B. Campbell. 150. Discussion: Donald M. Burmister, A. J. Weinig, Carl H. Kadie, Jr., Carlton S. Proctor, T. T. Knappen, Jacob Feld, Howard F. Peckworth, Joel D. Justin, L. B. Olmstead, T. A. Middlebrooks, Frank E. Fahlquist and Waldo I. Kenerson, E. W. Lane, F. J. Sanger, F. Knapp, and Charles H. Lee. 156.

"Lateral Earth and Concrete Pressures." Lazarus White and George Paaswell. (With Discussion.) 1685.

"Principles Applying to Highway Road-Beds." Ira B. Mullis. (With Discussion.) 1381.

SPILLWAYS.

"Siphons as Water-Level Regulators." J. C. Stevens. (With Discussion.) 1787.

STEAM POWER.

Heat generated energy. 1060.

Progress in the generation of energy by heat engines. 943.

STEEL PLATES.

"Pin-Connected Plate Links." Bruce G. Johnston. 314. Discussion: Harold D. Hussey. 337.

STREAM CONTAMINATION.

See **WATER POLLUTION.**

STRESS AND STRAIN.

"Design of Pile Foundations." C. P. Vetter. (With Discussion.) 758.

"Earthquake Stresses in an Arch Dam." Ivan M. Nelidov and Harold E. von Bergen. (With Discussion.) 133.

"Pin-Connected Plate Links." Bruce G. Johnston. (With Discussion.) 314.

STRUCTURES, THEORY OF.

"Mechanical Structural Analysis by the Moment Indicator." Arthur C. Ruge and Ernst O. Schmidt. 1659. Discussion: John B. Wilbur, William J. Eney, and Otto Gottschalk. 1672.

"Preliminary Design of Suspension Bridges." Shortridge Hardesty and Harold E. Wessman. (With Discussion.) 579.

"Relative Flexure Factors for Analyzing Continuous Structures." Ralph W. Stewart. (With Discussion.) 521.

"Solution of Equations in Structural Analysis by Converging Increments." George H. Dell. 1543. Discussion: A. Floris, L. E. Grinter, and A. A. Eremin. 1555.

SURVEYING.

"The Three-Point Problem in a Co-Ordinated Field." R. Robinson Rowe. (With Discussion.) 1314.

TIDAL POWER.

See **WATER POWER.**

TRADE WASTE.

Progress in industrial wastes control. 854.

TRAFFIC.

Definitions as proposed by a Committee of the Institute of Traffic Engineers. 441.

"Increasing the Traffic Capacity and Safety of Thoroughfares": A Symposium. Lewis W. McIntyre, Donald M. McNeil, Arnold H. Vey, and E. L. Schmidt. (With Discussion.) 440.

TRAILERS.

Trailers and mobile homes. 1576.

TRANSPORTATION.

"Motor Transportation—A Forward View": A Symposium. C. F. Kettering, Thomas H. MacDonald, and Leroy C. Smith. 1562. Discussion: F. Lavis, Edgar Dow Gilman, George Hartley, Robert Kingery, R. L. Morrison, Roy F. Bessey, Bruce D. Greenshields, Robert B. Brooks, H. George Altvater, and W. W. Crosby. 1597.

TRIANGULATION.

"The Three-Point Problem in a Co-Ordinated Field." R. Robinson Rowe. 1314. Discussion: Oscar S. Adams, George D. Whitmore, and John R. Jahn. 1325.

TRUSSES.

"Beam Constants for Continuous Trusses and Beams." George L. Epps. (With Discussion.) 1522.

TURBINES (WATER).

Hydro-generation of energy. 952.

TURBULENCE.

"Laboratory Investigation of Flume Traction and Transportation." Y. L. Chang. (With Discussion.) 1246.

VALUATION.

Cost of energy generation—depreciation and obsolescence. 1127.

VIBRATION.

"Earthquake Stresses in an Arch Dam." Ivan M. Nelidov and Harold E. von Bergen. (With Discussion.) 133.

"Natural Periods of Uniform Cantilever Beams." Lydik S. Jacobsen. (With Discussion.) 402.

WATER.

"Water-Softening Plant Design." W. H. Knox. 1425. Discussion: Philip Burgess, A. Elliott Kimberly, L. R. Howson, Charles P. Hoover, M. H. Klegerman, Rollin F. MacDowell, and D. E. Davis. 1436.

WATER, FLOW OF, IN OPEN CHANNELS.

"A Theory of Silt Transportation." W. M. Griffith. (With Discussion.) 1733.

"Energy Mass Diagrams for Power Studies." John W. Hackney. (With Discussion.) 1644.

"Laboratory Investigation of Flume Traction and Transportation." Y. L. Chang. (With Discussion.) 1246.

"Observed Effects of Geometric Distortion in Hydraulic Models." Kenneth D. Nichols. (With Discussion.) 1488.

WATER, FLOW OF, IN PIPES.

"Economic Pipe Sizes for Water Distribution Systems." Thomas R. Camp. (With Discussion.) 190.

"Relation of Rainfall and Run-Off to Cost of Sewers." John A. Rousculp. 1473.

"Transportation of Sand and Gravel in a Four-Inch Pipe." G. W. Howard. (With Discussion.) 1334.

"Water-Hammer Pressures in Compound and Branched Pipe." Robert W. Angus. (With Discussion.) 340.

WATER, FLOW OF, THROUGH ORIFICES.

"Siphons as Water-Level Regulators." J. C. Stevens. (With Discussion.) 1787.

WATER HAMMER.

"Water-Hammer Pressures in Compound and Branched Pipe." Robert W. Angus. 340. Discussion: K. J. DeJuhasz, Harold A. Thomas, F. Knapp, Martin A. Mason and Pierre F. Danel and Antoine Craya, and A. A. Kalinske. 377.

WATER LAWS.

"Interstate Water Problems." Final Report of the Committee of the Irrigation Division on Interstate Water Rights. 1822.

WATER METERS.

See **METERS.**

WATER POLLUTION.

"Deoxygenation and Reoxygenation." C. J. Velz. (With Discussion.) 560.

Planning for pollution control at Pittsburgh, Pa. 860.

"Stream Pollution in the Ohio River Basin": A Symposium. George E. Barnes, Harold W. Streeter, Charles E. Ryder, W. L. Stevenson, E. S. Tisdale, D. E. Davis, J. E. Root, and Abel Wolman. 812. Discussion: D. E. Davis, L. S. Morgan, C. A. Holmquist, Robert Spurr Weston, Karl Imhoff, Henry D. Johnson, Jr., Charles M. Reppert, Edwin K. Morse, James LeVan, John C. H. Lee, W. H. Wisely, F. C. Dugan, G. M. Ridenour, Arthur W. Baum, Jr., G. R. Scott, and Don E. Bloodgood. 878.

Stream pollution problems at Cincinnati, Ohio. 868.

WATER POWER.

Characteristics and costs of hydro-electric plants. 1099.

Economic value of the Conowingo water power in the Philadelphia electric system. 1121.

"Energy Mass Diagrams for Power Studies." John W. Hackney. 1644. Discussion: Edgar E. Foster, and Lyle A. Whitsit. 1654.

Hydro-generation of energy. 952, 1088.

Passamaquoddy tidal power project. 670, 691.

Power in the northwest. 674.

WATER PRESSURES.

"Siphons as Water-Level Regulators." J. C. Stevens. (With Discussion.) 1787.

"Water-Hammer Pressures in Compound and Branched Pipe." Robert W. Angus. (With Discussion.) 340.

WATER RIGHTS.

"Interstate Water Problems." Final Report of the Committee of the Irrigation Division on Interstate Water Rights. 1822.

WATER SOFTENING.

See WATER.

WATER STORAGE.

"Energy Mass Diagrams for Power Studies." John W. Hackney. (With Discussion.) 1644.

"Flood Routing." Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder. (With Discussion.) 275.

WATER WORKS.

"Water-Softening Plant Design." W. H. Knox. (With Discussion.) 1425.

WHARVES.

"Reconstruction of a Pier in Boston, Massachusetts, Harbor." Charles M. Spofford. 1625. Discussion: William G. Atwood, John N. Ferguson, and John Ayer. 1639.

WIND PRESSURES.

"Aerodynamics of the Perisphere and Trylon at World's Fair." Alexander Klemin, Everett B. Schaefer, and J. G. Beerer, Jr. (With Discussion.) 1449.

ZONING OF CITIES.

"Practical Methods of Re-Zoning Urban Areas." Hugh E. Young. 904. Discussion: Robert D. Kohn, Myron D. Downs, Harland Bartholomew, Benjamin Saltzman, and G. H. Herrold. 928.

AUTHOR INDEX

ADAMS, OSCAR S.
The three-point problem. 1325.

AGATZ, A.
Design of pile foundations. 795.

ALBERT, ODD.
Design of pile foundations. 784.

ALDERSON, ALGERNON BROWN.
Memoir of. 1871.

ALDRICH, ELLWOOD H.
Economic pipe sizes. 205.

ALLEN, C. FRANK.
Public works. 706.

ALLEN, WALTER HINDS.
Memoir of. 1872.

ALLIN, RAY L.
Earthquake stresses in an arch dam. 148.

ALLISON, JOSEPH CHESTER.
Memoir of. 1875.

ALLISON, WILLIAM NOEL.
Memoir of. 2054.

ALTVATER, H. GEORGE.
Motor transportation. 1619.

AMMANN, O. H.
Design of suspension bridges. 624.

ANGUS, ROBERT W.
" Water-Hammer Pressures in Compound and Branched Pipes." 340.

ARNESON, EDWIN PERCIVAL.
Memoir of. 1878.

ARNSTEIN, KARL.
Aerodynamics of perisphere and trylon. 1470.

ATWOOD, WILLIAM G.
Reconstruction of a harbor pier. 1639.

AUSTIN, ROBERT EDWARD.
Memoir of. 2055.

AYER, JOHN.
Reconstruction of a harbor pier. 1640.

AYRES, LOUIS E.
Public Works, 742.

BABB, CYRUS CATES.
Memoir of. 1881.

BACH, HERMANN.
Multiple-stage sewage sludge digestion. 125.

BAKER, DONALD M.
Public works. 720.

BANTA, A. PERRY.
"Multiple-Stage Sewage Sludge Digestion." 93.

BARNES, GEORGE E.
"Stream Pollution in the Ohio River Basin: A Symposium: Foreword." 814.

BARROWS, H. K.
"Cost of Energy Generation: Second Symposium on Power Costs: Hydro-Generated Energy." 1088.
"Final Report of the Committee of the Irrigation Division on Interstate Water Rights: Interstate Water Problems." 1822.
Public works. 727.

BARTHOLET, C. J.
"Final Report of the Committee of the Irrigation Division on Interstate Water Rights: Interstate Water Problems." 1822.

BARTHOLOMEW, HARLAND.
Re-zoning urban areas. 930.

BASS, FREDERIC.
Economics of sewage treatment. 1242.

BATCHELDER, CHARLES LELAND.
Memoir of. 1884.

BAUER, S. G.
A theory of silt transportation. 1768.

BAUM, ARTHUR W., JR.
Stream pollution. 897.

BAUMANN, PAUL.
Design of rock-fill dams. 39.

BAUSHER, CARMI IRVING.
Memoir of. 2023.

BEERER, J. G., JR.
"Aerodynamics of the Perisphere and Trylon at World's Fair." 1449.

BENNETT, RALPH.
Economics of energy generation. 1015.

BENT, ARTHUR SAMUEL.
Memoir of. 2056.

BESSEY, ROY F.
Motor transportation. 1609.
Public works. 713.

BIRBECK, RICHARD.
Memoir of. 1886.

BLANEY, HARRY F.
A theory of silt transportation. 1754.

BLOG, LEON.
Design of suspension bridges. 626.
Flexure factors for continuous structures. 542.

BLOODGOOD, DON E.
Stream pollution. 901.

BOGART, JOHN ALLEYNE CALHOUN.
Memoir of. 2023.

BOGERT, CLINTON L.
Economic pipe sizes. 203.

BONBRIGHT, JAMES C.
Cost of energy generation. 1192.

BROOKS, ROBERT B.
Motor transportation. 1616.

BROWN, FRED R.
Transportation of sand and gravel. 1349.

BUCK, HENRY ROBINSON.
Memoir of. 1888.

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